20

VENTILATION

20.1 INTRODUCTION

Indoor air pollution is rapidly becoming a major health issue in the United States. Indoor pollutant levels are quite often higher than outdoors, particularly where buildings are tightly constructed to save energy. Since most people spend nearly 90% of their time indoors, exposure to unhealthy concentrations of indoor air pollutants is often inevitable. The degree of risk associated with exposure to indoor pollutants depends on how well buildings are ventilated and the type, mixture, and amounts of pollutants in the building.(1)

Industrial ventilation is the field of applied science concerned with controlling airborne contaminants to produce healthy conditions for workers and a clean environment for the manufacture of products. However, to claim that industrial ventilation will prevent contaminants from entering the workplace is naive and unachievable. More to the point, and within the realm of achievement, is the goal of controlling contaminant exposure within prescribed limits. To accomplish this goal, one must be able to describe the movement of contaminants in quantitative terms that take into account:

1. The spatial and temporal rate at which contaminants are generated;
2. The velocity field of the air in the workplace;
3. The spatial relationship between source, workers, and openings through which air is withdrawn or added;
4. Exposure limits (time-concentration relationships) that define unhealthy conditions.(1)

Fluid Flow for the Practicing Chemical Engineer. By J. Patrick Abulencia and Louis Theodore

In general, most control/recovery equipment are more economically efficient when handling higher concentrations of contaminants, all else being equal. Therefore, the gas handling system should be designed to concentrate contaminants in the smallest possible volume of air. This is important since, exclusive of the fan, the cost of the control equipment is based principally upon the volume of gas to be handled and not on the quantity of contaminant to be removed. The reduction of emissions by process and system control is an important adjunct to any cleaning technology.

Although ventilation does not remove the contaminants from the workplace, it does provide an opportunity to either recover/control or dilute (into the atmosphere) any problem emissions. Regarding manufacture and production, some unions and European nations either require or recommend that closed operations be employed.

The reader is referred to the work of Heinsohn(2) for an excellent treatment of ventilation. This topic is also addressed in Chapter 29. In addition, it should be noted that several of the Illustrative Examples at the end of the chapter were excerpted and/or adapted from publications(3,4) resulting from NSF sponsored faculty workshops.

20.2 INDOOR AIR QUALITY

Indoor air quality (IAQ) is a major concern because indoor air pollution may present a greater risk of illness than exposure to outdoor pollutants. People spend nearly 90% of their time indoors. This situation is compounded as sensitive populations the very young, the very old, and sick people—who are potentially more vulnerable to disease, spend many more hours indoors than the average population.

Indoor air quality problems have become more serious and of greater concern now than in the past because of a number of developments that are believed to have resulted in increased levels of harmful chemicals in indoor air. Some of those developments are the construction of more tightly sealed buildings to save on energy costs, the reduction of the ventilation rate standards to save still more energy, the increased use of synthetic building materials and synthetics in furniture and carpeting that can release harmful chemicals, and the widespread use of new pesticides, paints, and cleaners.

Some of the immediate health effects of indoor air quality problems are irritation of the eyes, nose and throat, headaches, dizziness and fatigue, asthma, pneumonitis, and "humidifier fever." Some of the long-term health effects of indoor air quality problems are respiratory diseases and cancer. These are most often associated with radon, asbestos, and second-hand tobacco smoke. The U.S. EPA (EPA), in a report to Congress in 1989, estimated that the costs of IAQ problems were in the tens of billions of dollar per year. The major types of costs from IAQ problems are direct medical costs, lost productivity due to absence from the job because of illness, decreased efficiency on the job, and damage to materials and equipment.

20.3 INDOOR AIR/AMBIENT AIR COMPARISON

Outdoor air pollution and indoor air pollution share many of the same pollutants, concerns, and problems. Both can have serious negative impacts on the health of the population. Not too many years ago, it was a common practice to advise people with respiratory problems to stay indoors on days when pollution outdoors was particularly bad. The assumption was that the indoor environment provided protection against outdoor pollutants. Recent studies conducted by the EPA have found, however, that the indoor levels of many pollutants are often two to five times, and occasionally more than 100 times, higher than corresponding outdoor levels. Such high levels of pollutants indoors are of even greater concern than outdoors because most people spend more time indoors than out. Indoors is where most people work, attend school, eat, sleep, and even where much of their recreational activity takes place.

