## 22

## INDUSTRIAL APPLICATIONS

The illustrative examples provided in this chapter primarily pertain to modulated applications. There are a total of 17 illustrative examples; the first three examples are qualitative in nature. Those readers preferring academic calculations should definitely consider reviewing the illustrative examples in the previous chapter.

Illustrative Example 22.1 List the various classifications of industrial piping.
Solution Industrial piping can be divided into several major classifications. These are as follows: ${ }^{(1)}$

1. Outside overhead main lines.
2. Outside underground main lines.
3. Outside overhead lateral or distribution lines.
4. Outside underground lateral or distribution lines.
5. Process headers within buildings or tank farms.
6. Lateral distribution of process lines within buildings or tank farms.
7. Service headers in process or manufacturing buildings or tank farms.
8. Sewers, plumbing, and drain lines.

Illustrative Example 22.2 List the various services that employ piping.

Solution Piping may be classified by the services which they perform. Included in this list are piping to transport water, steam, compressed air, gas, sewage, drains, sanitary plumbing, process and instrument lines.

Illustrative Example 22.3 Outline how to determine the optimum economic pipe diameter for a flow system. ${ }^{(2)}$

Solution The investment for piping can amount to an important part of the total cost for a chemical process. It is usually necessary to select pipe sizes that provide the minimum total cost of both capital and operating charges. For any given set of flow conditions, the use of an increased pipe diameter will result in an increase in the capital cost for the piping system and a decrease in the operating costs. (The operating cost is generally the energy costs associated with moving the fluid; that is, pumping the fluid of concern.) Thus, an optimum economic pipe diameter can be found by minimizing the sum of pumping (or energy) costs and capital charges of the piping system.

The usual calculational procedure is as follows:

1. Select a pipe diameter.
2. Obtain the annual operating cost.
3. Obtain the capital equipment cost.
4. Convert the capital cost to an annual basis.
5. Sum the two annual costs in steps 2 and 4.
6. Return to step 1.

The only variable that will appear in the resulting total-cost expressions is the pipe diameter. The optimum economic pipe diameter can be generated by taking the derivative of the total annual cost with respect to pipe diameter, setting the result equal to zero, and solving for the diameter. The derivative operation can be replaced by a trial-and-error procedure that involves calculating the total cost for various diameters and simply selecting the minimum.

Generally, the initial/capital cost (see Chapter 32 for more details) of the pipe and valves fittings is directly proportional to the diameter, as are the other factors of pipe operating cost, depreciation and maintenance, which are a constant percentage of the initial pipeline cost. The cost of pressure drop (i.e., cost of pumping or blowing), however, is inversely proportional to the diameter.

Illustrative Example 22.4 For a centrifugal pump operating at 1800 rpm , find the impeller diameter needed to develop a head of 200 ft .

Solution Calculate the velocity needed to develop 200 ft of head. Use the equation

$$
v^{2}=2 \mathrm{gh}
$$

Substituting gives

$$
\begin{aligned}
v^{2} & =(2)\left(32.2 \mathrm{ft} / \mathrm{s}^{2}\right)(200 \mathrm{ft}) \\
& =12,880 \\
v & =113.5 \mathrm{ft} / \mathrm{s}
\end{aligned}
$$

Next, calculate the number of feet that the impeller travels in one rotation:

$$
(113 \mathrm{ft} / \mathrm{s}) /(1800 \mathrm{rpm} / 60 \mathrm{~s})=3.77 \mathrm{ft} / \text { rotation }
$$

This represents the circumference of the impeller since it is equal to one rotation. The diameter of the impeller may now be calculated:

$$
\begin{aligned}
& D=\text { circumference } / \pi \\
& D=3.77 \mathrm{ft} / \pi \\
& D=1.2 \mathrm{ft}
\end{aligned}
$$

An impeller diameter of approximately 1.2 ft will therefore develop a head of 200 ft .
Illustrative Example 22.5 Water for a processing plant is required to be stored in a reservoir. It is believed that a constant supply of $1.2 \mathrm{~m}^{3} / \mathrm{min}$ pumped to the reservoir, which is 22 m above the water intake, would be sufficient. The length of the pipe is about 120 m and there is 15 cm diameter galvanized iron piping available. The line would need to include eight regular elbows. Calculate the total energy required, the theoretical power, and the head to accomplish the above task.

Solution Assume the properties of water at $20^{\circ} \mathrm{C}$ are:

$$
\begin{aligned}
\rho & =998 \mathrm{~kg} / \mathrm{m}^{3} \\
\mu & =0.001 \mathrm{~N} \cdot \mathrm{~s} / \mathrm{m}^{2}
\end{aligned}
$$

Cross-sectional area of pipe:

$$
\begin{aligned}
S & =(\pi / 4) D^{2} \\
& =(0.785)(0.15)^{2} \\
& =0.0177 \mathrm{~m}^{2}
\end{aligned}
$$

Volume flow rate:

$$
\begin{aligned}
q & =1.2 \mathrm{~m}^{3} / \mathrm{min} \\
& =(1.2) /(60) \\
& =0.02 \mathrm{~m}^{3} / \mathrm{s}
\end{aligned}
$$

Velocity in the pipe:

$$
\begin{aligned}
v & =(1.2 / 60) / 0.0177 \\
& =1.13 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Calculate the Reynolds number.

$$
\begin{aligned}
\operatorname{Re} & =D v \rho / \mu \\
& =(0.15)(1.13)(998) / 0.001 \\
& =1.7 \times 10^{5}
\end{aligned}
$$

The flow is clearly turbulent.
There are three contributions to the energy load. These may be calculated individually or from the Bernoulli equation. Individual calculations are provided below.

