23

PARTICLE DYNAMICS

23.1 INTRODUCTION

It should be noted that this chapter will primarily address gas-particle rather than liquid-particle behavior. The treatment of liquid–solid behavior/separation will appear in Chapter 24—Sedimentation, Centrifugation, Flotation. The general subject of particle classification and measurement receives treatment at the end of the chapter.

23.2 PARTICLE CLASSIFICATION AND MEASUREMENT

Particle size is uniquely defined by particle diameter only for the case of spherical particles. Unfortunately, except for liquid droplets, certain metallurgical fumes, and combustion emissions, particles are usually not spherical. This may also be the case with nanoparticles. To deal with nonspherical particles, it becomes necessary to define an *equivalent diameter* term that depends upon the various geometrical and/or physical properties of the particles.

Some of the methods used to express the size of a nonspherical particle measured by microscopy are illustrated in Fig. 23.1. With reference to this figure, Ferret's diameter is the mean length between two tangents on opposite sides of the particle perpendicular to the fixed direction of the microscopic scan. Martin's diameter measures the diameter of the particle parallel to the microscope scan that divides

Fluid Flow for the Practicing Chemical Engineer. By J. Patrick Abulencia and Louis Theodore

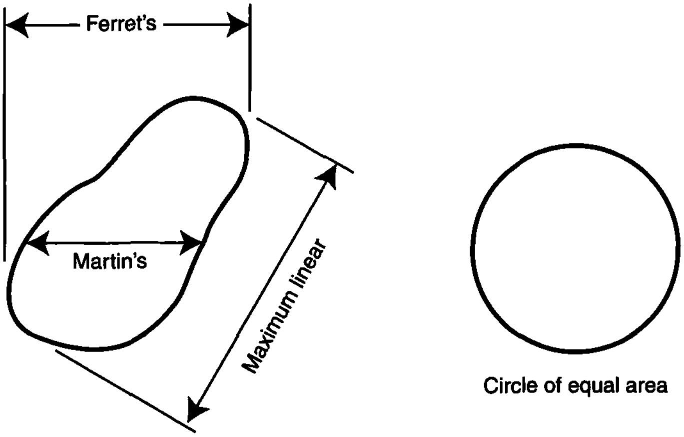

Figure 23.1 Diameters of nonspherical particles.

the particle into two equal areas. The diameter of a circle of equal area is obtained by estimating the projected area of the particle and comparing it with a sphere that approximates its size.

The most popular choice is that sphere diameter (of the same density) that will settle with the same velocity as the particle in question under the influence of gravity. Other diameters that are occasionally/rarely employed are listed in Table 23.1.

The *aerodynamic* diameter of a particle is defined as the diameter of a sphere of unit density (specific gravity = 1.0) having the same falling speed in air as the particle. It is most useful in evaluating particle motion in a fluid. The aerodynamic diameter is a function of the physical size, shape, and density of the particle. The aerodynamic diameter is useful when designing certain recovery/control devices and is usually measured by a device called an impactor.

The aerodynamic diameter ($d_{p,a}$) is defined by:

$$d_{p,a} = d_p\sqrt{\rho_p C} \tag{23.1}$$

Table 23.1 Equivalent diameters of particles

Name	Definition
Surface diameter	The diameter of a sphere having the same surface area as the particle
Volume diameter	The diameter of a sphere having the same volume as the particle
Drag diameter	The diameter of a sphere having the same resistance to motion as the particle in a fluid of the same viscosity and at the same velocity
Specific surface diameter	The diameter of a sphere having the same ratio of surface area to volume as the particle

where $d_{p,a}$ = aerodynamic diameter, consistent units; d_p = actual (equivalent) diameter, consistent units; ρ_p = particle specific gravity, dimensionless; and C = Cunningham correction factor (CCF), dimensionless (to be discussed shortly).

Illustrative Example 23.1 Calculate the aerodynamic diameter (μm) for the following two particles:

1. Solid sphere, equivalent diameter = 1.4 μm, specific gravity = 2.0.
2. Hollow sphere, equivalent diameter = 2.8 μm, specific gravity = 0.5.

Solution Employ Equation (23.1).

1. For the solid sphere,

$$\begin{aligned} d_{p,a} &= 1.4(2.0)^{0.5} \\ &= 1.98\ \mu\text{m} \end{aligned}$$

2. For the hollow sphere,

$$\begin{aligned} d_{p,a} &= 2.80(0.51)^{0.5} \\ &= 2.0\ \mu\text{m} \end{aligned}$$

Illustrative Example 23.2 Calculate the aerodynamic diameter (μm) of an irregular-shaped "sphere" with an equivalent diameter = 1.3 μm and specific gravity = 2.35.

Solution Once again employ Equation (23.1). For the irregular shape,

$$\begin{aligned} d_{p,a} &= 1.3(2.35)^{0.5} \\ &= 1.99\ \mu\text{m} \end{aligned}$$

Based on the results of this and the previous illustrative example, one concludes that particles with different specific gravity, but the same equivalent size, can have different aerodynamic diameters. For example, if $d_p = 2$ μm, the reader is left the exercise of showing that the aerodynamic diameter for particles with specific gravity 1.0, 2.0, 4.0, and 8.0 is 2.00 μm, 2.83 μm, 4.00 μm, and 5.66 μm, respectively. Thus, particles of different size and shape can have the same aerodynamic diameter while particles of the same size can have different aerodynamic diameters.

