24

SEDIMENTATION, CENTRIFUGATION, FLOTATION

This chapter examines three industrial separation techniques that exploit the density difference between a liquid and a solid. The driving force in these processes is usually the result of gravity, centrifugal action, and/or buoyant effects. Unlike the previous chapter that primarily treated gas-particle dynamics, this presentation will address the three above titled topics, with the bulk of the material keying on liquid–solid separation. The above captioned headings will be addressed in this chapter. Filtration is treated separately in Chapter 27.

24.1 SEDIMENTATION

Gravity sedimentation is a process of liquid–solid separation that separates, under the effect of gravity, a feed slurry into an underflow slurry of higher solids concentration and an overflow of substantially clearer liquid. A difference in density between the solids and the suspended liquid is, as indicated, a necessary prerequisite.

Nearly all commercial equipment for continuous sedimentation is built with relatively simple settling tanks. Distinction is commonly made depending on the purpose of the separation. If the clarity of the overflow is of primary importance, the process is called clarification and the feed slurry is usually dilute. If a thick underflow is the primary aim, the process is called thickening and the feed slurry is usually more concentrated.(1)

Fluid Flow for the Practicing Chemical Engineer. By J. Patrick Abulencia and Louis Theodore

The most commonly used thickener is the circular-basin type (shown in Fig. 24.1). The flocculant-treated feedstream enters the central feedwell, which dissipates the stream's kinetic energy and disperses it gently into the thickener. The feed finds its height in the basin, where its density matches the density of the suspension inside (the concentration increases downward in an operating thickener, giving stability to the process) and spreads out at that level. The settling solids move downward as does some liquid, the liquid amount being determined by the underflow withdrawal rate. Most of the liquid goes upward and into the overflow. Typically, a thickener has three operating zones: clarification, zone settling and compression (see Fig. 24.1).[1]

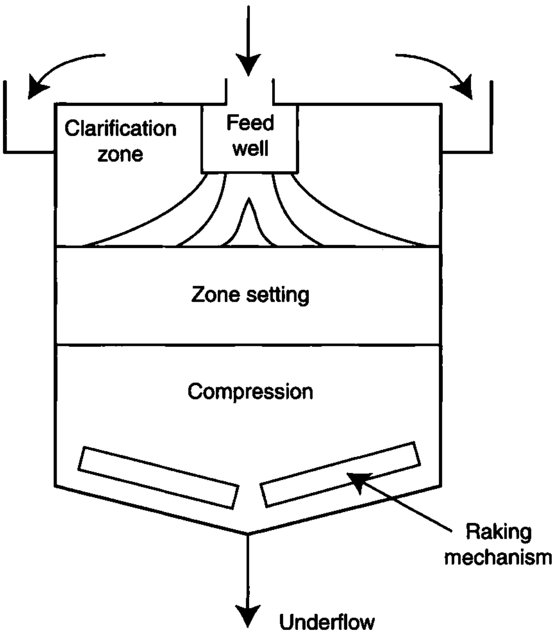

Figure 24.1 The circular-basin continuous thickener.

Gravity clarifiers sometimes resemble circular thickeners but more often are rectangular basins, with feed at one end and overflow at the other. Settled solids are pushed to a discharge trench by paddles or blades on a chain mechanism. Flocculent may be added prior to the clarifier. Conventional thickeners are also used for clarification, but the typically low feed concentrations hinder the benefits of zone settling, so the basin area is based on clarification-zone demands.[1] Thus, in a general sense, sedimentation is the process of removing solid particles heavier than water by gravity settling. It is the oldest and most widely used unit operation in water and wastewater treatment. The terms sedimentation, settling, and clarification are often used interchangeably. The sedimentation basin unit may also be referred to as a sedimentation tank, clarifier, settling basin, or settling tank.

In wastewater treatment (to be discussed in Part VI, Chapter 28), sedimentation is used to remove both inorganic and organic materials which are settleable in continuous flow conditions. It removes grit, particulate matter in the primary settling tank, and chemical flocculants from a chemical precipitation unit. Sedimentation is also used for solids concentration in sludge thickeners.[2]

Details on discrete, or individual, particle settling is available in Chapter 23. Here, Stokes' law assumes that particles are present in relatively dilute concentrations and in a fluid medium of relatively large cross-section. When there are a large number of particles, the particles in close proximity will retard other particles. This is termed hindered settling. The effect is not significant at volumetric concentrations below 0.1%. However, when the particle diameter becomes appreciable with respect to the diameter of the container in which it is settling, the container walls will exert an additional retarding effect known as the wall effect.

