14

TURBULENT FLOW IN PIPES

14.1 INTRODUCTION

The development now proceeds to turbulent flow. In the previous chapter, the Fanning friction factor was defined and presented in Equation (13.6)

$$f = \frac{16}{\text{Re}} \tag{14.1}$$

However, this only applies to laminar flow. Unlike laminar flow, the friction factor for turbulent flow cannot be derived from basic principles. Fortunately, extensive experimental data is available and this permits numerical evaluation of the friction factor for turbulent flow. Comments on turbulent flow now follow.

As described earlier, as the Reynolds number is increased above 2100 for flow in pipes, eddies and turbulence start to develop in the flowing fluid. From Re equal to 2100 to about 4000, the flow becomes more unstable. As the Reynolds number is increased to values above 4000, the turbulent state of the fluid core becomes well developed and the velocity distribution across a diameter of the pipe becomes similar to that of a flattened parabola (see profile D in Fig. 14.1 as well as Fig. 12.1). The equation of this flattened parabola can be approximately described by the following equation

$$v = v_{\max}\left(\frac{2n^2}{(n+1)(2n+1)}\right) \tag{14.2}$$

Fluid Flow for the Practicing Chemical Engineer. By J. Patrick Abulencia and Louis Theodore

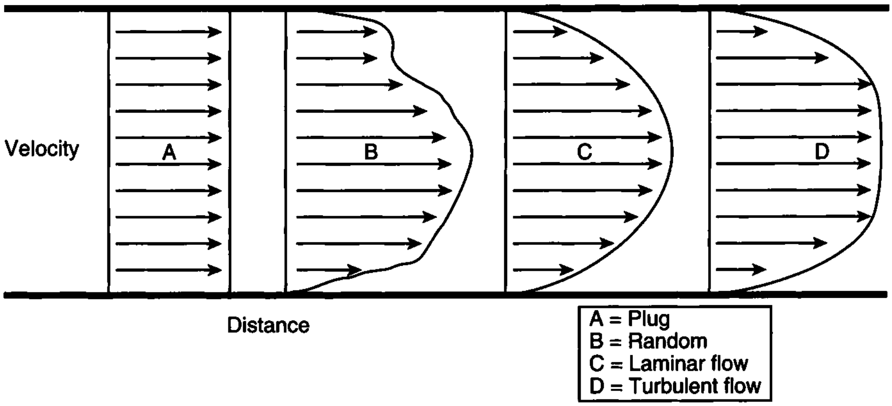

Figure 14.1 Velocity profiles.

where v_{max} is the centerline (maximum) velocity and n is 7 (the one-seventh power law applies).

Illustrative Example 14.1 A liquid with a viscosity of 0.78 cP and a density of $1.50\ g/cm^3$ flows through a 1-in. diameter tube at 20 cm/s. Calculate the Reynolds number. Is the flow laminar or turbulent?

Solution By definition, the Reynolds number (Re) is equal to:

$$\mathrm{Re} = Dv\rho/\mu$$

Since

$$1\ \mathrm{cP} = 10^{-2}\ \mathrm{g/(cm \cdot s)}$$

$$\mu = 0.78 \times 10^{-2}\ \mathrm{g/(cm \cdot s)}$$

$$1\ \mathrm{in} = 2.54\ \mathrm{cm}$$

$$\begin{aligned}\mathrm{Re} &= (2.54)(20)(1.50)/(0.78 \times 10^{-2}) \\ &= 9769.23 \approx 9800\end{aligned}$$

The flow is turbulent since Re > 2100.

Illustrative Example 14.2 A fluid is moving in laminar flow through a cylinder whose inside radius is 0.5 in. The viscosity and density of the fluid are 1.03 cP and $62.4\ lb/ft^3$, respectively. The velocity is then increased to higher values until turbulence appears. Determine the minimum velocity at which turbulence will appear (i.e., Re = 2100).

Solution Since

$$\mathrm{Re} = \frac{Dv\rho}{\mu}$$

$$2100 = \frac{(1.0\,\mathrm{in})(62.4\,\mathrm{lb/ft^3})(V)}{1.03\,\mathrm{cP}} \times \frac{1\,\mathrm{ft}}{12\,\mathrm{in}} \times \frac{1\,\mathrm{cP}}{6.72\times10^{-4}\,\mathrm{lb/ft\cdot s}}$$

$$v = \frac{(2100)(1.03)(6.72\times10^{-4})(12)}{62.4}$$

$$= 0.280\,\mathrm{ft/s}$$

14.2 DESCRIBING EQUATIONS

It is important to note that almost all the key fluid flow equations presented in Chapter 13 for laminar flow apply as well to turbulent flow, provided the appropriate friction factor is employed. These key equations (Eqs. (13.7), (13.10) and (13.11)) are again provided below. Note once again that v (the average velocity) is given by $q/(\pi d^2/4)$.

