UNITED ARAB EMIRATES
MINISTRY OF EDUCATION

YEAR OF
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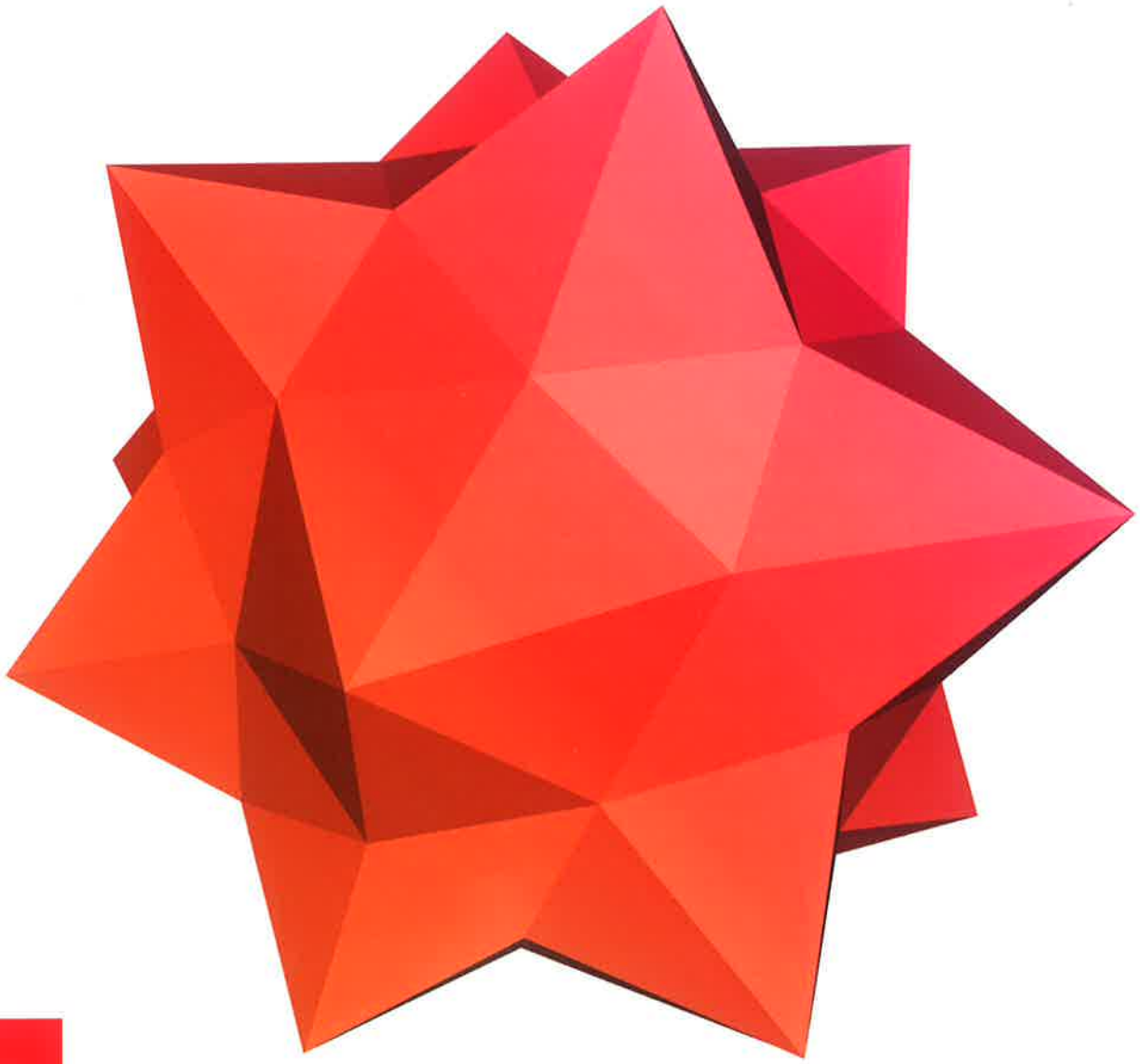
MATH

8

McGraw-Hill Education

Integrated Math

United Arab Emirates Edition



Mc Graw Hill Education

United Arab Emirates
Ministry of Education

رؤيـــة
VISION
2021

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Integrated Math

United Arab Emirates Edition

GRADE 8 • VOLUME 3

Mc Graw Hill Education

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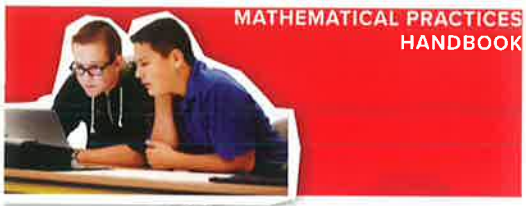
"Extensive knowledge and modern science must be acquired. The educational process we see today is in an ongoing and escalating challenge which requires hard work.
We succeeded in entering the third millennium, while we are more confident in ourselves."

H.H. Sheikh Khalifa Bin Zayed Al Nahyan
President of the United Arab Emirates

CONTENTS IN BRIEF

Units organized by domain

This book is organized into units based on groups called domains. The Standards for MP Mathematical Practices are embedded throughout the course.



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Our lead authors ensure that the McGraw-Hill mathematics programs are truly vertically aligned by beginning with the end in mind - success in Algebra 1 and beyond. By "backmapping" the content from the high school programs, all of our mathematics programs are well articulated in their scope and sequence.

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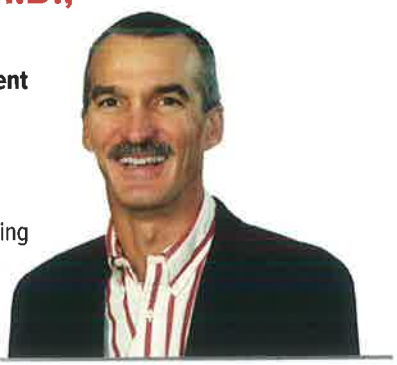
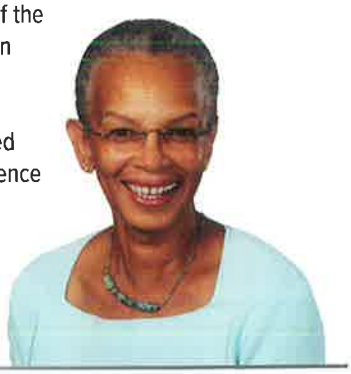


In Memoriam
Carol Malloy, Ph.D.

Dr. Carol Malloy was a fervent supporter of mathematics education. She was a Professor at the University of North Carolina, Chapel Hill, NCTM Board of Directors member, President of the Benjamin Banneker Association (BBA), and 2013 BBA Lifetime Achievement Award for Mathematics winner. She joined McGraw-Hill in 1996. Her influence significantly improved our programs' focus on real-world problem solving and equity. We will miss her inspiration and passion for education.



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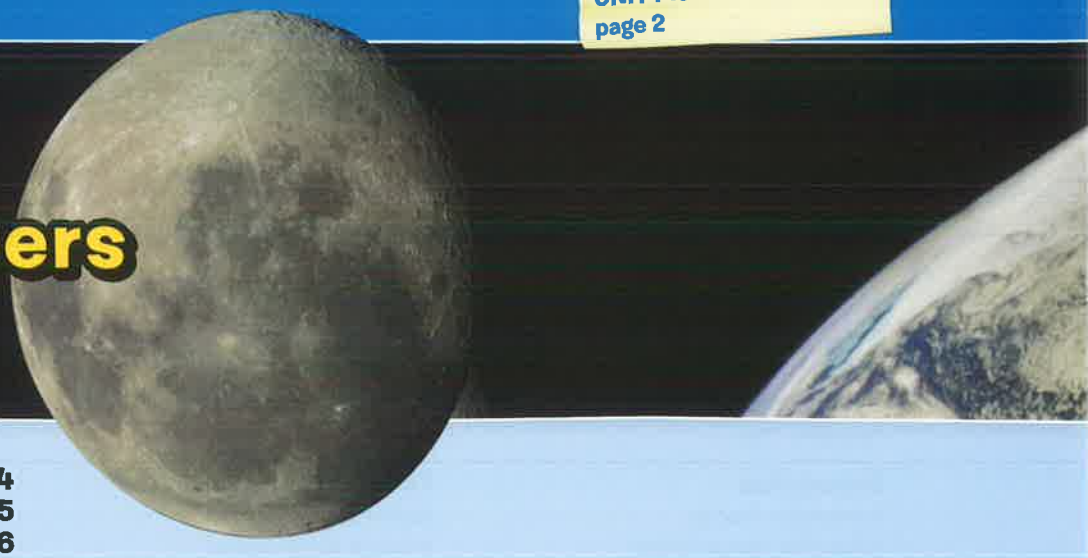
Chapter 1
Real Numbers



Essential Question

WHY is it helpful to write numbers in different ways?

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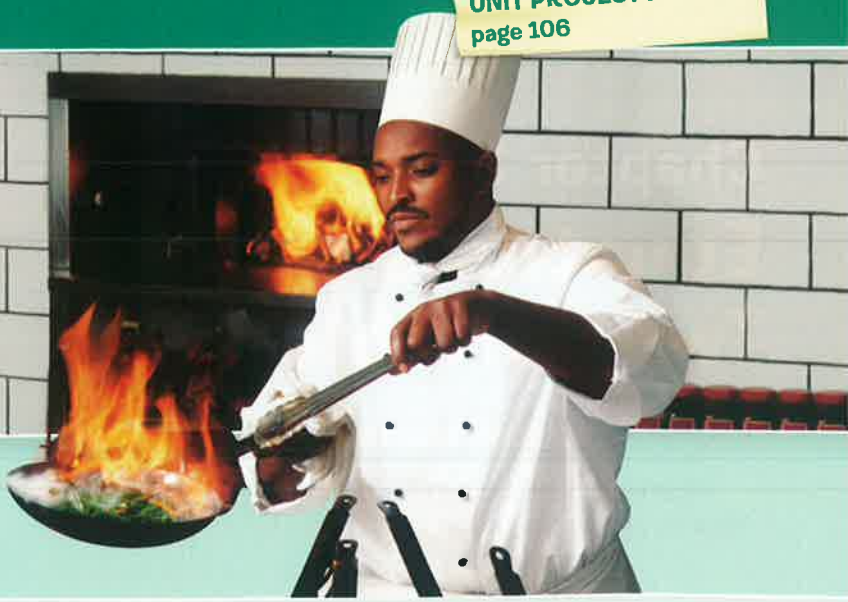
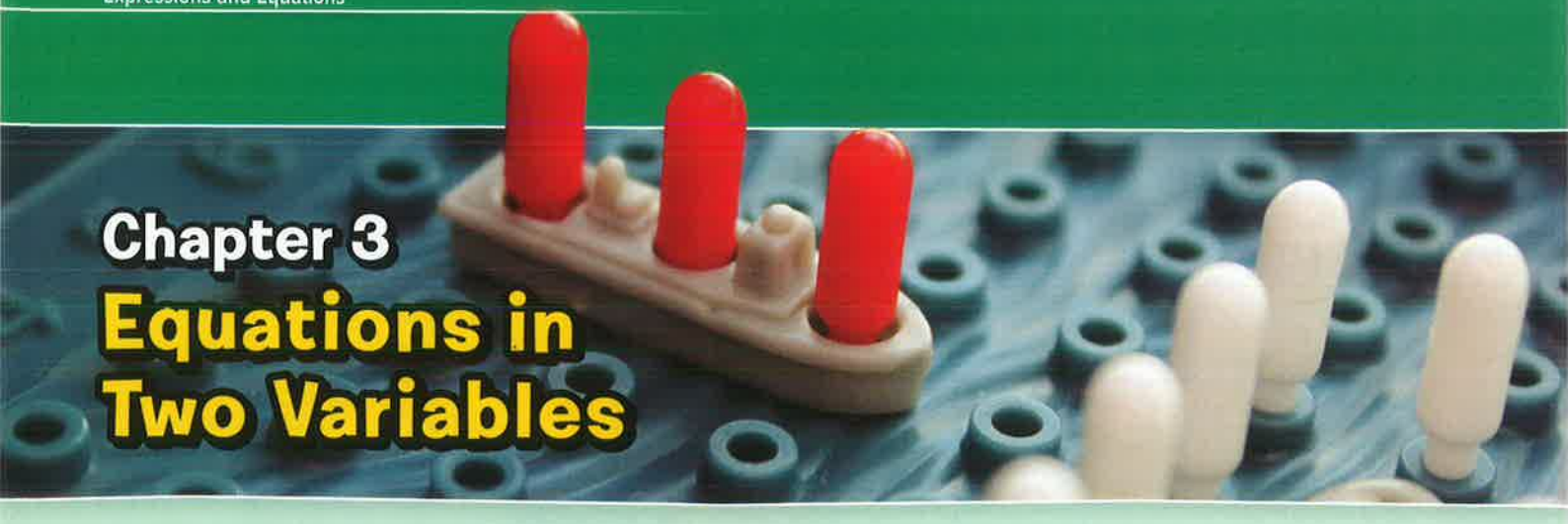
Chapter 2 Equations in One Variable



Essential Question

WHAT is equivalence?



Chapter 3 Equations in Two Variables

Essential Question

WHY are graphs helpful?

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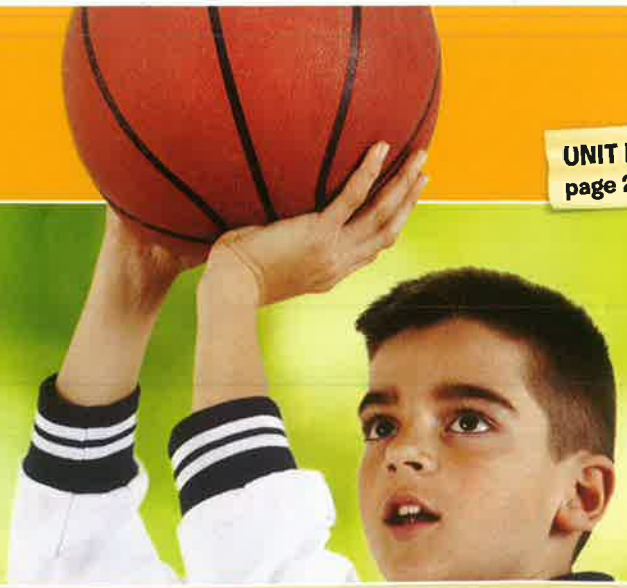
Chapter 4 Functions



Essential Question

HOW can we model relationships between quantities?

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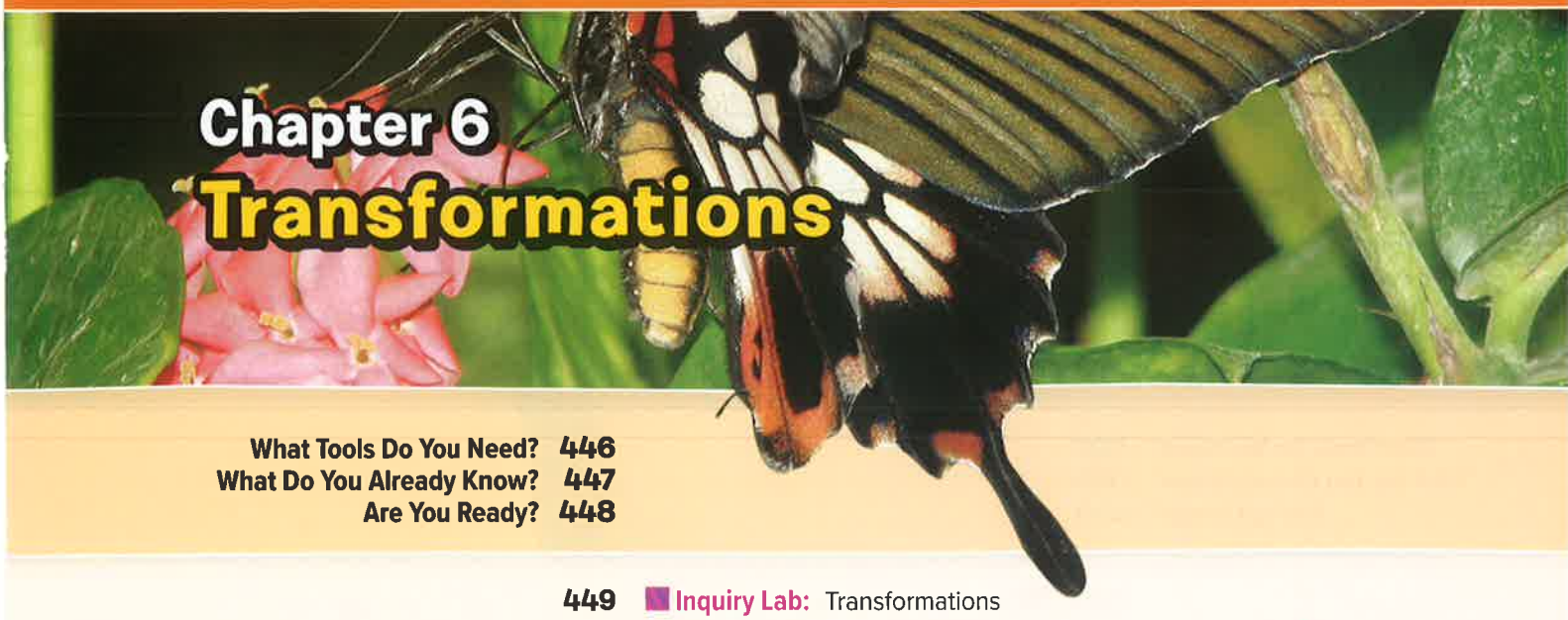
Chapter 5
Triangles and the Pythagorean Theorem

Essential Question

HOW can algebraic concepts be applied to geometry?



Chapter 6 Transformations

Essential Question

HOW can we best show or describe the change in position of a figure?

Chapter 7 Congruence and Similarity

Essential Question

HOW can you determine congruence and similarity?

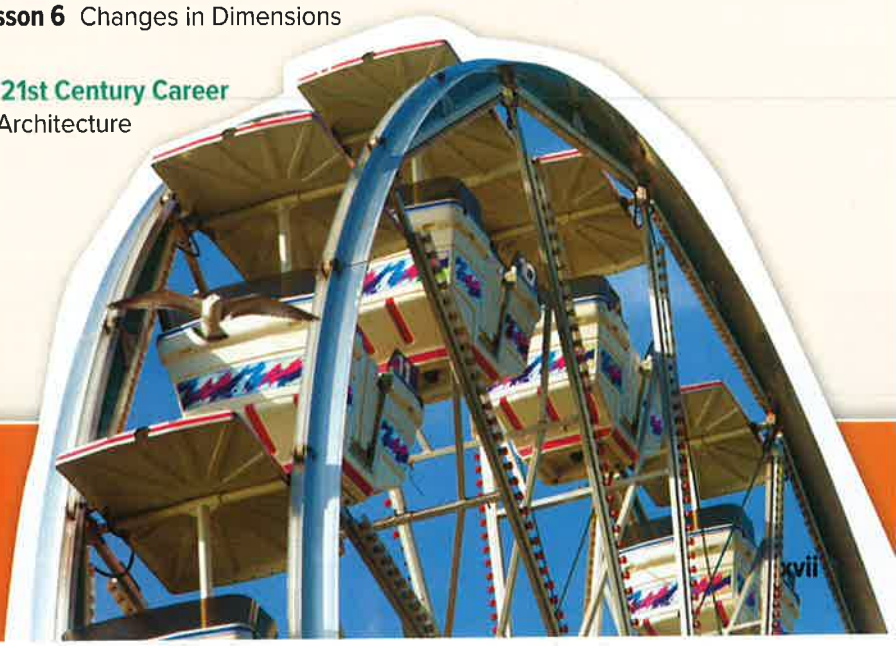
Chapter 8 Volume and Surface Area

Essential Question

WHY are formulas important in math and science?

UNIT PROJECT

Design That Ride



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Chapter 9 Scatter Plots and Data Analysis

Essential Question

HOW are patterns used when comparing two quantities?

UNIT PROJECT

Olympic Games

This book focuses on three critical areas: (1) applying equations in one and two variables; (2) understanding the concept of a function and using functions to describe quantitative relationships; (3) applying the Pythagorean Theorem and the concepts of similarity and congruence.

Content

The Number System

- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Geometry

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability

- Investigate patterns of association in bivariate data.

MP Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices Handbook

Essential Question

WHAT practices help me develop and demonstrate mathematical understanding?

Mathematical Practices

The standards for mathematical practice will help you become a successful problem solver and to use math effectively in your daily life.

What You'll Learn

MP Throughout this handbook, you will learn about each of these mathematical practices and how they are integrated in the chapters and lessons of this book.

① **Focus on Mathematical Practice**
Persevere with Problems

② **Focus on Mathematical Practice**
Reason Abstractly and Quantitatively

③ **Focus on Mathematical Practice**
Construct an Argument

④ **Focus on Mathematical Practice**
Model with Mathematics

⑤ **Focus on Mathematical Practice**
Use Math Tools

⑥ **Focus on Mathematical Practice**
Attend to Precision

⑦ **Focus on Mathematical Practice**
Make Use of Structure

⑧ **Focus on Mathematical Practice**
Use Repeated Reasoning

Place a checkmark below the face that expresses how much you know about each Mathematical Practice. Then explain in your own words what it means to you.

I have no clue. I've heard of it. I know it!

Mathematical Practices				
Mathematical Practice				What it Means to Me
①				
②				
③				
④				
⑤				
⑥				
⑦				
⑧				

Persevere with Problems

How do I make sense of a problem?

Making and using a step-by-step plan to solve a problem is like using directions to build a piece of furniture. If you follow the directions correctly, there is a good chance you will end up with a solid piece of furniture. Once you understand the meaning of the problem, you can decide what strategy will work best to solve it. You might try several strategies and then ask yourself, "Does this make sense?"

You have already used the four-step problem-solving plan in previous courses. Complete the graphic organizer that shows the four steps to solve the given problem.

MP Mathematical Practice 1

Make sense of problems and persevere in solving them.

Of the 480 students at Lincoln Middle School, one third have traveled overseas. Of these, 15% have been to Australia. How many students have not been to Australia?

Step 1. Understand

What are the facts?

Step 2. Plan

What strategy will you use to sol the problem above?

Step 3. Solve

Solve the problem. Show your steps below.

Step 4. Check

How do you know your answer i reasonable?

It's Your Turn!

Solve each problem by using the four-step problem-solving model.

1. About fifty percent of the population of Alaska lives within a 50-mile radius of Anchorage. If the total area of Alaska is 586,412 square miles, about what percent of the total land area is within 50 miles of Anchorage?

Understand What are you asked to find? Is there any information you will not use?

Plan How will you solve this problem?

Solve Solve the problem. Show your steps below. What is the solution?

Check Does your answer make sense?

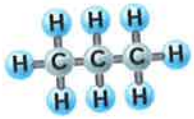
Check

Solve the problem using a different strategy to check your work.

2. The first three molecules for a certain family of hydrocarbons are shown. How many hydrogen atoms (H) are in a molecule containing 6 carbon atoms (C)?

CH_4—الميثان

C_2H_6—إيثان



C_3H_8—البروبان

Find it in Your Book!

MP Persevere with Problems

Look at Chapter 1. Give an example of where Mathematical Practice 1 is used. Then explain why your example represents this practice.

Reason Abstractly and Quantitatively

What does it mean to reason abstractly and quantitatively?

Mathematical Practice 2

Reason abstractly and quantitatively.

In math, we solve real-world problems where numbers and variables in an equation represent concrete objects. This involves thinking quantitatively.

Suppose you are given a AED25 gift card to an online music store. Each song costs AED1.95 to purchase and download. How many songs can you buy?

1. What values in the problem do we already know?

2. What are we trying to find?

3. What symbol can we use to represent the unknown value?

Now that the problem is broken down into known and unknown values, we can manipulate the symbols in order to solve the problem. This is thinking abstractly.

4. Write an equation to solve the problem. Explain what each quantity or symbol represents.

5. Use your equation to solve the problem and label your solution. Explain the meaning of the solution.

It's Your Turn!

Write and solve an equation for each of the following.

6. You are in the pit crew for a driver at a Nascar race. The gas weighs 5.92 pounds per gallon. Your driver uses 0.25 gallon per lap. With 42 laps to go, you put 60 pounds of fuel in the tank of the car. Will your driver finish the race at the same rate without more gas?

a. What values do we already know? What are we trying to find?

b. Write an equation to find the number of gallons in 60 pounds of fuel.

c. Use the equation to solve the problem and explain the meaning of the solution.

7. A class trip is scheduled for an amusement park. Group admission prices are AED31 per student. Parking is AED18 per bus.

Number of Students, s	Total Cost, c (AED)

a. Complete the table to show the total cost of 10, 20, 30, and 40 students and two buses.

b. Write an equation to show the total cost c if two buses transport s students to the park.

c. There are a total of 78 students attending on two buses. What is the total cost? Label your solution and explain its meaning.

Find it in Your Book!

MP Reason Abstractly

Look at Chapter 2. Give an example of where Mathematical Practice 2 is used. Then explain why your example represents this practice.

Construct an Argument

How do I construct a viable argument in math class?

Mathematical Practice 3

Construct viable arguments and critique the reasoning of others.

Suppose your friend told you that his rectangular flatscreen T.V. has congruent diagonals, simply because it was rectangular. How could you ask your friend to justify his argument? You could use inductive reasoning or deductive reasoning. *Inductive reasoning* uses examples to draw conclusions, while *deductive reasoning* uses definitions, rules, or facts.

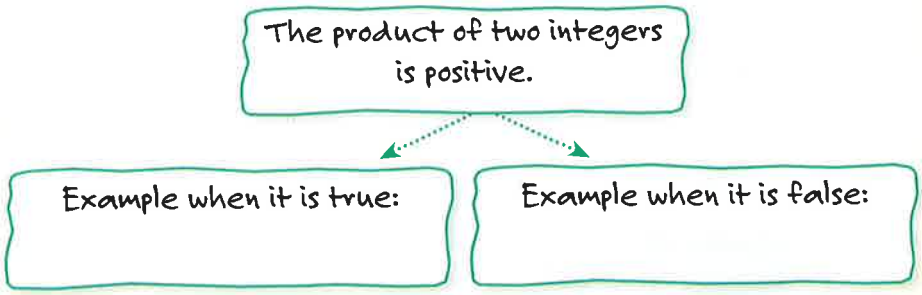
1. How could you use *inductive reasoning* to justify why the following statement is true?

   All rectangles have diagonals that are congruent.

2. How could you use *deductive reasoning* to justify why the following statement is false?

   Each angle of an equilateral triangle measures 90°.

3. Complete the graphic organizer to show that the statement below is *sometimes* true.



It's Your Turn!

For each of the following statements, determine if the statement is *always*, *sometimes*, or *never* true. Justify your response using examples or counterexamples.

4. The sum of two rational numbers is a rational number.

5. The sum of two odd numbers is an odd number.

6. The volume of a pyramid is less than the volume of a prism with the same size base.

Find it in Your Book!

MP **Construct an Argument**

Look at Chapter 1. Give an example of where Mathematical Practice 3 is used. Then explain why your example represents this practice.

Model with Mathematics

How does math fit into your future?

Mathematical Practice 4

Model with mathematics.

No matter what career path you choose, you are sure to use math in your job or career. Graphic organizers arrange ideas so that you can make informed decisions. Using and understanding models such as graphs, tables, and diagrams helps you to simplify a complicated situation and to identify important quantities in a real-life situation.

Suppose you are a doctor or a nurse. A prescription directs a patient to take 2.5 cc (cubic centimeters) of a medicine per 50 pounds of body weight.

1. What skill(s) would you use to see how much medicine you should give to a 125 pound person?

2. How much medicine would the 125 pound patient need?

3. What career path interests you? Research that career and complete the graphic organizer below.

Education Required

Career: _____

How is math used in this career?

It's Your Turn!

Use the given tools to solve each problem.

4. You are saving money to buy a new game system. You received AED 50 as a graduation gift from your grandparents. You want to save AED 25 a week from mowing lawns.

Week, w	Total Saved, s ($)

a. Tables Complete the table to show the total amount saved after 1, 2, 3, 4, and 5 weeks.

b. Symbols Write an equation to show the total amount saved s after w weeks. _____

c. Algebra Use the equation to determine the total amount saved after 17 weeks. _____

Use the table for Exercises 5 and 6.

5. Mrs. Fatima hired a party planner to plan Nouha's dinner party. There will be 125 guests and she wants to offer appetizers and a buffet dinner. What is the cost, before tax, for the party?

_____

Polly's Perfect Parties			
Cost of Food (per person)		**Cost of Extras**	
Appetizers	AED 9.20	Hall	AED 250
Buffet	AED 18.30	Linens	AED 15 per table
Sit-down Dinner	AED 25.75	Table and Chair Rental (seats 8)	AED 60 per table

6. There is a $7\frac{1}{2}\%$ sales tax added to the party bill. Mrs. Fatima also wants to add an 18% tip for the servers. This will be figured before tax is added. What will be the total cost of the party?

_____

Find it in Your Book!

MP Model with Mathematics

Look at Chapter 1. Give an example of where Mathematical Practice 4 is used. Then explain why your example represents this practice.

_____

_____

_____

_____

Use Math Tools

How do I use tools and strategies in math class?

Mathematical Practice 5

Use appropriate tools strategically.

Sometimes using math tools and strategies helps make solving problems easier if you know which tool to use in a given situation. Math tools are physical objects you use when solving problems. Paper and pencil, technology, or calculators are examples of tools.

1. List three other tools you can use to solve math problems.

Math strategies are more like skills or the ability to apply your math knowledge. Some math strategies are mental math, number sense, estimation, drawing a diagram, or solving a simpler problem.

2. List three other strategies you can use to solve math problems.

3. Complete the graphic organizer.

Problem	Tool	Strategy
You want to leave a 20% tip for your server.		
You want to determine how long it will take to drive from Austin to Dallas.		
You are stuck while in the middle of solving an equation.		

It's Your Turn!

List the tools or strategies you would use to solve each problem. Then solve the problem.

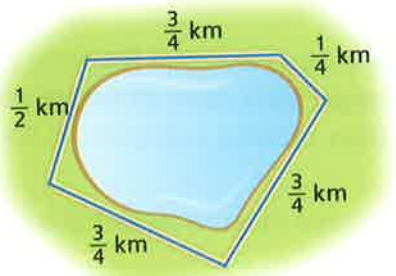
4. A pre-election survey was taken in Ms. Nouha's homeroom. The results for class president are shown in the table.

Class President	
Marwa	10
Karam	8
Asma	20
Sara	12

a. Based on the survey, if there are 850 students in the 8th grade, how many votes will Asma get?

b. A candidate needs to receive at least 51% of the votes to win the election. If every student votes, how many more votes would Asma need to win?

5. A walking path around a lake is in the shape of a pentagon like the one shown. If Shaima wants to walk $4\frac{1}{2}$ kilometers, how many times does she need to walk around the lake?



6. Write a word problem that requires the use of a protractor, a calculator, and one strategy, like mental math or estimation. Find the solution to your problem and explain how you used the tools to solve it.

Find it in Your Book!

MP Use Math Tools

Look at Chapter 1. Give an example of where Mathematical Practice 5 is used. Then explain why your example represents this practice.

Attend to Precision

What does it mean to be precise?

Mathematical Practice 6

Attend to precision.

Communication is important to our daily life, whether it's in school, sports, at home, or hanging out with friends. If you can't clearly express your thoughts, no one will understand what you mean! Math also requires clear and precise communication by using labels, appropriate symbols, and clear definitions.

Suppose you and your brother want to paint two walls in your bedroom a new color. Your bedroom is 3.78 meters long, 4.47 meters wide, and has a 2.44 meter ceiling height.

1. What skill(s) would you use to see how much paint you need?

2. What information do you need to know in order to make your calculations?

You are painting two walls that are perpendicular to each other. They do not have doors or windows on them. A can of paint covers about 32.52 square meters.

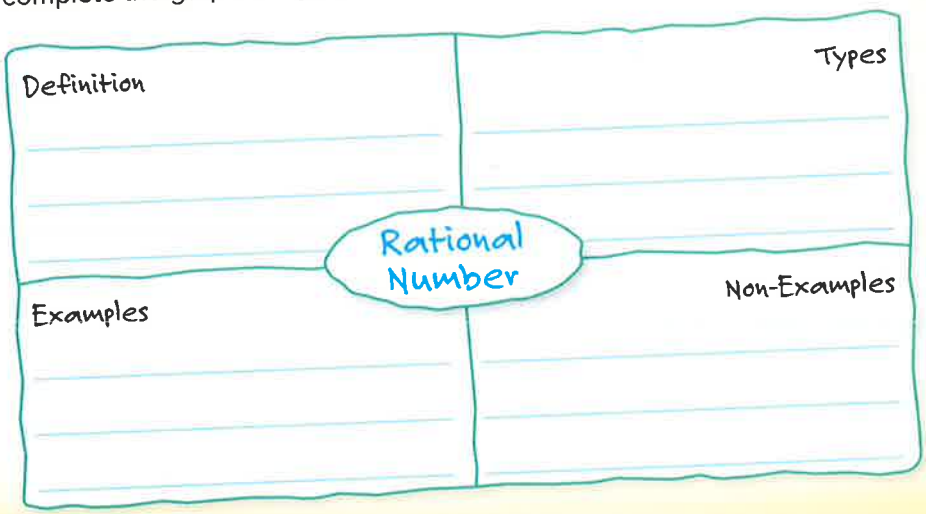
3. What is the area of wall space you will be painting? Label your answer.

4. How precise does the area need to be to determine how much paint you will need? Round the area and explain why you rounded to the place value you chose.

5. How many cans of paint do you need? Round to an appropriate place value and label your answer. Explain your rounding.

It's Your Turn!

6. Turn to page 7 in your text. Find the vocabulary term *rational number* and complete the graphic organizer for that term.



7. Model trains come in different scales. The ratio for an HO scale train is 1:87, while the ratio for a Z scale train is 1:220. Suppose a Z scale model of a steam engine is 62 millimeters long. What is the length of the HO scale model of the same engine? To what place value should you round? Explain your reasoning.

Find it in Your Book!

MP Attend to Precision

Look at Chapter 1. Give an example of where Mathematical Practice 6 is used. Then explain why your example represents this practice.

Make Use of Structure

What does it mean to use structure in math?

MP Mathematical Practice 7

Look for and make use of structure.

When you use structure in math, you might apply properties to solve equations or you might examine patterns in tables and graphs to describe relationships.

1. The table shows the diameters of several flying discs. Use the relationship between the radius and diameter of a circle to complete the table. Round to the nearest tenth.

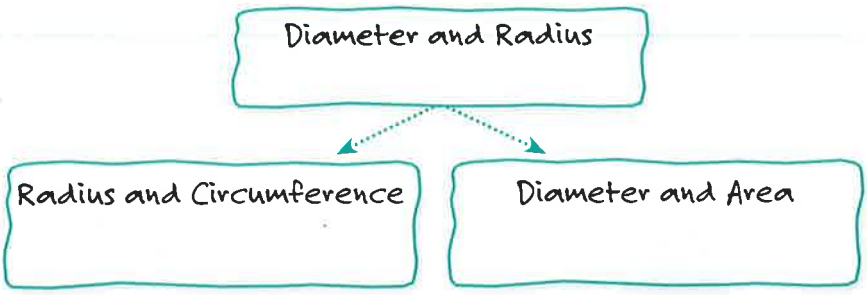
Diameter (cm)	Radius (cm)	Circumference (cm)	Area (cm^2)
20			
22			
25			

2. Describe the relationship between the diameter and radius of a circle. _____

3. Describe the relationship between the circumference and diameter of a circle. _____

_____

4. Complete the graphic organizer by writing a formula in each box that shows the relationship between each term.



It's Your Turn!

Suppose you are training for a marathon. A marathon is 26.2 miles long. You can run 3 miles in 16 minutes.

5. At this rate, how many miles can you run in one hour?

6. Complete the table and plot the points to make a line graph.

Time (h)	Distance (mi)
1	
2	
3	

7. Write an equation that shows the relationship between distance and time.

8. Estimate how long it will take to complete the marathon.

Find it in Your Book!

MP Make Use of Structure

Look at Chapter 1. Give an example of where Mathematical Practice 7 is used. Then explain why your example represents this practice.

Use Repeated Reasoning

What does it mean to look for repeated reasoning?

Mathematical Practice 8

Look for and express regularity in repeated reasoning.

Problems can often be solved by finding patterns or repeated processes. Sometimes you can even create shortcuts to solve a problem once you understand the pattern. For example, multiplication is a shortcut for repeating the same addition over and over.

Suppose you have a garden with a length of 1.83 meters and a width of 1.22 meters and you want to increase its size. Before making any changes, do some math!

1. What is the perimeter of the garden? _____ the area? _____
2. If you double the dimensions of the garden, what is the new perimeter? _____ new area? _____
3. What number can you multiply the original perimeter by to find the new perimeter? _____ What number can you multiply the original area by to find the new area? _____

Oh no, the increased size of the garden is too big! Using the original dimensions of the garden, you increase the length to 2.74 meters and the width to 1.83 meters

4. What is the new perimeter? _____ new area? _____
5. What number can you multiply the original perimeter by to find the new perimeter? _____ What number can you multiply the original area by to find the new area? _____
6. Try other changes in the dimensions of the garden to find the new perimeter and area of the garden.

_____

_____

It's Your Turn!

7. Ahmed is mixing orange juice and apple juice in a ratio of 3 to 4 to make a fruit punch. He wants to make 35 cups of the punch. To determine how many cups of each juice he needs, he started making a table. Complete the table to find how many cups of each juice he will need. Then explain a shortcut you could use to solve the problem.

Orange Juice	Apple Juice	Total Cups
3	4	7
6	8	14
9	12	21

8. Ameenah's parents are going to pay her for doing chores 6 days a week and they offer her two payment plans.

Option A						
Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Total
AED3	AED6	AED9				

Option B						
Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Total
AED 0.75	AED1.50	AED3.00				

Complete the table to determine which is the better option for Savannah to choose. Explain the pattern for each option.

Find it in Your Book!

MP Use Repeated Reasoning

Look at Chapter 1. Give an example of where Mathematical Practice 8 is used. Then explain why your example represents this practice.

Use the Mathematical Practices

Solve.

You are boxing and wrapping gifts for a club fundraiser. The charge to wrap a gift in the shape of a rectangular prism is shown in the table.

Total Surface Area	Cost
up to 35 in^2	AED5
36–54 in^2	AED8
over 55 in^2	AED12

a. Marina wrapped three different boxes with measurements shown in the table. Complete the table with the cost per box and the cost per square inch. Which box has the least cost per square inch? _____

Box	height in.	width in.	length in.	Cost to Wrap	Cost per Square Inch
A	2	4	3		
B	2	5	6		
C	2	3	2		

b. Which of those boxes has the least cost per cubic inch? Explain.

_____

_____

_____

_____

Determine which mathematical practices you used to determine the solution. Shade the circles that apply.

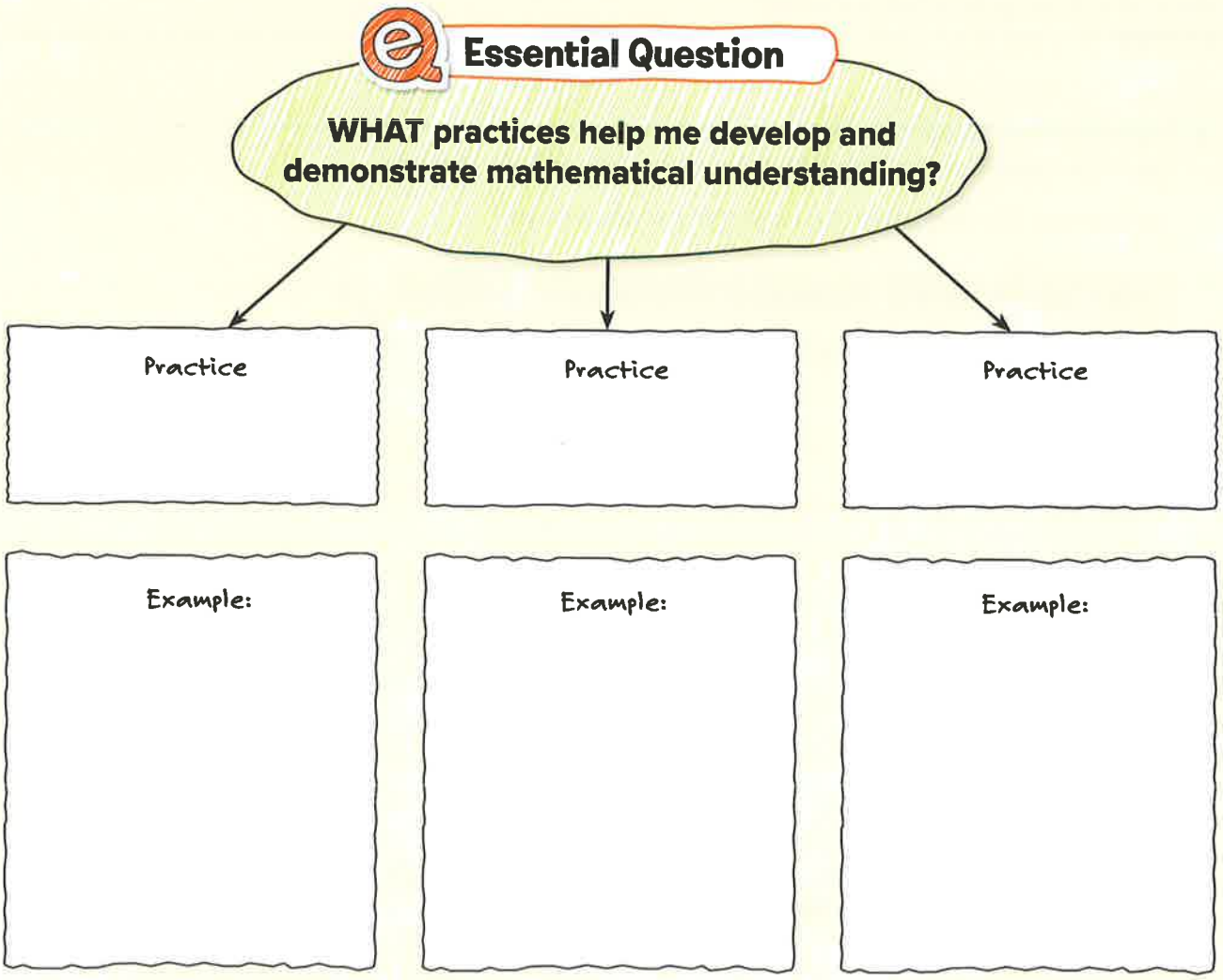
Which MP **Mathematical Practices** did you use?
Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Reflect

Answering the Essential Question

Use what you learned about the mathematical practices to complete the graphic organizer. List three practices that help you best demonstrate mathematical understanding. Then give an example for each practice.



Answer the Essential Question. WHAT practices help me develop and demonstrate mathematical understanding?

Chapter 7
Congruence and Similarity

Essential Question

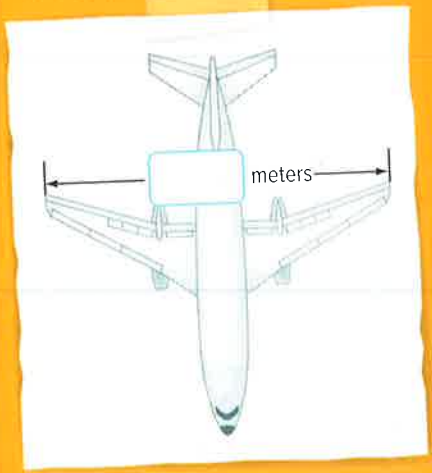
HOW can you determine congruence and similarity?

MP Mathematical Practices
1, 2, 3, 4, 5, 7

Math in the Real World

Models The wingspan of a model of a 737 commercial aircraft is 17 centimeters. The scale for the model is 1 cm = 200 cm.

Use the scale to find the wingspan in centimeters of the actual 737 aircraft. Then convert the centimeters to meters.



FOLDABLES® Study Organizer

1. Cut out the Foldable on page FL7 of this book.
2. Place your Foldable on page 580.
3. Use the Foldable throughout this chapter to help you learn about congruence and similarity.

What Tools Do You Need?

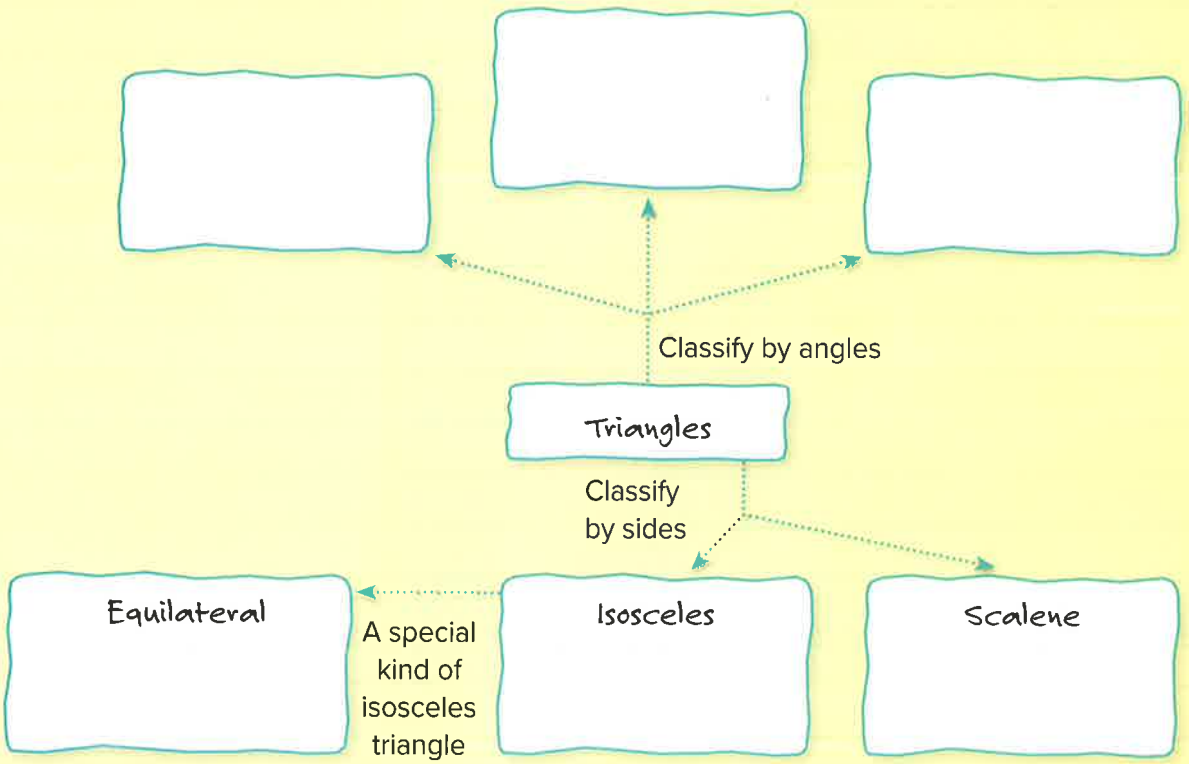
Vocabulary

composition of transformations	scale factor
corresponding parts	similar
indirect measurement	similar polygons

Study Skill: Use a Web

Use a Web A *web* can help you understand how math concepts are related to each other. To make a web, write the major topic in the center of a piece of paper. Then, draw "arms" from the center for as many categories as you need.

Here is a partial web for the major topic of triangles. Complete the web by adding descriptions for the classifications by sides. Then add the classifications by angles.



What Do You Already Know?

List three things you already know about congruence and similarity in the first section. Then list three things you would like to learn about congruence and similarity in the second section.

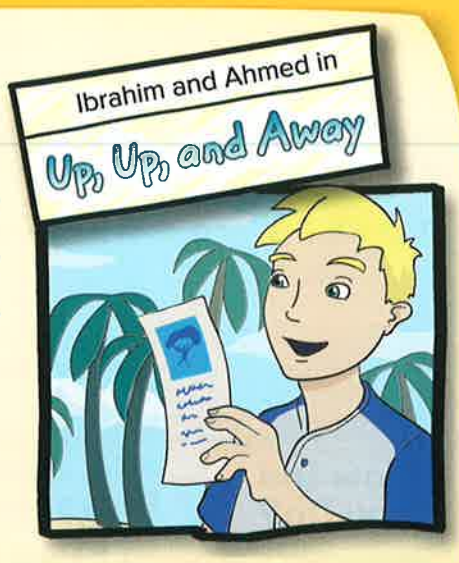
Congruence and Similarity

What I know	What I want to find out

When Will You Use This?

Here is an example of how triangles are used in the real world.

Activity Have you ever flown a kite? How much string did you have on the spool?



Are You Ready?

Try the Quick Check below.

Check

Quick Review

Example 1

Solve $\frac{w}{12} = \frac{5}{6}$.

$\frac{w}{12} = \frac{5}{6}$ Write the proportion.

$6 \times w = 12 \times 5$ Find cross products.

$6w = 60$ Simplify.

$w = 10$ Division Property of Equality

Example 2

Find the slope of the line that passes through (3, 8) and (−1, 0).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \frac{0 - 8}{-1 - 3}$ $(x_1, y_1) = (3, 8); (x_2, y_2) = (-1, 0)$

$m = \frac{-8}{-4}$ or 2 Simplify.

Quick Check

Proportions **Solve each proportion.**

1. $\frac{x}{15} = \frac{7}{30}$ _____

2. $\frac{4}{9} = \frac{14}{y}$ _____

3. $\frac{12}{z} = \frac{30}{37}$ _____

Show your work.

4. $\frac{8}{15} = \frac{m}{21}$ _____

5. $\frac{n}{5} = \frac{18}{45}$ _____

6. $\frac{3}{7} = \frac{21}{p}$ _____

Find Slope **Find the slope of the line that passes through each pair of points.**

7. (−1, 1), (−3, 7) _____

8. (2, 0), (0, 2) _____

9. (−6, −1), (−3, 4) _____

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4 5 6 7 8 9

Inquiry Lab
Composition of Transformations

Inquiry **HOW does a combination of transformations differ from a single transformation? How are they the same?**

MP **Mathematical Practices** 1, 3

Graphic artists often use several transformations to create designs. When a transformation is applied to a figure and then another transformation is applied to the image, the result is called a **composition of transformations.**

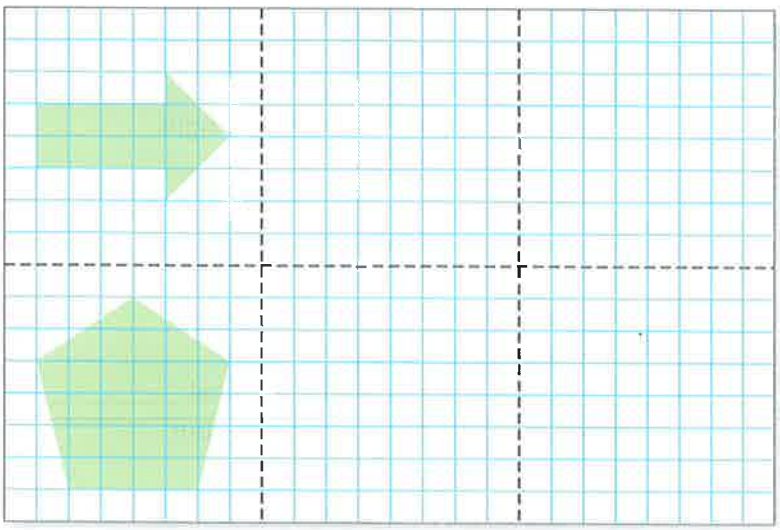
Hands-On Activity 1

Step 1 Fold the page in your book vertically into three sections along the dotted lines.

Step 2 Draw the reflection of the arrow over the fold in the middle section.

Step 3 Draw a reflection of the 2nd arrow over the fold in the right-hand section.

Step 4 Repeat Steps 2 and 3 with the pentagon.



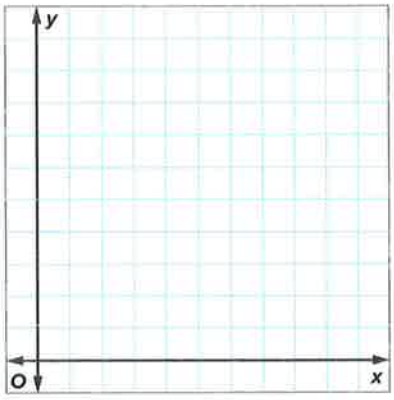
How are the original figures and the final figures related?

Would the final images be the same as the original figure if the second reflection was reflected over the horizontal line? Explain.

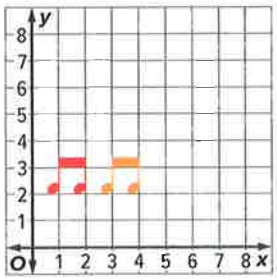
Hands-On Activity 2

In this Activity, you will use a translation and a reflection to create a decorative border.

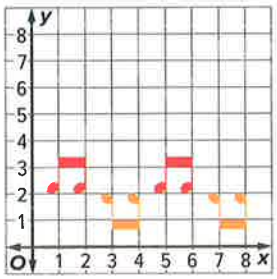
Step 1 Draw a figure on the coordinate plane shown, close to the origin.



Step 2 On the coordinate plane in Step 1, translate your figure. Lightly draw the image since it will not be in its final location. In this example, the red figure is translated 2 units to the right.



Step 3 On the coordinate plane in Step 1, reflect the drawn image across a horizontal line. This will be the final location so you can draw this in your book. In this example, the image is reflected across the line $y = 2$.



Step 4 Repeat the process to create your border.

How are the size and shape of the original figure related to the size and shape of the images?

Suppose you wanted your border to run up the side of the page instead of across the bottom of the page. Describe what transformations you might use to do this.

Investigate

Collaborate

Work with a partner. Describe the transformations combined to create the outlined patterns shown in Exercises 1–4.

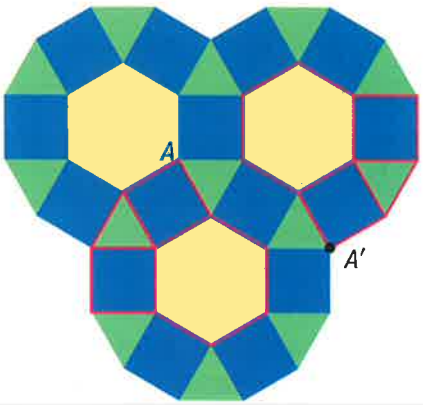
1.

2.

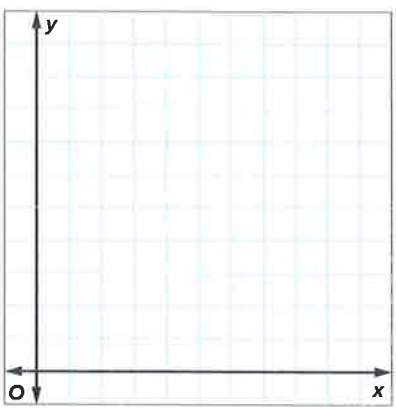
Show your work.

3.

4.



5. Draw a figure on the coordinate plane shown. Use a reflection and a rotation to create a logo for a company.

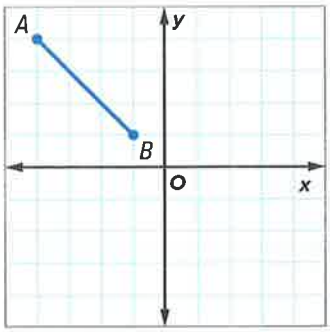


Analyze and Reflect

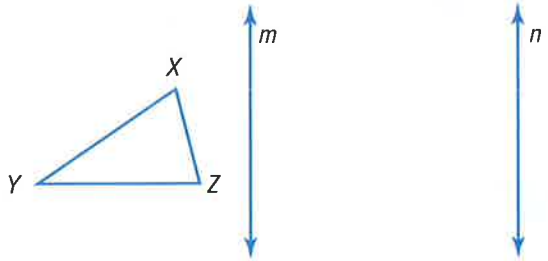
Collaborate

In some cases, a composition of transformations is the same as a single transformation. Draw the composition of transformations described. Then identify the single transformation that would produce the same image as each composition.

6. $\overline{AB}$ is reflected across the *y*-axis, then reflected across the *x*-axis.



7. $\triangle XYZ$ is reflected across line *m* and then reflected across line *n*.



Create

On Your Own

8. MP **Make a Conjecture** The transformations in the Activities and Exercises have been translations, reflections, and rotations which preserve distance. Make a conjecture about the position, size, and shape of a figure if a composition of transformations included a dilation.

9. Inquiry HOW does a combination of transformations differ from a single transformation? How are they the same?

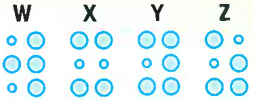
Lesson 1

Congruence and Transformations

Real-World Link

Braille The letter R in the Braille alphabet consists of four large dots and 2 smaller dots in the pattern shown. Circle the letter with the same shape as the letter R.

R:



Essential Question

HOW can you determine congruence and similarity?

MP Mathematical Practices

1, 3, 4

Collaborate

How can you determine whether two figures are the same size and shape?

Step 1 Copy the figure shown on tracing paper two times. Cut out both figures. Label the figures *A* and *B*.

Step 2 Place Figure *B* on top of Figure *A*. Are the side lengths the same? the angle measures?

Are the figures the same size and shape? _____

Step 3 Translate Figure *B* up and over on your desk. How can you move Figure *A* on top of Figure *B* so all sides and angles match?

Step 4 Flip Figure *B* over. How can you move Figure *A* on top of Figure *B* so all sides and angles match?

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

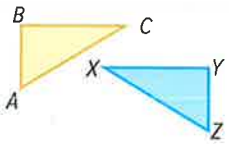
Identify Congruence

On the previous page, you matched Figure *A* to Figure *B* by a translation and a reflection. Two figures are congruent if the second can be obtained from the first by a series of rotations, reflections, and/or translations.

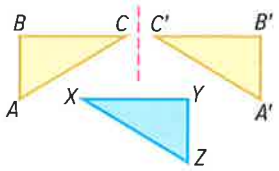
Examples

Determine if the two figures are congruent by using transformations. Explain your reasoning.

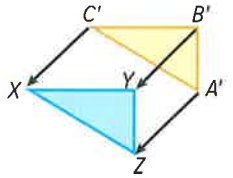
1.



Step 1 Reflect $\triangle ABC$ over a vertical line. Label the vertices of the image A', B', and C'.

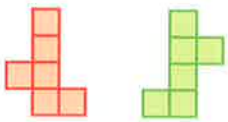


Step 2 Translate $\triangle A'B'C'$ until all sides and angles match $\triangle XYZ$.

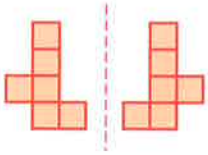


So, the two triangles are congruent because a reflection followed by a translation will map $\triangle ABC$ onto $\triangle ZYX$.

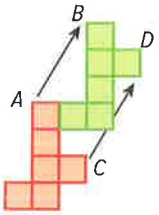
2.



Reflect the red figure over a vertical line.



Even if the reflected figure is translated up and over, it will not match the green figure exactly. The two figures are not congruent.



Transformations

Translations, reflections and rotations are called isometries. In an isometry, the distance between two points in an image is the same as the distance in the preimage.

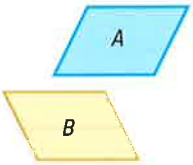
iso / metry

↓ ↓

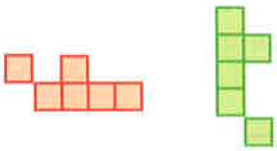
same distance

Got it? Do these problems to find out.

a.



b.



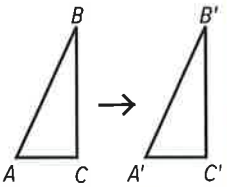
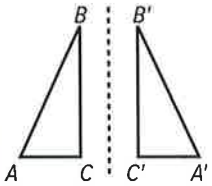
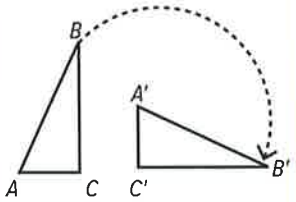
a. _____

b. _____

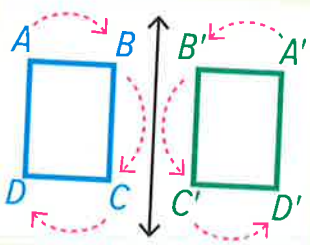
Determine the Transformations

If you have two congruent figures, you can determine the transformation, or series of transformations, that maps one figure onto the other by analyzing the orientation or relative position of the figures.

Show your work.

