# 8

# CONSERVATION LAW FOR ENERGY

## 8.1 INTRODUCTION

This chapter is concerned with the conservation law for energy. The presentation to follow once again includes a review of some key pressure terms. A general introduction to the conservation of energy is in turn followed by the development of a general total energy balance for steady-state flow. The chapter concludes by extending the total energy equation to include mechanical energy; this has come to be defined as the *mechanical energy balance equation.* It is this equation that is employed in the solution of most real-world fluid flow problems.

One of the most critical parameters in fluid flow is pressure. This was briefly defined in Chapter 3. Three additional pressure terms should be defined before proceeding to the body of this chapter. These are the *static pressure* $(P_s)$, the *velocity pressure* $(P_v)$, and the sum of the two—the total pressure $(P_t)$.

Any fluid confined in a stationary enclosure has static pressure simply because the molecules of that fluid are in constant random motion and are continually colliding with the container walls. The bulk velocity of this stationary fluid is zero, and the total pressure is then equal to the static pressure. If the same fluid is flowing and the temperature has not changed, it possesses the same static pressure since its molecules still have the same degree of random motion. Its total pressure is now higher, however, because it also possesses the second pressure component, velocity pressure. If the fluid flow were to suddenly change direction because of a solid obstruction (e.g., a plate), an extra pressure on the plate (over and above the static pressure) would be exerted because of the momentum of the bulk flow against the

plate. This extra pressure is the velocity pressure and the total fluid pressure is the sum of the static and velocity pressures. Static pressure is therefore the result of motion on the molecular level, while velocity pressure is due to motion at the macroscopic or bulk level.[1]

The difference in total pressure between two different points along the stream is called the pressure loss or the pressure drop. Pressure losses from fluid flow are due to any effect that can change fluid momentum at either the molecular or macroscopic levels; the two main contributing factors are skin friction and form friction. *Skin friction* losses are caused by fluid moving along (parallel to) a solid surface such as a pipe or duct wall. The layers of fluid immediately adjacent to the wall are in laminar flow and moving much slower than the bulk of the fluid. The pressure drop caused by the drag effect of the wall on the fluid is due to skin friction. *Form friction* losses are due to the acceleration or deceleration of the fluid. These include changes in bulk fluid velocity that occur because of changes in either flow direction or flow speed. An example of a change in flow direction is fluid flowing through a 90° elbow; alternatively, a change in flow speed occurs when the cross-section of a conduit changes. Besides changes in bulk fluid velocity, form friction losses also include changes in velocity that occur locally, i.e., internal to the bulk motion of the fluid. This occurs in turbulent flow (see Chapter 14 for more details), which is characterized by rapidly swirling masses of fluid called *eddies*.[1]

## 8.2 CONSERVATION OF ENERGY

A presentation of the conservation law for energy would be incomplete without a brief review of some introductory thermodynamic principles. *Thermodynamics* is defined as that science that deals with the relationships among the various forms of energy. A system may possess energy due to its temperature (internal energy), velocity (kinetic energy), position (potential energy), molecular structure (chemical energy), surface (surface energy), etc. Engineering thermodynamics is founded on three basic laws. As described earlier, energy, like mass and momentum, is conserved. Application of the conservation law for energy gives rise to the first law of thermodynamics. This law for batch processes, is presented below.

For batch processes

$$\Delta E = Q + W \tag{8.1}$$

where potential, kinetic, and other energy effects have been neglected and $Q$ is energy in the form of heat transferred across the system boundaries, $W$ is energy in the form of work transferred across system boundaries, $E$ (often denoted as $U$), the internal energy of the system, and $\Delta E$ is the change in the internal energy of the system. In accordance with the recent change in convention, both $Q$ and $W$ are treated as *positive* terms if *added* to the system.