From Table 14.1, the roughness factor $k$ is 0.0005 for galvanized iron so that

$$
\text { roughness ratio, } k / D=0.0005 / 0.15=0.003
$$

From Fig. 14.2, the friction factor is:

$$
f=0.0053
$$

Therefore, the friction loss of energy from Equation (14.3) is

$$
\begin{aligned}
h_{f} & =4 f L v^{2} / 2 g_{c} D \\
& =(4)(0.0053)(120)(1.13)^{2} /(2)(0.15) \\
& =10.8 \mathrm{~J}
\end{aligned}
$$

For the eight elbows (from Table 18.1), the estimated value of $K$ for one regular $90^{\circ}$ elbow is 0.5 .

$$
\begin{aligned}
K & =8(0.5) \\
& =4.0
\end{aligned}
$$

The velocity head is

$$
\begin{gathered}
v^{2} / 2 g_{c} \\
\mathrm{VH}=(1.13)^{2} / 2 \\
=0.64 \mathrm{~J} / \mathrm{kg}
\end{gathered}
$$

The total loss from the elbows is therefore

$$
\begin{aligned}
& =(4)(0.64) \\
& =2.56 \mathrm{~J} / \mathrm{kg}
\end{aligned}
$$

The energy to move 1 kg of water against a head of 22 m of water is

$$
\begin{aligned}
\Delta(P E) & =\Delta z\left(g / g_{c}\right) ; \quad \Delta z=22 \mathrm{~m} \\
& =(22)(9.81) \\
& =215.8 \mathrm{~J} / \mathrm{kg}
\end{aligned}
$$

Total energy requirement per kg:

$$
\begin{aligned}
E_{\mathrm{tot}} & =10.8+2.56+215.8 \\
& =229.2 \mathrm{~J} / \mathrm{kg}
\end{aligned}
$$

The theoretical power requirement is

$$
\begin{aligned}
\dot{W}_{s} & =\left(E_{\text {tot }}\right)(q)(\rho) \\
& =(229.2)(0.02)(998) \\
& =4574 \mathrm{~J} / \mathrm{s}
\end{aligned}
$$

The head (height of liquid) equivalent to the energy requirement is then

$$
\begin{aligned}
h & =E_{\mathrm{tot}}\left(\frac{g_{c}}{g}\right) \\
& =229.2 / 9.81 \\
& =23.4 \mathrm{~m} \text { of water }
\end{aligned}
$$

Illustrative Example 22.6 Oil is flowing through a standard $1 \frac{1}{2}$-inch steel pipe containing a 1.00 -inch square-edged orifice. The pressure differential across the orifice is indicated by two parallel vertical open tubes into which the oil rises from the two pressure taps. The oil is at $100^{\circ} \mathrm{F}$; its specific gravity is 0.87 and its viscosity 20.6 cP . Calculate the reading on the gauge described, when the oil is flowing at a rate of $400 \mathrm{gal} / \mathrm{hr}$.

Solution Applying appropriate conversion factors, the orifice velocity is:

$$
\begin{aligned}
v_{0} & =(400)(144) /(0.785)(3600)(7.48) \\
& =2.72 \mathrm{ft} / \mathrm{s}
\end{aligned}
$$

The Reynolds number is

$$
\begin{aligned}
\mathrm{Re}_{0} & =D_{0} v_{0} \rho / \mu \\
& =(1 / 12)(2.72)(0.87 \times 62.4) /(20.6 \times 0.000672) \\
& =889
\end{aligned}
$$

Since

$$
\begin{aligned}
\frac{D_{0}}{D_{1}} & =1.00 / 1.61 \\
& =0.62
\end{aligned}
$$

proceed to Fig. 19.8 and note

$$
C_{\mathrm{o}}=0.76
$$

Equation (19.17) must be rearranged to solved for $h$.

$$
h=\frac{v_{0}^{2}}{2 g C_{0}^{2}}\left[1-\left(\frac{D_{0}}{D_{1}}\right)^{4}\right]
$$

Substituting,

$$
\begin{aligned}
& =\frac{(2.72)^{2}}{(64.4)(0.76)^{2}}\left[1-(0.62)^{4}\right] \\
& =0.170 \mathrm{ft} \text { of oil } \\
& =2.04 \text { in of oil }
\end{aligned}
$$

Illustrative Example 22.7 Natural gas consisting of essentially pure methane flows through a long straight standard 10 -inch steel pipe into which is inserted a squareedged orifice 2.50 inches in diameter, with pressure taps, each 5.0 inches from the orifice plate. A manometer attached across the orifice reads 1.60 in $\mathrm{H}_{2} \mathrm{O}$. What is the mass rate of flow of gas through this line if the gas density is $0.054 \mathrm{lb} / \mathrm{ft}$.

Solution The ratio of orifice to pipe diameter is:

$$
D_{0} / D_{1}=2.50 / 10.15=0.245
$$

Assuming the Reynolds number in the orifice to be over 30,000 . The coefficient $C_{0}$ is therefore 0.61 from Fig. 19.8. Then, by Equation (19.19) with ( $D_{0} / D_{1}$ ) assumed approximately zero,

$$
\begin{aligned}
v_{0} & =0.61 \sqrt{\frac{(64.4)(1.60)(62.4)}{(12)(0.054)}} \\
& =60.8 \mathrm{ft} / \mathrm{s}
\end{aligned}
$$

Using this result, the Reynolds number in the orifice is

$$
\mathrm{Re}_{0}=\frac{D_{0} v_{0} \rho}{\mu}=\frac{(2.50)(60.7)(0.0540)}{(12)(0.011)(0.000672)}=92,800
$$

The assumption of $C_{\mathrm{o}}=0.61$ is permissible.
The mass rate of flow is

$$
\begin{aligned}
\dot{m} & =(60.7)(0.785)(2.50)^{2}(0.0540)(3600) / 144 \\
& =403 \mathrm{lb} / \mathrm{hr}
\end{aligned}
$$

Illustrative Example 22.8 In the gradual contraction pictured in Fig. 22.1, the upstream diameter (at station 1) is 10 cm and the downstream diameter (at station 6) is 6 cm . The flowing fluid is air at $20^{\circ} \mathrm{C}$, which has a specific weight of $12 \mathrm{~N} / \mathrm{m}^{3}$ and $\mu=0.0018 \mathrm{cP}$. The manometer fluid is Meriam red oil ( $\mathrm{SG}=0.827$ ); it indicates a manometer head, $h$, of 8 cm . Assume steady-state operation, constant properties, and no head losses.