A common method of specifying large particle sizes is to designate the screen mesh that has an aperture corresponding to the particle diameter. Since various screen scales are in use, confusion may result unless the screen scale involved is specified. The screen mesh generally refers to the number of screen openings per unit of length or area. The aperture for a given mesh will depend on the wire size

Table 23.2 Tyler and U.S. standard screen scales

Tyler Mesh	Aperture, Microns	U.S. Mesh	Aperture, Microns
400	37	400	37
325	43	325	44
270	53	270	53
250	61	230	62
200	74	200	74
170	88	170	88
150	104	140	105
100	147	100	149
65	208	70	210
48	295	50	297
35	417	40	420
28	589	30	590
20	833	20	840
14	1168	16	1190
10	1651	12	1680
8	2362		
6	3327		
4	4699		
3	6680		

employed. The Tyler and the U.S. Standard Screen Scales in SI units (Table 23.2) are the most widely used in the United States. The screens are generally constructed of wire mesh cloth, with the diameters of the wire and the spacing of the wires being closely specified. These screens form the bottoms of metal pans about 8 in. in diameter and 2 in. high, whose sides are so fashioned that the bottom of one sieve nests snugly on the top of the next. Additional information is provided in Chapter 25—refer to Table 25.2.

The clear space between the individual wires of the screen is termed the screen aperture. As indicated above, the term *mesh* is applied to the number of apertures per linear inch; for example, a 10-mesh screen will have 10 openings per inch, and the aperture will be 0.1 in. minus the diameter of the wire.

A typical particulate size distribution analysis method of representation employed in the past is provided below in Table 23.3. The numbers in Table 23.3 mean that 40% of the particles by weight are greater than 5 μm (microns or micrometers) in size, 27% are less than 5 μm but greater than 2.5 μm, 20% are less than 2.5 μm but greater than 1.5 μm, and the remainder (13%) are less than 1.5 μm.

Table 23.3 Particle size distribution

	>5.0	μm	40%
<5	>2.5	μm	27%
<2.5	>1.5	μm	20%
<1.5		μm	13%
			100%

Most industrial techniques used for the separation of particles from gases involve the relative motion of the two phases under the action of various external forces. The collection methods for particulates are based on the movement of solid particles (or liquid droplets) through a gas. The final objective is their removal and/or recovery for economic reasons. In order to accomplish this, the particle is subjected to external forces—forces large enough to separate the particle from the gas stream during its residence time in the unit. Perhaps the most important of these forces is the drag force.

23.3 DRAG FORCE

Whenever a difference in velocity exists between a particle and its surrounding fluid, the fluid will exert a resistive force upon the particle. Either the fluid (gas) may be at rest with the particle moving through it or the particle may be at rest with the gas flowing past it. It is generally immaterial which phase (solid or gas) is assumed to be at rest; it is the *relative* velocity between the two that is important. The resistive force exerted on the particle by the gas is called the *drag*.

In treating fluid flow through pipes, a friction factor term is used in many engineering calculations. An analogous factor, called the drag coefficient, is employed in drag force calculations for flow past particles. Consider a fluid flowing past a stationary solid sphere. If F_{D} is the drag force and ρ is the density of the gas, the drag coefficient, C_{D}, is defined as

$$C_{\mathrm{D}} = \frac{F_{\mathrm{D}}}{A_p}\frac{2g_c}{\rho v^2} \tag{23.2}$$

From dimensional analysis, one can then show that the drag coefficient is solely a function of the particle Reynolds number, Re, that is,

$$C_{\mathrm{D}} = C_{\mathrm{D}}(\mathrm{Re}) \tag{23.3}$$

where

$$\mathrm{Re} = \frac{d_p v \rho}{\mu}$$

The quantitative use of the equation of particle motion presented in the next section requires numerical and/or graphical values of the drag coefficient as a function of the Reynolds number. These are presented in Fig. 23.2 and Table 24.3 respectively.

In the following analysis, it is assumed that:

1. The particle is a rigid sphere (with a diameter d_p) surrounded by gas in an infinite medium (no wall or multiparticle effects).
2. The particle or fluid is not accelerating.

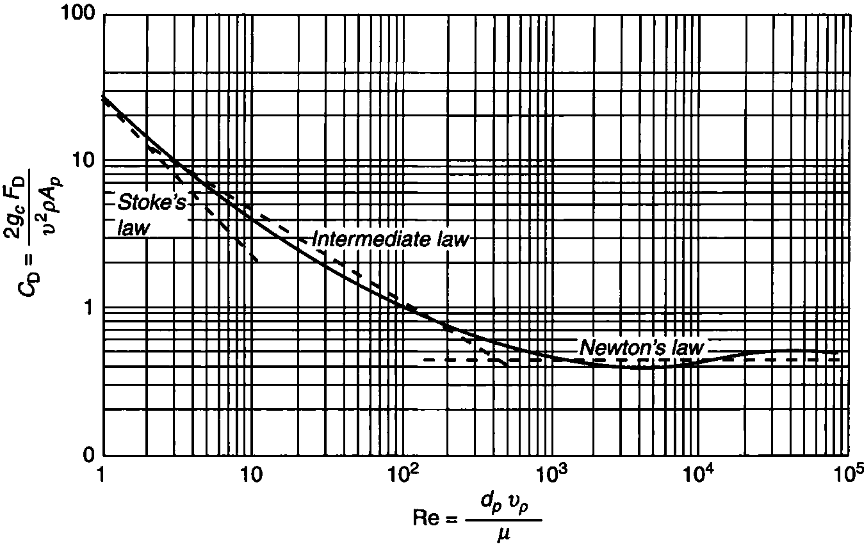

Figure 23.2 Drag coefficient for spheres.

A brief discussion of fundamentals is appropriate here because of the importance of air flow around particulates. No attempt will be made to develop the expressions for the distribution of momentum flux, pressure, and velocity. However, these expressions will be applied to develop some of the more important relationships.

The drag force, F_D, exerted on a particle by a gas at low Reynolds numbers is given by

$$F_D = \frac{6\pi\mu va}{g_c} = \frac{3\pi\mu vd_p}{g_c} \tag{23.4}$$

Equation 23.4 is known as *Stokes' law* and can be derived theoretically. However, keep in mind that Stokes' equation is valid only for very low Reynolds numbers—up to Re ≈ 0.1; at Re $= 1$, it predicts a value for the drag force that is nearly 10% too low. In practical applications, Stokes' law is generally assumed applicable up to a Reynolds number of 2.0. By rearranging Stokes' law in the form of Equation 23.2, the drag coefficient becomes

$$C_D = \frac{6\pi\mu va/\pi a^2}{\rho v^2/2}; \quad d_p = 2a \tag{23.5}$$

where a equals the particle radius. Hence, for *creeping flow* around a particle, Equation (23.5) reduces to

$$C_D = \frac{24}{\text{Re}} \tag{23.6}$$

This is the straight-line portion of the log-log plot of C_D vs. Re (Fig. 23.1). For higher values of the Reynolds number, it is almost impossible to perform purely theoretical calculations. However, several investigators have managed to estimate, with a considerable amount of effort, the drag and/or drag coefficient at higher Reynolds numbers.

