16

TWO-PHASE FLOW

16.1 INTRODUCTION

The simultaneous flow of two phases in pipes (as well as other conduits) is complicated by the fact that the action of gravity tends to cause settling and "slip" of the heavier phase with the result that the lighter phase flows at a different velocity in the pipe than does the heavier phase. The results of this phenomena are different depending on the classification of the two phases, the flow regime, and the inclination of the pipe (conduit).

As one might suppose, the major industrial application in this area is gas (G)–liquid (L) flow in pipes. Therefore, the subjects addressed in this chapter key on a G–L flow in pipes. The extension of much of the material to follow to flow in various conduits can be accomplished by employing the equivalent diameter of the conduit in question.

The general subject of liquid–solid flow in pipes is not considered in this chapter. Suspensions of solids in liquids fall into two general classes, Newtonian and non-Newtonian. Newtonian suspensions are characterized by a constant viscosity, independent of the rate of shear. In the case of non-Newtonian suspensions, the viscosity is a variable that is a function of the rate of shear and (in some cases) a function of the duration or period of shear for viscous flow. If the suspension is found to be Newtonian in character, the pressure drop can be calculated by standard equations available for both viscous flow and turbulent flow by employing the average density and viscosity of the mixture (see Chapters 11 and 12). The procedures for computing the pressure drop for non-Newtonian suspensions are more involved but received treatment in Chapter 6. Details on liquid–solid flow is also available from Perry and Green.[1]

Fluid Flow for the Practicing Chemical Engineer. By J. Patrick Abulencia and Louis Theodore

The general subject of flashing and boiling liquids is also not considered in this chapter. However, when a saturated liquid flows in a pipeline from a given point at a given pressure to another point at a lower pressure, several processes can take place. As the pressure decreases, the saturation or boiling temperature decreases, leading to the evaporation of a portion of the liquid. The net results that a one-phase flowing mixture is transformed into a two-phase mixture with a corresponding increase in frictional resistance in the pipe. Boiling liquids arise when liquids are vaporized in pipelines at approximately constant pressure. Alternatively, the flow of condensing vapors in pipes is complicated due to the properties of the mixture constantly changing with changes in pressure, temperature, and fraction condensed. Further, the condensate, which forms on the walls, requires energy in order to be transformed into spray, and this energy must be obtained from the main vapor stream, resulting in an additional pressure drop. An analytical treatment of these topics is beyond the scope of this book. However, information is available in the literature.[1]

The remainder of the chapter examines the following topics:

- Gas (G)–Liquid (L) Flow Principles: Generalized Approach
- Gas (Turbulent) Flow–Liquid (Turbulent) Flow
- Gas (Turbulent) Flow–Liquid (Viscous) Flow
- Gas (Viscous) Flow–Liquid (Viscous) Flow
- Gas–Solid Flow

16.2 GAS (G)–LIQUID (L) FLOW PRINCIPLES: GENERALIZED APPROACH

The suggested method of calculating the pressure drop of gas–liquid mixtures flowing in pipes is essentially that originally proposed by Lockhart and Martinelli[2] nearly 60 years ago. The basis of their correlation is that the two-phase pressure drop is equal to the single-phase pressure drop for either phase (G or L) multiplied by a factor that is a function of the single-phase pressure drops of the two phases. The equations for the total pressure drop per unit length Z $(\Delta P/Z)_{\mathrm{T}}$ are written as:

$$(\Delta P/Z)_{\mathrm{T}} = Y_{\mathrm{G}}(\Delta P/Z)_{\mathrm{G}} \tag{16.1}$$

$$(\Delta P/Z)_{\mathrm{T}} = Y_{\mathrm{L}}(\Delta P/Z)_{\mathrm{L}} \tag{16.2}$$

The terms Y_{L} and Y_{G} are functions of the variable X:

$$Y_{\mathrm{G}} = F_{\mathrm{G}}(X) \tag{16.3}$$

$$Y_{\mathrm{L}} = F_{\mathrm{L}}(X) \tag{16.4}$$

where

$$X = \left[\frac{(\Delta P/Z)_{\mathrm{L}}}{(\Delta P/Z)_{\mathrm{G}}}\right]^{0.5} \tag{16.5}$$