Farag[3] has reviewed and developed equations for the above situation. For hindered flow, the settling velocity is less than would be calculated from Stokes' law. The density of the fluid phase becomes the bulk density of the slurry, ρ_M, which is defined as follows:

$$\rho_M = \varepsilon_f \rho_f + (1 - \varepsilon_f)\rho_s \tag{24.1}$$

where ρ_M is the density of slurry in kg (solid plus liquid)/m^3 and ε_f is the volume fraction of the liquid in the slurry mixture. (See Chapter 25 for more details on void volume.) The density difference is then:

$$\rho_s - \rho_M = \rho_s - [(\varepsilon_f \rho_f + (1 - \varepsilon_f)\rho_s] = \varepsilon_f(\rho_s - \rho_f) \tag{24.2}$$

The effective viscosity of the mixture, μ_m, is defined as:

$$\mu_m = \mu_f b \tag{24.3}$$

where μ_f is the pure fluid viscosity and b is a dimensionless correction factor which is a function of ε_f. The term b can be evaluated from Equation (24.4)

$$b = 10^{1.82(1-\varepsilon_f)}$$
$$\log_{10} b = 1.82(1 - \varepsilon_f) \tag{24.4}$$

The settling velocity, v, with respect to the unit is ε_f times the velocity calculated by Stokes' law. Substituting Equations (24.2) and (24.3) into Stokes' law terminal velocity equation and multiplying by ε_f for the relative velocity effect leads to:

$$v = \frac{g d_p^{\,2}(\rho_s - \rho_f)}{18\mu_f}\frac{\varepsilon_f^{\,2}}{b} \tag{24.5}$$

The Reynolds number, based on the settling velocity relative to the fluid is then:

$$\mathrm{Re} = \frac{\rho_m v d_p}{\mu_m \varepsilon_f} = \frac{g d_p^{\,2}(\rho_s - \rho)\rho_m}{18\mu_f^{\,2}} \frac{\varepsilon_f}{b^2} \tag{24.6}$$

When the Reynolds number is less than 1.0, the settling is in the Stokes' law regime.

When the diameter, d_p, of the particle becomes appreciable with respect to the diameter of the container, D_w, the terminal velocity is reduced. This is termed the aforementioned wall effect. In the case of settling in the Stokes' law regime, the calculated terminal velocity is multiplied by a correction factor, k_w. This k_w is approximated as follows:

$$k_w = \frac{1}{1 + 2.1(d_p/D_w)^{1.5}} \quad \text{for } d_p/D_w < 0.05 \tag{24.7}$$

For $0.5 < d_p/D_w < 0.8$, employ the values provided in Table 24.1. For the turbulent flow regime, Equation (24.8) should be used

$$k_w = 1 - (d_p/D_w)^{1.5} \tag{24.8}$$

Table 24.1 Wall correction factors

d_p/d_w	k_w
0.1	0.792
0.2	0.596
0.3	0.422
0.4	0.279
0.5	0.170
0.6	0.0945
0.7	0.0468
0.8	0.0205

Illustrative Example 24.1 Glass spheres are settling in water at 20°C. The slurry contains 60 wt% solids and the particle diameter is 0.1554 mm. The glass density is 2467 kg/m^3. Find the volume fraction, ε_f, of the liquid, the bulk density of the slurry, ρ_m, the terminal velocity, and the effective mixture viscosity, μ_m. For water at 20°C, $\rho_f = 998\ \mathrm{kg/m^3}$ and $\mu_f = 0.001\ \mathrm{kg/m \cdot s}$.

