5

NEWTONIAN FLUIDS

5.1 INTRODUCTION

This chapter is introduced by examining the units of some of the pertinent quantities that will be encountered below. The momentum of a system is defined as the product of the mass and velocity of the system.

$$\text{Momentum} = (\text{Mass})(\text{Velocity}) \tag{5.1}$$

One set of units for momentum are, therefore, $\text{lb} \cdot \text{ft/s}$. The units of time rate of change of momentum (hereafter referred to as rate of momentum) are simply the units of momentum divided by time, i.e.,

$$\text{Rate of momentum} \equiv \frac{\text{lb} \cdot \text{ft}}{\text{s}^2} \tag{5.2}$$

The above units can be converted to lb_f if multiplied by an appropriate constant. The conversion constant in this case is a term that was developed in Chapter 2.

$$g_c = 32.2\frac{(\text{lb} \cdot \text{ft})}{(\text{lb}_\text{f} \cdot \text{s}^2)} \tag{5.3}$$

Fluid Flow for the Practicing Chemical Engineer. By J. Patrick Abulencia and Louis Theodore

This serves to define the conversion constant g_c. If the rate of momentum is divided by g_c as 32.2 $(\text{lb} \cdot \text{ft})/(\text{lb}_\text{f} \cdot \text{s}^2)$—the following units result:

$$\text{Rate of momentum} \equiv \left(\frac{\text{lb} \cdot \text{ft}}{\text{s}^2}\right)\left(\frac{\text{lb}_\text{f} \cdot \text{s}^2}{\text{lb} \cdot \text{ft}}\right)$$
$$\equiv \text{lb}_\text{f} \tag{5.4}$$

One may conclude from the above dimensional analysis that a force is equivalent to a rate of momentum.

5.2 NEWTON'S LAW OF VISCOSITY

The above development is now extended to Newton's law of viscosity. Consider a fluid flowing between the region bounded by two infinite parallel horizontal plates separated by a distance h. The flow is steady and only in the y-direction. Part of the system is represented in Fig. 5.1. A sufficient force F is being applied to the upper plate at $z = h$ to maintain the upper plate in motion with a velocity $v_y = V_h$. If the fluid density is constant and the flow is everywhere isothermal and laminar, the linear velocity gradient in the two-dimensional representation in Fig. 5.2 will result.

It has been shown by experiment that the applied force per unit area F/A required to maintain the upper plate in motion with velocity V_h is proportional to the velocity gradient, i.e.,

$$\frac{F}{A} \propto \frac{V_h}{h}$$

For a slightly more general form, one may write

$$\frac{F}{A} \propto \frac{\Delta v_y}{\Delta z} \tag{5.5}$$

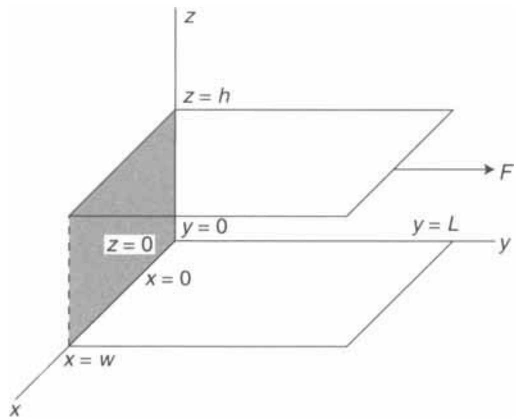

Figure 5.1 Fluid/two-plate system.

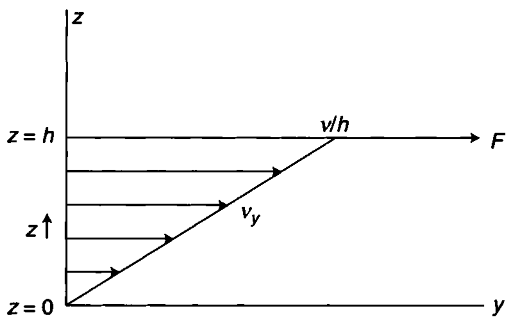

Figure 5.2 Velocity profile.