The relationship between Y_L and Y_G is therefore given by

$$Y_G = X^2 Y_L \tag{16.6}$$

The single-phase pressure-drop gradients $(\Delta P/Z)_L$ and $(\Delta P/Z)_G$ can be calculated by assuming that each phase is flowing alone in the pipeline, and the phase in question is traveling at its superficial velocity. The superficial velocities are therefore based on the full cross-sectional area, S, of the pipe so that

$$v_L = q_L/S \tag{16.7}$$

and

$$v_G = q_G/S \tag{16.8}$$

where v_L = liquid-phase superficial velocity,
v_G = gas-phase superficial velocity,
q_L = liquid-phase volume flow rate,
q_G = gas-phase volume flow rate,
S = pipe cross-sectional area.

Note that either Equation (16.1) or (16.2) can be employed to calculate the pressure drop.

The functional relationships for Y_L and Y_G in Equations (16.3) and (16.4) in terms of X were also provided by Lockhart and Martinelli[2] for the phase classification under different flow conditions. (These relationships are provided later in this chapter.) For gas–liquid flows, semi-empirical data were provided for the following three flow categories:

gas (turbulent flow, t)–liquid (turbulent flow)
gas (turbulent flow)–liquid (viscous flow, v)
gas (viscous flow)–liquid (viscous flow)

The next three subsections address each of the above topics. Note that applications involving gas (viscous flow)–liquid (viscous flow) do not receive treatment since this type of flow rarely occurs in practice; the low viscosity of a gas (or vapor) virtually eliminates the possibility of gas moving in a laminar flow.

A variety of the above flow phenomena is possible with the two-phase flow of gases and liquids in horizontal pipes ranging from parallel (two-layer) flow at low velocities to dispersed flow at high velocities (gas carried as bubbles in a continuous liquid phase or liquid carried as spray in the gas). The pressure drop is greater in liquid–gas flow than that for the single-phase flow of either gas or liquid for several reasons. These include the irreversible work done on the liquid by the gas and that the effective cross-sectional area of flow for either phase is reduced by the flow of the other phase in the area.

The basis for the Martinelli correlations[2,3] assumes that the pressure drop for the liquid phase must equal the pressure drop for the gas phase for all types of flow, provided that no appreciable pressure differences exist across any pipe diameter and that the volume occupied by the liquid and by the gas at any instant of time must equal the total volume of the pipe. Using these assumptions, the pressure drop due to the liquid flow and that due to the gas flow was expressed in each case by standard pressure drop equations using unknown "hydraulic diameters." The hydraulic diameters were then expressed in terms of the actual cross-sectional area of flow and the ratio of the actual cross-sectional area of flow to the area of a circle of diameter equal to the unknown hydraulic diameter. The unknown hydraulic diameter for the liquid flow was eliminated in the analysis and an expression was obtained for the pressure drop as a function of the single-phase pressure drop for gas alone. The function, expressed as ϕ^2 in their study, was introduced in order to reduce the range of the variables when providing ϕ^2 vs $\sqrt{X}$. Isothermal flow in smooth pipes was assumed.

It is important to know what type of flow is occurring, although this can obviously be a difficult task. In order to establish which flow mechanisms applied, Martinelli et al.[2,3] used a set of flow conditions (as noted above) that were functions of the Reynolds number

$$\mathrm{Re} = 4w/\pi D\mu \tag{16.9}$$

where w is the mass flow rate. Martinelli et al.[2,3] computed the Reynolds number for each using the *actual* pipe diameter; i.e., a superficial velocity was employed. For $\mathrm{Re} < 2000$, the flow for that phase was assumed to be viscous (laminar); for $\mathrm{Re} > 2000$, the flow is assumed to be turbulent.