Among the consumer and commercial products that release pollutants into the indoor air are pesticides, adhesives, cosmetics, cleaners, waxes, paints, automotive products, paper products, printed materials, air fresheners, dry cleaned fabrics, and furniture. In addition to the "active" ingredient in all the products mentioned, many products contain so-called inert ingredients that are also contaminants when released into indoor air. Examples include solvents, propellants, dyes, curing agents, flame retardants, mineral spirits, plasticizers, perfumes, hardeners, resins, binders, stabilizers, and preservatives. Aerosol products produce droplets, which remain in the air long enough to be inhaled. This allows the inhalation of some chemicals that would not be volatile enough to be inhaled otherwise. One consumer product that produces indoor air contaminants and merits special mention is tobacco smoke.

The single most important building or structural source of contaminant is formaldehyde contained in building materials such as plywood, adhesives, insulating materials such as urea formaldehyde foam, floor, and wall coverings. Depending on the individual type of construction and maintenance practices, there can be many other building sources of IAQ problems. Damp or wet wood, insulation, walls, and ceilings can be breeding places for allergens and pathogens that can become airborne. Allergens and pathogens can also originate from poorly maintained humidifiers, dehumidifiers, and air conditioners. If the building has openings to the soil, radon can enter the building in those areas where radon occurs. The building's heating plant can be the source of contaminants such as CO and NO_x. Automobile exhaust from attached garages is another source of carbon monoxide and nitrogen oxides. Particulates such as asbestos from crumbling insulation and lead form the sanding of lead-based paints are additional contaminants that can become part of the indoor air.

Personal activities can be sources of indoor air contaminants such as pathogenic viruses and bacteria, and a number of harmful chemicals such as products of human and animal (pet) respiration. Houseplants can release allergenic spores. Pet products, such as flea powder, can be sources of pesticides and pets produce allergenic dander when they lick themselves.

Outdoor sources of indoor air contaminants are widely varied. Polluted outdoor air can enter the indoor space through open windows, doors, and ventilation intakes. Most outdoor air is less contaminated than indoor air. In some cases, however, such as with a nearby smokestack, a parking lot, heavy street traffic, or an underground garage, outdoor air can be a significant source of indoor contaminants.

Outdoor pesticide applications, barbecue grills, and garbage storage areas can also bring outdoor contaminants into the building if placed close to a window or door, or the intake of a ventilation system. Improper placement of the intake of a ventilation system near a loading dock, parking lots, the exhaust from restrooms, laboratories, manufacturing spaces, and other exhausts of contaminated air is a major source of indoor air pollution. Other outdoor sources of indoor air pollutants are hazardous chemicals entering the structure from the soil. Examples are the aforementioned radon gas, methane and other gases from sanitary landfills, and vapors from leaking underground storage tanks of gasoline, oil, and other chemicals penetrating into basements. Polluted water can give off substantial quantities of harmful chemicals during showering, dishwashing, and similar activities.

20.4 INDUSTRIAL VENTILATION SYSTEMS

The major components of an industrial ventilation system include the following:

1. Exhaust hood
2. Ductwork
3. Contaminant control device
4. Exhaust fan
5. Exhaust vent or stack

Several types of hoods are available. One must select the appropriate hood for a specific operation to effectively remove contaminants from a work area and transport them into the ductwork. The ductwork must be sized such that the contaminant is transported without being deposited within the duct; adequate velocity must be maintained in the duct to accomplish this. Selecting a control device that is appropriate for the contaminant removal is important to meet certain pollution control removal efficiency requirements. The exhaust fan is the workhorse of the ventilation system. The fan must provide the volumetric flow at the required static pressure, and must be capable of handling contaminated air characteristics such as dustiness, corrosivity, and moisture in the air stream. Properly venting the exhaust out of the building is equally necessary to avoid contaminant recirculation into the air intake or into the building through other openings. Such problems can be minimized by properly locating the vent pipe in relation to the aerodynamic characteristics of the building. In addition, all or a portion of the cleaned air may be recirculated to the workplace. Primary (outside) air may be added to the workplace and is referred to as makeup air; the temperature and humidity of the makeup air may have to be controlled. It also may be necessary to exhaust a portion of the room air.