Solution Start by assuming a basis of 100 kg of slurry. To determine the volume fraction of the liquid, divide the mass of liquid by its density. Since 100 kg of slurry is the basis and the slurry contains 60 wt% solid,

$$m_f = 40\,\mathrm{kg}$$

The volume of the fluid (water) is

$$V_f = \frac{m_f}{\rho_f} = \frac{40}{998} = 0.040\,\text{m}^3$$

Similarly,

$$m_s = 60\,\text{kg}$$

$$V_s = \frac{m_s}{\rho_s} = \frac{60}{2467}$$
$$= 0.0243\,\text{m}^3$$

Therefore,

$$V = V_f + V_s = 0.040 + 0.0243$$
$$= 0.0643\,\text{m}^3$$

and

$$\varepsilon_f = \frac{V_f}{V} = \frac{0.040}{0.643}$$
$$= 0.622$$

For the glass particles,

$$\varepsilon_p = 1 - \varepsilon_f$$
$$= 0.378$$

Calculate ρ_m from Equation (24.1)

$$\rho_m = \varepsilon_f \rho_f + \varepsilon_p \rho_p = 0.622(998) + 0.378(2467)$$
$$= 1553\,\text{kg/m}^3$$

Calculate b and the terminal velocity. See Equations (24.4) and (24.5).

$$b = 10^{1.82(1-\varepsilon_f)} = 10^{1.82(0.378)}$$
$$= 4.875$$

$$v = \frac{g d_p{}^2(\rho_p - \rho_f)\varepsilon_f{}^2}{18\mu_f b} = \frac{9.807(0.0001554)^2(2467 - 998)(0.622)^2}{18(0.001)(4.875)}$$
$$= 0.00153\,\text{m/s}$$

Compute μ_m by employing Equations (24.3) and (24.6).

$$\begin{aligned}\mu_m &= \mu_f b \qquad (24.3)\\ &= 0.001(4.875)\\ &= 0.0049\,\text{kg/m}\cdot\text{s}\end{aligned}$$

Illustrative Example 24.2 Refer to Illustration Example (24.1). Calculate the Reynolds number.

Solution Employ Equation (24.6):

$$\text{Re} = \frac{\rho_m v d_p}{\mu_m \varepsilon_f} \qquad (24.6)$$

Substituting,

$$\begin{aligned}\text{Re} &= \frac{1553(0.00153)(1.554 \times 10^{-4})}{(0.0049)(0.622)}\\ &= 0.121\end{aligned}$$

Illustrative Example 24.3 Classify small spherical particles of charcoal with a specific gravity of 2.2. The particles are falling in a vertical tower against a rising current of air at 25°C and atmospheric pressure. Calculate the minimum size of charcoal that will settle to the bottom of the tower if the air is rising through the tower at the rate of 15 ft/s.

Solution The particles that have the settling velocity differentially greater than the rising air velocity will be the smallest diameter particles to settle. First note that

$$\rho = 0.075\,\text{lb/ft}^3$$

and

$$\mu = 1.23 \times 10^{-5}\,\text{lb/ft}\cdot\text{s}$$

Assume Stokes' Law to apply

$$d_p = \left(\frac{18\mu v}{g\rho_p}\right)^{0.5}$$

Substituting the data (in consistent units),

$$d_p = \frac{(18)(1.23 \times 10^{-5})(15)}{(32.2)2.2 \times 62.4}$$

$$d_p = 8.67 \times 10^{-4}\,\text{ft} = 264.19\,\mu\text{m}$$

Check K,

$$K = d_p \left[\frac{g \rho \rho_p}{\mu^2} \right]^{1/3}$$

Substituting the data gives

$$K = 8.67 \times 10^{-4}\,\text{ft} \left[\frac{(32.2\,\text{ft/sec}^2)(0.074\,\text{lb/ft}^3)(2.2 \times 62.4\,\text{lb/ft}^3)}{(1.23 \times 10^{-5}\,\text{lb/ft-s})^2} \right]^{1/3}$$

$$= 11.2$$

Stokes' law does not apply. Therefore, assume the Intermediate range law applies:

$$v_t = \frac{0.153\, g^{0.71} {d_p}^{1.14} {\rho_p}^{0.71}}{\rho^{0.29} \mu^{0.43}}$$