In addition to the analytical equation (Eq. 23.6), one may use

$$C_D = 18.5/\mathrm{Re}^{0.6}; \quad 2 < \mathrm{Re} < 500 \tag{23.7}$$

for the *intermediate* range. This indicates a lesser dependence than Stokes' law on Re; it is less accurate than Stokes' law for Re < 2. At higher Re, the drag coefficient is approximately constant. This is the *Newton's* law range, for which

$$C_D \approx 0.44; \quad 500 < \mathrm{Re} < 200{,}000 \tag{23.8}$$

In this region the drag force on the sphere is proportional to the square of the gas velocity. (Note that Newton's law for the drag force is not to be confused with Newton's law of viscosity or Newton's laws of motion.) A simple two-coefficient model of the form

$$C_D = \alpha\, \mathrm{Re}^{-\beta} \tag{23.9}$$

can therefore be used over the three Reynolds-number ranges given in Equations (23.6)–(23.8). The numerical values of α and β are given below:

	α	β
Stokes range	24.0	1.0
Intermediate range	18.5	0.6
Newton range	0.44	0.0

Using the above model in Equation (23.2), the drag force becomes

$$F_D = \frac{\alpha\pi(d_p v)^{2-\beta}\mu^{\beta}\rho^{1-\beta}}{8g_c} \tag{23.10}$$

The above equation reduces to

$$F_D = \frac{3\pi\mu v d_p}{g_c} \tag{23.11}$$

for the Stokes' law range (Re < 2),

$$F_D = \frac{2.31\pi(d_p v)^{1.4}\mu^{0.6}\rho^{0.4}}{g_c} \tag{23.12}$$

for the intermediate range (2 < Re < 500), and

$$F_D = \frac{0.055\pi(d_p v)^{2}\rho}{g_c} \tag{23.13}$$

Table 23.4 Calculated vs. experimental values for the drag coefficient as a function of the Reynolds number

		C_D					
Range	Re	Equation (23.6)	Equation (23.7)	Equation (23.8)	Equation (23.14)	Equation (23.15)	Experiment
Stokes' law	0.01	2400				2364	2100
	0.02	1200				1195	1050
	0.03	820				803	700
	0.05	490				488	420
	0.07	350				352	300
	0.10	290				249	240
	0.20	120				129	120
	0.30	82				88.3	80
	0.50	49				55.0	49.5
	0.70	35				40.5	36.5
	1.00	24			22.4	29.5	26.5
Intermediate	2		12.0			16.2	14.4
	3		9.5		10.5	11.6	10.4
	4		7.8			9.2	8.2
	5		6.9			7.7	6.9
	6				6.23		5.9
	7		5.4			5.97	5.4
	10		4.5		4.26	4.61	4.1
	20		3.0			2.90	2.55
	30		2.4			2.26	2.0
	40		2.0		1.71	1.93	1.6
	50		1.8			1.71	1.5
	70		1.5			1.45	1.27

	90			1.11		1.14
	100	1.2			1.23	1.07
	200	0.80		0.785	0.93	0.77
	300	0.68			0.82	0.65
	400	0.60			0.76	0.57
	500	0.56		0.568	0.71	0.55
Newton's law	700		0.44	—	0.66	0.50
	900		0.44	0.482	—	0.46
	1000		0.44	—	0.61	0.45
	2000		0.44	—	0.54	0.42
	3000		0.44	0.393	0.51	0.40
	4000		0.44	—	0.49	0.39
	5000		0.44	—	0.48	0.385
	7000		0.44	0.386	0.47	0.39
	9000		0.44	0.398	—	0.40
	10,000		0.44	—	0.46	0.405
	20,000		0.44	0.451	0.44	0.45
	30,000		0.44	—	0.43	0.47
	40,000		0.44	—	0.42	0.48
	50,000		0.44	—	0.42	0.49
	60,000		0.44	0.520	—	0.49
	70,000		0.44	—	0.42	0.50
	100,000		0.44	0.466	0.42	0.48
	200,000		0.44	—	0.41	0.42
	300,000		—	—	—	0.20
	400,000		—	—	—	0.084
	600,000		—	—	—	0.10
	1,000,000		—	—		0.13
	3,000,000		—	—	—	0.20

for the Newton's law range ($500 < \mathrm{Re} < 200{,}000$). This two-coefficient, three-Reynolds-number range model will be used for drag force calculations in this and subsequent chapters. Numerical and experimental values for the drag coefficient from the model, Equations (23.6)–(23.8) are presented in Table 23.4. A comparison between the two indicates that these three equations are fairly consistent with the experimental values found in the literature.

Another empirical drag coefficient model[1] is given by Equation (23.14):

$$\begin{aligned} \log C_{\mathrm{D}} = 1.35237 &- 0.60810(\log \mathrm{Re}) - 0.22961(\log \mathrm{Re})^2 \\ &+ 0.098938(\log \mathrm{Re})^3 + 0.041528(\log \mathrm{Re})^4 \\ &- 0.032717(\log \mathrm{Re})^5 + 0.007329(\log \mathrm{Re})^6 \\ &- 0.0005568(\log \mathrm{Re})^7 \end{aligned} \tag{23.14}$$

This is an empirical equation which has been obtained by the use of a statistical fitting technique. As is evident from Table 23.4, this correlation gives excellent results over the entire range of Reynolds numbers. An advantage of using this correlation is that it is not partitioned for application only to a specific Reynolds number range. However, the lengthy calculation warrants its use only as a subroutine in a computer program.