Illustrative Example 16.1 Air and oil are in concurrent flow through a horizontal pipe. The following pressure drop calculations were obtained from Theodore Consultants (a group of engineers with limited technical capabilities):

$$(\Delta P/Z)_G = 2.71\ \mathrm{psft}/100\ \mathrm{ft}$$
$$(\Delta P/Z)_L = 7.50\ \mathrm{psft}/100\ \mathrm{ft}$$

Calculate the dimensional parameter X.

Solution Refer to Equation (16.5):

$$X = [(\Delta P/Z)_L/(\Delta P/Z)_G]^{0.5}$$

Substitute

$$X = (7.50/2.71)^{0.5} = 1.66$$

The volume fraction or holdup of a phase for two-phase flow in a horizontal pipe is also available[1]:

$$\varepsilon_L = F_3(X) \tag{16.10}$$

and

$$\varepsilon_G = F_4(X) \tag{16.11}$$

where $\varepsilon_G + \varepsilon_L = 1$ and ε_G and ε_L are, the fraction (dimensionless) of pipe volume occupied by the liquid phase and gas phase, respectively; X is the aforementioned variable defined by Equation (16.5). The relationship between ε_L and X is approximately provided by[4]:

$$\varepsilon_L = 0.298 + 0.117 \ln(X) \tag{16.12}$$

Gas–liquid flow usually occurs in horizontal pipes. However, when gas–liquid mixtures flow in vertical pipes, there is an increase in liquid concentration or build-up of liquid due to the density difference in the case of upward flow, and a decrease in liquid concentration in the case of downward flow. Since information is available on the upward flow of gas–liquid mixtures, a variety of flow phenomena are possible including gas as the dispersed phase in a continuous liquid phase to gas as the continuous phase with liquid carried as spray. One of the intermediate types of flow is where the liquid flows as an annulus and the gas as a central core. The major applications are gas lifts. A gas lift is a vertical pipe (known as an eduction pipe) open at both ends, part of which is submerged below the surface of the liquid to be pumped. Compressed gas is admitted through a foot-piece inside the lower end; a mixture of liquid and gas is thus formed within the pipe. The gas reduces the average density of the mixture in the eduction pipe to a point where the weight of the mixture is less than equivalent to the pressure at the foot-piece. With the gas and liquid being supplied at a sufficient rate, the mixture rises upward through the pipe and is discharged at the upper end. Industrial use occurs with the operation of flowing oil wells. Considerable operating and experimental data have been reported but little attempt has been made to correlate them.

16.3 GAS (TURBULENT) FLOW–LIQUID (TURBULENT) FLOW

This section provides additional details of the original work of Lockhart and Martinelli.[2,3] This is followed by a simpler approach for predicting pressure drop. The simpler approach is recommended for industrial applications.

In the original work (with most of the notation retained), the ratio of the actual cross-sectional area of flow to the area of a circle of diameter equal to the unknown equivalent (or hydraulic) diameter for the gas phase was assumed to be unity and the ratio for the liquid phase was determined from experimental data. The following correlations were obtained from the ratio for the liquid phase and the properties of the liquid and gas

$$\left(\frac{\Delta P}{Z}\right)_{tt} = \phi_{tt}\left(\frac{\Delta P}{Z}\right)_G \tag{16.13}$$

where ϕ_{tt} is a function of a dimensionless group, X_{tt}; and,

$$X_{tt} = \left(\frac{\mu_L}{\mu_G}\right)^{0.111}\left(\frac{\rho_G}{\rho_L}\right)^{0.555}\left(\frac{\dot{m}_L}{\dot{m}_G}\right) \tag{16.14}$$

The magnitude of ϕ_{tt} for values of X_{tt} is given in Table 16.1.