Translation	Reflection	Rotation
• length is the same • orientation is the same	• length is the same • orientation is reversed	• length is the same • orientation is the same
		

Orientation
The order in which the vertices of a figure are named determines the figure's orientation. In the reflection shown, the vertices of the preimage are named in a clockwise direction, but the vertices of the image are named in a counterclockwise direction. The orientation has been reversed.



Real World Example

3. **Eiman created the logo shown. What transformations did she use if the letter "d" is the preimage and the letter "p" is the image? Are the two figures congruent?**

Step 1 Start with the preimage. Rotate the letter "d" 180° about point *A*.

Step 2 Translate the new image up.

Eiman used a rotation and translation to create the logo. The letters are congruent because images produced by a rotation and translation have the same shape and size.

Show your work.

c. _____

Got it? **Do this problem to find out.**

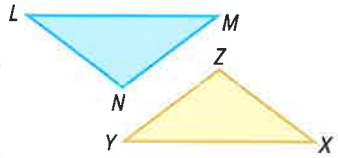
c. What transformations could be used if the letter "W" is the preimage and the letter "M" is the image in the logo shown? Are the two figures congruent? Explain.

Guided Practice

Check

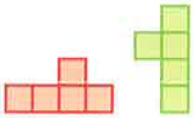
Determine if the two figures are congruent by using transformations. Explain your reasoning. (Examples 1 and 2)

1.

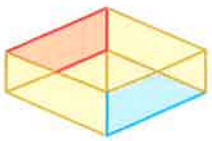


Show your work.

2.



3. The Boyd Box Company uses the logo shown. What transformations could be used if the red trapezoid is the preimage and the blue trapezoid is the image? Are the two figures congruent? Explain. (Example 3)



4. **Building on the Essential Question** Why do translations, reflections, and rotations create congruent images?

Rate Yourself!

How confident are you about the relationship between congruence and transformations? Check the box that applies.

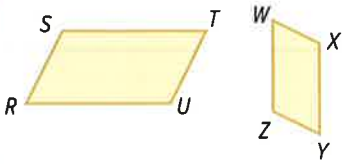
FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

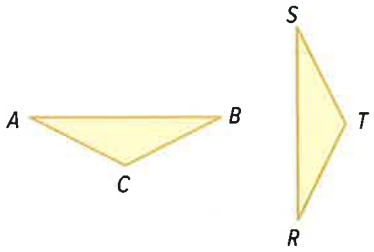
Determine if the two figures are congruent by using transformations. Explain your reasoning. (Examples 1 and 2)

1



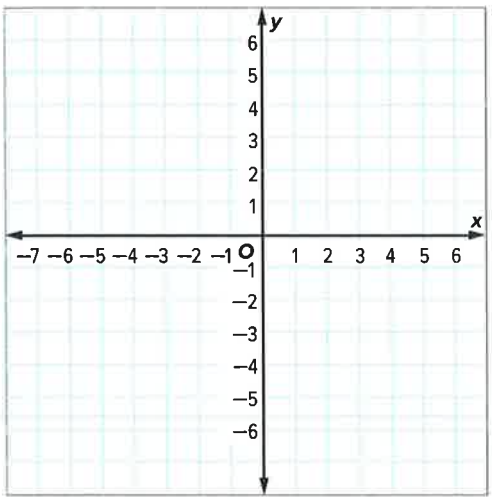
Show your work.

2.



3 Asma purchased some custom printed stationery with her initials. What transformations could be used if the letter "Z" is the preimage and the letter "N" is the image in the design shown? Are the two figures congruent? Explain. (Example 3)

4. MP **Multiple Representations** One way to identify congruent triangles is to prove their matching sides have the same measure. Triangle *CDE* has vertices at (1, 4), (1, 1), and (5, 1).



a. **Graphs** Graph △*CDE*.

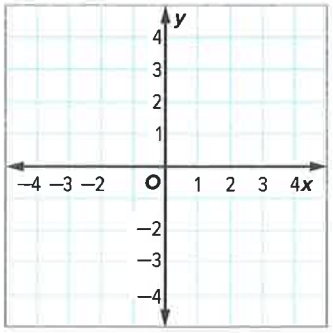
b. **Numbers** Find the lengths of the sides of △*CDE*.

c. **Geometry** Reflect △*CDE* over the *y*-axis, then translate it 2 units left. Label the vertices of the image *C′D′E′*. Write the coordinates of △*C′D′E′* below.

d. **Numbers** Find the lengths of the sides of △*C′D′E′*.

e. **Words** Are the two triangles congruent? Justify your response.

5. Graph $\triangle GHJ$ with vertices at $G(0, 1)$, $H(4, 0)$, and $J(4, 1)$. Then graph the image of the triangle after a translation of 3 units up followed by a reflection over the y-axis. Find the lengths of each side of the preimage and the image. Then determine if the two figures are congruent.



H.O.T. Problems Higher Order Thinking

6. MP **Model with Mathematics** Create a design in the space at the right, using a series of transformations that produce congruent figures. Exchange designs with a classmate and determine what transformations were used to create their design.

7. MP **Persevere with Problems** Triangle $A'B'C'$ has vertices $A'(-4, 5)$, $B'(-1, 4)$, and $C'(-2, 0)$. Triangle ABC was rotated 90° in a clockwise direction about the origin, translated 2 units up, and reflected over the y-axis. What were the coordinates of the vertices of triangle ABC?

8. MP **Persevere with Problems** Line segment XY has endpoints at $X(3, 1)$ and $Y(-2, 0)$. Its image after a series of transformations has endpoints at $X'(0, 1)$ and $Y'(5, 0)$. Find the series of transformations that maps $\overline{XY}$ onto $\overline{X'Y'}$. Then find the exact length of both segments.

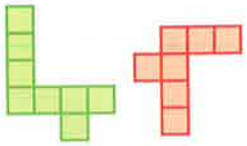
9. MP **Justify Conclusions** A line segment has endpoints at (a, b) and (c, d). Determine whether the following statements are *true* or *false*. Justify your reasoning.
   a. The line segment with endpoints at $(a + x, b)$ and $(c + x, d)$ is congruent to the original segment.
   b. The line segment with endpoints at $\left(\frac{2}{3}a, \frac{2}{3}b\right)$ and $\left(\frac{2}{3}c, \frac{2}{3}d\right)$ is congruent to the original segment.

Name _____ My Homework _____

Extra Practice

Determine if the two figures are congruent by using transformations. Explain your reasoning.

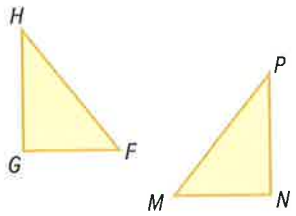
10.



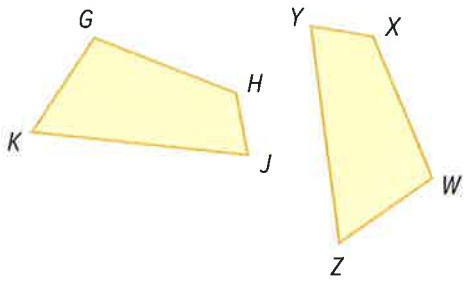
Homework Help

The two figures are not congruent because no sequence of transformations will map the green figure onto the red figure exactly.

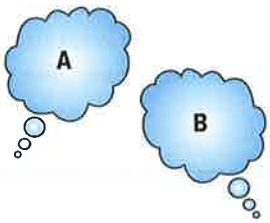
11.



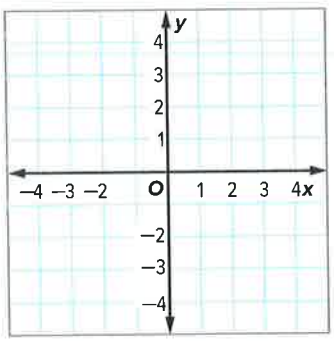
12.



13. Ismail is illustrating a graphic novel for a friend. He is using the two thought bubbles shown. What transformations did he use if Figure A is the preimage and Figure B is the image?

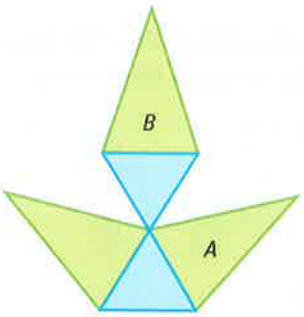


14. MP **Model with Mathematics** Graph $\triangle PQR$ with vertices at $P(0, 0)$, $Q(2, 0)$, and $R(0, 2)$. Then graph the image of the triangle after a reflection over the x-axis followed by a dilation with a scale factor of 2. Find the lengths of each side of the preimage and the image. Then determine if the two figures are congruent.



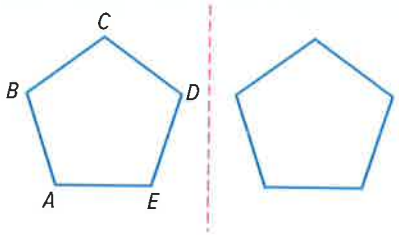
Power Up! Test Practice

15. Usama is creating a mosaic for art class. He started by using triangular tiles as shown.



Triangles *A* and *B* are congruent. Describe possible transformations he could have used if triangle *A* is the preimage and triangle *B* is the image?

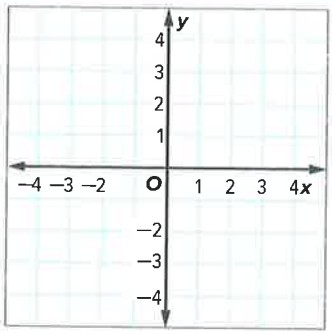
16. Pentagon *ABCDE* is reflected across the line shown and then rotated 72° clockwise about its center to create congruent pentagon *FGHIJ*. Label the vertices of *FGHIJ* in the correct positions on the image.



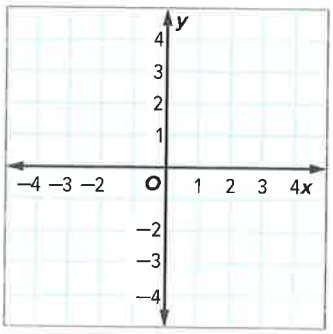
Spiral Review

Graph each figure with the given vertices and its image after the indicated transformation. Then give the coordinates of the final image.

17. $\overline{CD}$: $C(-2, 4)$, $D(0, 0)$; translation of 3 units right and 2 units down



18. $\triangle XYZ$: $X(-1, 1)$, $Y(3, 1)$, $Z(1, 3)$; reflection over the *y*-axis



Inquiry Lab

Investigate Congruent Triangles

Inquiry **WHICH three pairs of corresponding parts can be used to show that two triangles are congruent?**

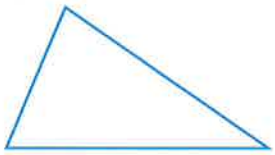
MP **Mathematical Practices** 1, 3

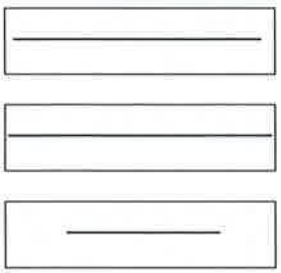
While driving past a bridge with his family, Ayman noticed that the bridge truss was made up of congruent triangles.

Hands-On Activity 1

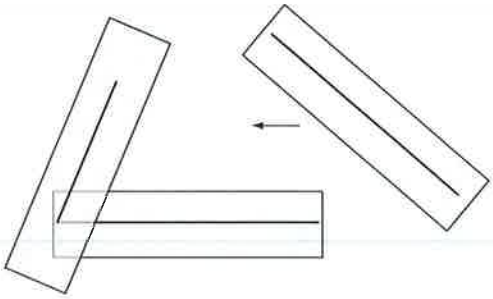
In this Activity you will investigate whether it is possible to show that two triangles are congruent without showing that all six pairs of corresponding parts are congruent.

Step 1 Copy the sides of the triangle shown onto a piece of patty paper and cut them out.





Step 2 Arrange and tape the pieces together so that they form a triangle.



Is the triangle you formed congruent to the original triangle?

Explain. _____

_____

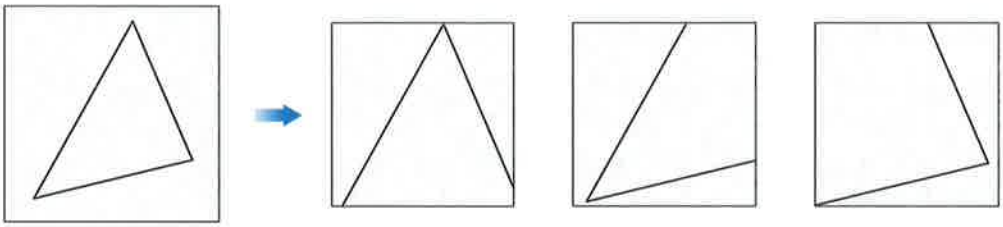
Rotate the triangle you formed 180°. Is the triangle congruent to the original triangle? Explain. _____

_____

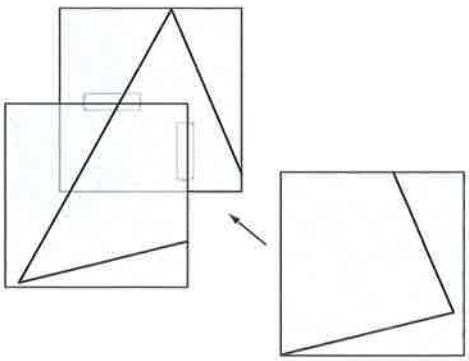
Try to form another triangle with the given sides. Is it congruent to the original triangle? _____

Hands-On Activity 2

Step 1 Draw a triangle on a piece of patty paper. Copy each angle of the triangle onto a separate piece of patty paper. Extend each side of each angle to the edge of the patty paper.



Step 2 Arrange and tape the pieces together so that they form a triangle.



Is the triangle you formed congruent to the original triangle? Explain.

Try to form another triangle with the given angles. Is it congruent to the original triangle?

A *counterexample* disproves a statement by showing an example of when the statement is not true. Based on this activity, is the following statement true? If not, provide a counterexample.

If the angles of one triangle are congruent to the angles of another triangle, the two triangles are congruent.

Collaborate

Investigate

1. Draw a triangle on a piece of tracing paper. Copy two sides of the triangle and the angle between them onto separate pieces of tracing paper and cut them out. Arrange and tape pieces together so that the two sides are joined to form the rays of the angle. Connect the two rays to form a triangle.

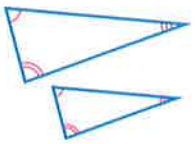
   a. Is the triangle you formed congruent to the original triangle?

   Explain. _____

   _____

   b. Try to form another triangle with the given sides and angle. Is it congruent to the original triangle? _____

2. Determine if two triangles with the following congruent parts are congruent. If not, draw a counterexample.

Various Parts	Congruent?	Counterexample
3 angles	No	
2 sides		
2 angles and 1 side		
2 angles and the side between the 2 angles		
2 angles		
3 sides		

Show your work.

Analyze and Reflect

Collaborate

3. Based on Activity 1, can three pairs of congruent sides be used to show that two triangles are congruent? _____

4. Based on Activity 2, can three pairs of congruent angles be used to show that two triangles are congruent? _____

5. Based on Exercise 1, can two pairs of congruent sides and the pair of congruent angles between them be used to show that two triangles are congruent? _____

Create

On Your Own

6. MP **Make a Conjecture** Use patty paper to investigate the relationship between two triangles with the given information. Make a conjecture about whether each of these cases can be used to show that two triangles are congruent.

Case 1 two pairs of congruent sides and a pair of congruent angles not between them _____

Case 2 two pairs of congruent angles and the pair of congruent sides between them _____

Case 3 two pairs of congruent angles and a pair of congruent sides not between them _____

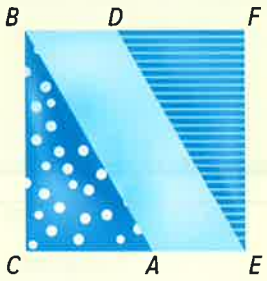
7. Inquiry WHICH three pairs of corresponding parts can be used to show that two triangles are congruent?

Lesson 2

Congruence

Real-World Link

Crafts Buthaina is creating a quilt using the geometric pattern shown. She wants to make sure that all of the triangles in the pattern are the same shape and size.



1. What would Buthaina need to do to show the two triangles are congruent?

2. Complete the lists of the parts of $\triangle ABC$ and $\triangle DEF$. Then draw lines between the corresponding parts of each triangle.

$\overline{CB}$ ____ $\overline{BA}$ $\angle BAC$ $\angle ABC$ $\angle$ ____

____ $\overline{ED}$ ____ $\angle$ ____ $\angle$ ____ $\angle EDF$

3. Suppose you cut out the two triangles and laid one on top of the other so the parts of the same measures were matched up. What is true about the triangles?

Essential Question

HOW can you determine congruence and similarity?

Vocabulary

corresponding parts

Math Symbols

$\cong$ is congruent to

MP Mathematical Practices

1, 2, 3, 4

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Key Concept

Corresponding Parts of Congruent Figures

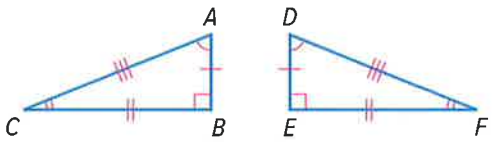
Work Zone

Congruence

To indicate that sides are congruent, an equal number of tick marks is drawn on the corresponding sides. To show that angles are congruent, an equal number of arcs is drawn on the corresponding angles.

Words If two figures are congruent, their corresponding sides are congruent and their corresponding angles are congruent.

Model

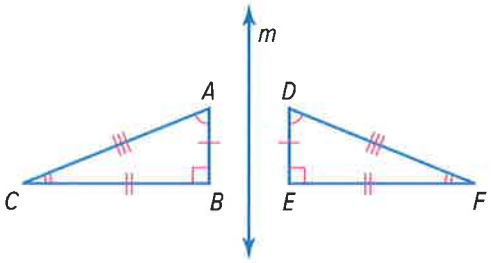


Symbols $\triangle ABC \cong \triangle DEF$

Congruent Angles: $\angle A \cong \angle D$; $\angle B \cong \angle E$; $\angle C \cong \angle F$

Congruent Sides: $\overline{AB} \cong \overline{DE}$; $\overline{BC} \cong \overline{EF}$; $\overline{CA} \cong \overline{FD}$

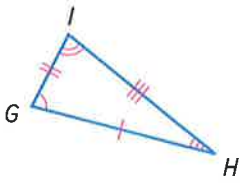
In the figure below, the two triangles are congruent because $\triangle DEF$ is the image of $\triangle ABC$ reflected over line m. The notation $\triangle ABC \cong \triangle DEF$ is read *triangle ABC is congruent to triangle DEF*.

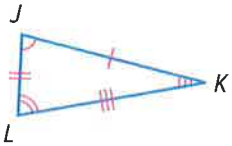


The parts of congruent figures that *match* or correspond, are called **corresponding parts**.

Example

1. **Write congruence statements comparing the corresponding parts in the congruent triangles shown.**



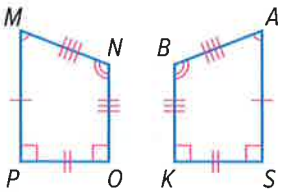


Use the matching arcs and tick marks to identify the corresponding parts.

Corresponding angles:
$\angle J \cong \angle G$, $\angle L \cong \angle I$, $\angle K \cong \angle H$

Corresponding sides:
$\overline{JK} \cong \overline{GH}$, $\overline{KL} \cong \overline{HI}$, $\overline{LJ} \cong \overline{IG}$

Got it? Do this problem to find out.

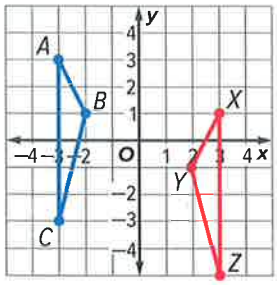
a. 

Show your work.

a. _____

Example

2. **Triangle *ABC* is congruent to $\triangle XYZ$. Write congruence statements comparing the corresponding parts. Then determine which transformations map $\triangle ABC$ onto $\triangle XYZ$.**



Step 1 Analyze the figures to determine which angles and sides of the figures correspond.

Corresponding angles: $\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$
Corresponding sides: $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, $\overline{CA} \cong \overline{ZX}$

Step 2 Determine any changes in the orientation of the triangles. The orientation is reversed so at least one of the transformations is a reflection. If you reflect $\triangle ABC$ over the *y*-axis and then translate it down 2 units, it coincides with $\triangle XYZ$.

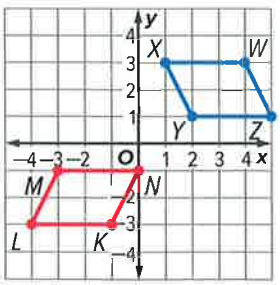
The transformations that map $\triangle ABC$ onto $\triangle XYZ$ consist of a reflection over the *y*-axis followed by a translation of 2 units down.

STOP and Reflect

When writing congruence statements, why is it important to match up corresponding points in the statement?

Got it? Do this problem to find out.

b. Parallelogram *WXYZ* is congruent to parallelogram *KLMN*. Write congruence statements comparing the corresponding parts. Then determine which transformation(s) map parallelogram *WXYZ* onto parallelogram *KLMN*.



b. _____

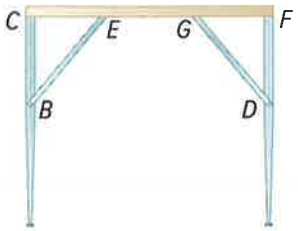
Find Missing Measures

You can use properties of congruent figures to find the missing measures of angles and sides in a figure.

Congruence
Congruent angles have the same measure and congruent sides have equal length.

Example

3. **Badria is using a brace to support a tabletop. In the figure, $\triangle BCE \cong DFG$. If $m\angle CEB = 50°$, what is the measure of $\angle FGD$?**



Since $\angle CEB$ and $\angle FGD$ are corresponding parts in congruent figures, they are congruent. So, $\angle FGD$ measures 50°.

Show your work.

Got it? **Do this problem to find out.**

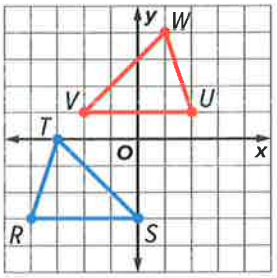
c. _____

c. In the figure shown above, the length of $\overline{CE}$ is 0.6 meters. What is the length of $\overline{FG}$?

Guided Practice

Check

1. Triangle *RST* is congruent to $\triangle UVW$. Write congruence statements comparing the corresponding parts. Then determine which transformation(s) map $\triangle RST$ onto $\triangle UVW$. (Examples 1 and 2)



Show your work.

2. In the table design shown in Example 3, suppose $BE = 45$ centimeters. What is DG? (Example 3)

3. **Building on the Essential Question** How can the coordinate plane help you determine that corresponding sides are congruent?

Rate Yourself!

How confident are you about congruence? Check the box that applies.

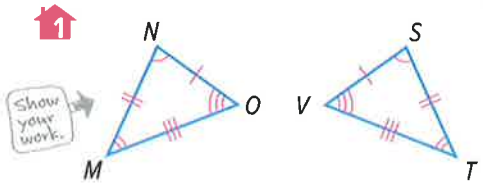
FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

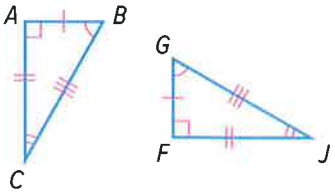
Independent Practice

Write congruence statements comparing the corresponding parts in each set of congruent figures. (Example 1)

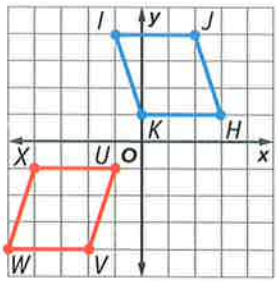
1



2.



3 Parallelograms *UVWX* and *HJIK* are congruent. Write congruence statements comparing the corresponding parts. Then determine which transformation(s) map parallelogram *UVWX* onto parallelogram *HJIK*. (Example 2)



4. In the umbrella shown at the right, $\triangle JLK \cong \triangle NLM$. (Example 3)

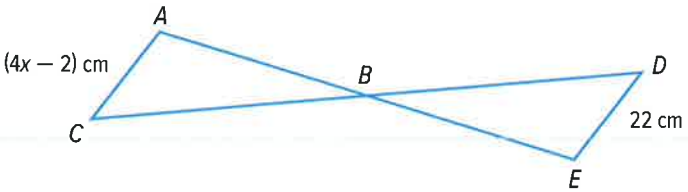
a. If $m\angle JKL = 66°$, then $m\angle NML =$ _____.

b. If $MN = 35$ centimeters, then $KJ =$ _____.

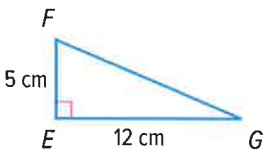
5. MP **Reason Abstractly** In the figure, $\triangle ABC \cong \triangle EBD$.

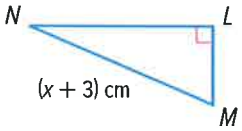
a. On the figure, draw arc and tic marks to identify the corresponding parts.

b. Find the value of x.

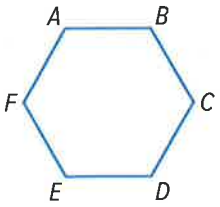


6. In the figure at the right, $\triangle EFG \cong \triangle LMN$. Find the value of x. Then describe the transformations that map $\triangle EFG$ onto $\triangle LMN$.





7. MP **Make a Conjecture** Hexagon *ABCDEF* has six congruent sides.



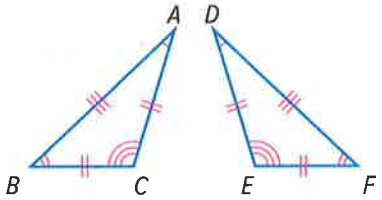
   a. Draw $\overline{CA}$, $\overline{CF}$, and $\overline{CE}$.

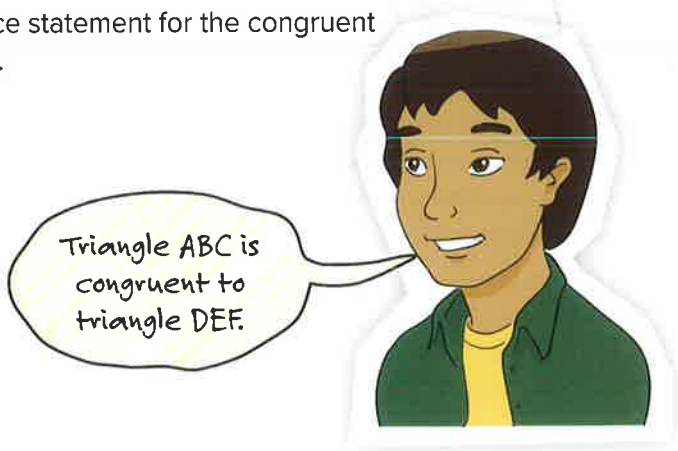
   b. How many triangles were formed? _____

   c. Make a conjecture about which triangles are congruent. Test your conjecture by measuring the sides and angles of the triangles.

H.O.T. Problems Higher Order Thinking

8. MP **Find the Error** Bilal is making a congruence statement for the congruent triangles shown. Find his mistake and correct it.





9. MP **Persevere with Problems** Determine whether each statement is *true* or *false*. If true, explain your reasoning. If false, give a counterexample.

   a. If two figures are congruent, their perimeters are equal.

   b. If two figures have the same perimeter, they are congruent.

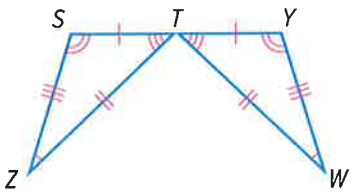
10. MP **Model with Mathematics** Write and solve a real-world problem that involves using the properties of congruent figures to find a missing measure.

Name _____ My Homework _____

Extra Practice

Write congruence statements comparing the corresponding parts in each set of congruent figures.

11.



Homework Help

Use the matching arcs and tick marks to identify the corresponding parts.

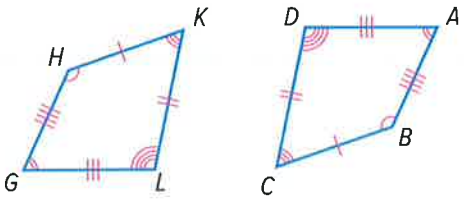
Corresponding angles:

$\angle S \cong \angle Y$, $\angle STZ \cong \angle YTW$, $\angle Z \cong \angle W$

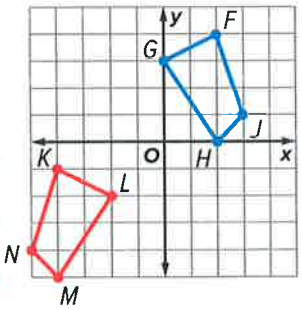
Corresponding sides:

$\overline{SZ} \cong \overline{YW}$, $\overline{ZT} \cong \overline{WT}$, $\overline{TS} \cong \overline{TY}$

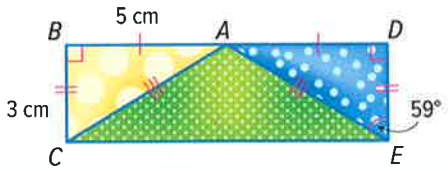
12.



13. Quadrilaterals *KLMN* and *FGHJ* are congruent. Write congruence statements comparing the corresponding parts. Then determine which transformation(s) map quadrilateral *KLMN* onto quadrilateral *FGHJ*.



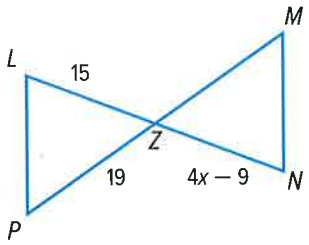
14. In the quilt design shown, $\triangle ABC \cong \triangle ADE$. What is the measure of $\angle BCA$?



15. MP **Reason Abstractly** In the figure, $\triangle LZP \cong \triangle NZM$.

a. On the figure, draw arc and tic marks to identify the corresponding parts.

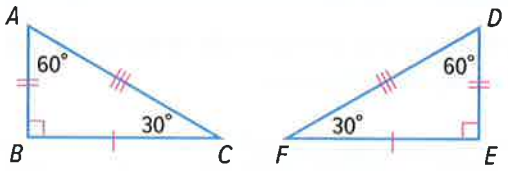
b. Find the value of *x*. _____



Power Up! Test Practice

16. The triangles shown are congruent.

Complete the congruence statements to compare the corresponding parts.



a. $\angle A \cong$ ☐ **b.** $\angle B \cong$ ☐

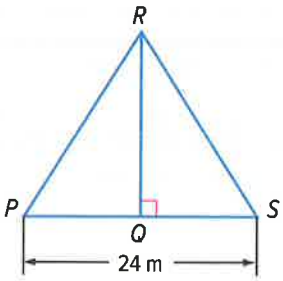
c. $\angle C \cong$ ☐ **d.** $\overline{AB} \cong$ ☐

e. $\overline{BC} \cong$ ☐ **f.** $\overline{AC} \cong$ ☐

$\angle A$	$\angle D$	$\overline{AB}$	$\overline{DE}$
$\angle B$	$\angle E$	$\overline{AC}$	$\overline{DF}$
$\angle C$	$\angle F$	$\overline{BC}$	$\overline{EF}$

17. In the figure, $\triangle PQR \cong \triangle SQR$. Which of the following represent a congruence statement for the corresponding parts? Select all that apply.

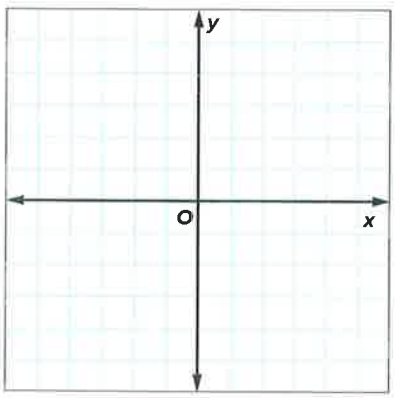
- ☐ $\angle RQP \cong \angle QSR$
- ☐ $\overline{PQ} \cong \overline{RQ}$
- ☐ $\overline{RP} \cong \overline{RS}$
- ☐ $\angle SRQ \cong \angle PRQ$



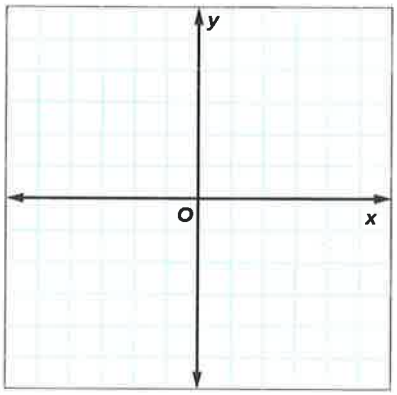
Spiral Review

Graph each figure with the given vertices and its image after the indicated transformations. Then give the coordinates of the final image.

18. $\triangle ABC$: $A(-4, 2)$, $B(-2, -3)$, $C(-4, -3)$; 90° counterclockwise rotation about A followed by a translation of 4 units to the right



19. quadrilateral $RSTU$: $R(4, 3)$, $S(5, -1)$, $T(4, -3)$, $U(3, -1)$; reflection over the x-axis followed by a reflection over the y-axis



MP Problem-Solving Investigation
Draw a Diagram

MP Mathematical Practices 1, 3, 4

Case #1 Hammer Time

Hasan wants to make shelves to store her game system and other electronics in her room. He will make brackets in the shape of right triangles to hold the shelves. Since it is a right triangle, one of the angles measure 90°.

What is the relationship of the other two angles in a right triangle?

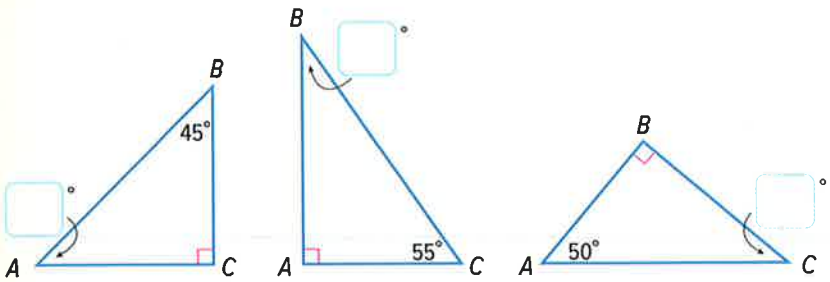
1 **Understand** ***What are the facts?***

The bracket is in the shape of a right triangle, so one of the angles measures 90°.

2 **Plan** ***What is your strategy to solve this problem?***

Draw several right triangles, measure each angle, and look for a pattern.

3 **Solve** ***How can you apply the strategy?***



It appears that the sum of the measures of the acute angles of a right triangle is ____. So, the acute angles are _____.

4 **Check** ***Does the answer make sense?***

You can try several more examples to see whether your conjecture appears to be true. But at this point, it is just a conjecture, not an actual proof.

Analyze the Strategy

MP **Justify Conclusions** Inductive reasoning is the process of making a conjecture after observing several examples. Did Hasan use inductive reasoning? Explain. _____

Mid-Chapter Check

Vocabulary Check

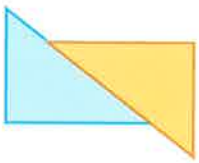
1. What transformations can be used to show two figures are congruent? (Lesson 1)

2. List two attributes of two congruent polygons. (Lesson 2)

Skills Check and Problem Solving

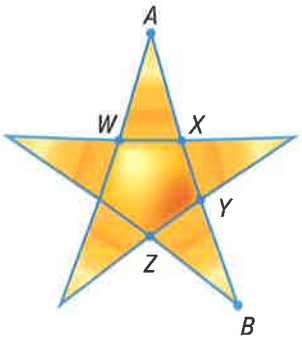
Determine if the two figures are congruent by using transformations. Explain your reasoning. (Lesson 1)

3. 

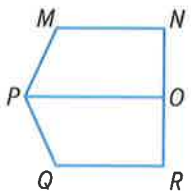
4. 

5. 

6. Husam is creating the logo shown using a pentagon and five congruent triangles. Triangle *WAX* is congruent to triangle *YBZ*. Describe the transformations that map $\triangle WAX$ onto $\triangle YBZ$. If *WX* measures 5 centimeters, what is the length of *YZ*? (Lesson 2)



7. **Persevere with Problems** Trapezoid *MNOP* is congruent to trapezoid *QROP*. Which transformation maps *MNOP* onto *QROP*? (Lesson 2)



Inquiry Lab
Similar Triangles

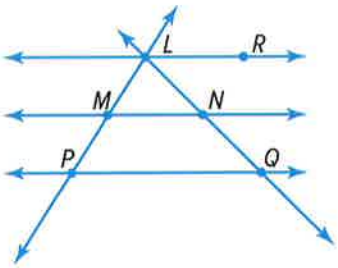
Inquiry **HOW are two triangles related if they have the same shape but different sizes?**

MP **Mathematical Practices** 1, 3

While flying in an airplane, Houriyya looked out the window and saw roads and a field like the one shown. She wondered if there was a relationship between the two triangles she saw.

Hands-On Activity

To determine if there is a relationship between the two triangles, use the diagram shown.



$\overleftrightarrow{LR} \parallel \overleftrightarrow{MN} \parallel \overleftrightarrow{PQ}$
$\overleftrightarrow{LP}$ and $\overleftrightarrow{LQ}$ are transversals.

Step 1 Measure and record the lengths of the line segments in millimeters and angles in degrees in the table.

$\triangle LPQ$		$\triangle LMN$	
$LP =$	$m\angle L =$ °	$LM =$	$m\angle L =$ °
$LQ =$	$m\angle P =$ °	$LN =$	$m\angle M =$ °
$PQ =$	$m\angle Q =$ °	$MN =$	$m\angle N =$ °

What do you notice about the measure of the corresponding angles of the triangles? _____

Step 2 Express the lengths of the corresponding sides of the triangles as ratios.

$\frac{LP}{LM} =$ _____ $\frac{LQ}{LN} =$ _____ $\frac{PQ}{MN} =$ _____

What do you notice about the ratios of the corresponding sides of the triangles? _____

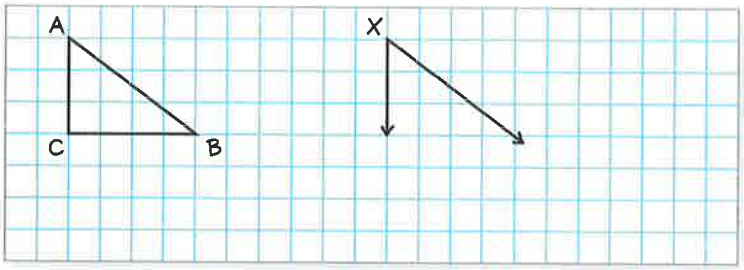
Investigate

Collaborate

Work with a partner.

1. MP **Model with Mathematics** Triangle ABC is a right triangle with $m\angle A = 53°$. On the grid, draw and label a different right triangle, XYZ, using the given angle X, which also measures 53°.

Show your work.



What do you notice about the shape of the triangles? _____

_____

Analyze and Reflect

Collaborate

For Exercises 2–4, refer to the triangles in Exercise 1.

2. What is the measure of $\angle B$? the measure of the angle that corresponds to $\angle B$ in $\triangle XYZ$? _____

3. Express the lengths of the corresponding sides of the triangles as ratios.

$$\frac{AC}{\square} = \frac{\square}{\square} \quad \frac{\square}{\square} \qquad \frac{CB}{\square} = \frac{\square}{\square} \quad \frac{\square}{\square} \qquad \frac{AB}{\square} = \frac{\square}{\square} \quad \frac{\square}{\square}$$

4. What do you notice about the ratios? _____

Create

On Your Own

5. MP **Reason Inductively** The two triangles in the Activity and in Exercise 1 are called *similar triangles*. Based on your discoveries, make a conjecture about the properties of similar triangles.

_____

_____

6. Inquiry HOW are two triangles related if they have the same shape but different sizes?

_____

Lesson 3

Similarity and Transformations

Vocabulary Start-Up

Recall that a dilation changes the size of a figure by a scale factor, but does not change the shape of the figure. Since the size is changed, the image and the preimage are not congruent.

Complete the graphic organizer. Consider each word on the Rating Scale and place a check ✓ in the appropriate column next to the word. If you do not know the meaning of a word, find the meaning in the glossary or on the Internet.

	Rating Scale			
Word	**Know it well**	**Have seen or heard it**	**No clue**	**What it means**
dilation				
scale factor				
similar figures				

Essential Question

HOW can you determine congruence and similarity?

Vocabulary

similar

MP Mathematical Practices
1, 3, 4, 7

Real-World Link

A *fractal* is a geometric image that can be divided into parts that are smaller copies of the whole. The photo at the right is an example of a fractal.

1. Circle two different size parts of the figure that are smaller copies of the whole.

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

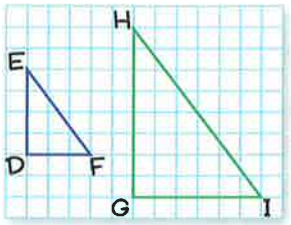
Work Zone

Identify Similarity

Two figures are **similar** if the second can be obtained from the first by a sequence of transformations and dilations.

Examples

1. **Determine if the two triangles are similar by using transformations.**



Since the orientation of the figures is the same, one of the transformations might be a translation.

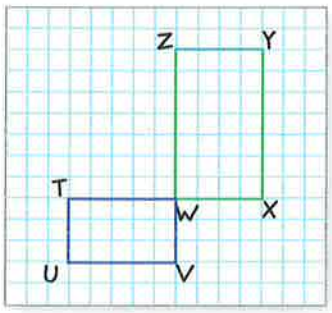
Step 1 Translate $\triangle DEF$ down 2 units and 5 units to the right so D maps onto G.

Step 2 Write ratios comparing the lengths of each side.

$$\frac{HG}{ED} = \frac{8}{4} \text{ or } \frac{2}{1} \qquad \frac{GI}{DF} = \frac{6}{3} \text{ or } \frac{2}{1} \qquad \frac{IH}{FE} = \frac{10}{5} \text{ or } \frac{2}{1}$$

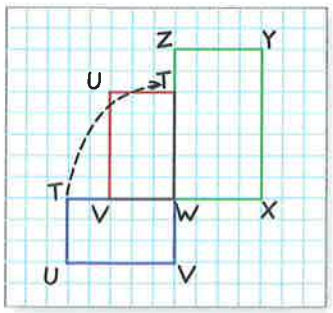
Since the ratios are equal, $\triangle HGI$ is the dilated image of $\triangle EDF$. So, the two triangles are similar because a translation and a dilation maps $\triangle EDF$ onto $\triangle HGI$.

2. **Determine if the two rectangles are similar by using transformations.**



The orientation of the figures is the same, so one of the transformations might be a rotation.

Step 1 Rotate rectangle $VWTU$ 90° clockwise about W so that it is oriented the same way as rectangle $WXYZ$.



Step 2 Write ratios comparing the lengths of each side.

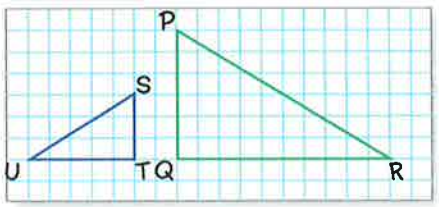
$$\frac{WT}{XY} = \frac{5}{7} \qquad \frac{TU}{YZ} = \frac{3}{4}$$

$$\frac{UV}{ZW} = \frac{5}{7} \qquad \frac{VW}{WX} = \frac{3}{4}$$

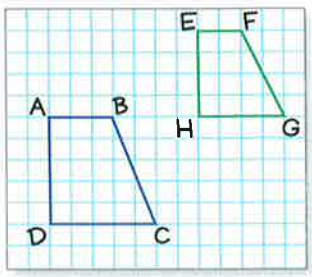
The ratios are not equal. So, the two rectangles are not similar since a dilation did not occur.

Got it? Do these problems to find out.

a.



b.



Show your work.

a. _____

b. _____

Use the Scale Factor

Similar figures have the same shape, but may have different sizes. The sizes of the two figures are related to the scale factor of the dilation.

If the scale factor of the dilation is ...	then the dilated figure is ...
between 0 and 1	smaller than the original
equal to 1	the same size as the original
greater than 1	larger than the original

Example

3. **Hamad enlarges the photo shown by a scale factor of 2 for his webpage. He then enlarges the webpage photo by a scale factor of 1.5 to print. If the original photo is 5 centimeters by 7.5 centimeters, what are the dimensions of the print? Are the enlarged photos similar to the original?**

Multiply each dimension of the original photo by 2 to find the dimensions of the webpage photo.

$5 \text{ cm} \times 2 = 10 \text{ cm}$ $\quad$ $7.5 \text{ cm} \times 2 = 15 \text{ cm}$

So, the webpage photo will be 10 centimeters by 15 centimeters. Multiply the dimensions of that photo by 1.5 to find the dimensions of the print.

$10 \text{ cm} \times 1.5 = 15 \text{ cm}$ $\quad$ $15 \text{ cm} \times 1.5 = 22.5 \text{ cm}$

The printed photo will be 15 centimeters by 22.5 centimeters. All three photos are similar since each enlargement was the result of a dilation.

STOP and Reflect

List below at least two topics in mathematics that use a scale factor.

Show your work.

c. _____

Got it? **Do this problem to find out.**

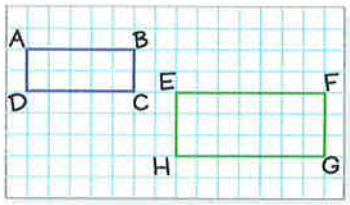
c. An art show offers different size prints of the same painting. The original print measures 24 centimeters by 30 centimeters. A printer enlarges the original by a scale factor of 1.5, and then enlarges the second image by a scale factor of 3. What are the dimensions of the largest print? Are both of the enlarged prints similar to the original?

Guided Practice

Check

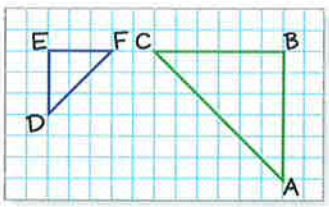
Determine if the two figures are similar by using transformations. Explain your reasoning. (Examples 1 and 2)

1.



Show your work.

2.



3. A T-shirt iron-on measures 5 cm by 2.5 cm. It is enlarged by a scale factor of 3 for the back of the shirt. The second iron-on is enlarged by a scale factor of 2 for the front of the shirt. What are the dimensions of the largest iron-on? Are both of the enlarged iron-ons similar to the original? (Example 3) _____

4. **Building on the Essential Question** What is the difference between using transformations to create similar figures versus using transformations to create congruent figures?

Rate Yourself!

How confident are you about similar figures? Shade the ring on the target.

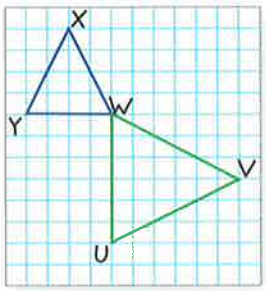
FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

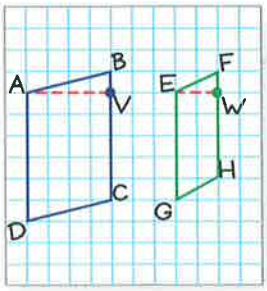
Determine if the two figures are similar by using transformations. Explain your reasoning. (Examples 1 and 2)

1



Show your work.

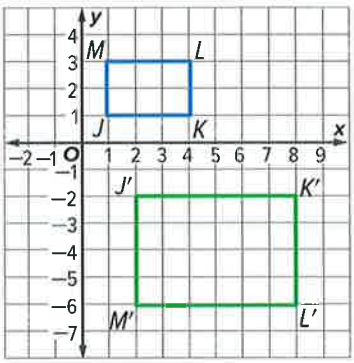
2.



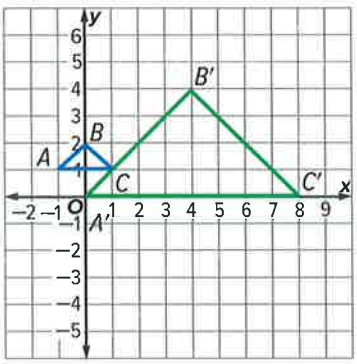
3 Rana is creating a scrapbook of her family. A photo of her grandmother measures 6 centimeters by 10 centimeters. She enlarges it by a scale factor of 1.5 to place in the scrapbook. Then she enlarges the second photo by a scale factor of 1.5 to place on the cover of the scrapbook. What are the dimensions of the photo for the cover of the scrapbook? Are all of the photos similar? (Example 3) _____

MP **Persevere with Problems** **Each preimage and image are similar. Describe a sequence of transformations that maps the preimage onto the image.**

4.



5.



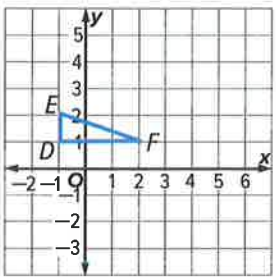
6. **Identify Structure** Use the graphic organizer to compare and contrast similar and congruent figures.

	Similar Figures	Congruent Figures
Side Measures		
Angle Measures		
Transformations Used		

H.O.T. Problems Higher Order Thinking

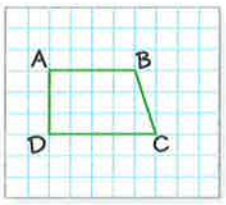
7. **Persevere with Problems** Using at least one dilation, describe a series of transformations where the image is congruent to the preimage.

8. **Model with Mathematics** The image of $\triangle DEF$ after two transformations has vertices at $D'(3, 3)$, $E'(6, 3)$ and $F'(3, -6)$. If the two triangles are similar, determine what two transformations map $\triangle DEF$ onto $\triangle D'E'F'$.



9. **Construct an Argument** *True* or *false*. If a dilation is in a composition of transformations, the order in which you perform the composition does not matter. Explain your reasoning.

10. **Model with Mathematics** Trapezoid $ABCD$ is shown at the right. Perform a series of transformations on the trapezoid and draw the image on the coordinate plane. List the transformations used below.

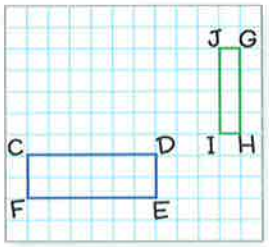


Name _____ My Homework _____

Extra Practice

Determine if the two figures are similar by using transformations. Explain your reasoning.

11.



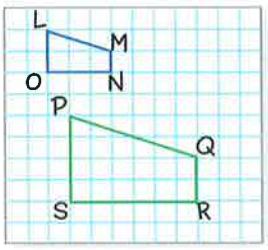
Homework Help

no; The ratios of the side lengths are not equal.

Find the ratios of the side lengths.

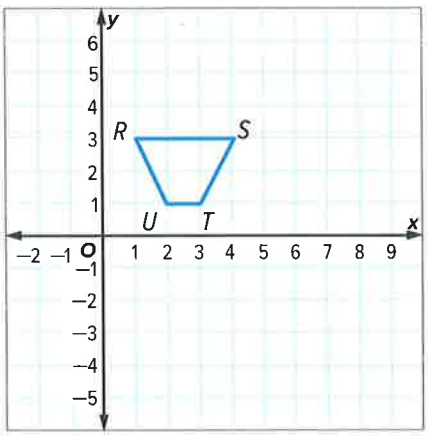
$\frac{CD}{GH} = \frac{6}{4}$ and $\frac{DE}{JG} = \frac{2}{1}$; $\frac{6}{4} \neq \frac{2}{1}$, so the two figures are not similar.

12.



13. Reham is making three different sizes of blankets from the same material. The first measures 1 meter by 0.6 meters. She wants to enlarge it by a scale factor of 2 to make the second blanket. Then she will enlarge the second one by a scale factor of 1.5 to make the third blanket. What are the dimensions of the third blanket? Are all of the blankets similar?

14. MP **Model with Mathematics** In the figure shown, trapezoid *RSTU* has vertices *R*(1, 3), *S*(4, 3), *T*(3, 1), and *U*(2, 1).



a. Draw the image of *RSTU* after a translation of 2 units down followed by a dilation with a scale factor of 2. Label the vertices *ABCD*.

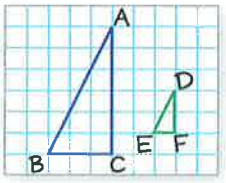
b. Draw the image of *RSTU* after a dilation with a scale factor of 2, followed by a translation of 2 units down. Label the vertices *EFGH*.

c. Which figures are similar? Which figures are congruent?

d. Are *ABCD* and *EFGH* in the same location? If they are not, what transformation would map *ABCD* onto *EFGH*?

Power Up! Test Practice

15. Triangle *DEF* is the image of triangle *ABC* after a sequence of transformations.



Determine if each statement is true or false.

a. $\triangle ABC$ was dilated by a scale factor of 3 to create $\triangle DEF$. ☐ True ☐ False

b. The transformation represents a reduction. ☐ True ☐ False

c. The ratios $\frac{AB}{DF}$ and $\frac{AC}{DE}$ are equal. ☐ True ☐ False

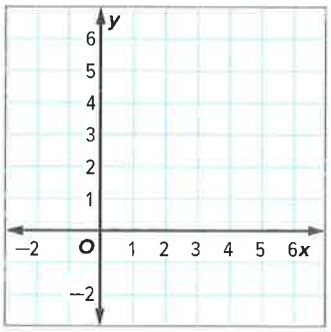
16. Which sequences of transformations would result in similar figures that are enlargements or reductions? Select all that apply.

- ☐ translation, dilation, rotation, reflection
- ☐ reflection, translation, rotation
- ☐ translation, reflection, rotation, reflection
- ☐ rotation, translation, dilation

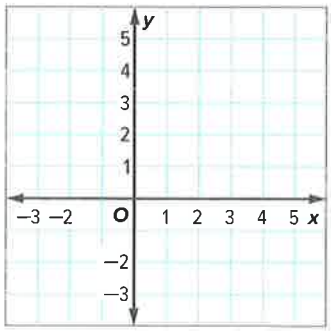
Spiral Review

Find the coordinates of the vertices of each figure after a dilation with the given scale factor *k*. Then graph the original image and the dilation.

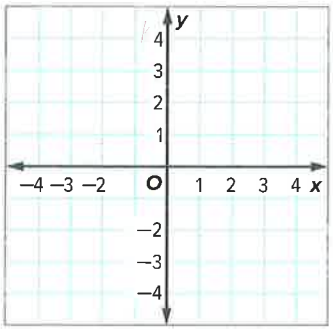
17. $M(0, 0)$, $N(-1, 1)$, $O(2, 3)$; $k = 2$



18. $A(-3, 3)$, $B(3, 3)$, $C(3, -3)$; $k = \frac{2}{3}$



19. $G(4, 4)$, $H(2, -4)$, $I(-4, -4)$, $J(0, 2)$; $k = \frac{1}{2}$



Lesson 4

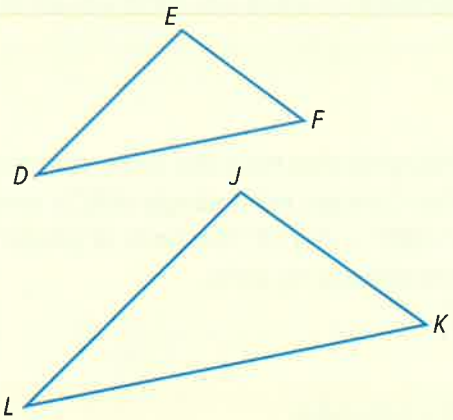
Properties of Similar Polygons

Real-World Link

Photos Salma is printing pictures at a photo kiosk in the store. She can choose between 4 × 6 prints or 5 × 7 prints. Are the side lengths of the two prints proportional? Explain.

Collaborate

Follow the steps to discover how the triangles are related.



1. Using a centimeter ruler, measure the sides of the two triangles. Then, use a protractor to measure the angles. Write the results in the table.

Figure	Side Length (cm)			Angle Measure (°)		
△EFD	DE	EF	FD	∠D	∠E	∠F
△LJK	LJ	JK	KL	∠L	∠J	∠K

2. Are the side lengths proportional? Explain.

3. What do you notice about the angles of the two triangles?

Essential Question

HOW can you determine congruence and similarity?

Vocabulary

similar polygons
scale factor

Math Symbols

~ is similar to

MP Mathematical Practices

1, 2, 3, 4

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

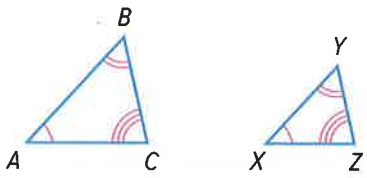
1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Key Concept

Similar Polygons

Words If two polygons are similar, then
- their corresponding angles are congruent and
- the measures of their corresponding sides are proportional.

Model



$\triangle ABC \sim \triangle XYZ$

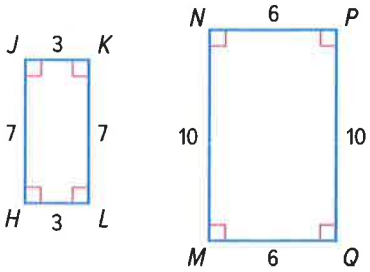
Symbols $\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$, and $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$

Work Zone

Polygons that have the same shape are called **similar polygons**. In the Key Concept box, triangle ABC is similar to triangle XYZ. This is written as $\triangle ABC \sim \triangle XYZ$. The parts of similar figures that "match" are called corresponding parts.

Example

1. Determine whether rectangle *HJKL* is similar to rectangle *MNPQ*. Explain.



First, check to see if corresponding angles are congruent.

Since the two polygons are rectangles, all of their angles are right angles. Therefore, all corresponding angles are congruent.

Next, check to see if corresponding sides are proportional.

$$\frac{HJ}{MN} = \frac{7}{10} \quad \frac{JK}{NP} = \frac{3}{6} \text{ or } \frac{1}{2} \quad \frac{KL}{PQ} = \frac{7}{10} \quad \frac{LH}{QM} = \frac{3}{6} \text{ or } \frac{1}{2}$$

Since $\frac{7}{10}$ and $\frac{1}{2}$ are not equivalent, the rectangles are *not* similar.

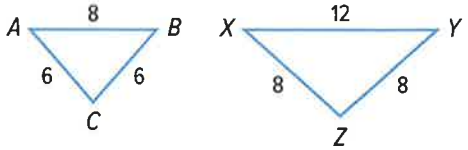
Common Error
Do not assume that two rectangles are similar just because their corresponding angles are congruent. Their corresponding sides must also be proportional.

Show your work.

Got it? Do this problem to find out.

a. _____

a. Determine whether $\triangle ABC$ is similar to $\triangle XYZ$. Explain.

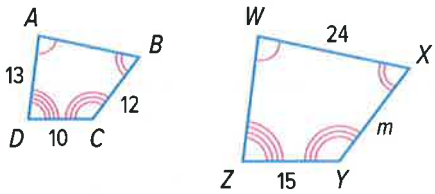


Find Missing Measures

Scale factor is the ratio of the lengths of two corresponding sides of two similar polygons. You can use the scale factor of similar figures to find missing measures.

Example

2. **Quadrilateral *WXYZ* is similar to quadrilateral *ABCD*.**



a. Describe the transformations that map *WXYZ* onto *ABCD*.

Since the figures are similar, they are not the same size. Choose two corresponding sides and determine what transformations will map one onto the other. A translation followed by a dilation will map $\overline{AB}$ onto $\overline{WX}$.

b. Find the missing measure.

Method 1

Find the scale factor from quadrilateral *ABCD* to quadrilateral *WXYZ*.

$$\text{scale factor: } \frac{YZ}{CD} = \frac{15}{10} \text{ or } \frac{3}{2}$$

So, a length on polygon *WXYZ* is $\frac{3}{2}$ times as long as the corresponding length on polygon *ABCD*. Let m represent the measure of $\overline{XY}$.

$m = \frac{3}{2}(12)$ Write the equation.

$m = 18$ Multiply.

Method 2

Set up a proportion to find the missing measure.

$\frac{XY}{BC} = \frac{YZ}{CD}$ Write the proportion.

$\frac{m}{12} = \frac{15}{10}$ $XY = m$, $BC = 12$, $YZ = 15$, $CD = 10$

$m \cdot 10 = 12 \cdot 15$ Find the cross products.

$10m = 180$ Simplify.

$m = 18$ Division Property of Equality

Show your work.

b. _____

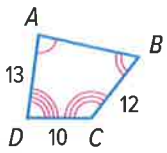
c. _____

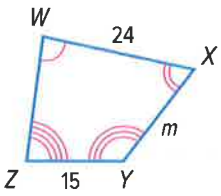
Got it? **Do these problems to find out.**

Find each missing measure.

b. WZ

c. AB



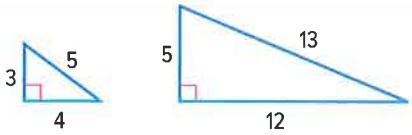


Guided Practice

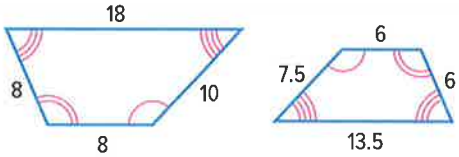
Check

Determine whether each pair of polygons is similar. Explain. (Example 1)

1.