$$h_f = \frac{4fLv^2}{2g_cD}$$

This equation may also be written as

$$h_f = \frac{32fLq^2}{\pi^2 g_c D^5} \tag{14.3}$$

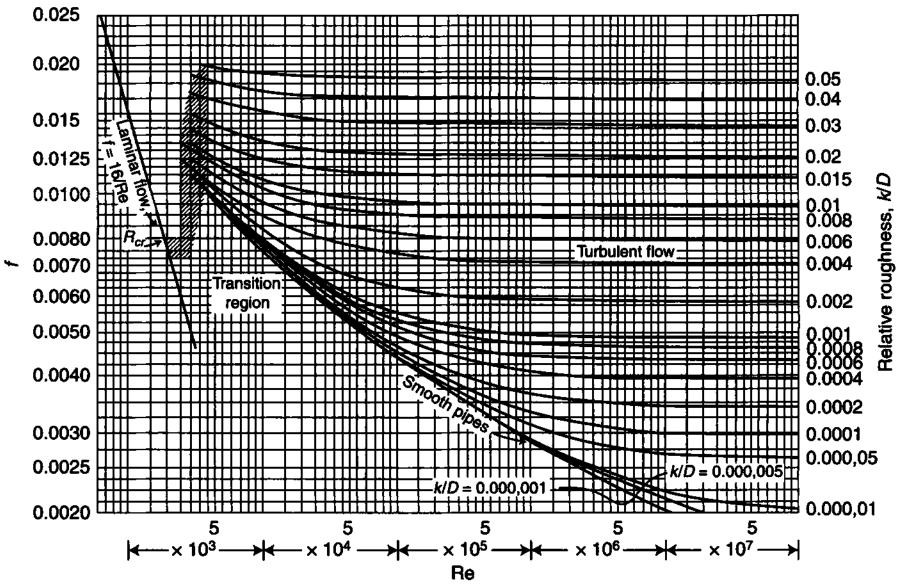

Figure 14.2 Fanning friction factor; pipe flow.

$$\frac{\Delta P}{\rho}+\frac{\Delta v^2}{2g_c}+\Delta z\frac{g}{g_c}-h_s+\sum\frac{4fLv^2}{2g_cD}=0 \tag{14.4}$$

$$\frac{\Delta P}{\rho}+\frac{\Delta v^2}{2g_c}+\Delta z\frac{g}{g_c}-h_s+\sum\frac{4fLv^2}{2g_cD}+\sum h_c+\sum h_e=0 \tag{14.5}$$

The effect of the Reynolds number on the Fanning friction factor is provided in Fig. 14.2. Note that Equation (14.1) appears on the far left-hand side of Fig. 14.2.

14.3 RELATIVE ROUGHNESS IN PIPES

In the turbulent regime, the "roughness" of the pipe becomes a consideration. In his original work on the friction factor, Moody[1] defined the term k as the roughness and the ratio, k/D, as the relative roughness. Thus, for rough pipes/tubes in turbulent flow

$$f = f(\mathrm{Re},\ k/D) \tag{14.6}$$

This equation reads that the friction factor is a function of *both* the Re and k/D. However, as noted above, the dependency on the Reynolds number is a weak one. Moody[1] provided one of the original friction factor charts. His data and results, as applied to the Fanning friction factor, are presented in Fig. 14.2 and covers the laminar, transition, and turbulent flow regimes. It should be noted that the laminar flow friction factor is independent of the relative roughness. Figure 14.2 also contains friction factor data for various relative roughness values.

The reader should note the following:

1. Moody's original work included a plot of the Darcy (or Moody) friction factor, not the Fanning friction factor. His chart has been adjusted to provide the Fanning friction factor, i.e., the plot in Fig. 14.2 is for the Fanning friction factor. Those choosing to work with the Darcy friction factor need only multiply the Fanning friction factor by 4, since

$$f_D = 4f \tag{14.7}$$

2. The intermediate regime of Re between 2100 and 4000 is indicated by the shaded area in Fig. 14.2.
3. The average "roughness" of commercial pipes is given in Table 14.1.[2]
4. Notice in Fig. 14.2 the relative roughness lines are nearly horizontal in the fully turbulent regime to the right of the dashed lines.
5. Roughness is a function of a variety of effects—some of which are difficult, if not impossible, to quantify. In effect, the roughness of a pipe resembling a smooth sine wave exhibits different frictional effects than a sharp sawtooth or step function.

Table 14.1 Average roughness of commerical pipes

Material (new)	Roughness, k	
	ft	mm
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stove	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Asphalted cast iron	0.0004	0.12
Commercial steel (Wrought iron)	0.00015	0.046
Drawn tubing	0.000005	0.0015
Glass	"smooth"	"smooth"

In summary, for Reynolds numbers below 2100, the flow will always be laminar and the value of f should be taken from the line at the left in Fig. 14.2. For Reynolds numbers above 4000, the flow will practically always be turbulent and the values of f should be read from the lines at the right. Between Re = 2100 and Re = 4000, no accurate calculations can be made because it is generally impossible to predict flow type in this range. If an estimate of friction loss must be made in this range, it is recommended that the figures for turbulent flow should be used, as that provides an estimate on the high side.[3]

