6

NON-NEWTONIAN FLOW

6.1 INTRODUCTION

The study of the mechanics of the flow of liquids and suspensions comes under the science of *Rheology*. The name Rheology was chosen by Prof. John R. Crawford of Lafayette College, PA, and is defined as the study of the flow and deformation of matter. (The name is a combination of the Greek words "Rheo"-flow and "Logos"-theory.)

The shear-stress equations developed in the previous chapter were written for fluids with a viscosity that is constant at constant temperature and independent of the rate of shear and the time of application of shear. Fluids with this property were defined as Newtonian fluids. All gases and pure low-molecular-weight liquids are Newtonian. Miscible mixtures of low-molecular-weight liquids are also Newtonian. On the other hand, high-viscosity liquids as well as polymers, colloids, gels, concentrated slurries and solutions of macromolecules generally do not exhibit Newtonian properties; i.e., a strict proportionality between stress and strain rate. Interestingly, non-Newtonian properties are sometimes desirable. For example, non-Newtonian behavior is exhibited in many paints. During brush working, certain paints flow readily to cover the surface, but upon standing, the original highly viscous condition returns and the paint will not run.

The study of non-Newtonian fluids has not progressed far enough to develop many useful theoretical approaches. As noted in the previous chapter, if the liquid or suspension is found to be Newtonian, the pressure drop can be calculated from the "Poiseuille" equation for laminar flow (see Chapter 13) and the Fanning equation for turbulent flow (see Chapter 14), using the density and viscosity of the liquid or

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suspension. For non-Newtonian liquids and suspensions, the viscosity is a variable, and the procedure for computing the pressure drop is more involved.

The remainder of this section will discuss non-Newtonian liquids and suspensions. Useful engineering design procedures and prediction equations receive treatment that are limited to isothermal laminar (viscous) flow. The turbulent flow of non-Newtonian fluids (as with Newtonian ones) is characterized by the presence of random eddies and whirls of fluid that cause the instantaneous values of velocity and pressure at any point in the system to fluctuate wildly. Because of these fluctuations, flow problems cannot be easily solved. Since non-Newtonian turbulent flow rarely occurs, it has not received much attention.

6.2 CLASSIFICATION OF NON-NEWTONIAN FLUIDS

Fluids can be classified based on their viscosity. An imaginary fluid of zero viscosity is called a *Pascal fluid.* The flow of a Pascal fluid is termed *inviscid* (or non-viscous) flow. Viscous fluids are classified based on their rheological (viscous) properties. These are detailed below:

1. Newtonian fluids, as described in the previous chapter, obey Newton's law of viscosity (i.e., the fluid shear stress is linearly proportional to the velocity gradient). All gases are considered Newtonian fluids. Newtonian liquid examples are water, benzene, ethyl alcohol, hexane and sugar solutions. All liquids of a simple chemical formula are normally considered Newtonian fluids.
2. Non-Newtonian fluids do not obey Newton's law of viscosity. Generally they are complex mixtures (e.g., polymer solutions, slurries, and so on). Non-Newtonian fluids are classified into three types:
   a. Time-independent fluids are fluids in which the viscous properties do not vary with time.
   b. Time-dependent fluids are fluids in which the viscous properties vary with time.
   c. Visco-elastic or memory fluids are fluids with elastic properties that allow them to "spring back" after the release of a shear force. Examples include egg-white and rubber cement.

Additional details on the first two classes of fluids follow.

3. Time-independent, non-Newtonian fluids are further classified into three types.
   a. *Pseudoplastic* or shear thinning fluids are characterized by a fluid resistance decrease with increasing stress (e.g., polymers).
   b. *Dilatant* or shear thickening fluids increase resistance with increasing velocity gradient or applied stress. These are uncommon, but an example is quicksand.

c. *Bingham plastics* are fluids that resist a small shearing stress. At low shear stress these fluids do not move. At high shear the fluids move. The fluid just starts moving when sufficient stress is applied. This stress is termed the *yield stress*. When the applied stress exceeds the yield stress, the Bingham plastic flows. Examples are toothpaste, jelly, and bread-dough.

4. Time-dependent, non-Newtonian fluids are further classified into two types.
   a. *Rheopectic* fluids are characterized by an increasing viscosity with time. Rubber cement is an example.
   b. *Thixotropic* fluids have a decreasing viscosity with time. Examples are slurries or solutions of polymers.