Table 16.1 ϕ_{tt} vs $\sqrt{X_{tt}}$

$\sqrt{X_{tt}}$	ϕ_{tt}
0	1.00
0.10	1.50
0.20	1.68
0.40	2.13
0.70	3.03
1.00	4.08
2.00	8.30
4.00	19.6
7.00	42.3
10.0	71.0
20.0	222
40.0	770
46.2	1000
for $\sqrt{X_{tt}} > 46$	$\phi_{tt} = (\sqrt{X_{tt}})^{1.8}$

Results were later expressed in terms of Y_L and Y_G, both of which are functions of X_{tt}; see Equations (16.3) and (16.4) for more details. Van Vliet[5] subsequently regressed the data to a model of the form

$$Y_G = a + bX + cX^2 + dX^3$$

and

$$Y_L = aX^b \tag{16.15}$$

The final results for Y_L and Y_G for tt flow are presented in Equations (16.16) and (16.17):

$$Y_G(\text{tt}) = 1.7172 + 15.431X + 3.9314X^2 - 2.2952X^3; \quad X < 1 \tag{16.16a}$$

$$= 5.80 + 6.7143X + 6.9643X^2 - 0.75X^3; \quad 1 < X < 10 \tag{16.16b}$$

$$= 131 + 1.4105X + 1.9362X^2 - 0.0087X^3; \quad X > 10 \tag{16.16c}$$

$$Y_L(\text{tt}) = 11.745X^{-1.4901}; \quad X < 1 \tag{16.17a}$$

$$= 18.219X^{-0.8192}; \quad 1 < X < 10 \tag{16.17b}$$

$$= 6.3479X^{-0.3518}; \quad X > 10 \tag{16.17c}$$

As noted earlier, the pressure drop for two-phase flow can be calculated using either Equation (16.1) or (16.2). Longhand calculations have shown[5] that the two equations can, in some cases, produce different results. The authors recommend using either the gas phase value or the average of the two for design purposes.

However, if the volume fraction of one phase predominates (see Eq. (16.12)), the authors suggest employing the pressure drop calculation for that phase.

Illustrative Example 16.2 Refer to Illustrative Example (16.1). Calculate the pressure drop (total) if the flow for both phases is turbulent. Base the calculation on:

a. Y_G
b. Y_L

Solution

a. Since the flow is tt and $1 < X < 10$, apply Equation (16.16b) to obtain Y_G while noting $X = 1.66$:

$$Y_G(\text{tt}) = 5.80 + 6.7143X + 6.9643X^2 - 0.75X^3$$

Substituting

$$\begin{aligned} Y_G(\text{tt}) &= 5.80 + 6.7143(1.66) + 6.9643(1.66)^2 - 0.75(1.66)^3 \\ Y_G &= 5.80 + 11.145 + 19.19 - 3.431 \\ &= 32.7 \end{aligned}$$

This value is an excellent agreement with the values provided by Lockhart and Martinelli.[3] The pressure drop is therefore (from Eq. 16.1):

$$\begin{aligned} \Delta P/Z &= (Y_G)(\Delta P/Z)_G \\ &= (32.7)(2.71) \\ &= 88.6 \text{ psf}/100 \text{ ft} \end{aligned}$$

b. Apply Equation (16.17b) to generate Y_L

$$\begin{aligned} Y_L(\text{tt}) &= 18.219X^{-0.8192} \\ &= 18.219(1.66)^{-0.8192} \\ &= 12.0 \end{aligned}$$

The literature value is approximately 12.[3] The pressure drop is therefore (from Eq. 16.2):

$$\begin{aligned} \Delta P/Z &= (Y_L)(\Delta P/Z)_L \\ &= (12.0)(7.50) \\ &= 90.2 \text{ psf}/100 \text{ ft} \end{aligned}$$

As expected, both results are in reasonable agreement.

16.4 GAS (TURBULENT) FLOW–LIQUID (VISCOUS) FLOW

The original work of Lockhart and Martinelli[2] is once again reviewed for the turbulent–viscous (tv) case. Using the same procedures as that for the turbulent–turbulent (tt) case, the final correlation took the form:

$$\left(\frac{\Delta P}{Z}\right)_{tv} = \phi_{tv}^2 \left(\frac{\Delta P}{Z}\right)_G \tag{16.18}$$

where ϕ_{tv} is a function of a dimensionless group, X_{tv}. The magnitude of ϕ_{tv} for values of X_{tv} is provided in Table 16.2.