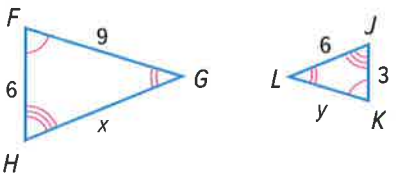
2.



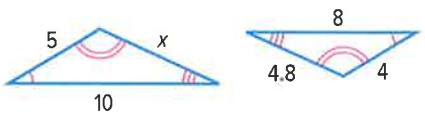
3. The two triangles are similar. (Example 2)

a. Determine the transformations that map one figure onto the other.

b. Find the missing side measures.



4. The two triangles are similar. (Example 2)



a. Determine the transformations that map one figure onto the other.

b. Find the missing side measure.

5. **Building on the Essential Question** How does the scale factor of a dilation relate to the ratio of two of the corresponding sides of the preimage and the image?

Rate Yourself!

Are you ready to move on? Shade the section that applies.

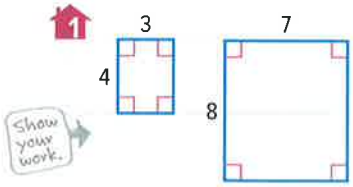
FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

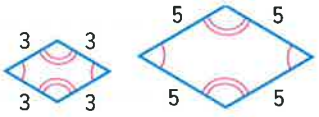
Independent Practice

Determine whether each pair of polygons is similar. Explain. (Example 1)

1

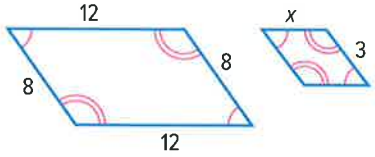


2.

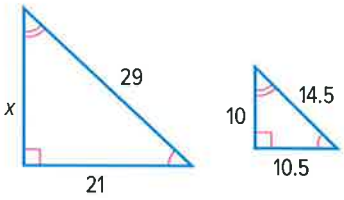


Each pair of polygons is similar. Determine the transformations that map one figure onto the other. Then find the missing side measures. (Example 2)

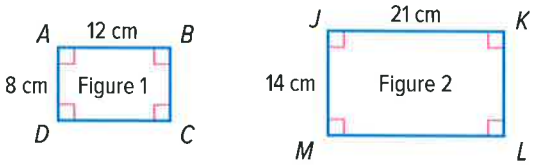
3



4.



5. MP **Persevere with Problems** The figures at the right are similar.

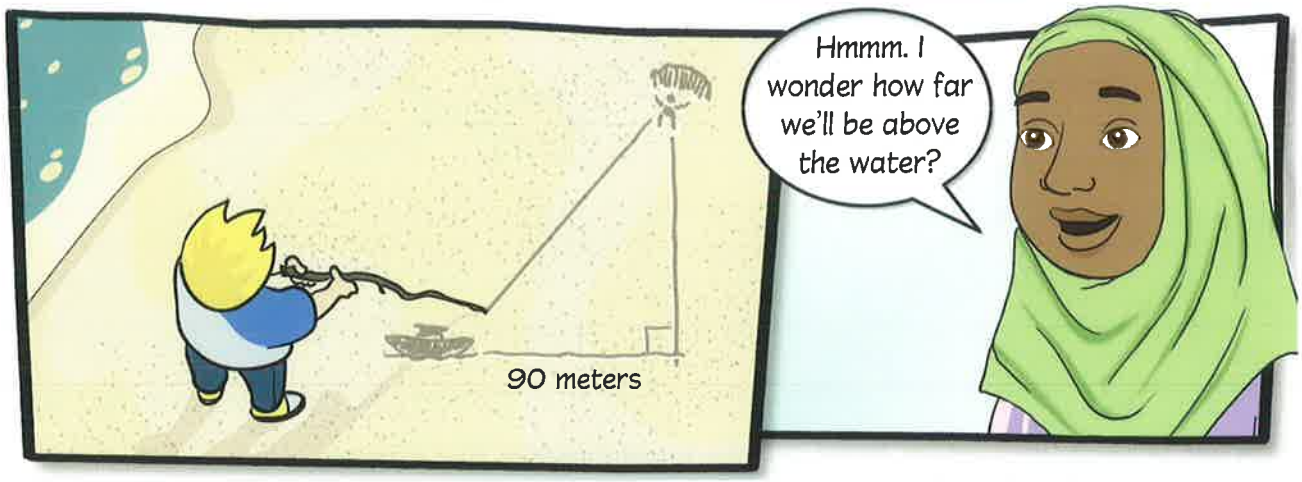


a. Find the area of both figures.

b. Compare the scale factor of the side lengths and the ratio of the areas.

6. STEM The scale factor from the model of a human inner ear to the actual ear is 55:2. If one of the bones of the model is 8.25 centimeters long, how long is the actual bone in a human ear? _____

7. **Model with Mathematics** Refer to the graphic novel frame below. The brochure says that the rope is 150 meters long. Use the properties of similar triangles to find the parasailer's height above the water.



H.O.T. Problems Higher Order Thinking

8. **Persevere with Problems** Suppose two rectangles are similar with a scale factor of 2. What is the ratio of their areas? Explain.

Justify Conclusions Determine whether each statement is *true* or *false*. If true, explain your reasoning. If false, provide a counterexample.

9. All rectangles are similar.

10. All squares are similar.

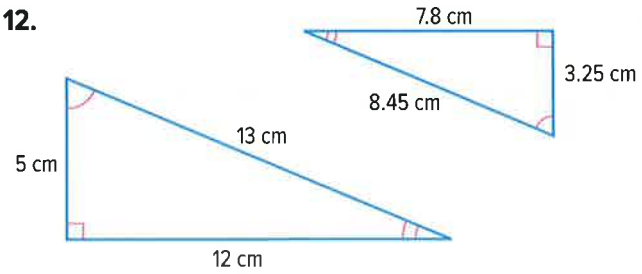
11. **Model with Mathematics** Draw two similar polygons in the space provided. Include the measures of the sides on your drawing, and identify the scale factor.

Name _____ My Homework _____

Extra Practice

Determine whether each pair of polygons is similar. Explain.

12.



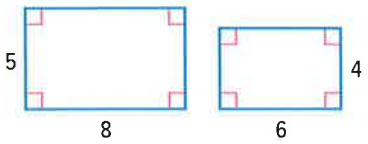
Homework Help

As indicated by the arc marks, corresponding angles are congruent. Check to see if the corresponding sides are proportional.

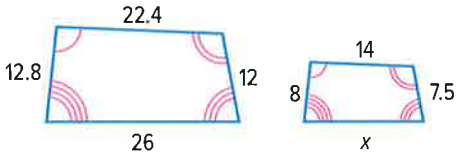
$\frac{3.25}{5} = \frac{8.45}{13} = \frac{7.8}{12}$

The sides are proportional so the triangles are similar.

13.

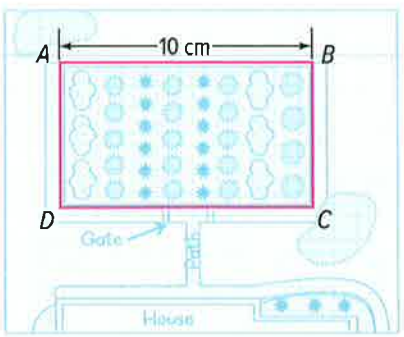


14. The two figures are similar. Determine the transformations that map one figure onto the other. Then find the missing side length.

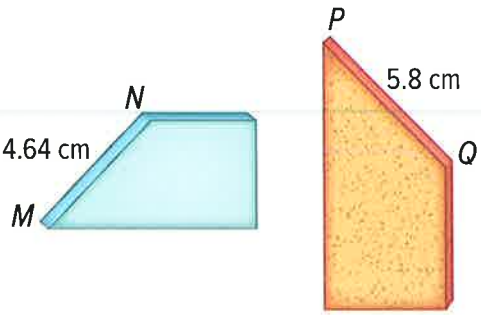


15. **Model with Mathematics**

Manal wants to build a fence around the rectangular garden in her backyard. In the scale drawing, the perimeter of the garden is 34 centimeters. If the actual length of $\overline{AB}$ is 6 meters, how many meters of fencing will she need?



16. Abdulaziz is making a mosaic using different pieces of tile. The tiles shown at the right are similar. If the perimeter of the larger tile is 23 centimeters, what is the perimeter of the smaller tile?



Power Up! Test Practice

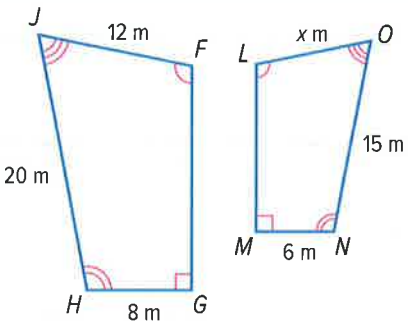
17. Quadrilateral *FGHJ* was transformed to create similar quadrilateral *LMNO*.

Determine if each statement is true or false.

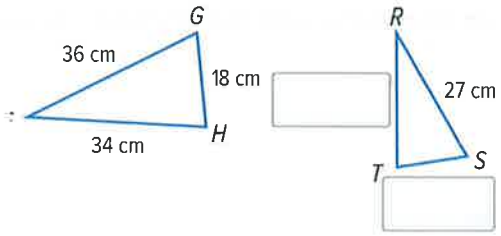
a. *FGHJ* was reflected and dilated to create *LMNO*. ☐ True ☐ False

b. The scale factor of the dilation is $\frac{3}{4}$. ☐ True ☐ False

c. The value of x is 16. ☐ True ☐ False



18. Triangle *FGH* is similar to triangle *RST*. Select the correct values to label the missing side lengths of triangle *RST*.



$13\frac{1}{2}$ cm

$14\frac{5}{7}$ cm

$22\frac{2}{3}$ cm

24 cm

$25\frac{1}{2}$ cm

Spiral Review

Find the scale factor for each scale drawing.

19. 6 cm = 1.44 m _____

20. 20 cm = 10 m _____

21. 15 cm =0.3 m _____

22. 8 cm = 2.5 mm _____

23. 2 cm = 0.5 km _____

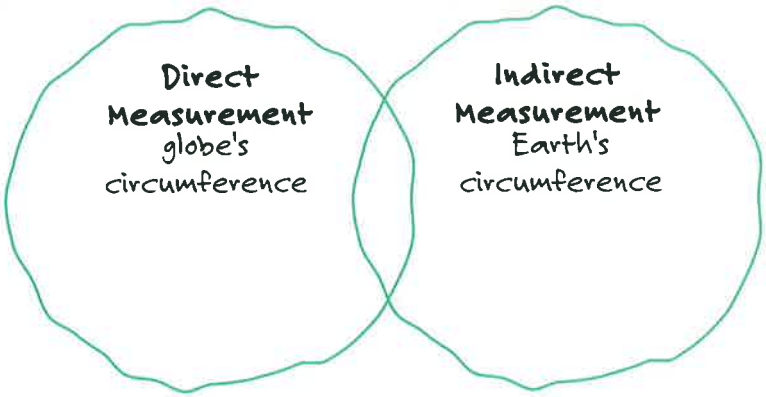
24. 5 m = 5 km _____

Similar Triangles and Indirect Measurement

Vocabulary Start-Up

Indirect measurement allows you to use properties of similar polygons to find distances or lengths that are difficult to measure directly.

Complete the graphic organizer. List three real-world examples in the Venn diagram for each method of measurement.



Write the name of an object that could be measured by either method. _____

Essential Question

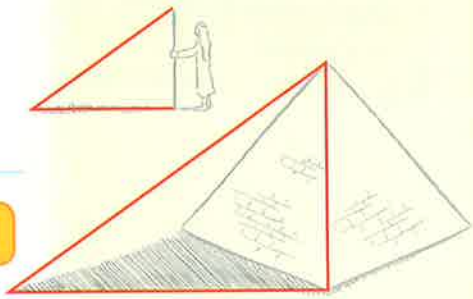
HOW can you determine congruence and similarity?

Vocabulary

indirect measurement

Mathematical Practices

1, 3, 4, 7



Real-World Link

Shadows Legend says that Thales, the first Greek mathematician, was the first to determine the height of the pyramids by examining the shadows made by the Sun.

1. What appears to be true about the corresponding angles in the two triangles? _____
2. If the corresponding sides are proportional, what could you conclude about the triangles? _____

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

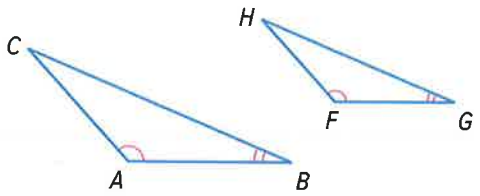
Key Concept

Angle-Angle (AA) Similarity

Words If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Symbols If $\angle A \cong \angle F$ and $\angle B \cong \angle G$, then $\triangle ABC \sim \triangle FGH$.

Model



Work Zone

In the figure below, $\angle X \cong \angle P$ and $\angle Y \cong \angle Q$. If you extend the sides of each figure to form a triangle, you can see the two triangles are similar. So, triangle similarity can be proven by showing two pairs of corresponding angles are congruent.

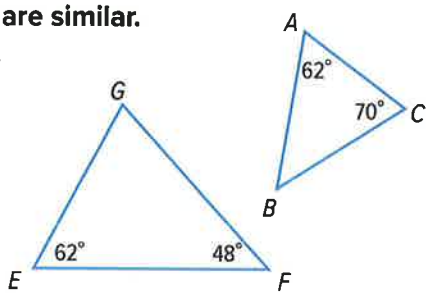
STOP and Reflect

What do you know about the third pair of angles in the triangle?

Example

1. **Determine whether the triangles are similar. If so, write a similarity statement.**

Angle A and $\angle E$ have the same measure, so they are congruent. Since $180 - 62 - 48 = 70$, $\angle G$ measures 70°. Two angles of $\triangle EFG$ are congruent to two angles of $\triangle ABC$, so $\triangle ABC \sim \triangle EFG$.

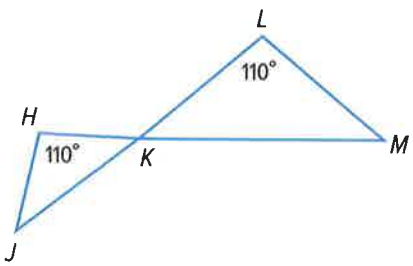


Show your work.

Got it? **Do this problem to find out.**

a. _____

a.

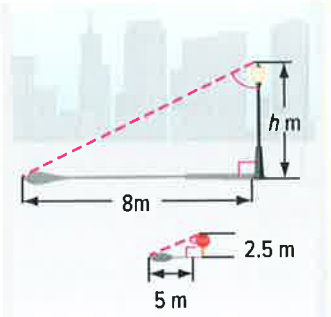


Use Indirect Measurement

One type of indirect measurement is *shadow reckoning*. Two objects and their shadows form two sides of right triangles. In shadow problems, you can assume that the angles formed by the Sun's rays with two objects at the same location are congruent. Since two pairs of corresponding angles are congruent, the two right triangles are similar. You can also use similar triangles that do not involve shadows to find missing measures.

Examples

2. **A road sign 2.5 meters high casts a 5-meter shadow. How tall is a street light that casts an 8-meter shadow at the same time? Let h represent the height of the street light.**



Shadow		Height
road sign →	$\frac{5}{8} = \frac{2.5}{h}$	← road sign
street light →		← street light

$5h = 8 \cdot 2.5$ Find the cross products.

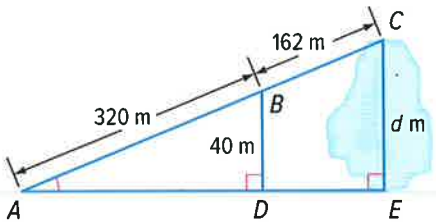
$5h = 20$ Multiply.

$\frac{5h}{5} = \frac{20}{5}$ Divide each side by 5.

$h = 4$

The street light is 4 meters tall.

3. **In the figure at the right, triangle *DBA* is similar to triangle *ECA*. Ramon wants to know the distance across the lake.**



$\frac{AB}{AC} = \frac{BD}{CE}$ $\overline{AB}$ corresponds to $\overline{AC}$ and $\overline{BD}$ corresponds to $\overline{CE}$.

$\frac{320}{482} = \frac{40}{d}$ Replace AB with 320, AC with 482, and BD with 40.

$320d = 482 \cdot 40$ Find the cross products.

$\frac{320d}{320} = \frac{19{,}280}{320}$ Multiply. Then divide each side by 320.

$d = 60.25$

The distance across the lake is 60.25 meters.

b. _____

Got it? **Do this problem to find out.**

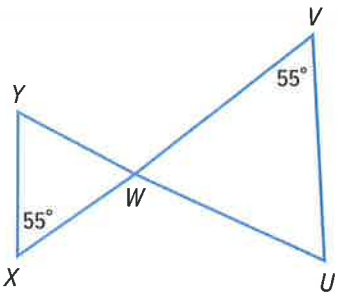
b. At the same time a 2-meter street sign casts a 3-meter shadow, a nearby telephone pole casts a 12.3-meter shadow. How tall is the telephone pole?

Guided Practice

Check

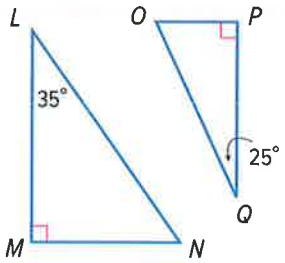
Determine whether the triangles are similar. If so, write a similarity statement. (Example 1)

1.

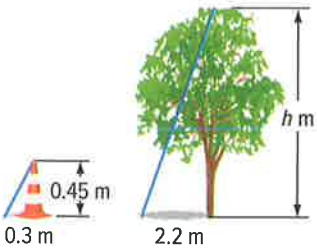


Show your work.

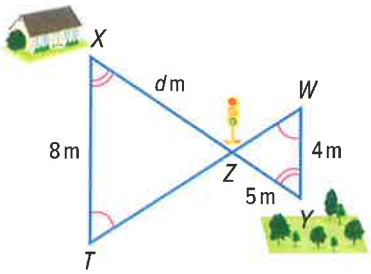
2.



3. How tall is the tree? (Example 2) _____



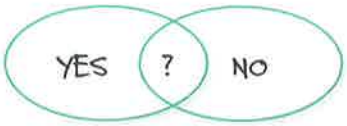
4. Find the distance from the house to the street light. (Example 3) _____



5. **Building on the Essential Question** How do similar triangles make it easier to measure very tall objects?

Rate Yourself!

Are you ready to move on? Shade the section that applies.

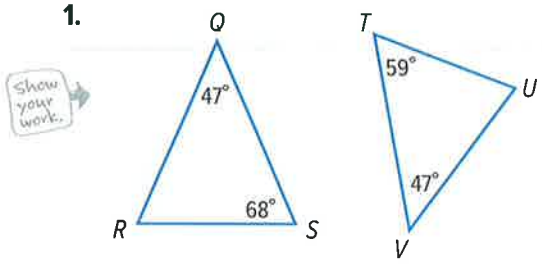


Name _____ My Homework _____

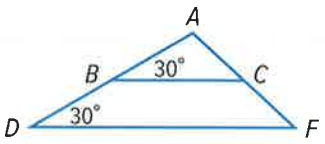
Independent Practice

Determine whether the triangles are similar. If so, write a similarity statement. (Example 1)

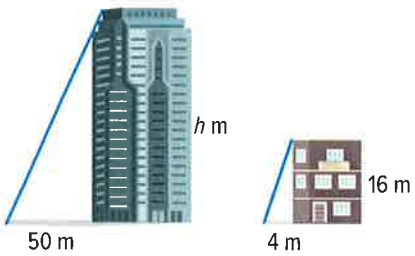
1.



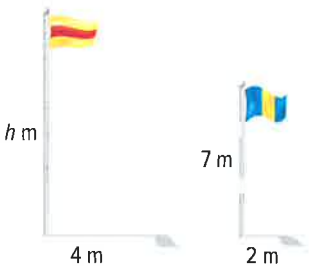
2.



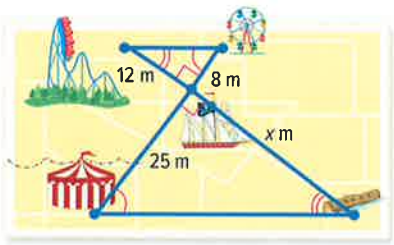
3. How tall is the building? (Example 2)



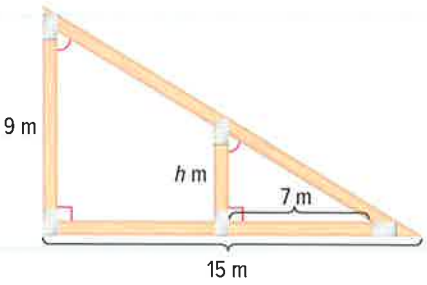
4. How tall is the taller flagpole? (Example 2)



5 How far is it from the log ride to the pirate ship? (Example 3)



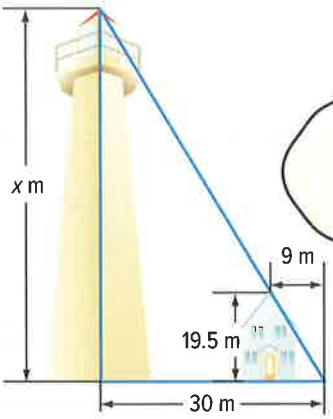
6. Find the height of the brace. (Example 3)

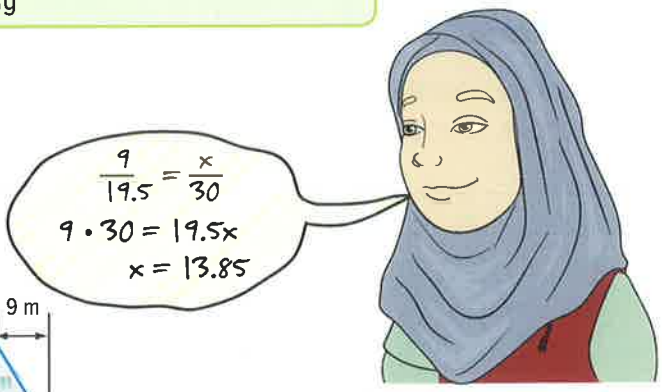


7 MP **Reason Abstractly** A giant ferris wheel is 136 meters tall. If the ferris wheel casts a 34-meter shadow, write and solve a proportion to find the height of a nearby street light that casts a $1\frac{1}{2}$-meter shadow.

H.O.T. Problems Higher Order Thinking

8. MP Find the Error Metha is finding the height of the lighthouse shown in the diagram. Find her mistake and correct it.



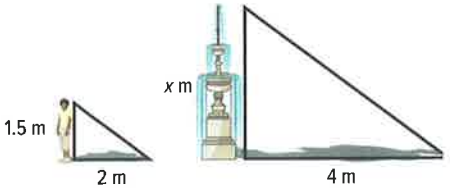


9. MP Model with Mathematics On a separate sheet of paper, draw two different triangles so that each one contains both of the angles shown. Then verify that they are similar by determining which transformation will map one onto the other.

10. MP Persevere with Problems You cut a circular hole $\frac{1}{2}$-centimeter in diameter in a piece of cardboard. With the cardboard 60 centimeters from your face, the Moon fits exactly into the hole. The Moon is about 390,000 kilometers from Earth. Is the Moon's diameter more than 2,500 kilometers? Justify your reasoning.

11. MP Identify Structure What measures must be known in order to calculate the height of tall objects using shadow reckoning?

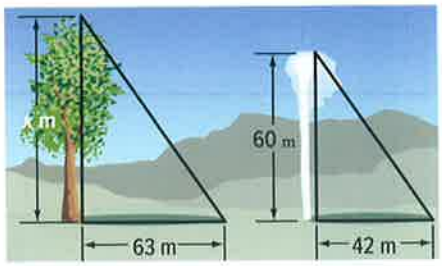
12. MP Reason Inductively Ali wants to estimate the height of a fountain in a local park. Ali's height and both shadow lengths are shown in the diagram. Is an estimate of 5 meters reasonable for the fountain's height? Explain your reasoning.



Name _____ My Homework _____

Extra Practice

13. What is the height of the tree? 90 m



Homework Help

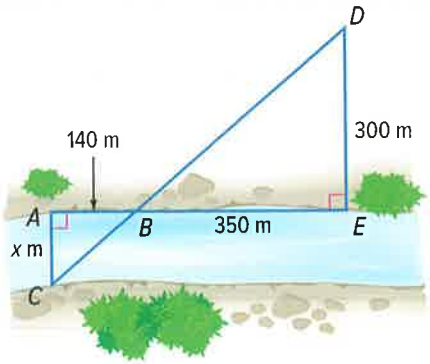
The triangles are similar. Write and solve a proportion.

$\frac{63}{42} = \frac{x}{60}$

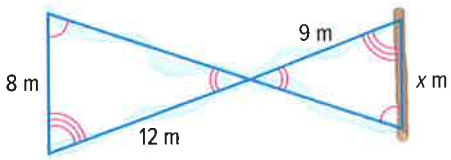
$63 \cdot 60 = 42x$

$90 = x$

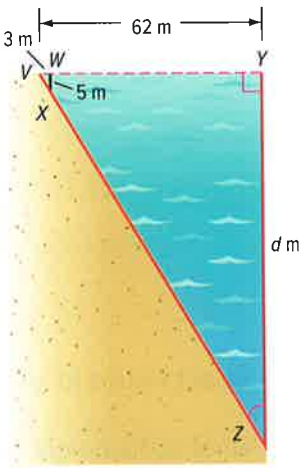
14. Find the distance across the river.



15. About how long is the log that goes across the creeks?



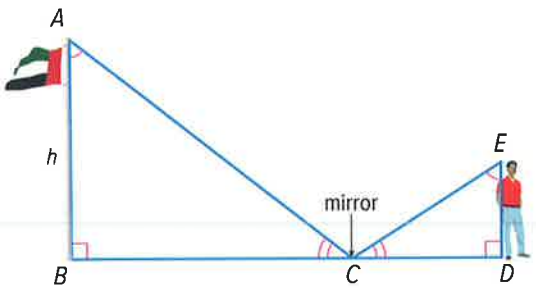
16. How deep is the water 62 meters from the shore?



17. In the diagram shown at the right, $\triangle ABC \sim \triangle EDC$.

a. Write a proportion that could be used to solve for the height h of the flag pole.

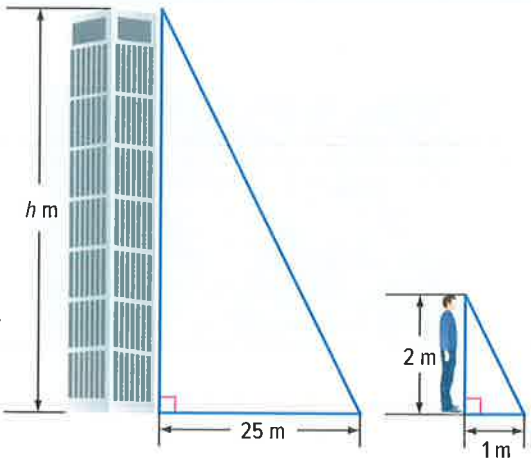
b. What information would you need to know in order to solve this proportion?



18. MP **Model with Mathematics** A 72 centimeter-tall child casts a shadow that is 48 centimeters long. At the same time, a nearby building casts a 16-meter-long shadow. Write and solve a proportion to find the height of the building.

Power Up! Test Practice

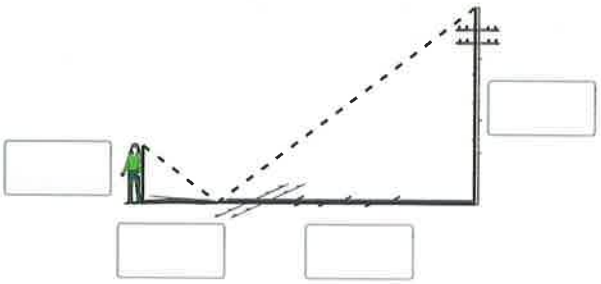
19. Omar is 2 meters tall and casts a shadow 1 meter long. At the same time, a nearby tower casts a shadow that is 25 meters long.



Write a proportion Omar can use to find the height of the tower. ☐

Using the proportion, the tower is ☐ meters tall.

20. Eissa is 1.6 meters tall and is using similar triangles and a mirror to find the height of a telephone pole. The horizontal distance between Eissa and the telephone pole is 9 meters. He places the mirror on the ground 2.4 meters from himself so that he can see the top of the pole in the mirror's reflection as shown in the figure below.



1.6 m	2.4 m	4.0 m	4.5 m
6.6 m	9 m	*h* m	

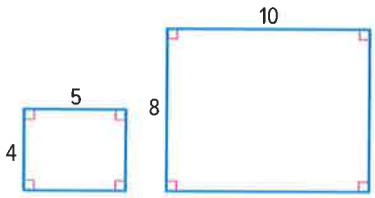
Select values to label the diagram with the correct dimensions.

What is the height of the telephone pole? ☐

Spiral Review

Determine whether each pair of polygons is similar. Explain.

21.



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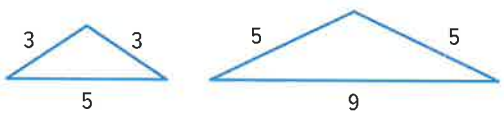
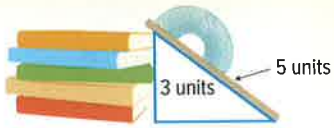


Lesson 6

Slope and Similar Triangles

Real-World Link

Physics In an experiment using a coiled spring toy, Faris and Faleh determined they needed to raise one side of a 5-unit board 2 units for the toy to move.



1. Find the slope of the board. (*Hint:* Use the Pythagorean Theorem to find how far the end of the board is from the books.)

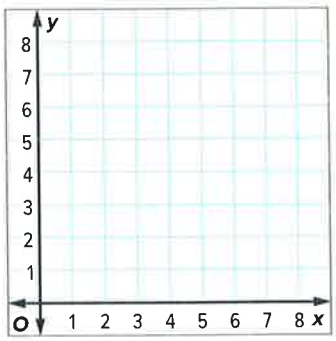
Essential Question

HOW can you determine congruence and similarity?

MP Mathematical Practices
1, 2, 3, 4

Collaborate

Work with a partner. Use the graph to discover how slope triangles are related.

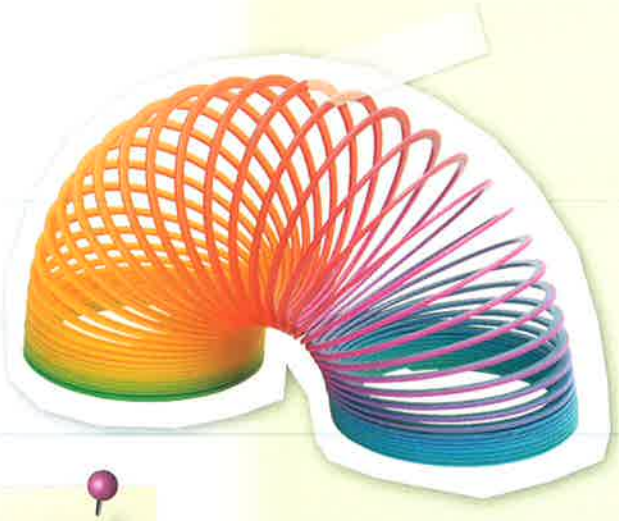


1. Draw the triangle formed by $A(0, 2)$, $B(0, 4)$, and $C(3, 4)$. What kind of triangle did you draw?

2. Draw the triangle formed by $D(6, 6)$, $F(6, 8)$, and $G(9, 8)$. How is $\triangle DFG$ related to $\triangle ABC$?

3. Draw the triangle formed by $A(0,2)$, $K(0, 6)$, and $D(6, 6)$. How is $\triangle AKD$ related to $\triangle ABC$?

4. What is true about the hypotenuses of the three triangles in Steps 1, 2, and 3?



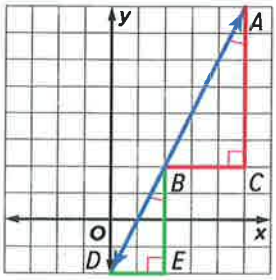
Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Work Zone

Similar Triangles and the Coordinate Plane

In the figure shown, $\triangle ABC$ and $\triangle BDE$ are slope triangles. Slope triangles are similar.



$\angle BAC \cong \angle DBE$	Given
$\angle ACB \cong \angle BED$	Given
$\triangle ABC \sim \triangle BDE$	Angle-Angle Similarity

You can use the properties of similar triangles to show the ratios of the rise to the run for each right triangle are equal.

Example

1. **Write a proportion comparing the rise to the run for each of the similar slope triangles shown above. Then find the numeric value.**

$\frac{AC}{BE} = \frac{BC}{DE}$	Corresponding sides of similar triangles are proportional.
$AC \cdot DE = BE \cdot BC$	Find the cross products.
$\frac{AC \cdot DE}{BC \cdot DE} = \frac{BE \cdot BC}{BC \cdot DE}$	Division Property of Equality
$\frac{AC}{BC} = \frac{BE}{DE}$	Simplify.
$\frac{6}{3} = \frac{4}{2}$	$AC = 6, BC = 3, BE = 4, DE = 2$

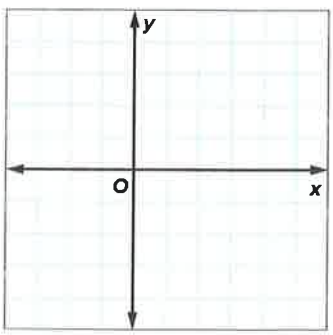
So, $\frac{AC}{BC} = \frac{BE}{DE}$, or $\frac{6}{3} = \frac{4}{2}$.

Show your work.

Got it? **Do this problem to find out.**

a. _____

a. Graph $\triangle MNO$ with vertices $M(3, 1)$, $N(1, 0)$, and $O(3, 0)$, and $\triangle PQR$ with vertices $P(5, 2)$, $Q(-1, -1)$, and $R(5, -1)$. Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value.

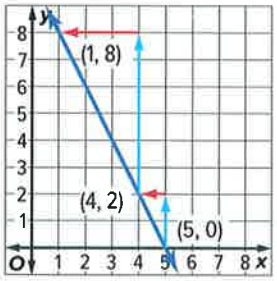


Similar Triangles and Slope

Key Concept

Words The ratio of the rise to the run of two slope triangles formed by a line is equal to the slope of the line.

Example



Larger Triangle

$\frac{\text{rise}}{\text{run}} = \frac{6}{-3} = -2$

Smaller Triangle

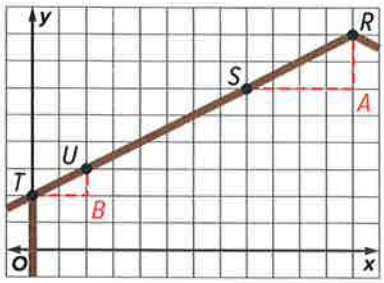
$\frac{\text{rise}}{\text{run}} = \frac{2}{-1} = -2$

$\text{slope} = \frac{-2}{1} = -2$

The ratios of the rise to the run of the two similar slope triangles in Example 1 are the same as the slope of the line. Since the ratios are equal, the slope m of a line is the same between any two distinct points on a non-vertical line in the coordinate plane.

Example

2. **The pitch of a roof refers to the slope of the roof line. Choose two points on the roof and find the pitch of the roof shown. Then verify that the pitch is the same by choosing a different set of points.**



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Formula for slope

$m = \frac{8 - 6}{12 - 8}$ Use the points S and R. $(x_1, y_1) = (8, 6)$ and $(x_2, y_2) = (12, 8)$

$m = \frac{2}{4} = \frac{1}{2}$ Simplify.

The pitch of the roof is $\frac{1}{2}$. Verify that the pitch is the same using two other points.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Formula for slope

$m = \frac{2 - 3}{0 - 2}$ Use the points U and T. $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (0, 2)$

$m = \frac{-1}{-2} = \frac{1}{2}$ Simplify. The pitch is the same.

STOP and Reflect

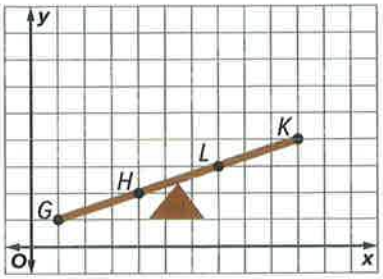
Is the statement $\Delta RAS \sim \Delta UBT$ true? Explain below.

Show your work.

b. _____

Got it? **Do this problem to find out.**

b. The plans for a teeter-totter are shown at the right. Using points G and L, find the slope of the teeter-totter. Then verify that the slope is the same at a different location by choosing a different set of points.

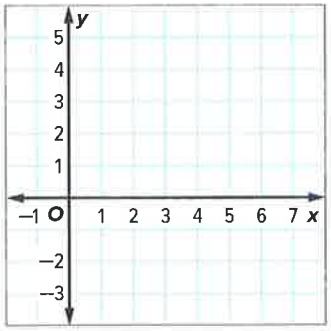


Guided Practice

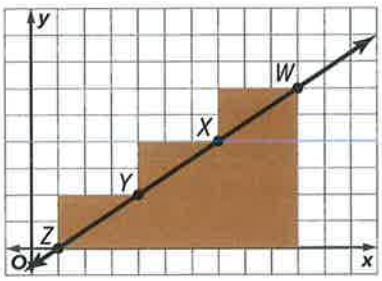
Check

1. Graph $\triangle ACG$ with vertices $A(1, 4)$, $C(3, -2)$, and $G(1, -2)$, and $\triangle BCF$ with vertices $B(2, 1)$, $C(3, -2)$, and $F(2, -2)$. Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value. (Example 1)

Show your work.



2. The plans for a set of stairs are shown below. Using points X and Z, find the slope of the line down the stairs. Then verify that the slope is the same at a different location by choosing a different set of points. (Example 2)



3. **Building on the Essential Question** How is the slope of a line related to the similar slope triangles formed by the line?

Rate Yourself!

How confident are you about slope and similar triangles? Check the box that applies.

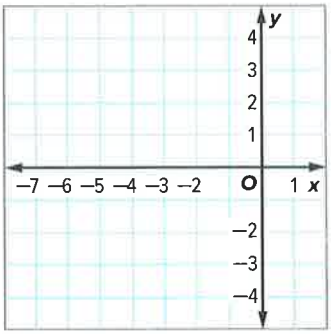
Name _____ My Homework _____

Independent Practice

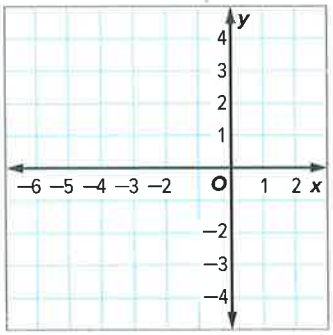
Graph each pair of similar triangles. Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value. (Example 1)

1. $\triangle ABC$ with vertices $A(-6, -1)$, $B(-4, -1)$, and $C(-6, -3)$; $\triangle NLM$ with vertices $N(-3, 3)$, $L(0, 3)$, and $M(-3, 0)$

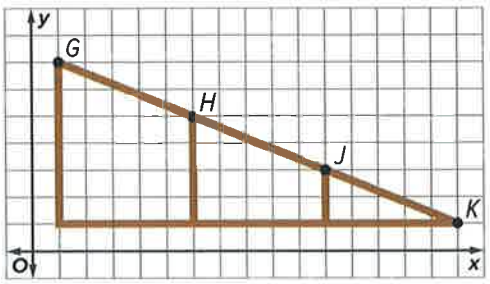
Show your work.



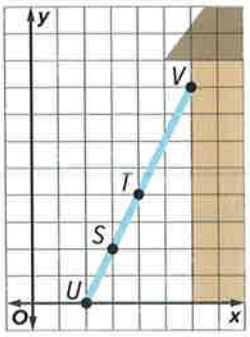
2. $\triangle FGH$ with vertices $F(2, 3)$, $G(2, -1)$, and $H(-6, 3)$; $\triangle JKL$ with vertices $J(0, 2)$, $K(0, 0)$, and $L(-4, 2)$



3. The plans for a skateboard ramp are shown. Use two points to find the slope of the ramp. Then verify that the slope is the same at a different location by choosing a different set of points. (Example 2)



4. A ladder is leaning up against the side of a house. Use two points to find the slope of the ladder. Then verify that the slope is the same at a different location by choosing a different set of points. (Example 2)



5. **Reason Abstractly** Triangle XYZ has vertices $X(0, 0)$, $Y(10, 0)$, and $Z(0, 6)$. Triangle MYP has vertices $M(5, 0)$, $Y(10, 0)$, and $P(x, y)$. Find the missing coordinates for P if $\triangle MYP \sim \triangle XYZ$.

6. **Model with Mathematics** Refer to the graphic novel frame below. On the beach, a cable is attached to the pier. The line formed by the cable has a slope of $\frac{3}{5}$. Is the triangle formed by the pier, the beach, and the cable similar to the triangle formed by the boat, the parasailer, and the rope? Explain.

H.O.T. Problems Higher Order Thinking

7. **Model with Mathematics** On a separate piece of grid paper, draw the graph of a line with a positive slope. Draw two slope triangles formed by the line. Demonstrate that the simplified ratio of the rise to the run of each triangle is equivalent to the slope.

8. **Persevere with Problems** The slope of a line is -3.5. Find two possible measurements for the legs of similar slope triangles. Explain your reasoning.

9. **Reason Inductively** Triangle JKL has vertices $J(0, 0)$, $K(1, 0)$, and $L(1, 2)$. Determine if each triangle is similar to and/or a slope triangle with $\triangle JKL$.

   a. $\triangle ABC$: $A(1, 2)$, $B(1, 6)$, $C(3, 6)$

   b. $\triangle MNP$: $M(3, 1)$, $N(6, 1)$, $P(6, 7)$

   c. $\triangle RST$: $R(1, 2)$, $S(4, 2)$, $T(4, 5)$

   d. $\triangle WXY$: $W(0, 0)$, $X(-1, -2)$, $Y(0, -2)$

Name _____ My Homework _____

Extra Practice

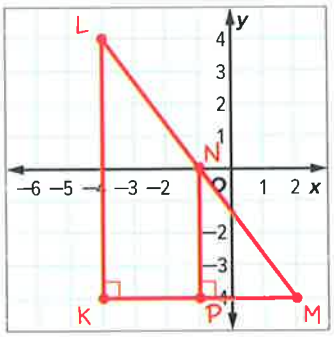
Graph each pair of similar triangles. Then write a proportion comparing the rise to the run for each of the similar slope triangles and find the numeric value.

10. $\triangle LKM$ with vertices $L(-4, 4)$, $K(-4, -4)$, and $M(2, -4)$; $\triangle NPM$ with vertices $N(-1, 0)$, and $P(-1, -4)$

$\frac{LK}{KM} = \frac{NP}{PM}$, or $-\frac{4}{3}$

Graph and label each triangle.

Homework Help

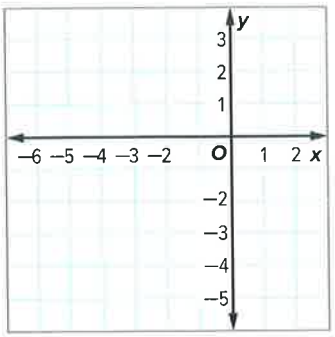


Write the proportion using the side labels.

$\frac{LK}{KM} = \frac{NP}{PM}$

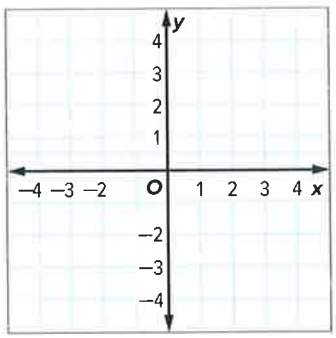
$-\frac{8}{6} = -\frac{4}{3}$

11. $\triangle ABC$ with vertices $A(-5, -6)$, $B(1, -6)$, and $C(1, 3)$; $\triangle GFD$ with vertices $G(-3, -3)$, $F(-1, -3)$, and $D(-1, 0)$

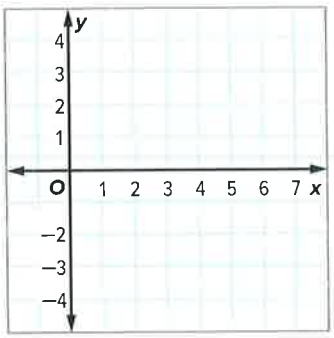


MP Model with Mathematics **Use a graph to find the missing coordinates for point Z if $\triangle MNP \sim \triangle XYZ$.**

12. $M(-2, -3)$, $N(2, -3)$, $P(2, 3)$, $X(0, 0)$, $Y(2, 0)$



13. $M(5, 0)$, $N(5, -3)$, $P(2, -3)$, $X(7, 2)$, $Y(1, 2)$



Copy and Solve **Find the missing coordinates for point D if $\triangle ABC \sim \triangle DEF$. Show your work on a separate sheet of paper.**

14. $A(-1, 3)$, $B(1, 3)$, $C(1, 6)$, $E(-4, -7)$, $F(-4, -1)$

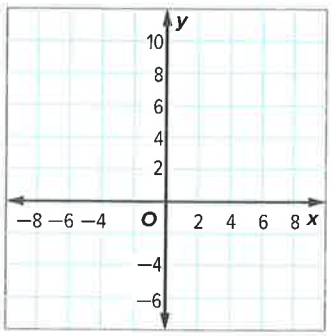
15. $A(1, 11)$, $B(1, 6)$, $C(3, 6)$, $E(1, 1)$, $F(5, 1)$

Power Up! Test Practice

16. Triangle *ABC* with vertices *A*(−3, 7), *B*(−3, 5), and *C*(0, 5), and triangle *CDE* with vertices *C*(0, 5), *D*(0, 1), and *E*(6, 1) are slope triangles.

Draw the triangles and the line that they represent on the coordinate plane.

Find the slope of the line. Then describe the relationship between the slope triangles and the slope of the line.



17. The statements below refer to any non-vertical line on the coordinate plane. Determine if each statement is true or false

a. All of the slope triangles on the line are similar. ☐ True ☐ False

b. The slope is the same between any two distinct points on the line. ☐ True ☐ False

c. In the slope triangles, the ratios of the rise to the run are equal to the absolute value of the slope. ☐ True ☐ False

Spiral Review

Find the slope of the line that passes through each pair of points.

18. (2, 2), (−2, −2) _____

19. (5, −4), (9, −4) _____

20. (4, 3), (−1, 6) _____

21. (3, 3), (3, 5) _____

22. (0, 0), (3, −6) _____

23. (−8, −15), (−2,−5) _____

24. (−3, 5), (3, 6) _____

25. (0.2, 0.7), (1.7, 1.2) _____

26. (−5, 0), (3, −2) _____

Lesson 7

Area and Perimeter of Similar Figures

Real-World Link

Watch

Games Four square is a ball game played on a hard surface. The court is a 4-meter by 4-meter square divided into four equal squares.

Essential Question

HOW can you determine congruence and similarity?

MP Mathematical Practices

1, 2, 3, 4

1. Use the figure to draw a four square court. Divide each side in half. Draw lines to divide the court into four equal squares. Is each smaller square similar or congruent to the larger square? Explain.

2. What is the perimeter of the larger square drawn above?

   ____ meters; the smaller square? ____ meters

3. How is the perimeter of one of the smaller squares related to the perimeter of the larger square and the scale factor?

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Key Concept

Perimeter and Area of Similar Figures

Work Zone

Perimeter

Words If figure B is similar to figure A by a scale factor, then the perimeter of B is equal to the perimeter of A times the scale factor.

Symbols perimeter of figure B = perimeter of figure A · scale factor

Area

Words If figure B is similar to figure A by a scale factor, then the area of B is equal to the area of A times the square of the scale factor.

Symbols area of figure B = area of figure A · (scale factor)^2

Models

a

Figure A

b

Figure B

In similar figures, the perimeters are related by the scale factor, k. What about area? The area of one similar figure is equal to the area of the other similar figure times the *square* of the scale factor, or k^2.

Example

1. Two rectangles are similar. One has a length of 6 centimeters and a perimeter of 24 centimeters. The other has a length of 7 centimeters. What is the perimeter of this rectangle?

The scale factor is $\frac{7}{6}$. The perimeter of the original is 24 centimeters.

$x = 24\left(\frac{7}{6}\right)$ Multiply by the scale factor.

$x = \frac{\cancel{24}^{4}}{1}\left(\frac{7}{\cancel{6}_{1}}\right)$ Divide out common factors.

$x = 28$ Simplify.

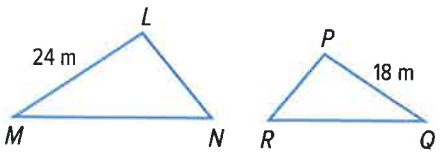
So, the perimeter of the new rectangle is 28 centimeters.

Show your work.

Got it? **Do this problem to find out.**

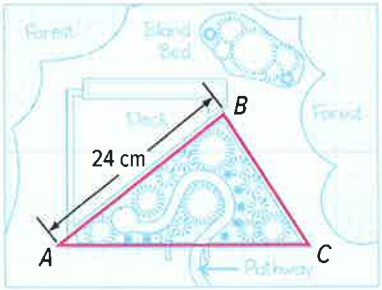
a. Triangle LMN is similar to triangle PQR. If the perimeter of $\triangle LMN$ is 64 meters, what is the perimeter of $\triangle PQR$?

a. _____



Example

2. **In a scale drawing, the perimeter of the garden is 64 centimeters. The actual length of $\overline{AB}$ is 18 meters. What is the perimeter of the actual garden?**



Step 1 The actual length is proportional to the length in the drawing with a ratio of $\frac{18 \text{ m}}{24 \text{ cm}}$. Find the scale factor.

$\frac{18 \text{ m}}{24 \text{ cm}} = \frac{1{,}800 \text{ cm}}{24 \text{ cm}}$ or $\frac{75}{1}$ Convert meters to centimeters and divide out units.

Step 2 Find the perimeter of the actual garden.

perimeter of garden = perimeter of drawing • scale factor

$P = 64 \cdot 75$ or $4{,}800$ Substitute. Then simplify.

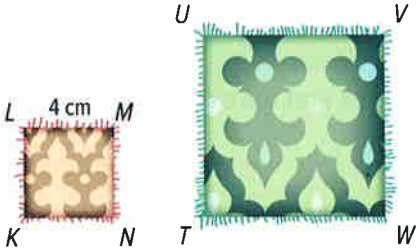
The perimeter of the actual garden is 4,800 cm or 48 meters.

Scale Factor

When finding the scale factor from one figure to another, the units must be the same.

Got it? **Do this problem to find out.**

b. Two quilting squares are shown. The scale factor is 3:2. What is the perimeter of square *TUVW*?

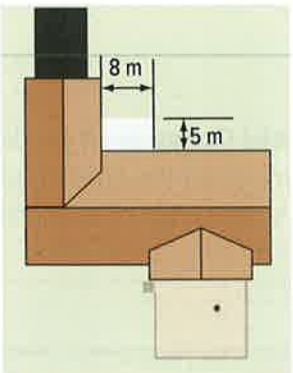


Show your work.

b. _____

Example

3. **The Jamals have a 5-meter by 8-meter patio at the front of their house. They are building a similar patio at the back with double the dimensions. Find the area of the back patio.**



The scale factor is 2.

The area of the front patio is (5)(8) or 40 square meters.

$x = 40(2)^2$ Multiply by the square of the scale factor.

$x = 40(4)$ or 160 Evaluate the power.

The back patio will have an area of 160 square meters.

Show your work.

c. _____

Got it? **Do this problem to find out.**

c. Noura is painting a mural on her bedroom wall. The image she is reproducing is 4.8 centimeters by 7.2 centimeters. If the dimensions of the mural are 10 times the dimensions of the image, find the area of the mural in square centimeters.

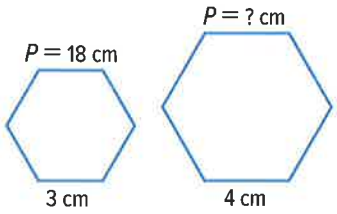
Guided Practice

Check

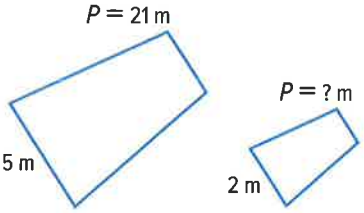
For each pair of similar figures, find the perimeter of the second figure.

(Example 1)

1.



2.



Show your work.

3. Hidaya is enlarging a digital photograph on her computer. The original photograph is 5 centimeters by 7 centimeters. If she enlarges the dimensions 1.5 times, what will be the perimeter and area of the new image? (Examples 2 and 3)

4. Mahmoud is flying a kite that is made up of three similar rectangles. The sides of the three rectangles are in the ratio 1:2:3. If the area of the smallest rectangle is 72 square centimeters, what are the areas of the other two rectangles? (Example 3)

5. **Building on the Essential Question** If you know two figures are similar and you are given the area of both figures, how can you determine the scale factor of the similarity?

Rate Yourself!

☐ I understand how to find the perimeter and area of similar figures.

Great! You're ready to move on!

☐ I still have some questions about the perimeter and area of similar figures.

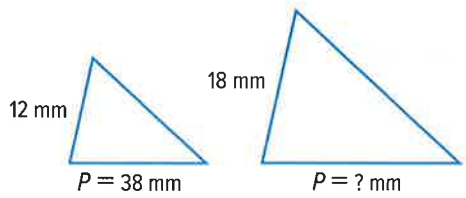
Name _____ My Homework _____

Independent Practice

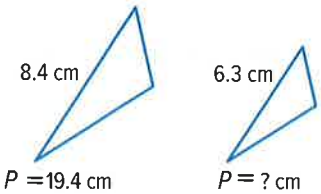
For each pair of similar figures, find the perimeter of the second figure.

(Example 1)

1.

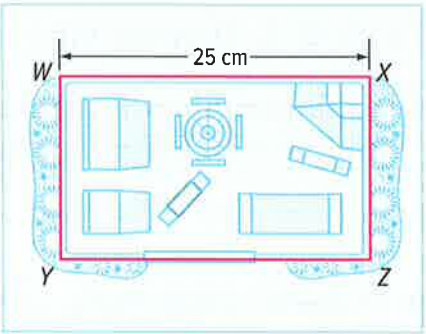


2.



Show your work.

3. A city is planning to build a skate park. An architect designed the area shown at the right. In the plan, the perimeter of the park is 80 centimeters. If the actual length of $\overline{WX}$ is 50 meters, what will be the perimeter of the actual skate park? (Example 2)



4. A child's desk is made so that the dimensions are two-thirds the dimensions of a full-size adult desk. Suppose the top of the full-size desk measures 135 centimeters long by 90 centimeters wide. What is the perimeter and area of the top of the child's desk? (Examples 2 and 3)

5. Mansour is constructing a miniature putting green in his backyard. He wants it to be similar to a putting green at the local golf course, but one third the dimensions. The area of the putting green at the golf course is 378 square meters. What will be the area of the putting green Mansour constructs?

6. Nasser is making a model version of his neighborhood that uses model trains. The ratio of the model train to the actual train is 1:64. His neighborhood covers an area of 18,432 square meters. What will be the area of the model neighborhood?

7. **Identify Structure** Complete the graphic organizer to compare how the scale factor affects the side lengths, perimeter, and area of similar rectangles.

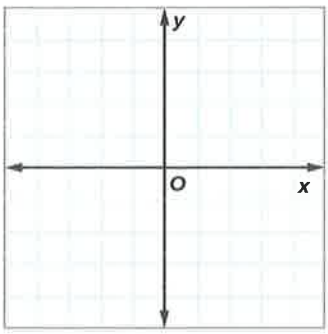
If the scale factor is...	Multiply the ...			
	Length by	Width by	Perimeter by	Area by
2	2	2	2	4
4	4	4	4	16
0.5	0.5	0.5	0.5	0.25
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{4}{9}$
k	k	k	k	k^2

H.O.T. Problems Higher Order Thinking

8. **Persevere with Problems** Two circles have circumferences of π and 3π. What is the ratio of the area of the circles? the diameters? the radii?

9. **Justify Conclusions** A company wants to reduce the dimensions of its logo from 15 centimeters by 10 centimeters to 7.5 centimeters by 5 centimeters to use on business cards. Yousif thinks that the new logo is $\frac{1}{4}$ the size of the original logo. Saeed thinks that it's $\frac{1}{2}$ of the original size. Explain their thinking to a classmate.

10. **Use Math Tools** Use the coordinate plane to draw a rectangle. Dilate the rectangle and draw the dilation. Then determine the perimeter and area of each rectangle to model the effect of the dilation.



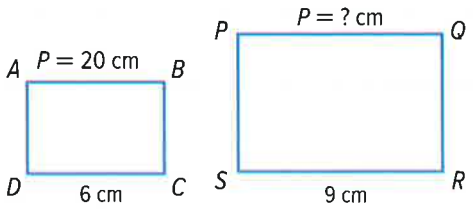
Show your work.

Name _____ My Homework _____

Extra Practice

For each pair of similar figures, find the unknown perimeter.

11.

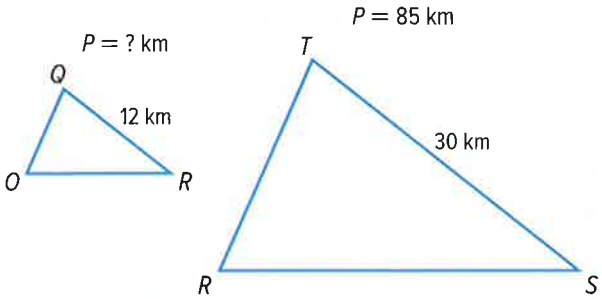


Homework Help

The scale factor is $\frac{3}{2}$. Multiply the perimeter of ABCD by $\frac{3}{2}$.

$P = 20 \cdot \frac{3}{2}$ or 30
30 cm

12.



13. For your dinner party, you make a map to your house on a 3-centimeter-wide by 5-centimeter-long index card. What will be the perimeter and area of your map if you use a copier to enlarge it so it is 8 centimeters long?

14. A company wants to reduce the dimensions of its logo by one fourth to use on business cards. If the area of the original logo is 16 square centimeters, what is the area of the logo that will be used on the business cards?

15. Two picture frames are similar. The ratio of the perimeters of the two pieces is 3:5. If the area of the smaller frame is 108 square centimeters, what is the area of the larger frame?

16. MP **Persevere with Problems** Mr. Ayoub is enlarging a logo for printing on the back of a T-shirt. He wants to enlarge a logo that is 3 centimeters by 5 centimeters so that the dimensions are 3 times larger than the original. How many times as large as the original logo will the area of the printing be?

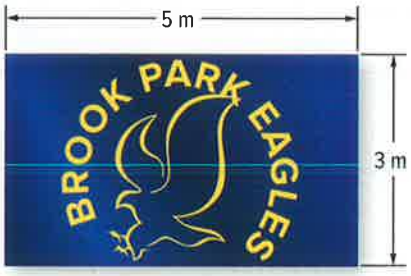
Power Up! Test Practice

17. A photograph is enlarged to 3 times the size of the original. Fill in the boxes to complete each statement.

The area of the enlargement is ☐ times the original area.

The perimeter of the enlargement is ☐ times the original perimeter.

18. A smaller version of this school banner is being made to appear on the front of the students' homework agenda books.



The perimeter of the smaller version of the flag is 2 meters. Select the correct values to complete each statement.

1	2	4	8	16
$\frac{1}{64}$	$\frac{1}{8}$	$\frac{15}{8}$	$\frac{15}{64}$	

a. The perimeter of the full size flag is ☐ meters

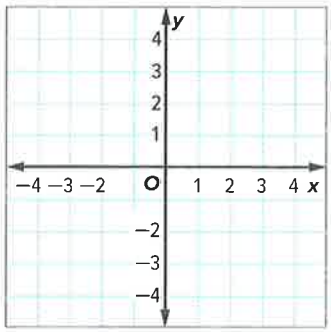
b. The scale factor of the reduction is ☐.

c. The area of the smaller version of the flag is ☐ square meters.

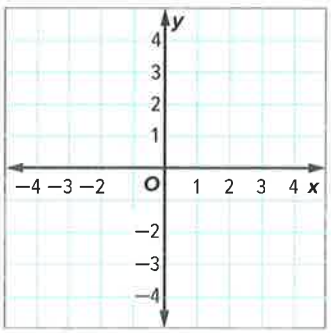
Spiral Review

Graph each figure with the given vertices and its image after the indicated transformation.