Is there a stagnation point in the flow? Where? Compute the flow rate. Is the air flow incompressible? If the static pressure of the upstream air is $130,000 \mathrm{~Pa}$ absolute, calculate the static pressure of the air in the 6 cm pipe.

Solution Calculate the density of air from the ideal gas law

$$
\rho=1.22 \mathrm{~kg} / \mathrm{m}^{3}
$$

Is there a stagnation point? Yes, at station number 6.
Express the velocity at point 1 in terms of the velocity at point 2 by applying the continuity equation

$$
\begin{aligned}
\left(\pi D_{1}^{2} / 4\right) v_{1} & =\left(\pi D_{2}^{2} / 4\right) v_{2} \\
v_{1} & =v_{2}\left(\frac{D_{2}}{D_{1}}\right)^{2}
\end{aligned}
$$



Figure 22.1 Gradual contraction.

Apply Bernoulli's equation between points 1 and 2, noting that $z_{1}=z_{2}$ :

$$
\begin{aligned}
P_{1}+\frac{\rho v_{1}^{2}}{2 g_{c}} & =P_{2}+\frac{\rho v_{2}^{2}}{2 g_{c}} \\
P_{1}-P_{2} & =\frac{\rho\left(v_{2}^{2}-v_{1}^{2}\right)}{2 g_{c}}
\end{aligned}
$$

Substituting for the velocity, $v_{1}$, leads to

$$
P_{1}-P_{2}=\frac{\rho v_{2}^{2}\left[1-\left(D_{2} / D_{1}\right)^{4}\right]}{2 g_{c}}
$$

Apply Bernoulli's equation between points 1 and 6 , noting that $z_{1}=z_{6}$ :

$$
\frac{P_{6}-P_{1}}{\rho}=\frac{v_{1}{ }^{2}}{2 g_{c}}+0
$$

Replacing $v_{1}$ by $v_{2}$ gives

$$
\frac{P_{6}-P_{1}}{\rho}=\frac{v_{2}^{2}}{2 g_{c}}\left(\frac{D_{2}}{D_{1}}\right)^{4}
$$

Combining the above two equations gives

$$
P_{6}-P_{2}=\frac{\rho v_{2}^{2}}{2 g_{c}}
$$

Equate the pressure on both sides of the manometer.

$$
\begin{aligned}
& P_{3}=P_{4} \\
& P_{3}=P_{6}+\rho \frac{g}{g_{c}}(L+h)=P_{4}=P_{2}+\rho \frac{g}{g_{c}} L+\rho_{M} \frac{g}{g_{c}} h
\end{aligned}
$$

Therefore,

$$
P_{6}-P_{2}=\left(\rho_{M}-\rho\right) \frac{g}{g_{c}} h
$$

This is essentially the manometer equation. Equating $P_{6}-P_{2}$ from the last two equations yields

$$
\frac{v_{2}^{2}}{2 g_{c}}=h\left(\frac{\rho_{M}}{\rho}-1\right) \frac{g}{g_{c}}
$$

or

$$
\begin{aligned}
v_{2} & =\sqrt{2 g h\left(\frac{\rho_{M}}{\rho}-1\right)}=\sqrt{2(0.08)(9.8)\left(\frac{827}{1.22}-1\right)}=32.58 \mathrm{~m} / \mathrm{s} \\
v_{1} & =v_{2}\left(\frac{D_{2}}{D_{1}}\right)^{2}=32.58(0.6)^{2}=11.73 \mathrm{~m} / \mathrm{s} \\
q & =v_{2} S_{2}=(32.58)(\pi)(0.06)^{2} / 4=0.092 \mathrm{~m}^{3} / \mathrm{s}
\end{aligned}
$$

Calculate the Mach number from Equation (15.1)

$$
\begin{aligned}
c & =20 \sqrt{T\left({ }^{\circ} \mathrm{K}\right)}=20 \sqrt{293}=342.4 \mathrm{~m} / \mathrm{s} \\
M a & =\frac{v_{2}}{c}=\frac{32.58}{342.4}=0.095
\end{aligned}
$$

Noting that $0.095<0.3$, one can conclude that the flow is incompressible.
Given that $P_{1}=130,000 \mathrm{~Pa}$ absolute, calculate $P_{2}$. Use Bernoulli's equation once again

$$
\begin{aligned}
P_{1}-P_{2} & =\frac{\rho v_{2}^{2}\left[1-\left(D_{2} / D_{1}\right)^{4}\right]}{2 g_{c}}=\frac{(1.22)(32.54)^{2}\left[1-(0.6)^{4}\right]}{2}=562.2 \mathrm{~Pa} \\
P_{2} & =130,000-562.2=129,438 \mathrm{~Pa} \text { absolute }
\end{aligned}
$$

Illustrative Example 22.9 Water is flowing from an elevated reservoir through a conduit to a turbine at a lower level and out of the turbine through a similar conduit. At a point in the conduit 300 ft above the turbine, the pressure is 30 psia ; at a point in the conduit 10 ft below the turbine, the pressure is 18 psia . The water is flowing at 3600 tons $/ \mathrm{hr}$, and the output at the shaft of the turbine is 1000 hp . If the efficiency of the turbine is known to be $90 \%$, calculate the friction loss in the conduit in $\mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{lb}$.