By definition, a flow process involves material streams entering and exiting a system. Work is done on the system at the stream entrance when the fluid is

pushed into the system. Work is performed by the system to push the fluid out at the stream exit. The net work on the system is called flow work, $W_f$, and is given by

$$W_f = \sum P_{out} V_{out} - \sum P_{in} V_{in} = -\Delta(PV) \tag{8.2}$$

where $P_{out}$ is the pressure of the outlet stream, $P_{in}$ is the pressure of the inlet stream, $V_{out}$ is the volume of fluid exiting the system during a given time interval, and $V_{in}$ is the volume of fluid entering the system during a given time interval. If the volume term is represented as the specific volume (i.e., volume/mass), the work term carries the units of energy/mass.

For practical purposes, the total work term, $W$, in the first law may be regarded as the sum of shaft work, $W_s$, and flow work, $W_f$

$$W = W_s + W_f \tag{8.3}$$

where $W_s$ is work done on the fluid by some moving solid part within the system such as the rotating vanes of a centrifugal pump. Note that in Equation (8.3), all other forms of work such as electrical, surface tension, and so on are neglected. The first law for steady-state flow processes is then:

$$\Delta H = Q + W_s \tag{8.4}$$

where $H$ is the enthalpy of the system and $\Delta H$ is the change in the system's enthalpy.

The internal energy and enthalpy in Equations (8.1) and (8.2), as well as other equations in this section may be on a mass basis, on a mole basis, or represent the total internal energy and enthalpy of the entire system. They may also be written on a time-rate basis as long as these equations are dimensionally consistent—it makes no difference. For the sake of clarity, upper case letters (e.g., $H$, $E$) represent properties on a mole basis, while lower-case letters (e.g., $h$, $e$) represent properties on a mass basis. Properties for the entire system will rarely be used and therefore require no special symbols.

Perhaps the most important thermodynamic function the engineer works with is the above mentioned *enthalpy*. This is a term that requires additional discussion. The enthalpy is defined by the equation

$$H = E + PV \tag{8.5}$$

where $P$ is once again the pressure of the system and $V$ is the volume of the system. The terms $E$ and $H$ are state or point functions. By fixing a certain number of variables upon which the function depends, the numerical value of the function is automatically fixed; that is, it is single-valued. For example, fixing the temperature and pressure of a one-component single-phase system immediately specifies the enthalpy and internal energy. This last statement can be verified by Gibbs's Phase Rule.[2]

The change in enthalpy as it undergoes a change in state from $(T_1, P_1)$ to $(T_2, P_2)$ is given by

$$\Delta H = H_2 - H_1 \tag{8.6}$$

Note that $H$ and $\Delta H$ are independent of the path. This is a characteristic of all state or point functions; that is, the state of the system is independent of the path by which the state is reached. The terms $Q$, $W$, and $W_s$ in Equations (8.4) and (8.5) are path functions; their values depend on the path used between the two states.

The following can be written for a mathematical representation of this important thermodynamic point function

$$H = H(T, P)$$

By the rules of partial differentiation, a differential change in $H$ is given by

$$dH = \left(\frac{\partial H}{\partial T}\right)_P dT + \left(\frac{\partial H}{\partial P}\right)_T dP \tag{8.7}$$

The term $(\partial H/\partial P)_T$ is assumed to be negligible in most engineering applications. It is exactly zero for an ideal gas and is small for solids and liquids, and gases near ambient conditions. The term $(\partial H/\partial T)_P$ is defined as the *heat capacity at constant pressure*

$$C_P = \left(\frac{\partial H}{\partial T}\right)_P \tag{8.8}$$

Equation (8.8) may also be written as

$$dH = C_P \, dT \tag{8.9}$$

If average molar heat capacity data are available, this equation may be integrated to yield

$$\Delta H = \overline{C_P} \, \Delta T \tag{8.10}$$

where $\overline{C_P}$ is the average value of $C_P$ in the temperature range $\Delta T$. Calculations involving enthalpy changes also finds extensive application in heat transfer.

