13

LAMINAR FLOW IN PIPES

13.1 INTRODUCTION

The following equation was derived in Chapter 8 (see Eq. (8.18))

$$\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2g_c} + \frac{g}{g_c}\Delta z - \eta W_s + \sum F = 0 \tag{13.1}$$

This was defined as the mechanical energy equation. The above equation was later rewritten without the work and friction terms

$$\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2g_c} + \frac{g}{g_c}\Delta z = 0 \tag{13.2}$$

This equation was defined as the basic form of the Bernoulli equation.

In applying the Bernoulli equation to a prime mover (e.g., a centrifugal pump application), Equation (8.18) was written as

$$\frac{P_1}{\rho}\frac{g_c}{g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho}\frac{g_c}{g} + \frac{v_2^2}{2g} + z_2 - h_s\frac{g_c}{g} + h_f\frac{g_c}{g} \tag{13.3}$$

where h_s and h_f have effectively replaced ηW_s and $\sum F$, respectively, in Equation (13.1). Note that h_s is a positive term as is h_f. The units of both h_s and h_f are therefore ft-lb$_f$/lb. However, each term in this equation, as written, has units of ft of flowing

Fluid Flow for the Practicing Chemical Engineer. By J. Patrick Abulencia and Louis Theodore fluid (a pressure term). If the equation is multiplied by g/g_c, each term returns to units of energy/mass (e.g., ft · lb_f/lb). The result is provided in Equation (13.4)

$$\frac{\Delta P}{\rho}+\frac{\Delta v^2}{2g_c}+\Delta z\frac{g}{g_c}-h_s+h_f=0 \tag{13.4}$$

Note that the prime notation is retained if the units of h_s' and h_f' are in height of flowing fluid, e.g., ft H_2O. The choice of units for h_s and h_f is, of course, optional and/or arbitrary.

The reader should note that in the process of converting Equation (13.3) to (13.4), the Δ term (representing a difference) applies to outlet minus inlet conditions (i.e., $P_2 - P_1$, etc.). One can now examine Equation (13.4) at the following extreme or limiting conditions.

1. If only the first ($\Delta P/\rho$) and last term (h_f) are present, an increase in the latter's frictional losses would result in a corresponding decrease in the former pressure term P_2. Thus, a large pressure drop results, which is to be expected.
2. If only the first ($\Delta P/\rho$) and fourth term (h_s) are present, an increase in the latter's input mechanical (shaft) work term would result in a corresponding increase in the frame pressure term P_2. Thus, a smaller pressure drop results, and again, this is in agreement with what one would expect.
3. If only the latter two terms are present, $h_s = h_f$, and this too agrees with one's expectation since both terms are positive and any frictional effect is compensated by the mechanical (shaft) work introduced to the system.

Care should be exercised in the interpretation of the term ΔP. Although the notation Δ represents difference, ΔP can be used to describe the difference between the inlet minus the outlet pressure (i.e., $P_1 - P_2$), or it can describe the difference between the outlet minus the inlet pressure (i.e., $P_2 - P_1$). When a fluid is flowing in the $1 \rightarrow 2$ direction, the term $P_1 - P_2$ is a positive and represents a decrease in pressure that is defined as the pressure drop. The term $P_2 - P_1$, however, also represents a pressure change whose difference is negative and is also defined as a pressure drop. One's wording and interpretation of this pressure change is obviously a choice that is left to the user.

13.2 FRICTION LOSSES

As indicated above, the h_f term was included to represent the loss of energy due to friction in the system. These frictional losses can take several forms. An important engineering problem is the calculation of these losses. It has been shown (earlier) that the fluid can flow in either of two modes—laminar or turbulent. For laminar flow, an equation is available from basic theory to calculate friction loss in a pipe. In practice, however, fluids (particularly gases) are rarely moving in laminar flow.