A line diagram of a typical industrial ventilation system is provided in Fig. 20.1. Note that either the control device or the fan (or both) can be located in the room/workplace.

Exposure to contaminants in a workplace can be reduced by proper ventilation. Ventilation can be provided either by *dilution ventilation* or by a *local exhaust*

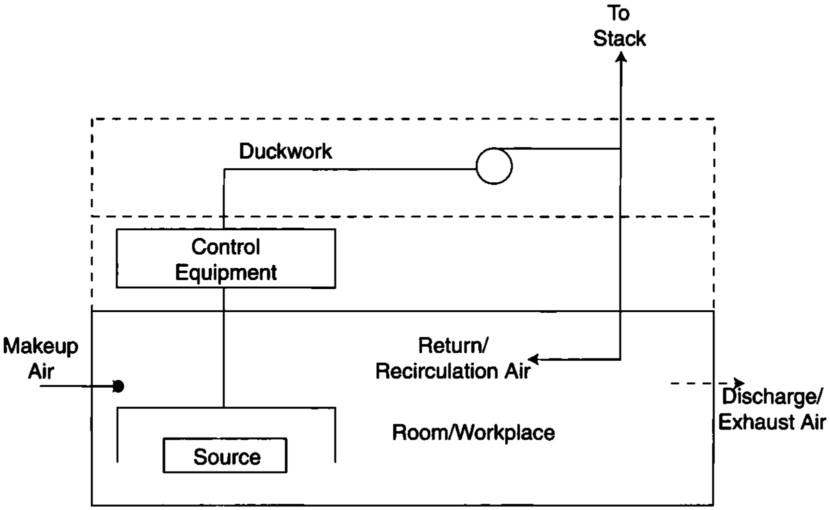

Figure 20.1 Industrial ventilation system components.

system. In dilution ventilation, air is brought into the work area to dilute the contaminant sufficiently to minimize its concentration and subsequently reduce worker exposure. In a local exhaust system, the contaminant itself is removed from the source through hoods.

A local exhaust is generally preferred over a dilution ventilation system for health hazard control because a local exhaust system removes the contaminants directly from the source, whereas dilution ventilation merely mixes the contaminant with uncontaminated air to reduce the contaminant concentration. Dilution ventilation may be acceptable when the contaminant concentration has a low toxicity, and the rate of contaminant emission is constant and low enough that the quantity of required dilution air is not prohibitively large. However, dilution ventilation is generally not acceptable when the acceptable concentration is less than 100 ppm.

In determining the quantity of dilution air required, one must also consider mixing characteristics of the work area in addition to the quantity (mass or volume) of contaminant to be diluted. Thus, the amount of air required in a dilution ventilation system is much higher than the amount required in a local exhaust system. In addition, if the replacement air requires heating or cooling to maintain an acceptable workplace temperature, then the operating cost of a dilution ventilation system may further exceed the cost of a local exhaust system.

The amount of dilution air required in a dilution ventilation system can be estimated using the following expression:

$$q = K(q_c/C_a) \tag{20.1}$$

where q = dilution air flowrate; K = dimensionless mixing factor; q_c = flowrate of pure contaminant vapor; and C_a = acceptable contaminant concentration. For more details, see Illustrative Example 20.2.

Illustrative Example 20.1 Discuss in some detail the term "air exchange rate." The answer should explain what is meant by the term "air exchange rate" and then discuss some of the variables that affect it. The discussion should address the following terms: infiltration, exfiltration, wind effects, stack effects, combustion effects, natural ventilation, and forced ventilation.(5)

Solution The term "air exchange rate" is the rate at which indoor air is replaced with outdoor air. The units of the air exchange rate are "air changes per hour" or "ach." If the volume of air in a building is replaced twice in 1 hour, the air exchange rate would be two. If the volume of air in a building is replaced once in 2 hours, the air exchange rate would be 0.5. The air exchange rate can be calculated by dividing the rate at which outdoor air enters the building in m^3/hr (or ft^3/hr) by the volume of the building in m^3 (or ft^3). If the air exchange rate were 1 ach, it would not mean that every molecule of indoor air would have been replaced at the end of 1 hour. Just which molecules were replaced would depend on a number of factors. Some of those factors are infiltration, exfiltration, wind effects, stack effects, combustion effects, natural ventilation, and forced ventilation.