Solution A pictural representation of the system is given in Fig. 22.2.
Since the diameter of the conduit is the same at location 1 and 2, kinetic energy effects can be neglected and Bernoulli's equation takes the form of

$$
\frac{\Delta P}{\rho}+\Delta z \frac{g}{g_{c}}-h_{s}+h_{f}=0
$$



Figure 22.2 Turbine problem.

The term $h_{s}$ is calculated using the following equation

$$
\begin{aligned}
h_{s} & =\frac{\dot{W}_{s}}{\eta \dot{m}} \\
E & =\frac{1000}{(0.9)(3600)}\left(\frac{550 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{s}}{\mathrm{hp}}\right)\left(\frac{3600 \mathrm{~s}}{\mathrm{hr}}\right)\left(\frac{\text { tons }}{2000 \mathrm{lb}}\right) \\
& =305.6 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{lb}
\end{aligned}
$$

Substituting back into the modified Bernoulli's equation yields:

$$
\frac{(18-30)(144)}{62.4}-310(1)-305.6+h_{f}=0
$$

Solving for the friction loss, $h_{f}$, gives

$$
h_{f}=-643.29 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{lb}
$$

Illustrative Example 22.10 Benzene is pumped from a large tank to a delivery station at a rate of $0.003 \mathrm{~m}^{3} / \mathrm{s}$. The tank is at atmospheric pressure. The pressure at the delivery station is 350 kPa gauge. The pump station is 1.8 m above the level in the tank. The delivery station is 3.8 m above the benzene level in the tank. The diameter of the suction and discharge line is 0.03 m . The head loss in the system is
estimated to be 8 m of benzene. The density of benzene is $865 \mathrm{~kg} / \mathrm{m}^{3}$ and its vapor pressure is $26,200 \mathrm{~Pa}$.

Determine the discharge, the lowest pressure in the system, and the NPSH based on the data given. If the pump manufacturer requires an NPSH of 8 m of benzene, is the height ( 1.8 m ) adequate? If not, determine the desired height.

Solution Refer to Fig. 22.3. Calculate the discharge velocity, $v_{2}$

$$
v_{2}^{2}=\frac{\dot{m}}{\pi D^{2} / 4}=\frac{0.003}{\pi(0.03)^{2} / 4}=4.24 \mathrm{~m} / \mathrm{s}
$$

Calculate the lowest pressure in the system. Note that the lowest pressure occurs at the pump suction point (station 3). Since all the line diameters are the same ( $D_{3}=D_{4}=D_{2}$ ), the velocities are likewise the same. First calculate the dynamic or velocity head at station 3 :

$$
\text { dynamic head } \frac{v_{3}^{2}}{2 g}=\frac{(4.24)^{2}}{2(9.807)}=0.917 \mathrm{~m} ;
$$

Set $\left(z_{1}=0\right)$ so that

$$
z_{3}=1.8 \mathrm{~m}
$$



Figure 22.3 Pump system.

Apply Bernoulli's equation between the top of the tank (open to the atmosphere) and the inlet to the pump (station 3)

$$
\begin{aligned}
0+0+0 & =\frac{P_{3}}{\rho g}+0.917+1.8 \\
\frac{P_{3}}{\rho g} & =-2.717 \mathrm{~m} \text { of benzene } \\
P_{3} & =101,325-(2.717)(865)(9.807)=78,277 \mathrm{~Pa}
\end{aligned}
$$

Calculate the NPSH employing Equation (17.12)

$$
\begin{aligned}
\mathrm{NPSH} & =\frac{P g_{c}}{\rho g}+\frac{v^{2}}{2 g}-\frac{p^{\prime} g_{c}}{\rho g}=\frac{P-p^{\prime} g_{c}}{\rho g}+\frac{v^{2}}{2 g} \\
& =\frac{78,277-26,200}{(865)(9.807)}+0.917=7.06 \mathrm{~m} \text { benzene }
\end{aligned}
$$

The manufacturer NPSH is 8 m , which is greater than the calculated NPSH of 7.06 m . Therefore, the suction point of the pump must be lowered.

Calculate the new pressure.

$$
\begin{aligned}
\mathrm{NPSH} & =8=\frac{P g_{c}}{\rho g}+0.917-3.09 \\
\frac{P g_{c}}{\rho g} & =10.173 \mathrm{~m} \text { of benzene }
\end{aligned}
$$

$P=10.173(865)(9.807)=86,300 \mathrm{~Pa}$ absolute $=-15,025 \mathrm{Pag}=-1.77 \mathrm{~m}$ of benzene

Apply Bernoulli's equation to determine the height $z$.

$$
\begin{aligned}
0 & =-1.77+0.917+z_{3} \\
z_{3} & =0.853 \mathrm{~m}
\end{aligned}
$$

Illustrative Example 22.11 A storage tank on top of a building pumps $60^{\circ} \mathrm{F}$ water through an open pipe to it from a reservoir. The reservoir's water level is 10 ft above the pipe outlet, and 200 feet below the water level in the tank. Both tanks are open to the atmosphere. A 4 in . ID piping system ( $k=0.0018 \mathrm{in}$.) contains two gate valves, five regular $90^{\circ}$ elbows, and is 525 ft long. A flow rate of $610 \mathrm{gal} / \mathrm{min}$ is desired. Calculate the pump requirement (in hp ) if it is rated as $60 \%$ efficient. Also, provide the pump requirements in units of $\mathrm{kW}, \mathrm{W}$, and $\mathrm{N} \cdot \mathrm{m} / \mathrm{s}$. Assume the density and viscosity of the water to be $62.37 \mathrm{lb} / \mathrm{ft}^{3}$ and 1.129 cP , respectively.

Solution A line drawing of the system is shown in Fig. 22.4.


Figure 22.4 Line diagram of the system.