Rearranging gives

$${d_p}^{1.14} = \frac{v_t \rho^{0.29} \mu^{0.43}}{0.153 (g \rho_p)^{0.71}}$$

Substituting the data,

$${d_p}^{1.14} = \frac{(1.5\,\text{ft/s})(0.074\,\text{lb/ft}^3)^{0.29}(1.23 \times 10^{-5}\,\text{lb/ft-s})^{0.43}}{0.153(32.2\,\text{ft/s}^2 \times 2.2 \times 62.4\,\text{lb/ft}^3)^{0.71}}$$

$$= 9.20 \times 10^{-4}\,(\text{ft})^{1.14}$$

or

$$d_p = 2.17 \times 10^{-3}\,\text{ft} = 662\,\mu\text{m}$$

Checking K,

$$K = \frac{2.17 \times 10^{-3}\,\text{ft}}{8.6677 \times 10^{-4}\,\text{ft}}(11.2081) = 28.077$$

Then, $3.3 < K < 43.6$, and the result is correct for the intermediate range

$$d_p = 2.17 \times 10^{-3}\,\text{ft} = 662\,\mu\text{m}$$

24.2 CENTRIFUGATION

Centrifugal force is widely used when a force greater than that of gravity as in settling, is desired for the separation of solids and fluids of different densities. Two terms need to be defined. A centrifugal force is created by moving a mass in a curved path and is exerted in the direction away from the center of curvature of the path. The centripetal force is the force applied to the moving mass in the direction toward the center of curvature that causes the mass to travel in a curved path. If these forces are equal, the particle continues to rotate in a circular path around the center.[4]

Centrifugation is therefore another process that uses density differences to separate solids from liquids (or an immiscible liquid from other liquids). The feed is subjected to centrifugal forces that make the solids move radially through the liquid (outward if heavier, inward if lighter). In a sense, centrifugation is an extension of gravity sedimentation to particle sizes and to emulsions that are normally stable in a gravity field. The describing equations developed earlier in the previous chapter again apply. The gravity force is replaced by a centrifugal force, F_c (force/mass) where

$$F_c = \frac{r\omega^2}{g_c} = \frac{v_t^2}{g_c r} \tag{24.9}$$

where r is the radius of curvature of the particle or heavier phase, ω the angular velocity and v_t the tangential velocity at the point in question.

As indicated above, centrifugation attempts to increase the particle "settling" velocity many times higher than that due to gravity by applying a centrifugal force. The ratio of these two forces has been defined by some as the number of "Gs", where

$$G = \frac{r\omega^2}{g} \tag{24.10}$$

Note that the g term in the denominator in Equation (24.10) is the acceleration due to gravity and not the term g_c (the conversion constant) that appears in Equation (24.9).

There are two main classes of centrifugation equipment: cyclones and centrifuges. Cyclones are generally classified as air recovery/control equipment. The unit is primarily used for separating gas–solid systems. Centrifuges are primarily employed for liquid–solid separation; units in this category include basket, tubular, scroll-type, dish and multiple chamber. The units may operate in the batch or continuous mode. Details on this equipment are available in the literature.[4–6]

Illustrative Example 24.4 A particle is spinning in a 3-inch ID centrifuge with an angular velocity of 30 rad/s. Calculate the number of Gs for the particle.

Solution Employ Equation (24.10).

$$G = \frac{r\omega^2}{g}$$

Substituting yields:

$$G = \frac{r\omega^2}{g} = \frac{(3/12)(30)^2}{32.2} = 7.0$$

24.3 HYDROSTATIC EQUILIBRIUM IN CENTRIFUGATION

Another extended topic of interest in centrifugation is hydrostatic centrifugation equilibrium. When a contained liquid of constant ρ and μ is rotated around the vertical z-axis at a constant angular speed, ω, it is thrown outward from the axis by centrifugal force. The free surface of the liquid develops a paraboloid of revolution, the cross-section of which has a parabolic shape. When the fluid container has been rotated long enough, the whole volume of fluid will be rotating at the same angular velocity. Then, there is no sliding of one layer of liquid over the other. This condition is termed rigid-body rotation. The pressure distribution can be calculated from the principles of fluid statics. Further, one may determine the shape of the surface and the pressure distribution. Figure 24.2 shows a cross-section of this system.