Since two methods of flow are so widely different, a different equation describing frictional resistance is to be expected in the case of turbulent flow from that which applies in the case of laminar flow. On the other hand, it will be shown in the next chapter that both cases may be handled by one relationship in such a way that it is not necessary to make a preliminary calculation to determine whether the flow is taking place above the critical Reynolds number or below it.(1)

One can theoretically derive the h_f term for laminar flow.(2,3) The equation takes the form

$$h_f = \frac{32\mu v L}{\rho g_c D^2} \tag{13.5}$$

for a fluid flowing through a straight cylinder of diameter D and length L. A friction factor, f, that is dimensionless may now be defined as (for laminar flow)

$$f = \frac{16}{\text{Re}} \tag{13.6}$$

so that Equation (13.5) takes the form

$$h_f = \frac{4fLv^2}{2g_c D} \tag{13.7}$$

Although the above equation describes friction loss or the pressure drop across a conduit of length L, it can also be used to provide the pressure drop due to friction per unit length of conduit, for example, $\Delta P/L$ by simply dividing the above equation by L.

It should also be noted that another friction factor term exists that differs from that of Equation (13.6). In this other case, f_D is defined as

$$f_D = \frac{64}{\text{Re}} \tag{13.8}$$

The f_D is used to distinguish the value of Equation (13.6) from that of Equation (13.8). In essence, then

$$f_D = 4f \tag{13.9}$$

The term f is defined as the *Fanning friction* factor while f_D is defined as the *Darcy* or *Moody friction factor*. Care should be taken as to which of the friction factors are being used and this will become more apparent in the next chapter. In general, chemical engineers employ the Fanning friction factor; other engineers prefer the Darcy (or Moody) factor. This book employs the Fanning friction factor.

With reference to Equation (13.6), one should note that this is an equation of a straight line with a slope of -1 if f is plotted versus Re on log-log coordinates.

Note (once again) that the equation for f applies only to laminar flow, i.e., when Re is <2100 for pipe flow.

Employing Equation (13.7), Equation (13.4) may be extended and rewritten as

$$\frac{\Delta P}{\rho}+\frac{\Delta v^2}{2g_c}+\Delta z\frac{g}{g_c}-h_s+\sum\frac{4fLv^2}{2g_cD}=0 \qquad (13.10)$$

Note that a summation sign has been inserted before the new term for the loss in a straight pipe because this loss may result from flow through several sections in a series of pipes of various lengths and diameters. The symbols $\sum h_c$ and $\sum h_e$, representing the sum of the contraction and expansion losses, respectively, may also be added to the equation, as provided in Equation (13.11). (These effects will be discussed in the next part of this book.)

$$\frac{\Delta P}{\rho}+\frac{\Delta v^2}{2g_c}+\Delta z\frac{g}{g_c}-h_s+\sum\frac{4fLv^2}{2g_cD}+\sum h_c+\sum h_e=0 \qquad (13.11)$$

13.3 TUBE SIZE

Fluids are usually transported in pipes or tubes. Generally speaking, pipes are heavy-walled and have a relatively large diameter. Tubes are thin-walled and often come in coils.

Pipes are specified in terms of their diameter and wall thickness. The nominal diameters range from 1/8 to 30 inches for steel pipe. Standard dimensions of steel pipe are provided in Table A.5 in the Appendix and are known as IPS (iron pipe size) or NPS (nominal pipe size). The wall thickness of the pipe is indicated by the schedule number, which can be approximated from $1000(P/S)$ where P is the maximum internal service pressure (psi) and S is the allowable bursting stress in the pipe material (psi). (The S value varies by material, grade of material and temperature; allowable S values may be found in Piping Handbooks).

Tube sizes are indicated by the outside diameter. The wall thickness is usually given a BWG (Birmingham Wire Gauge) number. The smaller the BWG, the heavier the tube. Table A.6 in the Appendix lists the sizes and wall thicknesses of condenser and heat exchanger tubes. For example, a 3/4-inch 16 BWG tube has an outside diameter (OD) of 0.75 in, an inside diameter (ID) of 0.62 in, a wall thickness of 0.065 in, and a weight of 0.476 lb/ft.

Illustrative Example 13.1 Consider the following three cases. Calculate the average velocities below for which the flow will be viscous (laminar).

1. Water at 60°F in a 2-inch standard pipe.
2. Air at 60°F and 5 psig in a 2-inch standard pipe.
3. Oil of a viscosity of 300 cP and SG of 0.92 in a 4-inch standard pipe.

