UNITED ARAB EMIRATES
MINISTRY OF EDUCATION

YEAR OF
ZAYED

TEACHER EDITION

MATH

8

McGraw-Hill Education

Integrated Math

United Arab Emirates Edition

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Ministry of Education

Teacher Edition

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Integrated Math

United Arab Emirates Edition

GRADE **8** • VOLUME 2

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5. Triangles and Pythagorean Theorem, from Glencoe Math Course 3 Vol 2 Chapter 05 © 2015
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"Extensive knowledge and modern science must be acquired. The educational process we see today is in an ongoing and escalating challenge which requires hard work.
We succeeded in entering the third millennium, while we are more confident in ourselves."

H.H. Sheikh Khalifa Bin Zayed Al Nahyan
President of the United Arab Emirates

CONTENTS IN BRIEF

Units organized by domain

This book is organized into units based on groups called domains.
MP Mathematical Practices are embedded throughout the course.

Mathematical Practices Handbook

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UNIT 1
Domain 8.NS

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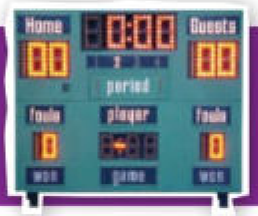

UNIT 5
Domain 8.SP

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Our lead authors ensure that the McGraw-Hill mathematics programs are truly vertically aligned by beginning with the end in mind - success in Algebra 1 and beyond. By "backmapping" the content from the high school programs, all of our mathematics programs are well articulated in their scope and sequence.

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Dr. Carol Malloy was a fervent supporter of mathematics education. She was a Professor at the University of North Carolina, Chapel Hill, NCTM Board of Directors member, President of the Benjamin Banneker Association (BBA), and 2013 BBA Lifetime Achievement Award for Mathematics winner. She joined McGraw-Hill in 1996. Her influence significantly improved our programs' focus on real-world problem solving and equity. We will miss her inspiration and passion for education.

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Mathematical Practices

MP Mathematical Practices	Student Edition
MP1 **Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	A strong problem-solving strand is present throughout the textbook with an emphasis on strategies in the Problem-Solving Investigation lessons. Look for the **Persevere with Problems** head in the exercises.
MP2 **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	Students are routinely asked to write an equation or an expression in order to solve a real-world problem. Exercises that emphasize this practice are labeled as **Reason Abstractly**.

Mathematical Practices	Student Edition
MP3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	Students are required to justify their reasoning in problems and to find the errors in samples of other's work. Look for these heads in the exercises: **Justify Conclusions** **Reason Inductively** **Make a Conjecture** **Use a Counterexample** **Find the Error** **Which One Doesn't Belong** **Make a Prediction** **Multiple Representations** **Construct an Argument**
MP4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	Real-world applications in problem solving are woven throughout every lesson. In addition to the real-world examples in each lesson, look for **Model with Mathematics** heads in the exercises.
MP5 Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	In addition to the traditional mathematical tools like estimating, using mental math, or measuring, students are encouraged to use software and the Internet in problem solving. Exercises utilizing this strategy are labeled with **Use Math Tools**.

Mathematical Practices	Student Edition
MP6 **Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	Solutions are not just numbers, but include measurements to give the solution meaning. Look for **Be Precise** heads in the exercises.
MP7 **Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	Emphasizing the structure of mathematics is present through use of classifying, explaining, giving examples as well as nonexamples. Exercises that emphasize this practice are labeled with **Identify Structure**.
MP8 **Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	Patterns in reasoning are demonstrated throughout leading students to sound mathematical conclusions. Exercises with **Identify Repeated Reasoning** heads exemplify this practice.

UNIT 1 The Number System

UNIT PROJECT PREVIEW page 2

Chapter 1

Real Numbers

Essential Question

WHY is it helpful to write numbers in different ways?

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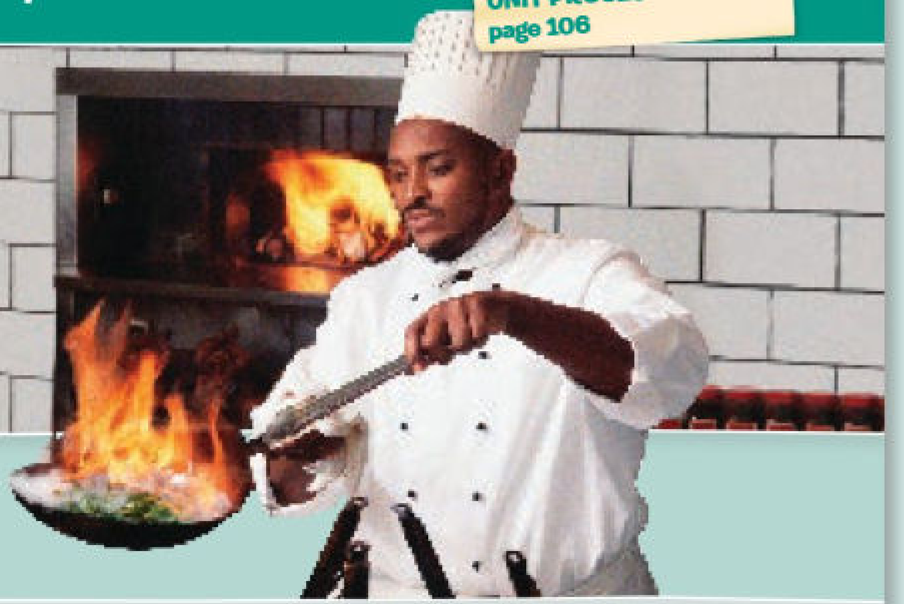
UNIT 2 Expressions and Equations

UNIT PROJECT PREVIEW page 106



Chapter 2
Equations in One Variable

Essential Question

WHAT is equivalence?

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Chapter 3

Equations in Two Variables

Essential Question

WHY are graphs helpful?

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UNIT 3 Functions

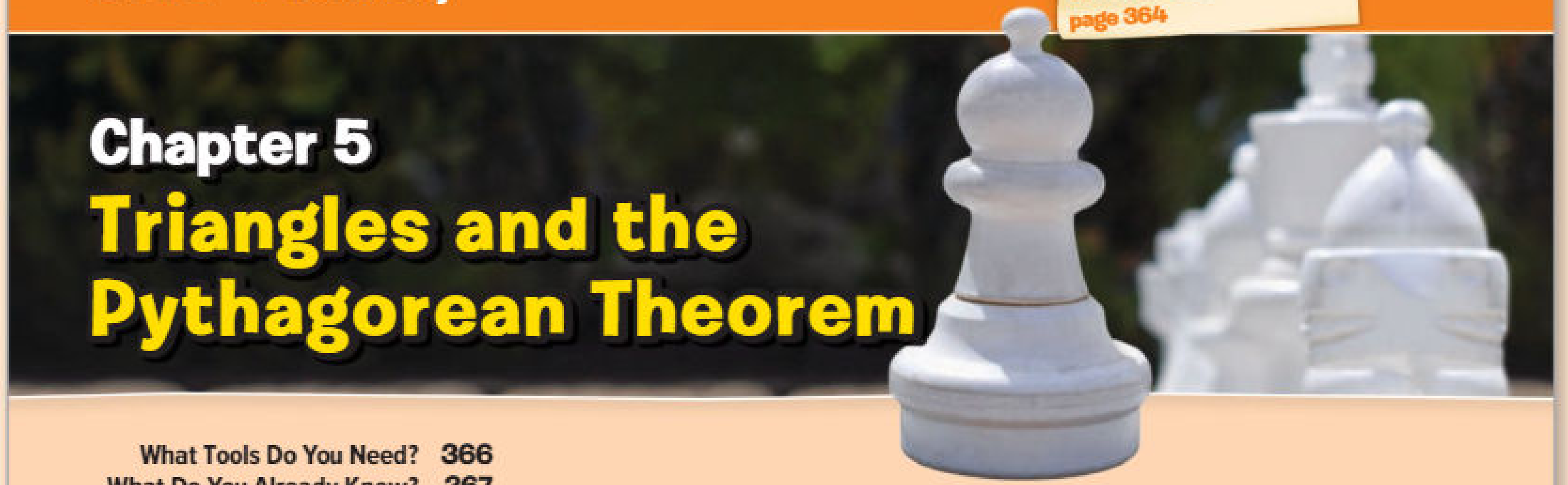
Chapter 4
Functions

Essential Question

HOW can we model relationships between quantities?

UNIT 4 Geometry

UNIT PROJECT PREVIEW
page 364



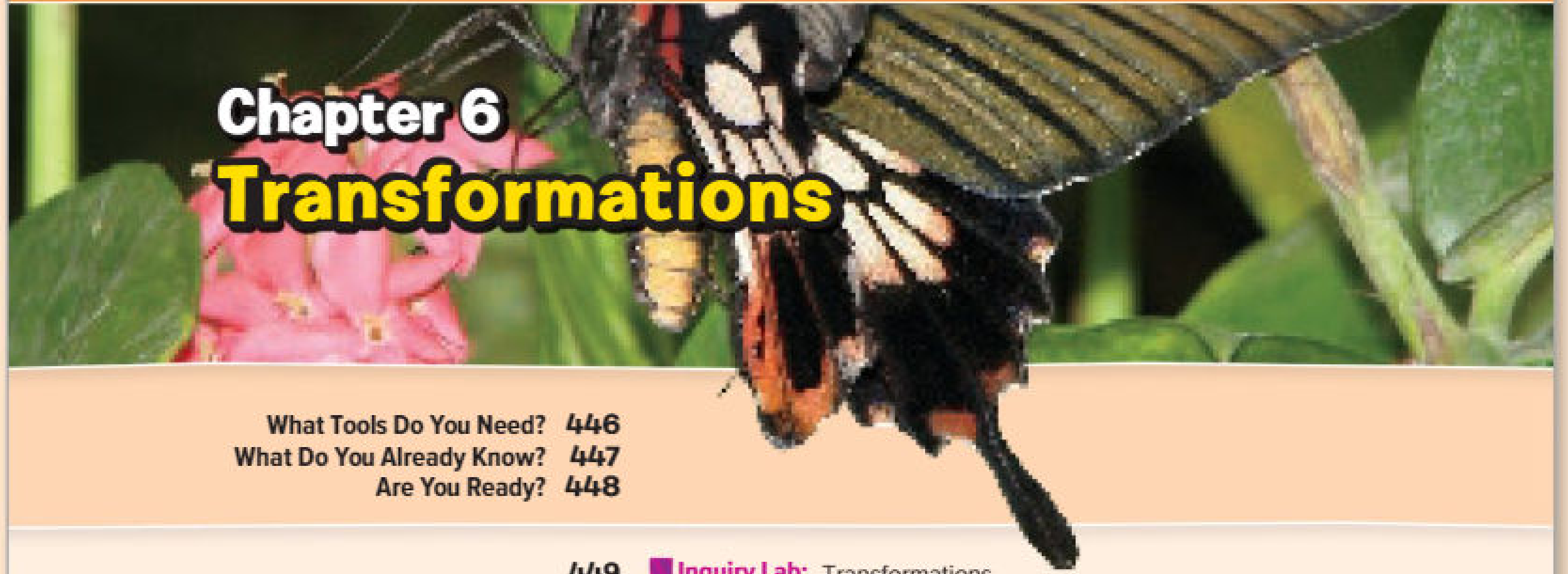
Chapter 5

Triangles and the Pythagorean Theorem

Essential Question

HOW can algebraic concepts be applied to geometry?

Valentyna Chukhlyebova/Alamy



Chapter 6

Transformations

Essential Question

HOW can we best show or describe the change in position of a figure?

Chapter 7
Congruence and Similarity

Essential Question

HOW can you determine congruence and similarity?

Carlos Davila/Photographer's Choice RF/Getty Images

Chapter 8
Volume and Surface Area

Essential Question

WHY are formulas important in math and science?

UNIT PROJECT

Design That Ride!

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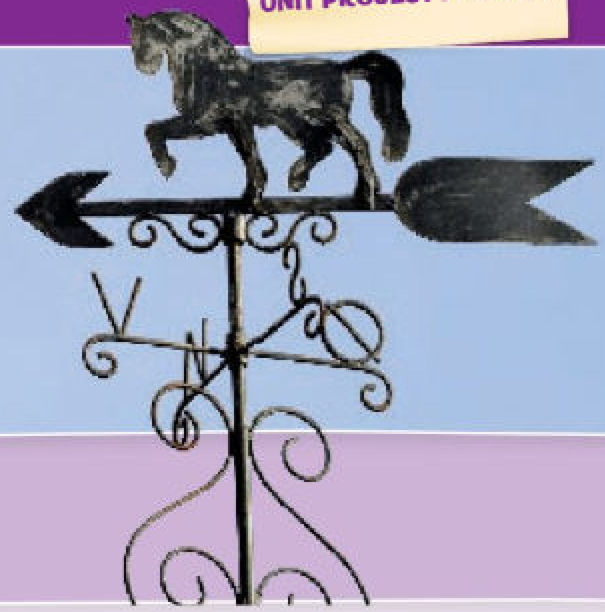
UNIT 5 Statistics and Probability

UNIT PROJECT PREVIEW



Chapter 9

Scatter Plots and Data Analysis

Essential Question

HOW are patterns used when comparing two quantities?

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This book focuses on three critical areas: (1) applying equations in one and two variables; (2) understanding the concept of a function and using functions to describe quantitative relationships; (3) applying the Pythagorean Theorem and the concepts of similarity and congruence.

Content

The Number System

- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous equations.

Functions

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Geometry

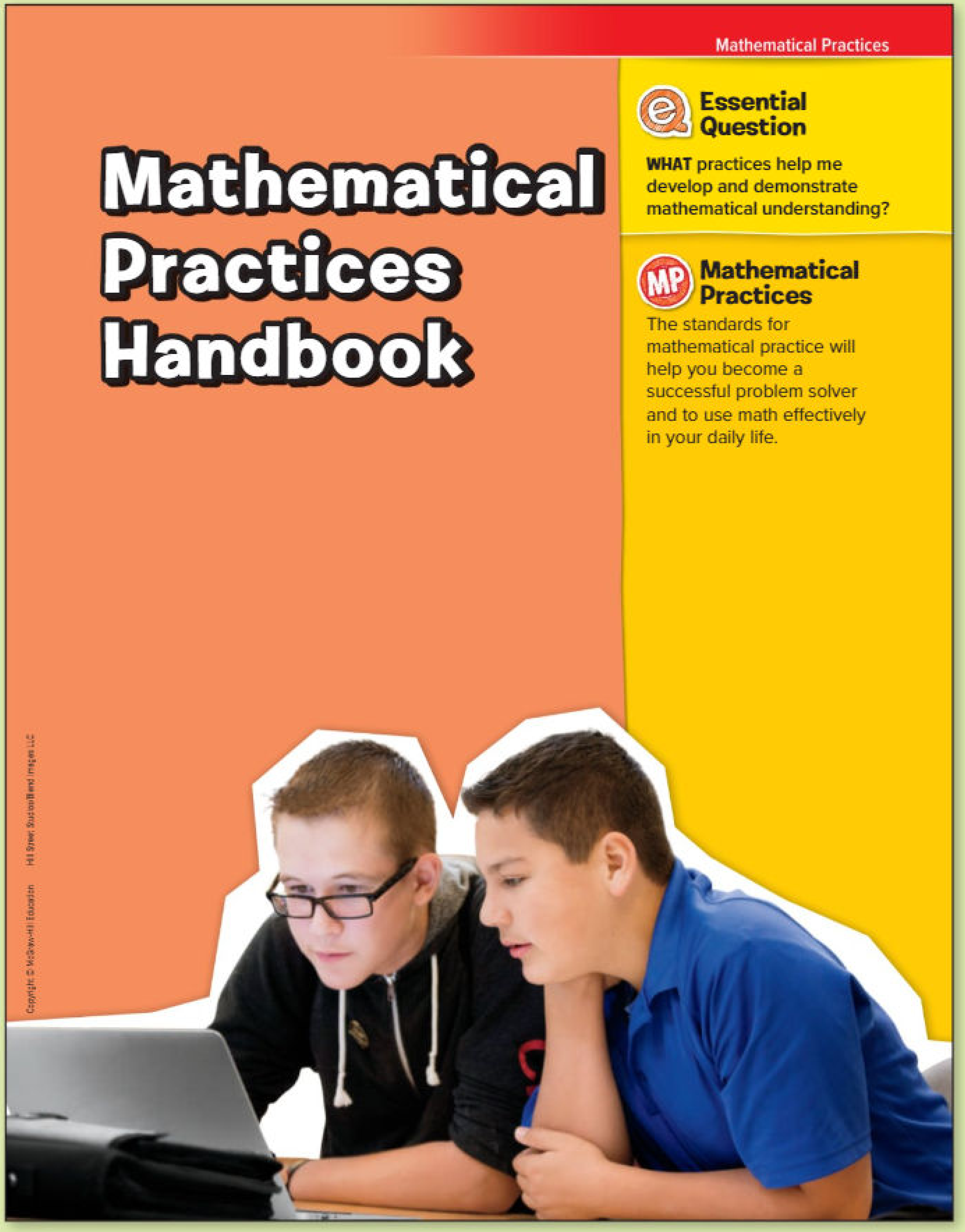
- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability

- Investigate patterns of association in bivariate data.

MP Mathematical Practices

1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
3 Construct viable arguments and critique the reasoning of others.
4 Model with mathematics.
5 Use appropriate tools strategically.
6 Attend to precision.
7 Look for and make use of structure.
8 Look for and express regularity in repeated reasoning.



Mathematical Practices Handbook

Essential Question

WHAT practices help me develop and demonstrate mathematical understanding?

Mathematical Practices

The standards for mathematical practice will help you become a successful problem solver and to use math effectively in your daily life.

What is the MP Mathematical Practices Handbook?

Use the Mathematical Practices Handbook to introduce students to the **Mathematical Practices**.

The Standards for Mathematical Practice describe how students should approach mathematics. The goal of the practice standards is to instill in all students the abilities to be mathematically literate and to create a positive disposition for the importance of using math effectively.

Included in this handbook are activities and exercises that allow students to become familiar with the Mathematical Practices below. Throughout this text, students will see MP to remind them that they are using these Mathematical Practices.

- MP Persevere with Problems
- MP Reason Abstractly and Quantitatively
- MP Construct an Argument
- MP Model with Mathematics
- MP Use Math Tools
- MP Attend to Precision
- MP Make Use of Structure
- MP Use Repeated Reasoning

Building on the Essential Question

At the end of the Mathematical Practices Handbook, students should be able to answer "WHAT practices help me develop and demonstrate mathematical understanding?"

Throughout this text, refer to the following icons to find differentiated strategies to meet the needs of all learners.

- AL Approaching-Level Learners
- OL On-Level Learners
- BL Beyond-Level Learners
- LA Language Acquisition

What You'll Learn

MP **Throughout this handbook, you will learn about each of these mathematical practices and how they are integrated in the chapters and lessons of this book.**

① **Focus on Mathematical Practice**
Persevere with Problems

② **Focus on Mathematical Practice**
Reason Abstractly and Quantitatively

③ **Focus on Mathematical Practice**
Construct an Argument

④ **Focus on Mathematical Practice**
Model with Mathematics

⑤ **Focus on Mathematical Practice**
Use Math Tools

⑥ **Focus on Mathematical Practice**
Attend to Precision

⑦ **Focus on Mathematical Practice**
Make Use of Structure

⑧ **Focus on Mathematical Practice**
Use Repeated Reasoning

Place a checkmark below the face that expresses how much you know about each Mathematical Practice. Then explain in your own words what it means to you.

🙁 I have no clue. 😐 I've heard of it. 🙂 I know it!

Mathematical Practices				
Mathematical Practice	🙁	😐	🙂	What it means to me
①				
②				
③				
④				
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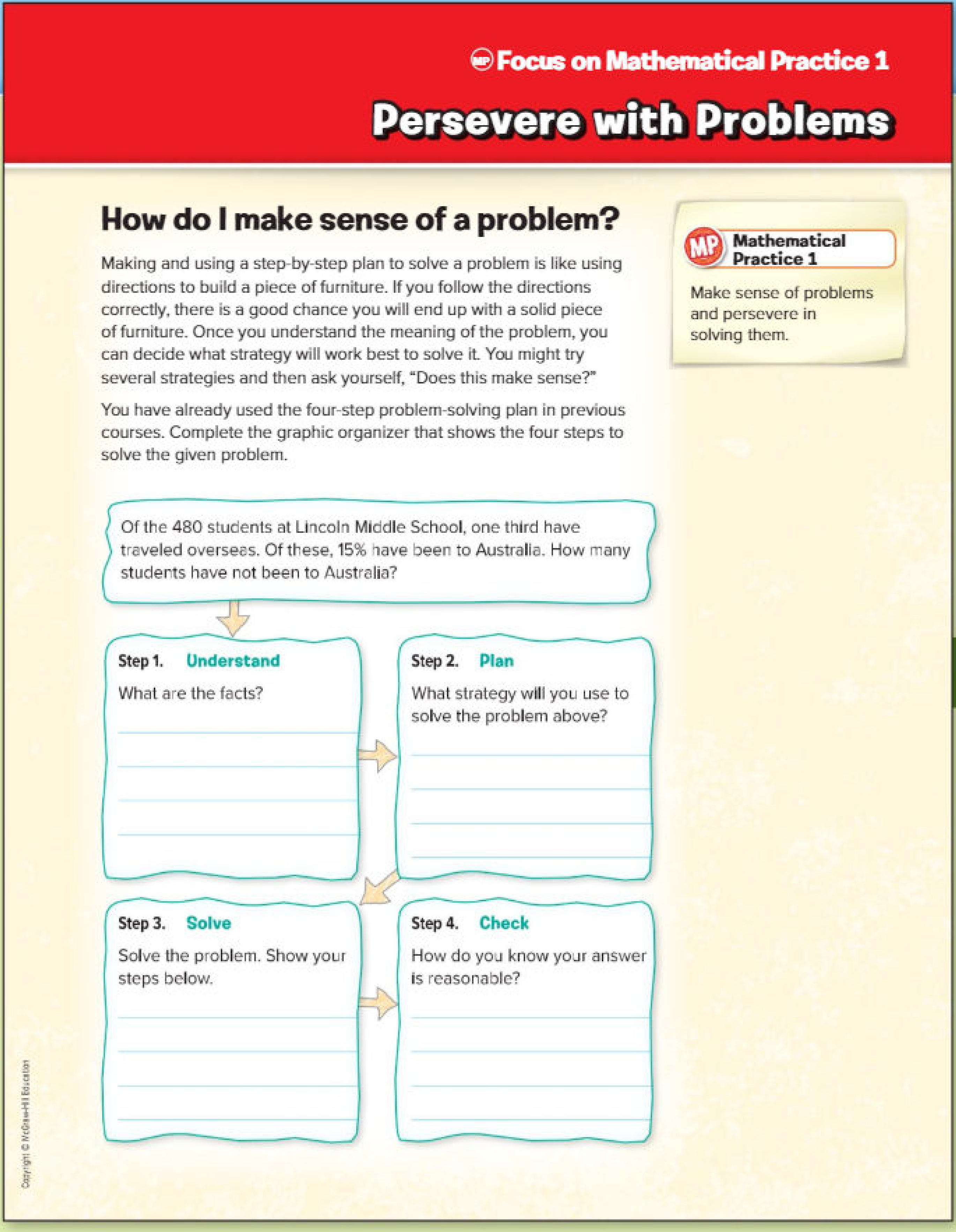


Focus on Mathematical Practice 1

Persevere with Problems

How do I make sense of a problem?

Making and using a step-by-step plan to solve a problem is like using directions to build a piece of furniture. If you follow the directions correctly, there is a good chance you will end up with a solid piece of furniture. Once you understand the meaning of the problem, you can decide what strategy will work best to solve it. You might try several strategies and then ask yourself, "Does this make sense?"

You have already used the four-step problem-solving plan in previous courses. Complete the graphic organizer that shows the four steps to solve the given problem.

Mathematical Practice 1

Make sense of problems and persevere in solving them.

Of the 480 students at Lincoln Middle School, one third have traveled overseas. Of these, 15% have been to Australia. How many students have not been to Australia?

Step 1. Understand

What are the facts?

Step 2. Plan

What strategy will you use to solve the problem above?

Step 3. Solve

Solve the problem. Show your steps below.

Step 4. Check

How do you know your answer is reasonable?

Focus narrowing the scope

Objective Persevere with problems.

Coherence connecting within and across grades

Previous	Now	Next
Students found percent of a number.	Students make sense of problems and persevere in solving them.	Students will reason abstractly and quantitatively.

Rigor pursuing concepts, fluency, and applications

Mathematically proficient students use a logical process to make sense of problems, understand that there may be more than one way to solve a problem, and alter the process if needed.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Ideas for Use

LA **Paired Heads Together** Have students complete the graphic organizer individually. Then have students pair up with a partner and share their answers. If either answer is incorrect, have the students alternate to go back through the steps to check their answers. MP 1, 3, 5, 6, 7

Alternate Strategy

AL Before solving the problem on page MP3, have students make a list of all of the problem-solving strategies they have used in previous math classes. Then randomly call on students to share their strategies. MP 1, 6

2 Practice and Apply

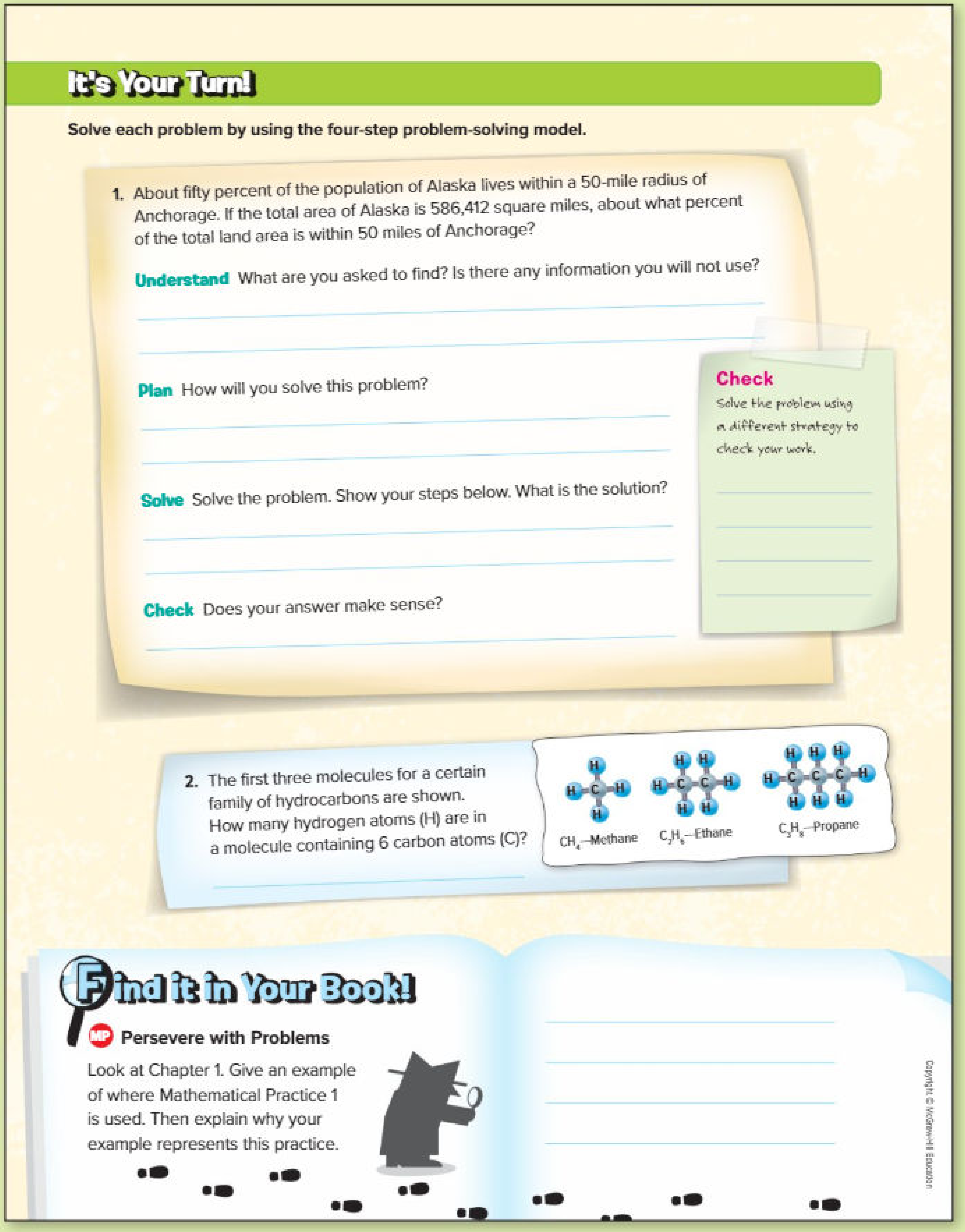
Ideas for Use

LA **Circle the Sage** Poll the class to see which students have a solid understanding of how to solve the problem presented in Exercise 1. These students (the sages) spread around the room. Have the rest of the class divide into small groups. Have each group member report to a different sage, if possible. The sages lead the discussion in how to solve the problem. Then have all students return to their groups to compare what was discussed with each sage. MP 1, 6, 7

Alternate Strategies

AL Review the formula for finding the area of a circle, $A = \pi r^2$. Have students describe the formula using multiple representations, such as verbal descriptions, models, tables, and graphs of ordered pairs. MP 1, 2, 4, 6, 7

BL LA Have students research a hydrocarbon molecule not shown in Exercise 2. Students should draw the molecular diagram, write the chemical formula, and include at least one other characteristic. MP 1, 2, 5, 6, 8



It's Your Turn!

Solve each problem by using the four-step problem-solving model.

1. About fifty percent of the population of Alaska lives within a 50-mile radius of Anchorage. If the total area of Alaska is 586,412 square miles, about what percent of the total land area is within 50 miles of Anchorage?

Understand What are you asked to find? Is there any information you will not use?

Plan How will you solve this problem?

Solve Solve the problem. Show your steps below. What is the solution?

Check Does your answer make sense?

Check
Solve the problem using a different strategy to check your work.

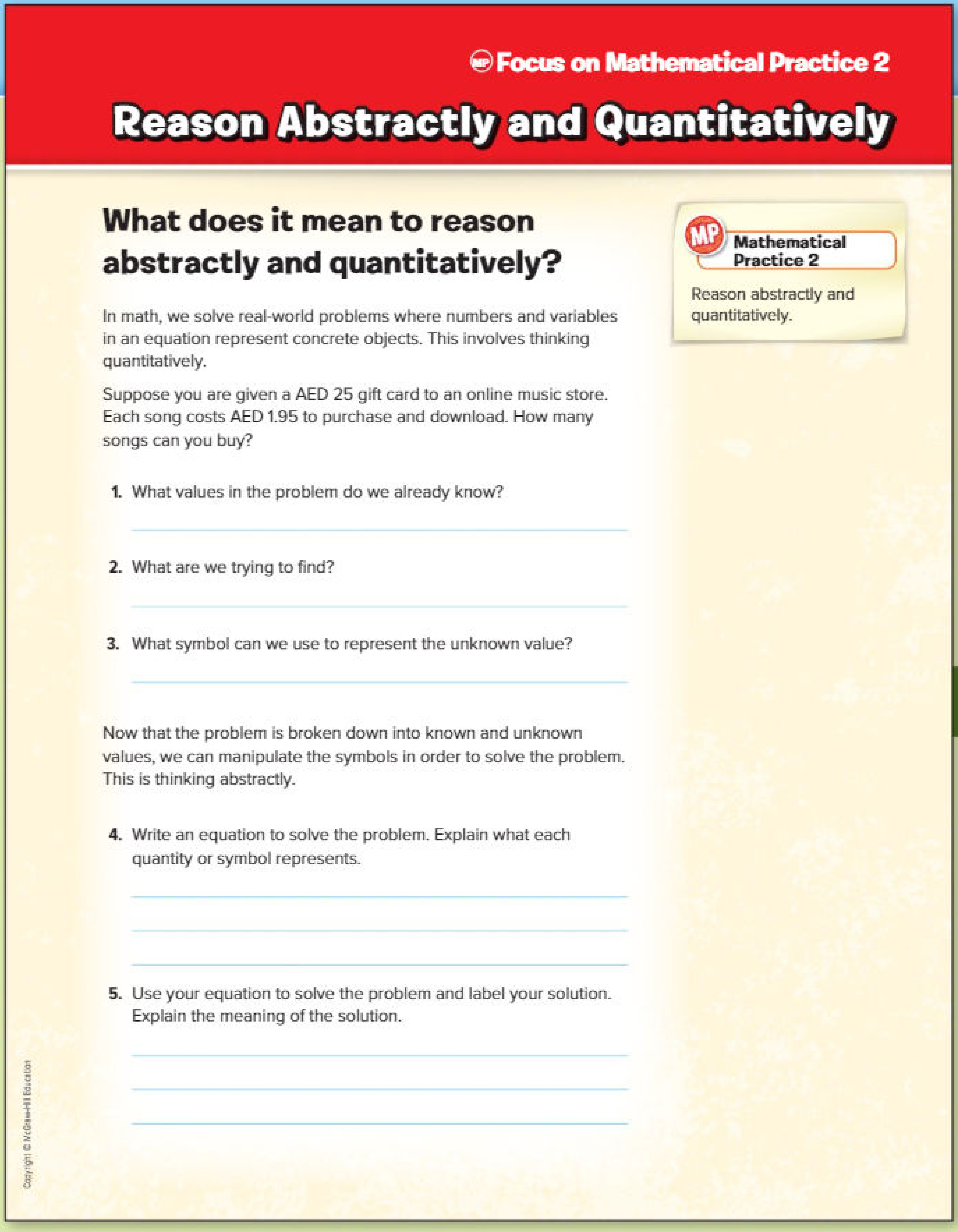
2. The first three molecules for a certain family of hydrocarbons are shown. How many hydrogen atoms (H) are in a molecule containing 6 carbon atoms (C)?

CH_4—Methane C_2H_6—Ethane C_3H_8—Propane

Find it in Your Book!

MP **Persevere with Problems**

Look at Chapter 1. Give an example of where Mathematical Practice 1 is used. Then explain why your example represents this practice.



Focus on Mathematical Practice 2

Reason Abstractly and Quantitatively

What does it mean to reason abstractly and quantitatively?

Mathematical Practice 2

Reason abstractly and quantitatively.

In math, we solve real-world problems where numbers and variables in an equation represent concrete objects. This involves thinking quantitatively.

Suppose you are given a AED 25 gift card to an online music store. Each song costs AED 1.95 to purchase and download. How many songs can you buy?

1. What values in the problem do we already know?
2. What are we trying to find?
3. What symbol can we use to represent the unknown value?

Now that the problem is broken down into known and unknown values, we can manipulate the symbols in order to solve the problem. This is thinking abstractly.

4. Write an equation to solve the problem. Explain what each quantity or symbol represents.
5. Use your equation to solve the problem and label your solution. Explain the meaning of the solution.

Focus narrowing the scope

Objective Reason abstractly and quantitatively.

Coherence connecting within and across grades

Previous
Students made sense of problems and persevered in solving them.

Now
Students reason abstractly and quantitatively.

Next
Students will construct viable arguments and critique the reasoning of others.

Rigor pursuing concepts, fluency, and applications

Mathematically proficient students can start with a concrete or real-world context and then represent it with abstract numbers or symbols (decontextualize), find a solution, then refer back to the context to check that the solution makes sense (contextualize).

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Ideas for Use

LA **Team-Pair-Solo** Have students work in small groups to complete Exercises 1–3. Then have them divide into pairs to complete Exercise 4. Have students work individually to complete Exercise 5. Have them regroup to discuss their responses to Exercises 4 and 5. MP 1, 2, 4, 6

Alternate Strategy

BL Tell students that there is a one-time $5 fee to create an account before you can purchase and download any songs. Have them write a new equation given this information and find how many songs they can purchase after creating an account. MP 1, 2, 4, 7

2 Practice and Apply

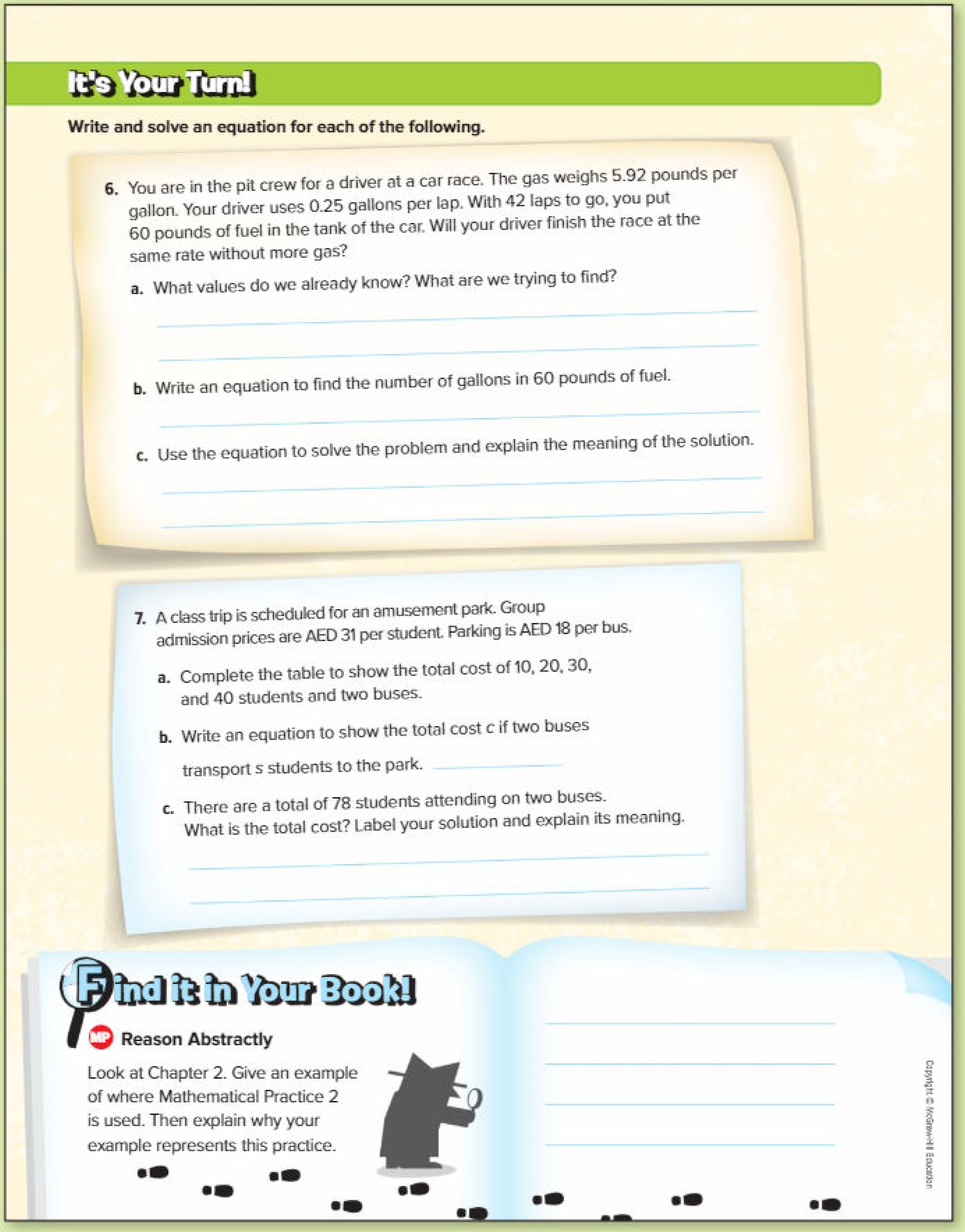
Ideas for Use

LA **Rally Coach** Have students work in pairs to complete Exercises 6 and 7. Have one partner complete Exercise 6 while the second partner watches, listens, coaches, and praises. Have them trade turns for Exercise 7. **MP 1, 2, 3, 4, 6, 8**

Alternate Strategies

AL **LA** If students need help completing the table, have them make a three column table (input, equation, and output) to organize their work. Remind students that by using an equation, they are using symbols to communicate mathematical ideas. **MP 1, 2, 4, 6, 7**

BL Have students determine how many pounds of fuel the car will need in order to finish the race. **MP 1, 5, 6**



It's Your Turn!

Write and solve an equation for each of the following.

6. You are in the pit crew for a driver at a car race. The gas weighs 5.92 pounds per gallon. Your driver uses 0.25 gallons per lap. With 42 laps to go, you put 60 pounds of fuel in the tank of the car. Will your driver finish the race at the same rate without more gas?

a. What values do we already know? What are we trying to find?

b. Write an equation to find the number of gallons in 60 pounds of fuel.

c. Use the equation to solve the problem and explain the meaning of the solution.

7. A class trip is scheduled for an amusement park. Group admission prices are AED 31 per student. Parking is AED 18 per bus.

a. Complete the table to show the total cost of 10, 20, 30, and 40 students and two buses.

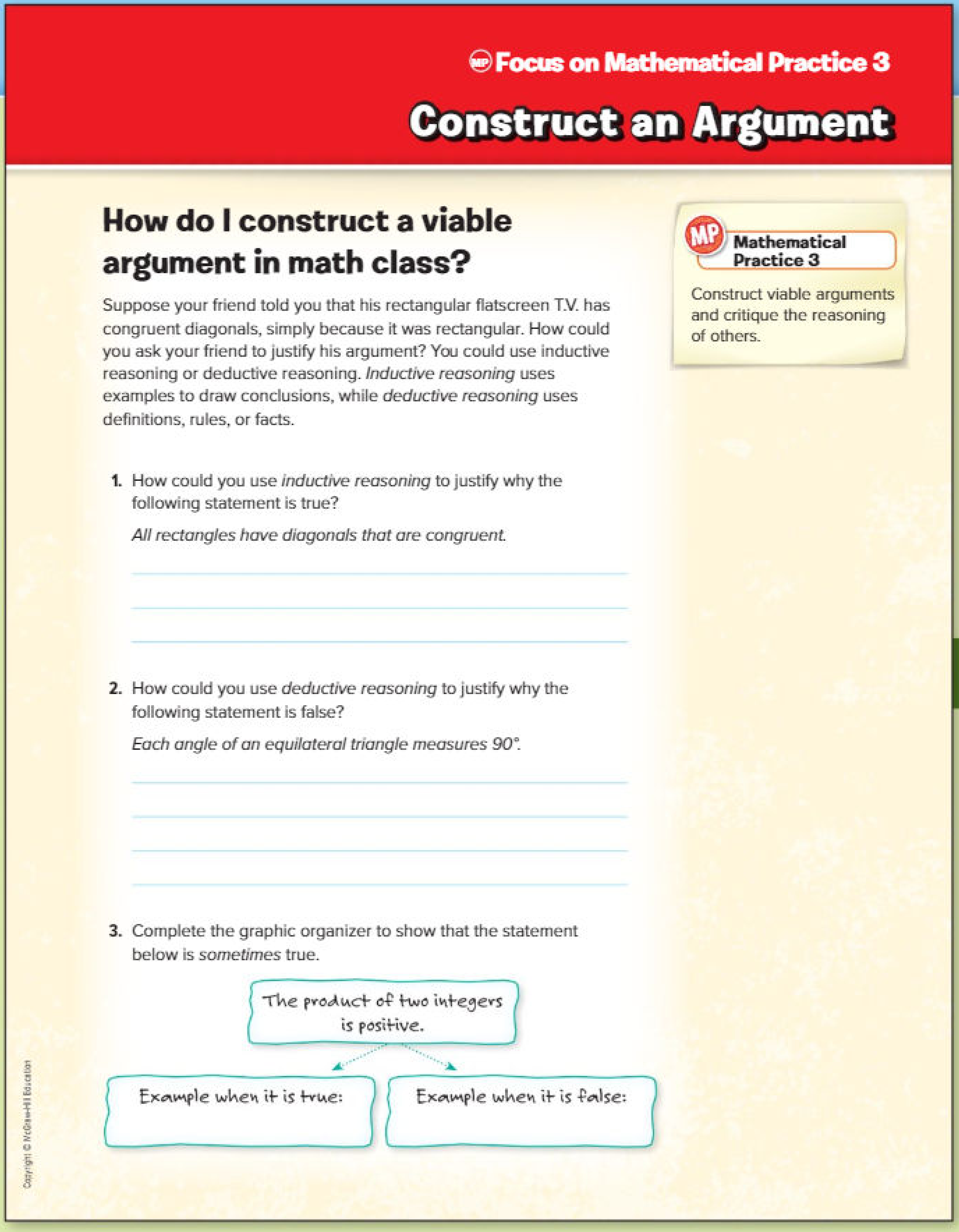
b. Write an equation to show the total cost *c* if two buses transport *s* students to the park. ______

c. There are a total of 78 students attending on two buses. What is the total cost? Label your solution and explain its meaning.

Find it in Your Book!

MP Reason Abstractly

Look at Chapter 2. Give an example of where Mathematical Practice 2 is used. Then explain why your example represents this practice.



MP Focus on Mathematical Practice 3

Construct an Argument

How do I construct a viable argument in math class?

MP Mathematical Practice 3

Construct viable arguments and critique the reasoning of others.

Suppose your friend told you that his rectangular flatscreen T.V. has congruent diagonals, simply because it was rectangular. How could you ask your friend to justify his argument? You could use inductive reasoning or deductive reasoning. *Inductive reasoning* uses examples to draw conclusions, while *deductive reasoning* uses definitions, rules, or facts.

1. How could you use *inductive reasoning* to justify why the following statement is true?

   All rectangles have diagonals that are congruent.

2. How could you use *deductive reasoning* to justify why the following statement is false?

   Each angle of an equilateral triangle measures 90°.

3. Complete the graphic organizer to show that the statement below is *sometimes* true.

   The product of two integers is positive.

   Example when it is true:

   Example when it is false:

Focus narrowing the scope

Objective Construct viable arguments and critique the reasoning of others.

Coherence connecting within and across grades

Previous	Now	Next
Students reasoned abstractly and quantitatively.	Students construct viable arguments and critique the reasoning of others.	Students will model with mathematics.

Rigor pursuing concepts, fluency, and applications

Mathematically proficient students can clearly communicate their thoughts and defend them using sound mathematical arguments.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Ideas for Use

LA **Numbered Heads Together** Have students work in groups of 3–4 and work as a team to solve each problem, making sure that everyone understands. Then call on a random student to explain the solution. MP 1, 3, 5, 6, 7

Alternate Strategy

AL Provide students with several real-world examples in which either inductive or deductive reasoning were used. Have them classify each example as *inductive reasoning* or *deductive reasoning*. MP 1, 6

2 Practice and Apply

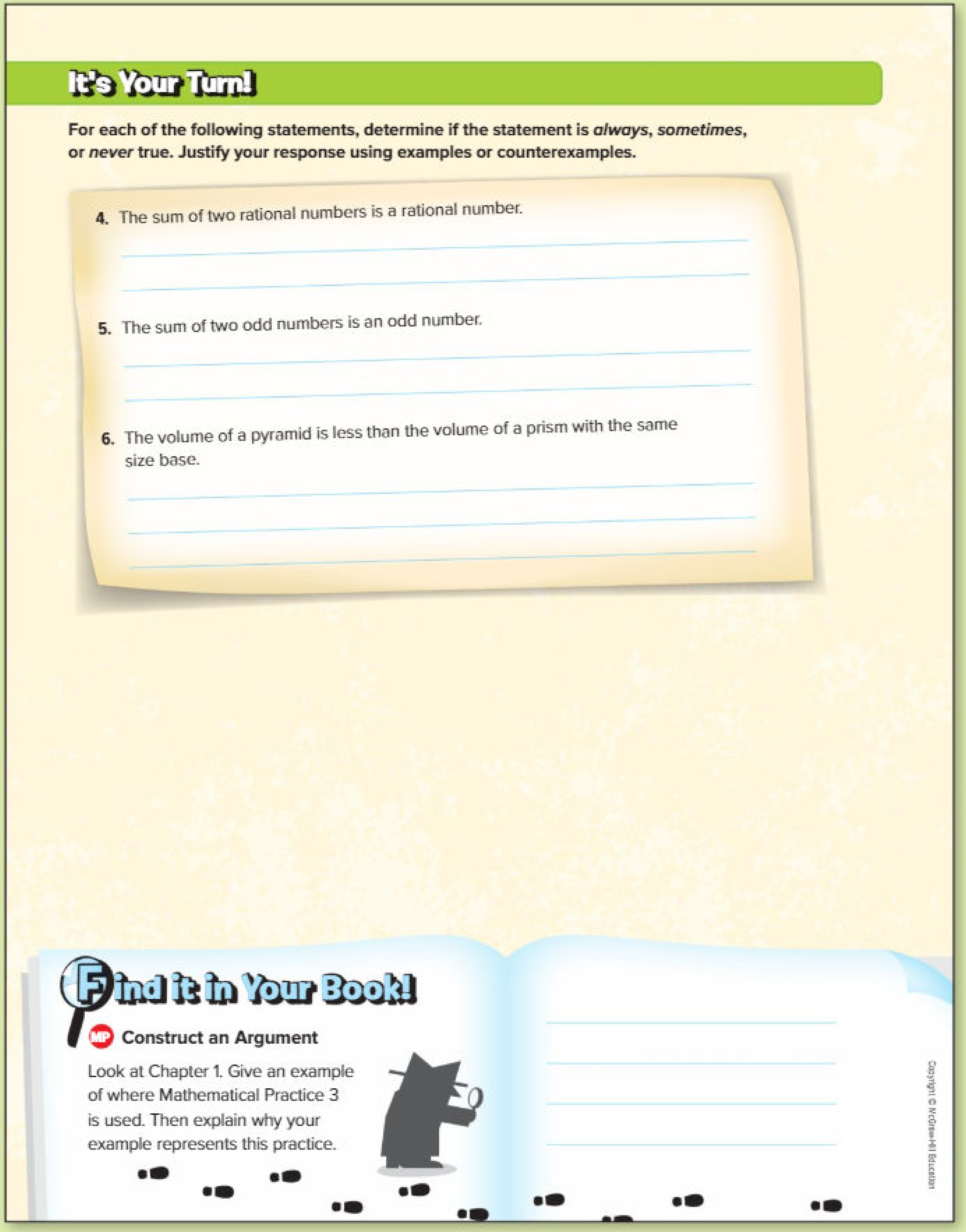
Ideas for Use

LA **Group-Pair-Share** Have students work in groups to complete Exercises 4 and 5, ensuring that each member understands. Then have the groups divide into pairs to complete Exercise 6. Have them reconvene into the larger group to share responses and resolve any discrepancies. MP 1, 3, 6, 7

Alternate Strategies

AL Create several statements similar to those in Exercises 4–6. Have the students determine if the statement is *always true*, *sometimes true*, or *never true*, and justify their choice. Guide students in providing convincing justifications. For example, if a statement is sometimes true, they should be able to give an example showing it is true and a counterexample showing it is false. MP 1, 3, 6, 7

BL LA **Gallery Walk** Have students work with a partner to create a problem similar to those in Exercises 4–6. Post the problems around the room. Students walk around the room and select a problem, not their own. Working with their partner, they determine the solution and write a convincing argument supporting their solution. Have them locate the pair of students who wrote the problem to share their solution and discuss and resolve any errors. MP 1, 3, 4, 6



It's Your Turn!

For each of the following statements, determine if the statement is *always*, *sometimes*, or *never* true. Justify your response using examples or counterexamples.

4. The sum of two rational numbers is a rational number.

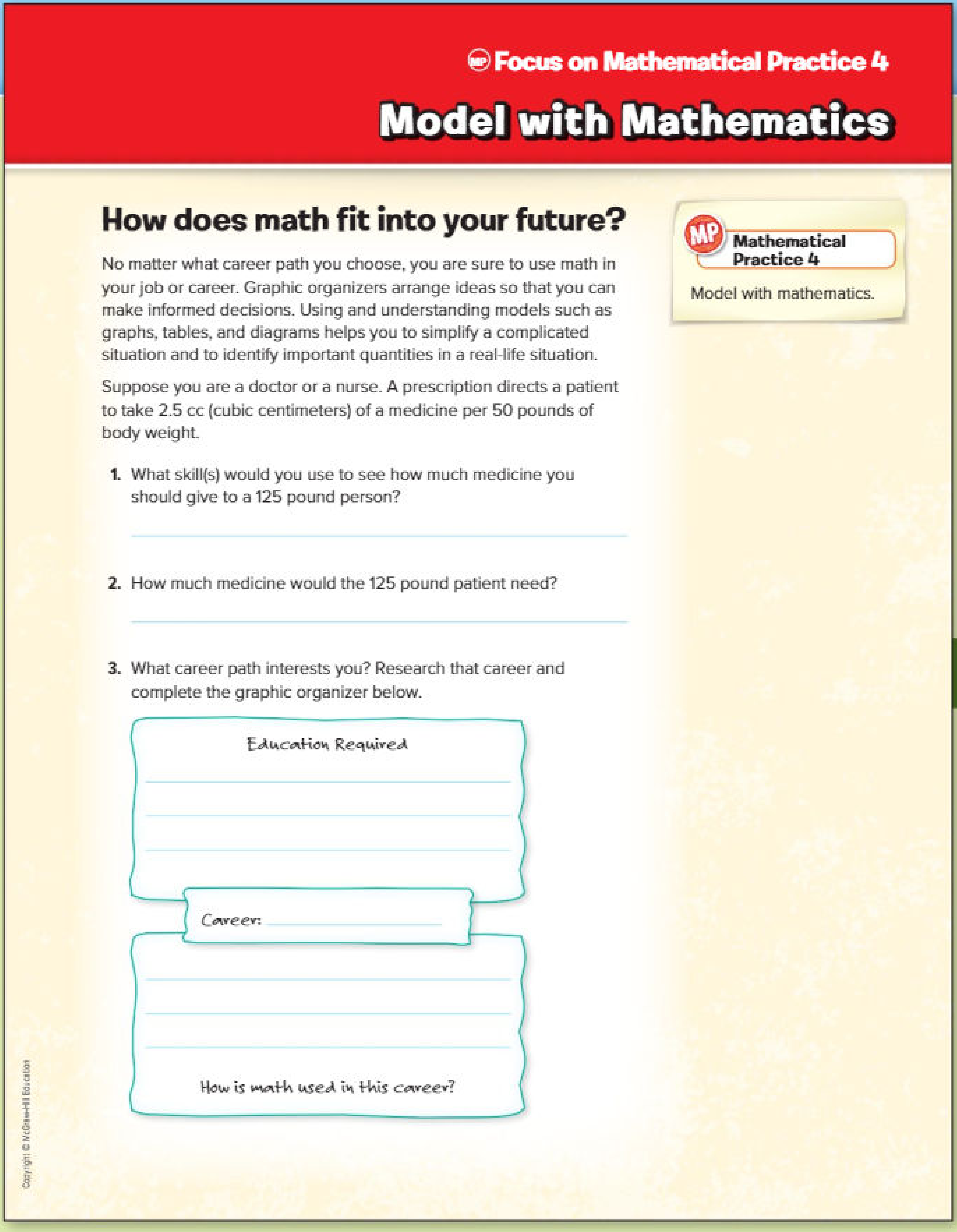
5. The sum of two odd numbers is an odd number.

6. The volume of a pyramid is less than the volume of a prism with the same size base.

Find it in Your Book!

MP Construct an Argument

Look at Chapter 1. Give an example of where Mathematical Practice 3 is used. Then explain why your example represents this practice.



MP Focus on Mathematical Practice 4

Model with Mathematics

How does math fit into your future?

No matter what career path you choose, you are sure to use math in your job or career. Graphic organizers arrange ideas so that you can make informed decisions. Using and understanding models such as graphs, tables, and diagrams helps you to simplify a complicated situation and to identify important quantities in a real-life situation.

Suppose you are a doctor or a nurse. A prescription directs a patient to take 2.5 cc (cubic centimeters) of a medicine per 50 pounds of body weight.

1. What skill(s) would you use to see how much medicine you should give to a 125 pound person?

2. How much medicine would the 125 pound patient need?

3. What career path interests you? Research that career and complete the graphic organizer below.

Education Required

Career: ________________

How is math used in this career?

MP Mathematical Practice 4

Model with mathematics.

Focus narrowing the scope

Objective Model with mathematics.

Coherence connecting within and across grades

Previous
Students constructed viable arguments and critiqued the reasoning of others.

Now
Students model with mathematics.

Next
Students will use math tools.

Rigor pursuing concepts, fluency, and applications

Mathematically proficient students explain their thinking or search for patterns using models such as diagrams, drawings, classroom objects, and manipulatives, or geometric, graphical, algebraic, tabular, and statistical models.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Ideas for Use

LA **Pairs Discussion** Have students work in pairs to complete Exercises 1 and 2. Have them trade their solutions with another pair of students to discuss and resolve any differences. MP 1, 4, 6, 7

Alternate Strategy

AL If students have difficulty deciding on a career, have them first take a career interest survey before completing Exercise 3. MP 1, 5

2 Practice and Apply

Ideas for Use

LA **Numbered Heads Together** Assign students to a 3- or 4-person learning team. Each member is assigned a number from 1 to 4. Each team completes Exercises 4–6, making sure every team member understands the solution. Call on a specific number from one team to present the team's solution to the class. MP 1, 2, 4, 6, 7

Alternate Strategies

AL Have students practice finding the percent of a number using either the percent proportion or the percent equation. MP 1, 2, 6, 8

BL Have students use the Internet, or another source, to research city, county, and state sales tax rates for their area. Have them recalculate the total cost of the party in Exercise 6 based on their findings. MP 1, 5, 6

It's Your Turn!

Use the given tools to solve each problem.

Week, w	Total Saved, s ($)

4. You are saving money to buy a new game system. You received AED 50 as a graduation gift from your grandparents. You want to save AED 25 a week from mowing lawns.

a. **Tables** Complete the table to show the total amount saved after 1, 2, 3, 4, and 5 weeks.

b. **Symbols** Write an equation to show the total amount saved s after w weeks. ____________

c. **Algebra** Use the equation to determine the total amount saved after 17 weeks. ____________

Use the table for Exercises 5 and 6.

5. Mrs. Fatma hired a party planner to plan Noha's dinner party. There will be 125 guests and she wants to offer appetizers and a buffet dinner. What is the cost, before tax, for the party?

____

Polly's Perfect Parties			
Cost of Food (per person)		Cost of Extras	
Appetizers	AED 9.20	Hall	AED 250
Buffet	AED 18.30	Linens	AED 15 per table
Sit-down Dinner	AED 25.75	Table and Chair Rental (seats 8)	AED 60 per table

6. There is a $7\frac{1}{2}$% sales tax added to the party bill. Mrs. Fatma also wants to add an 18% tip for the servers. This will be figured before tax is added. What will be the total cost of the party?

____

Find it in Your Book!

MP **Model with Mathematics**

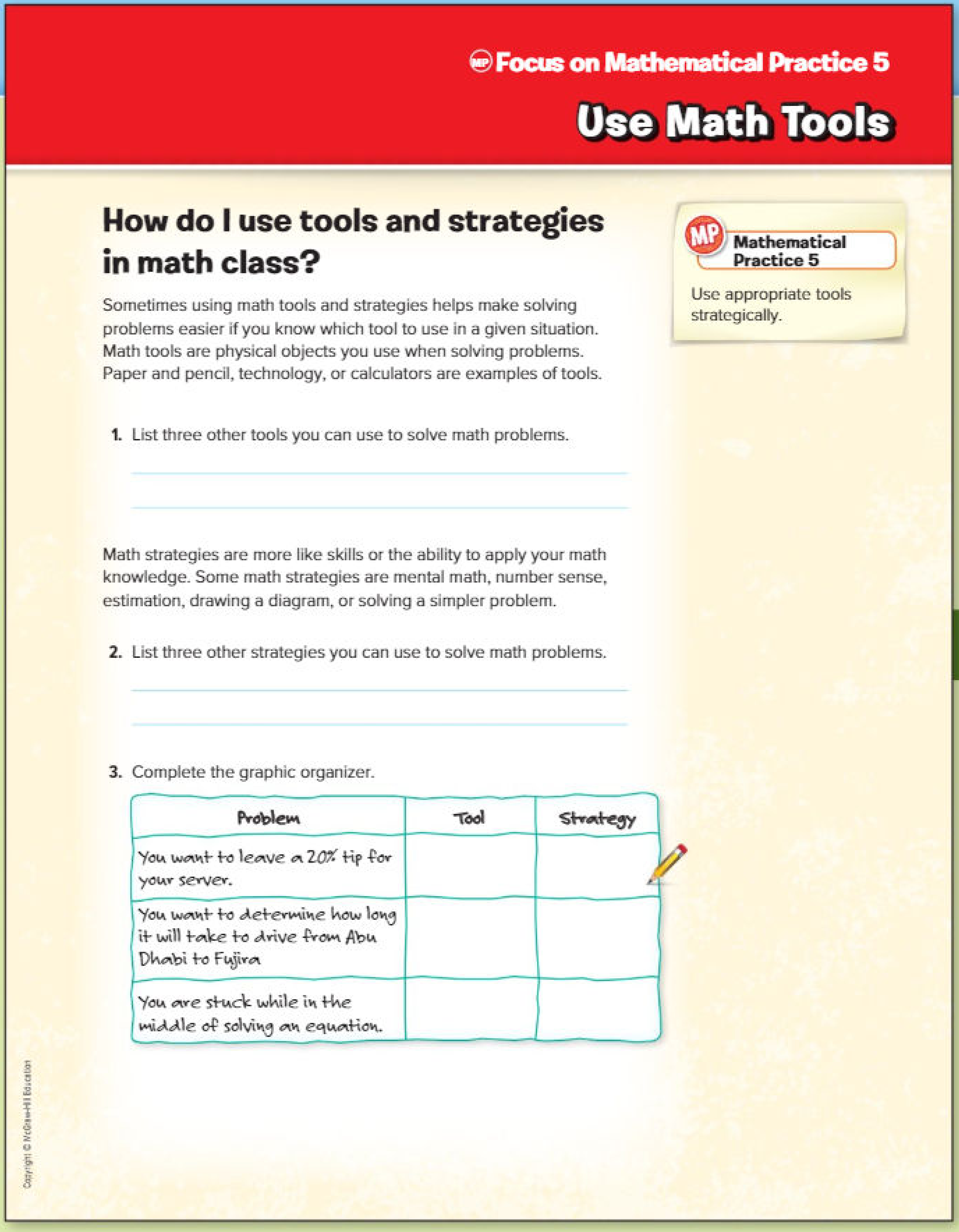
Look at Chapter 1. Give an example of where Mathematical Practice 4 is used. Then explain why your example represents this practice.

____

____

____

____



Focus on Mathematical Practice 5

Use Math Tools

How do I use tools and strategies in math class?

Mathematical Practice 5

Use appropriate tools strategically.

Sometimes using math tools and strategies helps make solving problems easier if you know which tool to use in a given situation. Math tools are physical objects you use when solving problems. Paper and pencil, technology, or calculators are examples of tools.

1. List three other tools you can use to solve math problems.

Math strategies are more like skills or the ability to apply your math knowledge. Some math strategies are mental math, number sense, estimation, drawing a diagram, or solving a simpler problem.

2. List three other strategies you can use to solve math problems.

3. Complete the graphic organizer.

Problem	Tool	Strategy
You want to leave a 20% tip for your server.		
You want to determine how long it will take to drive from Abu Dhabi to Fujira		
You are stuck while in the middle of solving an equation.		

Focus narrowing the scope

Objective Select tools and strategies to solve problems.

Coherence connecting within and across grades

Previous	Now	Next
Students modeled with mathematics.	Students use math tools and strategies.	Students will attend to precision.

Rigor pursuing concepts, fluency, and applications

Mathematically proficient students understand the benefits and limitations of using mathematics tools, including estimation and virtual tools, and use them appropriately.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Ideas for Use

LA **Rally Robin** In groups, assign one student as the Rally Robin Leader, who poses questions to help complete Exercise 3. The rest of the group takes turns responding orally to each question. MP 1, 5

Alternate Strategy

AL LA Have students work in small groups to create a graphic organizer that lists as many tools and strategies they can brainstorm. MP 1, 5

2 Practice and Apply

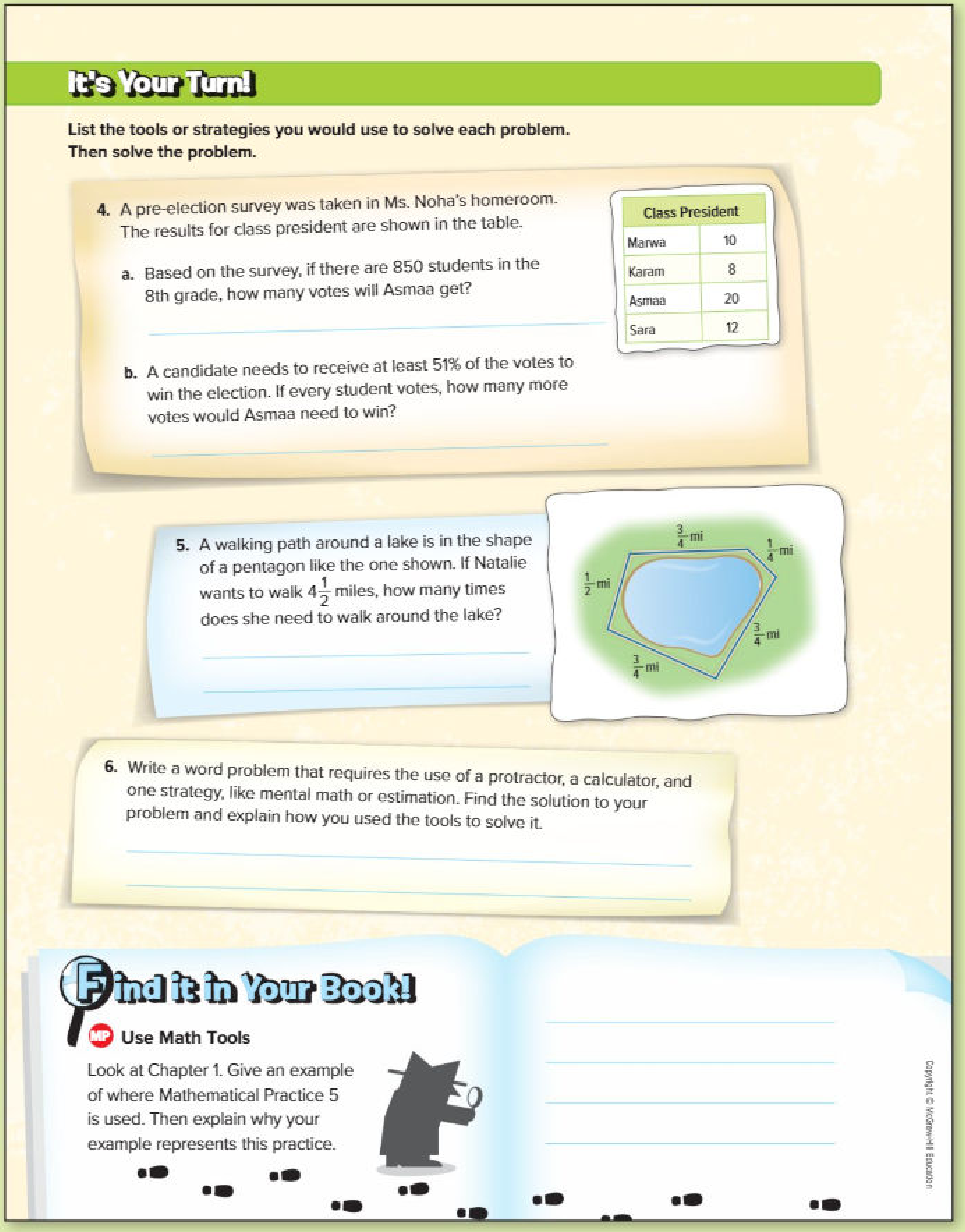
Ideas for Use

LA **Teammates Consult** Place students in teams of three to complete Exercises 4–6. Each student is given a number, 4–6, which represents the exercise discussion they are leading. Teammates discuss Exercise 4 with Student 4 leading the discussion. All members of the team contribute, but all have to agree upon one answer. Continue by rotating the leader role until all the exercises are completed. MP 1, 3, 4, 5, 6

Alternate Strategies

AL LA If students have difficulty solving the problems, have them make a template of the four-step problem-solving plan to use for each exercise. MP 1, 5

BL LA **Pairs Discussion** Have students choose one of the exercises and write an extension of that problem. For example, for Exercise 5, students may choose to determine how long it will take Natalie to walk the $7\frac{1}{2}$ miles for a chosen rate. Have them discuss with a partner which tools or strategies can be used to solve their extension problem. MP 1, 3, 4, 5



It's Your Turn!

List the tools or strategies you would use to solve each problem. Then solve the problem.

4. A pre-election survey was taken in Ms. Noha's homeroom. The results for class president are shown in the table.

Class President	
Marwa	10
Karam	8
Asmaa	20
Sara	12

a. Based on the survey, if there are 850 students in the 8th grade, how many votes will Asmaa get?

b. A candidate needs to receive at least 51% of the votes to win the election. If every student votes, how many more votes would Asmaa need to win?

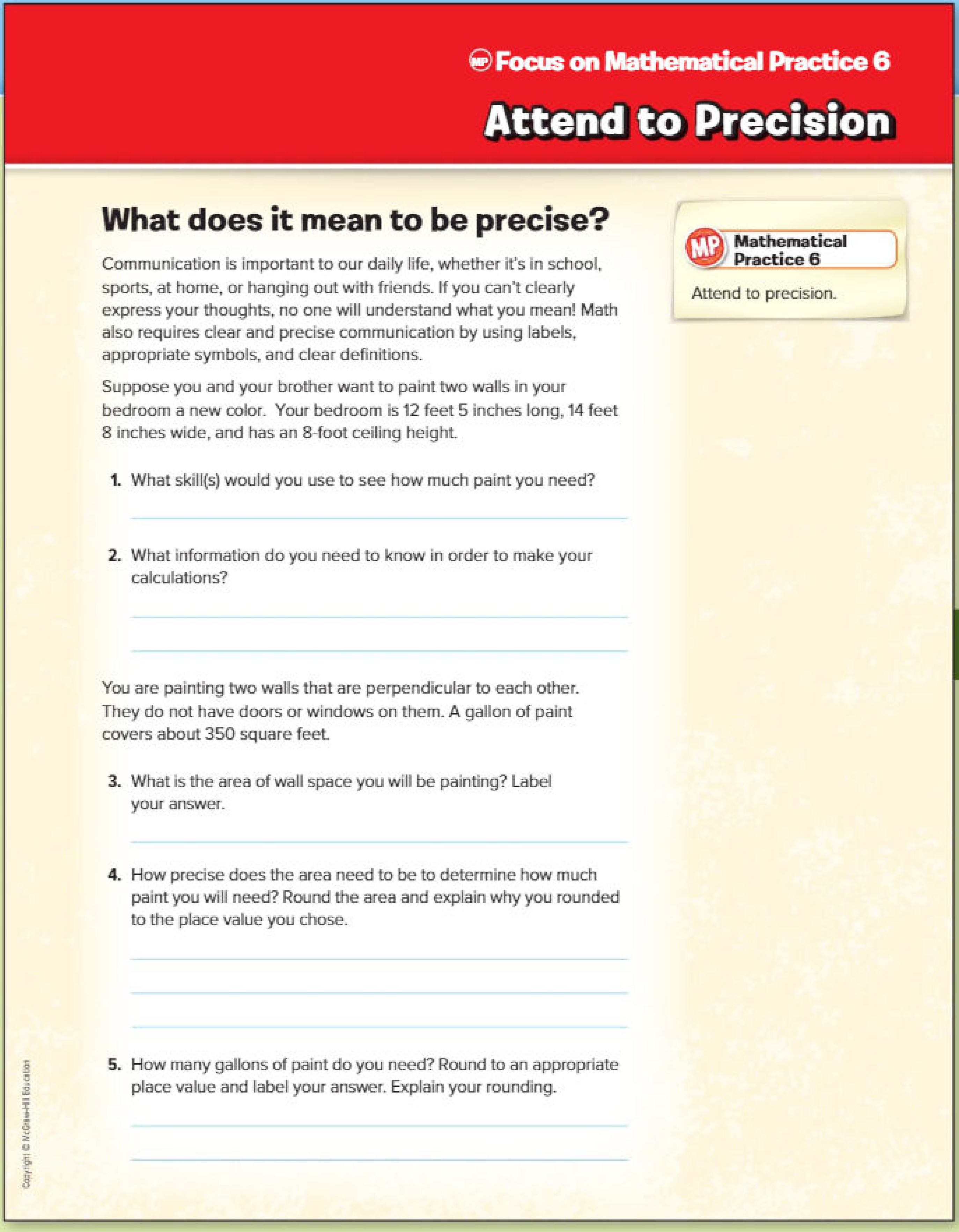
5. A walking path around a lake is in the shape of a pentagon like the one shown. If Natalie wants to walk $4\frac{1}{2}$ miles, how many times does she need to walk around the lake?

6. Write a word problem that requires the use of a protractor, a calculator, and one strategy, like mental math or estimation. Find the solution to your problem and explain how you used the tools to solve it.

Find it in Your Book!

MP **Use Math Tools**

Look at Chapter 1. Give an example of where Mathematical Practice 5 is used. Then explain why your example represents this practice.



MP Focus on Mathematical Practice 6

Attend to Precision

What does it mean to be precise?

MP Mathematical Practice 6

Attend to precision.

Communication is important to our daily life, whether it's in school, sports, at home, or hanging out with friends. If you can't clearly express your thoughts, no one will understand what you mean! Math also requires clear and precise communication by using labels, appropriate symbols, and clear definitions.

Suppose you and your brother want to paint two walls in your bedroom a new color. Your bedroom is 12 feet 5 inches long, 14 feet 8 inches wide, and has an 8-foot ceiling height.

1. What skill(s) would you use to see how much paint you need?
2. What information do you need to know in order to make your calculations?

You are painting two walls that are perpendicular to each other. They do not have doors or windows on them. A gallon of paint covers about 350 square feet.

3. What is the area of wall space you will be painting? Label your answer.
4. How precise does the area need to be to determine how much paint you will need? Round the area and explain why you rounded to the place value you chose.
5. How many gallons of paint do you need? Round to an appropriate place value and label your answer. Explain your rounding.

Focus narrowing the scope

Objective Attend to precision.

Coherence connecting within and across grades

Previous	Now	Next
Students used math tools and strategies.	Students attend to precision.	Students will look for and make use of structure.

Rigor pursuing concepts, fluency, and applications

Mathematically proficient students communicate the language of mathematics precisely, as well as calculate efficiently and accurately.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Ideas for Use

LA **Pairs Discussion** Have students work in pairs to complete Exercises 1 and 2. Then have them work individually to complete Exercises 3–5. Have them reconvene with their partner to check their answers and resolve any differences. MP 1, 3, 4, 6

Alternate Strategy

AL Review the formula for area of a rectangle and ask students which measurements they need to know to determine how many gallons of paint they need. MP 1, 6, 7

2 Practice and Apply

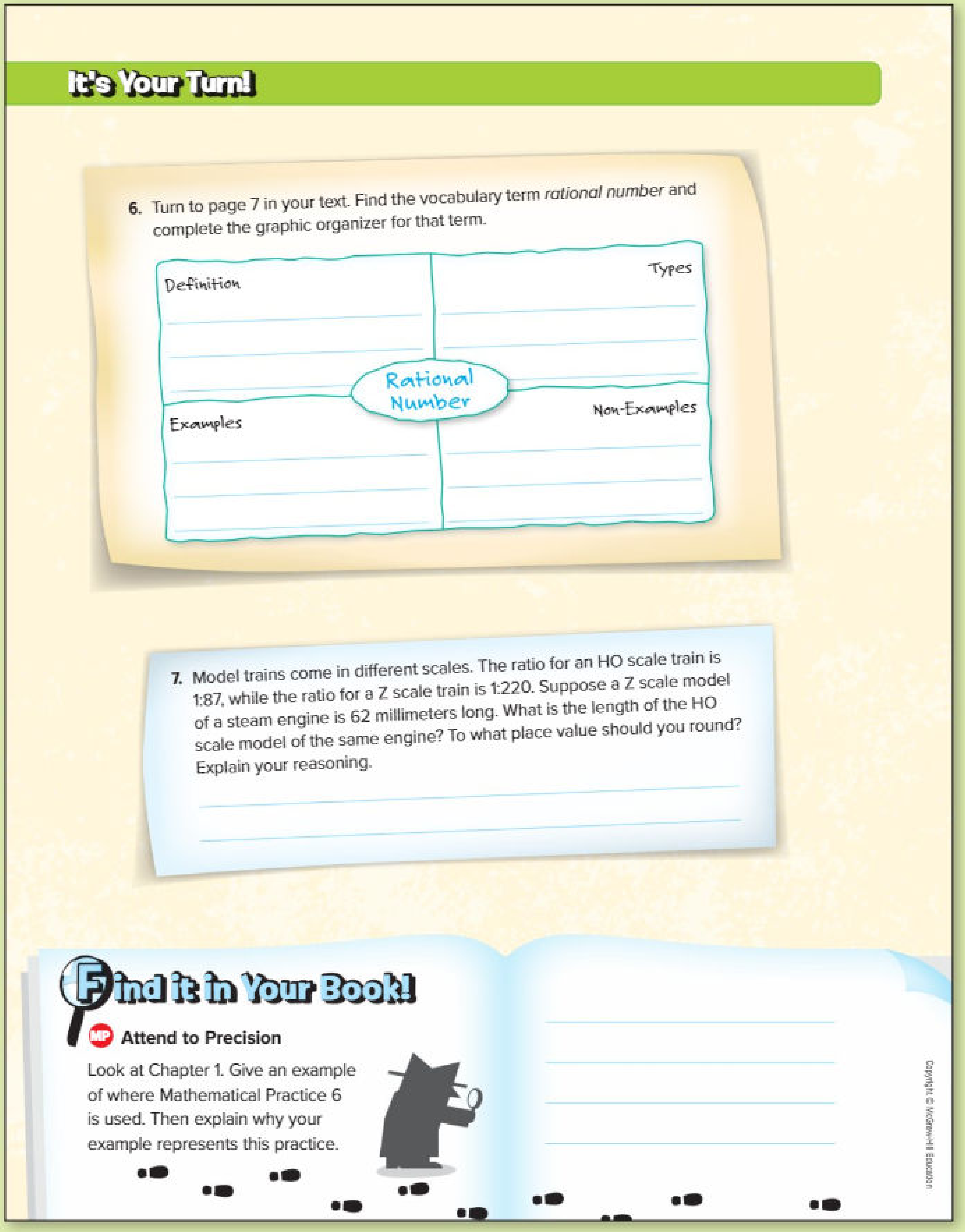
Ideas for Use

LA **Think-Pair-Share** Give students one to two minutes to think through their responses to Exercises 6 and 7. Then have them share their responses with their partner first, then with the entire class. MP 1, 3

Alternate Strategies

AL Before students complete Exercise 6, complete a similar graphic organizer for another concept, such as equations, as a whole group. MP 1, 5, 6

BL **LA** **Find the Fib** Have students work with a partner to write two facts and one fib for Exercise 7. Then have them exchange facts and fibs with another pair of students. Each pair identifies the other pair's facts and fib. MP 1, 3, 7



It's Your Turn!

6. Turn to page 7 in your text. Find the vocabulary term *rational number* and complete the graphic organizer for that term.

Definition

Types

Rational Number

Examples

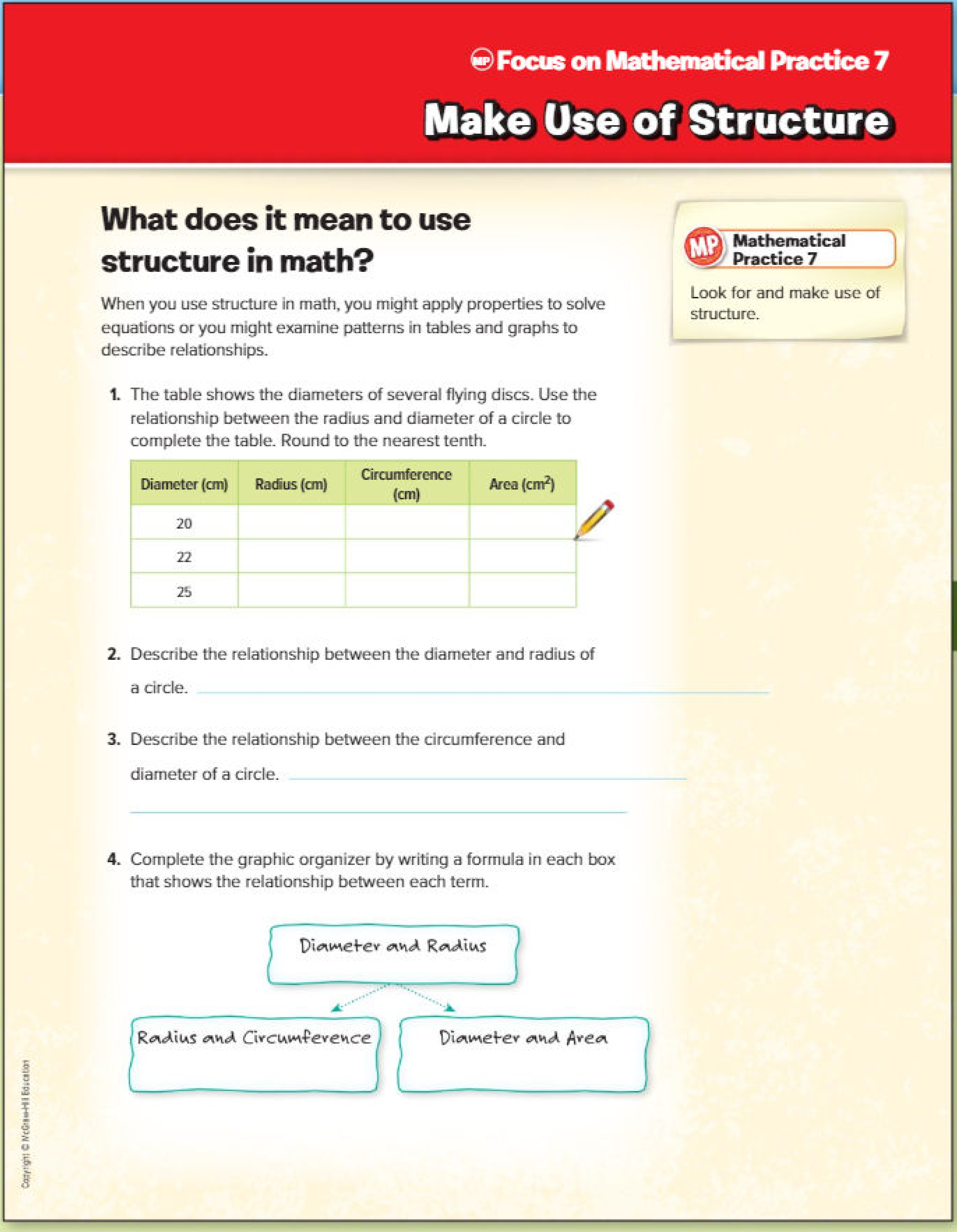
Non-Examples

7. Model trains come in different scales. The ratio for an HO scale train is 1:87, while the ratio for a Z scale train is 1:220. Suppose a Z scale model of a steam engine is 62 millimeters long. What is the length of the HO scale model of the same engine? To what place value should you round? Explain your reasoning.

Find it in Your Book!

MP **Attend to Precision**

Look at Chapter 1. Give an example of where Mathematical Practice 6 is used. Then explain why your example represents this practice.



Focus on Mathematical Practice 7

Make Use of Structure

What does it mean to use structure in math?

Mathematical Practice 7

Look for and make use of structure.

When you use structure in math, you might apply properties to solve equations or you might examine patterns in tables and graphs to describe relationships.

1. The table shows the diameters of several flying discs. Use the relationship between the radius and diameter of a circle to complete the table. Round to the nearest tenth.

Diameter (cm)	Radius (cm)	Circumference (cm)	Area (cm^2)
20			
22			
25			

2. Describe the relationship between the diameter and radius of a circle. ____________

3. Describe the relationship between the circumference and diameter of a circle. ____________

____

4. Complete the graphic organizer by writing a formula in each box that shows the relationship between each term.

Diameter and Radius

Radius and Circumference

Diameter and Area

Focus narrowing the scope

Objective Look for and make use of structure.

Coherence connecting within and across grades

Previous	Now	Next
Students attended to precision.	Students look for and make use of structure.	Students will look for and express regularity in repeated reasoning.

Rigor pursuing concepts, fluency, and applications

Mathematically proficient students look for structure to find easier ways to solve problems.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Ideas for Use

LA **Talking Chips** Have students work in small groups to complete Exercises 1–4. Give each student 5 chips. Students must place a chip in the center of the table each time they contribute to the discussion. After they have used all of their chips, they may no longer contribute to the discussion. All students must use all of their chips.
MP 1, 2, 4, 5, 6, 7, 8

Alternate Strategy

AL Review the formulas for the circumference and area of a circle with students before completing Exercises 1–4.
MP 1, 2, 6

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

2 Practice and Apply

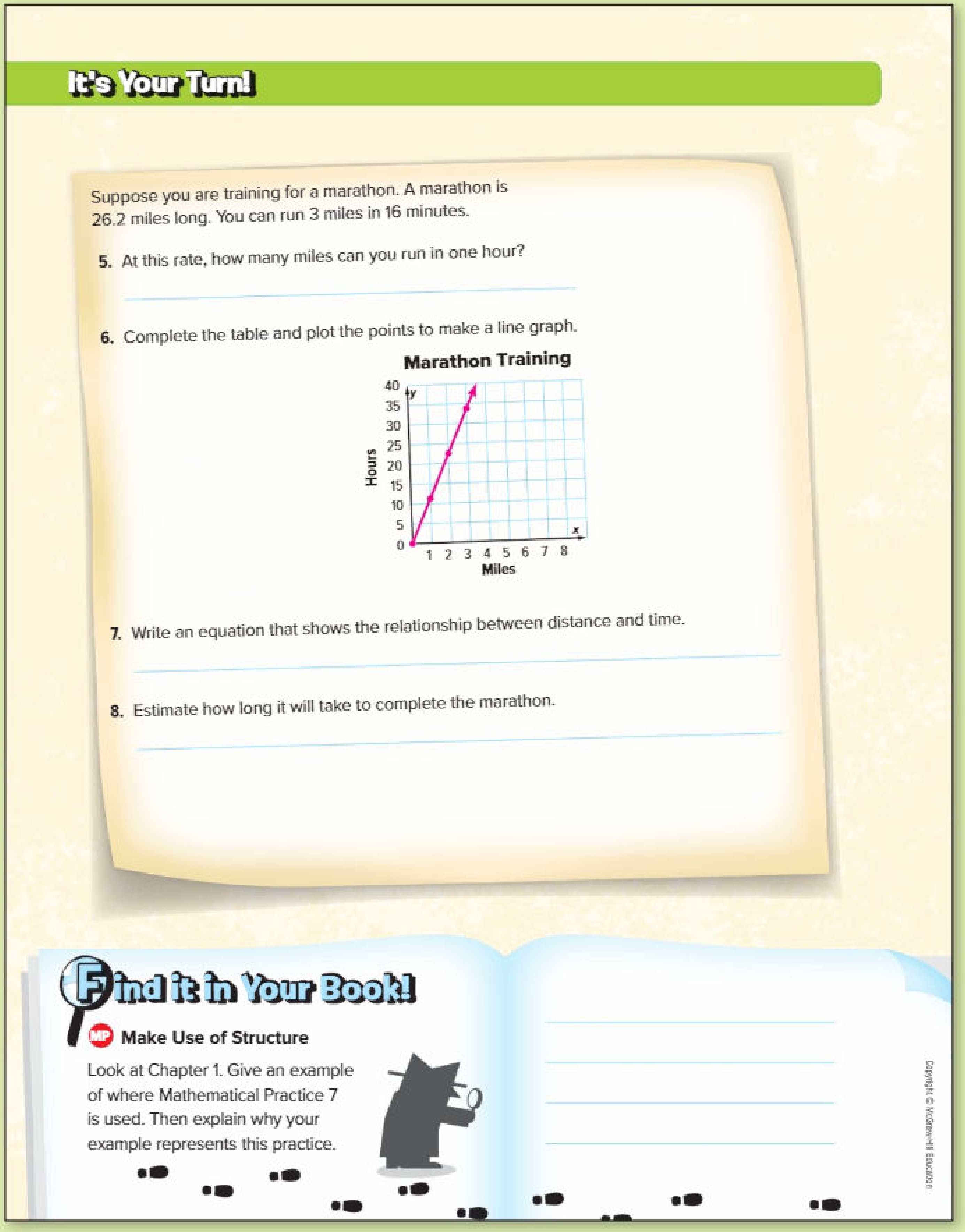
Ideas for Use

LA **Rally Coach** Have students work in pairs on Exercises 5-8. Have Partner A work through Exercise 5 while Partner B listens, coaches, and praises. Partners switch roles for Exercise 6. Continue this process with Exercises 7 and 8. **MP 1, 2, 4, 6, 7**

Alternate Strategies

AL Before students complete Exercise 5, review rate of change and what the rate means in the context of the situation. **MP 1, 6**

BL Have students research the average time that runners completed marathons in three different cities and compare the averages to their answer for Exercise 8. Have them explain if their answer is reasonable. **MP 1, 3, 5, 6**



It's Your Turn!

Suppose you are training for a marathon. A marathon is 26.2 miles long. You can run 3 miles in 16 minutes.

5. At this rate, how many miles can you run in one hour?

6. Complete the table and plot the points to make a line graph.

Marathon Training

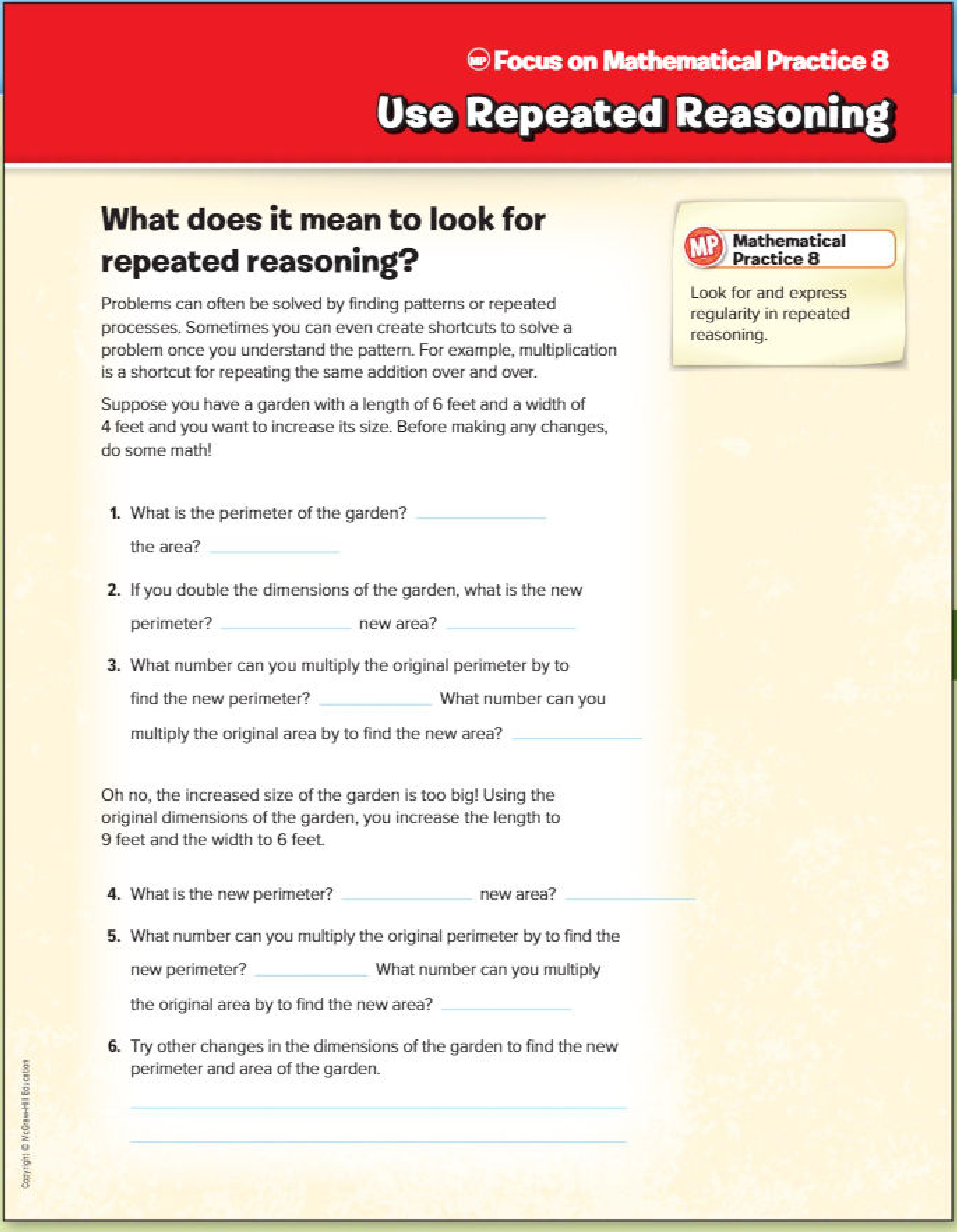
7. Write an equation that shows the relationship between distance and time.

8. Estimate how long it will take to complete the marathon.

Find it in Your Book!

MP Make Use of Structure

Look at Chapter 1. Give an example of where Mathematical Practice 7 is used. Then explain why your example represents this practice.



MP Focus on Mathematical Practice 8

Use Repeated Reasoning

What does it mean to look for repeated reasoning?

Problems can often be solved by finding patterns or repeated processes. Sometimes you can even create shortcuts to solve a problem once you understand the pattern. For example, multiplication is a shortcut for repeating the same addition over and over.

Suppose you have a garden with a length of 6 feet and a width of 4 feet and you want to increase its size. Before making any changes, do some math!

MP Mathematical Practice 8

Look for and express regularity in repeated reasoning.

1. What is the perimeter of the garden? __________ the area? __________
2. If you double the dimensions of the garden, what is the new perimeter? __________ new area? __________
3. What number can you multiply the original perimeter by to find the new perimeter? __________ What number can you multiply the original area by to find the new area? __________

Oh no, the increased size of the garden is too big! Using the original dimensions of the garden, you increase the length to 9 feet and the width to 6 feet.

4. What is the new perimeter? __________ new area? __________
5. What number can you multiply the original perimeter by to find the new perimeter? __________ What number can you multiply the original area by to find the new area? __________
6. Try other changes in the dimensions of the garden to find the new perimeter and area of the garden.

____

____

Focus narrowing the scope

Objective Look for and express regularity in repeated reasoning.

Coherence connecting within and across grades

Previous
Students looked for and made use of structure.

Now
Students look for and express regularity in repeated reasoning.

Next
Students will write rational numbers as fractions, mixed numbers, and decimals.

Rigor pursuing concepts, fluency, and applications

Mathematically proficient students recognize and use patterns that can lead to results more quickly and efficiently.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Ideas for Use

LA **Pairs Discussion** Have students work in pairs to complete Exercises 1–3. Then have them work individually to complete Exercises 4–6. Have them reconvene with their partner to check their answers and resolve any differences. MP 1, 4, 5, 6, 7, 8

Alternate Strategy

AL Review how to find perimeter and area. Then complete Exercises 1 and 2 as a whole group. MP 1, 4, 5, 6, 7

2 Practice and Apply

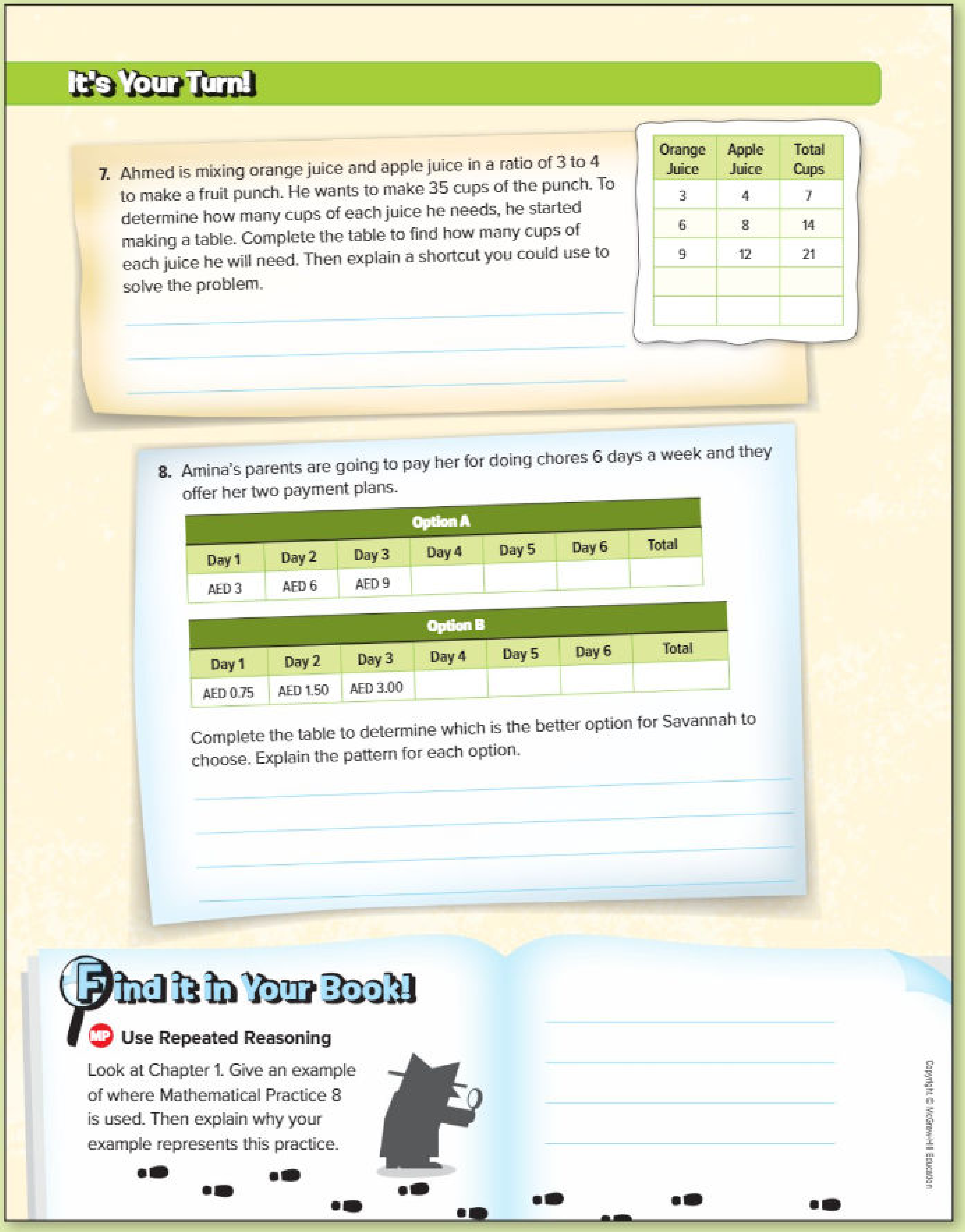
Ideas for Use

LA **Circle the Sage** Poll the class to see which students have a good understanding of how to solve Exercises 7 and 8. These students (the sages) spread around the room. Have the rest of the class divide into small groups. Have each group member report to a different sage if possible. The sages lead the discussion in how to solve the problems. Then have the students return to their groups to compare what was discussed with each sage. MP 1, 3, 4, 6, 7, 8

Alternate Strategies

AL **LA** **Act it Out** In small groups or as a class, have students act out pouring and combining cups of orange juice and apple juice for Exercise 7. Then have them continue this individually to fill in the table. MP 1, 4, 6, 8

BL For Exercise 8, have students make an "Option C" table that is better than Options A and B. Have them explain the pattern and why it is the best choice. MP 1, 3, 4



It's Your Turn!

7. Ahmed is mixing orange juice and apple juice in a ratio of 3 to 4 to make a fruit punch. He wants to make 35 cups of the punch. To determine how many cups of each juice he needs, he started making a table. Complete the table to find how many cups of each juice he will need. Then explain a shortcut you could use to solve the problem.

Orange Juice	Apple Juice	Total Cups
3	4	7
6	8	14
9	12	21

8. Amina's parents are going to pay her for doing chores 6 days a week and they offer her two payment plans.

Option A

Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Total
AED 3	AED 6	AED 9				

Option B

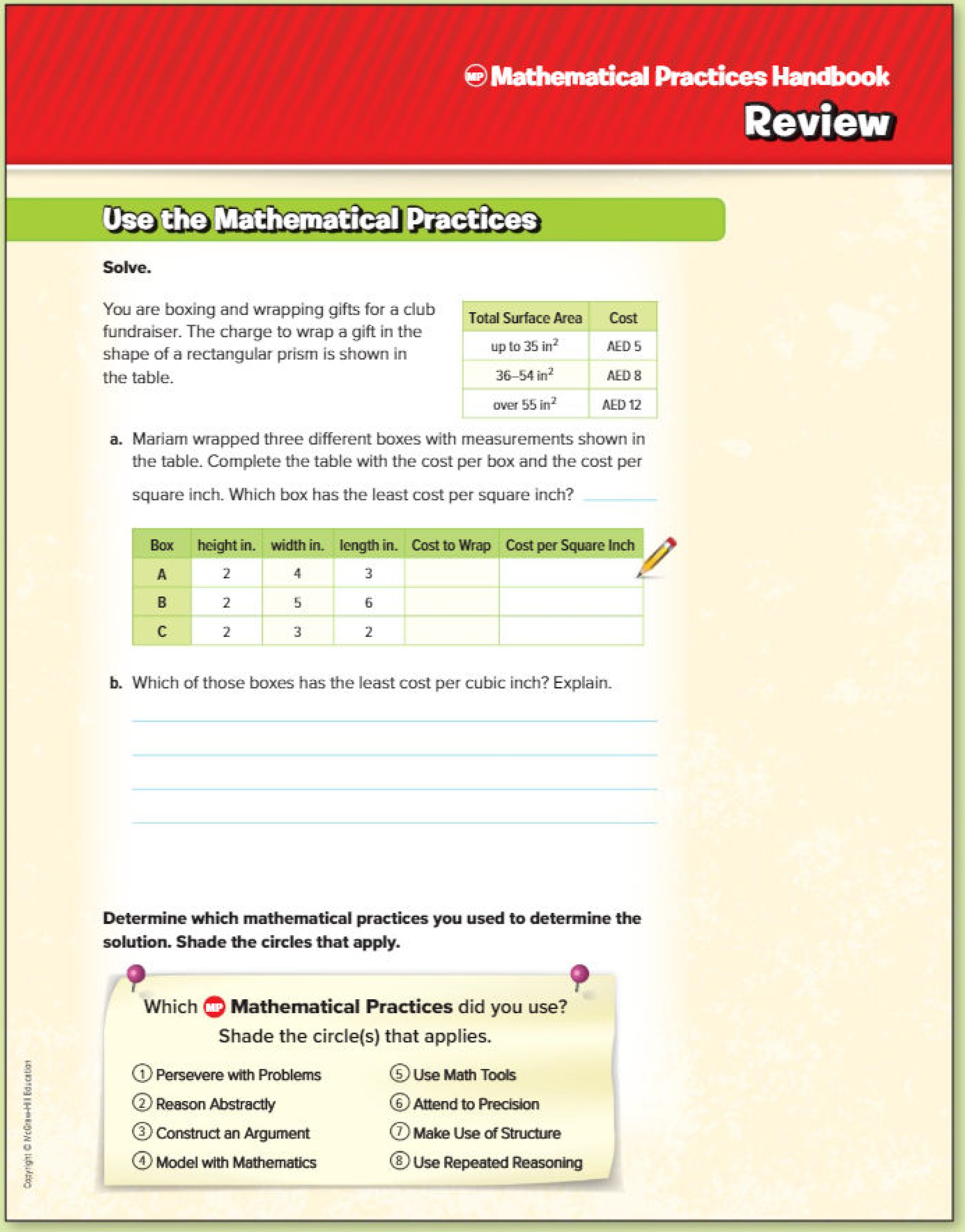
Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Total
AED 0.75	AED 1.50	AED 3.00				

Complete the table to determine which is the better option for Savannah to choose. Explain the pattern for each option.

Find it in Your Book!

MP **Use Repeated Reasoning**

Look at Chapter 1. Give an example of where Mathematical Practice 8 is used. Then explain why your example represents this practice.



MP **Mathematical Practices Handbook**

Review

Use the Mathematical Practices

Solve.

You are boxing and wrapping gifts for a club fundraiser. The charge to wrap a gift in the shape of a rectangular prism is shown in the table.

Total Surface Area	Cost
up to 35 in^2	AED 5
36–54 in^2	AED 8
over 55 in^2	AED 12

a. Mariam wrapped three different boxes with measurements shown in the table. Complete the table with the cost per box and the cost per square inch. Which box has the least cost per square inch? ____________

Box	height in.	width in.	length in.	Cost to Wrap	Cost per Square Inch
A	2	4	3		
B	2	5	6		
C	2	3	2		

b. Which of those boxes has the least cost per cubic inch? Explain.

Determine which mathematical practices you used to determine the solution. Shade the circles that apply.

Which MP **Mathematical Practices** did you use?
Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Use the Mathematical Practices

Use the next two pages to review each of the **Mathematical Practices** and how students will use them to engage in the content they will encounter throughout this text.

Ideas for Use

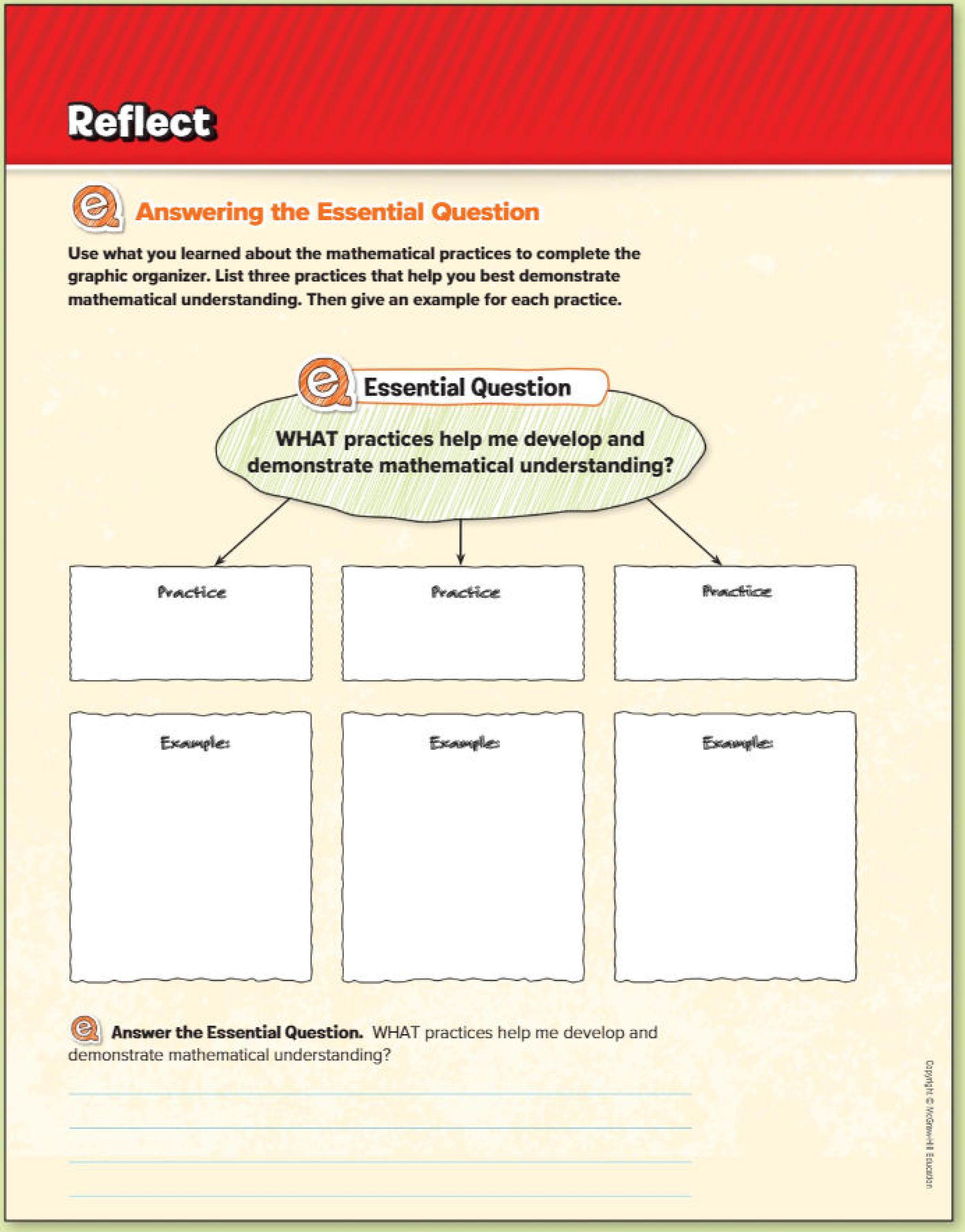
LA **Round Table Consensus** Have students work in small groups to complete the problem. Each student is responsible for contributing their individual response to the group for each step. Group members show agreement (thumbs up) or disagreement (thumbs down). If there is any disagreement, group members discuss to resolve it.
MP 1, 3, 4, 6

Answering the Essential Question

At the end of the Mathematical Practices Handbook, students should be able to answer "WHAT practices help me develop and demonstrate mathematical understanding?"

Ideas for Use

LA **Think-Pair-Share** Have students work in pairs. Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer. Then have them share their responses with their classmate before they complete the graphic organizer. **MP 1, 5, 6**



Reflect

Answering the Essential Question

Use what you learned about the mathematical practices to complete the graphic organizer. List three practices that help you best demonstrate mathematical understanding. Then give an example for each practice.

Essential Question

WHAT practices help me develop and demonstrate mathematical understanding?

Practice

Practice

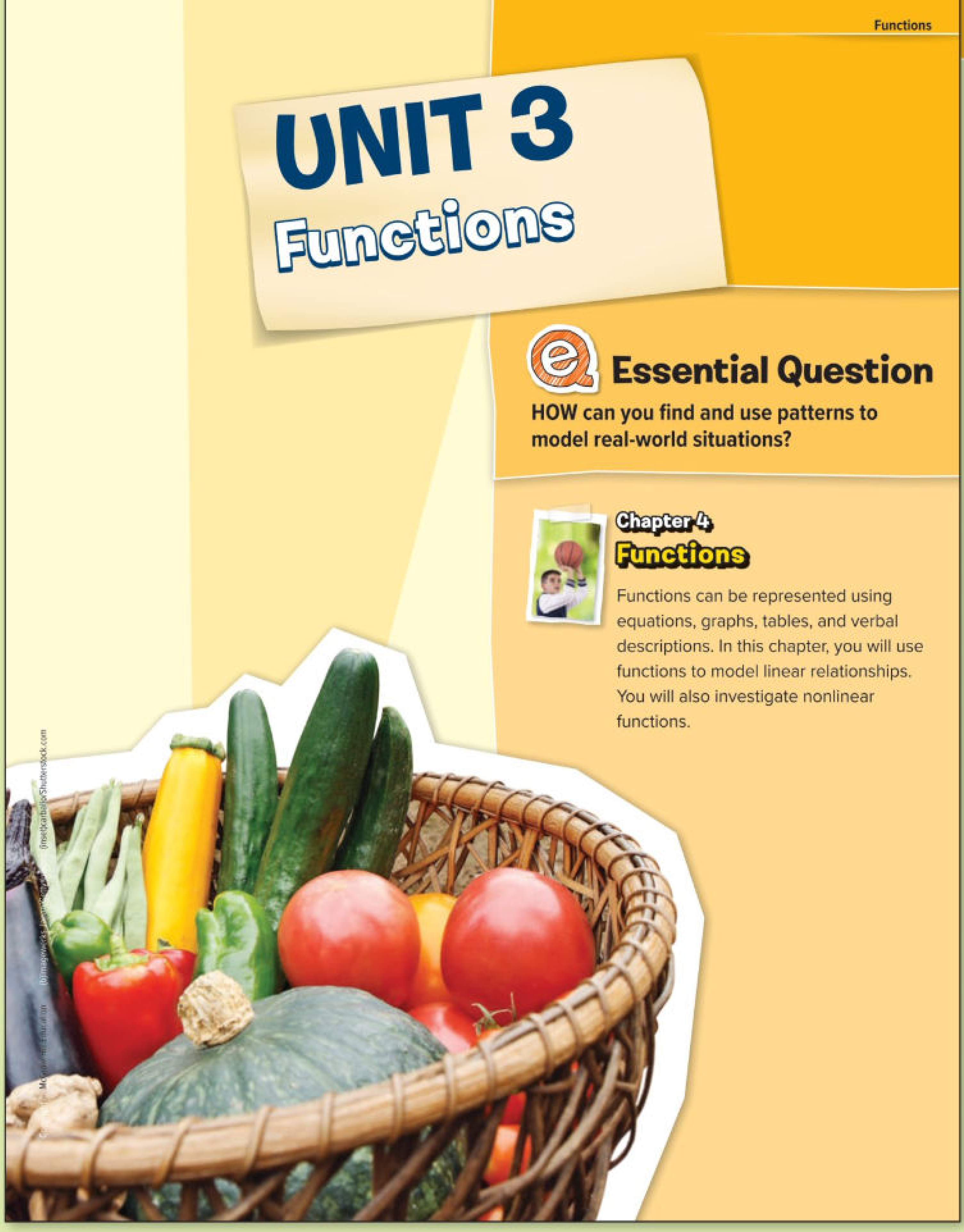
Practice

Example:

Example:

Example:

Answer the Essential Question. WHAT practices help me develop and demonstrate mathematical understanding?



UNIT 3
Functions

Essential Question

HOW can you find and use patterns to model real-world situations?

Chapter 4
Functions

Functions can be represented using equations, graphs, tables, and verbal descriptions. In this chapter, you will use functions to model linear relationships. You will also investigate nonlinear functions.

(inset)Karballo/Shutterstock.com

Essential Question

At the end of this unit, students should be able to answer “How can you find and use patterns to model real-world situations?”

Chapter 4 explores an essential question that assists students in answering the unit question. The lessons in this chapter include exercises that lead students to various aspects of the essential question.

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

continued on page 262

Use functions to model relationships between quantities.

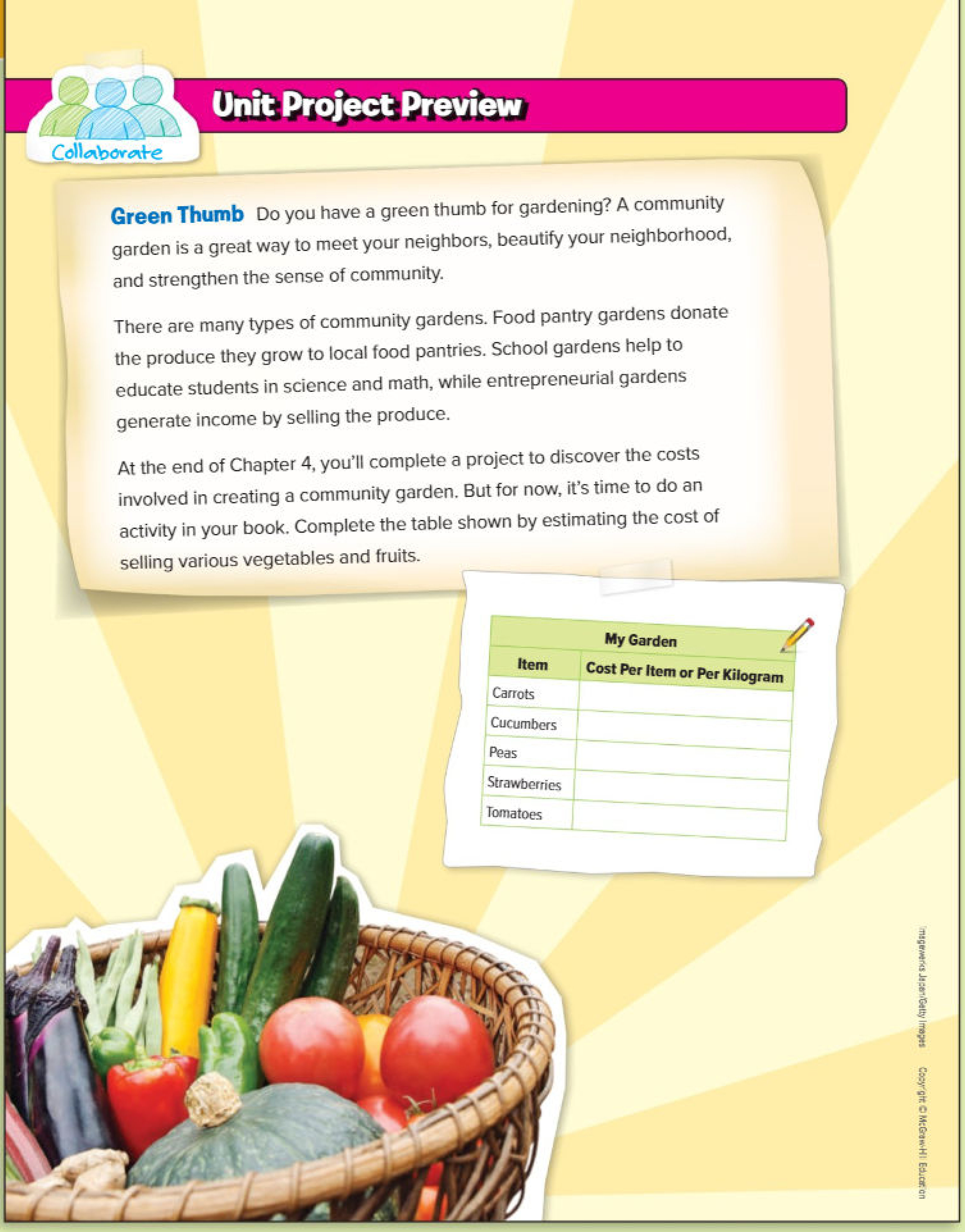
4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Unit Project Preview

Ask students what they know about gardening.

Remind students to label their costs by unit or by Kilogram.

The Unit Project can be found on pages 361–362.



Unit Project Preview

Green Thumb Do you have a green thumb for gardening? A community garden is a great way to meet your neighbors, beautify your neighborhood, and strengthen the sense of community.

There are many types of community gardens. Food pantry gardens donate the produce they grow to local food pantries. School gardens help to educate students in science and math, while entrepreneurial gardens generate income by selling the produce.

At the end of Chapter 4, you'll complete a project to discover the costs involved in creating a community garden. But for now, it's time to do an activity in your book. Complete the table shown by estimating the cost of selling various vegetables and fruits.

My Garden

Item	Cost Per Item or Per Kilogram
Carrots	
Cucumbers	
Peas	
Strawberries	
Tomatoes	

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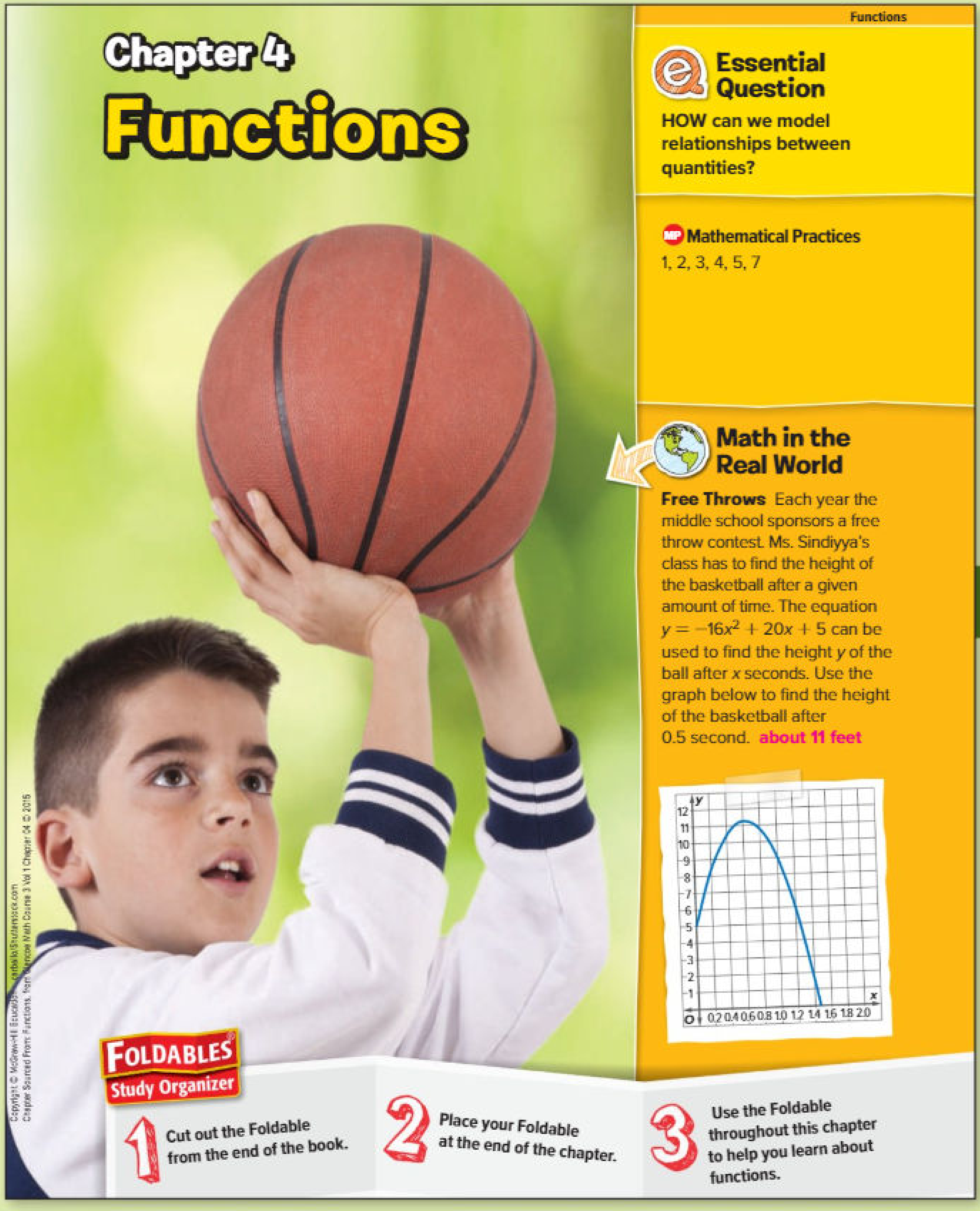

Chapter 4

Functions

Essential Question

HOW can we model relationships between quantities?