Still another empirical equation[2] is

$$C_{\mathrm{D}} = \left[0.63 + (4.80/\sqrt{\mathrm{Re}})\right]^2 \tag{23.15}$$

This correlation is also valid over the entire spectrum of Reynolds numbers. Its agreement with literature values, as seen from Table 23.4, is generally good. However, in the range of $30 < \mathrm{Re} < 10{,}000$, there is considerable deviation. For $\mathrm{Re} < 30$ or $\mathrm{Re} > 10{,}000$, the agreement is excellent. This correlation lends itself easily to manual calculations.

23.4 PARTICLE FORCE BALANCE

Consider now a solid spherical particle located in a gas stream and moving in one direction with a velocity, v, relative to the gas. The net or resultant force experienced by the particle is given by the summation of all the forces acting on the particle. These forces include drag, buoyancy, and one or more external forces (such as gravity, centrifugal, and electrostatic). In order to simplify the presentation, the direction of particle movement relative to the gas is always assumed to be positive. Newton's law of motion is then

$$F_{\mathrm{R}} = F - F_{\mathrm{B}} - F_{\mathrm{D}} \tag{23.16}$$

where F_R is the resultant or net force; F is the external force; F_B is the buoyant force; and F_D is the drag force. The net force results in acceleration of the particle, given by

$$F_R = \frac{m}{g_c}\left(\frac{dv}{dt}\right) \tag{23.17}$$

where m is the mass of the particle ($\pi d_p{}^3\rho_p/6$); and ρ_p is the particle density. The external force per unit mass is denoted as f. The external force, F, on the particle is then

$$F = mf \tag{23.18}$$

Unless the particle is in a vacuum, it will experience a buoyant force in conjunction with the external force(s); this is given by

$$F_B = m_f f \tag{23.19}$$

where m_f is the mass of gas (fluid) displaced by the particle. The equation of motion now becomes

$$\begin{aligned}\left(\frac{dv}{dt}\right)\Big/g_c &= f - \left(\frac{m_f}{m}\right)f - \left(\frac{F_D}{m}\right)\\ &= f\left(1 - \frac{m_f}{m}\right) - \left(\frac{F_D}{m}\right)\\ &= f\left(\frac{m - m_f}{m}\right) - \left(\frac{F_D}{m}\right)\end{aligned} \tag{23.20}$$

This equation may also be written as

$$\left(\frac{dv}{dt}\right)\Big/g_c = \left(\frac{fm_{eq}}{m}\right) - \left(\frac{F_D}{m}\right) \tag{23.21}$$

where $m_{eq} = (m - m_f)$, or

$$\begin{aligned}\left(\frac{dv}{dt}\right)\Big/g_c &= f\left(1 - \frac{\rho}{\rho_p}\right) - \left(\frac{F_D}{m}\right)\\ &= f\left(\frac{\rho_p - \rho}{\rho_p}\right) - \left(\frac{F_D}{m}\right)\end{aligned} \tag{23.22}$$

For gases, $\rho_p >>> \rho$, so that the bracketed terms in Equations (23.20) and (23.22) reduce to unity.

The particle may also be acted upon by one or more external forces. If the external force is gravity

$$f_g = \frac{g}{g_c}$$

with

$$F_g = m\left(\frac{g}{g_c}\right)$$

The describing equation for particle motion then becomes

$$\left(\frac{dv}{dt}\right)\Big/g_c = \left(\frac{g}{g_c}\right) - \left(\frac{F_D}{m}\right) \tag{23.23}$$

If the particle experiences another type of force, for example, an electrostatic force, F_E, then

$$F_E = mf_E$$

so that

$$\left(\frac{dv}{dt}\right)\Big/g_c = f_E - \left(\frac{F_D}{m}\right) \tag{23.24}$$

where f_E is the electrostatic force per unit mass of particle. If, for example, the external force is from a centrifugal field

$$f_C = \frac{r\omega^2}{g_c} = \frac{v_\phi{}^2}{g_c r}$$

where r is the radius of the path of the particle, f_C is the centrifugal force per unit mass of particle, ω is the angular velocity, and v_ϕ is the tangential velocity at that point. The centrifugal force, F_C, is then

$$F_C = mf_C$$

The describing equation becomes

$$\left(\frac{dv}{dt}\right)\Big/g_c = \left(\frac{r\omega^2}{g_c}\right) - \left(\frac{F_D}{m}\right) \tag{23.25}$$

or

$$\left(\frac{dv}{dt}\right)\Big/g_c = \left(\frac{v_\phi{}^2}{g_c r}\right) - \left(\frac{F_D}{m}\right) \tag{23.26}$$

The reader is reminded on the use of g_c. Any term or group of terms in the above equations may be indiscriminately multiplied or divided by this conversion constant.

If a particle is initially at rest in a stationary gas and is then set in motion by the application of a constant external force or forces, the resulting motion occurs in two stages. The first period involves acceleration, during which time the particle velocity increases from zero to some maximum velocity. The second stage occurs when the particle achieves this maximum velocity and remains constant. During the second stage, the particle is not accelerating. The left-hand side of Equations (23.20) and (23.26) are, therefore, zero. The final, constant, and maximum velocity attained is defined as the *terminal settling velocity* of the particle. Most particles reach their terminal settling velocity almost instantaneously.

