7

CONSERVATION LAW FOR MASS

7.1 INTRODUCTION

The three principles underlying the analysis of fluid flow are: the conservation law of mass, the conservation law of energy, and thirdly the conservation law of momentum. The first is expressed in the form of a material (mass) balance, the second in the form of an energy balance, and the third in the form of a momentum balance. These three topics are treated in this and the following two chapters.

7.2 CONSERVATION OF MASS

The conservation law for mass can be applied to any process or system. The general form of this law is given by Equation (7.1):

$$\{\text{mass in}\} - \{\text{mass out}\} + \{\text{mass generated}\} = \{\text{mass accumulated}\} \qquad (7.1)$$

or on a time rate basis

$$\{\text{rate of mass in}\} - \{\text{rate of mass out}\} + \{\text{rate of mass generated}\} = \{\text{rate of mass accumulated}\} \qquad (7.2)$$

This has also come to be defined as the continuity equation, a topic which also receives treatment in this chapter.

Fluid Flow for the Practicing Chemical Engineer. By J. Patrick Abulencia and Louis Theodore

This equation may be applied either to the total mass involved or to a particular species, on either a mole or mass basis. In many processes, it is often necessary to obtain quantitative relationships by writing mass balances on the various elements in the system.

In order to isolate a system for study, it is separated from the surroundings by a boundary or envelope. This boundary may be real (e.g., the walls of an incinerator) or imaginary. Mass crossing the boundary and entering the system is part of the *mass in* term in Equation (7.2), while that crossing the boundary and leaving the system is part of the *mass out* term.

Equation (7.2) may be written for any compound, the quantity of which is not changed by chemical reaction, and for any chemical element whether or not it has participated in a chemical reaction. It may be written for one piece of equipment, around several pieces of equipment, or around an entire process. It may be used to calculate an unknown quantity directly, to check the validity of experimental data, or to express one or more of the independent relationships among the unknown quantities in a particular problem situation.

This law can be applied to steady-state or unsteady state (transient) processes and to batch or continuous systems. A steady-state process is one in which there is no change in conditions (pressure, temperature, composition, etc.) or rates of flow with time at any given point in the system. The accumulation term in Equation (7.2) is then zero. (If there is no chemical or nuclear reaction, the generation term is also zero.) All other processes are unsteady state. In a batch process, a given quantity of reactants is placed in a container, and by chemical and/or physical means, a change is made to occur. At the end of the process, the container (or adjacent containers to which material may have been transferred) holds the product or products. In a continuous process, reactants are continuously removed from one or more points. A continuous process may or may not be steady-state. A coal-fired power plant, for example, operates continuously. However, because of the wide variation in power demand between peak and slack periods, there is an equally wide variation in the rate at which the coal is fired. For this reason, power plant problems may require the use of average data over long periods of time. Most industrial operations are assumed to be steady-state and continuous.

As indicated previously, Equation (7.2) may be applied to the total mass of each stream (referred to as an *overall* or *total* material balance) or to the individual component(s) of the streams (referred to as a *componential* or *component* material balance). The primary task in preparing a material balance in engineering calculations is often to develop the quantitative relationships among the streams. The primary factors, therefore, are those that tie the streams together. An element, compound, or unreactive mass (ash, for example) that enters or leaves in a single stream or passes through a process unchanged is so convenient for this purpose that it may be considered a key to the calculations. If sufficient data is given about this component, it can be used in a component balance to determine the total masses of the entering and exiting streams. Such a component is sometimes referred to as a *key component*. Since a key component does not react in a process, it must retain its identity as it passes through the process. Obviously, except for nuclear reactions, elements may always be used as key components because they do not change identity even though they may be involved in a chemical reaction. Thus, CO (carbon monoxide) may be used as a key component only when it does not react, but C (carbon) may always be used as a key component. A component that enters the system in only one stream and leaves in only one stream is usually the most convenient choice for a key component.