19. $\triangle ABC$: $A(0,-1)$, $B(0,3)$, $C(3, 3)$
90° clockwise rotation about the origin



20. $\overline{XY}$: $X(1, 1)$, $Y(-2, -3)$
translation of 1 unit right and 3 units up



21ST CENTURY CAREER in Car Design

Car Designer

Do you like drawing? Are you technical and precise in your drawings? You should consider a career as a car designer. Car designers use Computer Aided Design to create technical drawings that are used in manufacturing and construction. Information from architects and engineers is used to create highly specialized drawings that show how to construct everything from a nightstand to the space shuttle.

Is This the Career for You?

Are you interested in a career as a car designer? Take some of the following courses in high school.

- Geometry
- Mechanical Drawing
- Computer Graphics
- Design

Turn the page to find out how math relates to a career in Car Design.

MP Drive Yourself to Success

Use the information on the drawing to solve each problem.

1. What transformation maps the drawing to the actual car? _____

2. Are the views of the drawing of the car similar to views of the actual car? Explain.

   _____

   _____

3. If the scale factor is $\frac{1}{25}$, find the following:

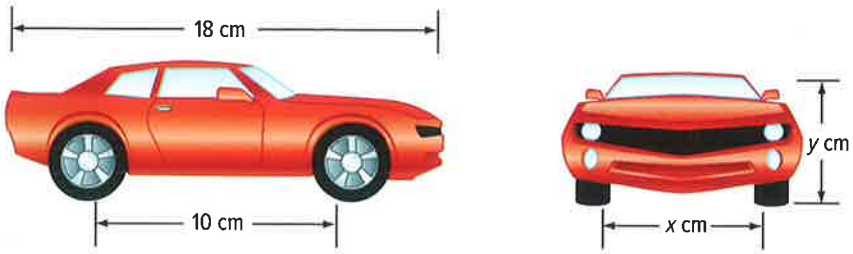
   a. the length of the actual car _____

   b. the distance from the front wheel to the rear wheel of the actual car _____

4. If the actual height of the car is 150 centimeters, what is y? _____

5. If $x = 7$ centimeters, what is the actual distance between the tires on the car?

   _____



MP Career Project

It's time to update your career portfolio! Describe the features that you, as a car designer, would include in a new car design. Determine whether these features already exist in cars today.

_____

_____

_____

_____

_____

_____

List several challenges associated with this career.

- _____
- _____
- _____
- _____
- _____

Chapter Review

Vocabulary Check

Reconstruct the vocabulary word and definition from the letters under the grid. The letters for each column are scrambled directly under that column.

	S	I	M	I	L	A	R		
P	O	L	Y	G	O	N	S	:	
			T	W	O				
	P	O	L	Y	G	O	N	S	
	W	I	T	H		T	H	E	
S	A	M	E		S	H	A	P	E

T

A L M I G O S

P I T W S H R

O O E H O T H S

S S I L G O A A P

P W M Y Y L N N E E

Complete each sentence using vocabulary from the chapter.

1. Two figures are _____ if one can be obtained from the other by a series of rotations, reflections, or translations.
2. _____ uses properties of similar polygons to find distances or lengths that are difficult to measure directly.
3. The parts of congruent figures that match are called _____.
4. Two figures are _____ if one can be obtained from the other by a series of transformations and dilations.
5. When a transformation is applied to a figure and then another transformation is applied to the image, the result is called a _____.

Key Concept Check

Use Your FOLDABLES

Use your Foldable to help review the chapter.

Tape here

Tab 1 **Congruent Figures**

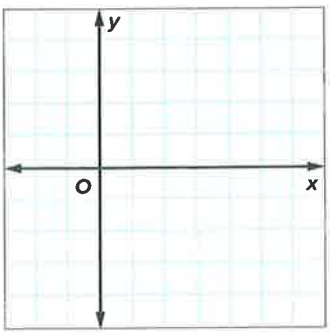
Draw	Draw
Draw	Draw

Tab 2 **Similar Figures**

Tape here

Got it?

Triangle *ABC* has vertices *A*(0, 0), *B*(2, 4), *C*(6, 0). Match each image with the description of its transformation.



1. $A'(0, 0), B'(2, -4), C'(6, 0)$
2. $A'(0, 0), B'(1, 2), C'(3, 0)$
3. $A'(0, 0), B'(4, -2), C'(0, -6)$
4. $A'(2, -6), B'(6, 2), C'(14, -6)$

a. similar; a dilation with a scale factor of $\frac{1}{2}$

b. congruent; a 90° clockwise rotation about the origin

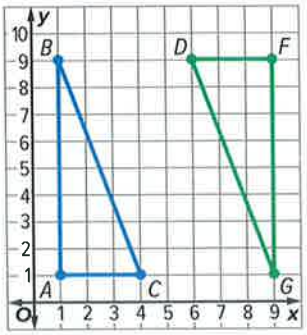
c. congruent; a reflection over the *x*-axis

d. similar; a translation of $(x + 1, y - 3)$ followed by a dilation with a scale factor of 2

Power Up! Performance Task

Can Triangles Model Maps?

Mazen is investigating the relationship between triangles and how they can model a city street map. On the graph, one unit is equal to one centimeter. The grid shows two right triangles. $\overline{DG} \cong \overline{CB}$, $m\angle B = 20.6°$, and $m\angle D = 69.4°$.



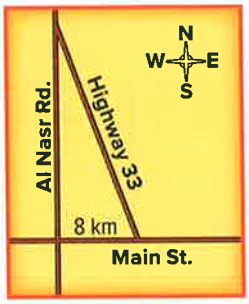
Write your answers on another piece of paper. Show all of your work to receive full credit.

Part A

Are the two triangles congruent? Explain your reasoning. What transformation(s) could be used to help determine that the two triangles are congruent?

Part B

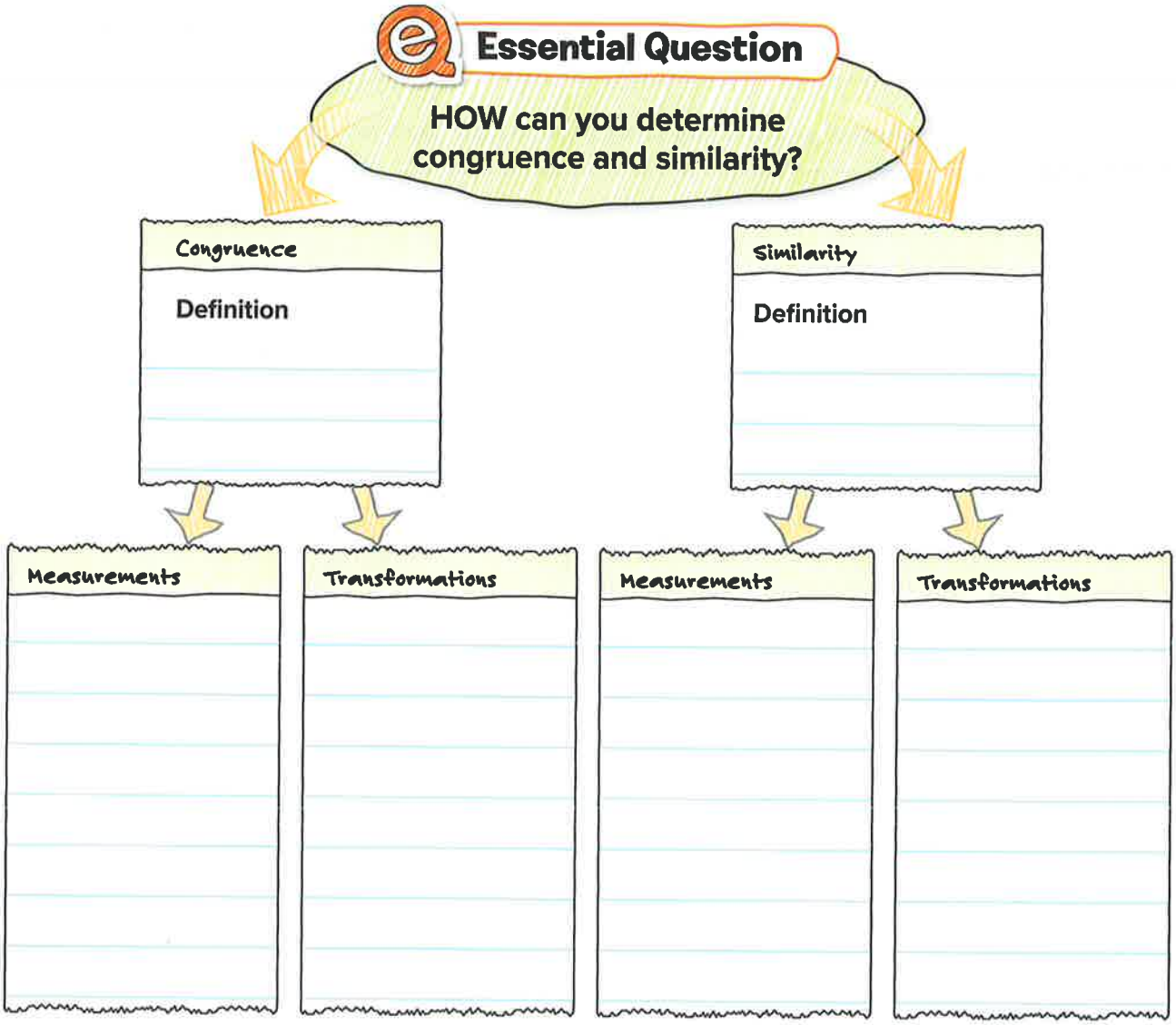
The triangle formed by the intersections of Al Nasr Road, Main Street, and Highway 33 is similar to triangle *ABC*. The section of Main Street between Oak Road and Highway 33 is eight kilometers long. Based on the information given, what is the length of Oak Road from Main Street north to the intersection of Highway 33? Set up a proportion and solve. Round your answer to the nearest tenth.



Reflect

Answering the Essential Question

Use what you learned about congruence and similarity to complete the graphic organizer. Describe how you would show congruence or similarity using measurements and transformations.



Answer the Essential Question. HOW can you determine congruence and similarity?

Chapter 8
Volume and Surface Area

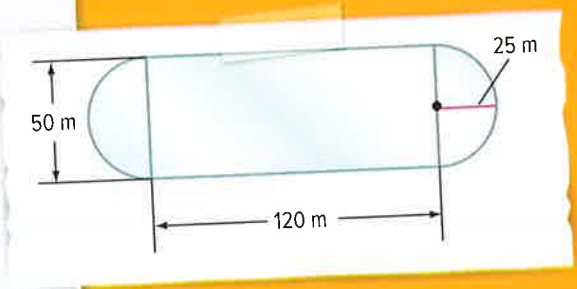
Essential Question

WHY are formulas important in math and science?

MP Mathematical Practices
1, 2, 3, 4, 6, 7

Math in the Real World

Ice Skating During the winter, Reham and her friends watch speed skating races at a local park. The ice skating rink is made up of two semi-circles and a rectangle. What is the area of the rink?



FOLDABLES® Study Organizer

1. Cut out the Foldable on page FL9 of this book.
2. Place your Foldable on page 652.
3. Use the Foldable throughout this chapter to help you learn about volume and surface area.

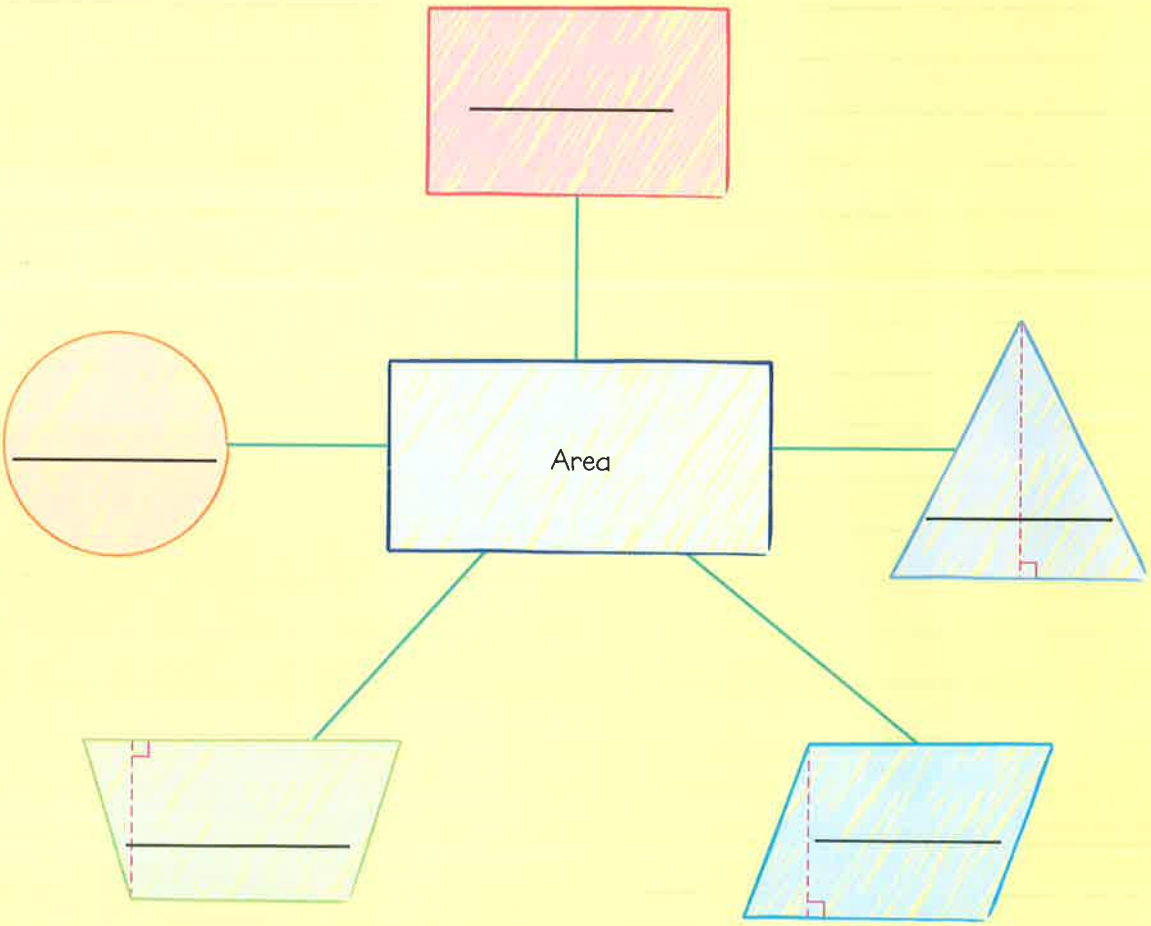
What Tools Do You Need?

Vocabulary

composite solids	lateral area	sphere
cone	nets	total surface area
cylinder	polyhedron	volume
hemisphere	similar solids	

Review Vocabulary

Area The area of a geometric figure is the measure of the surface enclosed by the figure. Write the correct area formula in each shape.



What Do You Already Know?

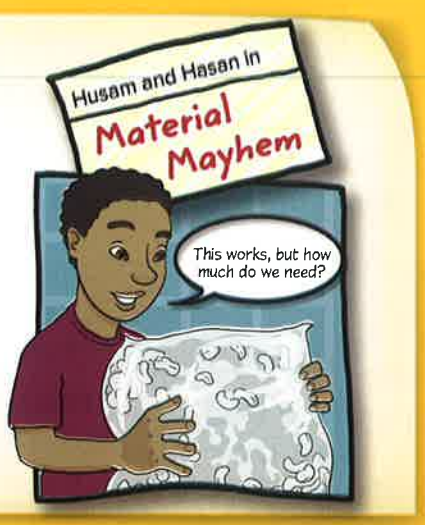
Read each statement. Decide whether you agree (A) or disagree (D). Place a checkmark in the appropriate column and then justify your reasoning.

Volume and Surface Area			
Statement	A	D	Why?
To find the volume of a cylinder, multiply the area of the base by the height of the cylinder.			
The formula to find the volume of a cone is the same as the formula to find the volume of a cylinder.			
A sphere is a set of all points in space that are a given distance from a given point called the center.			
The lateral area of a three-dimensional figure is the sum of the areas of all its surfaces.			
The net of a cylinder consists of two circles and a rectangle.			
To find the lateral area of a cone, you multiply π by the radius of the base by the height of the cone.			

When Will You Use This?

Here is an example of how unit rates are used in the real world.

Activity 1 Khawla's mom made soup in a large pot and is going to pour it into 1-liter jars. Name some ways her mom could estimate how many jars she would need.



Are You Ready?

Try the Quick Check below.

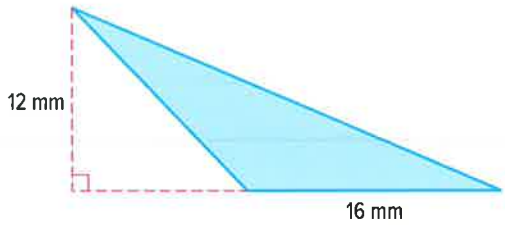
Check

Quick Review

Review

Example 1

Find the area of the triangle.



$A = \frac{1}{2}bh$ Formula for area of a triangle

$A = \frac{1}{2} \cdot 16 \cdot 12$ Replace b with 16 and h with 12.

$A = 96$ Simplify.

The area is 96 square millimeters.

Example 2

Evaluate $\pi \cdot 16^2$. Use 3.14 for π. Round to the nearest tenth.

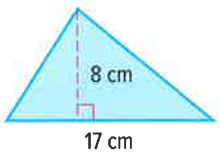
$\pi \cdot 16^2 \approx 3.14 \cdot 256$ Evaluate 16^2.

≈ 803.8 Multiply.

Quick Check

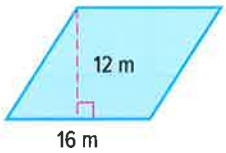
Area **Find the area of each figure.**

1.

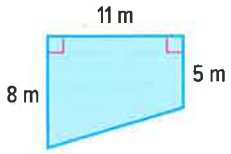


Show your work.

2.

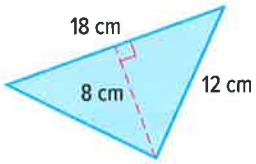


3.

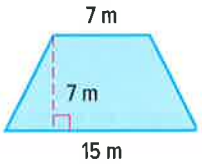


4.

5.



6.



Evaluate **Find the value of each expression. Use 3.14 for π. Round to the nearest tenth.**

7. $\pi \cdot 15 \approx$ _____

8. $2 \cdot \pi \cdot 3.2 \approx$ _____

9. $\pi \cdot (19 \div 2)^2 \approx$ _____

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

(1) (2) (3) (4) (5) (6) (7) (8) (9)

Inquiry Lab
Three-Dimensional Figures

Inquiry **HOW are some three-dimensional figures related to circles?**

MP **Mathematical Practices** 1, 3

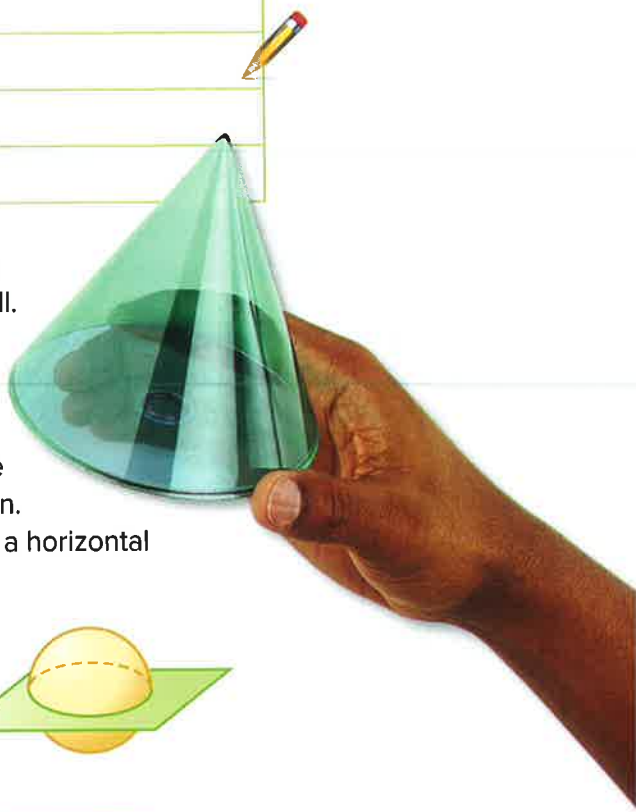
Bilal is training on an obstacle course. One of the activities in the course is the tunnel, an open tube through which he runs.

Hands-On Activity

A three-dimensional figure with faces that are polygons is called a **polyhedron**. There are three-dimensional figures that are *not* polyhedrons. Some examples of these figures are *cylinders*, *cones*, and *spheres*.

Step 1 For each figure, list three real-world items that represent the figure.

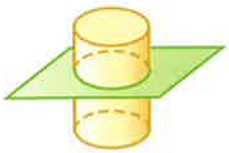
Cylinder	Cone	Sphere
		



Step 2 Just as a rectangular prism and a pyramid have bases, a cylinder and a cone have bases as well. What is the shape of the base of a cylinder?

_____ a cone? _____

Step 3 Interesting shapes can occur when you find the cross section of a figure that is not a polyhedron. Describe the shape of the figure resulting from a horizontal cross section of each of the following.

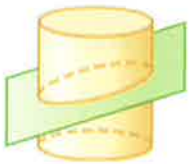


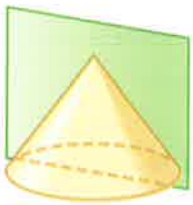
_____ _____ _____

Investigate

Collaborate

Work with a partner. Draw and describe the shape resulting from each cross section.

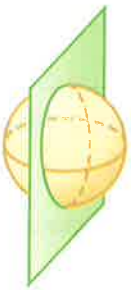
1. 

2. 

3. 

Show your work.

4. 

5. 

6. 

Create

On Your Own

7. MP **Use a Counterexample** *True* or *false*: The cross section of a cylinder, a cone, and a sphere will *always* be a circle or an oval. If false, provide a counterexample.

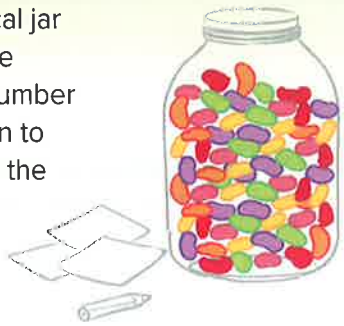
8. Inquiry HOW are some three-dimensional figures related to circles?

Lesson 1

Volume of Cylinders

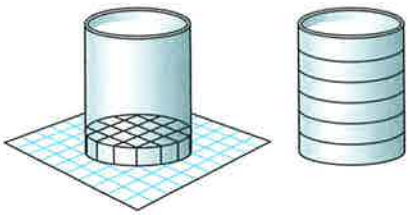
Real-World Link

Jelly Beans Muna's teacher filled a cylindrical jar with jelly beans. She is awarding a prize to the student who most accurately estimates the number of jelly beans in the jar. Muna used a soup can to model the jar and centimeter cubes to model the jelly beans.



Collaborate **Work with a partner.**

1. Set a soup can on a piece of grid paper. Trace the area around the base as shown.



About how many centimeter cubes would fit at the bottom of the container? Remember to include partial cubes in your total. _____

2. Suppose each layer is 1 centimeter high. How many layers would it take to fill the cylinder? ☐

3. MP **Be Precise** Write a formula that allows you to find the volume of the container. _____

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Essential Question

WHY are formulas important in math and science?

Vocabulary

volume
cylinder
composite solids

MP Mathematical Practices
1, 3, 4, 6

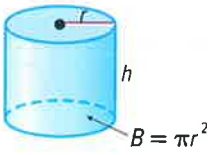
Key Concept

Volume of a Cylinder

Words The volume V of a cylinder with radius r is the area of the base B times the height h.

Symbols $V = Bh$, where $B = \pi r^2$ or $V = \pi r^2 h$

Model



Work Zone

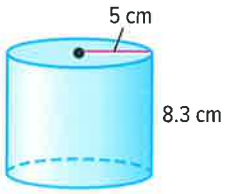
STOP and Reflect

What formula do you use to find the area of the base of a cylinder?

Volume is the measure of the space occupied by a solid. Volume is measured in cubic units. A **cylinder** is a three-dimensional figure with two parallel congruent circular bases connected by a curved surface. The area of the base of a cylinder tells the number of cubic units in one layer. The height tells how many layers there are in the cylinder.

Examples

1. **Find the volume of the cylinder. Round to the nearest tenth.**



$V = \pi r^2 h$ Volume of a cylinder

$V = \pi(5)^2(8.3)$ Replace r with 5 and h with 8.3.

Use a calculator.

2nd [π] × 5 x^2 × 8.3 ENTER 651.8804756

The volume is about 651.9 cubic centimeters.

Circles

Recall that the radius is half the diameter.

2. **Find the volume of a cylinder with a diameter of 16 centimeters and a height of 20 centimeters. Round to the nearest tenth.**

$V = \pi r^2 h$ Volume of a cylinder

$V = \pi(8)^2(20)$ The diameter is 16 so the radius is 8. Replace h with 20.

$V \approx 4{,}021.2$ Use a calculator.

The volume is about 4,021.2 cubic centimeters.

Got it? **Do these problems to find out.**

Find the volume of each cylinder. Round to the nearest tenth.

Show your work.

a. _____

b. _____

a.

b. diameter: 12 mm

height: 5 mm

Example

3. **A metal paperweight is in the shape of a cylinder. The paperweight has a height of 1.5 centimeters and a diameter of 2 centimeters. How much does the paperweight weigh if 1 cubic centimeters weighs 50 grams? Round to the nearest tenth.**

First find the volume of the paperweight.

$V = \pi r^2 h$ Volume of a cylinder

$V = \pi(1)^2 1.5$ Replace r with 1 and h with 1.5.

$V \approx 4.7$ Simplify.

To find the weight of the paperweight, multiply the volume by 50.

$4.7(50) = 235$

So, the weight of the paperweight is about 235 grams.

Got it? **Do this problem to find out.**

Show your work.

c. The Faris family uses a container shaped like a cylinder to recycle aluminum cans. It has a height of 130 centimeters and a diameter of 50 centimeters. The container is full. How much do the contents weigh if the average weight of aluminum cans is 0.037 grams per cubic centimeter? Round to the nearest tenth of a kilogram.

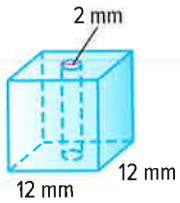
c. _____

Volume of a Composite Solid

Objects made up of more than one type of solid are called **composite solids.** To find the volume of a composite solid, decompose the figure into solids whose volumes you know how to find.

Example

4. **Badria uses cube-shaped beads to make jewelry. Each bead has a circular hole through the middle. Find the volume of each bead.**



The bead is made of one rectangular prism and one cylinder. Find the volume of each solid. Then subtract to find the volume of the bead.

Rectangular Prism	Cylinder
$V = Bh$	$V = Bh$
$V = (12 \cdot 12)12$ or 1,728	$V = (\pi \cdot 1^2)12$ or 37.7

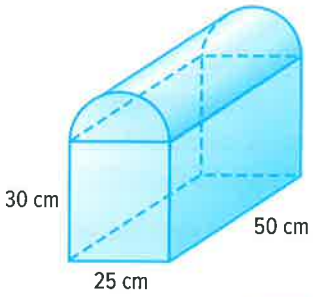
The volume of the bead is 1,728 − 37.7 or 1,690.3 cubic millimeters.

Show your work.

d. _____

Got it? **Do this problem to find out.**

d. The Service Club is building models of storage chests, like the one shown, to donate to a charity. Find the volume of the chest to the nearest tenth.

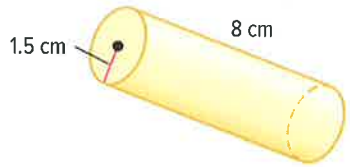


Guided Practice

Check

Find the volume of each cylinder. Round to the nearest tenth. (Examples 1 and 2)

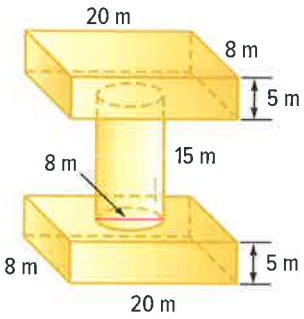
1. _____



2. _____

diameter: 8 cm.
height: 8 cm.

3. A platform like the one shown was built to hold a sculpture for an art exhibit. What is the volume of the figure? (Example 4)



4. A scented candle is in the shape of a cylinder. The radius is 4 centimeters and the height is 12 centimeters. Find the mass of the wax needed to make the candle if 1 cubic centimeter of wax has a mass of 3.5 grams. Round to the nearest tenth. (Example 3)

5. **Building on the Essential Question** How is the formula for the volume of a cylinder similar to the formula for the volume of a rectangular prism?

Rate Yourself!

How confident are you about volume of cylinders? Check the box that applies.

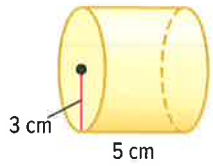
FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

Find the volume of each cylinder. Round to the nearest tenth. (Examples 1 and 2)

1. _____

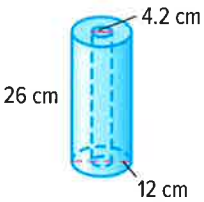


2. _____

diameter 4.5 m
height 6.5 m

3. Asma's parents have a cylindrical oak tree stump that has a diameter of 1.3 meters and a height of 2 meters. How much does the stump weigh if the average weight of oak is 946 kilograms per cubic meter? Round to the nearest tenth. (Example 3) _____

4. An unused roll of paper towels is shown. What is the volume of the unused roll? (Example 4) _____



5. MP **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–c.

a. Find the volume of the bag and candle. Round to the nearest tenth.

_____

b. How much packing material is needed to fill the empty space in the bag after the candle is placed in the bag? _____

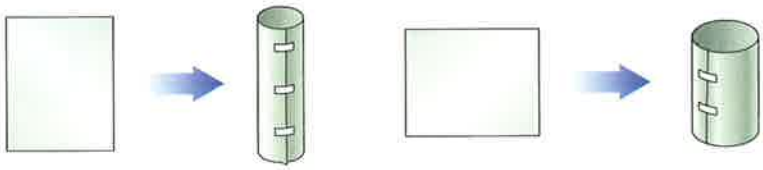
c. There are 70 teachers in the school. If each package of packing material contains 11,000 cubic centimeters of material, how many packages do they need to buy to fill all of the gift bags? _____

6. MP **Use Math Tools** Match each cylinder with its approximate volume.

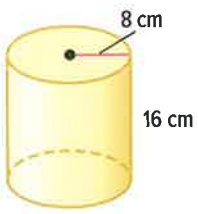
radius = 4.1 m, height = 5 m	91 m^3
diameter = 8 m, height = 2.2 m	111 m^3
diameter = 6.2 m, height = 3 m	264 m^3

H.O.T. Problems Higher Order Thinking

7. MP **Persevere with Problems** Two equally-sized sheets of construction paper are rolled; one along the length and the other along the width, as shown. Which cylinder has the greater volume? Explain.



8. MP **Model with Mathematics** Draw and label a cylinder that has a larger radius but less volume than the cylinder shown below.



Show your work.

9. MP **Reason Abstractly** Find the ratios of the volume of cylinder A to cylinder B.

   a. Cylinder A has the same radius but twice the height of cylinder B.

   b. Cylinder A has the same height but twice the radius of cylinder B.

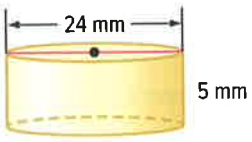
Name _____ My Homework _____

Extra Practice

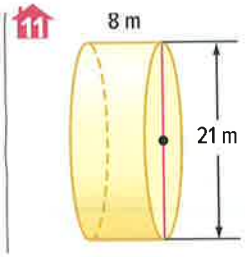
Copy and Solve **For Exercises 10–27, show your work and answers on a separate piece of paper.**

Find the volume of each cylinder. Round to the nearest tenth.

10.



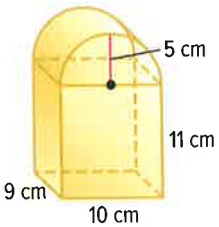
11.



12.

13. Ahmed has a container of flour in the shape of a cylinder. The container has a diameter of 10 centimeters and a height of 8 centimeters. If the container is full, how much will the flour weigh if the average weight of flour is 0.23 grams per cubic centimeter? Round to the nearest tenth.

14. Amna wants to make a box like the one shown. What is the volume of the box? Round to the nearest tenth.



15. Cylinder A has a radius of 4 centimeters and a height of 2 centimeters. Cylinder B has a radius of 2 centimeters. What is the height of Cylinder B to the nearest centimeter if both cylinders have the same volume?

16. Which will hold more cake batter, the rectangular pan or two round pans? Explain your reasoning to a classmate.

17. MP **Multiple Representations** The dimensions for four cylinders are shown in the table.

a. **Symbols** Write an equation to find the volume of each cylinder.

b. **Words** Compare the dimensions of Cylinder A with the dimensions of Cylinders B, C, and D.

c. **Numbers** Complete the table.

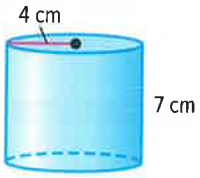
d. **Words** Explain how changing the dimensions of a cylinder affects the cylinder's volume.

	Radius (cm)	Height (cm)	Volume (cm^3)
Cylinder A	1	1	
Cylinder B	1	2	
Cylinder C	2	1	
Cylinder D	2	2	

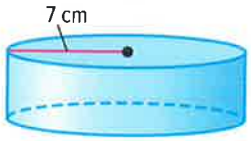
Power Up! Test Practice

18. Without doing any calculations, do you think Cylinder 1 and Cylinder 2 will have the same volume? Explain your reasoning.

Cylinder 1



Cylinder 2



Fill in each box to complete the following statements.

To the nearest tenth, the volume of Cylinder 1 is ☐.

To the nearest tenth, the volume of Cylinder 2 is ☐.

19. The oatmeal container shown has a diameter of 3 centimeters and a height of 9 centimeters. Which of the following statements are true? Select all that apply.

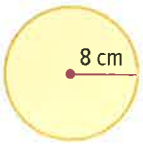
- ☐ The area of each base is exactly 9π square centimeters.
- ☐ The volume of the container is exactly 20.25π cubic centimeters.
- ☐ The volume of the container to the nearest tenth is about 63.6 cubic centimeters.

Spiral Review

Find the area of each circle. Round to the nearest tenth.

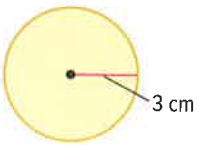
20.

21.



22.

23.



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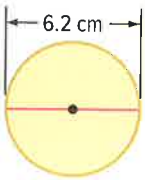

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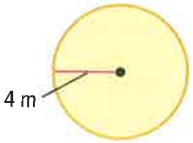
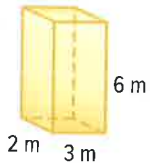
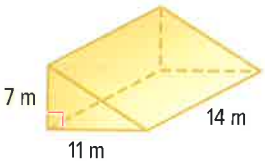


Find the volume of each prism.

26.



27.



Lesson 2

Volume of Cones

Real-World Link

Carnival Hala and Hidaya are making snow cones for the school carnival. They want to know how much ice goes into a paper cone that has a radius of 4 centimeters and a height of 10 centimeters.

1. Recall the formula for finding the volume of a rectangular pyramid is $V = \frac{1}{3}Bh$. How does the volume of a pyramid compare to the volume of a prism with the same base and height?

2. What is the formula for finding the volume of a cylinder?

3. What is the volume of a cylinder with a radius of 4 centimeters and a height of 10 centimeters? Use $\pi = 3.14$.

4. The volume of the cones Hala and Hidaya are using is about 167.5 cubic centimeters. Write a ratio in simplest form comparing the volume of the cone to the volume of the cylinder. $\frac{\square}{\square}$

5. MP **Make a Conjecture** What is the formula for the volume of a cone?

Essential Question

WHY are formulas important in math and science?

Vocabulary

cone

Mathematical Practices

1, 2, 3, 4

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

(1) Persevere with Problems	(5) Use Math Tools
(2) Reason Abstractly	(6) Attend to Precision
(3) Construct an Argument	(7) Make Use of Structure
(4) Model with Mathematics	(8) Use Repeated Reasoning

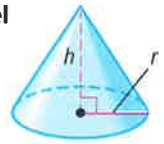
Key Concept

Volume of a Cone

Words The volume V of a cone with radius r is one third the area of the base B times the height h.

Symbol $V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$

Model



Work Zone

A **cone** is a three-dimensional figure with one circular base connected by a curved surface to a single vertex.

Example

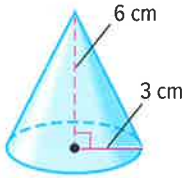
1. **Find the volume of the cone. Round to the nearest tenth.**

$V = \frac{1}{3}\pi r^2 h$ Volume of a cone

$V = \frac{1}{3} \cdot \pi \cdot 3^2 \cdot 6$ $r = 3, h = 6$

$V \approx 56.5$ Simplify.

The volume is about 56.5 cubic centimeters.



Show your work.

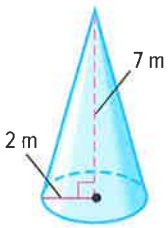
Got it? **Do these problems to find out.**

Find the volume of each cone. Round to the nearest tenth.

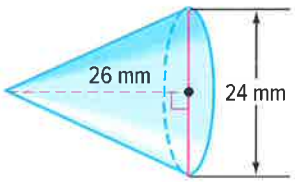
a. _____

b. _____

a.



b.



Real World

Example

2. **A cone-shaped paper cup is filled with water. The height of the cup is 10 centimeters and the diameter is 8 centimeters. What is the volume of the paper cup? Round to the nearest tenth.**

$V = \frac{1}{3}\pi r^2 h$ Volume of a cone

$V = \frac{1}{3} \cdot \pi \cdot 4^2 \cdot 10$ $r = 4, h = 10$

$V \approx 167.6$ Simplify.

The volume of the paper cup is about 167.6 cubic centimeters.

Got it? Do this problem to find out.

c. Sumayya is filling six identical cones for her competition. Each cone has a radius of 3.6 centimeters and a height of 21 centimeters. What is the total volume of the cones? Round to the nearest tenth.

Show your work.

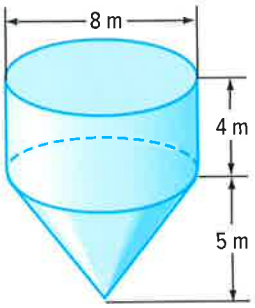
c. _____

Volume of Composite Solids

When a composite solid includes cylinders and cones, you can find the volume by decomposing it into solids whose volumes you know how to find.

Example

3. **Find the volume of the solid. Round to the nearest tenth.**



Step 1 Find the volume of the cylinder.

$V = \pi r^2 h$ Volume of a cylinder

$V = \pi \cdot 4^2 \cdot 4$ $r = 4$, $h = 4$

$V = \pi \cdot 16 \cdot 4$ Simplify.

$V \approx 201.1$ Simplify.

Step 2 Find the volume of the cone.

$V = \frac{1}{3}\pi r^2 h$ Volume of a cone

$V = \frac{1}{3} \cdot \pi \cdot 4^2 \cdot 5$ $r = 4$, $h = 5$

$V = \frac{1}{3} \cdot \pi \cdot 16 \cdot 5$ Simplify.

$V \approx 83.8$ Simplify.

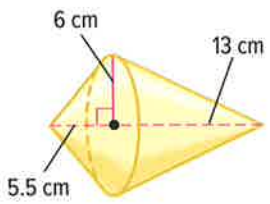
So, the volume of the solid is about 201.1 + 83.8 or 284.9 cubic meters.

STOP and Reflect

Hareb and Hamad are simplifying $\pi \cdot 5^2$. Hareb rounds π to 3.14 and Hamad uses the π key on her calculator. Which student's calculation is closer to the exact value? Explain below.

Got it? Do this problem to find out.

d. Find the volume of the solid.



d. _____

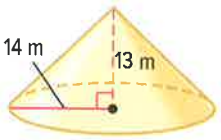
Guided Practice

Check

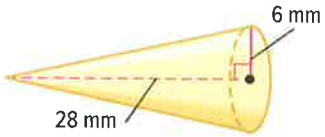
Find the volume of each cone. Round to the nearest tenth. (Examples 1 and 2)

1.

Show your work.



2.



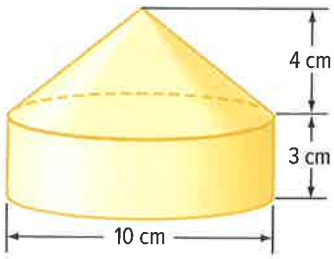
3. height: 9 m

diameter: 10 m

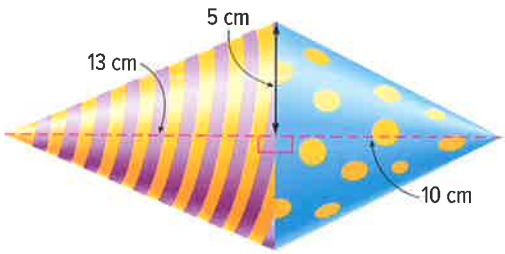
4. height: 120 millimeters

radius: 45 millimeters

5. Find the volume of the solid at the right. Round to the nearest tenth. (Example 3)



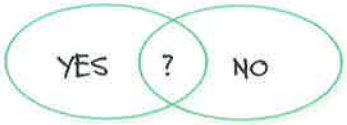
6. Find the volume of the pair of cones shown. Round to the nearest tenth. (Example 3)



7. **Building on the Essential Question** What would have a greater effect on the volume of a cone: doubling its radius or doubling its height? Explain.

Rate Yourself!

How confident are you about volume of cones? Shade the section that applies.



FOLDABLES Time to update your Foldable!

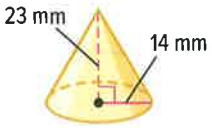
Name _____ My Homework _____

Independent Practice

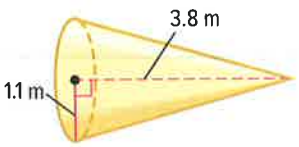
Go online for Step-by-Step Solutions

Find the volume of each cone. Round to the nearest tenth. (Example 1)

1.



2.



Show your work.

3 height: 8.4 meters

diameter: 3.5 meters

4. height: 3.9 meters

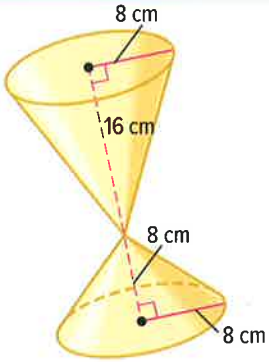
radius: 1.7 meters

5. A cone like the one at the right is going to be filled with candy. What is the volume of the cone? Round to the nearest tenth. (Example 2)

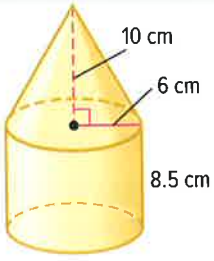
6. Mr. Ibrahim is building a storage shed in a conical shape. The base of the shed is 4 meters in diameter and the height of the shed is 3.8 meters. What is the volume of the shed? Round to the nearest tenth. (Example 2)

Find the volume of each solid. Round to the nearest tenth. (Example 3)

7.



8.



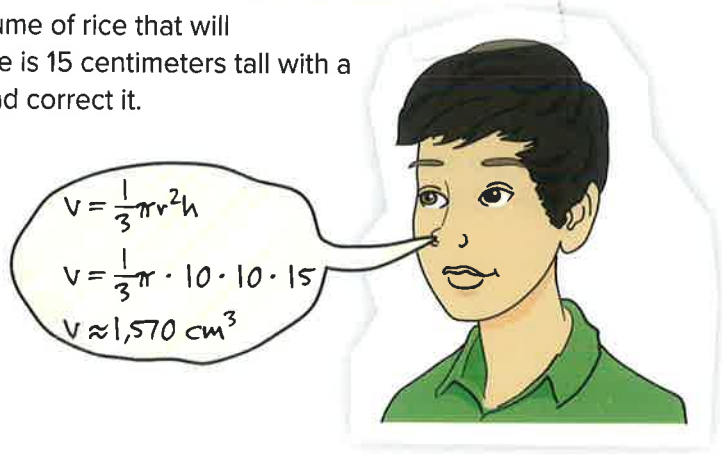
9. A cylinder has a radius of 5 centimeters and a height of 12 centimeters. What would the height of a cone need to be if it has the same volume and radius? Round to the nearest centimeter.

10. MP **Reason Abstractly** Eissa is making cone-shaped ice cubes by using a mold. The radius of the mold is 1.5 centimeters and the height is 2 centimeters. If one cubic centimeter is about 1 gram, how many grams will ten ice cubes weigh? Round to the nearest tenth.

11. The volume of a cone with a 30-millimeter radius is 9,420 cubic millimeters. What is the height of the cone to the nearest millimeter?

H.O.T. Problems Higher Order Thinking

12. MP **Find the Error** Faleh is finding the volume of rice that will fill a cone-shaped decorative vase. The vase is 15 centimeters tall with a 10-centimeter diameter. Find her mistake and correct it.



13. MP **Persevere with Problems** Draw and label two cones with different dimensions but the same volume.

14. MP **Reason Inductively** Determine whether the following statement is *always*, *sometimes*, or *never true*. Explain your reasoning.

The volume of a rectangular-based pyramid and a cone with the same height and equal areas of the base are equal.

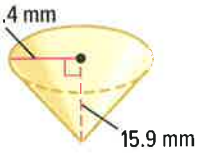
Name _____ My Homework _____

Extra Practice

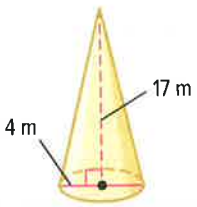
Copy and Solve **For Exercises 15–33, show your work and answers on a separate piece of paper.**

Find the volume of each cone. Round to the nearest tenth.

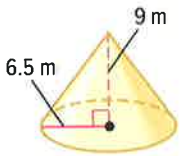
15.



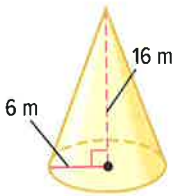
16.



17.



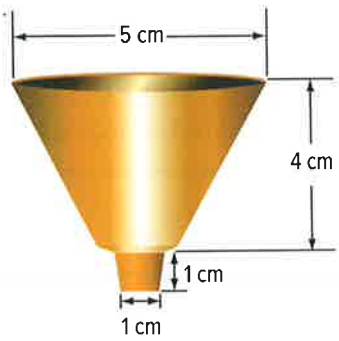
18.



19. height: 24 centimeters
diameter: 8 centimeters

20. height: 9 centimeters
diameter: $7\frac{1}{2}$ centimeters

21. Usama is using the funnel shown to fill a glass bottle with colored sand. Estimate the volume of the funnel.



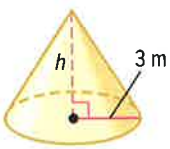
22. Mount Rainier, a cone-shaped volcano in Washington, is about 4.4 kilometers tall and about 18 kilometers across its base. Find the volume of Mount Rainier to the nearest whole number.

23. The volume of a cone is 471.24 cubic centimeters and the height is 8 centimeters. What is the diameter?

24. The volume of a cone is 593.46 cubic centimeters. The radius is 9 centimeters. Find the height of the cone to the nearest centimeter.

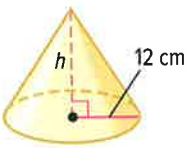
MP Persevere with Problems **Find the height of each cone. Round to the nearest tenth.**

25.



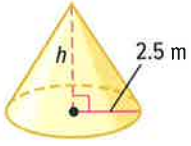
Volume: 42.39 m^3

26.



Volume: 1,205.76 cm^3

27.



Volume: 19.625 m^3

Power Up! Test Practice

28. Four cones have the dimensions shown below.

Figure 1

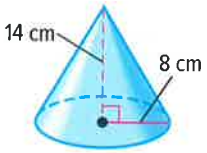


Figure 2

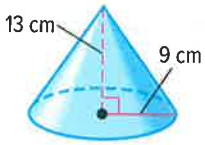


Figure 3

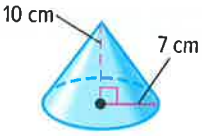
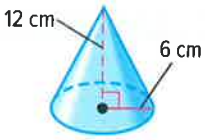


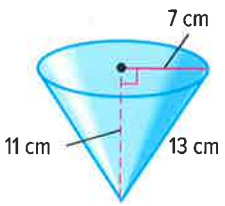
Figure 4



Sort the cones from least to greatest volume. Round to the nearest tenth.

	Figure	Volume (cm^3)
Least		
Greatest		

29. Refer to the cone shown at the right. Determine if each statement is true or false.



a. The approximate area of the base is 153.9 square centimeters. ☐ True ☐ False

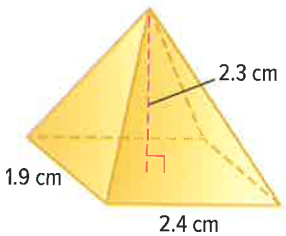
b. The approximate volume of the cone is 886.5 cubic centimeters. ☐ True ☐ False

c. The volume of a cylinder with the same height and radius would be 3 times the volume of the cone. ☐ True ☐ False

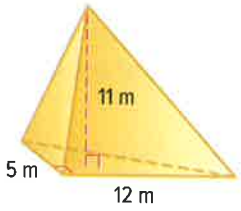
Spiral Review

Find the volume of each pyramid. Round to the nearest tenth if necessary. 7.G.6

30.



31.



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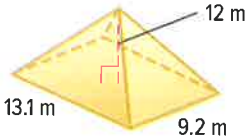

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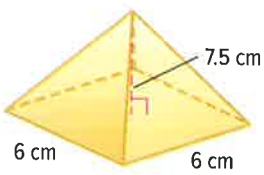


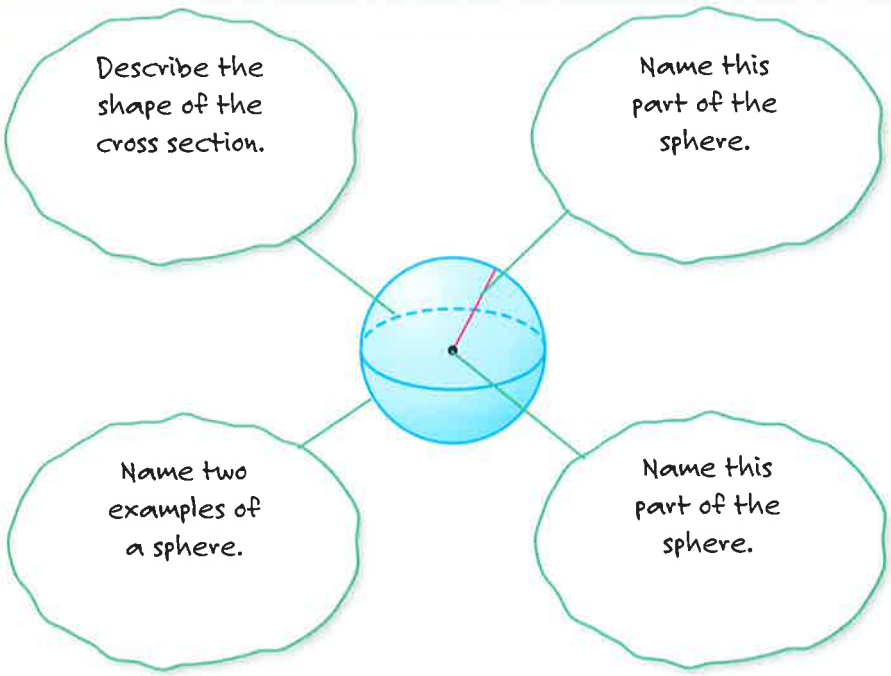
Lesson 3

Volume of Spheres

Vocabulary Start-Up

A **sphere** is a set of all points in space that are a given distance, known as the radius, from a given point, known as the center.

Complete the graphic organizer.



Real-World Link

Buthaina purchased a necklace that contained a round pearl with a diameter of 7.5 millimeters. What is the circumference of the largest circle around the outside of the pearl? Round to the nearest tenth.

Essential Question

WHY are formulas important in math and science?

Vocabulary

sphere
hemisphere

Mathematical Practices
1, 3, 4

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Key Concept

Volume of a Sphere

Words The volume V of a sphere is four thirds the product of π and the cube of the radius r.

Symbols $V = \frac{4}{3}\pi r^3$

Model

Work Zone

Exact and Approximate

Whenever you round or use 3.14 for π, you are finding the approximate value. An answer left in terms of π, such as $\frac{256}{3}\pi$, is an exact value.

You can use the formula for the volume of a sphere to solve mathematical and real-world problems.

Example

1. **Find the volume of the sphere. Round to the nearest tenth.**

$V = \frac{4}{3}\pi r^3$ Volume of a sphere

$V = \frac{4}{3} \cdot \pi \cdot 6^3$ Replace r with 6.

$V \approx 904.8$ Simplify. Use a calculator.

The volume of the sphere is about 904.8 cubic millimeters.

Show your work.

Got it? **Do these problems to find out.**

Find the volume of each sphere. Round to the nearest tenth.

a. _____

b. _____

a.

b.

Example

2. **A spherical giant balloon has a diameter of about 8 meters. Find the volume of the spherical balloon. Round to the nearest tenth.**

$V = \frac{4}{3}\pi r^3$ Volume of a sphere

$V = \frac{4}{3} \cdot \pi \cdot 4^3$ Replace r with 4.

$V \approx 268.1$ Simplify. Use a calculator.

The volume of the giant balloon is about 268.1 cubic meters.

Got it? Do this problem to find out.

c. A dish contains a spherical scoop of vanilla ice cream with a radius of 3 centimeters. What is the volume of the ice cream?

Show your work.

c. _____

Example

3. **A ball has a diameter of 10 centimeters. A pump can inflate the ball at a rate of 325 cubic centimeters per minute. How long will it take to inflate the ball? Round to the nearest tenth.**

Find the volume of the ball. Then use a proportion.

$V = \frac{4}{3}\pi r^3$ Volume of a sphere

$V = \frac{4}{3} \cdot \pi \cdot 5^3$ or 523.6 Replace r with 5.

$\frac{325 \text{ cm}^3}{1 \text{ min}} = \frac{523.6 \text{ cm}^3}{x \text{ min}}$ Write the proportion.

$325x = 523.6$ Cross multiply.

$x = 1.6$ Simplify.

So, it will take about 1.6 minutes to inflate the ball.

Got it? Do this problem to find out.

d. A snowball has a diameter of 6 centimeters. How long would it take the snowball to melt if it melts at a rate of 1.8 cubic centimeters per minute? Round to the nearest tenth.

d. _____

Volume of a Hemisphere

A circle separates a sphere into two congruent halves each called a **hemisphere**.

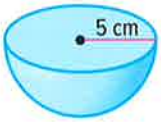
Example

4. **Find the volume of the hemisphere. Round to the nearest tenth.**

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ Volume of a hemisphere

$V = \frac{1}{2}\left(\frac{4}{3} \cdot \pi \cdot 5^3\right)$ Replace r with 5.

$V \approx 261.8$ Simplify. Use a calculator.



The volume of the hemisphere is about 261.8 cubic centimeters.

Hemisphere

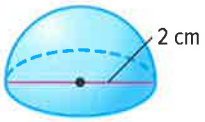
The volume of a hemisphere is $\frac{1}{2}$ the volume of a sphere.

Show your work.

e. _____

f. _____

Got it? Do these problems to find out.

e. 

f. 

Guided Practice

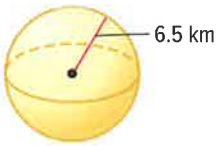
Check

Find the volume of each sphere. Round to the nearest tenth. (Example 1)

1. _____

Show your work.

2. _____



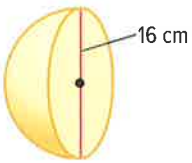
3. Sindiyya is blowing up spherical balloons for her brother's graduation party. One of the balloons has a radius of 7.5 centimeters Round to the nearest tenth.
(Examples 2 and 3)

   a. What is the volume of the balloon? _____

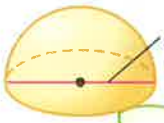
   b. Suppose Sindiyya can inflate the balloon at a rate of 3,000 cubic centimeters per minute. How long will it take her to inflate the ballon? _____

Find the volume of each hemisphere. Round to the nearest tenth. (Example 4)

4. _____



5. _____



6. **Building on the Essential Question** *True* or *false*? The volume of a sphere is two-thirds the volume of a cylinder with the same radius r and height of $2r$. Explain your reasoning.

Rate Yourself!

How well do you understand volume of spheres? Circle the image that applies.

Clear | Somewhat Clear | Not So Clear

Name _____ My Homework _____

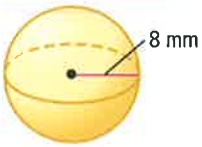
Independent Practice

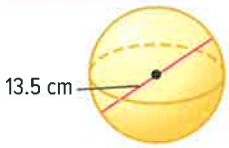
Find the volume of each sphere. Round to the nearest tenth. (Example 1)

1. 

Show your work.

2. 

3. 

4. 

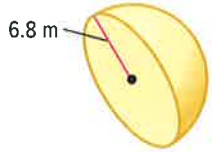
5. The radius of a ball is 4.7 centimeters. What is the volume of the basketball? Round to the nearest tenth. (Example 2) _____

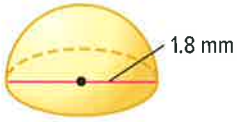
6. Najat bought a game that contained a ball and 10 jacks. The ball had a radius of 2 centimeters. What is the volume of the ball? Round to the nearest tenth. (Example 2) _____

7. A spherical ball has a diameter of 8 centimeters. The ball has a slow leak in which the air escapes at the rate of 20 cubic centimeters per second. How long it would take the ball to deflate? Round to the nearest tenth. (Example 3)

_____

Find the volume of each hemisphere. Round to the nearest tenth. (Example 4)

8. 

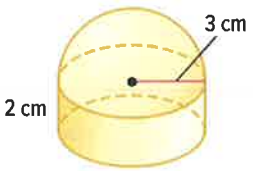
9. 

MP Persevere with Problems **Find the radius of each figure. Round to the nearest tenth.**

10. sphere with a volume of 1,767.1 m^3

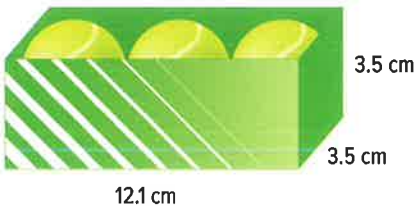
11. hemisphere with a volume of 2,712.3 cm^3

12. Find the volume of the composite solid shown. Round to the nearest tenth.



H.O.T. Problems Higher Order Thinking

13. **MP Persevere with Problems** Three tennis balls are packaged in a box as shown below. The box is 12.1 centimeters long, 3.5 centimeters wide, and 3.5 centimeters tall. Each ball is 3.3 centimeters in diameter. What is the volume of the empty space in the box?



14. **MP Reason Abstractly** A cylinder contains 150.8 cubic units of water. What is the minimum radius of a sphere that will hold the water? Round to the nearest tenth.

15. **MP Reason Inductively** Determine whether the following statement is *true* or *false*. Explain your reasoning.

Doubling a sphere's radius doubles its volume.

Name _____ My Homework _____

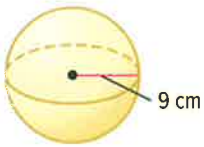
Extra Practice

Copy and Solve **For Exercises 16–36, show your work and answers on a separate piece of paper.**

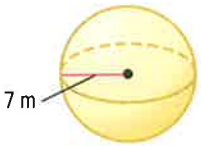
Find the volume of each figure. Round to the nearest tenth.

16.

17



18.



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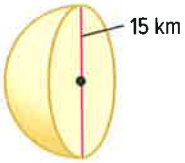

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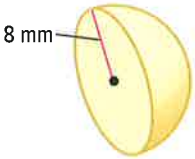

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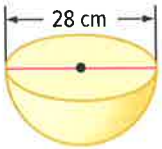


22. Ayesha is purchasing a ring that contains a 7.5 millimeter diameter round pearl. Find the volume of the pearl to the nearest tenth.

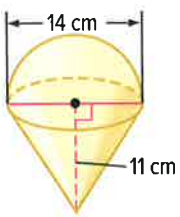
23. Amal is purchasing balloons for a party. Each spherical balloon is inflated with helium. How much helium is in the balloon if the balloon has a radius of 11 centimeters? Round to the nearest tenth.

24. MP **Persevere with Problems** The volume of a ball is about 13.39 cubic centimeters. What is the diameter? Round to the nearest tenth.