6.2.1 Non-Newtonian Fluids: Shear Stress[1]

There are the aforementioned class of fluids that do not obey Newton's law of viscosity. These were defined as *non-Newtonian* and several different types of these fluids exist. The shear stress equation equivalent to Equation (5.7) for one of the more common types of non-Newtonian fluids is given by the so-called "power law" equation:

$$\tau_{zy} = -\frac{K}{g_c}\left(\frac{\mathrm{d}v_y}{\mathrm{d}z}\right)^n \tag{6.1}$$

K is defined as the *consistency number* and may in special cases equal μ. The exponent n is defined as the *flow-behavior index* and is a real number that usually assumes a value other than unity. Although n is considered a physical property of the fluid, it is not necessarily a constant; rather, it may vary with the shear rate, $\mathrm{d}v_y/\mathrm{d}z$. Equation (6.1) may be written in terms of the apparent viscosity μ_a for non-Newtonian fluids (most non-Newtonian fluids have apparent viscosities that are relatively high compared with the viscosity of water)

$$\frac{\mu_a}{g_c} = -\frac{\tau_{zy}}{\mathrm{d}v_y/\mathrm{d}z}$$

or

$$\mu_a = K\left(\frac{\mathrm{d}v_y}{\mathrm{d}z}\right)^{n-1} \tag{6.2}$$

In order to remove the problem arising when the velocity gradient is a negative quantity, Equation (6.1) is rewritten as

$$\tau_{zy} = -\frac{K}{g_c}\frac{\mathrm{d}v_y}{\mathrm{d}z}\left|\frac{\mathrm{d}v_y}{\mathrm{d}z}\right|^{n-1} \tag{6.3}$$

A typical shear stress vs. shear rate ($\mathrm{d}v_y/\mathrm{d}z$) curve (often referred to as a rheogram), is shown for a non-Newtonian fluid in Fig. 6.1 on arithmetic coordinates. Newtonian

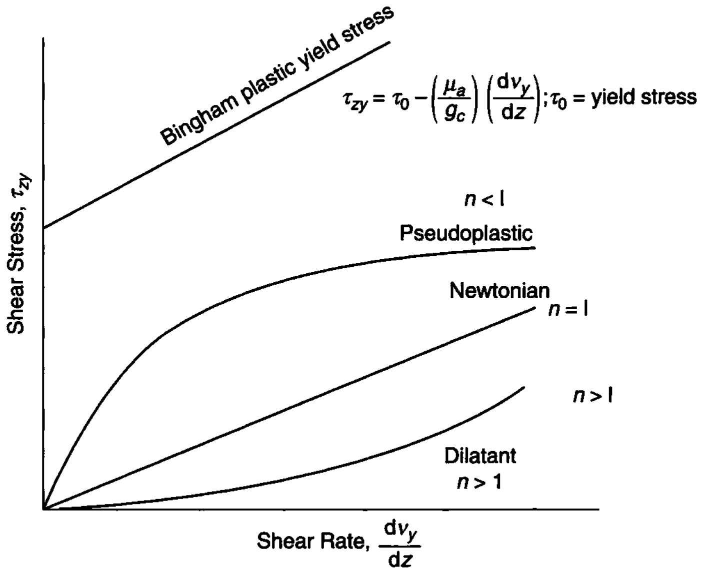

Figure 6.1 Fluid shear diagrams.

behavior is also depicted in the diagram. Due to the exponential nature of the shear rate of this type of non-Newtonian fluid, a straight line would be obtained on a log-log plot as demonstrated in Equations (6.4) and (6.5):

$$\tau_{zy} = -\frac{K}{g_c}\left(\frac{dv_y}{dz}\right)^n \tag{6.4}$$

$$\log \tau_{zy} = -\log\left(\frac{K}{g_c}\right) + n\log\left(\frac{dv_y}{dz}\right) \tag{6.5}$$

One notes that a Newtonian fluid yields a slope of 1.0 on log-log coordinates. The slope of a non-Newtonian fluid generally differs from unity. The slope, n, can be thought of as an index to the degree of non-Newtonian behavior in that the farther that n is from unity (above or below), the more pronounced is the non-Newtonian characteristics of the fluid.

Illustrative Example 6.1 For each of the following four classes of fluids, indicate the line type (straight or curved) on a logarithmic shear diagram, i.e., shear stress versus shear rate.

1. Newtonian
2. Pseudoplastic
3. Dilatant
4. Bingham plastic

Solution Refer to Fig. 6.1.