Mathematical Practices

1, 2, 3, 4, 5, 7

Math in the Real World

Free Throws Each year the middle school sponsors a free throw contest. Ms. Sindiyya's class has to find the height of the basketball after a given amount of time. The equation $y = -16x^2 + 20x + 5$ can be used to find the height y of the ball after x seconds. Use the graph below to find the height of the basketball after 0.5 second. **about 11 feet**

FOLDABLES Study Organizer

1 Cut out the Foldable from the end of the book.

2 Place your Foldable at the end of the chapter.

3 Use the Foldable throughout this chapter to help you learn about functions.

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Focus narrowing the scope

This chapter focuses on content from the **Functions** (F) domain.

Mathematical Background

Coherence connecting within and across grades

Previous	Now	Next
Students solved and graphed equations in one and two variables.	Students use words, tables, equations, and graphs to represent linear and nonlinear functions.	Students will apply expressions, equations, and functions to geometric concepts.

Rigor pursuing concepts, fluency, and applications

The Levels of Complexity charts located throughout this chapter indicate how the exercises progress from conceptual understanding and procedural skills and fluency, to application and critical thinking.

Launch the Chapter

Math in the Real World

Free Throws Students can find the actual height of the basketball after 0.5 second if they replace x with 0.5 in the equation and simplify.

What Tools Do You Need?

Vocabulary Activity

LA As you proceed through the chapter, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define: A function is a relation in which each member of the domain (input value) is paired with exactly one member of the range (output value).

Example:

Input	Output
1	3
2	5
3	7

Ask:

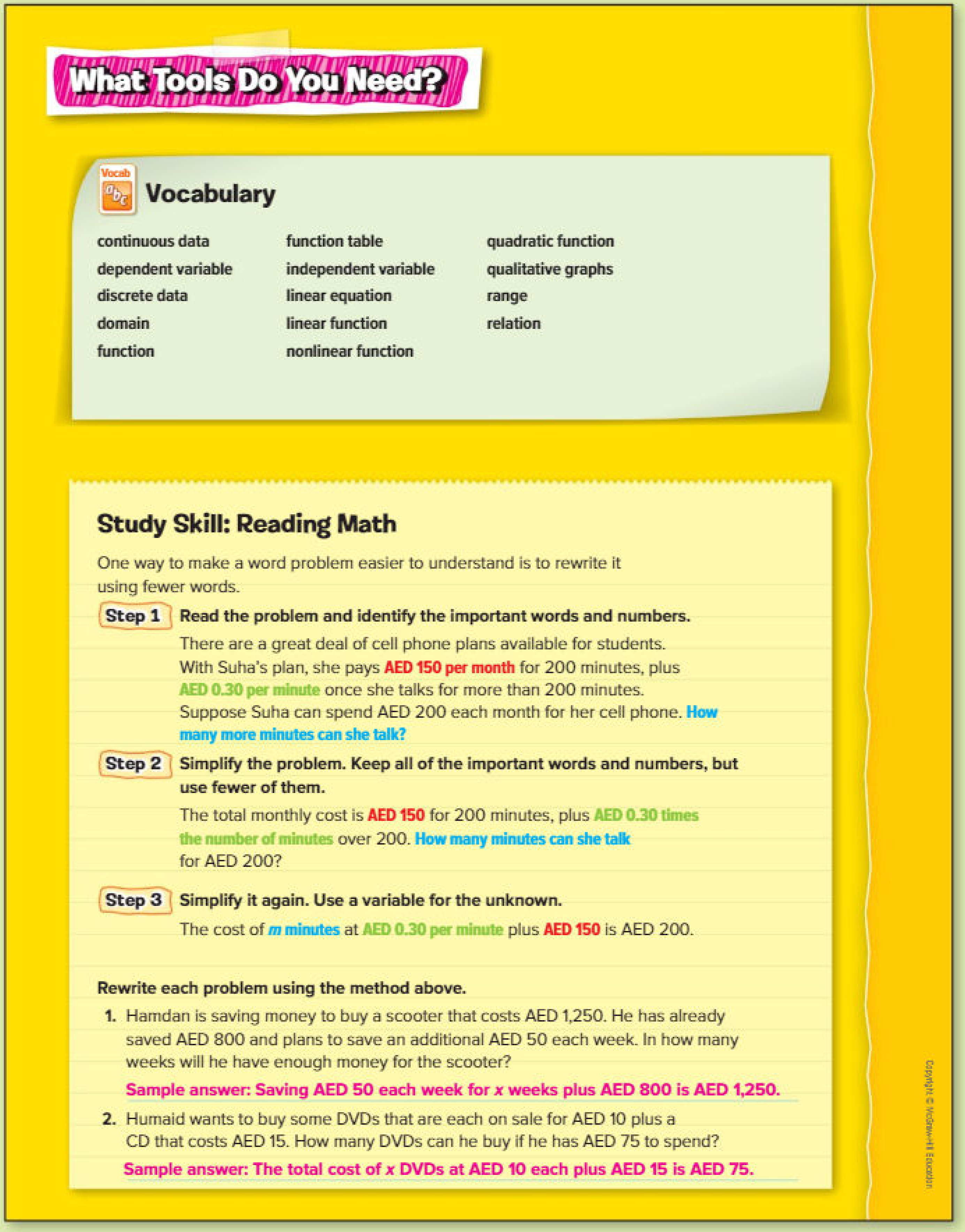
- *What are the domain and range of the function?* **D: {1, 2, 3}; R: {3, 5, 7}**

Reading Math

LA Have students read the Reading Math section.

Ask:

- *How does rewriting the problem in fewer words help in solving it?* **Sample answer: It helps to sort out what you know and what you need to determine.**
- *Have students read Exercise 1. Then ask, what are the important words and numbers in the problem?* **Sample answer: costs AED125, saved AED80, additional AED5 each week, in how many weeks will he have enough money for the scooter**
- *Have students read Exercise 2. Then ask, what are the important words and numbers in the problem?* **Sample answer: some DVDs are AED10 each, plus a CD that costs AED15, AED75 to spend**



What Tools Do You Need?

Vocabulary

continuous data
dependent variable
discrete data
domain
function
function table
independent variable
linear equation
linear function
nonlinear function
quadratic function
qualitative graphs
range
relation

Study Skill: Reading Math

One way to make a word problem easier to understand is to rewrite it using fewer words.

Step 1 **Read the problem and identify the important words and numbers.**

There are a great deal of cell phone plans available for students. With Suha's plan, she pays **AED 150 per month** for 200 minutes, plus **AED 0.30 per minute** once she talks for more than 200 minutes. Suppose Suha can spend AED 200 each month for her cell phone. **How many more minutes can she talk?**

Step 2 **Simplify the problem. Keep all of the important words and numbers, but use fewer of them.**

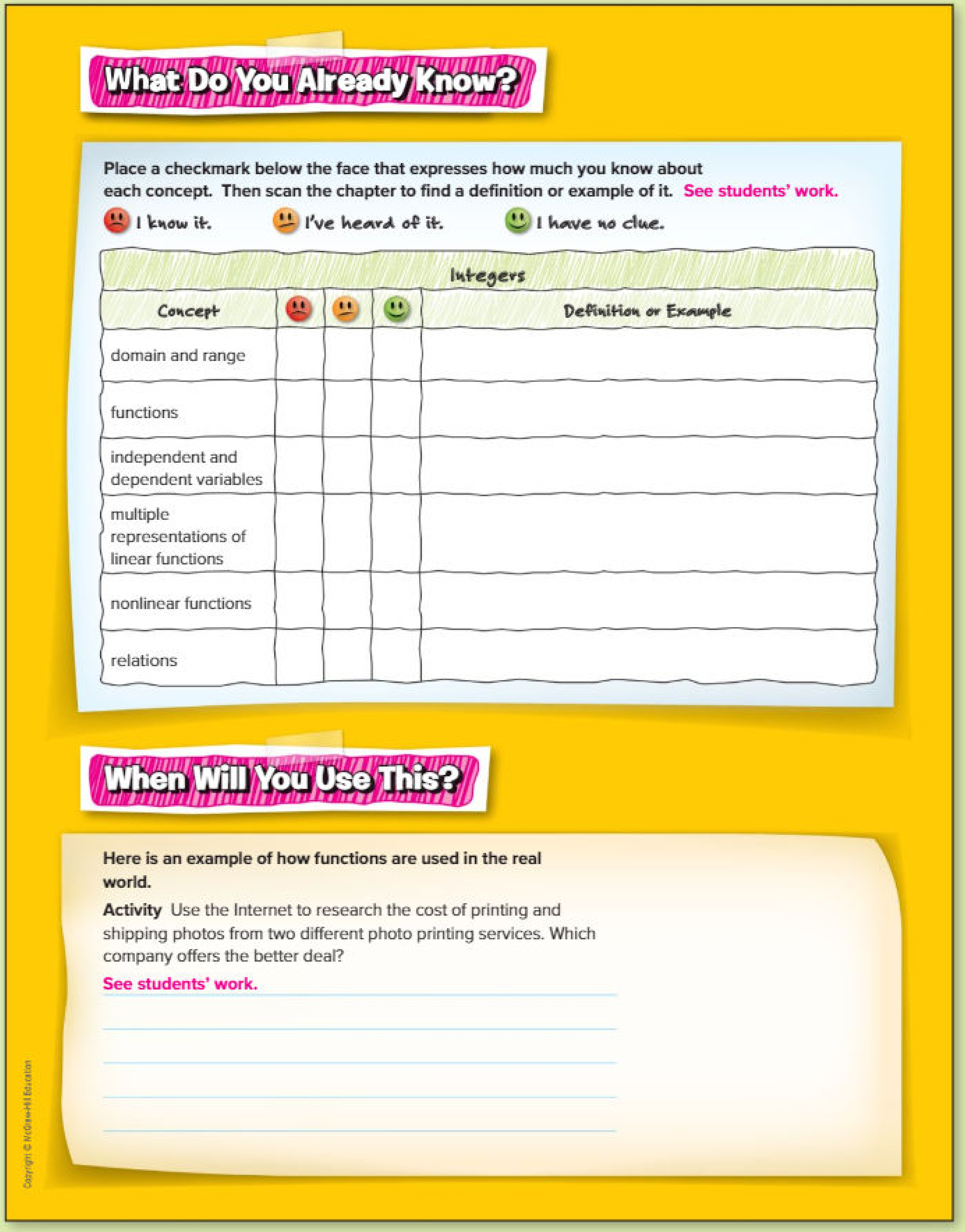
The total monthly cost is **AED 150** for 200 minutes, plus **AED 0.30 times the number of minutes** over 200. **How many minutes can she talk** for AED 200?

Step 3 **Simplify it again. Use a variable for the unknown.**

The cost of ***m*** **minutes** at **AED 0.30 per minute** plus **AED 150** is AED 200.

Rewrite each problem using the method above.

1. Hamdan is saving money to buy a scooter that costs AED 1,250. He has already saved AED 800 and plans to save an additional AED 50 each week. In how many weeks will he have enough money for the scooter?
Sample answer: Saving AED 50 each week for *x* weeks plus AED 800 is AED 1,250.
2. Humaid wants to buy some DVDs that are each on sale for AED 10 plus a CD that costs AED 15. How many DVDs can he buy if he has AED 75 to spend?
Sample answer: The total cost of *x* DVDs at AED 10 each plus AED 15 is AED 75.



What Do You Already Know?

Place a checkmark below the face that expresses how much you know about each concept. Then scan the chapter to find a definition or example of it. See students' work.

☹ I know it. 😐 I've heard of it. ☺ I have no clue.

Integers				
Concept	☹	😐	☺	Definition or Example
domain and range				
functions				
independent and dependent variables				
multiple representations of linear functions				
nonlinear functions				
relations				

When Will You Use This?

Here is an example of how functions are used in the real world.

Activity Use the Internet to research the cost of printing and shipping photos from two different photo printing services. Which company offers the better deal?

See students' work.

What Do You Already Know?

In this activity students assess their prior knowledge choosing a face to represent their knowledge about concepts in the chapter.

After completing the chapter, have students return to this page and have them reevaluate their knowledge level about the content.

When Will You Use This?

Activity

Students may not realize how functions help to represent real-world situations and reveal comparisons.

Are You Ready?

Use this page to determine if students have skills that are needed for the chapter.

Quick Review

Students with strong math backgrounds may opt to go directly to the Quick Check.

REVIEW	
Example	Skill
1	Graph on the coordinate plane.
2	Evaluate expressions.

Quick Check

If students have difficulty with the exercises, present an additional example to clarify any misconceptions they may have.

Exercises 1–6

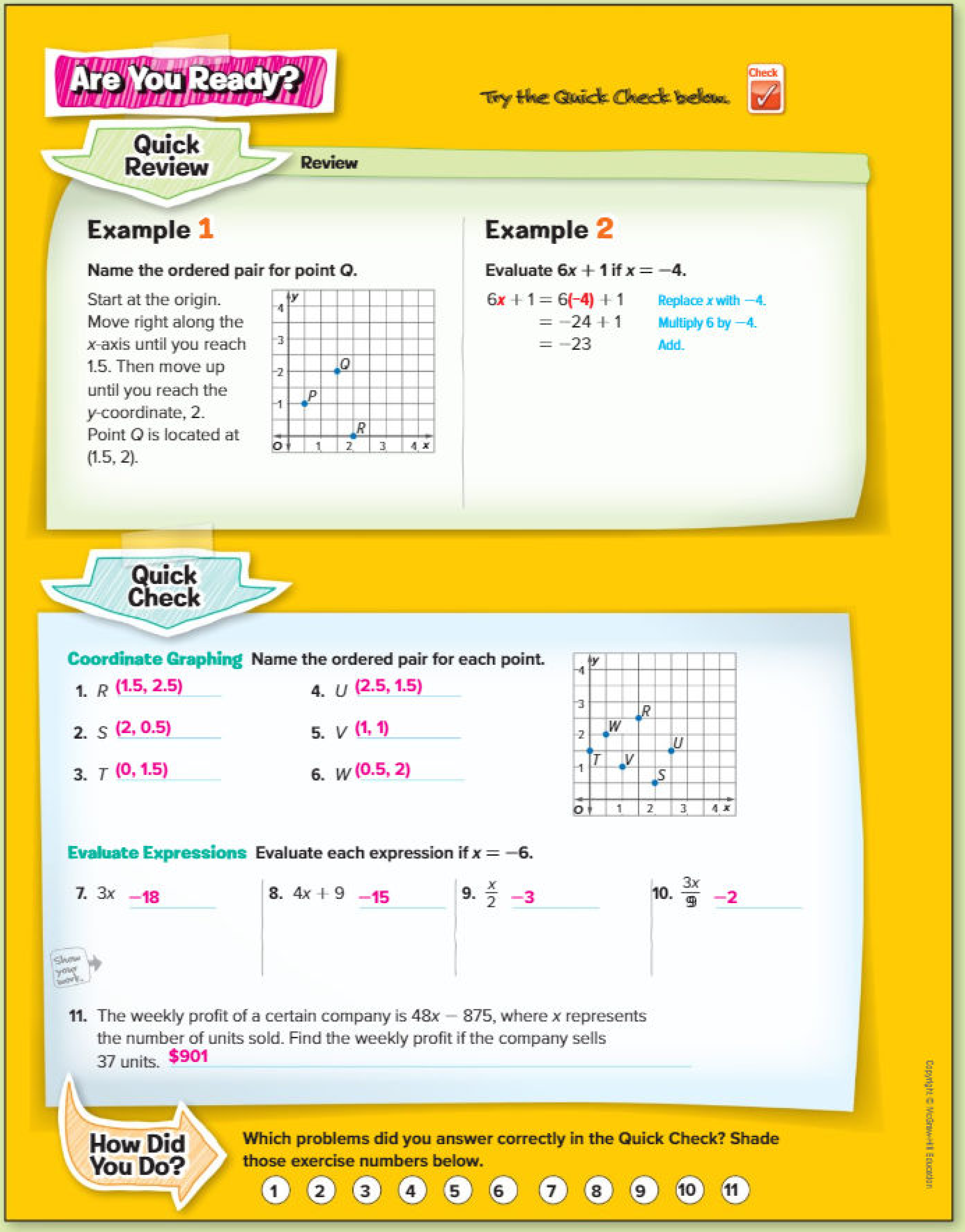
Refer to Example 1 on the student page. Name the ordered pair for point *R*. (2, 0)

Exercises 7–11

Evaluate $6x + 2$ if $x = -2$. -10

Track Your Progress

Prior to beginning this chapter, have your students rate their knowledge of the objectives it addresses. At the end of the chapter, have your students return to rate their knowledge again. They should see that their knowledge of the key ideas increased.



Are You Ready?

Try the Quick Check below.

Quick Review

Review

Example 1

Name the ordered pair for point *Q*.

Start at the origin. Move right along the *x*-axis until you reach 1.5. Then move up until you reach the *y*-coordinate, 2. Point *Q* is located at (1.5, 2).

Example 2

Evaluate $6x + 1$ if $x = -4$.

$$\begin{aligned} 6x + 1 &= 6(-4) + 1 && \text{Replace } x \text{ with } -4. \\ &= -24 + 1 && \text{Multiply 6 by } -4. \\ &= -23 && \text{Add.} \end{aligned}$$

Quick Check

Coordinate Graphing **Name the ordered pair for each point.**

1. *R* (1.5, 2.5)
2. *S* (2, 0.5)
3. *T* (0, 1.5)
4. *U* (2.5, 1.5)
5. *V* (1, 1)
6. *W* (0.5, 2)

Evaluate Expressions **Evaluate each expression if $x = -6$.**

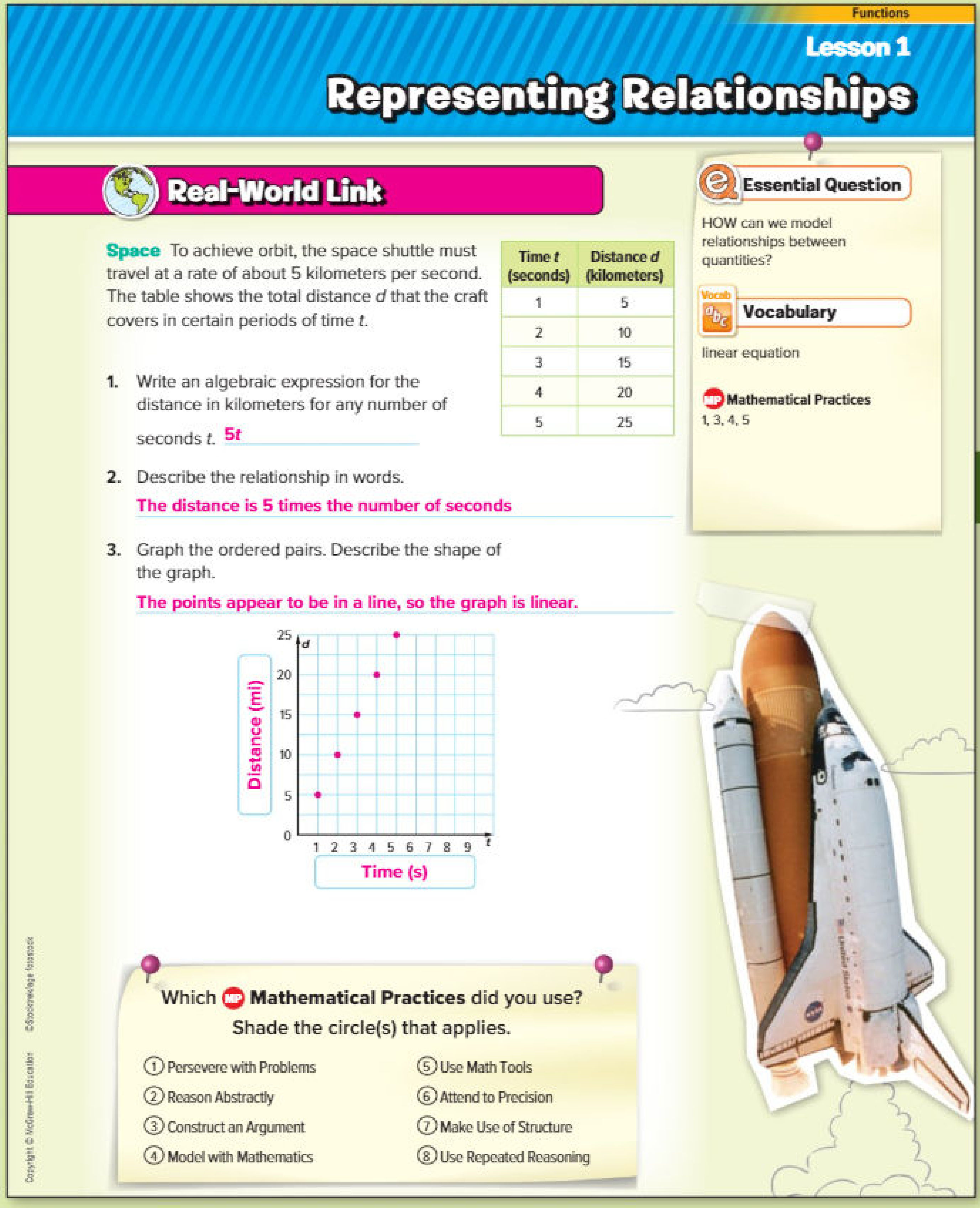
7. $3x$ -18
8. $4x + 9$ -15
9. $\frac{x}{2}$ -3
10. $\frac{3x}{9}$ -2

Show your work.

11. The weekly profit of a certain company is $48x - 875$, where x represents the number of units sold. Find the weekly profit if the company sells 37 units. $901

How Did You Do? **Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.**

① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨ ⑩ ⑪



Lesson 1

Representing Relationships

Real-World Link

Space To achieve orbit, the space shuttle must travel at a rate of about 5 kilometers per second. The table shows the total distance *d* that the craft covers in certain periods of time *t*.

Time *t* (seconds)	Distance *d* (kilometers)
1	5
2	10
3	15
4	20
5	25

1. Write an algebraic expression for the distance in kilometers for any number of seconds *t*. $5t$

2. Describe the relationship in words.
The distance is 5 times the number of seconds

3. Graph the ordered pairs. Describe the shape of the graph.
The points appear to be in a line, so the graph is linear.

Essential Question

HOW can we model relationships between quantities?

Vocab

Vocabulary

linear equation

Mathematical Practices
1, 3, 4, 5

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Translate tables and graphs into linear equations.

Coherence connecting within and across grades

Previous	Now	Next
Students wrote an equation in the form $y = mx + b$ to model linear relationships.	Students will translate tables and graphs into linear equations.	Students will use tables and graphs to represent relations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 273.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Small Group Grocery** Introduce the lesson by having small groups act out a grocery store purchase. Set up a produce section with potatoes or another item that is purchased by the kilogram Have one student play the role of the grocer and choose several students to play customers. Have the first student ask to buy 3 kilograms of potatoes. The grocer says that will cost AED6. The next student asks to buy 5 kilograms at a cost of AED10. The last student asks for 1 kilogram at a cost of AED2. Ask the class if they see a relationship between the number of pounds and the cost. Have them make a table of values. When the activity is complete, have students work in groups to complete Exercises 1–3. MP 1, 2, 4

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Translate a table into an equation.

AL
- *How do the values for quarts change?* They increase by 1.
- *How do the values for liters change?* They increase by 0.95.

OL
- *Is the rate of change constant?* yes

BL
- *What is another word for the constant rate of change?* slope

2. Use an equation to solve a real-world problem.

AL
- *How can you verify that the equation from Example 1 is correct?* Substitute the value for each number of quarts from the table for q in the equation and determine the value for liters. Check that the solution matches the corresponding value for the number of liters in the table.

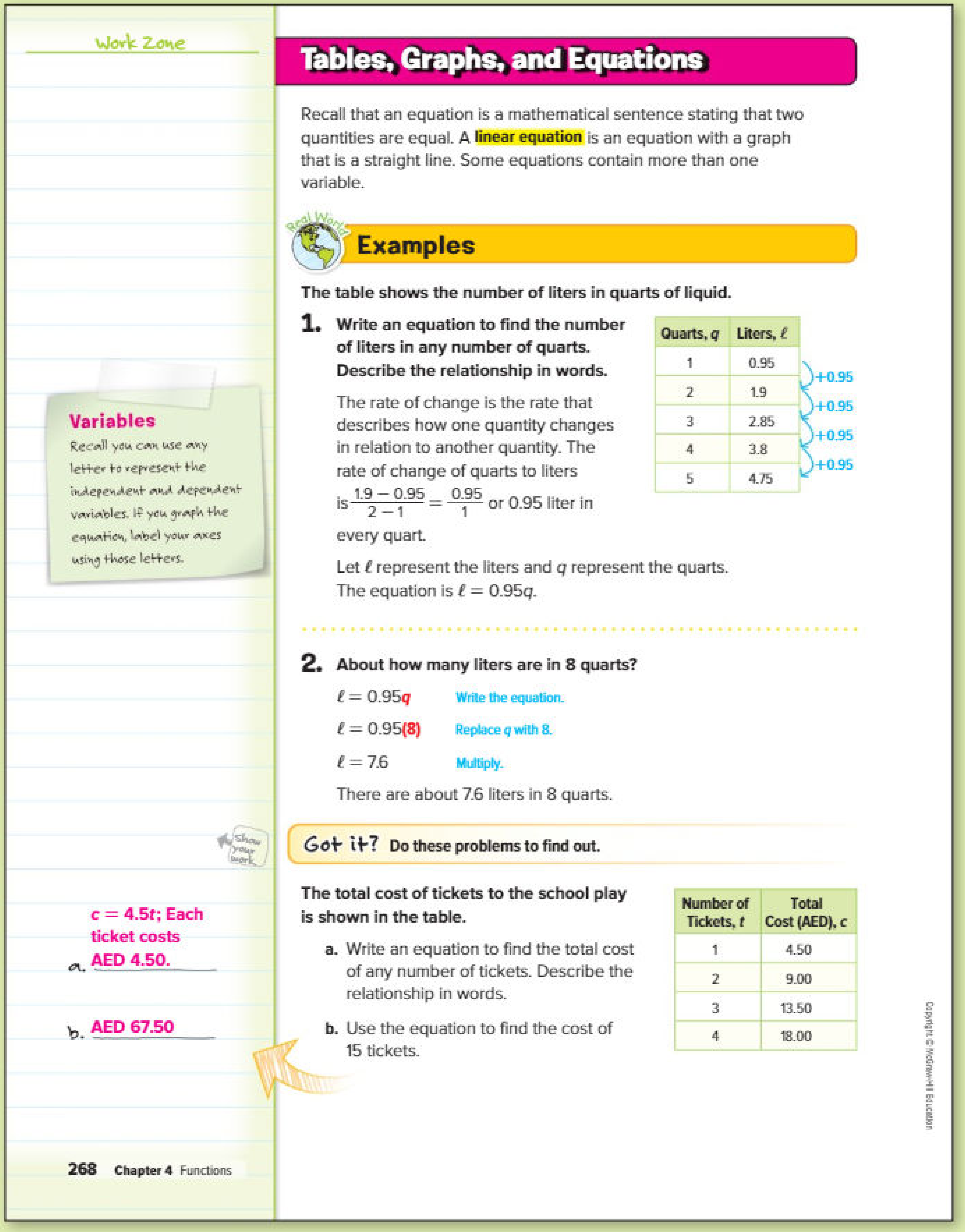
OL
- *How would you find the number of liters in 8 quarts?* Replace q with 8 in the equation and multiply.
- *About how many liters are in 8 quarts?* about 7.6 liters

BL
- *Is the solution for the number of liters in 8 quarts reasonable? How do you know?* yes; Sample answer: The number of liters in 1 quart is 0.95, which is a little less than 1. So, the number of liters in 8 quarts should be a little less than 8.

Need Another Example?

Kilometers, k	1	2	3	4
Miles, m	0.62	1.24	1.86	2.48

The table shows the relationship between miles and kilometers. Write an equation to find the number of miles in any number of kilometers. Describe the relationship in words then use the equation to find the number of miles in 20 kilometers. $m = 0.62k$; There is 0.62 mile in one kilometer; 12.4 mi



Work Zone

Variables

Recall you can use any letter to represent the independent and dependent variables. If you graph the equation, label your axes using those letters.

Tables, Graphs, and Equations

Recall that an equation is a mathematical sentence stating that two quantities are equal. A **linear equation** is an equation with a graph that is a straight line. Some equations contain more than one variable.

Examples

The table shows the number of liters in quarts of liquid.

1. Write an equation to find the number of liters in any number of quarts. Describe the relationship in words.

Quarts, q	Liters, ℓ
1	0.95
2	1.9
3	2.85
4	3.8
5	4.75

+0.95
+0.95
+0.95
+0.95

The rate of change is the rate that describes how one quantity changes in relation to another quantity. The rate of change of quarts to liters is $\frac{1.9 - 0.95}{2 - 1} = \frac{0.95}{1}$ or 0.95 liter in every quart.

Let ℓ represent the liters and q represent the quarts. The equation is $\ell = 0.95q$.

2. About how many liters are in 8 quarts?

$\ell = 0.95q$ Write the equation.
$\ell = 0.95(8)$ Replace q with 8.
$\ell = 7.6$ Multiply.

There are about 7.6 liters in 8 quarts.

Got it? Do these problems to find out.

The total cost of tickets to the school play is shown in the table.

Number of Tickets, t	Total Cost (AED), c
1	4.50
2	9.00
3	13.50
4	18.00

a. Write an equation to find the total cost of any number of tickets. Describe the relationship in words.

b. Use the equation to find the cost of 15 tickets.

Show your work

a. $c = 4.5t$; Each ticket costs AED 4.50.

b. AED 67.50

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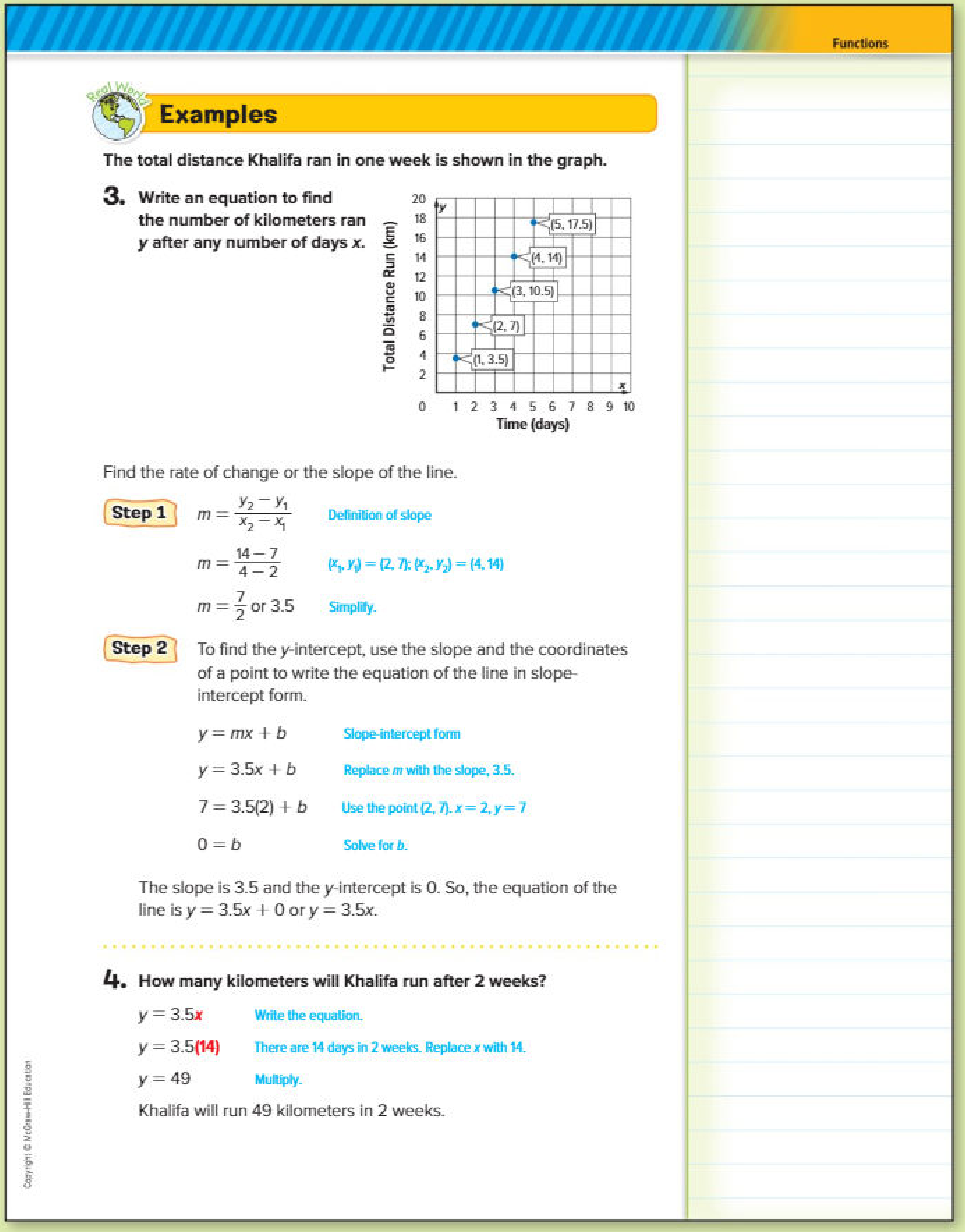


Examples

The total distance Khalifa ran in one week is shown in the graph.

3. **Write an equation to find the number of kilometers ran y after any number of days x.**

Find the rate of change or the slope of the line.

Step 1 $m = \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \frac{14 - 7}{4 - 2}$ $(x_1, y_1) = (2, 7); (x_2, y_2) = (4, 14)$

$m = \frac{7}{2}$ or 3.5 Simplify.

Step 2 To find the y-intercept, use the slope and the coordinates of a point to write the equation of the line in slope-intercept form.

$y = mx + b$ Slope-intercept form

$y = 3.5x + b$ Replace m with the slope, 3.5.

$7 = 3.5(2) + b$ Use the point (2, 7). $x = 2, y = 7$

$0 = b$ Solve for b.

The slope is 3.5 and the y-intercept is 0. So, the equation of the line is $y = 3.5x + 0$ or $y = 3.5x$.

4. **How many kilometers will Khalifa run after 2 weeks?**

$y = 3.5x$ Write the equation.

$y = 3.5(14)$ There are 14 days in 2 weeks. Replace x with 14.

$y = 49$ Multiply.

Khalifa will run 49 kilometers in 2 weeks.

Examples

3–4. Translate a graph into an equation. Use an equation to solve a real-world problem.

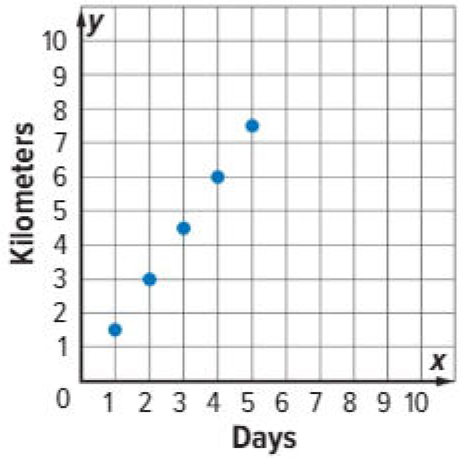
AL
- *Example 3: Is there a constant rate of change? Explain.* yes; Sample answer: The rise over run is the same for any two points.
- *Example 4: How many days are in two weeks?* 14 days

OL
- *Example 3: How would you find the slope of the line that runs through the points?* Choose two points on the line and find $\frac{\text{change in } y}{\text{change in } x}$.
- *Example 3: Using the slope, how could you write an equation for the line? What is the equation?* Sample answer: Use the slope and a point on the line to find the y-intercept. Then write the equation in slope-intercept form; $y = 3.5x$
- *Example 3: What is the y-intercept?* 0
- *Example 4: How would you find the number of kilometers ran after two weeks?* Replace x with 14 in the equation and simplify.

BL
- *Is this relationship proportional or non-proportional? Explain.* proportional; Sample answer: The equation can be written in the form $y = mx$.

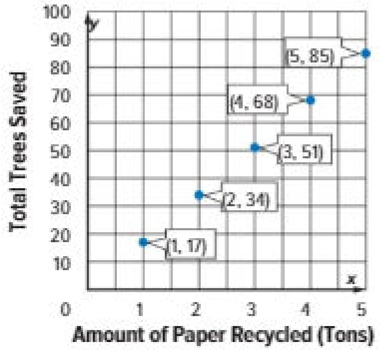
Need Another Example?

The graph shows the total number of kilometers Hala hiked. Write an equation to find the number of kilometers hiked after any number of days. Then use the equation to find the number of kilometers Hala will hike after 1 week. $y = 1.5x$; 10.5 km



Got it? Do these problems to find out.

The number of trees saved by recycling paper is shown.



c. Write an equation to find the total number of trees y that can be saved for any number of tons of paper x.

d. Use the equation to find how many trees could be saved if 500 tons of paper are recycled.

Show your work.

c. $y = 17x$

d. 8,500 trees

Key Concept

Multiple Representations of Linear Equations

Words

Distance traveled is equal to 12 kilometers per second times the number of seconds.

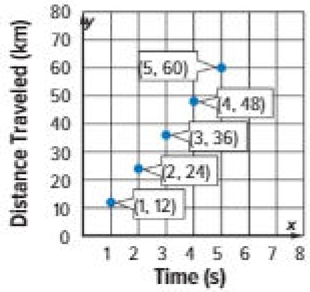
Equation

$d = 12s$

Table

Time (seconds)	Distance (kilometers)
1	12
2	24
3	36
4	48
5	60

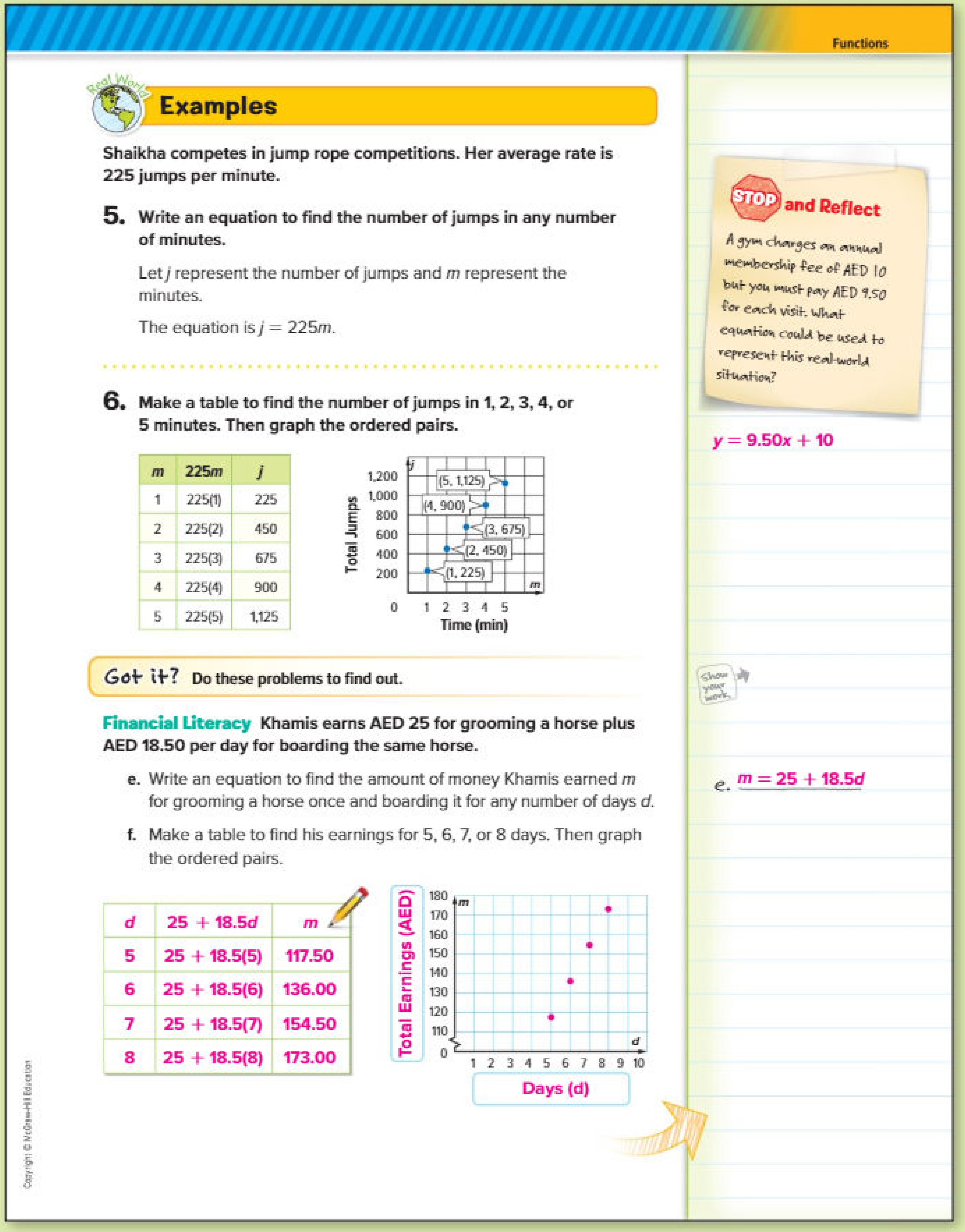
Graph



Words, equations, tables, and graphs can be used to represent linear relationships.

Watch Out!

Common Error Students may have trouble writing an equation given a graph. Remind them to find the rate of change between points and substitute that number for m in $y = mx$ or $y = mx + b$ form.



Functions

Examples

Shaikha competes in jump rope competitions. Her average rate is 225 jumps per minute.

5. Write an equation to find the number of jumps in any number of minutes.

Let j represent the number of jumps and m represent the minutes.

The equation is $j = 225m$.

6. Make a table to find the number of jumps in 1, 2, 3, 4, or 5 minutes. Then graph the ordered pairs.

m	$225m$	j
1	225(1)	225
2	225(2)	450
3	225(3)	675
4	225(4)	900
5	225(5)	1,125

Got it? Do these problems to find out.

Financial Literacy Khamis earns AED 25 for grooming a horse plus AED 18.50 per day for boarding the same horse.

e. Write an equation to find the amount of money Khamis earned m for grooming a horse once and boarding it for any number of days d.

f. Make a table to find his earnings for 5, 6, 7, or 8 days. Then graph the ordered pairs.

d	$25 + 18.5d$	m
5	25 + 18.5(5)	117.50
6	25 + 18.5(6)	136.00
7	25 + 18.5(7)	154.50
8	25 + 18.5(8)	173.00

STOP and Reflect

A gym charges an annual membership fee of AED 10 but you must pay AED 9.50 for each visit. What equation could be used to represent this real-world situation?

$y = 9.50x + 10$

Show your work.

e. $m = 25 + 18.5d$

Examples

5. Represent a relationship with an equation.

- **AL** • *What does the variable j represent?* the number of jumps *m?* minutes
- **OL** • *What is the equation that represents this situation?* $j = 225m$
- **BL** • *Why can we use j instead of y and m instead of x?* Sample answer: The letter we choose for our variable does not matter. In this case, it makes sense to use j for jumps and m for minutes.

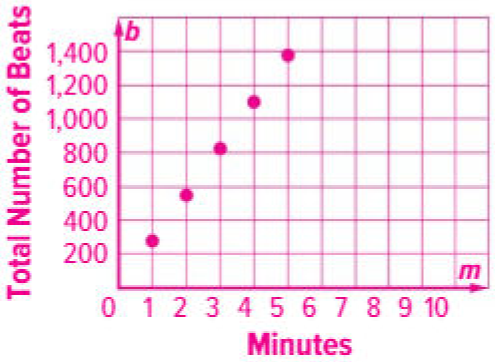
6. Represent a relationship with a table and graph.

- **AL** • *How many jumps will she have in two minutes?* 450 *three?* 675 *four?* 900 *five?* 1,125
- **OL** • *What ordered pairs could be used to represent the number of jumps in 1, 2, 3, 4, and 5 minutes?* (1, 225), (2, 450), (3, 675), (4, 900), (5, 1,125)
- **BL** • *Describe two different ways you can complete the j-values in the table.* Sample answer: Start at 225 and add 225 for each successive row or multiply each number of minutes by 225.

Need Another Example?

The average heart rate of a chicken is 275 beats per minute. Write an equation to find the number of beats in any number of minutes. Make a table to find the number of beats in 1, 2, 3, 4, or 5 minutes. Then graph the ordered pairs. $b = 275m$

m	$275m$	b
1	275(1)	275
2	275(2)	550
3	275(3)	825
4	275(4)	1,100
5	275(5)	1,375



Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL **LA** **Team-Pair-Solo** Have students complete Exercise 1 in a group of 4 students. Each student is responsible to ensure that every other group member understands how to write an equation in $y = mx$ form from a table of values and how to use that equation to determine an unknown value. Then have the group split into pairs to complete Exercise 2. Students should ask for support, if needed, prior to having students work individually to complete Exercises 3 and 4. Then have them share their responses with a partner. MP 1, 2, 4

BL **LA** **Trade-a-Problem** Have students write a real-world problem similar to Exercise 3. Have them trade their problems with a partner. Each partner should create a table of values, create a graph, and write an equation that represents the relationship. Have students check each other's work, and discuss and resolve any differences. MP 1, 2, 4

Guided Practice

1. The table shows the total number of text messages that Omar sent over 4 days. (Examples 1 and 2)

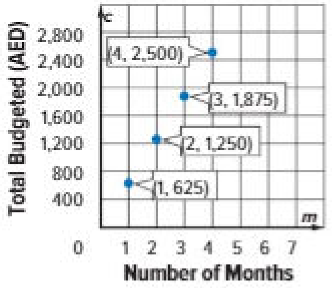
Number of Days, d	1	2	3	4
Total Messages, m	50	100	150	200

a. Write an equation to find the total number of messages sent in any number of days. Describe the relationship in words.

$m = 50d$; He sends an averages of 50 messages each day.

b. Use the equation to find how many text messages Omar would send in 30 days. 1,500 messages

2. **Financial Literacy** The graph shows the amount of money Rasheed's family budgets for food each month. Write an equation to find the total amount of money c budgeted in any number of months m. Use the equation to determine how much money Rasheed's family should budget for 12 months. (Examples 3 and 4) $c = 625m$; AED 7,500

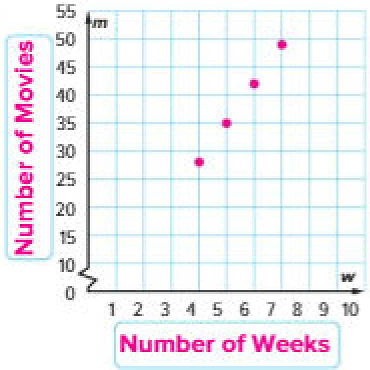


3. A store receives an average of 7 new movies per week. (Examples 5 and 6)

a. Write an equation to find the number of new movies m in any number of weeks w. $m = 7w$

b. Make a table to find the number of new movies received in 4, 5, 6, or 7 weeks. Then graph the ordered pairs.

w	$7w$	m
4	7(4)	28
5	7(5)	35
6	7(6)	42
7	7(7)	49



4. **Building on the Essential Question** How can you use a graph to write an equation?

Sample answer: Choose two points on the graph and find the slope. Then use the slope and one point in the slope-intercept form of an equation to find the y-intercept. Then write the equation.

Rate Yourself!

Are you ready to move on?
Shade the section that applies.

YES ? NO

Name ______________________ My Homework ______________________

Independent Practice

1. The number of baskets a company produces each day is shown in the table. (Examples 1 and 2)

Number of Days, d	Total Baskets, b
1	45
2	90
3	135
4	180

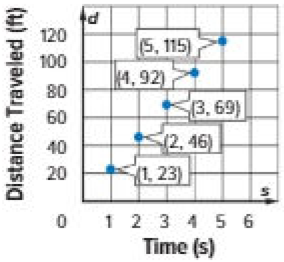
Show your work.

a. Write an equation to find the total number of baskets crafted in any number of days. Describe the relationship in words.

$b = 45d$; Forty-five baskets are produced every day.

b. Use the equation to determine how many baskets the company makes in one non-leap year. 16,425 baskets

2. A type of dragonfly is the fastest insect. The graph shows how far the dragonfly can travel. (Examples 3 and 4)



a. Write an equation to find how far the dragonfly can travel d in any number of seconds s. $d = 23s$

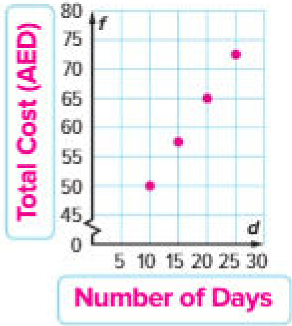
b. Use the equation to determine how far the dragonfly can travel in one minute. 1,380 ft

3. A library charges a late return fee of AED 35 plus AED 1.50 per day that a book is returned late. (Examples 5 and 6)

a. Write an equation to find the total late fee f for any number of days late d. $f = 3.5 + 0.15d$

b. Make a table to find the total fee if a book is 10, 15, 20, or 25 days late. Then graph the ordered pairs.

d	$35 + 1.5d$	f
10	$35 + 1.5(10)$	50
15	$35 + 1.5(15)$	57.5
20	$35 + 1.5(20)$	65
25	$35 + 1.5(25)$	72.5



ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–3, 8–10	4, 11, 12	5–7
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levles to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–3, 5, 7, 11, 12
OL	On Level	1, 3–5, 7, 11, 12
BL	Beyond Level	4–7, 11, 12

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	6
3 Construct viable arguments and critique the reasoning of others.	4
4 Model with mathematics.	5, 7
5 Use appropriate tools strategically.	10

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

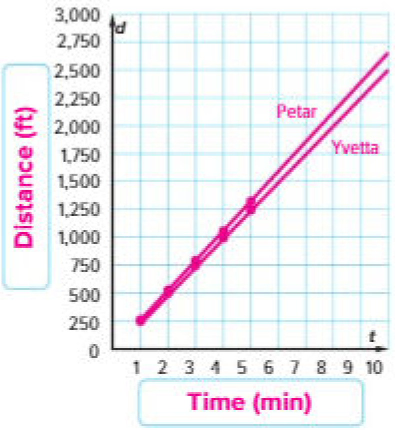
There are 1,000 meters in every kilometer. Have students write an equation to find the number of meters m in any number of kilometers k. Then have them find the number of feet in 2.5 kilometers. $m = 1{,}000k$; 2,500 m

4. **Multiple Representations** The two fastest times for swimming the English Channel belong to Petar Stoychev and Yvetta Hlaváčová. Petar's average speed was 80 meters per minute. Yvetta's average speed was 75 meters per minute.

Time (min)	Petar	Yvetta
1	80	75
2	530	498
3	795	747
4	1,060	996
5	1,325	1,245

a. **Tables** Complete the table of ordered pairs in which the x-coordinate represents the time and the y-coordinate represents the total distance swum in 1, 2, 3, 4, and 5 minutes.

b. **Graphs** Graph each set of ordered pairs on the coordinate plane.



c. **Algebra** Write an equation for each swimmer to find the number of meters swum d in any number of minutes t. Petar: $d = 265t$; Yvetta: $d = 249t$

d. **Numbers** If Petar Stoychev swam the Channel in 6 hours, 57 minutes, and 50 seconds, approximately how wide in kilometers is the English Channel? (*Hint:* 1 km = 1,000 m) about 34 kilometers

H.O.T. Problems Higher Order Thinking

5. **Model with Mathematics** Write an equation with two variables that represents a real-world situation. Sample answer: $d = 60t$; A car is traveling at a rate of 60 kilometers per hour.

6. **Persevere with Problems** The table shows the areas of circles with radii from 1 through 3 m.

Radius (m), r	1	2	3
Area (m^2), A	π	4π	9π

Recall that π has a value of about 3. Write an equation in two variables to represent the relation in the table. $A = \pi \cdot r \cdot r$ or $A = \pi r^2$

7. **Model with Mathematics** Write about a real-world situation that can be represented by the equation $y = 4x$.

See students' work.

Name ____________________ My Homework ____________________

Extra Practice

8. The table shows the number of square inches per square foot.

Square Feet, f	Square Inches, i
1	144
2	288
3	432
4	576

a. Write an equation to find the number of square inches i in any number of square feet f. Describe the relationship in words.

$i = 144f$; There are 144 square inches in every square foot.

Homework Help

The rate of change is $\frac{288 - 144}{2 - 1}$ or 144 square inches for every square foot. So the equation is $i = 144f$.

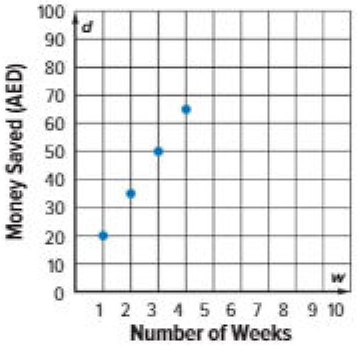
b. Use the equation to determine how many square inches are in 15 square feet. 2,160 square inches

Use the equation $i = 144f$.

$i = 144(15)$

$i = 2,160$

9. **Financial Literacy** Shaima is saving money for a school trip. The graph shows how much money she has saved over 4 weeks.



a. Write an equation to find how much money d Shaima can save over w weeks. $d = 15w + 5$

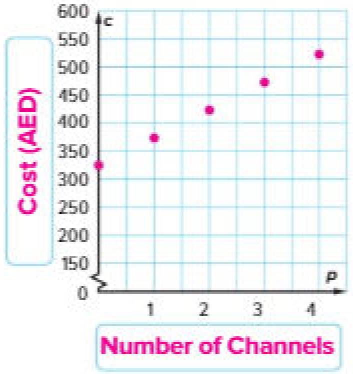
b. Use the equation to determine how much money Shaima can save in 24 weeks. AED 365

10. **Use Math Tools** Clearview Cable charges AED 325 a month for basic cable television. Each premium channel selected costs an additional AED 49.50 per month.

a. Write an equation to find the total monthly cost c for any number of premium channels p. $c = 325 + 49.5p$

b. Make a table to show the monthly cost for 0, 1, 2, 3, and 4 premium channels. Then graph the ordered pairs.

p	$325 + 49.5p$	c
0	325 + 495(0)	325.00
1	325 + 49.5(1)	374.50
2	325 + 49.5(2)	424.00
3	325 + 49.5(3)	473.50
4	325 + 49.5(4)	523.00



Power Up! Test Practice

Exercises 11 and 12 prepare students for more rigorous thinking needed for assessment.

11. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

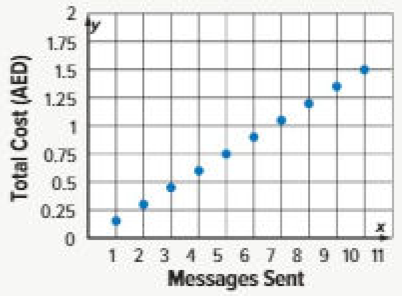
12. This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.

Depth of Knowledge	DOK3
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly graph the points and state the number of attendees.
1 point	Students correctly graph the points OR state the number of attendees.

Power Up! Test Practice

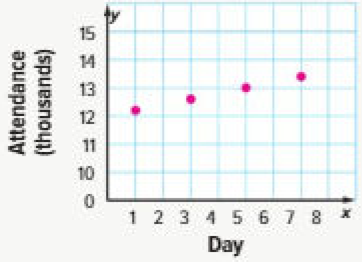
11. The graph represents the total cost to send text messages. Based on the graph, which of the following costs are correct? Select all that apply.

- ☑ 32 text messages cost AED 4.80
- ☑ 50 text messages cost AED 7.50
- ☐ 60 text messages cost AED 9.50
- ☑ 70 text messages cost AED 10.50



12. The table shows the number of people who attended a new movie over the course of a week. Graph the relationship on the coordinate plane.

Day	1	3	5	7
Attendance	12,200	12,600	13,000	13,400



If the pattern shown in the graph continues, how many people will attend the new movie on the 8th day?

13,600 people

Spiral Review

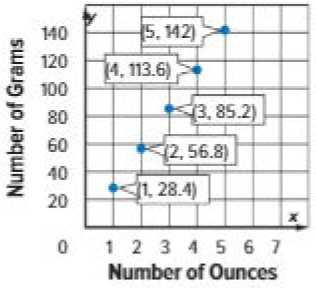
13. The graph at the right shows the approximate number of grams in one ounce.

a. Write an algebraic equation to represent the data in the graph.

$y = 28.4x$

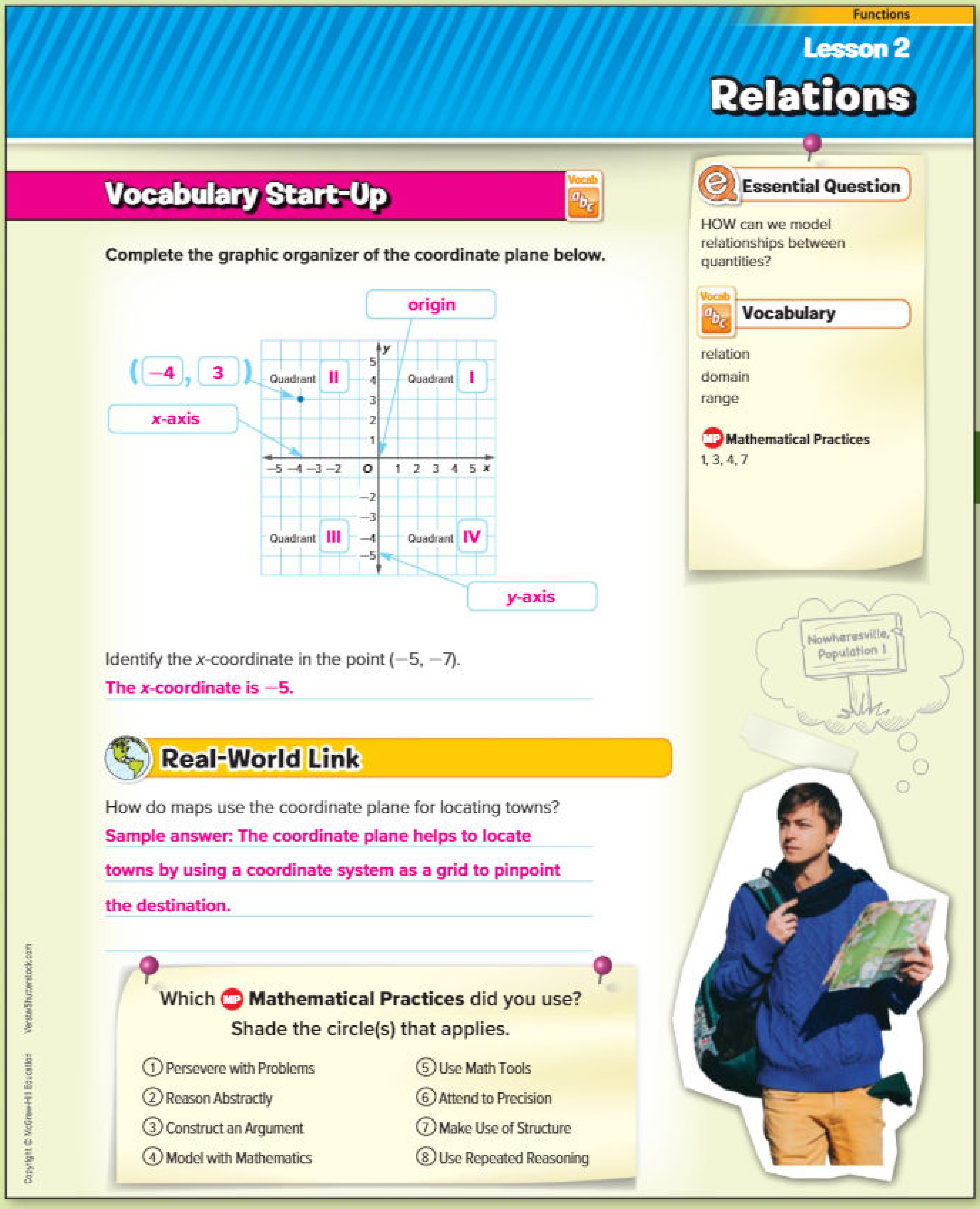
b. Use the expression to find the number of grams in 150 ounces.

4,260 g



14. Write an algebraic expression to represent the phrase *the difference between two times b and eleven.*

$2b - 11$



Lesson 2

Relations

Vocabulary Start-Up

Complete the graphic organizer of the coordinate plane below.

Identify the *x*-coordinate in the point (−5, −7).

The *x*-coordinate is −5.

Real-World Link

How do maps use the coordinate plane for locating towns?

Sample answer: The coordinate plane helps to locate towns by using a coordinate system as a grid to pinpoint the destination.

Which **MP** **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Essential Question

HOW can we model relationships between quantities?

Vocabulary

relation
domain
range

MP Mathematical Practices
1, 3, 4, 7

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Focus narrowing the scope

Objective Represent relations using tables and graphs.

Coherence connecting within and across grades

Previous	Now	Next
Students translated tables and graphs into linear equations.	Students will represent relations using tables and graphs.	Students will determine whether a relation is a function.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 281.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Find the Fib** Have students work in groups of 3–4 to label a copy of the coordinate plane with at least one error. Then have students trade papers with another group. Each group should determine the error(s). Have students correct the errors. MP 1, 2, 3, 4

Alternate Strategies

AL LA Provide students with a word bank of terms: origin, *x*-axis, *y*-axis, and quadrants I, II, III, and IV.

BL Have students determine whether the sign of the *x*- and *y*-coordinates of any point will be positive, negative, or zero, depending upon its location in the coordinate plane.

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Represent a relation as a table and a graph.

AL
- *In the ordered pairs, which value is the x-coordinate?* the first value
- *In the ordered pairs, which value is the y-coordinate?* the second value

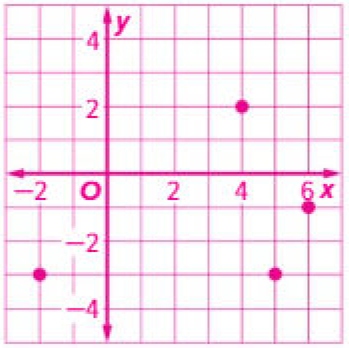
OL
- *Which values make up the domain of the relation?* the *x*-values
- *Which values make up the range of the relation?* the *y*-values

BL
- *Which representation makes it easier for you to see the relationship between the x- and y-coordinates?* See students' preferences.

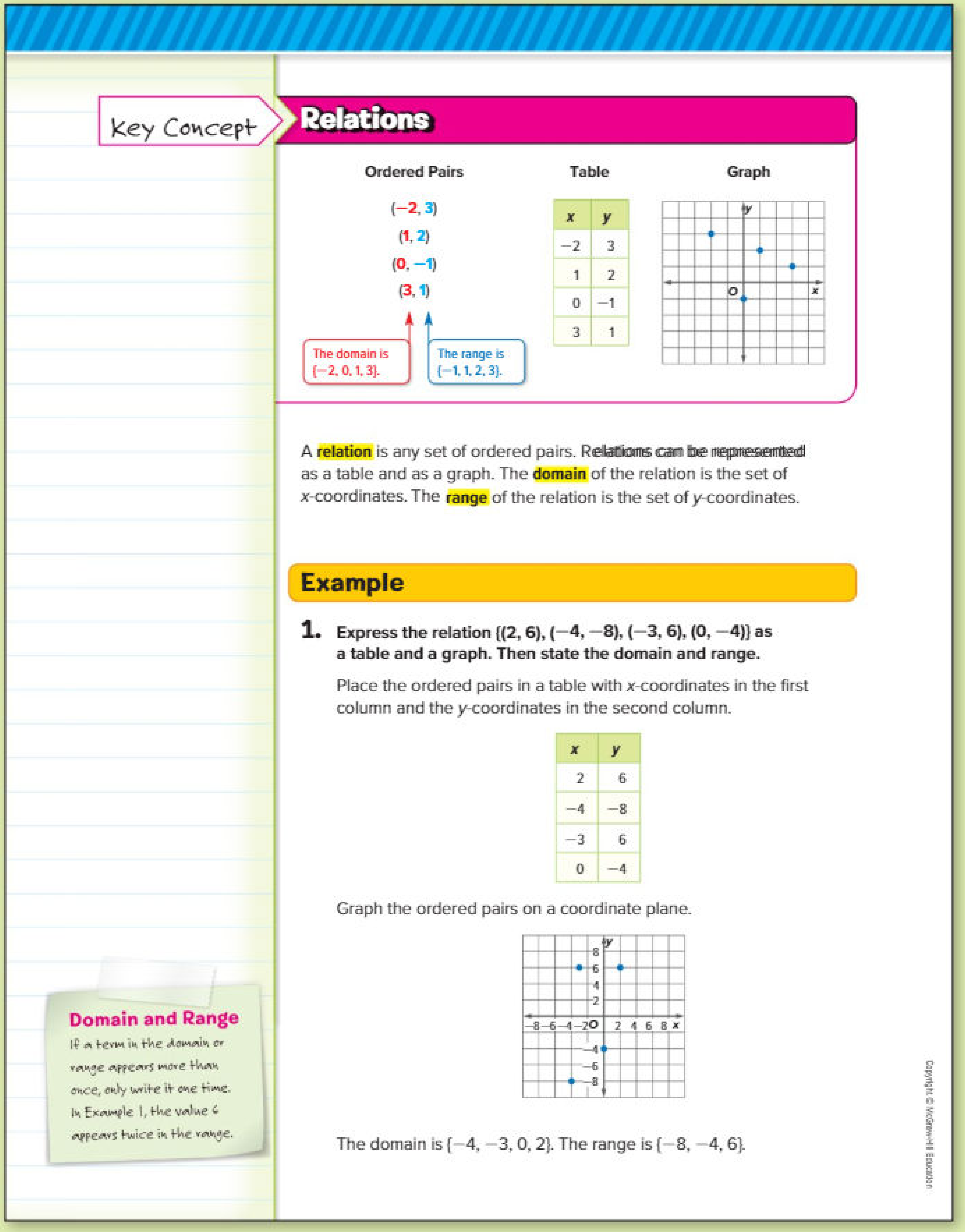
Need Another Example?

Express the relation {(4, 2), (6, −1), (5, −3), (−2, −3)} as a table and a graph. Then state the domain and range.

x	*y*
4	2
6	−1
5	−3
−2	−3



D: {−2, 4, 5, 6}; R: {−3, −1, 2}



Key Concept

Relations

Ordered Pairs
(−2, 3)
(1, 2)
(0, −1)
(3, 1)

The domain is {−2, 0, 1, 3}.

The range is {−1, 1, 2, 3}.

Table

x	*y*
−2	3
1	2
0	−1
3	1

Graph

A **relation** is any set of ordered pairs. Relations can be represented as a table and as a graph. The **domain** of the relation is the set of *x*-coordinates. The **range** of the relation is the set of *y*-coordinates.

Example

1. Express the relation {(2, 6), (−4, −8), (−3, 6), (0, −4)} as a table and a graph. Then state the domain and range.

Place the ordered pairs in a table with *x*-coordinates in the first column and the *y*-coordinates in the second column.

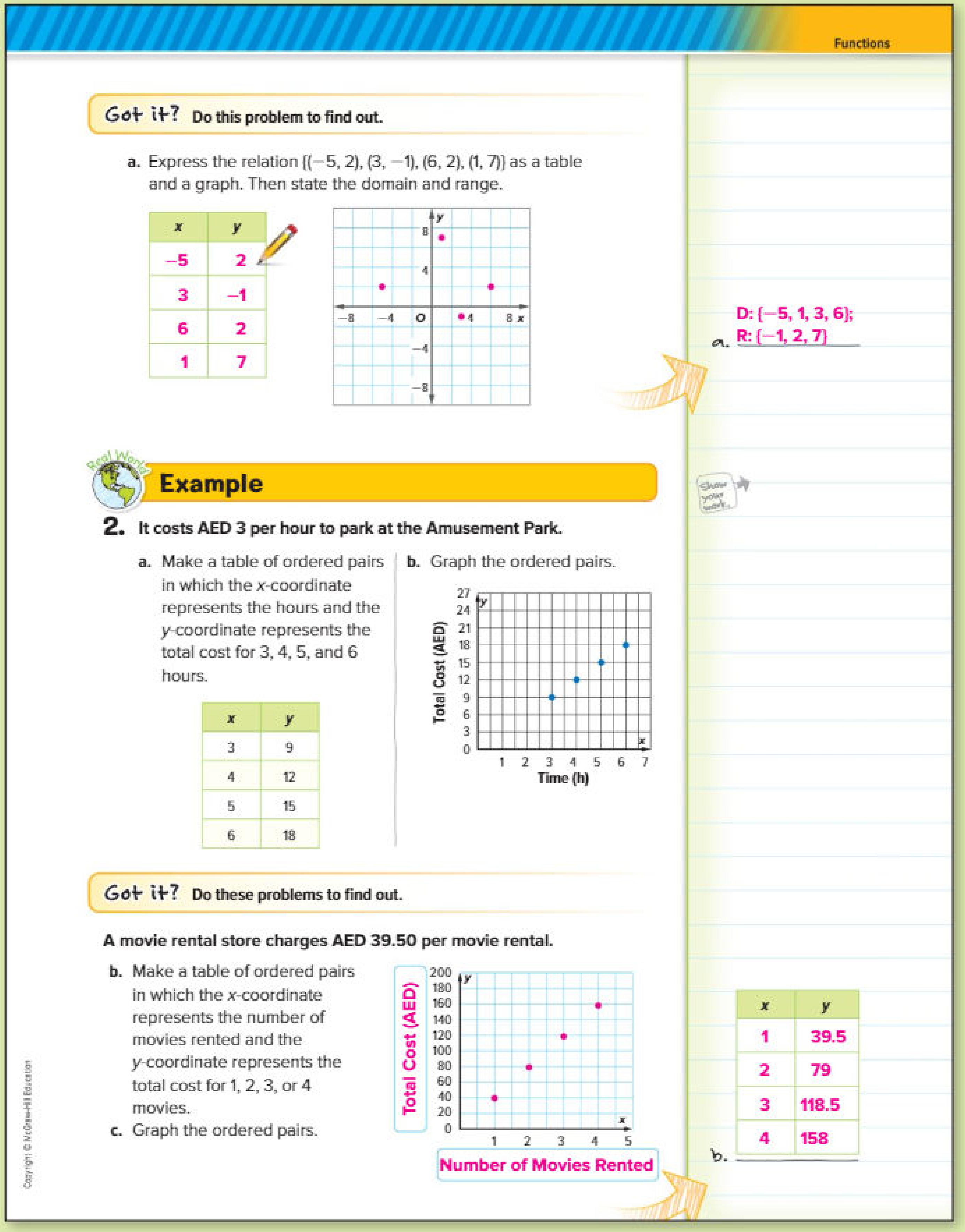
x	*y*
2	6
−4	−8
−3	6
0	−4

Graph the ordered pairs on a coordinate plane.

The domain is {−4, −3, 0, 2}. The range is {−8, −4, 6}.

Domain and Range

If a term in the domain or range appears more than once, only write it one time. In Example 1, the value 6 appears twice in the range.



Got it? Do this problem to find out.

a. Express the relation {(−5, 2), (3, −1), (6, 2), (1, 7)} as a table and a graph. Then state the domain and range.

x	y
−5	2
3	−1
6	2
1	7

a. D: {−5, 1, 3, 6}; R: {−1, 2, 7}

Real World

Example

2. It costs AED 3 per hour to park at the Amusement Park.

a. Make a table of ordered pairs in which the x-coordinate represents the hours and the y-coordinate represents the total cost for 3, 4, 5, and 6 hours.

x	y
3	9
4	12
5	15
6	18

b. Graph the ordered pairs.

Got it? Do these problems to find out.

A movie rental store charges AED 39.50 per movie rental.

b. Make a table of ordered pairs in which the x-coordinate represents the number of movies rented and the y-coordinate represents the total cost for 1, 2, 3, or 4 movies.

c. Graph the ordered pairs.

b.

x	y
1	39.5
2	79
3	118.5
4	158

Examples

2. Represent a relation using a table and a graph.

AL
- *What is the constant rate of change?* **AED3**
- *How can you complete the table?* **Multiply the number of hours by AED3.**

OL
- *What equation could be used to show the total cost y to park for any number of hours x?* **y = 3x**
- *Using the equation, how much would it cost to park for 3, 4, 5, and 6 hours?* **AED9, AED12, AED15, AED18**
- *What ordered pairs represent these values?* **(3, 9), (4, 12), (5, 15), (6, 18)**

BL
- *How much would it cost to park for a full day?* **AED72**

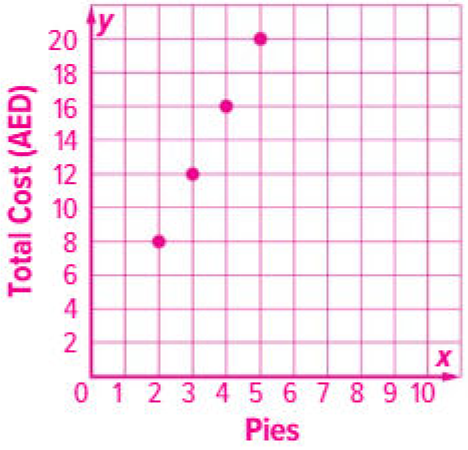
Need Another Example?

Mohammad is buying pies for an event. The price for one pie is AED4.

a. Make a table of ordered pairs in which the x-coordinate represents the number of pies and the y-coordinate represents the total cost for 2, 3, 4, and 5 pies.

Pies	Total Cost (AED)
2	8
3	12
4	16
5	20

b. Graph the ordered pairs.



Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA Gallery Walk Have students work in small groups to create a table, set of ordered pairs, and a graph for one of Exercises 1–3. Each representation should be created using a separate piece of paper. Students should not label the representation with the exercise number. Have them post the representations around the room. Students should walk around the room and determine which representation matches which exercise. MP 1, 2, 4

BL LA Pairs Consult Have students use the Internet or another source to research a real-world situation in which a relationship was represented using a table or a graph. Have them represent the relationship using another form (table, graph, or set of ordered pairs). MP 1, 2, 5

Watch Out!

Common Error Students may think that because the graph of the ordered pairs does not form a line there is no relation. Remind them that a relation is any set of ordered pairs and it does not have to form a line.

Guided Practice

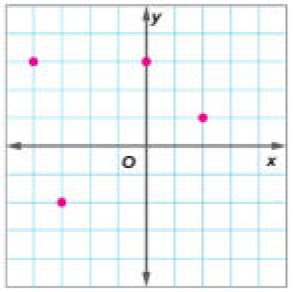
Check ✓

Express each relation as a table and a graph. Then state the domain and range. (Example 1)

1. {(−4, 3), (2, 1), (0, 3), (−3, −2)}

D: {−4, −3, 0, 2}; R: {−2, 1, 3}

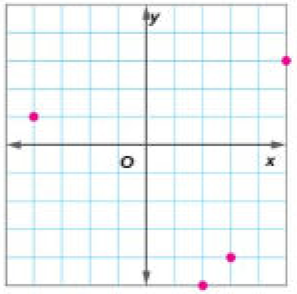
x	*y*
−4	3
2	1
0	3
−3	−2



2. {(5, 3), (−4, 1), (2, −5), (3, −4)}

D: {−4, 2, 3, 5}; R: {−5, −4, 1, 3}

x	*y*
5	3
−4	1
2	−5
3	−4

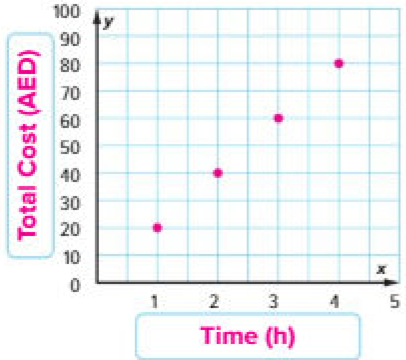


3. At a vacation resort, you can rent a personal watercraft for AED 20 per hour. (Example 2)

a. Make a table of ordered pairs in which the *x*-coordinate represents the number of hours and the *y*-coordinate represents the total cost for 1, 2, 3, or 4 hours.

x	*y*
1	20
2	40
3	60
4	80

b. Graph the ordered pairs.



4. **Building on the Essential Question** How do tables and graphs represent relations?

Sample answer: A set of ordered pairs is a relation. They can be represented by a table with a column for the x-values and a column for the y-values. The ordered pairs can also be graphed on a coordinate plane.

Rate Yourself!

How confident are you about relations? Check the box that applies.

☐ ☐ ☐ ☐ ☐

FOLDABLES Time to update your Foldable!

Name ______________________ My Homework ______________________

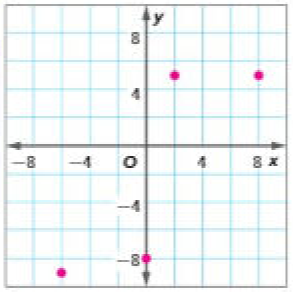
Independent Practice

Express each relation as a table and a graph. Then state the domain and range. (Example 1)

1. {(8, 5), (−6, −9), (2, 5), (0, −8)}

D: {−6, 0, 2, 8}; R: {−9, −8, 5}

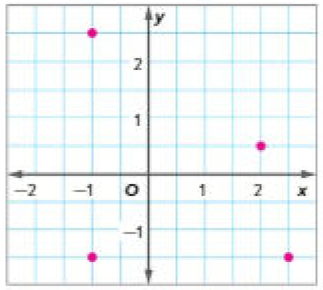
x	*y*
8	5
−6	−9
2	5
0	−8



2. $\left\{\left(2\frac{1}{2}, -1\frac{1}{2}\right), \left(2, \frac{1}{2}\right), \left(-1, 2\frac{1}{2}\right), \left(-1, -1\frac{1}{2}\right)\right\}$

D: $\{-1, 2, 2\frac{1}{2}\}$; R: $\{-1\frac{1}{2}, \frac{1}{2}, 2\frac{1}{2}\}$

x	*y*
$2\frac{1}{2}$	$-1\frac{1}{2}$
2	$\frac{1}{2}$
−1	$2\frac{1}{2}$
−1	$-1\frac{1}{2}$



Copy and Solve **Draw the table and graph on a separate sheet of paper. A company can manufacture 825 small cars per day.** (Example 2) **3–4. See Answer Appendix.**

3. Make a table of ordered pairs in which the *x*-coordinate represents the number of days and the *y*-coordinate represents the total number of cars produced in 1, 2, 3, 4, and 5 days.

4. Graph the ordered pairs.

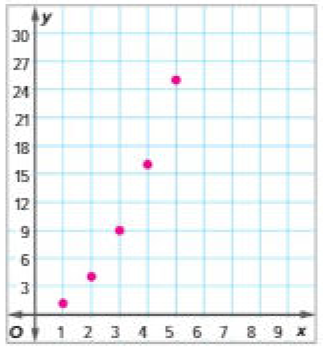
5. **Multiple Representations** Refer to the table at the right.

x	*y*
1	1
2	4
3	9
4	16
5	25

a. **Words** Describe the pattern, if any, in the table. **To get the *y*-value, the *x*-value was multiplied by itself.**

b. **Numbers** Write the ordered pairs (*x*, *y*). **(1, 1), (2, 4), (3, 9), (4, 16), (5, 25)**

c. **Graphs** Graph the ordered pairs on a coordinate plane.



d. **Words** Describe the graph. How is it different from the other real-world graphs in this lesson?

Sample answer: This graph curves upward. The points in all of the other graphs in the lesson lie in a straight line.

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

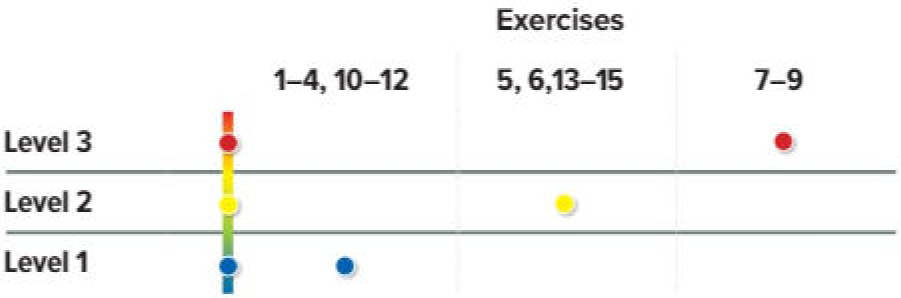
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 8, 9, 14, 15
OL	On Level	1, 3–6, 8, 9, 14, 15
BL	Beyond Level	5–9, 14, 15

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	7
3 Construct viable arguments and critique the reasoning of others.	5, 9
4 Model with mathematics.	6, 8
7 Look for and make use of structure.	13

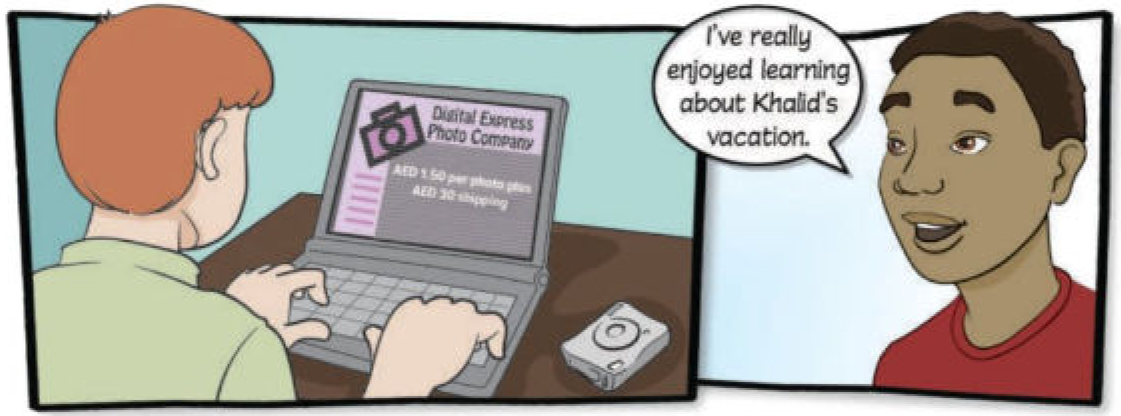
Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students find the domain and range of the relation {(3, 7), (−2, 5), (−3, −3), (4, −1)}. **D: {−3, −2, 3, 4}; R: {−3, −1, 5, 7}**

6. **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–c. Show your work on a separate sheet of paper. **a–c. See Answer Appendix.**

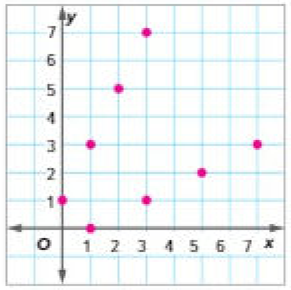


- **a.** Make a table to find the cost to print 10, 20, 30, 40 pictures.
- **b.** Graph the ordered pairs.
- **c.** How much would it cost for Khalid to print and ship 75 pictures? 100?

H.O.T. Problems Higher Order Thinking

7. **Persevere with Problems** Refer to the table at the right.

x	y
0	1
1	3
2	5
3	7



- **a.** Graph the ordered pairs.
- **b.** Reverse the y-coordinates and x-coordinates in each ordered pair.
  (1, 0), (3, 1), (5, 2), (7, 3)
- **c.** Graph the new ordered pairs on the same coordinate plane in part a.
- **d.** Describe the relationship between the two sets of ordered pairs.
  Sample answer: The distance between each point in the original table and the x-axis is the same as the distance between the points with the reversed ordered pairs and the y-axis.

8. **Model with Mathematics** Describe a real-world situation that can be represented using a table and a graph. **Sample answer: The number of movie tickets bought and the total cost of the tickets can be represented using a table and graph.**

9. **Find the Error** Ayesha says that the domain of the relation {(2, 3), (−4, 2), (0, −4), (1, 5)} is {−4, 2, 3, 5}. Find her mistake and correct it. **Sample answer: The domain is the set of x-coordinates. Ayesha listed the set of y-coordinates; {−4, 0, 1, 2}**

Name ______________________ My Homework ______________________

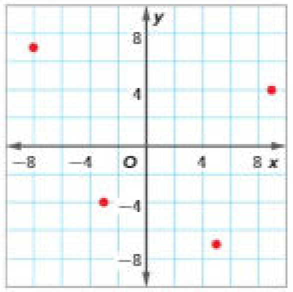
Extra Practice

Express each relation as a table and a graph. Then state the domain and range.

10. {(9, 4), (5, −7), (−3, −4), (−8, 7)}

D: {−8, −3, 5, 9}; R: {−7, −4, 4, 7}

x	*y*
9	4
5	−7
−3	−4
−8	7

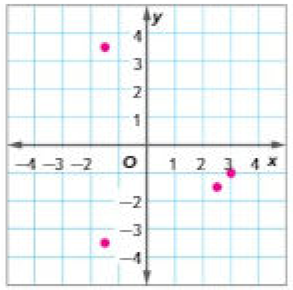


11. {(−1.5, 3.5), (2.5, −1.5), (3, −1), (−1.5, −3.5)}

D: {−1.5, 2.5, 3};

R: {−3.5, −1.5, −1, 3.5}

x	*y*
−1.5	**3.5**
2.5	**−1.5**
3	**−1**
−1.5	**−3.5**

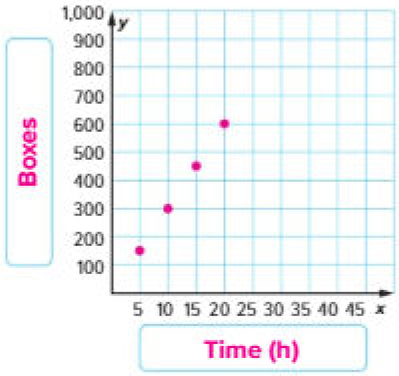


12. A candy company produces 30 boxes of candy per hour.

a. Make a table of ordered pairs in which the *x*-coordinate represents the number of hours and the *y*-coordinate represent the number of boxes of candy in 5, 10, 15, and 20 hours.

x	*y*
5	**150**
10	**300**
15	**450**
20	**600**

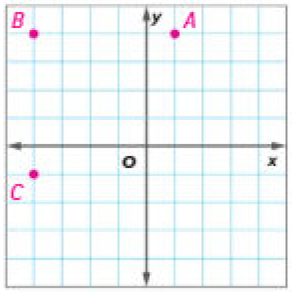
b. Graph the ordered pairs.



13. **Identify Structure** Graph the points in the table on a coordinate plane. Label the points *A*, *B*, and *C*. What are the coordinates of point *D* if points *A*, *B*, *C*, and *D* form a square?

***D*(1, −1)**

x	*y*
1	4
−4	4
−4	−1



Power Up! Test Practice

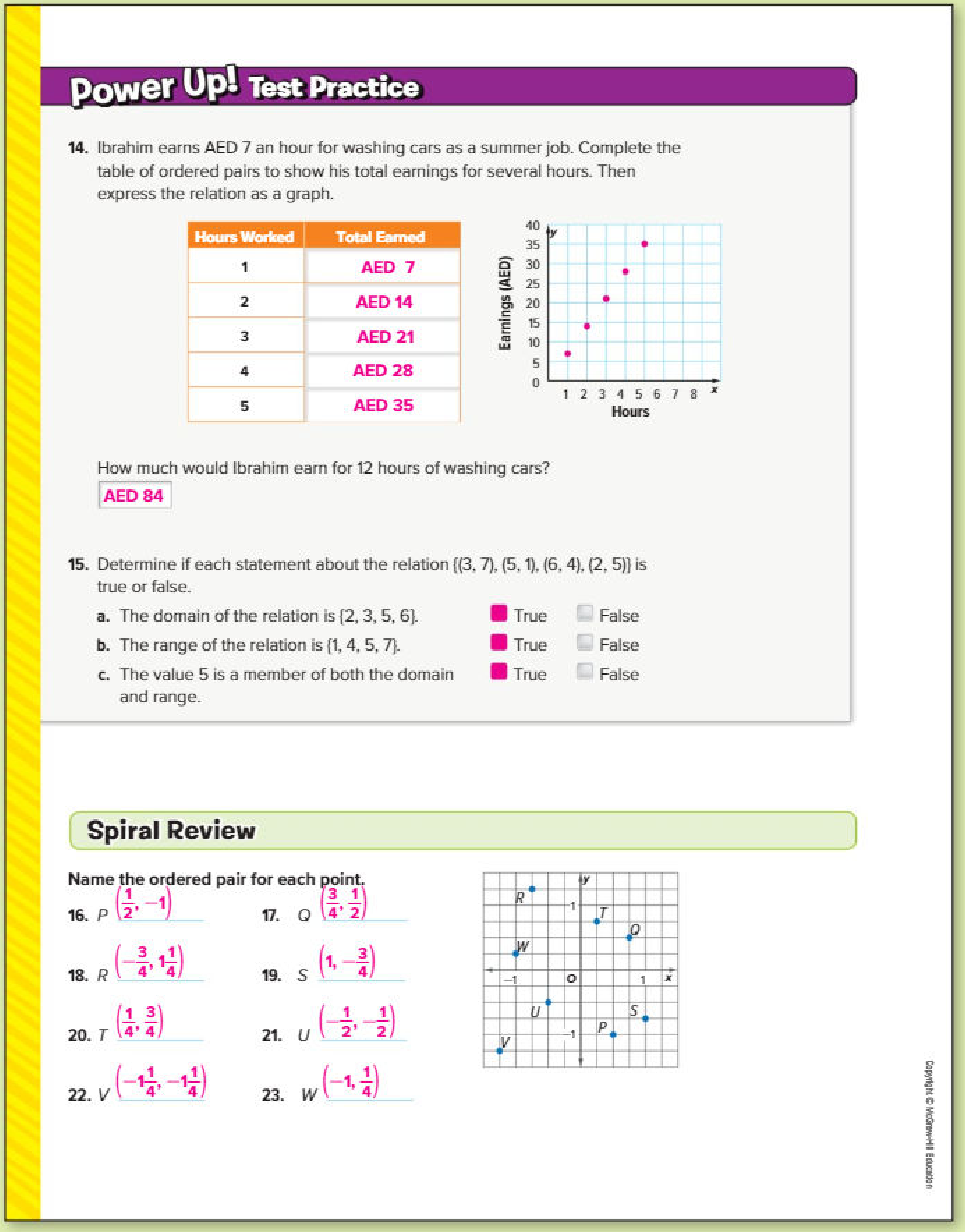
Exercises 14 and 15 prepare students for more rigorous thinking needed for assessment.

14.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK3
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly complete the table, the graph, and find the earnings.
1 point	Students correctly complete the table and find the earnings but have errors in the graph OR students incorrectly complete the table, but graph the points listed and find the earnings based on the incorrect table OR students correctly complete the table and the graph but fail to find the earnings.

15.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part.



Power Up! Test Practice

14. Ibrahim earns AED 7 an hour for washing cars as a summer job. Complete the table of ordered pairs to show his total earnings for several hours. Then express the relation as a graph.

Hours Worked	Total Earned
1	AED 7
2	AED 14
3	AED 21
4	AED 28
5	AED 35

How much would Ibrahim earn for 12 hours of washing cars?

AED 84

15. Determine if each statement about the relation {(3, 7), (5, 1), (6, 4), (2, 5)} is true or false.

- **a.** The domain of the relation is {2, 3, 5, 6}. ■ True ☐ False
- **b.** The range of the relation is {1, 4, 5, 7}. ■ True ☐ False
- **c.** The value 5 is a member of both the domain and range. ■ True ☐ False

Spiral Review

Name the ordered pair for each point.

16. P $\left(\frac{1}{2}, -1\right)$

17. Q $\left(\frac{3}{4}, \frac{1}{2}\right)$

18. R $\left(-\frac{3}{4}, 1\frac{1}{4}\right)$

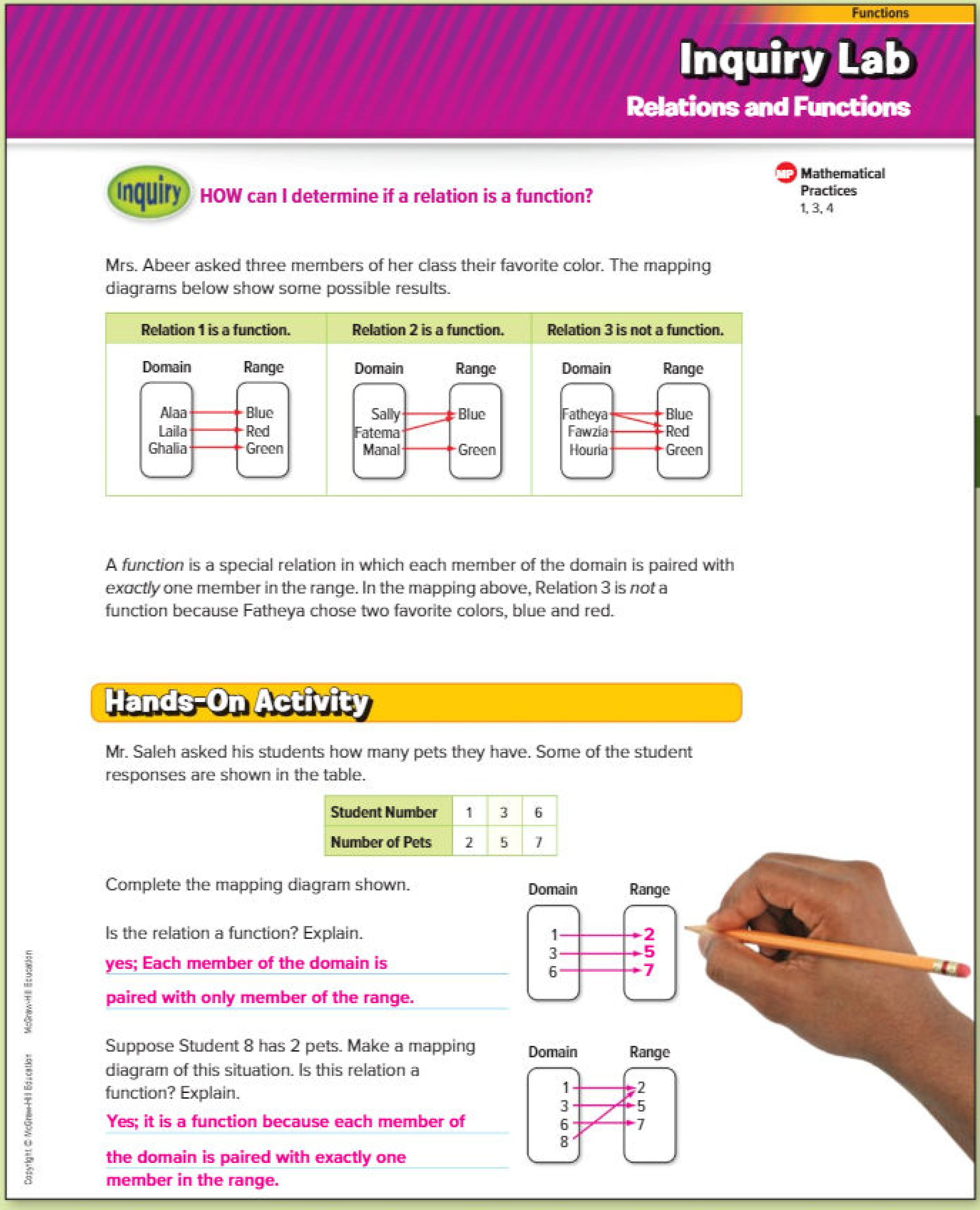
19. S $\left(1, -\frac{3}{4}\right)$

20. T $\left(\frac{1}{4}, \frac{3}{4}\right)$

21. U $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

22. V $\left(-1\frac{1}{4}, -1\frac{1}{4}\right)$

23. W $\left(-1, \frac{1}{4}\right)$



Inquiry Lab

Relations and Functions

Inquiry HOW can I determine if a relation is a function?

Mathematical Practices 1, 3, 4

Mrs. Abeer asked three members of her class their favorite color. The mapping diagrams below show some possible results.

Relation 1 is a function.	Relation 2 is a function.	Relation 3 is not a function.

A *function* is a special relation in which each member of the domain is paired with *exactly* one member in the range. In the mapping above, Relation 3 is *not* a function because Fatheya chose two favorite colors, blue and red.

Hands-On Activity

Mr. Saleh asked his students how many pets they have. Some of the student responses are shown in the table.

Student Number	1	3	6
Number of Pets	2	5	7

Complete the mapping diagram shown.

Is the relation a function? Explain.

yes; Each member of the domain is paired with only member of the range.

Suppose Student 8 has 2 pets. Make a mapping diagram of this situation. Is this relation a function? Explain.

Yes; it is a function because each member of the domain is paired with exactly one member in the range.

Focus narrowing the scope

Objective Determine whether a relation is a function.

Coherence connecting within and across grades

Now
Students will use mapping diagrams to identify functions.

Next
Students will find function values and complete function tables.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 286.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as whole-group activity.

Hands-On Activity

AL **LA** **Think-Pair-Draw** Have students create reference mappings to display around the room for the following scenarios. Then have them determine if each relation is a function in each instance. Display the mappings with their identification as a function or not a function in the classroom so students can refer to them. **MP 1, 2, 4, 6**

- Each element of the domain is paired with exactly one element of the range. (function)
- Some elements of the domain are paired with more than one element of the range. (not a function)

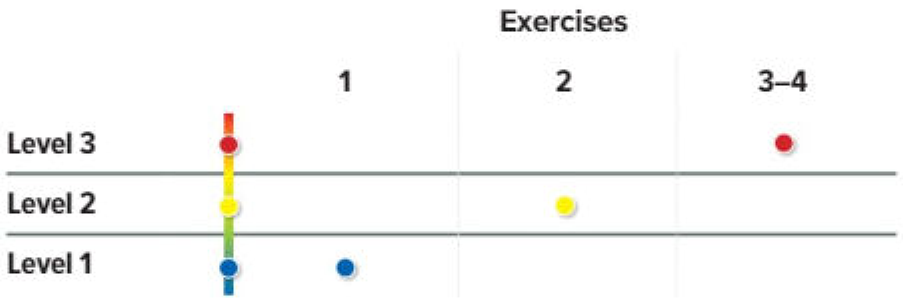
BL Have students survey the class on a topic of their choice. Have them create their own domain and range and then determine if the relation is a function. Have them display their mappings around the room.

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Analyze and Reflect

AL **LA** **Think-Pair-Share** Give pairs of students one minute to respond to Exercise 2, and then have them share their response with their partner. Then call on one pair to share with the class. MP 1, 3

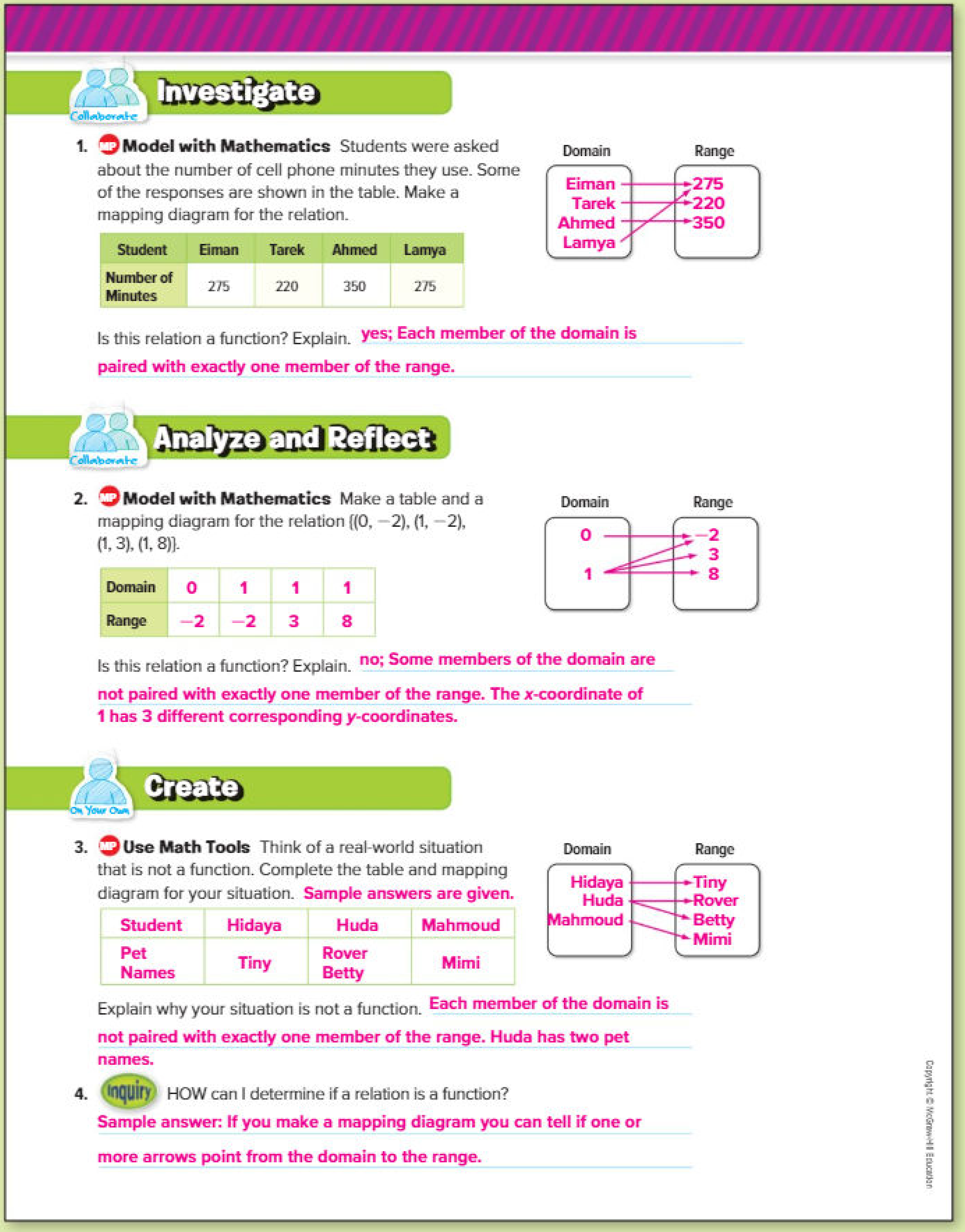
Ask:

- *If the domain and range were reversed, would the relation now be a function? Draw a mapping diagram.* no; Some elements of the domain are still paired with more than one element of the range; (−2, 0) and (−2, 1); See students' mappings.

Create

BL **LA** **Trade-a-Problem** For Exercise 3, have students trade their problem with a partner. The partner should check to make sure the relation is not a function. Have them justify their response using the mapping and their own words. MP 1, 3

Inquiry Students should be able to answer "HOW can I determine if a relation is a function?" Check for student understanding and provide guidance, if needed.



Investigate

1. MP **Model with Mathematics** Students were asked about the number of cell phone minutes they use. Some of the responses are shown in the table. Make a mapping diagram for the relation.

Student	Eiman	Tarek	Ahmed	Lamya
Number of Minutes	275	220	350	275

Is this relation a function? Explain. yes; Each member of the domain is paired with exactly one member of the range.

Analyze and Reflect

2. MP **Model with Mathematics** Make a table and a mapping diagram for the relation {(0, −2), (1, −2), (1, 3), (1, 8)}.

Domain	0	1	1	1
Range	−2	−2	3	8

Is this relation a function? Explain. no; Some members of the domain are not paired with exactly one member of the range. The *x*-coordinate of 1 has 3 different corresponding *y*-coordinates.

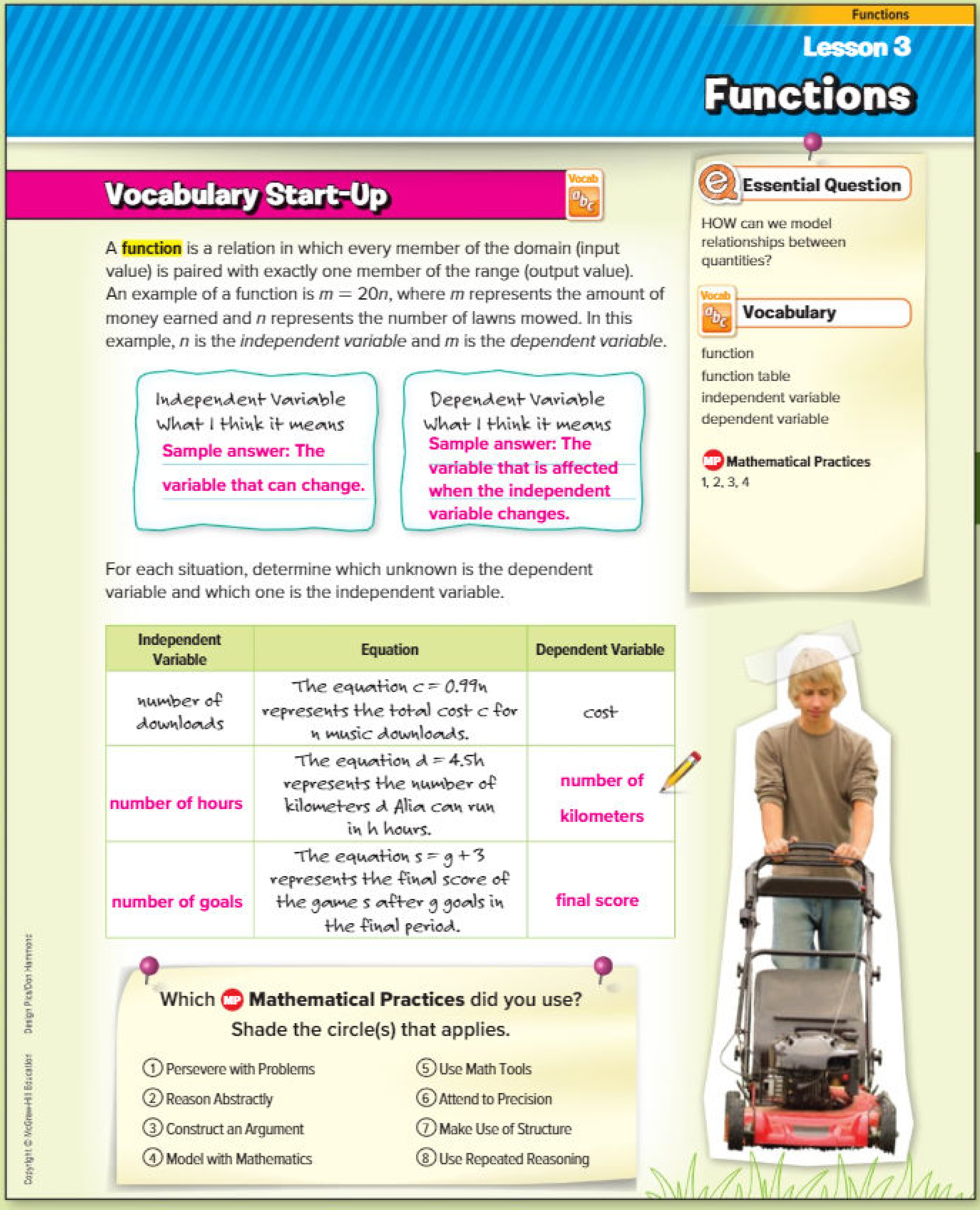
Create

3. MP **Use Math Tools** Think of a real-world situation that is not a function. Complete the table and mapping diagram for your situation. Sample answers are given.

Student	Hidaya	Huda	Mahmoud
Pet Names	Tiny	Rover Betty	Mimi

Explain why your situation is not a function. Each member of the domain is not paired with exactly one member of the range. Huda has two pet names.

4. Inquiry HOW can I determine if a relation is a function? Sample answer: If you make a mapping diagram you can tell if one or more arrows point from the domain to the range.



Lesson 3

Functions

Vocabulary Start-Up

A **function** is a relation in which every member of the domain (input value) is paired with exactly one member of the range (output value). An example of a function is $m = 20n$, where m represents the amount of money earned and n represents the number of lawns mowed. In this example, n is the *independent variable* and m is the *dependent variable*.

Independent Variable
What I think it means
Sample answer: The variable that can change.

Dependent Variable
What I think it means
Sample answer: The variable that is affected when the independent variable changes.

For each situation, determine which unknown is the dependent variable and which one is the independent variable.

Independent Variable	Equation	Dependent Variable
number of downloads	The equation $c = 0.99n$ represents the total cost c for n music downloads.	cost
number of hours	The equation $d = 4.5h$ represents the number of kilometers d Alia can run in h hours.	number of kilometers
number of goals	The equation $s = g + 3$ represents the final score of the game s after g goals in the final period.	final score

Which **MP** **Mathematical Practices** did you use?
Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Essential Question

HOW can we model relationships between quantities?

Vocabulary

function
function table
independent variable
dependent variable

MP Mathematical Practices
1, 2, 3, 4

Focus narrowing the scope

Objective Find function values and complete function tables.

Coherence connecting within and across grades

Previous	Now	Next
Students represented relationships using multiple representations.	Students use function tables to determine the domain and range of a function.	Students will use multiple representations to represent linear functions.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 291.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL LA **Find a Fib** Students work in pairs. Divide the pairs into two groups, one to write two facts and one fib about the dependent variable and one to write two facts and one fib about the independent variable. Pairs then trade their facts and fib with another pair in a different group. Each pair of students identifies the facts and fibs. Then the two pairs of students come together as a team to resolve any differences. MP 1, 3

Alternate Strategy

AL Tell students that the dependent variable is related to the output value and the independent variable is related to the input value, which may help differentiate between independent and dependent variables.

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Find function values.

AL
- *What does f(−3) mean?* find the value of the function when $x = -3$

OL
- *What value do you need to substitute for x?* −3
- *How would you find f(−3)?* replace x with −3 in the expression $2x + 1$ and then simplify

BL
- *What does f(x) mean?* Sample answer: the value of the function that corresponds to the number x.

Need Another Example?
Find $f(-6)$ if $f(x) = 3x + 4$. −14

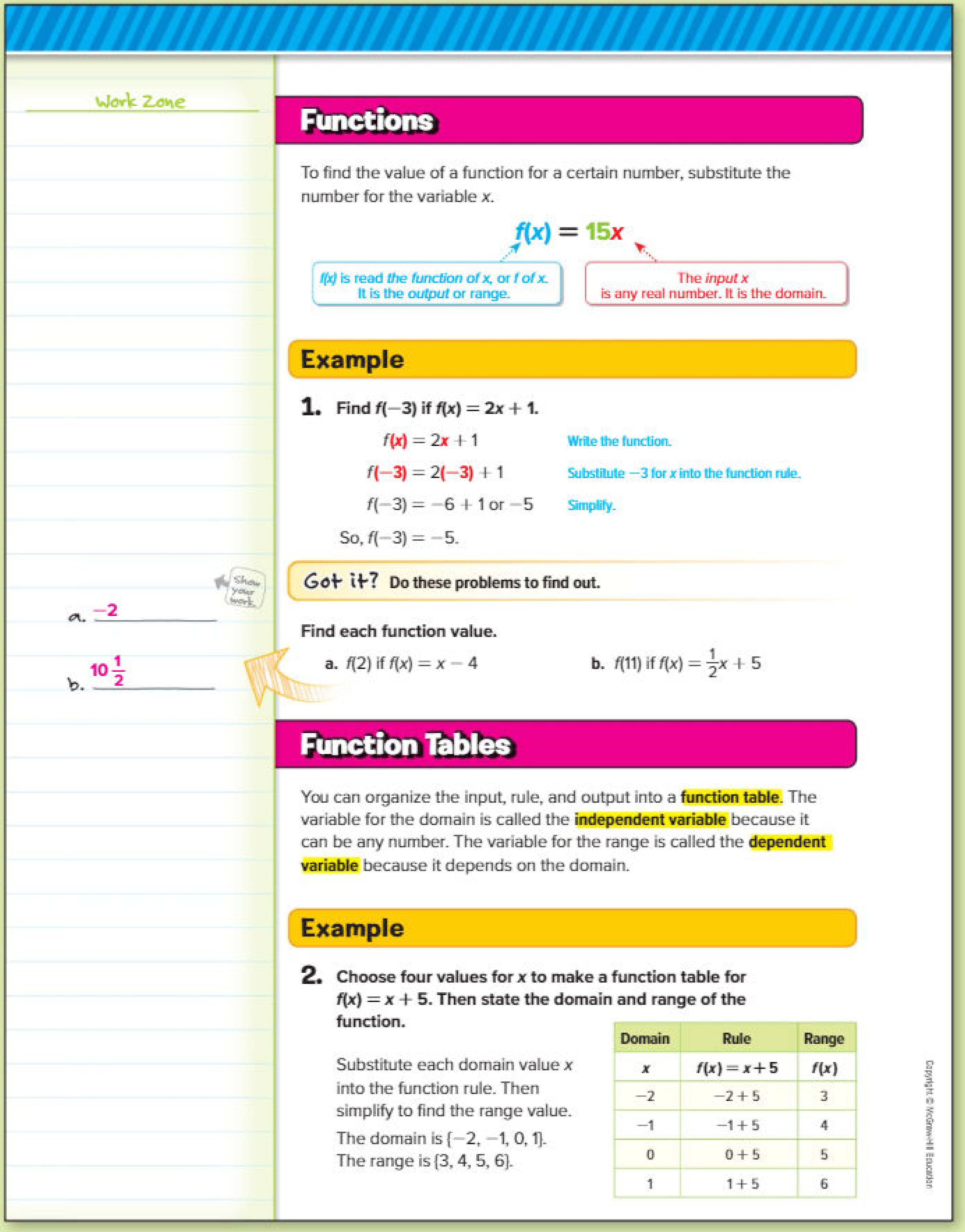
2. Make a function table.

AL
- *Which variable represents the input value in the function table?* x
- *What is used to represent the output value in the function table?* $f(x)$

OL
- *How would you find each output value for the input values of −2, −1, 0, and 1?* Replace x with each value in the expression $x + 5$ and simplify.
- *Does the set of input values or the set of output values give the domain of the function?* input values

BL
- *What represents the independent variable?* x *the dependent variable?* $f(x)$
- *Why do you think the values of −2, −1, 0, and 1 were chosen?* Sample answer: The numbers are in order and represent negative numbers, zero, and a positive number.

Need Another Example?
Choose four values for x to make a function table for $f(x) = 4x - 1$. Then state the domain and range of the function. See Answer Appendix.



Work Zone

a. −2

b. $10\frac{1}{2}$

Functions

To find the value of a function for a certain number, substitute the number for the variable *x*.

$$f(x) = 15x$$

f(*x*) is read *the function of x*, or *f of x*. It is the *output* or range.

The *input x* is any real number. It is the domain.

Example

1. Find $f(-3)$ if $f(x) = 2x + 1$.

$f(x) = 2x + 1$ Write the function.

$f(-3) = 2(-3) + 1$ Substitute −3 for *x* into the function rule.

$f(-3) = -6 + 1$ or -5 Simplify.

So, $f(-3) = -5$.

Got it? **Do these problems to find out.**

Show your work.

Find each function value.

a. $f(2)$ if $f(x) = x - 4$

b. $f(11)$ if $f(x) = \frac{1}{2}x + 5$

Function Tables

You can organize the input, rule, and output into a **function table**. The variable for the domain is called the **independent variable** because it can be any number. The variable for the range is called the **dependent variable** because it depends on the domain.

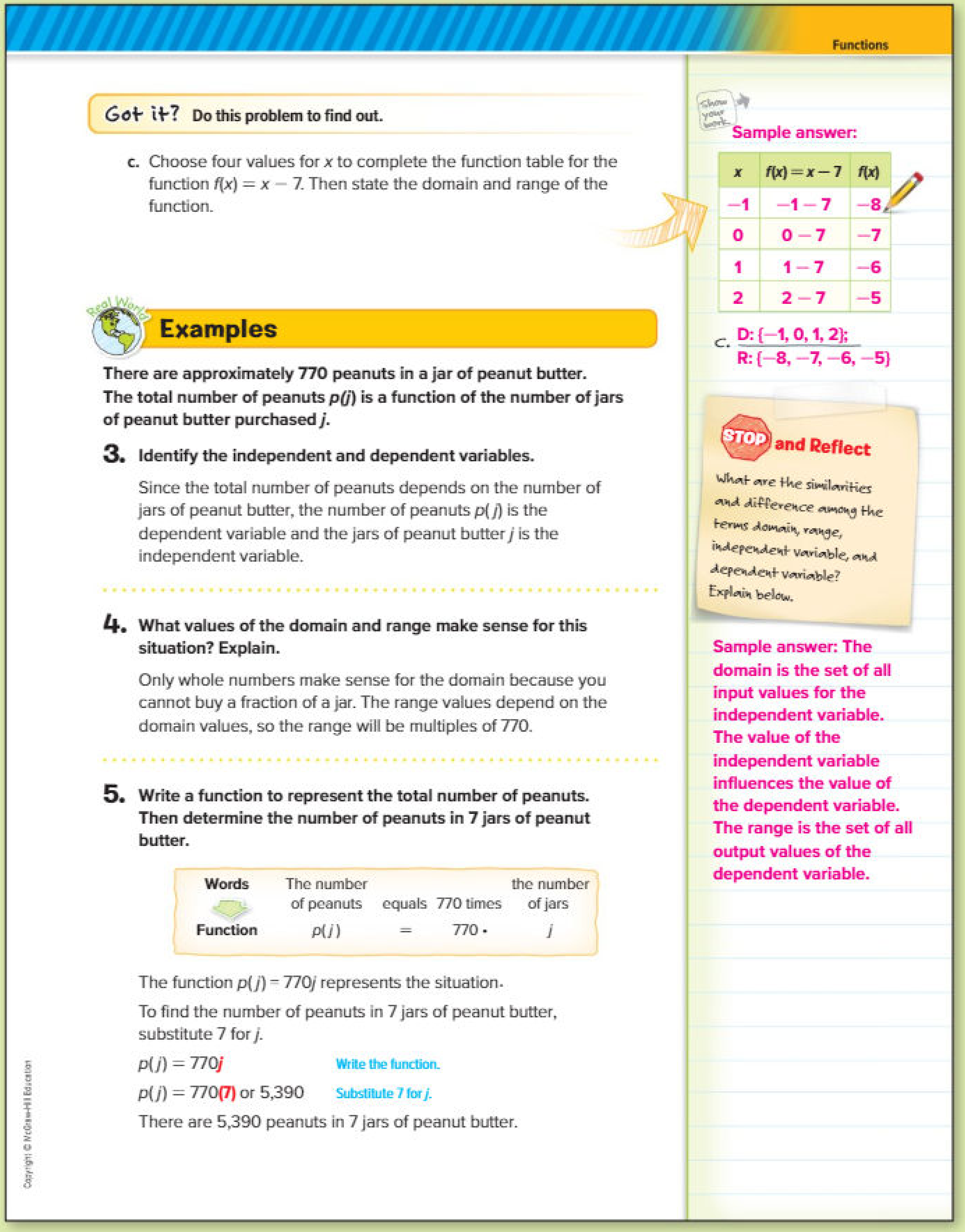
Example

2. Choose four values for *x* to make a function table for $f(x) = x + 5$. Then state the domain and range of the function.

Substitute each domain value *x* into the function rule. Then simplify to find the range value.

The domain is {−2, −1, 0, 1}. The range is {3, 4, 5, 6}.