14.4 FRICTION FACTOR EQUATIONS

Approximate equations for smooth pipe, turbulent flow, Fanning friction factors are:[2]

$$f = \frac{0.0786}{\text{Re}^{0.25}} \quad \text{for } 5000 < \text{Re} < 50{,}000, \text{ and} \tag{14.8}$$

$$f = \frac{0.046}{\text{Re}^{0.20}} \quad \text{for } 30{,}000 < \text{Re} < 1{,}000{,}000 \tag{14.9}$$

Farag[2] has also provided two formulas for the Fanning friction factor, valid for $10^{-5} < k/D < 0.02$ and $4000 < \text{Re} < 10^8$.

$$f = \frac{1}{4\left(1.8\log_{10}\left[0.135\left(\frac{k}{D}\right) + \frac{0.8775}{\text{Re}}\right]\right)^2} \tag{14.10}$$

The other approximate equation of the Fanning friction factor in the completely turbulent region is:

$$f = \frac{1}{4\left[1.14 - 2.0\log_{10}\left(\frac{k}{D}\right)\right]^2} \tag{14.11}$$

In a classic review of these equations, Churchill[4] provided a host of models for estimating the "Churchill" friction factor, f_c. The key equation is given below:

$$f_c = \left[\left(\frac{8}{\text{Re}}\right)^{12} + \frac{1}{(A+B)^{3/2}}\right]^{1/12} \qquad (14.12)$$

where

$$A = \left[2.457 \ln\left(\frac{1}{\left(\frac{7}{\text{Re}}\right)^{0.9} + \frac{0.27k}{D}}\right)\right]^{16}$$

$$B = \left(\frac{37{,}530}{\text{Re}}\right)^{16}$$

Equation (14.12) is valid for all Re and k/D. A trial-and-error solution is necessary if the pressure drop rather than the flow rate is specified. Churchill also notes that the equation is a convenient and accurate replacement for all of the friction-factor plots in the literature. The equation not only reproduces the friction factor plot but also avoids interpolation and provides unique values in the transition region. These values are, of course, subject to some uncertainty because of the physical instability inherent in this region. One of the drawbacks to Churchill's work is that all the presented equations for f_c need to be converted to the Fanning or Darcy friction factors by multiplying f by 2 or 8, respectively.

Perhaps the most accurate of all the equations appearing in the literature is that attributed to Jain.[5] This equation is as follows:

$$\frac{1}{f^{0.5}} = 2.28 - 4\log_{10}\left[\frac{k}{D} + \frac{21.25}{(\text{Re})^{0.9}}\right] \qquad (14.13)$$

Illustrative Example 14.3 PAT (Patrick, Abulencia, and Theodore) Consultants have proposed the following equation to predict the Fanning friction factor:

$$f = 0.0015 + [(8)(\text{Re})^{0.30}]^{-1}$$

For a Re of 14,080 and k/D of 0.004, calculate/obtain the friction factor using

1. The above equation.
2. Equations (14.8) and (14.9).
3. Equation (14.11).
4. Equation (14.13).
5. Figure 14.2.

Comment on the results.

Solution

1. $f = 0.0015 + [(8)(14{,}080)^{0.30}]^{-1}$
   $= 0.0015 + 0.0071$
   $= 0.0086$
2. $f = 0.0786/(\text{Re})^{0.25}$
   $= 0.0786/(14{,}080)^{0.25}$
   $= 0.0072$

   and

   $f = 0.046/(\text{Re})^{0.2}$
   $= 0.046/(14{,}080)^{0.2}$
   $= 0.0068$
3. $f = 1/4[1.14 - 2.0\log(k/D)]^2$
   $= (1)/4[1.14 - 2.0\log(0.004)]^2$
   $= 0.0071$
4. $\sqrt{f} = 1\bigg/\left[2.28 - 4\log_{10}\left(\frac{k}{D} + \frac{21.25}{(\text{Re})^{0.9}}\right)\right]$

   $\sqrt{f} = 1\bigg/\left[2.28 - 4\log_{10}\left(0.004 + \frac{21.25}{4.28}\right)\right]$

   $= 0.00875$
5. $f = 0.0085$

As can be seen from the above five results, there is some modest but acceptable scatter; the average value is 0.00782.

14.5 OTHER CONSIDERATIONS

Two "other" considerations are discussed in this subsection: flow in non-circular conduits and flow in parallel pipe/conduit systems.

Flow in non-circular conduits was discussed earlier but is reviewed again because of its importance in fluid flow studies. As noted in Chapter 13, the approach employed is to represent any conduit by a pipe or cylinder with an "equivalent" diameter, D_{eq}. Key equations include:

$$D_{eq} = \frac{4S}{P_p} = 4r_h \tag{14.14}$$

where S is the cross-sectional area of the conduit, P_p is the wetted perimeter and r_h is the hydraulic radius. For flow in the annular space between two concentric pipes of diameter D_1 and D_2,

$$D_{eq} = \frac{4\pi(D_1{}^2 - D_2{}^2)}{4\pi(D_1 + D_2)} = D_1 - D_2 \tag{14.15}$$

Although this approach strictly applies to turbulent flow, it may be employed for laminar flow situations if no other approaches are available.