The pipe velocity is first calculated

$$
\begin{aligned}
& q=610 \mathrm{gal} / \mathrm{min}=1.36 \mathrm{ft}^{3} / \mathrm{s} \\
& v=\frac{q}{S}=\frac{1.36}{\pi(4 / 12)^{2} / 4}=15.6 \mathrm{ft} / \mathrm{s}
\end{aligned}
$$

The Reynolds number is

$$
\operatorname{Re}=\frac{D v \rho}{\mu}=\frac{(0.333)(15.6)(62.37)}{1.129\left(6.72 \times 10^{-4}\right)}=427,480 ; \quad \text { turbulent flow }
$$

In addition,

$$
\begin{aligned}
k / D & =0.0018 / 4 \\
& =0.00045
\end{aligned}
$$

From Fig. 14.2

$$
f=0.0046
$$

The friction loss due to the length of pipe is:

$$
\begin{aligned}
h_{f p} & =4 f \frac{L}{D} \frac{v^{2}}{2 g_{c}}=4(0.0046) \frac{525}{(4 / 12)} \frac{(15.6)^{2}}{2(32.174)} \\
& =110 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{lb}
\end{aligned}
$$

The friction due to the fittings (see Table 18.1) is:

$$
\begin{aligned}
K_{\mathrm{ff}}(\mathrm{gate}) & =2(0.11)=0.22 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{lb} \\
K_{\mathrm{ff}}(\mathrm{elbows}) & =5(0.64)=3.2 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{lb}
\end{aligned}
$$

The friction due to the sudden contraction is obtained from Equation (18.10). Note that $D_{1} / D_{2}=0$, since the upstream diameter is significantly larger than the downstream diameter.

$$
\begin{aligned}
K_{c} & =0.42\left[1-\left(\frac{D_{1}}{D_{2}}\right)\right]=0.42\left[1-\frac{S_{1}}{S_{2}}\right]^{2} \\
& =0.42
\end{aligned}
$$

The friction due to a sudden expansion to the atmosphere (from Equation (18.8)) is

$$
\begin{aligned}
K_{e} & =\left[1-\left(\frac{S_{1}}{S_{2}}\right)\right]^{2}\left[1-\left(\frac{D_{1}}{D_{2}}\right)^{2}\right]^{2} \\
& =1.0
\end{aligned}
$$

The sum of the loss coefficients, $\sum K$, may now be calculated

$$
\sum K=0.22+3.2+0.42+1.0=4.84
$$

These friction losses are therefore

$$
h_{f}=\sum K \frac{v^{2}}{2 g_{c}}=4.84 \frac{(15.6)^{2}}{2(32.174)}=18.3 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{lb}
$$

and

$$
h_{f, \text { total }}=110.2+18.3=128.5 \mathrm{ft}-\mathrm{lb}_{\mathrm{f}} / \mathrm{lb}
$$

Applying Bernoulli's equation:

$$
W_{s}=\frac{\Delta P}{\rho}+\frac{v_{2}^{2}-v_{1}^{2}}{2 g_{c}}+\left(z_{2}-z_{1}\right) \frac{g}{g_{c}}+h_{f, \text { total }}
$$

Since both tanks are open to the atmosphere, $\Delta P=0$ and $v_{2}=15.6 \mathrm{ft} / \mathrm{s}$ (with $v_{1}=0$ ),

$$
\begin{aligned}
W_{s} & =0+\frac{15.6^{2}}{2(32.174)}+200+128.5 \\
& =332.2 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{lb}
\end{aligned}
$$

The mass flow rate is

$$
\begin{aligned}
\dot{m} & =q \rho=1.36(62.37) \\
& =84.82 \mathrm{lb} / \mathrm{s}
\end{aligned}
$$

and the actual horsepower requirement is

$$
\begin{aligned}
\dot{W}_{s} & =\frac{\dot{m} W_{s}}{\eta}=\frac{332.3(84.82)}{550(0.6)} \\
& =85.4 \mathrm{hp}
\end{aligned}
$$

In other units:

$$
\begin{aligned}
\dot{W}_{s} & =85.4(0.7457)=63.7 \mathrm{~kW} \\
& =63,700 \mathrm{~W}=63,700 \mathrm{~N} \cdot \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Illustrative Example 22.12 Turpentine is being moved from a large storage tank to a blender through a 700 ft pipeline. The temperature of the turpentine is $50^{\circ} \mathrm{F}$ and its specific gravity is 0.872 . The top surface of the turpentine in the storage tank is 20 ft above floor-level and the discharge end of the pipe (directly over the blender) is 90 ft above floor level. Both the tank and pipe discharge end are open to the atmosphere. The line contains five $90^{\circ}$ elbows, six wide open gate valves and one return bend. The average energy delivered by the pump is $401.9 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{lb}$ of turpentine, the efficiency of the pump is $74 \%$, and the average velocity of the turpentine in the line is $12.66 \mathrm{ft} / \mathrm{s}$. The friction loss coefficients of contraction, elbows, bends and valves are to be assumed equal to $0.9,2.2$ and 0.2 , respectively. Draw a diagram of this system and clearly show the location of the beginning point and the end point to be used in the solution of the problem. Determine the inside diameter of the pipeline, the volumetric flow rate in $\mathrm{gal} / \mathrm{min}$, and the brake horsepower of the pump.

Solution A line diagram of the system is provided in Fig. 22.5.
Write the modified Bernoulli equation. See Equation (13.4) or (17.11).

$$
\frac{\Delta P}{\rho}+\frac{\Delta\left(v^{2}\right)}{2 g_{c}}+\Delta z \frac{g}{g_{c}}=\eta_{p} W_{s}-h_{f}
$$

Calculate the friction loss in $\mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{lb}$, noting that there is no pressure drop in the system.

$$
\begin{aligned}
& 0+\frac{(12.66)^{2}}{2(32.174)}+\frac{(32.174)(90-20)}{32.174}=0.74(401.9)-h_{f} \\
& h_{f}=224.9 \mathrm{ft} \cdot \mathrm{lb}_{\mathrm{f}} / \mathrm{lb}
\end{aligned}
$$

The friction can be determined from the friction loss coefficient due to the fittings. The friction loss is expressed in terms of Fanning friction factor and the diameter of the tube.