Solution For laminar flow, Re < 2100, so the equation

$$\mathrm{Re} = \frac{Dv\rho}{\mu} = 2100$$

can be solved for the velocity term

$$v = \frac{2100\mu}{D\rho}$$

1. For water, $\mu = 6.72 \times 10^{-4}\,\mathrm{lb/(ft \cdot s)}$, $\rho = 62.4\,\mathrm{lb/ft^3}$. In addition, from Table A.5 in the Appendix, $D = 2.067$ in. Therefore,

$$v = \frac{2100(6.72 \times 10^{-4})}{(2.067/12)(62.4)} = 0.13\,\mathrm{ft/s}$$

2. For air, $\mu = 12.1 \times 10^{-6}\,\mathrm{lb/ft \cdot s}$, $\rho = 0.1024\,\mathrm{lb/ft^3}$ (from ideal gas law), and $D = 2.067$ in. Therefore,

$$v = \frac{2100(12.1 \times 10^{-6})}{(2.067/12)(0.1024)} = 1.44\,\mathrm{ft/s}$$

3. For oil, $\mu = 300(6.72 \times 10^{-4})\,\mathrm{lb/ft \cdot s}$, $\rho = 0.92(62.4)\,\mathrm{lb/ft^3}$, and, from Table A.5 in the Appendix, $D = 4.026$ in. Therefore,

$$v = \frac{2100(300)(6.72 \times 10^{-4})}{(4.026/12)(0.92)(62.4)} = 22.0\,\mathrm{ft/s}$$

Illustrative Example 13.2 Refer to Illustrative Example 13.1. Determine the pressure drop per unit length of pipe for part (1).

Solution To determine the pressure drop per unit length of pipe, $\Delta P/L$, in psf/ft, apply a modified form of the Hagen–Poiseuille equation. For these units, Equation (13.5) reduces to:

$$\frac{\Delta P}{L} = \frac{32\mu g v}{g_c D^2}$$

Substituting

$$\frac{\Delta P}{L} = \frac{32(6.72 \times 10^{-4})(0.13)}{(2.067/12)^2}$$

$$= 0.0293\,\mathrm{psf/ft}$$

Illustrative Example 13.3 Given the nominal size and schedule number of a 1-inch schedule 80 steel pipe, determine its inside diameter (ID), outside diameter (OD), wall thickness and pipe weight (lb/ft).

Solution Using Table A.5 in the Appendix, obtain the pipe inside diameter, outside diameter, wall thickness and weight

$$\text{ID} = 0.957\ \text{in.}$$
$$\text{OD} = 1.315\ \text{in.}$$
$$\text{Wall thickness} = 0.179\ \text{in.}$$
$$\text{Pipe weight} = 2.17\ \text{lb/ft}$$

13.4 OTHER CONSIDERATIONS

As discussed earlier, flow in conduits that are not cylindrical (e.g., a rectangular parallel pipe), are treated as if the flow occurs in a pipe. For this situation, a hydraulic radius, r_h, is defined as:

$$r_h = \frac{\text{Cross-sectional area perpendicular to flow}}{\text{Wetted perimeter}} \tag{13.12}$$

For flow in a circular tube

$$r_h = \frac{(\pi D^2/4)}{\pi D} = \frac{D}{4}$$

and

$$D = 4r_h \tag{13.13}$$

One may extend this concept to any cross-section such that

$$D_{\text{eq}} = 4r_h \tag{13.14}$$

It is then possible to use this equivalent diameter in the circular pipe expression presented in Equation (13.7) for pressure drop

$$h_f = \frac{4fLv^2}{2g_c D_{\text{eq}}} \tag{13.15}$$

The hydraulic diameter approach is usually valid for laminar flow and always valid for turbulent flow.

Illustrative Example 13.4 An air-conditioning duct has a rectangular cross-section of 1 m by 0.25 m. If the kinematic viscosity of the air is approximately $1 \times 10^{-5}\ \text{m}^2/\text{s}$, determine the maximum air velocity before the flow becomes turbulent. Assume the critical Reynolds number is 2300.