Table 16.2 ϕ_{tv} vs $\sqrt{X_{tv}}$

$\sqrt{X_{tv}}$	ϕ_{tv}
0	1.00
0.07	2.00
0.10	2.14
0.20	2.46
0.40	2.96
0.70	3.42
1.00	3.85
2.00	5.30
4.00	7.87
7.00	11.3
10.0	14.8
20.0	25.4
40.0	46.0
70.0	75.8
100	105
200	203
400	400
1000	1000
for $\sqrt{X_{tv}} > 1000$	$\phi_{tv} = \sqrt{X_{tv}}$

The results of Table 16.2, which are functions of X_{tv} were expressed in terms of Y_G and Y_L. Van Vliet[5] subsequently regressed the data to a model of the form as in Equations (16.16) and (16.17). The final results for Y_L and Y_G are presented in Equations (16.19) and (16.20)

$$Y_G(\text{tv}) = 1.6204 + 1.1825X + 34.778X^2 - 30.522X^3; \quad X < 1 \tag{16.19a}$$

$$= 20 - 21.81X + 16.357X^2 - 1.8333X^3; \quad 1 < X < 10 \tag{16.19b}$$

$$= 50.333 + 2.9782X + 1.9395X^2 - 0.0088X^3; \quad X > 10 \tag{16.19c}$$

$$Y_L(\text{tv}) = 6.7147X^{-1.5757}; \quad X < 1 \tag{16.20a}$$

$$= 11.702X^{-0.7334}; \quad 1 < X < 10 \tag{16.20b}$$

$$= 5.5873X^{-0.3215}; \quad X > 10 \tag{16.20c}$$

Illustrative Example 16.3 Refer to Illustrative Example (16.1). Calculate the pressure drop (total) if the flow for the gas phase is turbulent and the liquid phase is viscous. Base the calculation on:

a. Y_G
b. Y_L

Solution

a. Since the flow is tv and $1 < X < 10$, apply Equation (16.19b) to obtain Y_G while noting once again $X = 1.66$:

$$Y_G(\text{tv}) = 20 - 21.81X + 16.357X^2 - 1.8333X^3$$

Substituting

$$\begin{aligned} Y_G(\text{tv}) &= 20 - 21.81(1.66) + 16.357(1.66)^2 - 1.8333(1.66)^3 \\ Y_G &= 20 + 3.62 + 45.1 - 8.39 \\ &= 20.5 \end{aligned}$$

The literature value is approximately 22.[3] The pressure drop is therefore (from Eq. 16.1):

$$\begin{aligned} \Delta P/Z &= (Y_G)(\Delta P/Z)_G \\ &= (32.7)(2.71) \\ &= 55.6\,\text{psf}/100\,\text{ft} \end{aligned}$$

b. Apply Equation (16.20b) to generate Y_L.

$$\begin{aligned} Y_L(\text{tv}) &= 11.702X^{-0.7334} \\ &= 11.702(1.66)^{-0.7334} \\ &= 8.07 \end{aligned}$$

The literature value is approximately 7.5.[3] The pressure drop is therefore (from Eq. 16.2):

$$\begin{aligned}\Delta P/Z &= (Y_L)(\Delta P/Z)_L \\ &= (8.07)(7.50) \\ &= 60.5\,\mathrm{psf}/100\,\mathrm{ft}\end{aligned}$$

As expected, both results are in reasonable agreement.

16.5 GAS (VISCOUS) FLOW–LIQUID (VISCOUS) FLOW

The same procedure was employed by Lockhart and Martinelli[2,3] as in the preceding two cases except that both liquid and gas ratios of the actual cross-sectional area of flow to the area of a circle of diameter equal to the unknown hydraulic diameter for the gas phase were determined experimentally in capillary tubes. Their correlation was expressed as

$$\left(\frac{\Delta P}{Z}\right)_{vv} = \phi_{vv}^2 \left(\frac{\Delta P}{Z}\right)_G \tag{16.21}$$

where ϕ_{vv} is a function of a dimensionless group, X_{vv}. The magnitude of ϕ_{vv} for values of X_{vv} is provided in Table 16.3.