Many industrial applications operate in a steady-state flow mode with no significant mechanical or shaft work added (or withdrawn) from the system. For this condition Equation (8.4) reduces to

$$Q = \Delta H \tag{8.11}$$

This equation is routinely used in many calculations. If a unit or system is operated adiabatically, $Q = 0$ and Equation (8.11) becomes

$$\Delta H = 0 \tag{8.12}$$

Although the topics of material and energy balances have been covered separately in this and the previous chapter, it should be emphasized that this segregation does

not exist in reality. Many processes are accompanied by heat effects, and one must work with both energy and material balances simultaneously.[3]

**Illustrative Example 8.1**   5.5 MW of heat is transferred from a gas as it flows through a cooler. The average heat capacity of the gas is 1090 J/(kg · °C), the gas mass flow rate, $\dot{m}$, is 9 kg/s and the gas inlet temperature, $T_1$, is 650°C. For this example, kinetic and potential energy effects are neglected. Furthermore, there is no shaft work. Determine the gas outlet temperature.

*Solution*   Since there are no kinetic, potential, or shaft work effects in this flow process, Equations (8.10) and (8.11) applies

$$Q = \Delta H$$

where $\Delta H = \dot{m}\overline{C}_P \Delta T = \dot{m}\overline{C}_P(T_2 - T_1)$.
   Solving for the gas outlet temperature, $T_2$,

$$T_2 = \frac{Q}{\dot{m}\overline{C}_P} + T_1 = \frac{-5.5 \times 10^{-6}}{9(1090)} + 650 = 89°C$$

Note that the sign of $Q$ is negative since the heat is transferred out from the gas.

## 8.3  TOTAL ENERGY BALANCE EQUATION

Equations (8.1) and (8.4) find application in many chemical process units such as heat exchangers, reactors, and distillation columns, where shaft work plus kinetic and potential energy changes are negligible compared with heat flows and either internal energy or enthalpy changes. Energy balances on such units therefore reduce to $Q = \Delta E$ (closed system) or $\dot{Q} = \Delta \dot{H}$ (open system).
   Another important class of operations is one for which the opposite is true—heat flows and internal energy changes are secondary in importance to kinetic and potential energy changes and shaft work. Most of these operations involve the flow of fluids to, from, and between tanks, reservoirs, wells, and process units. Accounting for energy flows in such processes is most conveniently accomplished with mechanical energy balances.[4] Details of this approach follow.
   Consider the steady-state flow of a fluid in the process pictured in Fig. 8.1. The mass entering at location 1 brings in with it a certain amount of energy, existing in various forms. Thus, because of its elevation, $z_1$ ft above any arbitrarily chosen horizontal reference plane, for example, $z = 0$, it possesses a potential energy $(g/g_c)z_1$ (which can be recovered by allowing the fluid to fall from the height at location 1 to that of the reference point). Because of its velocity, $v_1$, the mass possesses and brings into location 1 of the system an amount of kinetic energy, $v_1^2/2g_c$. It also brings its so-called internal energy, $E_1$, because of its temperature. Furthermore, the mass of fluid in question entering at point 1 is forced into the section by the
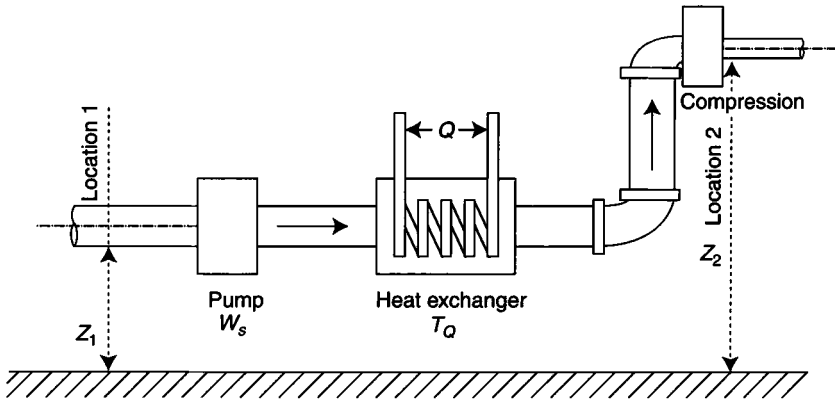
**Figure 8.1** Process flow.

pressure of the fluid behind it and this form of flow energy must also be included. The amount of this energy is given by the force exerted by the flowing fluid times the distance through which it acts, and this force is clearly the pressure per unit area, $P_1$, times the area $S_1$ of the cross-section. The distance through which the force acts is the volume, $V_1$ of the fluid divided by the cross-sectional area $S_1$. Since the work is the force times the distance, that is, $(P_1 S_1)(V_1/S_1) = P_1 V_1$, the energy expended is the product of the pressure times the volume of the fluid. This was referred to earlier as *flow work* (see Eq. (8.2)).