Infiltration and exfiltration refer to the uncontrolled leakage of air into or out of the building through cracks and other unintended openings in the outer shell of the building. In addition to leakage around windows and doors, infiltration and exfiltration can occur at points such as openings for pipes, wires, and ducts. The rate of infiltration and exfiltration can vary greatly depending on such factors as wind temperature differences between indoors and outdoors, as well as the operation of stacks and exhaust fans.

Wind effects result from wind striking one side of a building causing positive pressure on that side and lower pressure on the opposite side (the leeward side). Air is forced into the building on the windward side and out the leeward side. Some buildings may be somewhat protected from wind effects by terrain, trees, and other buildings.

The tendency of warm air to rise in a room or through the levels of a multilevel building result in what is known as stack effects. In winter, when there is a large temperature difference between indoor and outdoor air, rising warm air escapes through openings at the top of the building and outdoor air is drawn in at the bottom of the building. The effect is usually less pronounced in the summer because of smaller temperature differences and the direction of the flow may be reversed.

Combustion effects often arise from fires in fireplaces, stoves, and heating systems. The combustion uses up indoor air (oxygen), which causes pressure in the building to drop. Outdoor air is then drawn in. This effect can double infiltration rates. Use of outdoor air in a heating system or fireplace substantially reduces this effect.

Natural ventilation is air that is drawn into a building through windows, doors, and other controlled openings. Natural ventilation results from wind striking the building and/or temperature differences between the outdoor air and the indoor air.

Forced ventilation refers to drawing air into a building through fans and ducts. The effectiveness in removing contaminants from indoor air by the use of forced ventilation can vary widely. Fans used to exhaust specific sources of pollutants (such as the kitchen stove) can be very effective. Most forced ventilation systems are used to circulate air-conditioned air. Whole house fans and the forced ducted ventilation systems of large buildings must be carefully balanced by the air supply to prevent backdrafts of contaminants from stacks and heating plants.

Illustrative Example 20.2: Estimate the dilution ventilation required in an indoor work area where a toluene-containing adhesive in a nanotechnology process is used at a rate of 3 gal/8 h workday. Assume that the specific gravity of toluene (C_7H_8) is 0.87, that the adhesive contains 4 vol% toluene, and that 100% of the toluene is evaporated into the room air at 20°C. The plant manager has specified that the concentration of toluene must not exceed 80% of its threshold limit value (TLV) of 100 ppm.$^{(7)}$

As described earlier, the following equation can be used to estimate the dilution air requirement:

$$q = K(q_c/C_a) \qquad (20.1)$$

where q = dilution air flowrate; K = dimensionless mixing factor that accounts for less than complete mixing characteristics of the contaminant in the room, the contaminant toxicity level and the number of potentially exposed workers; usually, the value of K varies from 3 to 10, where 10 is used under poor mixing conditions and when the contaminant is relatively toxic; q_c = volumetric flowrate of pure contaminant vapor, c; C_a = acceptable contaminant concentration in the room, volume or mole fraction (ppm × 10^{-6}).

Solution The dilution air can be estimated from (see Problem statement)

$$q = K(q_c/C_a) \qquad (20.1)$$

Since the TLV for toluene is 100 ppm and C_a is 80% of the TLV,

$$C_a = [0.80(100)] \times 10^{-6} = 80 \times 10^{-6} \quad \text{(volume fraction)}$$

The mass flowrate of toluene is

$$\begin{aligned}\dot{m}_{\text{tol}} &= \left(\frac{3\ \text{gal}_{\text{adhesive}}}{8\ \text{h}}\right)\left(0.4\frac{\text{gal}_{\text{toluene}}}{1\ \text{gal}_{\text{adhesive}}}\right)\left[\frac{(0.87)(8.34\ \text{lb})}{1\ \text{gal}_{\text{toluene}}}\right]\\ &= 1.09\ \text{lb/h}\\ &= \left(\frac{1.09\ \text{lb}}{1\ \text{h}}\right)\left(\frac{454\ \text{g}}{1\ \text{lb}}\right)\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\\ &= 8.24\ \text{g/min}\end{aligned}$$

Since the molecular weight of toluene is 92,

$$\begin{aligned}\dot{n}_{\text{tol}} &= 8.24/92\\ &= 0.0896\,\text{gmol/min}\end{aligned}$$