Four important processing concepts are *bypass, recycle, purge*, and *makeup*. With *bypass*, part of the inlet stream is diverted around the equipment to rejoin the (main) stream after the unit (see Fig. 7.1). This stream effectively moves in parallel with the stream passing through the equipment. In *recycle*, part of the product stream is sent back to mix with the feed. If a small quantity of nonreactive material is present in the feed to a process that includes recycle, it may be necessary to remove the nonreactive material in a *purge* stream to prevent its building up above a maximum tolerable value. This can also occur in a process without recycle; if a nonreactive material is added in the feed and not totally removed in the products, it will accumulate until *purged*. The *purging* process is sometimes referred to as *blowdown*. *Makeup*, as its name implies, involves adding or making up part of a stream that has been removed from a process. *Makeup* may be thought of, in a final sense, as the opposite of *purge* and/or *blowdown*.[1,2]

The conservation law for mass may be applied to a flowing fluid in a process. Consider the equipment depicted in Fig. 7.2, focusing attention on the part between Sections 1 and 2. No fluid is entering or leaving the process between these sections. Under steady conditions, the mass flow rates at each section are identical since there would otherwise be a progressive accumulation or depletion of fluid within the unit between the two sections. The result can be expressed in the following form:

$$\dot{m} = \rho_1 v_1 S_1 = \rho_2 v_2 S_2 = G_1 S_1 = G_2 S_2 \qquad (7.3)$$

where $\dot{m}$ represents the mass flow rate; S, the cross-section area; v, the average velocity (equal to the volumetric flow rate, q, divided by the cross-section area); ρ, the mass density (i.e., mass per unit volume); and, G, the mass velocity or mass

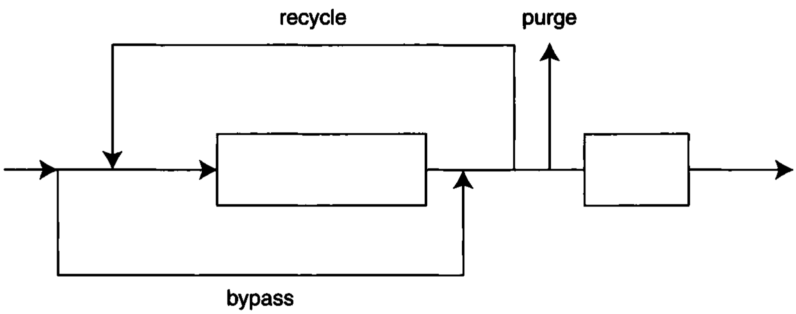

Figure 7.1 Recycle, bypass, and purge.

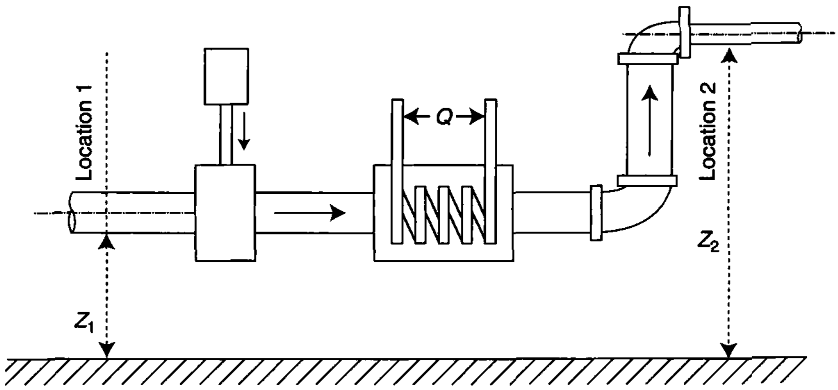

Figure 7.2 Process flow.

flux, equal to the mass flow rate divided by the cross-section, that is, the *mass flux*, G, is the mass flow rate of fluid, $\dot{m}$ per unit area of the duct, S:

$$G = \frac{\dot{m}}{S} \tag{7.4}$$

This is also equal to

$$G = \rho v \tag{7.5}$$

Illustrative Example 7.1 A gaseous waste is fed into an incinerator at a rate of 4000 kg/hr in the presence of 6000 kg/hr of air. Due to the low heating value of the waste, 550 kg/hr of methane is added to assist the combustion of the pollutants in the waste stream. Determine the rate of product gases exiting the incinerator in kg/hr. Assume steady-state operation.