Domain	Rule	Range
x	$f(x) = x + 5$	$f(x)$
−2	−2 + 5	3
−1	−1 + 5	4
0	0 + 5	5
1	1 + 5	6



Got it? **Do this problem to find out.**

c. Choose four values for x to complete the function table for the function $f(x) = x - 7$. Then state the domain and range of the function.

Show your work.

Sample answer:

x	$f(x) = x - 7$	$f(x)$
-1	$-1 - 7$	-8
0	$0 - 7$	-7
1	$1 - 7$	-6
2	$2 - 7$	-5

c. D: $\{-1, 0, 1, 2\}$; R: $\{-8, -7, -6, -5\}$

Examples

Real World

There are approximately 770 peanuts in a jar of peanut butter. The total number of peanuts $p(j)$ is a function of the number of jars of peanut butter purchased j.

3. **Identify the independent and dependent variables.**

Since the total number of peanuts depends on the number of jars of peanut butter, the number of peanuts $p(j)$ is the dependent variable and the jars of peanut butter j is the independent variable.

4. **What values of the domain and range make sense for this situation? Explain.**

Only whole numbers make sense for the domain because you cannot buy a fraction of a jar. The range values depend on the domain values, so the range will be multiples of 770.

5. **Write a function to represent the total number of peanuts. Then determine the number of peanuts in 7 jars of peanut butter.**

Words	The number of peanuts	equals	770 times	the number of jars
Function	$p(j)$	$=$	$770 \cdot$	j

The function $p(j) = 770j$ represents the situation.

To find the number of peanuts in 7 jars of peanut butter, substitute 7 for j.

$p(j) = 770j$ Write the function.

$p(j) = 770(7)$ or 5,390 Substitute 7 for j.

There are 5,390 peanuts in 7 jars of peanut butter.

STOP and Reflect

What are the similarities and difference among the terms domain, range, independent variable, and dependent variable? Explain below.

Sample answer: The domain is the set of all input values for the independent variable. The value of the independent variable influences the value of the dependent variable. The range is the set of all output values of the dependent variable.

Examples

3–5. Write a function.

AL
- *How can you determine the independent variable?* The independent variable is the input variable.
- *Do negative values make sense for the independent variable? Explain.* No; you cannot have a negative number of jars of peanut butter.
- *What does j represent?* the number of jars of peanut butter
- *What does p(j) represent?* the number of peanuts in j jars of peanut butter

OL
- *What is the independent variable?* the number of jars of peanut butter *the dependent variable?* the number of peanuts
- *What function is used to represent the situation?* $p(j) = 770j$
- *How can you find the number of peanuts in 7 jars of peanut butter?* Replace j with 7 and simplify.

BL
- *Would fractions ever make sense for this function? Explain.* Sample answer: Yes, you could have a fraction of a jar of peanut butter at your house.
- *How many peanuts are in $(x + 5)$ jars of peanut butter?* $770x + 3{,}850$

Need Another Example?

Fatema buys a can of tuna fish that weighs 120 grams. The total weight $w(c)$ is a function of the number of cans of tuna fish c.

a. Identify the independent and dependent variables. The number of cans c is the independent variable. The total weight w is the dependent variable.

b. Explain what values of the domain and range make sense for this situation. Only whole numbers make sense for the domain because you cannot buy a fraction of a can of tuna fish. The range will be multiples of 120.

c. Write a function to represent the total weight. Then determine the ounces in 8 cans of tuna fish. $w(c) = 120c$; 960 g

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL **LA** **Think-Pair-Share** Have students work in pairs. Give students one minute to think through their responses to each part of Exercise 3. Have them exchange thoughts on how to answer each part, then record their collaborative solutions. Have them trade their solutions with another pair of students and discuss any differences. **MP** **1, 3, 6**

BL **LA** **Trade-a-Problem** Have students create their own problem, similar to Exercise 3. Challenge students to create a problem where a negative number or fractions make up the domain and/or range. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. **MP** **1, 3**

Got it? **Do these problems to find out.**

A scrapbooking store is selling rubber stamps for AED 4.95 each. The total sales $f(n)$ is a function of the number of rubber stamps n sold.

d. Identify the independent and dependent variables.

e. What values of the domain and range make sense for this situation? Explain.

f. Write a function to represent the total sales. Then determine the total sales for 5 stamps.

d. **The number of stamps n is the independent variable and the total sales $f(n)$ is the dependent variable.**

Show your work.

e. **Only whole numbers make sense for the domain because you cannot buy a fraction of a stamp. The range will be multiples of 4.95.**

f. **$f(n) = 4.95n$; AED 24.75**

Guided Practice

Check

1. Find $f(4)$ if $f(x) = x - 6$. (Example 1) **−2**

2. Choose four values for x to make a function table for $f(x) = 8 - x$. Then state the domain and range of the function. (Example 2)

Sample answer: D: {−3, −1, 2, 4}; R: {11, 9, 6, 4}

x	$8 - x$	$f(x)$
−3	**$8 - (-3)$**	**11**
−1	**$8 - (-1)$**	**9**
2	**$8 - 2$**	**6**
4	**$8 - 4$**	**4**

Show your work.

3. A hot air balloon can hold 2,500 cubic meters of air. It is being inflated at a rate of 170 cubic meters per minute. The total cubic meters of air $a(t)$ is a function of the time in minutes t. (Examples 3–5)

a. Identify the independent and dependent variables.

The air $a(t)$ is the dependent variable and the time t is the independent variable.

b. What values of the domain and range make sense for this situation? Explain. **The domain can be any positive number. Since the balloon only holds 2,500 cubic meters, the range values can be any rational number greater than 0 but less than 2,500.**

c. Write a function to represent the total amount of air. Then determine the total amount of air in 6 minutes.

$a(t) = 170t$; 1,020 m^3

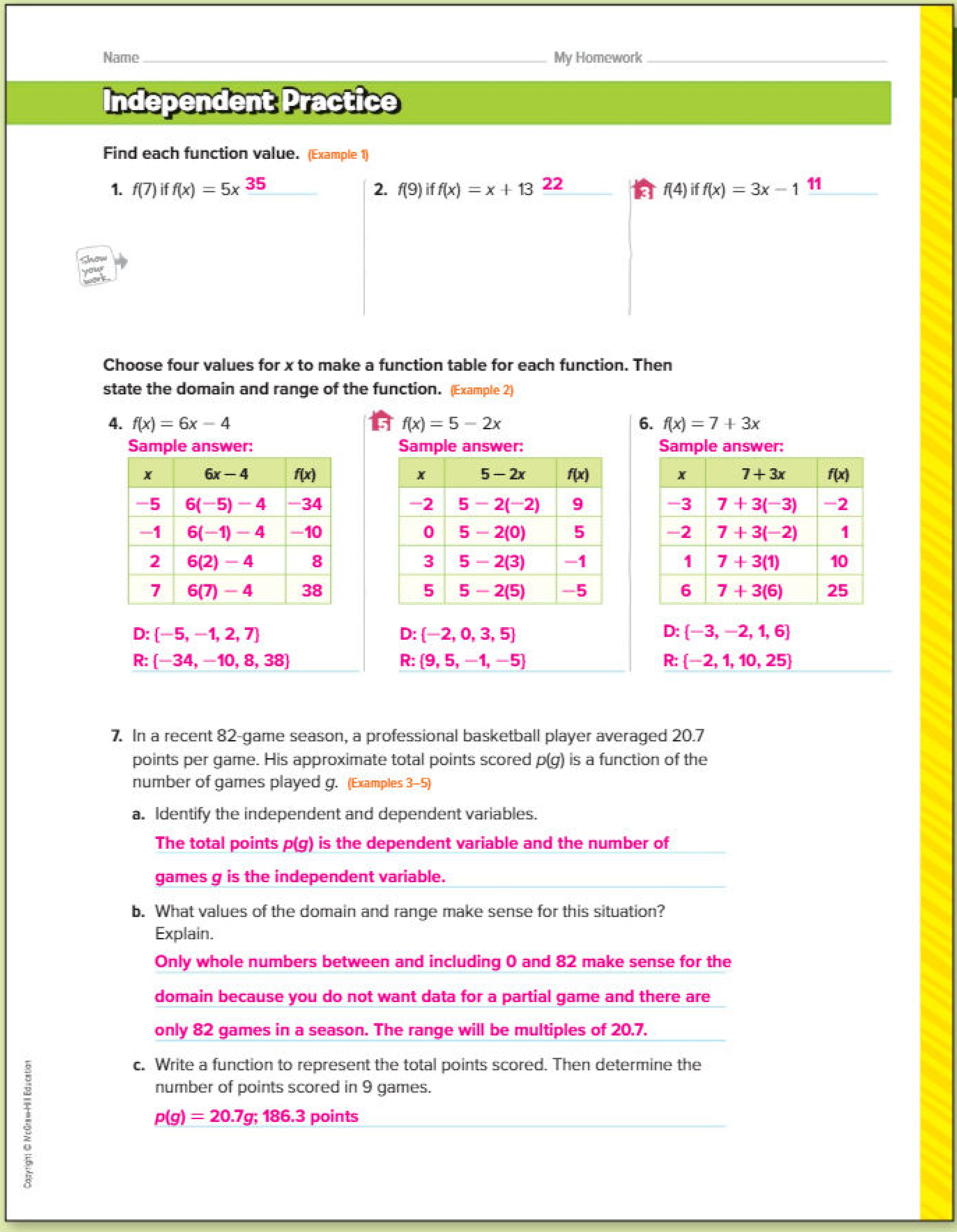
4. **Building on the Essential Question** How does the domain affect the range in a function?

Sample answer: Any number can be used for the domain. So, the range depends on the domain.

Rate Yourself!

How confident are you about functions? Check the box that applies.

☐ ☐ ☐ ☐ ☐



Name _______________ My Homework _______________

Independent Practice

Find each function value. (Example 1)

1. $f(7)$ if $f(x) = 5x$ 35

2. $f(9)$ if $f(x) = x + 13$ 22

3. $f(4)$ if $f(x) = 3x - 1$ 11

Choose four values for *x* to make a function table for each function. Then state the domain and range of the function. (Example 2)

4. $f(x) = 6x - 4$

Sample answer:

x	$6x - 4$	$f(x)$
−5	6(−5) − 4	−34
−1	6(−1) − 4	−10
2	6(2) − 4	8
7	6(7) − 4	38

D: {−5, −1, 2, 7}
R: {−34, −10, 8, 38}

5. $f(x) = 5 - 2x$

Sample answer:

x	$5 - 2x$	$f(x)$
−2	5 − 2(−2)	9
0	5 − 2(0)	5
3	5 − 2(3)	−1
5	5 − 2(5)	−5

D: {−2, 0, 3, 5}
R: {9, 5, −1, −5}

6. $f(x) = 7 + 3x$

Sample answer:

x	$7 + 3x$	$f(x)$
−3	7 + 3(−3)	−2
−2	7 + 3(−2)	1
1	7 + 3(1)	10
6	7 + 3(6)	25

D: {−3, −2, 1, 6}
R: {−2, 1, 10, 25}

7. In a recent 82-game season, a professional basketball player averaged 20.7 points per game. His approximate total points scored $p(g)$ is a function of the number of games played g. (Examples 3–5)

   a. Identify the independent and dependent variables.

   The total points $p(g)$ is the dependent variable and the number of games g is the independent variable.

   b. What values of the domain and range make sense for this situation? Explain.

   Only whole numbers between and including 0 and 82 make sense for the domain because you do not want data for a partial game and there are only 82 games in a season. The range will be multiples of 20.7.

   c. Write a function to represent the total points scored. Then determine the number of points scored in 9 games.

   $p(g) = 20.7g$; 186.3 points

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–7, 14–21	8–10, 22, 23	11–13
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 11, 22, 23
OL	On Level	1–7 odd, 8–11, 22, 23
BL	Beyond Level	8–13, 22, 23

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	12, 13
2 Reason abstractly and quantitatively.	11, 21
4 Model with mathematics.	8

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students explain how to find a function value given a rule and a value in the domain. Substitute the value in the function and evaluate. See students' work.

8. **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–c.
b–c. See Answer Appendix.

a. Write a function to represent the total cost c of printing and shipping any number of pictures p. $c(p) = 1.5p + 30$
b. Make a function table on a separate piece of paper to find the total cost of printing and shipping 25, 50, 75, and 100 pictures.
c. On a separate piece of paper, graph the ordered pairs on a coordinate plane. Can you determine how many pictures Khalid can ship for AED 250?

Copy and Solve **Find each function value. Show your work on a separate piece of paper.**

9. $f\left(\frac{5}{6}\right)$ if $f(x) = 2x + \frac{1}{3}$ **2**

10. $f\left(\frac{5}{8}\right)$ if $f(x) = 4x - \frac{1}{4}$ $2\frac{1}{4}$

H.O.T. Problems Higher Order Thinking

11. **Reason Abstractly** If $f(-3) = -8$, write a function rule and find the function values for zero, a negative, and a positive value of x.
Sample answer: $f(x) = 2x - 2$; $f(0) = -2$, $f(-4) = -10$, $f(3) = 4$

12. **Persevere with Problems** Write the function rule for each function.

a.

x	$f(x)$
−3	−30
−1	−10
2	20
6	60

$f(x) = 10x$

b.

x	$f(x)$
−5	−9
−1	−5
3	−1
7	3

$f(x) = x - 4$

c.

x	y
−2	−3
1	3
3	7
5	11

$y = 2x + 1$

d.

x	y
−2	−5
1	1
3	5
5	9

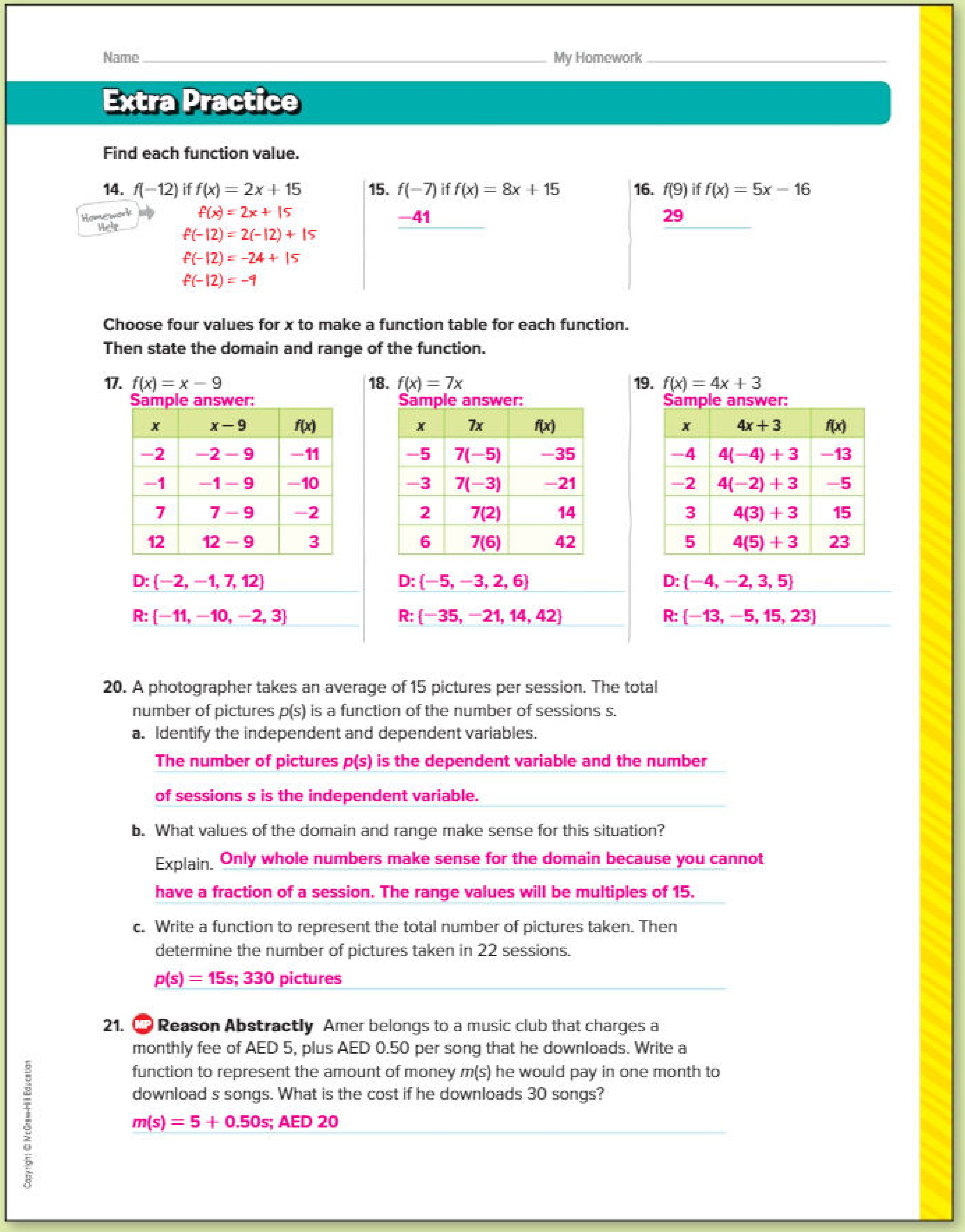
$y = 2x - 1$

13. **Persevere with Problems** If $f(x) = 4x - 3$ and $g(x) = 8x + 2$, find each function value.

a. $f[g(3)]$ 101

b. $g[f(5)]$ 138

c. $g[f[g(-4)]]$ −982



Name ______________________ My Homework ______________

Extra Practice

Find each function value.

14. $f(-12)$ if $f(x) = 2x + 15$

Homework Help

$f(x) = 2x + 15$
$f(-12) = 2(-12) + 15$
$f(-12) = -24 + 15$
$f(-12) = -9$

15. $f(-7)$ if $f(x) = 8x + 15$

−41

16. $f(9)$ if $f(x) = 5x - 16$

29

Choose four values for *x* to make a function table for each function. Then state the domain and range of the function.

17. $f(x) = x - 9$

Sample answer:

x	$x - 9$	$f(x)$
−2	−2 − 9	−11
−1	−1 − 9	−10
7	7 − 9	−2
12	12 − 9	3

D: {−2, −1, 7, 12}

R: {−11, −10, −2, 3}

18. $f(x) = 7x$

Sample answer:

x	$7x$	$f(x)$
−5	7(−5)	−35
−3	7(−3)	−21
2	7(2)	14
6	7(6)	42

D: {−5, −3, 2, 6}

R: {−35, −21, 14, 42}

19. $f(x) = 4x + 3$

Sample answer:

x	$4x + 3$	$f(x)$
−4	4(−4) + 3	−13
−2	4(−2) + 3	−5
3	4(3) + 3	15
5	4(5) + 3	23

D: {−4, −2, 3, 5}

R: {−13, −5, 15, 23}

20. A photographer takes an average of 15 pictures per session. The total number of pictures $p(s)$ is a function of the number of sessions s.

a. Identify the independent and dependent variables.

The number of pictures *p*(*s*) is the dependent variable and the number of sessions *s* is the independent variable.

b. What values of the domain and range make sense for this situation? Explain. **Only whole numbers make sense for the domain because you cannot have a fraction of a session. The range values will be multiples of 15.**

c. Write a function to represent the total number of pictures taken. Then determine the number of pictures taken in 22 sessions.

$p(s) = 15s$; 330 pictures

21. MP **Reason Abstractly** Amer belongs to a music club that charges a monthly fee of AED 5, plus AED 0.50 per song that he downloads. Write a function to represent the amount of money $m(s)$ he would pay in one month to download s songs. What is the cost if he downloads 30 songs?

$m(s) = 5 + 0.50s$; AED 20

Power Up! Test Practice

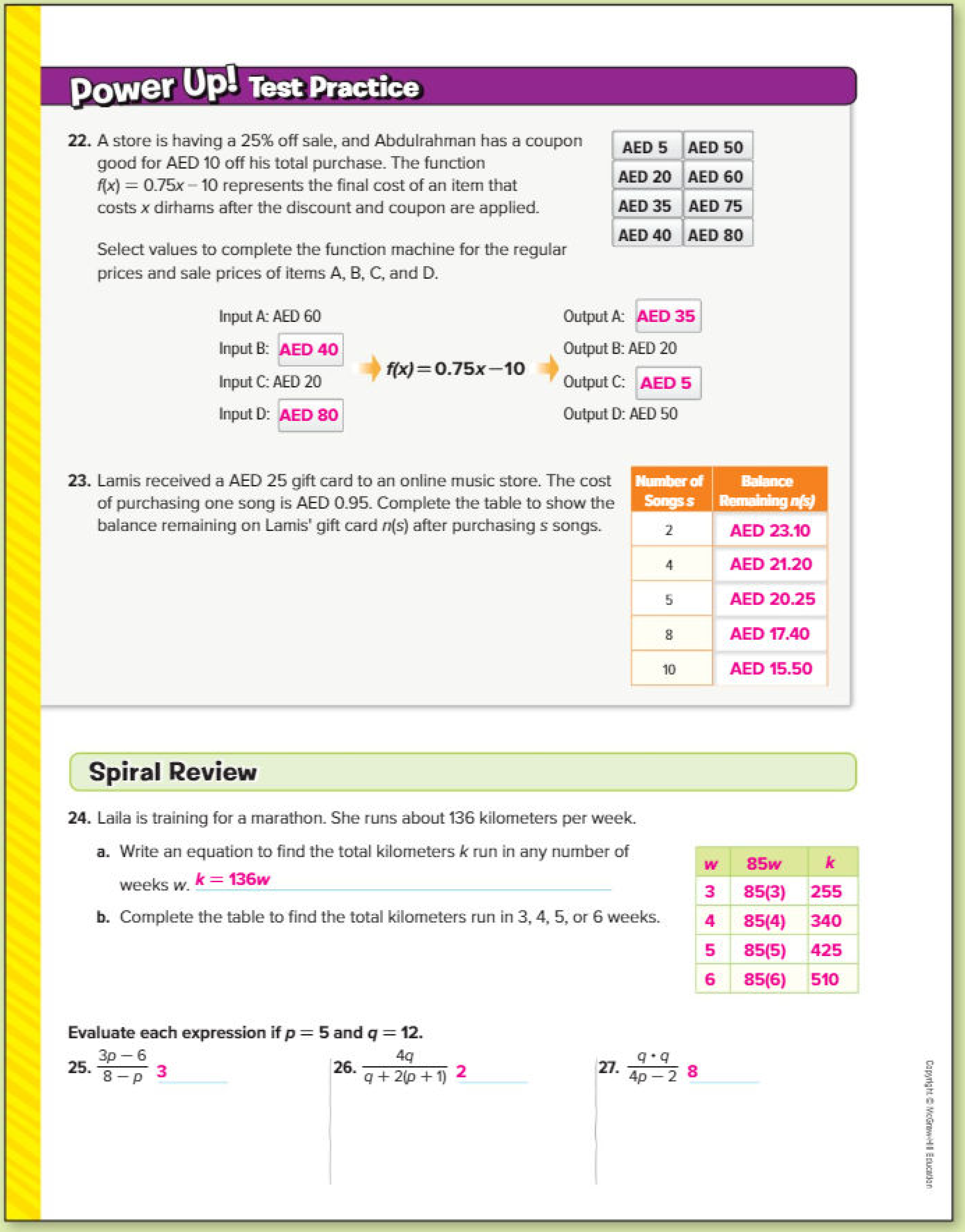
Exercises 22 and 23 prepare students for more rigorous thinking needed for assessment.

22.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly place all inputs with the corresponding outputs.
1 point	Students correctly place 2–3 inputs with the corresponding outputs.

23.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students complete the table correctly.
1 point	Students correctly type 3–4 of the 5 values.



Power Up! Test Practice

22. A store is having a 25% off sale, and Abdulrahman has a coupon good for AED 10 off his total purchase. The function $f(x) = 0.75x - 10$ represents the final cost of an item that costs x dirhams after the discount and coupon are applied.

AED 5	AED 50
AED 20	AED 60
AED 35	AED 75
AED 40	AED 80

Select values to complete the function machine for the regular prices and sale prices of items A, B, C, and D.

Input A: AED 60
Input B: **AED 40**
Input C: AED 20
Input D: **AED 80**

$f(x) = 0.75x - 10$

Output A: **AED 35**
Output B: AED 20
Output C: **AED 5**
Output D: AED 50

23. Lamis received a AED 25 gift card to an online music store. The cost of purchasing one song is AED 0.95. Complete the table to show the balance remaining on Lamis' gift card $n(s)$ after purchasing s songs.

Number of Songs s	Balance Remaining $n(s)$
2	AED 23.10
4	AED 21.20
5	AED 20.25
8	AED 17.40
10	AED 15.50

Spiral Review

24. Laila is training for a marathon. She runs about 136 kilometers per week.

a. Write an equation to find the total kilometers k run in any number of weeks w. $k = 136w$

b. Complete the table to find the total kilometers run in 3, 4, 5, or 6 weeks.

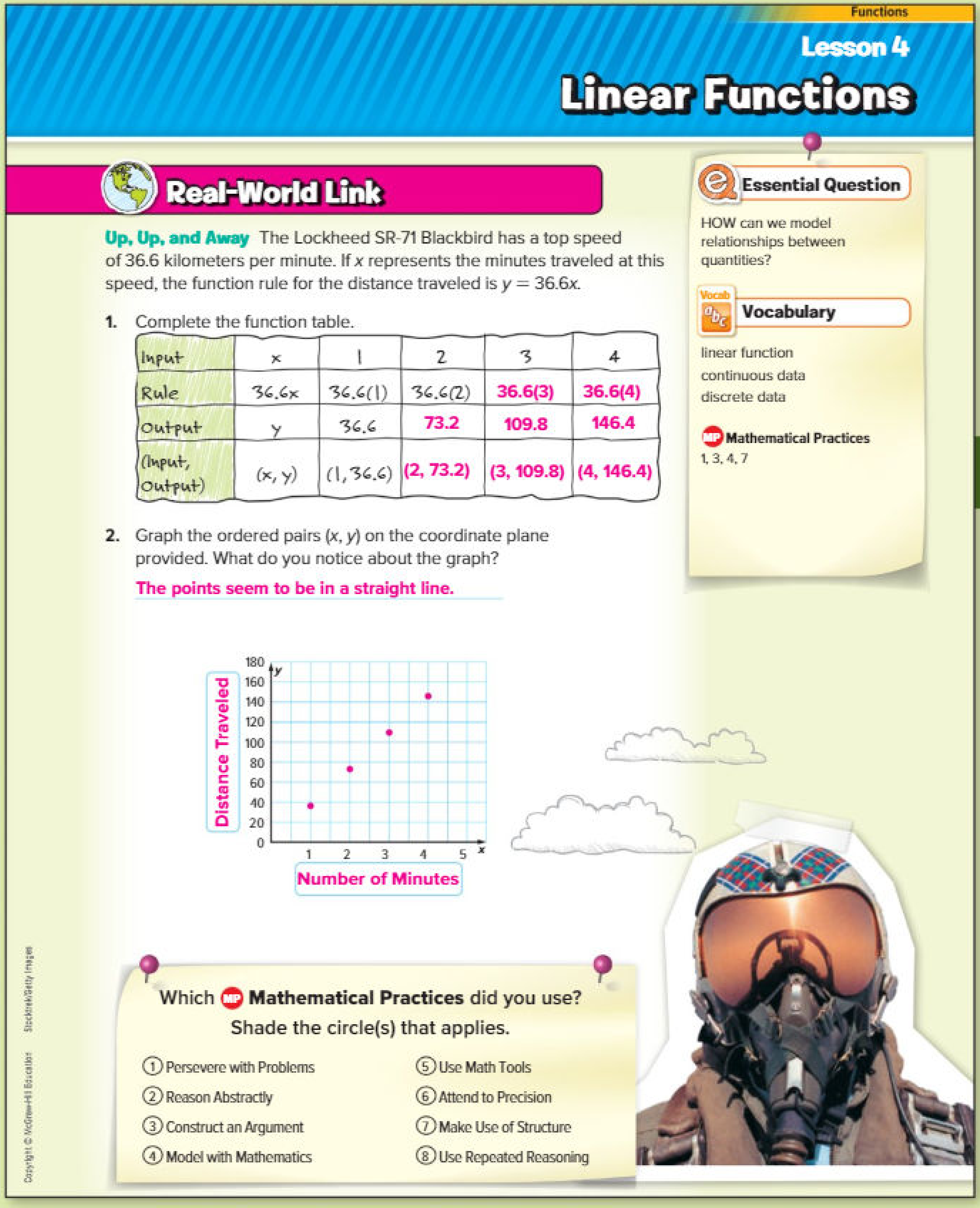
w	$85w$	k
3	85(3)	255
4	85(4)	340
5	85(5)	425
6	85(6)	510

Evaluate each expression if $p = 5$ and $q = 12$.

25. $\frac{3p - 6}{8 - p}$ 3

26. $\frac{4q}{q + 2(p + 1)}$ 2

27. $\frac{q \cdot q}{4p - 2}$ 8



Lesson 4

Linear Functions

Real-World Link

Up, Up, and Away The Lockheed SR-71 Blackbird has a top speed of 36.6 kilometers per minute. If *x* represents the minutes traveled at this speed, the function rule for the distance traveled is $y = 36.6x$.

1. Complete the function table.

Input	x	1	2	3	4
Rule	36.6x	36.6(1)	36.6(2)	36.6(3)	36.6(4)
Output	y	36.6	73.2	109.8	146.4
(Input, Output)	(x, y)	(1, 36.6)	(2, 73.2)	(3, 109.8)	(4, 146.4)

2. Graph the ordered pairs (*x*, *y*) on the coordinate plane provided. What do you notice about the graph?

The points seem to be in a straight line.

Essential Question

HOW can we model relationships between quantities?

Vocabulary

linear function
continuous data
discrete data

Mathematical Practices
1, 3, 4, 7

Which **Mathematical Practices** did you use?
Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Stocktrek/Getty Images

Focus narrowing the scope

Objective Represent linear functions using tables and graphs.

Coherence connecting within and across grades

Previous	Now	Next
Students used function tables to determine the domain and range of a function.	Students will represent linear functions using function tables and graphs.	Students will compare functions represented in different forms.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 301.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Circle the Sage** Poll students to determine who has a solid understanding of functions. Have those students (the sages) spread out around the room. Place the remaining students into teams. Have team members spread out to the sages, with no two team members going to the same sage. Have groups complete the exercises with sages leading the discussion. Once the exercises are complete, send students back to their original teams to discuss the exercises and any differences in how the sages explained the work. MP 1, 3

Alternate Strategy

AL Have students line up the points on the graph using a ruler to verify that the points fall in a straight line.

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Graph a function.

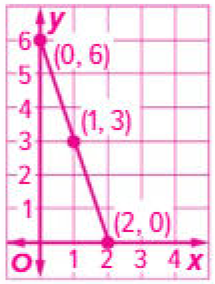
AL
- *What values of x would be reasonable to choose in this scenario?* **0, 1, 2, 3**
- *How can we determine each corresponding value of y in the table?* **Substitute each value of *x* into the expression 5 − 2*x*.**

OL
- *What do you notice about the points graphed in the coordinate plane?* **Sample answer: The points fall on a straight line.**
- *What do you notice about the constant rate of change of the graph?* **Sample answer: The constant rate of change is negative.**

BL
- *Why would you not choose a negative input value for x?* **Sample answer: She cannot buy a negative number of book covers, so a negative input value does not make sense in this situation.**

Need Another Example?

During a clearance sale, a store is selling DVDs for AED3 and CDs for AED1. Graph the function $y = 6 - 3x$ to find all possible values of DVDs x and CDs y that Mansour can buy with AED6.



0 DVDs and 6 CDs; 1 DVD and 3 CDs; 2 DVDs and 0 CDs

Work Zone

Graph a Function

Sometimes functions are written using two variables. One variable, usually *x*, represents the domain and the other, usually *y*, represents the range. When a function is written in this form it is an equation.

Like equations, functions can be represented in words, in a table, with a graph, and as ordered pairs. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

Function Notation

The equation $y = 5 - 3x$
can also be written in
function notation as
$f(x) = 5 - 3x$.

Real World

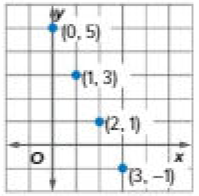
Example

1. The school store sells book covers for AED 2 each and notebooks for AED 1. Manal has AED 5 to spend. The function $y = 5 - 2x$ represents the number of book covers x and notebooks y she can buy. Graph the function. Interpret the points graphed.

Step 1 Choose values for *x* and substitute them in the function to find *y*.

x	5 − 2x	y
0	5 − 2(0)	5
1	5 − 2(1)	3
2	5 − 2(2)	1
3	5 − 2(3)	−1

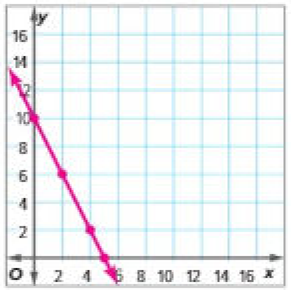
Step 2 Graph the ordered pairs (*x*, *y*).



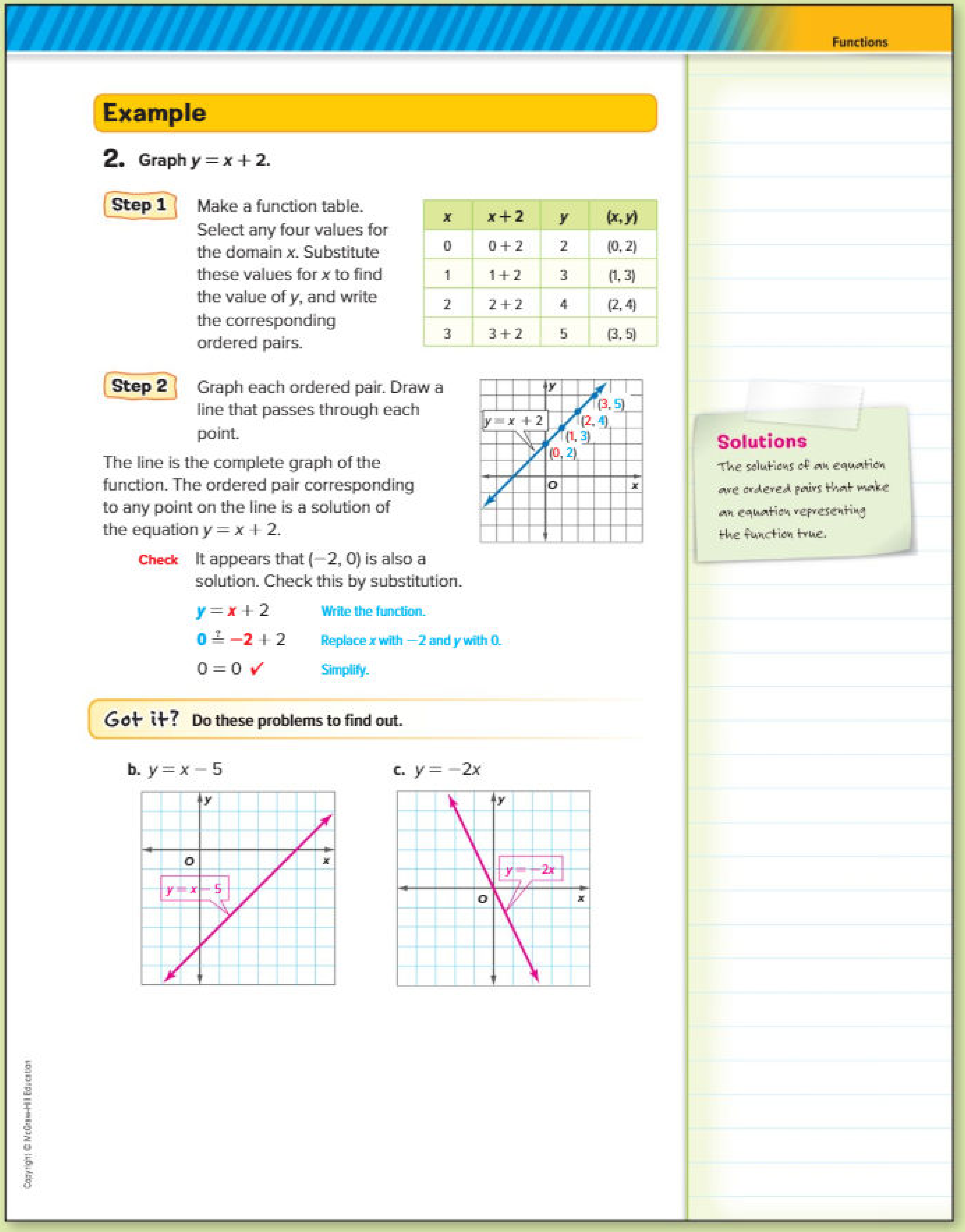
She cannot buy negative amounts. So she can buy 0 covers and 5 notebooks, 1 cover and 3 notebooks, or 2 covers and 1 notebook.

Got it? **Do this problem to find out.**

a. The farmer's market sells apples for AED 2 per kilogram and oranges for AED 1 per kilogram. Abdulraheem has AED 10 to spend. The function $y = 10 - 2x$ represents the number of apples x and oranges y Abdulraheem can purchase. Graph the function and interpret the points graphed.



a. **Sample answer: Abdulraheem can purchase 10 kilograms of oranges, 6 kilograms of oranges and 2 kilograms of apples, 2 kilograms of oranges and 4 kilograms of apples.**



Example

2. Graph $y = x + 2$.

Step 1 Make a function table. Select any four values for the domain x. Substitute these values for x to find the value of y, and write the corresponding ordered pairs.

x	$x + 2$	y	(x, y)
0	$0 + 2$	2	(0, 2)
1	$1 + 2$	3	(1, 3)
2	$2 + 2$	4	(2, 4)
3	$3 + 2$	5	(3, 5)

Step 2 Graph each ordered pair. Draw a line that passes through each point.

The line is the complete graph of the function. The ordered pair corresponding to any point on the line is a solution of the equation $y = x + 2$.

Check It appears that (−2, 0) is also a solution. Check this by substitution.

$y = x + 2$ Write the function.

$0 \overset{?}{=} -2 + 2$ Replace x with −2 and y with 0.

$0 = 0$ ✓ Simplify.

Solutions

The solutions of an equation are ordered pairs that make an equation representing the function true.

Got it? Do these problems to find out.

b. $y = x - 5$

c. $y = -2x$

Examples

2. Graph a function.

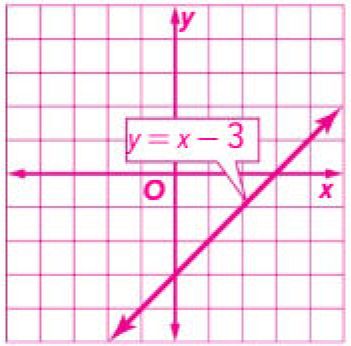
AL
- *What method could we use to help graph $y = x + 2$?* Sample answer: make a function table to find the x- and y-values
- *What input values could we use for the x-values?* Sample answer: 0, 1, 2, 3

OL
- *How can we check that our graph is correct?* Sample answer: Check a point on the line to see that it makes a true statement when the x- and y-coordinates are substituted into the equation.
- *How many solutions are there for the function $y = x + 2$? Explain.* There are an infinite number of solutions because there are an infinite number of points on the line.

BL
- *Is there another method you could use to graph the equation?* Sample answer: I could use the slope and y-intercept. The y-intercept is 2. Plot 2 on the y-axis. From there, use the slope, 1, to go up 1 and to the right 1 to plot the next point. Connect the points with a straight line.

Need Another Example?

Graph $y = x - 3$.



Examples

3. Write a function for a real-world problem.

AL
- *How much is the coupon worth?* AED5
- *If 6 people entered the store, what is the total value of coupons given out? How can you determine this value?* AED30; Multiply 6 by AED5 .

OL
- *What are the dependent and independent variables?* Sample answer: Because the total value of the coupons depends on the number of people, the value of the coupons is the dependent variable and the number of people is the independent variable.
- *What function can be used to represent the situation?* $y = 5x$

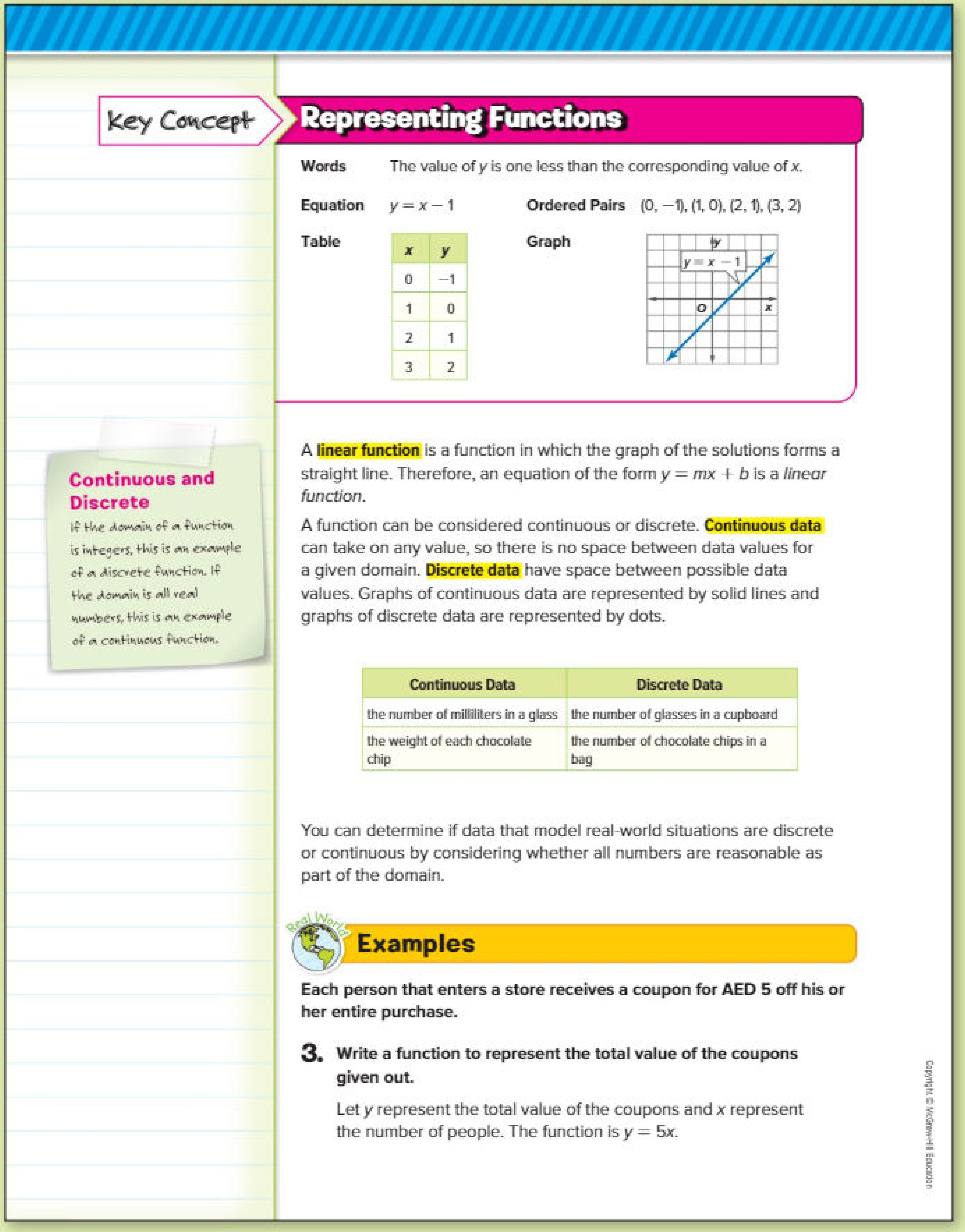
BL
- *What is the y-intercept? What does this mean in the context of the problem?* The y-intercept is 0; If 0 people enter the store, the total value of the coupons given out is AED0.

4. Make a function table for a real-world problem.

AL
- *How can we determine the y-values?* Multiply each x-value by 5.
- *What are the y-values?* 25, 50, 75, and 100

OL
- *How does the table show the constant rate of change?* The change in y-values divided by the change in x-values is 5.
- *Is this relationship proportional? Explain.* yes; Sample answer: The quantities are in a constant ratio.

BL
- *Determine the total value of the coupons given out to c customers.* AED5c



Key Concept

Representing Functions

Words The value of y is one less than the corresponding value of x.

Equation $y = x - 1$

Ordered Pairs (0, −1), (1, 0), (2, 1), (3, 2)

Table

x	y
0	−1
1	0
2	1
3	2

Graph

Continuous and Discrete

If the domain of a function is integers, this is an example of a discrete function. If the domain is all real numbers, this is an example of a continuous function.

A **linear function** is a function in which the graph of the solutions forms a straight line. Therefore, an equation of the form $y = mx + b$ is a *linear function*.

A function can be considered continuous or discrete. **Continuous data** can take on any value, so there is no space between data values for a given domain. **Discrete data** have space between possible data values. Graphs of continuous data are represented by solid lines and graphs of discrete data are represented by dots.

Continuous Data	Discrete Data
the number of milliliters in a glass	the number of glasses in a cupboard
the weight of each chocolate chip	the number of chocolate chips in a bag

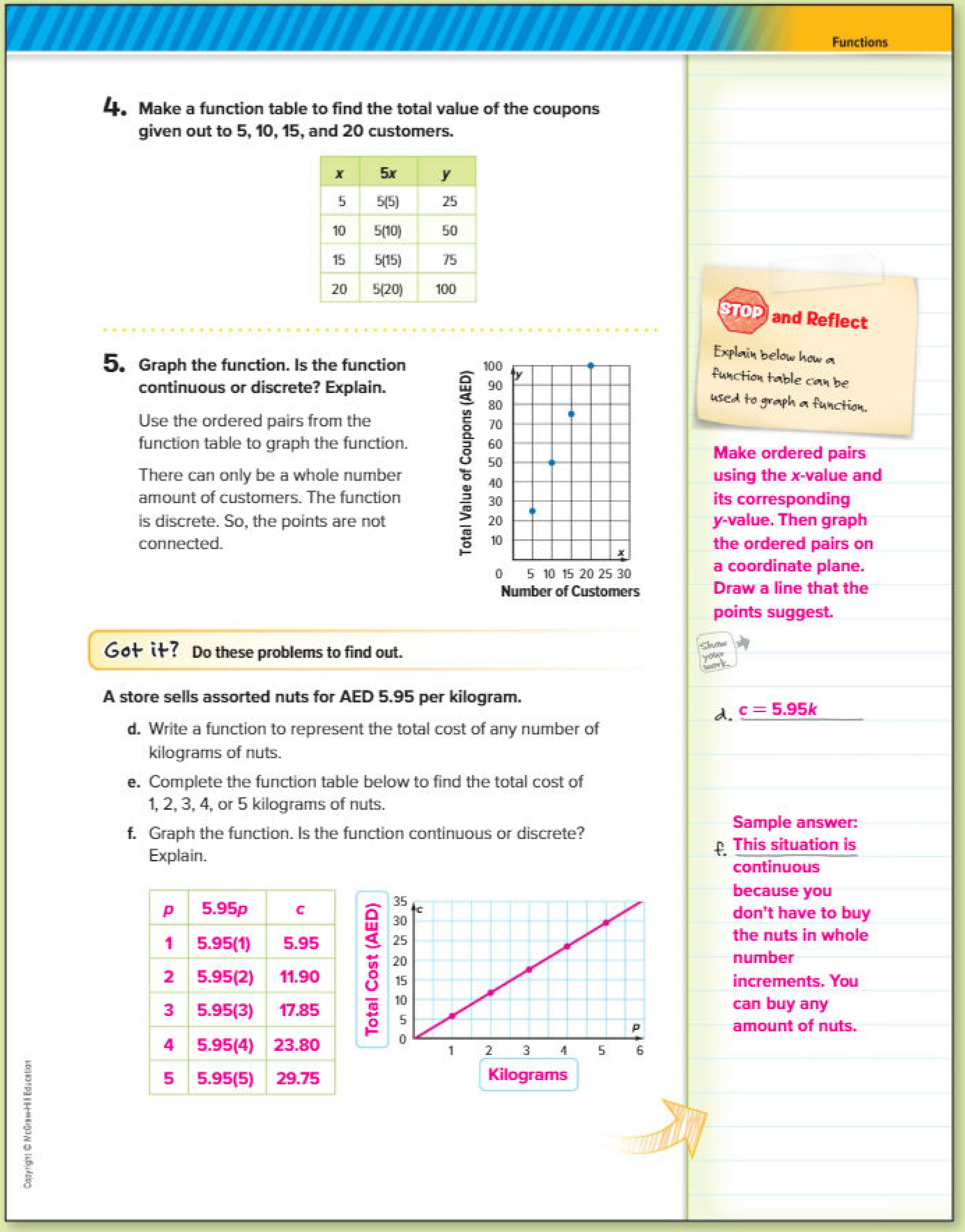
You can determine if data that model real-world situations are discrete or continuous by considering whether all numbers are reasonable as part of the domain.

Examples

Each person that enters a store receives a coupon for AED 5 off his or her entire purchase.

3. Write a function to represent the total value of the coupons given out.

Let y represent the total value of the coupons and x represent the number of people. The function is $y = 5x$.



4. Make a function table to find the total value of the coupons given out to 5, 10, 15, and 20 customers.

x	$5x$	y
5	5(5)	25
10	5(10)	50
15	5(15)	75
20	5(20)	100

5. Graph the function. Is the function continuous or discrete? Explain.

Use the ordered pairs from the function table to graph the function.

There can only be a whole number amount of customers. The function is discrete. So, the points are not connected.

STOP and Reflect

Explain below how a function table can be used to graph a function.

Make ordered pairs using the x-value and its corresponding y-value. Then graph the ordered pairs on a coordinate plane. Draw a line that the points suggest.

Got it? Do these problems to find out.

Show your work.

A store sells assorted nuts for AED 5.95 per kilogram.

d. Write a function to represent the total cost of any number of kilograms of nuts.

e. Complete the function table below to find the total cost of 1, 2, 3, 4, or 5 kilograms of nuts.

f. Graph the function. Is the function continuous or discrete? Explain.

p	$5.95p$	c
1	5.95(1)	5.95
2	5.95(2)	11.90
3	5.95(3)	17.85
4	5.95(4)	23.80
5	5.95(5)	29.75

d. $c = 5.95k$

f. Sample answer: This situation is continuous because you don't have to buy the nuts in whole number increments. You can buy any amount of nuts.

Examples

5. Graph a function and determine continuous or discrete data.

AL
- *How would you write the relationship as a set of ordered pairs?* (5, 25), (10, 50), (15, 75), and (20, 100)
- *Can you have a number of customers that is not a whole number?* no

OL
- *Since you cannot have part of a customer, is this function continuous or discrete?* discrete
- *Should we connect the points? Explain.* no; Sample answer: The function is discrete, so we leave the points unconnected.

BL
- *Does the ordered pair (2.5, 12.5) make sense in the context of the problem? Explain.* no; Sample answer: There cannot be 2.5 customers.

Need Another Example?

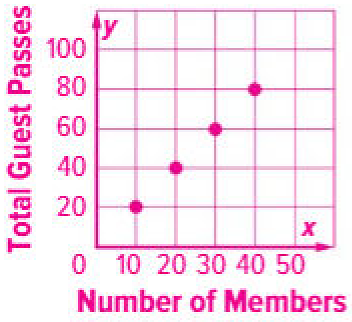
Each member of a health club receives two free guest passes.

a. Write a function to represent the situation. $y = 2x$

b. Make a function table to show the number of guest passes given out to 10, 20, 30, and 40 members.

x	$2x$	y
10	2(10)	20
20	2(20)	40
30	2(30)	60
40	2(40)	80

c. Graph the function. Is the function continuous or discrete? Explain. discrete; Sample answer: The number of health club members can only be represented by whole numbers.



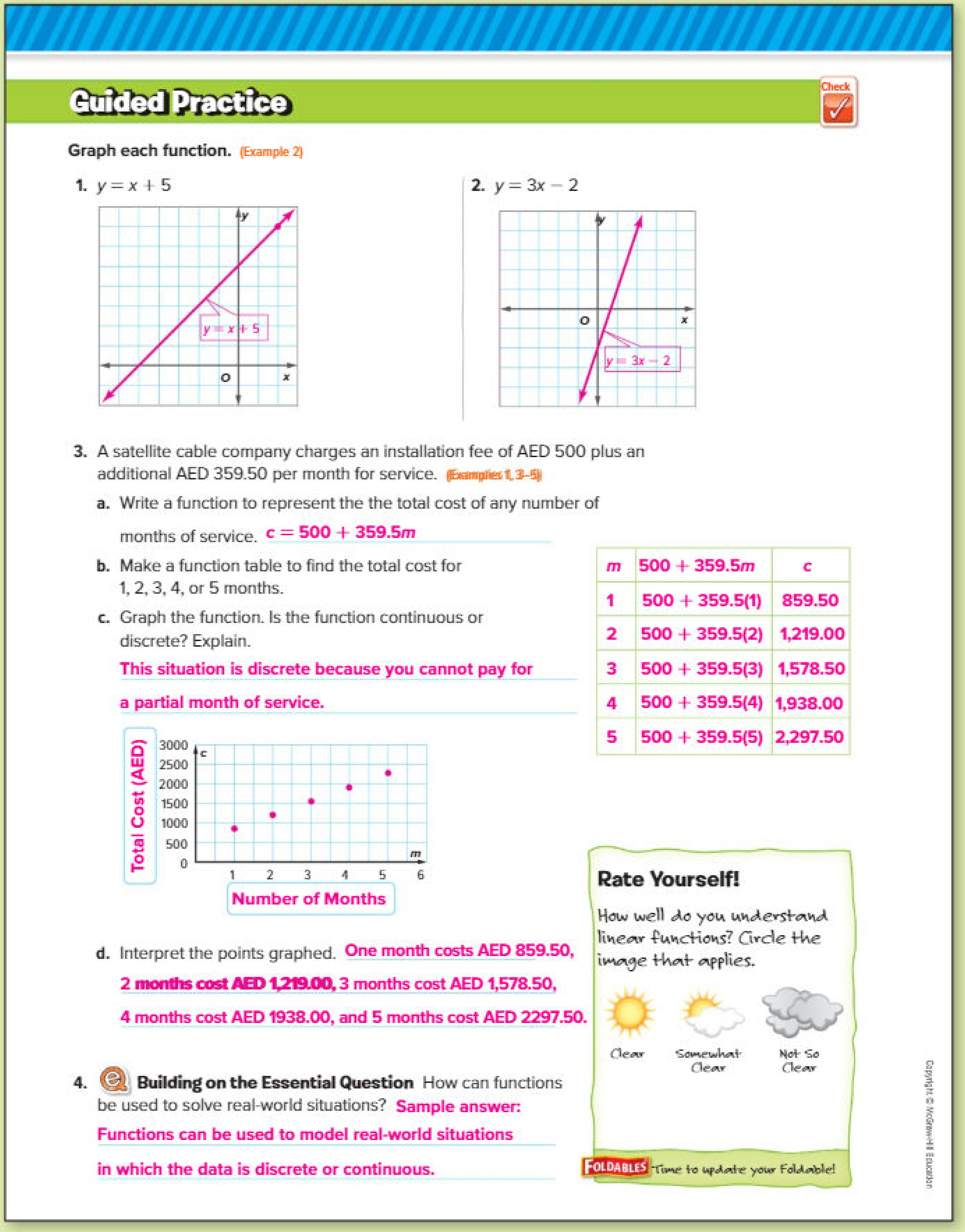
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL **LA** **Pairs Create** Have students work in pairs to create function machines out of construction paper or draw a function machine on paper. Have them use their machines to generate a table of values for Exercises 1 and 2. **MP 1, 5**

BL **LA** **Roundrobin** For each exercise, have students create a different representation than the one given. For example, in Exercise 1, students may create a table of values, set of ordered pairs, or a mapping diagram. **MP 1, 2, 4**



Guided Practice

Check ✓

Graph each function. (Example 2)

1. $y = x + 5$

2. $y = 3x - 2$

3. A satellite cable company charges an installation fee of AED 500 plus an additional AED 359.50 per month for service. (Examples 1, 3–5)

   a. Write a function to represent the the total cost of any number of months of service. **$c = 500 + 359.5m$**

   b. Make a function table to find the total cost for 1, 2, 3, 4, or 5 months.

m	$500 + 359.5m$	c
1	$500 + 359.5(1)$	859.50
2	$500 + 359.5(2)$	1,219.00
3	$500 + 359.5(3)$	1,578.50
4	$500 + 359.5(4)$	1,938.00
5	$500 + 359.5(5)$	2,297.50

   c. Graph the function. Is the function continuous or discrete? Explain.
   This situation is discrete because you cannot pay for a partial month of service.

   d. Interpret the points graphed. **One month costs AED 859.50, 2 months cost AED 1,219.00, 3 months cost AED 1,578.50, 4 months cost AED 1938.00, and 5 months cost AED 2297.50.**

4. **Building on the Essential Question** How can functions be used to solve real-world situations? **Sample answer: Functions can be used to model real-world situations in which the data is discrete or continuous.**

Rate Yourself!

How well do you understand linear functions? Circle the image that applies.

Clear Somewhat Clear Not So Clear

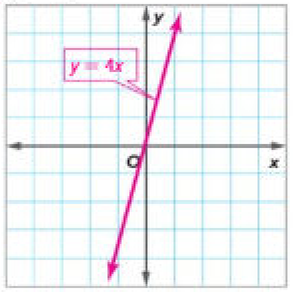
FOLDABLES Time to update your Foldable!

Name ______ My Homework ______

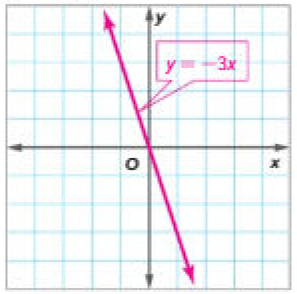
Independent Practice

Graph each function. (Example 2)

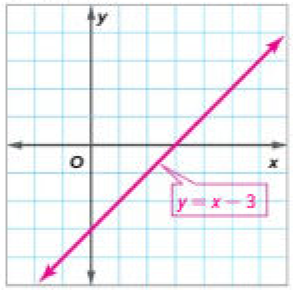
1. $y = 4x$



2. $y = -3x$



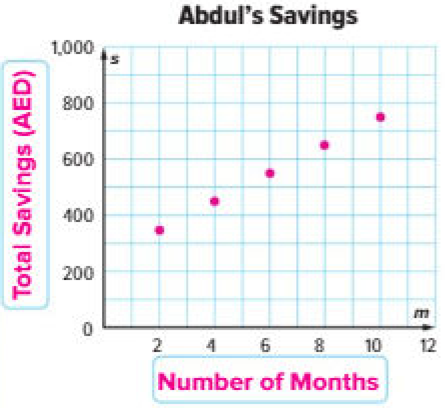
3. $y = x - 3$



4. **Financial Literacy** Abdul is saving money for college. He already has AED 250. He plans to save another AED 50 per month. (Examples 1, 3–5)
   a. Write a function to represent his savings for any number of months. $s = 250 + 50m$
   b. Make a function table to find his total savings for 2, 4, 6, 8 and 10 months.

m	$250 + 50m$	s
2	$250 + 50(2)$	350
4	$250 + 50(4)$	450
6	$250 + 50(6)$	550
8	$250 + 50(8)$	650
10	$250 + 50(10)$	750

   c. Graph the function. Is the function continuous or discrete? Explain. **discrete; You cannot find Abdul's total savings for part of a month.**
   d. Interpret the points graphed. **Abdul saved a total of AED 350 in 2 months, AED 450 in 4 months, AED 550 in 6 months, AED 650 in 8 months, and AED 750 in 10 months.**



5. **Copy and Solve** The table shows the cost to rent different items.

Item	Deposit (AED)	Cost per Hour (AED)
Mountain bike	150	42.50
Scooter	250	25.00

   a. Write a function to represent each situation. **bike: $c = 150 + 4.25h$; scooter: $c = 250 + 2.5h$**
   b. On a separate piece of paper, make a function table to find the total cost to rent each item for 2, 3, 4, or 5 hours. **See Answer Appendix.**
   c. On a separate piece of grid paper, graph the functions on the same coordinate plane. Are the functions continuous or discrete? Explain. **See Answer Appendix for graph. Both situations are discrete because you cannot rent either piece of equipment for a partial hour.**
   d. Will the mountain bike or the scooter cost more to rent for 8 hours? **mountain bike**
   e. How much is the cost to rent the mountain bike for 8 hours? **AED 490**

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7, 8, 10, 17, 18
OL	On Level	1–5 odd, 6–8, 10, 17, 18
BL	Beyond Level	5–10, 17, 18

Watch Out!

Common Error Some students may find that not all of the ordered pairs in their tables lie on a straight line. Suggest that they check each of their output calculations and pay special attention to negative signs in equations.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	9
4 Model with mathematics.	6, 10, 16
7 Look for and make use of structure.	8

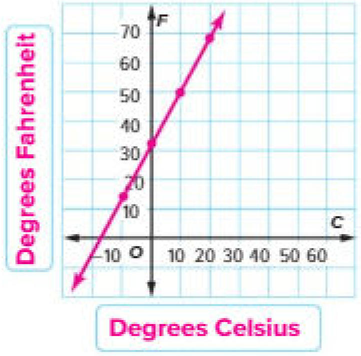
Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

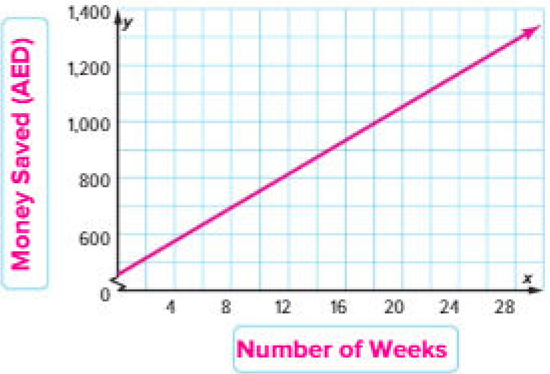
TICKET Out the Door

Ask students to write about how what they learned yesterday about functions helped them understand this lesson's topic of representing functions with tables, graphs, and equations. See students' work.

6. **Model with Mathematics** The formula $F = 1.8C + 32$ compares temperatures in degrees Celsius C to temperatures in degrees Fahrenheit F. Find four ordered pairs (C, F) that are solutions of the equation. Then graph the function.
Sample answer: (0, 32), (−10, 14), (10, 50), (20, 68)



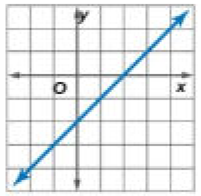
7. Abdulaziz is saving money to buy a new computer for AED 1,200. He already has AED 450 and plans to save AED 30 a week. The function $y = 30x + 450$ represents the amount Abdulaziz has saved after x weeks. Graph the function to determine the number of weeks it will take Abdulaziz to save enough money to buy the computer.
25 weeks



H.O.T. Problems Higher Order Thinking

8. **Identify Structure** Explain why a linear function that is continuous has an infinite number of solutions. Then determine which of the following representations shows all the solutions of the function: a table, a graph, or an equation. Explain. Sample answer: Since the function is continuous, an infinite number of values can be substituted for the domain. A table shows a finite number of solutions. An equation or a graph represent all the solutions of a function.

9. **Persevere with Problems** Name the coordinates of four points that satisfy the linear function shown. Then give the function rule.
Sample answer: (−2, −4), (0, −2), (2, 0), (4, 2); $y = x - 2$

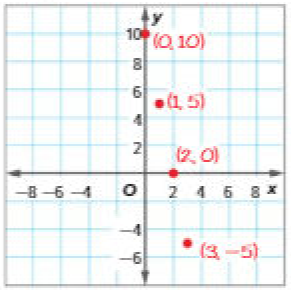


10. **Model with Mathematics** Write a set of four ordered pairs that represents a linear function. Then give the function rule.
Sample answer: (1, 6), (2, 11), (3, 16), (4, 21); $y = 5x + 1$

Name ______________________ My Homework ______________________

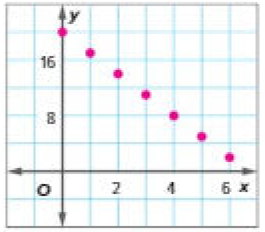
Extra Practice

11. A store sells T-shirts x in packs of 5 and regular shirts y individually. Graph the function $y = 10 - 5x$ to determine the number of each type of shirt Muna can have if she buys 10 shirts.



She cannot buy negative amounts. So, she can buy 0 T-shirt packs and 10 shirts individually, 1 T-shirt pack and 5 shirts individually, or 2 T-shirt packs and 0 shirts individually.

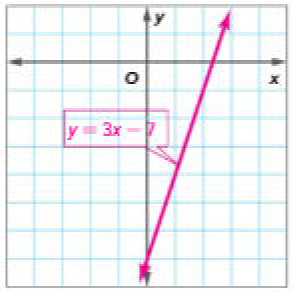
12. Fancy goldfish x cost AED 3 each and common goldfish y cost AED 1 each. Graph the function $y = 20 - 3x$ to determine how many of each type of goldfish Yasmin can buy for AED 20.



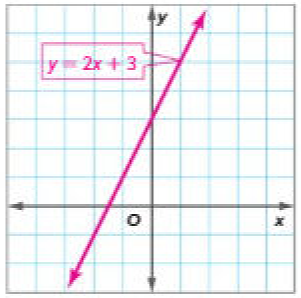
She can buy 20 common and 0 fancy, 17 common and 1 fancy, 14 common and 2 fancy, 11 common and 3 fancy, 8 common and 4 fancy, 5 common and 5 fancy or 2 common and 6 fancy.

Graph each function.

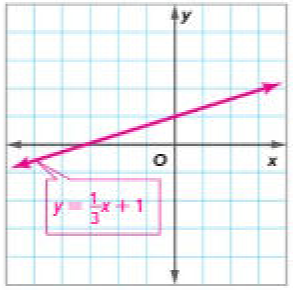
13. $y = 3x - 7$



14. $y = 2x + 3$



15. $y = \frac{1}{3}x + 1$



16. **Model with Mathematics** The equation $y = 1.09x$ describes the approximate number of yards y in x meters.

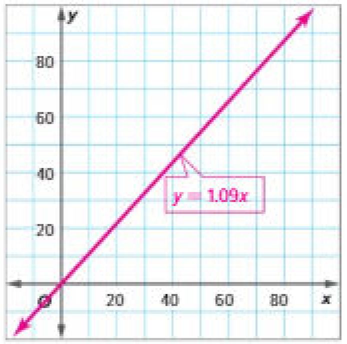
a. Would negative values of x have any meaning in this situation? Explain.

No; you cannot have a negative distance.

b. Graph the function.

c. About how many meters is a 40-yard race?

36.7 m



Power Up! Test Practice

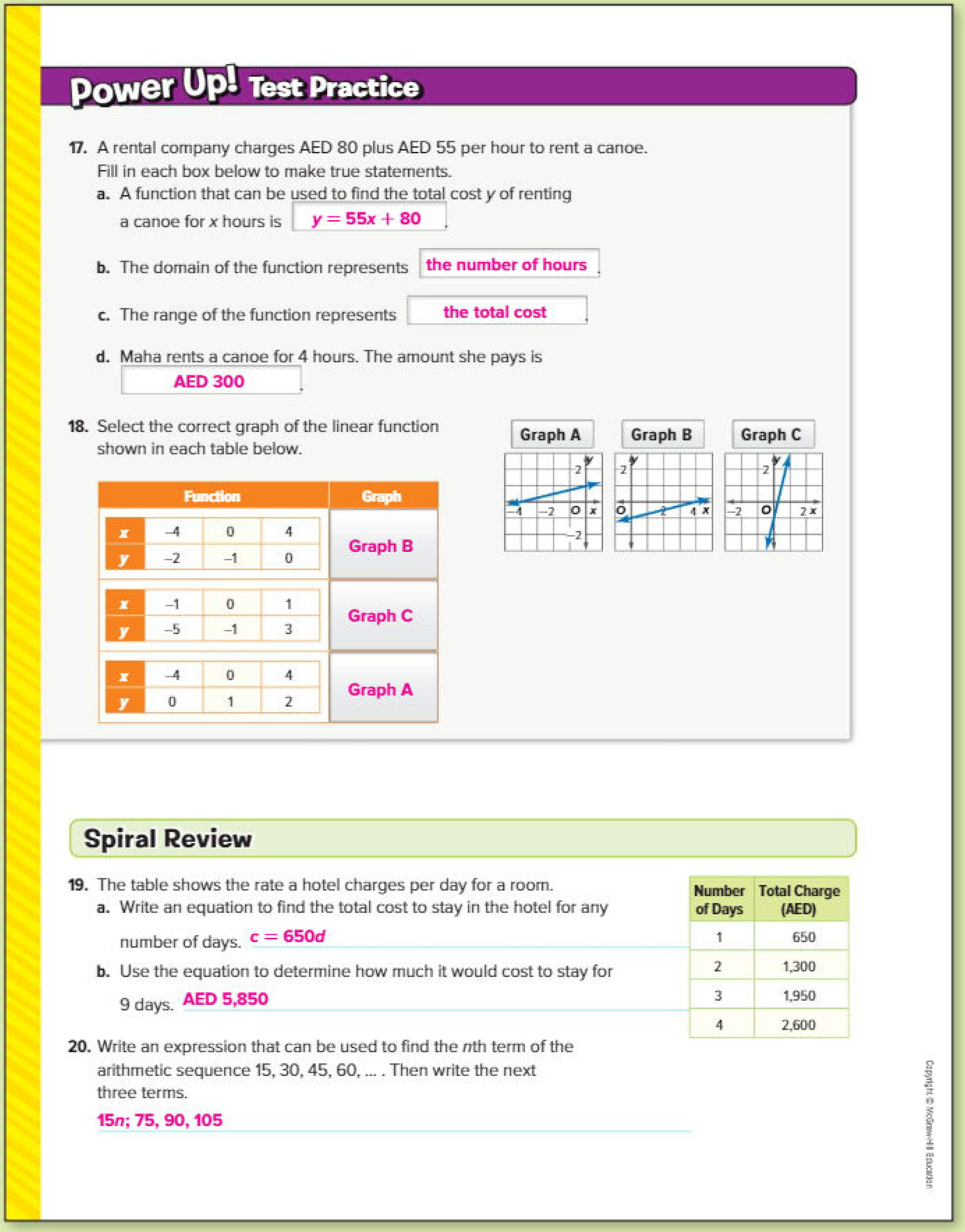
Exercises 17 and 18 prepare students for more rigorous thinking needed for assessment

17.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly complete each statement.
1 point	Students correctly complete 3 of the 4 statements.

18.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly assign each graph to the corresponding table.



Power Up! Test Practice

17. A rental company charges AED 80 plus AED 55 per hour to rent a canoe. Fill in each box below to make true statements.

a. A function that can be used to find the total cost y of renting a canoe for x hours is $y = 55x + 80$.

b. The domain of the function represents the number of hours.

c. The range of the function represents the total cost.

d. Maha rents a canoe for 4 hours. The amount she pays is AED 300.

18. Select the correct graph of the linear function shown in each table below.

Function				Graph
x	–4	0	4	Graph B
y	–2	–1	0	
x	–1	0	1	Graph C
y	–5	–1	3	
x	–4	0	4	Graph A
y	0	1	2	

Spiral Review

19. The table shows the rate a hotel charges per day for a room.

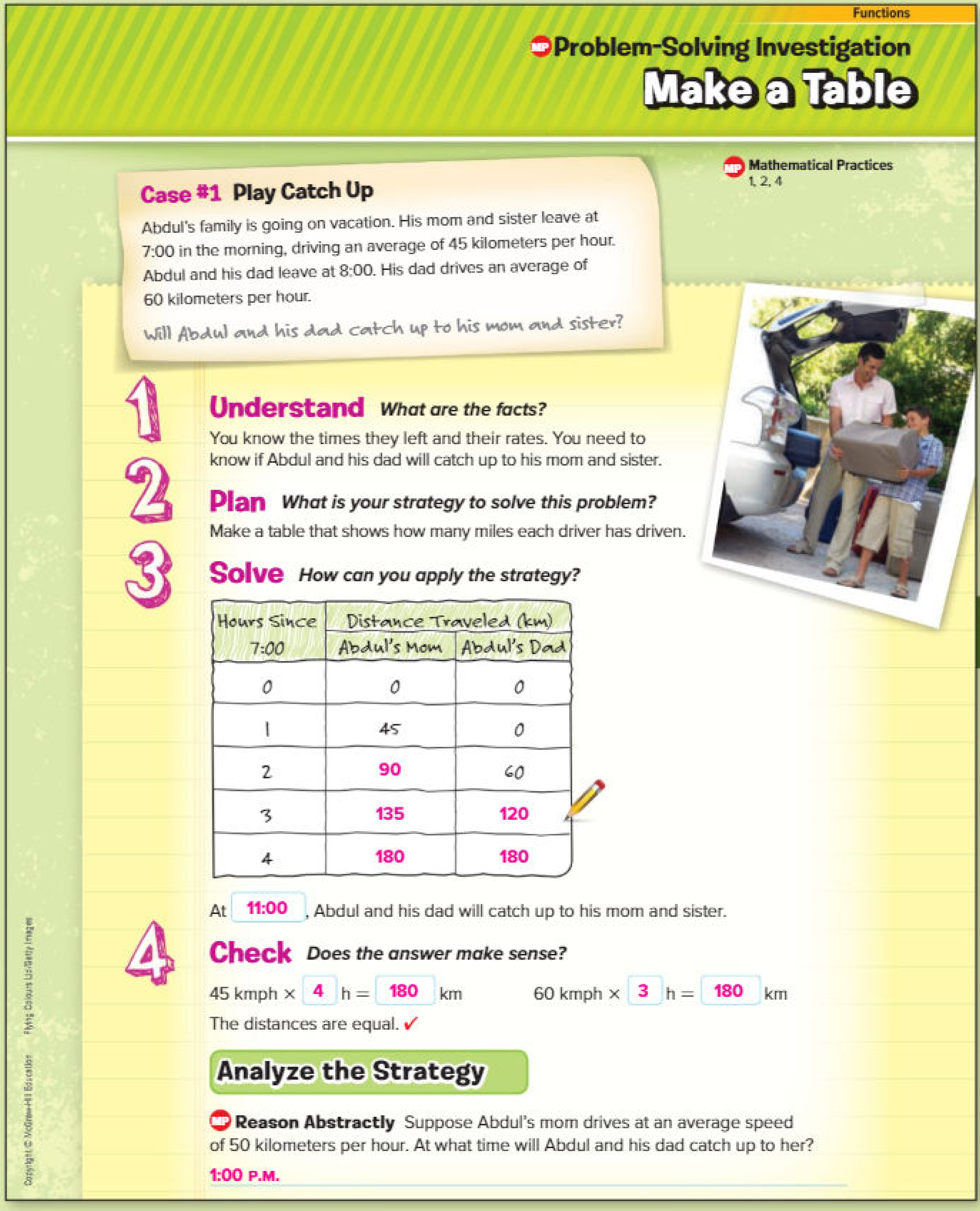
Number of Days	Total Charge (AED)
1	650
2	1,300
3	1,950
4	2,600

a. Write an equation to find the total cost to stay in the hotel for any number of days. $c = 650d$

b. Use the equation to determine how much it would cost to stay for 9 days. AED 5,850

20. Write an expression that can be used to find the nth term of the arithmetic sequence 15, 30, 45, 60, ... Then write the next three terms.

$15n$; 75, 90, 105



Problem-Solving Investigation
Make a Table

Mathematical Practices
1, 2, 4

Case #1 Play Catch Up

Abdul's family is going on vacation. His mom and sister leave at 7:00 in the morning, driving an average of 45 kilometers per hour. Abdul and his dad leave at 8:00. His dad drives an average of 60 kilometers per hour.

Will Abdul and his dad catch up to his mom and sister?

1 **Understand** *What are the facts?*

You know the times they left and their rates. You need to know if Abdul and his dad will catch up to his mom and sister.

2 **Plan** *What is your strategy to solve this problem?*

Make a table that shows how many miles each driver has driven.

3 **Solve** *How can you apply the strategy?*

Hours Since 7:00	Distance Traveled (km)	
	Abdul's Mom	Abdul's Dad
0	0	0
1	45	0
2	90	60
3	135	120
4	180	180

At 11:00, Abdul and his dad will catch up to his mom and sister.

4 **Check** *Does the answer make sense?*

45 kmph × 4 h = 180 km   60 kmph × 3 h = 180 km

The distances are equal. ✓

Analyze the Strategy

Reason Abstractly Suppose Abdul's mom drives at an average speed of 50 kilometers per hour. At what time will Abdul and his dad catch up to her?

1:00 P.M.

Focus narrowing the scope

Objective Solve problems by making a table. This lesson emphasizes **Mathematical Practice 2** Reason Abstractly.

Make a Table Students use the *make a table* strategy as a way to organize data given in a problem. By recording values that represent the relationships between changing quantities, students use tables to solve problems involving rates of change.

Coherence connecting within and across grades

Now	Next
Students apply the content standard to solve non-routine problems.	Students will make tables to solve real-world problems.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 307.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

The problems on pages 305 and 306 are intended to be used as a whole-group discussion on how to solve non-routine problems and are designed to provide scaffolded guidance. The problem on page 305 walks students through the solution, while the problem on page 306 asks students to come up with their own solutions.

Case #1 Play Catch-Up

BL Have students extend the problem by having them answer the question below.

Ask:

- *How many kilometers ahead of Abdulkarim's mom will Abdulkarim and his dad be at 1 P.M. if they both stay at their same average rates?* 30 kilometers ahead

Case #2 Karaoke Kid

AL **LA** **Roundrobin** Have students work in pairs to create a table for Step 3. Have them alternate completing the rows in the table. MP 1, 4

BL **LA** **Pairs Discussion** Have students work in pairs to answer the following extension question. MP 1

Ask:

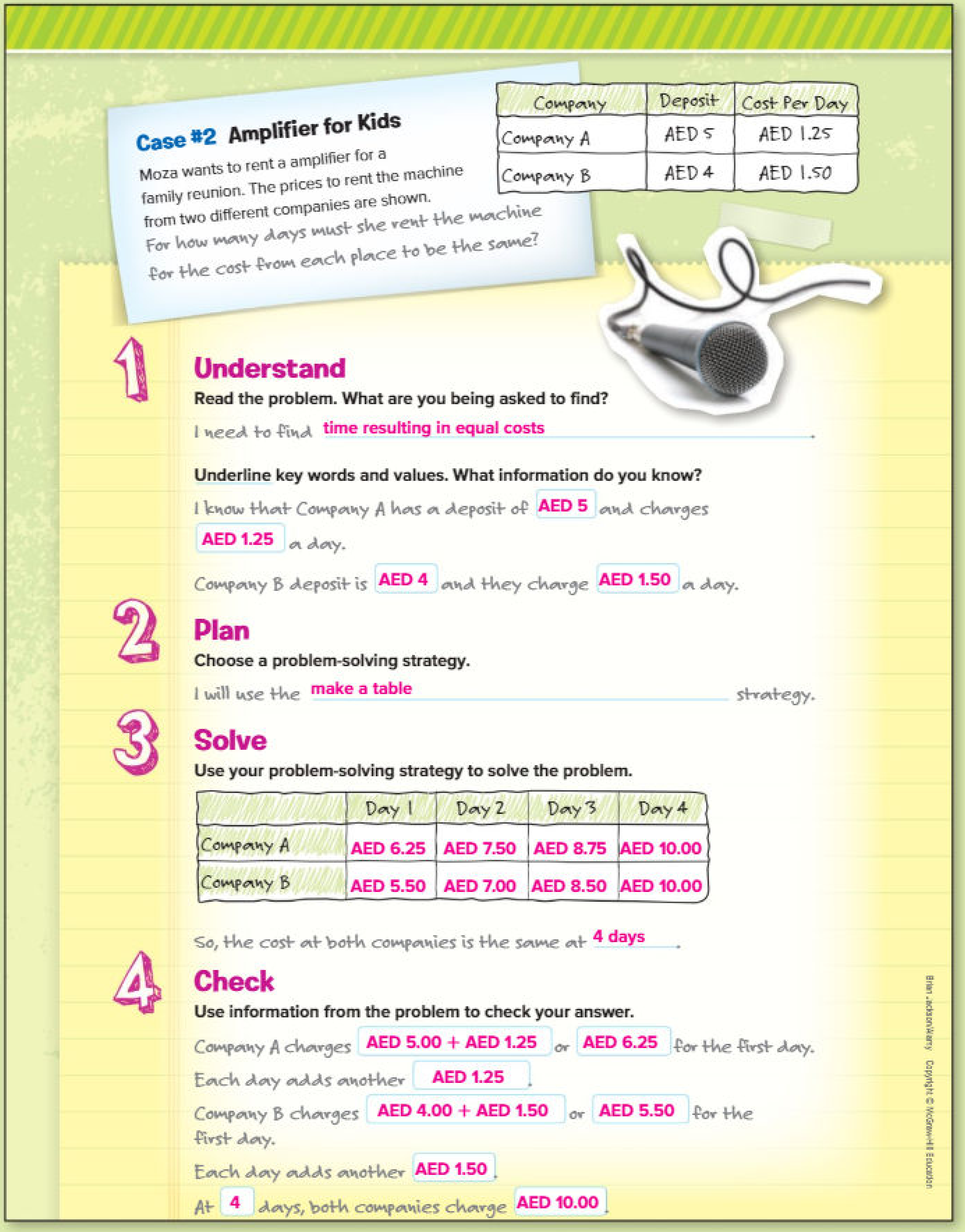
- *Sally's Songs has a daily rate of AED2.50 and no deposit. How many days would Moza need to rent from Sally's Songs for the price to be the same as at Karaoke Korner?* 4 days

Need Another Example?

Hana and Hessa are both riding their bikes on a trail that is 30 kilometers long. Hana rides an average of 3 kilometers per hour with stops and Hessa rides an average of 5 kilometers per hour with stops. If Hana starts at 9:00 A.M. and Hessa starts at 11:00 A.M., will Hessa catch up to Hana?

Hours Since 9:00 A.M.	Distance Traveled (km)	
	Hana	Hessa
0	0	0
1	3	0
2	6	0
3	9	5
4	12	10
5	15	15

Hessa will catch up to Hana 5 hours after 9:00 a.m., or 2:00 p.m.



Case #2 Amplifier for Kids

Moza wants to rent a amplifier for a family reunion. The prices to rent the machine from two different companies are shown. For how many days must she rent the machine for the cost from each place to be the same?

Company	Deposit	Cost Per Day
Company A	AED 5	AED 1.25
Company B	AED 4	AED 1.50

1 **Understand**

Read the problem. What are you being asked to find?

I need to find time resulting in equal costs.

Underline key words and values. What information do you know?

I know that Company A has a deposit of AED 5 and charges AED 1.25 a day.

Company B deposit is AED 4 and they charge AED 1.50 a day.

2 **Plan**

Choose a problem-solving strategy.

I will use the make a table strategy.

3 **Solve**

Use your problem-solving strategy to solve the problem.

	Day 1	Day 2	Day 3	Day 4
Company A	AED 6.25	AED 7.50	AED 8.75	AED 10.00
Company B	AED 5.50	AED 7.00	AED 8.50	AED 10.00

So, the cost at both companies is the same at 4 days.

4 **Check**

Use information from the problem to check your answer.

Company A charges AED 5.00 + AED 1.25 or AED 6.25 for the first day.

Each day adds another AED 1.25.

Company B charges AED 4.00 + AED 1.50 or AED 5.50 for the first day.

Each day adds another AED 1.50.

At 4 days, both companies charge AED 10.00.

Work with a small group to solve the following cases.
Show your work on a separate piece of paper.

Case #3 Plants

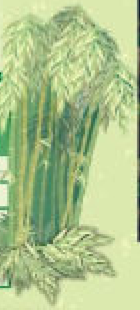
The table shows the height of a giant bamboo plant for Days 5-9. The bamboo grew at a steady rate each day, starting on Day 5.

From Day 5 to Day 9, the bamboo plant grew about what percent of its final height?

Sample answer: about 80%

Bamboo Growth

Number of Days	Total Growth (m)
5	?
6	?
7	3
8	4
9	5



Case #4 Financial Literacy

Maysa and her brother Abdalla each open a bank account with an initial deposit of AED 50 each. Abdalla planned to save 30% of his earnings from his after-school job. The job pays AED 8 per hour, and he works 25 hours each week. For four weeks, Maysa saved AED 45 per week. After that, she saved an additional AED 30 per week.

During which week will they have the same amount of money in their bank accounts?

Week 8

Case #5 Fitness

Maysoun created her own fitness plan to increase the amount of time she spends exercising each week. The total number of minutes in her exercise plan for each week is shown.

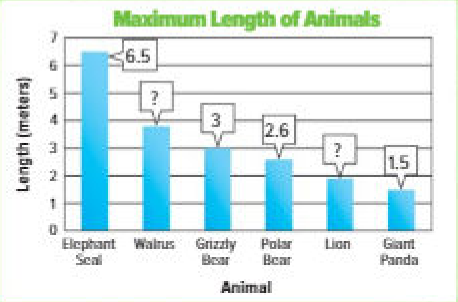
If she continues this pattern, for how many hours will she exercise in the 8th week?

$7\frac{7}{12}$ hours

Week	Exercise (min)
1	35
2	50
3	80
4	125
5	185

Case #6 Animals

The graph shows the maximum length of several animals. The maximum length of a walrus is twice the maximum length of a lion, which is 0.4 meter longer than the maximum length of a giant panda.



Find the maximum length of a walrus. 3.8 m

Use any strategy!

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** EVALUATE

2 Collaborate

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Exercises

	3, 4	5	6
Level 3			●
Level 2		●	
Level 1	●		

AL LA **Think-Pair-Share** Have students work in pairs. Give them one minute to think through their responses to Case 3. Then have them discuss their thoughts with a partner. Have them respond to the questions below. Upon completing Case 3, have them share their results with another pair. Repeat this process for Cases 4–6. MP 1, 3

Ask:

- *What do you need to determine?* the percent of growth from Day 5 to Day 9
- *What information do you need to solve the problem?* the final height on day 9 and the height on day 5

BL LA **Trade-a-Problem** Have students create their own real-world problem using the information in the table in Case 3. Students trade their problems, solve, and compare solutions. If the solutions do not agree, students work together to find any errors. MP 1, 4

Mid-Chapter Check

If students have trouble with Exercises 1–7, they may need help with the following concepts.

Concept	Exercise(s)
linear equations (Lesson 1)	1
graph relations (Lesson 2)	3
evaluate functions (Lesson 3)	4, 5, 6
graph functions (Lesson 4)	2, 7
domain and range (Lesson 2)	7

Vocabulary Activity

LA **Rally Coach** Have students work in pairs to complete Exercise 1. Have Student 1 speak aloud about what a linear equation means, while Student 2 listens, coaches, and encourages. If students are having trouble remembering the definition of a linear equation, have them discuss how the word line might help them. MP 1, 3

Alternate Strategies

AL LA Write four equations on the board, three of which are linear. Have students take turns coming to the board and circling the equations that are linear. When all equations are circled, have students define linear equation in their own words.

BL LA Give each student a number. Then have students write equations for which their number is a solution.

Mid-Chapter Check

Vocabulary Check

1. MP **Be Precise** Define *linear equation*. Give an example of a linear equation. (Lesson 1)

A linear equation is an equation with a graph that is a straight line.

Sample answer: $y = 8x$

2. Describe the difference between the graph of a set of discrete data and the graph of a set of continuous data. (Lesson 4) Sample answer: The graph of discrete data has space between the points while the graph of continuous data does not.

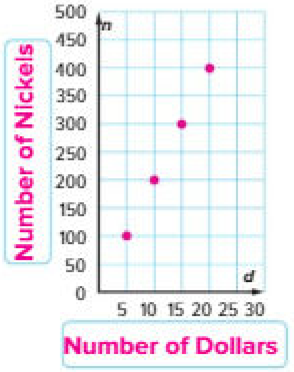
Skills Check and Problem Solving

3. There are 20 5-fils coins in one dirham. (Lesson 2)

a. Write an equation to find the number of 5-fils coins n in any number of dirhams d. $n = 20d$

b. Make a table to find the number of 5-fils coins in 5, 10, 15, or 20 dirhams. Then graph the ordered pairs.

d	$20d$	n
5	20(5)	100
10	20(5)	200
15	20(5)	300
20	20(5)	400



Find each function value. (Lesson 3)

4. $f(8)$ if $f(x) = 15x$

120

5. $f(2)$ if $f(x) = 2x - 5$

-1

6. $f(4)$ if $f(x) = -3x + 15$

3

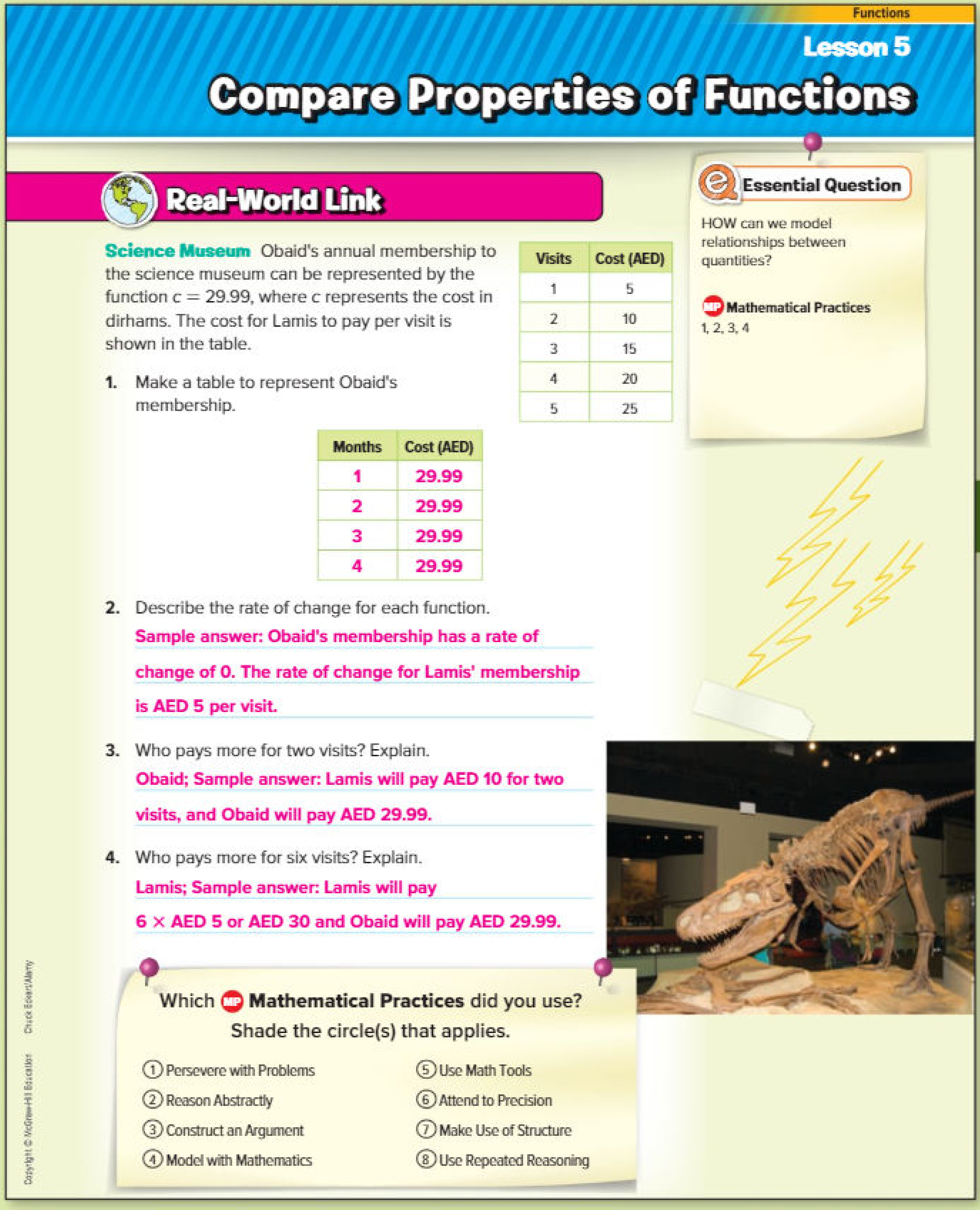
7. MP **Reason Inductively** A campground rents bicycles by the hour. The total cost y to rent a bicycle, including deposit, is presented by the function $y = \frac{1}{3}x + 12$. (Lessons 2 and 4)

a. Graph the function.

b. What do the domain and range of the function represent?

Sample answer: The domain is the hours that a bicycle is rented and the range is the total cost.

c. Is the function continuous or discrete? discrete



Functions

Lesson 5

Compare Properties of Functions

Real-World Link

Science Museum Obaid's annual membership to the science museum can be represented by the function $c = 29.99$, where c represents the cost in dirhams. The cost for Lamis to pay per visit is shown in the table.

Visits	Cost (AED)
1	5
2	10
3	15
4	20
5	25

1. Make a table to represent Obaid's membership.

Months	Cost (AED)
1	29.99
2	29.99
3	29.99
4	29.99

2. Describe the rate of change for each function.
Sample answer: Obaid's membership has a rate of change of 0. The rate of change for Lamis' membership is AED 5 per visit.

3. Who pays more for two visits? Explain.
Obaid; Sample answer: Lamis will pay AED 10 for two visits, and Obaid will pay AED 29.99.

4. Who pays more for six visits? Explain.
Lamis; Sample answer: Lamis will pay $6 \times$ AED 5 or AED 30 and Obaid will pay AED 29.99.

Essential Question

HOW can we model relationships between quantities?

Mathematical Practices
1, 2, 3, 4

Which **Mathematical Practices** did you use?
Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Compare properties of functions represented in different ways.

Coherence connecting within and across grades

Previous	Now	Next
Students represented relationships using multiple representations.	Students use different representations of two functions to compare the functions.	Students will use multiple representations to construct functions.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 315.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL LA **Trade-a-Problem** Have students create their own problem similar to the Real-World Link. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. MP 1, 4

Alternate Strategy

AL Explain that since there is no independent variable in the equation that represents Obaid's membership, the dependent value does not change. Ask students to give examples of other situations where the dependent value is a constant.

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Compare two functions.

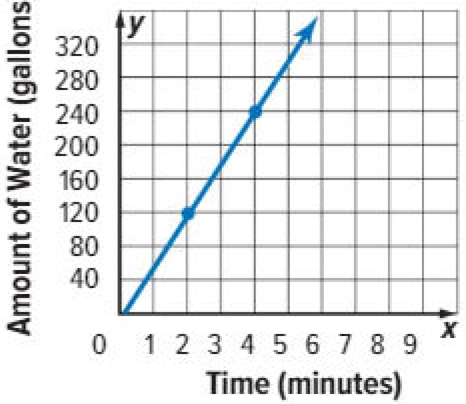
AL
- *What is the lion's rate of speed?* 16 meters per second
- *How is the speed of the zebra represented?* by a graph
- *Choose two points on the graph. What is the rate of change between the two points?* $\frac{18}{1}$
- *What is the zebra's speed?* 18 meters per second

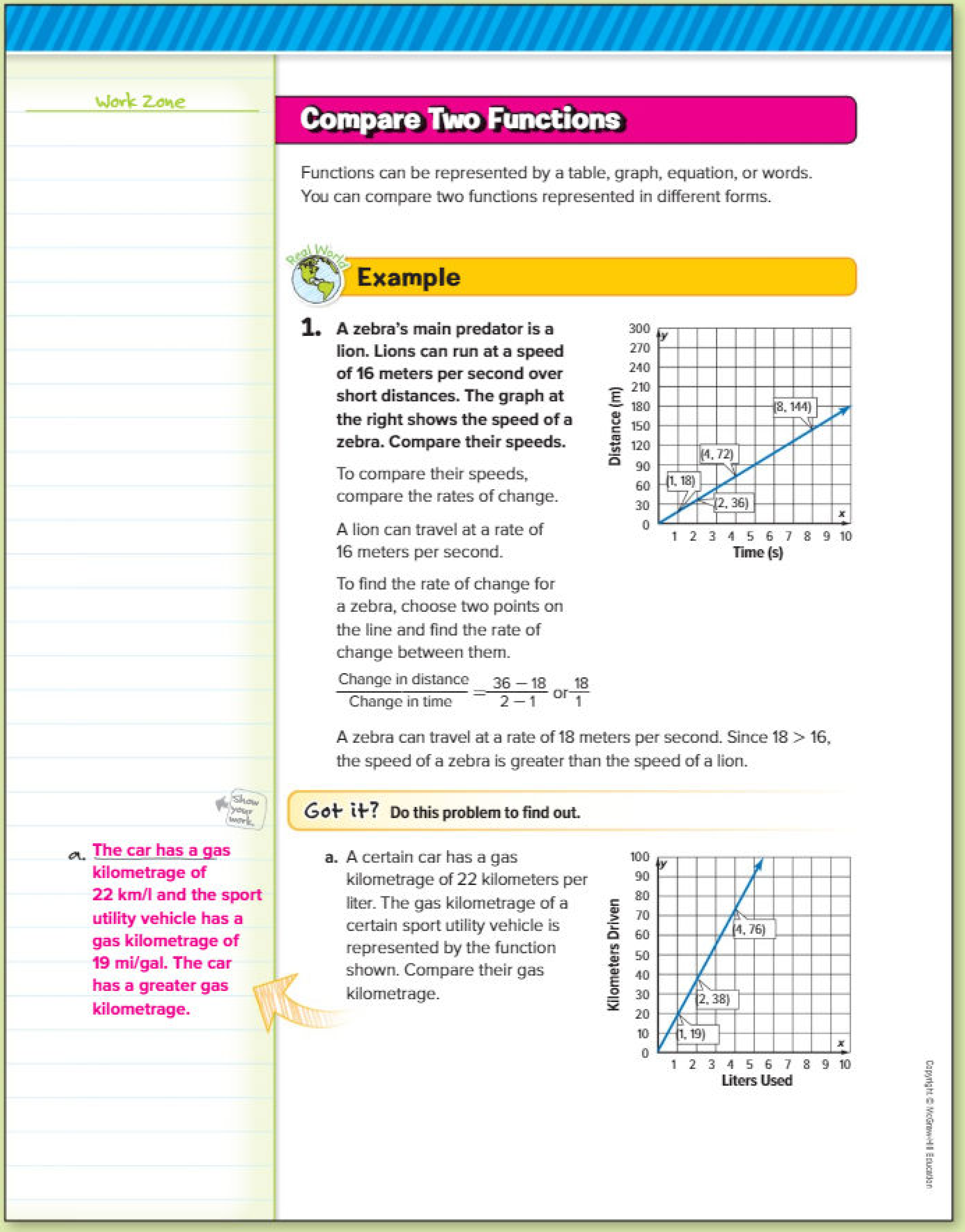
OL
- *In what forms are the functions in this Example given?* words and a graph
- *What is the rate of change of the lion's speed?* 16 meters per second
- *How would you find the rate of change of the zebra's speed?* Use two points on the line and find the rate of change between them.
- *What is the rate of change of the zebra's speed?* 18 meters per second

BL
- *How do the rates for the two animals compare? Interpret your answer in the context of the problem.* The rate of change for the zebra's speed is greater than the rate of change for the lion's speed. The zebra is faster than the lion.

Need Another Example?

The flow rate of water in a water garden is 52 liters per minute. The graph shows the flow rate of water in a koi pond. Compare the functions by comparing their rates of change. See Answer Appendix.





Work Zone

Compare Two Functions

Functions can be represented by a table, graph, equation, or words. You can compare two functions represented in different forms.

Real-World

Example

1. **A zebra's main predator is a lion. Lions can run at a speed of 16 meters per second over short distances. The graph at the right shows the speed of a zebra. Compare their speeds.**

To compare their speeds, compare the rates of change.

A lion can travel at a rate of 16 meters per second.

To find the rate of change for a zebra, choose two points on the line and find the rate of change between them.

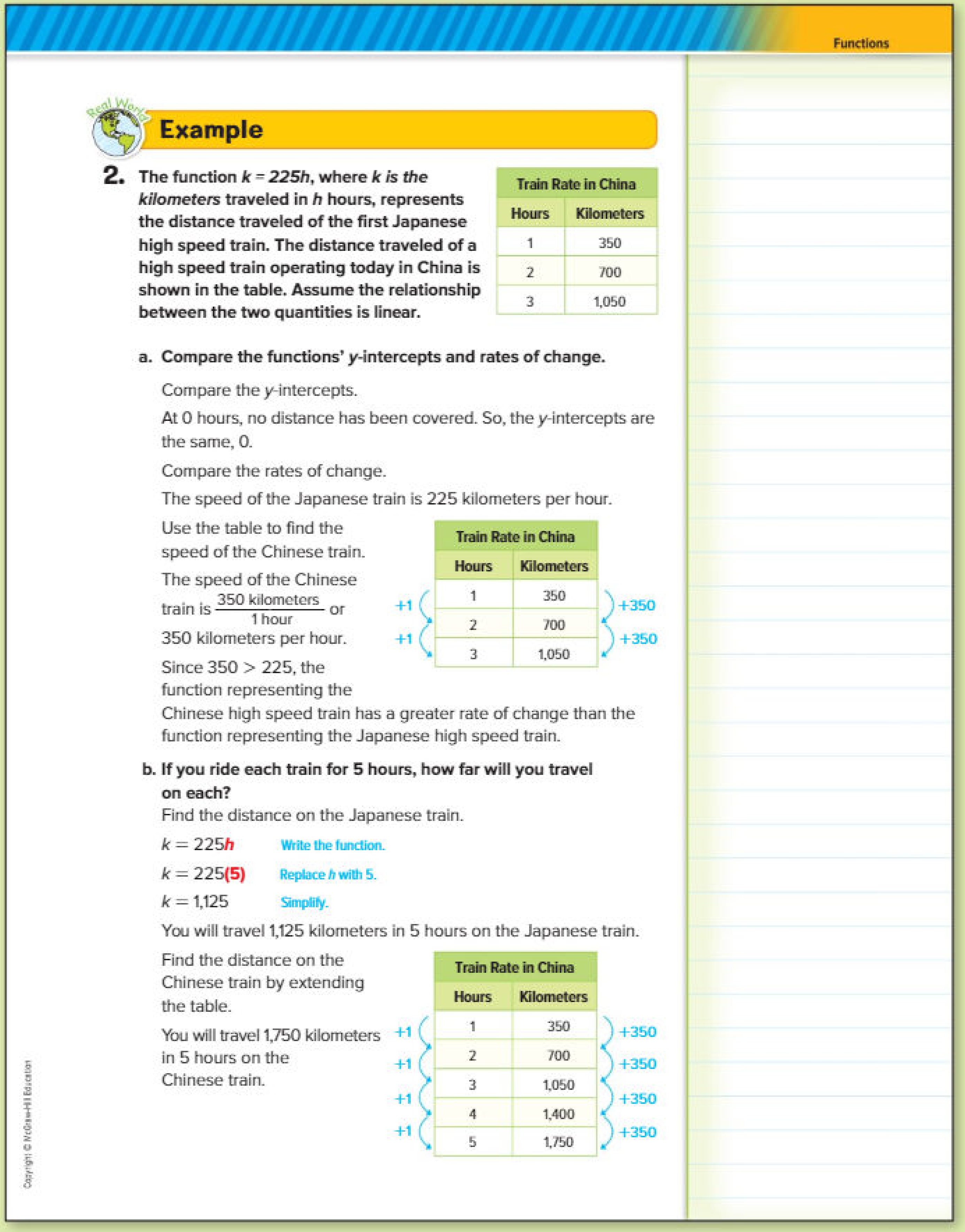
$$\frac{\text{Change in distance}}{\text{Change in time}} = \frac{36 - 18}{2 - 1} \text{ or } \frac{18}{1}$$

A zebra can travel at a rate of 18 meters per second. Since 18 > 16, the speed of a zebra is greater than the speed of a lion.

Got it? **Do this problem to find out.**

a. A certain car has a gas kilometrage of 22 kilometers per liter. The gas kilometrage of a certain sport utility vehicle is represented by the function shown. Compare their gas kilometrage.

a. The car has a gas kilometrage of 22 km/l and the sport utility vehicle has a gas kilometrage of 19 mi/gal. The car has a greater gas kilometrage.



Example

2. **The function $k = 225h$, where *k is the kilometers* traveled in *h* hours, represents the distance traveled of the first Japanese high speed train. The distance traveled of a high speed train operating today in China is shown in the table. Assume the relationship between the two quantities is linear.**

Train Rate in China	
Hours	**Kilometers**
1	350
2	700
3	1,050

a. Compare the functions' *y*-intercepts and rates of change.

Compare the *y*-intercepts.

At 0 hours, no distance has been covered. So, the *y*-intercepts are the same, 0.

Compare the rates of change.

The speed of the Japanese train is 225 kilometers per hour.

Use the table to find the speed of the Chinese train.

The speed of the Chinese train is $\frac{350 \text{ kilometers}}{1 \text{ hour}}$ or 350 kilometers per hour.

	Train Rate in China		
	Hours	**Kilometers**	
	1	350	
+1	2	700	+350
+1	3	1,050	+350

Since 350 > 225, the function representing the Chinese high speed train has a greater rate of change than the function representing the Japanese high speed train.

b. If you ride each train for 5 hours, how far will you travel on each?

Find the distance on the Japanese train.

$k = 225h$ Write the function.

$k = 225(5)$ Replace *h* with 5.

$k = 1{,}125$ Simplify.

You will travel 1,125 kilometers in 5 hours on the Japanese train.

Find the distance on the Chinese train by extending the table.

You will travel 1,750 kilometers in 5 hours on the Chinese train.

	Train Rate in China		
	Hours	**Kilometers**	
	1	350	
+1	2	700	+350
+1	3	1,050	+350
+1	4	1,400	+350
+1	5	1,750	+350

Example

2. Compare functions.

AL
- *In what forms are the functions in this Example given?* table and equation
- *What is a y-intercept?* where a function crosses the *y*-axis
- *If you ride the Japanese train for 5 hours, how far will you travel?* 1,125 km
- *If you ride the Chinese train for 5 hours, how far will you travel?* 1,750 km

OL
- *How could you find the y-intercepts of the functions?* Since the functions represent distances, there is no distance covered at 0 hours. You could also replace *h* with 0 in the equation and simplify. Either way, the *y*-intercept is 0 for both functions.
- *What is the rate of change for the Japanese train?* 225 kmph
- *What is the rate of change for the Chinese train?* 350 kmph

BL
- *What is the value of the independent variable in the y-intercept?* 0

Need Another Example?

A bowling alley offers different event packages. Package A is represented by the function $c = 7p + 5$, where *c* is the total cost and *p* is the number of people. Package B is represented in the table below.

Package B	
Number of People	**Total Cost (AED)**
1	9
2	18
3	27
4	36

a. Compare the functions by comparing their *y*-intercepts and rates of change. See Answer Appendix.

b. How much more will Package B cost than Package A if there are 12 people at the event? AED19 more

Example

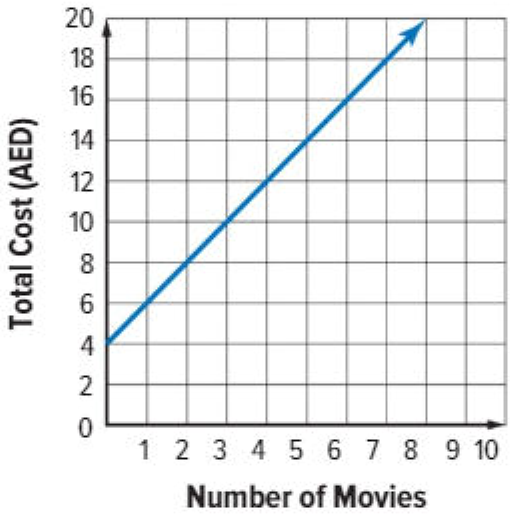
3. Compare functions.

AL
- *In what forms are the functions in this Example given?* equation and graph
- *What is the y-intercept for Nabila's bill?* 49
- *What is the y-intercept for Asma's bill?* 60
- *Compare what the two y-intercepts represent.* Asma has a greater initial cost in his plan.

OL
- *What is the rate of change for Nabila's bill?* 0.15
- *What does it represent?* the cost per minute
- *What is the rate of change for Asma's bill?* 0.10
- *Compare what the two rates of change represent.* Asma has a lower cost per minute than Nabila.

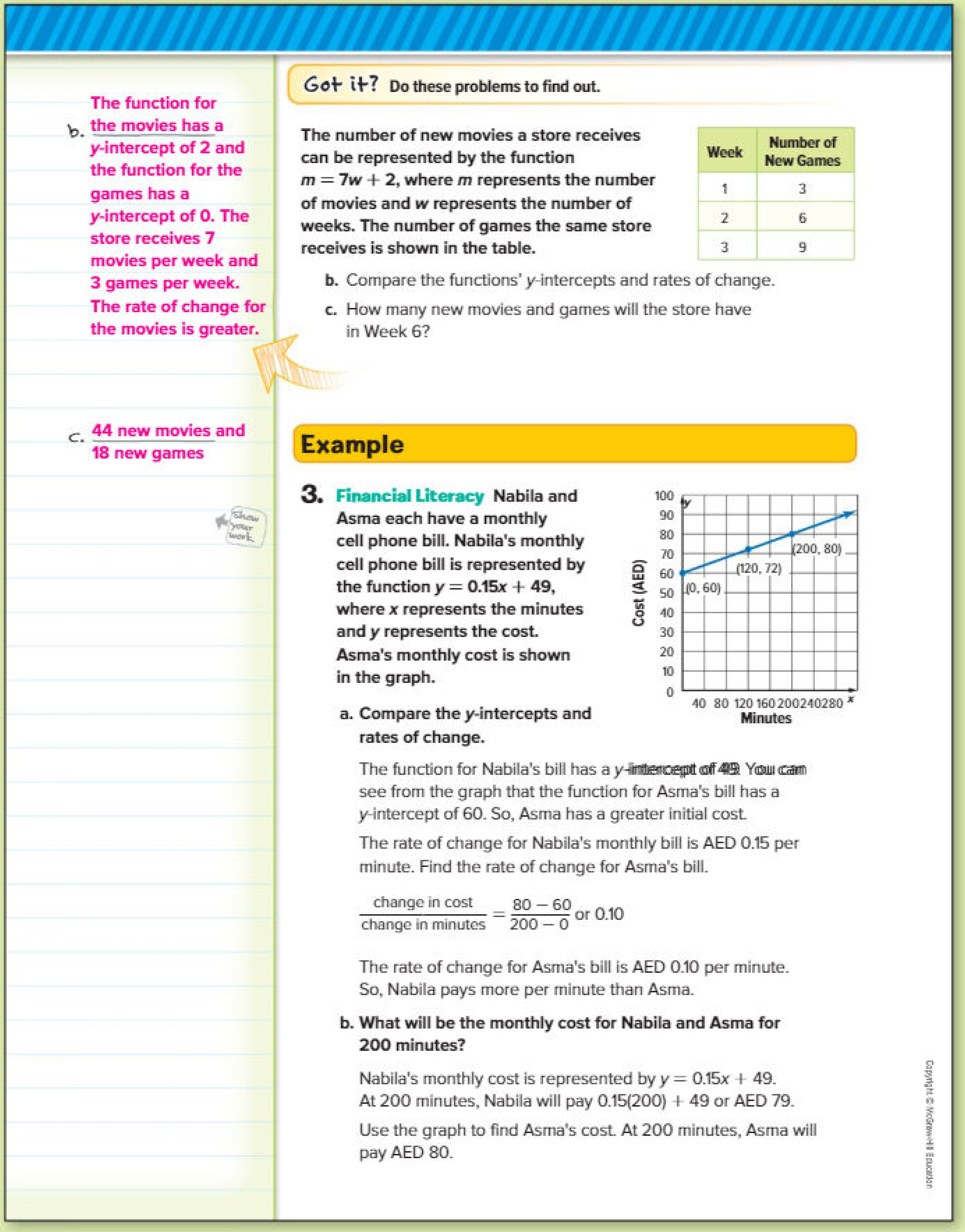
BL
- *After how many minutes would the plans cost the same?* 220

Need Another Example?
The total cost c to rent any number of movies m from an online movie rental company is represented by the function $c = 1.5m + 5$. The cost to rent movies from a different company is shown in the graph.



a. Compare the y-intercepts and rates of change. See Answer Appendix.

b. What will be the cost from each company if 15 movies are rented in one month? first company: AED27.50; second company: AED34



Got it? Do these problems to find out.

The number of new movies a store receives can be represented by the function $m = 7w + 2$, where m represents the number of movies and w represents the number of weeks. The number of games the same store receives is shown in the table.

Week	Number of New Games
1	3
2	6
3	9

b. Compare the functions' y-intercepts and rates of change.

c. How many new movies and games will the store have in Week 6?

b. The function for the movies has a y-intercept of 2 and the function for the games has a y-intercept of 0. The store receives 7 movies per week and 3 games per week. The rate of change for the movies is greater.

c. 44 new movies and 18 new games

Example

3. Financial Literacy Nabila and Asma each have a monthly cell phone bill. Nabila's monthly cell phone bill is represented by the function $y = 0.15x + 49$, where x represents the minutes and y represents the cost. Asma's monthly cost is shown in the graph.

a. Compare the y-intercepts and rates of change.

The function for Nabila's bill has a y-intercept of 49. You can see from the graph that the function for Asma's bill has a y-intercept of 60. So, Asma has a greater initial cost.

The rate of change for Nabila's monthly bill is AED 0.15 per minute. Find the rate of change for Asma's bill.

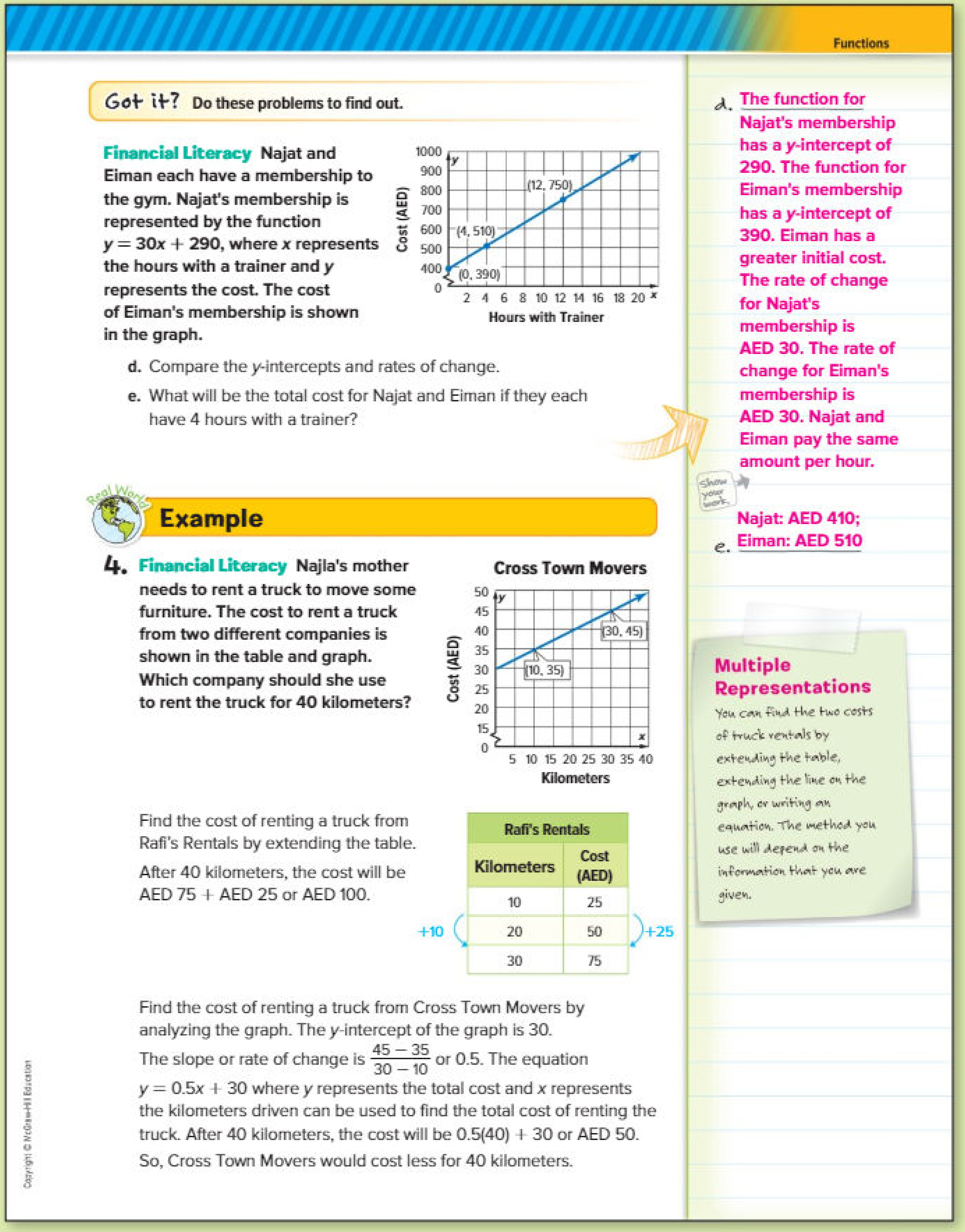
$$\frac{\text{change in cost}}{\text{change in minutes}} = \frac{80 - 60}{200 - 0} \text{ or } 0.10$$

The rate of change for Asma's bill is AED 0.10 per minute. So, Nabila pays more per minute than Asma.

b. What will be the monthly cost for Nabila and Asma for 200 minutes?

Nabila's monthly cost is represented by $y = 0.15x + 49$. At 200 minutes, Nabila will pay 0.15(200) + 49 or AED 79.

Use the graph to find Asma's cost. At 200 minutes, Asma will pay AED 80.



Got it? Do these problems to find out.

Financial Literacy Najat and Eiman each have a membership to the gym. Najat's membership is represented by the function $y = 30x + 290$, where x represents the hours with a trainer and y represents the cost. The cost of Eiman's membership is shown in the graph.

d. Compare the y-intercepts and rates of change.

e. What will be the total cost for Najat and Eiman if they each have 4 hours with a trainer?

d. The function for Najat's membership has a y-intercept of 290. The function for Eiman's membership has a y-intercept of 390. Eiman has a greater initial cost. The rate of change for Najat's membership is AED 30. The rate of change for Eiman's membership is AED 30. Najat and Eiman pay the same amount per hour.

Show your work.

e. Najat: AED 410; Eiman: AED 510

Example

4. Financial Literacy Najla's mother needs to rent a truck to move some furniture. The cost to rent a truck from two different companies is shown in the table and graph. Which company should she use to rent the truck for 40 kilometers?

Find the cost of renting a truck from Rafi's Rentals by extending the table.

After 40 kilometers, the cost will be AED 75 + AED 25 or AED 100.

Rafi's Rentals	
Kilometers	Cost (AED)
10	25
20	50
30	75

+10 +25

Find the cost of renting a truck from Cross Town Movers by analyzing the graph. The y-intercept of the graph is 30.

The slope or rate of change is $\frac{45 - 35}{30 - 10}$ or 0.5. The equation $y = 0.5x + 30$ where y represents the total cost and x represents the kilometers driven can be used to find the total cost of renting the truck. After 40 kilometers, the cost will be 0.5(40) + 30 or AED 50.

So, Cross Town Movers would cost less for 40 kilometers.

Multiple Representations

You can find the two costs of truck rentals by extending the table, extending the line on the graph, or writing an equation. The method you use will depend on the information that you are given.

Example

4. Compare functions.

AL
- *In what forms are the functions in this Example given?* table and graph
- *What are you being asked to find?* which company is less expensive for 40 kilometers
- *Which company costs less to rent for 10 kilometers?* Ron's Rentals

OL
- *How could you find the total cost to rent a truck for 40 kilometers from Ron's Rentals?* You can extend the pattern in the table or you could write and evaluate an equation.
- *How could you find the total cost to rent a truck for 40 kilometers from Cross Town Movers?* Extend the line on the graph to find the corresponding y value when the x value is 40.

BL
- *When is it cheaper to use Ron's Rentals?* when you rent the truck for less than 15 kilometers

Need Another Example?

The eighth grade class is selling pizzas and subs for a fundraiser. The amount of money they earn selling pizzas is shown in the table. The amount of money they earn selling sub sandwiches can be represented by the function $m = 4s$, where m is the total amount of money earned and s is the number of sub sandwiches sold. Which food will the students earn more money selling if they sell 100 of each item?

Selling Pizzas	
Number Sold	Total Earned (AED)
20	100
40	200
60	300
80	400

pizzas; The students will receive 4(100) or AED400 for selling 100 subs, but will earn AED500 for selling 100 pizzas.