Two additional energy terms need to be included in the analysis. These two involve energy exchange in the form of heat ($Q$) and work ($W$) between the fluid and the surroundings. In the development to follow, it will be assumed (consistent with the notation recently adopted by the scientific community) that any energy in the form of heat or work *added* to the system is treated as a *positive* term.

Applying the conservation law of energy mandates that all forms of energy entering the system equal that of those leaving. Expressing all terms in consistent units (e.g., energy per unit mass of fluid flowing), results in the total energy balance:

$$P_1 V_1 + \frac{v_1^2}{2g_c} + \frac{g}{g_c} z_1 + E_1 + Q + W_s = P_2 V_2 + \frac{v_2^2}{2g_c} + \frac{g}{g_c} z_2 + E_2 \qquad (8.13)$$

As written, each term in Equation (8.13) represents a mechanical energy effect. For this reason, it is defined as a form of the mechanical energy balance equation and is essentially a special application of the conservation law for energy. Also note that, as written, the volume term, $V$, (for necessity) is the specific volume. In terms of the density, the above equation becomes

$$\frac{P_1}{\rho} + \frac{v_1^2}{2g_c} + \frac{g}{g_c} z_1 + E_1 + Q + W_s = \frac{P_2}{\rho} + \frac{v_2^2}{2g_c} + \frac{g}{g_c} z_2 + E_2 \qquad (8.14)$$

and

$$V = 1/\rho \qquad (8.15)$$

Note once again that $Q$ and $W_s$ can be written on a time rate basis in the above equation by simply dividing by the mass flowrate though the system; the above equation then dimensionally reduces to an energy/mass balance.

Three points need to be made before leaving this subject.

1. The term $Q$ should represent the total net heat added to the fluid, but in this analysis it includes only the heat passing into the fluid across the walls of the containing walls from an external source. This excludes heat generated by friction, by the fluid or otherwise, within the unit. However, this effect can normally be safely neglected.

2. The work, $W_s$, similar to $Q$, must pass though the retaining walls. While it could conceivably enter in other ways, it is supplied in most applications by some form of moving mechanism, such as a pump, or a fan, and is often referred to as *shaft work*.

3. The internal energy term $E$ corresponds to the thermodynamic definition provided earlier. For convenience, the sum of $E$ and $PV$ may be treated as the single function defined above as the enthalpy, $H$,

$$H = E + PV \tag{8.5}$$

It too is a property of the fluid, uniquely determined by point conditions. Like $E$, its absolute value is arbitrary; differences in value are often given above a reference. With this revision, and assuming $\alpha = 1$, Equation (8.13) becomes:

$$\frac{v_1^2}{2g_c} + \frac{g}{g_c}z_1 + H_1 + Q + W_s = \frac{v_2^2}{2g_c} + \frac{g}{g_c}z_2 + H_2 \tag{8.16}$$

or simply

$$\frac{\Delta v^2}{2g_c} + \frac{g}{g_c}\Delta z + \Delta H = Q + W_s \tag{8.17}$$

As noted in the presentation of Equation (8.13), each term is dimensional with units of energy/mass. If this equation is multiplied by the fluid flow rate, that is, mass/time, the units of each term become energy/time. In the absence of both kinetic and potential energy effects, the above equation reduces to Equation (8.4). Also note that $\Delta$, the difference term, refers to a difference between the value at station 2 (the usual designation for the outlet) minus that at station 1 (the inlet).