The pressure at any point in the liquid has to be calculated in two directions: the axial (z) and radial (r) directions. In the vertical z-direction, the pressure gradient is given by

$$\frac{dP}{dz} = -\rho\frac{g}{g_c} \tag{24.11}$$

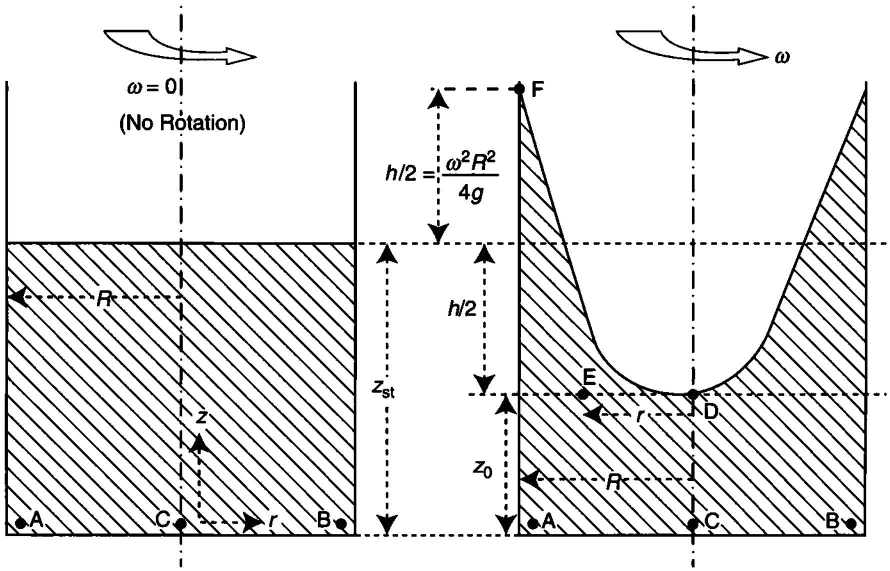

Figure 24.2 Rigid body rotation.

Therefore, the gauge pressure at any point in the liquid is

$$P_g = \rho \frac{g}{g_c} \Delta z \tag{24.12}$$

where Δz is the height of liquid above the point. Note that at point D (or at any point on the paraboloid surface), the gauge pressure is zero since it is exposed to the atmosphere. At point E, the gauge pressure, P_g, is

$$P_g = \rho \frac{g}{g_c}(z - z_0) \tag{24.13}$$

In the radial direction, the pressure gradient is given by

$$\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{-\rho a_r}{g_c} \tag{24.14}$$

where

$$a_r = \text{(radial centripetal acceleration)} = -\omega^2 r / g_c \tag{24.15}$$

Substitution of Equation (24.15) into Equation (24.14) yields

$$\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{\rho\omega^2 r}{g_c} \tag{24.16}$$

Integrating from point D (where $r = 0$ and $P_g = 0$) to any radius r (e.g., point E) gives

$$P_g = \frac{\rho\omega^2 r^2}{2g_c} \tag{24.17}$$

Equating Equations (24.13) and (24.17) and simplifying results in

$$\frac{\rho\omega^2 r^2}{2} = \rho g(z - z_0) \tag{24.18}$$

or

$$z - z_0 = \frac{\omega^2 r^2}{2g} \tag{24.19}$$

Equation (24.19) indicates that the free surface of the rotating liquid is a parabola. At point F on the wall surface, $r = R$, $z - z_0 = h$. Thus, Equation (24.19) becomes

$$h = \frac{\omega^2 R^2}{2g} \tag{24.20}$$

Since the volume of the liquid is conserved (remains the same), the volume of liquid in the cylinder of height $z_{st} - z_0$ is equal to the volume of liquid in the paraboloid of radius R and height h which reduces to

$$V_{cyl} = V_{para}$$

$$\pi R^2(z_{st} - z_0) = \frac{1}{2}\pi R^2 h \tag{24.21}$$

Solving of Equation (24.21) and replacing h by Equation (24.20) yields

$$z_{st} - z_0 = \frac{1}{2}h = \frac{\omega^2 R^2}{4g} \tag{24.22}$$

Equation (24.22) indicates that the increase in elevation at the container wall equals the decrease in elevation at the centerline, with both being equal to $\omega^2 R^2/4g$. Finally, the gauge pressure distribution at any point in the liquid can be shown to be

$$P = \rho\frac{g}{g_c}(z_0 - z) + \frac{\rho\omega^2 r^2}{2g_c} \tag{24.23}$$

Illustrative Example 24.5 Consider the case of a circular cylinder (diameter = 6 in., height = 1 ft), that is filled with water (density 62.4 lb/ft^3). If it is rotated at a uniform, steady angular speed about its central axis in rigid-body motion, calculate the rate in rpm for which the water will start spilling out. Also calculate the rpm for which one third of the water will spill out.