Solution Apply the conservation law for mass to the incinerator on a rate basis. See Equation (7.2)

$$\{\text{rate of mass in}\} - \{\text{rate of mass out}\} + \{\text{rate of mass generated}\}$$
$$= \{\text{rate of mass accumulated}\}$$

Rewrite the equation subject to the conditions in the example statement

$$\{\text{rate of mass in}\} = \{\text{rate of mass out}\}$$

or

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}}$$

Calculate $\dot{m}_{\text{in}}$

$$\dot{m}_{\text{in}} = 4000 + 8000 + 550 = 12{,}550\,\text{kg/hr}$$

Also, determine $\dot{m}_{out}$. For steady-state flow conditions,

$$\dot{m}_{in} = \dot{m}_{out} = 12{,}550\,\text{kg/hr}$$

Illustrative Example 7.2 Water ($\rho = 1000\,\text{kg/m}^3$) flows in a converging circular pipe (see Fig. 7.3). It enters at Section 1 and leaves at Section 2. At Section 1, the inside diameter is 14 cm and the velocity is 2 m/s. At Section 2, the inside diameter is 7 cm. Determine the mass and volumetric flow rates, the mass flux of water, and the velocity at Section 2. Assume steady-state flow.

Solution Calculate the flow rates, q and $\dot{m}$, based on the information at Section (station) 1:

$$S_1 = \frac{\pi D_1^2}{4} = \frac{\pi(0.14)^2}{4} = 0.0154\,\text{m}^2$$

$$q_1 = S_1 v_1 = 0.0154(2) = 0.031\,\text{m}^3/\text{s}$$

$$\dot{m}_1 = \rho q_1 = 1000(0.031) = 31\,\text{kg/s}$$

Obtain the mass flux, G, with units of $\text{kg/m}^2 \cdot \text{s}$—see Equation (7.4):

$$G = \frac{m_1}{S_1} = \frac{31}{0.0154} = 2013\,\text{kg/m}^2 \cdot \text{s}$$

Finally, calculate the velocity at cross-section 2, v_2. For steady flow,

$$q_1 = q_2 = 0.031\,\text{m}^3/\text{s}$$

Since

$$v_2 S_2 = v_1 S_1$$

$$v_2 = v_1 \frac{S_1}{S_2} = v_1 \frac{D_1{}^2}{D_2{}^2} = 2\frac{14^2}{7^2} = 8\,\text{m/s}$$

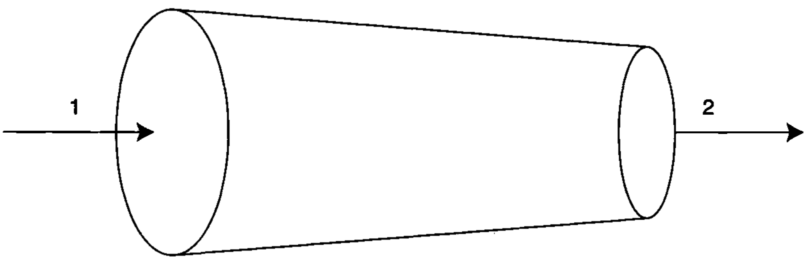

Figure 7.3 Converging pipe.

As expected, the decrease in cross-section area results in an increase in the flow velocity for steady-state flow of an incompressible fluid.

Illustrative Example 7.3 A fluid device has four openings, as shown in Fig. 7.4. The fluid has a constant density of $800\,\mathrm{kg/m^3}$. The steady-state conditions are listed in Fig. 7.4. Determine the magnitude and direction of the velocity, v_4. What is the mass flow rate at section 4?