The difference term Δ can be removed by applying Equation (5.5) to a differential width dz:

$$\frac{F}{A} \propto \frac{dv_y}{dz} \tag{5.6}$$

Equation (5.6) may be written in equation form by replacing the proportionality sign with a proportionality constant, $-\mu$:

$$\frac{F}{A} = -\mu \frac{dv_y}{dz} \tag{5.7}$$

The term μ is defined as the coefficient of viscosity, or simply the aforementioned viscosity of the fluid. The term F/A is a shear stress since F is exerted parallel to the direction of motion. This applied force per unit area is now designated by τ_{zy},

$$\tau_{zy} = -\mu \frac{dv_y}{dz} \tag{5.8}$$

A fluid whose shear stress is described by Equation (5.8) is defined as a Newtonian fluid.

A word of interpretation is in order for Equation (5.8). The applied force at $z = h$ has resulted in a velocity V_h at $z = h$. The fluid at this point possesses momentum due to this velocity. As z decreases the momentum of the fluid decreases since the velocity decreases in this direction. We have already shown that the force applied to a fluid is equivalent to the fluid receiving a rate of momentum. Part of the momentum imparted to the fluid at $z = h$ is transferred at the specified rate to the slower-moving fluid immediately below it. This momentum maintains the velocity of the fluid at that point, and is, in turn, transported to the slower fluid below it, and so on. This momentum transfer process is occurring in the z-direction throughout the fluid. One may therefore conclude the applied force in the positive y-direction has resulted in the transfer of momentum in the negative z-direction. The first subscript in τ_{zy} is retained as a reminder of this fact. The subscript y indicates the direction of motion. The negative sign in Equation (5.7) was introduced since momentum is transferred in the negative z-direction due to a positive velocity gradient.

The force per unit area term τ is equivalent to a rate of momentum per unit area. Therefore, the shear stress and its components are also defined as the momentum flux.

Referring once again to the shear stress component τ_{zy}, one may divide the RHS of Equation (5.8) by g_c,

$$\tau_{zy} = -\frac{\mu}{g_c}\frac{dv_y}{dz} \tag{5.8}$$

If τ_{zy} has the units lb_f/ft^2, the viscosity μ assumes the units $lb/ft \cdot s$.

A term that will frequently be employed in the text is the kinematic viscosity ν (see Chapter 3). This is defined as the ratio of the viscosity to the density of the fluid.

$$\nu = \frac{\mu}{\rho} \tag{5.9}$$

The units of v can be shown to be ft^2/s.

All components of the shear stress for a Newtonian fluid can be expressed in terms of the viscosity of the fluid and a velocity gradient. These are presented, but not derived, in Table 5.1[1,2] for rectangular, cylindrical, and spherical coordinates. The equations are applicable to all Newtonian fluids provided:

1. The system is isothermal.
2. Flow is laminar.
3. The fluid density is constant.

Procedures for predicting viscosity values from theory are beyond the scope of this text, but available in the literature.[1]

Illustrative Example 5.1 A fluid of viscosity μ is flowing in the y-direction between two infinite horizontal parallel plates. The velocity profile of the fluid is given by

$$v_y = V\left(\frac{z}{h} - z^2\right)$$

where V and h are constants.

Calculate the shear stress at the surface $z = 0$ in terms of μ, V, and h.

Solution This problem is solved using rectangular coordinates. First note that v_x and v_z equal zero and v_y is solely a function of z. From Table 5.1,

$$\begin{aligned}\tau_{zy} &= -\frac{\mu}{g_c}\left(\frac{\partial v_y}{\partial z} + \cancelto{0}{\frac{\partial v_z}{\partial y}}\right)\\ &= -\frac{\mu}{g_c}\frac{dv_y}{dz} \quad \text{(since } v_y \text{ is solely a function of } z\text{)}\end{aligned}$$