Solution Compute the equivalent or hydraulic diameter

$$D_h = \frac{2wh}{w+h} = \frac{2(1)(0.25)}{1+0.25}$$
$$= 0.4\,\text{m}$$

The equation for the "critical" Reynolds number is

$$\text{Re}_{\text{crit}} \approx 2300 = \frac{vD_h}{\nu}$$

Solve for v and substitute

$$v = 2300\frac{1\times10^{-5}}{0.4} = 0.0575\,\text{m/s}$$
$$= 5.8\,\text{cm/s}$$

Another important concept is that referred to as a "calming," "entrance," or "transition" length. This is the length of conduit required for a velocity profile to become fully developed following some form of disturbance in the conduit. This disturbance can arise because of a valve, a bend in the line, an expansion in the line, etc. This is an important concern when measurements are conducted in the cross-section of the pipe or conduit. An estimate of this "calming" length, L_c, for laminar flow is

$$\frac{L_c}{D} = 0.05\,\text{Re} \tag{13.16}$$

For turbulent flow (see the next chapter), one may employ

$$L_c = 50\,D \tag{13.17}$$

Illustrative Example 13.5 A circular 2-inch diameter horizontal tube contains liquid asphalt. For the purposes of this problem, assume asphalt to be a Newtonian fluid of density 70 lb/ft^3. The tube radius is 1 in. When a steady pressure gradient of 1.0 psi/ft is applied, the steady-state flow rate of the asphalt is 0.486 ft^3/s. Calculate the asphalt viscosity in cP and kg/m-s. Determine if the flow is laminar. Calculate the Darcy and Fanning friction factors. How long must the pipe be to ensure a fully developed flow?

Solution Apply the continuity equation to obtain the flow velocity

$$q = vS = v(\pi D^2/4)$$
$$v = \frac{4q}{\pi D^2} = \frac{4(0.486)}{\pi(0.1667)^2}$$
$$= 22.3\,\text{ft/s}$$

To determine the dynamic viscosity, assume laminar flow and use Equation (13.7) with $v = q/(\pi D^2/4)$ and $f = 16/\text{Re}$.

$$\mu = \frac{\pi \Delta P D^4 g_c}{128 q L} = \frac{\pi(144)(0.1667)^4(32.174)}{128(0.486)(1)}$$
$$= 0.1806\,\text{lb/ft} \cdot \text{s}$$

Check on the assumption of laminar flow:

$$\text{Re} = \frac{Dv\rho}{\mu} = \frac{(0.1667)(22.3)(70)}{0.1806} = 1440 < 2300$$

As expected, the flow is laminar. The Fanning friction factor is given by

$$f = \frac{16}{\text{Re}} = \frac{16}{1440}$$
$$= 0.0111$$

As expected, the friction factor is large.

The pipe must be longer than the entrance length to have fully developed flow. Calculate the entrance length from Equation (13.16):

$$L_c = 0.05D\,\text{Re} = 0.05(1440)(0.1667)$$
$$= 12.0\,\text{ft}$$

If $L > 12$ ft the flow may be assumed to be fully developed.

Illustrative Example 13.6 Liquid glycerin at 20°C flows in a tube of diameter 4 cm. The velocity profile of a slow moving fluid through a long circular tube is parabolic. Two possible expressions of the parabolic velocity distribution are available:

$$v = 16(1 - 2500r^2)$$

or

$$v = 16(1 - 8000r^2)$$

where v is the glycerin velocity at any radial distance and r is the radial distance measured from the center line. What is the correct velocity distribution and why? Using the correct velocity distribution, calculate the average velocity (m/s), volumetric flow rate (m^3/s and gpm), mass flow rate (kg/s), mass flux (kg/$m^2 \cdot$ s), and linear momentum flux ($\dot{M}$) of glycerin.

Solution Obtain the properties of liquid glycerine. Use Table A.2 in the Appendix

$$\rho = 1260\,\text{kg/m}^3$$

$$\mu = 1.49\,\text{kg/m-s}$$

$$\nu = \mu/\rho = 0.00118\,\text{m}^2/\text{s}$$

Because of no-slip conditions at the wall, the correct velocity distribution must produce $v = 0$ when $R = 0.02$ m. This is satisfied only for:

$$v = 16(1 - 2500r^2)$$

where, for $R = 0.02$ m

$$v = 16(1 - 2500(0.02)^2) = 0$$

This must be the correct velocity distribution.