**Illustrative Example 8.2**   A fluid flow device has three openings, as shown in Fig. 8.2. The flow within the control volume is steady and the fluid has a constant density of 800 kg/m³. The flow properties at each opening are provided below in Table 8.1. What is the rate of change of the system energy in the control volume? For steady-state adiabatic operation, is work being done on the system?
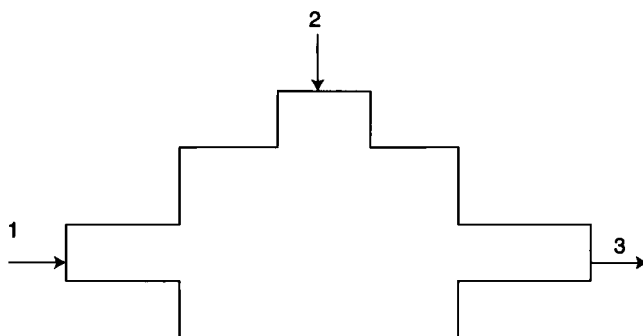
**Figure 8.2**  Fluid flow device for Illustrative Example 8.2.

**Table 8.1  Flow/energy data for Illustrative Example 8.2**

| Section | Flow Rate, $q$, m³/s | Enthalpy, $h$, J/kg | Flow Direction (Relative to the Device) |
|---------|---------------------|---------------------|------------------------------------------|
| 1 | 8 | 250 | In |
| 2 | 6 | 150 | In |
| 3 | 14 | 200 | Out |

*Solution*  Confirm the mass balance. For incompressible flow,

$$q_1 + q_2 = q_3$$
$$8 + 6 = 14 \, \text{m}^3/\text{s}$$

Apply the total energy balance, noting that only enthalpy effects need to be considered for this flow system

$$\Delta \dot{H} = 800[(8)(250) + (6)(150) - (14)(200)] = 80,000 \, \text{J/s}$$

For adiabatic steady operation

$$\dot{Q} = 0$$

so that

$$\Delta \dot{H} = \dot{W}_s = 80 \, \text{kW} = 107.2 \, \text{hp}$$

Since work is positive, the surroundings must be doing work on the system through some device.

### 8.3.1 The Mechanical Energy Balance Equation

As noted, the solutions to many fluid flow problems are based on the mechanical energy balance equation. This equation is derived, in part, from the general (or total) energy equation developed in the previous subsection. Equation (8.13) is shown again below:

$$P_1 V_1 + \frac{v_1^2}{2g_c} + \frac{g}{g_c} z_1 + E_1 + Q + W_s = P_2 V_2 + \frac{v_2^2}{2g_c} + \frac{g}{g_c} z_2 + E_2$$

Certain "changes" to the above equation are now made:

1. Assume adiabatic flow, that is, $Q = 0$.
2. For isothermal, or near isothermal, flow (valid in most applications), the internal energy is constant, so that $E_1 = E_2$.
3. A term $\sum F$, representing the total friction arising due to fluid flow, is added to the equation. This is treated as a positive term in Equation (8.18) below.
4. An efficiency (fractional) term, $\eta$, is combined with the shaft work term, $W_s$. If work is imparted on the system, the term becomes $\eta W_s$; if work is extracted (with an engine or turbine) the term appears as $W_s/\eta$. The efficiency term needs to be included since part of the work added to or extracted from the system is lost due to irreversibilities associated with the mechanical device. The notation $h_s$ will be employed for this term in Chapter 9.

Equation (8.14) now becomes

$$\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2g_c} + \frac{g}{g_c} \Delta z - \eta W_s + \sum F = 0 \tag{8.18}$$

This equation is defined as the mechanical energy balance equation; it will receive extensive attention later in the book.

### 8.3.2 The Bernoulli Equation

The Bernoulli equation has come to mean different things to different people. One definition of this equation is obtained by neglecting both work and friction effects. Under this condition, Equation (8.18) reduces to

$$\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2g_c} + \frac{g}{g_c} \Delta z = 0 \tag{8.19}$$

This is often referred to as the Field equation in other disciplines.