Table 5.1 Shear–stress components

Component	Rectangular Coordinates	Cylindrical Coordinates	Spherical Coordinates
τ_{11}	$\tau_{xx} = -\frac{\mu}{g_c}\left[2\left(\frac{\partial v_x}{\partial x}\right)\right]$	$\tau_{rr} = -\frac{\mu}{g_c}\left[2\left(\frac{\partial v_r}{\partial r}\right)\right]$	$\tau_{rr} = -\frac{\mu}{g_c}\left[2\left(\frac{\partial v_r}{\partial r}\right)\right]$
τ_{12}	$\tau_{xy} = -\frac{\mu}{g_c}\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]$	$\tau_{r\phi} = -\frac{\mu}{g_c}\left[r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{1}{r}\left(\frac{\partial v_r}{\partial \phi}\right)\right]$	$\tau_{r\theta} = -\frac{\mu}{g_c}\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\left(\frac{\partial v_r}{\partial \theta}\right)\right]$
τ_{13}	$\tau_{xz} = -\frac{\mu}{g_c}\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right]$	$\tau_{rz} = -\frac{\mu}{g_c}\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]$	$\tau_{r\phi} = -\frac{\mu}{g_c}\left[\frac{1}{r\sin\theta}\left(\frac{\partial v_r}{\partial \phi}\right) + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]$
τ_{21}	$\tau_{yx} = -\frac{\mu}{g_c}\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]$	$\tau_{\phi r} = -\frac{\mu}{g_c}\left[r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) + \frac{1}{r}\left(\frac{\partial v_r}{\partial \phi}\right)\right]$	$\tau_{\theta r} = -\frac{\mu}{g_c}\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\left(\frac{\partial v_r}{\partial \theta}\right)\right]$
τ_{22}	$\tau_{yy} = -\frac{\mu}{g_c}\left[2\left(\frac{\partial v_y}{\partial y}\right)\right]$	$\tau_{\phi\phi} = -\frac{\mu}{g_c}\left[2\left(\frac{1}{r}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r}\right)\right]$	$\tau_{\theta\theta} = -\frac{\mu}{g_c}\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)\right]$
τ_{23}	$\tau_{yz} = -\frac{\mu}{g_c}\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right]$	$\tau_{\phi r} = -\frac{\mu}{g_c}\left[\frac{\partial v_\phi}{\partial z} + \frac{1}{r}\left(\frac{\partial v_z}{\partial \phi}\right)\right]$	$\tau_{\theta\phi} = -\frac{\mu}{g_c}\left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\left(\frac{\partial v_\theta}{\partial_\phi}\right)\right]$
τ_{31}	$\tau_{zx} = -\frac{\mu}{g_c}\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right]$	$\tau_{zr} = -\frac{\mu}{g_c}\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]$	$\tau_{\phi r} = -\frac{\mu}{g_c}\left[\frac{1}{r\sin\theta}\left(\frac{\partial v_r}{\partial \phi}\right) + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]$
τ_{32}	$\tau_{zy} = -\frac{\mu}{g_c}\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right]$	$\tau_{z\phi} = -\frac{\mu}{g_c}\left[\frac{\partial v_\phi}{\partial z} + \frac{1}{r}\left(\frac{\partial v_z}{\partial \phi}\right)\right]$	$\tau_{\phi\theta} = -\frac{\mu}{g_c}\left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{\tau\sin\theta}\left(\frac{\partial v_\theta}{\partial \phi}\right)\right]$
τ_{33}	$\tau_{zz} = -\frac{\mu}{g_c}\left[2\left(\frac{\partial v_z}{\partial z}\right)\right]$	$\tau_{zz} = -\frac{\mu}{g_c}\left[2\left(\frac{\partial v_z}{\partial z}\right)\right]$	$\tau_{\phi\phi} = -\frac{\mu}{g_c}\left[2\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta\cot\theta}{r}\right)\right]$

The velocity profile is given as:

$$v_y = V\left[\frac{z}{h} - z^2\right]$$

so that,

$$\frac{dv_y}{dz} = \frac{V}{h} - 2Vz$$

$$-\mu\frac{dv_y}{dz} = 2\mu Vz - \frac{V\mu}{h}$$

The shear stress at the surface $z = 0$ is denoted by $\tau_{zy}|_{z=0}$:

$$\tau_{zy}\Big|_{z=0} = \frac{\mu}{g_c}\left[2V(0) - \frac{V}{h}\right]$$

$$= -\mu V/g_c h$$

Illustrative Example 5.2 Two vertical parallel plates are spaced 1 inch apart. The plate on the left side is moving at a velocity of 5 ft/min in the z-direction and the plate on the right side is stationary. The space between the plates contains a gas whose kinematic viscosity is 1.66 ft^2/hr and density is 0.08 lb/ft^3.

1. Calculate the force necessary to maintain the movement of the left plate.
2. Calculate the momentum flux at the surface of the left plate and at the surface of the right plate.

Solution Note that based on no slip conditions, the velocity of the gas at the surface of the moving plate is equal to the velocity of the plate and the velocity of the gas at the surface of the stationary plate is zero.