The resultant toluene vapor volumetric flowrate q_{tol} is calculated directly from the ideal gas law:

$$\begin{aligned}q_{\text{tol}} &= \frac{(0.0896\ \text{gmol/min})[0.08206\ \text{atm}\cdot\text{L/(gmol}\cdot\text{K)}](293\text{K})}{1\ \text{atm}}\\ &= 2.15\ \text{L/min}\end{aligned}$$

Therefore, the required diluent volumetric flowrate is (with $K = 5$)

$$\begin{aligned}q &= \frac{(5)(2.15\ \text{L/min})}{80 \times 10^{-6}}\\ &= 134{,}375\ \text{L/min}\\ &= \left(134{,}375\ \frac{\text{L}}{\text{min}}\right)\left(\frac{1\ \text{ft}^3}{28.36\ \text{L}}\right)\\ &= 4748\ \text{ft}^3/\text{min}\end{aligned}$$

Illustrative Example 20.3 A certain poorly-ventilated chemical storage room (10 ft × 20 ft × 8 ft) has a ceiling fan but no air conditioner. The air in the room is at 51°F and 1.0 atm pressure. Inside this room, a 1 lb bottle of iron (III) sulfide (Fe_2S_3) sits next to a bottle of sulfuric acid containing 1 lb H_2SO_4 in water. An earthquake (or perhaps the elbow of a passing technician) sends the bottles on the shelf crashing to the floor where the bottles break, and their contents mix and react to form iron (III) sulfate [$Fe_2(SO_3)$] and hydrogen sulfide (H_2S).

Calculate the maximum H_2S concentration that could be reached in the room assuming rapid complete mixing by the ceiling fan with no addition of outside air.$^{(6)}$

Solution Balance the chemical equation:

amount	Fe_2S_3	+	$3H_2SO_4$	$\longrightarrow$	$Fe_2(SO_4)_3$	+	$3H_2S$
before	1 lb		1 lb		0		0
reaction	0.0048 lbmol		0.010 lbmol		0		0

The molecular weights of Fe_2S_3 and H_2SO_4 are 208 and 98, respectively.

The terms *limiting reactant* and *excess reactant* refer to the actual number of moles present in relation to the stoichiometric proportion required for the reaction to proceed to completion. From the stoichiometry of the reaction, 3 lbmol of H_2SO_4 are required to react with each lbmol of Fe_2S_3. Therefore, sulfuric acid is the limiting reactant and the iron (III) sulfide is the excess reactant. In other words, 0.0144 lbmol of H_2SO_4 is required to react with 0.0048 lbmol of Fe_2S_3, or 0.030 lbmol of Fe_2S_3 is required to react with 0.010 lbmol of H_2SO_4.

Calculate the moles of H_2S generated n_{H_2S}:

$$\begin{aligned} n_{H_2S} &= (0.010\ \text{lbmol } H_2SO_4)(3H_2S/3H_2SO_4) \\ &= 0.010\ \text{lbmol} \end{aligned}$$

Next, convert the moles to mass:

$$\begin{aligned} m_{H_2S} &= (0.010\ \text{lbmol } H_2S)(34\ \text{lb/lbmol } H_2S) \\ &= 0.34\ \text{lb} \end{aligned}$$

The final H_2S concentration in the room in ppm, C_{H_2S}, can now be calculated. At 32° and 1 atm, 1 lbmol of an ideal gas occupies 359 ft^3; at 51° 1 lbmol occupies

$$V = 359\left(\frac{460+51}{460+32}\right) = 373\ \text{ft}^3$$

Therefore,

$$\begin{aligned} C_{H_2S} &= \frac{(0.34\ \text{lb})\left(\dfrac{373\ \text{ft}^3}{\text{lbmol air}}\right)\left(\dfrac{\text{lbmol air}}{29\ \text{lb}}\right)(10^6)}{1600\ \text{ft}^3} \\ &= 2733\ \text{ppm} \end{aligned}$$

This concentration of H_2S far exceeds an acceptable value.