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Numbered Heads Together** Assign students to 3-person teams. Each team completes Exercises 1–4 with a different team member leading each exercise and making sure each team member understands the problem before moving on. Call on a specific person from each team to present the team's solution to the class. MP 1, 3

BL LA **Pairs Discussion** Have students work in pairs to create a graphic organizer that answers Exercise 4. Have them extend the question by adding tables and words to the types of representations. Then have them display their graphic organizers around the room. MP 1, 2, 4

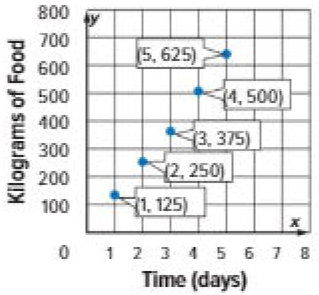
Guided Practice

Check

1. A tiger in captivity is fed 13.5 kilograms of food a day. The graph shows the kilograms of food an elephant in captivity eats per day. Compare the functions by comparing their rates of change. (Example 1)

Show your work.

A tiger is fed 13.5 kilograms per day. An elephant is fed 125 kilograms per day. Since 125 > 13.5, the function for the elephant has a greater rate of change.



2. Nisreen's profit at a craft fair is represented by the function $p = 5b - 15$, where p is the profit and b is the number of bracelets she sells. Nahla's profit is shown in the table. (Examples 2 and 3)

Bracelets Sold	Profit (AED)
1	5
2	10
3	15
4	20

a. Compare the y-intercepts and rates of change.

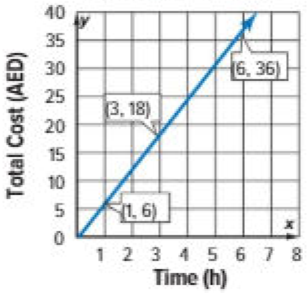
Nisreen makes AED 5 per bracelet, and Nahla makes AED 5 per bracelet. At 0 bracelets, Nisreen's profit is –AED 15 and Nahla's profit is AED 0.

b. How much will each girl make if she sells 30 bracelets?

Nisreen: AED 135; Nahla: AED 150

3. The cost to rent a raft from two different companies is shown. Which company should you use if you rent the raft for 9 hours? (Example 4) **Original Rafts; The cost to rent from Original Rafts is AED 33 and the cost to rent from Raft Center Rentals is AED 54.**

Raft Center Rafts

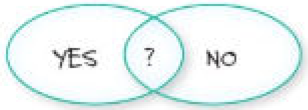


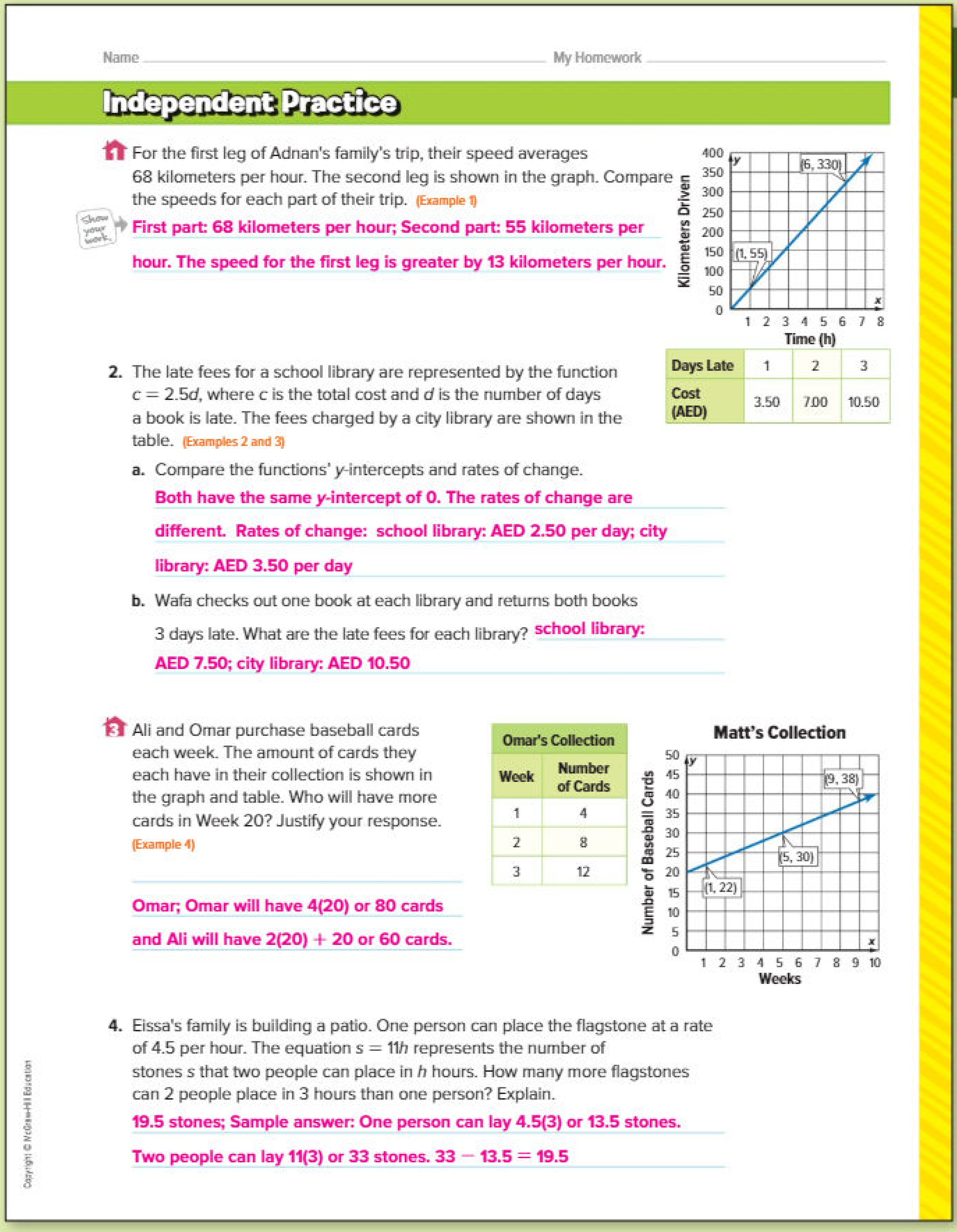
Original Rafts	
Time (h)	Total Cost (AED)
1	15.00
2	17.25
3	19.50
4	21.75
5	24.00

4. **Building on the Essential Question** What are the advantages and disadvantages to representing a function as an equation instead of a graph? **Sample answer: It is easy to see the rate of change for a function shown as an equation. In a graph, to find the rate of change, you have to determine the slope. A function shown as a graph displays the ordered pairs so you can easily see the relationship, but the relationship is not as easily shown in an equation.**

Rate Yourself!

Are you ready to move on?
Shade the section that applies.





Name ______________________ My Homework ______________________

Independent Practice

1. For the first leg of Adnan's family's trip, their speed averages 68 kilometers per hour. The second leg is shown in the graph. Compare the speeds for each part of their trip. (Example 1)

Show your work.

First part: 68 kilometers per hour; Second part: 55 kilometers per hour. The speed for the first leg is greater by 13 kilometers per hour.

2. The late fees for a school library are represented by the function $c = 2.5d$, where c is the total cost and d is the number of days a book is late. The fees charged by a city library are shown in the table. (Examples 2 and 3)

Days Late	1	2	3
Cost (AED)	3.50	7.00	10.50

a. Compare the functions' y-intercepts and rates of change.

Both have the same y-intercept of 0. The rates of change are different. Rates of change: school library: AED 2.50 per day; city library: AED 3.50 per day

b. Wafa checks out one book at each library and returns both books 3 days late. What are the late fees for each library? **school library: AED 7.50; city library: AED 10.50**

3. Ali and Omar purchase baseball cards each week. The amount of cards they each have in their collection is shown in the graph and table. Who will have more cards in Week 20? Justify your response. (Example 4)

Omar's Collection	
Week	Number of Cards
1	4
2	8
3	12

Omar; Omar will have 4(20) or 80 cards and Ali will have 2(20) + 20 or 60 cards.

4. Eissa's family is building a patio. One person can place the flagstone at a rate of 4.5 per hour. The equation $s = 11h$ represents the number of stones s that two people can place in h hours. How many more flagstones can 2 people place in 3 hours than one person? Explain.

19.5 stones; Sample answer: One person can lay 4.5(3) or 13.5 stones. Two people can lay 11(3) or 33 stones. 33 − 13.5 = 19.5

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–3, 9–11	4, 5, 12–15	6–8
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–3, 5, 6, 8, 14, 15
OL	On Level	1, 3, 5, 6, 8, 14, 15
BL	Beyond Level	4–8, 14, 15

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	7
2 Reason abstractly and quantitatively.	5
3 Construct viable arguments and critique the reasoning of others.	8, 11
4 Model with mathematics.	6

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET
Out the Door

Have students create a table of data that has a constant rate of change. Then write a linear function on the board. Have students compare their functions with the function on the board by comparing the rates of change. See students' work.

5. **Reason Abstractly** Refer to the conversions in the tables below.

Cups	Ounces
1	8
2	16
3	24
4	32

Pints	Ounces
1	16
2	32
3	48
4	64

Quarts	Ounces
1	32
2	64
3	96
4	128

a. Write a function for each table.
$z = 8c$; $z = 16p$; $z = 32q$

b. If you graph the points, the graph for which function would have the steepest slope? Justify your response.
the quart equation; Sample answer: The greater the rate of change, the steeper the slope of the graph.

c. Which function has the least rate of change? Explain.
The first function has the least rate of change because 8 is less than 16 and 32.

H.O.T. Problems Higher Order Thinking

6. **Model with Mathematics** Write a real-world problem where you would want to compare rates of change for two different functions.
See students' work.

7. **Persevere with Problems** Explain why the graph of the function $y = 3x + 40$ will never intersect the graph of the function $y = 3x + 35$.
Sample answer: Both functions have the same rate of change but because they have different y-intercepts, they are parallel lines and parallel lines will never intersect.

8. **Reason Inductively** The exchange rate to convert U.S. dollars to UAE dirhams is represented by the function $e = 3.67d$ where e is the amount in dirhams and d is the number of dollars. One U.S. dollar can also be exchanged for 0.94 European euro. If you exchange \$250 for dirhams and \$250 for euros,

I

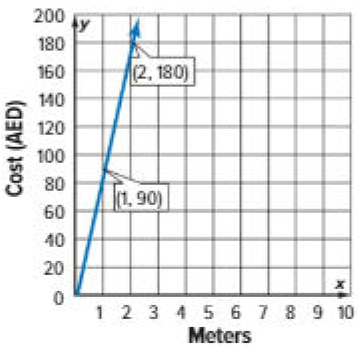
which of the following is true?

I You will receive AED 917.50 pounds and 235 euros.
II You will receive about AED 410 and about 362 euros.
III You will receive 250 dollars.
IV You will receive the same amount of dirhams and euros.

Name ______________________ My Homework ______________________

Extra Practice

9. A fabric store sells cotton for AED 20.00 a meter. The price of special occasion fabric is shown in the graph. Compare the functions' rates of change.



Homework Help

Cotton fabric: AED 20.00 per meter

Special occasion fabric: $\frac{180-90}{2-1} = \frac{90}{1}$ or AED 90.00 per meter.

The special occasion fabric has the greater rate of change.

10. Two players played a game. The first player's score is represented by the function $p = 5c - 3$, where p is the number of points scored and c is the number of correct answers. The second player's score is shown in the table.

Questions Answered	Score
1	5
2	10
3	15
4	20

a. Compare the functions by comparing their *y*-intercepts and rates of change. **Player 1: 5 points per question; Player 2: 5 points per question. Both have the same rate of change, but the function for Player 1 has a *y*-intercept of −3 and the function for Player 2 has a *y*-intercept of 0.**

b. How many points will the first player have if he or she correctly answers 30 questions? **147 points**

11. **Justify Conclusions** Usama and Faris each open savings accounts. The amounts in Usama's account are shown in the table. Faris saves AED 50 per week. Who will have more saved in 8 weeks? Explain.

Faris; Sample answer: In 8 weeks Faris will have 50(8) or AED 400. Usama will have saved AED 370.

Usama's Savings	
Week	Amount Saved (AED)
1	160
2	190
3	220
4	250
5	280

12. Canada Olympic Park features sports training and entertainment facilities. The Monster zip line produces average speeds of 120 kilometers per hour. A smaller line produces speeds represented by the function $d = 50h$ where d is the distance in kilometers after h hours. How much farther could you travel on the Monster zip line in 0.25 hours?

17.5 km

13. Faleh gets a 1.5 kilometers head start and runs at a rate of 4.5 kilometers per hour. Fahd's progress is represented by a graph that goes through the points (1, 10), (2, 20), and (3, 30). How long will Fahd need to run to catch up with Faleh? $\frac{3}{11}$ **h**

Power Up! Test Practice

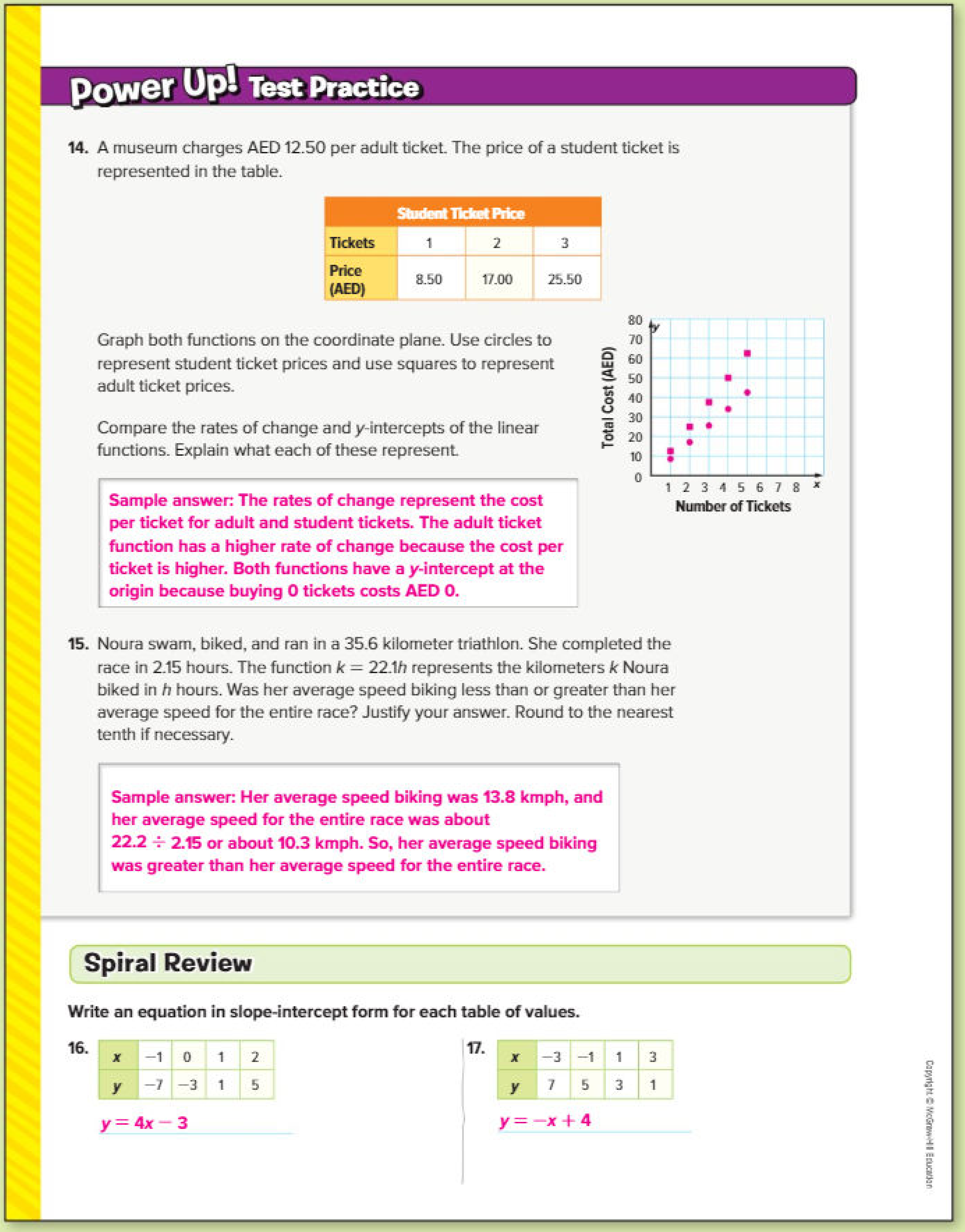
Exercises 14 and 15 prepare students for more rigorous thinking needed for assessment.

14.

This test item requires students to support their previous reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Kowledge	DOK3
Mathematical Practice	MP1, MP3, MP4
Scoring Rubric	
2 points	Students correctly graph both functions, compare the rates of change and *y*-intercepts, and explain what they represent.
1 point	Students correctly graph both functions but fail to correctly answer the question OR students correctly answer the question but fail to correctly graph either function.

15.

This test item requires students to support their previous reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Knowledge	DOK3
Mathematical Practice	MP1, MP3
Scoring Rubric	
2 points	Students correctly answer the question and justify their response.
1 point	Students correctly answer the question but fail to justify their response.



Power Up! Test Practice

14. A museum charges AED 12.50 per adult ticket. The price of a student ticket is represented in the table.

Student Ticket Price			
Tickets	1	2	3
Price (AED)	8.50	17.00	25.50

Graph both functions on the coordinate plane. Use circles to represent student ticket prices and use squares to represent adult ticket prices.

Compare the rates of change and *y*-intercepts of the linear functions. Explain what each of these represent.

Sample answer: The rates of change represent the cost per ticket for adult and student tickets. The adult ticket function has a higher rate of change because the cost per ticket is higher. Both functions have a *y*-intercept at the origin because buying 0 tickets costs AED 0.

15. Noura swam, biked, and ran in a 35.6 kilometer triathlon. She completed the race in 2.15 hours. The function $k = 22.1h$ represents the kilometers k Noura biked in h hours. Was her average speed biking less than or greater than her average speed for the entire race? Justify your answer. Round to the nearest tenth if necessary.

Sample answer: Her average speed biking was 13.8 kmph, and her average speed for the entire race was about 22.2 ÷ 2.15 or about 10.3 kmph. So, her average speed biking was greater than her average speed for the entire race.

Spiral Review

Write an equation in slope-intercept form for each table of values.

16.

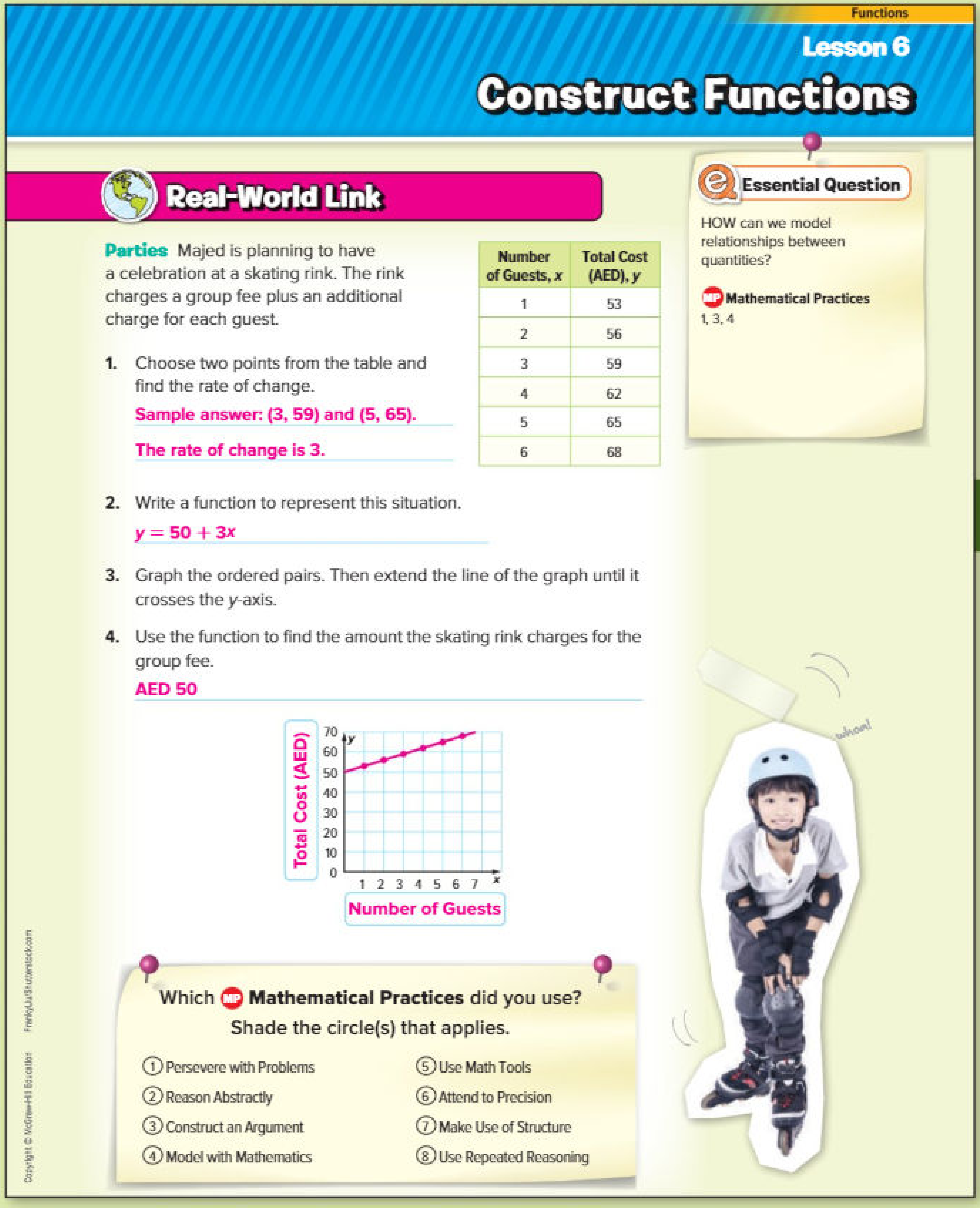
x	−1	0	1	2
y	−7	−3	1	5

$y = 4x - 3$

17.

x	−3	−1	1	3
y	7	5	3	1

$y = -x + 4$



Lesson 6

Construct Functions

Real-World Link

Parties Majed is planning to have a celebration at a skating rink. The rink charges a group fee plus an additional charge for each guest.

Number of Guests, *x*	Total Cost (AED), *y*
1	53
2	56
3	59
4	62
5	65
6	68

1. Choose two points from the table and find the rate of change.

Sample answer: (3, 59) and (5, 65).

The rate of change is 3.

2. Write a function to represent this situation.

$y = 50 + 3x$

3. Graph the ordered pairs. Then extend the line of the graph until it crosses the *y*-axis.

4. Use the function to find the amount the skating rink charges for the group fee.

AED 50

Essential Question

HOW can we model relationships between quantities?

Mathematical Practices
1, 3, 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

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Focus narrowing the scope

Objective Find and interpret the rate of change and initial value of a function.

Coherence connecting within and across grades

Previous	Now	Next
Students used different representations of two functions to compare the functions.	Students use multiple representations to construct functions.	Students will use multiple representations to identify linear and nonlinear functions.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 323.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Pairs Discussion** Have students work in pairs to complete Exercises 1–4. Ask them how they can determine the *y*-intercept from the table and the graph. Then ask them to interpret the *y*-intercept in the context of the real-world problem. MP 1

Alternate Strategy

AL Have students use the rate of change to count backward in the table to find the value of *y* when $x = 0$. Have them use the graph in Exercise 3 to verify that their function is correct.

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

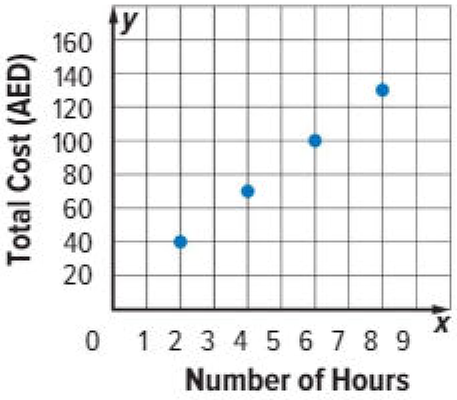
1. Find and interpret the rate of change and the initial value.

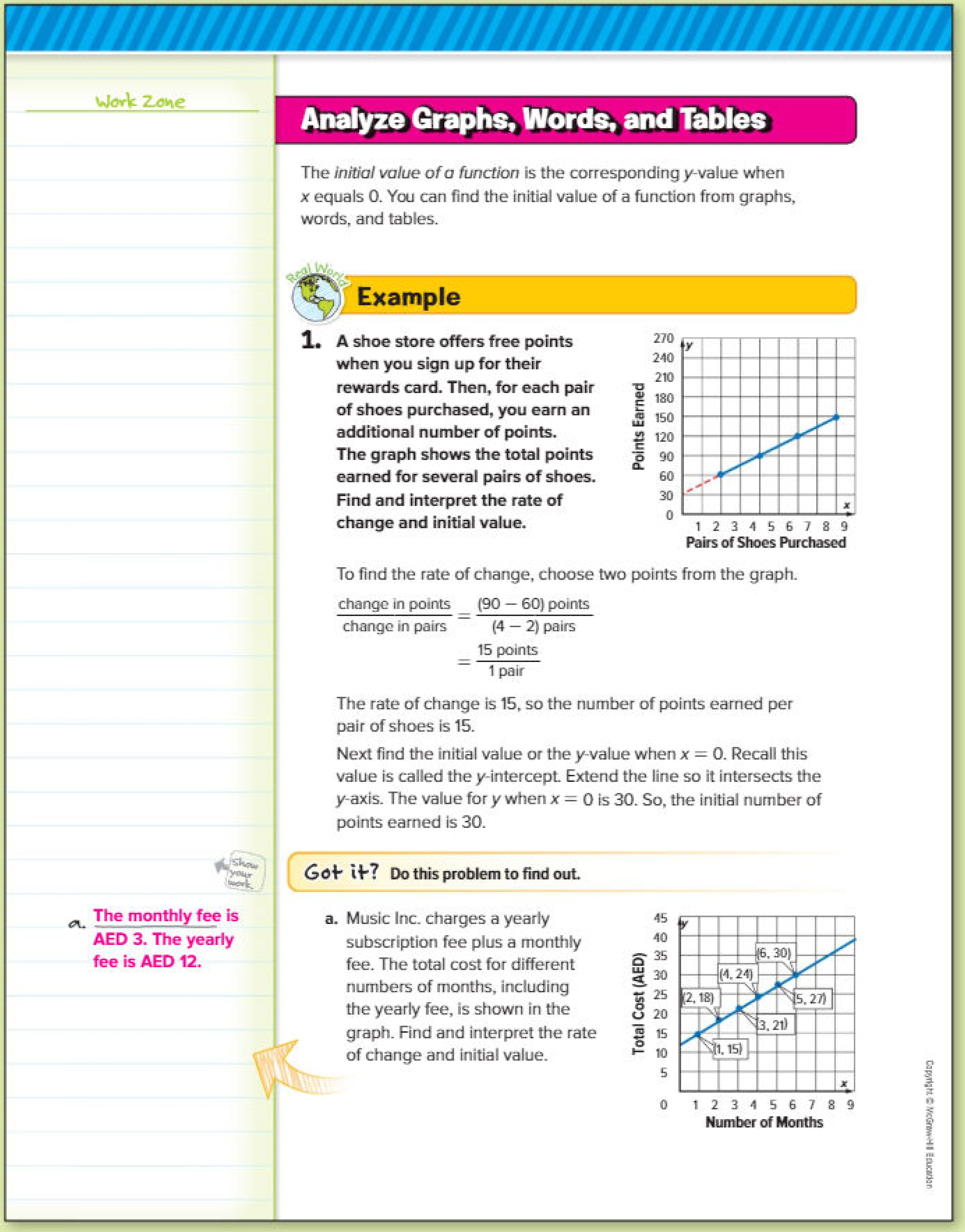
AL
- *How can you find the rate of change?* Determine the slope of the line by finding $\frac{\text{rise}}{\text{run}}$.
- *What two points can you choose?* Sample answer: (2, 60) and (4, 90)

OL
- *What is the rate of change? What does it mean in the context of the problem?* 15; The amount of points earned per pair of shoes is 15.
- *How can you determine the y-intercept? What is the y-intercept?* Extend the line so that it crosses the y-axis. The y-intercept is the y-coordinate of the point where the line intersects the y-axis. The y-intercept is 30.
- *What does the y-intercept mean in the context of the problem?* The initial number of points earned is 30.

BL
- *How is the slope different than the y-intercept?* The slope is the rate of how the y-values change as the x-values change. The y-intercept is the initial value.

Need Another Example?
A pottery studio charges a certain amount per hour, plus a firing fee to fire the pottery. The graph shows the total cost of using the studio for different amounts of time. Find and interpret the rate of change and initial value. The firing fee is AED10. The hourly fee is AED15.





Work Zone

Analyze Graphs, Words, and Tables

The *initial value of a function* is the corresponding y-value when x equals 0. You can find the initial value of a function from graphs, words, and tables.

Example

1. A shoe store offers free points when you sign up for their rewards card. Then, for each pair of shoes purchased, you earn an additional number of points. The graph shows the total points earned for several pairs of shoes. Find and interpret the rate of change and initial value.

To find the rate of change, choose two points from the graph.

$$\frac{\text{change in points}}{\text{change in pairs}} = \frac{(90-60)\text{ points}}{(4-2)\text{ pairs}}$$
$$= \frac{15\text{ points}}{1\text{ pair}}$$

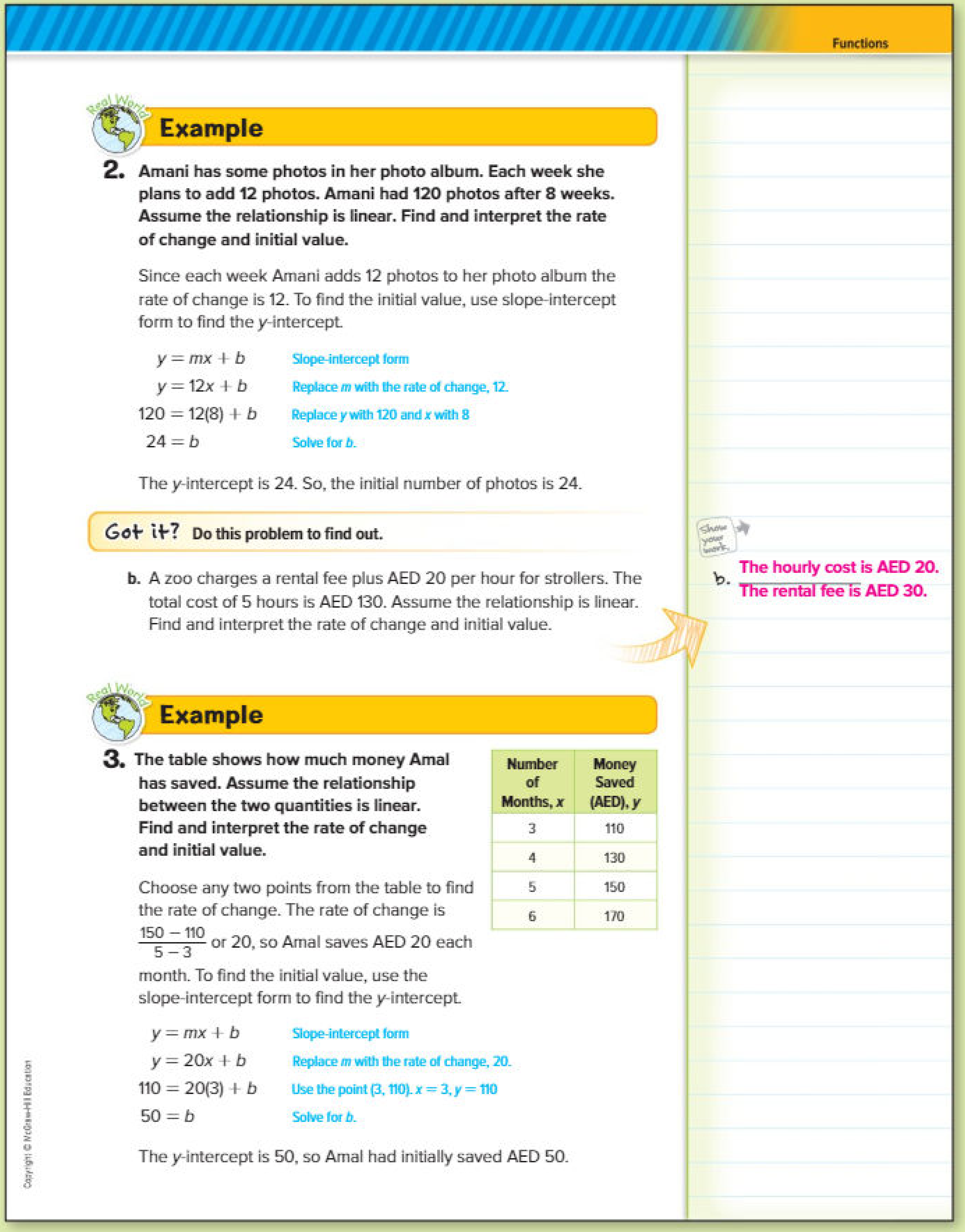
The rate of change is 15, so the number of points earned per pair of shoes is 15.

Next find the initial value or the y-value when $x = 0$. Recall this value is called the y-intercept. Extend the line so it intersects the y-axis. The value for y when $x = 0$ is 30. So, the initial number of points earned is 30.

Got it? **Do this problem to find out.**

a. Music Inc. charges a yearly subscription fee plus a monthly fee. The total cost for different numbers of months, including the yearly fee, is shown in the graph. Find and interpret the rate of change and initial value.

a. The monthly fee is AED 3. The yearly fee is AED 12.



Example

2. Amani has some photos in her photo album. Each week she plans to add 12 photos. Amani had 120 photos after 8 weeks. Assume the relationship is linear. Find and interpret the rate of change and initial value.

Since each week Amani adds 12 photos to her photo album the rate of change is 12. To find the initial value, use slope-intercept form to find the *y*-intercept.

$y = mx + b$ Slope-intercept form

$y = 12x + b$ Replace *m* with the rate of change, 12.

$120 = 12(8) + b$ Replace *y* with 120 and *x* with 8

$24 = b$ Solve for *b*.

The *y*-intercept is 24. So, the initial number of photos is 24.

Got it? **Do this problem to find out.**

b. A zoo charges a rental fee plus AED 20 per hour for strollers. The total cost of 5 hours is AED 130. Assume the relationship is linear. Find and interpret the rate of change and initial value.

Show your work.

b. The hourly cost is AED 20. The rental fee is AED 30.

Example

3. The table shows how much money Amal has saved. Assume the relationship between the two quantities is linear. Find and interpret the rate of change and initial value.

Number of Months, *x*	Money Saved (AED), *y*
3	110
4	130
5	150
6	170

Choose any two points from the table to find the rate of change. The rate of change is $\frac{150 - 110}{5 - 3}$ or 20, so Amal saves AED 20 each month. To find the initial value, use the slope-intercept form to find the *y*-intercept.

$y = mx + b$ Slope-intercept form

$y = 20x + b$ Replace *m* with the rate of change, 20.

$110 = 20(3) + b$ Use the point (3, 110). $x = 3$, $y = 110$

$50 = b$ Solve for *b*.

The *y*-intercept is 50, so Amal had initially saved AED 50.

Examples

2. Find and interpret the rate of change and the initial value.

AL
- *How many photos were added each week?* 12
- *How many photos were in the album after 8 weeks?* 120

OL
- *How can you determine the initial number of photos in the album?* You know that when the number of weeks *x* is 8, there are 120 photos *y*. You can use these values in the slope-intercept form equation to determine the initial value.

BL
- *What is the equation of the line in slope-intercept form?* $y = 12x + 24$

Need Another Example?

Each week, Hiyam works the same number of hours. Her first week included an orientation. The total number of hours that she worked the 2nd, 3rd, and 4th weeks was 20 hours, 28 hours, and 36 hours. Assume the relationship is linear. Find and interpret the rate of change and initial value. She works 8 hours each week. The orientation was 4 hours long.

3. Find and interpret the rate of change and the initial value.

AL
- *What two values do we need to determine?* slope and *y*-intercept

OL
- *How can you use the information in the table to find the initial amount?* You know one point on the line. You can find the slope using two points and then use the slope-intercept form equation to find the initial value.

BL
- *If the equation was $y = 20x + 45$, how much would Ava had initially saved?* AED45

Need Another Example?

The table shows the weight of a kitten in weeks 4, 5, 6, and 7. Assume the relationship between the two quantities is linear. Find and interpret the rate of change and initial value. The kitten gains 3 kilograms per week. At birth, the kitten weighed 5 kilograms.

Number of Weeks, *x*	4	5	6	7
Weight (kg), *y*	17	20	23	26

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL **LA** **Think-Pair-Share** Have students work in pairs. Give them one minute to think through their response to Exercise 1. Then have them share their responses with their partner and discuss any differences in their solutions. Have them repeat this process for Exercises 2–4. Call on one pair of students to share their responses with the class. MP 1, 3

BL **LA** **Trade-a-Problem** Have students create a real-world scenario similar to Exercises 2–3. Have them make a table of values or create a graph on the coordinate plane. Then have students trade their problem and write an equation for each other's graph or table. Discuss and resolve any differences in their solutions. MP 1, 2, 3, 4

Watch Out!

Common Error Students may use the first value given in the graph or table as the initial value. Remind them to extend the graph to an x-value of 0 to find the initial value.

Show your work.

c. Each text costs AED 0.10. The initial cost of the phone plan is AED 10.

Got it? **Do this problem to find out.**

c. The table shows the monthly cost of sending text messages. Assume the relationship between the two quantities is linear. Find and interpret the rate of change and initial value.

Number of Messages, x	Cost (AED), y
5	10.50
6	10.60
7	10.70

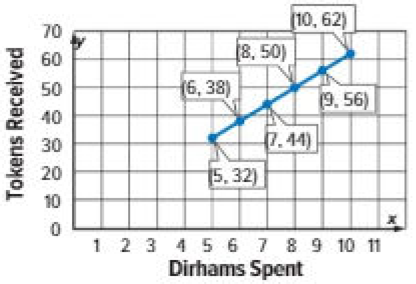
Guided Practice

Check ✓

1. As part of a grand opening, a funfair gave out free tokens to the first 100 customers. The graph shows the number of tokens customers received for each dirham spent at the funfair. Find and interpret rate of change and the initial value. (Example 1)

Each dirham buys 6 tokens. The initial number of tokens given out is 2.

Show your work.



2. A historic museum charges a rental fee plus AED 20 per hour for an audio tour guide. The total cost for 4 hours is AED 120. Find and interpret the rate of change and initial value. (Example 2)

The hourly cost is AED 20. The rental fee is AED 40.

3. A science center charges an initial membership fee. The total cost of the membership depends on the number of people on the membership as shown in the table. Assume the relationship between the two quantities is linear. Find and interpret the rate of change and the initial value. (Example 3) Each person pays an additional AED 15. The initial fee is AED 35.

Number of People, x	2	3	4	5
Additional Cost (AED), y	65	80	95	110

4. **Building on the Essential Question** How is the initial value of a function represented in a table and in a graph?

Sample answer: In a table, the initial value of a function is the corresponding y-value when $x = 0$. In a graph, the initial value is the y-intercept.

Rate Yourself!

☐ I understand how to construct functions.

Great! You're ready to move on!

☐ I still have questions about constructing functions.

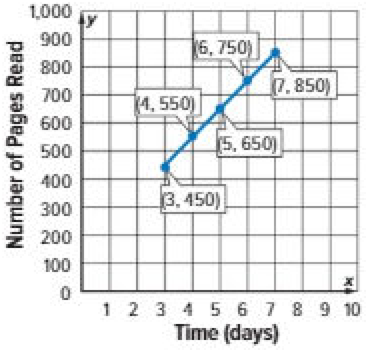
Name ______________________ My Homework ______________________

Independent Practice

1. A teacher read part of a book to a class. The graph shows the number of pages read by the teacher over the next several days. Find and interpret the rate of change and the initial value. (Example 1)

The teacher read 100 pages per day. The teacher initially read 150 pages before the graph begins.

Show your work



2. A water park charges a rental fee plus AED 15 per hour to rent inflatable rafts. The total cost to rent a raft for 6 hours is AED 150. Assume the relationship is linear. Find and interpret the rate of change and the initial value. (Example 2) **The hourly cost is AED 15. The rental fee is AED 60.**

3. A teacher already had a certain number of canned goods for the food drive. Each day of the food drive, the class plans to bring in 10 cans. The total number of canned goods for day 10 is 205. Assume the relationship is linear. Find and interpret the rate of change and the initial value. (Example 2)

The class brings in 10 cans per day. The teacher initially had 105 cans.

4. Hidaya frosted some cupcakes in the morning for a celebration. The table shows the total number of cupcakes frosted after she starts up after lunch. Assume the relationship between the two quantities is linear. Find and interpret the rate of change and the initial value. (Example 3)

Time (min), x	5	10	15	20
Number of Cupcakes, y	28	32	36	40

She can frost 0.8 cupcake per minute. She frosted 24 cupcakes in the morning.

5. Ayman has a certain number of DVDs in his collection. The table shows the total number of DVDs in his collection over several months. Assume the relationship between the two quantities is linear. Find and interpret the rate of change and the initial value. (Example 3)

Month, x	3	6	9	12
Number of DVDs, y	18	27	36	45

Each month Ayman adds 3 DVDs. He started with 9 DVDs.

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–5, 10, 11	6, 12–14	7–9
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 8, 9, 13, 14
OL	On Level	1–5 odd, 6, 8, 9, 13, 14
BL	Beyond Level	6–9, 13, 14

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	7
3 Construct viable arguments and critique the reasoning of others.	6, 9, 12
4 Model with mathematics.	8

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students describe a function that has an initial value of 30 by making a table or drawing a graph. Tell them to include the rate of change of their function. See students' work.

6. **Multiple Representations** Badr's family is driving from Boston to Chicago. The total distance of the trip is 1,600 kilometers and each hour they will drive 104 kilometers.

   a. **Algebra** Write an equation to represent the number of remaining kilometers y after driving any number of hours x.

   $y = 1{,}600 - 104x$

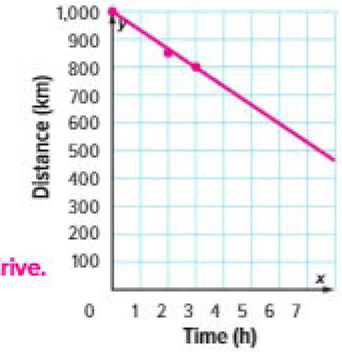
   b. **Graphs** Graph the equation from part **a** on a coordinate plane.

   c. **Numbers** What is the rate of change and y-intercept of the line? −104 kmph;1,600

   d. **Words** Explain why the line *slopes down* by 104 for each hour. Sample answer: The line slopes down by 104 because each hour they travel they have 104 less kilometers to drive.

   e. **Words** Why does the line cross the y-axis at 1,600?

   They began the trip with 1,600 kilometers to drive.



H.O.T. Problems Higher Order Thinking

7. **Persevere with Problems** Explain why a horizontal line has a rate of change of zero. Sample answer: The rate of change is represented by the ratio $\frac{\text{change in } y}{\text{change in } x}$. For a horizontal line, x can increase or decrease, but y does not change. The numerator will be 0, so the rate of change is 0.

8. **Model with Mathematics** Write and solve a real-world problem in which you need to find the initial value of a function. Then explain to a classmate how you solved your problem.

   See students' work.

9. **Justify Conclusions** Your teacher asks you to write a linear equation for a function in which the rate of change is −7 and the initial value is −2. You wrote the equation $y = -7x + (-2)$. Your classmate wrote the equation $y = -7x - 2$. Another classmate wrote the equation $y = (-2) + (-7)x$. Your teacher wrote the equation $y = -2 - 7x$. Who is correct? Justify your response.

   They are all correct; Sample answer: The properties of operations show that these four equations are equivalent.

Name ______________________ My Homework ______________________

Extra Practice

10. A snowboard instructor charges an initial fee plus AED 40 per hour for private snowboarding lessons. Amna paid AED 265 for six hours of instruction. Assume the relationship is linear. Find and interpret the rate of change and initial value.

The instructor charges AED 40 per hour. The initial fee is AED 25.

Since the instructor charges AED 40 per hour, the rate of change is 40. To find the initial value, use slope-intercept form to find the y-intercept.

Homework Help

$y = mx + b$

$y = 40x + b$

$265 = 40(6) + b$

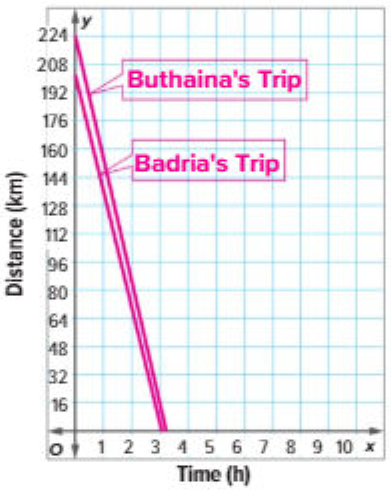
$25 = b$

The y-intercept is 25. So, the initial fee is AED 25.

11. A family drove to their grandmother's house. After that they averaged 200 kilometers per day for 8 days. They drove a total of 1,880 kilometers over the eight days. Assume the relationship is linear. Find and interpret the rate of change and initial value.

The family drove 200 kilometers per day. They drove 280 kilometers to their grandmother's house.

12. MP **Multiple Representations** Buthaina and Badria are traveling on the same highway to a family reunion at a park. Buthaina starts out 225 kilometers from the park and drives 70 kilometers per hour. Badria starts out 200 kilometers from the park and drives 65 kilometers per hour.



a. **Algebra** Write an equation for Buthaina's trip where *y* is the total distance from the park after *x* hours.

$y = 225 - 70x$

b. **Algebra** Write an equation for Badria's trip where *y* is the total distance from the park after *x* hours. **$y = 200 - 65x$**

c. **Graphs** Graph both equations on the same coordinate plane.

d. **Words** Will Buthaina's and Badria's trip overlap before they reach the park? Explain your reasoning.

No; the lines do not intersect.

e. **Numbers** Interpret the initial value of each function.

Buthaina begins her trip 225 kilometers from the park. Badria begins her trip 200 kilometers from the park.

Power Up! Test Practice

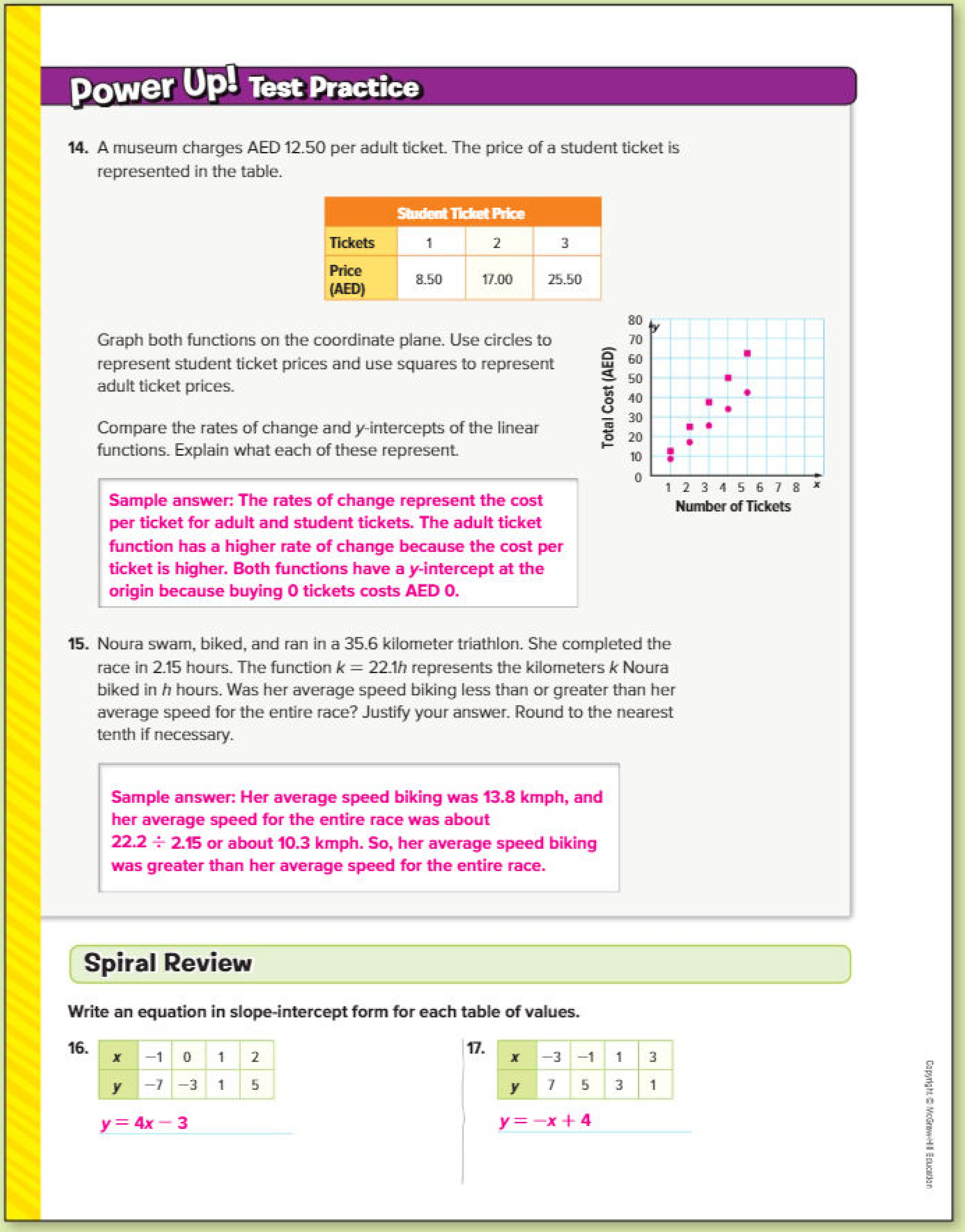
Exercises 13 and 14 prepare students for more rigorous thinking needed for assessment.

13.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

14.

This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Knowledge	DOK3
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly model the slope, *y*-intercept, and linear function and explain what the slope and *y*-intercept represent.
1 point	Students correctly model all three, but fail to explain what the slope and *y*-intercept represent OR students correctly model 2 of the 3 and may or may not explain what the slope and *y*-intercept represent OR students correctly model 1 and explain what slope and *y*-intercept represent.



Power Up! Test Practice

14. A museum charges AED 12.50 per adult ticket. The price of a student ticket is represented in the table.

Student Ticket Price			
Tickets	1	2	3
Price (AED)	8.50	17.00	25.50

Graph both functions on the coordinate plane. Use circles to represent student ticket prices and use squares to represent adult ticket prices.

Compare the rates of change and *y*-intercepts of the linear functions. Explain what each of these represent.

Sample answer: The rates of change represent the cost per ticket for adult and student tickets. The adult ticket function has a higher rate of change because the cost per ticket is higher. Both functions have a *y*-intercept at the origin because buying 0 tickets costs AED 0.

15. Noura swam, biked, and ran in a 35.6 kilometer triathlon. She completed the race in 2.15 hours. The function $k = 22.1h$ represents the kilometers *k* Noura biked in *h* hours. Was her average speed biking less than or greater than her average speed for the entire race? Justify your answer. Round to the nearest tenth if necessary.

Sample answer: Her average speed biking was 13.8 kmph, and her average speed for the entire race was about $22.2 \div 2.15$ or about 10.3 kmph. So, her average speed biking was greater than her average speed for the entire race.

Spiral Review

Write an equation in slope-intercept form for each table of values.

16.

x	−1	0	1	2
y	−7	−3	1	5

$y = 4x - 3$

17.

x	−3	−1	1	3
y	7	5	3	1

$y = -x + 4$

Lesson 7

Linear and Nonlinear Functions

Real-World Link

Football The table shows the approximate height and horizontal distance traveled by a football kicked at an angle of 30° with an initial velocity of 30 meters per second.

Time (s)	Height (m)	Distance (m)
0.0	0	0
0.5	6.2	13
1.0	9.7	26
1.5	10.5	39
2.0	8.7	52
2.5	4.2	65

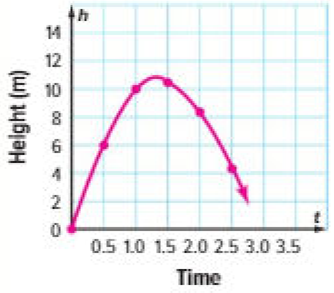
1. Is the rate of change for the height of the football constant? Explain.

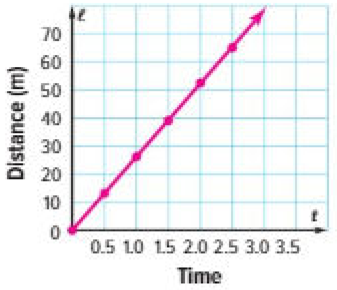
   no; Sample answer: In the first half second, the ball's change in height was 6.2 meters. In the next half-second, the ball's change in height was 9.7 − 6.2, or 3.5 meters.

2. Is the rate of change for the distance traveled constant? Explain.

   yes; Sample answer: The football travels a distance of 13 meters each half-second.

3. Graph the ordered pairs (time, height) and (time, distance) on separate grids. Connect the points with a straight line or smooth curve. Then compare the graphs.





Sample answer: The graph for the height of the football is a curve and the graph for the distance is linear.

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Essential Question

HOW can we model relationships between quantities?

Vocabulary

nonlinear function

Mathematical Practices

1, 3, 4, 7

Focus narrowing the scope

Objective Determine whether a function is linear or nonlinear.

Coherence connecting within and across grades

Previous Students used multiple representations to construct functions.

Now Students use multiple representations to identify linear and nonlinear functions.

Next Students will graph and analyze quadratic functions.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 331.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL LA **Research-Pair-Share** Have students work in pairs. Give students five minutes to research quadratic equations. Then have them share what they found with their partner and determine the maximum and minimum values for the height of the football over time. Then call on one student to share what they found within a small or large group. MP 1, 3, 5

Alternate Strategy

AL Tell students the graph of the height of the football should look like a curve. The graph of the distance traveled should be a straight line. Ask students how the rate of change differs in the two graphs.

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Identify linear or nonlinear functions using a table.

- AL • *What is the first step to find the rates of change for the values in the table?* find the change between the *x*-coordinates and the change between the *y*-coordinates
- OL • *Does the function have a constant rate of change?* yes
- • *Does this indicate a linear or nonlinear function?* linear
- BL • *Is this function increasing or decreasing? Explain.* decreasing, as *x* increases, *y* decreases

Need Another Example?
Determine whether the table represents a *linear* or *nonlinear* function. Explain. Nonlinear; the rate of change is not constant.

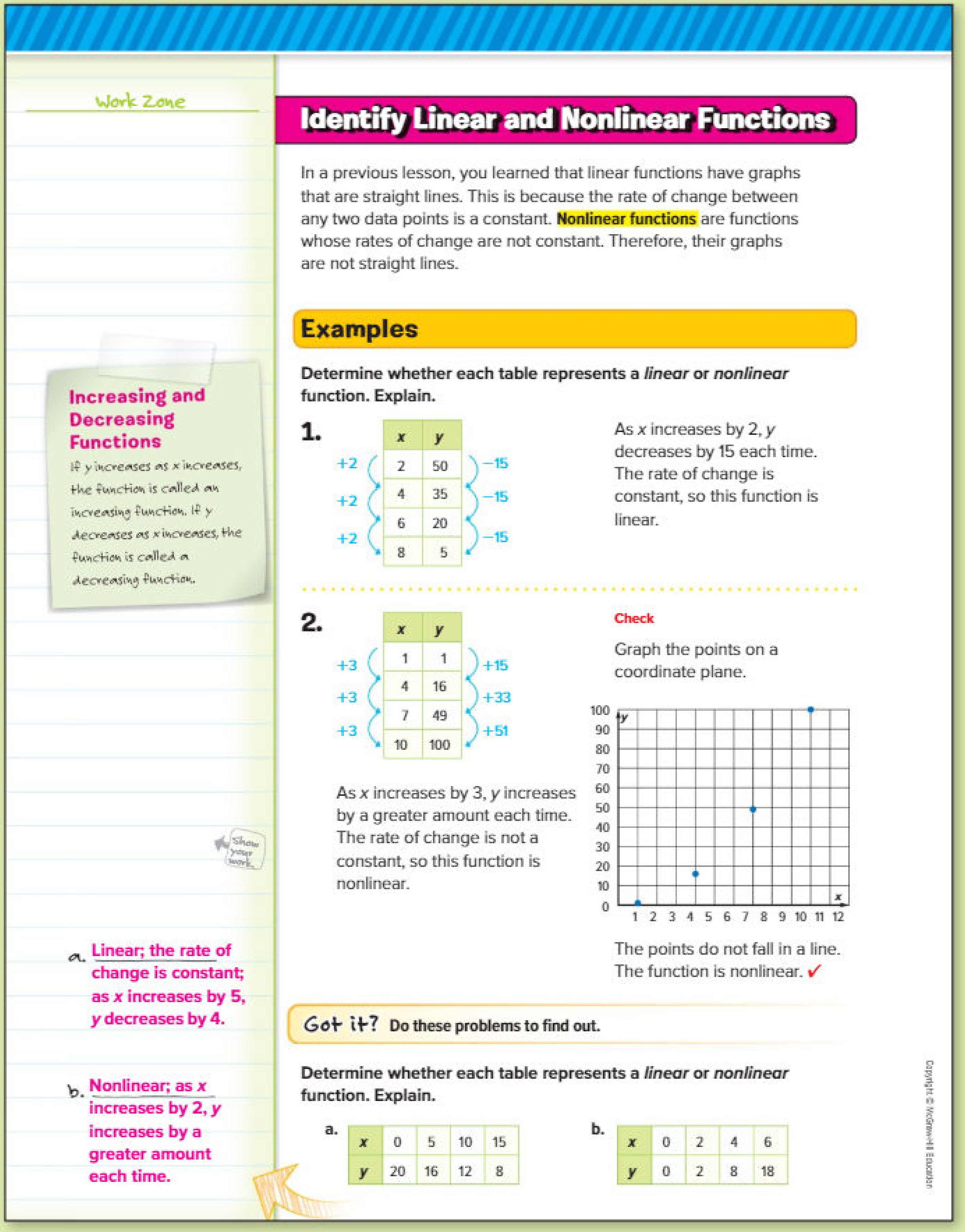
x	2	4	6	8
y	2	20	54	104

2. Identify linear or nonlinear functions using a table.

- AL • *Does the function have a constant rate of change?* no
- OL • *Is the function linear or nonlinear? Explain.* Nonlinear because the rate of change is not constant.
- BL • *Did you need to find the rate of change for the entire table to determine if the function is linear or nonlinear? Explain.* No, once you find the rate of change was different for the first few values, you know the function is nonlinear.

Need Another Example?
Determine whether the table represents a *linear* or *nonlinear* function. Explain. Linear; the rate of change is constant, as *x* increases by 3, *y* increases by 9.

x	1	4	7	10
y	0	9	18	27



Work Zone

Identify Linear and Nonlinear Functions

In a previous lesson, you learned that linear functions have graphs that are straight lines. This is because the rate of change between any two data points is a constant. **Nonlinear functions** are functions whose rates of change are not constant. Therefore, their graphs are not straight lines.

Increasing and Decreasing Functions

If y increases as x increases, the function is called an increasing function. If y decreases as x increases, the function is called a decreasing function.

Examples

Determine whether each table represents a *linear* or *nonlinear* function. Explain.

1.

	x	*y*	
	2	50	
+2	4	35	−15
+2	6	20	−15
+2	8	5	−15

As *x* increases by 2, *y* decreases by 15 each time. The rate of change is constant, so this function is linear.

2.

	x	*y*	
	1	1	
+3	4	16	+15
+3	7	49	+33
+3	10	100	+51

As *x* increases by 3, *y* increases by a greater amount each time. The rate of change is not a constant, so this function is nonlinear.

Check

Graph the points on a coordinate plane.

The points do not fall in a line. The function is nonlinear. ✓

Got it? Do these problems to find out.

Determine whether each table represents a *linear* or *nonlinear* function. Explain.

a.

x	0	5	10	15
y	20	16	12	8

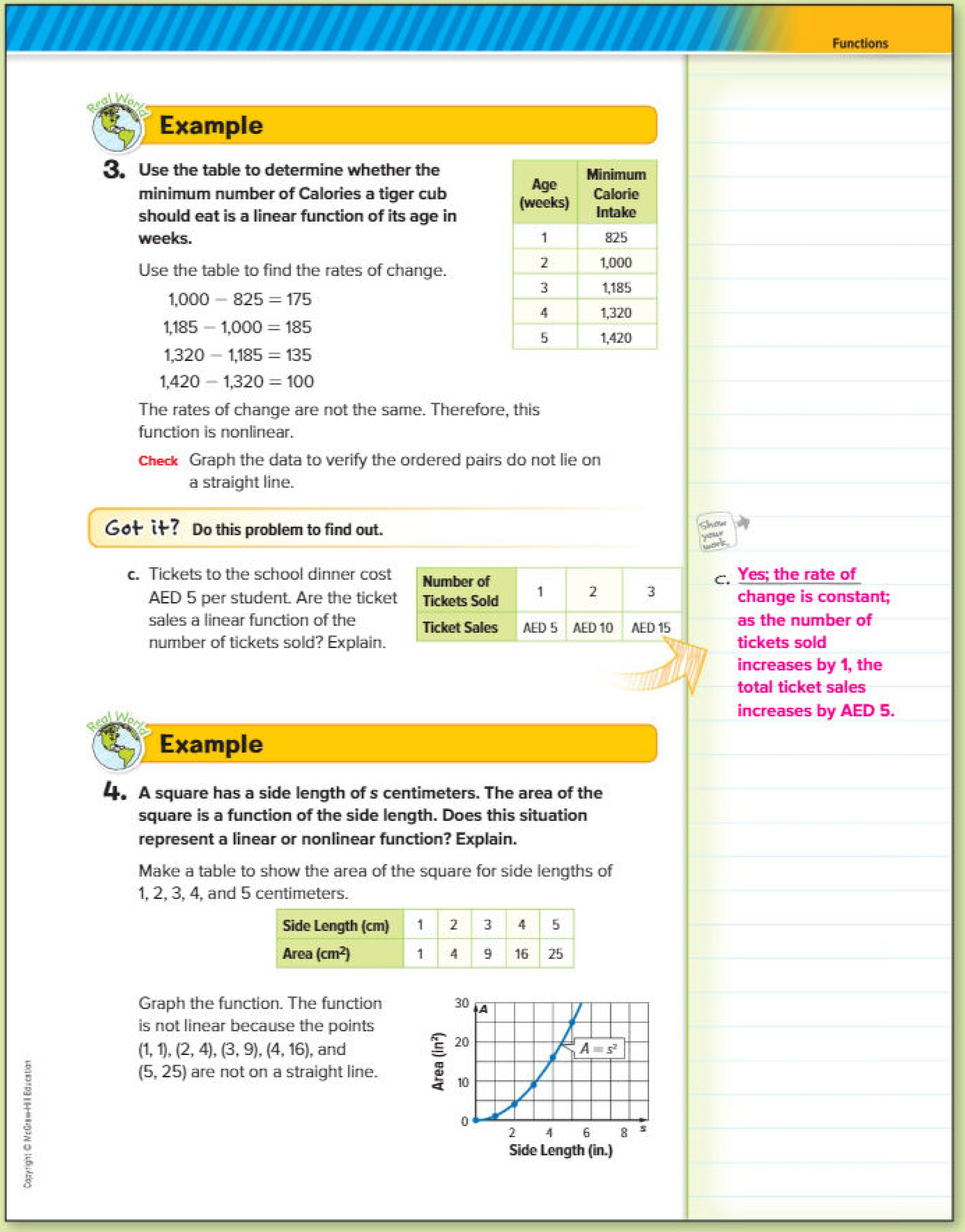
b.

x	0	2	4	6
y	0	2	8	18

Show your work.

a. Linear; the rate of change is constant; as *x* increases by 5, *y* decreases by 4.

b. Nonlinear; as *x* increases by 2, *y* increases by a greater amount each time.



Example

3. **Use the table to determine whether the minimum number of Calories a tiger cub should eat is a linear function of its age in weeks.**

Age (weeks)	Minimum Calorie Intake
1	825
2	1,000
3	1,185
4	1,320
5	1,420

Use the table to find the rates of change.

$1{,}000 - 825 = 175$

$1{,}185 - 1{,}000 = 185$

$1{,}320 - 1{,}185 = 135$

$1{,}420 - 1{,}320 = 100$

The rates of change are not the same. Therefore, this function is nonlinear.

Check Graph the data to verify the ordered pairs do not lie on a straight line.

Got it? **Do this problem to find out.**

c. Tickets to the school dinner cost AED 5 per student. Are the ticket sales a linear function of the number of tickets sold? Explain.

Number of Tickets Sold	1	2	3
Ticket Sales	AED 5	AED 10	AED 15

Show your work.

c. Yes; the rate of change is constant; as the number of tickets sold increases by 1, the total ticket sales increases by AED 5.

Example

4. **A square has a side length of *s* centimeters. The area of the square is a function of the side length. Does this situation represent a linear or nonlinear function? Explain.**

Make a table to show the area of the square for side lengths of 1, 2, 3, 4, and 5 centimeters.

Side Length (cm)	1	2	3	4	5
Area (cm^2)	1	4	9	16	25

Graph the function. The function is not linear because the points (1, 1), (2, 4), (3, 9), (4, 16), and (5, 25) are not on a straight line.

Examples

3. Identify linear and nonlinear functions.

AL
- *What is the rate of change from week 1 to week 2?* $\frac{175}{1}$
- *What is the rate of change from week 2 to week 3?* $\frac{185}{1}$

OL
- *Are the rates of change constant?* no
- *Is the function linear or nonlinear?* nonlinear

BL
- *Is the function increasing or decreasing?* increasing

Need Another Example?

Use the table to determine whether or not the number of revolutions per hour of a second hand on a clock is a linear function of the number of hours that pass. Linear; the rate of change is constant, as the number of hours increased by 1, the number of second hand revolutions increases by 60.

Hour	1	2	3	4	5
Second Hand Revolutions	60	120	180	240	300

4. Identify linear and nonlinear functions.

AL
- *How do you find the area of a square?* square the length of a side
- *What is the area of a square with a side length of 1 centimeter? 2 centimeters? 3 centimeters?* $1 cm^2$; $4 cm^2$; $9 cm^2$

OL
- *What points will you graph?* (1, 1), (2, 4), (3, 9), (4, 16), (5, 25)
- *Is there a constant rate of change?* no
- *Is the function linear or nonlinear?* nonlinear

BL
- *Can you look at the equation $A = s^2$ and determine if the function is linear or nonlinear? Explain.* Yes, the equation has a variable to a power other than 1 in it, so it is nonlinear.

Need Another Example?

At the first level of a maze, there are three possible paths that can be chosen. At the next level, each of those three paths have three more possible paths. Does this situation represent a linear or nonlinear function? Explain. Nonlinear; if you graph the function, the points do not lie on a straight line.

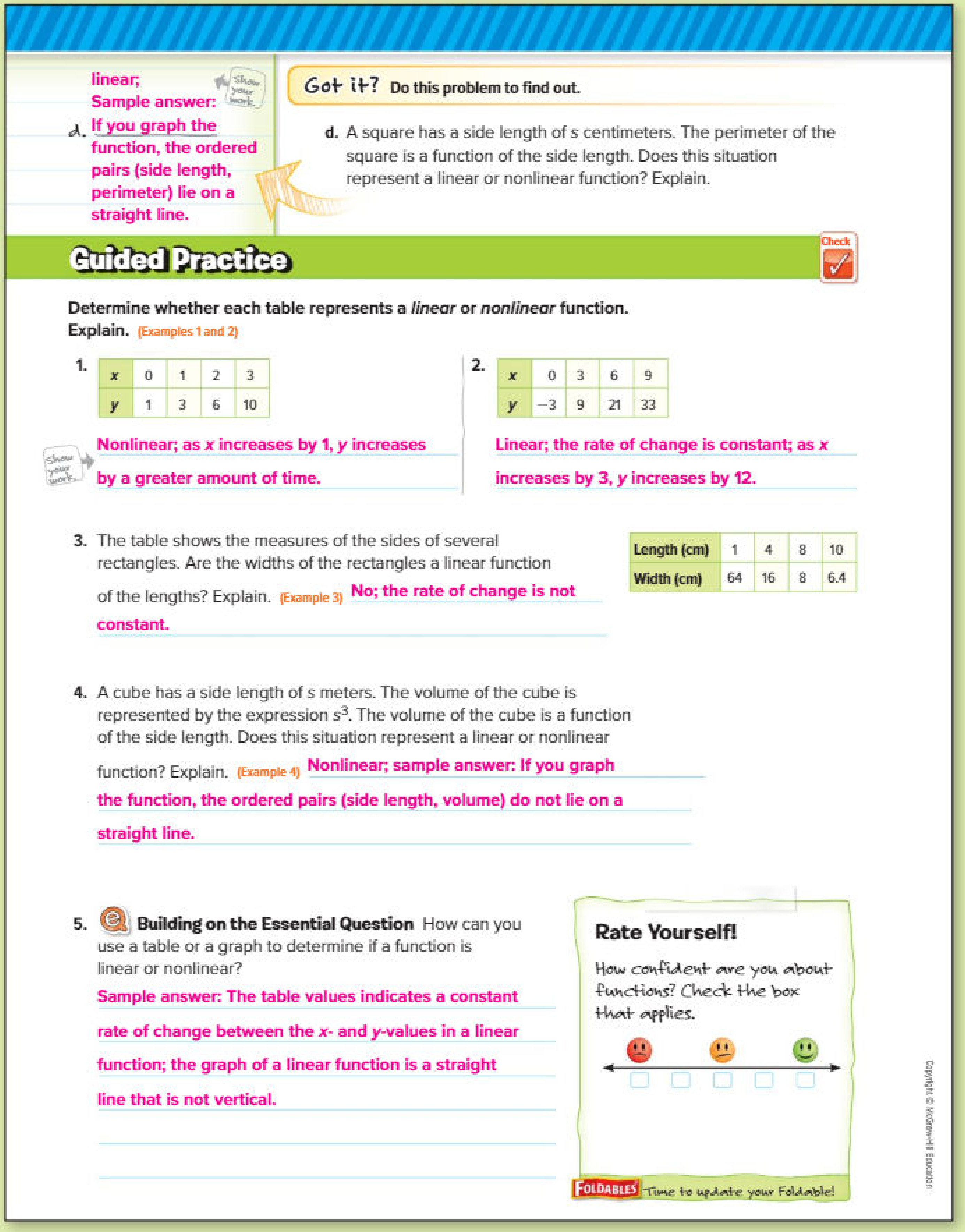
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL **LA** **Team-Pair-Solo** Have students work in teams of four to complete Exercise 1. Then each team splits into two pairs to complete Exercises 2 and 3. Finally, students work by themselves to solve Exercise 4. The team regroups to compare solutions and discuss any differences. MP 1, 3

BL **LA** **Find the Fib** Students write down two facts and one fib about linear functions or nonlinear functions. Students then form teams of three. The job of the team is to identify the fib in each group of statements. MP 1, 3



Got it? Do this problem to find out.

d. A square has a side length of *s* centimeters. The perimeter of the square is a function of the side length. Does this situation represent a linear or nonlinear function? Explain.

Show your work.

d. linear; Sample answer: If you graph the function, the ordered pairs (side length, perimeter) lie on a straight line.

Guided Practice

Check

Determine whether each table represents a *linear* or *nonlinear* function. Explain. (Examples 1 and 2)

1.

x	0	1	2	3
y	1	3	6	10

Show your work.

Nonlinear; as *x* increases by 1, *y* increases by a greater amount of time.

2.

x	0	3	6	9
y	−3	9	21	33

Linear; the rate of change is constant; as *x* increases by 3, *y* increases by 12.

3. The table shows the measures of the sides of several rectangles. Are the widths of the rectangles a linear function of the lengths? Explain. (Example 3) No; the rate of change is not constant.

Length (cm)	1	4	8	10
Width (cm)	64	16	8	6.4

4. A cube has a side length of *s* meters. The volume of the cube is represented by the expression s^3. The volume of the cube is a function of the side length. Does this situation represent a linear or nonlinear function? Explain. (Example 4) Nonlinear; sample answer: If you graph the function, the ordered pairs (side length, volume) do not lie on a straight line.

5. **Building on the Essential Question** How can you use a table or a graph to determine if a function is linear or nonlinear?

Sample answer: The table values indicates a constant rate of change between the *x*- and *y*-values in a linear function; the graph of a linear function is a straight line that is not vertical.

Rate Yourself!

How confident are you about functions? Check the box that applies.

☐ ☐ ☐ ☐ ☐

FOLDABLES Time to update your Foldable!

Name ______________________ My Homework ______________________

Independent Practice

Determine whether each table represents a *linear* or *nonlinear* function. Explain. (Examples 1 and 2)

1.

x	−2	0	2	4
y	−1	0	1	2

Show your work.

Linear; rate of change is constant; as x increases by 2, y increases by 1.

2.

x	1	2	3	4
y	1	4	9	16

Nonlinear; rate of change is not constant.

3.

x	5	10	15	20
y	13	28	43	58

Linear; rate of change is constant; as x increases by 5, y increases by 15.

4.

x	1	3	5	7
y	−2	−18	−50	−98

Nonlinear; rate of change is not constant.

5. Jassim's family drove from Sharjah to Ruwais. Use the table to determine whether the distance driven is a linear function of the hours traveled. Explain. (Example 3) Yes; the rate of change is constant; as the time increases by 1 hour, the distance increases by 65 kilometers.

Time (h)	1	2	3	4
Distance (km)	65	130	195	260

6. The table shows the height of several buildings in Sharjah. Use the table to determine whether the height of the building is a linear function of the number of stories. Explain. (Example 3)

No; the rate of change is not constant.

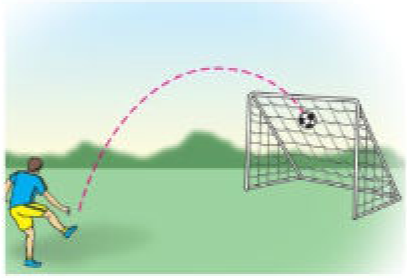
Building	Stories	Height (m)
United Bank	35	170
The Blue Tower	40	172
Department Store	45	182
Etisalat	50	194
Health Center	55	184

7. There are 3,600 seconds in one hour. The total seconds is a function of the hours. Does this situation represent a linear or nonlinear function? Explain. (Example 4)

Linear; sample answer: If you graph the function, the ordered pairs (hours, seconds) lie on a straight line.

8. A football is placed on the ground for a free kick. The height of the ball is a function of the time in seconds. Does the path the football follows after being kicked represent a linear or nonlinear function? Explain. (Example 4)

Nonlinear; sample answer: After being kicked, the ball will reach a maximum height and come back to the ground.



ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–8, 16–18	9–12, 19–21	13–15
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 11, 14, 15, 20, 21
OL	On Level	1–7 odd, 9–12, 14, 15, 20, 21
BL	Beyond Level	9–15, 20, 21

Watch Out!

Common Error Since the graphs of some nonlinear functions may appear linear because of the scales on the axes, students may incorrectly identify those functions as linear. Advise students to use a table to check that the rate of change is constant.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	13
3 Construct viable arguments and critique the reasoning of others.	15, 19
4 Model with mathematics.	14
7 Look for and make use of structure.	12

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

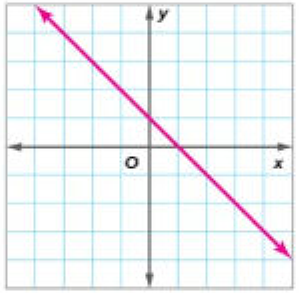
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students explain how to tell whether a table of *x*- and *y*-values describes a function. See students' work.

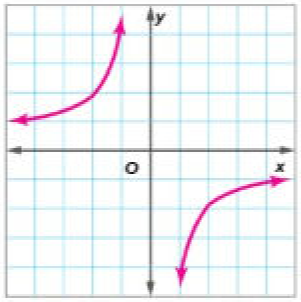
Graph each function by making a table of ordered pairs. Determine whether each function is *linear* or *nonlinear*. Explain.

9. $y = -x + 1$



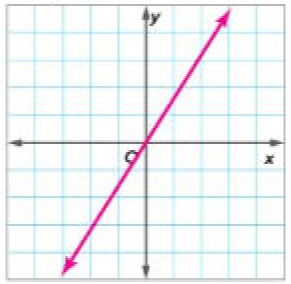
Linear; sample answer: The points lie on a straight line.

10. $y = \frac{-4}{x}$



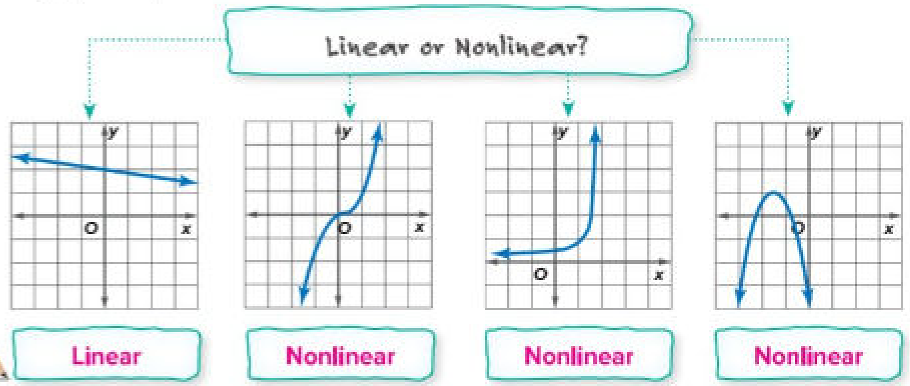
Nonlinear; sample answer: The graph is a curve.

11. $y = \frac{3x}{2}$

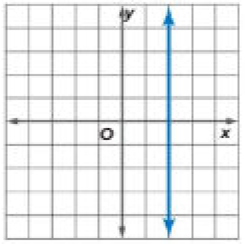


Linear; sample answer: The points lie on a straight line.

12. **Identify Structure** Complete the graphic organizer by determining if the graphs represent linear or nonlinear functions.



H.O.T. Problems Higher Order Thinking



13. **Persevere with Problems** Does the graph at the right represent a linear function? Explain. No; sample answer: the graphs of vertical lines are not functions because there is more than one value of *y* that corresponds to $x = 2$.

14. **Model with Mathematics** Give an example of a situation that can be represented by a nonlinear function. Sample answer: Every hour the number of bacteria in a petri dish doubles.

15. **Reason Inductively** Explain how you can use different representations to determine whether a function is linear. Sample answer: A non-vertical graph that is a straight line is linear. An equation that can be written in the form $y = mx + b$ is linear. If a table of values shows a constant rate of change, it is linear.

Name ______________________ My Homework ______________________

Extra Practice

Determine whether each table represents a *linear* or *nonlinear* function. Explain.

16.

	x	*y*	
	2	10	
+2	4	12	+2
+2	6	16	+4
+2	8	24	+8

Homework Help

Nonlinear; rate of change is not constant.

As x increases by 2, y increases by a greater amount each time. The rate of change is not constant, so this function is nonlinear.

17.

x	*y*
4	3
8	0
12	−3
16	−6

Linear; rate of change is constant; as *x* increases by 4, *y* decreases by 3.

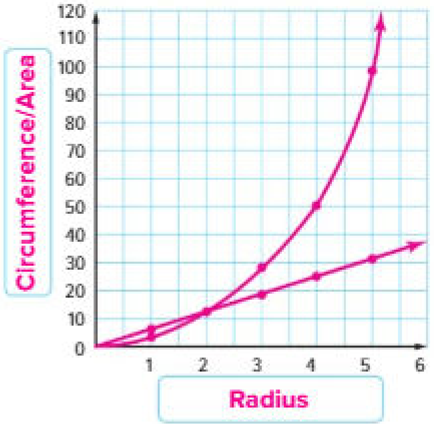
18. Copy and Solve The area of a square is a function of its perimeter. Graph the function on a separate sheet of grid paper. Explain whether the function is linear and if the graph is increasing or decreasing. **See margin.**

19. **Multiple Representations** Recall that the circumference of a circle is equal to two times π times its radius and that the area of a circle is equal to π times the square of the radius.

a. **Tables** Complete the table showing the circumference and area of circles with radius *r*.

Radius *r*	Circumference $2 \cdot \pi \cdot r$	Area πr^2
1	$2 \cdot \pi \cdot 1 \approx 6.28$	$\pi \cdot 1^2 \approx 3.14$
2	$2 \cdot \pi \cdot 2 \approx 12.57$	$\pi \cdot 2^2 \approx 12.57$
3	$2 \cdot \pi \cdot 3 \approx 18.85$	$\pi \cdot 3^2 \approx 28.27$
4	$2 \cdot \pi \cdot 4 \approx 25.13$	$\pi \cdot 4^2 \approx 50.27$
5	$2 \cdot \pi \cdot 5 \approx 31.42$	$\pi \cdot 5^2 \approx 78.54$

b. **Graphs** Graph the ordered pairs (radius, circumference) and (radius, area) for each function on the same coordinate plane.

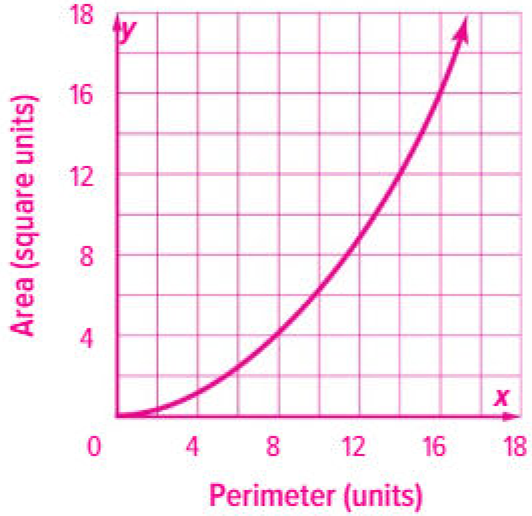


c. **Words** Is the circumference of a circle a linear or nonlinear function of its radius? the area?

Explain your reasoning. Circumference: linear; sample answer: When the ordered pairs are graphed, the points fall in a line. Area: nonlinear; sample answer: When the ordered pairs are graphed, the points do not fall in a line.

Additional Answer

18.



Sample answer: The function is nonlinear because the graph of the function is not a straight line and the line is increasing.

Power Up! Test Practice

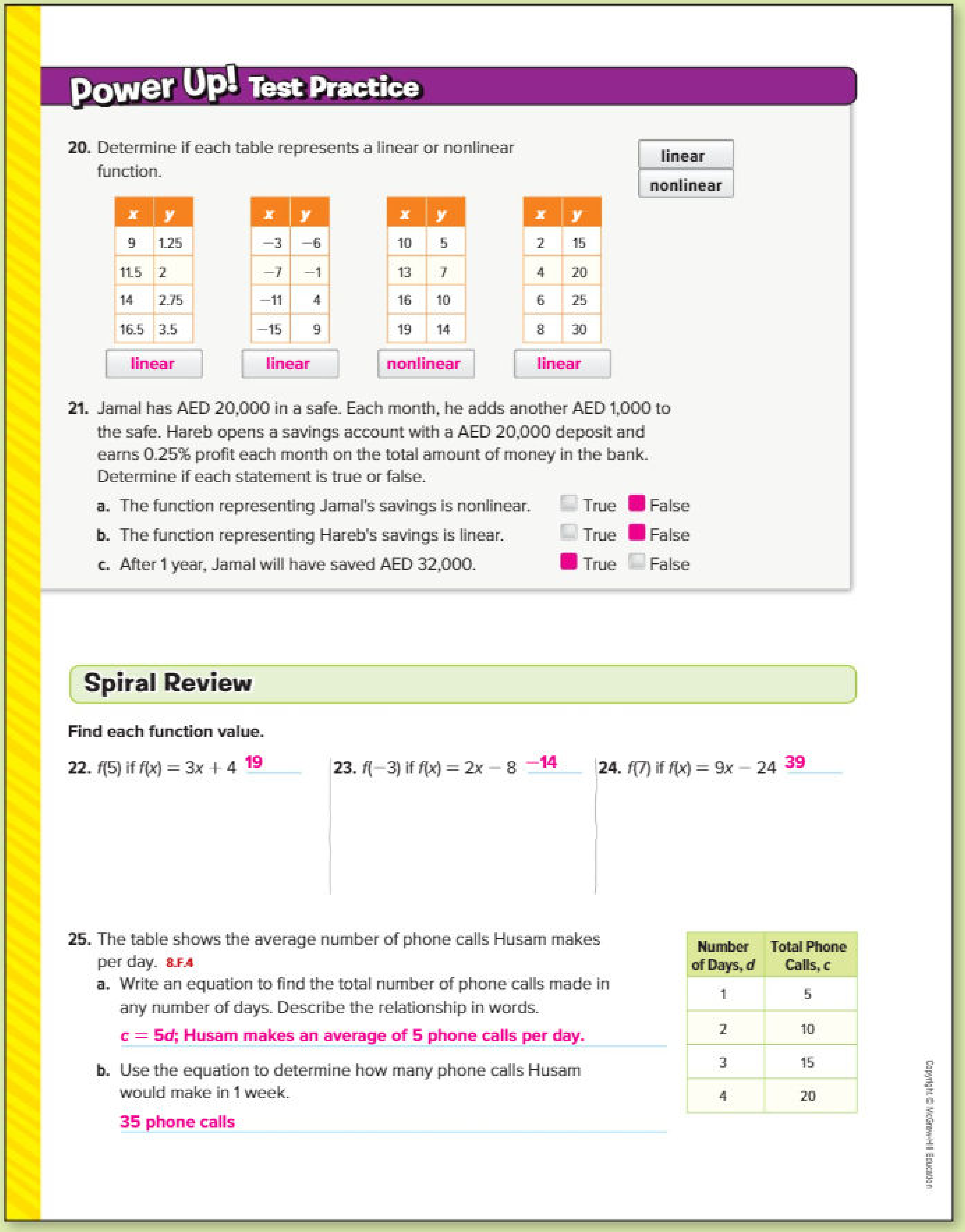
Exercises 20 and 21 prepare students for more rigorous thinking needed for assessment.

20.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

21.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.



Power Up! Test Practice

20. Determine if each table represents a linear or nonlinear function.

linear
nonlinear

x	y
9	1.25
11.5	2
14	2.75
16.5	3.5

linear

x	y
−3	−6
−7	−1
−11	4
−15	9

linear

x	y
10	5
13	7
16	10
19	14

nonlinear

x	y
2	15
4	20
6	25
8	30

linear

21. Jamal has AED 20,000 in a safe. Each month, he adds another AED 1,000 to the safe. Hareb opens a savings account with a AED 20,000 deposit and earns 0.25% profit each month on the total amount of money in the bank. Determine if each statement is true or false.

a. The function representing Jamal's savings is nonlinear. ☐ True ☑ False
b. The function representing Hareb's savings is linear. ☐ True ☑ False
c. After 1 year, Jamal will have saved AED 32,000. ☑ True ☐ False

Spiral Review

Find each function value.

22. $f(5)$ if $f(x) = 3x + 4$ 19

23. $f(-3)$ if $f(x) = 2x - 8$ −14

24. $f(7)$ if $f(x) = 9x - 24$ 39

25. The table shows the average number of phone calls Husam makes per day. 8.F.4

a. Write an equation to find the total number of phone calls made in any number of days. Describe the relationship in words.

$c = 5d$; Husam makes an average of 5 phone calls per day.

b. Use the equation to determine how many phone calls Husam would make in 1 week.

35 phone calls

Number of Days, d	Total Phone Calls, c
1	5
2	10
3	15
4	20

Lesson 8

Quadratic Functions

Vocabulary Start-Up

In the previous lesson, you learned about nonlinear functions. A special type of a nonlinear function is a quadratic function. A **quadratic function** is a function in which the greatest power of the variable is 2. Its graph is U-shaped, opening upward or downward.

Complete the graphic organizer by determining if the facts about quadratic functions are *true* or *false*. If false, give the true fact.

Quadratic Functions

A function in which the greatest power of the variable is 2. **true**

The graph is a straight line. **False, the graph is U-shaped.**

The graph of a quadratic function sometimes opens upward. **true**

The graph always opens downward. **False, the graph can open upward or downward.**

Essential Question

HOW can we model relationships between quantitites?

Vocabulary

quadratic function

Mathematical Practices

1, 3, 4, 7

Real-World Link

Hassan kicked a soccer ball straight into the air. The height h in meters of the ball after t seconds is found using the equation $h = -8t^2 + 16t + 2$. What is the height of the ball after 1.5 seconds? **8 meters**

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

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Focus narrowing the scope

Objective Graph quadratic functions.

Coherence connecting within and across grades

Previous	Now	Next
Students identified functions as linear or nonlinear.	Students graph quadratic functions and analyze the graphs of quadratic functions.	Students will use technology to analyze families of quadratic functions.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 339.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

AL LA **Pairs Consult** Have students work in pairs to complete the graphic organizer, and write a definition of quadratic function in their own words. Have them trade their definition with another pair and discuss any differences. MP 1, 2, 4

Alternate Strategy

BL Have students graph the functions $y = x^2 + 1$, $y = x^2 + 2$, and $y = x^2 - 1$. Then have them answer the following:

- Compare and contrast the graphs. Sample answer: The graphs have the same shape but different minimum points.
- Without graphing, describe the graph of $y = x^2 - 2$. Sample answer: The shape is the same; the minimum is at (0, −2).

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Graph quadratic functions.

AL
- *What method could you use to graph the function $y = x^2$?* Make a function table, and then plot the ordered pairs on a coordinate plane.
- *How do you square a number?* multiply it by itself
- *What is $(-2)^2$?* 4

OL
- *Without graphing, does the graph of the function open upward or downward? Explain.* Upward; the coefficient of x is positive.
- *What is the minimum point of the graph?* (0, 0)

BL
- *Why is it important to use varied numbers for x?* Sample answer: to get a good picture of the graph and when the graph changes direction

Need Another Example?
Graph $y = 5x^2$. See Answer Appendix.

2. Graph quadratic functions.

AL
- *Why is it helpful to use a table of values to graph the equation?* Sample answer: It helps to organize the input and output values into ordered pairs for graphing.
- *What is the value of $-x^2$ when $x = -2$?* −4

OL
- *Does the graph of the function open upward or downward? Explain.* Downward; the coefficient of x is negative

BL
- *Is there a minimum value or a maximum value for the graph? Explain.* Since the graph opens down, there is a maximum value for the graph.
- *What is the maximum value for the graph?* (0, 4)

Need Another Example?
Graph $y = -x^2 - 2$. See Answer Appendix.

Work Zone

STOP and Reflect

Is it possible for a function to be both increasing and decreasing? Explain below.

Yes; Sample answer: The graph of a quadratic function both increases and decreases.

Quadratic Functions

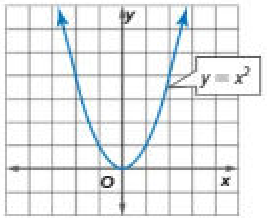
A quadratic function can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$. The graph of a quadratic function is called a parabola. The graph opens upward if the coefficient of the variable that is squared is positive, downward if it is negative.

Examples

1. Graph $y = x^2$.

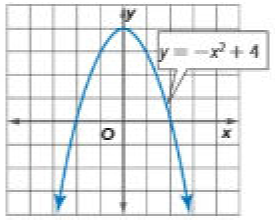
To graph a quadratic function, make a table of values, plot the ordered pairs, and connect the points with a smooth curve.

x	x^2	y	(x, y)
−2	$(-2)^2 = 4$	4	(−2, 4)
−1	$(-1)^2 = 1$	1	(−1, 1)
0	$(0)^2 = 0$	0	(0, 0)
1	$(1)^2 = 1$	1	(1, 1)
2	$(2)^2 = 4$	4	(2, 4)



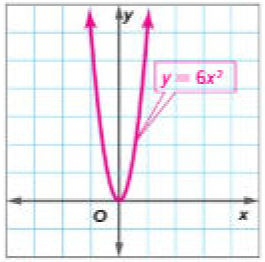
2. Graph $y = -x^2 + 4$.

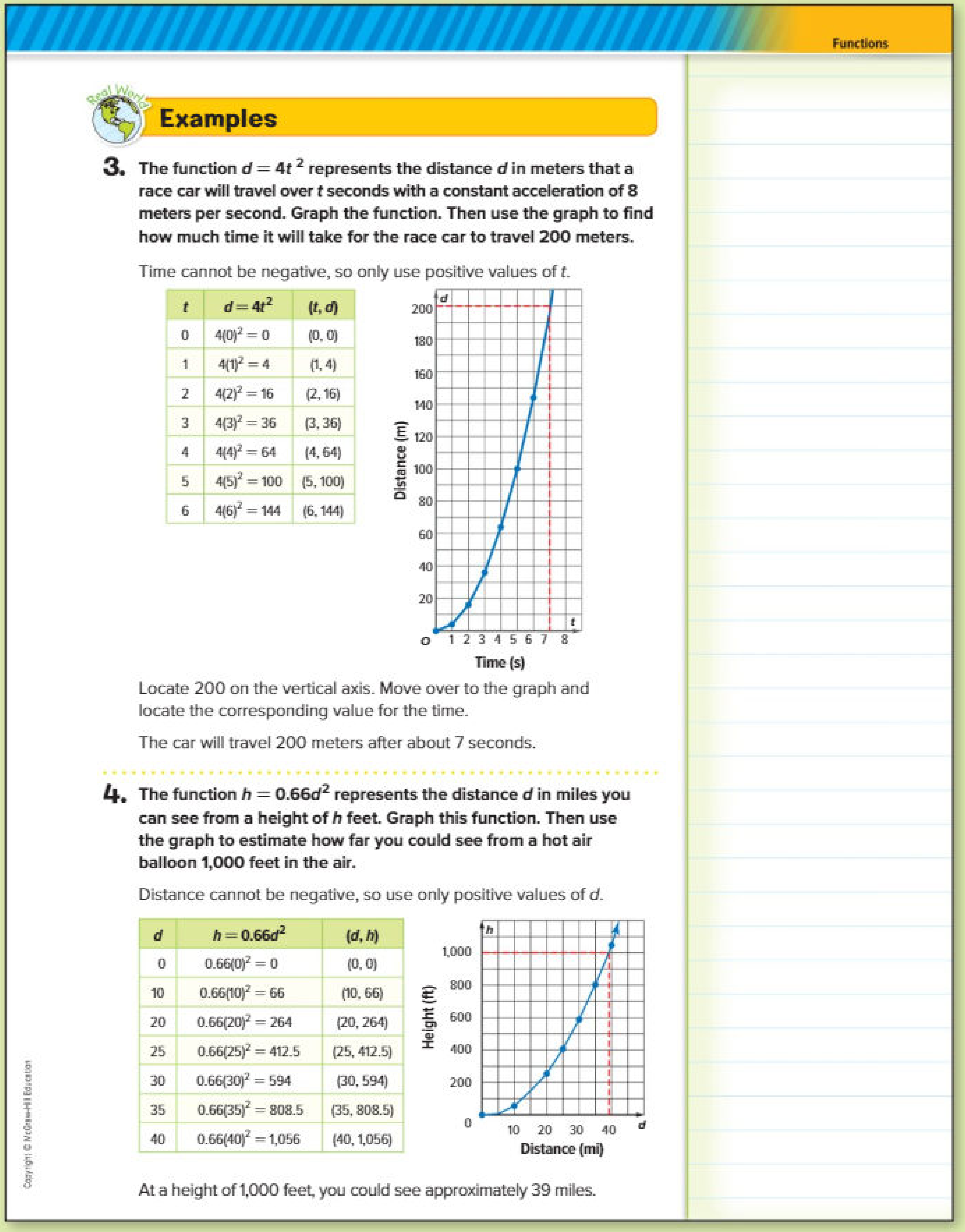
x	$-x^2 + 4$	y	(x, y)
−2	$-(-2)^2 + 4 = 0$	0	(−2, 0)
−1	$-(-1)^2 + 4 = 3$	3	(−1, 3)
0	$-(0)^2 + 4 = 4$	4	(0, 4)
1	$-(1)^2 + 4 = 3$	3	(1, 3)
2	$-(2)^2 + 4 = 0$	0	(2, 0)



Got it? Do this problem to find out.

a. Graph $y = 6x^2$.





Examples

3. **The function $d = 4t^2$ represents the distance d in meters that a race car will travel over t seconds with a constant acceleration of 8 meters per second. Graph the function. Then use the graph to find how much time it will take for the race car to travel 200 meters.**

Time cannot be negative, so only use positive values of t.

t	$d = 4t^2$	(t, d)
0	$4(0)^2 = 0$	(0, 0)
1	$4(1)^2 = 4$	(1, 4)
2	$4(2)^2 = 16$	(2, 16)
3	$4(3)^2 = 36$	(3, 36)
4	$4(4)^2 = 64$	(4, 64)
5	$4(5)^2 = 100$	(5, 100)
6	$4(6)^2 = 144$	(6, 144)

Locate 200 on the vertical axis. Move over to the graph and locate the corresponding value for the time.

The car will travel 200 meters after about 7 seconds.

4. **The function $h = 0.66d^2$ represents the distance d in miles you can see from a height of h feet. Graph this function. Then use the graph to estimate how far you could see from a hot air balloon 1,000 feet in the air.**

Distance cannot be negative, so use only positive values of d.

d	$h = 0.66d^2$	(d, h)
0	$0.66(0)^2 = 0$	(0, 0)
10	$0.66(10)^2 = 66$	(10, 66)
20	$0.66(20)^2 = 264$	(20, 264)
25	$0.66(25)^2 = 412.5$	(25, 412.5)
30	$0.66(30)^2 = 594$	(30, 594)
35	$0.66(35)^2 = 808.5$	(35, 808.5)
40	$0.66(40)^2 = 1{,}056$	(40, 1,056)

At a height of 1,000 feet, you could see approximately 39 miles.

Examples

3. Use quadratic functions.

AL
- *Is the graph of the function linear or nonlinear?* nonlinear
- *How do you know the graph is nonlinear?* The equation is a quadratic equation and the graph is nonlinear.

OL
- *How far has the race car traveled after 1 second?* 4 m *2 seconds?* 16 m *4 seconds?* 64 m
- *How long has the car been accelerating when it has traveled 200 meters?* about 7 seconds

BL
- *Why don't you use negative numbers for this graph?* time cannot be negative

Need Another Example?

The function $d = 16t^2$ represents the distance d in meters that a skydiver falls in t seconds. Graph the equation. Then use the graph to find how much time it will take for the skydiver to fall 400 meters. 5 s; See Answer Appendix for graph.

4. Use quadratic functions.

AL
- *What method could you use to graph the function?* Make a function table. Then plot the ordered pairs on a coordinate plane.
- *Is the graph of the function linear or nonlinear?* nonlinear

OL
- *Do negative input values make sense in this situation? Explain.* No; you cannot have a negative distance.
- *What is the corresponding x value on the graph for a y value of 1,000?* about 40

BL
- *Suppose you were 1,200 feet in the air. About how far could you see?* Sample answer: about 42 m

Need Another Example?

The equation $d = 16.065t^2$ describes the distance d in meters that a stone falls off a cliff over t seconds. Graph this function. Then use the graph to estimate how long it would take a stone to fall 200 meters. about 3.5 s; See Answer Appendix for graph.

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

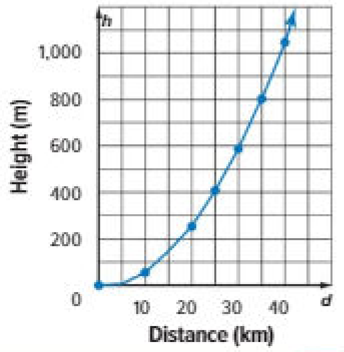
AL LA **Numbered Heads Together** Assign students to 3- or 4-person learning teams. Each member is assigned a number from 1 to 4. Each team completes Exercises 1–3, making sure that every member understands the steps to take in the solution. Call on a specific number from one team to present the team's solution to Exercise 1. Repeat for the remaining two Exercises. MP 1

BL LA **Pairs Discussion** Have students work in pairs to create a graphic organizer about quadratic functions. They should include information about the graphs, equations, and anything else they can find in their research. Display the graphic organizers around the room, have students circulate, and discuss any differences in the graphic organizers. MP 1, 2, 6

b. about 45 km

Got it? Do this problem to find out.

b. The outdoor observation deck of the Space Needle in Seattle, Washington, is 160 meters above ground level. Use the graph to estimate how far you could see from the observation deck.

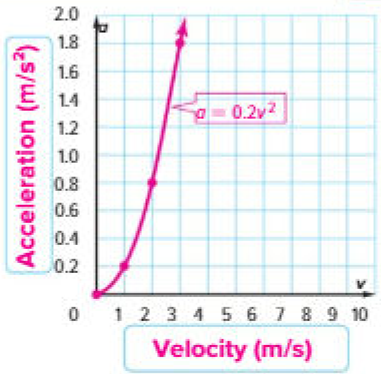


Guided Practice

Check

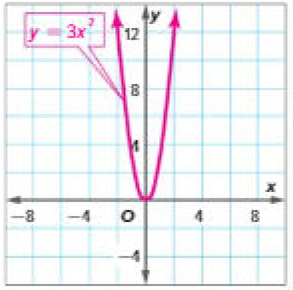
1. The function $a = 0.2v^2$ models the acceleration of a carnival ride, where a is the acceleration toward the center of the ride in meters per second every second and v is the velocity in meters per second. Graph this function. Then use your graph to estimate the velocity of the ride at an acceleration of 1 meter per second every second. (Examples 3 and 4)

   Sample answer: about 2.2 mps

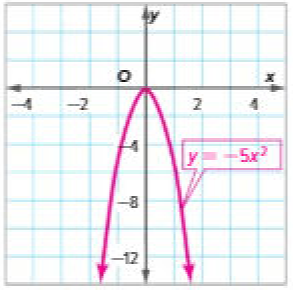


Graph each function. (Examples 1 and 2)

2. $y = 3x^2$



3. $y = -5x^2$



4. **Building on the Essential Question** When does the graph of a quadratic function open upward or downward?

   Sample answer: The graph opens upward if the coefficient of the variable that is squared is positive, downward if it is negative.

Rate Yourself!

How confident are you about quadratic functions? Check the box that applies.

☐ ☐ ☐ ☐ ☐

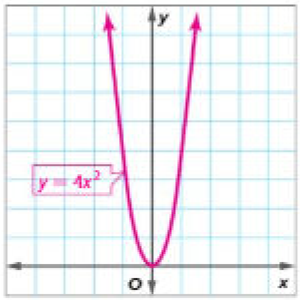
Name ______________________ My Homework ______________________

Independent Practice

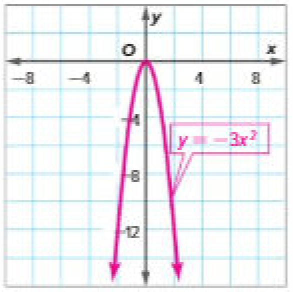
Graph each function. (Examples 1 and 2)

1. $y = 4x^2$

Show your work.



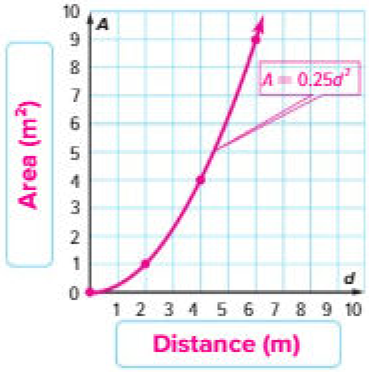
2. $y = -3x^2$



3. A coin is dropped from a height of 196 meters off a bridge. The function $d = -16t^2 + 196$ models the distance d in meters the coin is from the surface of the water at time t seconds. Graph this function. Then use your graph to estimate the time it will take for the coin to reach the water. (Examples 3 and 4) about 3.5 s

4. The area A in square meters of a projected movie on a movie screen can be represented by the equation $A = 0.25d^2$, where d represents the distance from a projector to the movie screen. Graph the function. Then use your graph to estimate the distance from the projector to a screen if the area of the movie is 7 square meters.

(Examples 3 and 4) about 5.2 m



5. Hessa has trim to make a rectangular border for a scrapbook page. The section inside the border is x centimeters long and $(12 - x)$ centimeters wide.
   a. Write a function to represent the area A of the section inside the border. $A = 12x - x^2$
   b. What should the dimensions of the section be to enclose the maximum area inside the border? (*Hint*: Graph the function and find the x-coordinate of the point at the peak of the graph.) 6 cm by 6 cm

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–4, 15–18	5–11, 19–21	12–14
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–4, 5–11, odd, 13, 14, 20, 21
OL	On Level	1, 3, 5–11, 13, 14, 20, 21
BL	Beyond Level	5–14, 20, 21

Watch Out!

Common Error If students graph a parabola in the wrong direction, remind them that it opens upward if the coefficient of x^2 is positive and downward if it is negative.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	12
3 Construct viable arguments and critique the reasoning of others.	14
4 Model with mathematics.	13, 19
7 Look for and make use of structure.	6–11

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students explain the steps they would use to graph a quadratic function. See students' work.

Identify Structure Without graphing, determine whether each equation represents a linear or nonlinear function. Explain.

6. $y = 3x$ linear; Sample answer: The equation is written in slope-intercept form so it is a straight line.

7. $y = 2x^2$ nonlinear; Sample answer: The function is quadratic.

8. $y = -3x^2$ nonlinear; Sample answer: The function is quadratic.

9. $y = -6x$ linear; Sample answer: The equation is written in slope-intercept form so it is a straight line.

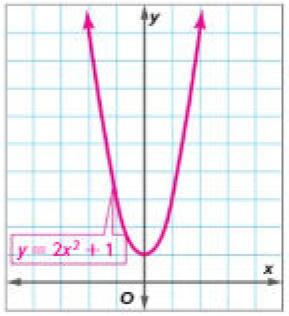
10. $5x + y = 7$ linear; Sample answer: The equation can be written in slope-intercept form so it is a straight line.

11. $7x^2 + y = 24$ nonlinear; Sample answer: The function is quadratic.

H.O.T. Problems Higher Order Thinking

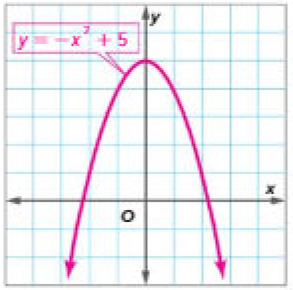
12. **Persevere with Problems** The graphs of quadratic functions may have exactly one highest point, called a *maximum*, or exactly one lowest point, called a *minimum*. Graph each quadratic equation. Determine whether each graph has a maximum or a minimum. If so, give the coordinates of each point.

a. $y = 2x^2 + 1$



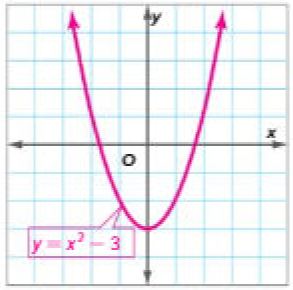
minimum; (0, 1)

b. $y = -x^2 + 5$



maximum; (0, 5)

c. $y = x^2 - 3$



minimum; (0, −3)

13. **Model with Mathematics** Write the equation of a quadratic function that opens upward and has its minimum at (0, −3.5). Sample answer: $y = x^2 - 3.5$

14. **Reason Inductively** The equation $y = ax^2 + bx + c$ represents a quadratic function. What does the constant c represent? Explain. the *y*-intercept; Sample answer: When a graph crosses the *y*-axis, $x = 0$. Substitute 0 for x in the equation, and $y = c$, so c represents the *y*-intercept.

Name ______________________ My Homework ______________________

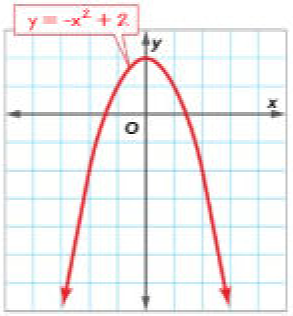
Extra Practice

Graph each function.

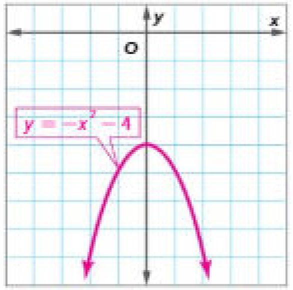
15. $y = -x^2 + 2$

Homework Help

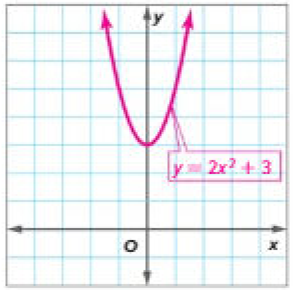
x	$-x^2 + 2$	y	(x, y)
-2	$-(-2)^2 + 2$	-2	$(-2, -2)$
-1	$-(-1)^2 + 2$	1	$(-1, 1)$
0	$-(0)^2 + 2$	2	$(0, 2)$
1	$-(1) + 2$	1	$(1, 1)$
2	$-(2)^2 + 2$	-2	$(2, -2)$



16. $y = -x^2 - 4$



17. $y = 2x^2 + 3$



18. The function $d = 0.006s^2$ represents the braking distance d in meters of a car traveling at a speed s in kilometers per second. Graph this function. Then use your graph to estimate the speed of the car if its braking distance is 12 meters. **about 45 km/s**

19. **MP** **Model with Mathematics** Halima is making a fabric memo board. The width of the board is x centimeters, and the length of the board is $(x + 4)$ centimeters.

a. Write a function that represents the area A of the memo board. $A = x^2 + 4x$

b. Graph the function.

c. If the width of the memo board is 8 centimeters, what is its area? **96 cm²**

Power Up! Test Practice

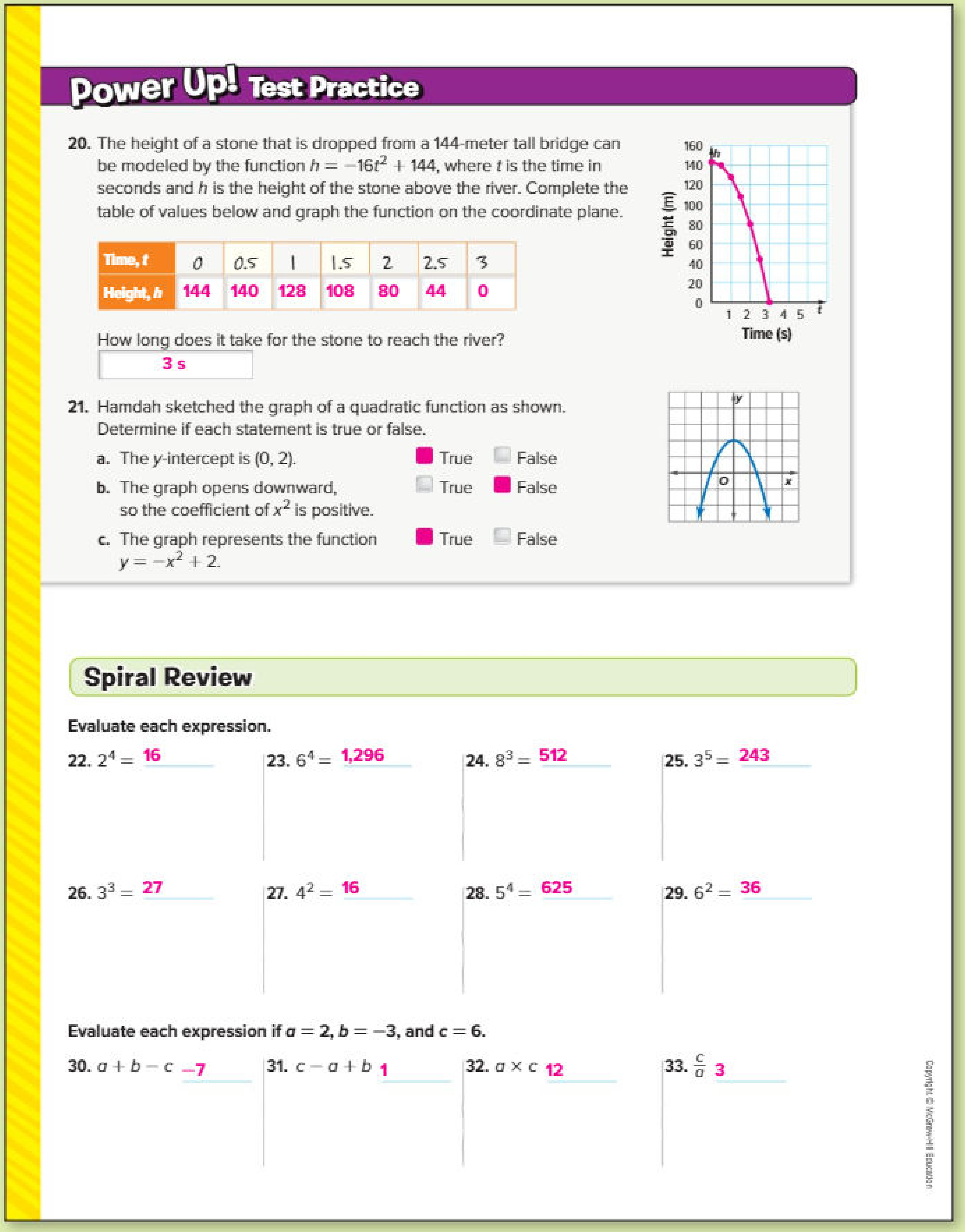
Exercises 20 and 21 prepare students for more rigorous thinking needed for assessment.

20.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK3
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly complete the table, graph the function, and find the time.
1 point	Students correctly complete the table and graph the function, but fail to find the time OR students correctly completely the table and find the time but fail to graph the function correctly OR students incorrectly complete the table, but the graph and time are correct according to the values in the table.

21.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part.



Power Up! Test Practice

20. The height of a stone that is dropped from a 144-meter tall bridge can be modeled by the function $h = -16t^2 + 144$, where t is the time in seconds and h is the height of the stone above the river. Complete the table of values below and graph the function on the coordinate plane.

Time, t	0	0.5	1	1.5	2	2.5	3
Height, h	144	140	128	108	80	44	0

How long does it take for the stone to reach the river?

3 s

21. Hamdah sketched the graph of a quadratic function as shown. Determine if each statement is true or false.

a. The y-intercept is (0, 2). ■ True ☐ False

b. The graph opens downward, so the coefficient of x^2 is positive. ☐ True ■ False

c. The graph represents the function $y = -x^2 + 2$. ■ True ☐ False

Spiral Review

Evaluate each expression.

22. $2^4 =$ 16

23. $6^4 =$ 1,296

24. $8^3 =$ 512

25. $3^5 =$ 243

26. $3^3 =$ 27

27. $4^2 =$ 16

28. $5^4 =$ 625

29. $6^2 =$ 36

Evaluate each expression if $a = 2$, $b = -3$, and $c = 6$.

30. $a + b - c$ −7

31. $c - a + b$ 1

32. $a \times c$ 12

33. $\frac{c}{a}$ 3

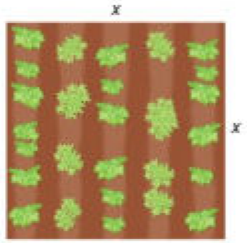
Inquiry Lab

Graphing Technology: Families of Nonlinear Functions

Inquiry **HOW are families of nonlinear functions the same as the parent function? How are families of nonlinear functions different from the parent function?**

MP Mathematical Practices
1, 3, 7

Houriyya is digging a vegetable garden and wants to know how much fertilizer to buy. The equation $y = x^2$ represents the area in square meters of the garden shown.



Hands-On Activity 1

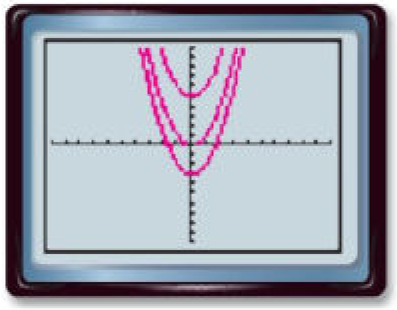
Families of nonlinear functions share a common characteristic based on a parent function. The parent function, or simplest function, of a family of quadratic functions is $y = x^2$. You can use a graphing calculator to investigate families of quadratic functions.

Graph $y = x^2$, $y = x^2 + 5$, and $y = x^2 - 3$ on the same screen.

Step 1 Clear any existing equations from the Y= list by pressing Y= CLEAR.

Step 2 Enter each equation. Press X,T,θ,n x^2 ENTER, X,T,θ,n x^2 + 5 ENTER, and X,T,θ,n x^2 − 3 ENTER.

Step 3 Press ZOOM 6.



Copy your calculator screen on the blank screen shown.

How are the three equations related?

Sample answer: All three equations contain an x^2. However, the number added to or subtracted from x^2 is different.

Describe how the graphs of the three equations are related.

Sample answer: All three are in the shape of a "U". However, the graphs have their vertices at different points.

Focus narrowing the scope

Objective Use a graphing calculator to graph families of nonlinear functions.

Coherence connecting within and across grades

Now
Students use graphing calculators to graph families of nonlinear functions.

Next
Students will model relationships by sketching and describing qualitative graphs.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 345.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance than Activity 2.

Materials: graphing calculators

Hands-On Activity 1

AL LA **Circle the Sage** Poll the class to see which students have some knowledge of graphing functions. Those students (the sages) spread out around the room. Assign the rest of the students to teams. Have the teams split up with each team member going to a different sage, if possible. Have the sages lead work for Activity 1. After completing the activity, students go back to their teams and compare solutions. Students discuss how the sage may have explained the steps differently. MP 1, 3, 4, 5

Hands-On Activity 2

AL LA **Pairs Check** Have students work in pairs to complete Steps 1–4 of Activity 2. One partner completes the odd steps, while the other completes the even steps. After each pair has recorded their results and stored their graphing results, have pairs share, discuss, and check their solutions with another pair. MP 1, 5

BL LA **Pairs Check** Have students work in pairs to complete Steps 1–4 of Activity 2. One partner completes the first step while the other coaches. Students switch roles for the next step. Students continue to switch roles for the remaining steps. After every two steps, the pairs check their solutions with another pair and make any necessary adjustments. MP 1, 5

Hands-On Activity 2

An **exponential function** is a nonlinear function in which the base is a constant and the exponent is an independent variable, *x*. The parent function for an exponential function is shown.

Exponent is a variable.

$$y = 2^x$$

Base is a constant.

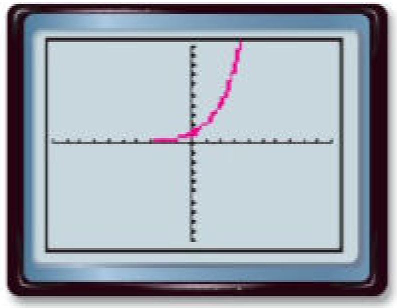
A certain type of bacteria doubles every hour. The function $y = 2^x$ represents the total number of bacteria y at the end of every hour x. Graph the function. Then find the number of bacteria at the end of 5 hours.

Step 1 Clear any existing equations from the Y= list by pressing Y= CLEAR.

Step 2 Enter the equation. Press Y= 2 ^ X,T,θ,n

Step 3 Graph the equation in the standard viewing window. Press ZOOM 6.

Copy your calculator screen on the blank screen shown.