1. Calculate the force per unit area of plate; this is the shear stress (τ_y) that can be evaluated from the appropriate equation in Table 5.1. For this application,

$$\tau_{xy} = -\frac{\mu}{g_c}\left(\frac{\partial v_y}{\partial x}\right)$$

Since $x_1 = 0$, $x_2 = 0.0833$ ft, $v_1 = (5)(60)$ ft/hr, $v_2 = 0$,

$$\tau_{xy} = -(1.66)(0.08)\left(\frac{0 - 300}{0.0833 - 0}\right)$$

$$= 478\frac{\text{lb}\cdot\text{ft/hr}}{\text{ft}^2\cdot\text{hr}}$$

Since,

$$g_c = 32.2 \frac{\text{ft} \cdot \text{lb/s}}{\text{lb}_\text{f} \cdot \text{s}} = 4.17 \times 10^8 \frac{\text{ft} \cdot \text{lb/hr}}{\text{lb}_\text{f} \cdot \text{hr}}$$

$$\tau_{xy} = \frac{478}{4.17 \times 10^8}$$

$$= 1.15 \times 10^{-6}\,\text{lb}_\text{f}/\text{ft}^2$$

5.3 VISCOSITY MEASUREMENTS

One of the simplest methods to measure viscosity is to time the discharge of a known volume of fluid through a nozzle. A vessel with a short capillary tube is employed. This equipment is known as the *Saybolt viscometer*. It has been used to determine the viscosities of oils and paints. Another common technique is to measure the torque required to rotate a torque element in a liquid (e.g., *Brookfield viscometer* and *coaxial cylindrical viscometers*). On a *Couette–Hatschek viscometer* (or *MacMichael viscometer*), the outer member of a pair of closely fitting coaxial cylinders is rotated, while in the *Stormer viscometer*, the inner member of a pair of closely fitting cylinders is rotated (see Figs. 5.3 and 5.4). The clearance between the two

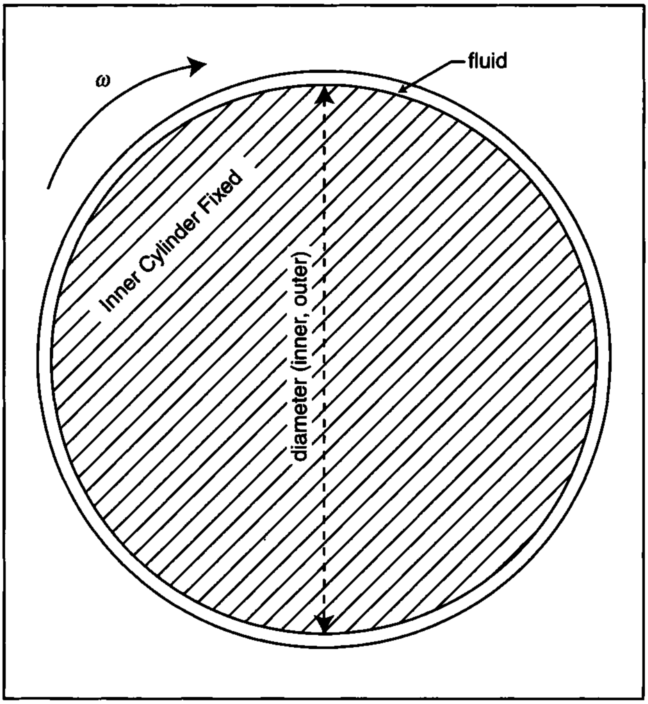

Figure 5.3 Couette-Hatschek viscometer.

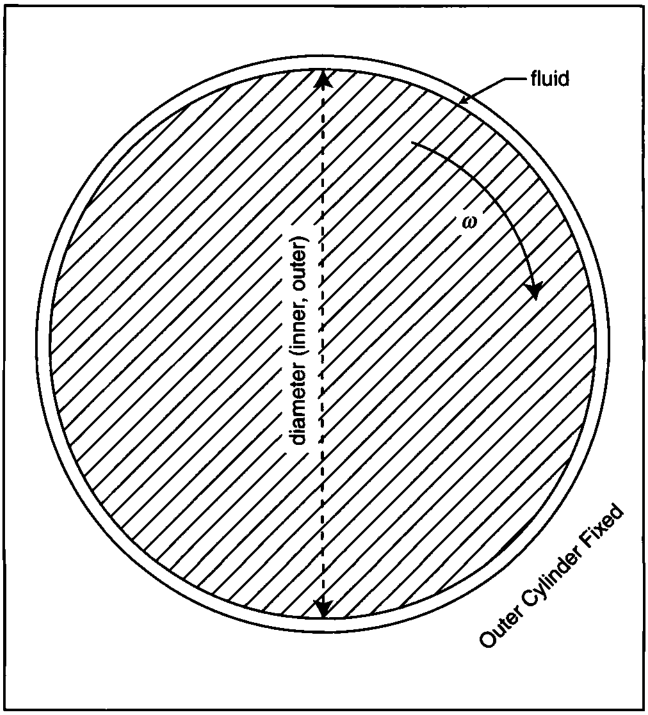

Figure 5.4 Stormer viscometer.