Illustrative Example 20.4: Vinyl Chloride Application The vinyl chloride fugitive emission rate in a process was estimated to be 10 g/min by a series of bag tests conducted for the major pieces of connections (i.e., flanges and valves) and pump seals. Determine the flowrate of air (25°C) necessary to maintain a level of 1.0 ppm by dilution ventilation. Correct for incomplete mixing by employing a safety factor of 10. Also consider partially enclosing the process and using local exhaust ventilation. Assume that the process can be carried out in a hood with an opening 30 inches wide by 25 inches high with a face velocity greater than 100 ft/min to ensure high capture efficiency. What will be the flowrate of air required for local exhaust ventilation? Which ventilation method seems better?

Solution Convert the mass flowrate of the vinyl chloride (VC) to volumetric flowrate q, in cm^3/min and acfm. First, use the ideal gas law to calculate the density.

$$\begin{aligned} \rho &= \frac{P(\text{MW})}{RT} \\ &= \frac{(1\ \text{atm})(78\ \text{g/gmol})}{\left(82.06\dfrac{\text{cm}^3\cdot\text{atm}}{\text{mol}\cdot\text{K}}\right)(298\text{K})} \\ &= 0.00319\ \text{g/cm}^3 \end{aligned}$$

$$\begin{aligned} q &= (\text{mass flowrate})/(\text{density}) \\ &= (10\,\text{g/min})/(0.00319\,\text{g/cm}^3) \\ &= 3135\,\text{cm}^3/\,\text{min} \\ &= 0.1107\,\text{acfm} \end{aligned}$$

Calculate the air flowrate in acfm, q_{air}, required to meet the 1.0 ppm constraint with the equation

$$\begin{aligned} q_{\text{air}} &= (0.1107\,\text{acfm})/10^{-6} \\ &= 1.107 \times 10^5\,\text{acfm} \end{aligned}$$

Apply the safety factor to calculate the actual air flowrate for dilution ventilation:

$$\begin{aligned} q_{\text{air,dil}} &= (10)(1.107 \times 10^5\,\text{acfm}) \\ &= 1.107 \times 10^6\,\text{acfm} \end{aligned}$$

Now consider the local exhaust ventilation by first calculating the face area of the hood S, in square feet:

$$\begin{aligned} S &= (\text{Height})(\text{Width}) \\ &= (30\,\text{in.})(25\,\text{in.})(\text{ft}^2/144\,\text{in.}^2) \\ &= 5.21\,\text{ft}^2 \end{aligned}$$

The air flowrate in acfm $q_{\text{air,exh}}$, required for a face velocity of 100 ft/min is then

$$\begin{aligned} q_{\text{air,exh}} &= (5.21\,\text{ft}^2)/(100\,\text{ft/min}) \\ &= 521\,\text{acfm} \end{aligned}$$

Since the air flowrate for dilution ventilation is approximately 2000 times higher than the local ventilation air flowrate requirement, and considering the high cost of large blowers to handle high air flowrates, the local ventilation method appears to be the better method for this case.

Illustrative Example 20.5(8,9) Your consulting firm has received a contract to develop, as part of an emergency preparation plan, mathematical models describing the concentration of a chemical in a medium-sized ventilated laboratory room. The following information/data (SI units) is provided:

V = volume of room, m^3
q_0 = volumetric flow rate of ventilation air, m^3/min
c_0 = concentration of the chemical in ventilation air, gmol/m^3

c = concentration of the chemical leaving ventilated room, gmol/m^3
c_1 = concentration of the chemical initially present in ventilated room, gmol/m^3
r = rate of disappearance of the chemical in the room due to reaction and/or other effects, gmol/m$^3 \cdot$ min.

As an authority in the field (having taken several Theodore courses), you have been requested to:

1. Obtain the equation describing the concentration in the room as a function of time if there are no "reaction" effects, that is, $r = 0$.
2. Obtain the equation describing the concentration in the room as a function of time if $r = -k$. Note once again that the minus sign is carried since the agent is disappearing.
3. Obtain the equation describing the concentration in the room as a function of time if $r = -kc$. Note once again that the minus sign is carried since the chemical is disappearing.
4. For Part 2, discuss the effect on the resultant equation if k is extremely small, that is, $k \to 0$.