Solution Since the cylinder is full, the water will spill the moment the cylinder starts to spin. Therefore, spilling occurs when $\omega > 0$ rpm. To determine the angular speed for $\frac{1}{3}$ of the water to spill, consider the cylinder at rest when $\frac{1}{3}$ of the water has already been spilled. The stationary height, z_{st}, is obviously $\frac{2}{3}$ ft. The increase in height, $h/2$, due to rigid-body motion is

$$h/2 = 1 - 0.667 = 0.333\,\text{ft}$$

Employ Equation (24.22) to calculated the angular velocity:

$$h/2 = \frac{\omega^2 R^2}{4g}$$

Substitution yields:

$$\omega = \sqrt{\frac{4g(h/2)}{R^2}} = \sqrt{\frac{4(32.174)(0.333)}{(0.25)^2}} = 26.2\,\text{rad/s} = 250\,\text{rpm}$$

Illustrative Example 24.6 Refer to Illustrative Example 24.5. Determine the equation describing the pressure distribution within the system.

Solution The (gauge) pressure distribution equation is obtained from Equation (24.23):

$$P = \rho \frac{g}{g_c}(z_0 - z) + \frac{\rho \omega^2 r^2}{2g_c}$$

since

$$z_0 = z_{st} - h/2 = 0.333\,\text{ft}$$

and substituting

$$P = \rho \frac{g}{g_c}(z_0 - z) + \frac{\rho \omega^2 r^2}{2g_c} = 62.4(0.333 - z) + \frac{62.4(26.2)^2 r^2}{2(32.174)}$$

$$P = 62.4(0.333 - z) + 665.3r^2;\ \text{psfg}$$

Illustrative Example 24.7 A cylindrical cup (diameter $= 6\,\text{cm}$, height $= 10\,\text{cm}$) open to the atmosphere is filled with a liquid (density $= 1010\,\text{kg/m}^3$) to a height of 7 cm. It is placed on a turn-table and rotated around its axis. Find:

1. The angular speed that causes the liquid to start spilling.
2. The gauge pressures at points A, B, and C along the bottom of the cup. Point C is at the center, and points A and B are at the wall (see Fig. 24.2).
3. The thickness of liquid film at the original height of the liquid during rotation.

Solution Calculate an angular velocity that will cause the liquid to start spilling

$$h = 10 - 7 = 3\,\text{cm} = 0.03\,\text{m}$$
$$R = \text{radius} = 0.03\,\text{m}$$

Applying Equation (24.22)

$$\omega^2 = \frac{(2)(0.03)(9.807)}{(0.03)^2}$$
$$\omega = 36.2\,\text{rad/s} = 345\,\text{rpm}$$

Calculate the pressure at A and B, that is, P_A and P_B, respectively

$$P_A = P_B \text{ (because of symmetry)}$$
$$P_A = P_B = \rho g\,\Delta z = (1010)(9.807)(0.1) = 990\,\text{Pa gauge (Pag)}$$

This may be converted to psig noting that 14.696 psig = 101,325 Pa. The liquid height above point C is

$$z_0 = z_{st} - h/2 = 7 - 3 = 4\,\text{cm} = 0.04\,\text{m}$$

The gauge pressure at point C, P_C is therefore

$$P_C = (1010)(9.807)(0.04) = 396\,\text{Pag}$$

To obtain the film thickness, determine the original height.

$$z_{st} = 0.07\,\text{m} = 7\,\text{cm}$$
$$z_0 = z_{st} - h/2 = 7 - 3 = 4\,\text{cm} = 0.04\,\text{m}$$

Substitute in Equation (24.18) to obtain r.