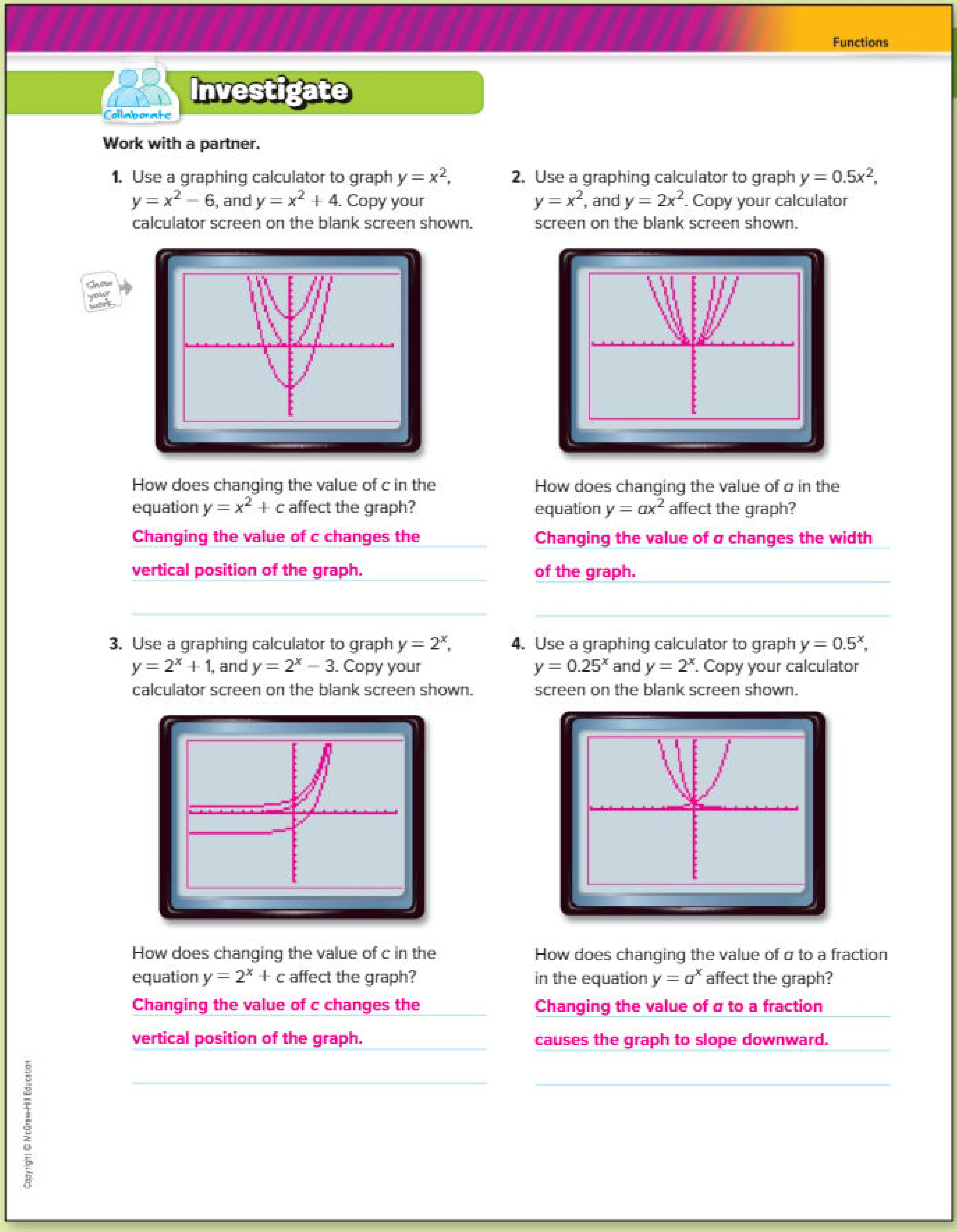


Describe the function by analyzing the graph.

Sample answer: The function is a nonlinear increasing function.

Step 4 Use the TABLE feature. Press 2nd GRAPH. What is the *y* value that corresponds to the *x* value of 5? 32

So, there are 32 bacteria at the end of 5 hours.



Investigate

Collaborate

Work with a partner.

1. Use a graphing calculator to graph $y = x^2$, $y = x^2 - 6$, and $y = x^2 + 4$. Copy your calculator screen on the blank screen shown.

Show your work.

How does changing the value of c in the equation $y = x^2 + c$ affect the graph?

Changing the value of c changes the vertical position of the graph.

2. Use a graphing calculator to graph $y = 0.5x^2$, $y = x^2$, and $y = 2x^2$. Copy your calculator screen on the blank screen shown.

How does changing the value of a in the equation $y = ax^2$ affect the graph?

Changing the value of a changes the width of the graph.

3. Use a graphing calculator to graph $y = 2^x$, $y = 2^x + 1$, and $y = 2^x - 3$. Copy your calculator screen on the blank screen shown.

How does changing the value of c in the equation $y = 2^x + c$ affect the graph?

Changing the value of c changes the vertical position of the graph.

4. Use a graphing calculator to graph $y = 0.5^x$, $y = 0.25^x$ and $y = 2^x$. Copy your calculator screen on the blank screen shown.

How does changing the value of a to a fraction in the equation $y = a^x$ affect the graph?

Changing the value of a to a fraction causes the graph to slope downward.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–4	5–9	10, 11
Level 3			●
Level 2		●	
Level 1	●		

Investigate

Collaborate

AL LA Group-Solo Work together as a class to complete Exercise 1 while you, or a student volunteer, demonstrates how to use a graphing calculator to graph the solution to Exercise 1. Then have students complete Exercises 2–4 individually. MP 1, 5

BL LA Think-Pair-Write Have students work with a partner to complete Exercises 1–4. Give students time to discuss the question in each exercise. After the students finish their discussions within the pairs, have each student write their own responses to the questions. If time permits, ask for volunteers to give their answers and how their partner influenced their answer. MP 1, 5

Analyze and Reflect

AL **LA** **Popcorn Share** Have students individually complete Exercises 5–9. Call out “Popcorn 5,” so that students can quickly pop out of their chairs one at a time to share their answer. Seated students write their responses on their piece of paper after a classroom consensus. Continue to call out Popcorn until all exercises are complete. MP 1, 5

BL **LA** **Pairs Discussion** Have students work in pairs to complete Exercises 5–9. Have them trade their solutions with another pair of students and discuss any differences. MP 1, 5

Create

BL **LA** **Trade-a-Problem** Have students work in pairs. Each partner writes a real-world word problem that uses graphing calculator technology. Have students trade with their partners and solve each other’s word problem. MP 1, 4, 5

Inquiry Students should be able to answer “HOW are families of nonlinear functions the same as the parent function? How are families of nonlinear functions different from the parent function?” Check for student understanding and provide guidance, if needed.

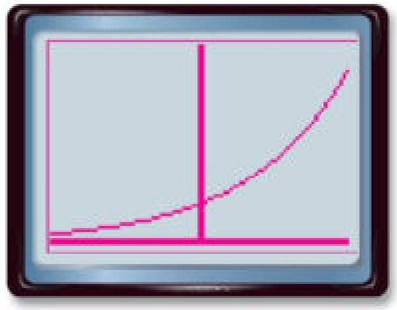
Analyze and Reflect

MP **Identify Structure** **Work with a partner to complete the table. Without graphing, determine which graph is wider.**

	Equation 1	Equation 2	Which graph is wider?
5.	$y = 5x^2$	$y = x^2$	$y = x^2$
6.	$y = \frac{1}{3}x^2$	$y = 3x^2$	$y = \frac{1}{3}x^2$
7.	$y = 2^x$	$y = 4^x$	$y = 2^x$
8.	$y = 0.25^x$	$y = 0.75^x$	$y = 0.75^x$

9. Khadija’s parents started a savings account when she was born by depositing AED 100 into the account. The account makes an annual profit of 3%. The balance y in the account can be represented by the function $y = 100(1.03)^x$, where x is the number of years.

a. Graph the function on your graphing calculator. Copy your calculator screen on the blank screen shown. (*Hint:* Use the x-scale -50 to 50 and the y-scale 0 to 500.)



b. Use the TABLE feature. How much money is in the account after 13 years? **AED 146.85**

c. Describe the function by analyzing the graph.

Sample answer: The function is a nonlinear increasing function.

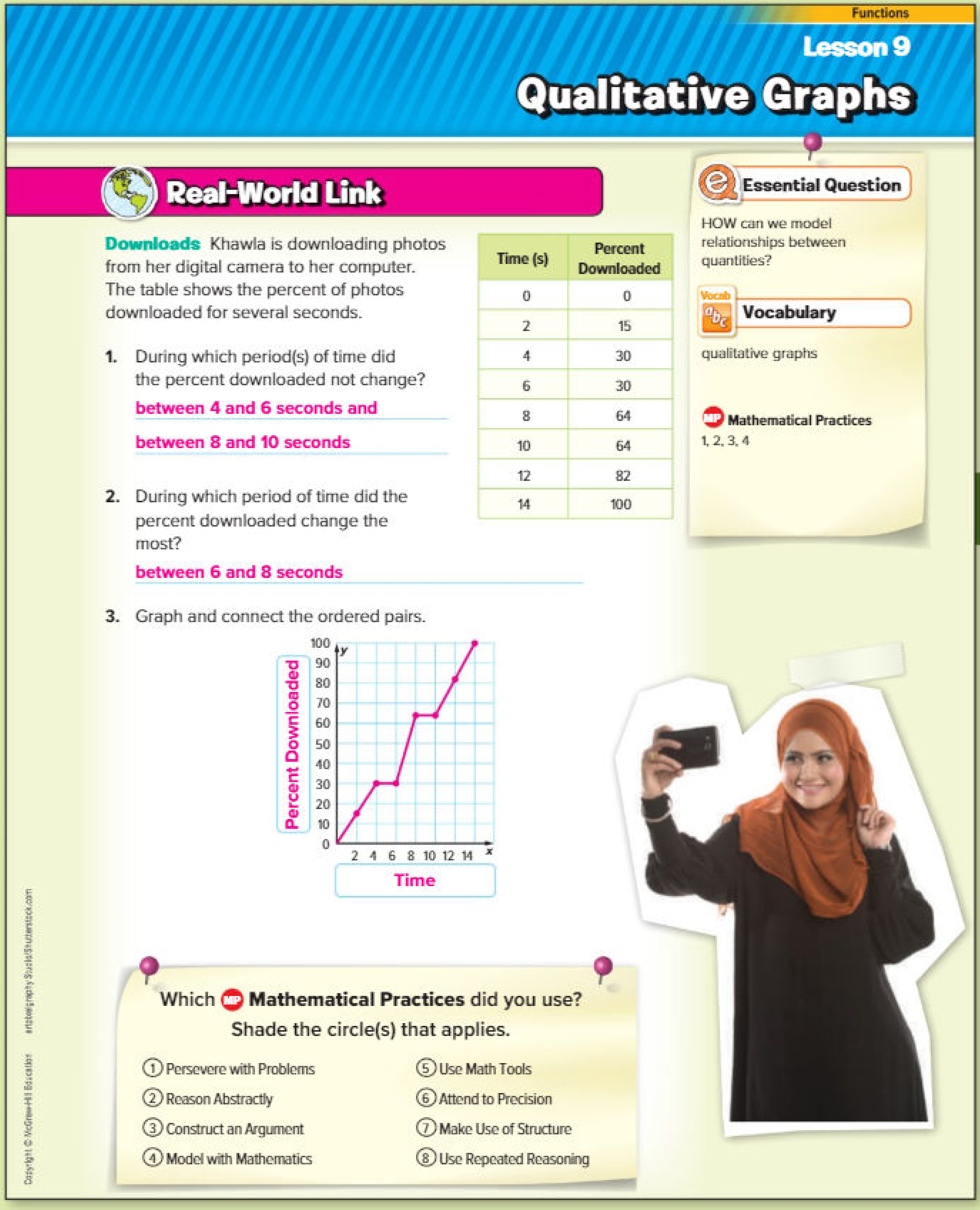
Create

10. MP **Model with Mathematics** Write the equation of a quadratic function that is wider than $y = \frac{2}{3}x^2$. Explain how you know it is wider.

Sample answer: $y = \frac{1}{3}x^2$; The value of the absolute value of the coefficient is a smaller number so the graph is wider.

11. Inquiry HOW are families of nonlinear functions the same as the parent function? How are families of nonlinear functions different from the parent function?

Sample answer: The shape of the graph is the same in a family of nonlinear functions. The location on the axes may change from one to another. The width and/or orientation may also change.



Functions

Lesson 9

Qualitative Graphs

Real-World Link

Downloads Khawla is downloading photos from her digital camera to her computer. The table shows the percent of photos downloaded for several seconds.

Time (s)	Percent Downloaded
0	0
2	15
4	30
6	30
8	64
10	64
12	82
14	100

1. During which period(s) of time did the percent downloaded not change?
   between 4 and 6 seconds and
   between 8 and 10 seconds
2. During which period of time did the percent downloaded change the most?
   between 6 and 8 seconds
3. Graph and connect the ordered pairs.

Essential Question

HOW can we model relationships between quantities?

Vocabulary

qualitative graphs

Mathematical Practices
1, 2, 3, 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Sketch and describe qualitative graphs.

Coherence connecting within and across grades

Previous	Now	Next
Students graphed and analyzed linear and quadratic graphs.	Students analyze qualitative graphs.	Students will analyze graphical displays of data and make predictions from the displays.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 351.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL LA **Think-Pair-Share** Give students 1 minute to think about the ordered pairs in the table and what a graph of the ordered pairs might look like. Have them share their responses with a partner. Then call on one student to share their response within a small group or large group discussion. MP1

Alternate Strategy

AL Present a measuring cup and a bucket to the class. Tell the class that each is going to be used to fill a small pool. Have a class discussion about how the water level of the pool will change over time in each case. Then sketch a graph on the board to represent each situation.

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. **Analyze a qualitative graph.**

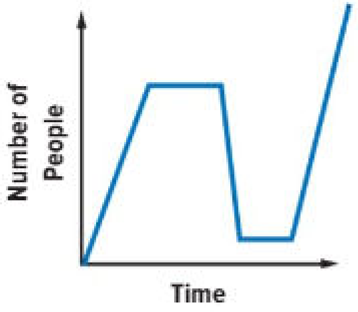
AL
- *Describe the graph.* The graph increases, then stays the same, then decreases.
- *Describe the graph in the context of the problem.* The water level rises, stays the same, then falls.
- *Finally, create a scenario where this might happen.* You close the drain in a bathtub, turn the water on in a bathtub, turn the water off and let the water stand, then open the drain and let the bathtub drain.

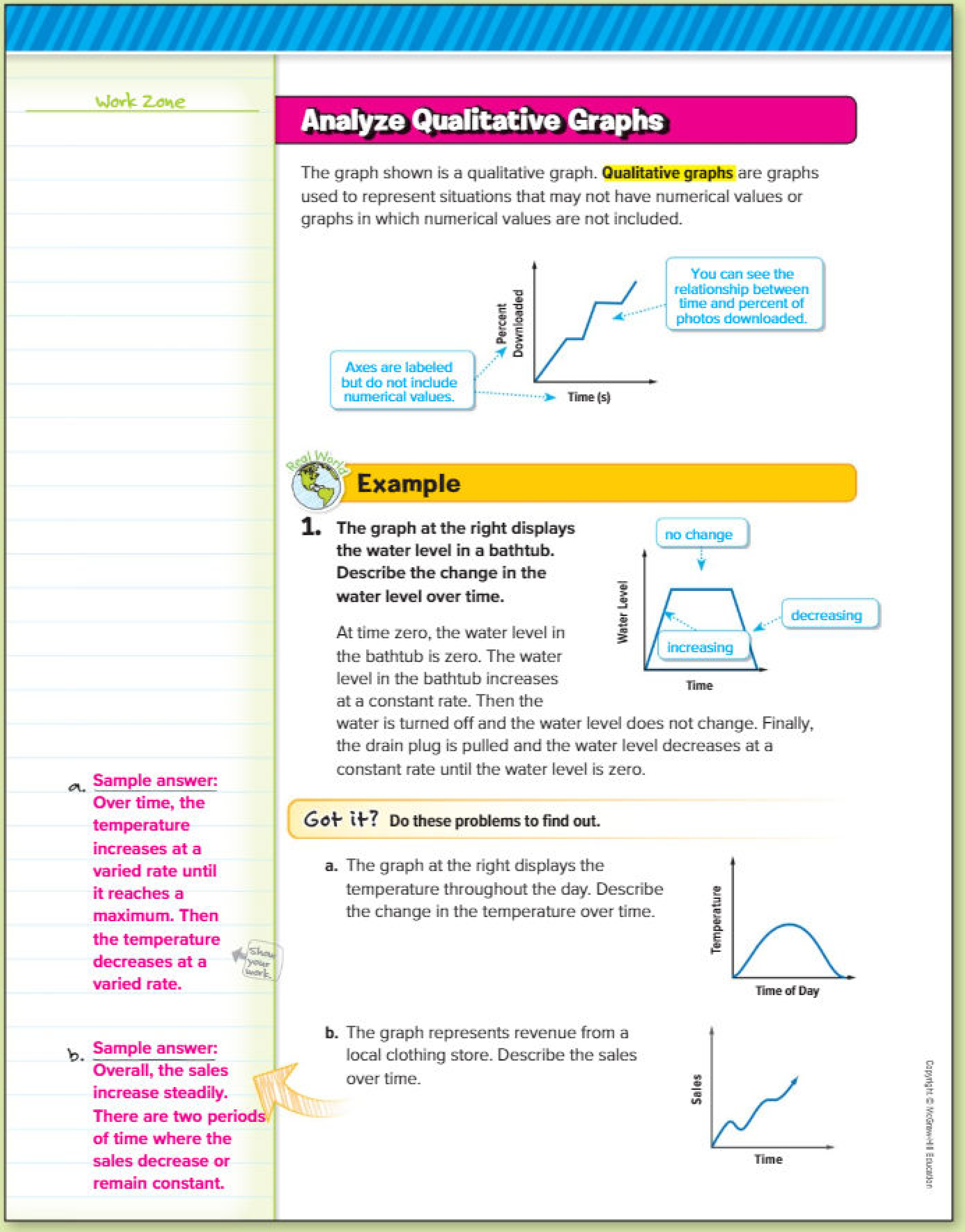
OL
- *Describe what is happening to the water in the first part of the graph.* As time increases, the water level increases at a constant rate.
- *Describe what is happening to the water in the second part of the graph.* As time increases, the water level stays the same.
- *Describe what is happening to the water in the third part of the graph.* As time increases, the water level decreases at a constant rate.

BL
- *Suppose the vertical axis is labeled Temperature. Create a scenario that would describe the graph.* Sample answer: Food is heated in a microwave, placed in an oven to keep it warm, then left out on a table to cool.

Need Another Example?

The graph displays the number of people at a restaurant during the morning and afternoon. Describe the change in the number of people over time. Sample answer: The number of people increases during breakfast and then stays the same. After breakfast, the number of people decreases and then stays the same. Finally, the number of people increases during lunch.





Work Zone

Analyze Qualitative Graphs

The graph shown is a qualitative graph. **Qualitative graphs** are graphs used to represent situations that may not have numerical values or graphs in which numerical values are not included.

Real World

Example

1. **The graph at the right displays the water level in a bathtub. Describe the change in the water level over time.**

At time zero, the water level in the bathtub is zero. The water level in the bathtub increases at a constant rate. Then the water is turned off and the water level does not change. Finally, the drain plug is pulled and the water level decreases at a constant rate until the water level is zero.

Got it? **Do these problems to find out.**

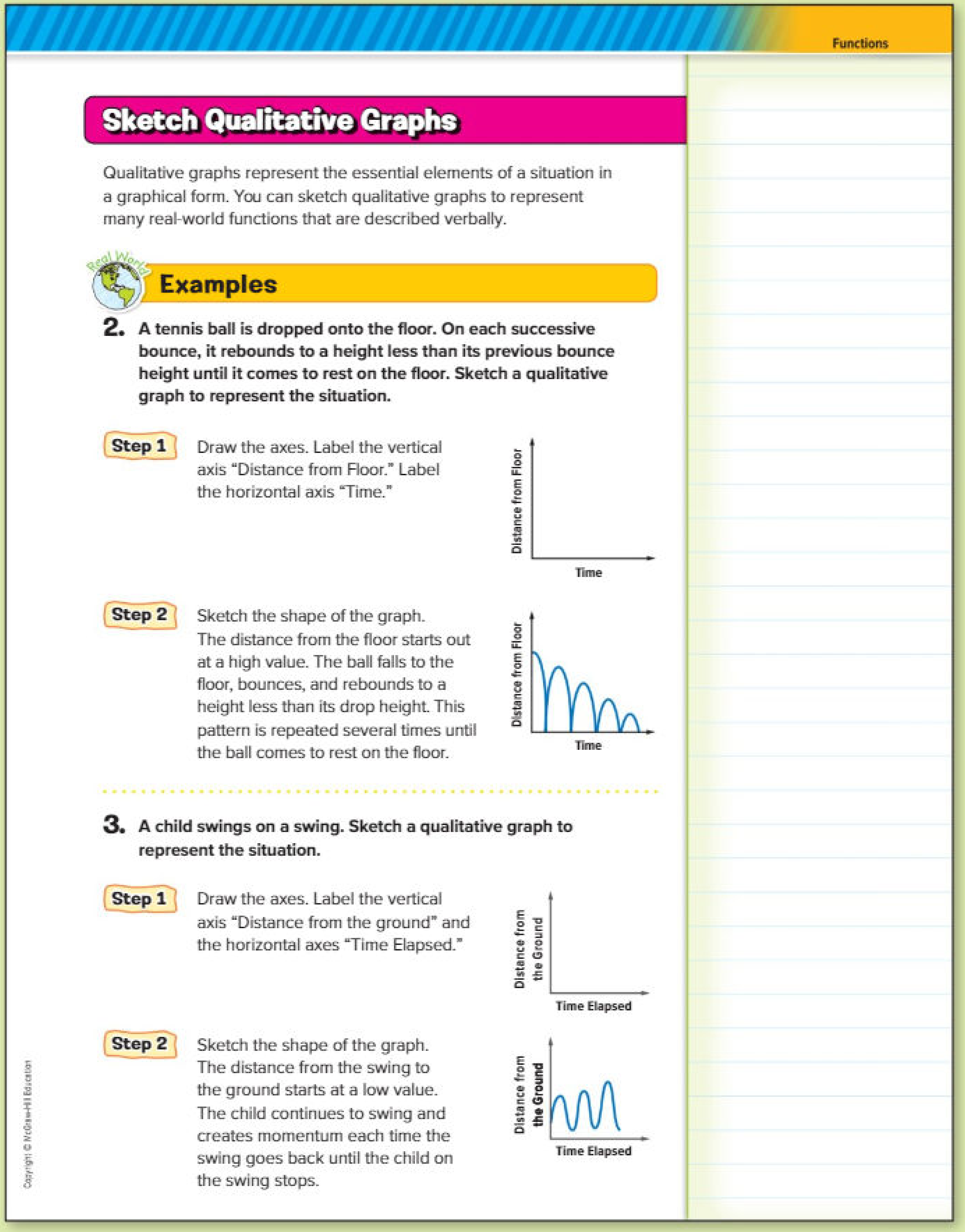
a. The graph at the right displays the temperature throughout the day. Describe the change in the temperature over time.

b. The graph represents revenue from a local clothing store. Describe the sales over time.

a. Sample answer: Over time, the temperature increases at a varied rate until it reaches a maximum. Then the temperature decreases at a varied rate.

Show your work.

b. Sample answer: Overall, the sales increase steadily. There are two periods of time where the sales decrease or remain constant.



Functions

Sketch Qualitative Graphs

Qualitative graphs represent the essential elements of a situation in a graphical form. You can sketch qualitative graphs to represent many real-world functions that are described verbally.

Examples

2. **A tennis ball is dropped onto the floor. On each successive bounce, it rebounds to a height less than its previous bounce height until it comes to rest on the floor. Sketch a qualitative graph to represent the situation.**

Step 1 Draw the axes. Label the vertical axis "Distance from Floor." Label the horizontal axis "Time."

Step 2 Sketch the shape of the graph. The distance from the floor starts out at a high value. The ball falls to the floor, bounces, and rebounds to a height less than its drop height. This pattern is repeated several times until the ball comes to rest on the floor.

3. **A child swings on a swing. Sketch a qualitative graph to represent the situation.**

Step 1 Draw the axes. Label the vertical axis "Distance from the ground" and the horizontal axes "Time Elapsed."

Step 2 Sketch the shape of the graph. The distance from the swing to the ground starts at a low value. The child continues to swing and creates momentum each time the swing goes back until the child on the swing stops.

Examples

2. **Draw a qualitative graph.**

AL • *Picture a tennis ball after it has dropped. How does the height of the ball after it bounces once compare to when you dropped it?* The height is lower.

OL • *Picture a tennis ball after it is dropped. What happens to it with each bounce?* With each bounce, the ball bounces lower and lower.

• *In this situation, what are the dependent and independent variables?* Time is the independent variable, and the height of the ball is the dependent variable.

BL • *What happens to the height of the ball as the time increases?* The height of the ball rises and falls, but the maximum height is lower with each bounce.

Need Another Example?

A train slowly increases speed, then maintains a constant speed, then quickly speeds up. Sketch a qualitative graph to represent the sitration. See Answer Appendix.

3. **Draw a qualitative graph.**

AL • *In this situation, what are the independent and dependent variables?* The elapsed time is the independent variable, and the distance from the ground is the dependent variable.

• *Is the graph of the function linear or nonlinear?* nonlinear

OL • *When a child is swinging, what happens to the child's distance from the ground?* The distance from the ground increases and decreases continually.

BL • *On the graph, why does each curve get closer together?* The speed of the swing increases with momentum so the time that passes is shortened with each swing.

Need Another Example?

Nasser is playing fetch with his cat. As he throws the ball away from him, the cat runs and gets the ball and brings it back. Sketch a qualitative graph to represent the situation.
See Answer Appendix.

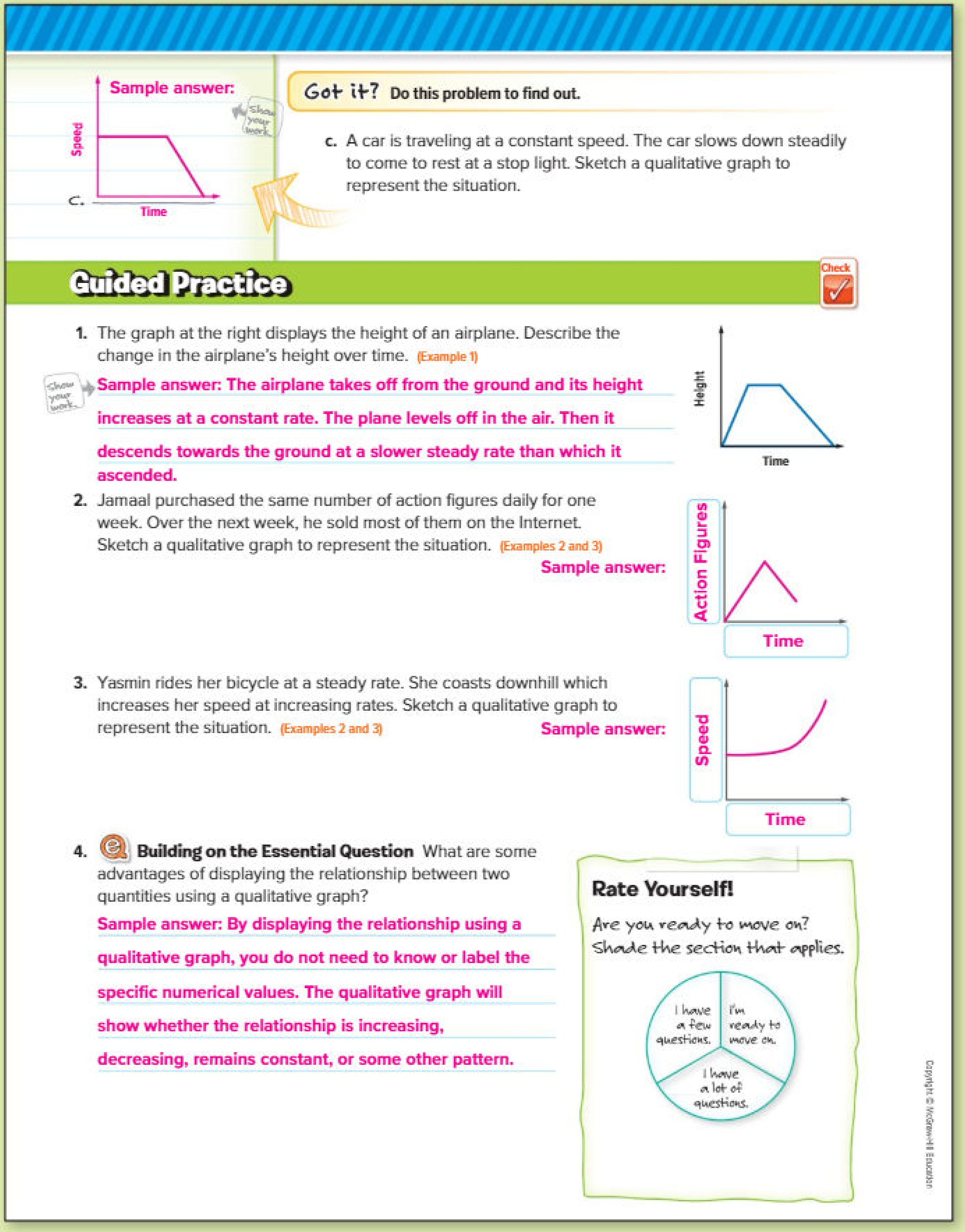
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL **LA** **Pairs Discussion** Have students work in pairs to complete Exercises 1–3. Encourage discussion of how different interpretations of the situation can happen. Students trade their solutions with another pair of students and discuss the differences. **MP** **1, 4**

BL **LA** **Trade-a-Problem** Have students create their own problem, similar to Exercises 2 and 3. Challenge them to have at least four changes in their problem. Students trade their problems, create a graph for each other's problem, and compare graphs. If the graphs are not similar, students discuss the differences. Remind students that different interpretations of the situation are possible, especially if they are not specific in their descriptions. **MP** **1, 3, 4**



Got it? Do this problem to find out.

c. A car is traveling at a constant speed. The car slows down steadily to come to rest at a stop light. Sketch a qualitative graph to represent the situation.

Guided Practice

Check

1. The graph at the right displays the height of an airplane. Describe the change in the airplane's height over time. (Example 1)

Sample answer: The airplane takes off from the ground and its height increases at a constant rate. The plane levels off in the air. Then it descends towards the ground at a slower steady rate than which it ascended.

2. Jamaal purchased the same number of action figures daily for one week. Over the next week, he sold most of them on the Internet. Sketch a qualitative graph to represent the situation. (Examples 2 and 3)

Sample answer:

3. Yasmin rides her bicycle at a steady rate. She coasts downhill which increases her speed at increasing rates. Sketch a qualitative graph to represent the situation. (Examples 2 and 3)

Sample answer:

4. **Building on the Essential Question** What are some advantages of displaying the relationship between two quantities using a qualitative graph?

Sample answer: By displaying the relationship using a qualitative graph, you do not need to know or label the specific numerical values. The qualitative graph will show whether the relationship is increasing, decreasing, remains constant, or some other pattern.

Rate Yourself!

Are you ready to move on?
Shade the section that applies.

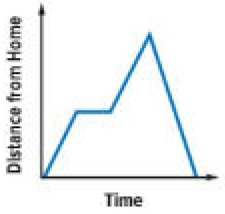
I have a few questions.
I'm ready to move on.
I have a lot of questions.

Name ____________________ My Homework ____________________

Independent Practice

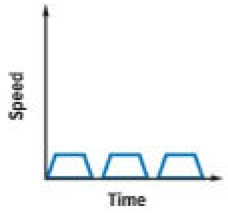
1. The graph below displays the distance from Hasan's home as he walks in his neighborhood. Describe the change in the distance from his home over time. (Example 1)

Show your work



Sample answer: Hasan starts out from his home. He walks away from his home, stops, and walks further away from home. Then he walks towards home.

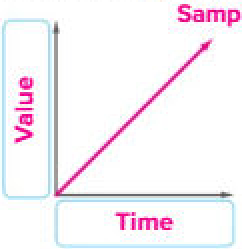
2. The graph below displays the speed of a city bus as it stops frequently to pick up passengers. Describe the change in the speed over time. (Example 1)



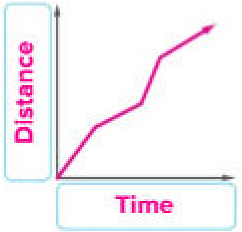
Sample answer: The speed of the bus increases at a constant rate, then remains constant, and **then slows** down. As it is picking up passengers, the speed is zero. This pattern continues.

3. A grand piano that is over 100 years old has increased in value rapidly from when it was first purchased. Sketch a qualitative graph to represent this situation. (Examples 2 and 3)

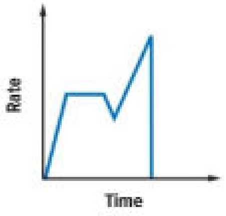
Sample answer:



4. An athlete alternates between running and walking during a workout. Sketch a qualitative graph to represent this situation. (Examples 2 and 3) Sample answer:



5. **Reason Abstractly** Use the graph at the right which displays the rate at which Hamad hiked along a path.



a. What situation could the horizontal line segment represent?
Sample answer: Hamad hikes at a steady rate.

b. What situation could the vertical line segment represent?
Sample answer: Hamad suddenly stops hiking.

c. Did Hamad's rate increase or decrease during the first portion of his hike? Explain your reasoning.
increase; Sample answer: The graph rises from left to right at the beginning.

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–4, 9–14	5, 15, 16	6–8
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7, 8, 15, 16
OL	On Level	1–7 odd, 8, 15, 16
BL	Beyond Level	5–8, 15, 16

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	6
2 Reason abstractly and quantitatively.	5
3 Construct viable arguments and critique the reasoning of others.	7, 8
4 Model with mathematics.	13

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

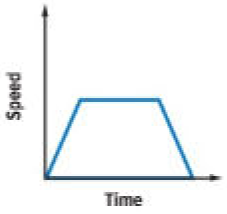
Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

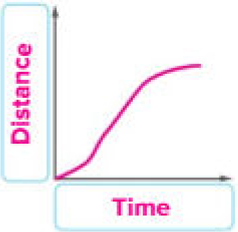
Have students draw a quantitative graph on a small piece of paper. Be sure they label the axes. Then instruct the students to choose one part of their graph and write a brief description of what it represents. See students' work.

H.O.T. Problems Higher Order Thinking



6. **Persevere with Problems** The graph at the right displays the speed of a car as time increases.

   a. Draw a qualitative graph that represets the distance the car travels as time increases.

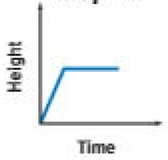


   b. Describe how the distance changes as time passes.

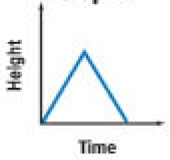
   Sample answer: As the time increases, the distance increases at a varied rate and then levels off when the car stops.

7. **Reason Inductively** A tree grows steadily. When it reaches a specific height, it stops growing. Which graph displays this relationship? Explain your reasoning to a classmate.

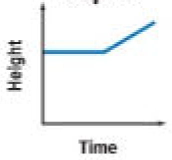
Graph A



Graph B

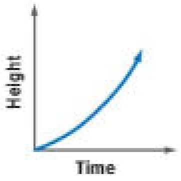


Graph C



Graph A; Sample answer: Graph A increases from left to right at a constant rate then levels off. This represents a tree growing steadily before it stops growing.

8. **Reason Inductively** The graph below represents the height of a rocket after being launched.



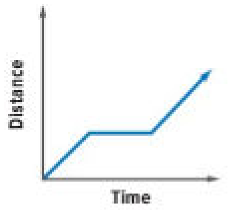
Describe the change in the height of the rocket over time.

Sample answer: The height of the rocket increases as the time after launch increases.

Name ______________________ My Homework ______________________

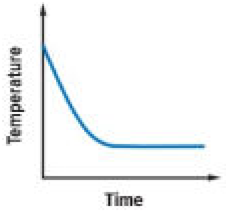
Extra Practice

9. The graph below displays the distance Rana rode on her bike. Describe the change in the distance over time.



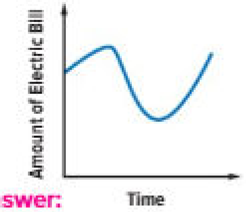
Rana rode her bike at a constant rate in the beginning. She then stopped riding for a period of time. Then she continued riding at a constant rate.

10. The graph below displays the temperature of a cup of hot chocolate. Describe the change in the temperature over time.



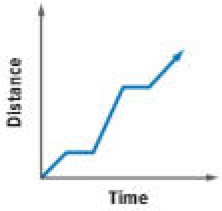
Sample answer: The temperature cools down rapidly in the beginning. Then it cools down at a slower rate and levels off.

11. A graph of Mrs. Reham's electric bill throughout the year, starting in June, is shown below. Describe the change in the bill over time.



Sample answer: Mrs. Reham's electric bill starts out high in June, increases until about August, and then decreases throughout the autumn and winter. In the spring, the electric bill increases again.

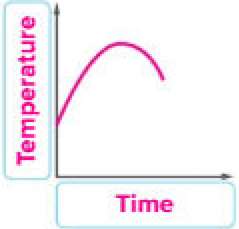
12. The graph below displays the distance covered on a long road trip. Describe the change in the distance over time.



Sample answer: The graph shows the car traveling at a constant speed then stopping, and then moving at a faster speed. The car stops a second time, then continues traveling.

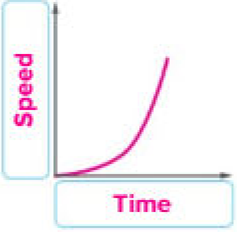
13. **Model with Mathematics** The outside temperature rises throughout the day at varied rates, then drops at night. Sketch a qualitative graph to represent the situation.

Sample answer:



14. A lion cub is resting in the grass. He sees another lion cub nearby and races after it, picking up speed as it runs. Sketch a qualitative graph to represent the situation.

Sample answer:



Power Up! Test Practice

Exercises 15 and 16 prepare students for more rigorous thinking needed for the assessment.

15.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

16.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly answer the each part of the question.
1 point	Students correctly select 3 or 4 of the segments.

Power Up! Test Practice

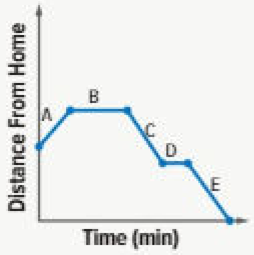
15. The graph represents the amount of pies sold by a bakery over the course of one day. Determine if each of the following statements is true or false.

a. The red segment represents sales increasing at a constant rate. ☑ True ☐ False

b. The blue segment represents sales, decreasing at a constant rate. ☐ True ☑ False

c. The green segment represents sales increasing, but not at a constant rate. ☑ True ☐ False

16. The graph represents Sally's activities on her way home from school on a given day. Match each statement with its corresponding segment on the graph.



Sally rides her bike from the park to Sumayya's house. Segment C

Sally rides her bike home from Sumayya's house. Segment E

Sally plays at the park. Segment B

Sally visits Sumayya at her house. Segment D

Sally rides her bike from school to the park. Segment A

Spiral Review

Simplify each expression.

17. $2(p + 8) + 4 =$ $2p + 20$

18. $(18 + t)(-3) + 9 =$ $-3t - 45$

19. $30q(2) =$ $60q$

20. $-5(n + 16) - 7 =$ $-5n - 87$

21ST CENTURY CAREER in Physical Therapy

Physical Therapist

Are you a compassionate person? Do you have a strong desire to help others? If so, a career as a physical therapist might be a good choice for you. Physical therapists help restore function, improve mobility, and relieve pain of patients suffering from injuries or disease. One of their jobs is to teach exercises or recommend activities to help patients regain balance, flexibility, endurance, and strength.

Is This the Career for You?

Are you interested in a career as a physical therapist? Take some of the following courses in high school.

- Algebra
- Biology
- Chemistry
- Introduction To Physical Therapy

Turn the page to find out how math relates to a career in Physical Therapy.

(t)UpperCut Images/SuperStock, (b)Tyler Olson/Shutterstock.com

Focus narrowing the scope

Objective Apply mathematics to problems arising in the workplace.

This lesson emphasizes MP **Mathematical Practice 4** Model with Mathematics.

Coherence connecting within and across grades

Previous
Students used words, tables, equations, and graphs to represent functions.

Now
Students apply the content standard to solve problems in the workplace.

Rigor pursuing concepts, fluency, and applications

See the Career Project on page 356.

ENGAGE **EXPLORE** EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ask students to read the information on the student page about physical therapists and answer the following questions.

Ask:

- *What do physical therapists do?* Sample answer: They help people regain balance, flexibility, endurance, and strength by teaching patients to do specific exercises.
- *What kinds of classes should you take if you want to become a physical therapist?* Sample answer: Algebra, Biology, Chemistry, and Introduction to Physical Therapy

2 Collaborate

AL LA **Pairs Discussion** Have students work in pairs to complete the following questions relating to Exercise 3. Have them trade their solutions with another pair of students and discuss any differences. MP 1, 2, 4

Ask:

- *What information do you need to solve this problem?* Sample answer: I need to find the rate of change using information in the table.
- *How far will the runner travel after 1 hour 15 minutes?* 11.25 miles

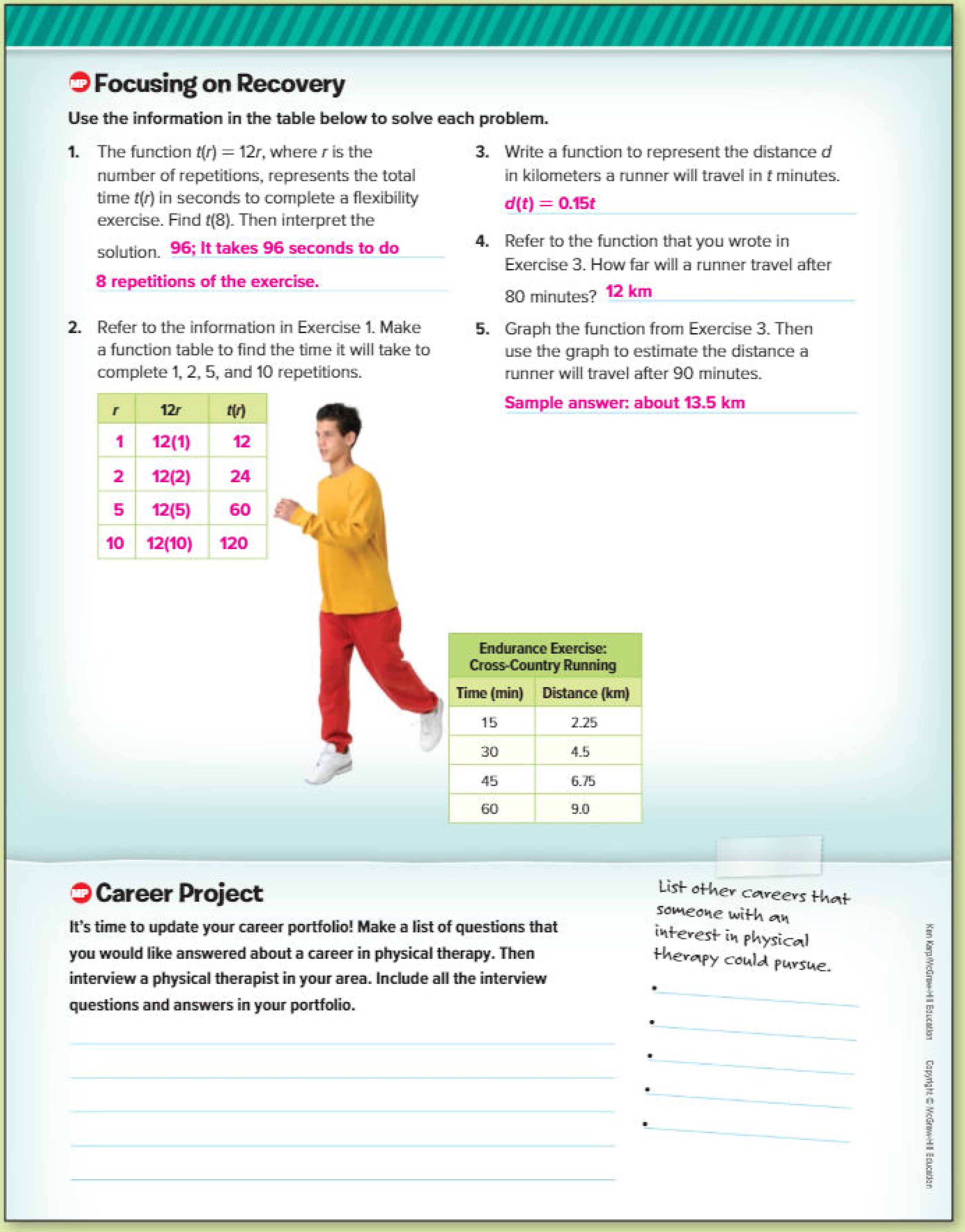
BL LA **Trade-a-Problem** Have students create their own real-world problem similar to Exercise 1. Students trade their problems and solve. If the solutions do not agree, have students work together to find the errors. MP 1, 4

Career Portfolio

When students complete this page, have them add it to their Career Portfolio.

Career Facts

According to the U.S. Bureau of Labor Statistics, employment of physical therapists is expected to grow significantly over the next few years as the population ages and people live longer.



MP Focusing on Recovery

Use the information in the table below to solve each problem.

1. The function $t(r) = 12r$, where r is the number of repetitions, represents the total time $t(r)$ in seconds to complete a flexibility exercise. Find $t(8)$. Then interpret the solution. **96; It takes 96 seconds to do 8 repetitions of the exercise.**

2. Refer to the information in Exercise 1. Make a function table to find the time it will take to complete 1, 2, 5, and 10 repetitions.

r	$12r$	$t(r)$
1	12(1)	12
2	12(2)	24
5	12(5)	60
10	12(10)	120

3. Write a function to represent the distance d in kilometers a runner will travel in t minutes. **$d(t) = 0.15t$**

4. Refer to the function that you wrote in Exercise 3. How far will a runner travel after 80 minutes? **12 km**

5. Graph the function from Exercise 3. Then use the graph to estimate the distance a runner will travel after 90 minutes. **Sample answer: about 13.5 km**

Endurance Exercise: Cross-Country Running

Time (min)	Distance (km)
15	2.25
30	4.5
45	6.75
60	9.0

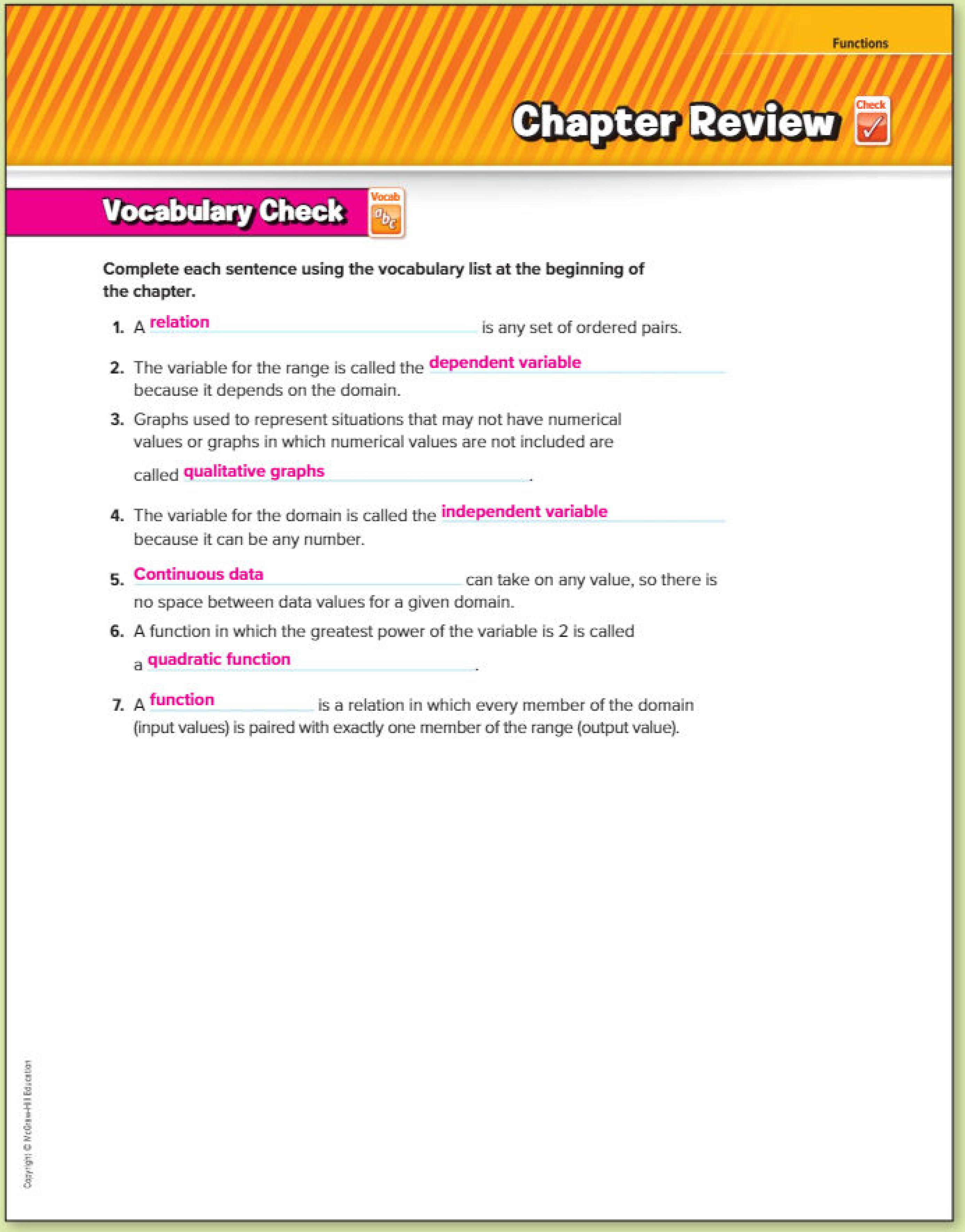
MP Career Project

It's time to update your career portfolio! Make a list of questions that you would like answered about a career in physical therapy. Then interview a physical therapist in your area. Include all the interview questions and answers in your portfolio.

List other careers that someone with an interest in physical therapy could pursue.

-
-
-
-
-

Ken Karp/McGraw-Hill Education



Chapter Review

Vocabulary Check

Complete each sentence using the vocabulary list at the beginning of the chapter.

1. A relation is any set of ordered pairs.
2. The variable for the range is called the dependent variable because it depends on the domain.
3. Graphs used to represent situations that may not have numerical values or graphs in which numerical values are not included are called qualitative graphs.
4. The variable for the domain is called the independent variable because it can be any number.
5. Continuous data can take on any value, so there is no space between data values for a given domain.
6. A function in which the greatest power of the variable is 2 is called a quadratic function.
7. A function is a relation in which every member of the domain (input values) is paired with exactly one member of the range (output value).

Vocabulary Check

LA **Roundrobin** Have students work in small groups to complete the Vocabulary Check. Have Student 1 respond to Exercise 1, reading the word and sentence aloud. Have the rest of the group listen, ask any clarifying questions, and state whether they agree or disagree. If there are any disagreements, have students work together to resolve them. Repeat for each successive exercise, having students take turns. MP1

Alternate Strategy

AL LA To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include the following terms.

- continuous data (Lesson 4)
- dependent variable (Lesson 3)
- function (Lesson 3)
- independent variable (Lesson 3)
- quadratic function (Lesson 8)
- qualitative graphs (Lesson 9)
- relation (Lesson 2)

Key Concept Check

FOLDABLES **LA** A completed Foldable for this chapter should include a review of relations and functions.

If you choose not to use this Foldable, have students write a brief review of the Key Concepts found throughout the chapter and give an example of each.

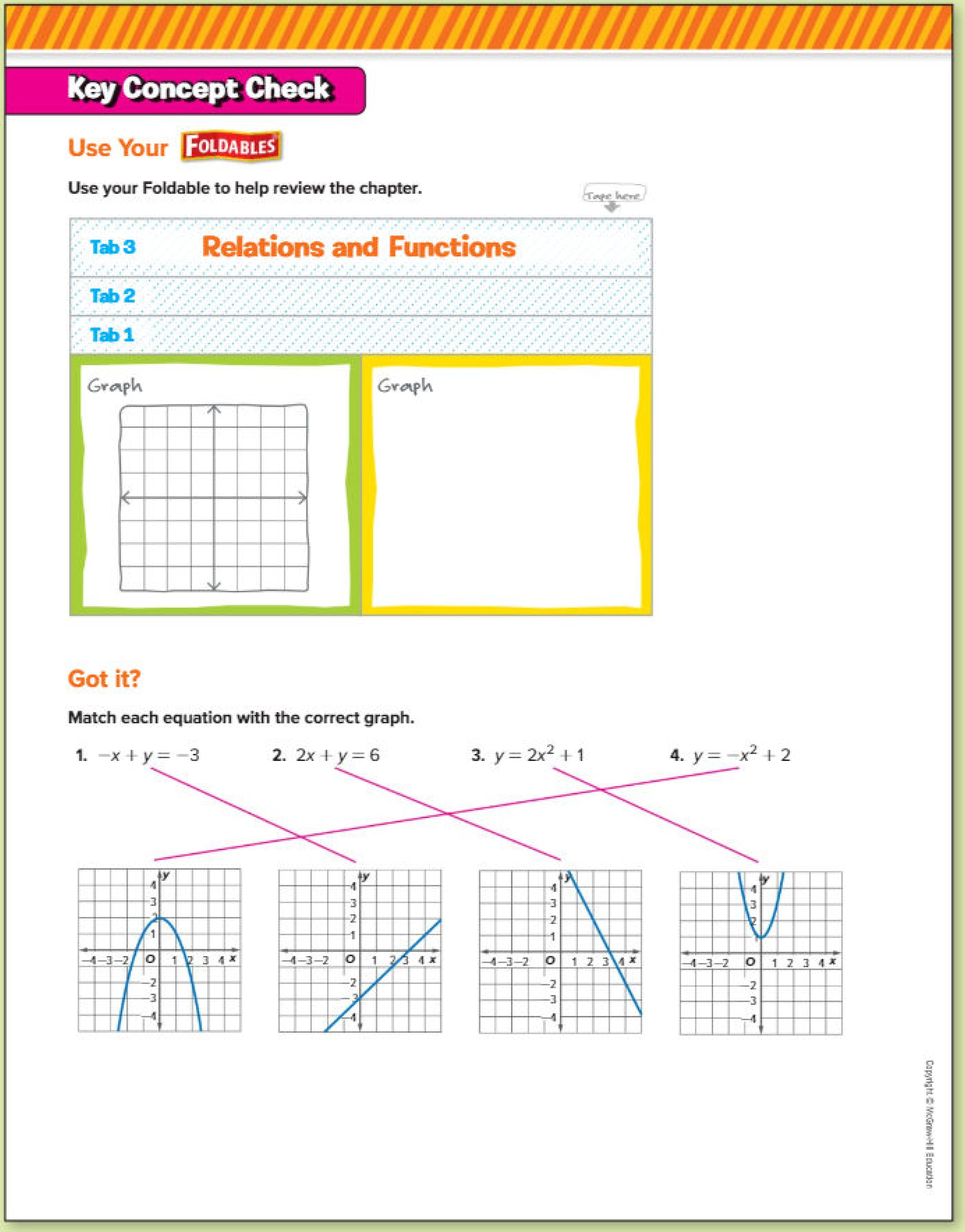
Ideas for Use

LA **Solo-Pair-Solo** Have students complete their Foldable for the chapter, if they have not already. Then have them meet with a partner to discuss how they have completed their Foldable and how it can be used to help them review the material they learned in the chapter. Have them discuss any similarities and differences in how each of them completed their Foldable. Then have students work individually to make any desired modifications to their own Foldable. **MP 1, 6**

Got It?

If students have trouble with Exercises 1–4, they may need help with the following concepts.

Concept	Exercise(s)
graphs of linear functions (Lesson 4)	1, 2
graphs of quadratic functions (Lesson 8)	3, 4



Key Concept Check

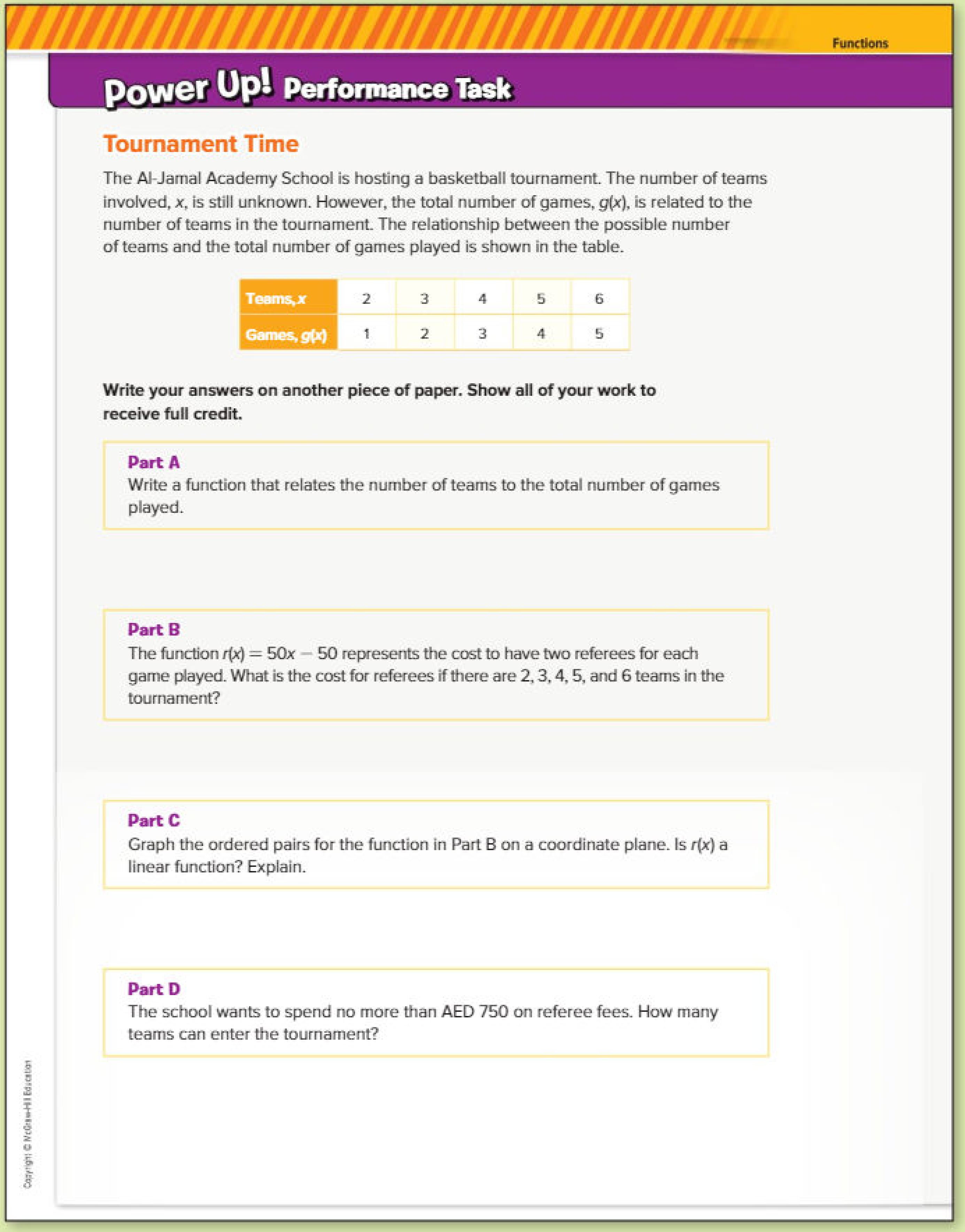
Use Your FOLDABLES

Use your Foldable to help review the chapter.

Got it?

Match each equation with the correct graph.

1. $-x + y = -3$
2. $2x + y = 6$
3. $y = 2x^2 + 1$
4. $y = -x^2 + 2$



Functions

Power Up! Performance Task

Tournament Time

The Al-Jamal Academy School is hosting a basketball tournament. The number of teams involved, *x*, is still unknown. However, the total number of games, $g(x)$, is related to the number of teams in the tournament. The relationship between the possible number of teams and the total number of games played is shown in the table.

Teams, x	2	3	4	5	6
Games, $g(x)$	1	2	3	4	5

Write your answers on another piece of paper. Show all of your work to receive full credit.

Part A
Write a function that relates the number of teams to the total number of games played.

Part B
The function $r(x) = 50x - 50$ represents the cost to have two referees for each game played. What is the cost for referees if there are 2, 3, 4, 5, and 6 teams in the tournament?

Part C
Graph the ordered pairs for the function in Part B on a coordinate plane. Is $r(x)$ a linear function? Explain.

Part D
The school wants to spend no more than AED 750 on referee fees. How many teams can enter the tournament?

Power Up! Performance Task

This Performance-Based Assessment requires students to solve multi-step problems through abstract reasoning, precision, and perseverance. This practice scenario can be used to help students prepare for the thinking skills that will be used during assessment.

A complete scoring rubric with answers to the Exercises can be found on page PT4.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the **Building on the Essential Question** exercises found in each lesson of the chapter.

- How can you use a graph to write an equation? (p. 272)
- How do tables and graphs represent relations? (p. 280)
- How does the domain affect the range in a function? (p. 290)
- How can functions be used to solve real-world situations? (p. 300)
- What are the advantages and disadvantages to representing a function as an equation instead of a graph? (p. 314)
- How is the initial value of a function represented in a table and in a graph? (p. 322)
- How can you use a table or a graph to determine if a function is linear or nonlinear? (p. 330)
- When does the graph of a quadratic function open upward or downward? (p. 338)
- What are some advantages of displaying the relationship between two quantities using a qualitative graph? (p. 350)

Ideas for Use

LA Jigsaw Assign students to a 4-person learning team. Provide students with sample topic ideas for each box in the graphic organizer, such as ordered pairs, table, graph, and domain and range. Each group member is assigned to one of these topic ideas. Then have all of the students who are assigned to each topic (from varying groups) meet together to discuss how that representation can be used to model a relationship between two quantities. Have students return to their original groups and explain what they learned in their topic groups. MP 1, 2, 4, 5

Track Your Progress

Return to the beginning of the chapter to review the objectives that it addressed. Students should see that their knowledge of the key ideas has increased now that they have completed this chapter.

Reflect

Answering the Essential Question

Use what you learned about functions to complete the graphic organizer. List four ways functions are expressed in this chapter. Sample answers are given.

Ordered Pairs

(2, 3), (−2, 4), (−3, −4), (0, 5)

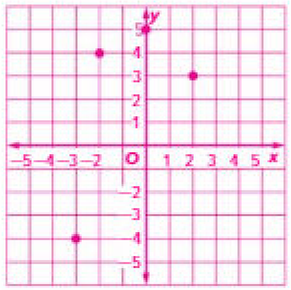
Table

x	y
2	3
−2	4
−3	−4
0	5

Essential Question

HOW can we model relationships between quantities?

Graph

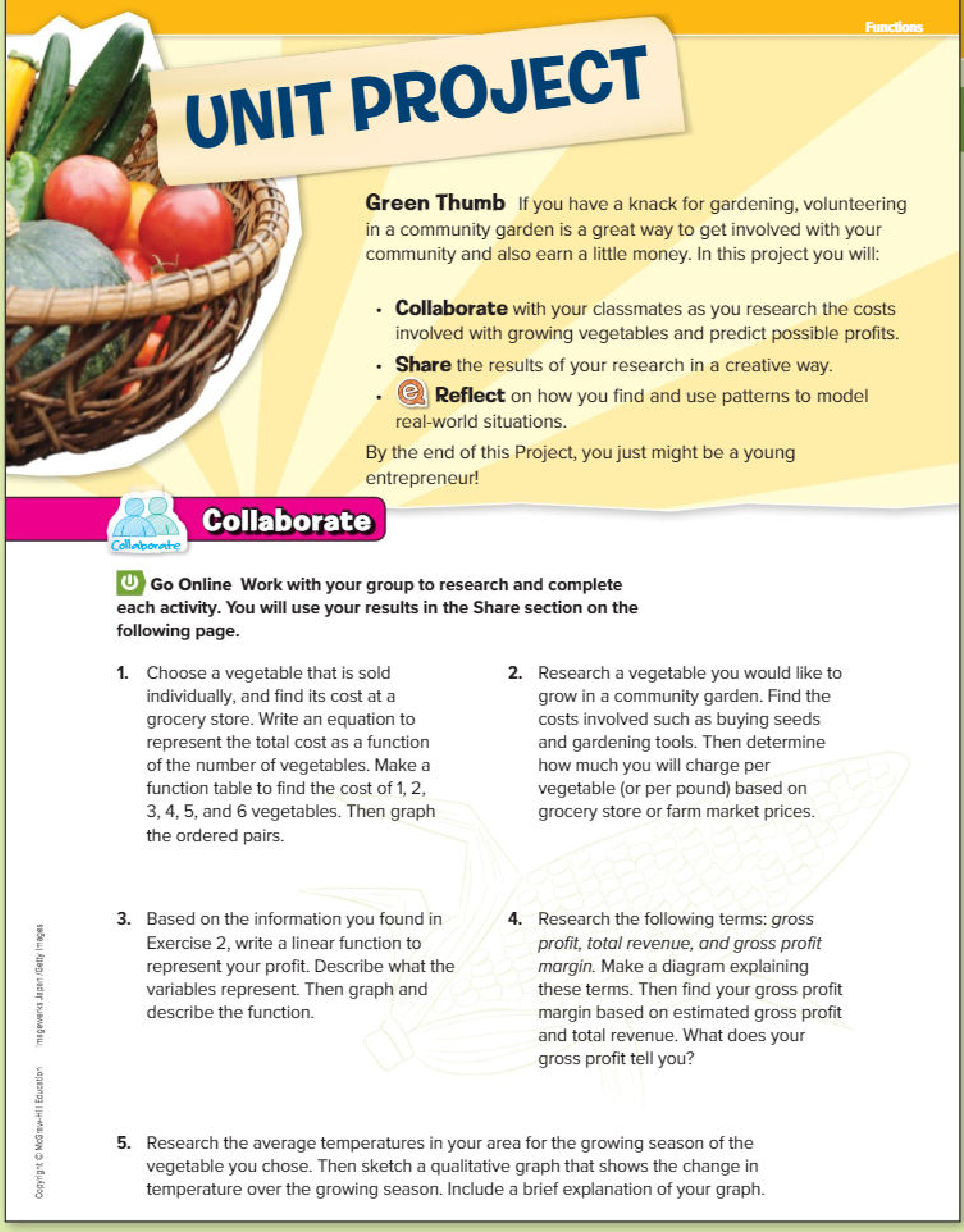


Domain and Range

D: {−3, −2, 0, 2}
R: {−4, 3, 4, 5}

Answer the Essential Question. HOW can we model relationships between quantities?

See students' work.



UNIT PROJECT

Green Thumb If you have a knack for gardening, volunteering in a community garden is a great way to get involved with your community and also earn a little money. In this project you will:

- **Collaborate** with your classmates as you research the costs involved with growing vegetables and predict possible profits.
- **Share** the results of your research in a creative way.
- **Reflect** on how you find and use patterns to model real-world situations.

By the end of this Project, you just might be a young entrepreneur!

Collaborate

Go Online Work with your group to research and complete each activity. You will use your results in the Share section on the following page.

1. Choose a vegetable that is sold individually, and find its cost at a grocery store. Write an equation to represent the total cost as a function of the number of vegetables. Make a function table to find the cost of 1, 2, 3, 4, 5, and 6 vegetables. Then graph the ordered pairs.
2. Research a vegetable you would like to grow in a community garden. Find the costs involved such as buying seeds and gardening tools. Then determine how much you will charge per vegetable (or per pound) based on grocery store or farm market prices.
3. Based on the information you found in Exercise 2, write a linear function to represent your profit. Describe what the variables represent. Then graph and describe the function.
4. Research the following terms: *gross profit, total revenue, and gross profit margin.* Make a diagram explaining these terms. Then find your gross profit margin based on estimated gross profit and total revenue. What does your gross profit tell you?
5. Research the average temperatures in your area for the growing season of the vegetable you chose. Then sketch a qualitative graph that shows the change in temperature over the growing season. Include a brief explanation of your graph.

Launch the Project

Objective Research the costs involved with growing vegetables and predict possible profits from selling vegetables.

Green Thumb

This project is designed to be completed by a group of 4 or 5 students over several days or several weeks. It utilizes concepts from the Functions domain. You may choose to complete this project after completing the chapters within this domain.

Collaborate

Have students work in teams to research information about growing and selling vegetables. Together, they should be able to gather the necessary information to complete Exercises 1–5. Students should show their work on a separate piece of paper.

Collaborate

Share

After each group gives their presentation, have students compare the costs and profits from growing and selling different types of vegetables.

21st Century Skills

You may want your students to connect their projects to a 21st century skill. Check out the suggestion below and on the student page.

connect with Language Arts

Financial Literacy Suppose you decide to sell a variety of produce from your garden. Create a flyer that advertises what produce you have available, as well as price.

On Your Own

Reflect

Students should work on their own to reflect on how the chapter from this unit and the objective of the project relate to the Essential Question.

Collaborate

Share

With your group, decide on a way to share what you have learned about growing and selling vegetables. Some suggestions are listed below, but you can also think of other creative ways to present your information. Remember to show how you used mathematics to complete each of the activities in this project!

- Imagine you sell your vegetables at a farmer's market. Describe your experience in a blog.
- Use a budget spreadsheet to show how your vegetable can generate a profit. Include tables, equations, and graphs.

Check out the note on the right to connect this project with other subjects.

connect with Science

Environmental Literacy Research information about Earth's soil and the qualities needed to grow plants. Some questions to consider are:

- What type of soil allows fruits and vegetables to grow well?
- What type of soil is typically found in your area?
- What could you add to the soil to make it better for growing your plants?

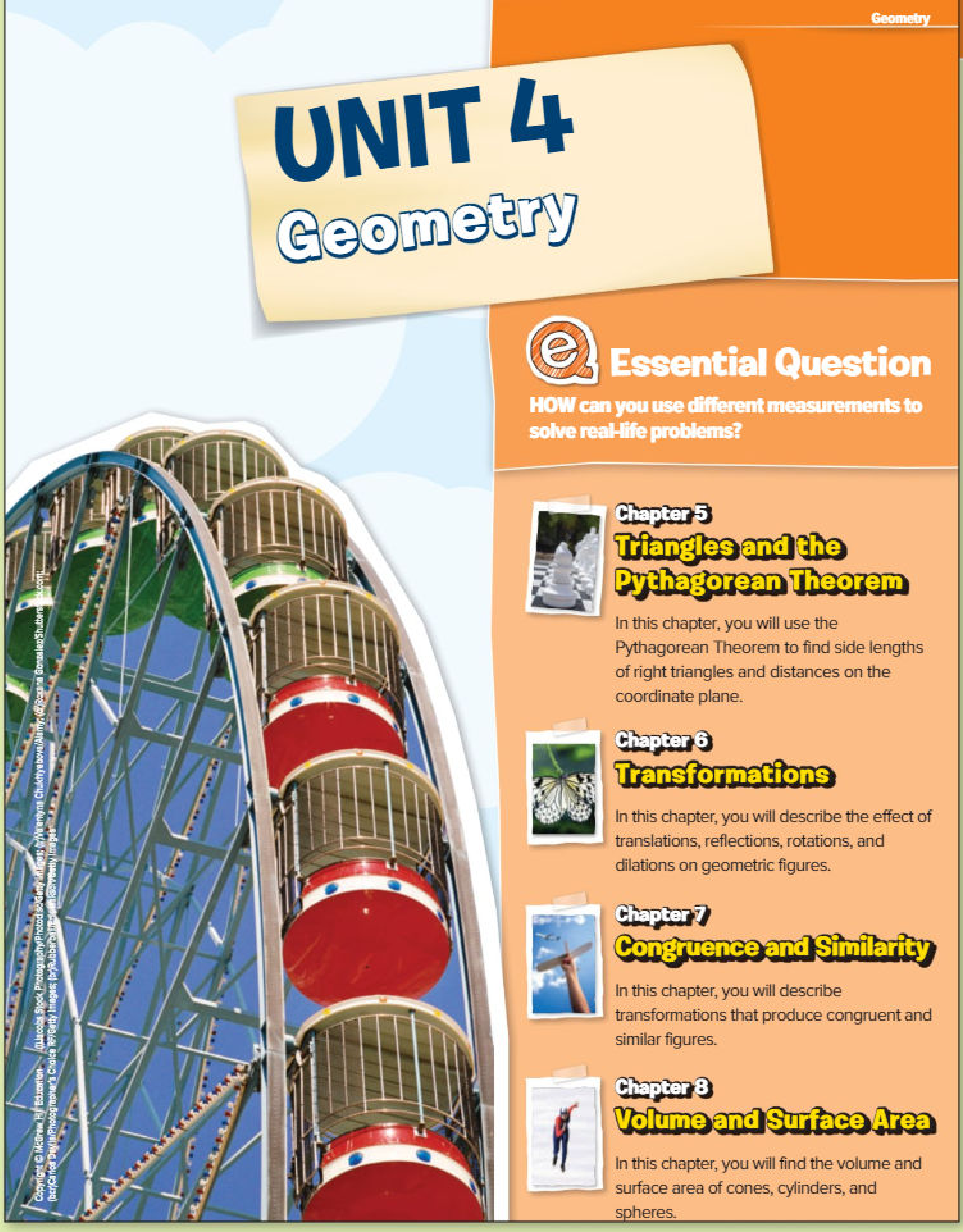
On Your Own

Reflect

6. **Answer the Essential Question** How can you find and use patterns to model real-world situations? **See students' work.**

a. How did you use what you learned about constructing functions in this chapter to find and use patterns to model real-world situations in this project?

b. How did you use what you learned about different representations of functions in this chapter to find and use patterns to model real-world situations in this project?



UNIT 4

Geometry

Essential Question

HOW can you use different measurements to solve real-life problems?

Chapter 5

Triangles and the Pythagorean Theorem

In this chapter, you will use the Pythagorean Theorem to find side lengths of right triangles and distances on the coordinate plane.

Chapter 6

Transformations

In this chapter, you will describe the effect of translations, reflections, rotations, and dilations on geometric figures.

Chapter 7

Congruence and Similarity

In this chapter, you will describe transformations that produce congruent and similar figures.

Chapter 8

Volume and Surface Area

In this chapter, you will find the volume and surface area of cones, cylinders, and spheres.

Essential Question

At the end of this unit, students should be able to answer "How can you use different measurements to solve real-life problems?"

Each chapter explores a different essential question that assists students in answering the unit question. The lessons in each chapter include exercises that lead students to various aspects of the essential question.

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
   - **a.** Lines are taken to lines, and line segments to line segments of the same length.
   - **b.** Angles are taken to angles of the same measure.
   - **c.** Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

continued on page 364

3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Unit Project Preview

Have students brainstorm different types of rides before they make their sketch.

The Unit Project can be found at the end of Chapter 8.

Unit Project Preview

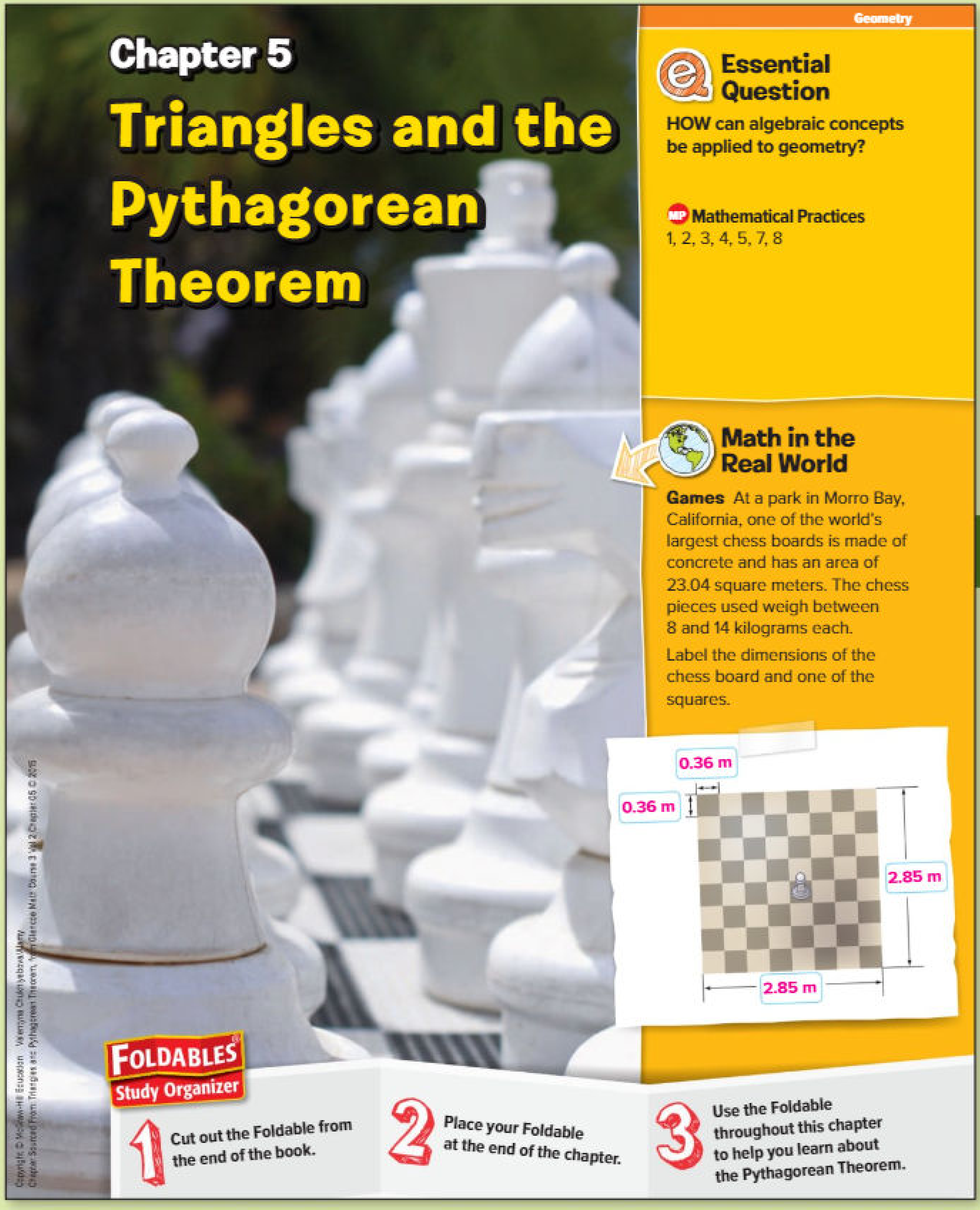
Design That Ride Do you remember how you felt the first time you were on a roller coaster? Scared, excited, terrified? Amusement park rides come in all shapes and sizes and are thrilling to ride.

Although amusement park rides are fun, they are also designed to be very safe. Engineers apply geometric concepts and use precise measurements when designing these rides.

Later in this course, you'll complete a project to find how mathematical concepts are used to design an amusement park ride. But for now, it's time to do an activity in your book. Sketch an amusement park ride. Label some geometric shapes that can be found within your ride.

Amusement Park Ride

Jacobs Stock Photography/Photodisc/Getty Images



Chapter 5

Triangles and the Pythagorean Theorem

Essential Question

HOW can algebraic concepts be applied to geometry?

Mathematical Practices
1, 2, 3, 4, 5, 7, 8

Math in the Real World

Games At a park in Morro Bay, California, one of the world's largest chess boards is made of concrete and has an area of 23.04 square meters. The chess pieces used weigh between 8 and 14 kilograms each.

Label the dimensions of the chess board and one of the squares.

FOLDABLES® Study Organizer

1. Cut out the Foldable from the end of the book.
2. Place your Foldable at the end of the chapter.
3. Use the Foldable throughout this chapter to help you learn about the Pythagorean Theorem.

Focus narrowing the scope

This chapter focuses on content from the **Geometry** (G) domain.

Coherence connecting within and across grades

Previous	Now	Next
Students solved multi-step equations.	Students use algebraic concepts to find relationships between lines, angles, and triangles.	Students will perform transformations on geometric figures.

Rigor pursuing concepts, fluency, and applications

At the end of the chapter, students should be able to answer “HOW can algebraic concepts be applied to geometry?”

Launch the Chapter

Math in the Real World

Games The chessboard is in the shape of a square. Remind students that to find the length of one side of the square, they need to find the square root of the area of the square.

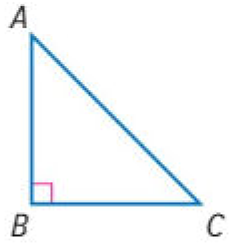
What Tools Do You Need?

Vocabulary Activity

LA As you proceed through the chapter, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define: The hypotenuse is the side opposite the right angle in a right triangle.

Example:



Ask:

- *Which side of the above triangle is the hypotenuse?* $\overline{AC}$

The Structure of Math

LA Have students read The Structure of Math section.

Ask:

- *What kinds of questions are asked in the diamonds in a flowchart?* **Sample answer: In a flowchart, the questions only have yes or no answers.**
- *What do the answers to the questions determine?* **Sample answer: They determine the path to follow.**
- *Where should all of the paths lead?* **the end oval**
- *What other uses can you think of for flowcharts?* **Sample answer: A flowchart can be used to clarify any process such as a computer program, the way a company produces goods, how to cook, how to write a paper in English class, and so on.**

What Tools Do You Need?

Vocabulary

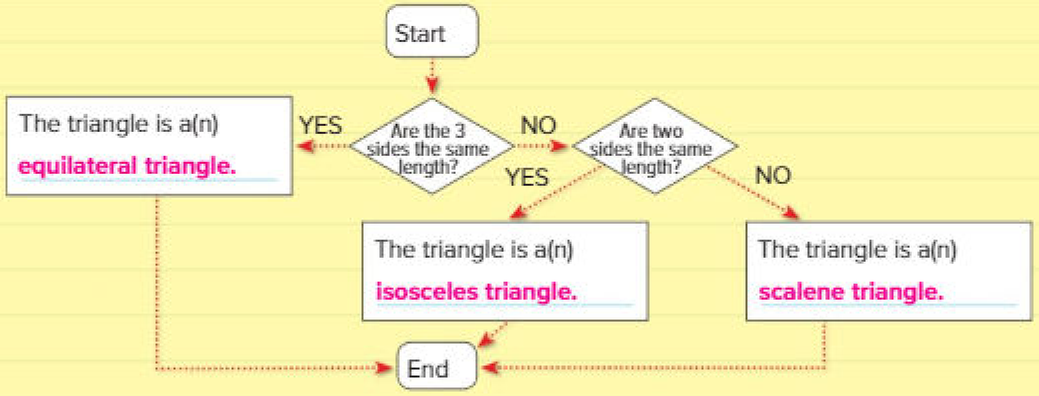
alternate exterior angles
alternate interior angles
converse
corresponding angles
deductive reasoning
Distance Formula
equiangular
exterior angles
formal proof
hypotenuse
inductive reasoning
informal proof
interior angles
legs
paragraph proof
parallel lines
perpendicular lines
polygon
proof
Pythagorean Theorem
regular polygon
remote interior angles
theorem
transversal
triangle
two-column proof

Study Skill: The Structure of Math

Use a Flowchart A *flowchart* is like a map that tells you how to get from the beginning of a problem to the end.

Flowchart Symbols	
◇	A diamond contains a question. You need to stop and make a decision.
▭	A rectangle tells you what to do.
⬭	An oval indicates the beginning or end.

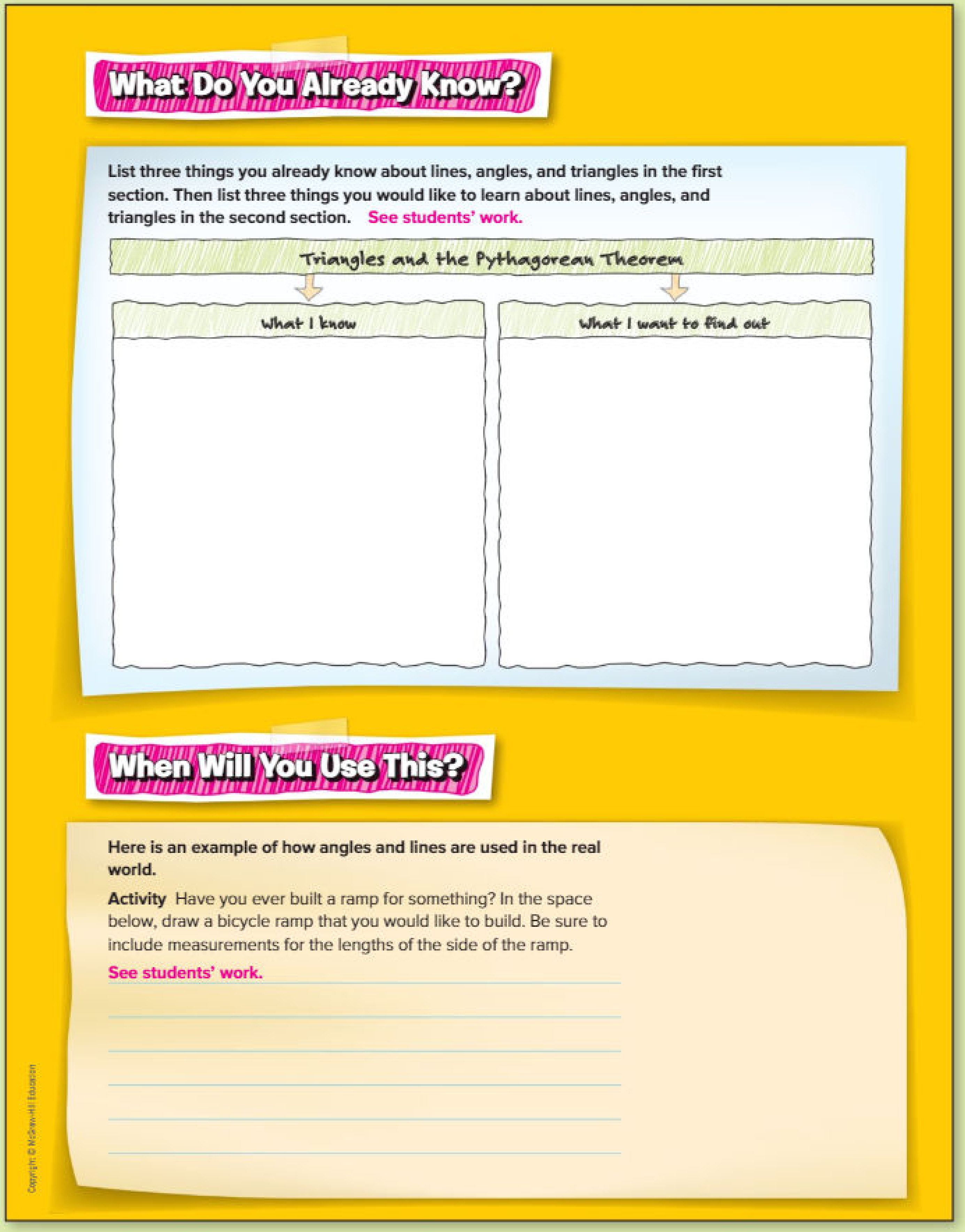
Complete the flowchart to classify triangles by their sides.



The triangle is a(n) **equilateral triangle.**

The triangle is a(n) **isosceles triangle.**

The triangle is a(n) **scalene triangle.**



What Do You Already Know?

List three things you already know about lines, angles, and triangles in the first section. Then list three things you would like to learn about lines, angles, and triangles in the second section. See students' work.

Triangles and the Pythagorean Theorem

What I know	What I want to find out

When Will You Use This?

Here is an example of how angles and lines are used in the real world.

Activity Have you ever built a ramp for something? In the space below, draw a bicycle ramp that you would like to build. Be sure to include measurements for the lengths of the side of the ramp.

See students' work.

What Do You Already Know?

In this activity students assess their prior knowledge by listing three things they already know and three things they would like to learn about concepts in the chapter.

- You may want to add a third option of "I don't know" for those students who do not have any prior knowledge of the topic.
- After completing the chapter, have students return to this page and have them add three new facts that they learned about the topic.

When Will You Use This?

Activity 1

Students use a real-world situation to understand polygons and angle relationships.

Activity 2

Use the Graphic Novel to help students learn about using polygons and parallel lines to solve problems.

Ask:

- *What should be the first steps for Mahmoud and Majed in finding the missing angle?* Sample answer: Identify the shape of the quadrilateral; identify the types of angles and lines in the ramp; set up a formula.

Are You Ready?

Use this page to determine if students have skills that are needed for the chapter.

Quick Review

Students with strong math backgrounds may opt to go directly to the Quick Check.

REVIEW	
Example	**Skill**
1	Solve Equations
2	Graph on the Coordinate Plane

Quick Check

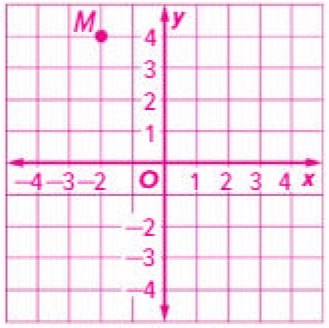
If students have difficulty with the exercises, present another example to clarify any misconceptions.

Exercises 1–3

Solve $90 + 62 + a = 180$. 28

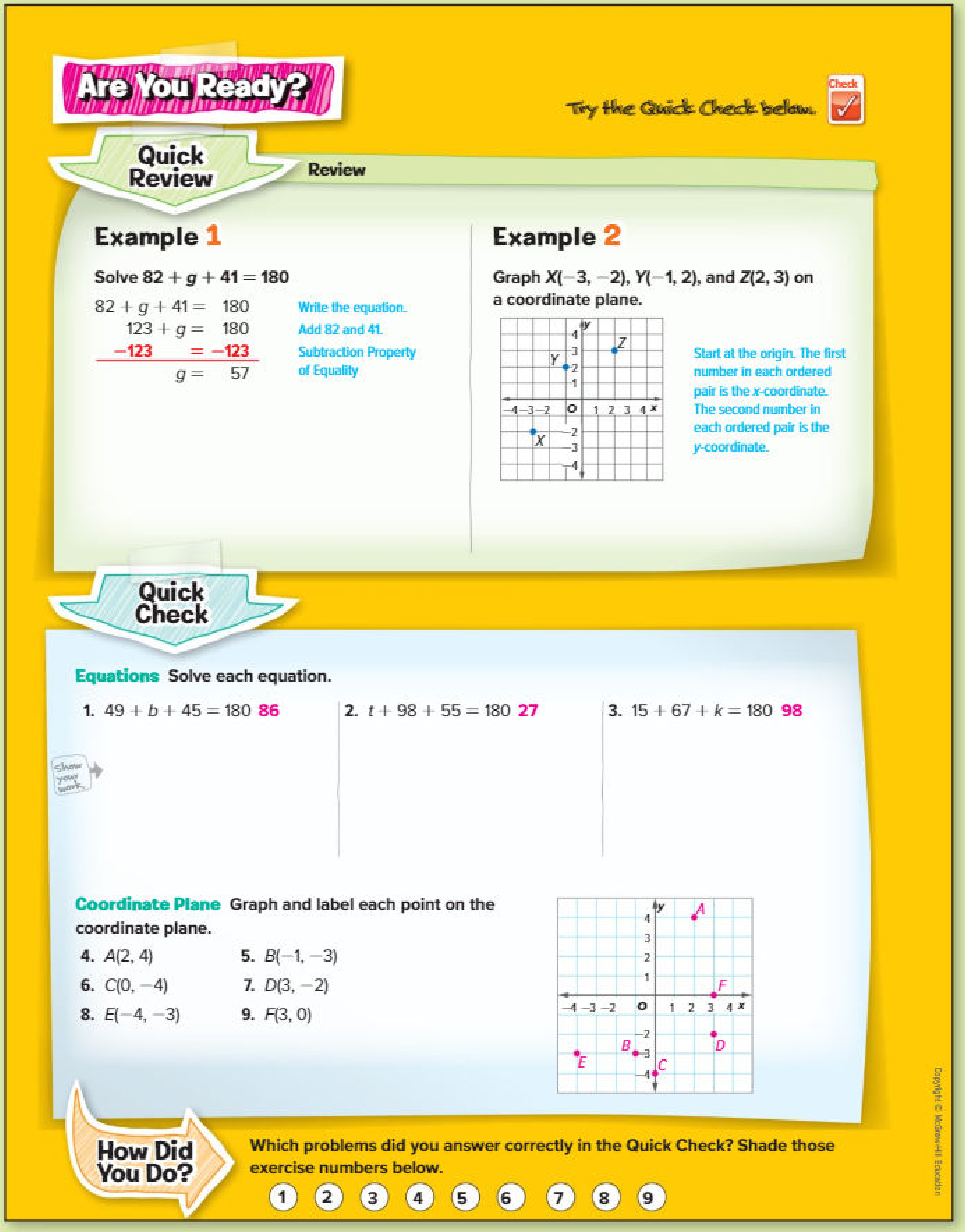
Exercises 4–9

Graph and label $M(-2, 4)$ on a coordinate plane.



Track Your Progress

Prior to beginning this chapter, have your students rate their knowledge of the objectives it addresses. At the end of the chapter, have your students return to these pages to rate their knowledge again. They should see that their knowledge of the key ideas has increased.



Are You Ready?

Try the Quick Check below. Check ✓

Quick Review

Review

Example 1

Solve $82 + g + 41 = 180$

$$\begin{aligned} 82 + g + 41 &= 180 && \text{Write the equation.} \\ 123 + g &= 180 && \text{Add 82 and 41.} \\ -123 \quad &= -123 && \text{Subtraction Property of Equality} \\ g &= 57 \end{aligned}$$

Example 2

Graph $X(-3, -2)$, $Y(-1, 2)$, and $Z(2, 3)$ on a coordinate plane.

Start at the origin. The first number in each ordered pair is the *x*-coordinate. The second number in each ordered pair is the *y*-coordinate.

Quick Check

Equations Solve each equation.

1. $49 + b + 45 = 180$ 86
2. $t + 98 + 55 = 180$ 27
3. $15 + 67 + k = 180$ 98

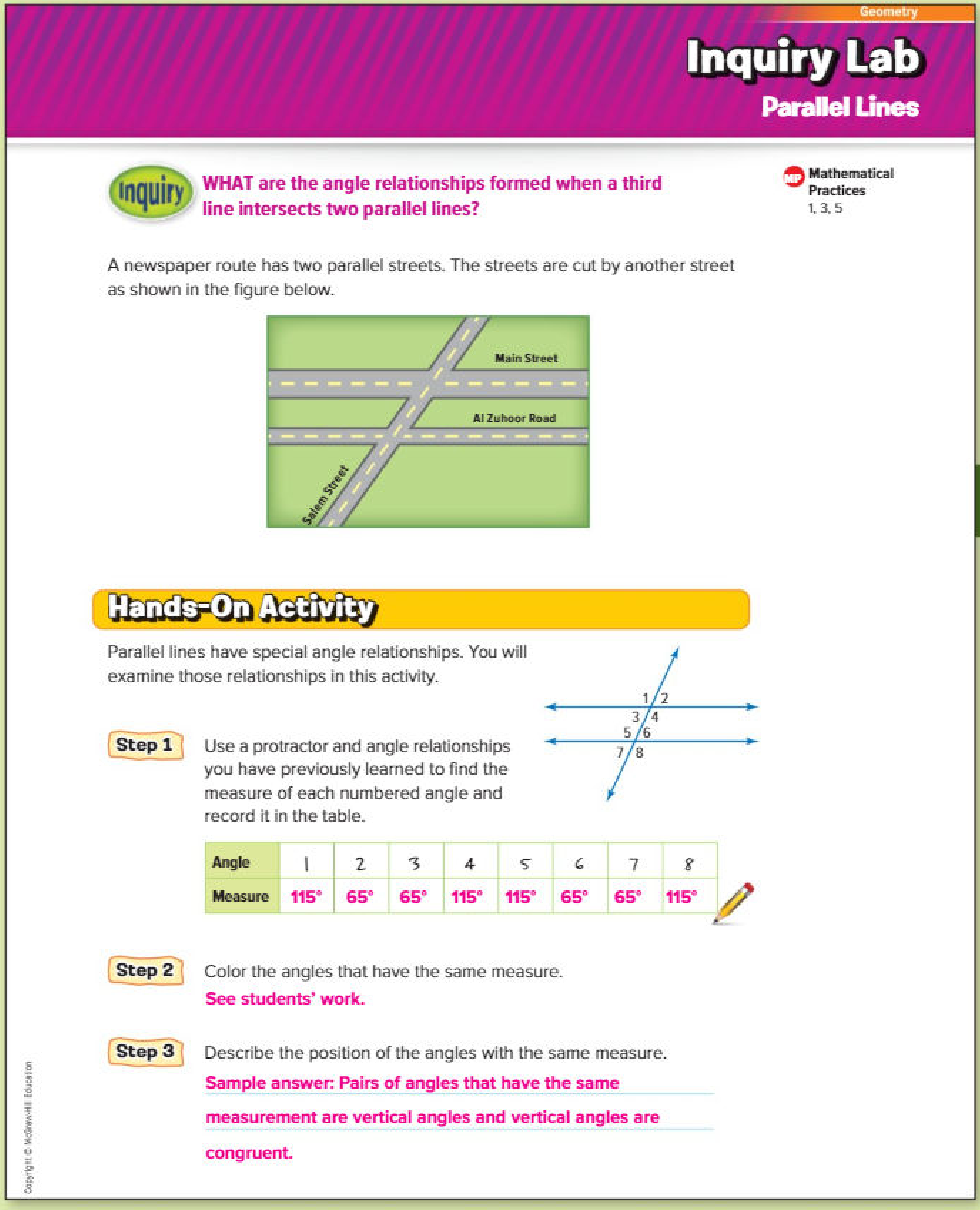
Show your work

Coordinate Plane Graph and label each point on the coordinate plane.

4. $A(2, 4)$
5. $B(-1, -3)$
6. $C(0, -4)$
7. $D(3, -2)$
8. $E(-4, -3)$
9. $F(3, 0)$

How Did You Do? Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨



Geometry

Inquiry Lab

Parallel Lines

Inquiry **WHAT are the angle relationships formed when a third line intersects two parallel lines?**

MP Mathematical Practices
1, 3, 5

A newspaper route has two parallel streets. The streets are cut by another street as shown in the figure below.

Hands-On Activity

Parallel lines have special angle relationships. You will examine those relationships in this activity.

Step 1 Use a protractor and angle relationships you have previously learned to find the measure of each numbered angle and record it in the table.

Angle	1	2	3	4	5	6	7	8
Measure	115°	65°	65°	115°	115°	65°	65°	115°

Step 2 Color the angles that have the same measure.
See students' work.

Step 3 Describe the position of the angles with the same measure.
Sample answer: Pairs of angles that have the same measurement are vertical angles and vertical angles are congruent.

Focus narrowing the scope

Objective Examine angle relationships formed when parallel lines are cut by a transversal.

Coherence connecting within and across grades

Now

Students will examine angle relationships formed when parallel lines are cut by a transversal.

Next

Students will use angle relationships to find missing angles when parallel lines are cut by a transversal.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 370.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Materials: protractor

Hands-On Activity

AL BL LA Roundrobin Have students work in groups of 3–4 to find the measure of an angle either by use of a protractor or by angle relationships. After each student gives an angle measure, the rest of the group gives a thumbs-up or thumbs-down to denote agreement or disagreement. If there is disagreement, students work together to resolve it. **MP 1, 3, 5, 6**

Ask:

- *What do you notice about the position of the angles that have the same measure?* **Sample answer: They are in the same location in relation to the top line and the bottom line. They are also opposite each other at the intersection of lines.**

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–7	8–10	11, 12
Level 3			●
Level 2		●	
Level 1	●		

Investigate

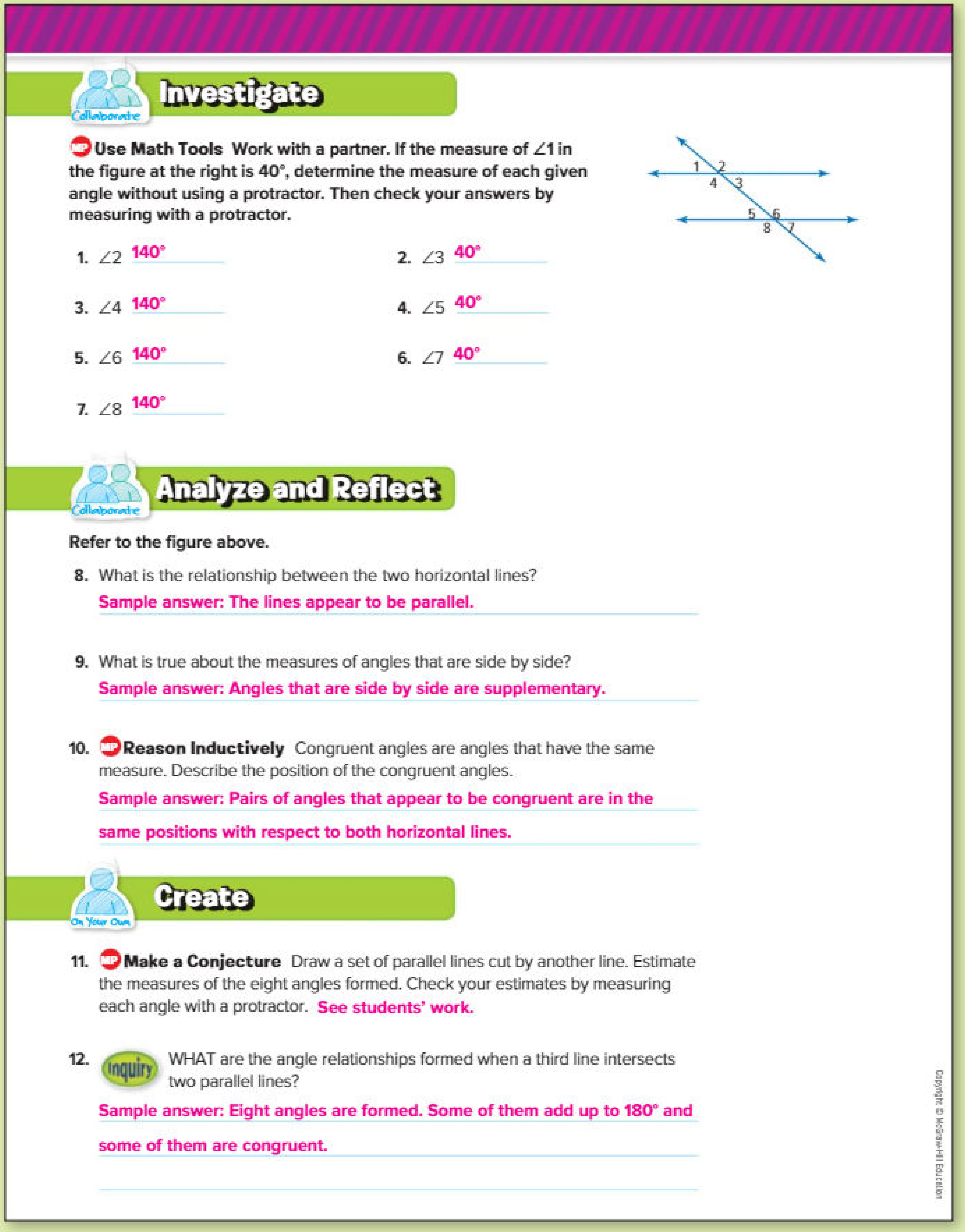
BL **EL** **Pairs Present** Have students explain how they can determine all of the angle measures if they are given only one angle measure. Have them prepare a brief oral presentation to share with the class, using illustrations. MP 1, 3, 5, 6, 7

Ask:

- *What do the obtuse angles have in common? the acute angles?* The obtuse angles have the same angle measure. The acute angles have the same measure.
- *What rule(s) can you generate to determine all of the missing angle measures when you are only given one angle measure?* Sample answer: Angles are either congruent or supplementary.

Create

Inquiry Students should be able to answer "WHAT are the angle relationships formed when a third line intersects two parallel lines?"



Investigate

MP Use Math Tools Work with a partner. If the measure of ∠1 in the figure at the right is 40°, determine the measure of each given angle without using a protractor. Then check your answers by measuring with a protractor.

1. ∠2 140°
2. ∠3 40°
3. ∠4 140°
4. ∠5 40°
5. ∠6 140°
6. ∠7 40°
7. ∠8 140°

Analyze and Reflect

Refer to the figure above.

8. What is the relationship between the two horizontal lines?
Sample answer: The lines appear to be parallel.

9. What is true about the measures of angles that are side by side?
Sample answer: Angles that are side by side are supplementary.

10. **MP Reason Inductively** Congruent angles are angles that have the same measure. Describe the position of the congruent angles.
Sample answer: Pairs of angles that appear to be congruent are in the same positions with respect to both horizontal lines.

Create

11. **MP Make a Conjecture** Draw a set of parallel lines cut by another line. Estimate the measures of the eight angles formed. Check your estimates by measuring each angle with a protractor. See students' work.

12. Inquiry WHAT are the angle relationships formed when a third line intersects two parallel lines?
Sample answer: Eight angles are formed. Some of them add up to 180° and some of them are congruent.

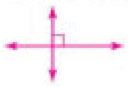
Lesson 1

Lines

Vocabulary Start-Up

When two lines intersect in a plane and form right angles they are called **perpendicular lines**. Two lines are called **parallel lines** when they are in the same plane and do not intersect.

Complete the graphic organizer. **Sample answers are given.**

	Parallel Lines	Perpendicular Lines
Symbols	ǁ	⊥
Define it in your own words	two lines that never cross	two lines that cross and form 90° angles
Draw it		
Describe a real-world example of it	railroad tracks	an intersection where two streets cross

Real-World Link

A gymnastic event in the Summer Olympics involves the parallel bars. The women compete on uneven parallel bars and the men compete on the parallel bars like the one shown. Circle the parallel lines shown in the photo at the right. **See students' work.**

Mathematical Practices did you use?
Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Essential Question

HOW can algebraic concepts be applied to geometry?

Vocabulary

perpendicular lines
parallel lines
transversal
interior angles
exterior angles
alternate interior angles
alternate exterior angles
corresponding angles

Math Symbols

ǁ is parallel to
⊥ is perpendicular to
$m\angle 1$ the measure of $\angle 1$

Mathematical Practices
1, 3, 4

Focus narrowing the scope

Objective Identify relationships of angles formed by two parallel lines cut by a transversal.

Coherence connecting within and across grades

Previous
Students examined angle relationships formed when parallel lines are cut by a transversal.

Now
Students will classify the angles formed and find missing angles when parallel lines are cut by a transversal.

Next
Students will explore the relationship among angles of a triangle.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 375.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Pairs Discussion** Have pairs complete the graphic organizer. Then have them share and revise their responses, if necessary, with another pair of students. Call on one pair to share their responses with the class. MP 1, 2, 3, 4, 5, 6

Alternate Strategy

AL LA Have students write *parallel lines* and *perpendicular lines* on the front of two index cards. On the back, have them draw an example and write the definition. Have them use these cards as a reference throughout the lesson. MP 1, 2, 4, 5, 6

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Classify angle pairs.

AL
- *What does the term interior mean?* inside
- *What does the term exterior mean?* outside

OL
- *Are $\angle 1$ and $\angle 7$ on the same side of the transversal or opposite sides?* opposite sides
- *Are the angles on the inside of the parallel lines or the outside?* outside
- *What type of angles are $\angle 1$ and $\angle 7$?* alternate exterior angles

BL
- *What is true about alternate exterior angles?* They are congruent if the lines are parallel.

2. Classify angle pairs.

AL
- *Look at the positions of $\angle 2$ and $\angle 6$ related to each of the lines. What do you notice?* Sample answer: The angles are in the same position.

OL
- *Are the angles on the same side of the transversal or opposite sides?* same side
- *What type of angles are $\angle 2$ and $\angle 6$?* corresponding

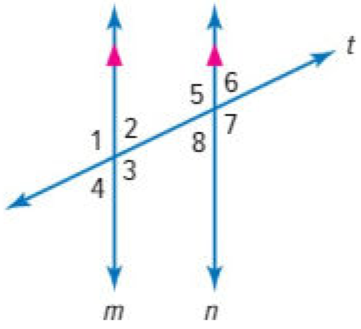
BL
- *What is true about corresponding angles?* They are congruent if the lines are parallel.

Need Another Example?

Classify each pair of angles as *alternate interior, alternate exterior,* or *corresponding.*

$\angle 3$ and $\angle 7$ corresponding

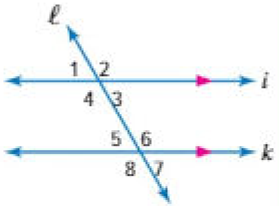
$\angle 2$ and $\angle 8$ alternate interior



Key Concept

Transversals and Angles

A line that intersects two or more lines is called a **transversal**, and eight angles are formed.



Interior angles lie inside the lines.
Examples: $\angle 3, \angle 4, \angle 5, \angle 6$

Exterior angles lie outside the lines.
Examples: $\angle 1, \angle 2, \angle 7, \angle 8$

Alternate interior angles are interior angles that lie on opposite sides of the transversal. When the lines are parallel, their measures are equal. **Examples:** $m\angle 4 = m\angle 6, m\angle 3 = m\angle 5$

Alternate exterior angles are exterior angles that lie on opposite sides of the transversal. When the lines are parallel, their measures are equal. **Examples:** $m\angle 1 = m\angle 7, m\angle 2 = m\angle 8$

Corresponding angles are those angles that are in the same position on the two lines in relation to the transversal. When the lines are parallel, their measures are equal. **Examples:** $m\angle 1 = m\angle 5, m\angle 2 = m\angle 6, m\angle 4 = m\angle 8, m\angle 3 = m\angle 7$

Work Zone

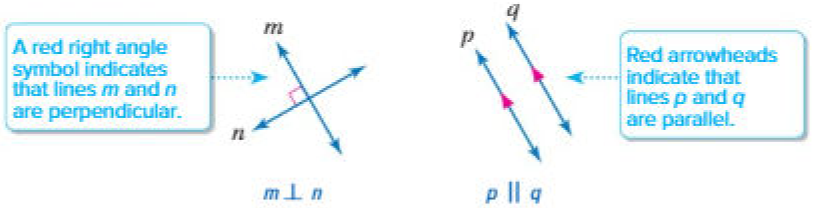
Angles

Read $m\angle 1$ as the measure of angle 1.

Parallel and Perpendicular Lines

Read $m \perp n$ as line m is perpendicular to line n. Read $p \parallel q$ as line p is parallel to line q.

Special notation is used to indicate perpendicular and parallel lines.



A red right angle symbol indicates that lines *m* and *n* are perpendicular.

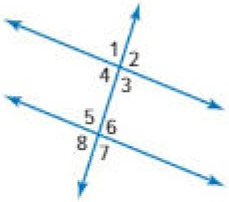
$m \perp n$

Red arrowheads indicate that lines *p* and *q* are parallel.

$p \parallel q$

Examples

Classify each pair of angles in the figure as *alternate interior, alternate exterior,* or *corresponding.*

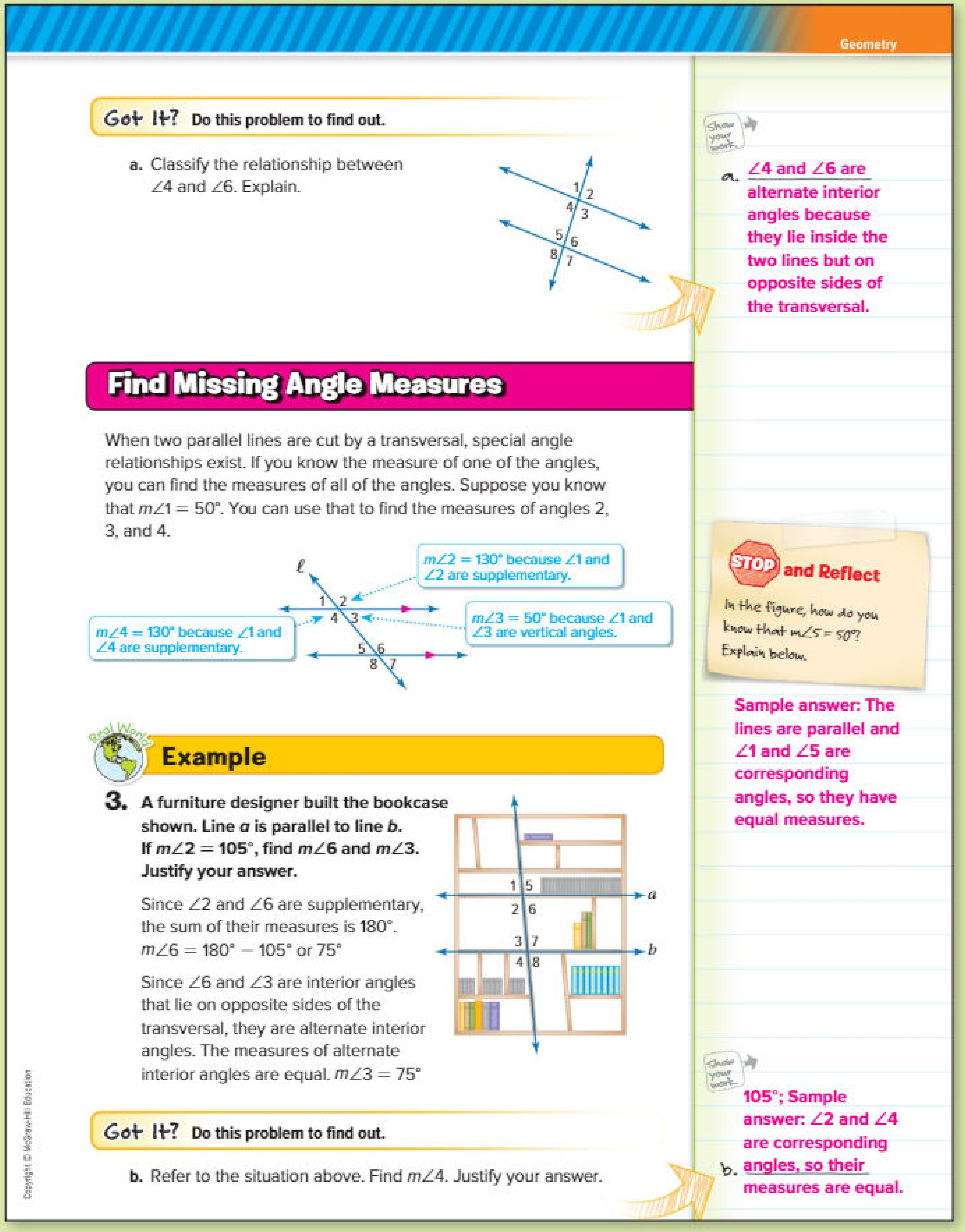


1. $\angle 1$ **and** $\angle 7$

$\angle 1$ and $\angle 7$ are exterior angles that lie on opposite sides of the transversal. They are alternate exterior angles.

2. $\angle 2$ **and** $\angle 6$

$\angle 2$ and $\angle 6$ are in the same position on the two lines. They are corresponding angles.



Got It? Do this problem to find out.

a. Classify the relationship between $\angle 4$ and $\angle 6$. Explain.

Show your work

a. $\angle 4$ and $\angle 6$ are alternate interior angles because they lie inside the two lines but on opposite sides of the transversal.

Find Missing Angle Measures

When two parallel lines are cut by a transversal, special angle relationships exist. If you know the measure of one of the angles, you can find the measures of all of the angles. Suppose you know that $m\angle 1 = 50^\circ$. You can use that to find the measures of angles 2, 3, and 4.

$m\angle 2 = 130^\circ$ because $\angle 1$ and $\angle 2$ are supplementary.

$m\angle 4 = 130^\circ$ because $\angle 1$ and $\angle 4$ are supplementary.

$m\angle 3 = 50^\circ$ because $\angle 1$ and $\angle 3$ are vertical angles.

STOP and Reflect

In the figure, how do you know that $m\angle 5 = 50^\circ$? Explain below.

Sample answer: The lines are parallel and $\angle 1$ and $\angle 5$ are corresponding angles, so they have equal measures.

Real World

Example

3. **A furniture designer built the bookcase shown. Line *a* is parallel to line *b*. If $m\angle 2 = 105^\circ$, find $m\angle 6$ and $m\angle 3$. Justify your answer.**

Since $\angle 2$ and $\angle 6$ are supplementary, the sum of their measures is 180°.
$m\angle 6 = 180^\circ - 105^\circ$ or 75°

Since $\angle 6$ and $\angle 3$ are interior angles that lie on opposite sides of the transversal, they are alternate interior angles. The measures of alternate interior angles are equal. $m\angle 3 = 75^\circ$

Got It? Do this problem to find out.

b. Refer to the situation above. Find $m\angle 4$. Justify your answer.

Show your work

b. 105°; Sample answer: $\angle 2$ and $\angle 4$ are corresponding angles, so their measures are equal.

Example

3. Find missing angle measures.

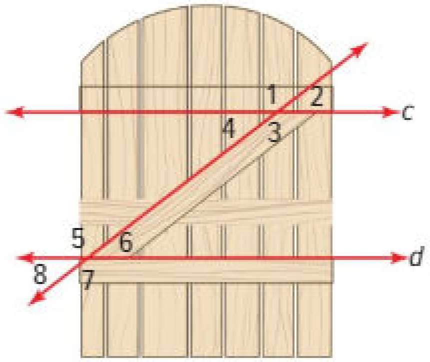
AL
- *How does $\angle 6$ relate to $\angle 2$?* $\angle 6$ and $\angle 2$ are supplementary.
- *If two angles are supplementary, what is the sum of their angle measures?* 180°
- *If I know one of the angle measures is 105°, how can I find the measure of the other angle?* Subtract 105° from 180°.

OL
- *What is the measure of $\angle 6$?* 75°
- *How does $\angle 3$ relate to $\angle 6$?* They are alternate interior angles.
- *What is true about the measures of alternate interior angles?* The angle measures are the same if the lines are parallel.
- *What is the measure of $\angle 3$?* 75°

BL
- *Explain how to find all of the missing angle measures in the diagram.* Sample answer: If $m\angle 2 = 105^\circ$, then $m\angle 4 = 105^\circ$, $m\angle 5 = 105^\circ$, and $m\angle 7 = 105^\circ$. The other four angles, $\angle 1$, $\angle 6$, $\angle 3$, and $\angle 8$ all have a measure of 75° using corresponding angles, vertical angles, alternate interior angles, and supplementary angles.

Need Another Example?

Mr. Mohammad installed the gate shown. Line *c* is parallel to line *d*. If $m\angle 4 = 40^\circ$, find $m\angle 6$ and $m\angle 7$. Justify your answer. $m\angle 6 = 40^\circ$ and $m\angle 7 = 140^\circ$; Sample answer: $\angle 4$ and $\angle 6$ are alternate interior angles, so they are congruent. $\angle 6$ and $\angle 7$ are supplementary. Since $m\angle 6 = 40^\circ$, $m\angle 7 = 140^\circ$.



Example

4. **Find missing angle measures.**

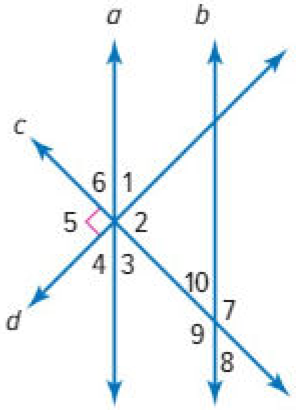
AL
- *If line q is perpendicular to line p, at what angle measure do they intersect?* 90°
- *What kind of angle do $\angle 8$, $\angle 7$, and $\angle 6$ form?* a straight angle
- *How many degrees are in a straight angle?* 180°

OL
- *If $m\angle 1 = 40°$, what is $m\angle 6$? Explain.* $m\angle 6 = 40°$; $\angle 1$ and $\angle 6$ are alternate exterior angles, so they have the same angle measure.

BL
- *How can you use the measure of $\angle 6$ to find $m\angle 7$?* $\angle 6$, $\angle 7$, and $\angle 8$ form a straight angle. Since a straight angle measures 180°, I can solve $40 + 90 + m\angle 7 = 180$ to find $m\angle 7$.

Need Another Example?

In the figure, line *a* is parallel to line *b*, and line *c* is perpendicular to line *d*. The measure of $\angle 7$ is 125°. What is the measure of $\angle 4$? 35°



Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

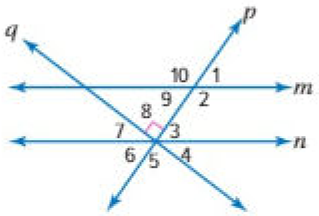
If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Whole Group Discussion** Work as a large group to complete Exercises 1–4. Have students highlight in different colors which angles are congruent, which are complementary, and which are supplementary. MP 1, 5

BL LA **Round Table** Students take turns completing the steps for Exercises 2 and 3, checking the previous person's work as they progress through each problem. MP 1, 3

Example

4. **In the figure, line *m* is parallel to line *n*, and line *q* is perpendicular to line *p*. The measure of $\angle 1$ is 40°. What is the measure of $\angle 7$?**



Since $\angle 1$ and $\angle 6$ are alternate exterior angles, $m\angle 6 = 40°$.

Since $\angle 6$, $\angle 7$, and $\angle 8$ form a straight line, the sum of their measures is 180°.

$40 + 90 + m\angle 7 = 180$

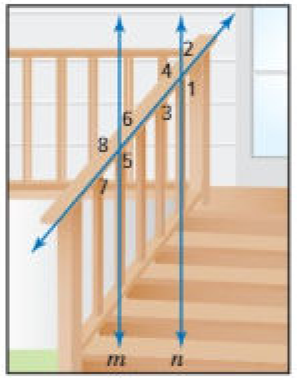
So, $m\angle 7$ is 50°.