Solution Use the laboratory room as the control volume. Apply the conservation law for mass to the chemical

$$\left\{\begin{matrix}\text{rate of mass}\\ \text{in}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of mass}\\ \text{out}\end{matrix}\right\} + \left\{\begin{matrix}\text{rate of mass}\\ \text{generated}\end{matrix}\right\} = \left\{\begin{matrix}\text{rate of mass}\\ \text{accumulated}\end{matrix}\right\}$$

Employing the notation specified in the problem statement gives:

$$\begin{aligned}\{\text{rate of mass in}\} &= q_0c_0\\ \{\text{rate of mass out}\} &= q_0c\\ \{\text{rate of mass generated}\} &= rV\\ \{\text{rate of mass accumulated}\} &= \frac{dV}{dt}\end{aligned}$$

Substituting above gives

$$q_0c_0 - q_0c + rV = \frac{dV}{dt}$$

Since the laboratory room is constant, V may be taken out of the derivative term. This leads to

$$\frac{q_0}{V}(c_0 - c) + r = \frac{dc}{dt}$$

The term V/q_0 represents the average residence time that the chemicals reside in the room and is usually designated as τ. The above equation may then be rewritten as

$$\frac{\mathrm{d}c}{\mathrm{d}t} = \frac{c_0 - c}{\tau} + r$$

1. If $r = 0$,

$$\frac{\mathrm{d}c}{\mathrm{d}t} = \frac{c_0 - c}{\tau}$$

separating variables

$$\frac{\mathrm{d}c}{c_0 - c} = \frac{\mathrm{d}t}{\tau}$$

$$\int_{c_i}^{c} \frac{\mathrm{d}c}{c_0 - c} = \int_{0}^{t} \frac{\mathrm{d}t}{\tau}$$

$$\ln\left(\frac{c_0 - c}{c_0 - c_i}\right) = \frac{t}{\tau}$$

$$\left(\frac{c_0 - c}{c_0 - c_i}\right) = e^{-t/\tau}$$

$$c = c_0 - (c_0 - c_i)e^{-t/\tau} = c_0 + (c_i - c_0)e^{-t/\tau}$$

2. If $r = -k$

$$\frac{\mathrm{d}c}{\mathrm{d}t} = \frac{c_0 - c}{\tau} - k = \frac{c_0}{\tau} - k - \frac{c}{\tau} = \left(\frac{c_0 - k\tau}{\tau}\right) - \frac{c}{\tau}$$

$$\frac{\mathrm{d}c}{\left[\left(\frac{c_0 - k\tau}{\tau}\right) - \frac{c}{\tau}\right]} = \mathrm{d}t$$

$$\frac{\mathrm{d}c}{[(c_0 - k\tau) - c]} = \frac{\mathrm{d}t}{\tau}$$

$$\int_{c_i}^{c} \frac{\mathrm{d}c}{[(c_0 - k\tau) - c]} = \int_{0}^{t} \frac{\mathrm{d}t}{\tau}$$

$$-\ln\left[\frac{(c_0 - k\tau) - c}{(c_0 - k\tau) - c_i}\right] = \tau$$

$$\frac{(c_0 - k\tau) - c}{(c_0 - k\tau) - c_i} = e^{-(t/\tau)}$$

$$c = c_i e^{-t/\tau} + (c_0 - k\tau)\left[1 - e^{-(t/\tau)}\right]$$

3. If $r = -kc$

$$\frac{dc}{dt} = \frac{c_0 - c}{\tau} - kc = \frac{c_0}{\tau} - \frac{c}{\tau} - kc$$

$$= \frac{c_0}{\tau} - c\left(\frac{k\tau + 1}{\tau}\right)$$

$$\frac{dc}{\left[\frac{c_0}{\tau} - \left(\frac{1 + k\tau}{\tau}\right)c\right]} = dt$$

$$\int_{c_i}^{c} \frac{dc}{[c_0 - (1 + k\tau)c]} = \int_0^t \frac{dt}{\tau}$$

$$-\left(\frac{1}{1 + k\tau}\right)\ln\left[\frac{c_0 - (1 + k\tau)c}{c_0(1 + k\tau)c_i}\right] = \frac{t}{\tau}$$

$$\left(\frac{c_0 - (1 + k\tau)c}{c_0 - (1 + k\tau)c_i}\right) = e^{-(t/\tau)(1+k\tau)}$$

$$c = c_i e^{-(t/\tau)(1+k\tau)} + \left(\frac{c_0}{1 + k\tau}\right)\left[1 - e^{-(\frac{t}{\tau})(1+k\tau)}\right]$$

4. If $k = 0$, see solution 2.

$$c = c_i e^{-t/\tau} + (c_0 - k\tau)[1 - e^{-t/\tau}]$$

$$= c_i e^{-t/\tau} + c_0 - c_0 e^{-t/\tau}$$

$$= c_0 + (c_i - c_0)e^{-t/\tau}$$

Illustrative Example 20.6 Refer to Illustrative Example 20.5. For Part 1, qualitatively discuss the effect on the final equation if the volumetric flow rate, v_0, varies sinusoidally. Also, qualitatively discuss the effect on the final equation if the inlet concentration, c_0, varies sinusoidally.