The equation for the friction loss is first written.

$$
\begin{equation*}
h_{f}=\left[4 f \frac{L}{D}+\sum K_{c}+\sum K_{e}+\sum K_{f}\right] \frac{v^{2}}{2 g_{c}} \tag{8.14}
\end{equation*}
$$



Figure 22.5 Line diagram of the system.

Substitution leads to

$$
h_{f}=\left[4 f\left(\frac{700}{D}\right)+0.4+5(0.9)+6(0.2)\right] \frac{(12.66)^{2}}{2(32.174)}=20.68+6975\left(\frac{f}{D}\right)
$$

Employing the friction loss calculated from the Bernoulli equation and substituting into the friction loss equation above leads to

$$
224.9=20.68+6975\left(\frac{f}{D}\right)
$$

so that

$$
f=0.0293 D
$$

The Reynolds number can also be expressed in terms of the tube diameter:

$$
\operatorname{Re}=\frac{\rho v D}{\mu}=\frac{0.872(62.4)(12.66) D}{1.76\left(6.72 \times 10^{-4}\right)}=582,250 D
$$

The tube diameter is determined by trial-and-error. First, guess the diameter in ft and then determine the Reynolds number from the equation above. Use the Reynolds number to determine the Fanning friction factor from the friction factor chart and then recalculate the diameter of the tube from the previous equation and compare the new diameter of the tube to the initial guess. Repeat the procedure until the assumed diameter and the calculated diameter are essentially the same. Note that the diameter calculated in each step may be used as the guess for the next step. The following table shows the trial-and-error calculation starting with a diameter of 1 ft .

The diameter of the tube is therefore 0.184 ft .

Table 22.1 Calculated results for Illustrative Example 22.12

| $D, \mathrm{ft}$ | Re | $k / D$ | $f$ | $D_{\text {new }}, \mathrm{ft}$ |
| :--- | ---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 582,250 | 0.00015 | 0.0037 | 0.126 |
| $\mathbf{0 . 1 2 6}$ | 73,270 | 0.00119 | 0.0052 | 0.177 |
| 0.177 | 103,000 | 0.00085 | 0.0054 | 0.184 |
| $\mathbf{0 . 1 8 4}$ | 106,900 | 0.00082 | 0.0054 | 0.184 |

Determine the cross-sectional area from the diameter obtained above

$$
S=\frac{\pi D^{2}}{4}=\frac{\pi(0.184)^{2}}{4}=0.0266 \mathrm{ft}^{2}
$$

The volumetric flow rate of the fluid is then:

$$
q=v S=12.66(0.0266)=0.337 \mathrm{ft}^{3} / \mathrm{s}=151,26 \mathrm{gal} / \mathrm{min}
$$

The mass flow rate of the fluid is

$$
\dot{m}=\rho v S=(0.872)(62.4)(12.66)(0.0266)=18.33 \mathrm{lb} / \mathrm{s}
$$

The brake horse power (bhp) is

$$
\begin{aligned}
\mathrm{bhp} & =\frac{\dot{m} W_{s}}{\eta}=(18.33)(401.9)\left(\frac{1}{550}\right)\left(\frac{1}{0.74}\right) \\
& =18.1 \mathrm{hp}
\end{aligned}
$$

Illustrative Example 22.13 Hydrogen at 1 atm and $20^{\circ} \mathrm{C}$ flows at $400 \mathrm{cc} / \mathrm{s}$ through an 80 mm diameter horizontal pipe. Assume the $z$-axis to be along the pipe axis. Calculate the average velocity and mass flow rate. Determine if the flow is laminar. What is the pressure drop per unit length of pipe (pressure gradient)? What are the velocities at $r=0, r=20 \mathrm{~mm}$, and $r=40 \mathrm{~mm}$ ? What is the ratio of the average velocity/maximum velocity? Calculate the Darcy and Fanning friction factors. Indicate the units. What is the friction loss, in m , and the friction power loss per unit length of pipe? If a pipe with twice the diameter is used instead, and the flow rate and pipe length remain the same, will the pressure drop increase, decrease, or remain the same? Why?

Solution Obtain the properties of hydrogen at $20^{\circ} \mathrm{C}$ from Table A. 3 in the Appendix.

$$
\begin{aligned}
\rho & =0.0838 \mathrm{~kg} / \mathrm{m}^{3} \\
\mu & =9.05 \times 10^{-6} \mathrm{~kg} / \mathrm{m} \cdot \mathrm{~s} \\
k & =1.41
\end{aligned}
$$

Convert all data into SI units (for convenience).

$$
\begin{aligned}
D & =80 \mathrm{~mm}=0.08 \mathrm{~m} \\
q & =400 \mathrm{cc} / \mathrm{s}=400 \mathrm{~mL} / \mathrm{s}=0.400 \mathrm{~L} / \mathrm{s} \\
& =0.0004 \mathrm{~m}^{3} / \mathrm{s} \\
S & =\pi D^{2} / 4=\pi(0.08)^{2} / 4 \\
& =0.000503 \mathrm{~m}^{2}
\end{aligned}
$$

Calculate the average velocity and mass flow rate.

$$
\begin{aligned}
v & =\frac{q}{S}=\frac{0.0004}{0.000503}=0.8 \mathrm{~m} / \mathrm{s} \\
\dot{m} & =\rho q=(0.0838)(0.0004)=33.52 \times 10^{-6} \mathrm{~kg} / \mathrm{s}
\end{aligned}
$$

Check the flow type using the Reynolds number.