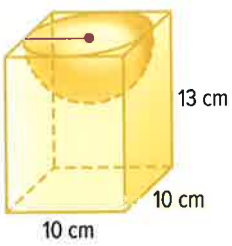
25. A golf ball has a diameter of 42.67 millimeters and a mass of 45.93 grams. What is the number of grams per cubic millimeter of the material used to make the golf ball? Round to the nearest ten-thousandth.

Find the volume of each composite solid. Round to the nearest tenth.

26.



27.



Power Up! Test Practice

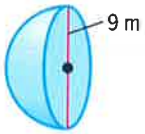
28. The volume of a golf ball is about 41.63 cm^3. Select the correct values to complete the formula below to find the radius of a golf ball.

2	6
3	9
4	41.63
	r

$$\square = \frac{\square}{\square}\pi\square^{\square}$$

To the nearest hundredth, what is the radius of the golf ball? ____

29. Refer to the hemisphere shown. Fill in each box to make a complete statement. Round to the nearest tenth if necessary.



a. The radius of the hemisphere is ____ meters.

b. The volume of a sphere with a diameter of 9 meters is ____ cubic meters.

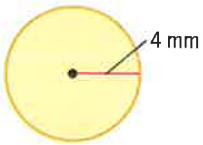
c. The volume of the hemisphere is ____ cubic meters.

Spiral Review

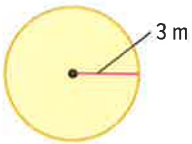
Find the circumference and area of each circle. Round to the nearest tenth.

30.

31.



32.



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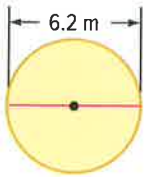


34. Find the area of a circle with a radius of 6 centimeters. Round to the nearest tenth.

35. Find the area of a circle with a diameter of 13.1 centimeters. Round to the nearest tenth.

36. An conical icicle has a volume of about 12 cubic centimeters. If the icicle has a height of 8 centimeters, what is the diameter of the icicle?

MP Problem-Solving Investigation

Solve a Simpler Problem

MP Mathematical Practices
1, 4, 7

Case #1 Spring Fling

Student government is decorating the school's gymnasium by hanging 100 of the containers shown from the ceiling. The cones have a radius of 3 centimeters and a height of 12 centimeters. The hemispheres have a radius of 3 centimeters.

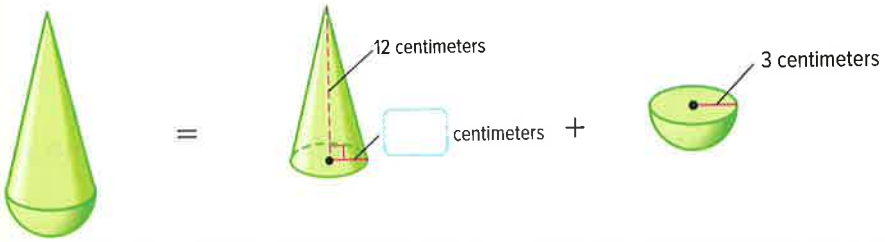
How much confetti will they need to fill one of the containers?

1 Understand *What are the facts?*

Each container is made up of a cone and a hemisphere.

2 Plan *What is your strategy to solve this problem?*

Solve a simpler problem by separating the container into a cone and a hemisphere. Find the volume of each, and then add the two volumes together. Round your answers to the nearest tenth.



3 Solve *How can you apply the strategy?*

Volume of the cone = _____ cm^3

Volume of the hemisphere = _____ cm^3

Volume of one container = _____ cm^3

So, one container will need _____ cubic centimeters of confetti.

4 Check *Does the answer make sense?*

The total volume is greater than each of the parts, so the answer is reasonable.

Analyze the Strategy

MP **Identify Structure** Suppose one bag contains 500 cubic centimeters of confetti. How many bags will Student Government need to fill 100 containers?

_____

Case #2 Carpenter's Riddle

Working separately, three carpenters can make three chairs in three days.

How many chairs can 7 carpenters working at the same rate make in 30 days?

1

Understand

Read the problem. What are you being asked to find?

I need to find _____.

Underline key words and values. What information do you know?

_____ carpenters make _____ chairs in _____ days.

2

Plan

Choose a problem-solving strategy.

I will use the _____ strategy.

3

Solve

Use your problem-solving strategy to solve the problem.

Use the information provided.

If three carpenters can make three chairs in three days, then one carpenter can make _____ chair in _____ days.

If one carpenter can make _____ chair in _____ days, then one carpenter can make ☐ chairs in 30 days. If one carpenter makes ☐ chairs in 30 days, then 7 carpenters make ☐ × ☐ = ☐ chairs in 30 days.

So, _____.

4

Check

Use information from the problem to check your answer.

_____

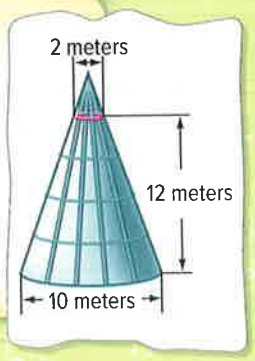
Collaborate

Work with a small group to solve the following cases. Show your work on a separate piece of paper.

Case #3 Storage

A 15-meter tall storage building is shown. Grain fills the storage building to a height of 12 meters.

What is the volume of the space filled with grain? Round your answer to the nearest tenth.



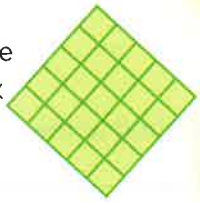
Case #4 School Play

Four students can sew four costumes in two days.

How many costumes can ten students sew in twelve days?

Case #5 Hidden Squares

Halima looked at the figure and decided there were 25 squares shown. Asma told her there were many more than that since there could be squares that measure 1×1, 2×2, 3×3, 4×4, and 5×5.



How many squares of any size are in the figure? (Hint: Count the number of squares in a 2×2 and a 3×3 square. Then look for a pattern.)

Use any strategy!

Case #6 Pizza

What is the largest number of pieces that can be cut from one pizza using five straight cuts?

3 cuts

4 cuts

Mid-Chapter Check

Vocabulary Check

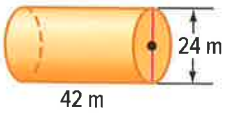
1. **Be Precise** Define *cylinder*. What are the symbols used to find the volume of a cylinder? (Lesson 1)

Fill in the blank.

2. The volume of a cone is _____ the volume of a cylinder with the same base and height. (Lesson 2)

Skills Check and Problem Solving

3. What is the volume of the cylinder shown at the right? Round to the nearest tenth. (Lesson 1) _____



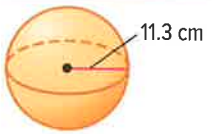
Show your work.

4. Find the height of a cone with a volume of 464.603 cubic centimeters and a diameter of 8 centimeters. (Lesson 2) _____

Find the volume of each sphere. Round to the nearest tenth. (Lesson 3)

5. _____

6. _____



7. _____

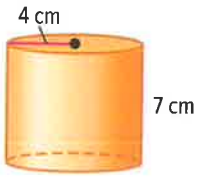
8. **Reason Inductively** Refer to the cylinders shown. If a cone has a base and height congruent to Cylinder 1, which statement is true? (Lesson 2) _____

I The volume of the cone is equal to the volume of Cylinder 1.

II The volume of the cone is equal to the volume of Cylinder 2.

III The cone has a greater volume than Cylinder 1.

IV The cone has one third the volume of Cylinder 1.



Cylinder 1

Cylinder 2

Inquiry Lab
Surface Area of Cylinders

MP **Mathematical Practices** 1, 3

Inquiry **HOW can the surface area of a cylinder be determined?**

Khawla used a clean frozen yogurt container for a school project. The container is shaped like a cylindrical tub that has a diameter of 20 centimeters and a height of 30 centimeters. She wants to know how much paper she will need to cover the entire container.

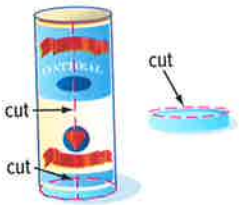
Hands-On Activity

Nets are two-dimensional patterns of three-dimensional figures. When you construct a net, you are decomposing the three-dimensional figure into separate shapes. You can use a net to find the area of each surface of a three-dimensional figure such as a cylinder.

Step 1 Use an empty cylinder shaped container that has a lid.

What is the height of the container? _____

Step 2 Take off the lid of the container and make 2 cuts as shown. Cut off the sides of the lid. Lay the lid, the curved side, and the bottom flat to form the net of the container. Sketch and label the parts of the net.



What are the shapes that make up the net of the container? _____

Step 3 Make a mark on the top of the lid. Place the mark at the top edge of the flattened curved side as shown. Roll the lid along the edge of the side until it completes one rotation.

Where does the lid stop? _____

How does the length of the curved side compare to the distance around the top? _____

Find the area of each shape.

Top _____ Bottom _____ Side _____

Find the sum of these areas. _____

Investigate

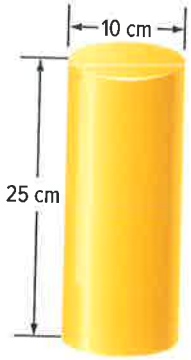
Collaborate

Work with a partner. Draw the net and label the parts of the cylinder and the measurements. Then complete the table to find the *total surface areas* for Exercises 1 and 2. Round to the nearest tenth.

1.

Show your work.

2.



	Area of top (πr^2)	Area of bottom (πr^2)	Curved Area	Total Surface Area
3.				
4.				

Analyze and Reflect

Collaborate

5. What is the total surface area of the container described at the beginning of the lesson? Round to the nearest tenth.

Create

On Your Own

6. MP **Reason Inductively** Describe how to find the area of the curved surface of a cylinder.

7. Inquiry How can the surface area of a cylinder be determined?

Lesson 4

Surface Area of Cylinders

Real-World Link

Bakery The Shiny Bright bakery is making a cake for Manal's wedding ceremony. The top tier of the cake will be in the shape of a cylinder with a height of 4 centimeters and a diameter of 14 centimeters.

1. What are the shapes that make up the net of the cake? Sketch the net in the space provided.

2. How is the length of the rectangle related to the circles that form the top and bottom of the cake?

3. Find the area of each part of the cake. Round to the nearest whole number.

   Top: ____ cm^2 Bottom: ____ cm^2 Side: ____ cm^2

4. Add the values from Exercise 3. What is the total surface area of the cake? ____ cm^2

Essential Question

WHY are formulas important in math and science?

Vocabulary

lateral area
total surface area

Mathematical Practices
1, 3, 4

Which Mathematical Practices did you use? Shade the circle(s) that applies.

(1) Persevere with Problems	(5) Use Math Tools
(2) Reason Abstractly	(6) Attend to Precision
(3) Construct an Argument	(7) Make Use of Structure
(4) Model with Mathematics	(8) Use Repeated Reasoning

Key Concept

Surface Area of a Cylinder

Work Zone

Lateral Area

Words The lateral area *L.A.* of a cylinder with height h and radius r is the circumference of the base times the height.

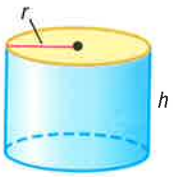
Symbols $L.A. = 2\pi rh$

Total Surface Area

Words The surface area *S.A.* of a cylinder with height h and radius r is the lateral area plus the area of the two circular bases.

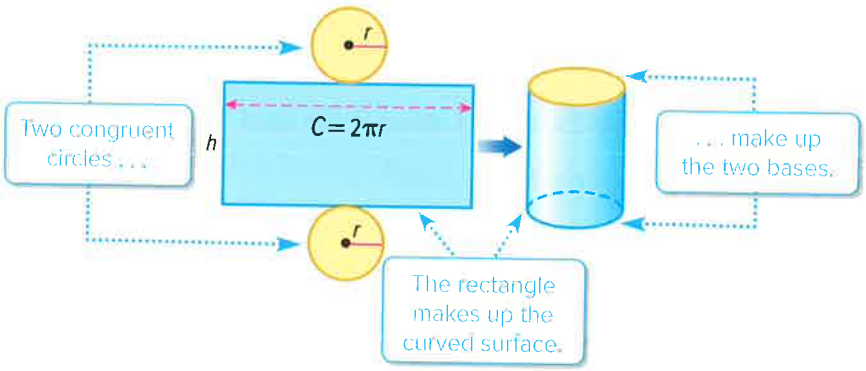
Symbols $S.A. = L.A. + 2\pi r^2$ or $S.A. = 2\pi rh + 2\pi r^2$

Model



area of base $= \pi r^2$

You can find the surface area of a cylinder using a net.



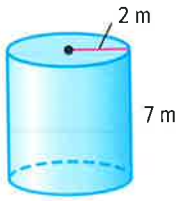
In the diagram above, the length of the rectangle is the same as the circumference of the circle, $2\pi r$. Also, the width of the rectangle is the same as the height of the cylinder.

The **lateral area** of a three-dimensional figure is the surface area of the figure, excluding the area of the base(s). So, the lateral area of a cylinder is the area of curved surface.

The **total surface area** of a three-dimensional figure is the sum of the areas of all its surfaces.

Example

1. **Find the surface area of the cylinder. Round to the nearest tenth.**



$S.A. = 2\pi rh + 2\pi r^2$ Surface area of a cylinder

$S.A. = 2\pi(2)(7) + 2\pi(2)^2$ Replace r with 2 and h with 7.

$S.A. \approx 113.1$ Simplify.

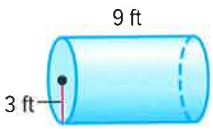
The surface area is about 113.1 square meters.

Got it? **Do these problems to find out.**

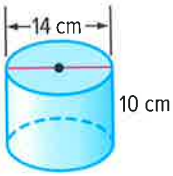
Show your work.

Find the surface area of each cylinder. Round to the nearest tenth.

a.



b.



a. _____

b. _____

Example

2. **A circular fence that is 2 meters high is to be built around the outside of a carousel. The distance from the center of the carousel to the edge of the fence will be 12 meters. What is the area of the fencing material that is needed to make the fence around the carousel?**

You need to find the lateral area. The radius of the circular fence is 12 meters. The height is 2 meters.

$L.A. = 2\pi rh$ Lateral area of a cylinder

$L.A. = 2\pi(12)(2)$ Replace r with 12 and h with 2.

$L.A. \approx 151$ Simplify.

So, about 151 square meters of material is needed to make the fence.

Got it? **Do these problems to find out.**

c. Find the area of the label of a can of tuna with a radius of 5.1 centimeters and a height of 2.9 centimeters. Round to the nearest tenth.

d. Find the total surface area of a cylindrical candle with a diameter of 4 centimeters and a height of 8 centimeters. Round to the nearest tenth.

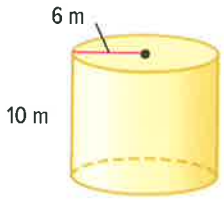
c. _____

d. _____

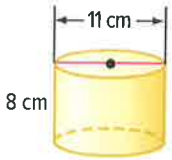
Guided Practice

Find the total surface area of each cylinder. Round to the nearest tenth. (Example 1)

1. Show your work.



2.



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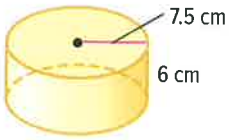

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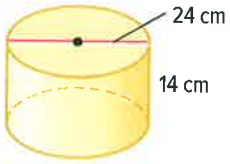
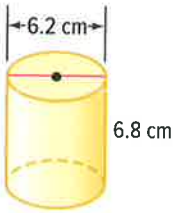


5. Find the total surface area of a water tank with a height of 10 meters and a diameter of 10 meters. Round to the nearest tenth. (Example 1)

Find the lateral area of each cylinder. Round to the nearest tenth. (Example 2)

6.



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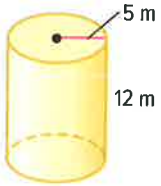


8. Find the area of the label of a cylindrical potato chip container with a radius of 7.5 cm and a height of 22 cm. Round to the nearest tenth. (Example 2)

9. **Building on the Essential Question** How is a calculation affected if you round π to 3.14 or use the π key on your calculator? Explain.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

FOLDABLES Time to update your Foldable!

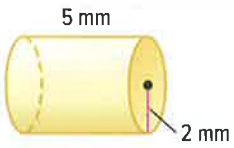
Name _____ My Homework _____

Independent Practice

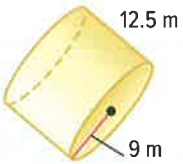
Find the total surface area of each cylinder. Round to the nearest tenth.

(Example 1)

1. Show your work.



2.



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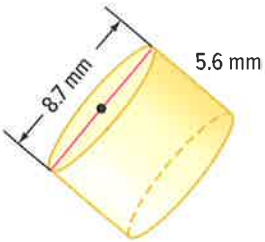
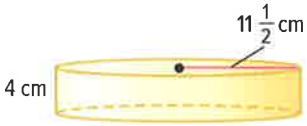


4.

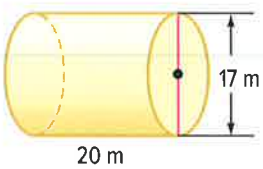


5. A cylindrical candle has a diameter of 4 centimeters and a height of 7 centimeters. To the nearest tenth, what is the total surface area of the candle? (Example 1) _____

6. Find the total surface area of an unsharpened cylindrical pencil that has a radius of 0.5 centimeter and a height of 19 centimeters. Round to the nearest tenth. (Example 1) _____

Find the lateral area of each cylinder. Round to the nearest tenth. (Example 2)

7.



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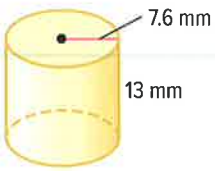


9. Find the lateral area of a cylindrical copper pipe that has a diameter of 6.4 centimeters and a height of 12 centimeters. Round to the nearest tenth.

(Example 2) _____

10. MP **Model with Mathematics** Refer to the graphic novel frame below.

a. What is the least amount of paper that will be needed to wrap one candle with no overlap? _____

b. How many square centimeters of wrapping paper will be needed to wrap all 70 candles? _____

H.O.T. Problems Higher Order Thinking

11. MP **Persevere with Problems** If the height of a cylinder is doubled, will its surface area also double? Explain your reasoning.

_____

_____

12. MP **Reason Inductively** Which has a greater surface area: a cylinder with radius 6 centimeters and height 3 centimeters or a cylinder with radius 3 centimeters and height 6 centimeters? Explain your reasoning.

_____

_____

_____

13. MP **Reason Inductively** A baker is icing a cylindrical cake with radius r and height h. The baker will ice the top and sides of the cake. Write an equation giving the total area A that the baker will ice. Explain why your equation is not the same as the formula for the total surface area of a cylinder.

_____

_____

_____

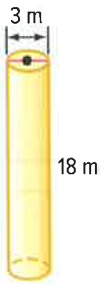
Name _____ My Homework _____

Extra Practice

Copy and Solve **For Exercises 14–27, show your work and answers on a separate piece of paper.**

Find the lateral area and the total surface area of each cylinder. Round to the nearest tenth.

14.



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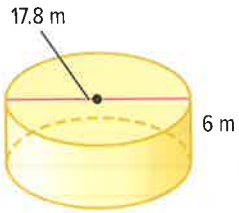
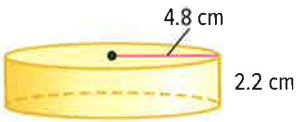
16.



17. A lamp shade is in the shape of a cylinder with a height of 18 centimeters and a radius of 6.75 centimeters. Fabric will cover the lateral area of the lamp shade. Find the area of the fabric needed. Round to the nearest tenth.

MP Use Math Tools **Estimate the surface area of each cylinder.**

18.



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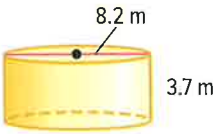

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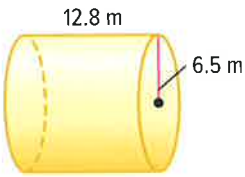
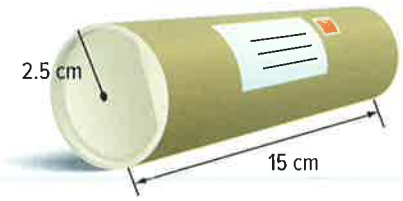


21. The mail tube shown is made of cardboard and has plastic end caps. Approximately what percent of the surface area of the mail tube is cardboard?



22. **MP Persevere with Problems** A hot cocoa canister is a cylinder with a height of 24.5 centimeters and a diameter of 13 centimeters.

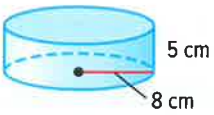
a. What is the lateral area of the hot cocoa canister to the nearest tenth?

b. How does the lateral area change if the height is divided by 2?

Power Up! Test Practice

23. Manal is comparing how much wrapping paper it would take to wrap the containers below.

Container I



Container II

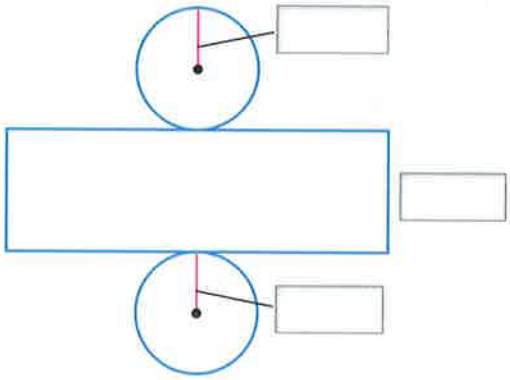
Complete each statement with the correct answers. Round answers to the nearest square centimeter.

a. The minimum amount of wrapping paper needed for Container I is _____ square centimeters.

b. The minimum amount of wrapping paper needed for Container II is _____ square centimeters.

c. Container _____ requires _____ more square centimeters of wrapping paper than Container _____.

24. Sally has a cylindrical paper clip holder with a diameter of 5 centimeters and a height of 4.2 centimeters. Label the net of the cylinder below with the correct dimensions.

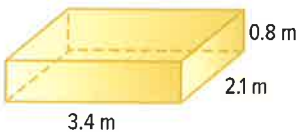


To the nearest tenth, what is the surface area of the paper clip holder? _____

Spiral Review

Find the surface area of each prism.

25.



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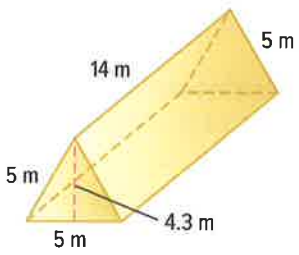


27. Najla is filling cone-shaped treat bags with candy. Each bag has a height of 15 cm and a radius of 2 cm. What is the volume of each bag? Round to the nearest tenth.

Inquiry Lab
Nets of Cones

Mathematical Practices
1, 3

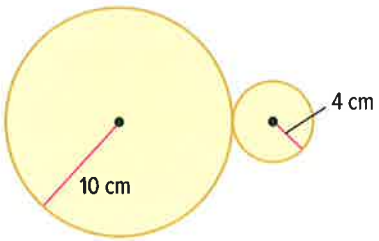
Inquiry **HOW can the surface area of a cone be found?**

Houriyya is making decorations for a celebration. They are in the shape of a cone and will be covered with tissue paper. What is the surface area of the cone that will be covered with tissue paper?

Hands-On Activity 1

In this Activity, you will construct a net of the cone for the party hats. The radius of the base is 4 centimeters. The *slant height* of the cone is 10 centimeters.

Step 1 On a separate sheet of paper, use a compass to draw two circles slightly touching, one with a radius of 10 centimeters and one with a radius of 4 centimeters.



Step 2 You need to find the portion of the circumference from the larger circle that will wrap around the outside of the smaller circle to make the cone. Use the proportion shown to find the central angle measure that represents the portion of the large circle you will use.

$$\frac{\text{circumference of smaller circle}}{\text{circumference of larger circle}} = \frac{\text{portion (in degrees) that is unknown}}{\text{total number of degrees in a circle}}$$

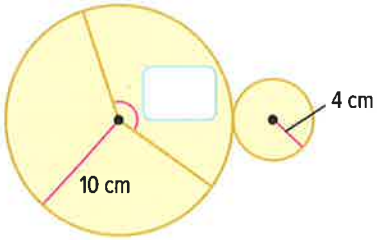
Write and solve the proportion. Round to the nearest whole number.

$$\frac{\square}{\square} = \frac{x}{360}$$

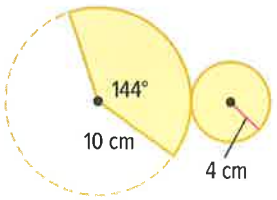
$$x \approx \square$$

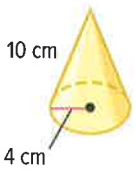
So, you will need $\square$ degrees of the larger circle.

Step 3 Use a protractor to draw the central angle in your larger circle to create the net of the cone.



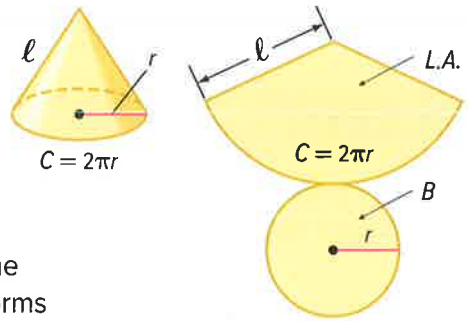
Step 4 The net of the cone is the portion of both circles that are shown by the solid lines. Cut out the net and make the cone.





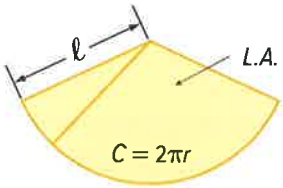
Hands-On Activity 2

The net shows that the surface area of a cone with slant height ℓ and radius r is the sum of its base B and its lateral area $L.A.$ The base B is a circle. The lateral area $L.A.$ is *part* of a larger circle.

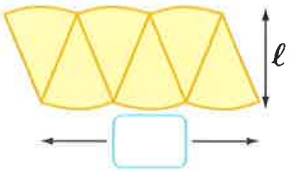


The circumference of the base B is the same length as the part of the larger circle that forms the lateral area of the cone.

Step 1 The figure represents the lateral area of the cone. Divide the figure into 6 equal sections. The first one is done for you.



Step 2 The parallelogram shows the 6 rearranged sections. Write an expression that represents the length of the parallelogram.



Step 3 Use the expression from Step 2 to write a formula for the area of the parallelogram, which is the lateral surface area of the cone. _____

Step 4 Write a formula for the total surface area of the cone. _____

Investigate

Work with a partner. Draw the net of a cone in the space provided with each of the following dimensions.

1. base radius: 2.4 centimeters

slant height: 3.6 centimeters

The angle measure needed to create the cone is _____°.

2. base radius: 2 centimeters

slant height: 4 centimeters

The angle measure needed to create the cone is _____°.

Analyze and Reflect

Collaborate

Work with a partner. Use the formula from Activity 2 to find the total surface area of each of the following cones given the radius of the base and the slant height. Round the measure of the central angle to the nearest whole number. Round the surface area to the nearest tenth.

	radius of base (r)	slant height (ℓ)	measure of central angle (°)	surface area ($\pi r\ell + \pi r^2$)
3.	2 m	5 m		
4.	5 cm	15 cm		
5.	3 cm	20 cm		

6. Refer to Activity 1. What is the lateral area of the party hat that Houriyya is covering with tissue paper? Round to the nearest tenth.

Create

On Your Own

7. MP **Make a Conjecture** Suppose the radius of the base of a cone is increased while the slant height stays the same. Make a conjecture about how the lateral surface area is affected.

8. MP **Make a Conjecture** Suppose a cone's slant height is decreased. Make a conjecture about which is affected more: the base or the lateral area. Justify your response.

9. Inquiry HOW can the surface area of a cone be found?

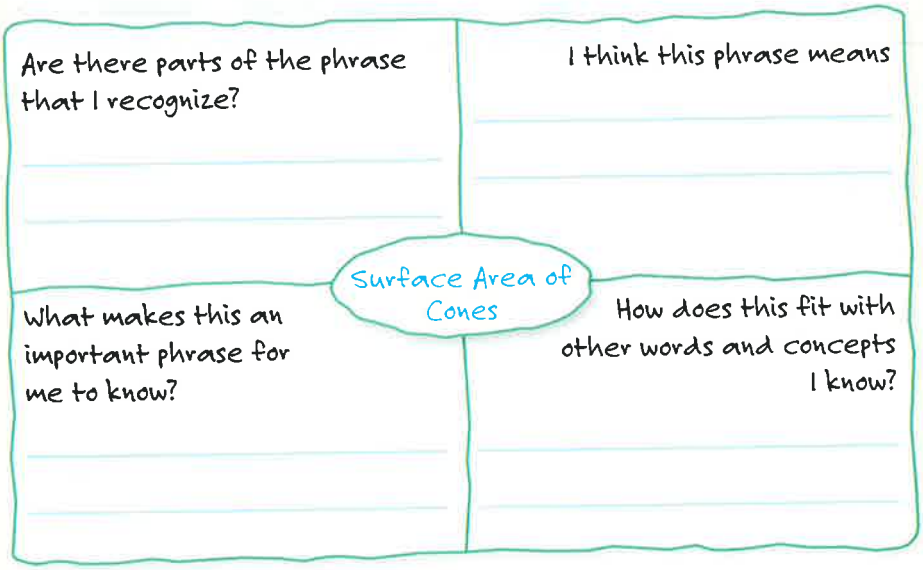
Lesson 5

Surface Area of Cones

Vocabulary Start-Up

Recall that a cone is a three-dimensional figure with one circular base. A curved surface connects the base and vertex.

Complete the graphic organizer.



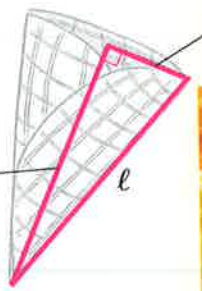
Essential Question

WHY are formulas important in math and science?

MP Mathematical Practices
1, 2, 3, 7

Real-World Link

Badr is making waffle cones from scratch. Use the Pythagorean Theorem to find the slant height ℓ of the cone if the radius is 5 centimeters and the height is 15 centimeters. Round to the nearest tenth. ____ cm.



Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

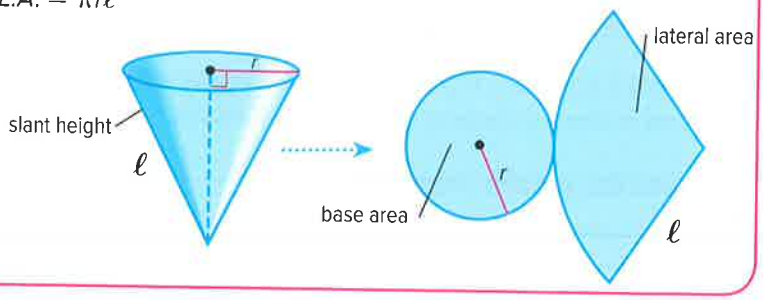
Key Concept

Lateral Area of a Cone

Words The lateral area *L.A.* of a cone is π times the radius times the slant height ℓ.

Symbols $L.A. = \pi r\ell$

Model



Work Zone

Lateral Area of a Cone

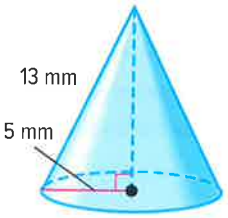
The lateral area of a cone is one-half the circumference of the base times the slant height.

$L.A. = \frac{1}{2}(2\pi r)l$

$L.A. = \pi rl$

Example

1. **Find the lateral area of the cone. Round to the nearest tenth.**



$L.A. = \pi r\ell$ Lateral area of a cone

$L.A. = \pi \cdot 5 \cdot 13$ Replace r with 5 and ℓ with 13.

$L.A. \approx 204.2$ Simplify.

The lateral area of the cone is about 204.2 square millimeters.

Got it? **Do these problems to find out.**

Show your work.

a. Find the lateral area of a cone with a radius of 4 centimeters and a slant height of 9.5 centimeters. Round to the nearest tenth.

a. _____

b. Find the lateral area of a cone with a diameter of 16 centimeters and a slant height of 10 centimeters. Round to the nearest tenth.

b. _____

Key Concept

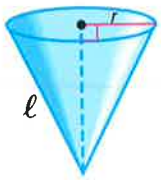
Surface Area of a Cone

Words The surface area *S.A.* of a cone with slant height ℓ and radius r is the lateral area plus the area of the base.

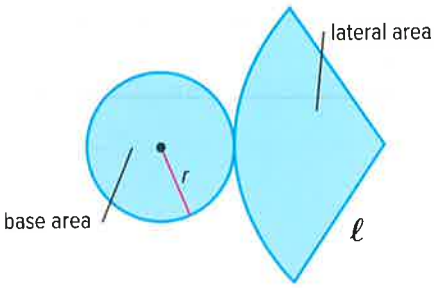
Symbols $S.A. = L.A. + \pi r^2$ or $S.A. = \pi r\ell + \pi r^2$

You can find the surface area of a cone using a net. The surface area of a cone is the sum of its lateral area and the area of its base.

Model of Cone

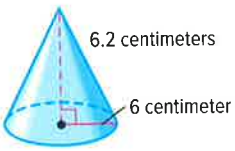


Net of Cone



Example

2. **Find the surface area of the cone. Round to the nearest tenth.**



$S.A. = \pi r\ell + \pi r^2$ Surface area of a cone

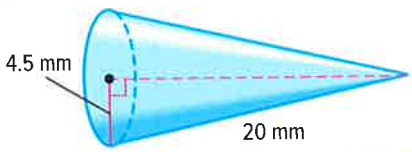
$S.A. = \pi \cdot 6 \cdot 6.2 + \pi \cdot 6^2$ Replace r with 6 and ℓ with 6.2.

$S.A. \approx 230.0$ Simplify.

The surface area of the cone is about 230.0 square centimeters.

Got it? **Do this problem to find out.**

c. Find the surface area of the cone. Round to the nearest tenth.



Show your work.

c. _____

Real World Example

3. **A conical tent has a radius of 5 meters and a slant height of 12 meters. Find the lateral area of the tent. Round to the nearest tenth.**

$L.A. = \pi r\ell$ Lateral area of a cone

$L.A. = \pi \cdot 5 \cdot 12$ Replace r with 5 and ℓ with 12.

$L.A. \approx 188.5$ Simplify.

The lateral area of the tent is about 188.5 square meters.

Got it? **Do this problem to find out.**

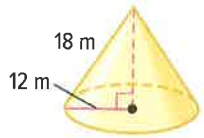
d. Rasheed bought candles that were in the shape of a cone. Each candle has a diameter of 8 centimeters and a slant height of 11 centimeters. Find the lateral area of one candle. Round to the nearest tenth.

d. _____

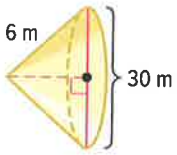
Guided Practice

Find the lateral area of each cone. Round to the nearest tenth. (Example 1)

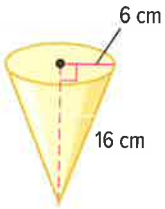
1.



2.



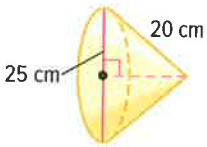
3.



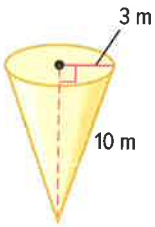
4.

Find the surface area of each cone. Round to the nearest tenth. (Example 2)

5.



6.



7. A local ice cream shop sells waffle cones dipped in chocolate. The waffle cone has a diameter 6.5 cm and a slant height of 15 cm. Find the lateral area of the waffle cone. Round to the nearest tenth. (Example 3)

8. **Building on the Essential Question** How does the volume of a three-dimensional figure differ from its surface area?

Rate Yourself!

- ☐ I understand the surface area of cones.

  Great! You're ready to move on!

- ☐ I still have some questions about the surface area of cones.

FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

Go online for Step-by-Step Solutions

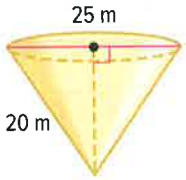
Find the lateral area of each cone. Round to the nearest tenth. (Example 1)

1.

Show your work.

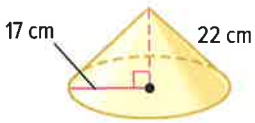
2.

3.



Find the surface area of each cone. Round to the nearest tenth. (Example 2)

4.



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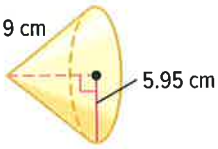

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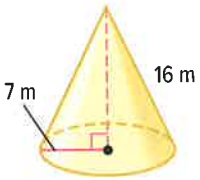


7. A snow cone has a diameter of 5 centimeters and a slant height of 12.7 centimeters. What is the lateral area of the snow cone? Round to the nearest tenth. (Example 3)

_____

8. An active conical volcano has a radius of about 2.5 kilometers and slant height of about 9.6 kilometers. What is the lateral area of the volcano? Round to the nearest tenth. (Example 3) _____

9. The lateral area of a cone with a diameter of 15 millimeters is about 333.5 square millimeters.

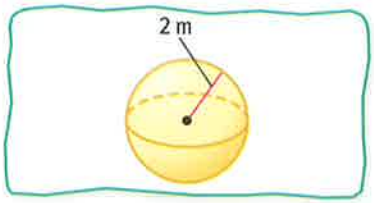
a. Find the surface area of the cone. Round to the nearest tenth.

_____

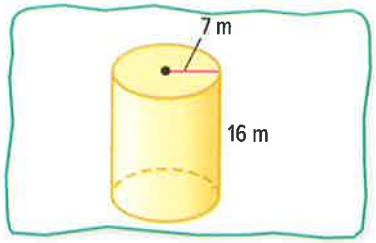
b. What is the slant height of the cone? Round to the nearest tenth.

_____

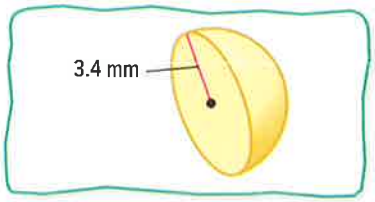
10. MP **Identify Structure** Match the figure with its correct volume or surface area formula.



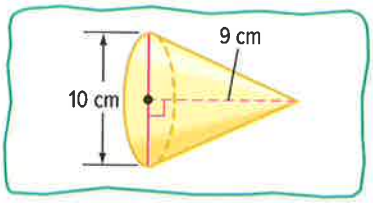
$SA = 2\pi r^2 + 2\pi rh$



$SA = \pi rl + \pi r^2$



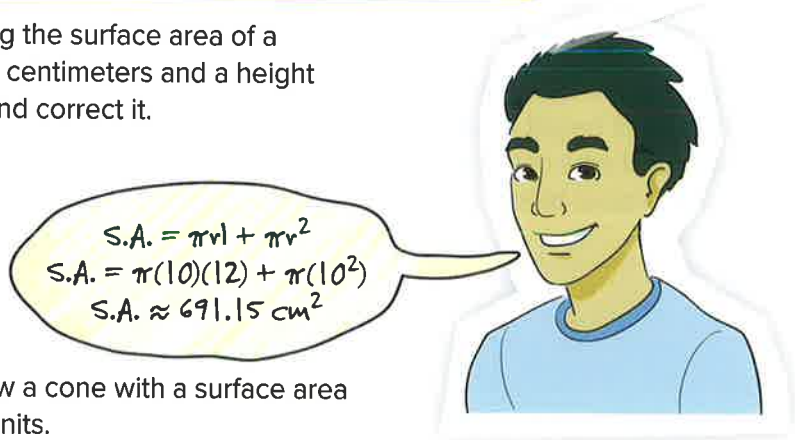
$V = \frac{4}{3}\pi r^3$



$V = \frac{2}{3}\pi r^3$

H.O.T. Problems Higher Order Thinking

11. MP **Find The Error** Enrique is finding the surface area of a cone. The cone has a diameter of 10 centimeters and a height of 12 centimeters. Find his mistake and correct it.



12. MP **Persevere with Problems** Draw a cone with a surface area that is between 50 and 100 square units.

13. MP **Justify Conclusions** Which has a greater surface area: a square pyramid with a base of x units and a slant height of ℓ units or a cone with a diameter of x units and a slant height of ℓ units? Explain your reasoning.

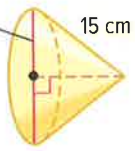
Name _____ My Homework _____

Extra Practice

Copy and Solve **For Exercises 14–35, show your work and answers on a separate piece of paper.**

Find the lateral area of each cone. Round to the nearest tenth.

14.



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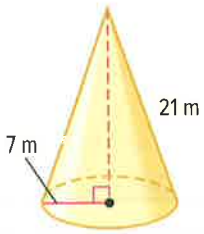

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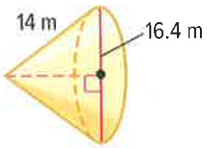

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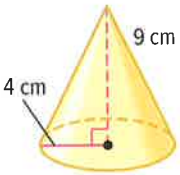

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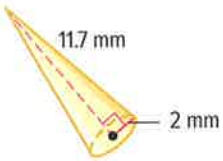

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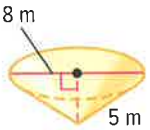
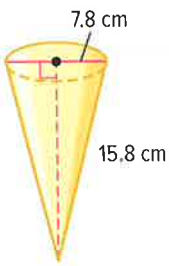


20. Find the lateral area of a cone with a radius of 3.5 millimeters and a slant height of 8 millimeters. Round to the nearest tenth.

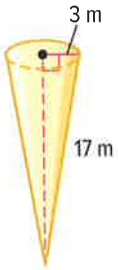
21. Find the lateral area of a cone with a radius of 9 centimeters and a slant height of 16 centimeters. Round to the nearest tenth.

Find the surface area of each cone. Round to the nearest tenth.

22.



23.



24. Find the surface area of a cone with a diameter of 20 millimeters and a slant height of 42 millimeters. Round to the nearest tenth.

25. Find the surface area of a cone with a radius of 5.1 meters and a slant height of 17 meters. Round to the nearest tenth.

26. MP **Reason Abstractly** A conical hat has a radius of 7 centimeters and a height of 14 centimeters. Find the slant height of the hat. Then find the lateral area. Round to the nearest tenth.

Power Up! Test Practice

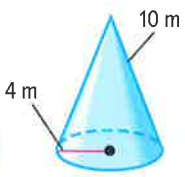
27. A cone has the radius and height shown. Which of the following statements are true? Select all that apply.

- ☐ The slant height of the cone is 13 cm.
- ☐ The lateral area of the cone is about 204 square centimeters.
- ☐ The total surface area of the cone is about 236 square centimeters.

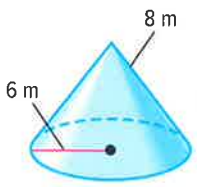
28. Four cones have the dimensions shown. Sort the cones from least to greatest lateral areas. Round to the nearest tenth.

Cone 1

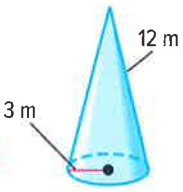
Cone 2



Cone 3



Cone 4

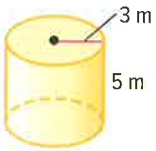


	Cone	Lateral Area (m^2)
Least		
Greatest		

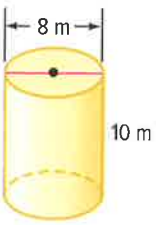
Spiral Review

Find the surface area of each cylinder. Round to the nearest tenth.

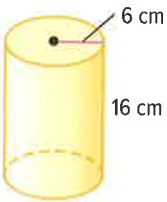
29.



30.



31.



32. diameter: 10 meters
height: 24 meters

33. radius: 12 meters
height: 9 meters

34. Find the volume of a cylinder with a radius of 2 centimeters and a height of 25 centimeters. Round to the nearest tenth.

35. Find the volume of a cone with a diameter of 16 meters and a height of 26 meters. Round the nearest tenth.

Inquiry Lab
Changes in Scale

Inquiry **HOW does multiplying the dimensions of a three-dimensional figure by a scale factor affect its volume and surface area?**

MP **Mathematical Practices** 1, 3

A cake decorator is making a large center cake surrounded by small cakes. The large cake has a diameter of 24 centimeters and is 8 centimeters tall. The small cakes' dimensions will be half the dimensions of the large cake. Use the Activity to determine how the volume and surface area of the large cake compare to the volume and surface area of one of the small cakes.

Hands-On Activity

In this Activity, you will determine how changes in the dimensions of a cylinder affect the volume and surface area.

Step 1 Complete the table with the measurements of the cakes.

	Diameter (cm)	Radius (cm)	Height (cm)
Large Cake			
Small Cake			

Step 2 Find the volume of each of the cakes. Round to the nearest whole number.

Volume of large cake = _____

Volume of small cake = _____

The volume of the large cake is about ____ times the volume of a small cake.

Step 3 Find the surface area of each cake. Round to the nearest whole number.

Surface area of large cake = _____

Surface area of small cake = _____

The surface area of the large cake is ____ times the surface area of a small cake.

Investigate

Collaborate

Work with a partner. Round your answers to the nearest whole number.

1. The radius of a cylinder is 25 centimeters and the height is 35 centimeters. The dimensions of a similar cylinder are one-fifth of the original cylinder. Complete the chart.

Show your work.

	Volume	Surface Area
original cylinder		
similar cylinder		
$\frac{\text{similar}}{\text{original}}$		

2. The diameter of a cone is 6 centimeters, the height is 4 centimeters, and the slant height is 5 centimeters. The dimensions of a similar cone are three times that of the original cone.

	Volume	Surface Area
original cone		
similar cone		
$\frac{\text{similar}}{\text{original}}$		

Analyze and Reflect

Collaborate

Work with a partner.

3. Find the volume and surface area for the cylinders shown in the table. Use 3.14 for π.

4. What happens to the volume of a cylinder when the radius and height are multiplied by two? by three?

Cylinder	Radius and Height	Volume	Surface Area
A	1		
B	2		
C	3		

5. What happens to the surface area of a cylinder when the radius and height are multiplied by two? by three?

Create

On Your Own

6. Suppose the dimensions of a rectangular prism are doubled. The volume of the new prism is 800 cubic units. What are the possible dimensions of the original prism?

7. Inquiry HOW does multiplying the dimensions of a three-dimensional figure by a scale factor affect its volume and surface area?

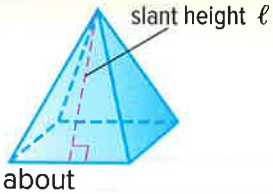
Lesson 6

Changes in Dimensions

Real-World Link

Watch

Monuments Bilal is creating a model of a monument. The model will be $\frac{1}{100}$ of the monument's actual size.



The square pyramid that sits atop the monument's obelisk shape has a slant height of about 17.5 meters. Each side of the pyramid's base is about 10.4 meters.

1. What is the area of one of the triangular faces of the actual pyramid? _____
2. What is the slant height of the pyramid on the model Stephen is creating? _____
3. What is the length of one side of the base of the pyramid on the model? _____
4. What is the area of one of the triangular faces of the model pyramid? _____
5. Write a ratio comparing the area of the triangular side of the model to the actual monument.

_____

6. MP **Make a Conjecture** Write a sentence about the surface area of the model pyramid compared with the actual pyramid.

_____

Essential Question

WHY are formulas important in math and science?

Vocab **Vocabulary**

similar solids

MP **Mathematical Practices**
1, 3, 4

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Key Concept

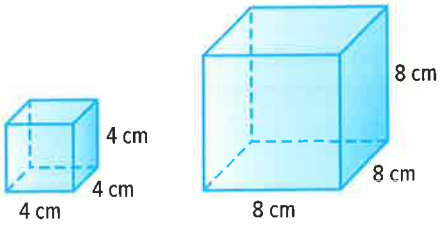
Surface Area of Similar Solids

If Solid X is similar to Solid Y by a scale factor, then the surface area of X is equal to the surface area of Y times the *square* of the scale factor.

Work Zone

Cubes are **similar solids** because they have the same shape and their corresponding linear measures are proportional.

The cubes at the right are similar. The ratio of their corresponding edge lengths is $\frac{8}{4}$ or 2. The scale factor is 2. How are their surface areas related?



S.A. of Small Cube

$S.A. = 6(4 \cdot 4)$

There are 6 faces.

S.A. of Large Cube

$S.A. = 6(2 \cdot 4)(2 \cdot 4)$

$= 2 \cdot 2(6)(4 \cdot 4)$

$= 2^2(6)(4 \cdot 4)$

To find the surface area of the large cube, multiply the surface area of the small cube by the *square* of the scale factor, 2^2 or 4. This relationship is true for any similar solids.

Example

1. **The surface area of a rectangular prism is 78 square centimeters. What is the surface area of a similar prism with dimensions that are 3 times as great as the dimensions of the original prism?**

$S.A. = 78 \times 3^2$ Multiply by the square of the scale factor.

$S.A. = 78 \times 9$ Square 3.

$S.A. = 702 \text{ cm}^2$ Simplify.

Show your work.

Got it? **Do these problems to find out.**

a. _____

b. _____

a. The surface area of a triangular prism is 34 square centimeters. What is the surface area of a similar prism with dimensions that are 3 times as great as the original prism?

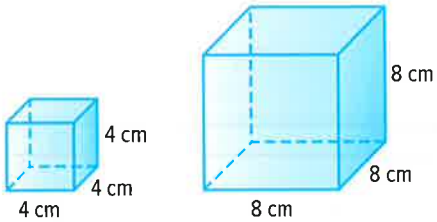
b. A giant box has a surface area of 352 square meters. If the dimensions of a similar box are smaller than the giant box by a scale factor of $\frac{1}{48}$, what is its surface area?

Volume of Similar Solids

Key Concept

If Solid X is similar to Solid Y by a scale factor, then the volume of X is equal to the volume of Y times the *cube* of the scale factor.

Refer to the cubes below.



Volume of Small Cube

$V = 4 \cdot 4 \cdot 4$

Volume of Large Cube

$V = (2 \cdot 4)(2 \cdot 4)(2 \cdot 4)$

$= 2 \cdot 2 \cdot 2(4 \cdot 4 \cdot 4)$

$= 2^3(4 \cdot 4 \cdot 4)$

The volumes of similar solids are related by the *cube* of the scale factor.

Example

2. **A triangular prism has a volume of 432 cubic meters. If the dimensions of the prism are reduced to one third of the original dimensions, what is the volume of the new prism?**

$V = 432 \times \left(\frac{1}{3}\right)^3$ Multiply by the cube of the scale factor.

$V = 432 \times \frac{1}{27}$ Cube $\frac{1}{3}$.

$V = 16 \text{ m}^3$ Simplify.

The volume of the new prism is 16 cubic meters.

Got it? **Do these problems to find out.**

Show your work.

c. A square pyramid has a volume of 512 cubic centimeters. What is the volume of a square pyramid with dimensions one-fourth of the original?

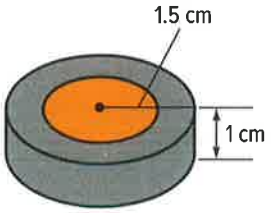
c. _____

d. A cylinder has a volume of 432 cubic meters. What is the volume of a cylinder with dimensions one-third of the original?

d. _____

Example

3. **The measurements for a toy truck wheel are shown at the right. The large truck wheel at the left has dimensions that are 40 times the dimensions of the toy wheel. Find the volume and surface area of the large wheel. Use 3.14 for π.**



Find the volume and surface area of the standard puck first.

$V = \pi r^2 h$
$\approx (3.14)(1.5)^2(1)$
$\approx 7.065 \text{ cm}^3$

$S.A. = 2(\pi r^2) + 2\pi rh$
$\approx 2(3.14)(1.5)^2 + 2(3.14)(1.5)(1)$
$\approx 14.13 + 9.42$
$\approx 23.55 \text{ cm}^2$

Find the volume and surface area of the giant puck using the computations for the standard puck and the scale factor.

$V = V(40)^3$
$= (7.065)(40)^3$
$= 452{,}160 \text{ cm}^3$

$S.A. = S.A.(40)^2$
$= (23.55)(40)^2$
$= 37{,}680 \text{ cm}^2$

The large wheel has a volume of about 452,160 cubic centimeters and a surface area of about 37,680 square centimeters.

STOP and Reflect

What happens to the surface area of a cylinder if its radius and its height are doubled?

Guided Practice

Check

1. The surface area of a rectangular prism is 35 square centimeters. What is the surface area of a similar solid with dimensions that have been enlarged by a scale factor of 7? (Example 1)

Show your work.

2. The volume of a cylinder is about 425 cubic centimeters. What is the volume, to the nearest tenth, of a similar solid with dimensions that are smaller by a scale factor of $\frac{1}{3}$? (Example 2)

3. A box with a sliding lid in Josh's art studio measures 16 centimeters by 15 centimeters by 6 centimeters. A second box used just for paintbrushes has a similar shape and is smaller by a scale factor of $\frac{1}{2}$. Find the volume and surface area of the second box. (Example 3)

4. **Building on the Essential Question** How is the volume of a prism affected when its dimensions are tripled?

Rate Yourself!

How confident are you about changes in dimensions? Check the box that applies.

Name _____ My Homework _____

Independent Practice

1. The surface area of a rectangular prism is 95 square centimeters. What is the surface area of a similar prism with dimensions that are 4 times as great as the original prism? (Example 1) _____

Show your work.

2. The surface area of a pyramid is 57.8 square centimeters. What is the surface area of a similar pyramid with dimensions that are 2 times as great as the original prism? (Example 1) _____

3. A cereal box has a surface area of 280 square centimeters. What is the surface area of a similar box that is larger by a scale factor of 1.4? (Example 1) _____

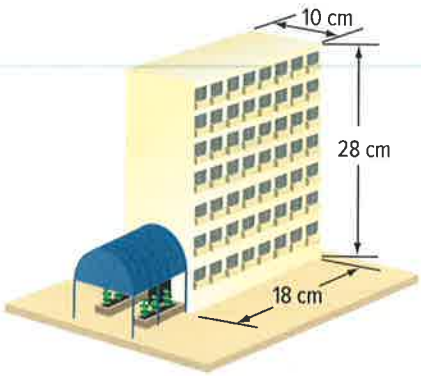
4. A glass display box has a surface area of 378 square centimeters. How many square centimeters of glass are used to create a glass display box with dimensions that are one-half those of the original? (Example 1) _____

5. A cone has a volume of 9,728 cubic millimeters. What is the volume of a similar cone with dimensions that are one-eighth the dimensions of the original? (Example 2) _____

6. A triangular prism has a volume of 350 cubic meters. If the dimensions are tripled, what is the volume of the new prism? (Example 2) _____

7. The model of a new apartment building is shown. The architect plans for the building to be 360 times the dimensions of the model. What will be the volume and surface area of the new building, in cubic meters and square meters, when it is completed? (Example 3)

_____

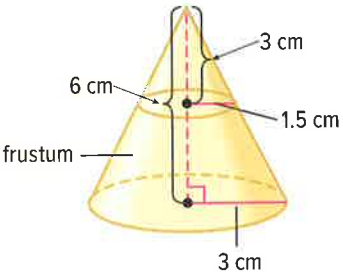


8. The world's largest cube puzzle is in Knoxville, Tennessee. It measures 180 centimeters on each side. The scale factor between a standard cube puzzle and the largest puzzle is $\frac{1}{24}$. Find the surface area and volume of the standard cube puzzle. (Example 3)

9. MP **Persevere with Problems** Two spheres are similar in shape. The scale factor between the smaller sphere and the larger sphere is $\frac{3}{4}$. If the volume of the smaller sphere is 126.9 cubic meters, what is the volume of the larger sphere?

H.O.T. Problems Higher Order Thinking

10. MP **Persevere with Problems** A *frustum* is the solid left after a cone is cut by a plane parallel to its base and the top cone is removed.



   a. Is the smaller cone that is removed similar to the original cone? Justify your response.

   b. What is the volume of the smaller cone? the larger cone? Use 3.14 for π.

   c. What is the ratio of the volume of the smaller cone to the volume of the larger cone?

   d. What is the volume of the frustum?

11. MP **Justify Conclusions** A cone has a volume of x cubic centimeters. If the dimensions of a second cone are one-sixth the original cone, what is the volume of the second cone? Explain your reasoning.

12. MP **Reason Inductively** Determine whether the following statement is *true* or *false*. Explain your reasoning.

All spheres are similar.

Name _____ My Homework _____

Extra Practice

Copy and Solve **For Exercises 13–29, show your work and answers on a separate piece of paper.**

13. The surface area of a triangular prism is 300 square meters. What is the surface area of a similar prism with dimensions that are 3 times greater than the dimensions of the original prism?

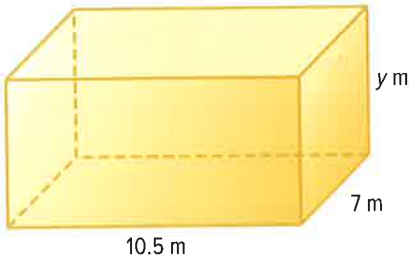
14. The surface area of a rectangular prism is 1,350 square centimeters. What is the surface area of a similar prism with dimensions that are 2 times greater than the dimensions of the original prism?

15. A pyramid has a volume of 640 cubic centimeters. If the dimensions of the pyramid are reduced to one-fourth of the original dimensions, what is the volume of the new pyramid?

16. The surface area of a rectangular prism is 1,300 square centimeters. Find the surface area of a similar solid that is larger by a scale factor of 3.

17. The surface area of a triangular prism is 10.4 square meters. What is the surface area of a similar solid that is smaller by a scale factor of $\frac{1}{4}$?

18. Find the missing measures for the pair of similar solids.



MP Reason Inductively **Determine whether each statement is *always*, *sometimes*, or *never* true.**

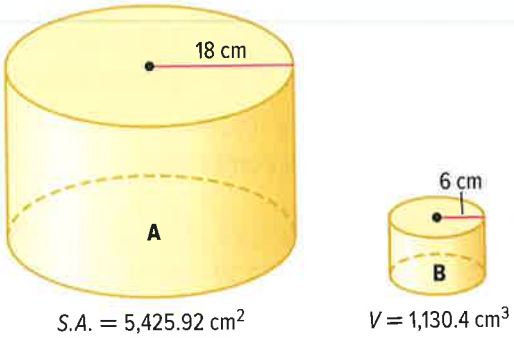
19. Two prisms with congruent bases are similar.

20. Similar solids have equal volumes.

21. Two cubes are similar.

22. A prism and pyramid are similar.

23. Two similar cylinders are shown.



   a. What is the ratio of their radii?

   b. What is the ratio of their surface areas? volumes?

   c. Find the surface area of Cylinder B.

   d. Find the volume of Cylinder A.

Power Up! Test Practice

24. The side lengths of Cube *A* are 3 times the side length of Cube *B*. The side lengths of Cube *B* are half the side lengths of Cube *C*. Select the correct values to complete the following statements.

$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$
2	3	4	6
8	9	16	27

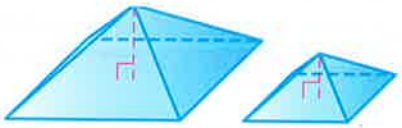
a. The surface area of Cube *A* is ☐ times the surface area of Cube *B*.

b. The volume of Cube *A* is ☐ times the volume of Cube *B*.

c. The volume of Cube *B* is ☐ the volume of Cube *C*.

d. The surface area of Cube *B* is ☐ the surface area of Cube *C*.

25. The two pyramids shown are similar.



Determine if each statement is true or false.

a. The scale factor from the smaller pyramid to the larger pyramid is $\frac{5}{3}$. ☐ True ☐ False

b. The surface area of the larger pyramid is $\frac{5}{3}$ times the surface area of the smaller pyramid. ☐ True ☐ False

c. The volume of the larger pyramid is $\frac{25}{9}$ times the volume of the smaller pyramid. ☐ True ☐ False

Common Core Spiral Review

26. Amira is making a clay figurine of a dog. The dog is 75 centimeters tall. If she uses a scale of 1 centimeter = 10 centimeters, how tall will the clay figurine be?

27. The table shows the approximate dimensions of the fields used in various sports.

Sport	Width (meters)	Length (meters)
Field hockey	60	100
Rugby	68	112
Lacrosse	60	110
Soccer	70	115

a. What is the area of the field hockey field in square meters?

b. What is the difference between the area of the soccer field and the area of the lacrosse field in square meters?

c. If a hectare is 10,000 square meters, about how many hectares are all four fields combined?