Consider the equations examined above under terminal settling conditions. Since

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 0$$

the general equation for particle motion becomes

$$0 = f - \left(\frac{F_{\mathrm{D}}}{m}\right)$$

or

$$f = \frac{F_{\mathrm{D}}}{m} \tag{23.27}$$

The units of f in this equation are those of acceleration, that is, length/(time)^2. The general equation for the terminal settling velocity is obtained by direct substitution of Equation (23.10) into Equation (23.27) and solving for v. Thus,

$$f = \frac{3\alpha v^2 \mu^\beta \rho}{4d_p(d_p v\rho)^\beta}$$

so that

$$v = \left[\frac{4f{d_p}^{1+\beta}\rho_p}{3\alpha\mu^\beta\rho^{1-\beta}}\right]^{1/(2-\beta)} \tag{23.28}$$

For the Stokes' law range, Equation (23.28) becomes

$$v = \frac{f{d_p}^2\rho_p}{18\mu} \tag{23.29}$$

For the intermediate range,

$$v = \frac{0.153f^{0.71}{d_p}^{1.14}{\rho_p}^{0.71}}{\mu^{0.43}\rho^{0.29}} \tag{23.30}$$

Finally, for Newton's law range

$$v = 1.74(fd_p\,\rho_p\rho)^{0.5} \tag{23.31}$$

Keep in mind that f denotes the external force per unit mass of particle. One consistent set of units (English) for the above equations is ft/s^2 for f, ft for d_p, lb/ft^3 for ρ, lb/ft · s for μ, and ft/s for v.

Ordinarily, determining the settling velocity of a particle of known diameter would require a trial-and-error calculation since the particle's Reynolds number is unknown. Thus, one cannot select the proper describing drag force equation. This iterative calculation can be circumvented by rearrangement of the drag force equations and solving for the settling velocity directly. Both sides of Equations (23.29) and (23.31) are multiplied by

$$\frac{d_p\rho}{\mu}$$

A dimensionless constant, K, is defined as

$$K = d_p\left(\frac{f\rho_p\rho}{\mu^2}\right)^{1/3} \tag{23.32}$$

Equations (24.29) and (24.31) can now be rewritten, respectively, as

$$\text{Re} = \frac{K^3}{18} \tag{23.33}$$

and

$$\text{Re} = 1.74K^{1.5} \tag{23.34}$$

Since K is not a function of the settling velocity, the choice of drag force equations may now be based on calculated K values. These new K range limits are given as follows:

$K < 3.3$	Stokes
$43.6 > K > 3.3$	Intermediate range
$2360 > K > 43.6$	Newton

If K is greater than 2360, the drag coefficient may change abruptly with small changes in fluid velocity.

Larocca[3] and Theodore[4], using the same approach employed above, defined a dimensionless term W that would enable one to calculate the diameter of a particle if the terminal velocity is known. This particular approach has found application in catalytic reactor particle size calculations. The term W—which does not depend on the particle diameter—is given by

$$W = \frac{v^3 \rho^2}{g \mu \rho_p} \tag{23.35}$$

The two key values of W that are employed in a manner similar to that for K are 0.2222 and 1514, that is, for $W < 0.2222$, the Stokes' law region applies, for $W > 1514$, the Newton's law region applies and in between, the intermediate law region applies.

23.5 CUNNINGHAM CORRECTION FACTOR

When particles approach sizes comparable to the mean free path of other fluid molecules, the medium can no longer be regarded as continuous since particles can fall between the molecules at a faster rate than predicted by aerodynamic theory. To allow for this "slip," Cunningham's correction factor[5] is introduced to Stokes' law

$$v = \frac{g d_p^2 \rho_p}{18 \mu} C \tag{23.36}$$

where C is the Cunningham correction factor (CCF), and

$$C = 1 + \frac{2A\lambda}{d_p} \tag{23.37}$$

The term A is $1.257 \times 10^{0.40} \exp(-1.10 d_p/2\lambda)$ and λ is the mean free path of the fluid molecules (6.53×10^{-6} cm for ambient air). The CCF is usually applied to particles equal to or smaller than 1 micron. Applications include particulate air pollution and nanotechnology[6] studies.

Illustrative Example 23.3 Calculate the CCF for particle size variation from 1.0 nm to 10^4 nm at temperatures of 70°F, 212°F, and 500°F. Include a sample calculation for a particle diameter of 400 nm (0.4 μm) at 70°F, 1 atm.

Solution Employ the equations presented above. The calculated results are provided in Table 23.5 along with a sample calculation.

Table 23.5 Cunningham correction factors

d_p (nm)	d_p (μm)	C (70°F)	C (212°F)	C (500°F)
1	0.001	216.966	274.0	405.32
10	0.01	22.218	27.92	39.90
100	0.1	2.867	3.61	5.14
250	0.25	1.682	1.952	2.528
500	0.5	1.330	1.446	1.711
1000	1	1.164	1.217	1.338
2500	2.5	1.066	1.087	1.133
5000	5	1.033	1.043	1.067
10,000	10	1.016	1.022	1.033

For a d_p of 0.4 μm, the CCF should be included. Employing the equation given above

$$\begin{aligned} A &= 1.257 + 0.40e^{-1.10d_p/2\lambda} \\ &= 1.257 + 0.40e^{-\left[\frac{(1.10)(0.4)}{(2)(6.53\times 10^{-2})}\right]} \\ &= 1.2708 \end{aligned}$$

Therefore

$$\begin{aligned} C &= 1 + \frac{2A\lambda}{d_p} \\ &= 1 + \frac{(2)(1.2708)(6.53 \times 10^{-2})}{0.4} \\ &= 1.415 \end{aligned}$$

The results clearly demonstrate that the CCFs become more pronounced for nano-sized particles in the 10–1000 nm range. In addition, an increase in temperature also leads to an increase in this effect.

The reader should also note that a comparable effect does not exist for particles settling in liquids until the diameter become less than 10 nm (0.01 μm).

Illustrative Example 23.4 Three different diameter sized fly-ash particles—0.4, 40, and 400 microns—settle through air. You are asked to calculate the particle terminal velocity and determine how far each will fall in 30 seconds. Assume the particles are spherical. The air temperature and pressure are 238°F and 1 atm, respectively. The specific gravity of fly-ash is 2.31.

Solution Calculate the particle density using the specific gravity given

$$\rho_p = \text{SG}(62.4) = 2.31(62.4) = 144.14\,\text{lb/ft}^3$$

Determine the properties of the air

$$\rho = \frac{PM}{RT} = \frac{(1)(29)}{(0.7302)(238 + 460)} = 0.0569\,\text{lb/ft}^3$$

For the viscosity of air (see Table A.9 in the Appendix)

$$\mu = 0.021\,\text{cP} = 1.41 \times 10^{-5}\,\text{lb/ft-s}$$

Determine the value for K for each fly-ash particle size settling in air.