To calculate q, v, $\dot{m}$, G, and $\dot{M}$, first write a differential equation for the volumetric flow rate, dq

$$\begin{aligned} \mathrm{d}q &= v\,\mathrm{d}S = 2\pi r v\,\mathrm{d}r = 2\pi r[16(1 - 2500r^2)]\,\mathrm{d}r \\ &= 32\pi(r - 2500r^3)\,\mathrm{d}r \end{aligned}$$

Integrating

$$\begin{aligned} q &= \int \mathrm{d}q = 32\pi \int_0^{R=0.02} (r - 2500r^3)\,\mathrm{d}r = 32\pi\left(\frac{R^2}{2} - \frac{2500}{4}R^4\right) \\ &= 32\pi\left[\frac{(0.02)^2}{2} - \frac{2500}{4}(0.02)^4\right] \\ &= 0.010\,\text{m}^3/\text{s} = 158.5\,\text{gpm} \end{aligned}$$

The average velocity for laminar flow (see next section) is given by

$$\bar{v} = \frac{v_{\text{max}}}{2} = \frac{16(1 - 2500r^2)}{2}$$

Substituting

$$\bar{v} = 8(1 - 2500(0)^2) = 8\,\text{m/s}$$

Finally,

$$\begin{aligned} \dot{m} &= q\rho = 0.01(1260) = 12.6\,\text{kg/s} \\ G &= \frac{\dot{m}}{S} = \rho v = (1260)(8) = 10{,}080\,\text{kg/m}^2\text{-s} \\ \dot{M} &= \dot{m}v = 12.6(8) = 100.8\,\text{N} \end{aligned}$$

Illustrative Example 13.7 Refer to Illustrative Example 13.6. Calculate the Reynolds number of the flow and determine how many hours it will take for 14,000 gallons to pass through the cross-section of the tube.

Solution Calculate the flow Reynolds number:

$$\text{Re} = \frac{\rho v D}{\mu} = \frac{1260(8)(0.02)}{1.49} = 135.3$$

Since 135.3 < 2100, the flow is laminar.

Calculate the time, t, to pass 14,000 gallons of glycerine through a cross-section of the tube

$$t = \frac{V}{q} = \frac{14{,}000}{159.6} = 87.7 \text{ min}$$

13.5 MICROSCOPIC APPROACH

It is important to note that the equations provided in Tables 9.1–9.3 in Chapter 9 are valid only for laminar flow. As such, they can be used to develop equations describing laminar flow for various systems under different conditions. For example, refer to Equation (5.16) provided in Chapter 5

$$v_z = \frac{g_c \Delta P}{4\mu L}\left(R^2 - r^2\right) \tag{5.16}$$

This equation can be derived from the aforementioned tables using the approach provided below. This system requires using cylindrical coordinates. The describing equations are now "extracted" from Table 9.2. Since the flow is one-dimensional

$$v_r = 0$$
$$v_\phi = 0$$
$$v_z \neq 0$$

The terms v_r, v_ϕ, and all their derivatives must be zero. From equation (2) in Table 9.1, one concludes

$$\frac{\partial v_z}{\partial z} = 0$$

Based on physical grounds

$$\frac{\partial v_z}{\partial \phi} = 0$$

Based on the problem statement

$$\partial v_z/\partial t = 0$$

it is reasonable to conclude that v_z might vary with r, that is

$$v_z = v_z(r)$$

This means

$$\frac{\partial v_z}{\partial r} \neq 0$$

or perhaps

$$\frac{\partial^2 v_z}{\partial r^2} \neq 0$$

Examining the equation of motion in cylindrical coordinates in Table 9.2, one notes that

$$\frac{\partial P}{\partial r} = 0$$

$$\frac{\partial P}{\partial \phi} = 0$$

$$\frac{\partial P}{\partial z} = \frac{\mu}{g_c}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right]$$

The last equation may be rewritten

$$\frac{\mathrm{d}P}{\mathrm{d}z} = \frac{\mu}{g_c}\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}v_z}{\mathrm{d}r}\right)\right]$$

The left-hand side (LHS) is a constant or a function of z. The right-hand side (RHS) is either a constant or a function of r. It is therefore concluded that both must equal a constant. Since $\Delta P/\mathrm{d}z$ is a constant, it is written in finite form:

$$\frac{\mathrm{d}P}{\mathrm{d}z} = +\frac{\Delta P}{\Delta z}$$

$$= -\frac{\Delta P}{L}$$

The negative sign appears because P decreases as z increases. The above equation is now written as