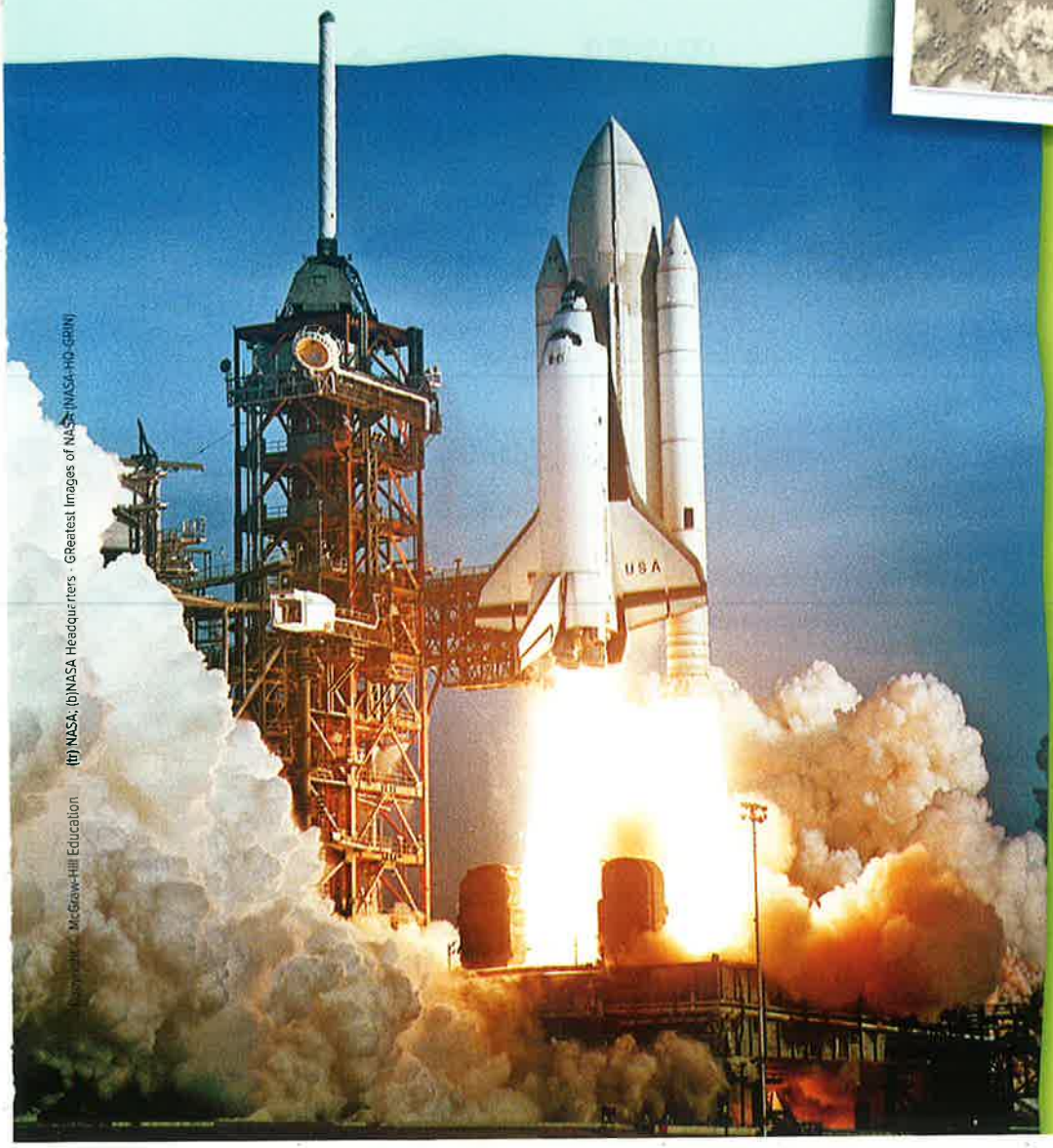
28. Find the surface area of a cylinder with a radius of 15 meters and a height of 5 meters. Round to the nearest tenth.

29. Find the surface area of a cone with a diameter of 4.5 centimeters and a slant height of 12 centimeters. Round to the nearest tenth.

21ST CENTURY CAREER in Architecture

Space Architect

Do you like building things? Are you an excellent problem solver? If so, you have what it takes to be a space architect. Space architects use principles from architecture, design, engineering, and science to create places for people to live and work in outer space. Their designs include transfer vehicles, lunar habitats, and Martian greenhouses. Because of the limitations, space architecture must be very efficient and functional. Every square centimeter of surface and every cubic centimeter of space must have a purpose.



(t) NASA; (b)NASA Headquarters - GReatest Images of NASA (NASA-HQ-GRIN)

Is This the Career for You?

Are you interested in a career as a space architect? Take some of the following courses in high school.

- Aerospace Technology
- Calculus
- Geometry
- Introductory Space Planning
- Intro to CAD

Turn the page to find out how math relates to a career in Architecture.

MP Out of this World Architecture

Use the space laboratories below to solve each problem. Round to the nearest tenth.

1. *Destiny* has one round window that is 50 centimeters in diameter. What is the circumference and area of the window?

2. What is the volume of *Destiny*?

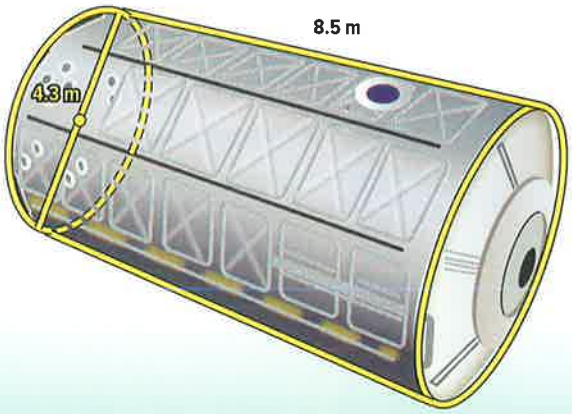
3. The internal volume of *Columbus*, or the space where the astronauts live and work, is about 34.7 cubic meters less than the total volume. What is the internal volume of *Columbus*?

4. Find the surface area of *Destiny*.

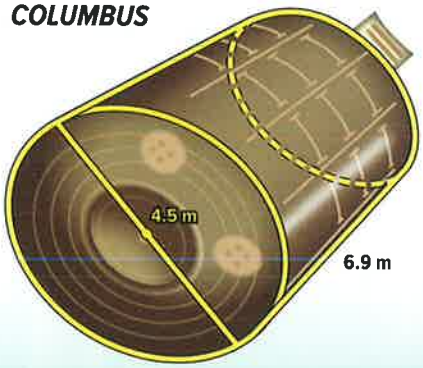
5. Without calculating, predict whether *Destiny* or *Columbus* has a greater surface area. Then test your prediction by calculating the solution.

6. *Kibo* is a Japanese laboratory on the International Space Station. It is a cylinder 11.2 meters long with a radius of 2.2 meters. Compare its volume to the volumes of *Destiny* and *Columbus*.

DESTINY



COLUMBUS



MP Career Project

It's time to update your career portfolio! Use the Internet or another source to research a career as a space architect. Write a paragraph that summarizes your findings.

What subject in school is the most important to you? How would you use that subject in this career?

Chapter Review

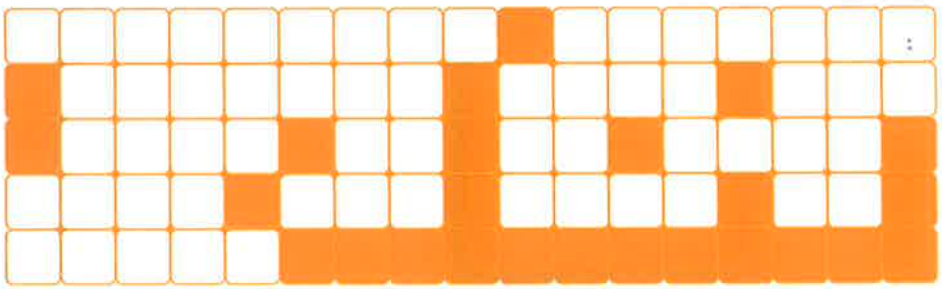
Vocabulary Check

Complete each sentence using the vocabulary list at the beginning of the chapter.

1. A set of all points in space that are a given distance from a given point is called a _____.
2. Cubes are _____ because they have the same shape and their corresponding linear measures are proportional.
3. A _____ is a three-dimensional figure with two parallel congruent circular bases connected by a curved surface.

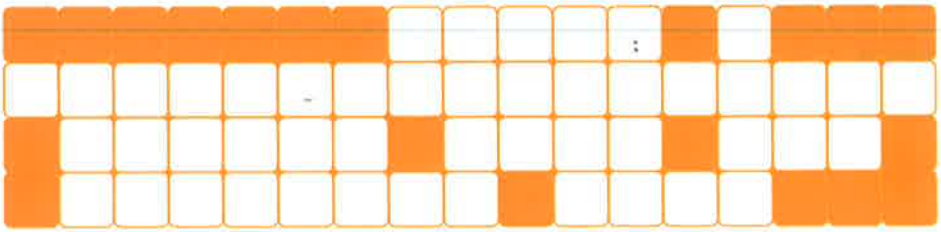
Reconstruct the vocabulary word and definition from the letters under the grid. The letters for each column are scrambled directly under that column.

4.



H A D
M A I D I E F E A E
C O B J E C N P O H A L D F
S O L N E O T S T S O M I O R
T O M P O S U T E T Y P T O R S E

5.



R E A
F I E E D C W E B A E
H I G U U L I M I T S I O N E
T C R R C R E A O N N H S O N A L

Key Concept Check

Use Your FOLDABLES

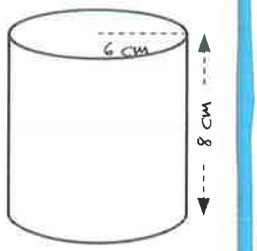
Use your Foldable to help review the chapter.

Tape here

Tape here

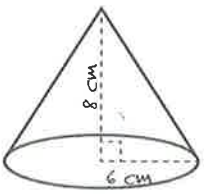
Volume

Volume =



Surface area =

Volume =



Surface area =

Tab 1

Surface Area

Tab 2

Got it?

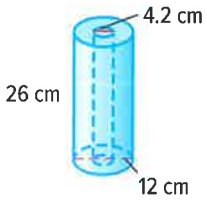
The problems below may or may not contain an error. If the problem is correct, write a "✓" by the answer. If the problem is not correct, write an "X" over the answer and correct the problem.

Find the volume of each figure.

1.

The volume of the sphere is 11,494.0 mm^3.

2.

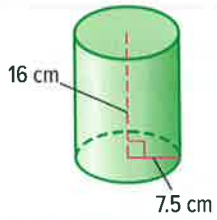


The volume of the composite figure is 3,300.7 cm^3.

Power Up! Performance Task

Popcorn Packaging

A movie theater is considering using cylinders or cones to serve popcorn. The measurements of the proposed cylinder are shown.



Write your answers on another piece of paper. Show all of your work to receive full credit. Use 3.14 for π.

Part A

The manager of the theater wants to sell the popcorn in a cone with a diameter of 17 centimeters. What does the height of the cone need to be in order to hold the same amount of popcorn as the cylinder? Round your answer to the nearest tenth.

Part B

The monthly popcorn special comes in a bowl that is shaped like a hemisphere. The diameter of the bowl is 20 centimeters. The theatre sells the cylindrical container of popcorn for AED5.25. If they sell the bowl of popcorn for AED5.00, is it a good deal? Explain your reasoning.

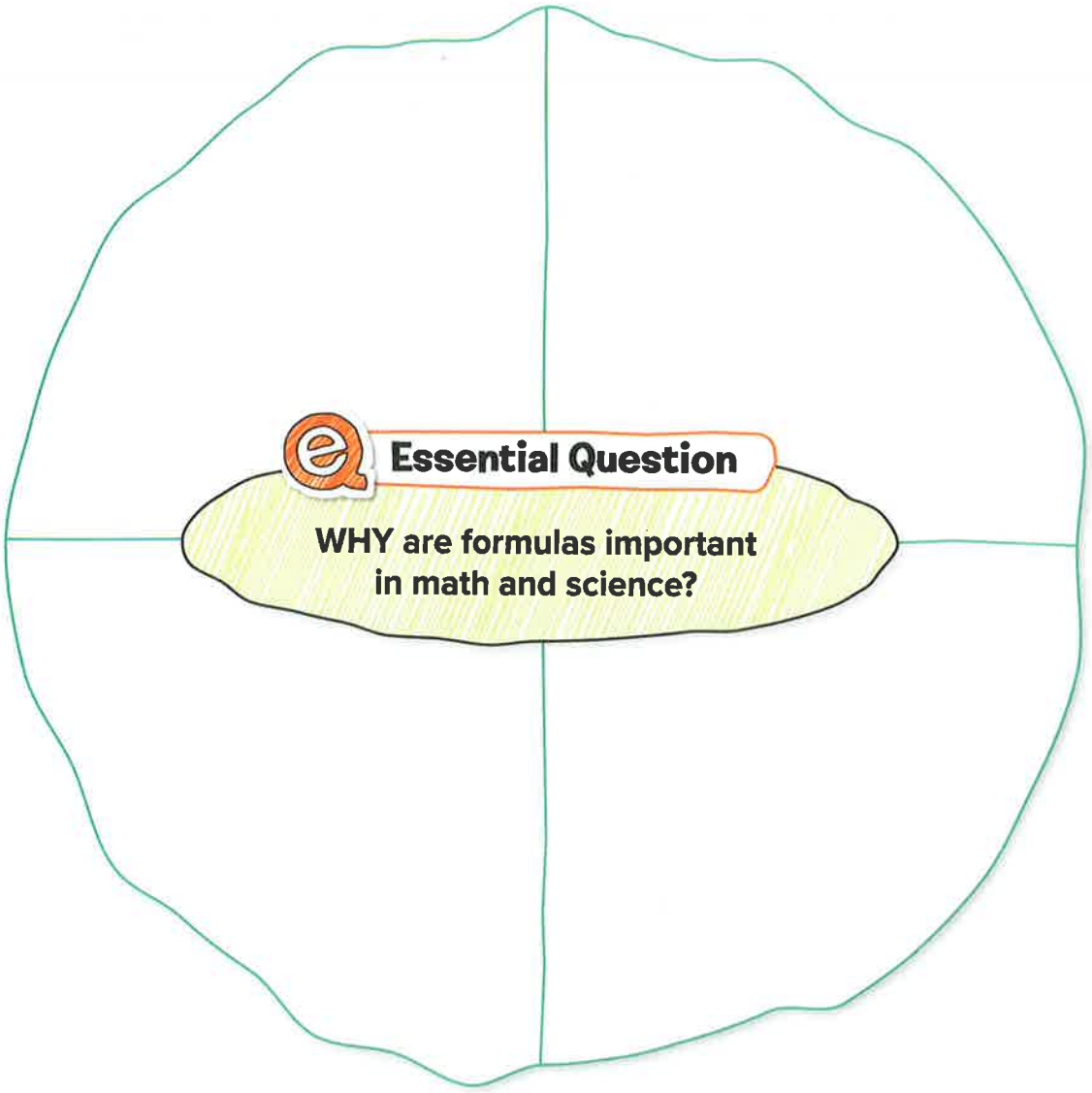
Part C

Suppose the popcorn containers cost AED0.002 per square centimeter to manufacture. Is it less expensive to produce the cone in Part A or the cylinder? Assume that neither container comes with a lid. Explain your reasoning.

Reflect

Answering the Essential Question

Use what you learned about volume and surface area formulas to complete the graphic organizer. Name four topics that use a formula to solve a problem.



Answer the Essential Question. WHY are formulas important in math and science?

UNIT PROJECT

Design That Ride Designers apply many geometric concepts to build exciting new rides. In this project you will:

- **Collaborate** with your classmates as you research amusement park rides.
- **Share** the results of your research in a creative way.
- **Reflect** on how you use different measurements to solve real-life problems.

By the end of this project, you'll be ready to design a new amusement park ride!

Collaborate

Go Online Work with your group to research and complete each activity. You will use your results in the Share section on the following page.

1. Find photos of several different types of amusement park rides. Then describe and label any parallel lines, angles, triangles, congruent figures, similar figures, and three-dimensional shapes you observe.

2. Make a sketch of a current amusement park ride. Research and label its dimensions. Then use what you know about angles, congruence, similarity, the Pythagorean Theorem, surface area, and volume to label as many of its other attributes as you can.

3. Find several examples of amusement park rides that use transformations. Explain the transformation(s) that the ride exhibits.

4. Research different types of trusses. Then explain why trusses are used in the design of some amusement park rides. Include drawings to justify your explanation.

5. Research *potential energy* and *kinetic energy* as they relate to roller coasters. Then create a drawing that explains the concepts.

Share

Collaborate

With your group, decide on a way to share what you have learned about designing an amusement park ride. Some suggestions are listed below, but you can also think of other creative ways to present your information. Remember to show how you used mathematics to complete each of the activities in this project.

- Design an amusement park ride using online simulations. Don't forget to give your ride a name.
- Imagine a nearby amusement park is seeking suggestions for a new ride. Write a proposal for your ride. Be sure to include drawings.

Check out the note on the right to connect this project with other subjects.

Connect with Social Studies

Global Literacy Research information about amusement parks in other countries. Some questions to consider are:

- How are the rides different from those in your country?
- When and where was the earliest amusement park constructed?

Reflect

On Your Own

6. **Answer the Essential Question** HOW can you use different measurements to solve real-life problems?

   a. How did you use what you learned about the measurements of triangles to solve real-life problems in this project?

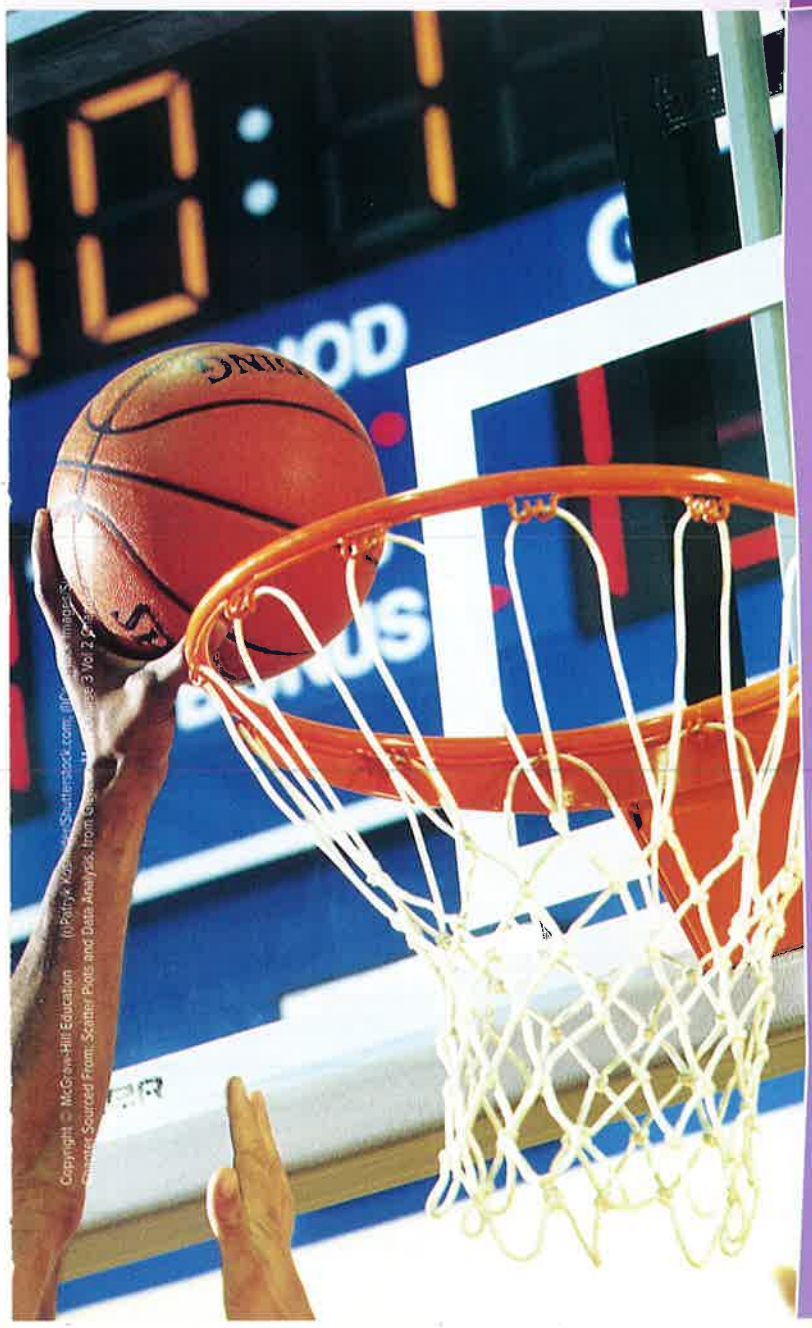
   b. How did you use what you learned about the measurements involved in congruence and similarity to solve real-life problems in this project?

   c. How did you use what you learned about volume and surface area to solve real-life problems in this project?

Statistics and Probability

Essential Question

WHY is learning mathematics important?



Chapter 9
Scatter Plots and Data Analysis

Scatter plots can be used to investigate patterns of association between two quantities. In this chapter, you will construct and interpret scatter plots. You will also draw lines of best fit to model data that suggest a linear association.

Collaborate

Unit Project Preview

Olympic Games Basketball has been a Summer Olympic sport since 1936. You can compare and analyze certain data and statistics based on the scores from various games. The scores from other sports in the Summer Olympics can also be compared and analyzed in the same way.

At the end of Chapter 9, you'll complete a project to learn about scoring in basketball and other Olympic sports. But for now, it's time to do an activity in your book. Name some Summer Olympic sports you enjoy watching and explain their scoring systems.

Summer Olympic Sports

Chapter 9

Scatter Plots and Data Analysis

Essential Question

HOW are patterns used when comparing two quantities?

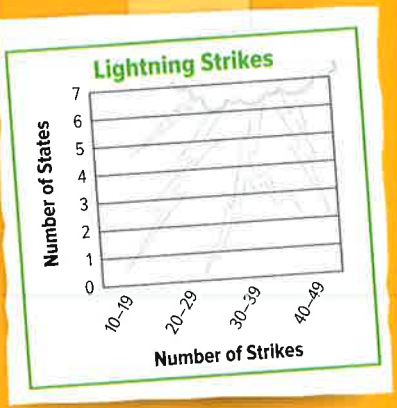
MP Mathematical Practices

1, 2, 3, 4, 5, 7

Math in the Real World

Weather During any given year, the Earth is struck by lightning more than 100 times every second. The numbers of lightning strikes for some states in a recent time period were 10, 10, 11, 12, 12, 14, 18, 20, 20, 21, 39, and 47.

Complete the histogram using the data.



FOLDABLES® Study Organizer

1. Cut out the Foldable on page FL11 of this book.
2. Place your Foldable on page 728.
3. Use the Foldable throughout this chapter to help you learn about scatter plots and two-way tables.

What Tools Do You Need?

Vocabulary

bivariate data	relative frequency
distribution	scatter plot
five-number summary	standard deviation
line of best fit	symmetric
mean absolute deviation	two-way table
qualitative data	univariate data
quantitative data	

Study Skill: Reading Math

Topic Sentences A topic sentence is a sentence that expresses the main idea in a paragraph. In a word problem, the "topic sentence" is usually found near the end. It is the sentence or question that tells you what you need to find. The "topic sentence" in the following example is underlined for you.

> Mrs. Khawla's math class was doing research about wild horses living on public lands. They found that there are about 30,000 wild horses living in Nevada, 4,000 living in Wyoming, and 2,000 living in California. Is the number of wild horses living on public lands in Nevada, Wyoming, and California greater than 35,000?

When you start to solve a word problem, follow these steps.

Step 1 Skim through the problem, looking for the "topic sentence."

Step 2 Go back and read the problem more carefully, looking for the supporting details you need to solve the problem.

For each exercise, underline the "topic sentence". Do not solve the problem.

1. Manal collected data for her science fair project on the relationship between a person's arm span and their height. She wanted to determine whether a relationship exists.
2. The two-way table shows the places that males and females volunteered in the past month. Do a higher percentage of males or females volunteer at the animal shelter?

What Do You Already Know?

Place a checkmark below the face that expresses how much you know about each concept. Then scan the chapter to find a definition or example of it.

I have no clue. I've heard of it. I know it!

Integers				
Concept				Definition or Example
bivariate data				
five-number summaries				
lines of best fit				
positive and negative associations				
scatter plots				
two-way tables				

When Will You Use This?

Here is an example of how unit rates are used in the real world.

Activity How much is the cost of a movie ticket at your local movie theater? Has the price gone up recently? Does it cost more to see a 3-D movie? If so, how much more?

Are You Ready?

Try the Quick Check below. Check

Quick Review

Review

Example 1

The ages of people in a play are shown in the histogram. Describe the histogram. Then find the number of play performers under the age of 30.

There are $4 + 5 + 5 + 1 + 1$ or 16 performers. Most of the performers are between the ages of 20 and 39.

There are $4 + 5$ or 9 performers under the age of 30.

Example 2

In a game of rugby, the Hawks completed passes for 15, 3, 8, 4, and 5 meters. What was the average number of meters per completed pass?

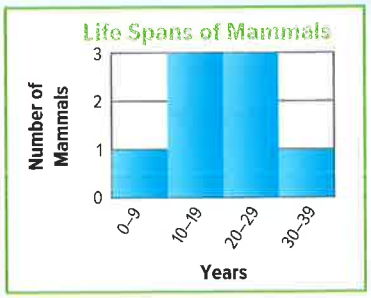
Find the sum of the numbers. Then divide by how many numbers are in the set.

$$\frac{15 + 3 + 8 + 4 + 5}{5} = \frac{35}{5}$$
$$= 7$$

The Hawks gained an average of 7 meters per completed pass.

Quick Check

1. **Graphs** Describe the histogram. Then find the number of mammals with a life span of more than 20 years.



Data Analysis **Find the mean (average) for each data set. Round to the nearest tenth if necessary.**

2. 14, 17, 20, 16, 13 _____

3. 52, 36, 17, 41, 18, 29, 28, 32 _____

4. In 12 games last season the school baseball team scored 5, 11, 2, 0, 4, 8, 9, 6, 7, 4, 1, and 2 runs. What is the average number of runs scored per game? Round to the nearest tenth. _____

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

(1) (2) (3) (4)

Inquiry Lab
Scatter Plots

Mathematical Practices 1, 3, 5

Inquiry **HOW can I use a graph to investigate the relationship or trends between two sets of data?**

Manal collected data for her science fair project on the relationship between a person's arm span and their height. She wanted to determine whether or not a relationship exists.

Hands-On Activity

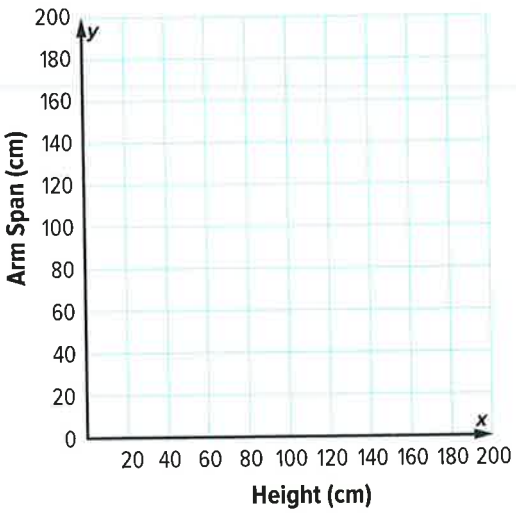
Sometimes it is difficult to determine whether a relationship exists between two sets of data simply by looking at them. You can write the data as a set of ordered pairs and graph them on a coordinate plane.

Step 1 Have a classmate measure your height and the length of your arm span with a meter stick to the nearest centimeter. Then write your height x and arm span y as an ordered pair. (,)

Step 2 Combine your data with that of your classmates. Write the ordered pairs (height, arm span) in the table.

Classmates' Data (height, arm span)							

Step 3 Graph the ordered pairs (height, arm span) on the coordinate plane.



Is there a noticeable trend in the data? If so, describe the trend as *positive* or *negative*.

Using your graph, estimate the arm spans of two people whose heights are 150 centimeters and 185 centimeters.

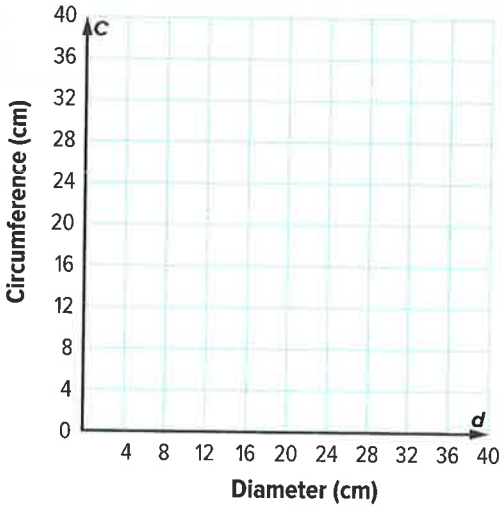
Investigate

Collaborate

Work with a partner.

1. To determine whether a relationship exists between the circumference and the diameter of a circle, find 6 different circular objects in the room.
   - **a.** Measure and record the diameter and circumference of each object in centimeters.
   - **b.** Write the measurements of each circle as an ordered pair (d, C). Graph the ordered pairs on the coordinate plane.

	diameter, d	circumference, C	(d, C)
1			
2			
3			
4			
5			
6			



Analyze and Reflect

Collaborate

2. MP **Reason Inductively** Is there a noticeable trend in the data? If so, describe the trend as *positive* or *negative*. Explain.

3. MP **Use Math Tools** Use your graph to estimate the circumference C of a circle with a diameter d of 10 centimeters.

Create

On Your Own

4. Write an example of a relationship with a negative association.

5. Inquiry HOW can I use a graph to investigate the relationship or trends between two sets of data?

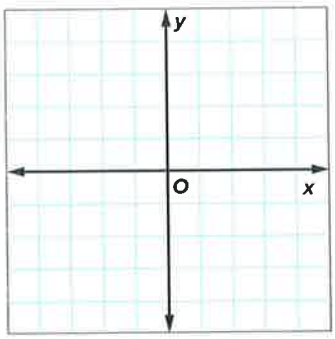
Lesson 1

Scatter Plots

Vocabulary Start-Up

Recall that the graph of a linear equation is a line on the coordinate plane. The slope of the line describes the direction and steepness of the line.

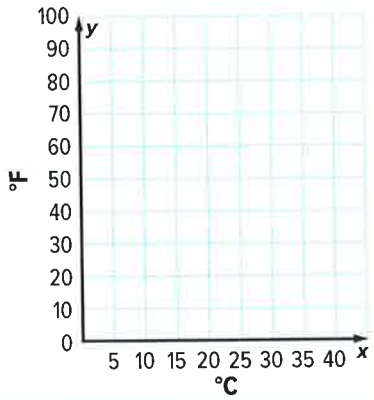
On the coordinate grid shown, graph and label two lines. One line should have a positive slope and one line should have a negative slope.



Real-World Link

Weather The table shows temperatures in degrees Celsius and the corresponding temperatures in Fahrenheit from a local weather station. Graph the ordered pairs (°C, °F). Is the slope of the line passing through the points positive or negative?

°C	0	5	10	15	20	25	30
°F	32	41	50	59	68	77	86



Essential Question

HOW are patterns used when comparing two quantities?

Vocabulary

bivariate data
scatter plot

MP Mathematical Practices
1, 3, 4

Which **MP Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Construct a Scatter Plot

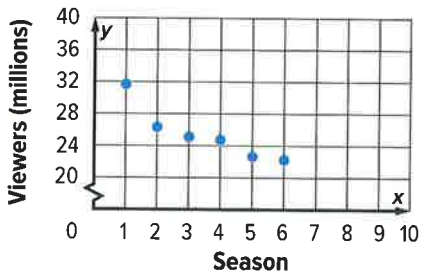
Data with two variables, or pairs of numerical observations, are called **bivariate data**. A **scatter plot** shows the relationship between bivariate data graphed as ordered pairs on a coordinate plane. For example, the bivariate data set year and number of visitors can be displayed as a scatter plot.

Example

1. **Construct a scatter plot of the number of viewers who watched new seasons of a certain television show.**

Let the horizontal axis, or x-axis, represent the number of seasons. Let the vertical axis, or y-axis, represent the number of viewers. Then graph the ordered pairs (season, viewers).

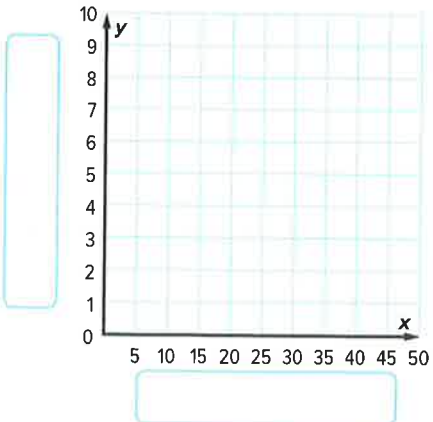
Television Ratings	
Season	Viewers (millions)
1	31.7
2	26.3
3	25.0
4	24.7
5	22.6
6	22.1



Got it? **Do this problem to find out.**

a. Construct a scatter plot of the weight of an alligator at various times after hatching.

Weeks	Weight (kilograms)
0	3
9	4.3
18	5
27	6.8
34	7.5
43	8.6
49	9.9

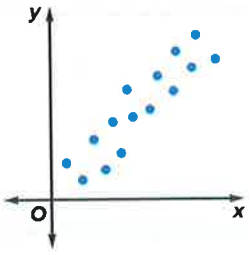


Types of Associations

Key Concept

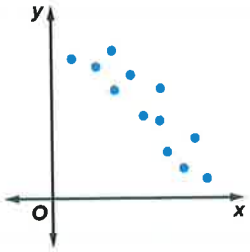
Variable Association

Positive Association



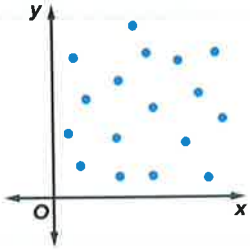
As x increases, y increases.

Negative Association



As x increases, y decreases.

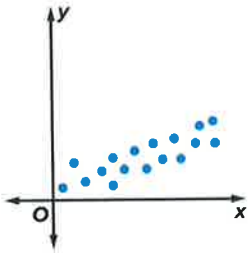
No Association



No obvious pattern.

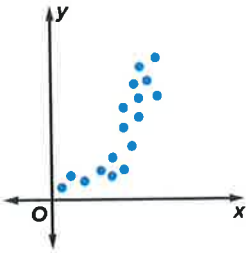
Linear Association

Linear



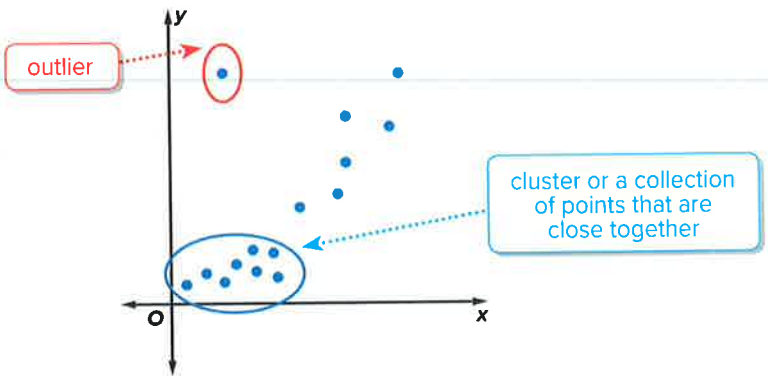
The data points lie close to a line.

Nonlinear



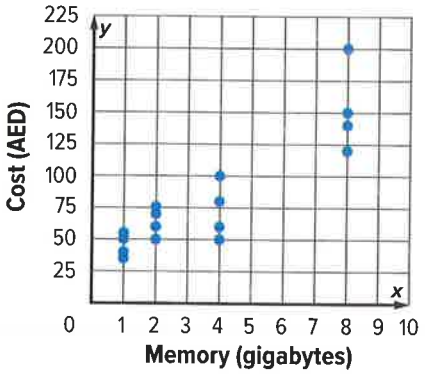
The data points lie in the shape of a curve.

You can analyze the shape of the distribution of a scatter plot to investigate patterns of association. If the distribution shows a positive or negative association, then the distribution can be classified as linear or nonlinear. The scatter plot below shows a positive nonlinear association. Clusters or outliers can also be identified.



Example

2. **Interpret the scatter plot of the data for the amount of memory in an MP3 player and the cost based on the shape of the distribution.**



Consider the different associations and patterns.

Variable Association As the amount of memory increases, the cost increases. Therefore, the scatter plot shows a positive association.

Linear Association The data appear to lie close to a line, so the association is linear.

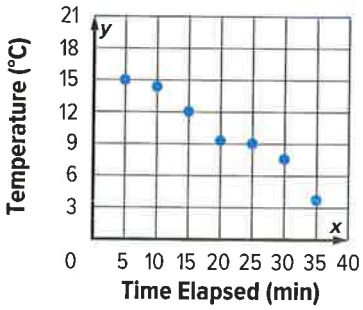
Other Patterns There appears to be a cluster of data. One to two gigabytes of memory costs between AED30 and AED75. There does not appear to be an outlier.

Show your work.

Got it? **Do these problems to find out.**

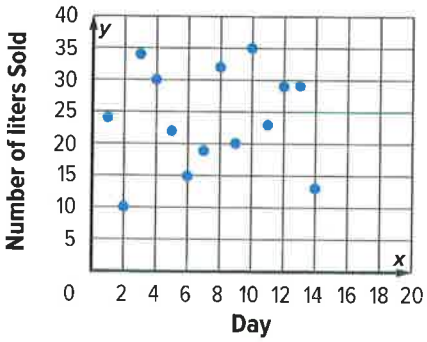
b. Interpret the scatter plot of the data for the time elapsed and temperature of water based on the shape of the distribution.

b. _____



c. Interpret the scatter plot of the data for two weeks in May and the amount of ice cream sold at a shop based on the shape of the distribution.

c. _____



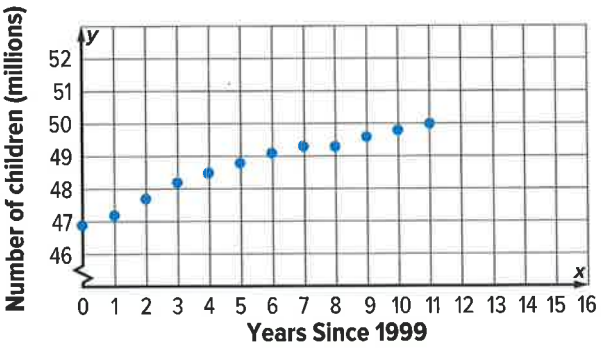
Example

3. **The table shows population of children under 18 in a particular country from 1999–2010.**

Years Since 1999	0	1	2	3	4	5
Number of Students (millions)	46.9	47.2	47.7	48.2	48.5	48.8
Years Since 1999	6	7	8	9	10	11
Number of Students (millions)	49.1	49.3	49.3	49.6	49.8	50.0

Construct and interpret a scatter plot of the data. If an association exists, make a conjecture about the number of children under 18 in the year 2015.

Construct a scatter plot of the data. Let the horizontal axis represent the year since 1999 and the vertical axis represent the number of children under 18.



Consider the different associations and patterns.

Variable Association As the years increase, the number of students increases. Therefore, the scatter plot shows a positive association.

Linear Association The data appear to lie close to a line, so the association is linear.

Other Patterns There are no clusters or outliers.

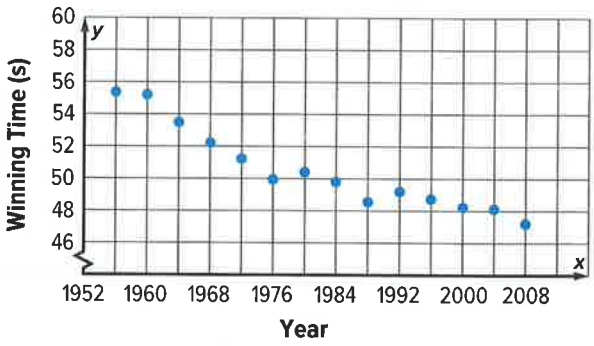
To make a conjecture about the number of children under 18 in the year 2015, follow the pattern until the x-value is 15. Then find the corresponding y-value.

So, there will be about 51 million children under 18 in 2015.

d. _____ Show your work.

Got it? **Do this problem to find out.**

d. Interpret the scatter plot shown for the men's Olympic 100-meter freestyle swim winning times. If an association exists, make a conjecture about the winning time in the 2016 Olympics.



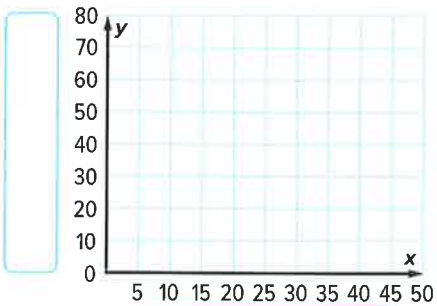
Guided Practice

Check

1. The table shows the number of units produced in a certain number of hours at a manufacturing plant. (Examples 1–3)

Time (h)	8	19	16	40	34	8	40	19	34
Units Produced	20	41	28	60	49	28	63	40	58

a. Construct a scatter plot of the data.



b. Interpret the scatter plot of the data.

_____

_____

_____

c. Make a conjecture about the number of units produced in 50 hours. _____

2. **Building on the Essential Question** What are the inferences that can be drawn from sets of data points having a positive association and a negative association?

_____

_____

_____

_____

_____

_____

_____

Rate Yourself!

How confident are you about creating and interpreting scatter plots? Check the box that applies.

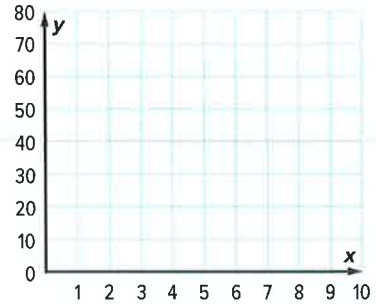
FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

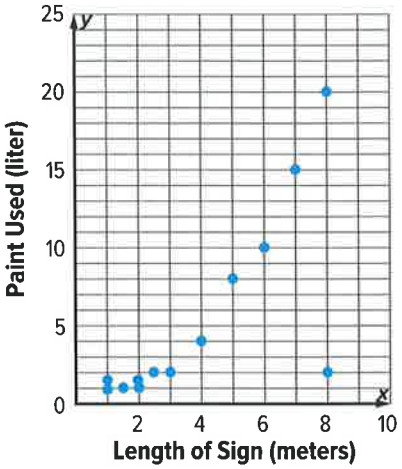
Independent Practice

1. Construct a scatter plot of the number of books donated over time. (Example 1)

Year	1	2	3	4	5	6	7	8
Number of Books	27	38	24	47	58	65	63	68



2. Interpret the scatter plot of the data for the amount of paint used to paint signs of various lengths based on the shape of the distribution. (Example 2)

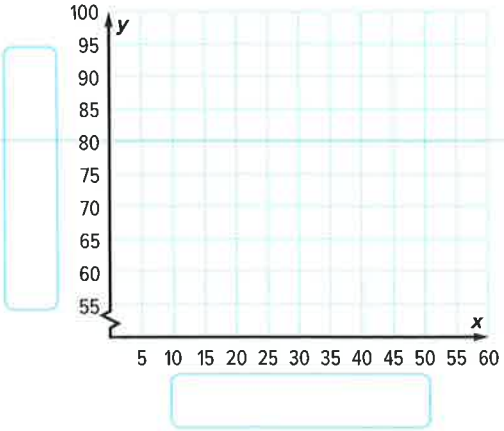


3. The table shows the amount of time different students studied for a test and their test scores. (Example 3)

Time (min)	10	15	20	25	30	35	40	45
Test Score	65	68	67	78	79	85	89	92

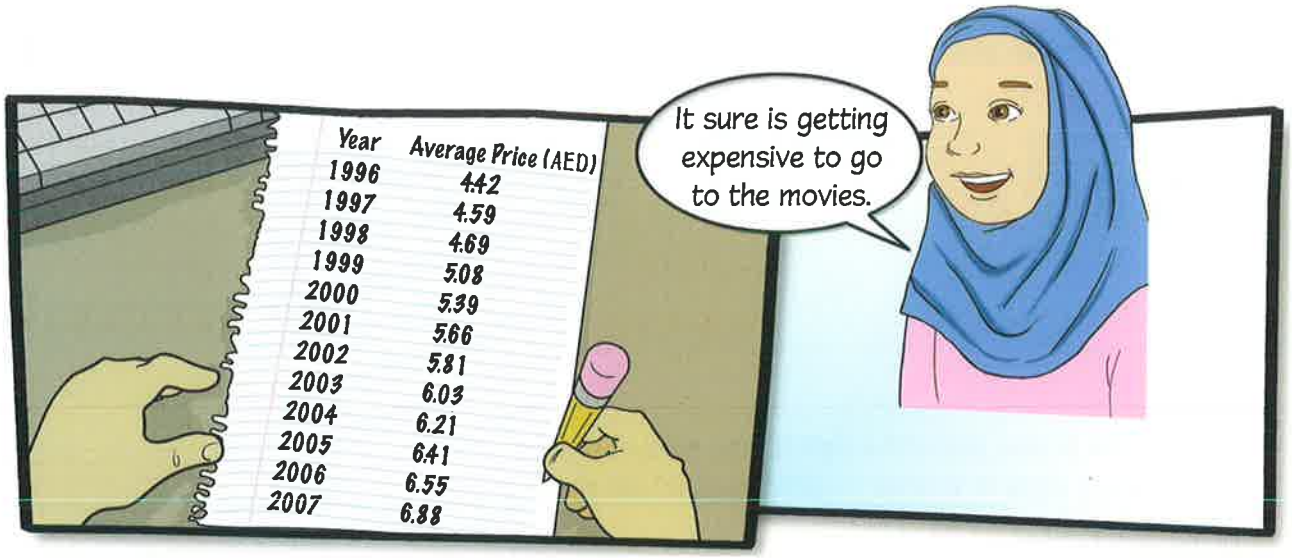
a. Construct a scatter plot of the data.

b. Interpret the scatter plot of the data based on the shape of the distribution.



c. If a relationship exists, make a conjecture about the test score for a student who studied for 60 minutes.

4. MP **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–b.



a. On a separate sheet of grid paper, construct a scatter plot of the data. The values for the horizontal axis should be years since 1995.

b. Do the data represent a *positive, negative,* or *no* association? Explain.

H.O.T. Problems Higher Order Thinking

5. MP **Make a Conjecture** Suppose a scatter plot shows that as the values of x decrease, the values of y decrease. Does the scatter plot show a *positive, negative,* or *no* association?

6. MP **Persevere with Problems** Is it *always*, *sometimes*, or *never* true that a scatter plot that shows a positive association suggests that the relationship is proportional? Justify your answer.

7. MP **Reason Inductively** Complete the table that shows the side lengths of a square related to its perimeter and area. Would a scatter plot of the side length and perimeter or the side length and area represent a linear relationship? Explain.

Side Length (units)	Perimeter (units)	Area (units^2)
1		
2		
3		
4		
5		
6		

Name _____ My Homework _____

Extra Practice

Copy and Solve **For Exercises 8–16, show your work and answers on a separate piece of paper.**

8. Construct and interpret a scatter plot of the data collected by a travel agency. If a relationship exists, make a conjecture about the number of visitors in month 12.

Month	1	2	3	4	5	6	7	8	9	10
Number of Visitors	208	245	423	432	412	626	647	620	402	356

9 The table shows the number of junk E-mails Reham received over the last 10 days.

Day	1	2	3	4	5	6	7	8	9	10
Number of E-Mails	10	12	15	10	11	8	20	10	10	9

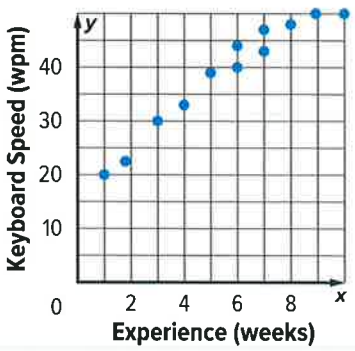
a. Construct a scatter plot of the data.

b. Interpret the scatter plot of the data based on the shape of the distribution.

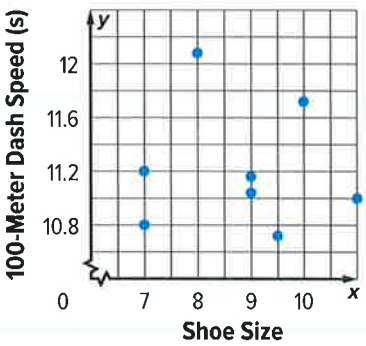
c. If a relationship exists, make a conjecture about the number of junk E-mails on Day 15.

Explain whether the scatter plot of the data for each of the following shows a *positive*, *negative*, or *no* association.

10.



11.



12. MP **Multiple Representations** A thirteen-year-old takes an average of 14 breaths per minute.

a. Tables Let x represent minutes and y represent the number of times the teenager breathes. Make a table using the x-values of 1, 2, 3, 4, 8, and 10.

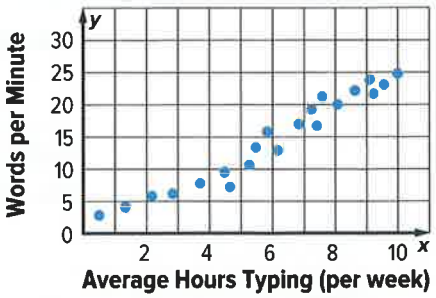
b. Graphs Make a scatter plot of the data. Describe the association between minutes and the number of times a person breathes.

c. Words Predict how many times a person would breathe in 25 minutes. Explain your reasoning.

Power Up! Test Practice

13. The scatter plot shows the relationship between the average number of hours spent typing per week and the number of words typed per minute. Circle the appropriate word in the statement below to draw an accurate conclusion about the relationship shown in the scatter plot.

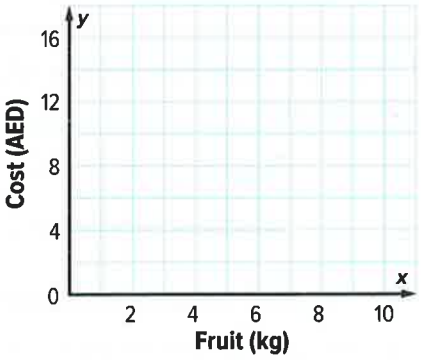
As the average number of hours spent typing per week (increases, decreases) the number of words typed per minute (increases, decreases).



14. The table shows the cost of fruit sold at a produce stand. Construct a scatter plot of the data.

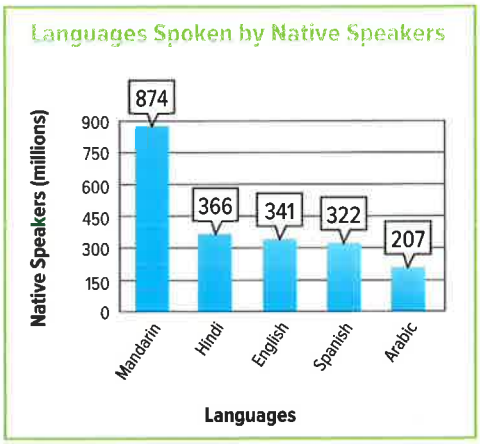
Fruit (kg)	2	4	6	8	10
Cost (AED)	5.00	10.00	12.00	15.00	16.00

Write a statement that is supported by the scatter plot.



Spiral Review

15. The graph shows the top five languages spoken by at least 100 million native speakers worldwide. What conclusions can you make about the number of Mandarin native speakers and the number of English native speakers?



16. For a school food drive, Homeroom 212 collected 8, 17, 4, 10, 8, 8, 12, 20, 10, 11, 12, 13, and 25 food items. Homeroom 215 collected 16, 24, 10, 15, 12, 14, 12, 30, 15, 10, 15, 20, and 14 food items.

a. Construct a double box plot for the data.

b. Compare the donations of the two homerooms.

Inquiry Lab
Lines of Best Fit

Inquiry **HOW can I use a data model to predict an outcome?**

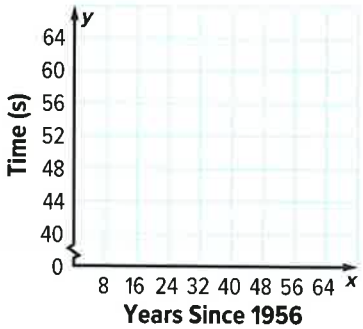
MP **Mathematical Practices** 1, 3

Huda and Muna found the data below showing the winning times in the Olympics for the women's 100-meter freestyle swim. They want to predict the winning time in the 2024 Olympics.

100-Meter Freestyle Winning Times							
Years Since 1956	0	4	8	12	16	20	24
Winning Time (s)	62.0	61.2	59.5	60.0	58.59	55.65	54.79
Years Since 1956	28	32	36	40	44	48	52
Winning Time (s)	55.92	54.93	54.65	54.5	53.83	53.84	53.12

Hands-On Activity

Step 1 Construct a scatter plot by graphing the points (years since 1956, time).



Step 2 Use a piece of uncooked spaghetti to make a line that goes through most of the data points.

How close are the other data points to the line you drew?

Step 3 Look at the point where the spaghetti has an x-value of 68. The corresponding y-value represents the projected winning time in 2024.

What is the projected winning time in 2024?

Refer to the line drawn in the scatter plot. Is this method always valid when making a prediction?

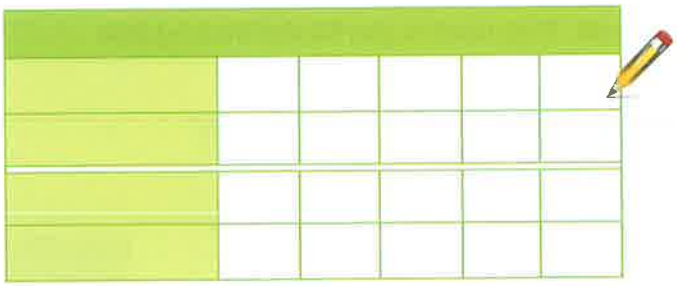
Investigate

Collaborate

Work with a partner.

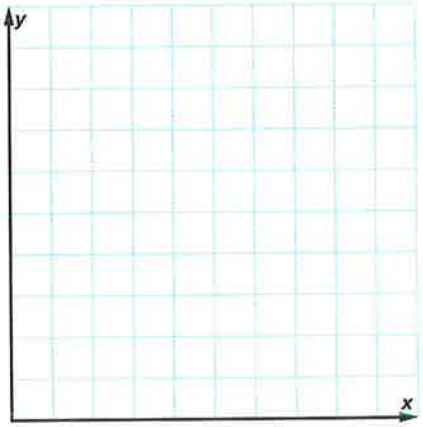
1. Research and collect a set of data from a newspaper or the Internet that has a positive or negative association.

   a. Compile your data in the table below. Space has been provided for ten sets of data values. Use a separate sheet of paper if you need more room. Be sure to label the rows.



   b. Construct a scatter plot for the data by graphing your data as ordered pairs.

   c. Draw a line that goes through most of the data points.



Analyze and Reflect

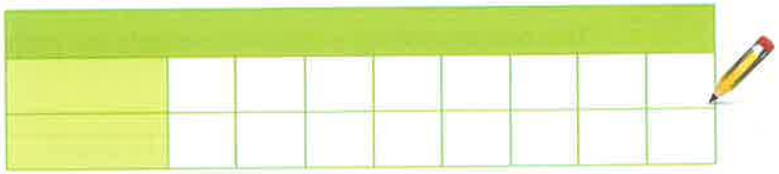
Collaborate

2. MP **Reason Inductively** Is it possible to make a prediction from your data? Explain.

Create

On Your Own

3. MP **Model with Mathematics** Create a set of data for which a prediction cannot be made.



4. Inquiry HOW can I use a data model to predict an outcome?

Lesson 2

Lines of Best Fit

Real-World Link

Cookies The table shows the average annual cost of one kilogram of chocolate chip cookies.

Years Since 2000	0	1	2	3	4	5	6	7	8	9
Average Cost (AED)	2.59	2.81	2.65	2.67	2.88	2.70	2.85	2.88	3.17	3.24

1. What year corresponds to 0 years since 2000? _____

9 years since 2000? _____

2. If the data were displayed in a scatter plot, would the scatter plot show a positive, negative, or no association? Explain.

_____

_____

3. Would a more reasonable prediction for the cost of cookies in 2015 be AED3.25 or AED4.00? Explain.

_____

_____

_____

_____

Essential Question

HOW are patterns used when comparing two quantities?

Vocabulary

line of best fit

MP Mathematical Practices
1, 3, 4, 5

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Line of Best Fit

When data are collected, the points graphed usually do not form a straight line, but may approximate a linear relationship. A **line of best fit** is a line that is very close to most of the data points.

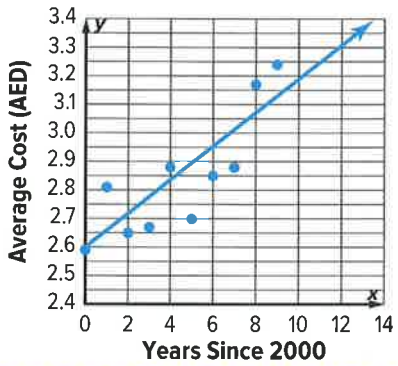
Examples

Refer to the information in the table about the cost of cookies.

1. Construct a scatter plot using the data. Then draw and assess a line that seems to best represent the data.

Years Since 2000	0	1	2	3	4	5	6	7	8	9
Average Cost (AED)	2.59	2.81	2.65	2.67	2.88	2.70	2.85	2.88	3.17	3.24

Graph each of the data points. Draw a line that fits the data. About half of the points are above the line and half of the points are below the line. Judge the closeness of the data points to the line. Most of the points are close to the line.



2. Use the line of best fit to make a conjecture about the cost of cookies in 2013.

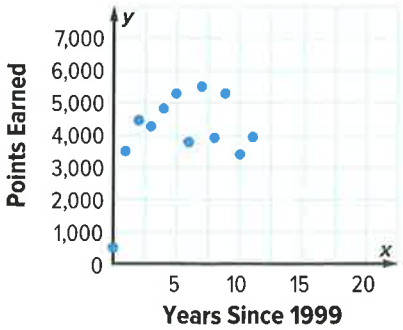
Extend the line so that you can estimate the y-value for an x-value of 2013 − 2000 or 13. The y-value for 13 is about AED3.35. We can predict that in 2013, a kilogram of chocolate chip cookies will cost AED3.35.

Got it? **Do these problems to find out.**

Refer to the scatter plot about yearly points scored by a certain race car driver.

a. Draw and assess a line that seems to best represent the data.

b. Use the line of best fit to make a conjecture about the points the driver will score in 2015.



STOP and Reflect

How do you determine how well a line of best fit models the data? Explain below.

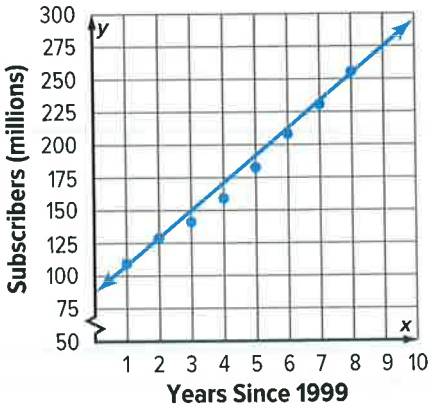
Show your work.

a. _____

b. _____

Examples

The scatter plot shows the number of cellular service subscribers in a large country.



3. Write an equation in slope-intercept form for the line of best fit that is drawn, and interpret the slope and *y*-intercept.

Choose any two points on the line. They may or may not be data points. The line passes through points (3, 150) and (9, 275). Use these points to find the slope, or rate of change, of the line.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \frac{275 - 150}{9 - 3}$ $(x_1, y_1) = (3, 150)$ and $(x_2, y_2) = (9, 275)$

$m = \frac{125}{6}$ or about 20.83 Simplify.

The slope is about 20.83. This means the number of cell phone subscribers increased by about 20.83 million people per year.

The *y*-intercept is 87.5 because the line of fit crosses the *y*-axis at about the point (0, 87.5). This means there were about 87.5 million cell phone subscribers in 1999.

$y = mx + b$ Slope-intercept form

$y = 20.83x + 87.5$ Replace m with 20.83 and b with 87.5.

The equation for the line of best fit is $y = 20.83x + 87.5$.

4. Use the equation to make a conjecture about the number of cellular subscribers in 2015.

The year 2015 is 16 years after 1999.

$y = 20.83x + 87.5$ Equation for the line of best fit

$y = 20.83(16) + 87.5$ Replace x with 16.

$y = 333.28 + 87.5$ Simplify.

So, in 2015, there will be about 420.83 million cellular subscribers.

Estimation

Drawing a line of best fit using the method in this lesson is an estimation. Therefore, it is possible to draw different lines to approximate the same data.