Illustrative Example 14.4 Calculate the equivalent (or hydraulic) diameter for turbulent fluid flow in a cross-section which has:

1. a 2 in × 10 in rectangle flowing full;
2. an annulus with outer diameter $D_o = 10$ cm and inner diameter $D_i = 8$ cm; and,
3. a 10 cm diameter circle (tube) flowing half full.

Solution First, write Equation (14.14) describing the calculation of the equivalent diameter:

$$D_{eq} = \frac{4S}{P_p} = 4r_h$$

1. Consider the 2 in × 10 in rectangle flowing full:

$$D_{eq} = \frac{4S}{P_p} = \frac{4(2)(10)}{(2 + 10 + 1 + 10)} = 3.33\text{ in}$$

2. Consider the annulus:

$$D_{eq} = \frac{4S}{P_p} = \frac{4(\pi/4)(D_o{}^2 - D_i{}^2)}{\pi(D_o - D_i)} = D_o - D_i = 2\text{ cm}$$

3. Consider the half-full circle:

$$D_{eq} = \frac{4S}{P_p} = 4\frac{\pi D^2/8}{\pi D/2} = D = 10\text{ cm}$$

Note the importance of using the wetted perimeter and the cross-sectional area of the fluid flow for situations where the conduit is not full of fluid.

14.6 FLOW THROUGH SEVERAL PIPES

Flow through a number of pipes or conduits often arise in engineering practice. If flow originates from the same source and exits at the same location, the pressure drop cross each conduit must be the same. Thus, for flow through conduits 1, 2, and 3, one may write:

$$\Delta P_1 = \Delta P_2 = \Delta P_3 \tag{14.16}$$

Solutions to this type of problem usually require a trial-and-error solution since several (in this case three) simultaneous, nonlinear equations may be involved.

14.7 GENERAL PREDICTIVE AND DESIGN APPROACHES

Almost any problem involving friction losses in long pipe flows can be solved using the Fanning friction factor charts or an equivalent equation. Unless otherwise indicated, the charts are employed in the solution to illustrative examples, applications and problems to follow. Some problems can be solved directly; however, others are trial-and-error, since a knowledge of the Reynolds number is required.

There are three important fundamental pipe flow problems. These are detailed below (see Fig. 14.3).

1. Head loss problem: Given D, L, and v or q, ρ, μ, and g. Compute the head loss (h_f) or pressure drop (ΔP).
2. Flow rate problem: Given D, L, h_f (or ΔP), ρ, μ, and g. Compute the velocity, v, or flow rate q.
3. Sizing problem: Given q, L, h_f (or ΔP), ρ, μ, and g. Compute the diameter of the pipe, D.

Only a Type 1 problem involves a direct application of the chart and does not require a trial-and-error calculation. The engineer has to iterate to compute the velocity (Type 2) or diameter (Type 3) because both D and v are contained in the ordinate and abscissa of the charts or equations. The iteration proceeds as follows:

1. Make an initial guess for v or D.
2. Calculate the corresponding Reynolds number.
3. If necessary, calculate the relative roughness.
4. Use the Fanning chart to find the corresponding friction factor.
5. From the data given, generate an improved v or D.
6. Use the improved v or D from step (5) and repeat steps (2–5).

The iteration converges when v or D stops changing significantly.

An approximate explicit formula to obtain the unknown volumetric flow rate is:[2]

$$q = -2.22D^{2.5}\sqrt{\frac{gh'_f}{L}}\log_{10}\left(\frac{k/D}{3.7} + \frac{1.78\mu}{D^{1.5}\rho\sqrt{gh'_f/L}}\right) \tag{14.17}$$