This equation, which applies to flow in the absence of friction, has some interesting ramifications. If one of the three terms is increased, either of the other two terms must decrease; alternately, both of the other two terms can change but the sum of the

two changes must decrease. For example, if the Bernoulli equation is applied along a horizontal streamline (path) of a fluid, an increase in the velocity results in a decrease in pressure. The phenomenon is "exploited" by birds during flight, and at the industrial level in the design of airplane wings.

The above effect can also explain why roofs are lifted off some buildings during a hurricane or tornado; the high velocity on top of the roof creates a lower pressure at the outer surface relative to the inner surface. This difference in pressure—force per unit area—across the roof's top surface produces a net upward force lifting the roof off its foundation. This can be prevented in many instants by simply opening all windows and doors; the high velocity within the structure produces a lower pressure and consequently a smaller or zero upward force.

The reader might like to test the validity of the proposed explanation by taking a sheet of $8\frac{1}{2}''$ by $11''$ paper and holding it by its sides while allowing the paper to droop. Blowing across the top of the paper does in fact result in the paper rising to a near horizontal level.

Bernoulli's equation is valid for steady-state flow. However, if the flow is not steady, but the changes in flow rate are slow enough to be ignored, then Bernoulli's equation may still be applied. In tank flow problems for example, the velocity of the fluid in the tank is taken to be the rate of change of liquid height with time, that is, $v = dz/dt$. When this velocity is combined with Bernoulli's equation, the result is a differential equation. The solution of the equation normally requires integration. To justify the assumption that the flow rate is slow, the flow acceleration, $a$, is calculated and compared to the gravity acceleration, $g$. If $a/g$ is $\ll 1$, the above assumption may be assumed valid.

**Illustrative Example 8.3**    Refer to Illustrative Example 3.5 in Chapter 3. What is the kinetic energy of the water just before it reaches ground level conditions, that is, strikes the bottom?

*Solution*    Apply the conservation law for energy to the 1 kg of water. During free fall, $\Delta E = 0$, and the only terms that remain are the $KE$ and $PE$. Therefore,

$$\Delta(PE) + \Delta(KE) = 0$$

since both

$$KE_1 = PE_2 = 0$$
$$KE_2 = PE_1 = 98\,\text{J}$$

**Illustrative Example 8.4**    Refer once again to Illustrative Example 3.5. If the 1 kg of water enters a river upon reaching ground conditions, what change has occurred to the water.

*Solution*    Physically, it is still liquid. Energy-wise, it has lost the $PE$ it started with. That $PE$ has been converted to the internal energy of the entire river; however, the temperature change of the river would be neglible.

**Illustrative Example 8.5**   A cylindrical tank (see Fig. 8.3) with a diameter of 3 m has an outlet hole of 0.3 m in diameter at its bottom. The initial water level is 9 m. How long does it take the liquid level to drop to 1 m above the tank outlet? Justify the assumption of using Bernoulli's equation. The density of water is 1000 kg/m³.

**Solution**   Assume the control volume (CV) to be the liquid in the tank. A mass balance on the CV yields

$$\rho S_1 v_1 = \rho S_2 v_2$$

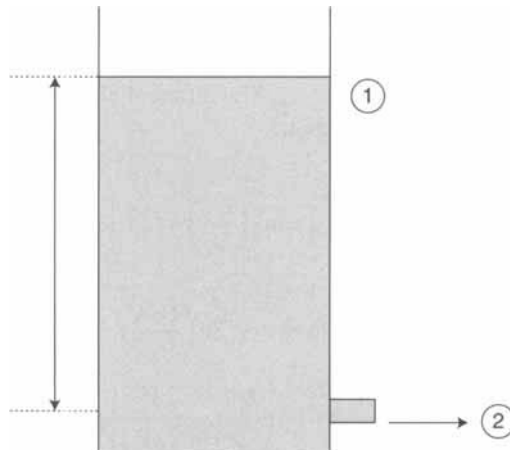$$v_2 = v_1 \frac{S_1}{S_2} = v_1 \left(\frac{D_1}{D_2}\right)^2$$