Solution Calculate the volumetric flow rate through each section

$$q_1 = v_1S_1 = 5(0.2) = 1\,\mathrm{m^3/s}$$

$$q_2 = v_2S_2 = 7(0.3) = 2.1\,\mathrm{m^3/s}$$

$$q_3 = v_3S_3 = 12(0.25) = 3\,\mathrm{m^3/s}$$

Apply the continuity equation assuming the flow is exiting section 4

$$q_1 + q_2 = q_3 + q_4$$

$$q_4 = q_1 + q_2 - q_3 = 1 + 2.1 - 3 = 0.1\,\mathrm{m^3/s}$$

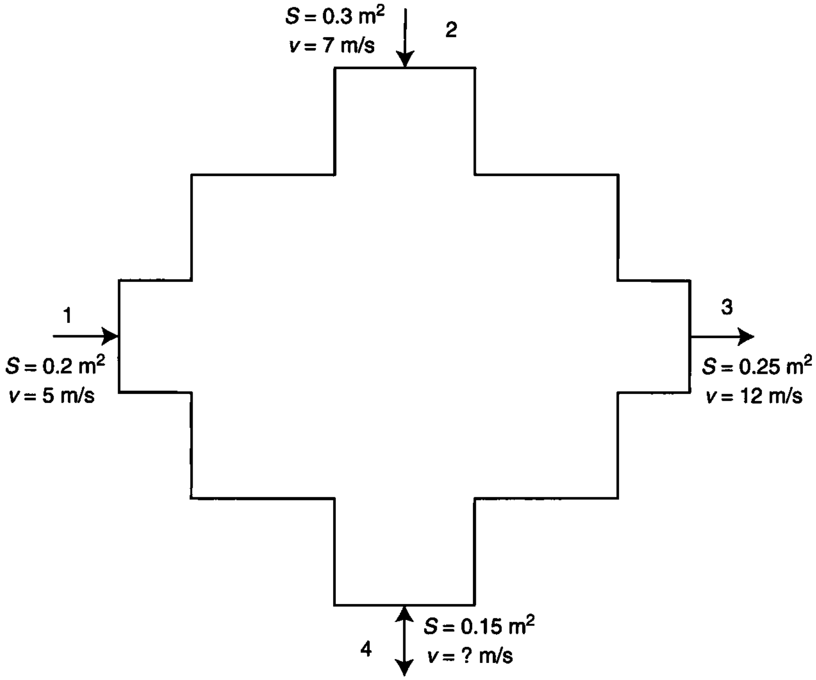

Figure 7.4 Fluid flow device.

The positive value of the calculated mass flow rate indicates the correctness of the assumption that the flow is out (and down) through section 4.

Calculate the velocity, v_4:

$$v_4 = \frac{q_4}{S_4} = \frac{0.1}{0.15} = 0.667\,\text{m/s}$$

Also calculate the mass flow rate at section 4

$$\dot{m} = \rho q_4 = 800(0.1) = 80\,\text{kg/s}$$

Illustrative Example 7.4 A liquid stream contaminated with a pollutant is being cleansed with a control device. If the liquid has 600 ppm (parts per million) of pollutant, and it is permissible to have 50 ppm of this pollutant in the discharge stream, what fraction of the liquid, B, can bypass the control device?

Solution Use a basis of 1 lb of liquid fed to the control device. The flow diagram in Fig. 7.5 applies to this system. Note that:

$$B = \text{fraction of liquid bypassed}$$
$$1 - B = \text{fraction of liquid treated}$$

Performing a pollutant mass balance around point 2 in Fig. 7.5 yields

$$(1 - B)(0) + 600B = (50)(1.0)$$

Solving gives

$$B = 0.0833 = 8.33\%$$

Illustrative Example 7.5 A vertical tank 1.4 m in diameter and 1.9 m high, contains water to a height of 1.5 m. Water flows into the tank through a 9 cm pipe