For d_p of 0.4 microns

$$K = d_p\left(\frac{g\rho_p\rho}{\mu^2}\right)^{1/3} = \frac{0.4}{25{,}400(12)}\left(\frac{32.174(144.14)(0.0569)}{(1.41 \times 10^{-5})^2}\right)^{1/3} = 0.0144$$

For d_p of 40 microns

$$K = \frac{40}{25{,}400(12)}\left(\frac{32.174(144.14)(0.0569)}{(1.41 \times 10^{-5})^2}\right)^{1/3} = 1.44$$

For d_p of 400 microns

$$K = \frac{400}{25{,}400(12)}\left(\frac{32.174(144.14)(0.0569)}{(1.41 \times 10^{-5})^2}\right)^{1/3} = 14.4$$

Determine which fluid-particle dynamic law applies for the above values of K.

For a d_p of 0.4 microns, Stokes' law applies; for a d_p of 40 microns, Stokes' law applies; for a d_p of 400 microns, the Intermediate law applies.

Calculate the terminal settling velocity for each particle size in ft/s using the appropriate velocity equation.

For a d_p of 0.4 microns

$$v = \frac{g{d_p}^2\rho_p}{18\mu} = \frac{32.2(0.4)^2 144.14}{(25{,}400(12))^2(18)(1.41 \times 10^{-5})} = 3.15 \times 10^{-5}\,\text{ft/s}$$

For a d_p of 40 microns

$$v = \frac{g{d_p}^2\rho_p}{18\mu} = \frac{32.2(40)^2 144.14}{(25{,}400(12))^2(18)(1.41 \times 10^{-5})} = 0.315\,\text{ft/s}$$

For a d_p of 400 microns

$$v = \frac{0.153 g^{0.71} d_p{}^{1.14} \rho_p{}^{0.71}}{\mu^{0.43} \rho^{0.29}} = \frac{0.153(32.2)^{0.71}[400/25{,}400(12)]^{1.14}(144.14)^{0.71}}{(1.41 \times 10^{-5})^{0.43}(0.0569)^{0.29}}$$
$$= 8.76\,\text{ft/s}$$

Calculate how far, x, the fly-ash particles will fall in 30 seconds.

For a d_p of 40 microns

$$x = vt = 0.315(30) = 9.45\,\text{ft}$$

For a d_p of 400 microns

$$x = vt = 8.76(30) = 262.8\,\text{ft}$$

For a d_p of 0.4 microns (0.4×10^{-6} m), $K = 0.0144$ and $v = 3.15 \times 10^{-5}$ ft/s, without the CCF. With the correction factor ($\lambda = 6.53 \times 10^{-8}$), one obtains

$$A = 1.257 + 0.40 \exp\left(\frac{-1.10 d_p}{2\lambda}\right)$$
$$= 1.257 + 0.40 \exp\left(\frac{-1.10(0.4 \times 10^{-6})}{2(6.53 \times 10^{-8})}\right) = 1.2708$$
$$C = 1 + \frac{2A\lambda}{d_p} = 1 + \frac{2(1.2708)(6.53 \times 10^{-8})}{0.4 \times 10^{-6}} = 1.415$$

Equation (23.36) may now be employed.

$$v_{\text{corrected}} = vC = 3.15 \times 10^{-5}(1.415) = 4.45 \times 10^{-5}\,\text{ft/s}$$
$$x = v_{\text{corrected}} t = 4.45 \times 10^{-5}(30) = 1.335 \times 10^{-3}\,\text{ft}$$

Illustrative Example 23.5 Refer to Illustrative Example 23.4. Calculate the size of the a fly-ash particle that will settle with a velocity of 1.384 ft/s.

Solution First calculate the dimensionless number, W, using Equation (24.35):

$$W = \frac{(1.384)^3(0.0569)^2}{32.2(144.14)(1.41 \times 10^{-5})} = 0.1312$$

Since $W < 0.2222$, Stokes' law applies

$$d_p = \sqrt{\frac{18\mu v}{g\rho_p}} = \sqrt{\frac{18(1.41 \times 10^{-5})(1.384)}{(32.2)(144.14)}} = 2.751 \times 10^{-4}\,\text{ft}$$

Illustrative Example 23.6 Appropriate Drag Force Equation In order to calculate the terminal settling velocity of a particle in a gravity field, one must decide which of the three approximate drag force equations (Stokes, Intermediate, or Newton) is applicable. Explain why, when all three equations are used to calculate values of the terminal velocities for a given Reynolds number, the correct value is always the smallest of the three.

Solution Refer to Fig. 23.3, which is a slight modification of Fig. 23.2. Irrespective of whether one is in region I, II, or III, the calculated drag coefficient from any of these describing equations produces the highest drag for the correct (and applicable) drag force equation. The higher drag provides greater resistance to flow, which in turn corresponds to a smaller (or lower) velocity.