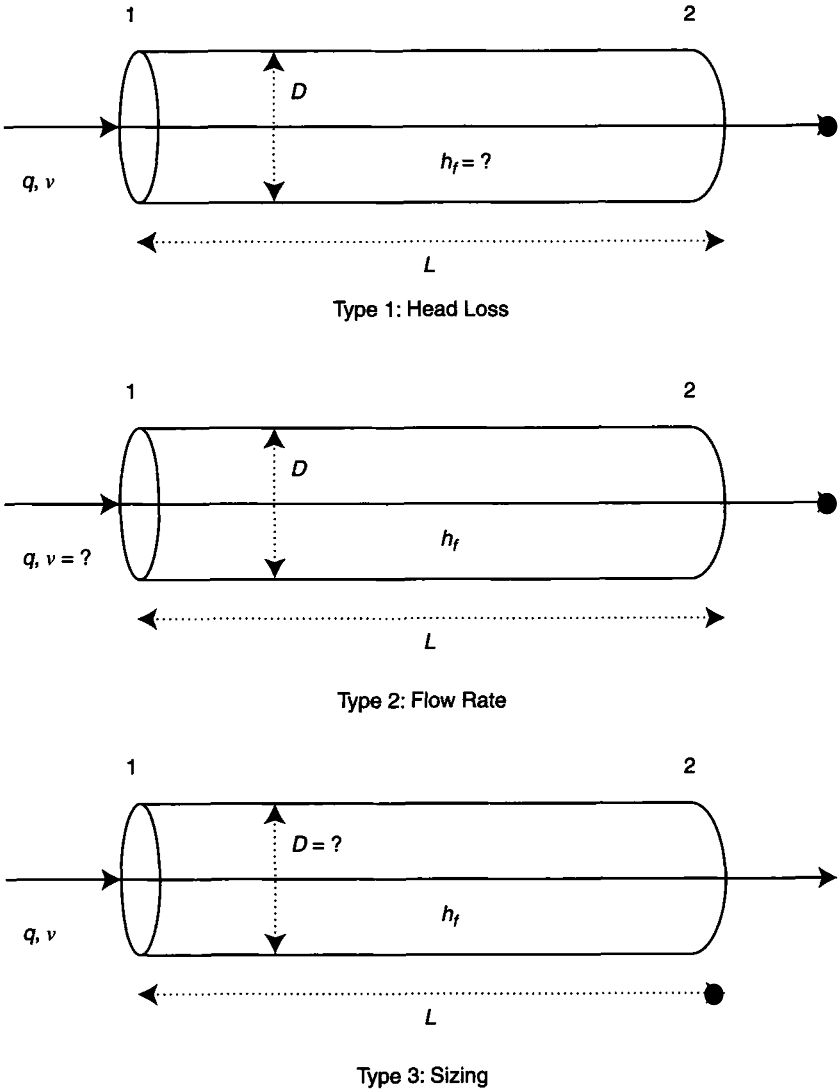

Figure 14.3 Three flow problems.

For a horizontal pipe, the flow rate equation simplifies to:[(2)]

$$q = -2.22D\sqrt{\frac{\Delta PD^3}{\rho L}}\log_{10}\left(\frac{k/D}{3.7}+\frac{1.78\mu}{D^{1.5}\rho\sqrt{\Delta PD^3/\rho L}}\right) \tag{14.18}$$

The units of h'_f are height of flowing fluid (e.g., ft H_2O), while ΔP is in force per unit area (e.g., lb_f/ft^2). The relationship (as noted earlier) is given by $gh'_f/g_c = \Delta P/\rho$.

Consistent units must be used in the above two equations. If SI units are used, then the volumetric flow rate, q, is in m^3/s. If engineering units are used, the pressure drop, ΔP, must be in psf, the density, ρ, must be in slug/ft^3, the kinematic viscosity in ft^2/s, and the volumetric flow rate in ft^3/s. Note 1.0 slug = 32.174 lb.

An approximate explicit correlation has been developed to calculate the pipe diameter, D:[(2)]

$$D = 0.66\left[k^{1.25}\left(\frac{Lq^2}{gh'_f}\right)^{4.75} + \frac{\mu}{q\rho}\left(\frac{Lq^2}{gh'_f}\right)^{5.2}\right]^{0.4} \tag{14.19}$$

For a horizontal pipe, $gh_f = \Delta P/\rho$, and the equation simplifies to

$$D = 0.66\left[k^{1.25}\left(\frac{\rho Lq^2}{\Delta P}\right)^{4.75} + \frac{v}{q}\left(\frac{\rho Lq^2}{\Delta P}\right)^{5.2}\right]^{0.4} \tag{14.20}$$

Consistent units with those provided above need to be used. If engineering units are employed, the pressure drop must be expressed in psf.

Illustrative Example 14.5 Air at a temperature of 40°F and pressure of $P_1 = 0.1$ psig is to be transported horizontally through a circular conduit of length $L = 800$ ft. At the delivery point, the air pressure, P_2, is 0.01 psig, and the air rate is 500 cfm. The circular duct is made of sheet metal and has a roughness of 0.00006 in. Find the pipe diameter, D, and the average air velocity, v.

Solution Calculate the air density using the ideal gas law and employing an average pressure of 14.75 psia.

$$\rho = \frac{P(\mathrm{MW})}{RT} = \frac{(14.75)(28.9)}{(10.73)(500)}$$
$$= 0.08\ \mathrm{lb/ft^3}$$

Obtain the air viscosity from Fig. B.2 in the Appendix (estimated at 40°F).

$$\mu = 0.0173\ \mathrm{cP} = 1.14 \times 10^{-5}\ \mathrm{lb/ft \cdot s}$$

Assume first the flow is laminar and calculate D using Equation (14.3) with $f = 16/\mathrm{Re}$ and $v = q/(\pi D^2/4)$.

$$q = 500\ \mathrm{ft^3/min} = 8.33\ \mathrm{ft^3/s}$$

$$D = \sqrt[4]{\frac{128\mu Lq}{\pi(P_1 - P_2)}} = \sqrt[4]{\frac{128(3.54 \times 10^{-7})(800)(8.33)}{\pi(0.09)(144)}} = 0.293\ \mathrm{ft}$$

Check the flow type.

$$\mathrm{Re} = \frac{Dv\rho}{\mu} = \frac{4q\rho}{\pi D\mu} = \frac{4(8.33)(0.08)}{\pi(0.293)(1.14 \times 10^{-5})} = 3.17 \times 10^6$$

Therefore, the assumption of laminar flow is not valid.