Table 16.3 ϕ_{vv} vs $\sqrt{X_{vv}}$

$\sqrt{X_{vv}}$	ϕ_{vv}
0.2	1.40
0.4	1.69
0.6	1.93
0.8	2.16
1	2.44
2	3.81
3	5.15
4	6.4
6	8.7 (limit of experimental data)
.	.
.	.
.	.
	∞
	$\phi_{vv} = \sqrt{X_{vv}}$

The results in Table 16.3 were later expressed in terms of Y_L and Y_G, both of which are functions of X_{vv} (see Equations (16.3) and (16.4) for more details). Van Vliet[5] subsequently regressed the data to a model of the form as in Equations

(16.16)–(16.17) and (16.19)–(16.20). The final results for Y_L and Y_G are presented in Equations (16.22) and (16.23):

$$Y_G(\text{vv}) = 1.1241 + 3.7085X + 6.7318X^2 - 11.541X^3; \quad X < 1 \tag{16.22a}$$

$$= 10 - 10.405X + 8.6786X^2 - 0.9167X^3; \quad 1 < X < 10 \tag{16.22b}$$

$$= -78.333 + 7.3223X + 1.8957X^2 - 0.0087X^3; \quad X > 10 \tag{16.22c}$$

$$Y_L(\text{vv}) = 3.9794X^{-1.6583}; \quad X < 1 \tag{16.23a}$$

$$= 6.4699X^{-0.556}; \quad 1 < X < 10 \tag{16.23b}$$

$$= 3.7013X^{-0.2226}; \quad X > 10 \tag{16.23c}$$

As noted earlier, this finds flow regime limited application in practice.

Illustrative Example 16.4 Refer to Illustrative Example (16.1). Calculate the pressure drop (total) if the flow for both phases is laminar. Base the calculation on:

a. Y_G
b. Y_L

Solution

a. Since the flow is tv and $1 < X < 10$, apply Equation (16.22b) to obtain Y_G, noting $X = 1.66$.

$$Y_G(\text{vv}) = 10 - 10.405X + 8.6786X^2 - 0.9167X^3$$

Substituting

$$\begin{aligned} Y_G(\text{vv}) &= 10 - 10.405(1.66) + 8.6786(1.66)^2 - 0.9167(1.66)^3 \\ Y_G &= 10 - 17.3 + 23.9 - 4.19 \\ &= 12.41 \end{aligned}$$

The literature value is approximately 12.5.[(3)] The pressure drop is therefore (from Eq. 16.1):

$$\begin{aligned} \Delta P/Z &= (Y_G)(\Delta P/Z)_G \\ &= (12.41)(2.71) \\ &= 33.6\,\text{psf}/100\,\text{ft} \end{aligned}$$

b. Apply Equation (16.23b) to generate Y_L:

$$\begin{aligned} Y_L(\text{vv}) &= 6.4699X^{-0.556} \\ &= 6.4699(1.66)^{-0.556} \\ &= 4.88 \end{aligned}$$

The literature value is approximately 5.0.[3] The pressure drop is therefore (from Eq. 16.2):

$$\begin{aligned} \Delta P/Z &= (Y_L)(\Delta P/Z)_L \\ &= (4.88)(7.50) \\ &= 36.1\ \text{psf}/100\ \text{ft} \end{aligned}$$

As expected, both results are in fair agreement.

16.6 GAS–SOLID FLOW

There are many gas–solid flow systems in pipes but this section solely addresses pneumatic conveying. The material to follow in this section will essentially be divided into five subsections:

1. Introduction
2. Solids Motion
3. Pressure Drop
4. Design Procedure
5. Pressure Drop Reduction in Gas Flow

16.6.1 Introduction

Conveying material pneumatically has been used for many years. The system can be either a pressure system or a suction system. The materials that have been handled include grain, wood shavings, pulverized coal, cement, staple, plastic chips, small metal parts, and money containers in department stores. Pneumatic conveyors are simple, quiet, convenient, and clean; however, pneumatic conveyors have a much lower efficiency than the belt or bucket type conveyor.