cylinders is so small (relative to the cylinder dimensions) that a linear velocity profile may be assumed in the fluid filling the gap. By measuring the torque, T, required to rotate the cylinder at a specified angular velocity, ω (rad/s), it is possible to calculate the fluid absolute viscosity and/or *fluidity*, where the *fluidity* is defined as the reciprocal of viscosity. In the SI system of units, the fluidity unit is known as the "rhe" (1 rhe = 1/poise = 1 s/g · cm).

The definitions and equations for the calculation of the fluid viscosity from these viscometers are given below:

$$\begin{aligned}
\text{Torque, } T &= (\text{force})(\text{cylinder radius}) = (\text{force})(\text{diameter}/2) \\
&= (F)(D/2) && (5.10) \\
\text{Force, } F &= (\text{shear stress})(\text{surface area of cylinder}) \\
&= (\tau)(\pi D L) && (5.11) \\
\text{Shear stress, } \tau &= (\text{absolute viscosity})(\text{velocity gradient}) = (\mu)(\mathrm{d}v/\mathrm{d}y) \\
&= (\text{viscosity})(\text{velocity at the rotating cylinder})/ \\
&\quad\ (\text{gap separation}) = (\mu)(v/D) && (5.12) \\
\text{Velocity, } v &= \text{velocity of the rotating cylinder} \\
&= (\omega)(R) = (\omega)(D/2) && (5.13)
\end{aligned}$$

$$\begin{aligned}
\text{Radius, } R &= \text{radius of inside (or outside) cylinder} \\
\text{Diameter, } D &= \text{diameter of inside (or outside) cylinder} \\
\text{Height, } L &= \text{height of cylinder} \\
\text{Friction power loss, } W_L &= (\text{force})(\text{velocity}) = (F)(v) \\
d &= \text{gap separation, clearance}
\end{aligned} \tag{5.14}$$

Illustrative Example 5.3 A Couette–Hatschek viscometer is used to measure the viscosity of an oil (SG = 0.97). The viscometer used has a fixed inner cylinder of 3 inches diameter and 6 inches height, and a rotating outer cylinder of the same height. The clearance, d, between the two cylinders is 0.001 inch. The measured torque is 15.3 ft $\cdot$ lb$_f$ at an angular rotation speed of 250 rpm. Determine the shear stress in the oil. Assume the viscometer clearance gap is so small that the velocity distribution is assumed linear, that is, $dv/dy = \Delta v/\Delta y = v/d$.

Solution Calculate the force, F, employing Equation (5.10). Since $D = 3$ in $= 0.25$ ft and $L = 6$ in $= 0.5$ ft.

$$F = \frac{2T}{D} = \frac{2(15.3)}{0.25} = 122.4\,\text{lb}_f = 544.5\,\text{N}$$

Calculate the shear stress, τ (force parallel to the surface), using Equation (5.11),

$$\tau = \frac{F}{\pi DL} = \frac{122.4}{\pi(0.25)(0.5)} = 311.7 \text{ psf} = 14.924 \text{ kPa}$$

Illustrative Example 5.4 Refer to Illustrative Example 5.3. Determine the dynamic and kinematic viscosities.

Solution Calculate the linear velocity of the oil, v, from its angular velocity ω. See Equation (5.13).

$$\omega = 250\,\text{rpm} = \left(250\frac{\text{rev}}{\text{min}}\right)\left(2\pi\frac{\text{rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60\,\text{sec}}\right) = 26.2\,\text{rad/s}$$

$$v = \frac{\omega D}{2} = \frac{26.2(0.25)}{2} = 3.27 \text{ ft/s}$$

Calculate the velocity gradient

$$\frac{dv}{dy} = \frac{v}{d} = \frac{3.27}{(0.001/12)} = 39{,}270\,\text{s}^{-1}$$

Assume Newton's law of viscosity to apply and calculate the viscosity, μ, noting that $\tau = (\mu/g_c)(dv/dy)$. Rearranging yields

$$\mu = \frac{g_c\tau}{dv/dy} = \frac{(32.174)(311.7)}{39{,}270} = 0.256\,\mathrm{lb/ft\cdot s}$$