Assume a pipe diameter of 1 ft (0.3048 m). The relative roughness is then

$$k/D = (0.00006/12)/1 = 0.000005$$

Calculate the Reynolds number.

$$\mathrm{Re} = \frac{Dv\rho}{\mu} = \frac{4q\rho}{\pi D\mu} = \frac{4(8.33)(0.08)}{\pi(1)(1.14 \times 10^{-5})} = 74{,}168$$

Obtain an estimate of the Fanning friction factor from Fig. 14.2.

$$f = 0.005$$

Write the pressure drop equation; see Equation (14.3).

$$\Delta P = P_1 - P_2 = \frac{32\rho f L q^2}{g_c \pi^2 D^5}$$

Solving for the diameter:

$$D = \left(\frac{32\rho f L q^2}{g_c \pi^2 \Delta P}\right)^{0.2} = \left(\frac{32(0.08)(0.005)(800)(8.33)^2}{(32.174)\pi^2(12.96)}\right)^{0.2} = 0.70\,\mathrm{ft}$$

Start the second iteration with the newly calculated D.

$$k/D = (0.00006/12)/0.70 = 0.0000071$$

Calculate the new Reynolds number.

$$\mathrm{Re} = \frac{Dv\rho}{\mu} = \frac{4q\rho}{\pi D\mu} = \frac{4(8.33)(0.08)}{\pi(0.7)(1.14 \times 10^{-5})}$$

$$= 106{,}000$$

Obtain the new Fanning friction factor.

$$f = 0.0045$$

Solving for the diameter:

$$D = \left(\frac{8\rho f L q^2}{g_c \pi^2 \Delta P}\right)^{0.2} = \left(\frac{8(0.08)(0.0045)(800)(8.33)^2}{g_c \pi^2(12.96)}\right)^{0.2} = 0.69\,\mathrm{ft}$$

The iteration may now be terminated.

Calculate the flow velocity using the last calculated diameter.

$$v = \frac{q}{S} = \frac{8.33}{\pi(0.69)^2/4} = 6.8\,\text{m/s}$$

Illustrative Example 14.6 If ethyl alcohol at 20°C is to be pumped through 60 m of horizontal drawn tubing at 10 m^3/h with a friction loss of 30 m, what tube diameter in cm must be employed? What is the alcohol velocity? Is the flow turbulent?

Solution Obtain the properties of ethyl alcohol at 20°C from Table A.2 in the Appendix.

$$\rho = 789\,\text{kg/m}^3$$

$$\mu = 1.1 \times 10^{-3}\,\text{kg/m-s}$$

Obtain the roughness of drawn tubing using Table 14.1.

$$k = 0.0015\,\text{mm}$$

Use the approximate explicit Equation (14.20) to calculate the diameter.

$$D = 0.66\left[k^{1.25}A^{4.75} + \frac{\mu}{q\rho}A^{5.2}\right]^{0.04}$$

where

$$A = \frac{Lq^2}{gh'_f} = \frac{60(2.778 \times 10^{-3})^2}{(9.807)(30)}$$

$$= 1.574 \times 10^{-6}\,\text{m}$$

Substituting gives

$$D = 0.66\left[(1.5 \times 10^{-6})^{1.25}(1.57 \times 10^{-6})^{4.75} + \frac{1.1 \times 10^{-3}}{2.778 \times 10^{-3}(789)}(1.57 \times 10^{-6})^{5.2}\right]^{0.04}$$

$$= 0.0303\,\text{m}$$

Take $D = 3$ cm. Next, calculate the velocity of alcohol in the tube.

$$v = \frac{q}{S} = \frac{4(2.778 \times 10^{-3})}{\pi(0.03)^2} = 3.93\,\text{m/s}$$

Characterize the flow.

$$\text{Re} = \frac{Dv}{\nu} = \frac{(3.93)(0.0377)}{1.395 \times 10^{-6}} = 106405 > 4000$$

The flow is turbulent since Re is greater than 4000.

Illustrative Example 14.7 Kerosene at 20°C (SG = 0.82, $\mu = 0.0016\,\text{kg/m}\cdot\text{s}$) flows in a 9 meter long, smooth, horizontal, 2-inch schedule 80 pipe. The flow Reynolds number is 60,000. Using SI units, calculate the kerosene density, the pipe inside diameter, the average velocity of kerosene, the volumetric and mass flow rate and the maximum kerosene velocity in the pipe assuming the one-seventh power-law applies. Where will the maximum velocity occur? How good is the assumption of fully developed flow? Assume

$$L_C/D = 4.4\,\text{Re}^{1/6}$$

Solution Calculate the kerosene density.

$$\rho = \text{SG}(1000) = 820\,\text{kg/m}^3$$

Obtain the pipe inside diameter from Table A.5 in the Appendix.

$$D = 1.939\,\text{in} = 0.0493\,\text{m}$$

Calculate the average velocity from the Reynolds number equation.