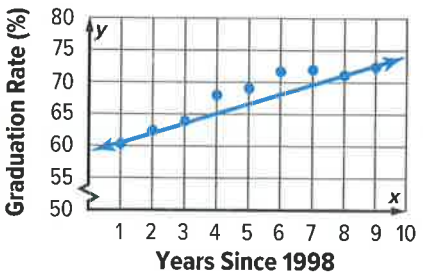
Show your work.

c. _____

d. _____

Got it? **Do these problems to find out.**

The scatter plot shows the graduation rate of high school students.



c. Write an equation in slope-intercept form for the line of best fit that is drawn, and interpret the slope and *y*-intercept.

d. Use the equation to make a conjecture about the graduation rate in 2020.

Guided Practice

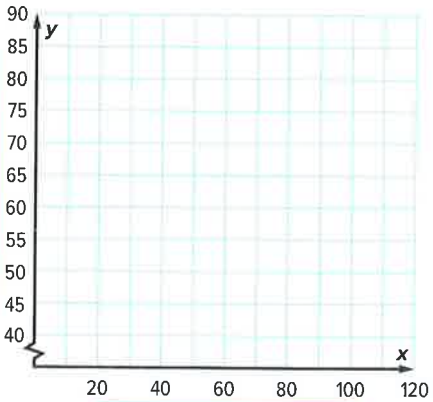
Check

1. The table shows the life expectancy, in years, for people born in certain years. (Examples 1–4)

Years Since 1900	0	10	20	30	40	50	60	70	80	90	100
Life Expectancy	47.3	50.0	54.1	59.7	62.9	68.2	69.7	70.8	73.7	75.4	77.1

a. Construct a scatter plot of the data. Then draw and assess a line that best represents the data.

Show your work.



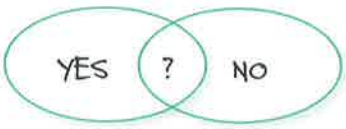
b. Write an equation in slope-intercept form for the line of fit and interpret the slope and *y*-intercept.

c. Use the equation to make a conjecture about the life expectancy for a person born in 2020.

2. **Building on the Essential Question** Why do we estimate a line of best fit for a scatter plot?

Rate Yourself!

Are you ready to move on? Shade the section that applies.



FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

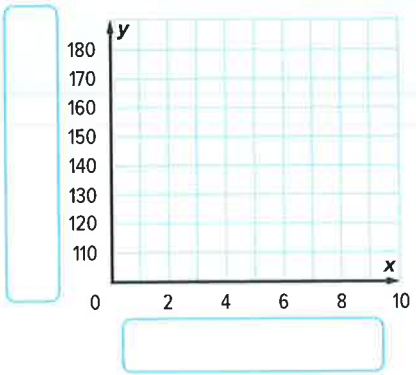
Independent Practice

1 The results of a survey about women's shoe sizes and heights are shown. (Examples 1 and 2)

Height (centimeters) and Shoe Size			
Shoe Size	**Height**	**Shoe Size**	**Height**
8	166	$6\frac{1}{2}$	165
8	165	9	168
$7\frac{1}{2}$	165	$7\frac{1}{2}$	163
7	162	7	164
7	162	$5\frac{1}{2}$	162
9	168	5	160
9	165	9	167
9	165	6	159

a. Construct a scatter plot of the data. Then draw and assess a line that best represents the data.

Show your work.



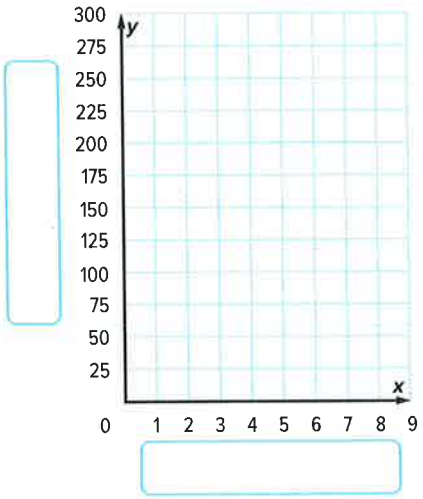
b. Use the line of best fit to make a conjecture about the height of a female who wears a size 5 shoe. _____

2. The table shows the number of Calories burned when walking laps around a track. (Examples 1-4)

Laps Completed	1	2	3	4	5	6	7
Calories Burned	30	70	80	112	150	170	225

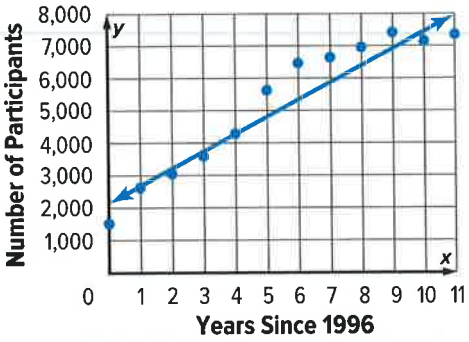
a. Construct a scatter plot of the data. Then draw a line that best represents the data.

b. Write an equation for the line of best fit. Use the equation to make a conjecture about the number of Calories burned if someone walks 15 laps.



_____

3 The scatter plot shows the number of girls who participate in basketball. (Examples 3 and 4)



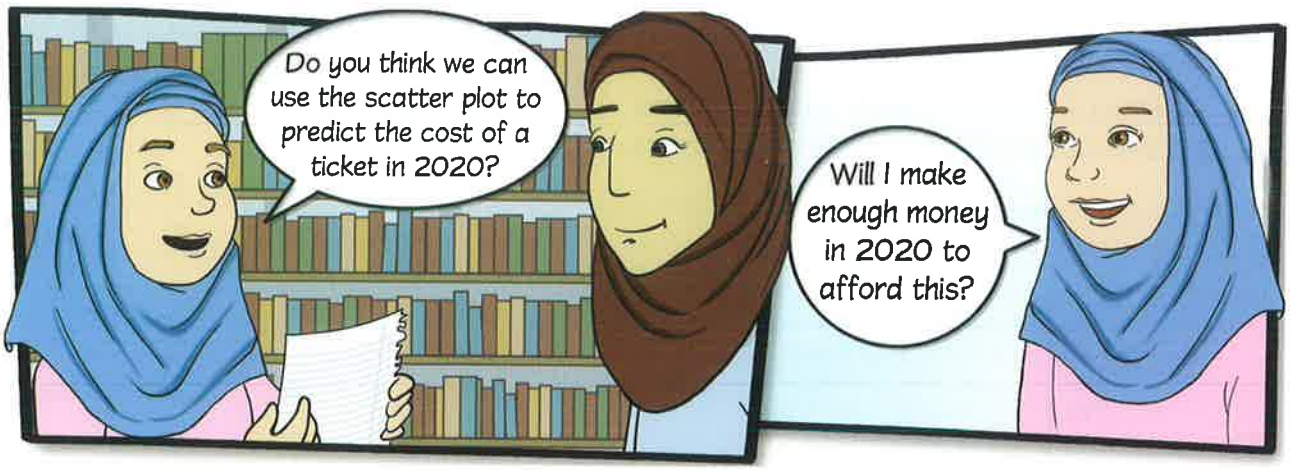
a. Write an equation in slope-intercept form for the line of best fit that is drawn, and interpret the slope and *y*-intercept. _____

_____

_____

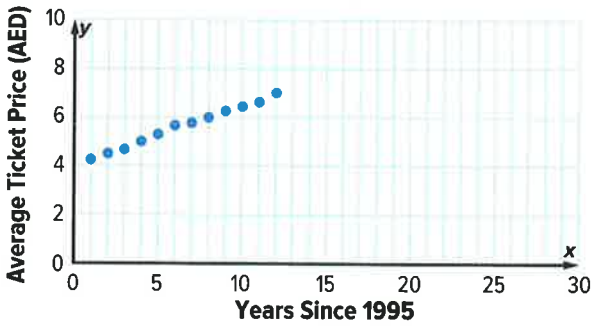
b. Use the equation to make a conjecture about the number of girls that will participate in basketball in 2020. _____

4. MP **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–b.



a. The scatter plot shows the average ticket prices since 1995. Draw a line that best represents the data in your scatter plot.

b. Write an equation in slope-intercept form for the line of best fit. Make a conjecture about the cost of a movie ticket in 2020.



H.O.T. Problems Higher Order Thinking

5. MP **Use Math Tools** Use a newspaper or the Internet to find a scatter plot that consists of at least seven data points. Tape the scatter plot to your book. Draw a line of best fit and write an equation for the line.

6. MP **Persevere with Problems** Describe or draw a scatter plot where a line of fit does not model the data. Explain your reasoning to a classmate.

7. MP **Justify Conclusions** Determine whether each statement is *always*, *sometimes*, or *never* true for data with a positive association. Justify your response.

a. The slope of the line of best fit is positive.

b. The y-intercept is positive.

Name _____ My Homework _____

Extra Practice

Copy and Solve **For Exercises 8–14, show your work and answers on a separate piece of paper.**

8. The table shows the storage amount and cost for flash drives at a local electronics store.

Storage (MB)	2	2	2	2	4	4	4	4	8	8	8	8	8	16	16	16	16
Cost (AED)	6.5	12	7	10	9	10	20	23	15	17	20	25	40	23	40	50	80

a. Construct a scatter plot of the data. Draw and assess a line that best represents the data.

b. Write an equation in slope-intercept form for the line of best fit, and interpret the slope and *y*-intercept.

c. Use the equation to make a conjecture about the cost of a flash drive with a storage capacity of 32 MB.

9 MP **Model with Mathematics** The table shows fat and Calories for fast food sandwiches.

Fat (grams)	21	10	14	21	30	34	32	37	27	26	18	7
Calories	490	280	330	430	530	590	540	590	550	470	450	340

a. Construct a scatter plot of the data. Draw and assess a line that best represents the data.

b. Write an equation in slope-intercept form for the line of best fit, and interpret the slope and *y*-intercept.

c. Use the equation to make a conjecture about the number of grams of fat in a sandwich with 350 Calories.

10. The table shows the cost per kilogram of apples for several years.

Years Since 1999	1	2	3	4	5	6	7	8	9	10	11
Cost per Kilogram (AED)	0.92	0.87	0.95	0.98	1.04	1.07	1.12	1.12	1.32	1.18	1.22

a. Construct a scatter plot of the data. Draw and assess a line that best represents the data.

b. Write an equation in slope-intercept form for the line of best fit, and interpret the slope and *y*-intercept.

c. Use the equation to make a conjecture about the cost of apples in the year 2025.

Power Up! Test Practice

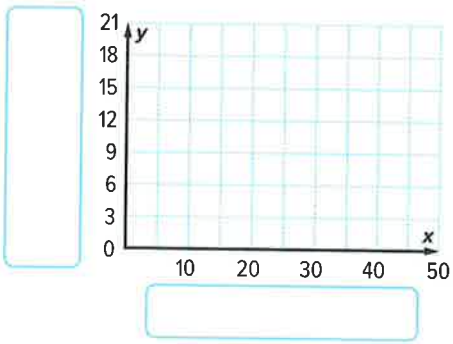
11. Suhaila recorded data on how many Calories she burned for different lengths of time while jogging on a treadmill. She plotted the data in a scatter plot and drew the line of best fit. The equation for the line is $C = 14.5m$, where C represents the number of Calories burned and m represents the number of minutes spent jogging. Determine if each statement is true or false.

a. The slope is positive because as Suhaila jogs more minutes, she burns more Calories. ☐ True ☐ False

b. According to the line of best fit, Suhaila will burn about 290 Calories if she jogs 20 minutes. ☐ True ☐ False

12. The table shows the wind chill temperatures for different wind speeds when the outside temperature is 17°C. Construct a scatter plot of the data. Then draw a line of best fit.

Wind Chill Temperatures at 17°C			
Wind Speed (km/h)	Temperature (°F)	Wind Speed (km/h)	Temperature
5	v	25	6
10	11	30	5
15	9	35	4
20	7	40	3



Does the line of best fit have a positive or negative slope? Explain what this represents.

Spiral Review

Determine whether a scatter plot of each of the following might show a *positive, negative,* or *no* association.

13. a student's age and how many siblings he or she has

14. the number of homeruns hit and the amount of time spent in batting practice

Inquiry Lab

Graphing Technology: Linear and Nonlinear Association

Inquiry **HOW can you use technology to describe associations in scatter plots?**

MP Mathematical Practices 1, 3, 5

The table shows the weekly number of hours spent watching television and the weekly number of hours spent exercising.

Weekly Television (h)	17	20	11	10	15	38	5	25
Weekly Exercise (h)	5	4.5	7.5	8	6.5	1	7.5	3
Weekly Television (h)	25	32	5	17	40	28	20	30
Weekly Exercise (h)	2.5	3.5	6	7	0.5	5	4	1.5

Hands-On Activity 1

You can use a graphing calculator to construct a scatter plot of the data and find and graph a line of best fit.

Step 1 Clear the existing data by pressing STAT ENTER ▲ CLEAR ENTER. Then enter the data. Input the number of weekly hours spent watching television in L_1 and press ENTER. Then enter the weekly hours spent exercising in L_2.

Step 2 Turn on the statistical plot by pressing 2nd STAT PLOT ENTER ENTER. Select the scatter plot and confirm L_1 as the Xlist, L_2 as the Ylist, and the square as the mark.

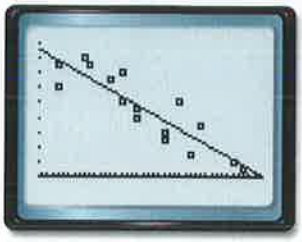
Step 3 Graph the data by pressing ZOOM 9. Use the Trace feature and the left and right arrow keys to move from one point to another.

Do the data suggest a linear association? _____

Step 4 Access the CALC menu by pressing STAT ▶. Select 4 to find a line of best fit in the form $y = ax + b$. Press 2nd L1 , 2nd L2 ENTER to find a line of best fit for the data in lists L_1 and L_2. What does the screen show for y, a, and b?

y = _____ a = _____ b = _____

Step 5 Graph the line of best fit in Y_1 by pressing [Y] and then [VARS] 5 to access the Statistics... menu. Use the [▶] and [ENTER] keys to select EQ. Then press 1 to select RegEQ, the line of best fit equation. Finally, press [GRAPH].



Use the TRACE feature to predict the average number of hours of exercise someone who watches 35 hours of television would get.

Hands-On Activity 2

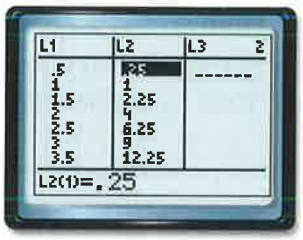
A nonlinear association is one where the pattern does not follow a linear trend.

The table shows the side length and the corresponding area for various squares. Construct a scatter plot of the data to determine what kind of relationship, if any, exists between the side of a square and its area.

Side Length (cm)	Area (cm^2)
0.5	0.25
1	1
1.5	2.25
2	4
2.5	6.25
3	9
3.5	12.25

Step 1 Clear the equation from Y_1 by pressing [Y] [CLEAR]. Clear the existing data from L_1 and L_2 by pressing [STAT] [ENTER] [▲] [CLEAR] [ENTER] [▶] [▲] [CLEAR] [ENTER].

Step 2 Next, enter the data. Input the side lengths in L_1 and press [ENTER]. Then enter the areas in L_2.



Step 3 Turn on the statistical plot by pressing [2nd] [STAT PLOT] [ENTER] [ENTER]. Select the scatter plot and confirm L_1 as the Xlist, L_2 as the Ylist, and the square as the mark.

Step 4 Graph the data by pressing [ZOOM] 9. Use the Trace feature and the left and right arrow keys to move from one point to another.

Does the scatter plot show a *linear* or *nonlinear* association? Explain.

Investigate

Collaborate

Work with a partner.

Salma lives in UAE. She kept track of how much her energy bill was every month for one year. She displayed it in the table shown at the right. Use your graphing calculator and the following steps to construct a scatter plot of the data.

Month	Bill (AED)
January	146
February	138
March	116
April	84
May	72
June	73
July	94
August	114
September	92
October	91
November	126
December	139

Step 1 Clear the existing data from L_1 and L_2.

Step 2 Enter the data. Input the month numbers in L_1 and the amounts of the electric bill in L_2.

Step 3 Turn on the statistical plot. Select the scatter plot and confirm L_1 as the Xlist, L_2 as the Ylist.

Step 4 Graph the data.

1. How is the scatter plot different from the scatter plot for Activity 2?

2. Does the scatter plot show a *linear* or *nonlinear* association? Explain.

3. What does a negative rate of change mean in the problem's context?

4. MP **Use Math Tools** Collect a set of data that can be represented in a scatter plot. Use a graphing calculator to determine whether the data have a *linear* or *nonlinear* association. If the association is linear, use the graphing calculator to find the line of best fit and to make a prediction. Show your data and results in the space provided.

Show your work.

Analyze and Reflect

Work with a partner.

The *correlation coefficient* measures the strength of the association between two sets of data, or how closely the data is clustered around the line of best fit.

You can use the graphing calculator to find the correlation coefficient for the data in Activity 1. Before you reenter the data, you need to make sure that you have the Diagnostics on. Press [2nd] [Catalog]. Scroll down until you see DiagnosticOn. Then press [ENTER] [ENTER].

Complete Steps 1–4 in Activity 1. This time, when you complete Step 4, you should see values for r^2 and r. The value for r is the correlation coefficient.

5. In Activity 1, what is the r value? _____

6. MP **Make a Conjecture** The chart shows how the r value reflects the strength of the association. For example, a strong negative association would indicate the data is tightly clustered around a line of fit with a negative slope.

If...	$-1 \le r \le -0.5$	$-0.5 < r < 0$	$r = 0$	$0 < r < 0.5$	$0.5 \le r \le 1$
...then the association is...	strong negative	weak negative	no association	weak positive	strong positive

How would you classify the association in Activity 1?

_____

Create

Write a correlation coefficient for each association. Explain why you selected each value.

7. strong positive

_____

_____

_____

8. weak negative

_____

_____

_____

9. Inquiry HOW can you use technology to describe associations in scatter plots?

_____

_____

_____

_____

Lesson 3

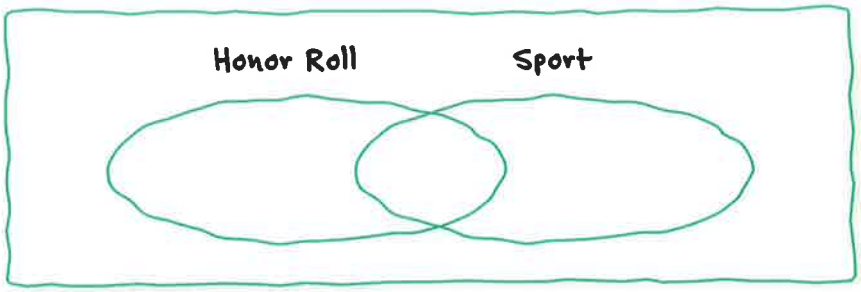
Two-Way Tables

Real-World Link

Student Athletes The data from a survey of 440 students are shown in the table. The students were asked whether or not they were on the honor roll and whether or not they played a sport.

Student Athlete Survey	
Only on the honor roll	115
Only play a sport	45
Play a sport and are on the honor roll	250

1. Complete the Venn diagram to represent the data.



2. **Relative frequency** is the ratio of the value of a subtotal to the value of the total. What is the relative frequency of a student that is on the honor roll and plays a sport to all students that are on the honor roll?

3. Is there evidence that students that play sports are also on the honor roll? Explain.

Essential Question

HOW are patterns used when comparing two quantities?

Vocabulary

relative frequency
two-way table

Mathematical Practices
1, 3, 4, 5

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Construct a Two-Way Table

A **two-way table** shows data from one sample group as it relates to two different categories.

The same information from the Venn diagram on the previous page is shown below as a two-way table, where one category is represented by rows and the other category is represented by columns. The two categories in the table shown are "Play a Sport" and "On the Honor Roll."

	Play a Sport	Do Not Play a Sport	Total
On the Honor Roll	250	115	365
Not On the Honor Roll	45	30	75
Total	250 + 45 = 295	115 + 30 = 145	440

Example

1. **Rasheed surveyed students at his school. He found that 78 students own a cell phone and 57 of those students own an MP3 player. There are 13 students that do not own a cell phone, but own an MP3 player. Nine students do not own either device. Construct a two-way table summarizing the data.**

Step 1 Create a table using the two categories: cell phones and MP3 players. Fill in the table with the given values.

	MP3 Player	No MP3 Player	Total
Cell Phone	57		78
No Cell Phone	13	9	
Total			

Step 2 Use reasoning to complete the table. Remember, the totals are for each row and column. The column labeled "Total" should have the same sum as the row labeled "Total."

	MP3 Player	No MP3 Player	Total
Cell Phone	57	21	78
No Cell Phone	13	9	22
Total	70	30	100

Got it? **Do this problem to find out.**

a. There are 150 children at summer camp and 71 signed up for swimming. There were a total of 62 children that signed up for canoeing and 28 of them also signed up for swimming. Construct a two-way table summarizing the data.

	Canoeing	No Canoeing	Total
Swimming			
No Swimming			
Total			

Interpret Relative Frequencies

A two-way table can show relative frequencies for rows or for columns, rather than the actual values. By analyzing the relative frequencies in a two-way table, you can determine possible associations between the two variables.

Example

2. **Find and interpret the relative frequencies of students in the survey from Example 1 by row.**

	MP3 Player	No MP3 Player	Total
Cell Phone	57	21	78
No Cell Phone	13	9	22
Total	70	30	100

To find the relative frequencies by row, write the ratios of each value to the total in that row. Round to the nearest hundredth.

	MP3 Player	No MP3 Player	Total
Cell Phone	$57; \frac{57}{78} \approx 0.73$	$21; \frac{21}{78} \approx 0.27$	78; 1.00
No Cell Phone	$13; \frac{13}{22} \approx 0.59$	$9; \frac{9}{22} \approx 0.41$	22; 1.00

Only the totals needed are shown in the table.

Based on the relative frequency value of 0.73 in one of the cells, you can imply that most students that own a cell phone also own an MP3 player. The data also suggest that over half of the students that do not own a cell phone will own an MP3 player.

STOP and Reflect

What relative frequency would you use to determine if there was an association between the two variables in a two-way table? Explain below.

b. _____

Got it? **Do this problem to find out.**

b. Find and interpret the relative frequencies of students in the survey by column. Round to the nearest hundredth if necessary.

	MP3 Player	No MP3 Player
Cell Phone	57;	21;
No Cell Phone	13;	9;
Total	70;	30;

Guided Practice

Check

1. Najat surveyed the people in her cafeteria and found that 38 teachers agree with the new cafeteria rules while 70 do not. There were 92 students surveyed and 41 of them agree with the new cafeteria rules. Construct a two-way table summarizing the data. (Example 1)

	Agree with Rules	Do Not Agree with Rules	Total
Teachers			
Students			
Total			

2. The two-way table shows how some students get their news. Find and interpret the relative frequencies of students in the survey by row. (Example 2)

	TV	Internet	Total
7th grade	13;	49;	
8th grade	20;	68;	
Total			

3. **Building on the Essential Question** How is a two-way table used when determining possible associations between two different categories from the same sample group?

Rate Yourself!

How well do you understand two-way tables? Circle the image that applies.

Clear Somewhat Clear Not So Clear

FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

1. One hundred customers in a restaurant were asked whether they liked chicken or beef and whether they liked rice or pasta. Out of 30 customers that liked rice, 20 liked chicken. There were 60 customers that liked chicken. Construct a two-way table summarizing the data. (Example 1)

	Chicken	Beef	Total
Rice			
Pasta			
Total			

2. The two-way table shows the number of students that do or do not do chores at home and whether they receive an allowance or not. Find and interpret the relative frequencies of students in the survey by columns. (Example 2)

	Allowance	No Allowance	Total
Chores	13;	3;	
No Chores	5;	4;	
Total			

3. The two-way table shows the number of students that message on a daily basis. Find and interpret the relative frequencies of students in the survey by rows. (Example 2)

	Text Message	Instant Message	Total
7th graders	59;	25;	
8th graders	59;	41;	
Total			

4. MP **Use Math Tools** The Venn diagram shows the number of students that exercise in different ways. Construct a two-way table that displays the data. Find and interpret the relative frequencies by column.

Show your work.

H.O.T. Problems Higher Order Thinking

5. MP Find the Error Najla is interpreting data about her classmates that have an after-school job and are on the honor role. Out of 100 students that do not have a job, 67 of them are on the honor roll. Find her mistake and correct it.

6. MP Use Math Tools Survey your classmates to find out if they have a curfew and if they have assigned chores at home. On a separate sheet of paper, make a two-way table that displays the results. Then interpret the results.

7. MP Persevere with Problems The two-way table below shows the number of students with each hair color and eye color.

		Hair Color				
		Black	Brown	Red	Blond	Total
Eye Color	Brown	7	12	3	1	23
	Blue	2	8	2	9	21
	Hazel	2	5	1	1	9
	Green	1	3	1	2	7
	Total	12	28	7	13	60

Which is greater: the percentage of the brown-haired students with blue eyes or the percentage of the red-haired students with brown eyes?

8. MP Model with Mathematics The two-way table at the right shows the number of hours students studied and whether they studied independently or with a study group. Write two questions that could be answered using the relative frequencies of the data in the table. Then ask a classmate to solve your questions.

	Studied Less Than 2 Hours	Studied More Than 2 Hours
Studied Independently	12	4
Studied with a Study Group	8	11

Name _____ My Homework _____

Extra Practice

Copy and Solve **For Exercises 9–17, show your work and answers on a separate piece of paper.**

9. As each person entered the theater, Hamdan counted how many of the 105 people had popcorn and how many had a drink. He found that out of 84 people that had popcorn, only 10 did not have a drink. Six people walked in without popcorn or a drink. Construct a two-way table summarizing the results.

10. The two-way table shows the number of Hala's soccer teammates that are in her Math class and English class.

	Math Class	Not in Math Class
English Class	4	2
Not in English Class	1	3

a. How many teammates does Hala have?

b. What is the relative frequency of teammates that are in both of Hala's classes to all of her teammates?

c. Of the teammates in her math class, which percentage is greater: the percentage of teammates that are in her English class or the percentage of teammates that are not in her English class?

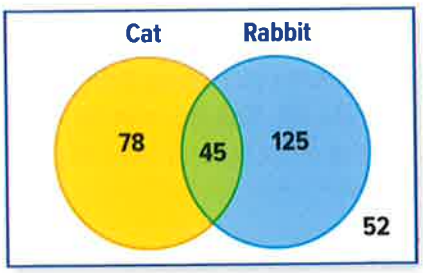
11. The two-way table shows the places that Class A and Class B volunteered in the past month. Do a greater percentage of Class A or Class B volunteer at the animal shelter? Justify your response.

	Class A	Class B
Animal Shelter	26	21
Hospital	13	17
Library	9	14

12. **Use Math Tools** Hiyam surveyed the students in two classes about the number of times they bring their lunch to school per month. The table shows her findings. Construct a two-way table that shows the relative frequencies by columns. What is the relative frequency of the number of Class B students that bring their lunch to school less than 6 times a month to the total number of students surveyed? Round to the nearest hundredth if necessary.

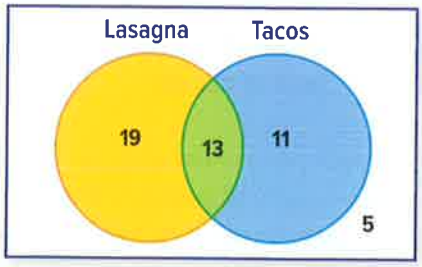
Number of Times per Month	Class A	Class B
0–5	35	25
6–10	23	16
11–15	22	13
16–20	18	8

13. An animal shelter conducted a survey on the types of pets sponsored. Construct a two-way table summarizing the data. Then find and interpret the relative frequencies by columns.



Power Up! Test Practice

14. The Pep Club was asked to vote for which dinner they would like for the banquet. Complete the two-way table based on the information shown in the Venn diagram.



	Lasagna	No Lasagna	Total
Tacos			
No Tacos			
Total			

What is the relative frequency of students who chose tacos and lasagna to the total number of students? Round to the nearest hundredth.

15. Amna surveyed the eighth grade students to find which school activities they attended last weekend. The results are shown in the two-way table. Which of the following are valid conclusions about the data? Select all that apply.

	Attended the School Play	Did Not Attend the School Play	Total
Attended the Basketball Game	55	63	118
Did Not Attend the Basketball Game	88	15	103
Total	143	78	221

- ☐ Of the students who attended the basketball game, fewer than half of them also attended the school play.
- ☐ More than half of the students who were surveyed attended the school play and did not attend the basketball game.
- ☐ Students who attended the school play were more likely not to attend the basketball game.

Spiral Review

16. The ages of people working in an office are shown in the stem-and-leaf plot. Find the mean, median, and mode of the data.

Ages of Office Workers

Stem	Leaf
2	3 5 8 8
3	1 2 3 3 6 9
4	2 5 7
5	1 3

2|3 = 23 years

17. In a golf league, the golfers are allowed to drop their highest score before calculating their average score. Sami has scores of 103, 98, 125, 96, 100, 95, and 98. Which measure of center will be affected the most by dropping the highest score? Explain.

MP Problem-Solving Investigation
Use a Graph

MP Mathematical Practices
1, 4, 7

Case #1 Up to Speed

Wafa ranked ten Web sites from 1 to 10 with a ranking of 1 being the most popular. Then, she created a graph showing the download times of these Web sites.

Does the most popular Web site have the fastest download time?

1 Understand *What are the facts?*

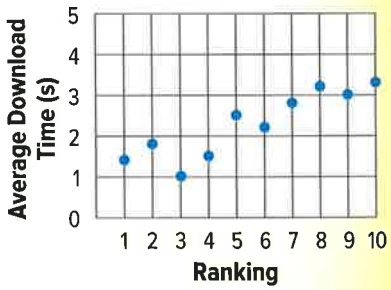
The graph shows the popularity of some Web sites and the download times for each.

2 Plan *What is your strategy to solve this problem?*

Study the data on the graph.

3 Solve *How can you apply the strategy?*

Use the graph to answer the following questions.
The graph shows that, in general, the more popular Web sites are _____ than the less popular Web sites.



The fastest Web site has what ranking? ☐

The slowest Web site has what ranking? ☐

4 Check *Does the answer make sense?*

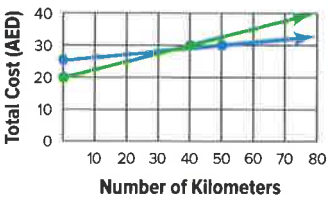
Look at the graph. Two Web sites have a higher rating than the fastest Web site.

Analyze the Strategy

MP **Identify Structure** Explain what the ordered pair (1, 1.4) represents in terms of the question posed. _____

Case #2 The Right Rental

The blue line shows the weekly cost of a car rental at Company A. The green line shows the weekly cost of a car rental at Company B.



If you wish to rent a car for one week and drive 60 kilometers which company charges the lesser amount?

1 Understand

Read the problem. What are you being asked to find?

I need to find _____

_____.

Underline key words and values. What information do you know?

The graph shows the _____ and the _____ for Companies A and B.

2 Plan

Choose a problem-solving strategy.

I will use the _____ strategy.

3 Solve

Use your problem-solving strategy to solve the problem.

The graph shows that at zero kilometers, Company A charges AED ☐ and Company B charges AED ☐; but at 60 kilometers, Company A charges about AED ☐ and Company B charges about AED ☐. So, Company ☐ is less expensive to rent for one week and 60 kilometers.

4 Check

Use information from the problem to check your answer.

Use the horizontal axis and find ☐ kilometers. Follow that vertical line up to the car rental graphs.

The _____ line represents the less expensive car rental company.

Collaborate

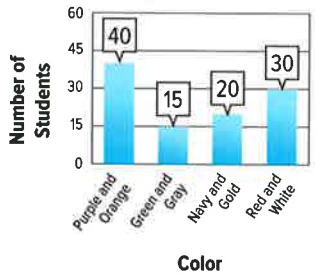
Work with a small group to solve the following cases. Show your work on a separate piece of paper.

Case #3 Blogs

The numbers of followers of a popular blog are shown in the table.

What is a reasonable estimate for the number of followers in Year 10 if this trend continues?

Year	Number of Followers
1	42,000
2	50,000
3	76,000
4	94,000
5	115,000



Case #4 School Colors

The graph shows the results of a favorite color survey.

To the nearest percent, what percent more of the students chose purple and orange than green and gray?

Case #5 Clubs

The table shows the math club membership in the years 2010 through 2015.

What is a reasonable prediction for the membership in 2020 if this trend continues?

Year	Number of Members
2010	20
2011	21
2012	30
2013	34
2014	38
2015	45

Use any strategy!

Case #6 Sales

A shirt regularly sells for AED125. The price is reduced by 10% each week for the next four weeks.

What is the mean price of the shirt during this five-week period?

Mid-Chapter Check

Vocabulary Check

1. **Be Precise** Define *bivariate data*. Give an example of a data set made up of bivariate data. (Lesson 1)

2. Fill in the blank with the correct term. (Lesson 3)

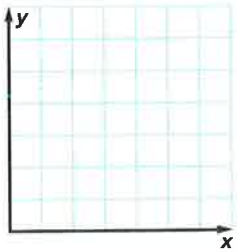
   The _____ in a two-way table is the ratio of the value of a subtotal to the value of the total.

Skills Check and Problem Solving

The table below shows the average cost to own a certain car over a period of five years. (Lessons 1 and 2)

Year	1	2	3	4	5
Cost (AED)	10,600	7,900	8,000	8,100	7,000

3. Construct and interpret a scatter plot of the data.



4. Draw a line of best fit.

5. Write an equation in slope-intercept form for the line of best fit and interpret the slope and *y*-intercept.

6. **Persevere with Problems** The two-way table shows the amount of time students studied for a test and the score they received. What is the relative frequency by column of the students that studied more than 30 minutes and received a score of 75% or more? (Lesson 3)

	Less than 30 minutes	More than 30 minutes
Score of 75% or more	20	45
Score below 75%	33	27

Lesson 4

Descriptive Statistics

Vocabulary Start-Up

Recall that measures of center represent the middle of the data. The most common measures of center are mean, median, and mode.

Complete the graphic organizer. Consider each word on the Knowledge Rating Scale and place a check ✓ in the appropriate column next to the word.

Knowledge Rating Scale			
Word	No clue	Have seen or heard it	Know it well
mean			
median			
mode			

Real-World Link

The data in the table represent the results of a survey about distance driven over spring break. Does the mean or the median best represent the data? Explain.

Distance Driven over Spring Break (km)				
749	312	302	296	293
277	257	256	219	209

Essential Question

HOW are patterns used when comparing two quantities?

Vocabulary

univariate data
quantitative data
five-number summary

Mathematical Practices

1, 2, 3, 4, 7

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Measures of Center and Variability

Data with one variable, such as test scores, are called **univariate data**. These data can be described by a measure of center.

Example

1. **The ages, in years, of the people seated in one row of a movie theater are 16, 15, 24, 33, 30, 56, 19, and 19. Find the mean, median, mode, and range of the data set.**

Mean $\frac{16 + 15 + 24 + 33 + 30 + 56 + 19 + 19}{8} = \frac{212}{8}$ or 26.5

Median 15, 16, 19, 19, 24, 30, 33, 56 — Arrange in order from least to greatest.

$\frac{19 + 24}{2} = 21.5$ years old

Mode The mode is 19, since it is the number that occurs most often.

Range $56 - 15 = 41$

Show your work.

Got it? **Do this problem to find out.**

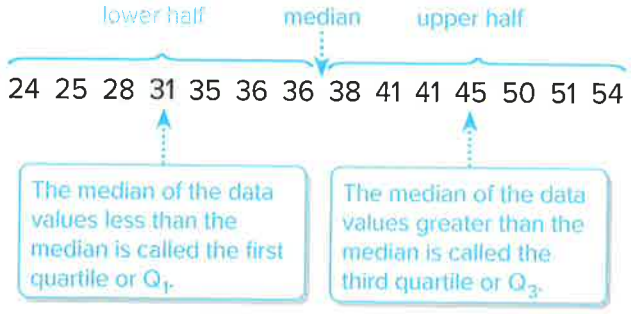
a. _____

a. Find the mean, median, mode, and range of the data set.

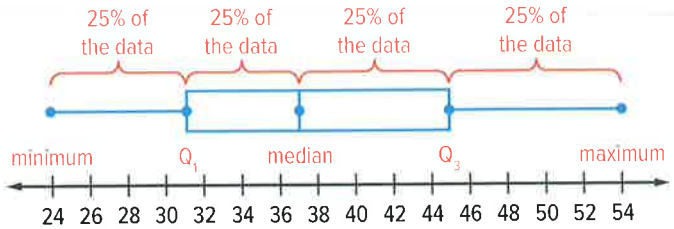
Monthly Rainfall (mm)				
66	72	70	74	64
65	60	62	66	67
68	71	70	72	73

Five-Number Summary

Quantitative data are data that can be measured. A set of quantitative data can be divided into four equal parts, called quartiles.



This **five-number summary,** which includes the minimum value, first quartile (Q_1), median, third quartile (Q_3), and the maximum value of a data set, provides a numerical way of characterizing a set of data. The five-number summary can be described visually with a box plot, as shown below.



Outliers

If an asterisk (*) appears on a box plot, it represents an outlier. Outliers are data that are more than 1.5 times the interquartile range from the first or third quartiles.

Example

2. **The data for monthly rainfall for 15 months in the UK are shown in the table.**

Monthly Rainfall (mm)				
68	73	70	71	74
72	75	69	76	75
72	75	76	75	76

a. Find the five-number summary of the data.

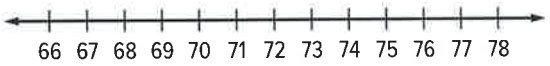
Write the data from least to greatest.

68 69 70 71 72 72 73 74 75 75 75 75 76 76 76

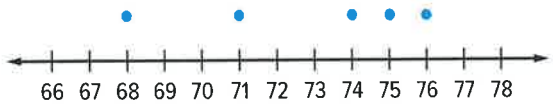
minimum: 68 | first quartile: 71 | median: 74 | third quartile: 75 | maximum: 76

b. Draw a box plot of the data.

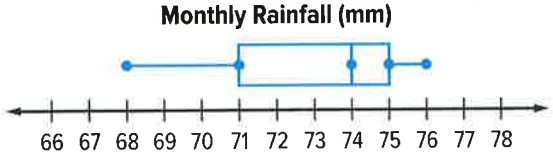
Step 1 Draw a number line that includes the least and greatest numbers in the data.



Step 2 Mark the minimum and maximum values, the median, and the first and third quartiles above the number line.



Step 3 Draw the box plot and assign a title to the graph.



b. _____

Got it? **Do these problems to find out.**

b. The points scored by a basketball team are shown in the table. Find the five-number summary of the data.

Game	1	2	3	4	5	6	7	8	9
Number of Points	34	20	83	36	37	44	40	35	36

c. Draw a box plot of the data.

Points Scored

Guided Practice

1. The points scored by each of seven basketball players is 12, 4, 18, 16, 21, 8, and 12. Find the mean, median, mode, and range of the data set. (Example 1) _____

2. The data for Calories burned per minute of exercise is in the table. (Example 2)

Exercise	Jogging	Jumping Rope	Basketball	Soccer	Bicycling	Downhill Skiing	Walking
Calories Burned	8	7	7	6	5	5	4

a. Find the five-number summary of the data. _____

b. Draw a box plot to represent the data.

Calories Burned

3. **Building on the Essential Question** What does the length of the "whiskers" in a box plot say about the data?

_____

Rate Yourself!

How confident are you about finding the five-number summary? Color the square that applies.

Name _____ My Homework _____

Independent Practice

Find the mean, median, mode, and range of each data set. Round to the nearest tenth if necessary. (Example 1)

1. Roller coaster speeds shown in the table at the right

Show your work.

Fastest Roller Coasters	
Coaster	**Speed (kmph)**
Dodonpa	107
Kingda Ka	128
Millennium Force	93
Phantom's Revenge	82
Steel Dragon 2000	95
Superman: The Escape	100
Top Thrill Dragster	120
Tower of Terror	100

2. Number of words in magazine articles: 115, 118, 115, 100, 97, 105

Find the five-number summary of each set of data. Then draw a box plot of the data. (Example 2)

3.

Number of Days of Incubation Periods for Pet Birds	
Australian King Parrot	20
Glossy Cockatoo	30
Major Mitchell's Cockatoo	26
Princess Parrot	21
Red-Tailed Cockatoo	30
Red-Winged Parrot	21
Regent Parrot	21
Superb Parrot	20
White-Tailed Cockatoo	29
Yellow-Tailed Cockatoo	29

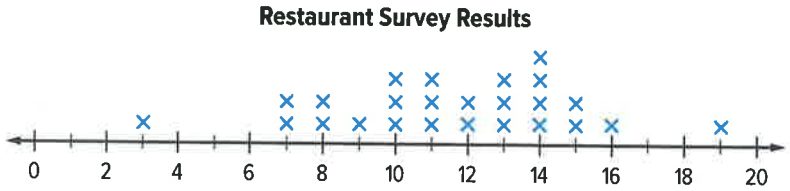
Incubation Period

4.

Top Ten Countries Average Daily Teen Spending	
Norway	AED49.70
Sweden	AED41.70
Brazil	AED41.30
Argentina	AED40.50
Hong Kong	AED38.00
United States	AED37.60
Denmark	AED37.40
Singapore	AED34.10
Greece	AED32.90
France	AED31.30

Teen Spending

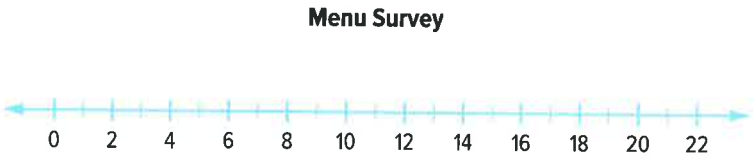
5. MP **Multiple Representations** A restaurant conducted a survey asking its customers to rate the new menu using a scale of 1 to 20. The results of the survey are shown in the line plot.



a. **Numbers** Find the mean, median, mode, and range of the data set. Round to the nearest tenth if necessary.

b. **Numbers** Find the five-number summary of the data set.

c. **Graphs** Draw a box plot to represent the set of data.



H.O.T. Problems Higher Order Thinking

6. MP **Reason Abstractly** Create a data set that contains 8 to 12 values such that the mean is greater than the median.

7. MP **Persevere with Problems** Create two different data sets that have the same median and same quartiles, but different ranges.

8. MP **Persevere with Problems** The ages of the students in a class at the community center are shown below.

25, 28, 36, 21, 28, 15, 24, 30

If the age of the teacher is added to the set of data, the mean age becomes 27. What is the age of the teacher?

Name _____ My Homework _____

Extra Practice

Copy and Solve **For Exercises 9–16, show your work and answers on a separate piece of paper.**

Find the mean, median, mode, and range of each data set.

9. Length in centimeters of spools of ribbon: 60, 48, 36, 144, 72

10. Cost in dirhams for medium pizza: 6, 6, 8, 10, 4, 6, 8, 9

11. Khalid's bowling scores are shown in the table.

Bowling Scores			
164	128	151	138
158	162	130	162

   a. Find the mean, median, mode, and range of the data. Round to the nearest tenth.

   b. Find the five-number summary of the data.

   c. Draw a box plot to represent the data.

12. The prices of video games are shown in the table.

Prices of video games (AED)					
100	180	250	200	130	180
250	280	90	300	300	750
150	130	200	180	100	350

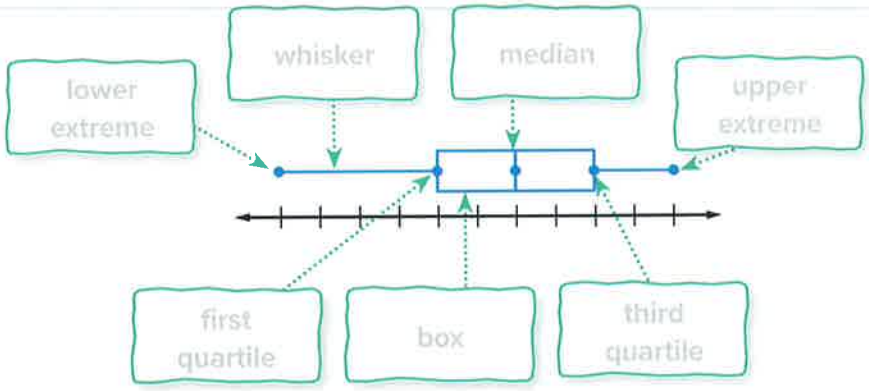
   a. Find the mean, median, mode, and range of the data. Round to the nearest tenth.

   b. Find the five-number summary of the data.

   c. Draw a box plot to represent the data.

   d. What conclusions can be drawn from the box plot?

13. MP **Identify Structure** Label the parts of the box plot.



Power Up! Test Practice

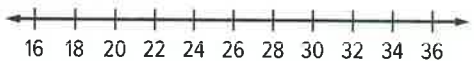
14. The table shows the ages of people standing in line for tickets to see a movie. Find each of the follow values for the data set.

Ages				
18	22	31	25	30
19	26	24	35	25

Minimum: ☐ First Quartile: ☐ Median: ☐

Third Quartile: ☐ Maximum: ☐

Draw a box plot to represent the set of data.



15. The speeds, in kilometers per hour, of several cars on a busy street are shown. Determine if each statement is true or false.

42 38 44 35 50 38

a. The range of the speeds is 12 kilometers per hour. ☐ True ☐ False

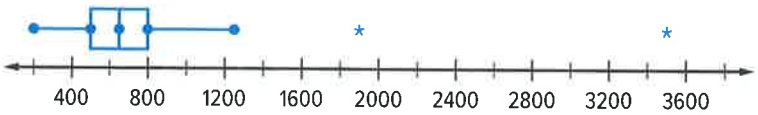
b. The mean is the measure of center that makes the speeds appear fastest. ☐ True ☐ False

c. The median is the measure of center that makes the speeds appear slowest. ☐ True ☐ False

Spiral Review

16. The box plot below shows the areas of the largest zoos in the United States.

Areas (acres) of the Ten Largest Zoos in the United States



a. How many outliers are in the data?

b. Describe the distribution of the data. What can you say about the areas of the major zoos in the U.S.?

Lesson 5

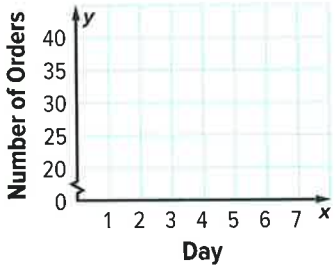
Measures of Variation

Real-World Link

Restaurant A restaurant asks the staff to record the number of people who order the special each day. The table shows the number of specials ordered per day.

Day	1	2	3	4	5	6
Number of Specials	26	25	30	32	27	28

1. Plot the data on the graph provided.



2. Find the mean of the data set.

3. Complete the table by finding the absolute value of the difference between the mean and each data value in the set.

Number of Specials	26	25	30	32	27	28
Difference from Mean						

4. Find the average of the values for the difference from the mean in the table.

Essential Question

HOW are patterns used when comparing two quantities?

Vocabulary

mean absolute deviation
standard deviation

Mathematical Practices
1, 3, 4, 7

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Mean Absolute Deviation

You have used measures of center to describe the middle of a set of data, and you have used range to describe the spread or *variation* of a set of data. Another way to describe the variation of a set of data is to use its mean absolute deviation. The **mean absolute deviation** of a set of data is the average distance between each data value and the mean.

Example

1. **The table shows the heights of eight sunflowers. Find the mean absolute deviation of the set of data. Describe what the mean absolute deviation represents.**

Heights (cm.)			
52	48	60	55
59	54	58	62

Step 1 Find the mean.

$$\frac{52 + 48 + 60 + 55 + 59 + 54 + 58 + 62}{8} = 56$$

Step 2 Find the absolute value of the differences between each value in the data set and the mean.

$|52 - 56| = 4$ $\quad$ $|59 - 56| = 3$

$|48 - 56| = 8$ $\quad$ $|54 - 56| = 2$

$|60 - 56| = 4$ $\quad$ $|58 - 56| = 2$

$|55 - 56| = 1$ $\quad$ $|62 - 56| = 6$

Step 3 Find the average of the absolute values of the differences between each value in the data set and the mean.

$$\frac{4 + 8 + 4 + 1 + 3 + 2 + 2 + 6}{8} = 3.75$$

The mean absolute deviation is 3.75. This means that the average distance each person's height is from the mean height is 3.75 inches.

Got it? **Do this problem to find out.**

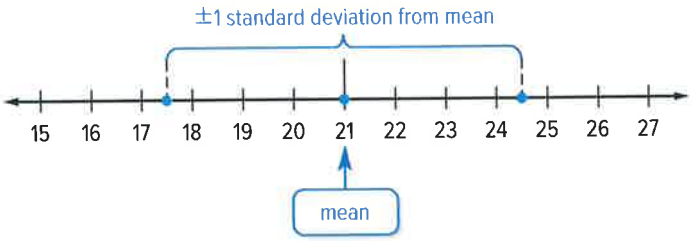
Show your work.

a. The number of points that Khadija scored in five basketball games was 8, 14, 10, 7, and 13. Find the mean absolute deviation of the set of data. Describe what the mean absolute deviation represents.

a. _____

Standard Deviation

The **standard deviation** of a set of data is a calculated value that shows how the data deviates from the mean of the data. In a given set of data, most of the values fall within one standard deviation of the mean. So, if the mean of a set of data is 21 and the standard deviation is 3.5, then most of the values fall between 21 − 3.5 or 17.5 and 21 + 3.5 or 24.5.



Real World Example

2. **The standard deviation of quiz scores for Class A is about 1.2. Describe the quiz scores that are within one standard deviation of the mean.**

Quiz Scores, Class A			
9	8	6	7
8	9	9	10
7	10	8	8

Step 1 Find the mean.

$$\text{mean} = \frac{9 + 8 + \ldots + 8}{12} = 8.25$$

Step 2 Find the range of values that are within one standard deviation of the mean.

$8.25 - 1.2 = 7.05$ Subtract the standard deviation from the mean.

$8.25 + 1.2 = 9.45$ Add the standard deviation to the mean.

Quiz scores that are between 7.05 and 9.45 points are within one standard deviation of the mean.

Show your work.

b. _____

Got it? **Do this problem to find out.**

b. The standard deviation of quiz scores for Class B is about 1.9. Describe the quiz scores that are within one standard deviation of the mean.

Quiz Scores, Class B			
8	5	3	7
7	9	7	9
7	8	10	10

Guided Practice

Check

1. The table shows the milligrams of caffeine per serving in certain types of tea. Find the mean absolute deviation of the set of data. Describe what the mean absolute deviation represents. (Example 1)

Amount of Caffeine in Tea (milligrams)				
9	46	18	35	30
12	56	24	38	32

Show your work.

2. The table shows the milligrams of caffeine per serving in certain types of coffee. Find the mean absolute deviation of the data. Describe what the mean absolute deviation represents. (Example 1)

Amount of Caffeine in Coffee (milligrams)		
145	170	150
90	100	100
165	135	106

3. Refer to the table in Exercise 1. The standard deviation of the amounts of caffeine is about 14 milligrams. Describe the data values that are within one standard deviation of the mean. (Example 2)

4. **Building on the Essential Question** How does the mean absolute deviation describe the variation of a set of data? _____

Rate Yourself!

How confident are you about measures of variation? Color the square that applies.

Name _____ My Homework _____

Independent Practice

Find the mean absolute deviation of each set of data. Round to the nearest tenth if necessary. Describe what the mean absolute deviation represents. (Example 1)

1.

Average Speeds of Selected Animals (kmph)		
70	40	45
42	40	36

2.

Average Numbers of Annual Vacation Days for Selected Countries						
34	26	37	35	42	25	25

Show your work.

3. Refer to the table in Exercise 1. The standard deviation of the average speeds of some animals is about 11.3 kilometers per hour. Describe the data values that are within one standard deviation of the mean. (Example 2)

4. MP **Justify Conclusions** The table shows the total points scored in beach volleyball matches.

Beach Volleyball Scores	
Home	**Away**
52	47
61	42
42	42
44	42
60	17
50	54
55	52
42	42
49	29
46	37

a. Find the mean absolute deviation for each set of data. Round to the nearest hundredth. Then write a few sentences comparing their variation.

b. The standard deviation of the Home teams' scores is 6.6 points. The standard deviation of the Away teams' scores is 10.3 points. Describe how this information supports your answer to part **a**.

H.O.T. Problems Higher Order Thinking

5. **Find the Error** Hassan is describing the data values that are within one standard deviation of the mean of a set of data. Find his mistake and correct it.

6. **Identify Structure** Create a list of data with at least five numbers that has a range of 40. Describe the mean absolute deviation.

7. **Persevere with Problems** The standard deviation of ribbon lengths is about 7.2 centimeters. Describe the lengths that are within two standard deviations of the mean. Explain your reasoning.

Lengths of Ribbon (cm)			
42	24	48	36
28	36	36	30

8. **Justify Conclusions** Determine whether the following statement is *always*, *sometimes*, or *never* true. Justify your response.

   A data set with a mean absolute deviation of 9 is more spread out than a data set with a mean absolute deviation of 3.

9. **Reason Inductively** Compare and contrast standard deviation and mean absolute deviation.

Name _____ My Homework _____

Extra Practice

Copy and Solve **For Exercises 10–16, show your work and answers on a separate piece of paper.**

10. The table shows the number of hours of sleep for selected animals, rounded to the nearest hour.

Daily Sleep						
Animal	brown bat	giant armadillo	cat	bottle-nosed dolphin	gray seal	horse
Amount of Sleep (h)	20	18	12	10	6	3

a. Find the mean absolute deviation of the set of data. Describe what the mean absolute deviation represents.

b. The standard deviation of the data is about 6 hours. Describe the data that are within one standard deviation of the mean.

11. The table shows the speeds of eight roller coasters in the United States.

Roller Coaster Speeds								
Coaster	Dodonpa	Kingda Ka	Millennium Force	Phantom's Revenge	Steel Dragon 2000	Superman: The Escape	Top Thrill Dragster	Tower of Terror
Speed (km/h)	107	128	93	82	95	100	120	100

a. Find the mean absolute deviation of the set of data. Round to the nearest hundredth if necessary. Describe what the mean absolute deviation represents.

b. The standard deviation of the data is about 13.9 kilometers per hour. Describe the data that are within one standard deviation of the mean. Round to the nearest hundredth if necessary.

12. The table shows the bids that some comic books received at an auction.

Comic Book Bids (AED)				
3.25	4.50	5.00	5.75	2.25
8.50	6.00	3.50	4.50	5.00

a. Find the mean absolute deviation of the set of data. Round to the nearest hundredth if necessary. Describe what the mean absolute deviation represents.

b. The standard deviation of the data is about AED1.64. Describe the data that are within one standard deviation of the mean. Round to the nearest hundredth if necessary.

Power Up! Test Practice

13. The table shows the lengths of four different boards. The standard deviation of the lengths is about 2.9 meters. Determine which statements are true. Select all that apply.

Length (m)			
12	15	15	20

- ☐ The mean absolute deviation is less than the standard deviation.
- ☐ The range is greater than the mean absolute deviation.
- ☐ The standard deviation is greater than the range.

14. The numbers of cans donated by five students during a canned food drive are shown in the table.

Number of Cans Donated				
8	10	14	22	16

What is the mean absolute deviation of the data? ☐

Spiral Review

15. The table shows lengths of rivers in two continents.

Length (kilometers) of Rivers			
Africa		South America	
4,160	700	4,000	1,300
2,900	660	2,485	1,100
2,590	500	2,100	1,000
1,700	1,100	2,013	1,000
1,300	1,020	1,988	1,000
1,100	1,000	1,750	956
1,000		1,677	910
		1,600	808
		1,584	400
		1,400	150

a. Which continent has a greater range of length of rivers?

b. Find the measures of center for each continent.

c. Select the appropriate measure of center or range to describe the lengths of rivers for each continent. Justify your response.

d. Find the measures of variation for each continent.

16. Find the measures of center for the set of data. Round to the nearest tenth if necessary.

Tallest Buildings in a City

Stem	Leaf
2	7 9 9
3	0 1 1 1 3 3 4 4 6 6 7
4	0 2 2 5 9
5	0 0 0 0 2 5 6 8
6	0
7	2

$217 = 27$

Lesson 6

Analyze Data Distributions

Vocabulary Start-Up

Recall that in statistical displays, peaks, gaps, clusters, and outliers are identified easily.

Complete the graphic organizer by matching the term with the correct description.

A gap is...	...the most frequently occurring value or interval of value.
A peak is...	...when many data values are grouped together.
An outlier is...	...a data value that is 1.5 times the interquartile range from the first or third quartiles.
A cluster is...	...where there are no data values.

Real-World Link

Find a photo of a mountain range. Describe clusters, gaps, and peaks in terms of the photo.

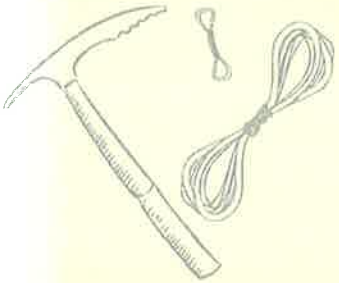
Essential Question

HOW are patterns used when comparing two quantities?

Vocabulary

distribution
symmetric

MP Mathematical Practices
1, 3, 4



Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

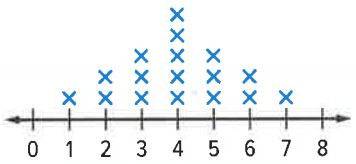
1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Describe a Distribution by Shape

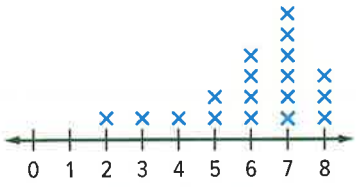
The **distribution** of a set of data shows the arrangement of data values. It can be described by its center, spread (variation), and overall shape. Determining the symmetry of the distribution is one way to describe shape. If the left side of a distribution looks like the right side, then the distribution is **symmetric**.

Symmetric



The left side looks like the right side.

Non-Symmetric



The right side is taller than the left side.

Another way to describe the shape of a distribution is to identify peaks, clusters, gaps, and outliers. If there is an outlier, the distribution is not symmetric.

Example

1. **The graph shows the weights of adult cats. Identify any symmetry, clusters, gaps, peaks, or outliers in the distribution.**

The distribution is non-symmetric. There is a cluster from 7–12 with a peak at 10. There is a gap between 12 and 14, and there are no outliers.

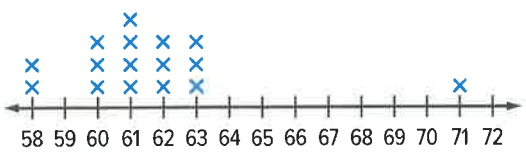
Show your work.

Got it? **Do this problem to find out.**

a. Identify any symmetry, clusters, gaps, peaks, or outliers in the distribution below.

a. _____

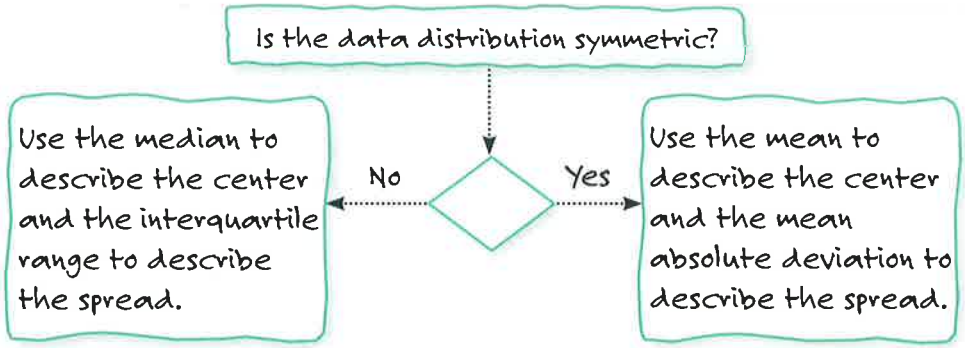
Height of Tomato Plants (cm)



Describe the Center and Spread of a Distribution

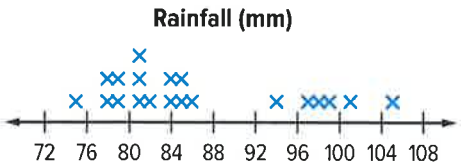
The shape of a distribution tells you which measures are most appropriate for describing the center and spread of a distribution. The mean and mean absolute deviation are affected by outliers, while the median and interquartile range are resistant to outliers.

Use the following graphic organizer to decide which measures of center and spread are most appropriate to describe a data distribution.



Example

2. **Mr. Omar's class charted the monthly rainfall in various cities. The results are shown in the line plot.**



Describe the center and spread of the distribution. Justify your response based on the shape of the distribution.