$$0.07 - 0.04 = \frac{36.2^2 r^2}{2(9.807)}$$
$$r = 0.0212\,\text{m} = 2.12\,\text{cm}$$

Therefore

$$\text{film thickness} = R - r = 3 - 2.12 = 0.88\,\text{cm} = 8.8\,\text{mm}$$

24.4 FLOTATION

Flotation processes are useful for the separation of a variety of species, ranging from molecular and ionic to microorganisms and mineral fines, from one another for the purpose of extraction of valuable products as well as cleaning of waste waters. These processes are particularly attractive for separation problems involving very dilute solutions where most other processes usually fail. The success of flotation processes is dependent primarily on the tendency of surface-active species to concentrate at the water–fluid interface and on their capability to make selected non-surface-active materials hydrophobic by means of adsorption on them or association with them. Under practical conditions, the amount of interfacial area available for such concentration is increased by generating air bubbles or oil droplets in the aqueous solution.[6]

Flotation is a gravity separation process based on the attachment of air or gas bubbles to solid (or liquid) particles that are then carried to the liquid surface where they accumulate as float material and can be skimmed off. The process consists of two stages: the production of suitably small bubbles, and their attachment to the particles. Depending on the method of bubble production, flotation is classified as dissolve-air, electrolytic, or dispersed-air, with dissolve-air primarily employed by industry.[1]

The separation of solid particles into several fractions based upon their terminal velocities is called hydraulic classification. By placing the particles of different densities in an upward-flowing stream of fluid (often the fluid is water) particles of materials of different densities but of the same size may also be separated by the method of hydraulic separation or classification. If the velocity of the water is adjusted so that it lies between the terminal falling velocities (or settling velocities) of the two particles, the slower particles will be carried upward and the particles of higher terminal velocity than the water velocity will move downward, and a separation is thereby attained.

Illustrative Example 24.8 It is desired to separate quartz (q) particles (SG = 2.65, with a size range of 40–90 μm) from galena (g) particles (SG = 7.5, with a similar size range of 40–90 μm). The mixture will be placed in a rising water (w) flow (density, $\rho = 1000\,\mathrm{kg/m^3}$; viscosity, $\mu = 0.001\,\mathrm{kg/m\cdot s}$). See Fig. 24.3. Calculate the water velocity to obtain pure galena. Will this pure galena be a top or bottom product?

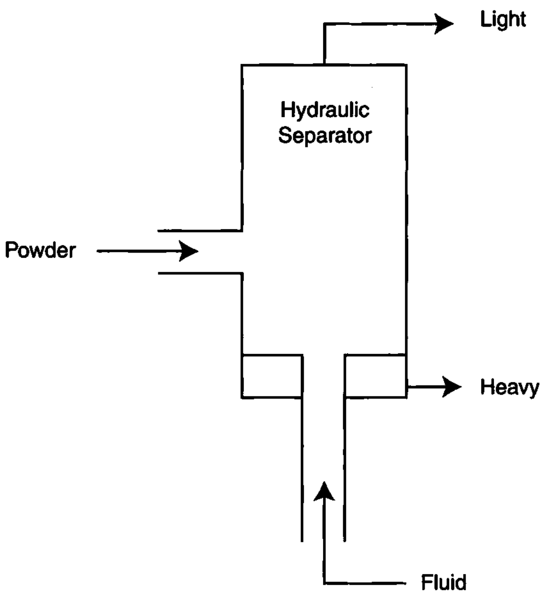

Figure 24.3 Rigid body rotation.

Solution Calculate the settling velocity of the largest quartz particle, with a diameter, $d_p = 90\,\mu\mathrm{m}$. Employ Equation (23.32) from Chapter 23.

$$K = d_p\left(\frac{g(\rho_s - \rho_f)\rho_f}{{\mu_f}^2}\right)^{1/3} = 9 \times 10^{-5}\left(\frac{9.807(2650 - 1000)1000}{0.001^2}\right)^{1/3} = 2.27 < 3.3$$

Stokes' flow regime applies. Therefore, from Equation (23.36),

$$v_q = \frac{g d_p{}^2(\rho_s - \rho_f)}{18\mu_f} = \frac{9.807(9 \times 10^{-5})^2(2650 - 1000)}{18(0.001)}$$
$$= 0.0073 \text{ m/s}$$