$$\text{Re} = \frac{Dv\rho}{\mu}$$

Solving for the average velocity, v,

$$v = \frac{(\text{Re})\mu}{D\rho} = \frac{(60{,}000)(0.0016)}{(0.0493)(820)} = 2.38\,\text{m/s}$$

Calculate the volumetric and mass flow rates.

$$q = \frac{v}{S} = \frac{v}{\pi D^2/4} = \frac{2.38}{\pi(0.0493)^2/4} = 0.00454\,\text{m}^3/\text{s}$$

$$\dot{m} = \rho q = (820)(0.00454) = 3.72\,\text{kg/s}$$

Calculate the maximum velocity assuming one-seventh power law applies (see Eq. 14.2).

$$\frac{v}{v_{\text{max}}} = \frac{2n^2}{(n+1)(2n+1)}$$

For $n = 7$

$$\frac{v}{v_{\text{max}}} = 0.817$$

$$v_{\text{max}} = 2.92\,\text{m/s}$$

As noted earlier, the maximum velocity will occur at the pipe center line.

Check on the assumption of fully developed flow.

$$\frac{L_C}{D} = 4.4\,\mathrm{Re}^{1/6} = 4.4(60{,}000^{1/6}) = 27.5$$

$$L_C = 27.5(0.0493) = 1.36\,\mathrm{m}$$

Since L_C is less than L (= 9 m), the assumption is valid.

Illustrative Example 14.8 Refer to Illustrative Example 14.7. Determine the Fanning friction factor, the friction loss, and the pressure drop (in Pa and atm) due to friction.

Solution Calculate the Fanning friction factor using Equation (14.9) since Re = 60,000.

$$f = \frac{0.046}{\mathrm{Re}^{0.2}} = \frac{0.046}{(60{,}000)^{0.2}}$$
$$= 0.0051$$

Calculate the friction loss due to friction.

$$h'_f = 4f\frac{L}{D}\frac{v^2}{2g} = 4(0.0051)\frac{9}{0.0493}\frac{(2.38)^2}{2(9.807)}$$
$$= 1.08\,\mathrm{m\ of\ kerosene}$$

Calculate the pressure drop using Bernoulli's equation or the hydrostatic equation, noting that $v_1 = v_2 = v$, $z_1 = z_2$, and $h_s = 0$.

$$\Delta P = \rho\frac{g}{g_c}h'_f = 820(9.807)(1.08)$$
$$= 8685\,\mathrm{Pa} = 0.086\,\mathrm{atm}$$

Illustrative Example 14.9 Refer to Illustrative Example 14.7. Calculate the force required to hold the pipe in place.

Solution The force required to hold the pipe is:

$$F = PS = \pi\frac{(0.0493)^2}{4}(8685) = 16.6\,\mathrm{N}$$

The force direction is opposite the flow.

14.8 MICROSCOPIC APPROACH

As noted in this and the previous two chapters, fluid-flow systems are classified as either laminar or turbulent flow. This last section is concerned with a microscopic treatment of turbulent flow. However, the reader should note that a complete and fundamental understanding of turbulent flow has yet to be developed.

A fixed point in space during a given finite time interval θ can be resolved into the average velocity over the same time interval and a fluctuation or disturbance velocity term that accounts for the turbulent motion

$$v = \bar{v} + v' \tag{14.21}$$

where v = instantaneous velocity,
$\bar{v}$ = average velocity over the time interval θ,
v' = instantaneous fluctuation velocity.

This system is represented pictorially in Fig. 14.4. One can conclude from the discussion above and the definition of average values that

$$\bar{v}' = \frac{\int_0^\theta v' \, dt}{\int_0^\theta dt} = 0 \tag{14.22}$$

where $\bar{v}'$ = time average value of the fluctuation velocity during θ.

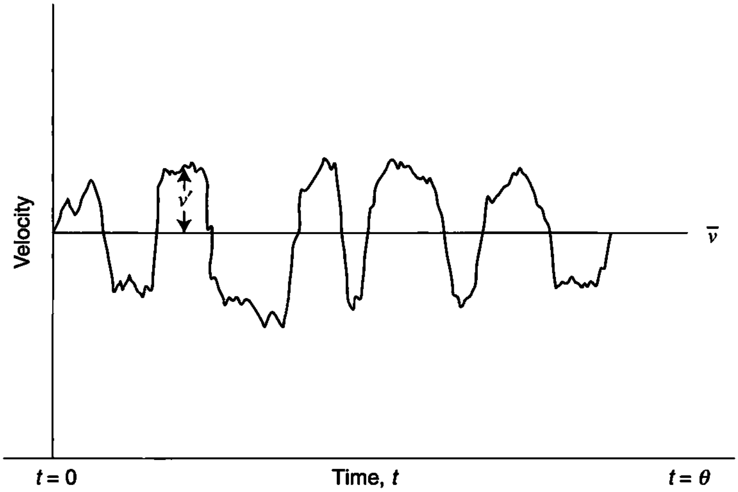

Figure 14.4 Velocity variation with time.