1. Newtonian: Straight
2. Pseudoplastic: Straight
3. Dilatant: Straight
4. Bingham plastic: Curved

Illustrative Example 6.2 For each of the following four classes of fluids, indicate the line slope (>1, 1, or <1) on a logarithmic shear diagram, i.e., shear stress versus shear rate.

1. Newtonian
2. Pseudoplastic
3. Dilatant
4. Bingham plastic

Solution Refer once again to Fig. 6.1.

1. Newtonian: 1
2. Pseudoplastic: <1
3. Dilatant: >1
4. Bingham plastic: not applicable

Illustrative Example 6.3 Classify the following substances according to their rheological behavior: paint, grease, toothpaste, tar, silly putty, and ordinary putty.

Solution The classification is tabulated below:

SUBSTANCE	EXPLANATION
Paint	Shear-thinning (pseudo-plastic). Also rheopectic (it hardens with time)
Grease	Bingham plastic (needs a yield stress before flowing). Visco-elastic
Toothpaste	Ideal Bingham plastic
Tar	Pseudoplastic at high temperature
Silly putty	Dilatant (shear thickening). Visco-elastic
Ordinary putty	Visco-elastic

6.3 MICROSCOPIC APPROACH

Most non-Newtonian fluids either follow the power law relationship provided in Equation (6.4) or may be approximated by it for engineering purposes. The presentation below is therefore limited to power-law applications.

6.3.1 Flow in Tubes

The reader is referred to the Microscopic Approach section in the previous chapter on Newtonian flow. One may now re-examine the flow of a fluid through a horizontal tube under the condition that it follows the power law relationship

$$\tau_{rz} = K\left(\frac{\mathrm{d}v_z}{\mathrm{d}r}\right)^n \tag{6.6}$$

Wohl[2] has shown that the velocity profile for the above system is given by

$$v_z = \left(\frac{n}{n+1}\right)\left(\frac{\Delta P}{2LK}\right)^{1/n}[R^{(n+1)n} - r^{(n+1)/n}] \tag{6.7}$$

Illustrative Example 6.4 Verify that Equation (6.7) reduces to the velocity profile relationship provided in Equation (5.16) for a Newtonian fluid.

Solution One notes that for $n = 1$, where the fluid is Newtonian, Equation (6.7) reduces to

$$v_z = \left(\frac{\Delta P}{4\mu L}\right)[R^2 - r^2]$$

This is, as one would expect, the same equation provided in the previous chapter for flow of a Newtonian fluid through a pipe (see Eq. (5.16)).

The above equation may also be written in terms of the maximum centerline velocity, v_{max}.

$$v_z = v_{\mathrm{max}}\left[1 - \left(\frac{r}{R}\right)^{(n+1)/n}\right] \tag{6.8}$$

Alternately, the local velocity can be expressed in terms of the average velocity, v_{av}.

$$v_z = v_{\mathrm{av}}\left(\frac{3n+1}{n+1}\right)\left[1 - \left(\frac{r}{R}\right)^{(n+1)n}\right] \tag{6.9}$$

For a Bingham plastic, the local velocity is given by

$$v_z = \frac{\Delta P}{4\mu L}(R^2 - r^2) - \frac{\tau_0}{\mu}(R - r) \tag{6.10}$$

For values of $r_p < r < R$ where

$$r_p = \frac{2L\tau_0}{\Delta P} \tag{6.11}$$

For $r_p > r > 0$, the describing equation

$$v_z = \frac{\Delta P}{2L\mu}(R - r)^2 \tag{6.12}$$

6.3.2 Flow Between Parallel Plates

For flow between parallel plates of height H, length L, and width W, Wohl[3] has shown that the local velocity is given by

$$v = \left(\frac{n}{n+1}\right)\left(\frac{\Delta P}{LK}\right)^{1/n}\left[\left(\frac{H}{2}\right)^{(n+1)/n} - (z)^{(n+1)/n}\right] \tag{6.13}$$

where z is the vertical Cartisian coordinate constrained by $z = \Delta \pm H/2$.

Illustrative Example 6.5 Refer to Equation (6.13). Generate an equation that describes the maximum velocity.

Solution For the maximum velocity, set $z = 0$ in Equation (6.13).

$$v = \left(\frac{n}{n+1}\right)\left(\frac{\Delta P}{LK}\right)^{1/n}\left[\left(\frac{H}{2}\right)^{(n+1)/n} - (0)^{(n+1)/n}\right]$$

$$= \left(\frac{n}{n+1}\right)\left(\frac{\Delta P}{LK}\right)^{1/n}\left(\frac{H}{2}\right)^{(n+1)/n}$$

Illustrative Example 6.6 Starting with Equation (6.13), obtain the equation for the velocity profile if the fluid is Newtonian.