Calculating the settling velocity of the smallest galena particle with a diameter of $d_p = 4 \times 10^{-5}$ m.

$$K = d_p \left(\frac{g(\rho_s - \rho_f)\rho_f}{\mu_f{}^2} \right)^{1/3} = 4 \times 10^{-5} \left(\frac{9.807(7500 - 1000)1000}{0.001^2} \right)^{1/3} = 1.6 < 3.3$$

Stokes' flow regime again applies. Therefore,

$$v_g = \frac{g d_p{}^2(\rho_s - \rho_f)}{18\mu_f} = \frac{9.807(4 \times 10^{-5})^2(7500 - 1000)}{18(0.001)} = 0.00567 \text{ m/s}$$

To obtain pure galena, the upward velocity of the water must be equal to or greater than the settling velocity of the largest quartz particle. Therefore,

$$v_w = 0.0073 \text{ m/s} = 7.3 \text{ mm/s}$$

Since the water velocity of 7.3 mm/s is greater than the settling velocity of the smallest galena particle, some galena will be washed up with the quartz. One may conclude that pure galena will be the bottom product; the top product will be the quartz plus some galena.

Illustrative Example 24.9 Refer to Illustrative Example 24.8. Determine the size range of the galena in the top product.

Solution To determine the size range of the galena product, calculate the galena particle size that has a settling velocity of 7.33 mm/s. Assume Stokes' law applies

$$v = \frac{g d_p{}^2(\rho_s - \rho_f)}{18\mu}$$

$$d_p = \sqrt{\frac{18\mu_f v}{g(\rho_s - \rho_f)}} = \sqrt{\frac{18(0.001)(0.0073)}{9.807(7500 - 1000)}} = 4.54 \times 10^{-5} \text{ m} = 45.4\ \mu\text{m}$$

Check on the validity of Stokes' flow by calculating the K factor.

$$K = d_p\left(\frac{g(\rho_s - \rho_f)\rho_f}{\mu_f{}^2}\right)^{1/3} = 4.54 \times 10^{-5}\left(\frac{9.807(7500 - 1000)1000}{0.001^2}\right)^{1/3}$$
$$= 1.82 < 3.3$$

Therefore, the flow is in the Stokes' law range.

The size ranges for the galena are 40–45.4 μm for the top washed product.

Illustrative Example 24.10 Air is being dried by bubbling (in very small bubbles) through concentrated NaOH (specific gravity of 1.34 and viscosity equal to 4.3 cP). The base fills a 4.5-ft tall, 2.5-in. ID tube to a depth of 1.0 ft. The air above the base is at ambient conditions. If the air rate is 4.0 ft^3/min, what is the maximum diameter of a base spray droplet that would be carried out of the apparatus by entrainment in the air stream?

Solution The velocity of air is

$$v = \frac{q}{S} = \frac{4\,\dfrac{\text{ft}^3}{\text{min}} \times \dfrac{\text{min}}{60\,\text{s}}}{\dfrac{\pi}{4}(2.5/12\,\text{ft})^2} = 1.956\,\text{ft/s}$$

Assuming that the Intermediate range applies, calculated $d_p(\max)$ by

$$d_p(\max)^{1.14} = \frac{v\rho^{0.29}\mu^{0.43}}{0.153(g\rho_p)^{0.71}}$$

Substituting the data (in consistent units) gives

$$d_p(\max)^{1.14} = \frac{(1.956)(0.0775)^{0.29}(1.23 \times 10^{-5})^{0.43}}{(0.153)(32.2 \times 1.34 \times 62.4)^{0.71}}$$
$$= 1.7291 \times 10^{-4}(\text{ft})^{1.14}$$

or

$$d_p(\max) = 5.01 \times 10^{-4}\,\text{ft}$$

Check the original assumption by calculating K.

$$K = d_p\left[\frac{g\rho\rho_p}{\mu^2}\right]^{1/3}$$

Substituting the data,

$$K = 5.01 \times 10^{-4} \left[\frac{(32.2)(0.0775)(1.34)(62.4)}{(1.23 \times 10^{-5})^2} \right]^{1/3}$$

$$= 5.58$$

Thus, the result for d_p(max) is correct.

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