The intensity of turbulence I for the velocity component v_y at a given point in space is defined by

$$I = \frac{(\overline{v}'_y)^2}{\overline{v}_y} \tag{14.23}$$

As its name implies, I is a measure of the intensity of turbulence at a point and is given by the ratio of the magnitude of the fluctuation and average velocities.

Illustrative Example 14.10 A fluid is moving in turbulent flow through a pipe. A hot-wire anemometer is inserted to measure the local velocity at a given point P in the system. The following readings were recorded at equal time intervals during a very short period of time:

Time Increment	1	2	3	4	5	6	7	8	9	10
Velocity v_z at P	43.4	42.1	42.0	40.8	38.5	37.0	37.5	38.0	39.0	41.7

Determine the intensity of turbulence at point P.

Solution The terms $\overline{v}'_z$ and $\left(\overline{v}'_z\right)^2$ are first calculated

$$\overline{v}_z = \frac{\sum_{i=1}^{i=n} v_z}{n} = \frac{400.0}{10} = 40$$

where n = number of time increments.

$$\left(\overline{v}'_z\right)^2 = \frac{\sum_{i=1}^{10} (v_z - \overline{v}_z)^2}{10}$$

$$= \frac{45.9}{10}$$

$$= 4.59$$

Substituting into Equation (14.23) gives

$$I = \frac{\sqrt{4.59}}{40}$$

$$= \frac{2.15}{40}$$

$$= 0.0538$$

The disturbance or fluctuation velocity components of a fluid in turbulent flow not only alters the transport equations for momentum, energy, and mass, but also gives rise to the aforementioned velocity profiles that are quite different from the corresponding profiles for laminar flow. A semiempirical equation describing the velocity profile in pipes for turbulent flow is:

$$v_z = v_{z_{max}} \left[1 - \left(\frac{r}{R}\right)\right]^{1/7} \tag{14.24}$$

where v_z = local velocity,
$v_{z_{max}}$ = maximum velocity,
R = radius of the pipe,
r = radial cylindrical coordinate.

Illustrative Example 14.11 A fluid is flowing through a pipe whose inside diameter is 2.0 in. The maximum velocity measured is 30 ft/min. Calculate the volumetric flow rate Q if the flow characteristics are (a) laminar, (b) plug, (c) turbulent.

Solution

a. Laminar flow:

$$\bar{v}_z = \frac{1}{2} v_{z_{max}}$$
$$= \frac{1}{2}(30\,\text{ft/min})$$
$$= 15\,\text{ft/min}$$

By definition,

$$q = \bar{v}A$$

where

$$A = \pi D^2/4$$
$$= \frac{\pi(2/12)^2}{4} = 0.0218\,\text{ft}^2$$

Therefore

$$q = 15\frac{\text{ft}}{\text{min}} \times .0218\,\text{ft}^2$$
$$= 0.327\,\text{ft}^3/\text{min}$$

b. Plug flow:

$$\bar{v}_z = v_{z_{max}}$$
$$q = (30\,\text{ft/min})(.0218\,\text{ft}^2)$$
$$= 0.654\,\text{ft}^3/\text{min}$$

c. Turbulent flow: For turbulent flow,

$$v_z = v_{z_{\max}}\left[1 - \left(\frac{r}{R}\right)\right]^{1/7} \tag{14.24}$$

By definition

$$\bar{v}_z = \frac{\int_0^{2\pi}\int_0^{R} v_z r\,\mathrm{d}r\,\mathrm{d}\phi}{\int_0^{2\pi}\int_0^{R} r\,\mathrm{d}r\,\mathrm{d}\phi}$$

$$= \frac{2\pi v_{z_{\max}}\int_0^{R}\left(1 - \frac{r}{R}\right)^{1/7} r\,\mathrm{d}r}{\pi R^2}$$

$$= \frac{2v_{z_{\max}}}{R^2}\int_0^{R}\left(1 - \frac{r}{R}\right)^{1/7} r\,\mathrm{d}r$$

(The above integral can be evaluated from a standard table of integrals.)

$$\int_0^{R}\left(1 - \frac{r}{R}\right)^{1/7} r\,\mathrm{d}r = +\frac{R^2}{\left[\frac{1}{7}+2\right]}\left(1 - \frac{r}{R}\right)^{(1/7)+2}\Bigg|_0^R$$

$$-\frac{R^2}{\left[\frac{1}{7}+1\right]}\left(1 - \frac{r}{R}\right)^{(1/7)+1}\Bigg|_0^R$$

$$= -\frac{7}{15}R^2 + \frac{7}{8}R^2$$

$$= \frac{49}{120}R^2$$

Therefore,

$$\bar{v}_z = \frac{2v_{z_{\max}}}{R^2}\left(\frac{49}{120}R^2\right)$$

$$= \frac{49}{60}v_{z_{\max}}$$

$$= (0.816)v_{z_{\max}}$$

(The 0.816 term is in agreement with the value presented earlier.)

Substitution

$$\bar{v}_z = 0(.816)(30)$$
$$= 24.5\,\text{ft/min}$$

and

$$q = (24.5)(0.0218)$$
$$= 0.535\,\text{ft}^3/\text{min}$$

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