Guided Practice

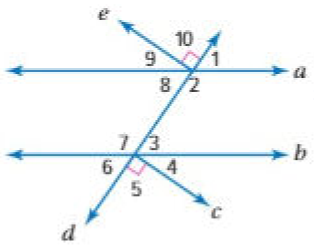
Check

1. Refer to the porch stairs shown. Line *m* is parallel to line *n* and $m\angle 7$ is 35°. Find the measure of $\angle 1$. Justify your answer. (Example 3)

145°; Sample answer: $\angle 7$ and $\angle 5$ are supplementary. So, $m\angle 5 = 180° - 35°$ or 145°. $\angle 5$ and $\angle 1$ are corresponding angles. Since corresponding angles are have the same measure, $m\angle 1 = 145°$.



Refer to the figure at the right. Line *a* is parallel to line *b* and $m\angle 2$ is 135°. Find each given angle measure. Justify your answer. (Examples 1, 2, and 4)



2. $m\angle 9$ 45°; Sample answer: $\angle 2$ and angles 9 and 10 are vertical angles. So, $m\angle 9 + m\angle 10 = 135°$. So, $m\angle 9 = 135° - 90°$ or 45°.

3. $m\angle 7$ 135°; Sample answer: $\angle 2$ and $\angle 7$ are alternate interior angles. So, $m\angle 7 = 135°$.

4. **Building on the Essential Question** How are the measures of the angles related when parallel lines are cut by a transversal?

Sample answer: The angles are either equal or supplementary.

Rate Yourself!

How confident are you about lines and angles? Check the box that applies.

☐ ☐ ☐ ☐ ☐

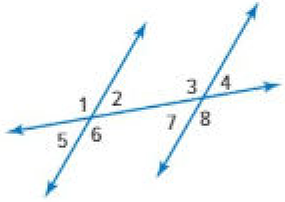
Name ____________ My Homework ____________

Independent Practice

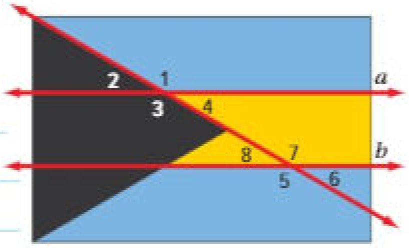
Classify each pair of angles as *alternate interior, alternate exterior,* or *corresponding.* (Examples 1 and 2)

1. ∠2 and ∠4 corresponding

2. ∠4 and ∠5 alternate exterior



3. In the flag shown at the right, line *a* is parallel to line *b*. If m∠1 = 150°, find m∠4 and m∠7. Justify your answers. (Example 3) m∠4 = 30°, m∠7 = 150°; Sample answer: ∠1 and ∠7 are corresponding angles so their measures are equal. ∠1 and ∠4 are supplementary. So, m∠4 = 180° − 150° or 30°.

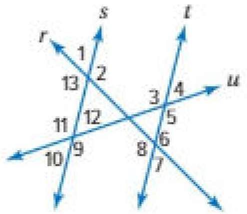


Refer to the figure at the right. Line *s* is parallel to line *t*, m∠2 is 110° and m∠11 is 137°. Find each given angle measure. Justify your answer. (Example 4)

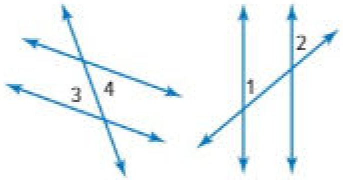
4. m∠7 70°; Sample answer: ∠2 and ∠6 are corresponding angles, so they have the same measure. ∠6 and ∠7 are supplementary. So, m∠7 = 180 − 110 or 70°.

5. m∠8 110°; Sample answer: ∠2 and ∠8 are alternate interior angles, so they have the same measure.

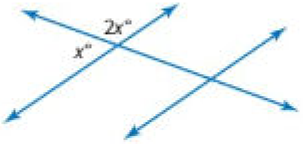
6. m∠3 137°; Sample answer: ∠11 and ∠3 are corresponding angles, so they have the same measure.



7. The parallel lines at the right are cut by a transversal. Find the value of *x*.
   a. Angles 1 and 2 are corresponding angles, m∠1 = 45°, and m∠2 = (x + 25)°. 20
   b. Angles 3 and 4 are alternate interior angles, m∠3 = 2x°, and m∠4 = 80°. 40



8. Describe a method you could use to find the value of *x* in the figure at the right without using a protractor. Sample answer: The two angles are supplementary. So, x + 2x = 180°; x = 60.



ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

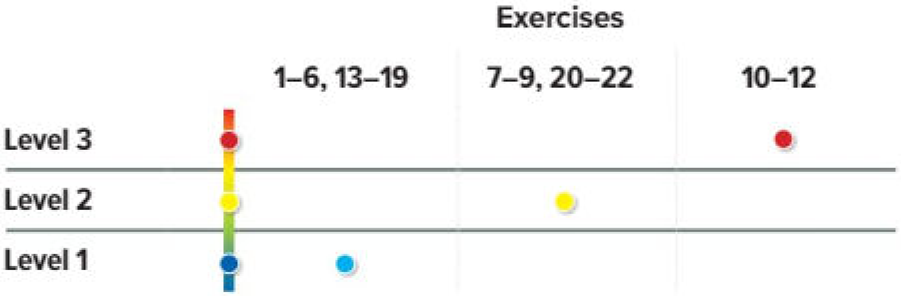
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 11, 12, 21, 22
OL	On Level	1–5 odd, 7–9, 12, 21, 22
BL	Beyond Level	7–12, 21, 22

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	10
3 Construct viable arguments and critique the reasoning of others.	11, 12
4 Model with mathematics.	9, 20

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

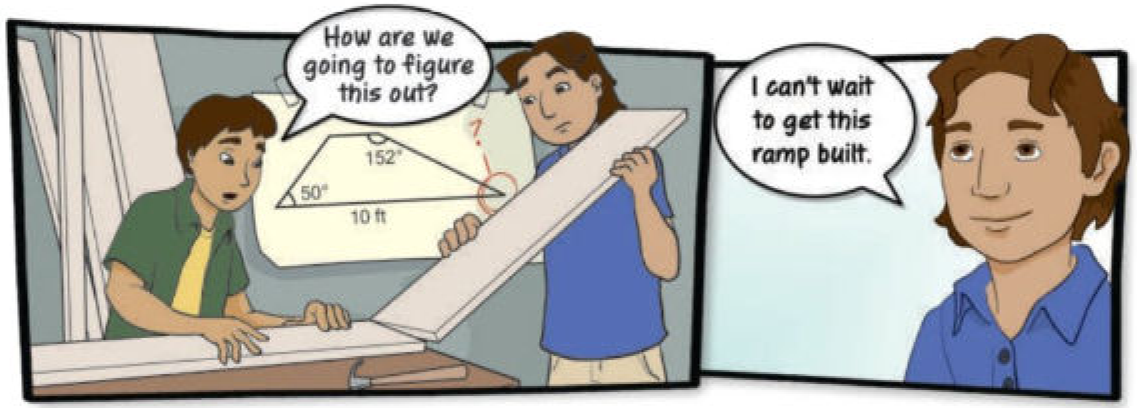
Ask students to describe the pairs of congruent angles in a set of parallel lines cut by a transversal.

See students' work.

Watch Out!

Common Error Students may make errors finding missing angle measures when there is more than one transversal. Encourage students to begin by locating the given angle measure and finding the angle measures that are adjacent and vertical to that angle. They can then identify the parallel lines and use the relationships between corresponding, alternate interior, and alternate exterior angles to find the remaining angle measures.

9. **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–b.

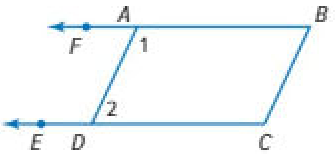


a. Describe a method you could use to find the missing angle. **The top and bottom of the ramp are parallel. The slanted part of the ramp can be considered a transversal. You can use angle relationships of parallel lines to find the measure of the missing angle.**

b. Use your method from part **a** to find the measure of the missing angle.
28°

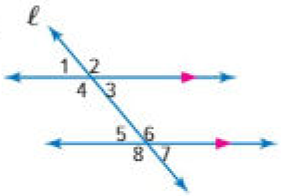
H.O.T. Problems Higher Order Thinking

10. **Persevere with Problems** Quadrilateral *ABCD* is a parallelogram. Make a conjecture about the relationship of ∠1 and ∠2. Justify your reasoning. **∠1 and ∠2 are supplementary. Sample answer: Since $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ are parallel, $m\angle 1 = m\angle ADE$ (alternate interior angles have the same measure). Since $\angle ADE$ and ∠2 lie on the same line, they are supplementary, and $m\angle ADE + m\angle 2 = 180°$. Substitute ∠1 for $\angle ADE$. Therefore, $m\angle 1 + m\angle 2 = 180°$.**



11. **Reason Inductively** If two parallel lines are cut by a transversal, what relationship exists between interior angles that are on the same side of the transversal? **They are supplementary.**

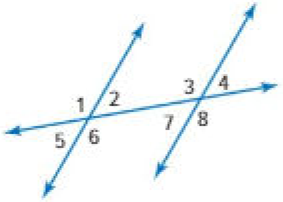
12. **Reason Inductively** Suppose $m\angle 1 = x°$. Use an informal argument to write an expression for the measure of ∠6 in the diagram at the right. **Sample answer: ∠1 and ∠2 are supplementary, so $m\angle 2 = 180° - x°$. ∠2 and ∠6 are corresponding angles. So, $m\angle 6 = 180° - x°$.**



Name ______________________ My Homework ______________________

Extra Practice

Classify each pair of angles as *alternate interior, alternate exterior,* or *corresponding*.



Homework Help

13. ∠3 and ∠6 alternate interior

∠3 and ∠6 are interior angles that lie on opposite sides of the transversal. They are alternate interior angles.

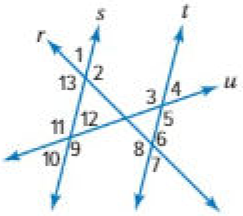
14. ∠1 and ∠3 corresponding

15. ∠2 and ∠7 alternate interior

16. In the quilt design at the right, line *a* is parallel to line *b*. If $m\angle 1 = 120°$, find $m\angle 2$ and $m\angle 3$.

Justify your answers. $m\angle 2 = 120°$, $m\angle 3 = 60°$;

Sample answer: ∠1 and ∠2 are alternate exterior angles, so they have the same measure. ∠2 and ∠3 are supplementary. So, $m\angle 3 = 180° - 120°$ or 60°.



Refer to the figure at the right. Line *s* is parallel to line *t*, *m*∠2 is 110° and *m*∠11 is 137°. Find each given angle measure. Justify your answer.

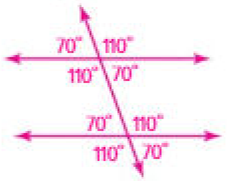
17. $m\angle 6$ 110°; Sample answer: ∠2 and ∠6 are corresponding angles, so they have the same measure.

18. $m\angle 13$ 110°; Sample answer: ∠2 and ∠13 are vertical angles, so they have the same measure.

19. $m\angle 4$ 43°; Sample answer: ∠11 and ∠3 are corresponding angles, so they have the same measure and ∠3 and ∠4 are supplementary. So, $m\angle 4 = 180 - 137$ or 43°.

20. **Model with Mathematics** Draw a pair of parallel lines cut by a transversal. Estimate the measure of one angle and label it. Without using a protractor, label all the other angles with their approximate measure.

Sample answer:



Power Up! Test Practice

Exercises 21 and 22 prepare students for more rigorous thinking needed for assessment.

21.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

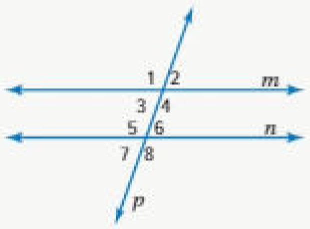
22.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly label all 7 of the angles.
1 point	Students correctly label 5–6 of the 7 angles.

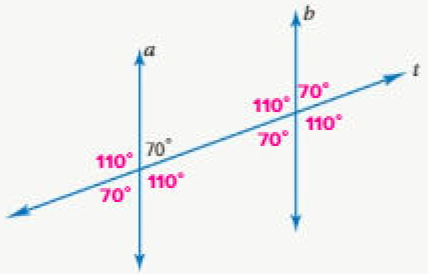
Power Up! Test Practice

21. Lines *m* and *n* are parallel and cut by the transversal *p*. Which of the following pairs of angles represent corresponding angles? Select all that apply.

☑ ∠2 and ∠6
☐ ∠4 and ∠6
☐ ∠3 and ∠4
☑ ∠1 and ∠5



22. Lines *a* and *b* are parallel and cut by the transversal *t*. Label each of the 7 unknown angles with the correct angle measure.



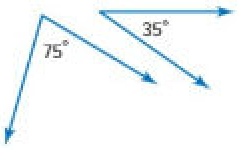
Spiral Review

23. A poster has a triangular image with a base that measure 10 centimeters, and a height that measures 20 centimeters. What is the area of the poster?

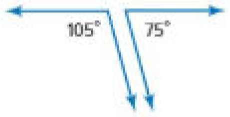
100 cm^2

Classify each pair of angles as *complementary*, *supplementary*, or *neither*.

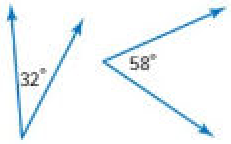
24. neither

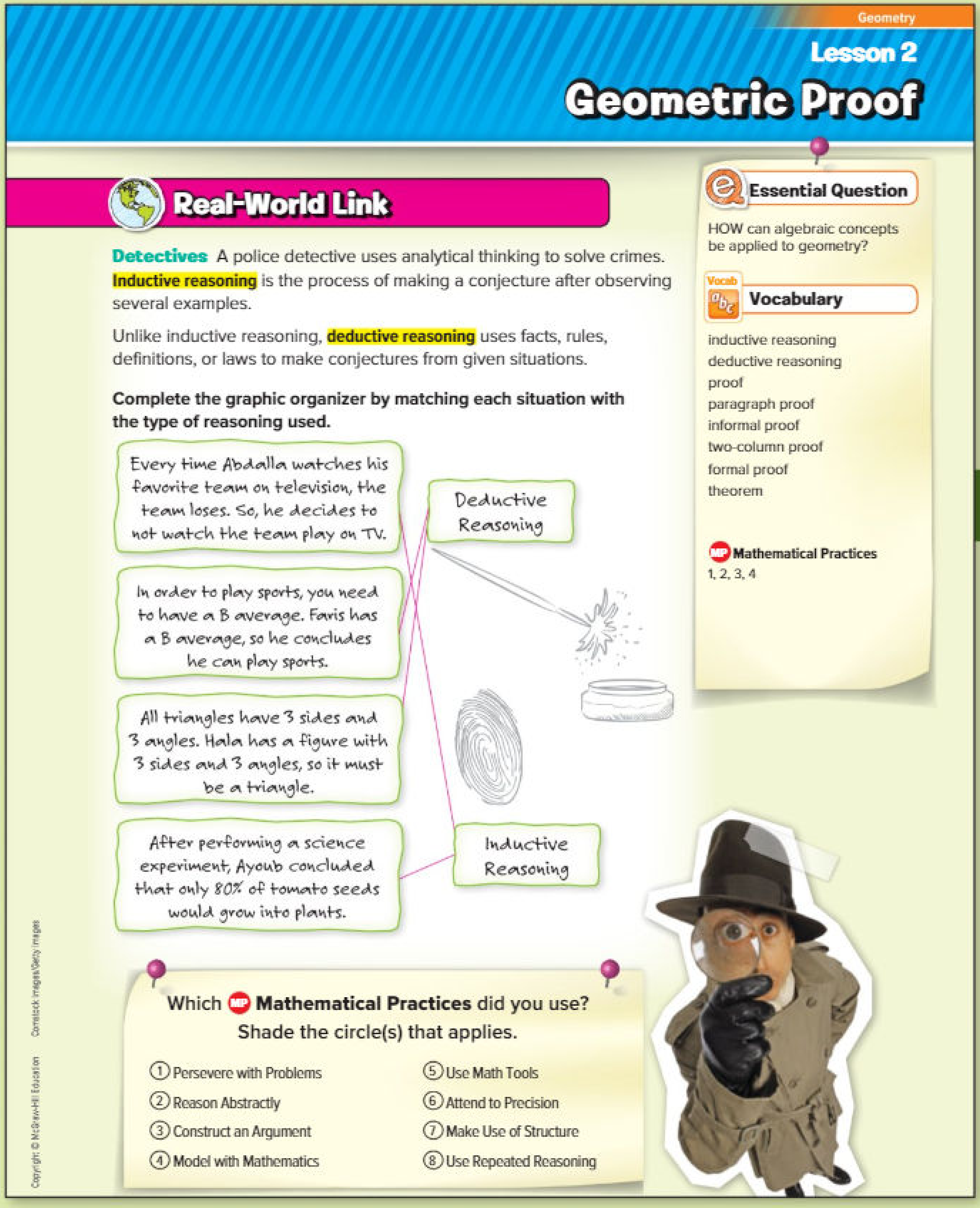


25. supplementary



26. complementary





Geometry

Lesson 2

Geometric Proof

Real-World Link

Detectives A police detective uses analytical thinking to solve crimes. **Inductive reasoning** is the process of making a conjecture after observing several examples.

Unlike inductive reasoning, **deductive reasoning** uses facts, rules, definitions, or laws to make conjectures from given situations.

Complete the graphic organizer by matching each situation with the type of reasoning used.

Every time Abdalla watches his favorite team on television, the team loses. So, he decides to not watch the team play on TV.

In order to play sports, you need to have a B average. Faris has a B average, so he concludes he can play sports.

All triangles have 3 sides and 3 angles. Hala has a figure with 3 sides and 3 angles, so it must be a triangle.

After performing a science experiment, Ayoub concluded that only 80% of tomato seeds would grow into plants.

Deductive Reasoning

Inductive Reasoning

Essential Question

HOW can algebraic concepts be applied to geometry?

Vocabulary

- inductive reasoning
- deductive reasoning
- proof
- paragraph proof
- informal proof
- two-column proof
- formal proof
- theorem

Mathematical Practices

1, 2, 3, 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Write geometric proofs.

Coherence connecting within and across grades

Previous	Now	Next
Students used the properties of mathematics to justify the steps to solve an equation.	Students use definitions, properties, and theorems to prove a hypothesis.	Students will prove the Pythagorean Theorem and its converse.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 383.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

AL LA **Pairs Consult** Have pairs write *inductive reasoning* and *deductive reasoning* on two index cards. Have them write the attributes of each kind of reasoning on the back of each card. Have them use their cards to determine which type of reasoning was used for each situation in the graphic organizer. MP 1, 5, 6

Alternate Strategy

BL Have students provide their own examples of when they have used inductive and deductive reasoning to solve a problem in everyday life. MP 6

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

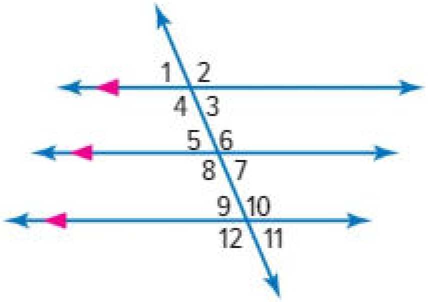
1. **Complete a paragraph proof.**

AL
- *What is a paragraph proof?* a proof that is written in paragraph form
- *What information do you know that is stated in the problem?* $m\angle 1 = m\angle 4$
- *What do you know about the relationship between $\angle 1$ and $\angle 2$ from the diagram?* They are vertical angles.
- *What do you know about the relationship between $\angle 3$ and $\angle 4$ from the diagram?* They are vertical angles.
- *What is true about the measures of vertical angles?* They are equal.

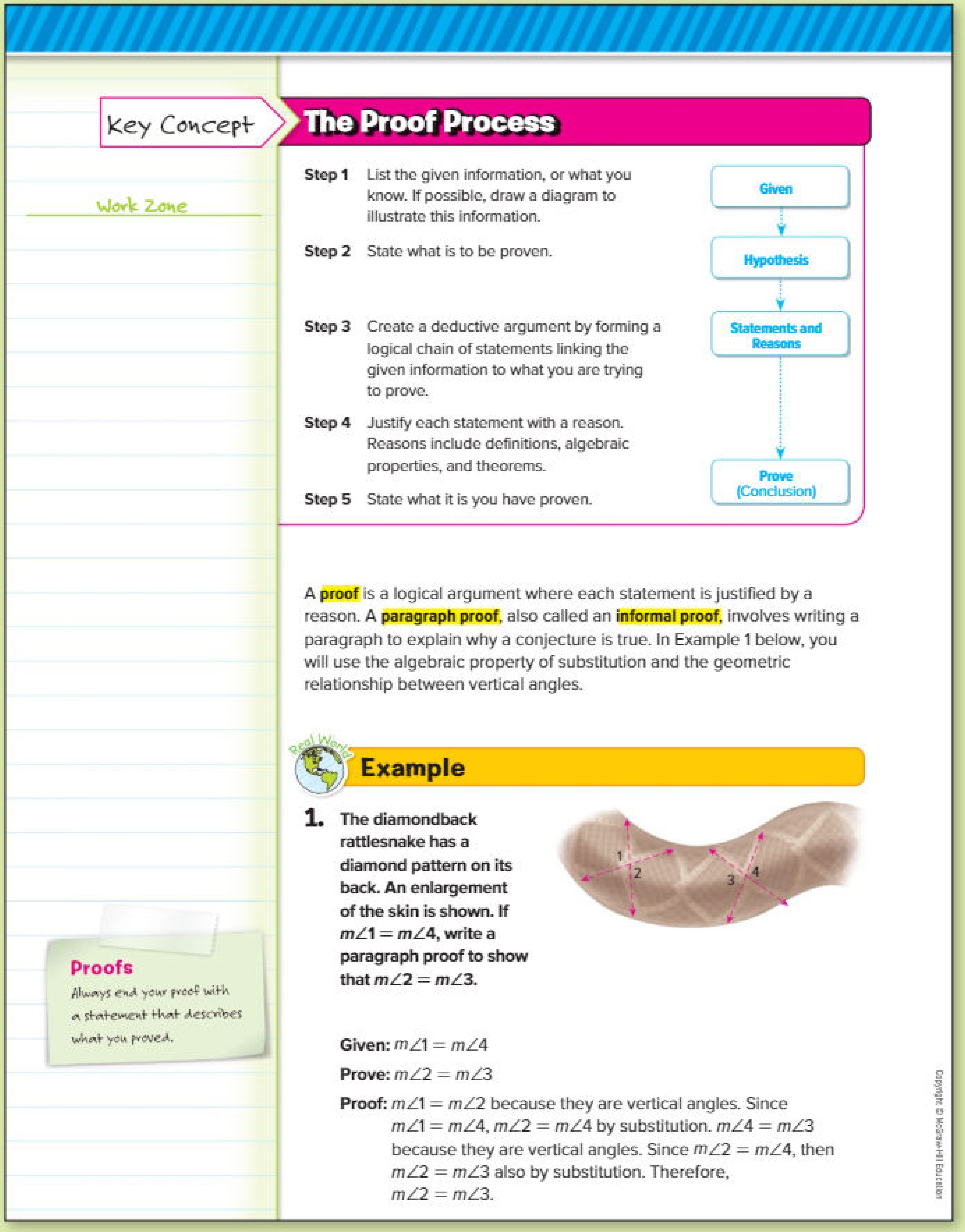
OL
- *If $m\angle 1 = m\angle 4$ and $m\angle 1 = m\angle 2$, what can you say about $m\angle 2$ and $m\angle 4$?* $m\angle 2 = m\angle 4$
- *If $m\angle 2 = m\angle 4$ and $m\angle 4 = m\angle 3$, what can you say about $m\angle 2$ and $m\angle 3$?* $m\angle 2 = m\angle 3$

BL
- *What definitions, properties, or relationships were used in this proof?* substitution; vertical angles have equal measures

Need Another Example?
Refer to the diagram. If $m\angle 1 = m\angle 5$, write a paragraph proof to show that $m\angle 1 = m\angle 11$.



$m\angle 1 = m\angle 9$ because they are corresponding angles. $m\angle 9 = m\angle 11$ because they are vertical angles. Since $m\angle 9 = m\angle 11$, then $m\angle 1 = m\angle 11$ by substitution.



Key Concept The Proof Process

Step 1 List the given information, or what you know. If possible, draw a diagram to illustrate this information.

Step 2 State what is to be proven.

Step 3 Create a deductive argument by forming a logical chain of statements linking the given information to what you are trying to prove.

Step 4 Justify each statement with a reason. Reasons include definitions, algebraic properties, and theorems.

Step 5 State what it is you have proven.

A **proof** is a logical argument where each statement is justified by a reason. A **paragraph proof**, also called an **informal proof**, involves writing a paragraph to explain why a conjecture is true. In Example 1 below, you will use the algebraic property of substitution and the geometric relationship between vertical angles.

Proofs
Always end your proof with a statement that describes what you proved.

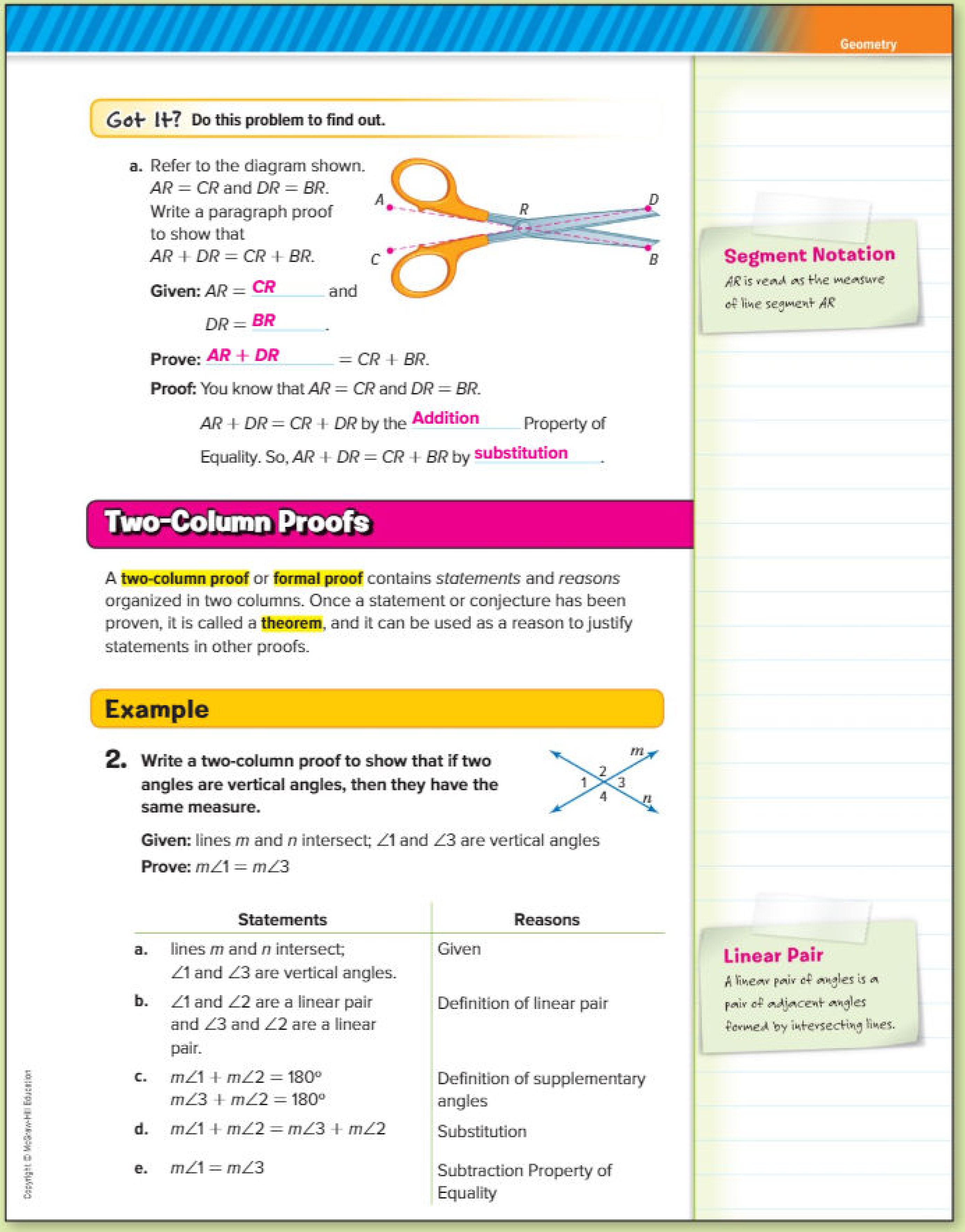
Example

1. **The diamondback rattlesnake has a diamond pattern on its back. An enlargement of the skin is shown. If $m\angle 1 = m\angle 4$, write a paragraph proof to show that $m\angle 2 = m\angle 3$.**

Given: $m\angle 1 = m\angle 4$

Prove: $m\angle 2 = m\angle 3$

Proof: $m\angle 1 = m\angle 2$ because they are vertical angles. Since $m\angle 1 = m\angle 4$, $m\angle 2 = m\angle 4$ by substitution. $m\angle 4 = m\angle 3$ because they are vertical angles. Since $m\angle 2 = m\angle 4$, then $m\angle 2 = m\angle 3$ also by substitution. Therefore, $m\angle 2 = m\angle 3$.



Got It? Do this problem to find out.

a. Refer to the diagram shown. $AR = CR$ and $DR = BR$. Write a paragraph proof to show that $AR + DR = CR + BR$.

Given: $AR =$ CR and

$DR =$ BR.

Prove: $AR + DR$ $= CR + BR$.

Proof: You know that $AR = CR$ and $DR = BR$.

$AR + DR = CR + DR$ by the Addition Property of Equality. So, $AR + DR = CR + BR$ by substitution.

Two-Column Proofs

A **two-column proof** or **formal proof** contains *statements* and *reasons* organized in two columns. Once a statement or conjecture has been proven, it is called a **theorem**, and it can be used as a reason to justify statements in other proofs.

Example

2. **Write a two-column proof to show that if two angles are vertical angles, then they have the same measure.**

Given: lines m and n intersect; $\angle 1$ and $\angle 3$ are vertical angles

Prove: $m\angle 1 = m\angle 3$

	Statements	Reasons
a.	lines m and n intersect; $\angle 1$ and $\angle 3$ are vertical angles.	Given
b.	$\angle 1$ and $\angle 2$ are a linear pair and $\angle 3$ and $\angle 2$ are a linear pair.	Definition of linear pair
c.	$m\angle 1 + m\angle 2 = 180°$ $m\angle 3 + m\angle 2 = 180°$	Definition of supplementary angles
d.	$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	Substitution
e.	$m\angle 1 = m\angle 3$	Subtraction Property of Equality

Segment Notation

AR is read as the measure of line segment AR

Linear Pair

A linear pair of angles is a pair of adjacent angles formed by intersecting lines.

Example

2. Complete a two-column proof.

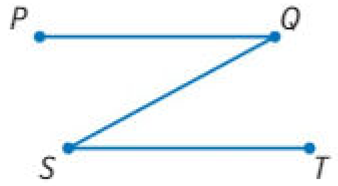
AL
- *What information are you given?* lines *m* and *n* intersect; $\angle 1$ and $\angle 3$ are vertical angles
- *What is the relationship between* $\angle 1$ *and* $\angle 2$*?* They are supplementary angles.
- *What is the relationship between* $\angle 2$ *and* $\angle 3$*?* They are supplementary angles.
- *If two angles are supplementary, what is the sum of their angle measures?* 180°

OL
- *If* $m\angle 1 + m\angle 2 = 180°$ *and* $m\angle 3 + m\angle 2 = 180°$*, what new equation can be written using substitution?* $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$
- *What property allows you to subtract* $m\angle 2$ *from each side?* Subtraction Property of Equality

BL
- *Do you prefer using a paragraph proof or a two-column proof? Explain.* See students' preferences.

Need Another Example?

Complete the two-column proof to show that if $PQ = QS$ and $QS = ST$, then $PQ = ST$.



Given: $PQ = QS$; $QS = ST$

Prove: $PQ = ST$

Statements	Reasons
a. $PQ = QS$ and $QS = ST$	Given
b. $PQ = ST$	Substitution

Guided Practice

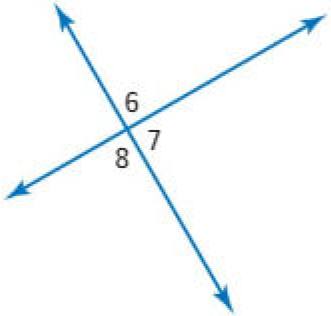
Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

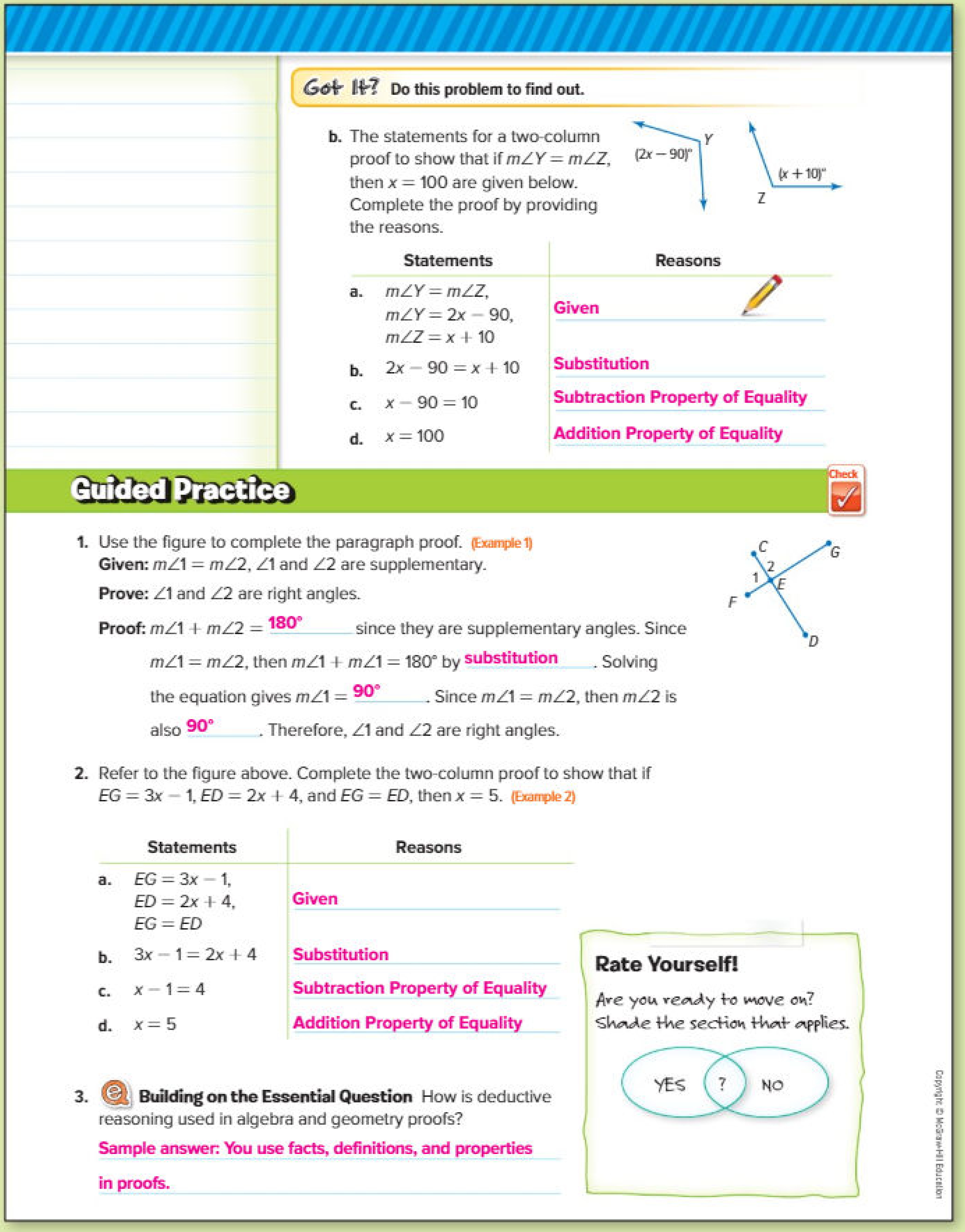
If some of your students are not ready for assignments, use the differentiated activities below.

AL **LA** **Roundrobin** Have students work in teams of three to complete Exercises 1 and 2. The first student should fill in the first blank and explain their answer, then move on to the second student, and so on. Have them trade their solutions with another team of students and discuss any differences. MP 1, 3, 5, 6, 7

BL **LA** **Pairs Consult** Have students work in pairs. Give them the following information and ask them to create a paragraph proof or a two-column proof. MP 1, 3, 5, 6, 7

In the figure, two lines intersect to form four angles. If ∠6 and ∠8 are supplementary angles, prove that ∠7 is a right angle. Have them trade their proofs with another pair of students and discuss any differences.





Got It? Do this problem to find out.

b. The statements for a two-column proof to show that if $m\angle Y = m\angle Z$, then $x = 100$ are given below. Complete the proof by providing the reasons.

	Statements	Reasons
a.	$m\angle Y = m\angle Z$, $m\angle Y = 2x - 90$, $m\angle Z = x + 10$	Given
b.	$2x - 90 = x + 10$	Substitution
c.	$x - 90 = 10$	Subtraction Property of Equality
d.	$x = 100$	Addition Property of Equality

Guided Practice

Check

1. Use the figure to complete the paragraph proof. (Example 1)
Given: $m\angle 1 = m\angle 2$, ∠1 and ∠2 are supplementary.
Prove: ∠1 and ∠2 are right angles.
Proof: $m\angle 1 + m\angle 2 =$ 180° since they are supplementary angles. Since $m\angle 1 = m\angle 2$, then $m\angle 1 + m\angle 1 = 180°$ by substitution. Solving the equation gives $m\angle 1 =$ 90°. Since $m\angle 1 = m\angle 2$, then $m\angle 2$ is also 90°. Therefore, ∠1 and ∠2 are right angles.

2. Refer to the figure above. Complete the two-column proof to show that if $EG = 3x - 1$, $ED = 2x + 4$, and $EG = ED$, then $x = 5$. (Example 2)

	Statements	Reasons
a.	$EG = 3x - 1$, $ED = 2x + 4$, $EG = ED$	Given
b.	$3x - 1 = 2x + 4$	Substitution
c.	$x - 1 = 4$	Subtraction Property of Equality
d.	$x = 5$	Addition Property of Equality

3. **Building on the Essential Question** How is deductive reasoning used in algebra and geometry proofs?
Sample answer: You use facts, definitions, and properties in proofs.

Rate Yourself!

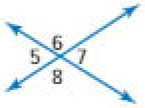
Are you ready to move on?
Shade the section that applies.

YES ? NO

Name ______________________ My Homework ______________________

Independent Practice

In the figure at the right, two lines intersect to form four angles. If $m\angle 7 = 9x$ and $m\angle 8 = 11x$, complete the paragraph proof to show that $x = 9$. (Example 1)



Given: Two intersecting lines with $m\angle 7 = 9x$ and $m\angle 8 = 11x$

Prove: $x = 9$

Proof: $\angle 7$ and $\angle 8$ form a straight angle so they are supplementary angles. So, $m\angle 7 + m\angle 8 =$ 180°, by the definition of supplementary angles. By substitution, 9x $+ 11x = 180$. So, $x =$ 9 by the Division Property of Equality.

2. **Construct an Argument** Four towns lie on a straight road. Town B is midway between Town A and Town C. Town C is midway between Town B and Town D. Write a paragraph proof to show the distance from Town A to Town B is the same as the distance from Town C to Town D. (Example 1)

Given: B is the midpoint of $\overline{AC}$ and C is the midpoint of $\overline{BD}$.

Prove: $AB = CD$.

Proof: By the definition of midpoint, AB $= BC$ and BC $= CD$.

Therefore, $AB = CD$ by substitution.

3. **Construct an Argument** Complete the two-column proof to show that if $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 = m\angle 2$, then $\angle 1$ and $\angle 2$ are right angles. (Example 2)

Given: $\angle 1$ and $\angle 2$ are supplementary; $m\angle 1 = m\angle 2$

Prove: $\angle 1$ and $\angle 2$ are right angles

	Statements	Reasons
a.	$\angle 1$ and $\angle 2$ are supplementary; $m\angle 1 = m\angle 2$	Given
b.	$m\angle 1 + m\angle 2 = 180°$	Definition of supplementary angles
c.	$m\angle 1 + m\angle 1 = 180°$	Substitution
d.	$2(m\angle 1) = 180°$	Simplify
e.	$m\angle 1 = 90°$	Division Property of Equality
f.	$m\angle 2 = 90°$	$m\angle 1 = m\angle 2$ (Given)
g.	$\angle 1$ and $\angle 2$ are right angles.	Definition of right angles

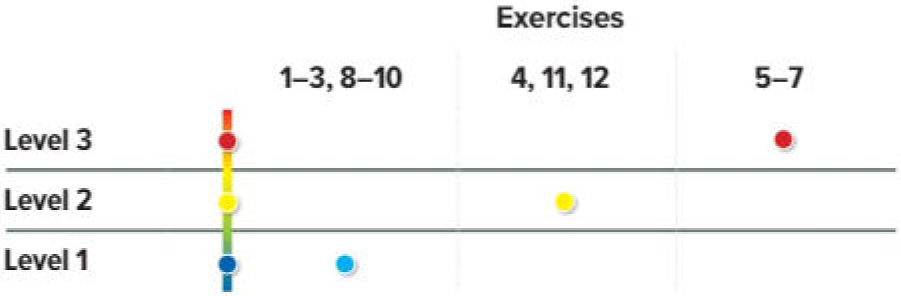
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–3, 5, 7, 11, 12
OL	On Level	1, 3–5, 7, 11, 12
BL	Beyond Level	4–7, 11, 12

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	6
2 Reason abstractly and quantitatively.	5
3 Construct viable arguments and critique the reasoning of others.	2–4, 7–10

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

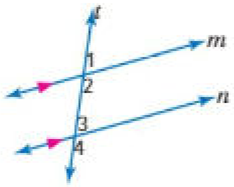
Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students describe the differences between a paragraph proof and a two-column proof. See students' work.

4. **Construct an Argument** Complete the two-column proof to show that when two parallel lines are cut by a transversal, consecutive interior angles are supplementary.



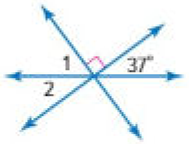
Given: parallel lines m and n cut by transversal t

Prove: $\angle 2$ and $\angle 3$ are supplementary.

	Statements	Reasons
a.	Lines m and n are parallel and cut by transversal t	Given
b.	$\angle 1$ and $\angle 2$ form a straight angle.	Definition of straight angle
c.	$m\angle 1 + m\angle 2 = 180$	Definition of supplementary angles
d.	$m\angle 1 = m\angle 3$	Corresponding $\angle$s have equal measures
e.	$m\angle 3 + m\angle 2 = 180$	Substitution
f.	$\angle 2$ and $\angle 3$ are supplementary angles	Definition of supplementary angles

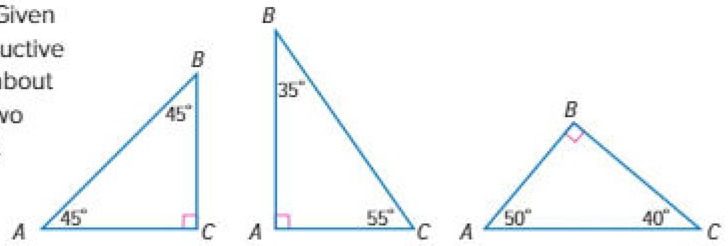
H.O.T. Problems Higher Order Thinking

5. **Reason Abstractly** Describe the theorem or definition you could use to find the measure of $\angle 2$.



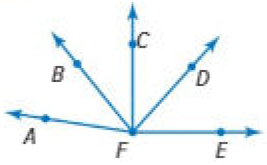
Sample answer: Vertical angles have the same measure.

6. **Persevere with Problems** Given the right triangles shown, use inductive reasoning to make a conjecture about the sum of the measures of the two acute angles of any right triangle.



Sample answer: The sum of the measures of the acute angles of a right triangle is 90°. So, the acute angles are complementary.

7. **Reason Inductively** In the diagram, $m\angle CFE = 90°$ and $m\angle AFB = m\angle CFD$. Which of the following conclusions does *not* have to be true? II



I $m\angle AFB + m\angle DFE = 90°$

II $\overline{BF}$ divides $\angle AFD$ in half

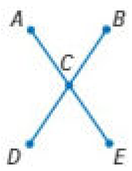
III $m\angle CFD = m\angle AFB$

IV $\angle CFE$ is a right angle.

Name ______________________ My Homework ______________________

Extra Practice

8. **Construct an Argument** In the figure at the right, $AE = DB$ and C is the midpoint of $\overline{AE}$ and $\overline{DB}$. Complete the proof to show that $AC = CB$.

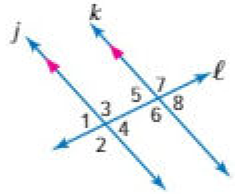


Given: $AE = DB$ and C is the midpoint of $\overline{AE}$ and $\overline{DB}$.

Prove: $AC = CB$

Proof: Since C is the midpoint of **$\overline{AE}$** and **$\overline{DB}$**, $AC = CE = \frac{1}{2}$ **AE** and $DC = CB = \frac{1}{2}$ **DB** by the definition of midpoint. We are given $AE = DB$. By the **Multiplication** Property of Equality, $\frac{1}{2}AE = \frac{1}{2}DB$. So, by **substitution**, $AC = CB$.

9. **Construct an Argument** Refer to the figure at the right. Complete the two-column proof to show if $m\angle 3 = 2x - 15$ and $m\angle 6 = x + 55$, then $x = 70$.

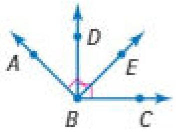


Given: $j \parallel k$, transversal ℓ; $m\angle 3 = 2x - 15$, $m\angle 6 = x + 55$

Prove: $x = 70$

Statements	Reasons
a. $j \parallel k$, transversal ℓ; $m\angle 3 = 2x - 15$, $m\angle 6 = x + 55$	**Given**
b. $m\angle 3 = m\angle 6$	**Alternate interior angles have the same measure.**
c. $2x - 15 = x + 55$	**Substitution**
d. $x - 15 = 55$	**Subtraction Property of Equality**
e. $x = 70$	**Addition Property of Equality**

10. **Construct an Argument** Refer to the figure at the right. Complete the two-column proof to show if $\angle ABE$ and $\angle DBC$ are right angles, then $m\angle ABD = m\angle EBC$.



Given: $\angle ABE$ and $\angle DBC$ are right angles.

Prove: $m\angle ABD = m\angle EBC$

Statements	Reasons
a. $\angle ABE$ and $\angle DBC$ are right angles.	**Given**
b. $m\angle ABE = 90$ and $m\angle DBC = 90$	**Definition of right angles**
c. $m\angle ABD + m\angle DBE = 90$ $m\angle DBE + m\angle EBC = 90$	**Angle addition**
d. $m\angle ABD + m\angle DBE = m\angle DBE + m\angle EBC$	**Substitution**
e. $m\angle ABD = m\angle EBC$	**Subtraction Property of Equality**

Power Up! Test Practice

Exercises 11 and 12 prepare students for more rigorous thinking needed for assessment.

11.

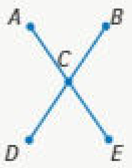
This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

12.

This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly identify all 4 steps in the proof.
1 point	Students correctly identify 3 of the 4 steps in the proof.

Power Up! Test Practice

11. In the diagram shown, $\overline{AE}$ intersects $\overline{DB}$ at C.



Determine if each of the following conclusions will always be true. Select yes or no.

a. $m\angle ACD = m\angle BCE$ ☑ Yes ☐ No

b. $\angle ACD$ and $\angle ECD$ form a linear pair. ☑ Yes ☐ No

c. $\angle DCE$ and $\angle ACB$ are vertical angles. ☑ Yes ☐ No

d. $\angle ACB$ and $\angle BCE$ are complementary angles. ☐ Yes ☑ No

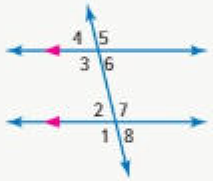
12. Select the appropriate reason for each statement of the geometric proof below.

Substitution	**Division Property of Equality**	**Vertical angles have equal measures.**
Given	**Alternate interior angles have equal measures.**	**Corresponding angles have equal measures.**

Given: two parallel lines cut by a transversal, $m\angle 1 = 2x$, $m\angle 3 = 94$

Prove: $x = 47$

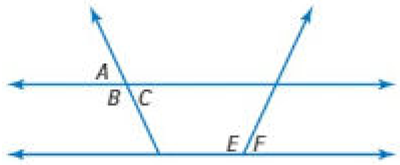
Proof:



Statements	Reasons
a. $m\angle 1 = 2x, m\angle 3 = 94$	**Given**
b. $m\angle 1 = m\angle 3$	**Corresponding angles have equal measures.**
c. $2x = 94$	**Substitution**
d. $x = 47$	**Division Property of Equality**

Spiral Review

Refer to the diagram. Identify each pair of angles as *adjacent*, *vertical*, or *neither*.

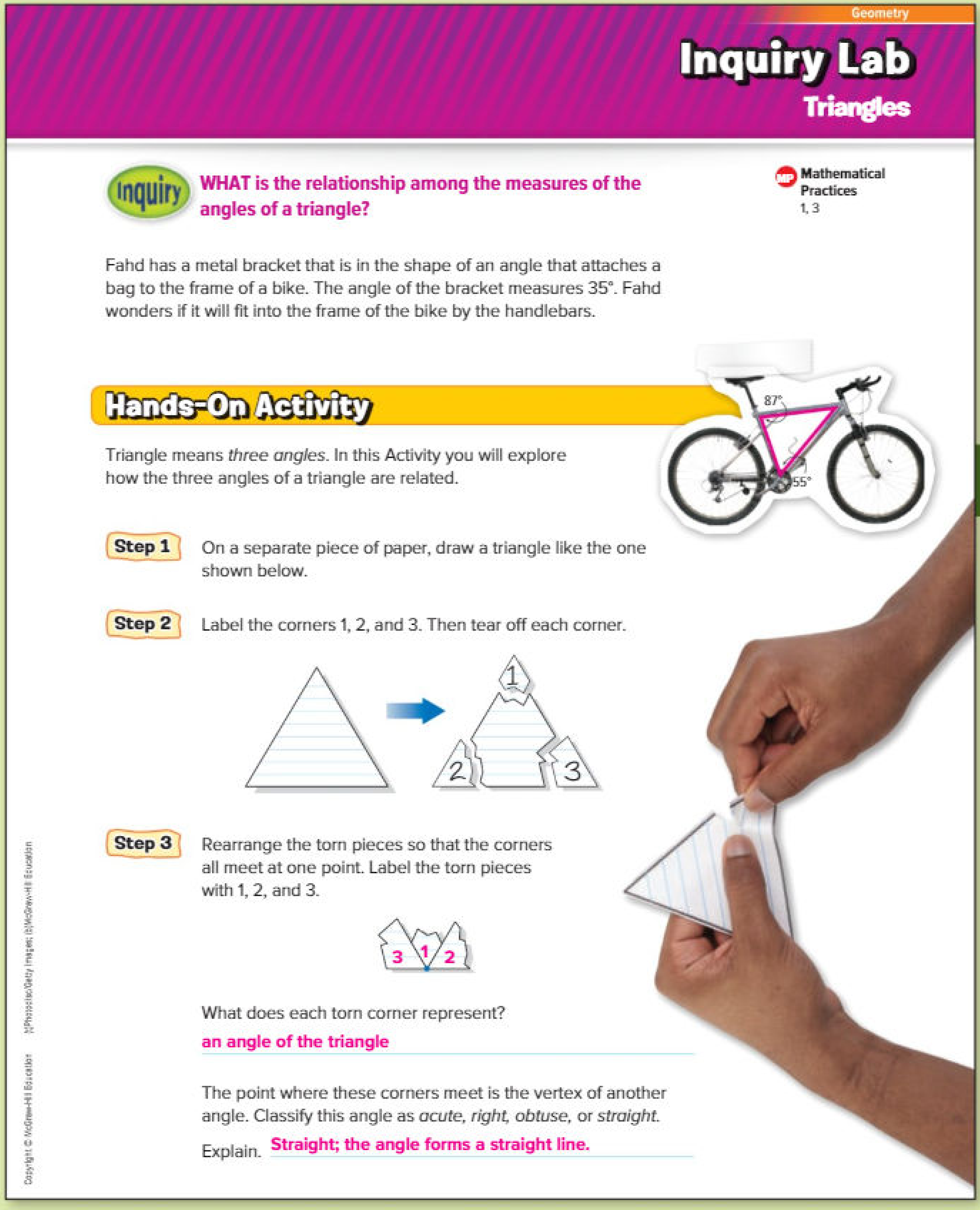


13. $\angle A$ and $\angle B$ **adjacent**

14. $\angle A$ and $\angle C$ **vertical**

15. $\angle C$ and $\angle E$ **neither**

16. $\angle E$ and $\angle F$ **adjacent**



Inquiry Lab

Triangles

Inquiry **WHAT is the relationship among the measures of the angles of a triangle?**

MP Mathematical Practices 1, 3

Fahd has a metal bracket that is in the shape of an angle that attaches a bag to the frame of a bike. The angle of the bracket measures 35°. Fahd wonders if it will fit into the frame of the bike by the handlebars.

Hands-On Activity

Triangle means *three angles*. In this Activity you will explore how the three angles of a triangle are related.

Step 1 On a separate piece of paper, draw a triangle like the one shown below.

Step 2 Label the corners 1, 2, and 3. Then tear off each corner.

Step 3 Rearrange the torn pieces so that the corners all meet at one point. Label the torn pieces with 1, 2, and 3.

What does each torn corner represent?

an angle of the triangle

The point where these corners meet is the vertex of another angle. Classify this angle as *acute, right, obtuse,* or *straight*.

Explain. Straight; the angle forms a straight line.

(t)Photodisc/Getty Images; (b)McGraw-Hill Education

Focus narrowing the scope

Objective Explore the relationship among the angles of a triangle.

Coherence connecting within and across grades

Now
Students explore the relationship among the angles of a triangle.

Next
Students will find missing angle measures in triangles.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 388.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Hands-On Activity

AL **LA** **Think-Pair-Share** Have students work in pairs to complete the Activity. Give students a few minutes to individually read through each step in the activity and think about how they would complete each step. Students then gather in pairs to discuss their responses and progress through completing each step in the activity. Students can then present their responses to the questions in Step 3 to the class. MP 1, 3, 4, 6, 7

BL **LA** **Pairs Consult** Have students work in pairs to alter the activity so that an obtuse or right triangle is used instead of an equilateral triangle. Have them note whether the type of triangle affects the angle that the three corners form. MP 1, 3, 4, 6, 7, 8

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1, 2	 3, 4	 5, 6
Level 3			●
Level 2		●	
Level 1	●		

Investigate

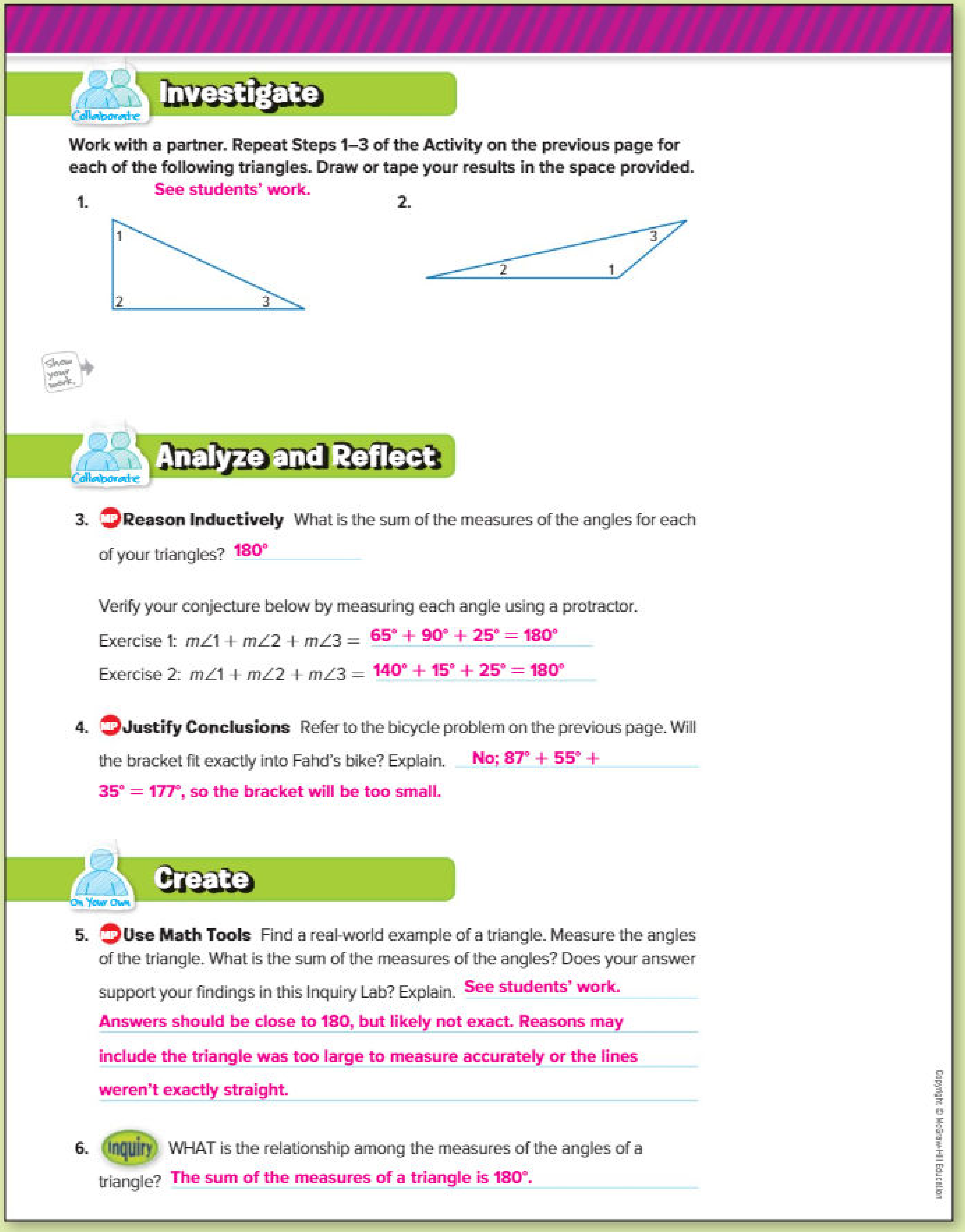
AL **LA** **Rally Coach** Have students work in pairs to complete Exercises 1 and 2. The pair has one piece of paper and pencil. Student 1 completes the first exercise while Student 2 watches and listens, coaches and praises. Students switch roles for the second exercise. **MP 1, 3, 4, 6, 8**

Analyze and Reflect

BL **LA** **Pairs Check** Have students work in pairs to complete Exercises 3 and 4 and trade their answers with another pair of students to check their work. **MP 1, 3, 5, 6, 7, 8**

Create

Inquiry Students should be able to answer "WHAT is the relationship among the measures of the angles of a triangle?" Check for student understanding and provide guidance, if needed.



Investigate

Work with a partner. Repeat Steps 1–3 of the Activity on the previous page for each of the following triangles. Draw or tape your results in the space provided.

See students' work.

1.

2.

Analyze and Reflect

3. **MP Reason Inductively** What is the sum of the measures of the angles for each of your triangles? **180°**

Verify your conjecture below by measuring each angle using a protractor.

Exercise 1: $m\angle 1 + m\angle 2 + m\angle 3 =$ **$65° + 90° + 25° = 180°$**

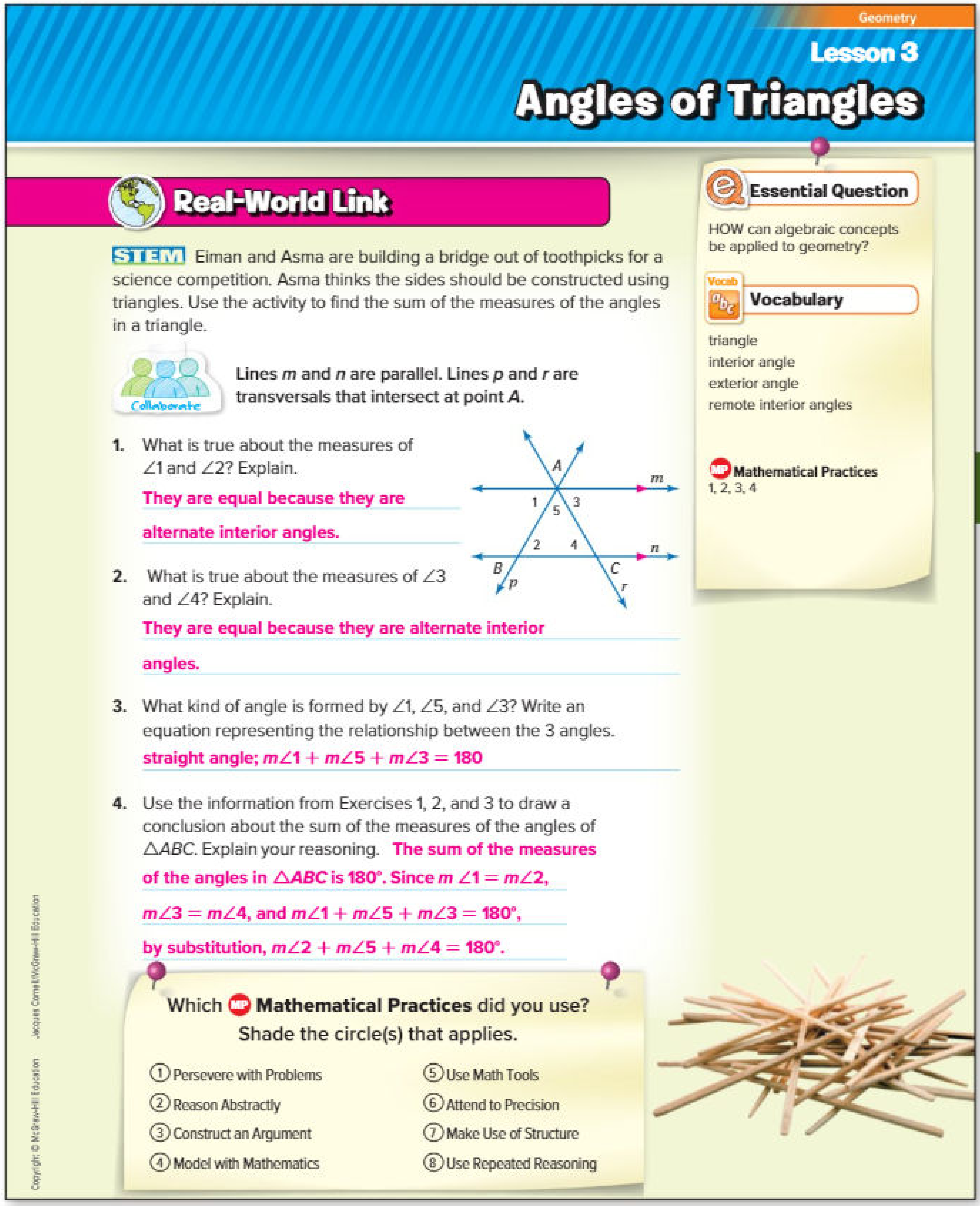
Exercise 2: $m\angle 1 + m\angle 2 + m\angle 3 =$ **$140° + 15° + 25° = 180°$**

4. **MP Justify Conclusions** Refer to the bicycle problem on the previous page. Will the bracket fit exactly into Fahd's bike? Explain. **No; $87° + 55° + 35° = 177°$, so the bracket will be too small.**

Create

5. **MP Use Math Tools** Find a real-world example of a triangle. Measure the angles of the triangle. What is the sum of the measures of the angles? Does your answer support your findings in this Inquiry Lab? Explain. **See students' work. Answers should be close to 180, but likely not exact. Reasons may include the triangle was too large to measure accurately or the lines weren't exactly straight.**

6. **Inquiry** WHAT is the relationship among the measures of the angles of a triangle? **The sum of the measures of a triangle is 180°.**



Geometry

Lesson 3

Angles of Triangles

Real-World Link

STEM Eiman and Asma are building a bridge out of toothpicks for a science competition. Asma thinks the sides should be constructed using triangles. Use the activity to find the sum of the measures of the angles in a triangle.

Collaborate

Lines *m* and *n* are parallel. Lines *p* and *r* are transversals that intersect at point *A*.

1. What is true about the measures of ∠1 and ∠2? Explain.
They are equal because they are alternate interior angles.

2. What is true about the measures of ∠3 and ∠4? Explain.
They are equal because they are alternate interior angles.

3. What kind of angle is formed by ∠1, ∠5, and ∠3? Write an equation representing the relationship between the 3 angles.
straight angle; $m\angle 1 + m\angle 5 + m\angle 3 = 180$

4. Use the information from Exercises 1, 2, and 3 to draw a conclusion about the sum of the measures of the angles of △*ABC*. Explain your reasoning. The sum of the measures of the angles in △*ABC* is 180°. Since $m\angle 1 = m\angle 2$, $m\angle 3 = m\angle 4$, and $m\angle 1 + m\angle 5 + m\angle 3 = 180°$, by substitution, $m\angle 2 + m\angle 5 + m\angle 4 = 180°$.

Which **MP Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Essential Question

HOW can algebraic concepts be applied to geometry?

Vocabulary

triangle
interior angle
exterior angle
remote interior angles

MP Mathematical Practices
1, 2, 3, 4

Jacques Cornell/McGraw-Hill Education

Focus narrowing the scope

Objective Find missing angle measures in triangles.

Coherence connecting within and across grades

Previous	Now	Next
Students explored the relationship of angle measures in triangles.	Students will find missing angle measures in triangles.	Students will examine the relationship among angles of regular polygons.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 393.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA Circle the Sage Poll students to see who has a solid understanding of angles formed when parallel lines are cut by a transversal. Have those students (the sages) spread out around the room. Place remaining students in teams. Have students report to different sages, with no two team members going to the same sage, if possible. Have the sages lead the work for the Real-World Link. Then have students report back to their teams and discuss any differences in solutions. MP 1, 2, 3, 4, 5, 6, 7

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

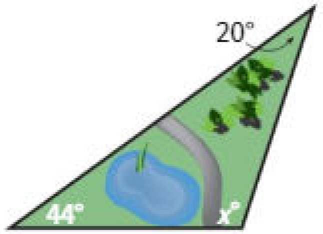
1. Find missing angle measures in triangles.

AL
- *What is the sum of the measures of the interior angles of a triangle?* 180°
- *What are the known angle measures?* 55° and 90°

OL
- *What equation could be used to determine the missing angle measure?* $x° + 55° + 90° = 180°$
- *What is the value of x?* 35

BL
- *If the measure of one known angle in a triangle is 90°, what is the sum of the other two angle measures?* 90°
- *What other equation could you use to solve for x in this example?* $55° + x° = 90°$

Need Another Example?
The city park is in the shape of a triangle. Find the value of x. 116



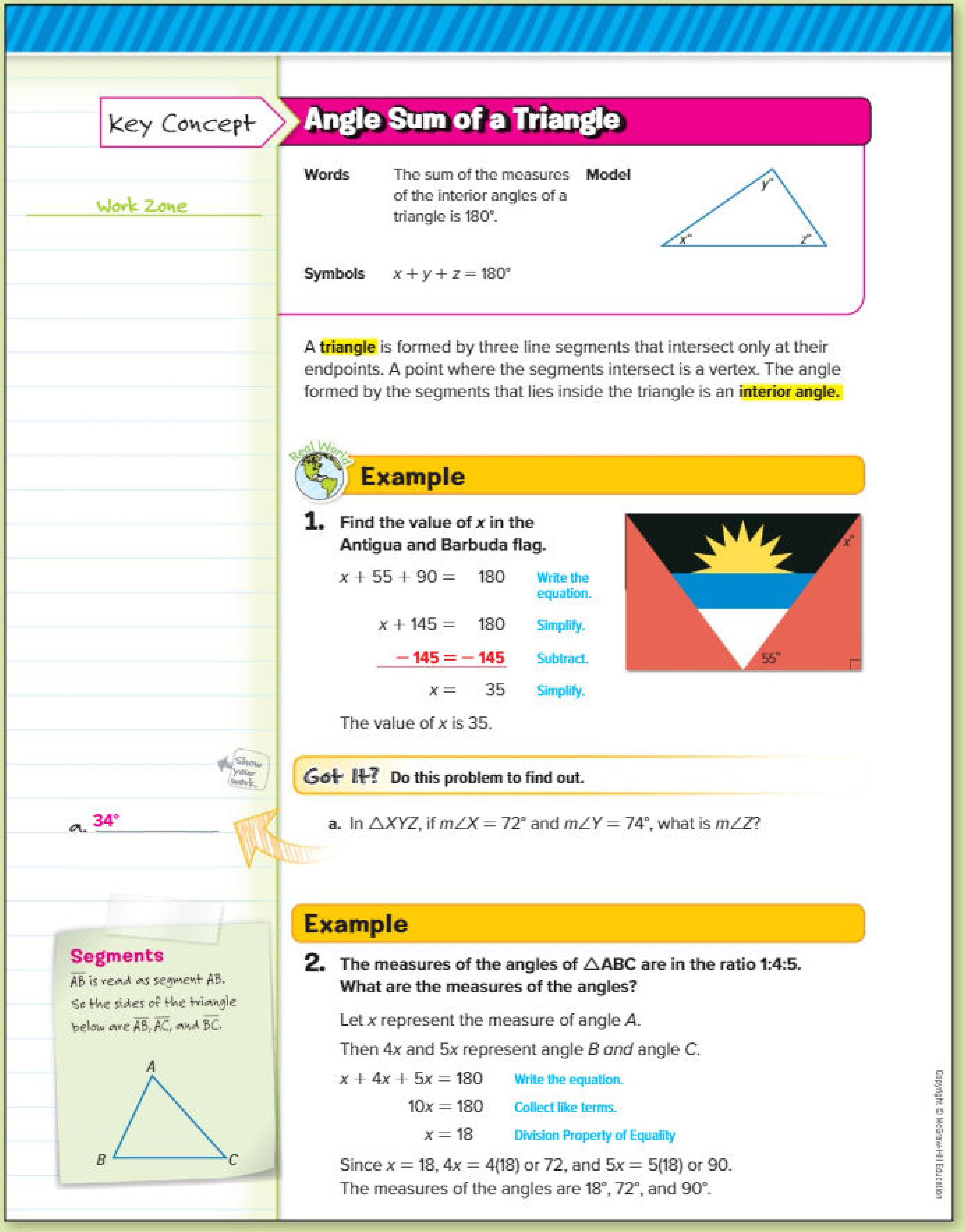
2. Use ratios to find angle measures.

AL
- *What expression could be used to represent the measure of the first angle?* x *the second angle?* 4x *the third angle?* 5x

OL
- *What equation could be used to find the value of x?* $x + 4x + 5x = 180$
- *Why can we write x + 4x + 5x as 10x?* They are like terms.

BL
- *How can we check our work?* Sample answer: Check that the measures 18°, 72°, and 90° are in the ratio 1:4:5.

Need Another Example?
The measures of the angles of triangle DEF are in the ratio 1:2:3. What are the measures of the angles? 30°, 60°, and 90°



Work Zone

Key Concept

Angle Sum of a Triangle

Words The sum of the measures of the interior angles of a triangle is 180°.

Model

Symbols $x + y + z = 180°$

A **triangle** is formed by three line segments that intersect only at their endpoints. A point where the segments intersect is a vertex. The angle formed by the segments that lies inside the triangle is an **interior angle.**

Real World

Example

1. Find the value of x in the Antigua and Barbuda flag.

$x + 55 + 90 = 180$ Write the equation.

$x + 145 = 180$ Simplify.

$-145 = -145$ Subtract.

$x = 35$ Simplify.

The value of x is 35.

Show your work.

Got It? **Do this problem to find out.**

a. In $\triangle XYZ$, if $m\angle X = 72°$ and $m\angle Y = 74°$, what is $m\angle Z$?

a. 34°

Segments

$\overline{AB}$ is read as segment AB. So the sides of the triangle below are $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$.

Example

2. The measures of the angles of △ABC are in the ratio 1:4:5. What are the measures of the angles?

Let x represent the measure of angle A.

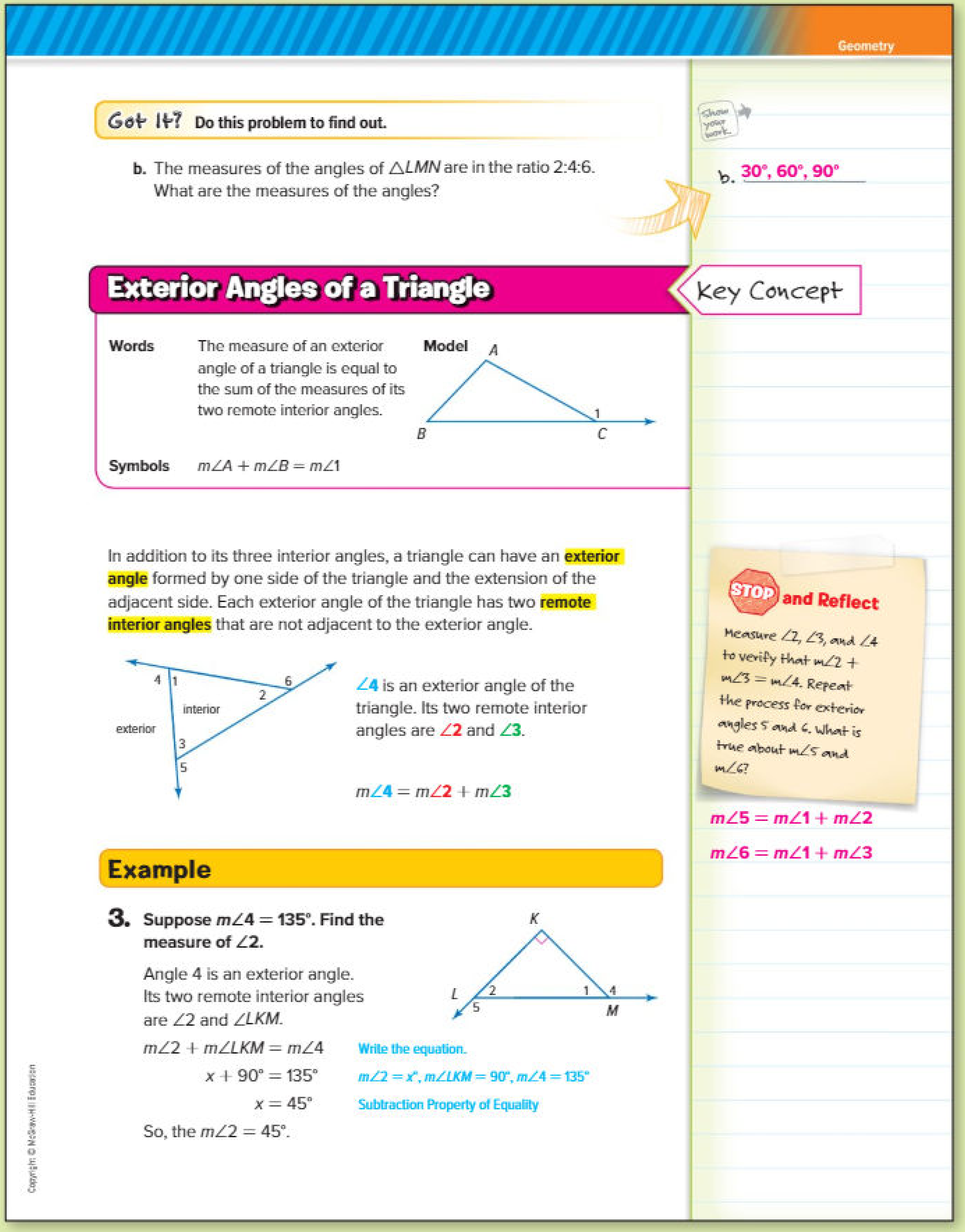
Then 4x and 5x represent angle B *and* angle C.

$x + 4x + 5x = 180$ Write the equation.

$10x = 180$ Collect like terms.

$x = 18$ Division Property of Equality

Since $x = 18$, $4x = 4(18)$ or 72, and $5x = 5(18)$ or 90.
The measures of the angles are 18°, 72°, and 90°.



Got It? Do this problem to find out.

b. The measures of the angles of $\triangle LMN$ are in the ratio 2:4:6. What are the measures of the angles?

Show your work.

b. 30°, 60°, 90°

Exterior Angles of a Triangle

Key Concept

Words The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

Model

Symbols $m\angle A + m\angle B = m\angle 1$

In addition to its three interior angles, a triangle can have an **exterior angle** formed by one side of the triangle and the extension of the adjacent side. Each exterior angle of the triangle has two **remote interior angles** that are not adjacent to the exterior angle.

$\angle 4$ is an exterior angle of the triangle. Its two remote interior angles are $\angle 2$ and $\angle 3$.

$m\angle 4 = m\angle 2 + m\angle 3$

STOP and Reflect

Measure ∠2, ∠3, and ∠4 to verify that m∠2 + m∠3 = m∠4. Repeat the process for exterior angles 5 and 6. What is true about m∠5 and m∠6?

$m\angle 5 = m\angle 1 + m\angle 2$

$m\angle 6 = m\angle 1 + m\angle 3$

Example

3. **Suppose $m\angle 4 = 135°$. Find the measure of $\angle 2$.**

Angle 4 is an exterior angle. Its two remote interior angles are $\angle 2$ and $\angle LKM$.

$m\angle 2 + m\angle LKM = m\angle 4$ Write the equation.

$x + 90° = 135°$ $m\angle 2 = x°$, $m\angle LKM = 90°$, $m\angle 4 = 135°$

$x = 45°$ Subtraction Property of Equality

So, the $m\angle 2 = 45°$.

Example

3. Use exterior angles to find a missing angle.

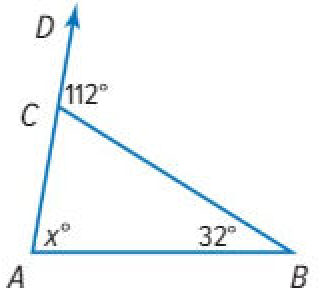
AL
- *What is the sum of the interior angle measures of a triangle?* 180°
- *What kind of angle is angle 4?* exterior
- *What are the two remote interior angles for angle 4?* $\angle 2$ and $\angle LKM$
- *What is the measure of $\angle LKM$?* 90°

OL
- *What is true about an exterior angle and its two remote interior angles?* The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.
- *What equation could be used to find the measure of $\angle 2$?* Sample answer: $m\angle 2 + 90 = 135$

BL
- *Explain another way to find the measure of $\angle 2$.* Sample answer: $\angle 4$ and $\angle 1$ form a straight line, which means that the sum of their angle measures is 180°. $135° + m\angle 1 = 180°$, so $m\angle 1 = 45°$. The total degrees in a triangle are 180°. $45° + 90° + m\angle 2 = 180°$, so $m\angle 2 = 45°$.

Need Another Example?

Find the value of x in the triangle. 80



Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA Team-Pair-Solo Have students complete Exercise 1 in a team of four students, then complete Exercises 2 and 3 in pairs. Have them complete Exercises 4 and 5 on their own and then compare answers with their original team. **MP 1, 3, 5, 6, 7**

BL LA Pairs Research Have students research or look for a real-world example of a triangle formed by transversals crossing parallel lines (a map, for example). Have them explain how known angle measures can help to find unknown angle measures in the triangle. **MP 1, 2, 3, 4, 5, 6, 7**

Watch Out!

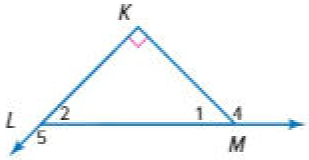
Common Error When working with angle ratios, students may solve for x and leave out calculating the measures of the three angles. Remind students to multiply x by each of the coefficients in the original equation to find the measures of each of the three angles. Then have them find the sum of the angles to check their work.

Show your work.

c. **57°**

Got It? Do this problem to find out.

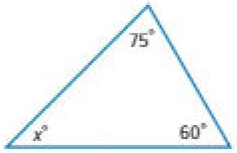
c. Refer to the figure at the right. Suppose $m\angle 5 = 147°$. Find $m\angle 1$.



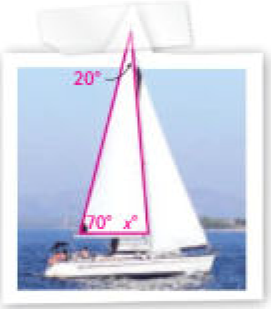
Guided Practice

Check

1. Find the value of x in the triangle. (Example 1) **45**



2. What is the value of x in the sail of the sailboat? (Example 1) **90**



3. The measures of the angles of $\triangle LMN$ are in the ratio 1:2:5. What are the measures of the angles? (Example 2)

22.5°, 45°, 112.5°

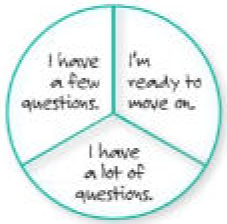
4. Find the value of x in the triangle. (Example 3) **31**

5. **Building on the Essential Question** How can you find the missing measure of an angle in a triangle if you know the measure of two of the interior angles?

Sample answer: If you know the measure of two of the interior angles, you can subtract the sum of those angles' measures from 180 to find the missing measure.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

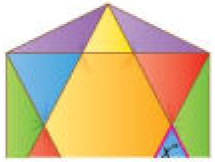


Maria Macrina/Alamy

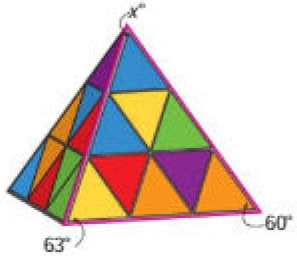
Name ____________________ My Homework ____________________

Independent Practice

1. The top of a kite is shown below. What is the value of x? (Example 1) 55



2. A popular toy puzzle is shown below. What is the value of x? (Example 1) 57



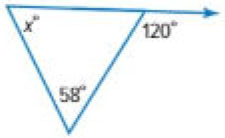
3. The measures of the angles of △RST are in the ratio 2:4:9. What are the measures of the angles? (Example 2) 24°, 48°, 108°

4. The measures of the angles of △XYZ are in the ratio 3:3:6. What are the measures of the angles? (Example 2) 45°, 45°, 90°

Find the value of x in each triangle. (Example 3)

5. 112

6. 62



7. 45

8. In △ABC the measure of angle A is $2x + 3$, the measure of angle B is $4x + 2$, and the measure of angle C is $2x - 1$. What are the measures of the angles? $m\angle A = 47°, m\angle B = 90°, m\angle C = 43°$

9. MP **Reason Abstractly** What is the measure of the third angle of a triangle if one angle measures 25° and the second angle measures 50°?

105°

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–7, 16–26	8–12, 27–29	13–15
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 11, 14, 15, 28, 29
OL	On Level	1–7 odd, 8–12, 14, 15, 28, 29
BL	Beyond Level	8–15, 28, 29

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	13
2 Reason abstractly and quantitatively.	9
3 Construct viable arguments and critique the reasoning of others.	14, 15, 27

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

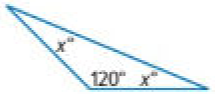
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

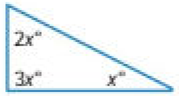
Draw a triangle on the board and label the measures of two of the three angles. Have students describe the steps they would take to find the missing angle measure.
See students' work.

Find the measures of the angles in each triangle.

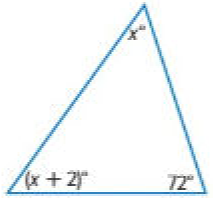
10. 120°, 30°, 30°



90°, 60°, 30°

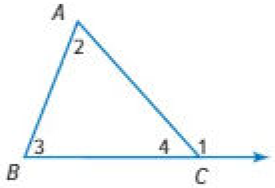


12. 53°, 55°, 72°



H.O.T. Problems Higher Order Thinking

13. **Persevere with Problems** Use the figure at the right to informally prove that an exterior angle of a triangle is equal to the sum of its two remote interior angles.



Given: $\triangle ABC$; $\angle 1$ is an exterior angle.

Prove: $m\angle 1 = m\angle 2 + m\angle 3$

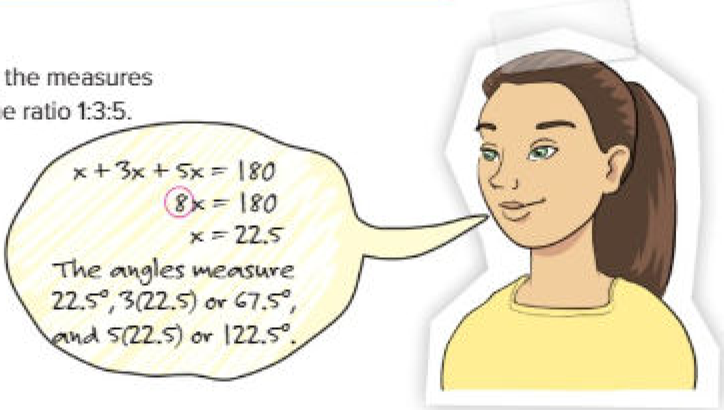
Proof: Sample answer: Since $\angle 1$ and $\angle 4$ form a straight angle , $m\angle 1 + m\angle 4 = 180°$. By the Subtraction Property of Equality, $m\angle 1 = 180 - m\angle 4$. Since ABC is a triangle, $m\angle 2 + m\angle 3 + m\angle 4 = 180$. By the Subtraction Property of Equality, $m\angle 2 + m\angle 3 = 180 - m\angle 4$. So by substitution, $m\angle 2 + m\angle 3 = m\angle 1$.

14. **Find the Error** Nisreen is finding the measures of the angles in a triangle that have the ratio 1:3:5. Circle her mistake and correct it.

$9x = 180$

$x = 20$

The angles measure 20°, 60°, and 100°.



15. **Justify Conclusions** Make a conjecture about the sum of the interior angles of a quadrilateral. Justify your reasoning.

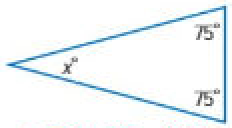
Sample answer: The sum is 360°. Drawing the diagonal of a quadrilateral forms two triangles. So, the sum of the interior angles is 2(180°), or 360°.

Name ______________________ My Homework ______________________

Extra Practice

Find the value of *x* in each triangle with the given angle measures.

16.



Homework Help

$x + 75 + 75 = 180$

$x + 150 = 180$

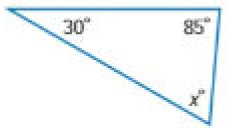
$x = 30$

30

17.

100

18.



65

19. 70°, 60°, x° 50

20. x°, 60°, 25° 95

21. x°, 35°, 25° 120

22. The measures of the angles of △*DEF* are in the ratio 2:4:4. What are the measures of the angles?

36°, 72°, 72°

23. The measures of the angles of △*XYZ* are in the ratio 4:5:6. What are the measures of the angles?

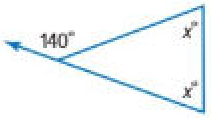
48°, 60°, 72°

Copy and Solve **Find the value of *x* in each triangle. Show your work on a separate sheet of paper.**

24.

75

25.



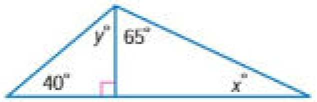
70

26.

125

27. **Reason Inductively** Apply what you know about angles and lines to find the values of *x* and *y* in the figure at the right.

$x =$ 25 $y =$ 50



Power Up! Test Practice

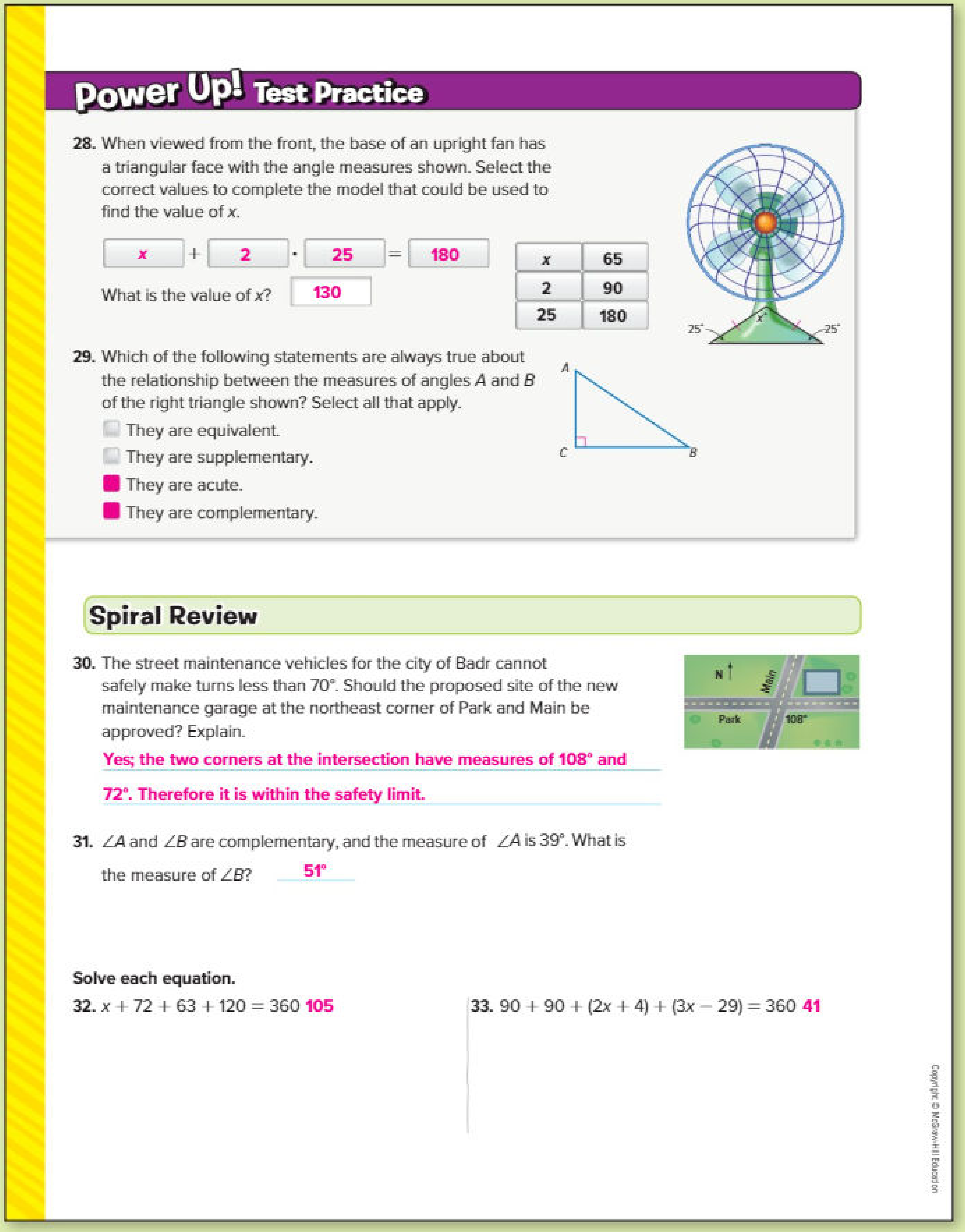
Exercises 28 and 29 prepare students for more rigorous thinking needed when taking assessment.

28.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly place all values to complete the model and find the value of *x*.
1 point	Students correctly place all values to complete the model OR find the value of *x*.

29.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.



Power Up! Test Practice

28. When viewed from the front, the base of an upright fan has a triangular face with the angle measures shown. Select the correct values to complete the model that could be used to find the value of *x*.

$\boxed{x} + \boxed{2} \cdot \boxed{25} = \boxed{180}$

What is the value of *x*? $\boxed{130}$

x	65
2	90
25	180

29. Which of the following statements are always true about the relationship between the measures of angles *A* and *B* of the right triangle shown? Select all that apply.

- ☐ They are equivalent.
- ☐ They are supplementary.
- ☑ They are acute.
- ☑ They are complementary.

Spiral Review

30. The street maintenance vehicles for the city of Badr cannot safely make turns less than 70°. Should the proposed site of the new maintenance garage at the northeast corner of Park and Main be approved? Explain.

Yes; the two corners at the intersection have measures of 108° and 72°. Therefore it is within the safety limit.

31. ∠A and ∠B are complementary, and the measure of ∠A is 39°. What is the measure of ∠B? 51°

Solve each equation.

32. $x + 72 + 63 + 120 = 360$ 105

33. $90 + 90 + (2x + 4) + (3x - 29) = 360$ 41

Lesson 4

Polygons and Angles

Vocabulary Start-Up

A **polygon** is a simple closed figure formed by three or more line segments. The segments intersect only at their endpoints.

A map of the United States is shown. List the states that are in the shape of a polygon. Then list the number of segments that form the polygon. **Sample answers are given. Some students may interpret some state borders as straight segments.**

State	Number of Segments
New Mexico	8
Utah	6
Colorado	4
North Dakota	4
Wyoming	4

Which Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Essential Question

HOW can algebraic concepts be applied to geometry?

Vocabulary

polygon
equiangular
regular polygon

Mathematical Practices
1, 3, 4

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Focus narrowing the scope

Objective Find the sum of the angle measures of a polygon and the measure of one interior angle of a regular polygon.

Coherence connecting within and across grades

Previous
Students used properties of triangles to find missing angle measures.

Now
Students find the measures of angles in polygons.

Next
Students use angle measures to show congruence and similarity of figures.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 401.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

AL LA **Team Consult** Organize students into three teams. Assign each team a two-dimensional figure to find in a picture: 5-sided, 6-sided, or 8-sided. You can provide the picture or they can research a picture using the Internet. Tell them that the figures should not have any curved sides. Have them present their pictures with the figures outlined to the class. MP 7

Alternate Strategy

BL Ask students to give a few examples of states that are not polygons. Then have them justify their response. MP 3, 6

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Find the sum of the interior angle measures of a polygon.

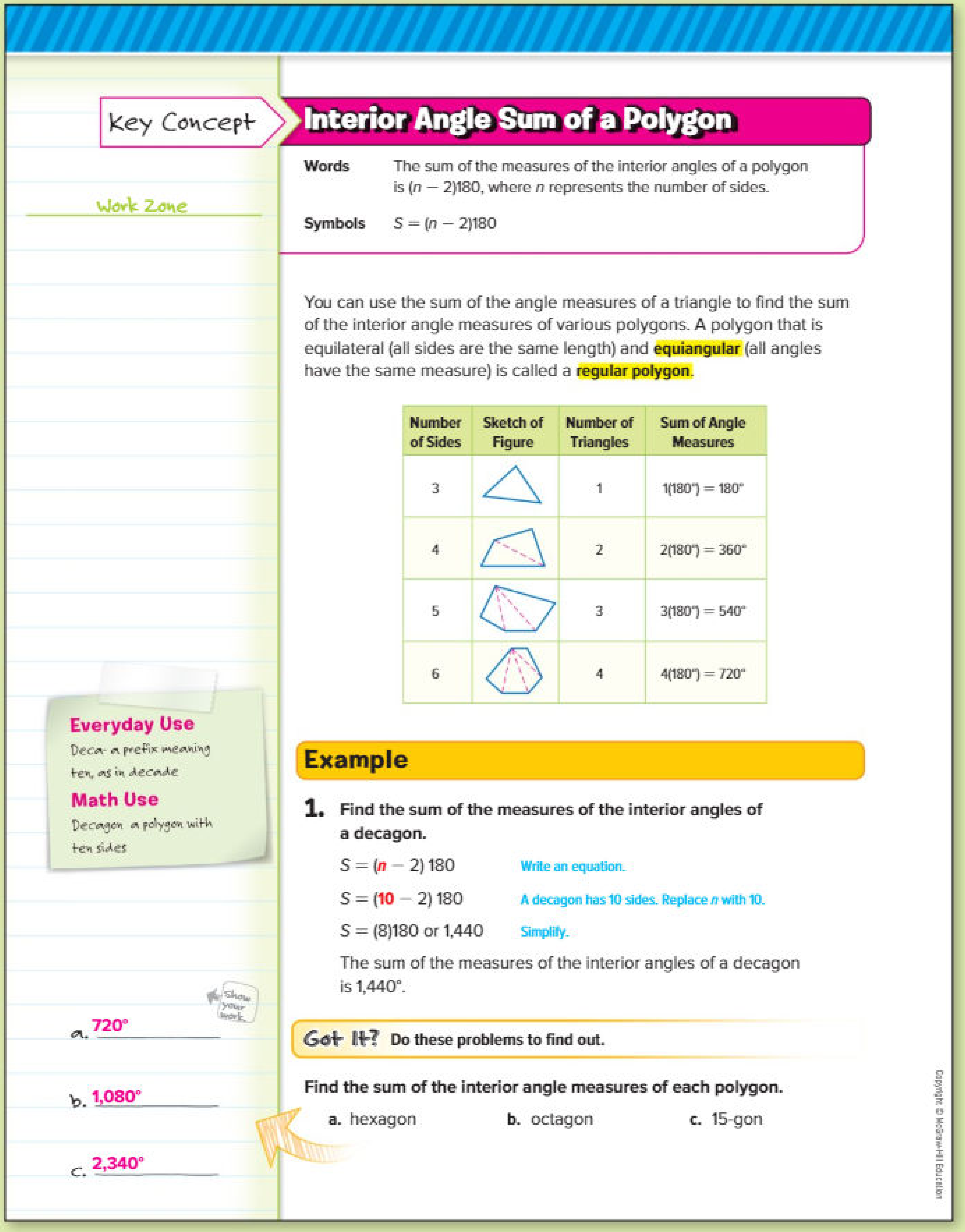
AL
- *How many years are in a decade?* 10
- *Decagon contains the same root word as decade. How many sides are in a decagon?* 10
- *Draw a decagon, then draw all of the diagonals from one vertex. How many triangles did you create?* 8
- *What is the sum of the angle measures for each triangle?* 180°
- *What is the sum of the interior angle measures of eight triangles? Explain how you found the sum.* 1,440°; Multiply 180° by 8.
- *What is the sum of the angle measures of a decagon?* 1,440°

OL
- *What equation can be used to find the sum of the interior angles measures of a polygon with n sides?* $S = (n - 2)180$
- *What equation can be used to find the sum of the interior angle measures of a decagon?* $S = (10 - 2)180$

BL
- *In the equation to find the sum of the measures of the interior angles of a polygon, what does $n - 2$ represent?* the number of triangles you get when you draw all of the diagonals from one vertex
- *Why does the equation use $n - 2$ instead of just n?* The number of triangles that a polygon can be separated into is not equal to the number of sides, n. It is equal to 2 less than the number of sides.

Need Another Example?

Find the sum of the measures of the interior angles of a 13-gon. 1,980°



Key Concept

Interior Angle Sum of a Polygon

Words The sum of the measures of the interior angles of a polygon is $(n - 2)180$, where n represents the number of sides.

Symbols $S = (n - 2)180$

Work Zone

You can use the sum of the angle measures of a triangle to find the sum of the interior angle measures of various polygons. A polygon that is equilateral (all sides are the same length) and **equiangular** (all angles have the same measure) is called a **regular polygon**.

Number of Sides	Sketch of Figure	Number of Triangles	Sum of Angle Measures
3		1	1(180°) = 180°
4		2	2(180°) = 360°
5		3	3(180°) = 540°
6		4	4(180°) = 720°

Everyday Use
Deca- a prefix meaning ten, as in decade
Math Use
Decagon a polygon with ten sides

Example

1. Find the sum of the measures of the interior angles of a decagon.

$S = (n - 2)\ 180$ Write an equation.

$S = (10 - 2)\ 180$ A decagon has 10 sides. Replace n with 10.

$S = (8)180$ or 1,440 Simplify.

The sum of the measures of the interior angles of a decagon is 1,440°.

Got It? Do these problems to find out.

Find the sum of the interior angle measures of each polygon.

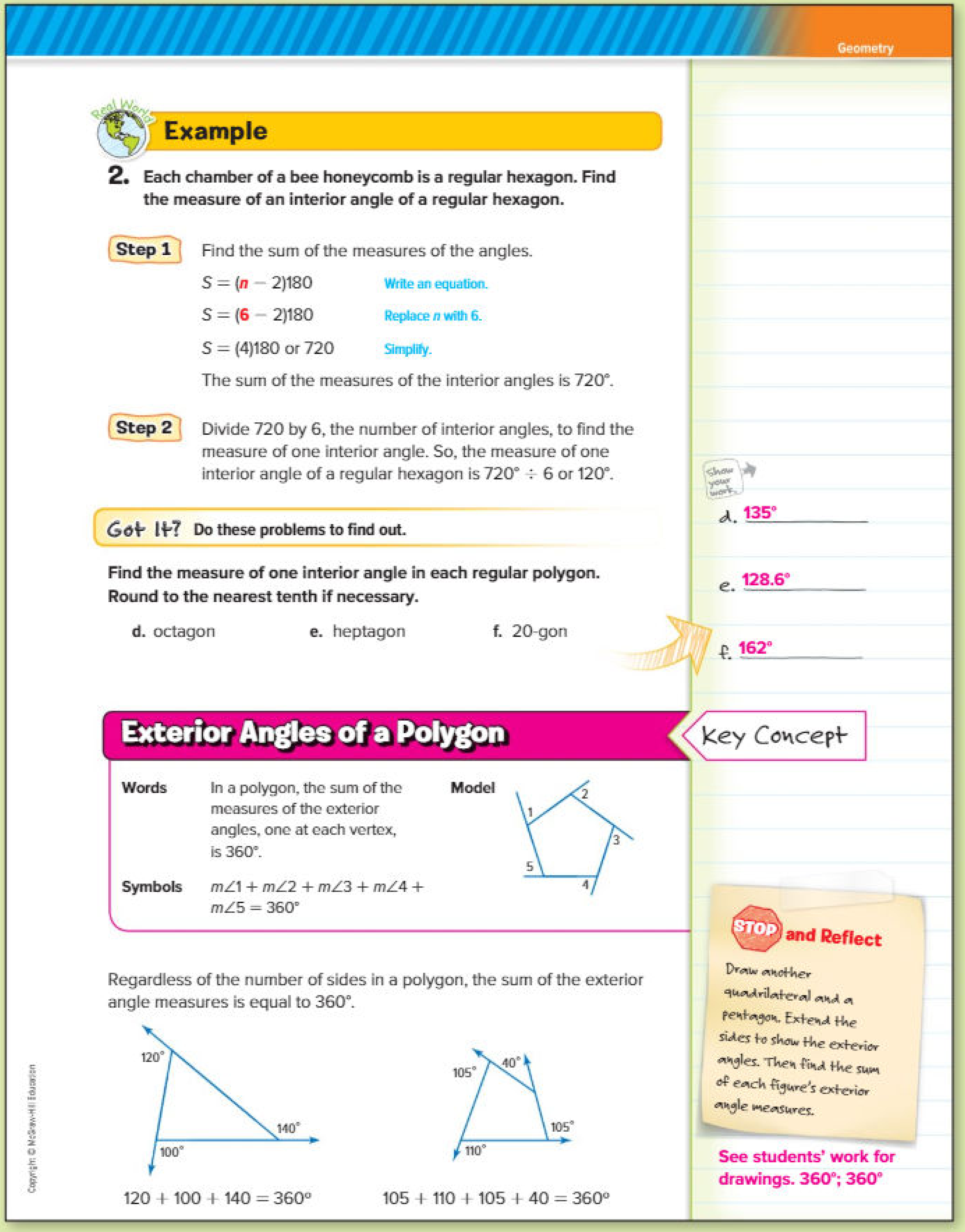
a. hexagon **b.** octagon **c.** 15-gon

Show your work

a. 720°

b. 1,080°

c. 2,340°



Example

2. **Each chamber of a bee honeycomb is a regular hexagon. Find the measure of an interior angle of a regular hexagon.**

Step 1 Find the sum of the measures of the angles.

$S = (n - 2)180$ Write an equation.

$S = (6 - 2)180$ Replace n with 6.

$S = (4)180$ or 720 Simplify.

The sum of the measures of the interior angles is 720°.

Step 2 Divide 720 by 6, the number of interior angles, to find the measure of one interior angle. So, the measure of one interior angle of a regular hexagon is 720° ÷ 6 or 120°.

Got It? **Do these problems to find out.**

Find the measure of one interior angle in each regular polygon. Round to the nearest tenth if necessary.

d. octagon **e.** heptagon **f.** 20-gon

Show your work.

d. 135°

e. 128.6°

f. 162°

Exterior Angles of a Polygon

Key Concept

Words In a polygon, the sum of the measures of the exterior angles, one at each vertex, is 360°.

Model

Symbols $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360°$

Regardless of the number of sides in a polygon, the sum of the exterior angle measures is equal to 360°.

120 + 100 + 140 = 360°

105 + 110 + 105 + 40 = 360°

STOP and Reflect

Draw another quadrilateral and a pentagon. Extend the sides to show the exterior angles. Then find the sum of each figure's exterior angle measures.

See students' work for drawings. 360°; 360°

Example

2. **Find the measure of one interior angle of a regular polygon.**

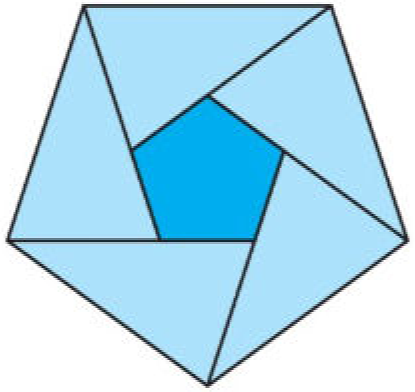
AL
- *What is a regular polygon?* a polygon with equal side lengths and equal angle measures
- *How many sides are in a hexagon?* 6

OL
- *What equation can be used to find the sum of the interior angle measures of a hexagon?* $S = (6 - 2)180$
- *What is the sum of the interior angle measures of a hexagon?* 720°
- *How can you find the measure of one of the interior angles of a regular hexagon?* Divide 720 by 6.

BL
- *Write an equation that can be used to find the measure of one angle M of a regular polygon with n sides.*
  $M = \frac{(n - 2) \cdot 180}{n}$

Need Another Example?

A designer is creating a new logo for a bank. The logo consists of a regular pentagon surrounded by isosceles triangles. Find the measure of an interior angle of a regular pentagon. 108°



Example

3. **Find the measure of one exterior angle in a regular polygon.**

AL
- *What is the sum of the measures of the exterior angles of any polygon?* 360°
- *How many exterior angles are in a hexagon?* 6
- *How would you find the measure of one exterior angle of a regular hexagon?* Find 360 ÷ 6.

OL
- *What is the measure of each exterior angle?* 60°
- *Write an equation to find the measure of one exterior angle m in a regular polygon with n sides.* $m = \frac{360}{n}$

BL
- *Can you use this method to find the measure of an exterior angle of a polygon that is not regular? Explain.* No, the exterior angles of a polygon that is not regular are not all the same.

Need Another Example?
Find the measure of an exterior angle in a regular 30-gon. 12°

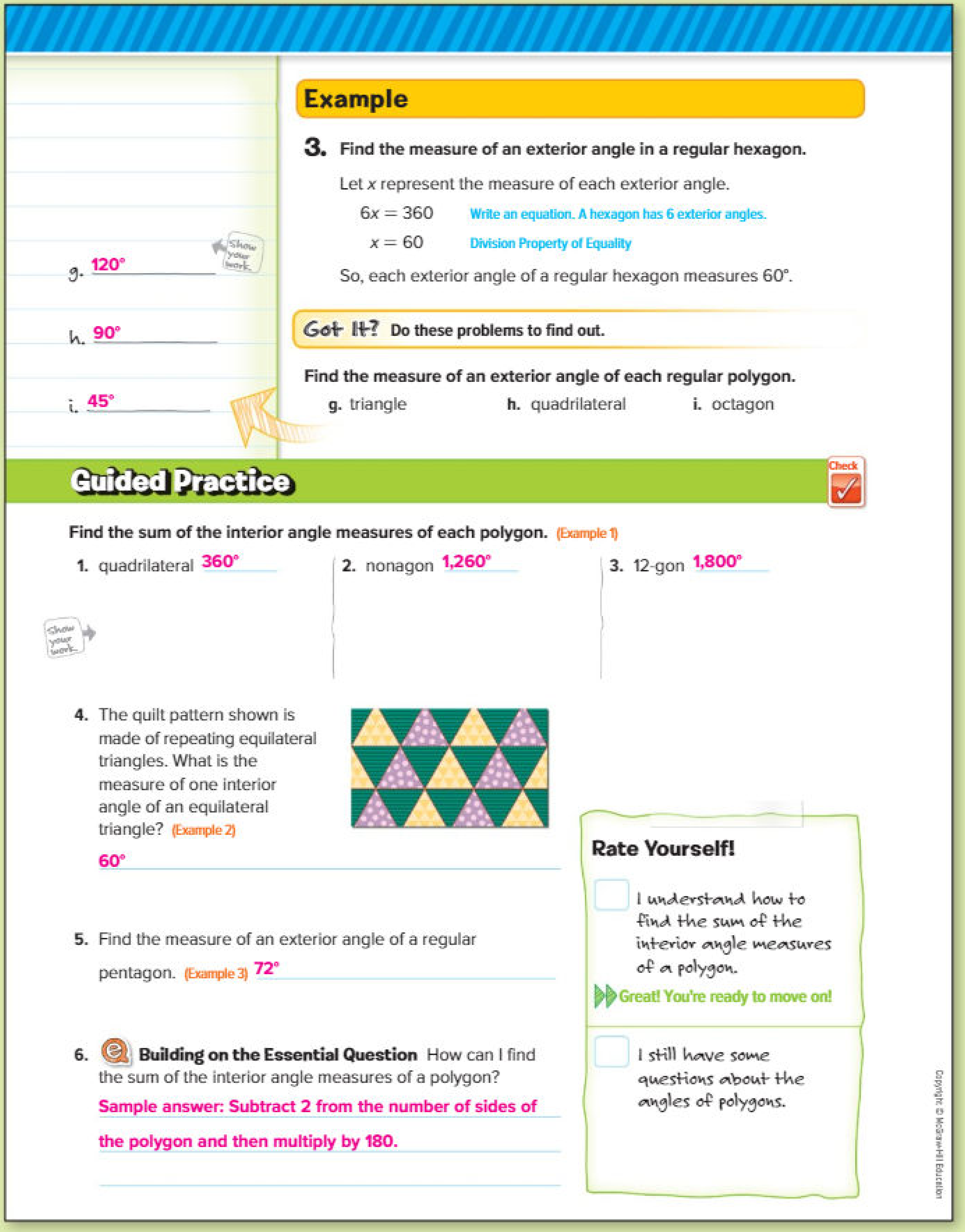
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL **LA** **Pairs Check** Have students work in pairs. Give each pair an index card. On the card, they should list the names of polygons and corresponding number of sides from triangle to decagon. Have students refer to their cards when completing Exercises 1–5. Then have them compare their answers with another pair of students. MP 7

BL **LA** **Pairs Consult** Have students read the information about tessellations in Exercises 8 and 9. Have students work in pairs to research M.C. Escher and tessellations. They should write a paragraph about the attributes of polygon tessellations. Give them sheets of colored paper or allow them to work on a computer to create their own tessellations. Display the tessellations throughout the room. MP 6



Example

3. **Find the measure of an exterior angle in a regular hexagon.**

Let x represent the measure of each exterior angle.

$6x = 360$ Write an equation. A hexagon has 6 exterior angles.

$x = 60$ Division Property of Equality

So, each exterior angle of a regular hexagon measures 60°.

Got It? **Do these problems to find out.**

Find the measure of an exterior angle of each regular polygon.

g. triangle **h.** quadrilateral **i.** octagon

Show your work.

g. 120°

h. 90°

i. 45°

Guided Practice

Check

Find the sum of the interior angle measures of each polygon. (Example 1)

1. quadrilateral 360°

2. nonagon 1,260°

3. 12-gon 1,800°

Show your work.

4. The quilt pattern shown is made of repeating equilateral triangles. What is the measure of one interior angle of an equilateral triangle? (Example 2)

60°

5. Find the measure of an exterior angle of a regular pentagon. (Example 3) 72°

6. **Building on the Essential Question** How can I find the sum of the interior angle measures of a polygon?

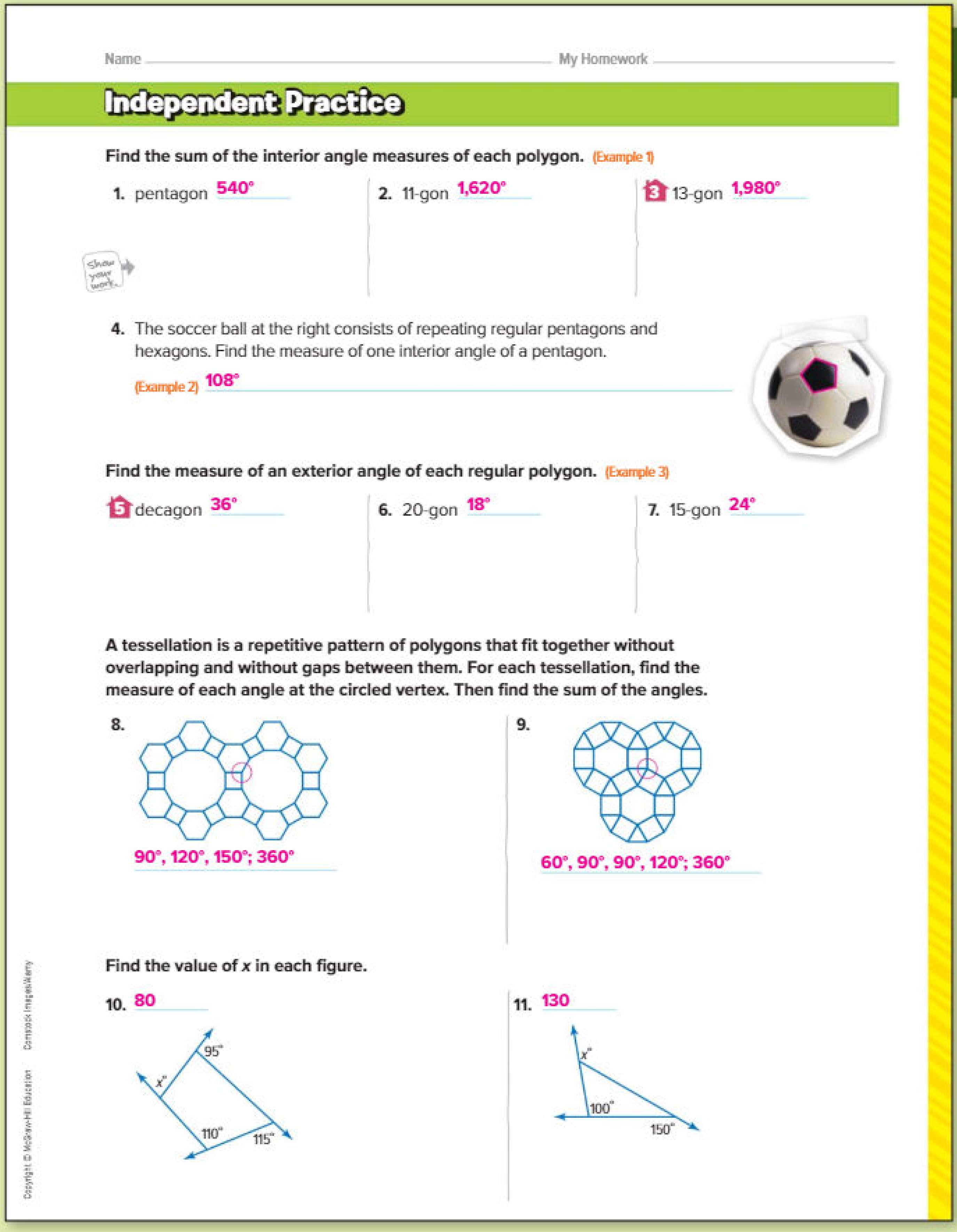
Sample answer: Subtract 2 from the number of sides of the polygon and then multiply by 180.

Rate Yourself!

☐ I understand how to find the sum of the interior angle measures of a polygon.

Great! You're ready to move on!

☐ I still have some questions about the angles of polygons.



Name ______________ My Homework

Independent Practice

Find the sum of the interior angle measures of each polygon. (Example 1)

1. pentagon 540°

2. 11-gon 1,620°

3. 13-gon 1,980°

Show your work.

4. The soccer ball at the right consists of repeating regular pentagons and hexagons. Find the measure of one interior angle of a pentagon.

(Example 2) 108°

Find the measure of an exterior angle of each regular polygon. (Example 3)

5. decagon 36°

6. 20-gon 18°

7. 15-gon 24°

A tessellation is a repetitive pattern of polygons that fit together without overlapping and without gaps between them. For each tessellation, find the measure of each angle at the circled vertex. Then find the sum of the angles.

8. 90°, 120°, 150°; 360°

9. 60°, 90°, 90°, 120°; 360°

Find the value of *x* in each figure.

10. 80

11. 130

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 11, 14, 15, 28, 29
OL	On Level	1–7 odd, 8–12, 14, 15, 28, 29
BL	Beyond Level	8–15, 28, 29

MP MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	13
3 Construct viable arguments and critique the reasoning of others.	14, 15, 27
4 Model with mathematics.	12

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students explain how to find the sum of the interior angles of a polygon when they know the number of sides of the polygon. See students' work.

12\. **Model with Mathematics** Refer to the graphic novel frame below. Find the measures of the two missing angles using properties of quadrilaterals and parallel lines. 130° and 28°

H.O.T. Problems Higher Order Thinking

13\. **Persevere with Problems** How many sides does a regular polygon have if the measure of an interior angle is 160°? Justify your answer.

18; $\frac{(n-2)180}{n} = 160$

$(n-2)180 = 160n$ Multiplication Property of Equality

$180n - 360 = 160n$ Distributive Property

$20n = 360$ Properties of Equality

$n = 18$ Division Property of Equality

14\. **Reason Inductively** If the number of sides of a polygon increases by 1, what happens to the sum of the measures of the interior angles?

It increases by 180°.

15\. **Reason Inductively** Jamal drew a regular polygon and measured one of its interior angles. Explain why it is impossible for his angle measure to be 145°.

Regular decagons have equal angles measuring 144° and regular 11-sided polygons have angles measuring 147.27°. 145° is between these two values so it cannot be the interior angle measure of a regular polygon.

Name ______________________ My Homework ______________________

Extra Practice

Find the sum of the interior angle measures of each polygon.

16. heptagon 900°

Homework Help

$s = (n - 2)180$

$s = (7 - 2)180$

$s = 5 \cdot 180$

$s = 900$

17. 14-gon 2,160°

18. 24-gon 3,960°

Find the measure of one interior angle in each regular polygon. Round to the nearest tenth if necessary.

19. nonagon 140°

20. decagon 144°

21. 19-gon 161.1°

22. 16-gon 157.5°

Find the measure of an exterior angle of each regular polygon.

23. nonagon 40°

24. 12-gon 30°

25. 18-gon 20°

26. The surface of the dome of Spaceship Earth in Orlando consists of repeating equilateral triangles as shown. Find the measure of each angle in each outlined triangle. Then make a conjecture about the interior angle measures in equilateral triangles of different sizes.

The measure of each angle in each outlined triangle is 60°. If a triangle is equilateral, the measure of each angle will be 60° regardless of the size of the triangle.

27. **MP Justify Conclusions** What is the sum of the interior angles of nonregular hexagons? Explain your reasoning to a classmate.

Sample answer: The sum of the interior angles will still be 720° because even though the figures are not regular, they are still hexagons.

Power Up! Test Practice

Exercises 28 and 29 prepare students for more rigorous thinking needed when taking assessment.

28.

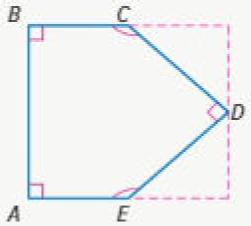
This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly complete the model and find the measure of angle *AED*.
1 point	Students correctly complete the model OR find the measure of angle *AED*.

29.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly answer all four parts of the question.
1 point	Students correctly answer three of the four parts of the question.

Power Up! Test Practice

28. After the first two folds of an origami paper design, the paper is shaped like a square with two isosceles triangles removed from two adjacent corners.



Angles *AED* and *BCD* are congruent. Select the correct values to complete the model below to find the measure of angle *AED*.

x	2	3	45	90
180	360	540	720	

$\boxed{2} \cdot \boxed{x} + \boxed{3} \cdot \boxed{90} = \boxed{540}$

What is $m\angle AED$? $\boxed{135^\circ}$

29. Fill in each box to make each statement true.

a. The sum of the interior angle measures of a quadrilateral is $\boxed{360^\circ}$.

b. The sum of the interior angle measures of a(n) $\boxed{\text{hexagon}}$ is 720°.

c. The sum of the interior angle measures of an octagon is $\boxed{1{,}080^\circ}$.

d. The sum of the interior angle measures of a(n) $\boxed{\text{11-gon}}$ is 1,620°.