$$
\begin{aligned}
\operatorname{Re} & =\frac{D v \rho}{\mu}=\frac{(0.08)(0.8)(0.0838)}{9.05 \times 10^{-6}} \\
& =593<2100 ; \quad \text { laminar }
\end{aligned}
$$

Calculate the pressure gradient. Since the tube is horizontal, $z_{1}=z_{2}$, and from Equation (14.3) with $v=\pi D^{2} / 4$

$$
\begin{aligned}
\frac{\Delta P}{L}=\frac{128 \mu q}{\pi D^{4}} & =\frac{128\left(9.05 \times 10^{-6}\right)(0.0004)}{\pi(0.08)^{4}} \\
& =3.60 \times 10^{-3} \mathrm{~Pa} / \mathrm{m}
\end{aligned}
$$

Note that $\Delta P \propto D^{-4}$.
Calculate the velocity at $r=0,0.02$, and 0.04 m using the parabolic (laminar flow) velocity equation.

$$
\begin{aligned}
v_{\max } & =2 v=1.6 \mathrm{~m} / \mathrm{s} \\
v & =1.6\left[1-(r / 0.04)^{2}\right] \\
\text { At } r & =0, \quad v=v_{\max }=1.6 \mathrm{~m} / \mathrm{s} \\
\text { At } r & =0.02, \quad v=1.2 \mathrm{~m} / \mathrm{s} \\
\text { At } r & =0.04, \quad v=0 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Calculate the Fanning friction factor. Since the flow is laminar, the Fanning friction factor is

$$
\begin{aligned}
f & =\frac{16}{\operatorname{Re}}=\frac{16}{593} \\
& =0.0269
\end{aligned}
$$

The Darcy friction factor is

$$
f_{D}=4 f=4(0.0269)=0.108
$$

Calculate the friction loss using Equation (14.3) employing the above Fanning friction factor

$$
h_{f}^{\prime}=4 f \frac{L}{D} \frac{v^{2}}{2 g}=\frac{4(0.0269)(1 / 0.08)(0.8)^{2}}{2(9.807)}=4.39 \times 10^{-2} \mathrm{~m} \text { of hydrogen }
$$

Calculate the friction power loss

$$
\begin{aligned}
\dot{W}_{f}= & \dot{m} g h_{f}^{\prime} \\
& =\left(33.52 \times 10^{-6}\right)(9.807)\left(4.39 \times 10^{-2}\right) \\
& =1.4 \times 10^{-5} \mathrm{~W}
\end{aligned}
$$

Examine the effect of doubling pipe diameter. For constant $q, \mu$, and $L$ :

$$
\Delta P \propto D^{-4}
$$

Thus, by doubling $D$, the pressure drop will decrease to $1 / 16$ of its original value.
Illustrative Example 22.14 Gasoline at $20^{\circ} \mathrm{C}$ is pumped at $0.3 \mathrm{~m}^{3} / \mathrm{s}$ through 30 m of $20-\mathrm{cm}$ diameter horizontal cast-iron pipe. Calculate the average velocity of gasoline. Compute the head loss and brake power required to pump the gasoline if the pump is $80 \%$ efficient. By what percentage are the head loss and power requirements increased due to the roughness of the tube? For gasoline at $20^{\circ} \mathrm{C}, \rho=680 \mathrm{~kg} / \mathrm{m}^{3}$, $\mu=2.92 \times 10^{-4} \mathrm{~kg} / \mathrm{m} \cdot \mathrm{s}$ (Table A. 2 in the Appendix).

Solution Calculate the average velocity.

$$
\begin{aligned}
& q=q_{1}=q_{2}=v S \\
& S=\frac{\pi D^{2}}{4}=\frac{\pi(0.2)^{2}}{4}=0.03142 \mathrm{~m}^{2} \\
& v=\frac{q}{S}=\frac{0.3}{0.03142}=9.5 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Check the flow regime

$$
\operatorname{Re}=\frac{D v \rho}{\mu}=\frac{(0.2)(9.5)(680)}{\left(2.92 \times 10^{-4}\right)}=4.42 \times 10^{6}>4000 ; \text { turbulent }
$$

Obtain the roughness, $k$, of cast iron pipe (see Table 14.1).

$$
k=0.26 \mathrm{~mm}=0.00026 \mathrm{~m}
$$

Calculate the relative roughness

$$
k / D=\frac{0.00026}{0.2}=0.0013
$$

Obtain the Fanning friction factor, $f$, from Fig. 14.2:

$$
f=0.00525
$$

Note that the flow corresponds to complete turbulence in the rough pipe. Calculate the head loss

$$
h_{f}^{\prime}=4 f \frac{L}{D} \frac{v^{2}}{2 g}=4(0.00525) \frac{30}{0.2} \frac{(9.5)^{2}}{2(9.807)}=14.50 \mathrm{~m} \text { of gasoline }
$$

Apply Bernoulli's equation to the fluid in the pipe. In the present case, the pipe is horizontal ( $z_{1}=z_{2}$ ) with constant diameter ( $v_{1}=v_{2}$ ) and no shaft head ( $h_{s}=0$ ). First convert the friction head to a pressure difference

$$
\Delta P=\rho g h_{f}^{\prime}=(680)(9.807)(14.647)=97.68 \times 10^{3} \mathrm{~Pa}=0.9640 \mathrm{~atm}
$$

Calculate the ideal shaft work.

$$
\dot{W}_{s, i d}=q \Delta P=0.3\left(97.68 \times 10^{3}\right)=29,304 \mathrm{~W}=39.297 \mathrm{hp}
$$

Calculate the actual shaft work rate.

$$
\dot{W}_{s}=\frac{\dot{W}_{s, i d}}{\eta}=\frac{29,304}{0.8}=36,630 \mathrm{~W}=49.121 \mathrm{hp}
$$

Calculate the increase in power requirement due to pipe roughness

$$
\begin{aligned}
f_{\text {smooth }} & =0.009 \\
f_{\text {rough }} / f_{\text {smooth }} & =0.021 / 0.009=2.333
\end{aligned}
$$