Solution If v_0 varies and τ varies, solving the equation becomes more complex. Variations need to be included in the describing equation

$$\frac{dc}{dt} = \frac{c_0 - c}{\tau} + r; \quad \tau = \tau(t)$$

This may require numerical solution. If both $c_0 = c_0(t)$ and $v_0 = v$ d(t), the solution again applies.

Illustrative Example 20.7 Refer to Illustrative Example 20.5.

1. Calculate minimum air ventilation flow rate into the room containing 10 ng/m^3 (ng = nanograms) of a toxic chemical to assure that the chemical concentration does not exceed 35.0 ng/m^3. The chemical is being generated in the laboratory at a rate of 250 ng/min. Assume steady-state conditions.
2. Calculate the steady-state concentration in the laboratory: the initial concentration of the chemical is 500 ng/m^3. There is no additional source of chemical generated and the ventilation air is essentially pure, i.e., there is no background chemical concentration.

Solution

1. The applicable model for this case is:

$$q_0(c_0 - c) + rV = V\frac{dc}{dt}$$

Under steady-state, dc/dt = 0. Pertinent information includes

$$rV = 250\,\text{ng/min}$$
$$c_0 = 10\,\text{ng/m}^3$$
$$c = 35\,\text{ng/m}^3$$

Substituting gives

$$\begin{aligned} q_0 &= \frac{-rV}{c_0 - c} \\ &= \frac{rv}{c - c_0} \\ &= \frac{250}{35 - 10} \\ &= 10\,\text{m}^3/\text{min} = 353\,\text{ft}^3/\text{min} \end{aligned}$$

2. The applicable model is:

$$q_0(c_0 - c) + rV = V\frac{dc}{dt}$$

Once again, for steady-state condition, $dc/dt = 0$. In addition, based on the information provided, $r = 0$ and $c_0 = 0$. Therefore, and as expected,

$$c = 0$$

Illustrative Example 20.8 Refer to Illustrative Examples 20.5 and 20.7. If the room volume is $142\ m^3$, the flowrate of the $10\ ng/m^3$ ventilation air is $12.1\ m^3/min$, and the chemical is being generated at a steady rate of 30 ng/min, calculate how long it would take for the concentration to reach $20.7\ ng/m^3$. The initial concentration in the laboratory is $85\ ng/m^3$. How long would it take to reach $12.2\ ng/m^3$?

Solution First note that

$$\tau = 142/12.1 = 11.73\ \text{min}$$

$$k = r/V = 30/142 = 0.211\ \text{ng}/(\text{m}^3 \,.\, \text{min})$$

The applicable describing equation is:

$$c = c_i e^{-t/\tau} + (c_0 + k\tau)[1 - e^{-(t/\tau)}]$$

Substituting gives

$$20.7 = 85e^{-(t/11.73)} + (10 + 2.48)[1 - e^{-(t/11.73)}]$$

Solving by trial-and-error gives (approximately)

$$t = 29\ \text{min}$$

Calculate the steady-state concentration for this condition. The applicable model is obtained from Part 2 of Illustrative Example 20.5, after setting $dc/dt = 0$:

$$\frac{c_0 + k\tau}{\tau} - \frac{c}{\tau} = 0$$

Solving and substituting gives

$$\begin{aligned} c &= c_0 + k\tau \\ &= 10 + (0.211)(11.73) \\ &= 12.48\ \text{ng/m}^3 \end{aligned}$$

Since this is the steady-state concentration, it will take an infinite period of times to reach this value. This steady-state concentration represents the minimum concentration achievable in the laboratory based on the conditions specified. Therefore, the concentration will never reach a value of 12.2 $\mathrm{ng/m^3}$.

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