Spiral Review

Classify each pair of angles as *complementary*, *supplementary*, or *neither*.

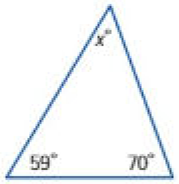
30. angle 1: 35° complementary
angle 2: 55°

Show your work

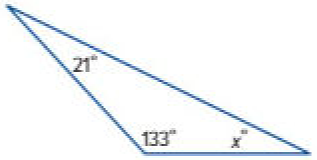
31. angle 1: 62° neither
angle 1: 108°

Find the value of *x* in each triangle.

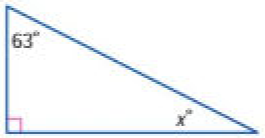
32. 51

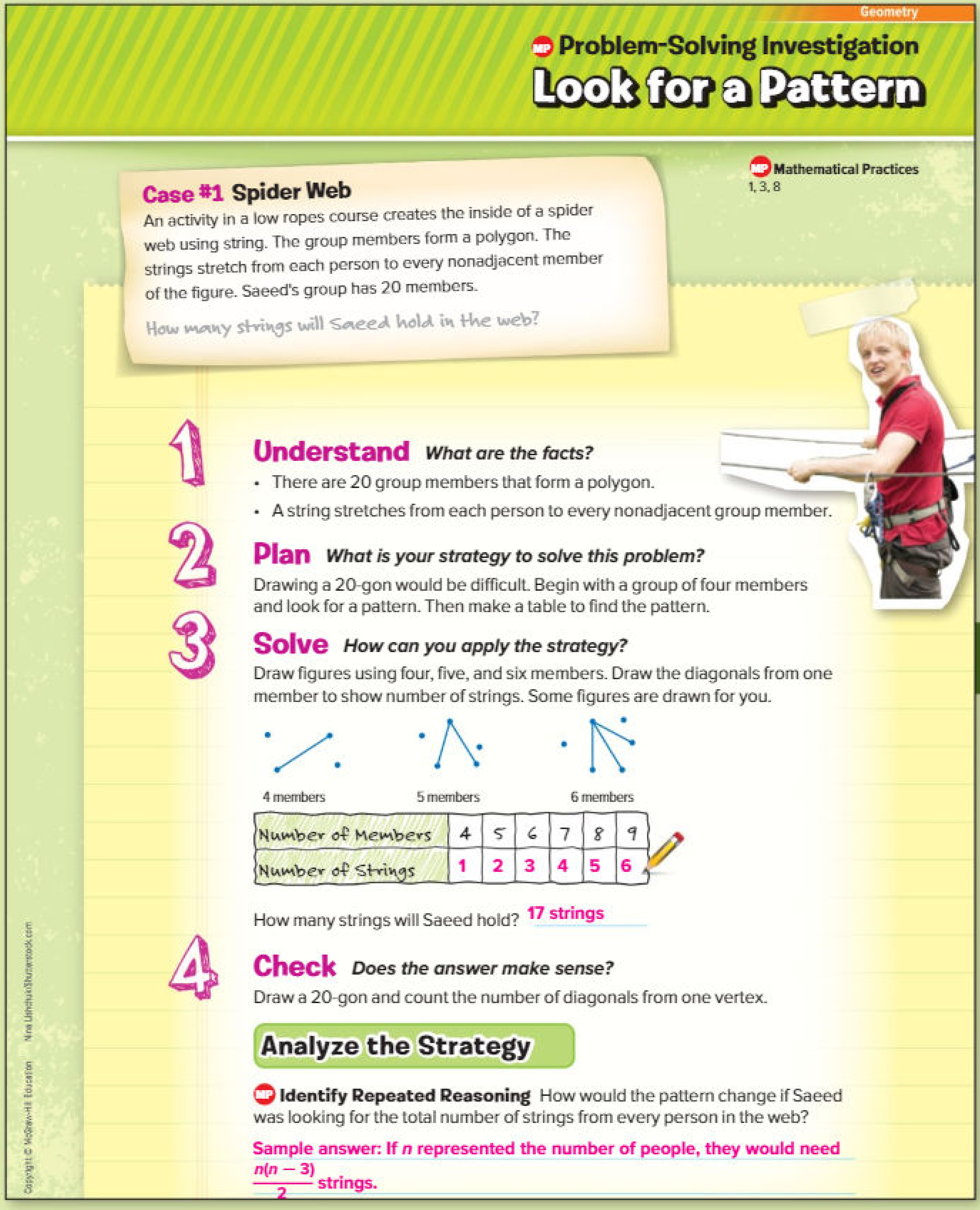


33. 26



34. 27





Problem-Solving Investigation

Look for a Pattern

Mathematical Practices
1, 3, 8

Case #1 Spider Web

An activity in a low ropes course creates the inside of a spider web using string. The group members form a polygon. The strings stretch from each person to every nonadjacent member of the figure. Saeed's group has 20 members.

How many strings will Saeed hold in the web?

1 **Understand** *What are the facts?*

- There are 20 group members that form a polygon.
- A string stretches from each person to every nonadjacent group member.

2 **Plan** *What is your strategy to solve this problem?*

Drawing a 20-gon would be difficult. Begin with a group of four members and look for a pattern. Then make a table to find the pattern.

3 **Solve** *How can you apply the strategy?*

Draw figures using four, five, and six members. Draw the diagonals from one member to show number of strings. Some figures are drawn for you.

4 members   5 members   6 members

Number of Members	4	5	6	7	8	9
Number of Strings	1	2	3	4	5	6

How many strings will Saeed hold? 17 strings

4 **Check** *Does the answer make sense?*

Draw a 20-gon and count the number of diagonals from one vertex.

Analyze the Strategy

Identify Repeated Reasoning How would the pattern change if Saeed was looking for the total number of strings from every person in the web?

Sample answer: If *n* represented the number of people, they would need $\frac{n(n-3)}{2}$ strings.

Focus narrowing the scope

Objective Solve problems by using the *look for a pattern* strategy. This lesson emphasizes **Mathematical Practice 8** Identify Repeated Reasoning.

Look for a Pattern Looking for a pattern is a good strategy for solving a variety of problems. When working with patterns, it is sometimes helpful to organize information in a table.

Coherence connecting within and across grades

Now	Next
Students solve non-routine problems.	Students will apply the look for a pattern strategy to analyze the relationship between the side lengths of a right triangle.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 407.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

The problems on pages 405 and 406 are intended to be used as a whole-group discussion on how to solve non-routine problems and are designed to provide scaffolded guidance. The problem on page 405 walks students through the solution, while the problem on page 406 asks students to come up with their own solutions.

Case #1 Spider Web

BL Have students extend the problem by answering the following question.

Ask:

- *Write an expression that can be used to find the number of strings Chen will hold for n members.* $n - 3$

Case #2 Follow the Bouncing Ball

AL LA **Rally Coach** Have students work in pairs to solve the problem. Have Student A complete the first step, speaking out loud, while Student B listens carefully, coaches, and praises. Next, have Student B complete the second step while Student A listens carefully, coaches, and praises. Partners take turns until they have solved the problem. MP 1, 2, 3, 4, 5, 6, 7, 8

BL LA **Pairs Discussion** Have students work in pairs to answer the following extension question. MP 1, 2, 3, 4, 5, 6, 7, 8

Ask:

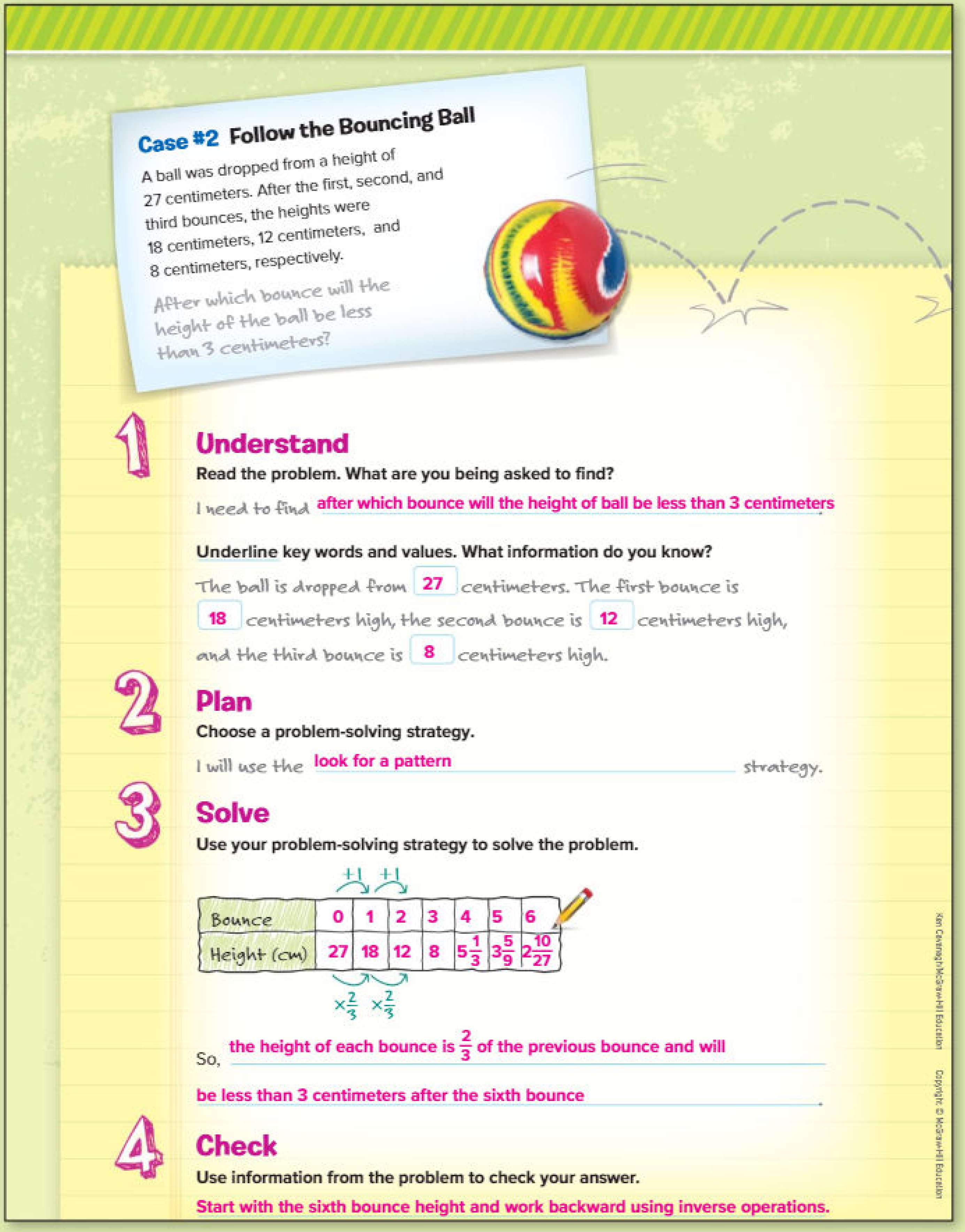
- *How could you solve this problem a different way?* Sample answer: The height of each bounce is $\frac{2}{3}$ the height of the previous bounce. I could draw a diagram to show the height of each bounce after the third bounce.

Need Another Example?

All new showerheads are required to restrict their water flow. Determine how long it will take Hidaya to use 18 liters of water.

Number of Minutes	1	2	3	4
Number of Liters	$2\frac{1}{2}$	5	$7\frac{1}{2}$	10

Each minute, she uses $2\frac{1}{2}$ liters of water. She will use 18 liters of water between the 7th and 8th minute.



Case #2 Follow the Bouncing Ball

A ball was dropped from a height of 27 centimeters. After the first, second, and third bounces, the heights were 18 centimeters, 12 centimeters, and 8 centimeters, respectively.

After which bounce will the height of the ball be less than 3 centimeters?

1 Understand

Read the problem. What are you being asked to find?

I need to find after which bounce will the height of ball be less than 3 centimeters.

Underline key words and values. What information do you know?

The ball is dropped from 27 centimeters. The first bounce is 18 centimeters high, the second bounce is 12 centimeters high, and the third bounce is 8 centimeters high.

2 Plan

Choose a problem-solving strategy.

I will use the look for a pattern strategy.

3 Solve

Use your problem-solving strategy to solve the problem.

+1 +1

Bounce	0	1	2	3	4	5	6
Height (cm)	27	18	12	8	$5\frac{1}{3}$	$3\frac{5}{9}$	$2\frac{10}{27}$

$\times\frac{2}{3}$ $\times\frac{2}{3}$

So, the height of each bounce is $\frac{2}{3}$ of the previous bounce and will be less than 3 centimeters after the sixth bounce.

4 Check

Use information from the problem to check your answer.

Start with the sixth bounce height and work backward using inverse operations.

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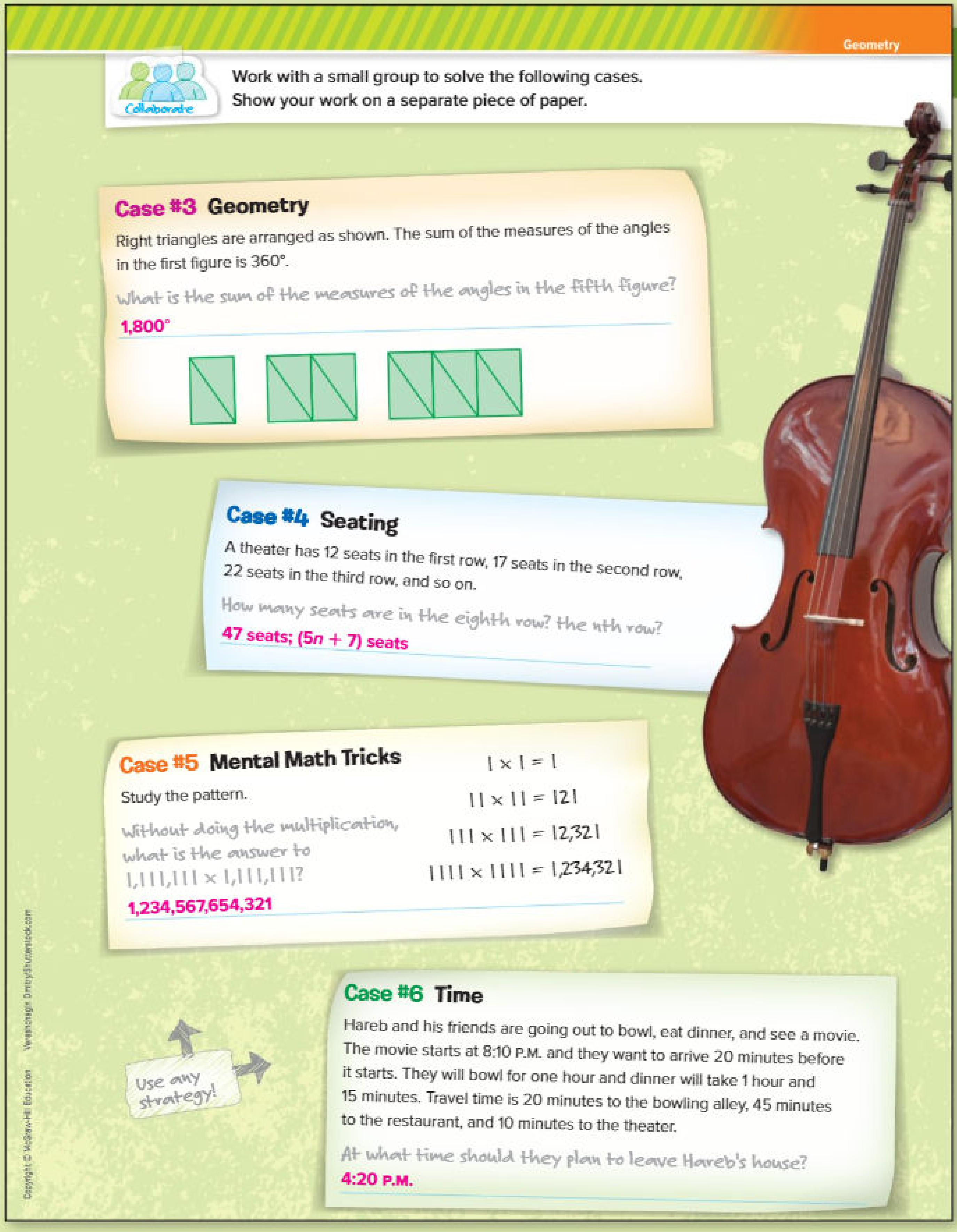

Work with a small group to solve the following cases.
Show your work on a separate piece of paper.

Case #3 Geometry

Right triangles are arranged as shown. The sum of the measures of the angles in the first figure is 360°.

What is the sum of the measures of the angles in the fifth figure?

1,800°

Case #4 Seating

A theater has 12 seats in the first row, 17 seats in the second row, 22 seats in the third row, and so on.

How many seats are in the eighth row? the *n*th row?

47 seats; (5*n* + 7) seats

Case #5 Mental Math Tricks

Study the pattern.

1 × 1 = 1
11 × 11 = 121
111 × 111 = 12,321
1111 × 1111 = 1,234,321

Without doing the multiplication, what is the answer to 1,111,111 × 1,111,111?

1,234,567,654,321

Case #6 Time

Hareb and his friends are going out to bowl, eat dinner, and see a movie. The movie starts at 8:10 P.M. and they want to arrive 20 minutes before it starts. They will bowl for one hour and dinner will take 1 hour and 15 minutes. Travel time is 20 minutes to the bowling alley, 45 minutes to the restaurant, and 10 minutes to the theater.

At what time should they plan to leave Hareb's house?

4:20 P.M.

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ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

2 Collaborate

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	3, 4	5	6
Level 3			●
Level 2		●	
Level 1	●		

AL **LA** **Roundrobin** Have students work in pairs to extend the pattern in Case 4 to find the number of seats in the fourth row, fifth row, sixth row, and so on, until they reach the eighth row. If they are struggling with the number of seats in the *n*th row, provide them with the beginning of the expression, ____*n* + 7. Have them find the coefficient of *n*. MP 1, 2, 3, 4, 5, 6, 7, 8

BL **LA** **Trade-a-Problem** Have students create their own real-world problem with a pattern. Then have them trade their problems with each other, solve, and compare solutions. If the solutions do not agree, students should work together to find the errors. MP 1, 2, 3, 4, 5, 6, 7, 8

Mid-Chapter Check

If students have trouble with Exercises 1–10, they may need help with the following concepts.

Concept	Exercise(s)
parallel lines and transversals (Lesson 1)	1, 3–8, 10
polygons and angles (Lesson 4)	2, 9

Vocabulary Activity

LA **Numbered Heads Together** Have students work in groups of 4 to complete Exercise 1. Each student is assigned a number from 1 to 4. Each student is also assigned one of the angle relationships, such as corresponding angles. Students are responsible to ensure that each group member understands the meaning of their angle relationship. Students should ask each other for clarification and assistance, as needed. Call on one numbered student to define their angle relationship for the class. Then have the groups complete Exercise 2. MP 1, 4, 6, 7

Alternate Strategies

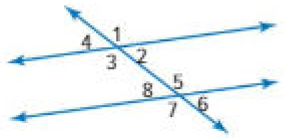
AL Have students use highlighters or colored pencils to identify examples of angle relationships shown on the diagram in Exercise 1.

BL Have students write equations that represent the angle relationships shown on the diagram in Exercise 1.

Mid-Chapter Check

Vocabulary Check

1. Name a pair of angles for each of the following. (Lesson 1) **Sample answers: 1a–1d.**



a. corresponding angles **2 and 6**

b. alternate interior angles **3 and 5**

c. vertical angles **4 and 2**

d. alternate exterior angles **1 and 7**

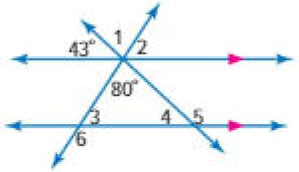
2. List two attributes of regular polygons. (Lesson 4)

all of the sides are the same length

all of the angles have the same measure

Skills Check and Problem Solving

Refer to the figure at the right. Find the missing measure of each angle. (Lessons 1 and 3)



3. $m\angle 1 =$ **80°**

4. $m\angle 2 =$ **57°**

5. $m\angle 3 =$ **57°**

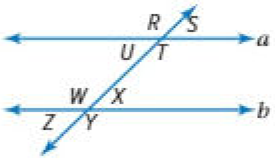
6. $m\angle 4 =$ **43°**

7. $m\angle 5 =$ **137°**

8. $m\angle 6 =$ **123°**

9. A building is in the shape of a regular polygon with five sides. What is the measure of one of the interior angles of the building? (Lesson 4) **108°**

10. MP **Use Math Tools** In the figure, line a is parallel to line b. Which of the following are equal to the measure of $\angle T$? (Lesson 1) **I, III, IV**

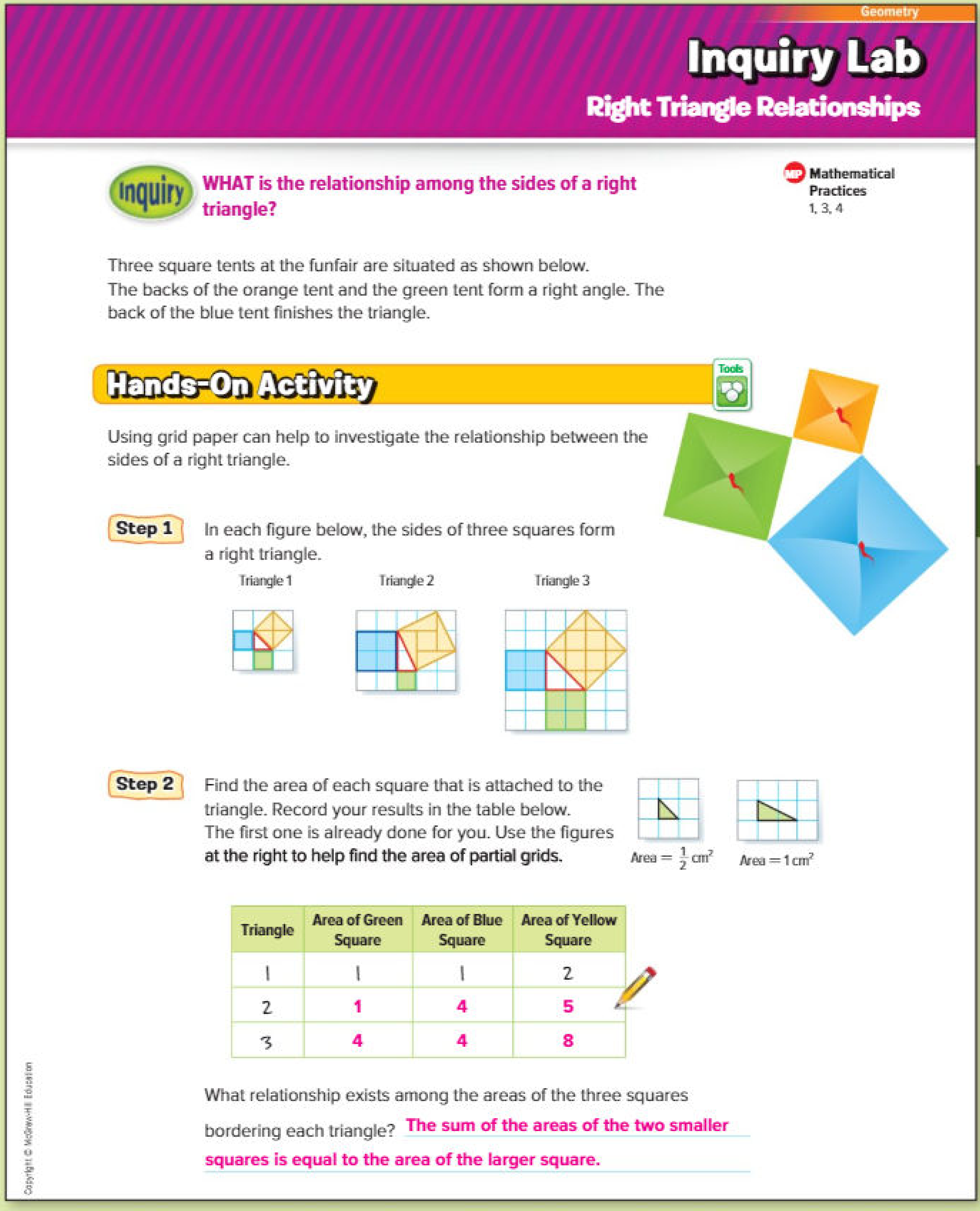


I the supplement of $\angle S$

II the complement of $\angle X$

III the angle adjacent to $\angle Z$

IV angle corresponding to $\angle R$



Inquiry Lab

Right Triangle Relationships

Inquiry WHAT is the relationship among the sides of a right triangle?

Mathematical Practices 1, 3, 4

Three square tents at the funfair are situated as shown below.
The backs of the orange tent and the green tent form a right angle. The back of the blue tent finishes the triangle.

Hands-On Activity

Tools

Using grid paper can help to investigate the relationship between the sides of a right triangle.

Step 1 In each figure below, the sides of three squares form a right triangle.

Triangle 1 Triangle 2 Triangle 3

Step 2 Find the area of each square that is attached to the triangle. Record your results in the table below.
The first one is already done for you. Use the figures at the right to help find the area of partial grids.

Area = $\frac{1}{2}$ cm^2 Area = 1 cm^2

Triangle	Area of Green Square	Area of Blue Square	Area of Yellow Square
1	1	1	2
2	1	4	5
3	4	4	8

What relationship exists among the areas of the three squares bordering each triangle? The sum of the areas of the two smaller squares is equal to the area of the larger square.

Focus narrowing the scope

Objective Model the relationship among the sides of a right triangle.

Coherence connecting within and across grades

Now
Students will model the relationship among the sides of a right triangle.

Next
Students will use the Pythagorean Theorem and its converse to solve problems.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 410.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Hands-On Activity

AL LA **Three-Step Interview** Have pairs complete the activity. Upon completion, have Student 1 interview Student 2, using the table and and question in Step 2 as interview questions. Then Student 1 asks Student 2 any clarification questions about the relationship and how Student 2 can verify the relationship using the values they generated in the table. MP 1, 3, 4, 6, 7, 8

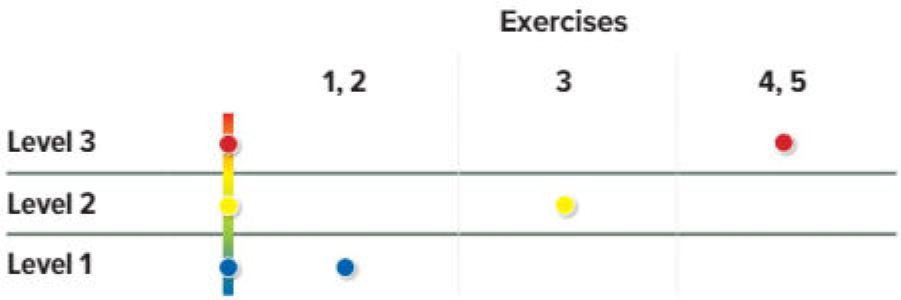
BL LA **Pairs Consult** Have students work with a partner to translate their verbal response to the question in Step 2 into an equation. Have them use the letters *a*, *b*, and *c* to represent the lengths of the sides of the triangle, letting *c* represent the length of the longest side. MP 1, 2, 4, 6, 7

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



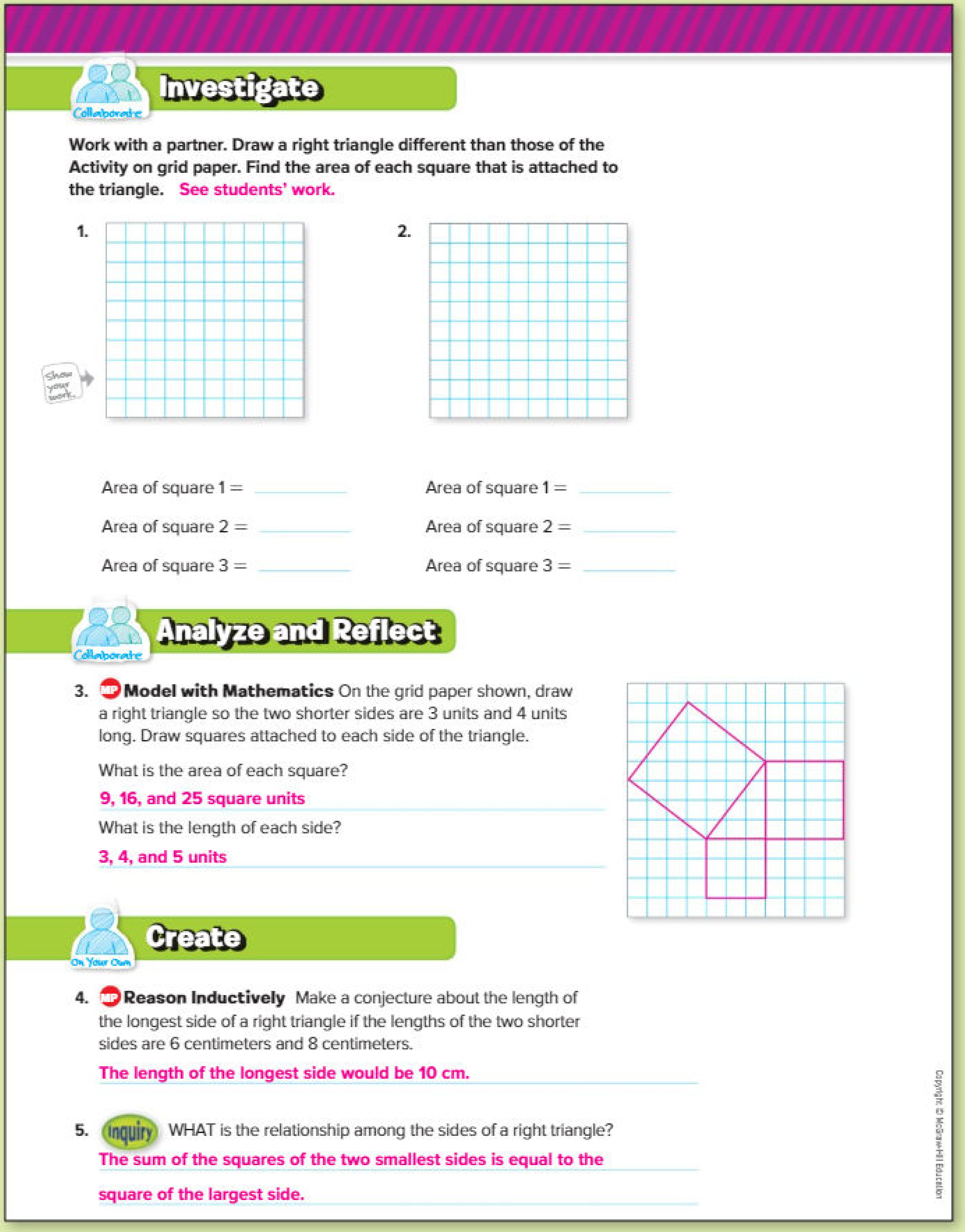
Investigate

AL **LA** **Pairs Discussion** Have students work in pairs. Have each student complete Exercise 1 on their own. Then have them trade their triangles and complete their partner's triangle for Exercise 2. Then have them compare solutions and discuss any differences. **MP** 1, 3, 4, 6

Create

BL Have students verify their conjecture in Exercise 4 by using grid paper to draw the triangle. Then have them make a conjecture about the length of the longest side of a triangle if the lengths of the two shorter sides are double that of the sides in Exercise 4. **MP** 1, 3, 4, 5, 6, 7

Inquiry Students should be able to answer "WHAT is the relationship among the sides of a right triangle?" Check for student understanding and provide guidance, if needed.



Investigate

Work with a partner. Draw a right triangle different than those of the Activity on grid paper. Find the area of each square that is attached to the triangle. See students' work.

1.

Area of square 1 = ______

Area of square 2 = ______

Area of square 3 = ______

2.

Area of square 1 = ______

Area of square 2 = ______

Area of square 3 = ______

Analyze and Reflect

3. **MP** **Model with Mathematics** On the grid paper shown, draw a right triangle so the two shorter sides are 3 units and 4 units long. Draw squares attached to each side of the triangle.

What is the area of each square?

9, 16, and 25 square units

What is the length of each side?

3, 4, and 5 units

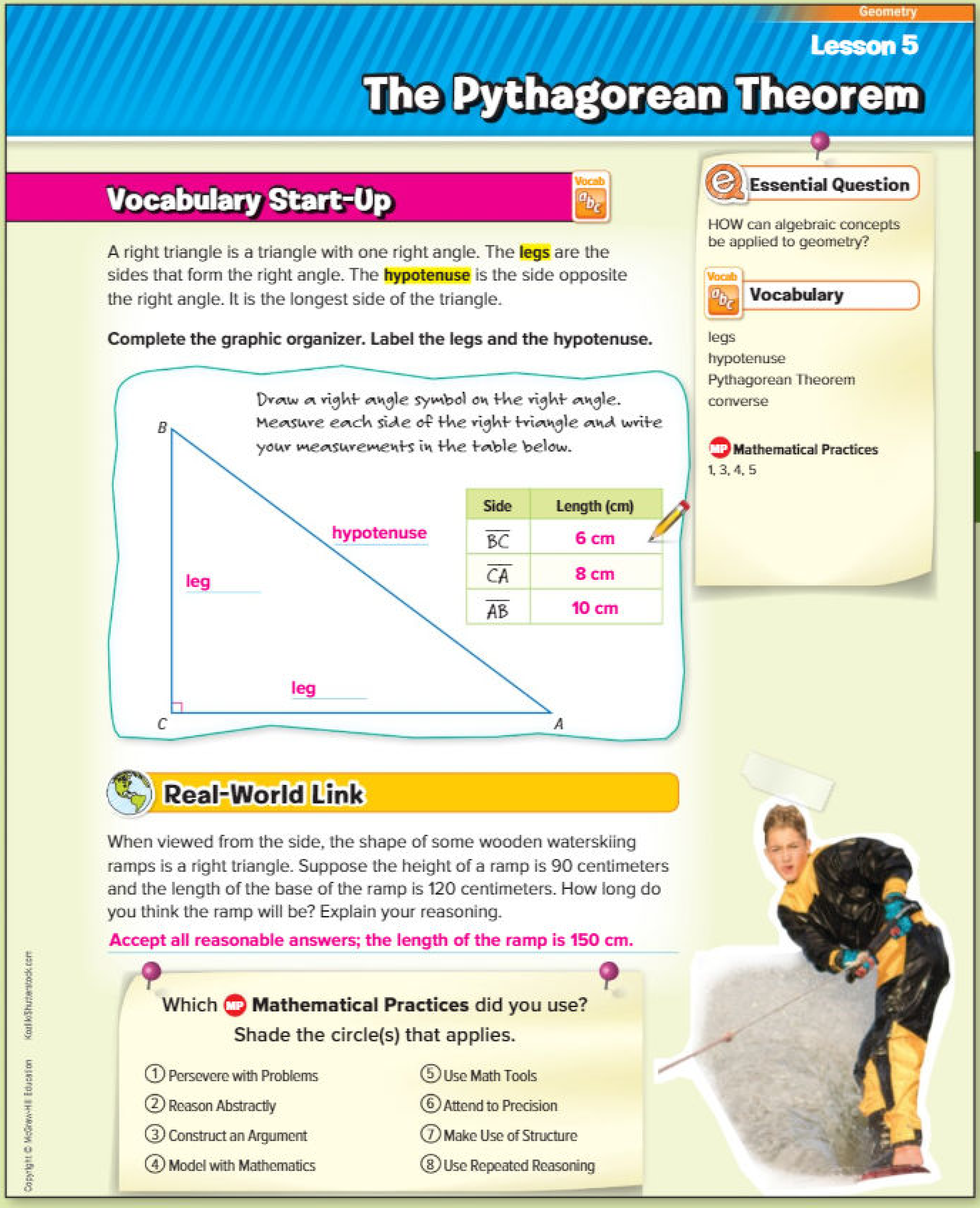
Create

4. **MP** **Reason Inductively** Make a conjecture about the length of the longest side of a right triangle if the lengths of the two shorter sides are 6 centimeters and 8 centimeters.

The length of the longest side would be 10 cm.

5. **Inquiry** WHAT is the relationship among the sides of a right triangle?

The sum of the squares of the two smallest sides is equal to the square of the largest side.



Geometry

Lesson 5

The Pythagorean Theorem

Vocabulary Start-Up

A right triangle is a triangle with one right angle. The **legs** are the sides that form the right angle. The **hypotenuse** is the side opposite the right angle. It is the longest side of the triangle.

Complete the graphic organizer. Label the legs and the hypotenuse.

Draw a right angle symbol on the right angle. Measure each side of the right triangle and write your measurements in the table below.

Side	Length (cm)
$\overline{BC}$	6 cm
$\overline{CA}$	8 cm
$\overline{AB}$	10 cm

Essential Question

HOW can algebraic concepts be applied to geometry?

Vocabulary

legs
hypotenuse
Pythagorean Theorem
converse

Mathematical Practices
1, 3, 4, 5

Real-World Link

When viewed from the side, the shape of some wooden waterskiing ramps is a right triangle. Suppose the height of a ramp is 90 centimeters and the length of the base of the ramp is 120 centimeters. How long do you think the ramp will be? Explain your reasoning.

Accept all reasonable answers; the length of the ramp is 150 cm.

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Use the Pythagorean Theorem.

Coherence connecting within and across grades

Previous	Now	Next
Students used a model to explore the relationship among the sides of a right triangle.	Students will use the Pythagorean Theorem and its converse to solve problems.	Students will explore proofs of the Pythagorean Theorem.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 415.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Pairs Discussion** Ask students to draw a rectangle on a piece of paper. Then have them draw the diagonal. Ask the questions below, then have students work in pairs to complete the Vocabulary Start-Up on the student page. MP 1, 2, 3, 4, 5, 6

Ask:

- *When a diagonal is drawn, what is formed?* two triangles
- *How would you classify the triangles?* right triangles
- *Is the diagonal longer or shorter than the sides of the rectangle?* longer

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Find a missing length in a right triangle.

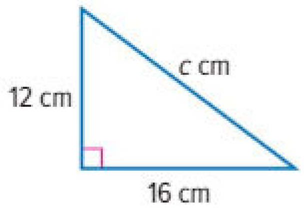
AL
- *Do you need to find the length of one of the legs or the hypotenuse?* hypotenuse
- *What are the lengths of the legs?* 9 cm and 12 cm

OL
- *What equation do we use to model the Pythagorean Theorem?* $a^2 + b^2 = c^2$
- *What value would you substitute for a in the equation?* 12
- *What value would you substitute for b in the equation?* 9

BL
- *Why can the value of c not be negative?* Length must be positive, so you need to use the positive square root.
- *How do we know that our answer is reasonable?* Sample answer: The hypotenuse is the longest side of a right triangle. Since $15 > 12 > 9$, the answer is reasonable.

Need Another Example?

Write an equation you could use to find the length of the missing side of the right triangle shown. Then find the missing length. Round to the nearest tenth if necessary.



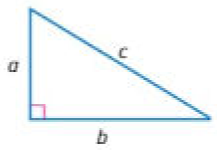
$12^2 + 16^2 = c^2$; 20 cm

Key Concept

Pythagorean Theorem

Words In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Model



Symbols $a^2 + b^2 = c^2$

Work Zone

The **Pythagorean Theorem** describes the relationship between the lengths of the legs and the hypotenuse for *any* right triangle.

You can use the Pythagorean Theorem to find the length of a side of a right triangle when you know the other two sides.

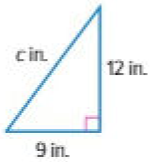
Right Angle

The symbol ⌐ indicates an angle with a measure of 90°.

Examples

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

1.



$a^2 + b^2 = c^2$ Pythagorean Theorem

$12^2 + 9^2 = c^2$ Replace a with 12 and b with 9.

$144 + 81 = c^2$ Evaluate 12^2 and 9^2.

$225 = c^2$ Add 81 and 144.

$\pm\sqrt{225} = c$ Definition of square root

$c = 15$ or -15 Simplify.

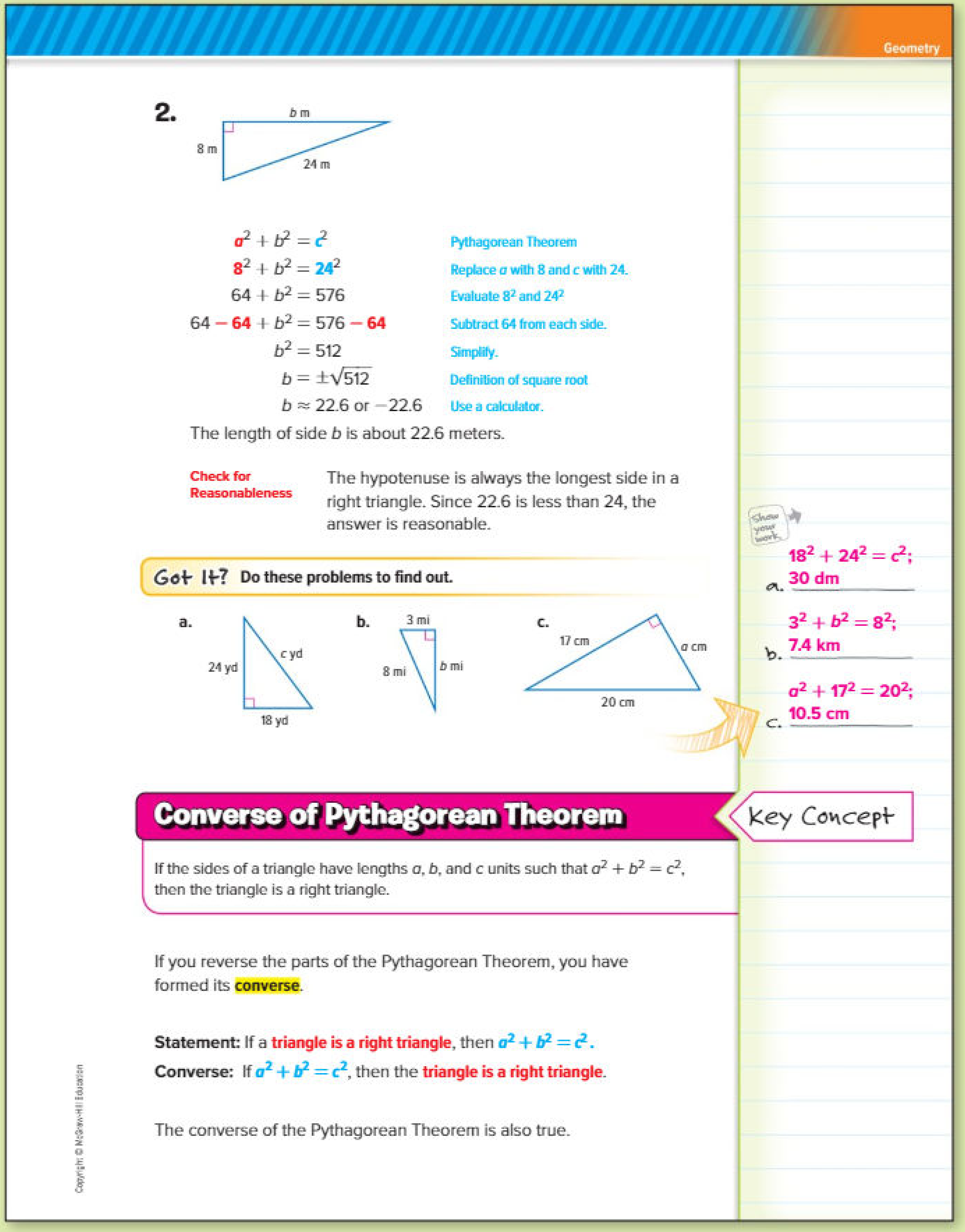
The equation has two solutions, 15 and −15. However, the length of a side must be positive. So, the hypotenuse is 15 centimeters long.

Check: $a^2 + b^2 = c^2$

$12^2 + 9^2 \stackrel{?}{=} 15^2$

$144 + 81 \stackrel{?}{=} 225$

$225 = 225$ ✓



2.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$8^2 + b^2 = 24^2$	Replace *a* with 8 and *c* with 24.
$64 + b^2 = 576$	Evaluate 8^2 and 24^2
$64 - 64 + b^2 = 576 - 64$	Subtract 64 from each side.
$b^2 = 512$	Simplify.
$b = \pm\sqrt{512}$	Definition of square root
$b \approx 22.6$ or -22.6	Use a calculator.

The length of side *b* is about 22.6 meters.

Check for Reasonableness The hypotenuse is always the longest side in a right triangle. Since 22.6 is less than 24, the answer is reasonable.

Got It? **Do these problems to find out.**

a.

b.

c.

Show your work

a. $18^2 + 24^2 = c^2$; 30 dm

b. $3^2 + b^2 = 8^2$; 7.4 km

c. $a^2 + 17^2 = 20^2$; 10.5 cm

Converse of Pythagorean Theorem

Key Concept

If the sides of a triangle have lengths *a*, *b*, and *c* units such that $a^2 + b^2 = c^2$, then the triangle is a right triangle.

If you reverse the parts of the Pythagorean Theorem, you have formed its **converse**.

Statement: If a triangle is a right triangle, then $a^2 + b^2 = c^2$.

Converse: If $a^2 + b^2 = c^2$, then the triangle is a right triangle.

The converse of the Pythagorean Theorem is also true.

Example

2. Find a missing length in a right triangle.

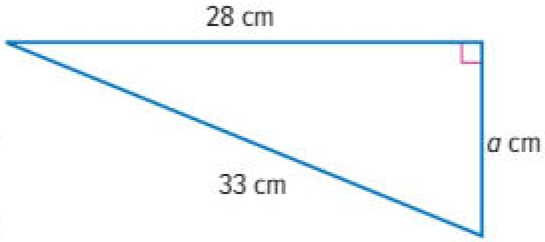
AL
- *Do you need to determine the length of one of the legs or the hypotenuse?* one of the legs
- *What is the length of the known leg?* 8 m
- *What is the length of the hypotenuse?* 24 m

OL
- *What equation do we use to model the Pythagorean Theorem?* $a^2 + b^2 = c^2$
- *What value would you substitute for a in the equation?* 8
- *What value would you substitute for c in the equation?* 24

BL
- *Why do we not consider the negative square root, −22.6?* The side length of a triangle cannot be negative.
- *How do we know if our answer is reasonable?* Sample answer: The hypotenuse is the longest side of a right triangle. Since 22.6 is less than 24, the answer is reasonable.

Need Another Example?

Write an equation you could use to find the length of the missing side of the right triangle shown. Then find the missing length. Round to the nearest tenth if necessary.



$a^2 + 28^2 = 33^2$; 17.5 cm

Watch Out!

Common Error Students may think that they always substitute the known side lengths for *a* and *b* and solve for *c* in the Pythagorean Theorem. Remind them that *c* is always the hypotenuse of the triangle, so if the hypotenuse is one of the known sides, they will need to substitute the known lengths for *a* and *c*, and then solve for *b*.

Example

3. Use the converse of the Pythagorean Theorem.

AL
- *What are the lengths of the sides of the triangle?* 5 cm, 12 cm, and 13 cm
- *Which side length is the longest?* 13 cm

OL
- *If this is a right triangle, which side would be the hypotenuse?* 13 cm
- *What equation do we use to determine if the triangle is a right triangle?* $a^2 + b^2 = c^2$
- *What value would you substitute for a in the equation?* 5 *b?* 12 *c?* 13

BL
- *Sets of numbers that work in the Pythagorean Theorem are called Pythagorean Triples. Can you think of other values for a, b, and c that are Pythagorean Triples?* Sample answer: 3, 4, and 5

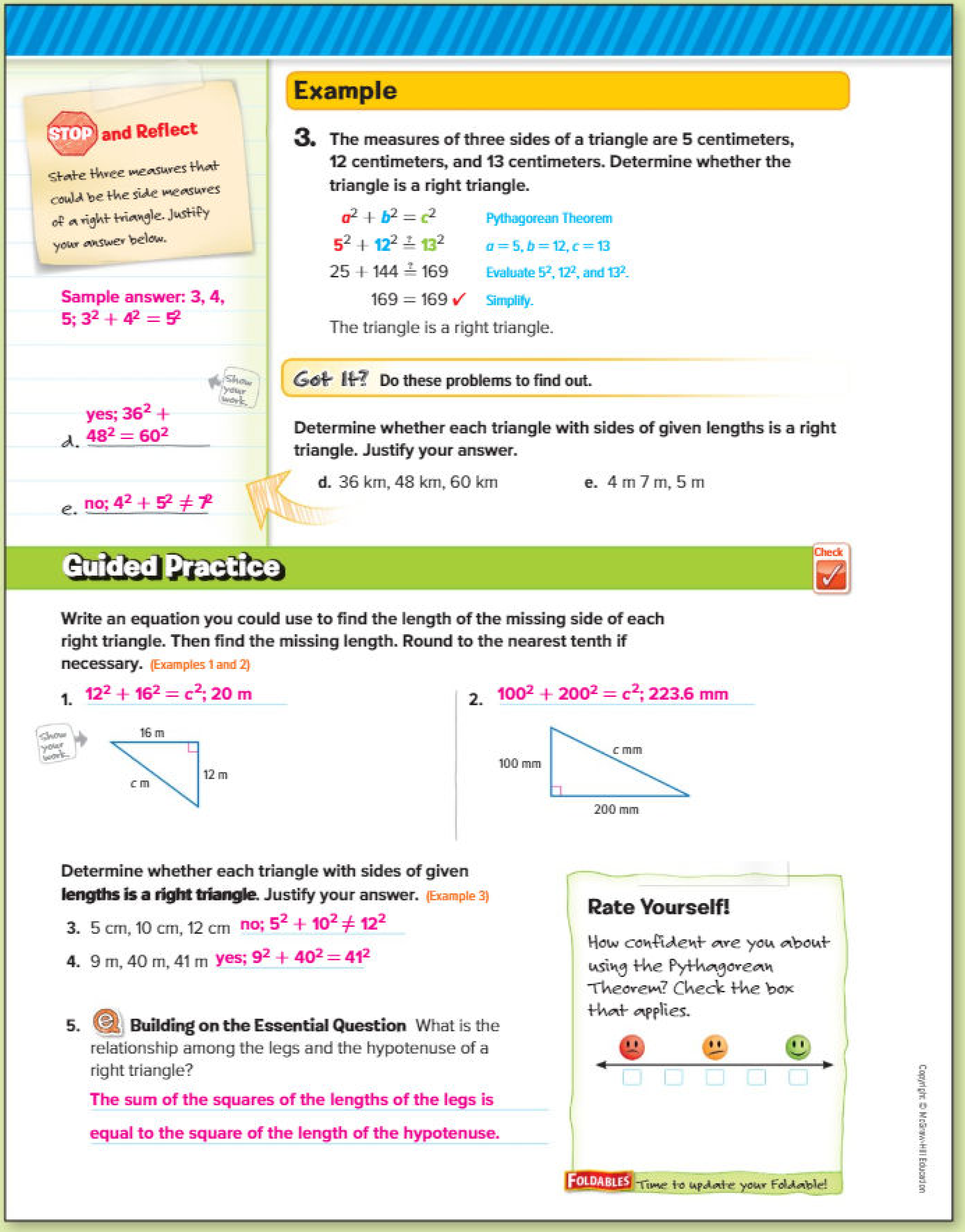
Need Another Example?
The measures of three sides of a triangle are 24 centimeters, 7 centimeters, and 25 centimeters. Determine whether the triangle is a right triangle. yes; $7^2 + 24^2 = 25^2$

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

AL BL LA **Team-Pair-Solo** Have students work in a team of 4 (making sure there are some approaching level students grouped with beyond level students) to complete Exercises 1 and 3. Then have them work in pairs to complete Exercises 2 and 4. Finally, have them work individually to complete Exercise 5. Upon completion, have them regroup with their original team to compare answers and resolve any differences.
MP 1, 2, 3, 4, 6



STOP and Reflect

State three measures that could be the side measures of a right triangle. Justify your answer below.

Sample answer: 3, 4, 5; $3^2 + 4^2 = 5^2$

Example

3. **The measures of three sides of a triangle are 5 centimeters, 12 centimeters, and 13 centimeters. Determine whether the triangle is a right triangle.**

$a^2 + b^2 = c^2$ Pythagorean Theorem
$5^2 + 12^2 \overset{?}{=} 13^2$ $a = 5, b = 12, c = 13$
$25 + 144 \overset{?}{=} 169$ Evaluate 5^2, 12^2, and 13^2.
$169 = 169$ ✓ Simplify.

The triangle is a right triangle.

Got It? Do these problems to find out.

Determine whether each triangle with sides of given lengths is a right triangle. Justify your answer.

d. 36 km, 48 km, 60 km **e.** 4 m 7 m, 5 m

Show your work.

d. yes; $36^2 + 48^2 = 60^2$

e. no; $4^2 + 5^2 \neq 7^2$

Guided Practice

Check

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary. (Examples 1 and 2)

1. $12^2 + 16^2 = c^2$; 20 m

Show your work.

2. $100^2 + 200^2 = c^2$; 223.6 mm

Determine whether each triangle with sides of given lengths is a right triangle. Justify your answer. (Example 3)

3. 5 cm, 10 cm, 12 cm no; $5^2 + 10^2 \neq 12^2$
4. 9 m, 40 m, 41 m yes; $9^2 + 40^2 = 41^2$

5. **Building on the Essential Question** What is the relationship among the legs and the hypotenuse of a right triangle?
The sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Rate Yourself!

How confident are you about using the Pythagorean Theorem? Check the box that applies.

☐ ☐ ☐ ☐ ☐

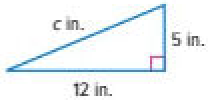
FOLDABLES Time to update your Foldable!

Name _______________ My Homework _______________

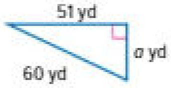
Independent Practice

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary. (Examples 1 and 2)

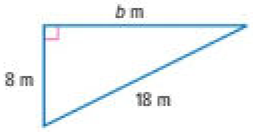
1. $5^2 + 12^2 = c^2$; 13 cm.



2. $a^2 + 51^2 = 60^2$; 31.6 m



3. $8^2 + b^2 = 18^2$; 16.1 m

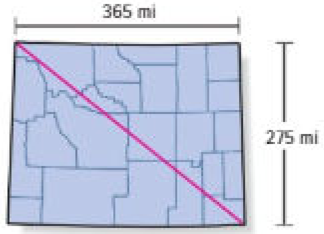


Determine whether each triangle with sides of given lengths is a right triangle. Justify your answer. (Example 3)

4. 28 m, 195 m, 197 m
yes; $28^2 + 195^2 = 197^2$

5. 30 cm, 122 cm, 125 cm
no; $30^2 + 122^2 \neq 125^2$

6. Calculate the length of the diagonal of the rectangle.
about 735 km



Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

7. $a = 48$ m; $b = 55$ m
$48^2 + 55^2 = c^2$; 73 m

8. $a = 23$ cm; $b = 18$ cm
$23^2 + 18^2 = c^2$; 29.2 cm

9. $b = 5.1$ m; $c = 12.3$ m
$a^2 + 5.1^2 = 12.3^2$; 11.2 m

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–6, 15–22	7–10, 23–25	11–14
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7, 9, 12–14, 24, 25
OL	On Level	1–5 odd, 6–10, 12–14, 24, 25
BL	Beyond Level	6–14, 24, 25

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	11
3 Construct viable arguments and critique the reasoning of others.	12–14, 23
5 Use appropriate tools strategically.	10

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students determine whether a triangle with side lengths of 1 centimeter, 3 centimeters, and 3 centimeters is a right triangle. Have them justify their response. no; Sample answer: $1^2 + 3^2 \neq 3^2$.

Watch Out!

Find the Error In Exercise 12, Amani identified the hypotenuse as one of the triangle's legs. Ask students to label the two legs of the right triangle *a* and *b* and the hypotenuse *c*. Point out that it is often helpful to write the formula before replacing variables with numbers.

10. **Use Math Tools** The whole numbers 3, 4, and 5 are called Pythagorean triples because they satisfy the Pythagorean Theorem. Complete the graphic organizer shown to list 4 additional sets of Pythagorean triples.

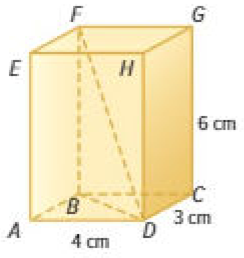
Pythagorean Triples

3	4	5
6	8	10
9	12	15
5	12	13
8	15	17

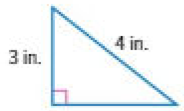
H.O.T. Problems Higher Order Thinking

11. **Persevere with Problems** In the figure, $\overline{BD}$ is the diagonal of the base and $\overline{FD}$ is the diagonal of the figure. Find FD to the nearest tenth.

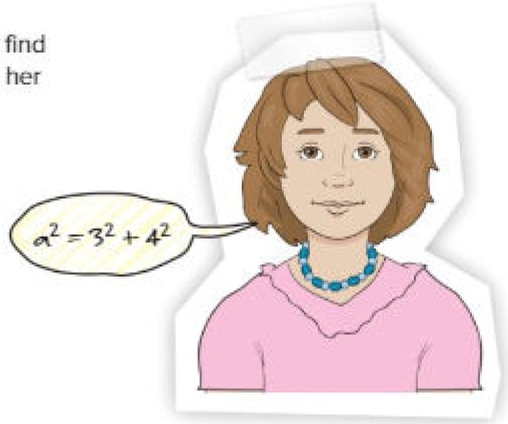
7.8 cm



12. **Find the Error** Amani is writing an equation to find the length of the third side of the right triangle. Find her mistake and correct it.

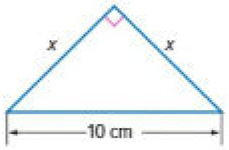


She used the sides given as legs, when one is a hypotenuse; $a^2 + 3^2 = 4^2$.



13. **Justify Conclusions** What does the value of *x* have to be for the figure to be classified as a right isosceles triangle? Justify your reasoning.

about 7.1 cm; Sample answer: The Pythagorean Theorem states that $c^2 = a^2 + b^2$. Since both legs are *x* inches, $c^2 = 2x^2$. When you replace c with 10 and simplify, $x \approx 7.1$.



14. **Justify Conclusions** The hypotenuse of a right triangle is 23 centimeters long. Find possible measures for the legs of the triangle. Round to the nearest hundredth. Justify your answer.

Sample answer: 15 cm and 17.44 cm; $23^2 = 529$ and $15^2 + 17.44^2 = 529.1536$. So, $23^2 \approx 15^2 + 17.44^2$.

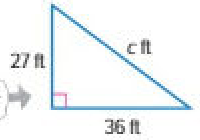
Name ______________________ My Homework ______________________

Extra Practice

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

15. $27^2 + 36^2 = c^2$; 45 meter

Homework Help



$27^2 + 36^2 = c^2$

$729 + 1{,}296 = c^2$

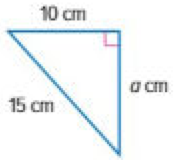
$2{,}025 = c^2$

$\pm\sqrt{2{,}025} = c$

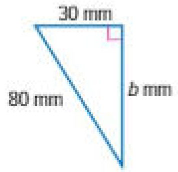
$\pm 45 = c$

Since length cannot be negative, the length of side c is 45 meters.

16. $a^2 + 10^2 = 15^2$; 11.2 cm



17. $30^2 + b^2 = 80^2$; 74.2 mm



Copy and Solve **Determine whether each triangle is a right triangle. Justify your answer. Show your work on a separate piece of paper.**

18. 24 m, 143 m, 145 m
yes; $24^2 + 143^2 = 145^2$

19. 135 cm, 140 cm, 175 cm
no; $135^2 + 140^2 \neq 175^2$

20. 56 m, 65 m, 16 m
no; $56^2 + 16^2 \neq 65^2$

21. 44 cm, 70 cm, 55 cm
no; $44^2 + 55^2 \neq 70^2$

22. A triangle is formed by three towns, as shown on the map. Is this triangle a right triangle? Explain.
no; $12^2 + 24^2 \neq 29^2$

23. **Construct an Argument** Explain to a classmate why you can use any two sides of a right triangle to find the third side. **Sample answer: If you know the lengths of two sides of a right triangle, you can substitute the values in the Pythagorean Theorem and find the missing side.**

Power Up! Test Practice

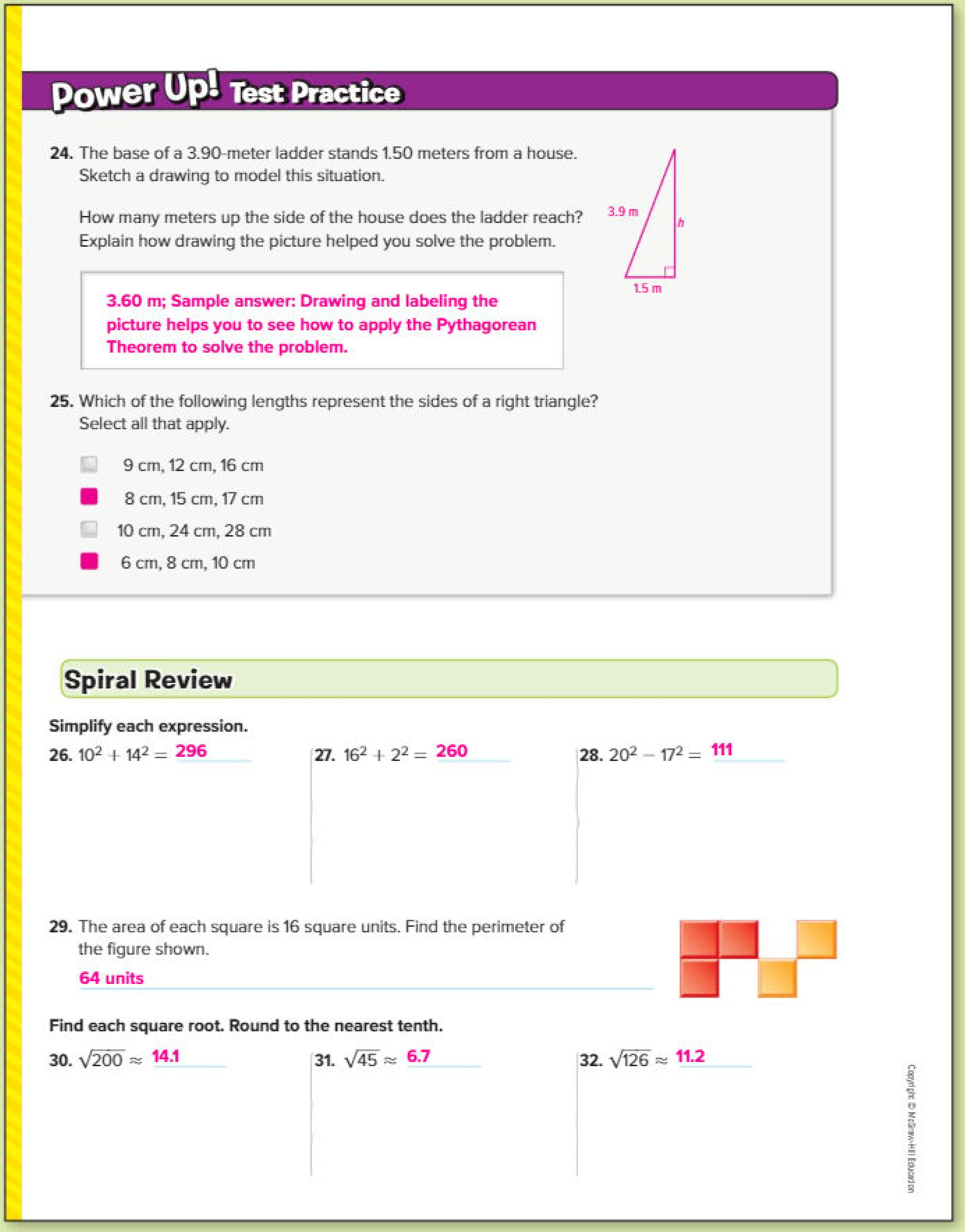
Exercises 24 and 25 prepare students for more rigorous thinking needed when taking assessment.

24.

This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Knowledge	DOK3
Mathematical Practices	MP1, MP3, MP4, MP5
Scoring Rubric	
2 points	Students correctly model the situation, find the height of the ladder and explain their response.
1 point	Students correctly model the situation OR find the height of the ladder and explain their response.

25.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP2, MP6
Scoring Rubric	
1 point	Students correctly answer the question.



Power Up! Test Practice

24. The base of a 3.90-meter ladder stands 1.50 meters from a house. Sketch a drawing to model this situation.

How many meters up the side of the house does the ladder reach? Explain how drawing the picture helped you solve the problem.

3.60 m; Sample answer: Drawing and labeling the picture helps you to see how to apply the Pythagorean Theorem to solve the problem.

25. Which of the following lengths represent the sides of a right triangle? Select all that apply.

☐ 9 cm, 12 cm, 16 cm
☑ 8 cm, 15 cm, 17 cm
☐ 10 cm, 24 cm, 28 cm
☑ 6 cm, 8 cm, 10 cm

Spiral Review

Simplify each expression.

26. $10^2 + 14^2 =$ **296**

27. $16^2 + 2^2 =$ **260**

28. $20^2 - 17^2 =$ **111**

29. The area of each square is 16 square units. Find the perimeter of the figure shown.

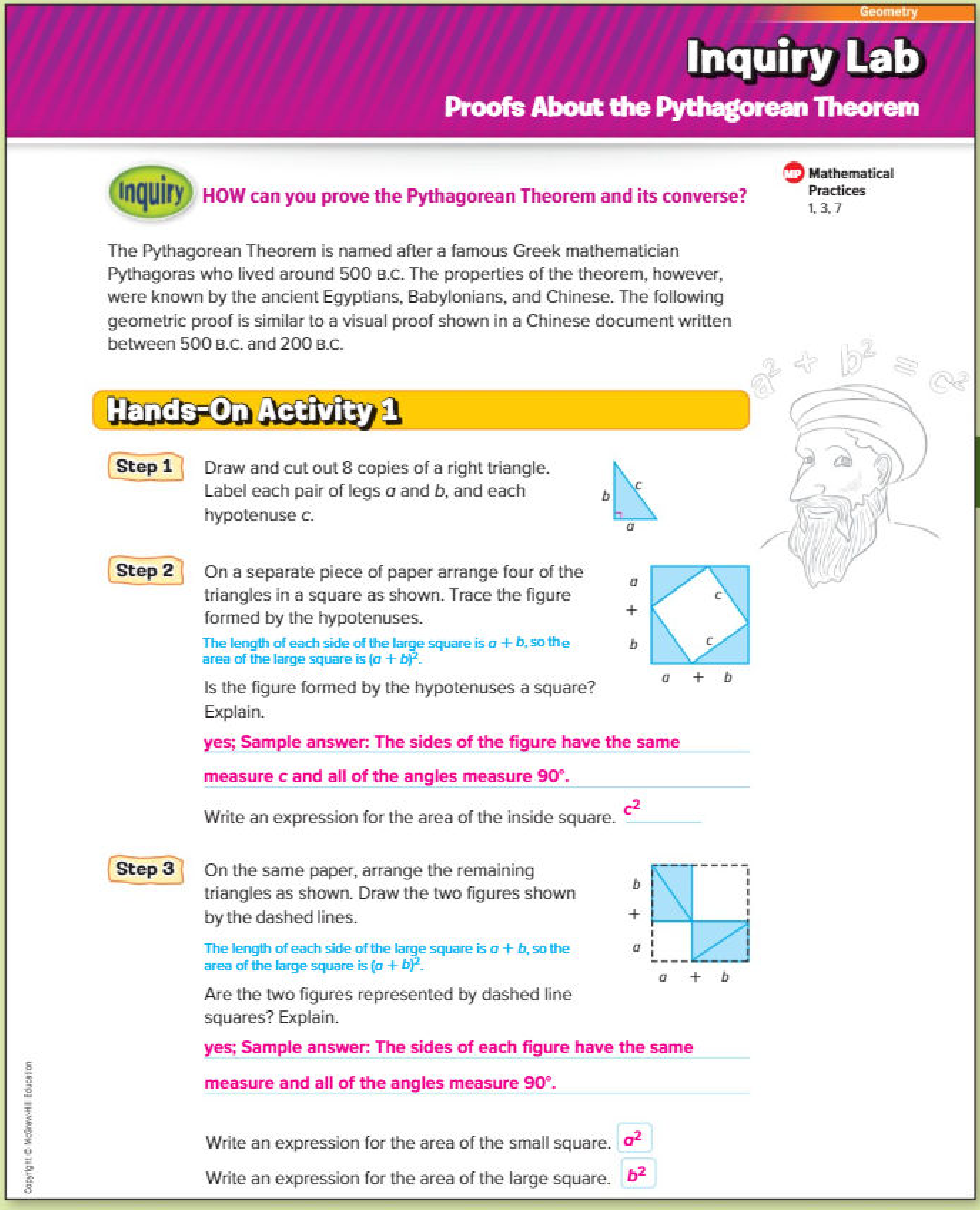
64 units

Find each square root. Round to the nearest tenth.

30. $\sqrt{200} \approx$ **14.1**

31. $\sqrt{45} \approx$ **6.7**

32. $\sqrt{126} \approx$ **11.2**



Inquiry Lab

Proofs About the Pythagorean Theorem

Inquiry HOW can you prove the Pythagorean Theorem and its converse?

Mathematical Practices 1, 3, 7

The Pythagorean Theorem is named after a famous Greek mathematician Pythagoras who lived around 500 B.C. The properties of the theorem, however, were known by the ancient Egyptians, Babylonians, and Chinese. The following geometric proof is similar to a visual proof shown in a Chinese document written between 500 B.C. and 200 B.C.

Hands-On Activity 1

Step 1 Draw and cut out 8 copies of a right triangle. Label each pair of legs a and b, and each hypotenuse c.

Step 2 On a separate piece of paper arrange four of the triangles in a square as shown. Trace the figure formed by the hypotenuses.

The length of each side of the large square is $a + b$, so the area of the large square is $(a + b)^2$.

Is the figure formed by the hypotenuses a square? Explain.

yes; Sample answer: The sides of the figure have the same measure c and all of the angles measure 90°.

Write an expression for the area of the inside square. c^2

Step 3 On the same paper, arrange the remaining triangles as shown. Draw the two figures shown by the dashed lines.

The length of each side of the large square is $a + b$, so the area of the large square is $(a + b)^2$.

Are the two figures represented by dashed line squares? Explain.

yes; Sample answer: The sides of each figure have the same measure and all of the angles measure 90°.

Write an expression for the area of the small square. a^2

Write an expression for the area of the large square. b^2

Focus narrowing the scope

Objective Prove the Pythagorean Theorem and its converse.

Coherence connecting within and across grades

Now
Students will use models and diagrams to prove the Pythagorean Thoerem.

Next
Students will use the Pythagorean Theorem and its converse to solve problems.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 420.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activity 2.

Hands-On Activity 1

AL BL LA **Teammates Consult** Have students work in small groups to complete the activity. Group one or more Approaching Level students and one or more Beyond Level students in the same groups, if possible. Have one student read each set of directions aloud, checking to see if the team understands what procedures to follow. Have students who understand the procedures check the work of those who have trouble or need clarification. For each step, have the student who is reading the directions pause and ask if students have questions. If no one on the team is able to answer a specific question, the team may ask you for help and assistance. MP 1, 4, 5

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

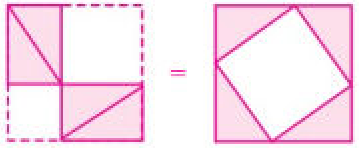
	Exercises		
	1–3	4–6	7, 8
Level 3			●
Level 2		●	
Level 1	●		

Investigate

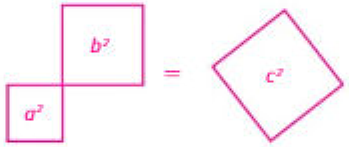
AL BL LA **Teammates Consult** Continue the groupings from Activity 1. After researching the Egyptian technique in Exercise 1, have students work in their small groups to demonstrate the procedure. Give groups ropes of different lengths with knots already tied. Have students trace their triangle on paper or on the board. MP 1, 4, 5

BL LA **Teammates Consult** Using the same activity above, give the groups ropes of different lengths that do not have any knots. Have students tie equally spaced knots and trace their triangle on paper or on the board. MP 1, 4, 5

Step 4 Since the area of each of the two composite figures you created is $(a + b)^2$, the areas are equal. Use the space provided to draw each figure from Step 2 and Step 3. Place an equal sign between the two figures to show the two areas are equal.



Step 5 Remove the triangles from each side. Use the space provided to draw the remaining figures.



What property justifies removing the triangles from each side of the equation? Subtraction Property of Equality

Write an algebraic equation that represents the relationship between the figures shown in Step 5. $a^2 + b^2 = c^2$

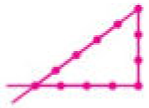
Summarize the relationship among the sides of a right triangle measuring *a* units, *b* units, and *c* units.

Sample answer: The sum of the squares of the two smaller sides is equal to the square of the largest side.

Investigate

Work with a partner.

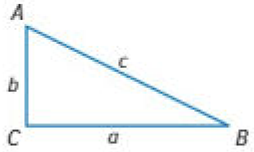
1. A legend states that the ancient Egyptians could create a right triangle by using a knotted rope. Research this on the Internet. Describe their technique in the space provided and draw a diagram to illustrate the technique.



Sample answer: The Egyptians tied 12 knots in a rope that were equally spaced. They then laid it out so one side had 3 units, another side had 4 units and the third side had 5 units. This made a right triangle.

Hands-On Activity 2

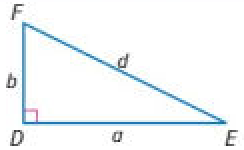
The converse of the Pythagorean Theorem states if a triangle has side lengths, a, b, and c units such that $a^2 + b^2 = c^2$, then the triangle is a right triangle. In this Activity, you will prove the converse of the Pythagorean Theorem by using a two-column proof.



Given: $\triangle ABC$ such that $a^2 + b^2 = c^2$.

Prove: $\triangle ABC$ is a right triangle.

Complete the proof with the correct reasons justifying each statement.

Statements	Reasons
a. Draw a right triangle DEF so that $\overline{DE}$ is a units long and $\overline{DF}$ is b units long. Label $\overline{FE}$ as d.	
b. Write an equation that describes the relationship between the side lengths of $\triangle DEF$. State the theorem that allows you to make that statement.	**$a^2 + b^2 = d^2$; Pythagorean Theorem**
c. $a^2 + b^2 = c^2$	Given
d. If $a^2 + b^2 = c^2$ and $a^2 + b^2 = d^2$, then $d^2 = c^2$.	**Substitute c^2 for $a^2 + b^2$ in the second equation.**
e. If $d^2 = c^2$, then $d = c$.	**Definition of square root**
f. If $d = c$, then $FE = AB$.	**Definition of equal segments**
g. If $AC = FD$, $CB = DE$, and $AB = FE$, the two triangles are the same shape and size.	If three sides of a triangle are the same length as the corresponding sides of another triangle, the triangles are the same shape and size.
h. $m\angle C = m\angle D$	Corresponding parts of the triangles with the same size and shape have the same measures.
i. $\angle C$ is a right angle.	**Definition of right angle**
j. $\triangle ABC$ is a right triangle	**Definition of right triangle**

So, if a triangle has side lengths, a, b, and c units such that $a^2 + b^2 = c^2$, then the triangle is a right triangle.

Hands-On Activity 2

AL **LA** **Think-Pair-Share** Have students work with a partner to complete the activity. For each step, give students about 10–20 seconds to think through how they would provide a reason for each step. Then have them share their responses with their partner, asking for support and clarification if needed from their partner, or you. You may wish to provide students with numerical measurements for the side lengths of the triangle instead of variable measurements, if students are struggling with the algebraic manipulation. **MP 1, 2, 3, 4, 5, 6, 7**

BL **LA** **Popcorn Share** Have students work in small groups to complete Activity 2. Have each student be responsible for providing the reason for each step. Have students take turns providing the reasons. When it is their turn to provide a reason, have that student stand up, verbally give the reason, and then explain the reason in their own words. **MP 1, 2, 3, 4, 5, 6, 7**

Investigate

AL **LA** **Group-Solo** Have students work as a small group to complete Exercise 2, ensuring that each group member understands how to determine whether the triangle is a right triangle. Then have students complete Exercise 3 individually. Upon completion, have them share their responses with their group to check their work. MP 1, 2, 3, 4, 5, 6, 7

Ask:

- *What equation can you use to determine if the triangle is a right triangle?* $a^2 + b^2 = c^2$
- *In Exercise 1, what length on the triangle must be substituted for c? Why?* 6; Sample answer: because it is the longest side of the triangle

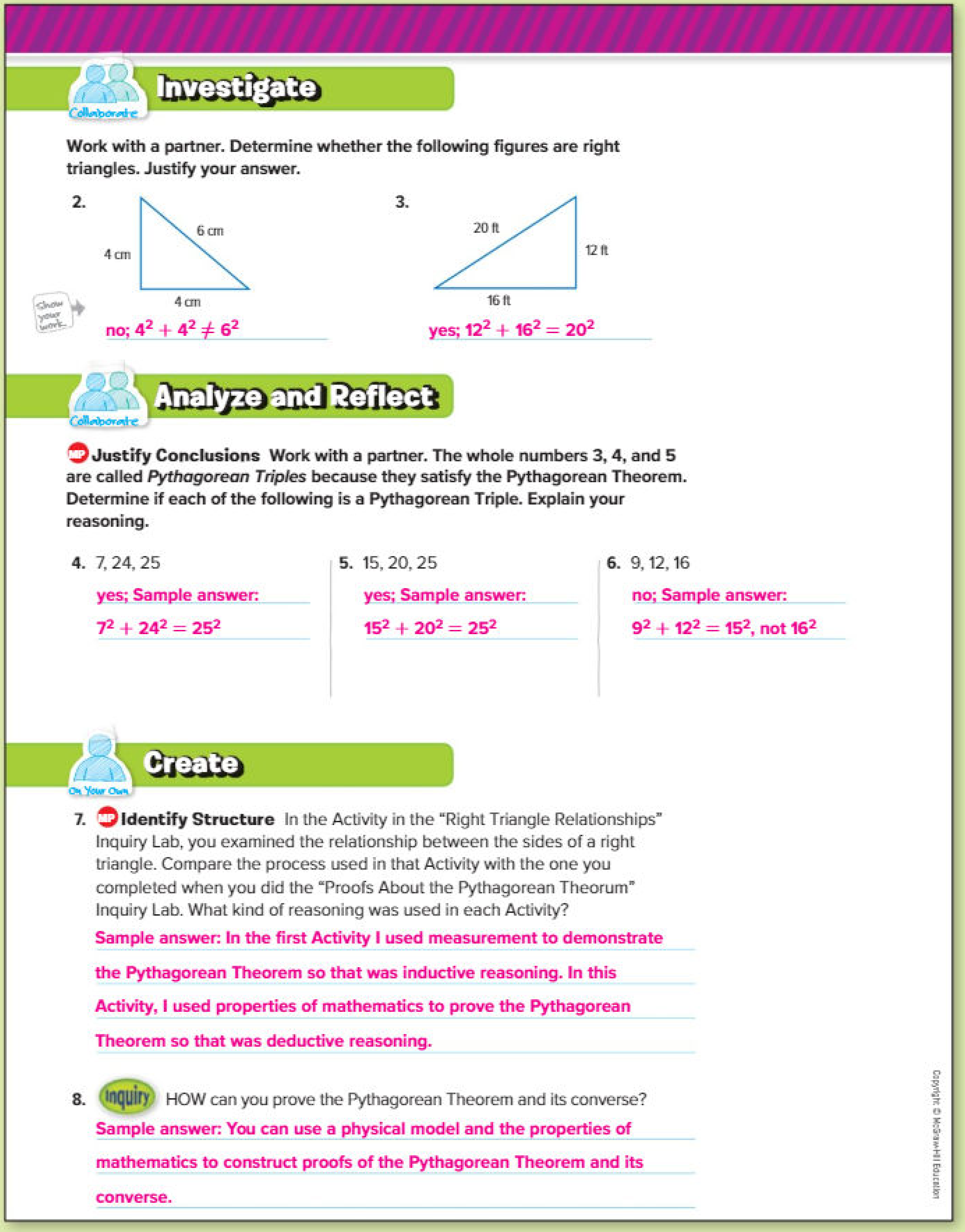
Analyze and Reflect

AL **BL** **LA** **Value Line** Students place themselves on a pretend line, using the number 10 to represent that they understand the Pythagorean Theorem and its converse completely and the number 1 to represent that they do not understand or have a lot of questions. Have students pair up with someone from the other side of the line to complete Exercises 4–6. MP 1, 2, 4, 5, 6, 7

Create

AL **LA** **Think-Pair-Share** Have students work in pairs. Give students several minutes to think through their responses to Exercises 7 and 8. Have them share their responses with their partner. Then call on one student to share their response within a small group or large group discussion. MP 1, 2, 4, 5, 6

Inquiry Students should be able to answer "HOW can you prove the Pythagorean Theorem and its converse?" Check for student understanding and provide guidance, if needed.



Investigate

Work with a partner. Determine whether the following figures are right triangles. Justify your answer.

2. no; $4^2 + 4^2 \neq 6^2$

3. yes; $12^2 + 16^2 = 20^2$

Analyze and Reflect

MP Justify Conclusions **Work with a partner. The whole numbers 3, 4, and 5 are called *Pythagorean Triples* because they satisfy the Pythagorean Theorem. Determine if each of the following is a Pythagorean Triple. Explain your reasoning.**

4. 7, 24, 25
yes; Sample answer:
$7^2 + 24^2 = 25^2$

5. 15, 20, 25
yes; Sample answer:
$15^2 + 20^2 = 25^2$

6. 9, 12, 16
no; Sample answer:
$9^2 + 12^2 = 15^2$, not 16^2

Create

7. MP **Identify Structure** In the Activity in the "Right Triangle Relationships" Inquiry Lab, you examined the relationship between the sides of a right triangle. Compare the process used in that Activity with the one you completed when you did the "Proofs About the Pythagorean Theorum" Inquiry Lab. What kind of reasoning was used in each Activity?

Sample answer: In the first Activity I used measurement to demonstrate the Pythagorean Theorem so that was inductive reasoning. In this Activity, I used properties of mathematics to prove the Pythagorean Theorem so that was deductive reasoning.

8. Inquiry HOW can you prove the Pythagorean Theorem and its converse?

Sample answer: You can use a physical model and the properties of mathematics to construct proofs of the Pythagorean Theorem and its converse.

Lesson 6

Use the Pythagorean Theorem

Real-World Link

Parasailing In parasailing, a towrope is used to attach a parasailer to a boat. Refer to the diagram below for Exercises 1–4.

1. What type of triangle is formed by the horizontal distance, the vertical height, and the length of the towrope? Explain.
   right triangle; Since the sum of the squares of two sides is equal to the square of the third side, the triangle is a right triangle; $21^2 + 72^2 = 75^2$.

2. Suppose the wind picks up and the parasailer rises to 17 m and remains 24 m behind the boat. Write an equation that will help you find how much towrope *c* the parasailer will need.
   $50^2 + 72^2 = c^2$

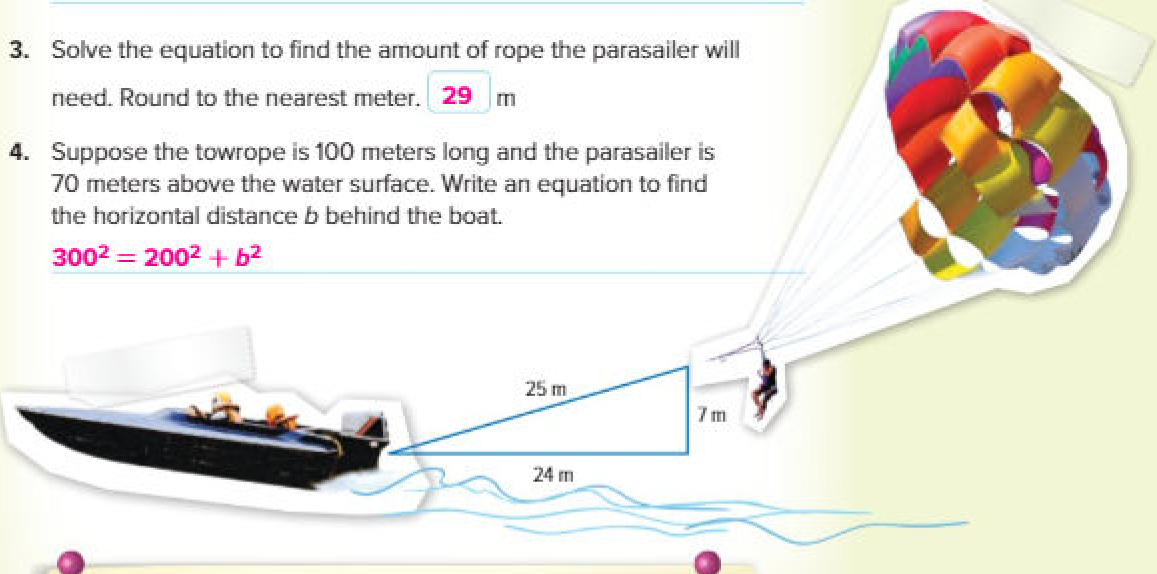
3. Solve the equation to find the amount of rope the parasailer will need. Round to the nearest meter. **29** m

4. Suppose the towrope is 100 meters long and the parasailer is 70 meters above the water surface. Write an equation to find the horizontal distance *b* behind the boat.
   $300^2 = 200^2 + b^2$

Essential Question

HOW can algebraic concepts be applied to geometry?

Mathematical Practices
1, 3, 4, 7



Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Solve problems using the Pythagorean Theorem.

Coherence connecting within and across grades

Previous	Now	Next
Students used models and diagrams to prove the Pythagorean Theorem.	Students will use the Pythagorean Theorem to solve problems.	Students will find the distance between two points on a coordinate plane.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 427.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Think-Pair-Solo** Have students work in pairs. Give them about 20 seconds to think through their response to Exercise 1 individually, then have them share their response with a partner, taking care to make sure they justify their response. Then have them work together to complete Exercises 2 and 3. Have them work individually to complete Exercise 4. MP 1, 2, 3, 4, 6

Alternate Strategy

AL Provide students with copies of a blank Pythagorean Theorem equation, $\_\_\_\_^2 + \_\_\_\_^2 = \_\_\_\_^2$ to use as they complete the exercises.

2 Teach the Concept

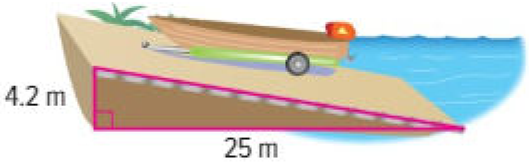
Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Solve a right triangle.

- AL • *How can you tell that the triangle is a right triangle?* There is a right angle symbol.
- OL • *What equation can we use to model the Pythagorean Theorem?* $a^2 + b^2 = c^2$
- BL • *Why do we not use the negative square root?* Sample answer: The length of a ladder cannot be negative.

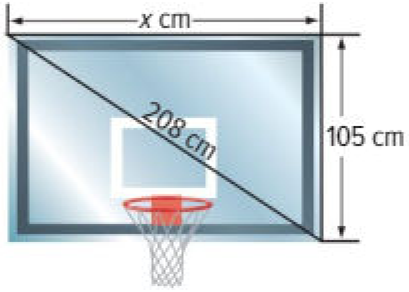
Need Another Example?
Write an equation that can be used to find the length of the boat ramp. Then solve. Round to the nearest tenth.
$4.2^2 + 25^2 = c^2$; 25.4 m

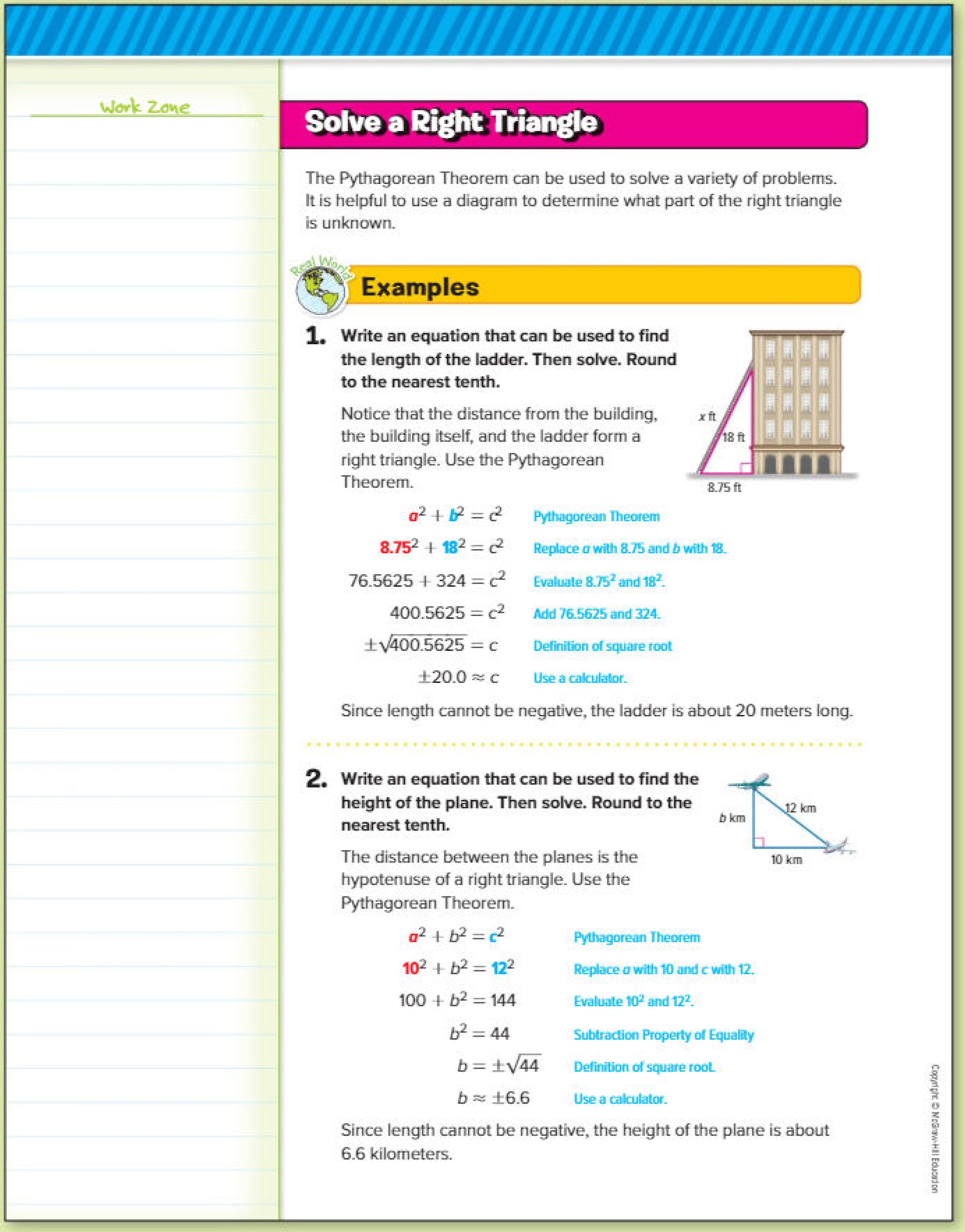


2. Solve a right triangle.

- AL • *Do you need to find the length of one of the legs or the hypotenuse?* one of the legs
- OL • *What equation can we use to model the Pythagorean Theorem?* $a^2 + b^2 = c^2$
- BL • *Derive a different equation that you can use to solve for the leg of a right triangle, when the other two sides are known.* Sample answer: When b is the unknown, you must subtract a^2 from both sides to get b^2 by itself; $b^2 = c^2 - a^2$.

Need Another Example?
Write an equation that can be used to find the length of the backboard. Then solve. Round to the nearest tenth.
$105^2 + x^2 = 208.5^2$; 180 cm





Work Zone

Solve a Right Triangle

The Pythagorean Theorem can be used to solve a variety of problems. It is helpful to use a diagram to determine what part of the right triangle is unknown.

Examples

1. Write an equation that can be used to find the length of the ladder. Then solve. Round to the nearest tenth.

Notice that the distance from the building, the building itself, and the ladder form a right triangle. Use the Pythagorean Theorem.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$8.75^2 + 18^2 = c^2$	Replace a with 8.75 and b with 18.
$76.5625 + 324 = c^2$	Evaluate 8.75^2 and 18^2.
$400.5625 = c^2$	Add 76.5625 and 324.
$\pm\sqrt{400.5625} = c$	Definition of square root
$\pm 20.0 \approx c$	Use a calculator.

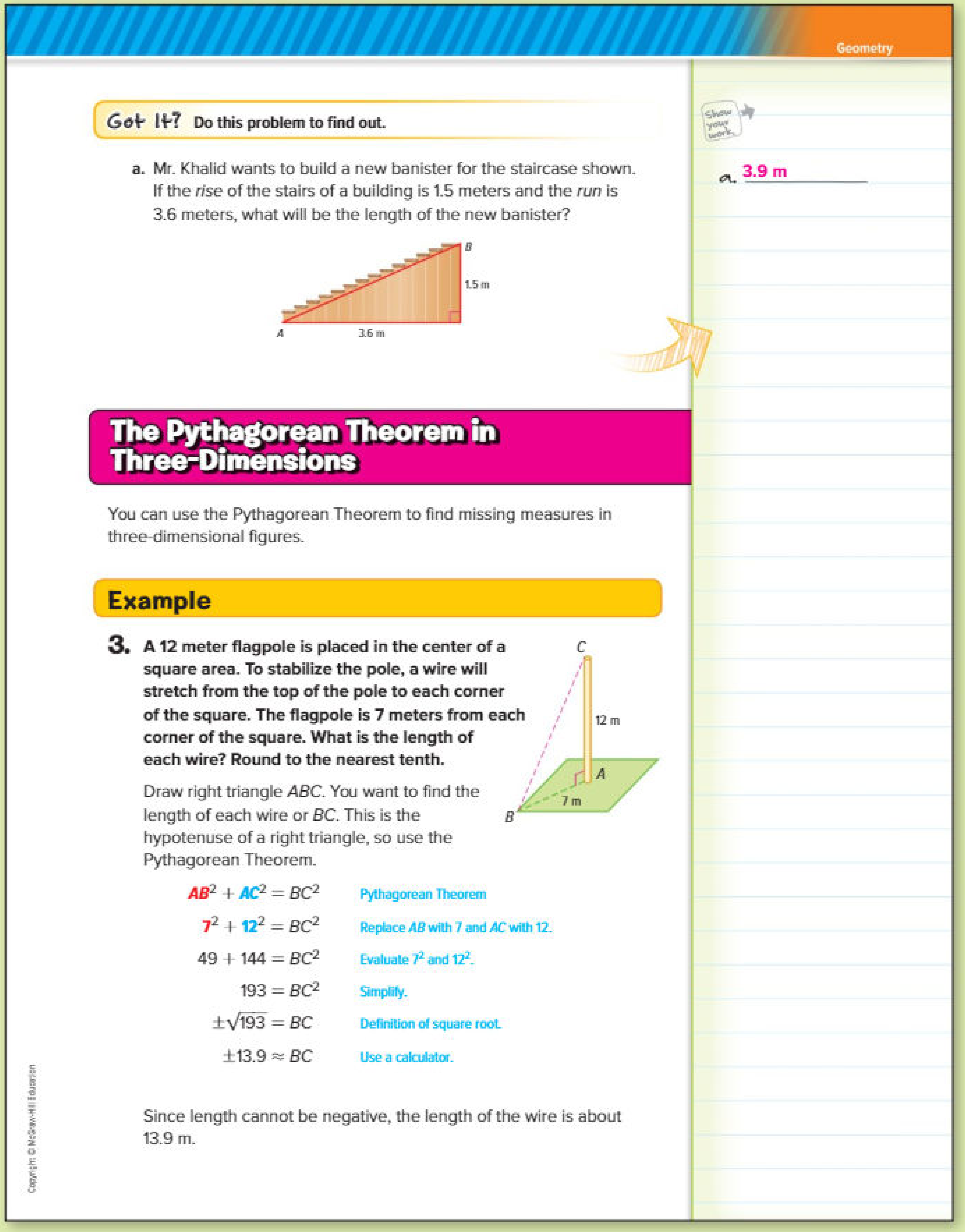
Since length cannot be negative, the ladder is about 20 meters long.

2. Write an equation that can be used to find the height of the plane. Then solve. Round to the nearest tenth.

The distance between the planes is the hypotenuse of a right triangle. Use the Pythagorean Theorem.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$10^2 + b^2 = 12^2$	Replace a with 10 and c with 12.
$100 + b^2 = 144$	Evaluate 10^2 and 12^2.
$b^2 = 44$	Subtraction Property of Equality
$b = \pm\sqrt{44}$	Definition of square root.
$b \approx \pm 6.6$	Use a calculator.

Since length cannot be negative, the height of the plane is about 6.6 kilometers.



Got It? Do this problem to find out.

a. Mr. Khalid wants to build a new banister for the staircase shown. If the *rise* of the stairs of a building is 1.5 meters and the *run* is 3.6 meters, what will be the length of the new banister?

a. 3.9 m

The Pythagorean Theorem in Three-Dimensions

You can use the Pythagorean Theorem to find missing measures in three-dimensional figures.

Example

3. A 12 meter flagpole is placed in the center of a square area. To stabilize the pole, a wire will stretch from the top of the pole to each corner of the square. The flagpole is 7 meters from each corner of the square. What is the length of each wire? Round to the nearest tenth.

Draw right triangle *ABC*. You want to find the length of each wire or *BC*. This is the hypotenuse of a right triangle, so use the Pythagorean Theorem.

$AB^2 + AC^2 = BC^2$ Pythagorean Theorem

$7^2 + 12^2 = BC^2$ Replace *AB* with 7 and *AC* with 12.

$49 + 144 = BC^2$ Evaluate 7^2 and 12^2.

$193 = BC^2$ Simplify.

$\pm\sqrt{193} = BC$ Definition of square root.

$\pm 13.9 \approx BC$ Use a calculator.

Since length cannot be negative, the length of the wire is about 13.9 m.

Example

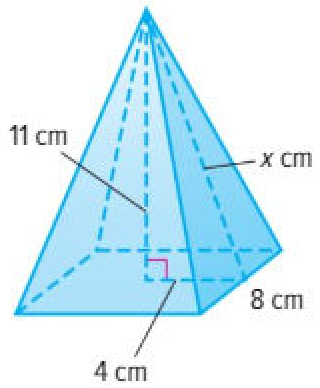
3. Use the Pythagorean Theorem in three dimensions.

AL
- *What do the wire and the pole form?* the hypotenuse and one leg of a right triangle
- *What could be used as the second leg of the right triangle?* the length from the pole to one corner of the square

OL
- *Do you need to find the length of one of the legs or the hypotenuse?* hypotenuse
- *What are the lengths of the legs?* 7 m and 12 m
- *Use AB and AC to denote the legs of the triangle. Use BC to denote the hypotenuse. What equation can be used to find the length of the wire?* $AB^2 + AC^2 = BC^2$
- *What value would you substitute for AB?* 7 *AC?* 12
- *Explain why we don't consider the negative square root to be a solution.* The length of the wire cannot be negative.

BL
- *Suppose your friend solved this problem and came up with an answer of approximately 9.7 meters. How would you know that this answer is incorrect without performing the calculations?* Sample answer: The length of the wire must be the longest side because it is the hypotenuse. Since 9.7 < 12, 9.7 cannot represent the hypotenuse. So, this answer is incorrect.

Need Another Example?
The *slant height* of a pyramid is the height of each lateral face. What is the slant height of the pyramid shown? Round to the nearest tenth. 11.7 cm



Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL **LA** **Numbered Heads Together** Assign students to 3- or 4-person learning teams. Each member is assigned a number from 1 to 4. Each team completes Exercises 1–4, making sure that every member understands. Call on a specific number from one team to present the team's solution to the class. Ask each student clarifying questions to ensure understanding. MP 1, 2, 4, 5, 6

Ask:

- *How did you know how to set up the equation?* See students' work.
- *What steps did you take to solve the equation?* See students' work.
- *How can you determine if your answer is reasonable?* See students' work.

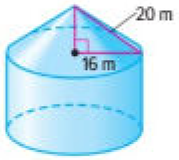
BL **LA** **Gallery Walk** Have students work in pairs to create a real-world problem in which the Pythagorean Theorem must be used to solve the problem. In their problem, they should create a drawing. Then have them post the problems around the room. Pairs of students should walk around the room and select a problem, not their own, and return to their seats to solve it. After every pair has solved their problem, have pairs of students re-post the solutions around the room. The original pair locates their problem and determines if the solution is correct. MP 1, 2, 3, 4, 5, 6

Show your work.

b. 12 m

Got It? Do this problem to find out.

b. The top part of a circus tent is in the shape of a cone. The tent has a radius of 16 meters. The distance from the top of the tent to the edge is 20 meters. How tall is the top part of the tent? Round to the nearest whole number.

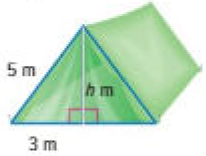


Guided Practice

Check

Write an equation that can be used to answer the question. Then solve. Round to the nearest tenth if necessary. (Examples 1 and 2)

1. What is the height of the tent?



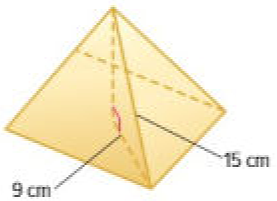
Show your work.

$3^2 + h^2 = 5^2$; 4 m

2. How high is the wheelchair ramp?

$36^2 + h^2 = 36.1^2$; 2.7 m

3. Nisreen made a model of a pyramid like the one shown for history class. What is the height of the model? (Example 3) 12 cm



4. **Building on the Essential Question** How do you solve a right triangle?

Sample answer: You need to determine what measurements represent the legs and the hypotenuse, and appropriately use the Pythagorean Theorem.

Rate Yourself!

☐ I understand how to apply the Pythagorean Theorem.

Great! You're ready to move on!

☐ I still have questions about how to apply the Pythagorean Theorem.

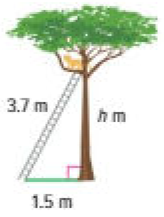
FOLDABLES Time to update your Foldable!

Name ______________________ My Homework ______________________

Independent Practice

Write an equation that can be used to answer the question. Then solve. Round to the nearest tenth if necessary. (Examples 1 and 2)

How far up the tree is the cat?



Show your work

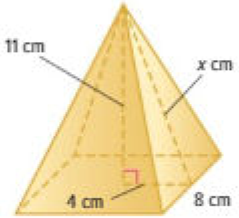
$1.5^2 + h^2 = 3.7^2$; 3.4 m

2. How deep is the water?

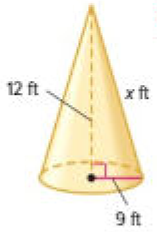
$6^2 + x^2 = 15^2$; 13.7 m

Find the missing measure in each figure below. Round to the nearest tenth if necessary. (Example 3)

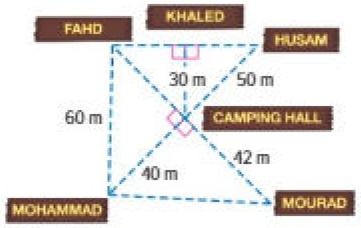
11.7 cm



4. 15 m



5. Refer to the map of the Scout's Camp at the right. Round your answers to the nearest tenth.



a. How far is it from Khaled's cabin to Husam's cabin? 40 m

b. A camper in Fahd's cabin wants to visit a friend in Mohammad's cabin. How much farther is it if he walks to the Camping Hall first? 24.7 m

6. **Justify Conclusions** Ibrahim is buying a 165-centimeter-long fishing rod for his father. He wants to put it in a box so that his dad will not be able to guess what is in the box. The box he wants to use is 120 centimeters long and 120 centimeters wide. Will the pole fit in the box? Justify your reasoning.

yes; Sample answer: The corner of the box is a right angle. Find the length of the diagonal using the Pythagorean Theorem. $120^2 + 120^2 = 28{,}800$, $\sqrt{28} \approx 170$. Since the fishing rod is 165 centimeters long, it will fit diagonally in the box.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

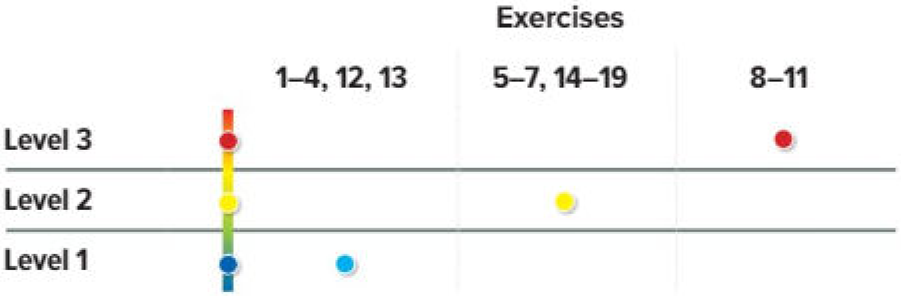
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7–9, 11, 18, 19
OL	On Level	1–5 odd, 6–9, 11, 18, 19
BL	Beyond Level	5–11, 18, 19

Watch Out!

Common Error Students may substitute the side lengths of the triangle for any variable and then solve. Remind them that the variables *a* and *b* must be legs of the triangle and *c* must be the hypotenuse.

MP MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	10, 16, 17
3 Construct viable arguments and critique the reasoning of others.	6, 9
4 Model with mathematics.	8, 11
7 Look for and make use of structure.	7

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

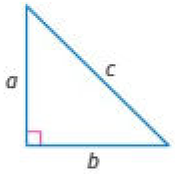
Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Ask students to write how they think using the Pythagorean Theorem will connect with the next lesson about finding the distance between two points on the coordinate plane. Use the writing prompt below. See students' work.

- The Pythagorean Theorem will help me find the distance between two points on the coordinate plane because ...

7. MP **Identify Structure** How do you use the Pythagorean Theorem?



Write the Pythagorean Theorem.

$a^2 + b^2 = c^2$

How do you find a when b and c are known?

See students' work.

How do you find c when a and b are known?

See students' work.

H.O.T. Problems Higher Order Thinking

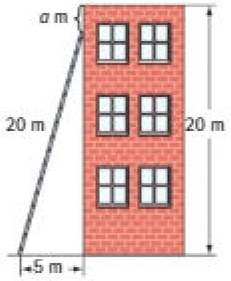
8. MP **Model with Mathematics** Write a real-world problem that can be solved by using the Pythagorean Theorem. Then explain how to solve the problem.
Sample answer: Nasser leaves his house. He walks 2 kilometers north, and then turns and walks 3 kilometers west. How far is Nasser from his house? Using the Pythagorean Theorem, $c^2 = 2^2 + 3^2$. Solving for c, Nasser is about 3.6 kilometers from his house.

9. MP **Which One Doesn't Belong?** Each set of numbers represents the side measures of a triangle. Identify the set that does not belong with the other three. Explain your reasoning.

3-4-5 | 12-35-37 | 3-5-7 | 6-8-10

3–5–7; $3^2 + 5^2 \neq 7^2$

10. MP **Persevere with Problems** Suppose a ladder 20 meters long is placed against a vertical wall 20 meters high. How far would the top of the ladder move down the wall by pulling out the bottom of the ladder 5 meters? Explain your reasoning.
about 0.6 m; By solving $20^2 = x^2 + 5^2$, you find that the ladder reaches approximately 19.4 m up the wall. Therefore, the top of the ladder would move down 20 m − 19.4 m or 0.6 m. by pulling out the bottom of the ladder 5 meters.



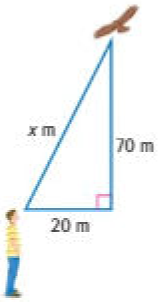
11. MP **Model with Mathematics** Write and solve a real-world problem that involves using the Pythagorean Theorem or its converse.
See students' work.

Name ______________________ My Homework ______________________

Extra Practice

12. Write an equation to find how far the bird is from the boy. Then solve the equation. Round to the nearest tenth.

$70^2 + 20^2 = x^2$; 72.8 m



Homework Help

$a = 70$, $b = 20$, and $c = x$

$70^2 + 20^2 = x^2$

$4{,}900 + 400 = x^2$

$5{,}300 = x^2$

$\sqrt{5{,}300} = x^2$

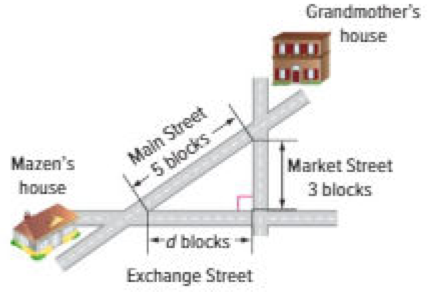
$72.8 \approx x$

13. A hat is in the shape of a cone with dimensions shown.

Find the height of the hat. Round to the nearest tenth. **22.5 centimeters**

14. Mazen wants to go from his house to his grandmother's house. How much distance is saved if he takes Main Street instead of Market and Exchange?

2 blocks

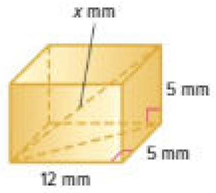


15. Suppose three towns form a right triangle. What is the distance between the two towns that form the hypotenuse?

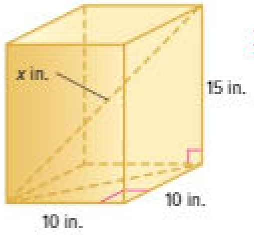
about 105 mi

Persevere with Problems Find the missing measure in each figure below. Round to the nearest tenth if necessary.

16. **13.9 mm**



17. **20.6 cm**



Power Up! Test Practice

Exercises 18 and 19 prepare students for more rigorous thinking needed when taking assessment.

18.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP7
Scoring Rubric	
2 points	Students correctly label the diagram and find the perimeter.
1 point	Students correctly label the diagram OR find the perimeter.

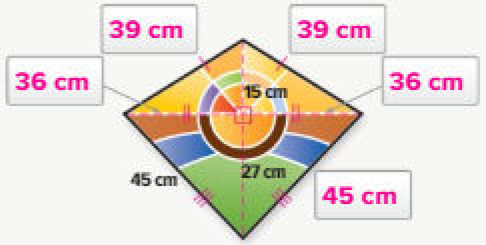
19.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

18. Suhaila designed a decorative glass window in the shape of a kite. Select the correct measures to label the dimensions of the window.

12 cm	42 cm
31 cm	45 cm
36 cm	60 cm
39 cm	

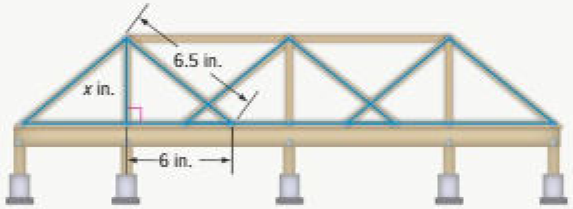


What is the perimeter of the window?

168 cm

19. Ayoub is building the model bridge shown. How long must he cut the piece of wood for one of the vertical support beams, represented by *x*?

2.5 cm



Spiral Review

20. Determine whether a triangle with sides 20 centimeters, 48 centimeters, and 52 centimeters long is a right triangle. Justify your answer.

yes; $20^2 + 48^2 = 52^2$

Estimate each of the following to the nearest whole number. Justify your reasoning.

21. $\sqrt{39} \approx$ 6

$6^2 = 36$ and $7^2 = 49$, since 39 is closer to 36 than 49, $\sqrt{39} \approx 6$.

22. $-\sqrt{146} \approx$ −12

$-(12^2) = -144$ and $-(13^2) = -169$. Since −146 is closer to −144 than −169, $-\sqrt{146} \approx -12$.

23. $\sqrt[3]{30} \approx$ 3

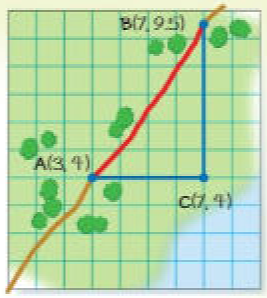
$3^3 = 27$ and $4^3 = 64$. Since 30 is closer to 27 than 64, $\sqrt[3]{30} \approx 3$.

Geometry

Lesson 7

Distance on the Coordinate Plane

Real-World Link



Mountain Biking Saeed was biking on a trail. A map of the trail is shown. His brother timed his ride from point *A* to point *B*.

1. What do the blue and red lines on the graph represent?
   The blue lines represent the horizontal and vertical distances between the two points. The red line represents the actual distance between the two points.
2. What type of triangle is formed by the lines?
   a right triangle
3. How can you find the length of $\overline{AC}$ and $\overline{BC}$ without counting the number of units?
   Sample answer: subtract the *x*-coordinates and subtract the *y*-coordinates
4. What are the lengths of the two blue lines?
   $AC = \boxed{4}$ units $BC = \boxed{5.5}$ units
5. Write an equation using the Pythagorean Theorem that you can use to find the length of $\overline{AB}$.
   $4^2 + 5.5^2 = c^2$

Essential Question

HOW can algebraic concepts be applied to geometry?

Vocabulary

Distance Formula

Mathematical Practices
1, 3, 4, 5

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Cernan Elias/Alamy

Focus narrowing the scope

Objective Find the distance between two points on the coordinate plane.

Coherence connecting within and across grades

Previous
Students used the Pythagorean Theorem to solve problems.

Now
Students will find the distance between two points on a coordinate plane.

Next
Students will apply distance on the coordinate plane to transformations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 435.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Numbered Heads Together** Have students work in groups of 3–4 to complete Exercises 1–5. Assign each student a number. Each student is responsible to ask for help or support and for ensuring that their teammates understand each exercise. Call on one numbered student to explain the group's responses to the class. MP 1, 4, 5, 6

Alternate Strategy

AL Students may benefit from a quick review of how the Pythagorean Theorem can be used to find the length of a missing side of a right triangle.

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

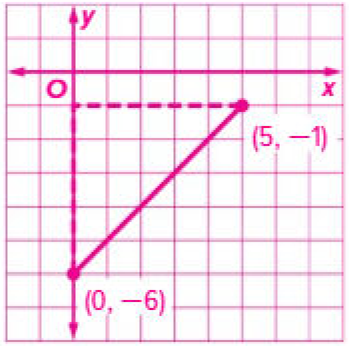
1. Find distance on the coordinate plane.

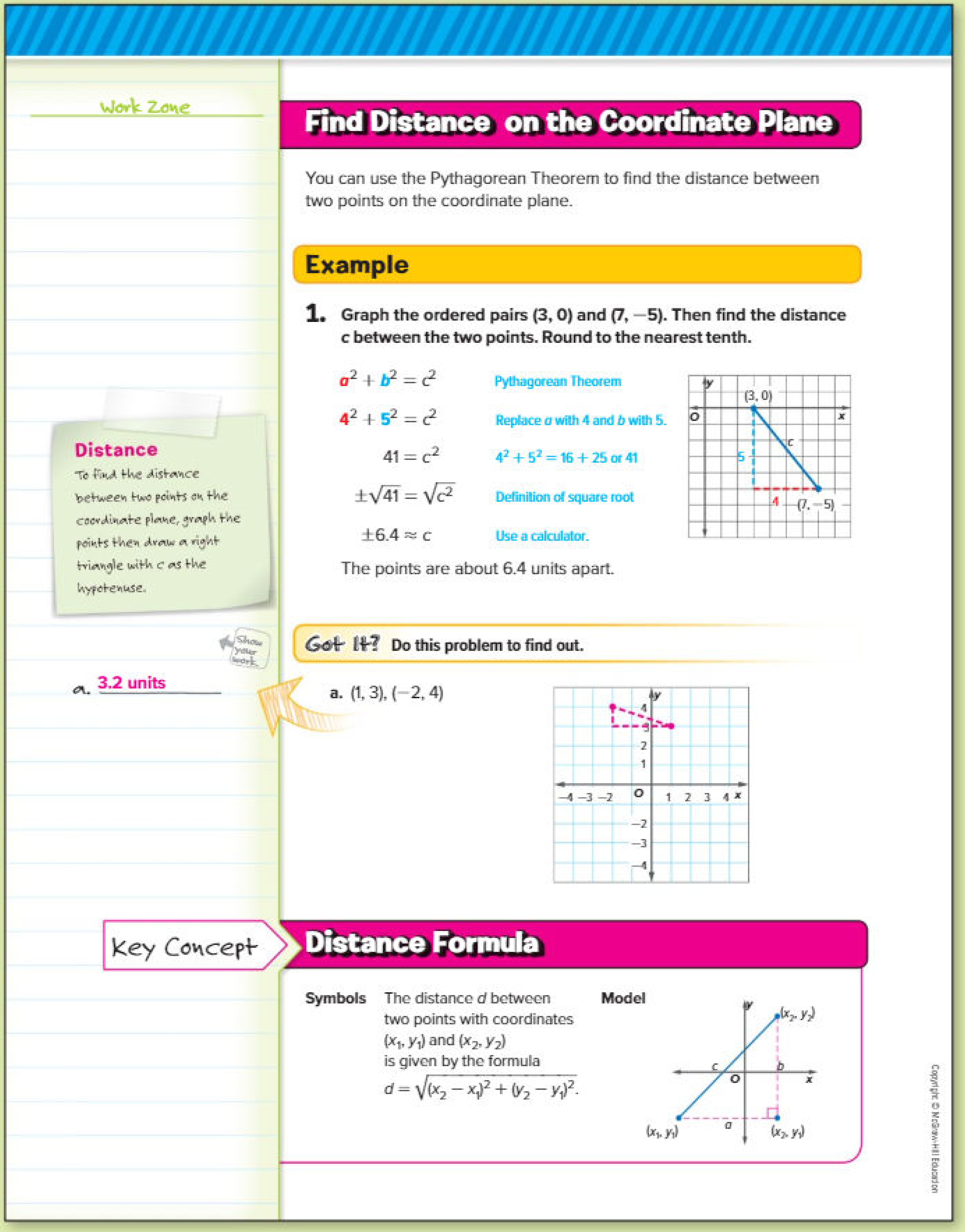
AL
- *A right triangle can be formed using these two points and a third point (3, −5). On the drawn triangle, which side represents the distance that we need to determine?* the hypotenuse, *c*
- *What is the length of the horizontal leg?* 4 units
- *What is the length of the vertical leg?* 5 units

OL
- *What equation can we use to model the Pythagorean Theorem?* $a^2 + b^2 = c^2$
- *What value will we use for a? b?* 4 units; 5 units
- *Could we have used 5 units for a and 4 units for b?* yes; The order in which you add does not matter.
- *Why do we not use the negative square root?* Distance cannot be negative.

BL
- *Could you draw a different triangle and still determine the same distance? Explain.* yes; You could draw a horizontal line to the right of (3, 0) and a vertical line up from (7, −5).

Need Another Example?
Graph the ordered pairs (0, −6) and (5, −1). Then find the distance *c* between the points. Round to the nearest tenth. 7.1 units





Work Zone

Find Distance on the Coordinate Plane

You can use the Pythagorean Theorem to find the distance between two points on the coordinate plane.

Example

1. **Graph the ordered pairs (3, 0) and (7, −5). Then find the distance *c* between the two points. Round to the nearest tenth.**

$a^2 + b^2 = c^2$	Pythagorean Theorem
$4^2 + 5^2 = c^2$	Replace *a* with 4 and *b* with 5.
$41 = c^2$	$4^2 + 5^2 = 16 + 25$ or 41
$\pm\sqrt{41} = \sqrt{c^2}$	Definition of square root
$\pm 6.4 \approx c$	Use a calculator.

The points are about 6.4 units apart.

Distance
To find the distance between two points on the coordinate plane, graph the points then draw a right triangle with c as the hypotenuse.

Got It? Do this problem to find out.

a. (1, 3), (−2, 4)

Show your work.