The \% increase in $f$ due to pipe roughness is:

$$
100(2.333-1)=133.3 \%
$$

Illustrative Example 22.15 Liquid benzene flows steady at $4000 \mathrm{gal} / \mathrm{min}$ (gpm) through a 480 ft long horizontal smooth iron pipe that has an inside diameter of 2.3 m . The density of benzene is $899 \mathrm{~kg} / \mathrm{m}^{3}\left(56.1 \mathrm{lb} / \mathrm{ft}^{3}\right)$ and the viscosity is $0.0008 \mathrm{~kg} / \mathrm{m} \cdot \mathrm{s}(0.000538 \mathrm{lb} / \mathrm{ft} \cdot \mathrm{s})$. What is the average velocity of the benzene? What is the Reynolds number? Is the flow laminar or turbulent? What is the Fanning friction factor for benzene. What is the pressure drop? What are the friction power losses?

Solution Calculate the cross-sectional area of the pipe

$$
\begin{aligned}
S & =\frac{\pi D^{2}}{4}=\frac{\pi(2.3)^{2}}{4} \\
& =4.155 \mathrm{~m}^{2}
\end{aligned}
$$

Calculate the average velocity

$$
\begin{aligned}
v & =\frac{q}{S}=\frac{4000}{(4.155)(264.17)(60)} \\
& =6.074 \times 10^{-2} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Calculate the Reynolds number

$$
\begin{aligned}
\operatorname{Re} & =\frac{D v \rho}{\mu}=\frac{(2.3)\left(6.074 \times 10^{-2}\right)(899)}{(0.0008)} \\
& =156,990
\end{aligned}
$$

Since the Reynolds number falls in the turbulent regime, determine the Fanning friction factor from Figure 14.2.

$$
f \approx 0.0032
$$

Calculate the pressure drop with the assumption of no height and velocity change, and no pump work. Since only frictional losses are to be considered, apply Equation (14.3).

$$
\begin{aligned}
& \frac{\Delta P}{\rho}=4 f \frac{L}{D} \frac{v^{2}}{2 g_{c}} \\
& \Delta P=4 f \frac{L}{D} \frac{v^{2}}{2 g_{c}} \rho=4(0.0032) \frac{(480)(0.3048)}{(2.3)} \frac{\left(6.074 \times 10^{-2}\right)^{2}}{2(1)}(899) \\
& \Delta P=1.35 \mathrm{~Pa}
\end{aligned}
$$

Write a friction power loss equation, employing both the volumetric flow rate and the pressure drop

$$
\begin{aligned}
\dot{W}_{f} & =q \Delta P \\
& =(4000)\left(\frac{1}{264.17}\right)\left(\frac{1}{60}\right)(1.35) \\
& =0.34 \mathrm{~W}
\end{aligned}
$$

Illustrative Examples 22.16 A power plant employs steam to generate power and operates with a steam flowrate of $450,000 \mathrm{lb} / \mathrm{h}$. For the adiabatic conditions listed below, determine the power produced in horsepower, kilowatts, Btu $/ \mathrm{h}$, and $\mathrm{Btu} / \mathrm{lb}$ of steam. Data are given in Table 22.2.

Table 22.2 Power plant data

|  | Inlet | Outlet |
| :--- | ---: | ---: |
| Pressure, psia | 100 | 1 |
| Temperature, ${ }^{\circ} \mathrm{F}$ | 1500 | 350 |
| Steam velocity, $\mathrm{ft} / \mathrm{s}$ | 120 | 330 |
| Steam vertical position, ft | 0 | -20 |

Solution Apply the following energy equation:

$$
z_{1}\left(\frac{g}{g_{c}}\right)+\frac{v_{1}{ }^{2}}{2 g_{c}}+H_{1}+Q=z_{2}\left(\frac{g}{g_{c}}\right)+\frac{v_{2}^{2}}{2 g_{c}}+H_{2}+W_{s}
$$

where $\quad z_{1}, z_{2}=$ vertical position at inlet/outlet, respectively
$v_{1}, v_{2}=$ steam velocity at inlet/outlet, respectively
$H_{1}, H_{2}=$ steam enthalpy at inlet/outlet, respectively
$W_{s} \quad=$ work extracted from system (a negative quantity as written)
For adiabatic conditions, $Q=0$. Substituting data from the problem statement yields

$$
\begin{aligned}
& 0+\frac{(120)^{2}}{2(32.17)(778)}+1505.4+0=\frac{-20}{778}+\frac{(330)^{2}}{2(32.17)(778)}+940.0+W_{s} \\
& 0+0.288+1505.4+0=-0.026+2.176+940.0+W_{s} \\
& W_{s}=563.54 \mathrm{Btu} / \mathrm{lb}
\end{aligned}
$$

The total power generated by the turbine is equal to

$$
\begin{aligned}
\dot{W}_{s} & =(450,000 \mathrm{lb} / \mathrm{h})(563.54 \mathrm{Btu} / \mathrm{lb}) \\
& =2.54 \times 10^{8} \mathrm{Btu} / \mathrm{h}
\end{aligned}
$$

Converting to horsepower gives

$$
\begin{aligned}
& \left(2.54 \times 10^{8} \mathrm{Btu} / \mathrm{h}\right)\left(3.927 \times 10^{-4} \mathrm{hp} \cdot \mathrm{~h} / \mathrm{Btu}\right) \\
& =9.98 \times 10^{4} \mathrm{hp}
\end{aligned}
$$

## REFERENCES

1. C. E. Lapple, "Fluid and Particle Dynamics," University of Delaware, Newark, Delaware, 1951.
2. J. Reynolds, J. Jeris, and L. Theodore, "Handbook of Chemical and Environmental Engineering Calculations," John Wiley \& Sons, Hoboken, NJ, 2004.

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