In a pressure system, the material can be fed by a screw conveyor or similar feeder and then forced through the system by compressed air or the material can be fed into a tank and then forced through the system by compressed air. In a suction system, a fan or blower is installed after the separating system thereby putting the entire system under vacuum. The material, with sufficient air to keep the material in suspension, is then drawn or "sucked" through the system.

Occasionally, it is more convenient to use a combination of pressure and suction systems. In a combination system, the material is drawn in, passes through the fan and then under pressure is forced through the remainder of the system. For cases where the material may damage the fan, an ejector may be used in place of the fan. Also, using an ejector, the material can be fed in the mixing throat of the ejector.

It should be noted that in the design of pneumatic conveying systems, neither a pneumatic or theoretical method has been developed; the design is based on practical operating experience and empirical correlations of test data. Earlier general treatments of the particle motion and pressure drop are available.[6–8]

16.6.2 Solids Motion

The path of the solids in a horizontal pipe is somewhat sinusoidal, the solids striking the bottom of the pipe at intervals and then rising again. The height and length of the rise appears to decrease as the air velocity decreases. The vertical distribution of the solids across the pipe diameter is fairly uniform at low concentrations but becomes more dense at the bottom of the pipe as the loading (ratio of weight rate of solids to weight rate of air) increases. Finally, at high loadings, a considerable portion of the solids have been reported along the bottom of the pipe. The difference between the final average velocity of the solids and that of the air stream is almost constant for both horizontal and vertical conveyors. This difference is the "slip" between the solids and the air and increases with increasing velocity of the air stream. This "slip" velocity is of the order of magnitude of the "choking" velocity and is essentially the minimum transport or conveying velocity. For estimating purposes, the "slip" velocity may be taken as equal to the "choking" velocity. It has been reported that the "choking" velocity is independent of loading for relatively large particles.

The minimum transport velocities of a material can be estimated by testing the solids in horizontal and vertical glass tubes by determining the minimum air velocity to convey the solids in a horizontal tube and the minimum air velocity to just suspend the solids in a vertical tube. The minimum transport velocity of the solids may be several times the free fall velocity.

The minimum velocity v_m to prevent the settling of some particles of diameter d_p (in inches) and specific gravity s can be estimated from the following correlation:

(16.24)

For horizontal pipes

$$k = 100$$

and

$$b = 0.40.$$

For vertical pipes

$$k = 205$$

and

$$b = 0.60.$$

See Chapter 22 for additional details.

16.6.3 Pressure Drop

The total pressure drop in the system can be considered to consist of the sum of the following pressure drops:

1. to accelerate the air to the carrying velocity
2. to overcome the friction of the air on the pipe walls
3. to supply the loss of momentum of the air in:
   a. accelerating the solids
   b. keeping the solids in suspension
4. to support the air (vertical pipes)
5. to support the solids (vertical pipes)

The total pressure drop for horizontal pipes, ΔP_{TH}, is given by

$$\Delta P_{\mathrm{TH}} = \Delta P_{\mathrm{AG}} + \Delta P_{\mathrm{AS}} + \Delta P_{\mathrm{F}} \tag{16.25}$$

where ΔP_{AG} = pressure drop to accelerate the air
ΔP_{AS} = pressure drop to accelerate the solids
ΔP_{F} = pressure drop due to the friction of moving air

For vertical pipes, the pressure drop ΔP_{TV} is

$$\begin{aligned} \Delta P_{\mathrm{TV}} &= \Delta P_{\mathrm{AG}} + \Delta P_{\mathrm{AS}} + \Delta P_{\mathrm{F}} + \Delta P_{\mathrm{V}} \\ &= \Delta P_{\mathrm{TH}} + \Delta P_{\mathrm{V}} \end{aligned} \tag{16.26}$$

where ΔP_{V} = pressure drop to support the air and solid. Details are available in the literature.[9]