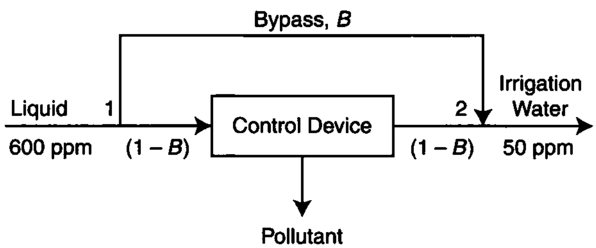

Figure 7.5 Flow diagram.

with a velocity of 4 m/s. Water leaves the tank through a 4 cm pipe at a velocity of 3 m/s. Is the level in the tank rising or falling?

Solution Select the control volume (CV). Take the CV to be the instantaneous mass (or volume V) of water in the tank and apply the continuity principle to the CV. Since the generation rate = 0,

$$\left(\frac{dm}{dt}\right) = \dot{m}_{in} - \dot{m}_{out}$$

For an incompressible fluid of volume V:

$$\frac{dV}{dt} = q_{in} - q_{out}$$

since

$$V = Sz; \quad \text{where} \quad z = \text{fluid height (water)}$$

$$\frac{dV}{dt} = S\frac{dz}{dt}$$

Therefore

$$S\frac{dz}{dt} = q_{in} - q_{out}$$

Calculate the cross-section area and volumetric flow rates

$$q_{in} = \frac{\pi D^2}{4}v = \frac{\pi(0.09\,\text{m})^2}{4}\,4\,\text{m/s}$$

$$q_{in} = 0.0255\,\text{m}^3/\text{s}$$

$$q_{out} = \frac{\pi D^2}{4}v = \frac{\pi(0.04\,\text{m})^2}{4}3\,\text{m/s}$$

$$q_{out} = 0.0038\,\text{m}^3/\text{s}$$

$$S = \pi(1.4\,\text{m})^2/4$$

$$S = 1.539\,\text{m}^2$$

Substitute in the above material balance differential equation

$$1.539\frac{dz}{dt} = 0.0255 - 0.0038$$

$$\frac{dz}{dt} = 0.0141\,\text{m/s}$$

$$\text{at } t = 0,\ z = 1.5\,\text{m}$$

Because dz/dt is positive, the water level is rising in the tank from its initial height of 1.5 m.

7.3 MICROSCOPIC APPROACH

The equation of continuity describes the variation of density with position and time in a moving or stationary fluid, and may be viewed as an extension of the conservation law for mass. It can also be used to simplify the equations of energy transfer and momentum transfer. In addition, it serves as an excellent warm-up for the presentation of the equation of momentum transfer in the next chapter. The continuity equation can be developed by applying the conservation law for mass to a fixed volume element in a moving one-component one-phase fluid. The derivation is available in the literature.[3,4]

The continuity equation is written in rectangular, cylindrical and spherical coordinates. The results are presented in Table 7.1.

Table 7.1 Equation of continuity for incompressible fluids

Rectangular coordinates (x, y, z):

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \qquad (1)$$

Cylindrical coordinates (r, ϕ, z):

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z} = 0 \qquad (2)$$

Spherical coordinates (r, θ, ϕ):

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\left(\frac{\partial v_\phi}{\partial\phi}\right) = 0 \qquad (3)$$

Illustrative Example 7.6 The velocity of incompressible fluid in a steady-state system is directed along the y rectangular coordinate. Prove that the velocity is not a function of y.

Solution For an incompressible fluid, see Equation (1) in Table 7.1:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

One concludes from the problem statement that

$$v_x = 0$$
$$v_z = 0$$

Therefore

$$\frac{\partial v_x}{\partial x} = 0 \quad \text{since } v_x = 0$$
$$\frac{\partial v_z}{\partial z} = 0 \quad \text{since } v_z = 0$$

Finally,

$$\frac{\partial v_y}{\partial y} = 0$$

If v_y is a function of y, then $\partial v_y/\partial y$ cannot equal zero. The velocity therefore, is not a function of y.

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