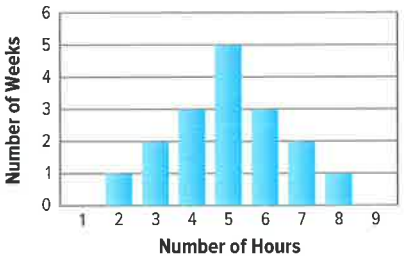
The distribution is not symmetric. So, the median and interquartile range are the appropriate measures to use. The data are centered around the median of 84 mm. The first quartile is 80 and the third quartile is 95.5. So, the interquartile range is 95.5 − 80 or 15.5 mm. The spread of the data around the center is 15.5 mm.

Show your work.

b. _____

Got it? **Do this problem to find out.**

b. The graph shows the hours per week that dance students practice their dances. Describe the center and spread of the distribution. Justify your response based on the shape of the distribution. Round to the nearest tenth if necessary.



Guided Practice

Check

1. The number of nachos sold at the football concession stand is shown in the line plot at the right.

Number of Nacho Bowls Sold Each Night at Concession Stand

a. Describe the shape of the distribution. Identify any clusters, gaps, peaks, or outliers. (Example 1)

b. Describe the center and spread of the distribution. Justify your response based on the shape of the distribution. (Example 2)

2. **Building on the Essential Question** Why is the median used to describe the center of a non-symmetric distribution instead of the mean?

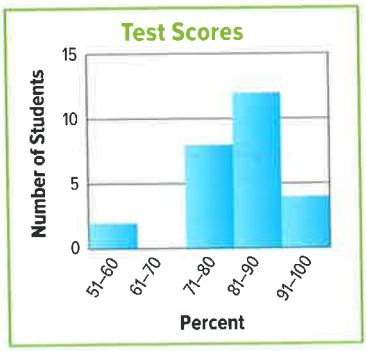
Rate Yourself!

How confident are you about describing data distributions? Check the box that applies.

Name _____ My Homework _____

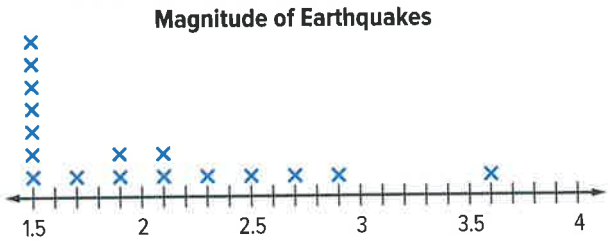
Independent Practice

1. The scores for Ms. Hessa math class are shown in the histogram. Describe the shape of the distribution shown. Identify any clusters, gaps, peaks, or outliers. (Example 1)



Show your work.

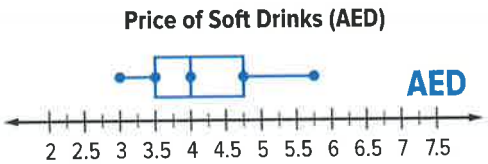
2. The magnitude of several earthquakes is shown in the line plot at the right.



   a. Describe the shape of the distribution shown. Identify any clusters, gaps, peaks, or outliers. (Example 1)

   b. Describe the center and spread of the distribution. Justify your response based on the shape of the distribution. (Example 2)

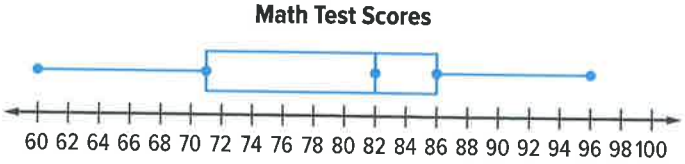
3. The box plot shows the prices of soft drinks at different restaurants.



   a. Describe the shape of the distribution using symmetry and outliers. (Example 1)

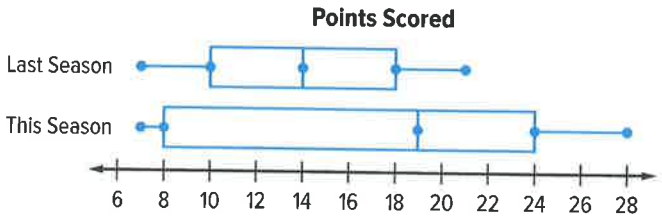
   b. Describe the center and spread of the distribution. Justify your response based on the shape of the distribution. (Example 2)

4. MP **Make a Conjecture** A distribution that is not symmetric is called *skewed*. A distribution can be skewed left or right. It is skewed left if the data are more spread out on the left side than the right side. Is the distribution shown skewed left or skewed right? Explain your reasoning to a classmate.



H.O.T. Problems Higher Order Thinking

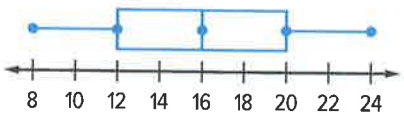
5. MP **Persevere with Problems** The double box plot shows scores for a football team.



a. Choose the measures that are appropriate to describe the center and spread of each box plot. Explain.

b. Is it possible to find each value? Explain.

6. MP **Persevere with Problems** Explain why you cannot describe the specific location of the center and spread of the box plot shown using the most appropriate measures.



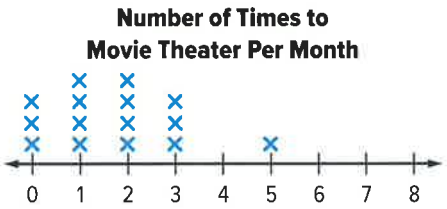
Name _____ My Homework _____

Extra Practice

Copy and Solve **For Exercises 7–17, show your work and answers on a separate piece of paper.**

7. The winning scores for twenty rugby games are shown in the histogram below. Describe the shape of the distribution. Identify any clusters, gaps, peaks, or outliers.

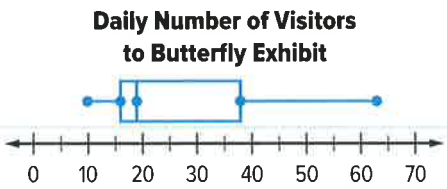
8. Khamis took a survey of the number of times his classmates when to the movies this month. The results are shown in the line plot below.



   a. Describe the shape of the distribution. Identify any clusters, gaps, peaks, or outliers.

   b. Describe the center and spread of the distribution. Justify your response based on the shape of the distribution.

9. The box plot shows the visitors to a butterfly exhibit each day for a month.



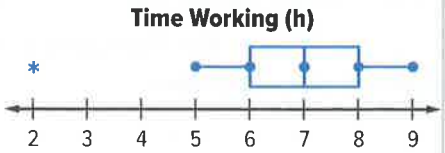
   a. Describe the shape of the distribution using symmetry and outliers.

   b. Describe the center and spread of the distribution. Justify your response based on the shape of the distribution.

10. MP **Justify Conclusions** Examine the data displays in Exercises 7–9. Determine if any of the distributions are skewed left or skewed right. Explain.

Power Up! Test Practice

11. The box plot shows the number of hours spent working on a science project by students. Determine if each statement is a valid conclusion based on the box plot. Select yes or no.



a. The distribution is symmetric. ☐ Yes ☐ No

b. The median is the best measure to describe the center. ☐ Yes ☐ No

c. There is an outlier at 2. ☐ Yes ☐ No

12. The list of data shows the number of students at different bus stops on Mr. Jamal's route. Construct a line plot of the data.

Number of Students at Bus Stops						
9	5	7	10	2	6	4
9	5	6	4	5	9	7

Number of Students at Bus Stops

Circle the correct term to make each statement true.

a. The distribution is (symmetric, not symmetric).

b. There is a (gap, cluster) between 4 and 7.

c. The (mean, median) is the best measure to describe the center.

Spiral Review

Select an appropriate display for each situation. Choose from the list shown.

Bar Graph	Line Graph
Box Plot	Line Plot
Circle Graph	Double Bar Graph
Scatter Plot	Double Line Graph
Histogram	Double Box Plot

13. the number of cell phone subscribers for the past 5 years

14. point totals for the top 10 NASCAR drivers

15. the portion of a family's budget assigned to each category

16. the median of the exam scores for one class

17. gas mileage for 2013 cars

21ST CENTURY CAREER in Marketing

Sports Marketer

Are you creative and competitive? Would you enjoy a job working in the sports business? If so, you should consider a career in sports marketing. Sports marketers use statistics to develop plans to promote sporting events, such as state athletic games. They also work for professional and college sports teams, Olympic athletes, and sporting event venues. Their job is to develop merchandise and plan events that promote an athlete's or team's popularity, thereby increasing sales.

Is This the Career for You?

Are you interested in a career as a sports marketer? Take some of the following courses in high school.

- Calculus for Business
- Principles of Marketing
- Entertainment Essentials
- Statistical Methods

Turn the page to find out how math relates to a career in Marketing.

MP Promoting the Games

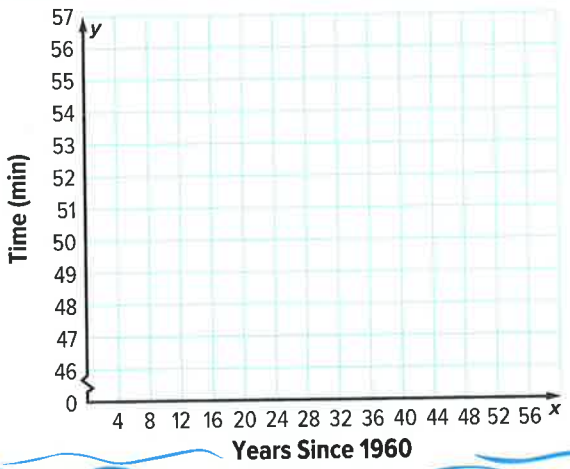
Use the information in the table to solve each problem.

1. On the coordinate grid, graph each point. Then draw a line of best fit. What two points did you use to draw the line?

2. Write an equation in slope-intercept form for the line of fit.

3. What do the slope and *y*-intercept of the line of fit represent?

Olympic Winning Times for 100-meter Men's Freestyle			
Years Since 1960	**Time (min)**	**Years Since 1960**	**Time (min)**
4	53.4	8	52.2
12	51.22	16	49.99
20	50.40	24	49.8
28	48.63	32	49.02
36	48.74	40	48.30
44	48.17	48	47.21



MP Career Project

It's time to update your career portfolio! Find sports marketing blogs on the Internet and use them to answer these questions: Which sport(s) did the bloggers write about? What did you learn about sports marketing? Were there any common themes or attitudes among the sports marketing bloggers?

List the strengths you have that would help you succeed in this career.

-
-
-
-
-

Chapter Review

Vocabulary Check

Complete each sentence using the vocabulary list at the beginning of the chapter. Then circle the word that completes the sentence in the word search.

1. Data with one variable are called _____.
2. _____ is the ratio of the value of a subtotal to the value of the total.
3. Data with two variables are called _____.
4. Data that can be measured are _____.
5. The _____ uses numbers to characterize a set of data.
6. A _____ shows the relationship between data graphed as ordered pairs on the coordinate plane.
7. In a scatter plot, the _____ fit is close to most of the data points on the coordinate plane.
8. The arrangement of data values is called a _____.
9. When the left side of a distribution looks like the right side, the distribution is _____.
10. Data that can be observed or described are _____.
11. A _____ shows data that pertain to two different categories.
12. The _____ is a numerical value that shows how the data deviated from the mean.
13. The average distance between each data value and the mean is called the _____.

L	V	P	R	M	U	V	B	P	W	L	R	J	Y	G	L	V	N	Y	E	K	S	A	T	T
N	A	J	N	N	M	E	A	Q	H	G	K	V	Y	C	T	E	B	O	F	L	C	T	U	I
N	O	I	T	A	I	V	E	D	D	R	A	D	N	A	T	S	J	F	O	T	A	A	N	F
U	G	U	H	R	D	I	S	T	R	I	B	U	T	I	O	N	L	E	Q	J	T	D	I	T
J	K	B	E	C	A	D	I	S	F	S	I	N	Q	C	N	G	T	F	A	N	T	E	V	S
E	N	O	R	M	S	D	T	Z	Y	H	W	E	G	E	M	Z	A	J	K	P	E	V	A	E
D	N	T	I	X	D	W	O	G	G	S	W	Y	O	X	A	K	A	U	Z	I	R	I	R	B
F	W	B	I	U	P	P	W	Z	U	F	W	S	V	S	U	X	F	S	L	A	P	T	I	F
M	E	A	N	A	B	S	O	L	U	T	E	D	E	V	I	A	T	I	O	N	L	A	A	O
A	A	Y	C	N	E	U	Q	E	R	F	E	V	I	T	A	L	E	R	L	O	O	T	T	E
Q	U	A	L	I	T	A	T	I	V	E	D	A	T	A	Z	Q	L	F	K	P	T	I	E	N
H	X	T	O	Q	J	E	D	P	E	L	B	A	T	Y	A	W	O	W	T	T	L	T	D	I
A	V	R	E	R	F	I	V	E	N	U	M	B	E	R	S	U	M	M	A	R	Y	N	A	L
T	J	Y	D	E	B	S	Y	M	M	E	T	R	I	C	H	A	X	V	J	A	W	A	T	M
A	T	A	D	E	T	A	I	R	A	V	I	B	B	G	J	S	P	H	N	P	Z	U	A	D
D	J	Z	P	S	F	F	U	N	P	P	C	L	S	Z	C	L	C	Y	Z	S	S	Q	B	D

Use Your FOLDABLES

Use your Foldable to help review the chapter.

Tape here

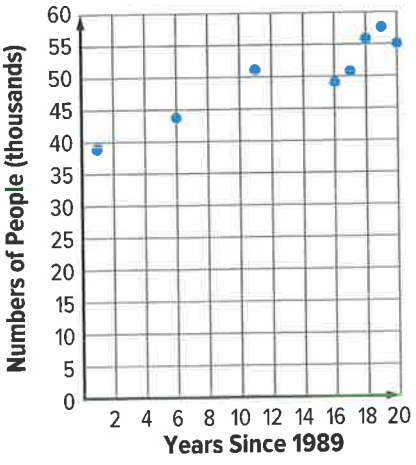
A Line of Best Fit is useful for:

A Two-Way Table is useful for:

A Scatter Plot is useful for:

Got it?

Number and perform the steps in the correct order to write an equation for the line of best fit for the scatter plot.



____ Write the equation in $y = mx + b$ form. _____

____ Find the y-intercept. _____

____ Draw the line.

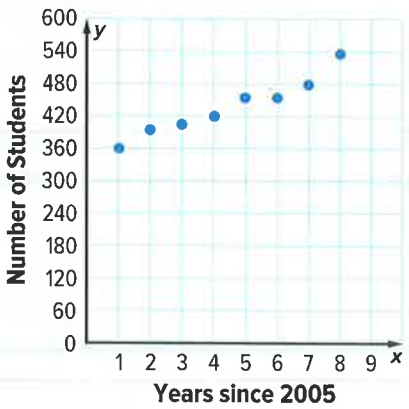
____ Choose two points. _____

____ Find the slope. _____

Power Up! Performance Task

Grand Expansion

A school is making plans to move to a new building to meet the increasing number of students in its middle school. The school's enrollment over the last several years is shown on the scatter plot.



Write your answers on another piece of paper. Show all of your work to receive full credit.

Part A

Draw a line of best fit on the graph. Interpret the scatter plot based on the shape of the distribution. Explain what this means in regard to enrollment.

Part B

Write an equation for the line of best fit. What do the slope and *y*-intercept represent?

Part C

The school must move to the new building once enrollment exceeds 690 students. If the enrollment continues to increase at the current rate, what year must the new building be ready?

Part D

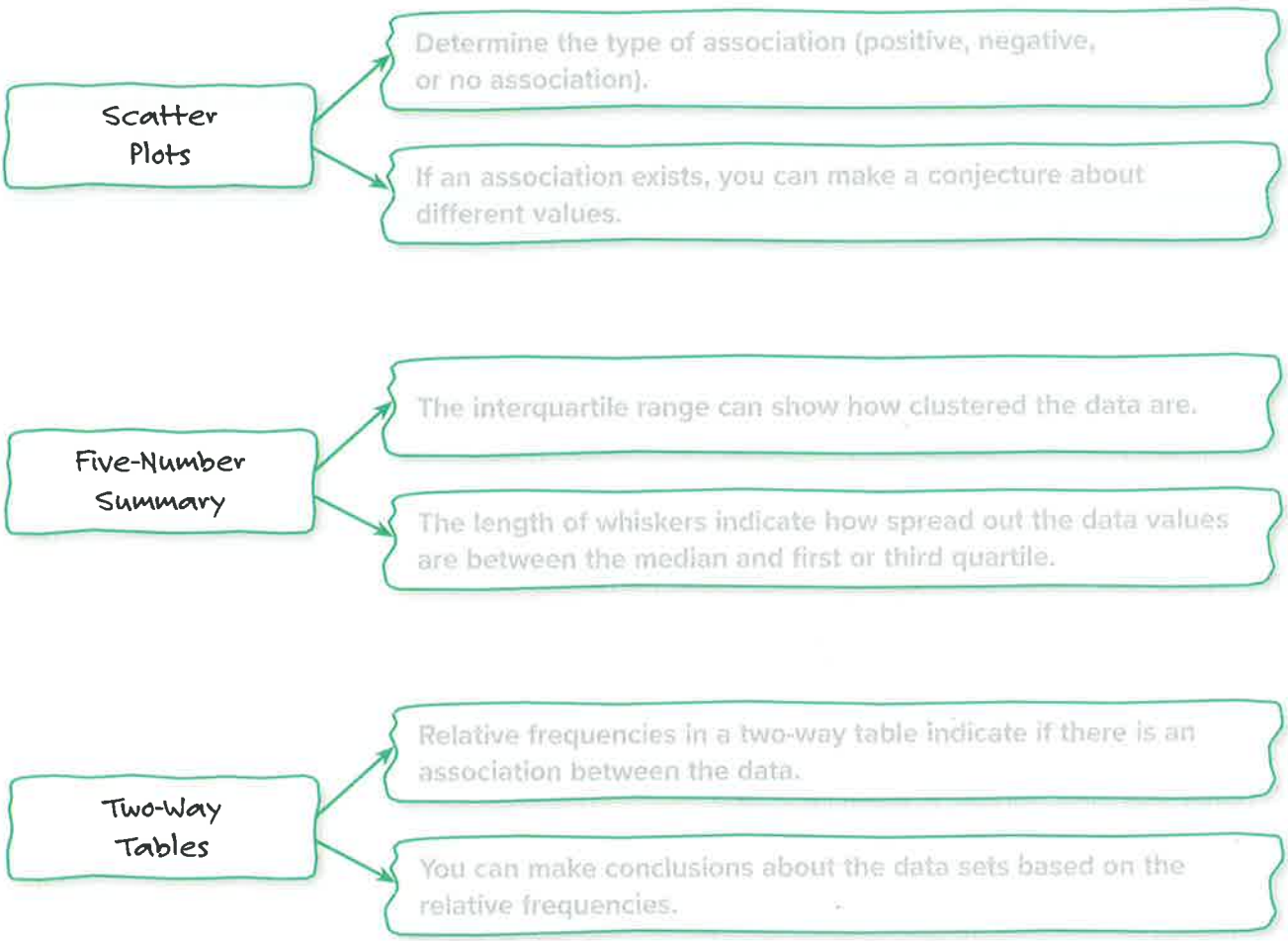
The two-way table shows the number of students that play water sports or field sports in the middle school and high school. Find the relative frequencies of the students by columns to the nearest percent. Two hundred fifty new lockers will be purchased. The new lockers will be placed in the locker rooms based on the current distribution of students playing sports. Determine how the lockers should be distributed. Explain.

	Water Sports	Field Sports	Total
Middle School	40	35	75
High School	30	45	75
Total	70	80	150

Reflect

Answering the Essential Question

Use what you learned about data analysis to complete the graphic organizer. Describe two ways patterns are used in each concept when analyzing data.



Answer the Essential Question. HOW are patterns used when comparing two quantities?

UNIT PROJECT

Olympic Games The Olympics consist of many types of sports. Many of these have a unique scoring process that determines the winner. In this project you will:

- **Collaborate** with your classmates as you gather Olympics statistics.
- **Share** the results of your research in a creative way.
- **Reflect** on why learning mathematics is important.

By the end of this project, you will understand how scatter plots and data analysis are involved in presenting Olympic Games statistics.

Collaborate

Go Online Work with your group to research and complete each activity. You will use your results in the Share section on the following page.

1. Choose a country that has participated in basketball in the Summer Olympics. Use the Internet to research the team. Find their average points per game over the past 10 Summer Olympic Games. Record the information in a table.

2. Make a scatter plot of the data from Exercise 1. Determine if the data can be used to predict the average number of points in the next Summer Olympics. If so, make a prediction.

3. Research the number of Olympic records that some country has received in the Olympic sport of your choice. Use a graph of your choice and interpret the graph.

4. Research the winning scores in archery over the past 10 Summer Olympics. Draw a histogram to display the data. Interpret the graph.

5. During the ranking round in archery each player will shoot a total of 72 arrows. Create a score card for one player in the first round. Summarize the data in a box plot and interpret the graph.

Share

Collaborate

With your group, decide on a way to share what you have learned about Olympic scoring. Some suggestions are listed below, but you can also think of other creative ways to present your information. Remember to show how you used mathematics in your project!

- Act as a television reporter for the Olympics and describe the scores and medals won in a few events. Include graphics that would appear on screen.
- Choose an Olympic sport that you do not know much about. Explain the scoring system in your sport. Then create tables and graphs to present real data from your sport in the most recent Olympics.

Check out the note on the right to connect this project with other subjects.

Connect with Physical Education

Global Awareness Research information on how to play or participate in a sport that is popular in another country. Some questions to consider are:

- What are the basic rules?
- What are the jobs of the offense and the defense?

Reflect

On Your Own

6. **Answer the Essential Question** Why is learning mathematics important?

   a. How did you use what you learned about scatter plots in this chapter to represent mathematical ideas in this project?

   b. How did you use what you learned about data analysis to communicate mathematical ideas effectively in this project?

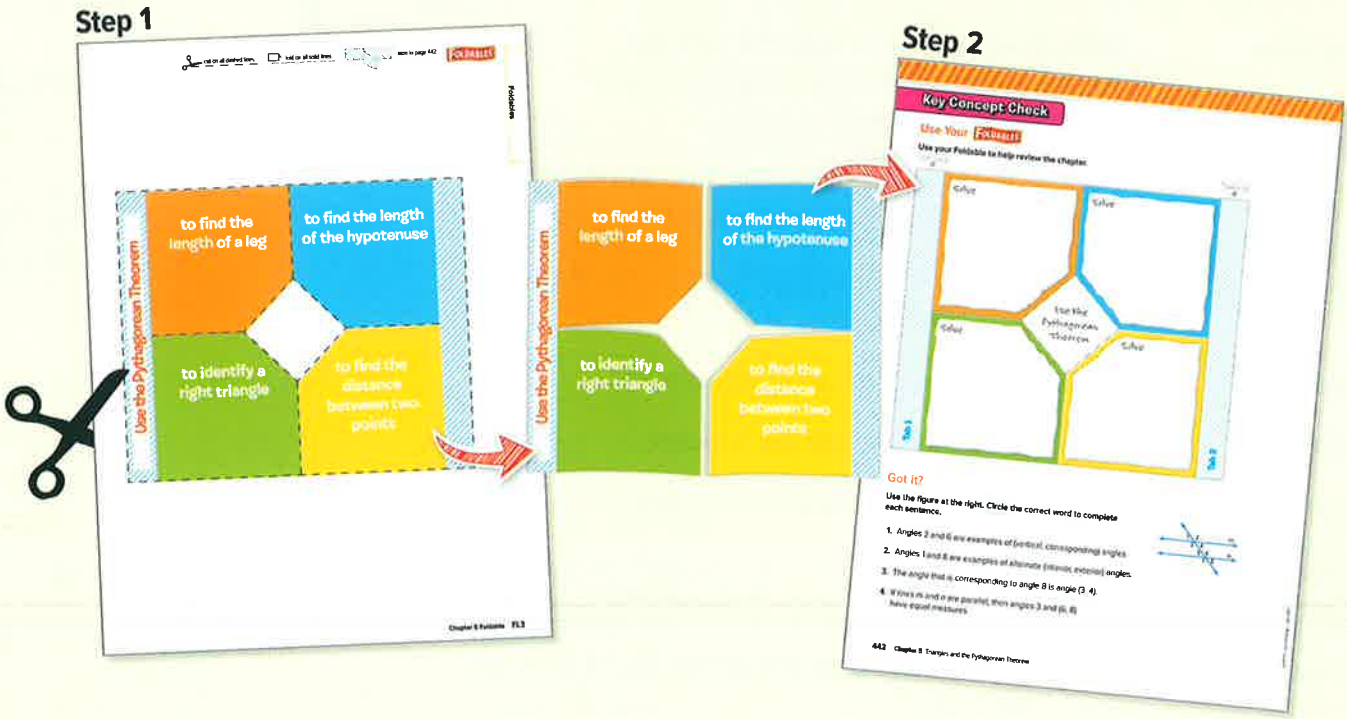
FOLDABLES® Study Organizers

What Are Foldables and How Do I Create Them?

Foldables are three-dimensional graphic organizers that help you create study guides for each chapter in your book.

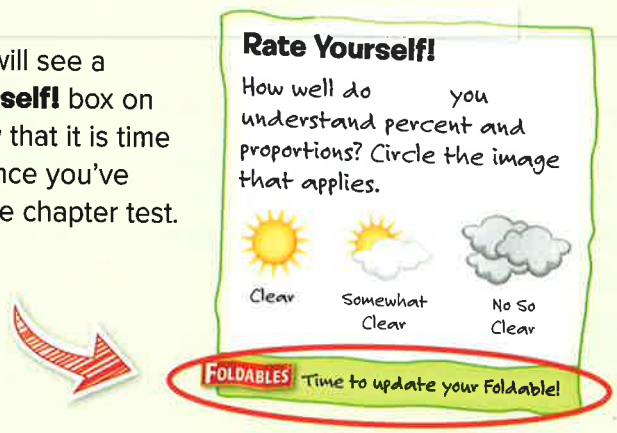
Step 1 Go to the back of your book to find the Foldable for the chapter you are currently studying. Follow the cutting and assembly instructions at the top of the page.

Step 2 Go to the Key Concept Check at the end of the chapter you are currently studying. Match up the tabs and attach your Foldable to this page. Dotted tabs show where to place your Foldable. Striped tabs indicate where to tape the Foldable.



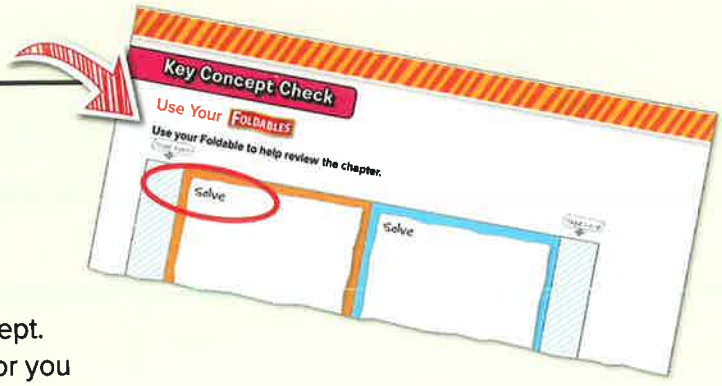
How Will I Know When to Use My Foldable?

When it's time to work on your Foldable, you will see a Foldables logo at the bottom of the **Rate Yourself!** box on the Guided Practice pages. This lets you know that it is time to update it with concepts from that lesson. Once you've completed your Foldable, use it to study for the chapter test.



How Do I Complete My Foldable?

No two Foldables in your book will look alike. However, some will ask you to fill in similar information. Below are some of the instructions you'll see as you complete your Foldable. **HAVE FUN** learning math using Foldables!



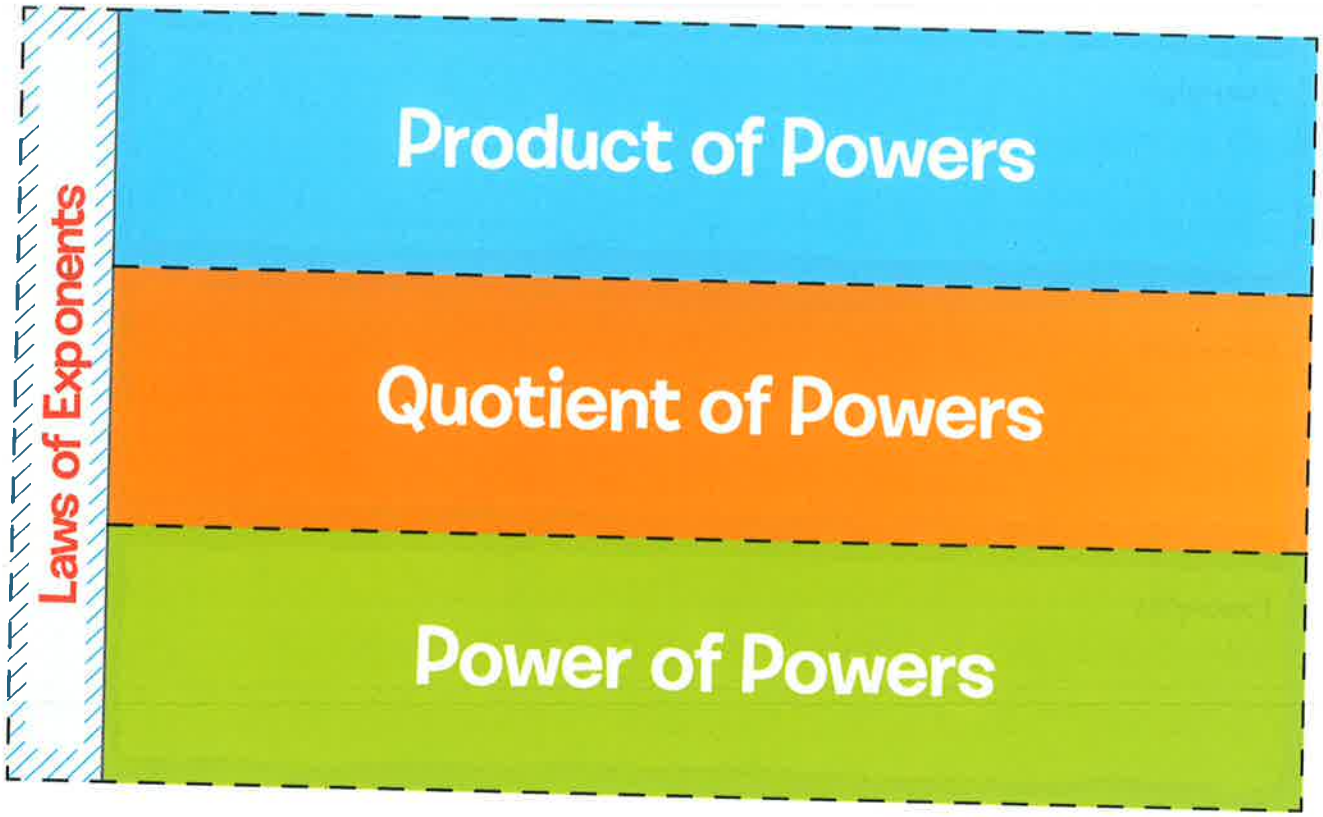
Instructions and what they mean

Best Used to...	Complete the sentence explaining when the concept should be used.
Definition	Write a definition in your own words.
Description	Describe the concept using words.
Equation	Write an equation that uses the concept. You may use one already in the text or you can make up your own.
Example	Write an example about the concept. You may use one already in the text or you can make up your own.
Formulas	Write a formula that uses the concept. You may use one already in the text.
How do I ...?	Explain the steps involved in the concept.
Models	Draw a model to illustrate the concept.
Picture	Draw a picture to illustrate the concept.
Solve Algebraically	Write and solve an equation that uses the concept.
Symbols	Write or use the symbols that pertain to the concept.
Write About It	Write a definition or description in your own words.
Words	Write the words that pertain to the concept.

Meet Foldables Author Dinah Zike

Dinah Zike is known for designing hands-on manipulatives that are used nationally and internationally by teachers and parents. She is an explosion of energy and ideas. Her excitement and joy for learning inspires everyone she touches.

Courtesy of Dinah Zike

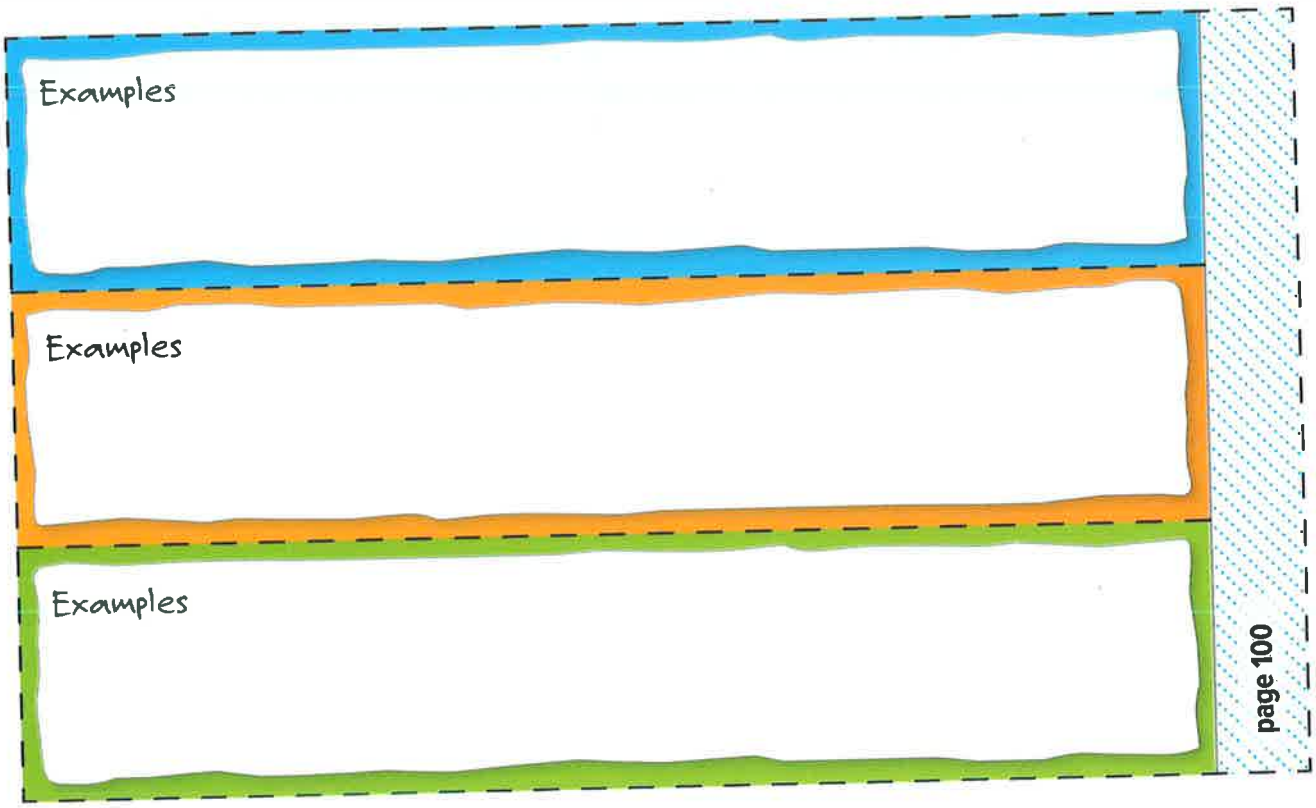


cut on all dashed lines

fold on all solid lines

tape to page 100

FOLDABLES



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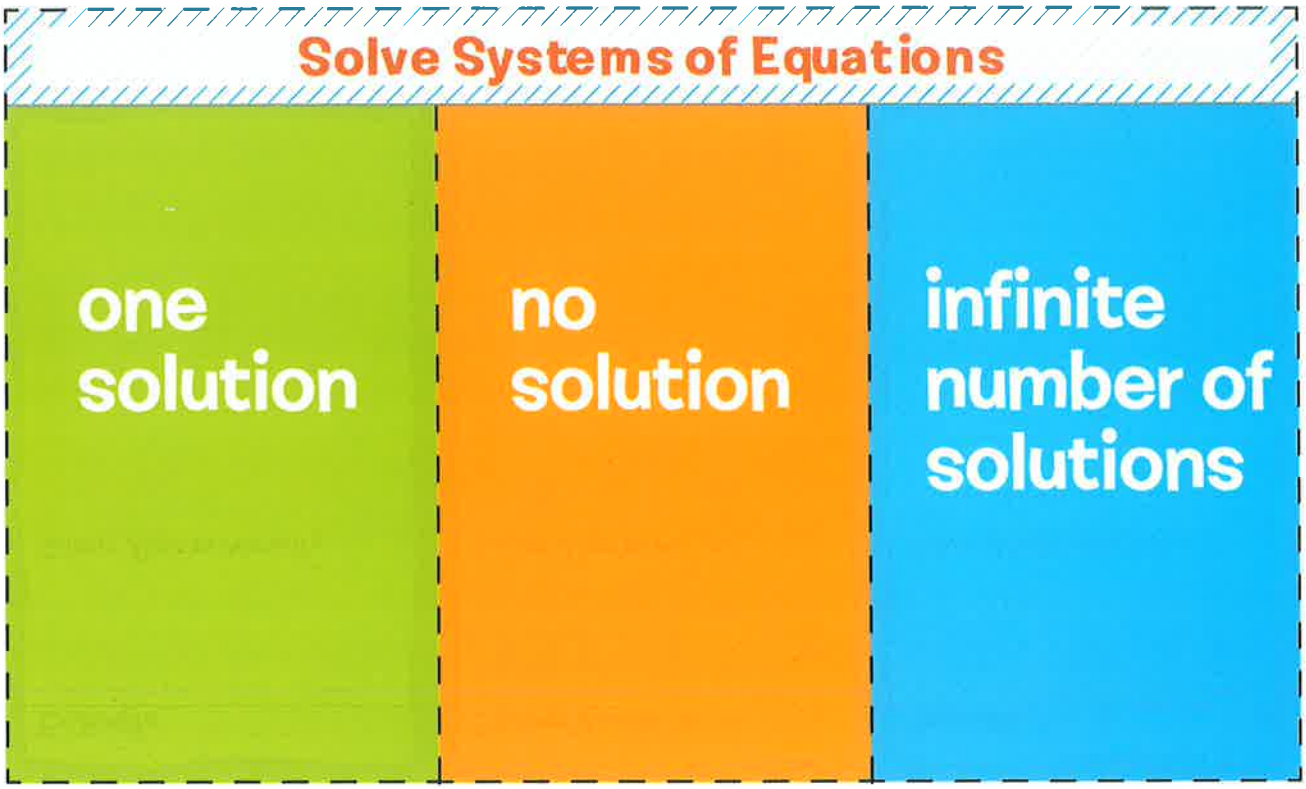

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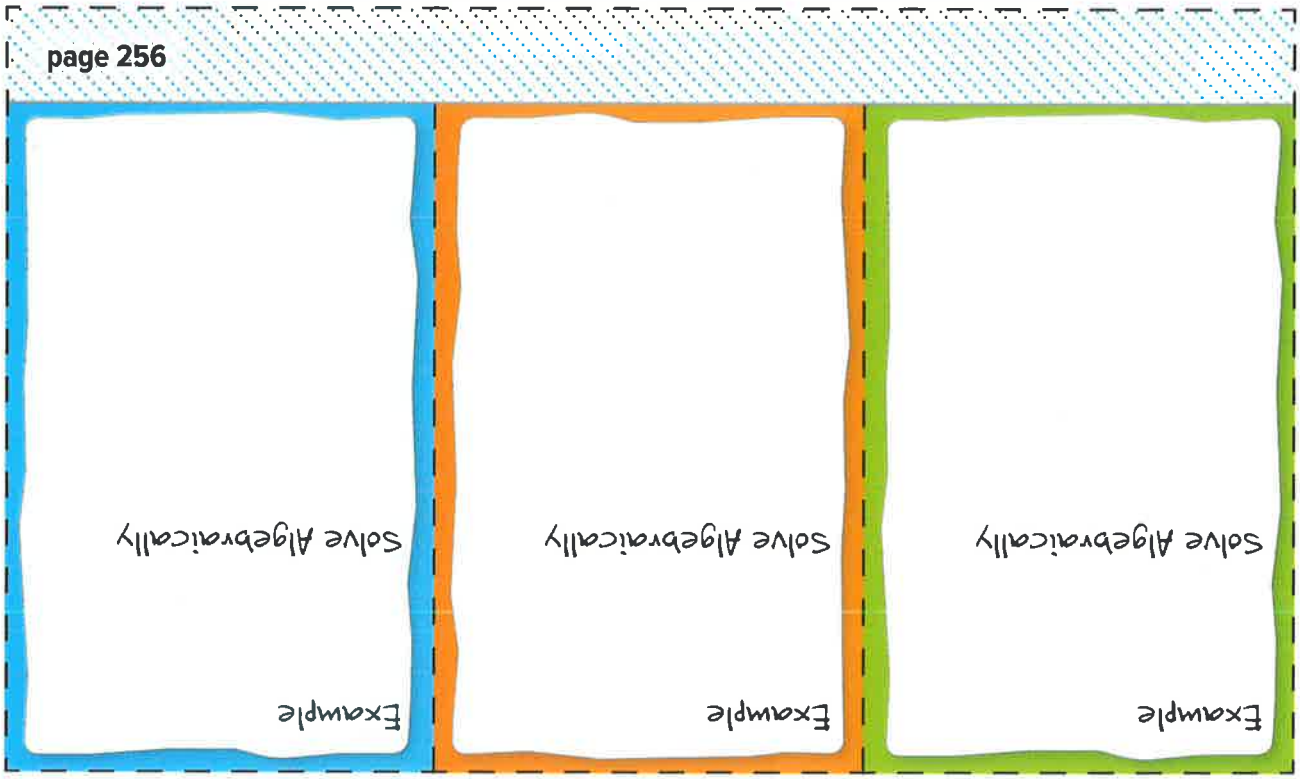

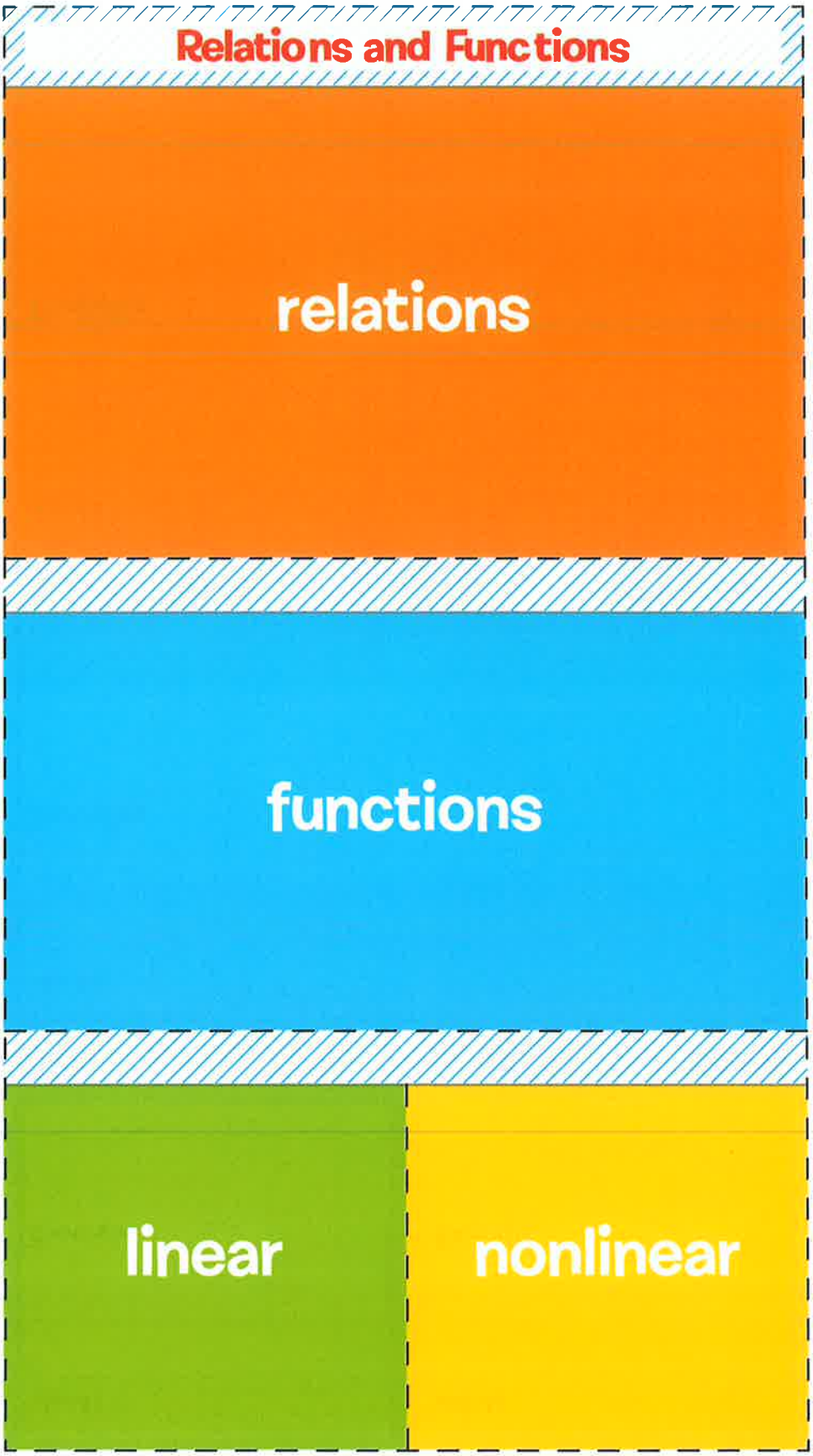

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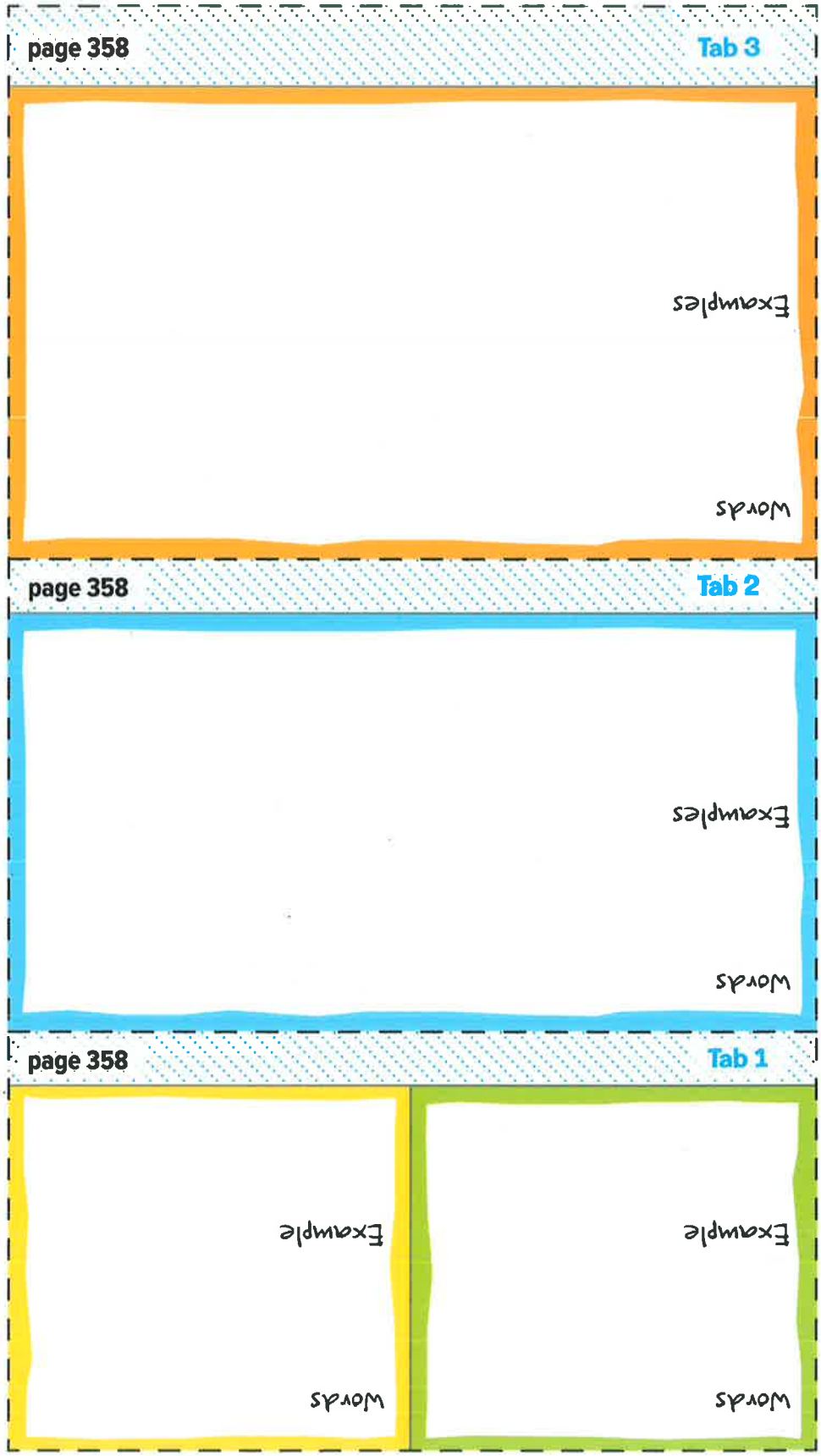

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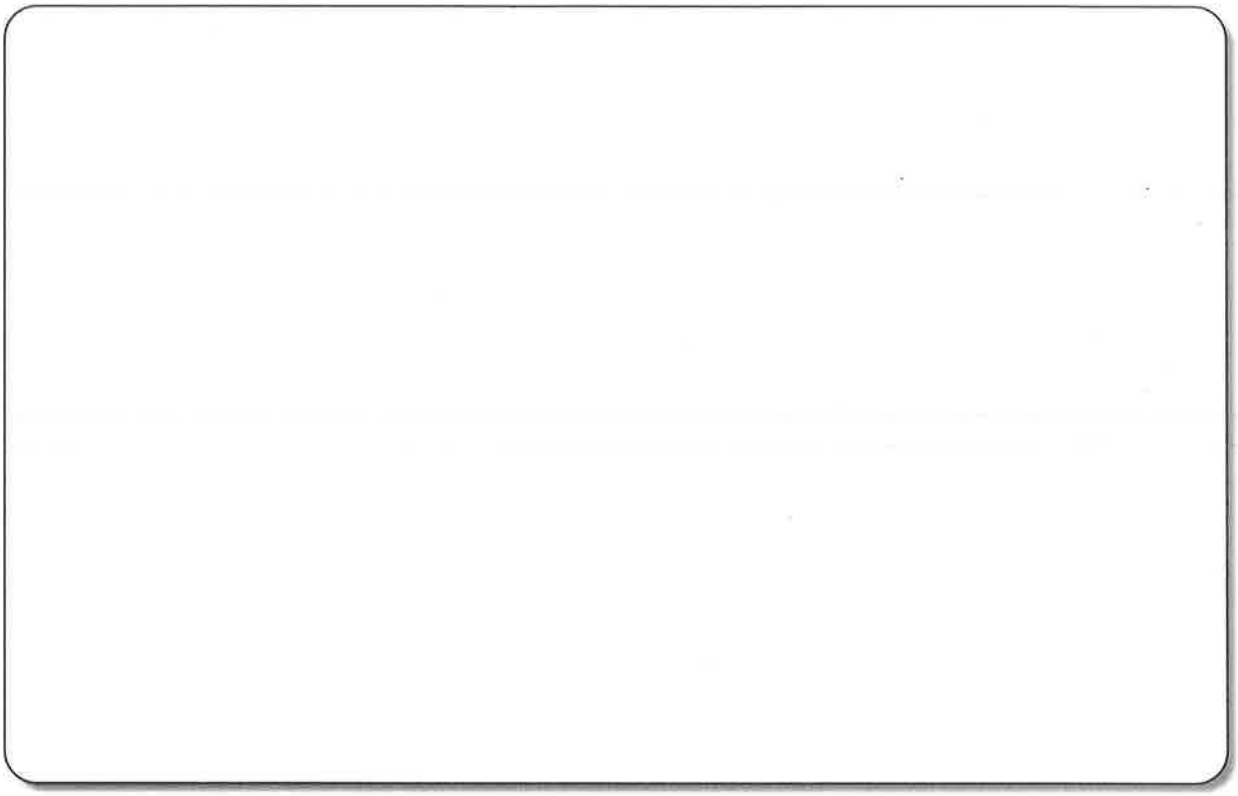

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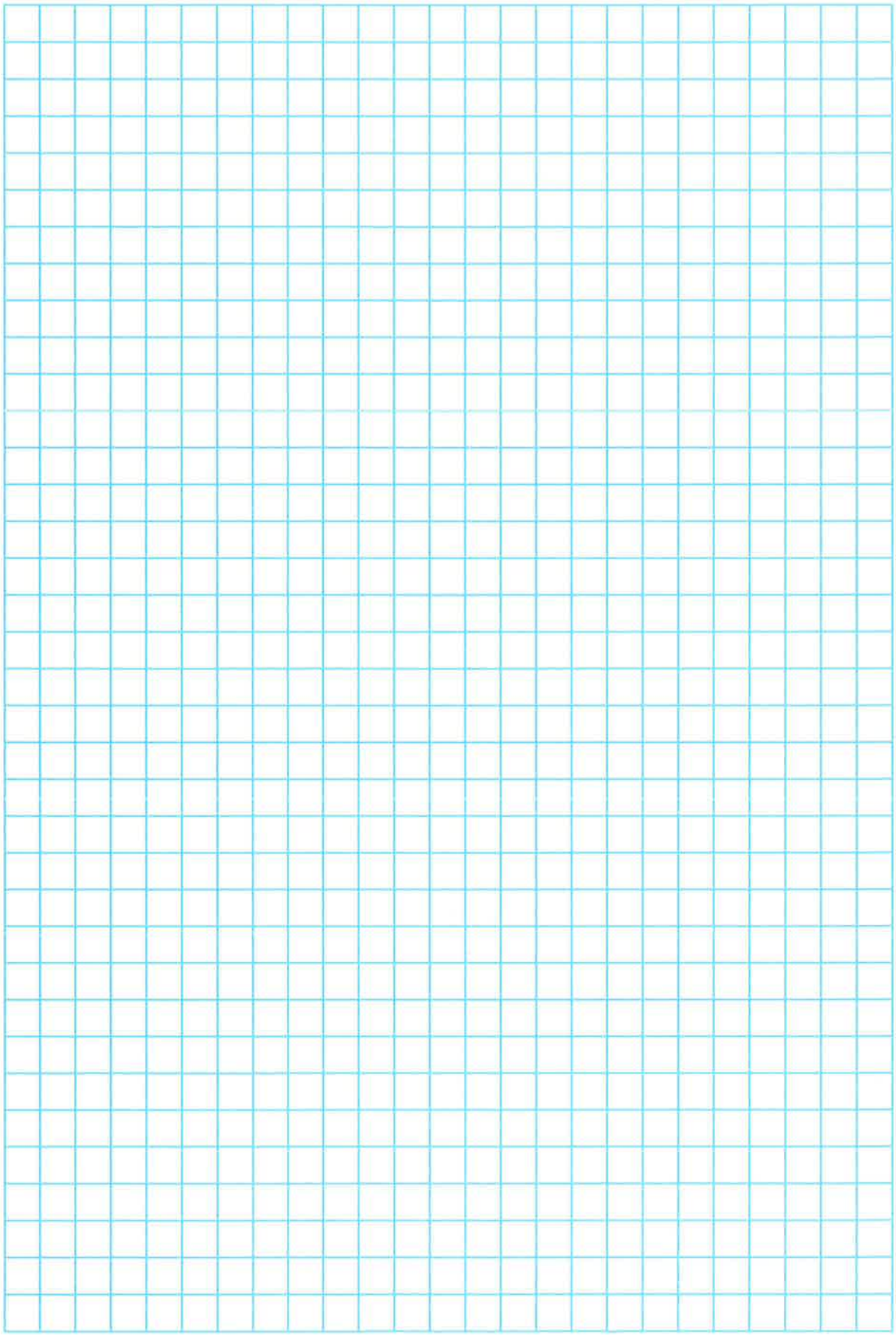
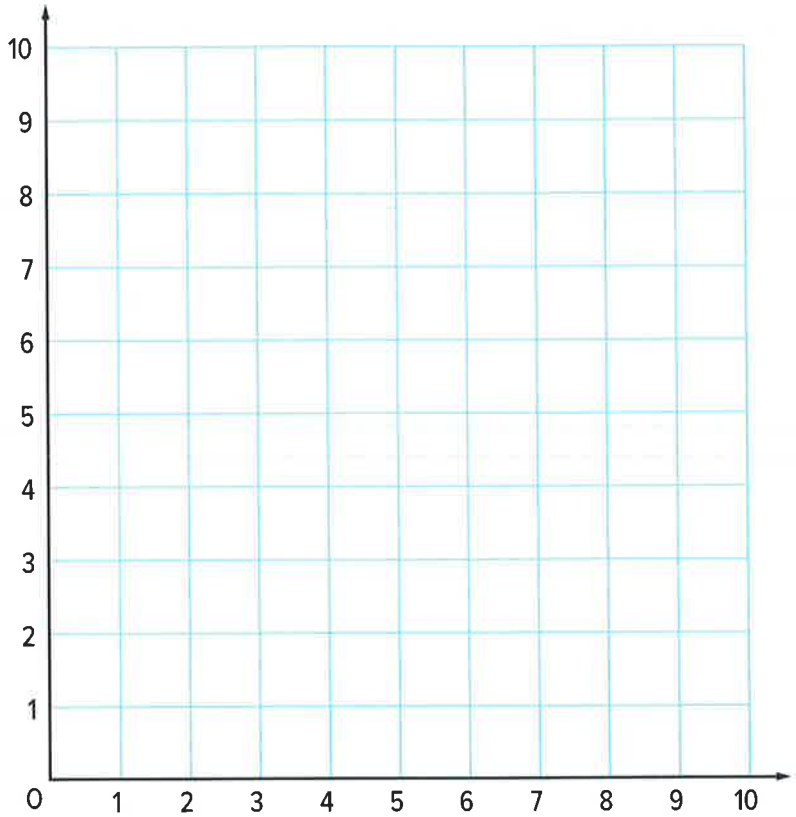
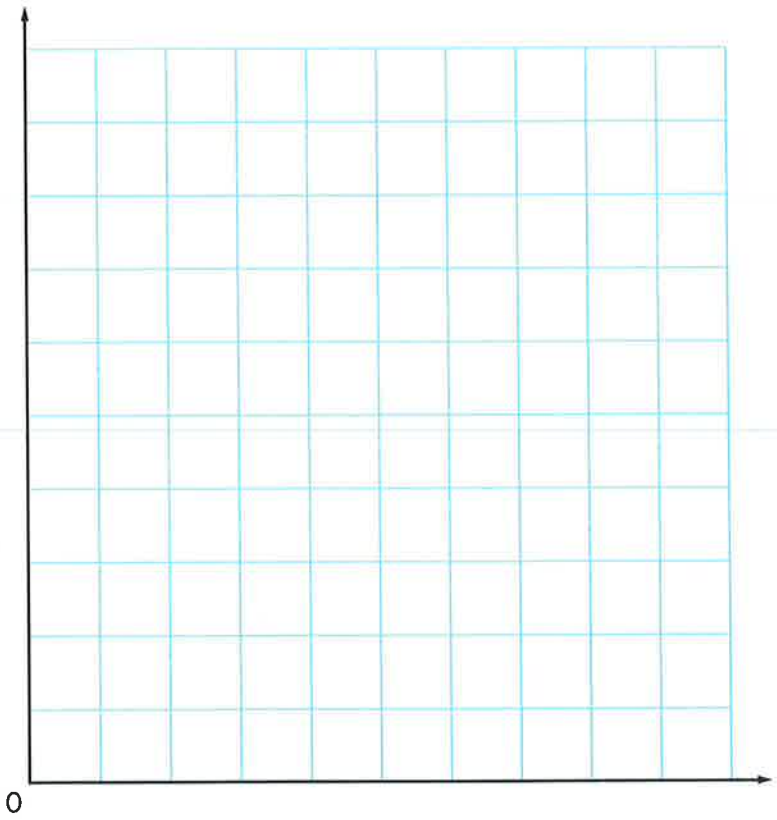
Name _____



WM2 **Quarter-Inch Grid**

Name _____





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