The kinematic viscosity, ν, is

$$\nu = \frac{\mu}{\rho} = \frac{0.256}{(62.4)(0.97)} = 0.00423\ \mathrm{ft^2/s}$$

5.4 MICROSCOPIC APPROACH

Consider the following application. A fluid is flowing through a long horizontal cylindrical duct of radius R under steady-state conditions (see Fig. 5.5). The general equation for the velocity profile in a pipe as a function of the pressure drop per unit length in the direction of motion has been shown to take the form[1,2]

$$v_z = -\frac{g_c\Delta P}{4\mu L}r^2 + A\ln r + B \tag{5.15}$$

where A and B are integration constants that are evaluated from the boundary and/or initial conditions (Ba/oICs) for the system in question. An equation describing the velocity profile in the tube can be generated. Referring to Fig. 5.5, one concludes

$$\text{BC(1): } v_z = 0 \quad \text{at } r = R$$

and

$$\left.\begin{aligned} &\text{BC(2): } v_z = \text{finite} \quad \text{at } r = 0 \\ &\text{or the equivalent: } \\ &\frac{dv_z}{dr} = 0 \quad \text{at } r = 0 \end{aligned}\right\}\text{based on physical grounds}$$

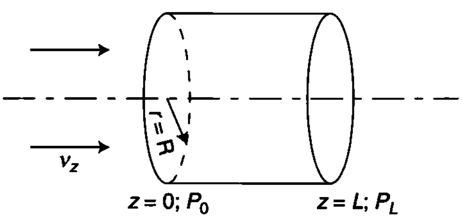

Figure 5.5 Horizontal flow in a tube.

Substituting BC(2) into Equation (5.15) yields

$$A = 0$$

BC(1) gives

$$0 = -\frac{g_c \Delta P}{4\mu L} R^2 + B$$

$$B = \frac{g_c \Delta P}{4\mu L} R^2$$

Equation (5.15) now becomes

$$v_z = \frac{g_c \Delta P}{4\mu L}(R^2 - r^2) \tag{5.16}$$

This equation will be derived and reviewed again in Chapters 9 and 13.

Another application involves fluid flowing between the region bounded by two infinite parallel horizontal plates separated by a distance h. The flow is steady and only in the y-direction. Part of the system is represented in Figs. 5.1 and 5.2. A sufficient force is applied to the upper plate to maintain a velocity V_h. The general equation for the velocity profile is given by[2]

$$v_y = \frac{g_c z^2}{2\mu} \frac{\Delta P}{\Delta y} + Bz + A \tag{5.17}$$

The boundary conditions (BC) are

$$\begin{aligned} &\text{BC(1)}: \quad v_y = 0 \quad \text{at } z = 0 \\ &\text{BC(2)}: \quad v_y = V_h \quad \text{at } z = h \end{aligned}$$

Substituting BC(1) into Equation (5.17) yields

$$0 = 0 + 0 + A$$

$$A = 0$$

Substituting BC(2) into Equation (5.17) gives

$$V_h = Bh + \frac{g_c h^2}{2\mu} \frac{\Delta P}{\Delta y}$$

$$B = \frac{V_h}{h} - \frac{g_c h}{2\mu} \frac{\Delta P}{\Delta y}$$

Therefore,

$$v_y = V_h\left(\frac{z}{h}\right) - \frac{g_c hz}{2\mu}\frac{\Delta P}{\Delta y} + \frac{g_c z^2}{2\mu}\frac{\Delta P}{\Delta y}$$

$$= V_h\left(\frac{z}{h}\right) - \frac{g_c z}{2\mu}(h - z)\frac{\Delta P}{\Delta y} \tag{5.18}$$

Since the fluid is not moving relative to fixed points on both plates, $\Delta P/\Delta y = 0$ and

$$v_y = V_h\left(\frac{z}{h}\right) \tag{5.19}$$

It would be wise at this point to verify that the above solution satisfies both the differential equation and BCs. We leave this exercise to the reader.

REFERENCES

1. R. Bird, W. Stewart, and E. Lightfoot, "Transport Phenomena", 2nd edition, John Wiley & Sons, Hoboken, NJ, 2002.
2. L. Theodore, "Transport Phenomena for Engineers", International Textbook Company, Scranton, PA, 1971.

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