16.6.4 Design Procedure

In an actual design, the quantity of material to be conveyed and the distance are generally known. One can then assume a loading and conveying velocity, and the diameter of the pipe can be computed. Finally, the pressure drop through the system is computed. If the pressure drop is excessive, a smaller loading can be taken and the above procedure is repeated until a reasonable pressure drop is obtained. There is no reliable method to accurately calculate the conveying velocity; however, a conveying velocity of 70 ft/s can be assumed in lieu of any information.

Some order of magnitude values of loading, conveying velocities, and pressure drops for various systems are outlined below[10]:

1. Fan system
   pressure drop = 10 to 30 in H_2O (50 in H_2O is about the maximum)
   loading = 0.1 to 2.0 (possibly 5.0) lbsolids/lbair

conveying velocity = 30 to 100 ft/s; usually 50 to 70 ft/s
The fan system is generally used for distances less than 200 ft.

2. Vacuum system
   pressure drop = 5 to 10 in Hg
   loading = 5 to 20 lbsolids/lbair
   conveying velocity = (same as above)
3. Pressure system
   pressure drop = 10 to 50 psia (possibly as high as 100 psia)
   loading = 5.0 to 40 lbsolids/lbair
   conveying velocity = (same as above)

Large radius bends are recommended as the pressure drop will be less than with tight bends and it will also be less likely for the solids to collect and choke the bend.

16.6.5 Pressure Drop Reduction in Gas Flow

Scattered statements in the literature seem to suggest that the pressure to convey a gas can be reduced by the addition of fine particles to the moving stream. This is an area that requires more research since the pressure drop reduction effect is a function of both the particle size (and/or particle size distribution) and concentration.

Illustrative Example 16.5 Illustrative Examples 16.2, 16.3, and 16.4 were solved using the Y_G and Y_L equations presented in Equations (16.16)–(16.17), Equations (16.19)–(16.20), and Equations (16.22)–(16.23), respectively. Comment on the similarity of the equations.

Solution A quick check of the Y_G values generated from the three equations shows little variation. The same applies to the three Y_L values. Because of the token variation of the values for each of the three equations, one might be justified to combine the three equations into one. This suggestion is addressed in one of the problems for this chapter.

Illustrative Example 16.6 A mixture of air (a) and kerosene (k) are flowing in a horizontal 2.3-inch ID pipe. Data for each component is provided below

$$\rho_a = 0.075\,\text{lb/ft}^3 \qquad \rho_k = 52.1\,\text{lb/ft}^3$$

$$\mu_a = 1.24 \times 10^{-5}\,\text{lb/ft}\cdot\text{s} \qquad \mu_k = 0.00168\,\text{lb/ft}\cdot\text{s}$$

$$q_a = 5.3125\,\text{ft}^3/\text{s} \qquad q_k = 1.790\,\text{ft}^3/\text{s}$$

Calculate the flow regime for both phases employing superficial velocities.

Solution Calculate the cross-sectional area of the pipe.

$$S = (\pi/4)(2.3/12)^2$$
$$= 0.0288\ \text{ft}^2$$

The superficial velocity of each phase can be obtained by applying either Equation (16.7) or (16.8).

$$v_a = 5.3125/(0.0288)(60)$$
$$= 3.07\ \text{ft/s}$$
$$v_k = 1.79/(0.0288)(60)$$
$$= 1.036\ \text{ft/s}$$

The Reynolds number can now be calculated by employing Equation (16.9) or the equivalent.

$$\text{Re}_a = (2.3/12)(3.07)(0.075)/(1.24 \times 10^{-5})$$
$$= 3570$$
$$\text{Re}_k = (2.3/12)(1.036)(52.1)/(0.00168)$$
$$= 6158$$

Turbulent flow exists for both phases based on the superficial velocities.

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