Solution If the fluid is Newtonian, $n = 1$. Therefore, set $n = 1$ and $K = \mu$ in Equation (6.13).

$$v = \left(\frac{\Delta P}{L\mu}\right)\left[\left(\frac{H}{2}\right) - z\right]$$

$$= \frac{\Delta P}{2\mu L}(H - 2z)$$

Illustrative Example 6.7 Refer to Equation (6.13). Obtain an equation describing the volumetric flow rate q.

Solution By definition, the integral below

$$q_{1/2} = \int_{z=0}^{z=H/2} Wv \, dz$$

provides the volumetric flow rate passing the upper half of the system. Substituting for v,

$$q_{1/2} = W \int_{z=0}^{z=H/2} \left(\frac{n}{n+1}\right)\left(\frac{\Delta P}{LK}\right)^{1/n} \left[\left(\frac{H}{2}\right)^{(n+1)/n} - (z)^{(n+1)/n}\right] dz$$

$$= W\left(\frac{n}{n+1}\right)\left(\frac{\Delta P}{LK}\right)^{1/n} \int_0^{H/2} \left(\frac{H}{2}\right)^{(n+1)/n} - (z)^{(n+1)/n} \Bigg] dz$$

$$= W\left(\frac{n}{n+1}\right)\left(\frac{\Delta P}{LK}\right)^{1/n} \left[\left(\frac{H}{2}\right)^{(n+1)/n}(z) - \frac{(z)^{[(n+1)/n]+1}}{[(n+1)/n]+1}\right]_0^{H/2}$$

$$= W\left(\frac{n}{n+1}\right)\left(\frac{\Delta P}{LK}\right)^{1/n} \left[\left(\frac{H}{2}\right)^{(n+1)/n}\left(\frac{H}{2}\right) - \frac{(H/2)^{(2n+1)/n}}{2n+1}\right]$$

$$= W\left(\frac{n}{n+1}\right)\left(\frac{\Delta P}{LK}\right)^{1/n} \left[\left(\frac{H}{2}\right)^{(2n+1)/n} - \frac{(H/2)^{(2n+1)/n}}{(2n+1)/n}\right]$$

$$= W\left(\frac{n}{n+1}\right)\left(\frac{\Delta P}{LK}\right)^{1/n}\left(\frac{H}{2}\right)^{1/n}\left(\frac{H}{2}\right)^2 \left[1 - \frac{1}{(2n+1)/n}\right]$$

$$= W\left(\frac{n}{2n+1}\right)\left(\frac{H\Delta P}{2LK}\right)^{1/n}\left(\frac{H}{4}\right)^2 \left[\frac{2n+1}{n}\right]$$

$$= \frac{WH^2}{4}\left(\frac{n}{2n+1}\right)\left(\frac{H\Delta P}{2LK}\right)^{1/n}$$

$$= \left(\frac{n}{8n+4}\right) WH^2 \left(\frac{H\Delta P}{2LK}\right)^{1/n}$$

The total volumetric flow rate q is

$$q = 2q_{1/2}$$

$$= \left(\frac{n}{4n+2}\right) WH^2 \left(\frac{H\Delta P}{LK}\right)^{1/n}$$

For $n = 0.5$

$$q = \left(\frac{0.5}{4}\right) WH^2 \left(\frac{H\Delta P}{LK}\right)^2$$

$$= \frac{1}{8} WH^2 \left(\frac{H\Delta P}{LK}\right)^2$$

$$= \frac{1}{8} WH^2 \left(\frac{H\Delta P}{2LK}\right)$$

$$= \left(\frac{1}{2}\right) WH^2 \left(\frac{H\Delta P}{2LK}\right)^2$$

6.3.3 Other Flow Geometries

Kozicki[4] has developed simple and useful expressions for the flow of several time-independent non-Newtonian fluids in ducts of various shapes. The equations contain two shape factors, and a function of the stress, which characterize the fluid. Numerical values of the shape factors have been determined for circular, slit, concentric annular, rectangular, elliptical, and isosceles triangular ducts. The reader is referred to Kozicki's work[4] for the formulas by which these shape factors are calculated, and for a tabulated list of values to four significant figures. The derived equations are for the average and the maximum velocities as functions of the shape factors, hydraulic radius, parameters of the constitutive equations, and average shear stress at the duct wall. This average shear stress is defined by:

$$\tau_0 = r_H \Delta P/L \tag{6.14}$$

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