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}v_z}{\mathrm{d}r}\right) = -\frac{g_c\Delta P}{\mu L}$$

It would be wise to multiply both sides of the equation by r dr; otherwise, some difficulty would be encountered upon integration.

$$\mathrm{d}\left(r\frac{\mathrm{d}v_z}{\mathrm{d}r}\right) = -\frac{g_c\Delta P}{\mu L} r\,\mathrm{d}r$$

Integrating once

$$r\frac{\mathrm{d}v_z}{\mathrm{d}r} = -\frac{g_c\Delta P}{2\mu L} r^2 + A$$

Multiplying both sides by dr/r

$$\mathrm{d}v_z = -\frac{g_c\Delta P}{2\mu L} r\,\mathrm{d}r + \frac{A}{r}\,\mathrm{d}r$$

and integrating

$$v_z = -\frac{g_c\Delta P}{4\mu L} r^2 + A\ln r + B$$

The integration constants were evaluated in Chapter 5. This ultimately resulted in

$$v_z = \frac{g_c\Delta P}{4\mu L}(R^2 - r^2) \tag{5.16}$$

Illustrative Example 13.8 Refer to Equation (5.16). Express the local velocity in terms of the volumetric flow rate q.

Solution The volumetric flow rate is given by

$$q = \iint_f v_z\,\mathrm{d}f: \quad f = \text{area available for flow}$$

where d$f = r$ dr dϕ = differential area in cylindrical coordinates. Substituting Equation (5.16) into the above equation leads to

$$q = \int_{\phi=0}^{2\pi}\int_{r=0}^{R} \frac{g_c\Delta P}{4\mu L}(R^2 - r^2) r\,\mathrm{d}r\,\mathrm{d}\phi$$

$$= \frac{\pi g_c\Delta P}{2\mu L}\int_0^R (R^2 r - r^3)\,\mathrm{d}r$$

$$= \frac{\pi g_c\Delta P}{2\mu L}\left[\frac{R^2 r^2}{2} - \frac{R^4}{4}\right]_0^a$$

$$= \frac{\pi g_c\Delta P R^4}{8\mu L}$$

and

$$\frac{\Delta P}{L} = \frac{8\mu q}{g_c \pi R^4}$$

Substituting the above equation into Equation (5.10) leads to

$$v_z = \frac{2q}{\pi R^2}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

The flow profile is therefore parabolic.

Illustrative Example 13.9 Refer to Illustrative Example (13.8). Calculate the maximum velocity, v_{max}, the average velocity, v_{av}, and the ratio of the average to the maximum velocity

Solution The maximum velocity is located at $r = 0$. Therefore

$$q_{max} = \frac{2q}{\pi R^2}$$

The average velocity is defined as

$$\bar{v} = v_{av} = \frac{\int_0^{2\pi}\int_0^{R} \frac{2q}{\pi R^2}\left[1 - \left(\frac{r}{R}\right)^2\right] r\,dr\,d\phi}{\int_0^{2\pi}\int_0^{R} r\,dr\,d\phi}$$

$$= \frac{\frac{2q}{\pi R^2}(2\pi)\int_0^{R}\left[1 - \left(\frac{r}{R}\right)^2\right] r\,dr}{\pi a^2}$$

$$= \frac{4q}{\pi R^4}\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R = \frac{q}{\pi R^2}$$

$$= \frac{q}{\pi R^2} = \frac{q}{S}; \text{ as expected}$$

Therefore

$$v_z = 2v_{av}\left[1 - \left(\frac{r}{R}\right)^2\right] = v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Finally

$$\frac{v_{\text{av}}}{v_{\max}} = \frac{1}{2}$$

REFERENCES

1. W. Badger and J. Banchero, "Introduction to Chemical Engineering," McGraw-Hill, New York, 1955.
2. C. Bennett and J. Meyers, "Momentum, Heat, and Mass Transfer," McGraw-Hill, New York, 1962.
3. L. Theodore, "Transport Phenomena for Engineers," International Textbook Company, Scranton, PA, 1971.

NOTE: Additional problems are available for all readers at www.wiley.com. Follow links for this title.