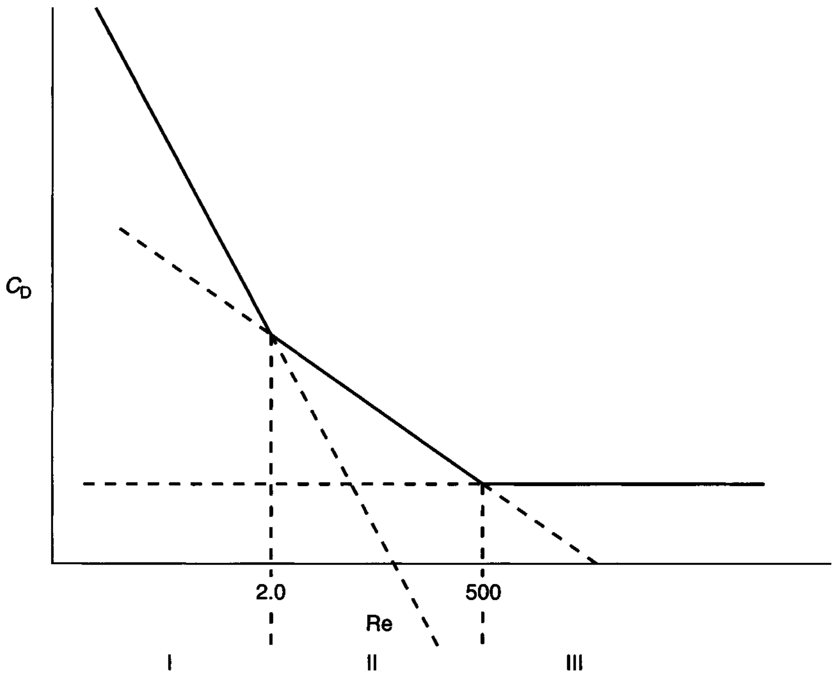

Figure 23.3 Drag force–Reynolds number regimes.

Illustrative Example 23.7 A plant manufacturing Ivory Soap detergent explodes one windy day. It disperses 100 tons of soap particles (SG = 0.8) into the atmosphere (70°F, $\rho = 0.0752\ \mathrm{lb/ft^3}$). If the wind is blowing 20 miles/h from the west and the particles range in diameter from 2.1 to 1000 μm, calculate the distance from the plant where the soap particles will start to deposit and where they will cease to deposit. Assume the particles are blown vertically 400 ft in the air before they start to settle. Also, assuming even ground-level distribution through an average 100 ft wide path of settling, calculate the average height of the soap particles on the ground in the settling area. Assume the bulk density of the settled particles equals half the actual density.

Solution The smallest particle will travel the greatest distance while the largest will travel the least distance. For the minimum distance, use the largest particle:

$$d_p = 1000\ \mu\mathrm{m} = 3.28 \times 10^{-3}\ \mathrm{ft}$$

$$K = d_p\left(\frac{g(\rho_p - \rho)\rho}{\mu^2}\right)^{1/3} = 3.28 \times 10^{-3}\left(\frac{32.174(0.8(62.4)) - (0.0752)0.0752}{(1.18 \times 10^{-5})^2}\right)^{1/3} = 31.3$$

The value of K indicates the intermediate range applies. The settling velocity is given by

$$v = \frac{0.153 g^{0.71} d_p{}^{1.14} \rho_p{}^{0.71}}{\mu^{0.43}\rho^{0.29}} = \frac{0.153(32.2)^{0.71}[3.28 \times 10^{-3}]^{1.14}(0.8(62.4))^{0.71}}{(1.18 \times 10^{-5})^{0.43}(0.0752)^{0.29}}$$

$$= 11.9\ \mathrm{ft/s}$$

The descent time is

$$t = \frac{H}{v} = \frac{400}{11.9} = 33.6\ \mathrm{s}$$

The horizontal distance traveled is

$$L = tv = 33.6\left(\frac{20}{3600}\right)5280 = 986\ \mathrm{ft}$$

For the maximum distance, use the smallest particle

$$d_p = 2.1\ \mu\mathrm{m} = 6.89 \times 10^{-6}\ \mathrm{ft}$$

$$K = d_p\left(\frac{g(\rho_p - \rho)\rho}{\mu^2}\right)^{1/3} = 6.89 \times 10^{-6}\left(\frac{32.174(0.8(62.4)) - (0.0752)0.0752}{(1.18 \times 10^{-5})^2}\right)^{1/3}$$

$$= 0.066$$

The velocity is in the Stokes regime and is given by

$$v = \frac{g d_p{}^2 \rho_p}{18\mu} = \frac{(32.2)(6.89 \times 10^{-6})^2 (0.8(62.4))}{(1.18 \times 10^{-5})} = 3.59 \times 10^{-4}\,\text{ft/s}$$

The descent time is

$$t = \frac{H}{v} = \frac{400}{3.59 \times 10^{-4}} = 1.11 \times 10^{6}\,\text{s}$$

The horizontal distance traveled is

$$L = tv = 1.11 \times 10^{6}\left(\frac{20}{3600}\right)5280 = 3.26 \times 10^{7}\,\text{ft}$$

To calculate the depth D, the volume of particles (actual), V_{act}, is first determined.

$$V_{\text{act}} = \frac{m}{\rho_p} = \frac{100(2000)}{0.8(62.4)} = 4006\,\text{ft}^3$$

The bulk volume is (with 50% voids)

$$V_b = \frac{V_{\text{act}}}{\varepsilon} = \frac{4006}{0.5} = 8012\,\text{ft}^3$$

The length of the drop area, L_{d}, is

$$L_{\text{d}} = 3.2 \times 10^{7} - 994 = 3.2 \times 10^{7}\,\text{ft}$$

Since the width is 100 ft, the deposition area A is

$$A = L_{\text{d}} W = (3.2 \times 10^{7})(100) = 3.2 \times 10^{9}\,\text{ft}^2$$

The deposition height H is then

$$H = \frac{V_{\text{b}}}{A} = \frac{8012}{3.2 \times 10^{9}} = 2.5 \times 10^{-6}\,\text{ft}$$

The deposition height can be, at best, described as a "sprinkling."

23.6 LIQUID-PARTICLE SYSTEMS

As indicated in the introduction to this chapter, the general treatment of liquid-particle dynamics, as it applies to liquid–solid separation, appears later in this Part (Chapter 24). However, the reader should note that the equations developed earlier in this chapter may be applied directly with only one minor change. Equation (23.38) contains the density ratio, DR,

$$\mathrm{DR} = \frac{\rho_p - \rho}{\rho} \tag{23.38}$$

For gases, $\rho_p > >> \rho$ so that the above term reduces to

$$\mathrm{DR} = \frac{\rho_p}{\rho} \tag{23.39}$$

since the ρ term can be neglected in comparison to ρ_p. However, if the fluid medium is a liquid rather than a gas, the ρ term must be retained in all the equations. This change is demonstrated in the next Illustrative Example.