Set

$$v_1 = \frac{dz}{dt}$$

Substituting into the above equation gives

$$v_2 = -\left(\frac{D_1}{D_2}\right)^2 \frac{dz}{dt}$$

From Bernoulli's equation

$$\frac{P_1}{\rho} + \frac{v_1^2}{2g_c} + \frac{g}{g_c} z_1 = \frac{P_2}{\rho} + \frac{v_2^2}{2g_c} + \frac{g}{g_c} z_2$$
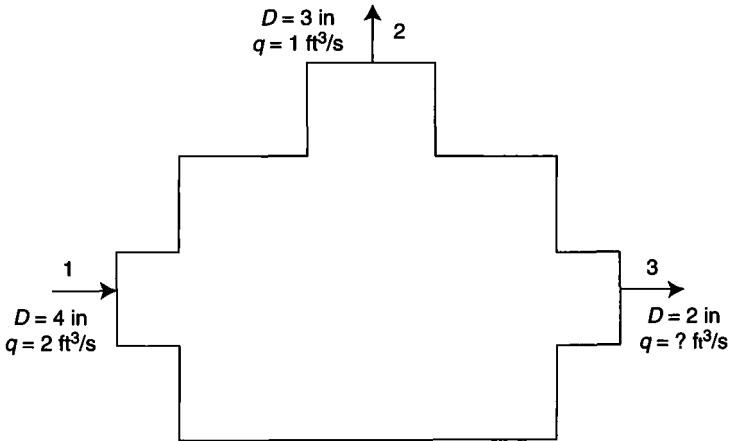


**Figure 8.3**  Tank drainage.

**Figure 8.4**   Fluid flow device.

From Fig. 8.3, it is clear that $P_1 = P_2$, since both ends are open to the atmosphere. Likewise, $v_1 = 0$, and $z_2$ is arbitrarily set to equal zero for convenience. Therefore,

$$\frac{v_2^2}{2g_c} = \frac{g}{g_c} z_1$$

or

$$v_2 = \sqrt{2gz_1}$$

where $z_1$ is the height at location 1. However, since $z_1$ varies, it is replaced by $h$, the height of water in the tank. Equating the two velocity terms for $v_2$ leads to

$$\frac{dh}{dt} = -\left(\frac{D_2}{D_1}\right)^2 \sqrt{2gh}$$

The initial conditions are at $t = 0$, $h = z_1$. The differential equation may be solved by separating the variables and integrating:

$$\int_{z_1}^{h_2} \frac{dh}{h^{1/2}} = -\left(\frac{D_2}{D_1}\right)^2 \sqrt{2g} \int_0^t dt$$

$$2(h_2^{1/2} - z_1^{1/2}) = -\sqrt{2g}\left(\frac{D_2}{D_1}\right)^2 t$$

$$t = \frac{2(z_1^{1/2} - h_2^{1/2})}{\sqrt{2g}\left(\frac{D_2}{D_1}\right)^2} = \frac{2(9^{1/2} - 1^{1/2})}{\sqrt{2(9.807)}(0.1)^2} = 90.3 \text{ s}$$

Justify the use of Bernoulli's equation.

$$a = \frac{dv_2}{dt} = \frac{d}{dt}(\sqrt{2gh}) = \frac{1}{2}\sqrt{\frac{2g}{h}\frac{dh}{dt}}$$

Substituting for $dh/dt$ and noting

$$D\sqrt{h} = \frac{dh}{2\sqrt{h}}$$

$$\frac{a}{g} = -\left(\frac{D_2}{D_1}\right)^2 - \left(\frac{0.3}{3.0}\right)^2 = -0.01$$

For this example, the maximum acceleration is 1% of $g$. One can therefore safely use Bernoulli's equation.

**Illustrative Example 8.6**   Explain in layman terms, why the $\sum F$ term in Equation (8.18) is positive.

*Solution*   This steady-state conservation law, as written, is derived on an

$$\text{out} - \text{in} = 0$$

basis. Thus, the work term is positive only if it is removed (lost) from the system. The same applies to any energy term. Frictional effects give rise to energy that is lost from the system and must therefore be retained as a positive term.

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