Illustrative Example 23.8 A small sphere (6 mm diameter) is observed to fall through castor oil at a terminal speed of 42 mm/s. At the operating temperature of 20°C, the densities of castor oil and water are 970 kg/m^3 and 1000 kg/m^3, respectively. The viscosities of castor oil and water are 900 cP and 1 cP, respectively. Determine the density of the spherical particle, compute the drag coefficient for the spherical particle, calculate the drag and buoyant forces and, if the same sphere is dropped in water, would the sphere fall slower or faster and why? Also, calculate the Reynolds number and the terminal settling velocity.

Solution Calculate the particle density, ρ_p, assuming Stokes' law to apply

$$v_t = \frac{g{d_p}^2(\rho_p - \rho_f)}{18\mu_f}$$

Solving for ρ_p

$$\rho_p = \frac{18\mu_f v_t}{g{d_p}^2} + \rho_f = \frac{18(0.9)(0.042)}{(9.807)(0.006)^2} + 970 = 2897\,\mathrm{kg/m^3}$$

Check on Stokes' law validity with $\mu/\rho = 9.28 \times 10^{-4}$.

$$\mathrm{Re} = \frac{d_p v_t}{\nu_f} = \frac{0.006(0.042)}{9.28 \times 10^{-4}} = 0.272 < 0.3$$

Alternatively, calculate the settling criterion factor, K.

$$K = d_p\left(\frac{g\rho_p(\rho_s - \rho_f)}{{\mu_f}^2}\right)^{1/3} = 0.006\left(\frac{9.807(970)(2897 - 970)}{(0.9)^2}\right)^{1/3} = 1.7$$

Since $1.7 < 3.3$, Stokes' law applies. The drag coefficient, C_D, for the Stokes' law regime is

$$C_D = \frac{24}{\text{Re}} = \frac{24}{0.272}$$
$$= 88.2$$

Calculate the drag force, F_D, for the Stokes' law regime using Equation (23.11).

$$F_D = 3\pi\mu_f d_p v_t = 3\pi(0.9)(0.006)(0.042)$$
$$= 0.00213\,\text{N}$$

Calculate the buoyancy force

$$F_b = V_p \rho_f g = \frac{\pi d_p{}^3}{6}\rho_f g = \frac{\pi(0.006)^3}{6}970(9.807)$$
$$= 0.001076\,\text{N}$$

Consider the case when the same spherical particle is dropped in water. For water, ρ_f is $1000\,\text{kg/m}^3$, and μ is $0.001\,\text{kg/m}\cdot\text{s}$. The particle will move faster because of the lower viscosity of water. Stokes' law will almost definitely not apply.

Calculate the settling criterion factor once again.

$$K = d_p\left(\frac{g\rho_p(\rho_s - \rho_f)}{\mu_f{}^2}\right)^{1/3} = 0.006\left(\frac{9.807(970)(2897 - 970)}{(0.001)^2}\right)^{1/3}$$
$$= 158$$

Since $158 > 43.6$, the flow is in the Newton's law regime. Employ Equation (23.31) but include the (buoyant) density ratio factor. Therefore,

$$v_t = 1.75\sqrt{\left(\frac{\rho_s - \rho_f}{\rho_f}\right)gd_p} = 1.75\sqrt{\left(\frac{2897 - 1000}{1000}\right)(9.807)(0.006)} = 0.58\,\text{m/s}$$

23.7 DRAG ON A FLAT PLATE

The previous sections treated fluid particle dynamics where the particle was assumed to be a sphere. The drag force and the development that followed keyed solely on spheres. However, there are other applications involving drag that address other bodies. One such body is a flat plate.

The drag on a body submerged in a moving fluid depends on the body shape and size, the speed of the flow, and the properties of the fluid (its viscosity and density). One of the simpler bodies to study is a flat plate aligned parallel to the flow. Several semi-empirical relationships for the drag coefficient for this geometry have been proposed. Two such equations are presented in Equations (23.40) and (23.41)

$$C_D = \frac{1.33}{\mathrm{Re}^{0.5}}; \quad 10^4 < \mathrm{Re} < 5 \times 10^5 \tag{23.40}$$

$$C_D = \frac{0.031}{\mathrm{Re}^{1/7}}; \quad 10^6 < \mathrm{Re} < 10^9 \tag{23.41}$$

The first equation applies to a laminar flow, and the second (note the 1/7th power) when the flow is turbulent. The Reynolds number for a flat plate is given by:

$$\mathrm{Re} = \frac{\rho v L}{\mu} \tag{23.42}$$

where L is the plate length parallel to the flow.

The drag coefficient can be used to find the drag on the plate using Equation (23.43)

$$F_D = \frac{1}{2} C_D \rho v^2 L W \tag{23.43}$$

where W is the plate width perpendicular to the flow.

Note that these formulas compute the force on one side of a plate. The drag based on these equations should be doubled to compute the drag on a plate in which both sides are exposed to the fluid. There are also other similar formulas available in the literature.

Illustrative Example 23.9 The bottom of a ship, moving at 12 m/s, can be modeled as a flat plate of length 20 m and width 5 m. The water density is 1000 kg/m^3, and the viscosity is $10^{-3}\,\mathrm{N \cdot s/m^2}$. Calculate the drag on the bottom of the ship.

Solution Compute the flow Reynolds number

$$\mathrm{Re} = \frac{\rho v L}{\mu} = \frac{(1000)(12)(20)}{(10^{-3})}$$
$$= 2.4 \times 10^8; \quad \text{turbulent flow}$$

Compute the drag coefficient employing the appropriate equation.

$$C_D = \frac{0.031}{\text{Re}^{1/7}} = \frac{0.031}{(2.4 \times 10^8)^{1/7}}$$
$$= 0.002$$

Calculate the drag on area LW is

$$F_D = \frac{1}{2} C_D \rho v^2 LW$$
$$= \frac{1}{2}(0.002)(1000)(12)^2(20)(5) = 14{,}180\,\text{N}$$
$$= 14.2\ \text{kN}$$

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