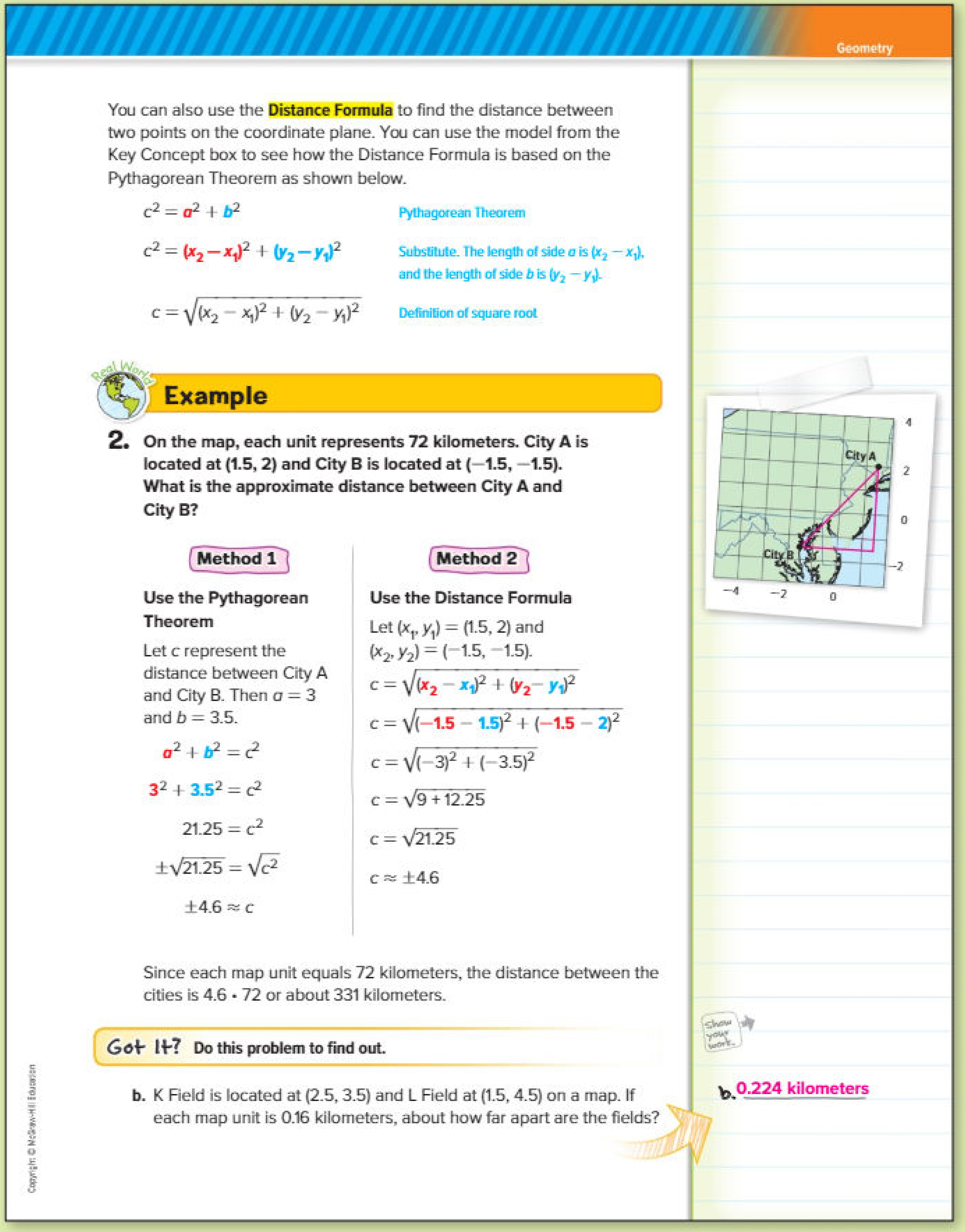
a. 3.2 units

Key Concept

Distance Formula

Symbols The distance *d* between two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Model



You can also use the **Distance Formula** to find the distance between two points on the coordinate plane. You can use the model from the Key Concept box to see how the Distance Formula is based on the Pythagorean Theorem as shown below.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ Substitute. The length of side a is $(x_2 - x_1)$, and the length of side b is $(y_2 - y_1)$.

$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Definition of square root

Example

2. **On the map, each unit represents 72 kilometers. City A is located at (1.5, 2) and City B is located at (–1.5, –1.5). What is the approximate distance between City A and City B?**

Method 1

Use the Pythagorean Theorem

Let c represent the distance between City A and City B. Then $a = 3$ and $b = 3.5$.

$$a^2 + b^2 = c^2$$

$$3^2 + 3.5^2 = c^2$$

$$21.25 = c^2$$

$$\pm\sqrt{21.25} = \sqrt{c^2}$$

$$\pm 4.6 \approx c$$

Method 2

Use the Distance Formula

Let $(x_1, y_1) = (1.5, 2)$ and $(x_2, y_2) = (-1.5, -1.5)$.

$$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$c = \sqrt{(-1.5 - 1.5)^2 + (-1.5 - 2)^2}$$

$$c = \sqrt{(-3)^2 + (-3.5)^2}$$

$$c = \sqrt{9 + 12.25}$$

$$c = \sqrt{21.25}$$

$$c \approx \pm 4.6$$

Since each map unit equals 72 kilometers, the distance between the cities is 4.6 • 72 or about 331 kilometers.

Got It? **Do this problem to find out.**

b. K Field is located at (2.5, 3.5) and L Field at (1.5, 4.5) on a map. If each map unit is 0.16 kilometers, about how far apart are the fields?

b. 0.224 kilometers

Example

2. Use the Distance Formula to find actual distances.

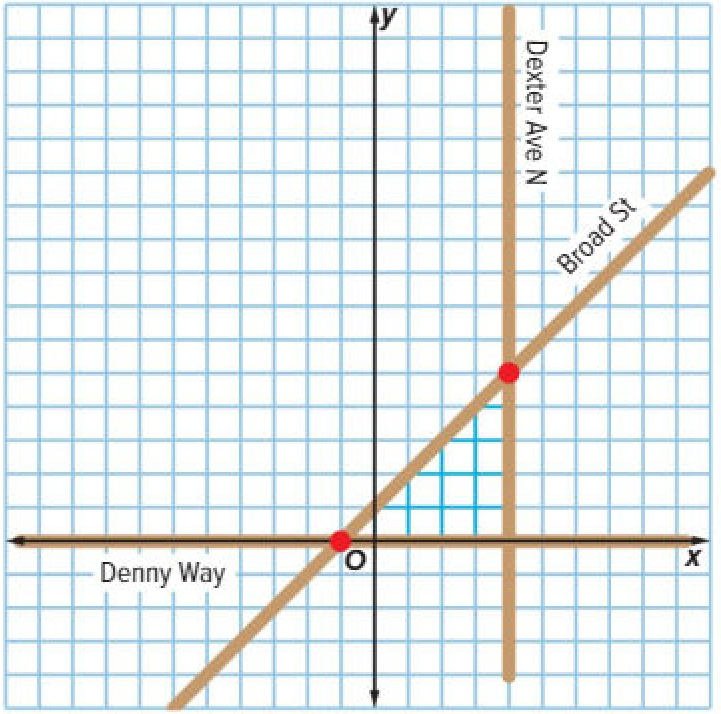
AL
- *What ordered pair represents City A?* (1.5, 2)
- *What ordered pair represents City B?* (–1.5, –1.5)

OL
- *If you make a right triangle to find the distance, what would be the length of each side?* 3 units and 3.5 units
- *If you were to use the Distance Formula, what point would you use for (x_1, y_1)?* (1.5, 2)
- *What point would you use for (x_2, y_2)?* (–1.5, –1.5)
- *Once you find the map distance, what do you need to do to find the actual distance?* Multiply 4.6 by 45.

BL
- *Does it matter whether we use the point (1.5, 2) for (x_1, y_1) or for (x_2, y_2)? Explain.* no; Sample answer: The distance between the two points will be the same, regardless of which point you consider, the first or second point.

Need Another Example?

Fahd lives in Seattle, Washington. One unit on this map is 0.08 mile. Find the approximate distance he drives between Broad Street at Denny Way (–1, 0) and Broad Street at Dexter Avenue North (4, 5). 0.57 mi



Example

3. Use the Distance Formula.

AL
- *What point would you use for (x_1, y_1)?* (5, −4)
- *What point would you use for (x_2, y_2)?* (−3, −2)

OL
- *When you substitute the values in the formula, what expression is inside the radical sign?* $(-3 - 5)^2 + [-2 - (-4)]^2$
- *How can you simplify this expression?* $(-8)^2 + 2^2$, or 68

BL
- *Why do we not consider the negative square root?* Distance is never negative.
- *Suppose you forgot the Distance Formula. How could you determine the distance between these points?* Sample answer: Use the Pythagorean Theorem.

Need Another Example?
Use the Distance Formula to find the distance between G(−3, −2) and H(−6, 5). Round to the nearest tenth, if necessary. 7.6 units

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL **LA** **Team-Pair-Solo** Have students work in a group of 3–4 students to complete Exercise 1, ensuring that every team member understands. Then have them work in pairs to complete Exercise 2. Individually, they should complete Exercise 3 and then share their responses with their partner. MP 1, 4, 5, 6

BL **LA** **Pairs Present** Have students work with a partner to prepare a brief oral presentation about how the Distance Formula is derived from the Pythagorean Theorem. Their presentation should include illustrations. Have them present to the class. MP 1, 4, 5, 6

STOP and Reflect

Explain below how to find the length of a non-vertical and a non-horizontal segment whose endpoints are (x_1, y_1) and (x_2, y_2).

Sample answer: substitute the values for x_2 and x_1 and y_2 and y_1 into the Distance Formula. Then simplify.

Example

3. **Use the Distance Formula to find the distance between X(5, −4) and Y(−3, −2). Round to the nearest tenth if necessary.**

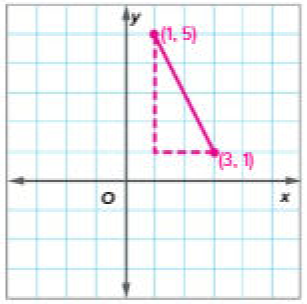
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$XY = \sqrt{(-3 - 5)^2 + [-2 - (-4)]^2}$	$(x_1, y_1) = (5, -4)$, $(x_2, y_2) = (-3, -2)$
$XY = \sqrt{(-8)^2 + 2^2}$	Simplify.
$XY = \sqrt{64 + 4}$	Evaluate $(-8)^2$ and 2^2.
$XY = \sqrt{68}$	Add 64 and 4.
$XY \approx \pm 8.2$	Simplify.

So, the distance between points X and Y is about 8.2 units.

Guided Practice

Check

1. Graph the ordered pairs (1, 5) and (3, 1). Then find the distance between the points. Round to the nearest tenth if necessary. (Example 1)
4.5 units



2. On a park map, the ranger station is located at (2.5, 3.5) and the nature center is located at (0.5, 4). Each unit in the map is equal to 0.8 kilometers. What is the approximate distance between the ranger station and the nature center? (Examples 2 and 3) 1.6 kilometers

3. **Building on the Essential Question** How can you use the Pythagorean Theorem to find the distance between two points on the coordinate plane?
Sample answer: After you plot the points, draw a right triangle. Use the Pythagorean Theorem to find the length of the hypotenuse which is the distance between the two points.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

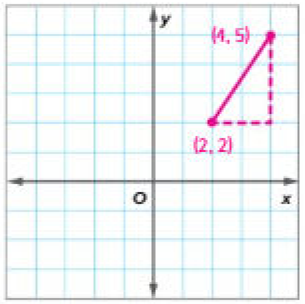
FOLDABLES Time to update your Foldable!

Name My Homework

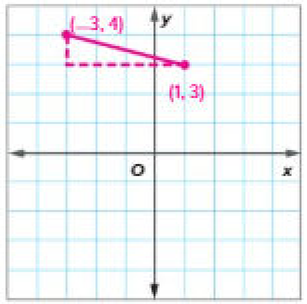
Independent Practice

Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary. (Example 1)

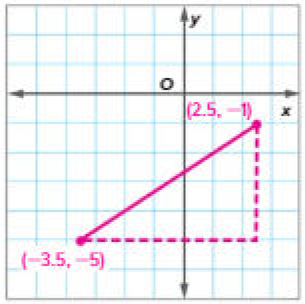
1. (4, 5), (2, 2) 3.6 units



2. (−3, 4), (1, 3) 4.1 units



3. (2.5, −1), (−3.5, −5) 7.2 units



Show your work

4. A ferry sets sail from an island located at (4, 12) on a map. Its destination is Ferry Landing *B* at (6, 2). How far will the ferry travel if each unit on the grid is 0.5 kilometer? (Example 2) about 5.1 km

Use the Distance Formula to find the distance between each pair of points. Round to the nearest tenth if necessary. (Example 3)

5. $C(-5, -3)$, $D(-4, -2)$ 1.4 units

6. $Y(3.5, 1)$, $Z(-4, 2.5)$ 7.6 units

7. $K\left(8\frac{1}{2}, 12\right), L\left(-6\frac{3}{4}, 7\frac{1}{2}\right)$ 15.9 units

8. Chicago, Illinois, has a longitude of 88°W and a latitude of 42°N. Indianapolis, Indiana, is located at 86°W and 40°N. At this longitude/latitude, each degree is about 85 kilometers. Find the distance between Chicago and Indianapolis. about 240 km

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–7, 13–19	8, 9, 20, 21	10–12
Level 3			●
Level 2		●	
Level 1	●		

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 10, 12, 20, 21
OL	On Level	1–7 odd, 8–10, 12, 20, 21
BL	Beyond Level	8–12, 20, 21

MP MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	11
2 Reason abstractly and quantitatively.	12
3 Construct viable arguments and critique the reasoning of others.	9
5 Use appropriate tools strategically.	10, 16

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

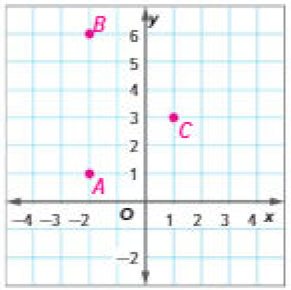
Have students explain how their knowledge of the Pythagorean Theorem helped them to find the distance between two points on a coordinate plane.
See students' work.

Watch Out!

Common Error When the Distance Formula calculations are done mentally, it is easy to incorrectly subtract a negative number from another number or replace a variable with an incorrect value. Encourage students to designate which values are x_1, x_2, y_1, and y_2, write the formula and all the steps, and then evaluate the formula.

9. MP **Multiple Representations** Points $A(-2, 1)$, $B(-2, 6)$, and $C(1, 3)$ are the vertices of a triangle.

   a. **Graphs** Graph the points $A(-2, 1)$, $B(-2, 6)$ and $C(1, 3)$.



   b. **Words** Explain how to find the length of segment *BC*.
   Sample answer: Use the Distance Formula and the points (−2, 6) and (1, 3).

   c. **Numbers** Find the length of each side of △*ABC*. Round to the nearest tenth if necessary.
   $AC \approx$ 3.6 units
   $AB =$ 5 units
   $BC \approx$ 4.2 units

   d. **Numbers** What is the perimeter of △*ABC*? Use the values from part *c*.
   perimeter = 12.8 units

H.O.T. Problems Higher Order Thinking

10. MP **Use Math Tools** Layla needs to find the distance between the points $A(-2.4, 3.7)$ and $B(4.5, -1.4)$. Suggest a tool she could use to find the length. Then find the length. Explain your reasoning.
Sample answer: Calculator; it will be most helpful when squaring and finding the square root involving decimals; about 8.6 units.

11. MP **Persevere with Problems** Apply what you have learned about distance on the coordinate plane to write the coordinates of two possible endpoints of a line segment that is neither horizontal nor vertical and has a length of 5 units.
Sample answer: (1, 2) and (4, 6)

12. MP **Reason Inductively** Compare the steps to find the distance between two points on the coordinate plane by first using the Pythagorean Theorem and then using the Distance Formula.
Sample answer: To use the Pythagorean Theorem, connect the points to form a right triangle. Then use the Pythagorean Theorem to find the length of the hypotenuse. To use the Distance Formula, replace (x_1, y_1) and (x_2, y_2) in the formula with the coordinates of the two endpoints and simplify.

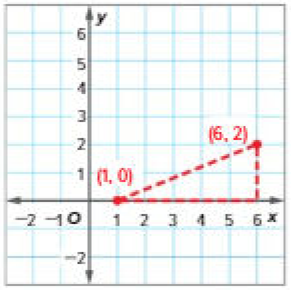
Name ______________________ My Homework ______________________

Extra Practice

Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary.

13. (6, 2), (1, 0) 5.4 units

Homework Help



$a = 2, b = 5$

$a^2 + b^2 = c^2$

$2^2 + 5^2 = c^2$

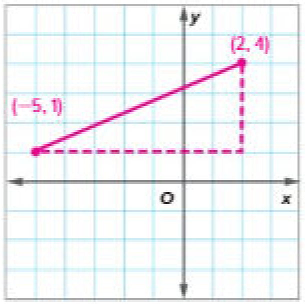
$4 + 25 = c^2$

$29 = c^2$

$\pm\sqrt{29} = \sqrt{c^2}$

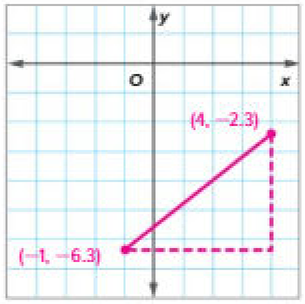
$\pm 5.4 \approx c$

14. (−5, 1), (2, 4) 7.6 units



15. (4, −2.3), (−1, −6.3)

6.4 units



16. **MP** **Use Math Tools** On a map, Al Jabar is located at (3, 2.5), and Dhaman is located at (8.5, 14.5). Each unit on the map equals 26.4 kilometers. What is the approximate distance between the cities?

about 348.5 km

Use the Distance Formula to find the distance between each pair of points. Round to the nearest tenth if necessary.

17. $W(1, 7), X(-2, -4)$

11.4 units

18. $G(-6.25, 5), H(-3.75, 2)$

3.9 units

19. $P\left(-9\frac{1}{4}, -7\frac{1}{2}\right), Q(-4, 5)$

13.6 units

Power Up! Test Practice

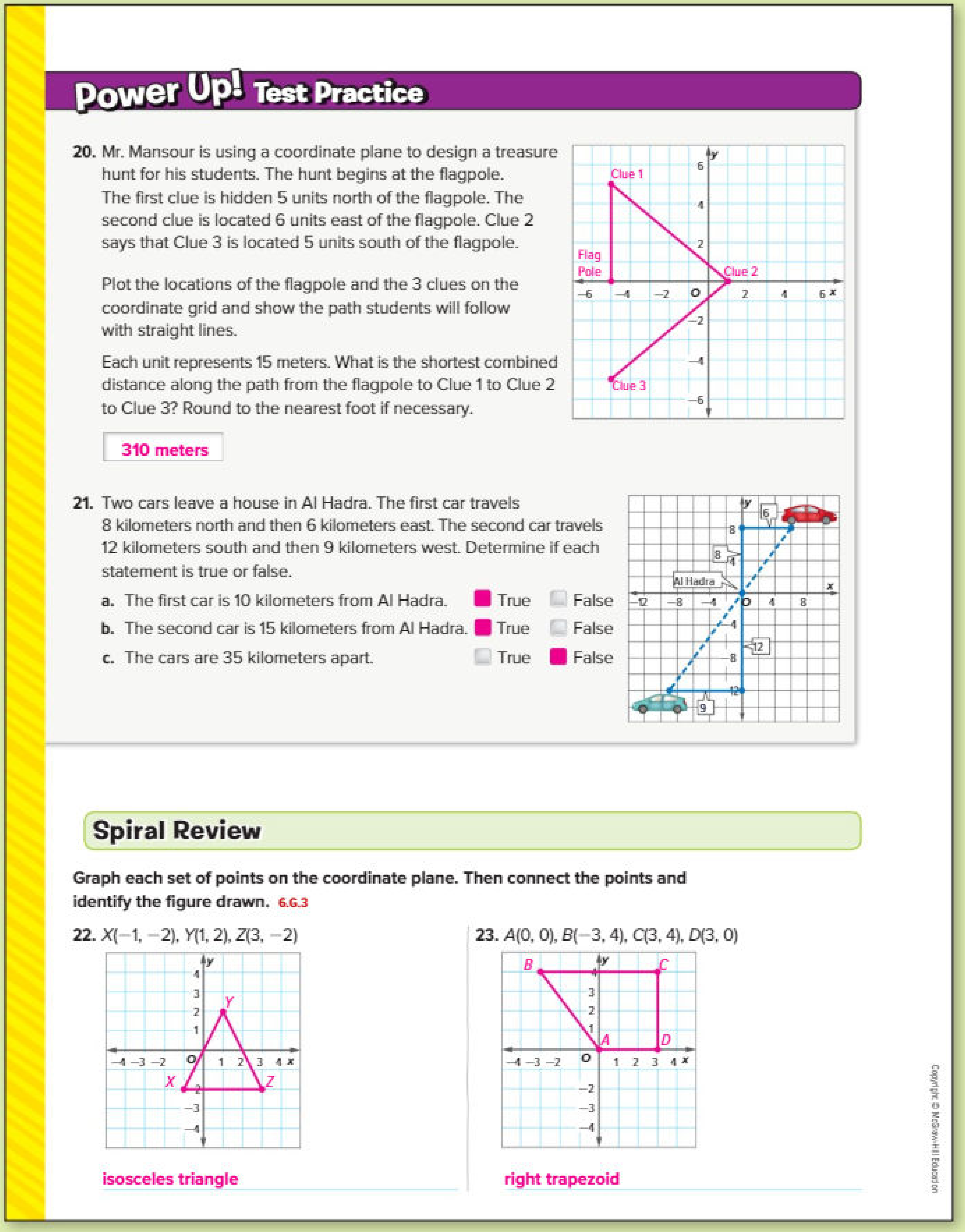
Exercises 20 and 21 prepare students for more rigorous thinking needed when taking assessment.

20.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK3
Mathematical Practices	MP1, MP4, MP5
Scoring Rubric	
2 points	Students correctly plot and connect the points and also find the shortest combined distance.
1 point	Students correctly plot and connect the points, but fail to find the shortest combined distance OR students incorrectly plot points but draw lines and find a distance based on the incorrect points.

21.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP2, MP5
Scoring Rubric	
1 point	Students correctly answer each part of the question.



Power Up! Test Practice

20. Mr. Mansour is using a coordinate plane to design a treasure hunt for his students. The hunt begins at the flagpole. The first clue is hidden 5 units north of the flagpole. The second clue is located 6 units east of the flagpole. Clue 2 says that Clue 3 is located 5 units south of the flagpole.

Plot the locations of the flagpole and the 3 clues on the coordinate grid and show the path students will follow with straight lines.

Each unit represents 15 meters. What is the shortest combined distance along the path from the flagpole to Clue 1 to Clue 2 to Clue 3? Round to the nearest foot if necessary.

310 meters

21. Two cars leave a house in Al Hadra. The first car travels 8 kilometers north and then 6 kilometers east. The second car travels 12 kilometers south and then 9 kilometers west. Determine if each statement is true or false.

a. The first car is 10 kilometers from Al Hadra. ☑ True ☐ False

b. The second car is 15 kilometers from Al Hadra. ☑ True ☐ False

c. The cars are 35 kilometers apart. ☐ True ☑ False

Spiral Review

Graph each set of points on the coordinate plane. Then connect the points and identify the figure drawn. 6.G.3

22. $X(-1, -2)$, $Y(1, 2)$, $Z(3, -2)$

isosceles triangle

23. $A(0, 0)$, $B(-3, 4)$, $C(3, 4)$, $D(3, 0)$

right trapezoid

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21ST CENTURY CAREER in Travel and Tourism

Travel Agent

Do you love to travel? Would you be interested in helping others plan their ideal vacation getaways? You should consider a career in travel and tourism. Travel agents offer advice on destination locations and make arrangements for transportation, lodging, car rentals, and tours for their clients. In addition to having personal travel experience and being knowledgeable about popular vacation destinations, travel agents also need to be detail-oriented and have excellent communication, math, and computer skills.

Is This the Career for You?

Are you interested in a career as a travel agent? Take some of the following courses in high school.

- Algebra
- Business Software Applications
- Computer Technology
- Geometry

Turn the page to find out how math relates to a career in Travel and Tourism.

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Focus narrowing the scope

Objective Apply mathematics to problems arising in the workplace.

This lesson emphasizes **MP Mathematical Practice 4** Model with Mathematics.

Coherence connecting within and across grades

Previous
Students used the Distance Formula to find lengths of sides of a triangle.

Now
Students apply the content standard to solve problems in the workplace.

Rigor pursuing concepts, fluency, and applications

See the Career Project on page 440.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ask students to read the information on the student page about travel agents and answer the following questions.

Ask:

- *What kind of classes should you take to be a travel agent?* Algebra, Business Software Applications, Computer Technology, Geometry
- *What is one aspect of travel that a travel agent needs to be knowledgeable about?* Sample answer: A travel agent needs to be knowledgeable about popular vacation destinations.

2 Collaborate

AL **LA** **Circle the Sage** Poll the class to see which students have knowledge about the Pythagorean Theorem. Those students (the sages) spread out around the room. Assign the rest of the students to teams. Have the teams split up with each team member going to a different sage, if possible. Have the sages lead work for Exercises 1–6. When the exercises are complete, students go back to their teams and compare solutions. Students discuss how the sages may have explained the steps differently. MP 1, 2, 3, 4, 5, 6, 7

Ask:

- *How do right triangles help in reading a map and finding distances?* **Sample answer: Right triangles help find distances that are not easily distinguishable.**

BL **LA** **Pairs Discussion** Have students work in pairs to extend the activity by answering the following question. MP 1, 4, 5, 6, 7

Ask:

- *Describe the locations of Marathon, Cudjoe Key, and Islamorada using ordered pairs. Why is Cudjoe Key closer to Marathon than Islamorada?* **Sample answer: There are about 17.68 miles between Cudjoe Key and Marathon and about 20.16 miles between Islamorada and Marathon.**

Career Portfolio

When students complete this page, have them add it to their Career Portfolio.

Career Facts

Agents with backgrounds in computer science, geography, communication, foreign languages, or world history often have a greater chance of being hired by travel agencies. These backgrounds show that the agents have an interest in travel and culture, which clients find appealing.

Time to Get Away!

Use the map to solve each problem. Round to the nearest tenth if necessary.

1. What is the approximate distance between Key Largo and Islamorada? **23 kilometers**

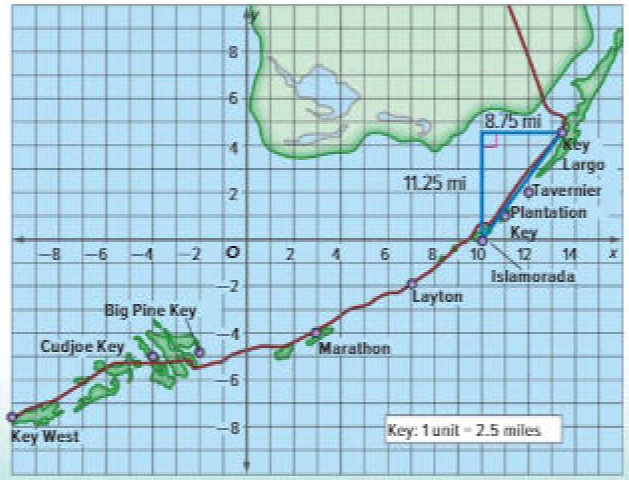
2. Draw and label a right triangle to find the distance between Plantation Key and Islamorada. Then find the approximate distance. **5.6 kilometers**

3. Describe the ordered pairs that represent Layton and Plantation Key. Then find the approximate distance between Layton and Plantation Key. **Layton: (7, −2); Plantation Key: (11, 1); 20 kilometers**

4. To the nearest 0.5 unit, name the ordered pairs that represent Key West and Cudjoe Key. Then use the ordered pairs to estimate the distance between the keys. **Sample answer: Key West (−10, −7.5); Cudjoe Key (−4, −5); 26 kilometers**

5. What is the approximate distance between Key West and Layton? **Sample answer: 71.5 kilometers**

6. What is the approximate distance between Tavernier and Big Pine Key? **Sample answer: 62.5 kilometers**

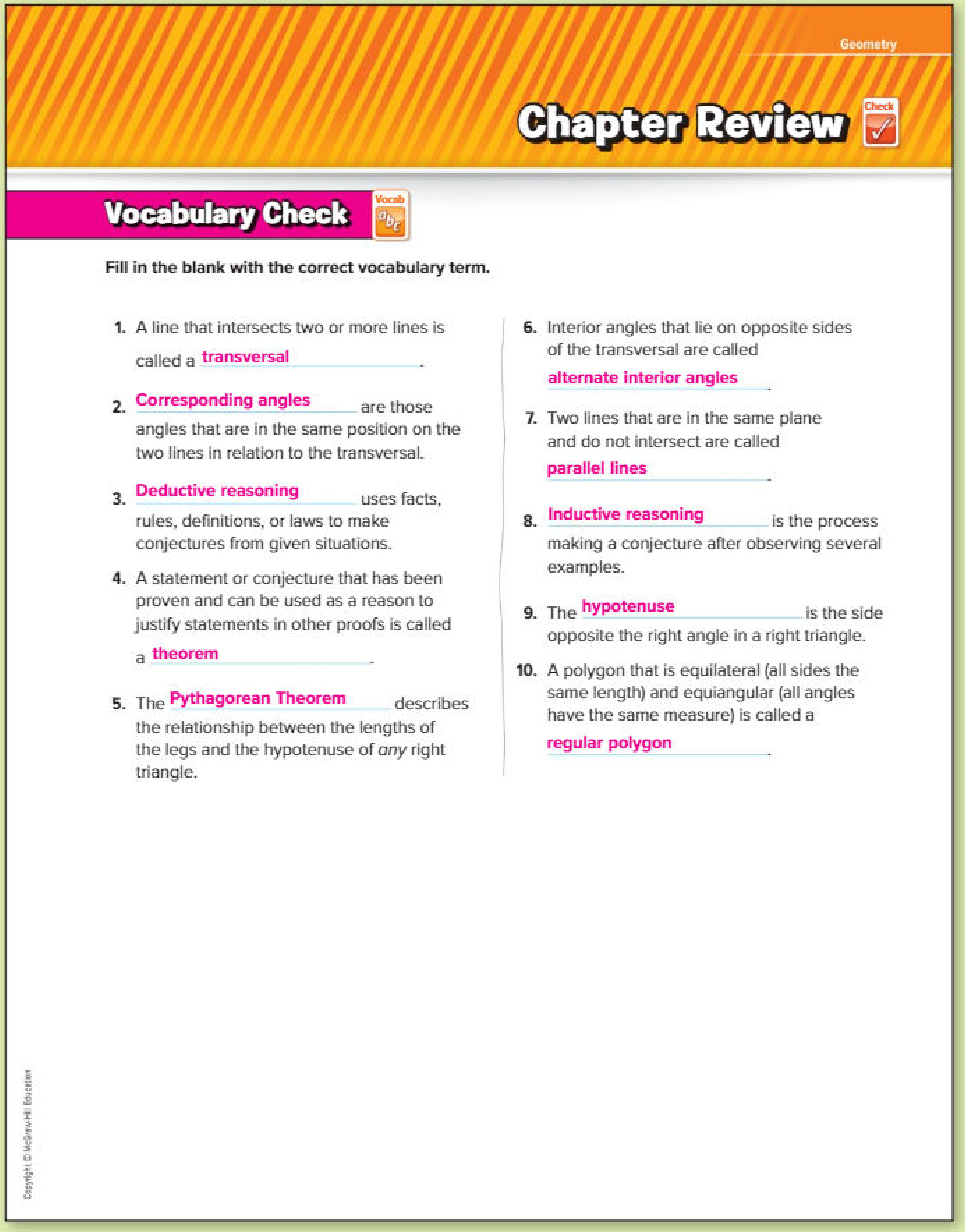


Career Project

It's time to update your career portfolio! Go online and research a career as a travel agent.

Describe three things that you learned about being a travel agent that you did not know.

List several jobs that are created by the travel and tourism industry.



Chapter Review

Vocabulary Check

Fill in the blank with the correct vocabulary term.

1. A line that intersects two or more lines is called a transversal.

2. Corresponding angles are those angles that are in the same position on the two lines in relation to the transversal.

3. Deductive reasoning uses facts, rules, definitions, or laws to make conjectures from given situations.

4. A statement or conjecture that has been proven and can be used as a reason to justify statements in other proofs is called a theorem.

5. The Pythagorean Theorem describes the relationship between the lengths of the legs and the hypotenuse of *any* right triangle.

6. Interior angles that lie on opposite sides of the transversal are called alternate interior angles.

7. Two lines that are in the same plane and do not intersect are called parallel lines.

8. Inductive reasoning is the process making a conjecture after observing several examples.

9. The hypotenuse is the side opposite the right angle in a right triangle.

10. A polygon that is equilateral (all sides the same length) and equiangular (all angles have the same measure) is called a regular polygon.

Vocabulary Check

LA **Rally Coach** Have students work in pairs to complete the Vocabulary Check. Have Student A complete the first exercise, speaking out loud, while Student B listens carefully, coaches, and praises. Next have Student B complete the second exercise while Student A listens carefully, coaches, and praises. Partners take turns until they have completed the Vocabulary Check. MP 1, 6

Alternate Strategy

AL LA To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include the following terms.

- alternate interior angles (Lesson 1)
- corresponding angles (Lesson 1)
- deductive reasoning (Lesson 2)
- hypotenuse (Lesson 3)
- inductive reasoning (Lesson 2)
- parallel lines (Lesson 1)
- Pythagorean Theorem (Lesson 3)
- regular polygon (Lesson 4)
- theorem (Lesson 3)
- transversal (Lesson1)

Key Concept Check

FOLDABLES LA A completed Foldable for this chapter should include a review of the Pythagorean Theorem and the Distance Formula.

If you choose not to use this Foldable, have students write a brief review of the Key Concepts found throughout the chapter and give an example of each.

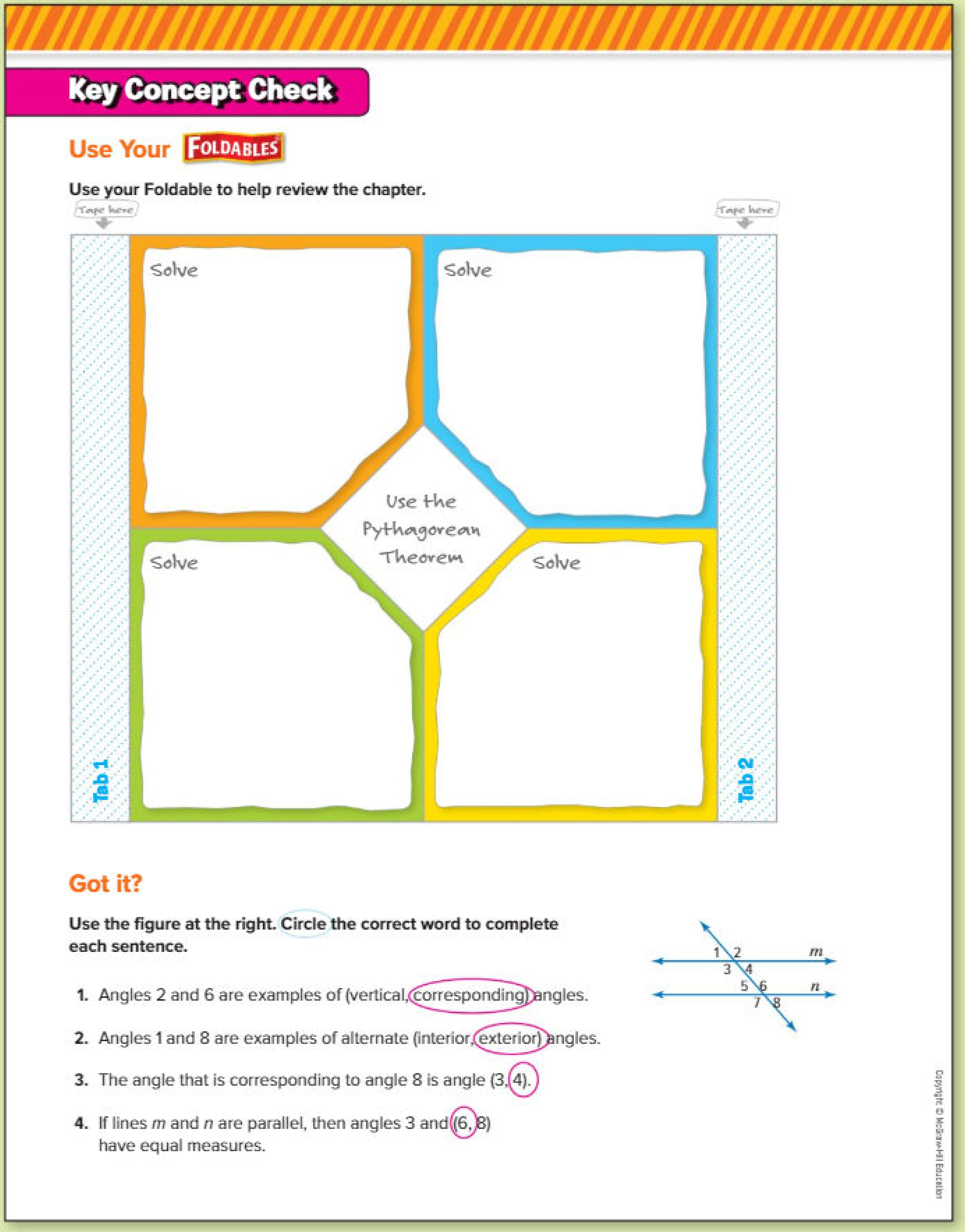
Ideas for Use

LA Have students work in pairs to discuss their Foldables. Have them practice speaking out loud in a collaborative setting by sharing how they have completed their Foldable thus far and how they could finish it. Have each student complete their Foldable and trade with their partner to discuss any similarities and differences. MP 1, 4

Got It?

If students have trouble with Exercises 1–4, they may need help with the following concept(s).

Concept	Exercise(s)
identifying angle relationships (Lesson 1)	1–4



Key Concept Check

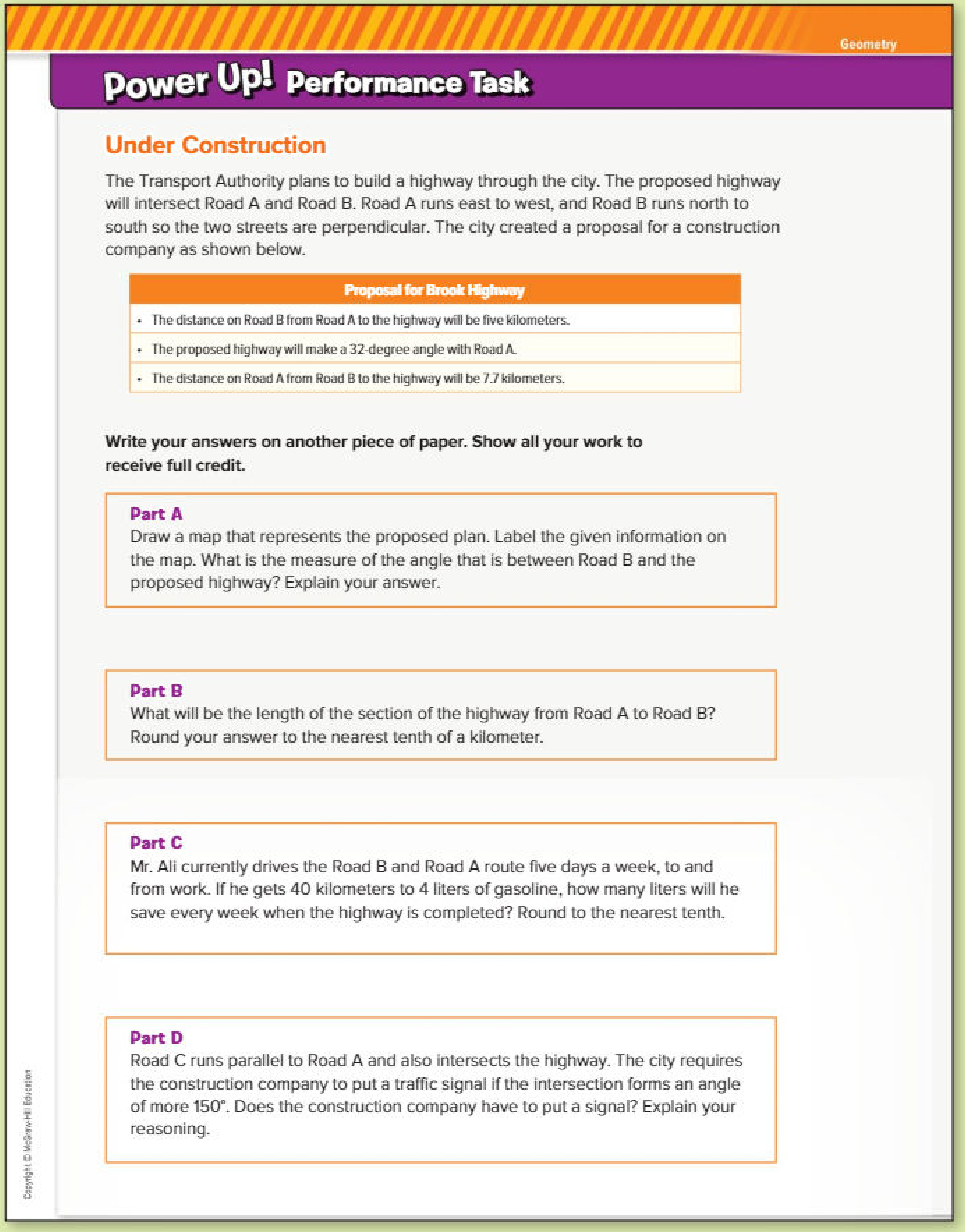
Use Your FOLDABLES

Use your Foldable to help review the chapter.

Got it?

Use the figure at the right. Circle the correct word to complete each sentence.

1. Angles 2 and 6 are examples of (vertical, [✓ corresponding]) angles.
2. Angles 1 and 8 are examples of alternate (interior, [✓ exterior]) angles.
3. The angle that is corresponding to angle 8 is angle (3, [✓ 4]).
4. If lines *m* and *n* are parallel, then angles 3 and ([✓ 6], 8) have equal measures.



Power Up! Performance Task

Under Construction

The Transport Authority plans to build a highway through the city. The proposed highway will intersect Road A and Road B. Road A runs east to west, and Road B runs north to south so the two streets are perpendicular. The city created a proposal for a construction company as shown below.

Proposal for Brook Highway
• The distance on Road B from Road A to the highway will be five kilometers.
• The proposed highway will make a 32-degree angle with Road A.
• The distance on Road A from Road B to the highway will be 7.7 kilometers.

Write your answers on another piece of paper. Show all your work to receive full credit.

Part A
Draw a map that represents the proposed plan. Label the given information on the map. What is the measure of the angle that is between Road B and the proposed highway? Explain your answer.

Part B
What will be the length of the section of the highway from Road A to Road B? Round your answer to the nearest tenth of a kilometer.

Part C
Mr. Ali currently drives the Road B and Road A route five days a week, to and from work. If he gets 40 kilometers to 4 liters of gasoline, how many liters will he save every week when the highway is completed? Round to the nearest tenth.

Part D
Road C runs parallel to Road A and also intersects the highway. The city requires the construction company to put a traffic signal if the intersection forms an angle of more 150°. Does the construction company have to put a signal? Explain your reasoning.

Power Up! Performance Task

This Performance-Based Assessment requires students to solve multi-step problems through abstract reasoning, precision, and perseverance. This practice scenario can be used to help students prepare for the thinking skills that will be used during assessment.

A complete scoring rubric with answers to the Exercises can be found on page PT1.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the **Building on the Essential Question** exercises found in each lesson of the chapter.

- How are the measures of angles related when parallel lines are cut by a transversal? (p. 374)
- How is deductive reasoning used in algebra and geometry proofs? (p. 382)
- How can you find the missing measure of an angle in a triangle if you know the measure of two of the interior angles? (p. 392)
- How can I find the sum of the interior angle measurse of a polygon? (p. 400)
- What is the relationship among the legs and the hypotenuse of a right triangle? (p. 414)
- How do you solve a right triangle? (p. 426)
- How can you use the Pythagorean Theorem to find the distance between two points on the coordinate plane? (p. 434)

Ideas for Use

Think-Pair-Share Have students work in pairs. Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer. Then have them share their responses with their classmate before they complete the graphic organizer. MP 1, 6

Track Your Progress

Return to the beginning of the chapter to review the objectives that it addressed. Students should see that their knowledge of the key ideas has increased now that they have completed this chapter.

Reflect

Answering the Essential Question

Use what you learned about triangles and the Pythagorean Theorem to complete the graphic organizer. List three ways you used algebra in this chapter. Draw a model to represent each way.

Sample answers are given. See students' work for models.

Essential Question

HOW can algebraic concepts be applied to geometry?

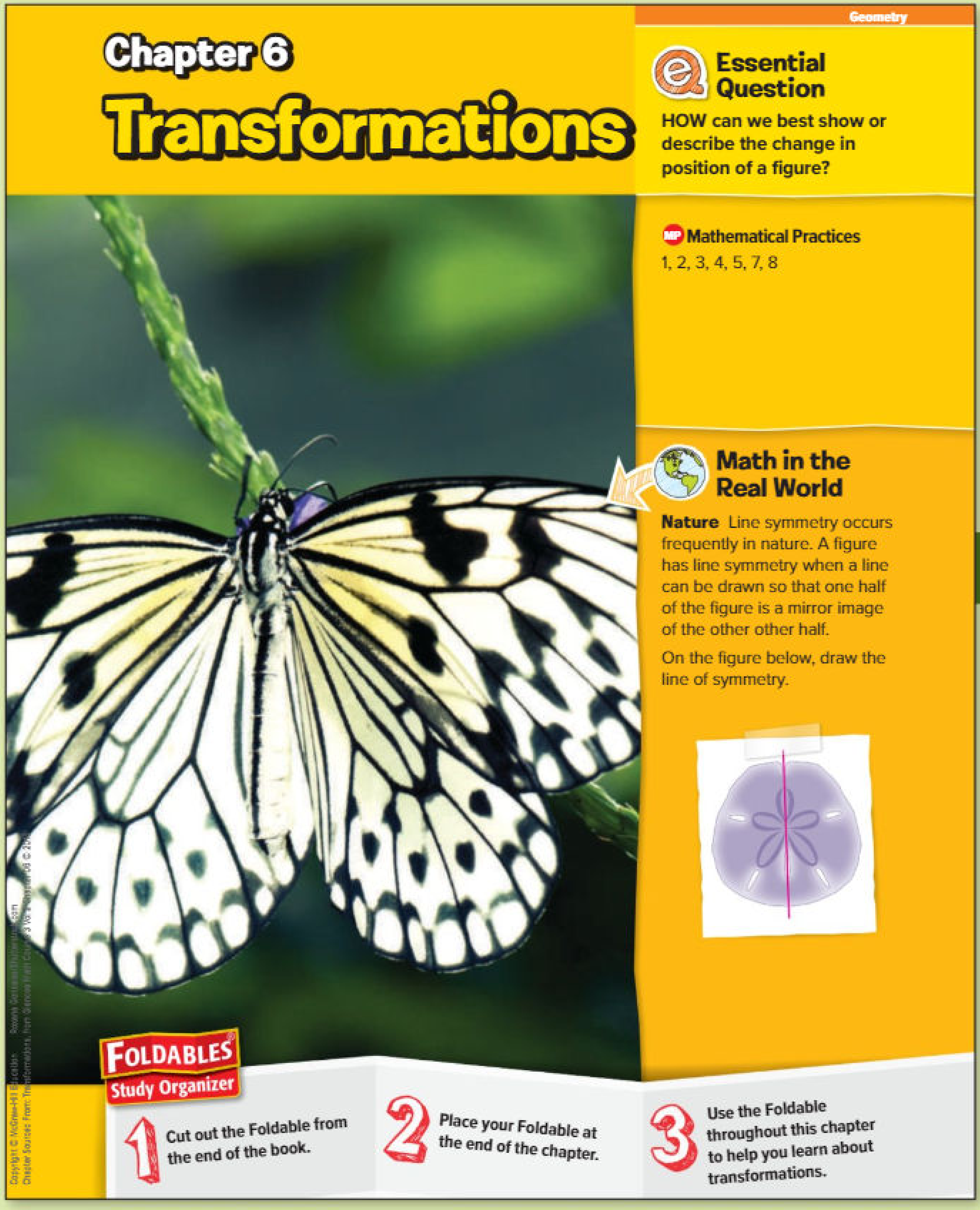
Find the missing value in a triangle.

Find the measure of an exterior angle in a regular hexagon.

Use the Pythagorean Theorem to find the length of the missing side.

Answer the Essential Question. HOW can algebraic concepts be applied to geometry?

See students' work.



Geometry

Chapter 6

Transformations

Essential Question

HOW can we best show or describe the change in position of a figure?

Mathematical Practices

1, 2, 3, 4, 5, 7, 8

Math in the Real World

Nature Line symmetry occurs frequently in nature. A figure has line symmetry when a line can be drawn so that one half of the figure is a mirror image of the other other half.

On the figure below, draw the line of symmetry.

FOLDABLES Study Organizer

1 Cut out the Foldable from the end of the book.

2 Place your Foldable at the end of the chapter.

3 Use the Foldable throughout this chapter to help you learn about transformations.

Focus narrowing the scope

This chapter focuses on content from the **Geometry** (G) domain.

Coherence connecting within and across grades

Previous	Now	Next
Students used the Pythagorean Theorem.	Students study the effects of various types of transformations.	Students will explore congruence and similarity of figures.

Rigor pursuing concepts, fluency, and applications

The Levels of Complexity charts located throughout this chapter indicate how the exercises progress from conceptual understanding and procedural skills and fluency, to application and critical thinking.

Launch the Chapter

Math in the Real World

Nature Remind students that a line of symmetry will always go through the center of an object.

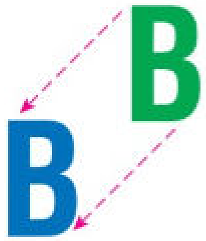
What Tools Do You Need?

Vocabulary Activity

LA As you proceed through the chapter, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define: A transformation is an operation that places an original figure, the preimage, onto a new figure, the image.

Example:



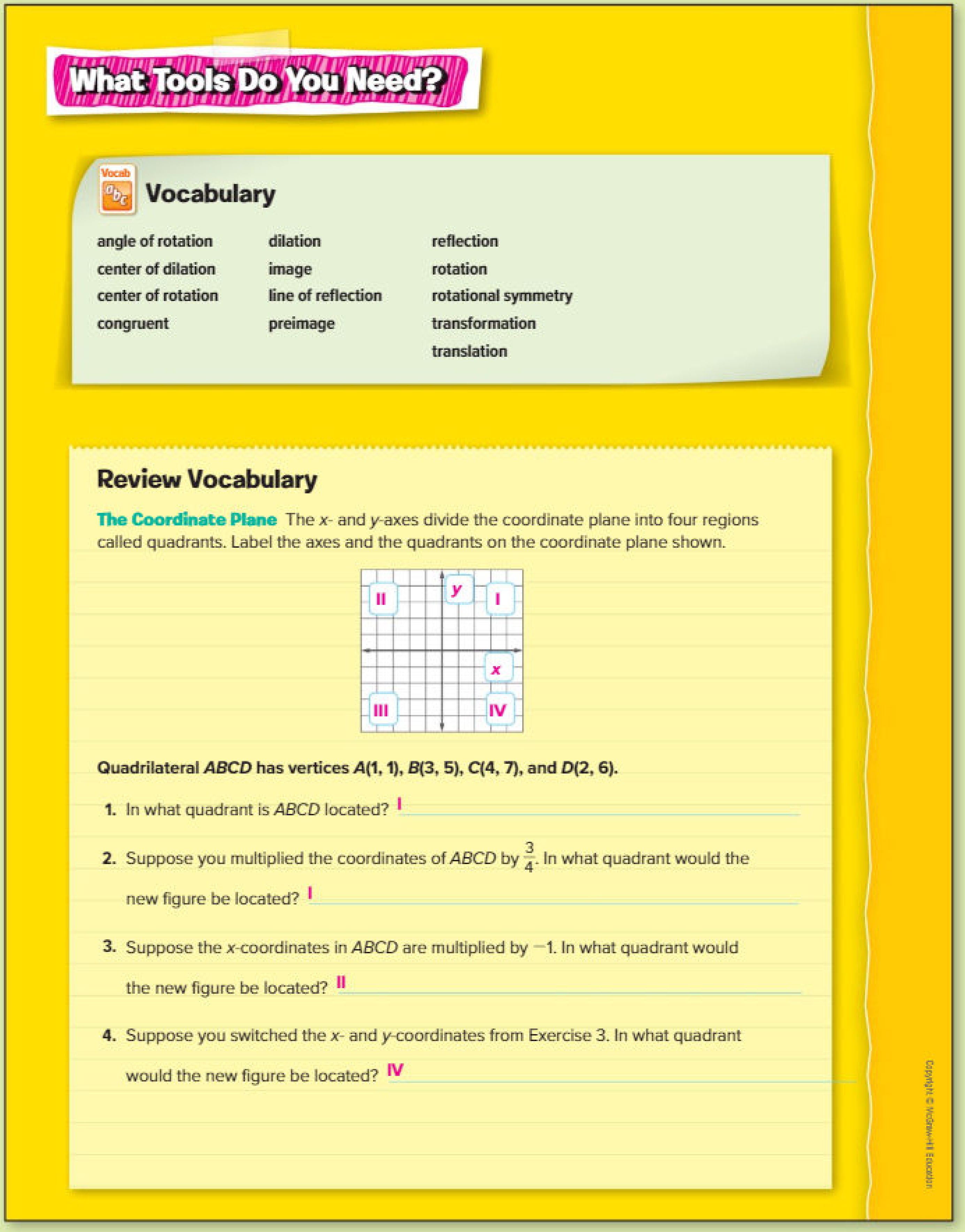
Ask:

- *What type of transformation is shown above?* translation

Review Vocabulary

LA Have students read the Review Vocabulary section.

Students should be able to graph ordered pairs in all four quadrants of the coordinate plane.
Remind students that the signs of the coordinates in an ordered pair indicate in which quadrant the point is located.
(+, +) Quadrant I
(−, +) Quadrant II
(−, −) Quadrant III
(+, −) Quadrant IV



What Tools Do You Need?

Vocabulary

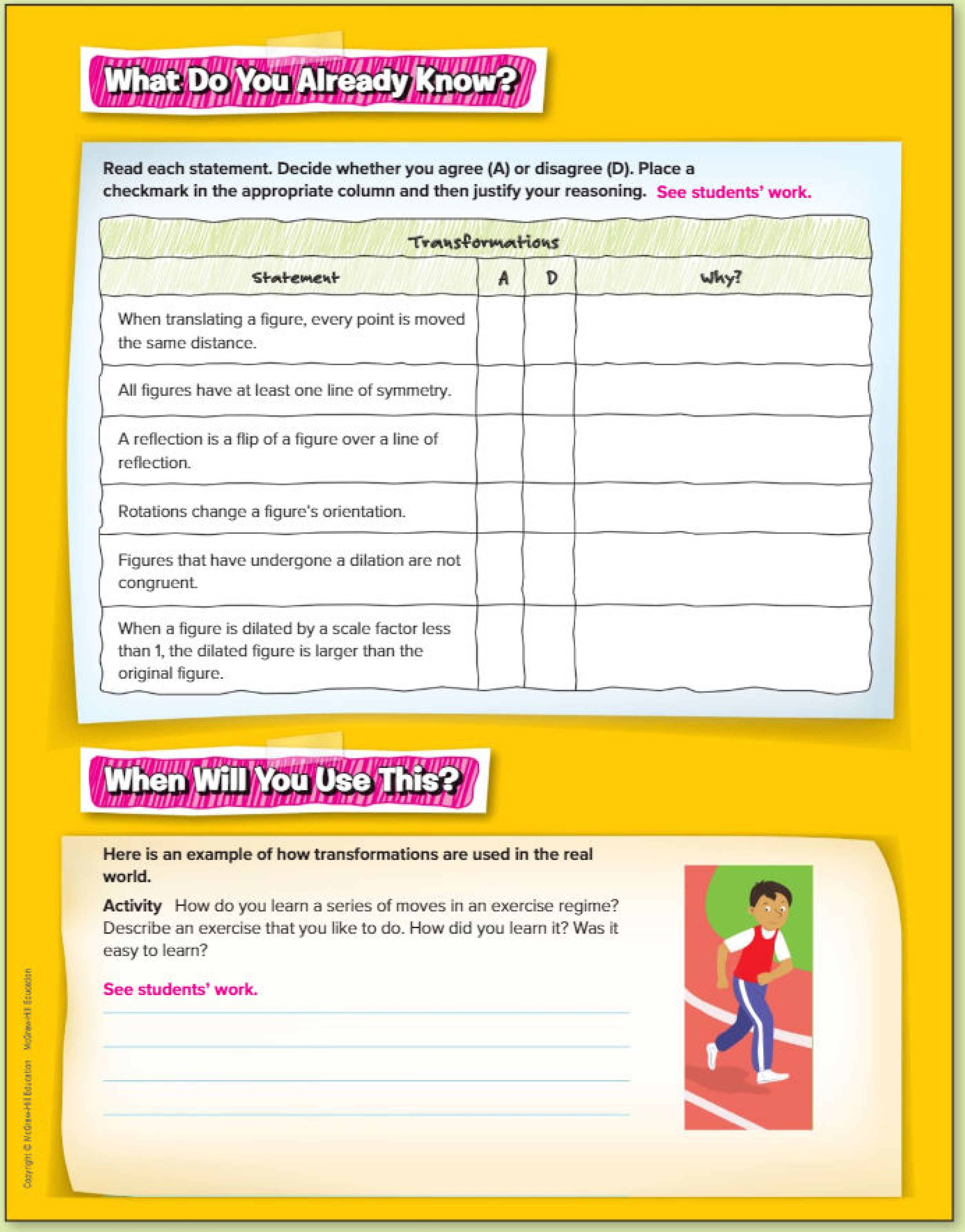
angle of rotation
center of dilation
center of rotation
congruent
dilation
image
line of reflection
preimage
reflection
rotation
rotational symmetry
transformation
translation

Review Vocabulary

The Coordinate Plane The *x*- and *y*-axes divide the coordinate plane into four regions called quadrants. Label the axes and the quadrants on the coordinate plane shown.

Quadrilateral *ABCD* has vertices *A*(1, 1), *B*(3, 5), *C*(4, 7), and *D*(2, 6).

1. In what quadrant is *ABCD* located? I
2. Suppose you multiplied the coordinates of *ABCD* by $\frac{3}{4}$. In what quadrant would the new figure be located? I
3. Suppose the *x*-coordinates in *ABCD* are multiplied by −1. In what quadrant would the new figure be located? II
4. Suppose you switched the *x*- and *y*-coordinates from Exercise 3. In what quadrant would the new figure be located? IV



What Do You Already Know?

Read each statement. Decide whether you agree (A) or disagree (D). Place a checkmark in the appropriate column and then justify your reasoning. **See students' work.**

Transformations

Statement	A	D	Why?
When translating a figure, every point is moved the same distance.			
All figures have at least one line of symmetry.			
A reflection is a flip of a figure over a line of reflection.			
Rotations change a figure's orientation.			
Figures that have undergone a dilation are not congruent.			
When a figure is dilated by a scale factor less than 1, the dilated figure is larger than the original figure.			

When Will You Use This?

Here is an example of how transformations are used in the real world.

Activity How do you learn a series of moves in an exercise regime? Describe an exercise that you like to do. How did you learn it? Was it easy to learn?

See students' work.

What Do You Already Know?

In this activity students assess their prior knowledge by determining whether they agree or disagree with each statement about concepts in this chapter.

- You may want to add a third option of "I don't know" for those students who do not have any prior knowledge of the content of the statement.
- After completing the chapter, have students return to this page and see if any of their responses would change now that they have finished the chapter.

When Will You Use This?

Activity

Students may not realize how much they already know about transformations. This activity shows the real-world connection with transformations.

Are You Ready?

Use this page to determine if students have skills that are needed for the chapter.

Quick Review

Students with strong math backgrounds may opt to go directly to the Quick Check.

REVIEW	
Example	**Skill**
1	Graph on the coordinate plane.
2	Add integers.

Quick Check

If students have difficulty with the exercises, present an additional example to clarify any misconceptions.

Exercises 1–3

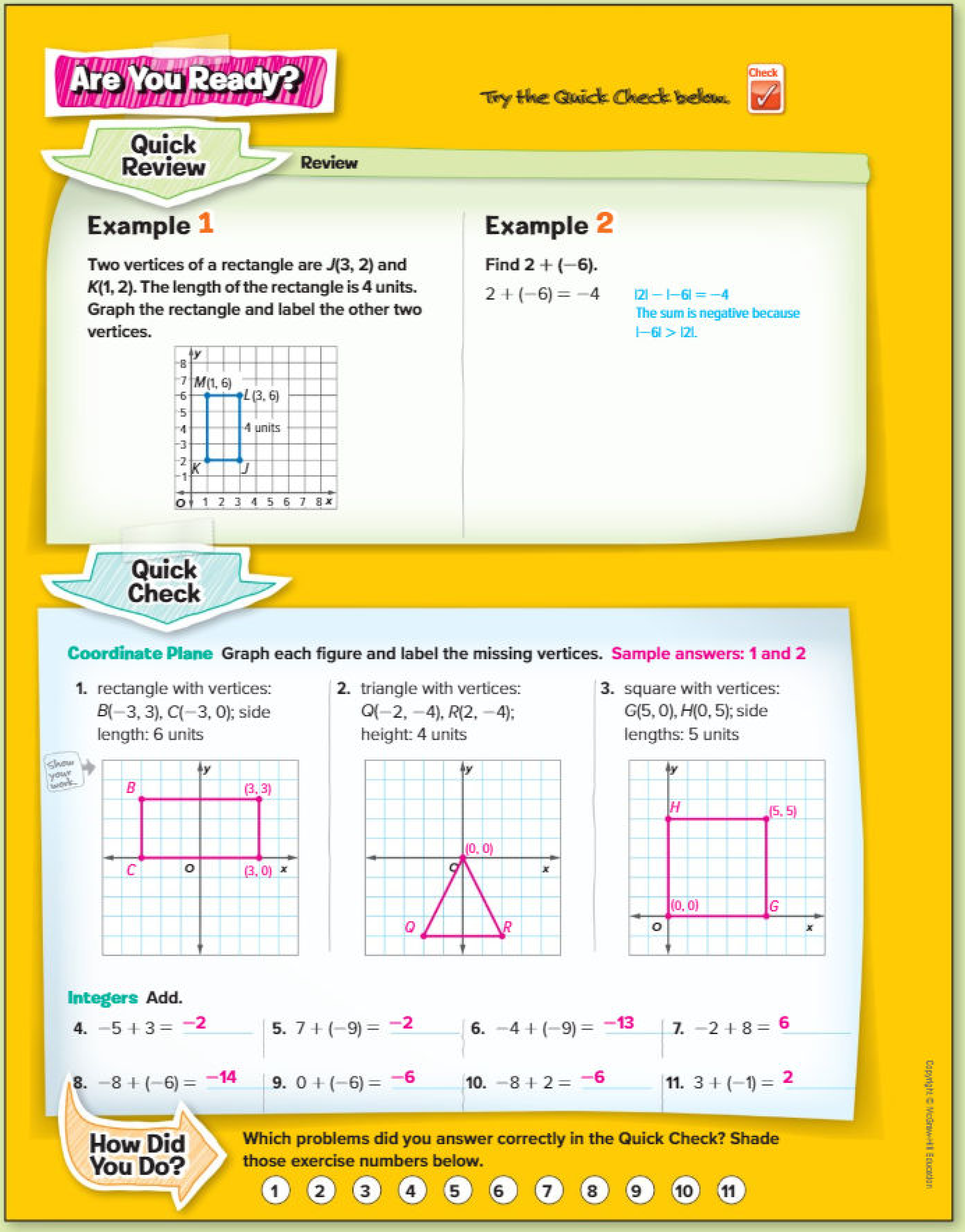
Two vertices of a rectangle are $A(-2, 4)$ and $B(3, 4)$. The height of the rectangle is 3 units. Graph the rectangle and label the other two vertices. **See Answer Appendix.**

Exercises 4–11

Find 12 + (−4). **8**

Track Your Progress

Prior to beginning this chapter, have your students rate their knowledge of the objectives it addresses. At the end of the chapter, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge of the key ideas has increased.



Are You Ready?

Try the Quick Check below.

Check

Quick Review

Review

Example 1

Two vertices of a rectangle are $J(3, 2)$ and $K(1, 2)$. The length of the rectangle is 4 units. Graph the rectangle and label the other two vertices.

Example 2

Find 2 + (−6).

2 + (−6) = −4

|2| − |−6| = −4
The sum is negative because |−6| > |2|.

Quick Check

Coordinate Plane **Graph each figure and label the missing vertices.** **Sample answers: 1 and 2**

1. rectangle with vertices: $B(-3, 3)$, $C(-3, 0)$; side length: 6 units

2. triangle with vertices: $Q(-2, -4)$, $R(2, -4)$; height: 4 units

3. square with vertices: $G(5, 0)$, $H(0, 5)$; side lengths: 5 units

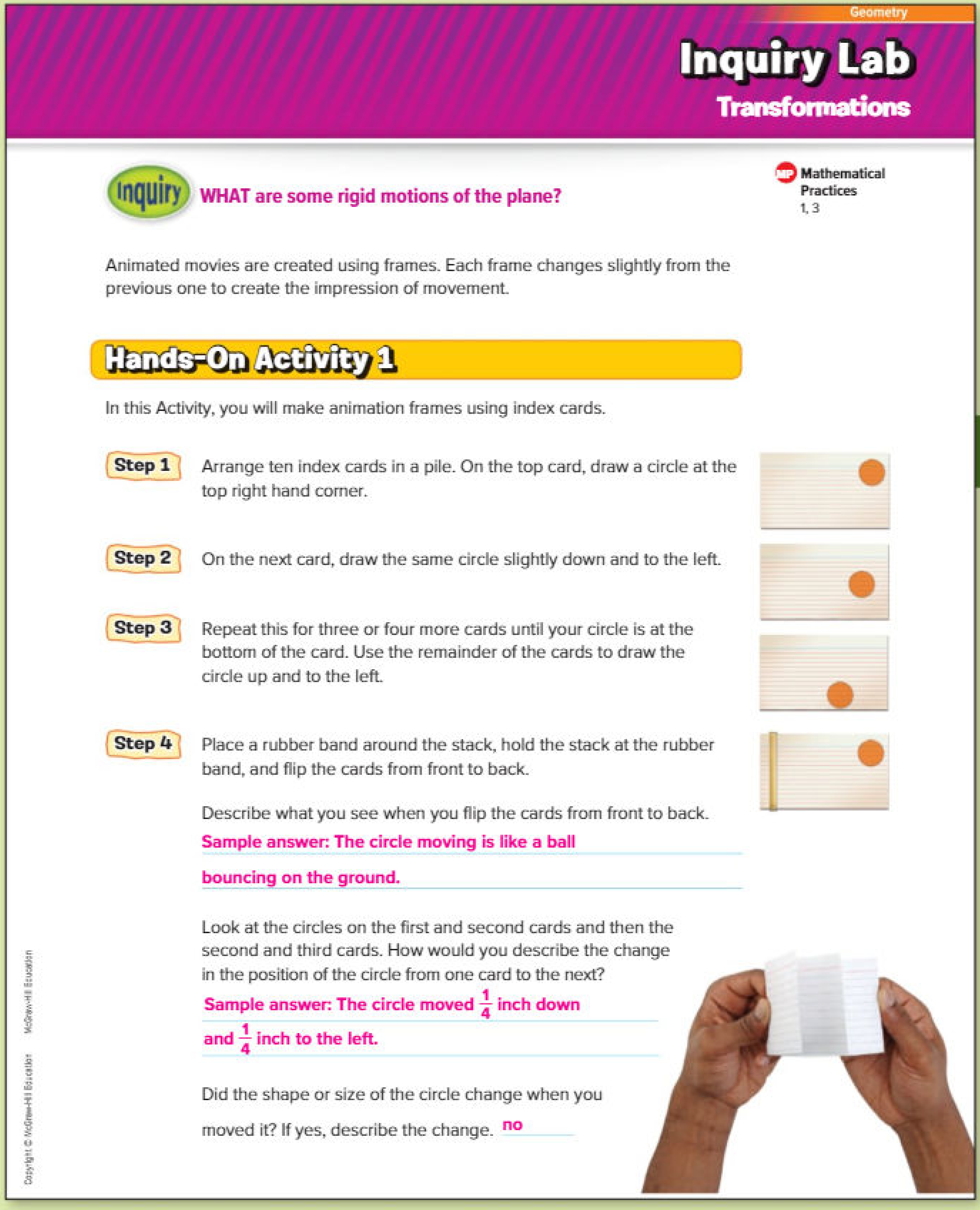
Integers **Add.**

4. −5 + 3 = **−2**
5. 7 + (−9) = **−2**
6. −4 + (−9) = **−13**
7. −2 + 8 = **6**
8. −8 + (−6) = **−14**
9. 0 + (−6) = **−6**
10. −8 + 2 = **−6**
11. 3 + (−1) = **2**

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨ ⑩ ⑪



Inquiry Lab

Transformations

Inquiry **WHAT are some rigid motions of the plane?**

MP Mathematical Practices
1, 3

Animated movies are created using frames. Each frame changes slightly from the previous one to create the impression of movement.

Hands-On Activity 1

In this Activity, you will make animation frames using index cards.

Step 1 Arrange ten index cards in a pile. On the top card, draw a circle at the top right hand corner.

Step 2 On the next card, draw the same circle slightly down and to the left.

Step 3 Repeat this for three or four more cards until your circle is at the bottom of the card. Use the remainder of the cards to draw the circle up and to the left.

Step 4 Place a rubber band around the stack, hold the stack at the rubber band, and flip the cards from front to back.

Describe what you see when you flip the cards from front to back.

Sample answer: The circle moving is like a ball bouncing on the ground.

Look at the circles on the first and second cards and then the second and third cards. How would you describe the change in the position of the circle from one card to the next?

Sample answer: The circle moved $\frac{1}{4}$ inch down and $\frac{1}{4}$ inch to the left.

Did the shape or size of the circle change when you moved it? If yes, describe the change. **no**

Focus narrowing the scope

Objective Identify and apply flips, slides, and turns.

Coherence connecting within and across grades

Now
Students will identify properties of reflections, translations, and rotations.

Next
Students will graph transformations on a coordinate plane.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 451.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1, 2, and 3 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activities 2 and 3.

Materials: index cards, circular chips, tracing paper

Hands-On Activity 1

AL LA **Pairs Discussion** Provide students with index cards and a circular chip or disk for them to trace. Have them work with a partner, following the directions in Steps 1–4 and answering the questions. Then have students compare drawings and responses to the questions with a partner. MP 1, 5

BL LA **Pairs Discussion** Students may wish to draw something more detailed than a circle. Have them work with a partner to draw a more complicated shape. Make sure that they can draw the shape in the same way on each index card, changing only the position of the shape. MP 1, 5

Hands-On Activity 2

AL LA **Paired Heads Together** Have students work with a partner to complete the activity. Each student is assigned a number. Provide students with tracing paper, a protractor, and a ruler. Have them follow the steps using the given tools. Then have them complete the questions. Upon completion of the activity, call on one numbered student to share their responses with the class. MP 1, 5, 6, 7

BL LA **Pairs Discussion** Provide students with tracing paper, a protractor, and a ruler. Have them draw an angle that is not a right angle and label the points as in the directions. Have them follow the steps using the given tools. Then have them work in pairs to check each other's work and answer the questions. MP 1, 5, 6, 7

Hands-On Activity 3

AL LA **Solo to Groups** Provide students with tracing paper. Have them work individually to follow the steps, making sure to rotate in the correct direction. Then have students work in small groups to discuss the questions. MP 1, 5, 6, 7

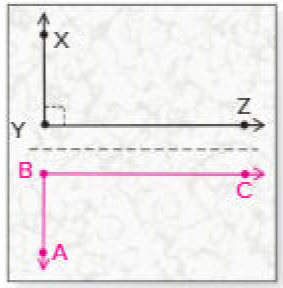
BL LA **Pairs Consult** Have students work in pairs to complete the activity and the questions. Then have them answer the following extension question. MP 1, 5, 6, 7

Ask:

- *The orientation of a figure is determined by the order in which the vertices are named. If you walk around the vertices W, Y, Z, and X in the original figure, you walk clockwise. What happens if you walk around the vertices W, Y, Z, and X after the turn? What does this tell you about the figure's orientation after the turn?* Sample answer: You still walk clockwise. So, orientation is preserved after the turn.

Hands-On Activity 2

Step 1 Draw right angle *XYZ* on a piece of tracing paper. Place a dashed line on the paper as shown.



Step 2 Fold the paper along the dashed line. Trace the angle onto the folded portion of the paper. Unfold and label the angle *ABC* so that *A* matches up with *X*, *B* matches up with *Y*, and *C* matches up with *Z*. Tape the paper to your book.

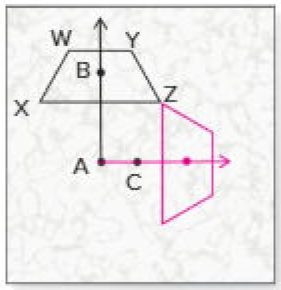
Use a protractor to find the measure of ∠*XYZ* and ∠*ABC*. Did the measure of the angle change after the flip? 90°; 90°; no

Use a centimeter ruler to measure the shortest distance from *X* and *A* to the dashed line. Repeat for *Y* and *B* and for *Z* and *C*. What do you notice?

See students' work; Sample answer: The distance from the original image to the dotted line is the same as the distance from the image to the dotted line.

Hands-On Activity 3

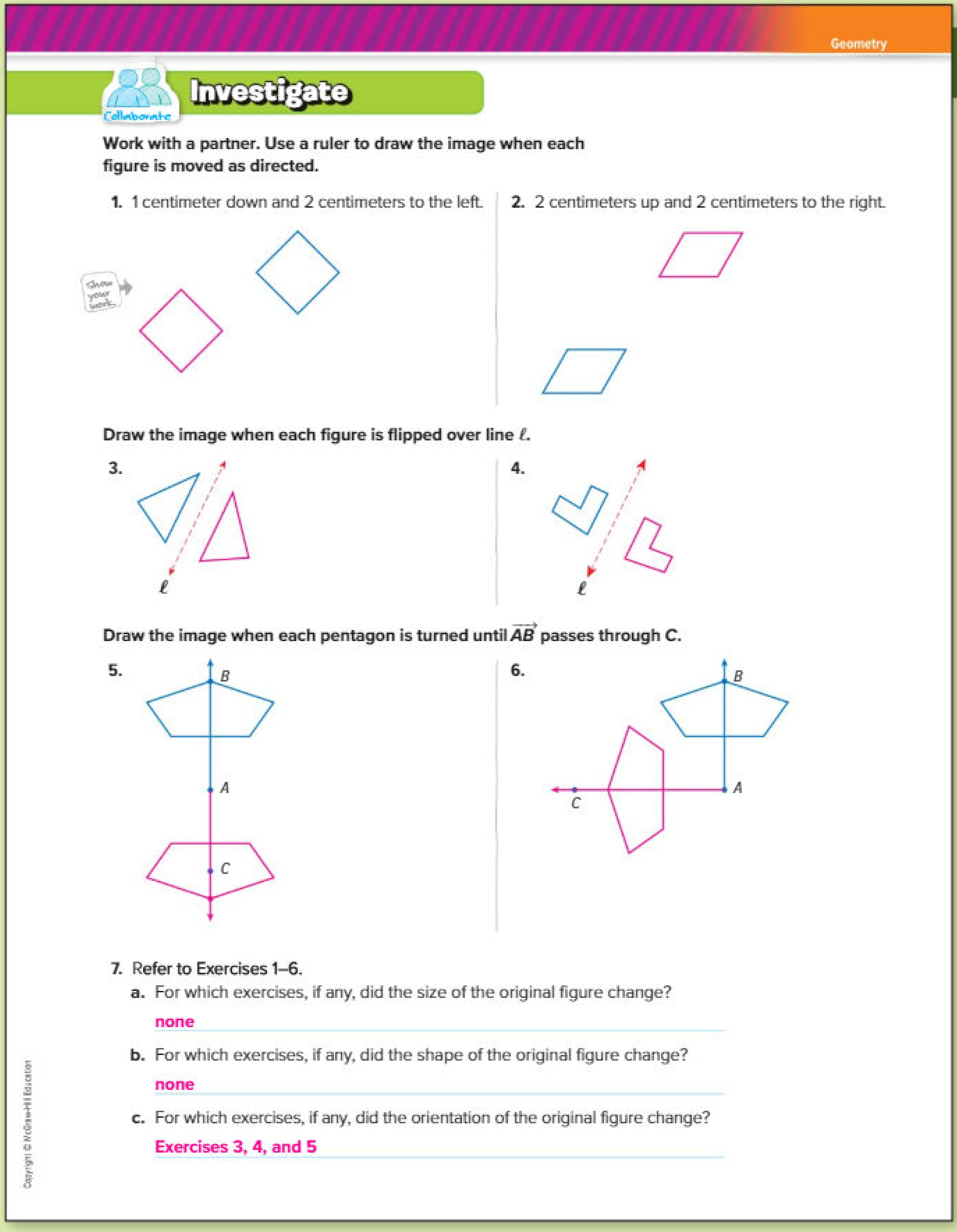
Step 1 Place a piece of tracing paper over the trapezoid shown. Copy the trapezoid. Draw points *A*, *B*, and *C*. Draw $\overrightarrow{AB}$.



Step 2 Place the eraser end of your pencil on *A*. Turn the tracing paper until $\overrightarrow{AB}$ passes through *C*. Tape the paper to your book.

Did the shape of the trapezoid change when you moved it? If yes, describe the change. no

Did the size of the trapezoid change when you moved it? If yes, describe the change. no



Geometry

Investigate

Work with a partner. Use a ruler to draw the image when each figure is moved as directed.

1. 1 centimeter down and 2 centimeters to the left.

2. 2 centimeters up and 2 centimeters to the right.

Draw the image when each figure is flipped over line ℓ.

3.

4.

Draw the image when each pentagon is turned until $\overrightarrow{AB}$ passes through *C*.

5.

6.

7. Refer to Exercises 1–6.
   - **a.** For which exercises, if any, did the size of the original figure change?
     none
   - **b.** For which exercises, if any, did the shape of the original figure change?
     none
   - **c.** For which exercises, if any, did the orientation of the original figure change?
     Exercises 3, 4, and 5

ENGAGE **EXPLORE** **EXPLAIN** **ELABORATE** **EVALUATE**

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–7	8–13	14, 15
Level 3			●
Level 2		●	
Level 1	●		

Investigate

AL **LA** **Group-Solo-Partner** Ask a volunteer to lead Exercise 1 in front of the class while students watch and listen carefully. Then have students complete Exercise 2 on their own. Continue this process for Exercises 3–6. Then have students discuss Exercise 7 with a partner. MP 1, 5, 6, 7

Ask:

- *For Exercises 3 and 4, what tool could we use in place of the dotted line to help draw the new image?* **a mirror**

BL **LA** **Roundrobin** Have students work in pairs. Have Student 1 complete Exercises 1, 3, and 5 while Student 2 completes Exercises 2, 4, and 6. Then have students exchange solutions and check each other's work. Have them discuss and respond to Exercise 7. MP 1, 5, 6, 7

Analyze and Reflect

AL **LA** **Pairs Discussion** Have students use tracing paper to complete Exercises 8 and 9 as they did in the activities. Then have them work with a partner to complete Exercises 10–12. MP 1, 5, 6, 7

BL **LA** **Category Sort** Have students draw a pair of figures in which either a slide, flip, or turn has taken place. Then have students place all of their drawings in the center of the room. Have students pop up out of their seats to randomly select one drawing and classify it as a slide, flip, or turn. Have them place the drawing into a pile labeled either *Slide, Flip,* or *Turn.* MP 1, 5, 6, 7

Create

BL **LA** **Pairs Present** Have pairs research online or look indoors and outdoors for real-world examples of slides, flips, and turns. Have them explain to the class what they discovered and how they determined the type of transformation. MP 1, 4, 5, 6, 7

Inquiry Students should be able to answer "WHAT are some rigid motions of the plane?" Check for student understanding and provide guidance, if needed.

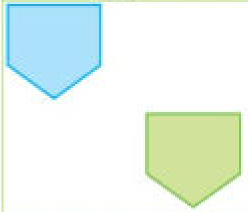
Analyze and Reflect

For each pair of figures, describe a movement or movements that will place the blue figure on top of the green figure. 8–9. Sample answers are given

8.

Figure	Movement(s)
	$\frac{1}{3}$ turn clockwise

9.

Figure	Movement(s)
	slide to the right and down

10. Refer to Activity 1 and Exercises 1 and 2. Circle the word that best describes the movement of the figures: **flip** [✓ **slide**] **turn**

11. Refer to Activity 2 and Exercises 3 and 4. Circle the word that best describes the movement of the figures: [✓ **flip**] **slide** **turn**

12. Refer to Activity 3 and Exercises 5 and 6. Measure one side of the original figures. Then measure that same side after the turn. Did the length of the side change after you turned it? If yes, describe the change.

no

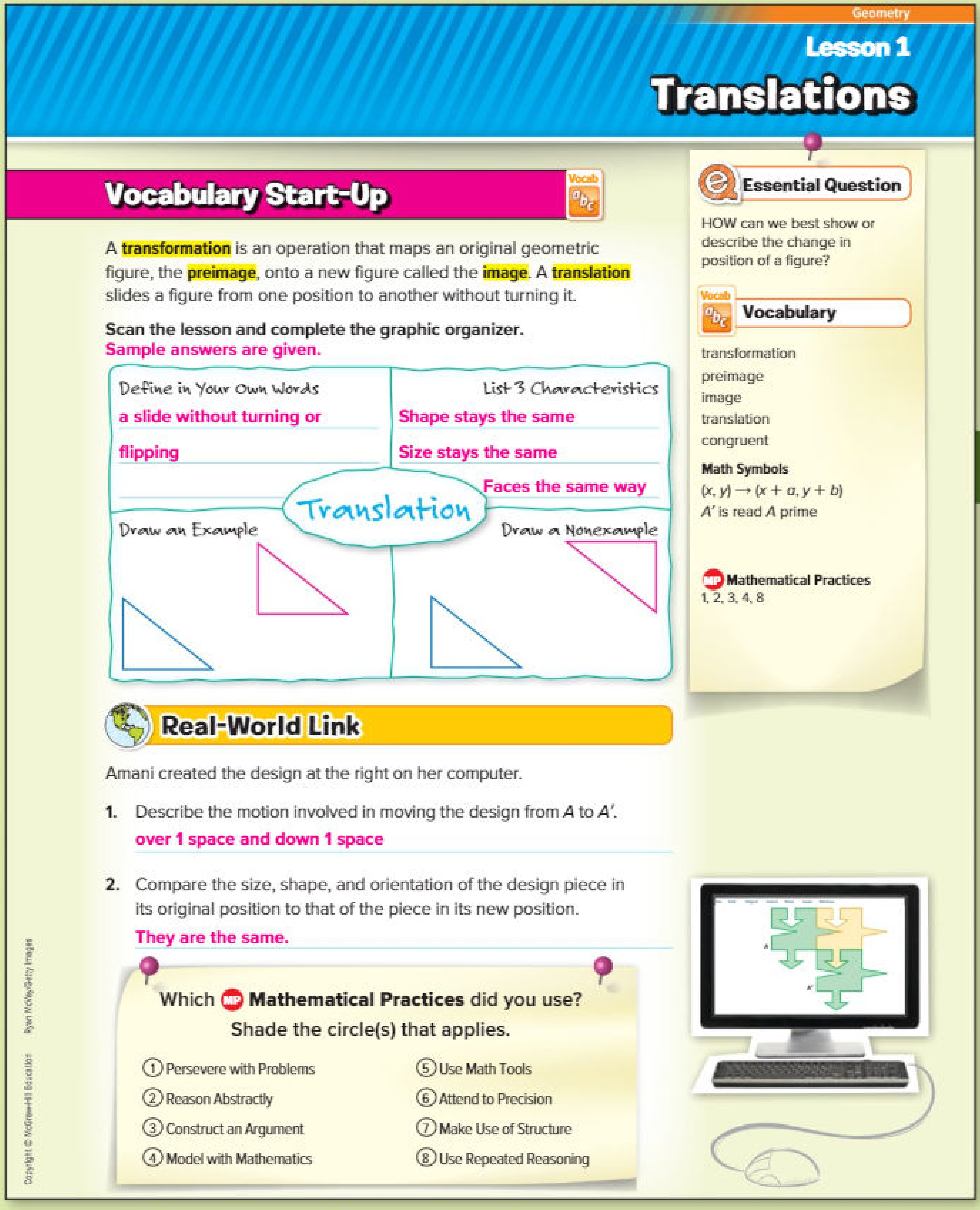
13. MP **Justify Conclusions** In Activity 3, $\overline{WY}$ and $\overline{XZ}$ are parallel. Were the segments still parallel after the turn? Would they still be parallel after a slide? flip? Explain.

yes; yes; yes; Sample answer: Since the size of the figure does not change in any of the movements, the distance between the two lines is the same, so the lines remain parallel.

Create

14. MP **Reason Inductively** Slides, flips, and turns are called *rigid motions of the plane*. Based on the Activities, describe two characteristics of a rigid motion of the plane. Sample answer: The shape and the size of a figure do not change in a rigid motion of the plane.

15. Inquiry WHAT are some rigid motions of the plane? Slides, flips, and turns are some rigid motions of the plane.



Geometry

Lesson 1

Translations

Vocabulary Start-Up

A **transformation** is an operation that maps an original geometric figure, the **preimage**, onto a new figure called the **image**. A **translation** slides a figure from one position to another without turning it.

Scan the lesson and complete the graphic organizer.
Sample answers are given.

Translation

Define in Your Own Words
a slide without turning or flipping

List 3 Characteristics
Shape stays the same
Size stays the same
Faces the same way

Draw an Example

Draw a Nonexample

Essential Question

HOW can we best show or describe the change in position of a figure?

Vocabulary

transformation
preimage
image
translation
congruent

Math Symbols

$(x, y) \rightarrow (x + a, y + b)$

A' is read A prime

Mathematical Practices
1, 2, 3, 4, 8

Real-World Link

Amani created the design at the right on her computer.

1. Describe the motion involved in moving the design from A to A'.
   over 1 space and down 1 space
2. Compare the size, shape, and orientation of the design piece in its original position to that of the piece in its new position.
   They are the same.

Which **Mathematical Practices** did you use?
Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Ryan McVay/Getty Images

Focus narrowing the scope

Objective Graph translations on the coordinate plane.

Coherence connecting within and across grades

Previous
Students identified the properties of translations.

Now
Students will graph translations on the coordinate plane.

Next
Students will graph reflections on the coordinate plane.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 457.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Think-Pair-Share** Give students about one minute to think through their responses to the graphic organizer on the student page. Then have them share their responses with a partner. MP 1, 5, 6

Alternate Strategy

AL LA Have students describe what a *translation* is in everyday life, such as the translation of a word from Spanish to English. Then have them explain the meaning of the term *translation* in mathematics. MP 1, 5, 6

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Translate a figure in the coordinate plane.

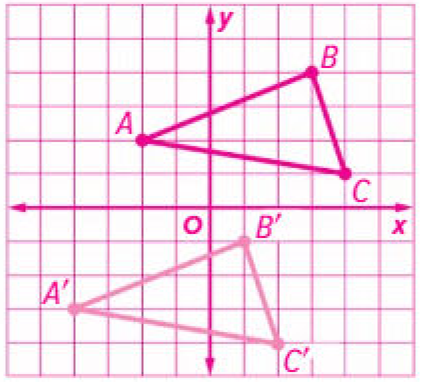
AL
- *How do we graph coordinate J?* Start at the origin and move 3 units left and then 4 units up. *coordinate K?* Start at the origin and move 1 unit right and 3 units up. *coordinate L?* Start at the origin, move 4 units left, then 1 unit up.

OL
- *If point J(−3, 4) is moved two units right and 5 units down, what are the coordinates of point J′?* (−1, −1)
- *If point K(1, 3) is moved two units right and 5 units down, what are the coordinates of point K′?* (3, −2)
- *If point L(−4, 1) is moved two units right and 5 units down, what are the coordinates of point L′?* (−2, −4)

BL
- *Was the orientation of the image changed?* no
- *Are the two images congruent?* yes

Need Another Example?

Graph △*ABC* with vertices *A*(−2, 2), *B*(3, 4), and *C*(4, 1). Then graph the image of △*ABC* after a translation 2 units left and 5 units down. Write the coordinates of its vertices. *A*′(−4, −3), *B*′(1, −1), *C*′(2, −4)



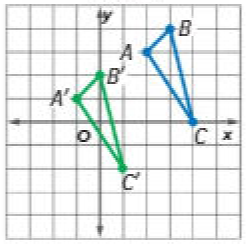
Work Zone

Key Concept **Translations in the Coordinate Plane**

Words When a figure is translated, the *x*-coordinate of the preimage changes by the value of the horizontal translation *a*. The *y*-coordinate of the preimage changes by the vertical translation *b*.

Symbols $(x, y) \longrightarrow (x + a, y + b)$

Model



Prime Symbols

Use prime symbols for vertices in a transformed image.

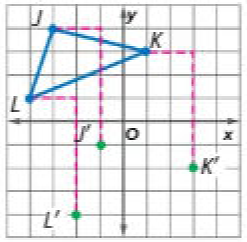
A → A′
B → B′
B → B′
A′ is read A prime.

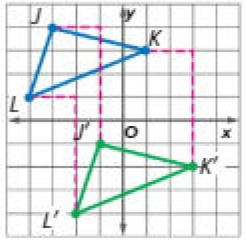
When translating a figure, every point of the preimage is moved the same distance and in the same direction. The image and the preimage are congruent. **Congruent** figures have the same shape and same size. So, line segments in the preimage have the same length as line segments in the image. Angles in the preimage have the same measure as angles in the image.

Example

1. **Graph △*JKL* with vertices *J*(−3, 4), *K*(1, 3), and *L*(−4, 1). Then graph the image of △*JKL* after a translation 2 units right and 5 units down. Write the coordinates of its vertices.**

Move each vertex of the triangle 2 units right and 5 units down. Use prime symbols for the vertices of the image.





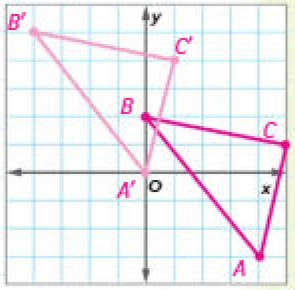
From the graph, the coordinates of the vertices of the image are *J*′(−1, −1), *K*′(3, −2), and *L*′(−2, −4).

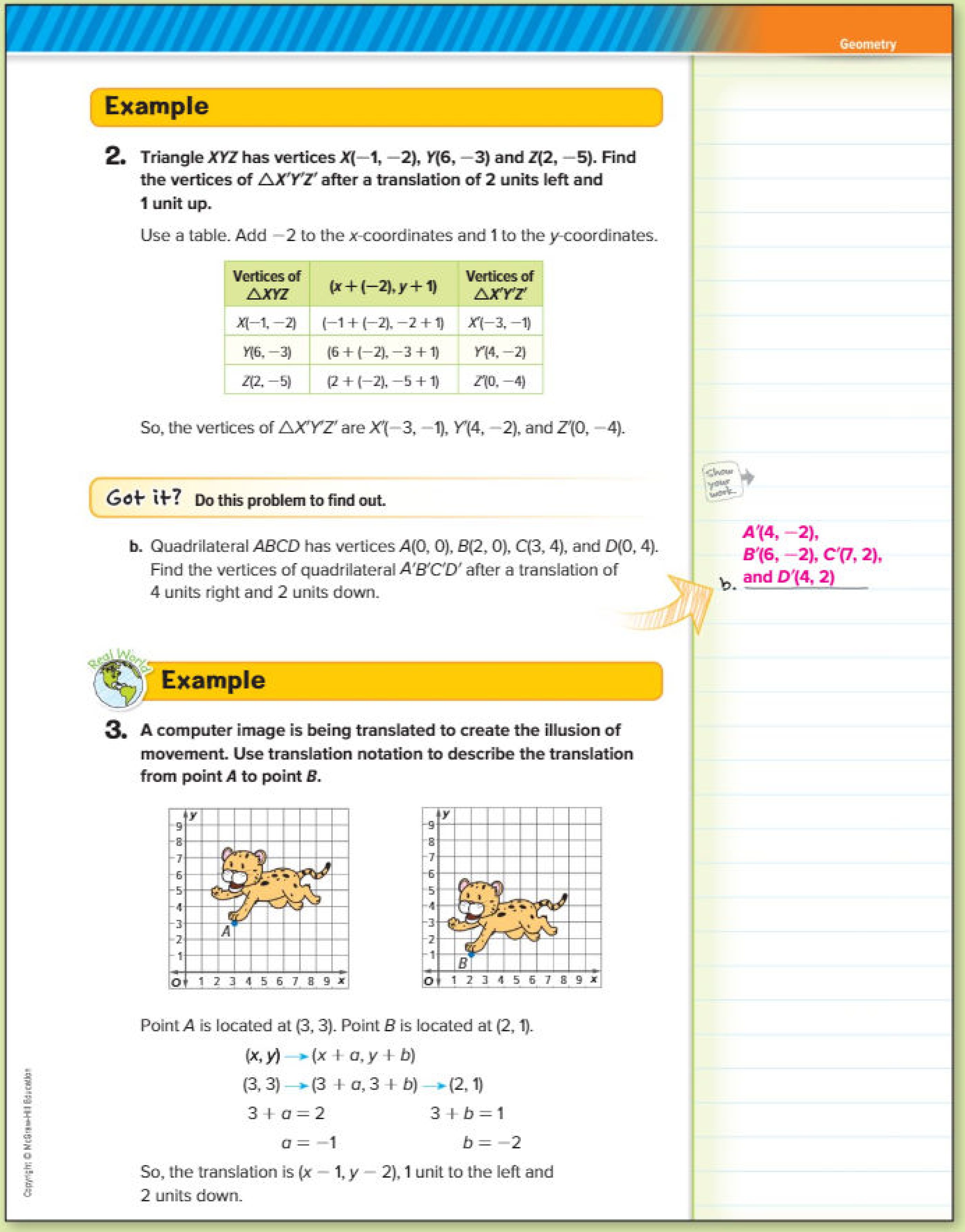
Got it? **Do this problem to find out.**

a. Graph △*ABC* with vertices *A*(4, −3), *B*(0, 2) and *C*(5, 1). Then graph its image after a translation of 4 units left and 3 units up. Write the coordinates of the image.

Show your work.

a. A′(0, 0), B′(−4, 5) and C′(1, 4)





Example

2. **Triangle *XYZ* has vertices *X*(−1, −2), *Y*(6, −3) and *Z*(2, −5). Find the vertices of △*X'Y'Z'* after a translation of 2 units left and 1 unit up.**

Use a table. Add −2 to the *x*-coordinates and 1 to the *y*-coordinates.

Vertices of △*XYZ*	$(x + (-2), y + 1)$	Vertices of △*X'Y'Z'*
$X(-1, -2)$	$(-1 + (-2), -2 + 1)$	$X'(-3, -1)$
$Y(6, -3)$	$(6 + (-2), -3 + 1)$	$Y'(4, -2)$
$Z(2, -5)$	$(2 + (-2), -5 + 1)$	$Z'(0, -4)$

So, the vertices of △*X'Y'Z'* are *X'*(−3, −1), *Y'*(4, −2), and *Z'*(0, −4).

Got it? **Do this problem to find out.**

b. Quadrilateral *ABCD* has vertices *A*(0, 0), *B*(2, 0), *C*(3, 4), and *D*(0, 4). Find the vertices of quadrilateral *A'B'C'D'* after a translation of 4 units right and 2 units down.

Show your work.

b. *A'*(4, −2), *B'*(6, −2), *C'*(7, 2), and *D'*(4, 2)

Real World

Example

3. **A computer image is being translated to create the illusion of movement. Use translation notation to describe the translation from point *A* to point *B*.**

Point *A* is located at (3, 3). Point *B* is located at (2, 1).

$$(x, y) \rightarrow (x + a, y + b)$$

$$(3, 3) \rightarrow (3 + a, 3 + b) \rightarrow (2, 1)$$

$$3 + a = 2 \qquad 3 + b = 1$$

$$a = -1 \qquad b = -2$$

So, the translation is (*x* − 1, *y* − 2), 1 unit to the left and 2 units down.

Examples

2. Find the coordinates after a translation.

- AL • *What do you need to do to the x-coordinates of the vertices of triangle XYZ to determine the x-coordinates of the vertices of triangle X'Y'Z'? the y-coordinates?* Subtract 2 from *x*-coordinates. Add 1 to *y*-coordinates.
- OL • *Why do you subtract 2 from the x-coordinates?* because translation is 2 units to the left
  - • *Why do you add 1 to the y-coordinates?* because the translation is 1 unit up
- BL • *If the translation was 1 unit down instead of 1 unit up, how would it affect the operation you perform on the y-coordinates?* We would subtract 1 instead of adding 1.

Need Another Example?

Rectangle *ABCD* has vertices *A*(−3, 2), *B*(2, 2), *C*(2, −3), and *D*(−3, −3). Find the vertices of rectangle *A'B'C'D'* after a translation of 4 units right and 2 units down. ***A'*(1, 0), *B'*(6, 0), *C'*(6, −5), *D'*(1, −5)**

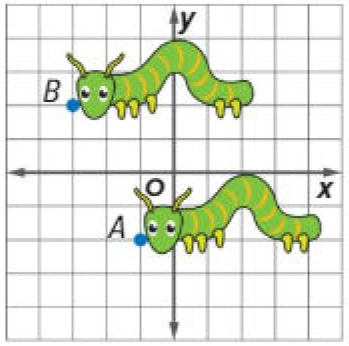
3. Describe translations.

- AL • *What are the coordinates of point A?* (3, 3) *point B?* (2, 1)
- OL • *What do you need to do to the x-coordinate of point A to get the x-coordinate of point B?* Subtract 1. *the y-coordinate?* Subtract 2.
- BL • *How can you check to make sure your solution is correct?* Sample answer: Check several other points on the picture to see if the movement is the same for each.

Need Another Example?

The character below was translated from point *A* to point *B*. Use translation notation to describe the translation.

(*x* − 2, *y* + 4)



Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL **LA** **Paired Heads Together** Have a student volunteer to lead a class discussion on how to translate the figure in Exercise 1. Then have students complete Exercise 2 with a partner, ensuring that each partner understands how to translate a figure. Have a student-volunteer draw the image on the board. If students are ready, have them complete Exercises 3–5 on their own. If not, have them work with a partner, while you walk around the room and monitor their progress. **MP 1, 5, 6, 7**

BL **LA** **Trade-a-Problem** Have students draw an image on a coordinate grid. Then have them write instructions in coordinate form, such as translation notation, for how to translate the image. Have students trade graph paper and translate each other's images using the translation notations only. **MP 1, 5, 6, 7**

Show your work.

c. $(x + 1, y - 4)$

Got it? **Do this problem to find out.**

c. Refer to the figure in Example 3. If point *A* was at (1, 5), use translation notation to describe the translation from point *A* to point *B*.

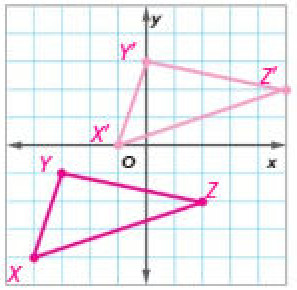
Guided Practice

Check

Graph $\triangle XYZ$ with vertices $X(-4, -4)$, $Y(-3, -1)$, and $Z(2, -2)$. Then graph the image of $\triangle XYZ$ after each translation, and write the coordinates of its vertices. (Example 1)

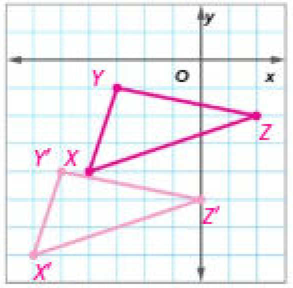
1. 3 units right and 4 units up

$X'(-1, 0)$, $Y'(0, 3)$, $Z'(5, 2)$



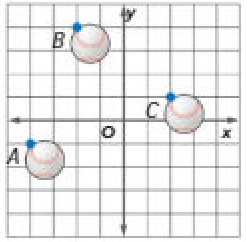
2. 2 units left and 3 units down

$X'(-6, -7)$, $Y'(-5, -4)$, $Z'(0, -5)$



3. The baseball at the right was filmed using stop-motion animation so it appears to be thrown in the air. Use translation notation to describe the translation from point *A* to point *B*. (Example 3)

$(x + 2, y + 5)$



4. Quadrilateral *DEFG* has vertices at $D(1, 0)$, $E(-2, -2)$, $F(2, 4)$, and $G(6, -3)$. Find the vertices of $D'E'F'G'$ after a translation of 4 units right and 5 units down. (Example 2)

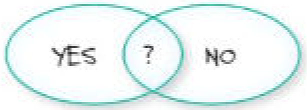
Sample answer: $D'(5, -5)$, $E'(2, -7)$, $F'(6, -1)$, and $G'(10, -8)$

5. **Building on the Essential Question** How are figures translated on the coordinate plane?

Sample answer: They are slid up or down and right or left.

Rate Yourself!

Are you ready to move on?
Shade the section that applies.



FOLDABLES Time to update your Foldable!

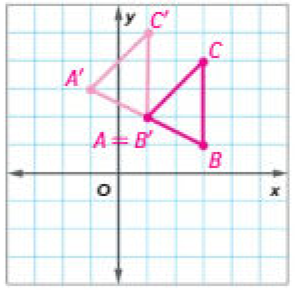
Name ______________________ My Homework ______________________

Independent Practice

Graph each figure with the given vertices. Then graph the image of the figure after the indicated translation, and write the coordinates of its vertices. (Example 1)

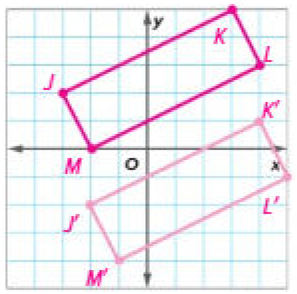
1. △*ABC* with vertices *A*(1, 2), *B*(3, 1), and *C*(3, 4) translated 2 units left and 1 unit up

   ***A*′(−1, 3), *B*′(1, 2), *C*′(1, 5)**



2. rectangle *JKLM* with vertices *J*(−3, 2), *K*(3, 5), *L*(4, 3), and *M*(−2, 0) translated 1 unit right and 4 units down

   ***J*′(−2, −2), *K*′(4, 1), *L*′(5, −1), *M*′(−1, −4)**

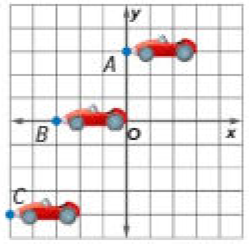


Triangle *PQR* has vertices *P*(0, 0), *Q*(5, −2), and *R*(−3, 6). Find the vertices of *P*′*Q*′*R*′ after each translation. (Example 2)

3. 6 units right and 5 units up **$P'(6, 5), Q'(11, 3), R'(3, 11)$**

4. 8 units left and 1 unit down **$P'(-8, -1), Q'(-3, -3), R'(-11, 5)$**

Use the image of the race car at the right. (Example 3)



5. Use translation notation to describe the translation from point *A* to point *B*. **$(x - 3, y - 3)$**

6. Use translation notation to describe the translation from point *B* to point *C*. **$(x - 2, y - 4)$**

7. Quadrilateral *KLMN* has vertices *K*(−2, −2), *L*(1, 1), *M*(0, 4), and *N*(−3, 5). It is first translated by $(x + 2, y - 1)$ and then translated by $(x - 3, y + 4)$. When a figure is translated twice, a double prime symbol is used. Find the coordinates of quadrilateral *K*″*L*″*M*″*N*″ after both translations.

   ***K*″(−3, 1), *L*″(0, 4), *M*″(−1, 7), *N*″(−4, 8)**

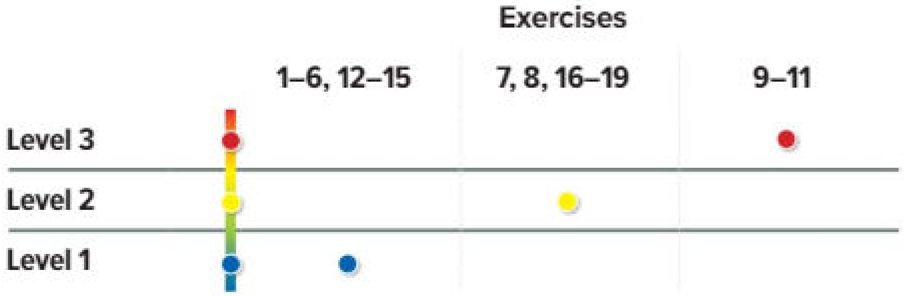
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 11, 18, 19
OL	On Level	1–5 odd, 7–9, 11, 18, 19
BL	Beyond Level	7–11, 18, 19

Watch Out!

Common Error Suggest to students that they graph the original figures in one color and the translated images in another color to avoid confusion.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	10
3 Construct viable arguments and critique the reasoning of others.	9, 11
4 Model with mathematics.	8
8 Look for and express regularity in repeated reasoning.	17

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

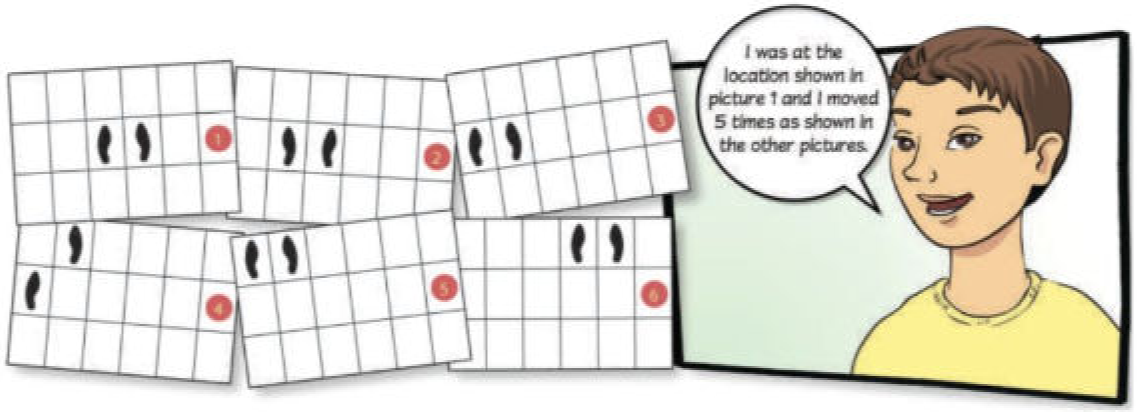
Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students answer the following question: If point $P(-3, 2)$ is translated 3 units right and 2 units down, what are the coordinates of P'? **(0, 0)**

8. **Model with Mathematics** Refer to the graphic novel frame below. List the five steps the girl took and identify any transformations used in the movements. **Sample answer: right crosses over left; left crosses behind right; right forward one step; left forward one step; both hop three to the right; Steps and hops are translations.**



H.O.T. Problems Higher Order Thinking

9. **Reason Inductively** A figure is translated by $(x - 5, y + 7)$, then by $(x + 5, y - 7)$. Without graphing, what is the final position of the figure? Explain your reasoning to a classmate. **the same as the original position of the figure; Sample answer: Since -5 and 5 are opposites, and -7 and 7 are opposites, the translations cancel each other out.**

10. **Persevere with Problems** What are the coordinates of the point (x, y) after being translated m units left and n units up? **$(x - m, y + n)$**

11. **Reason Inductively** Determine whether each of the following statements is *always*, *sometimes*, or *never* true. Justify your reasoning.
    a. A translation preserves orientation. **always; Sample answer: Each point moves the same distance and in the same direction.**
    b. A preimage and its translated image are the same size, but not the same shape. **never; Sample answer: A preimage and image in a translation will always have the same size and shape.**

Name ______________________ My Homework ______________________

Extra Practice

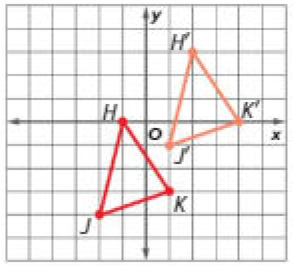
Graph each figure with the given vertices. Then graph the image of the figure after the indicated translation, and write the coordinates of its vertices.

12. △*HJK* with vertices *H*(−1, 0), *J*(−2, −4) and *K*(1, −3) translated 3 units right and 3 units up

H′(2, 3), J′(1, −1), K′(4, 0)

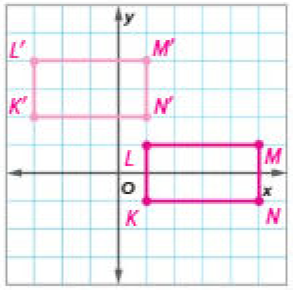
Graph each point and connect them to form a triangle. Then move each point 3 units to the right and then 3 units up. Connect them to form △H′J′K′.

Homework Help



13. Rectangle *KLMN* with vertices *K*(1, −1), *L*(1, 1), *M*(5, 1), and *N*(5, −1) translated 4 units left and 3 units up

***K*′(−3, 2), *L*′(−3, 4), *M*′(1, 4), *N*′(1, 2)**



Quadrilateral *ABCD* has vertices *A*(−5, −1), *B*(−3, 0), *C*(2, −2), and *D*(0, −6). Find the vertices of *A*′*B*′*C*′*D*′ after each translation.

14. 4 units up ***A*′(−5, 3), *B*′(−3, 4), *C*′(2, 2), *D*′(0, −2)**

15. 2 units right and 2 units down ***A*′(−3, −3), *B*′(−1, −2), *C*′(4, −4), *D*′(2, −8)**

16. Ismail is in Colorado exploring part of the Denver Zoo as shown. He starts at the Felines exhibit and travels 3 units to the right and 5 units up. At which exhibit is Ismail located? If the Felines exhibit is located at (3, 1), what are the coordinates of Ismail's new location?

Hoofed Animals; (6, 6)

17. MP **Identify Repeated Reasoning** A diagram of a DNA double helix is shown below. Look for a pattern. On the diagram indicate where this pattern repeats or is translated. Find how many translations of the original pattern are shown in the diagram. **1**

Power Up! Test Practice

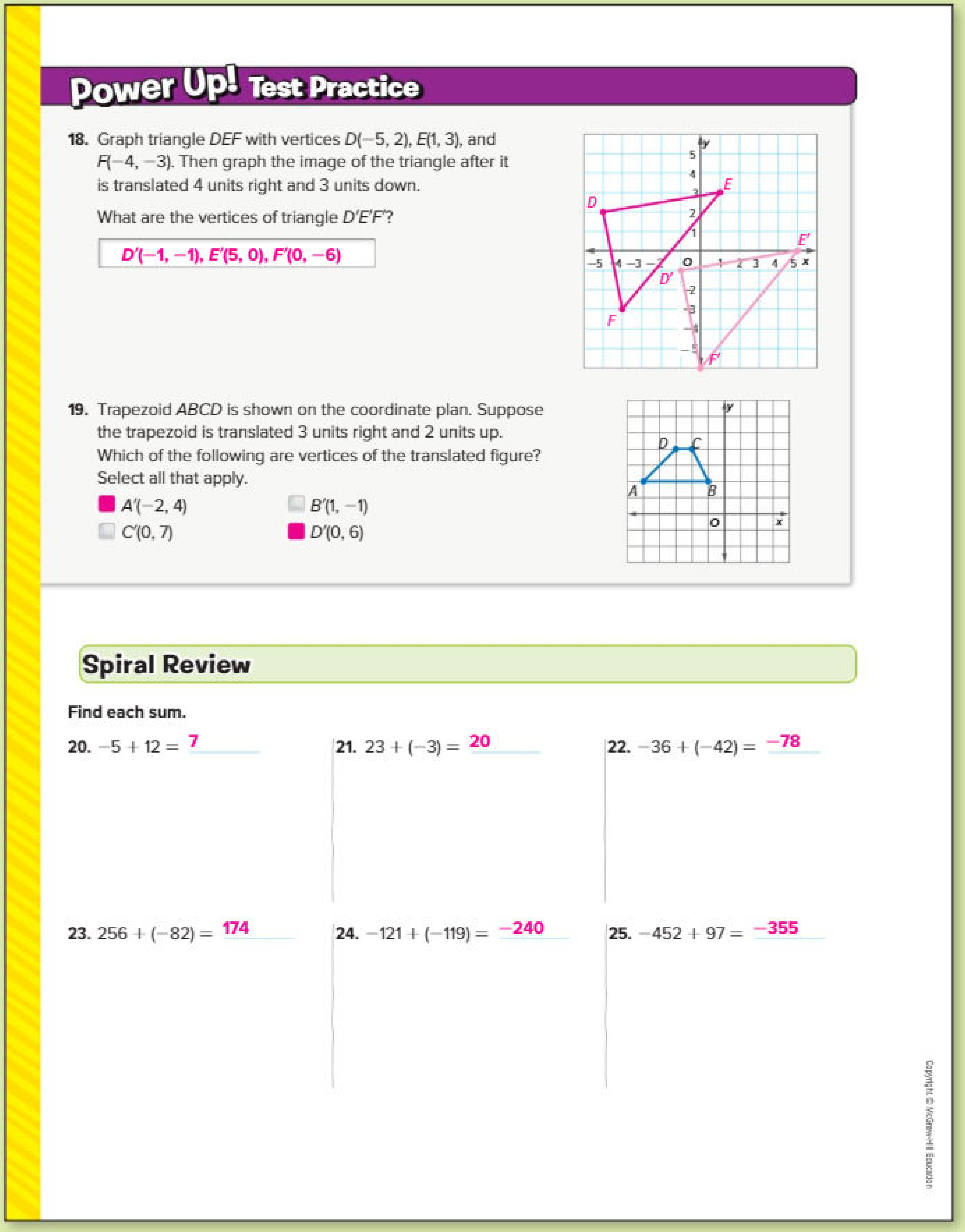
Exercises 18 and 19 prepare students for more rigorous thinking needed for assessment.

18.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK3
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly graph the figures and list the vertices.
1 point	Students correctly graph the figures but fail to list the vertices OR students correctly graph one figure and list the vertices.

19.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.



Power Up! Test Practice

18. Graph triangle *DEF* with vertices *D*(−5, 2), *E*(1, 3), and *F*(−4, −3). Then graph the image of the triangle after it is translated 4 units right and 3 units down.

What are the vertices of triangle *D′E′F′*?

D′(−1, −1), *E′*(5, 0), *F′*(0, −6)

19. Trapezoid *ABCD* is shown on the coordinate plan. Suppose the trapezoid is translated 3 units right and 2 units up. Which of the following are vertices of the translated figure? Select all that apply.

☑ *A′*(−2, 4) ☐ *B′*(1, −1)
☐ *C′*(0, 7) ☑ *D′*(0, 6)

Spiral Review

Find each sum.

20. −5 + 12 = 7

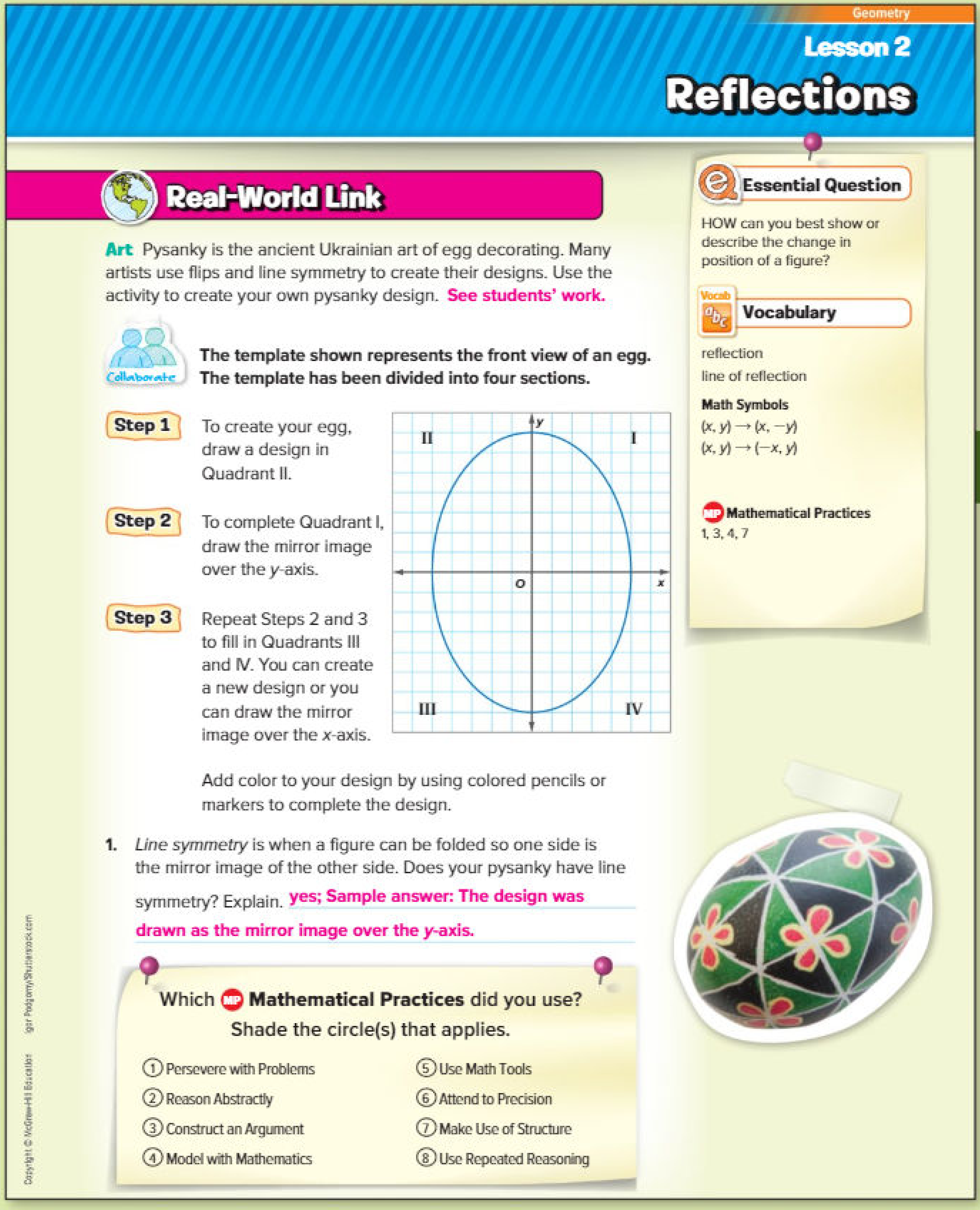
21. 23 + (−3) = 20

22. −36 + (−42) = −78

23. 256 + (−82) = 174

24. −121 + (−119) = −240

25. −452 + 97 = −355



Lesson 2

Reflections

Real-World Link

Art Pysanky is the ancient Ukrainian art of egg decorating. Many artists use flips and line symmetry to create their designs. Use the activity to create your own pysanky design. **See students' work.**

Collaborate

The template shown represents the front view of an egg. The template has been divided into four sections.

Step 1 To create your egg, draw a design in Quadrant II.

Step 2 To complete Quadrant I, draw the mirror image over the *y*-axis.

Step 3 Repeat Steps 2 and 3 to fill in Quadrants III and IV. You can create a new design or you can draw the mirror image over the *x*-axis.

Add color to your design by using colored pencils or markers to complete the design.

1. *Line symmetry* is when a figure can be folded so one side is the mirror image of the other side. Does your pysanky have line symmetry? Explain. **yes; Sample answer: The design was drawn as the mirror image over the *y*-axis.**

Which MP **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Essential Question

HOW can you best show or describe the change in position of a figure?

Vocabulary

reflection
line of reflection

Math Symbols

$(x, y) \rightarrow (x, -y)$
$(x, y) \rightarrow (-x, y)$

MP **Mathematical Practices**
1, 3, 4, 7

Focus narrowing the scope

Objective Graph reflections on the coordinate plane.

Coherence connecting within and across grades

Previous
Students identified the properties of reflections.

Now
Students will graph reflections on the coordinate plane.

Next
Students will graph rotations on the coordinate plane.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 465.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Think-Pair-Share** Have students work with a partner to complete the activity. Give students about one minute to think through how their design would look in Quadrants I, III, and IV. Then have them discuss their responses with a partner. Call on several pairs of students to share their drawings with the class. MP 1, 5, 6, 7

Alternate Strategy

AL Have students fold their paper in half to verify that their design has line symmetry.

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Reflect a figure over the *x*-axis.

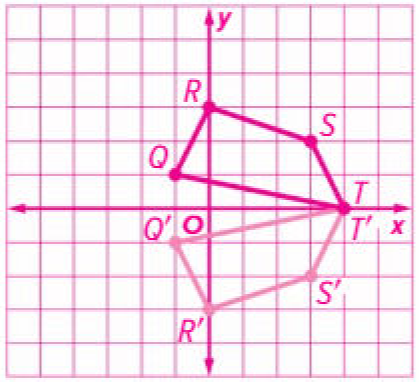
AL
- *What line in the coordinate plane are we using as the mirror?* *x*-axis
- *When reflecting a point over the x-axis, which coordinate remains the same?* *x*-coordinate *Which coordinate changes?* *y*-coordinate

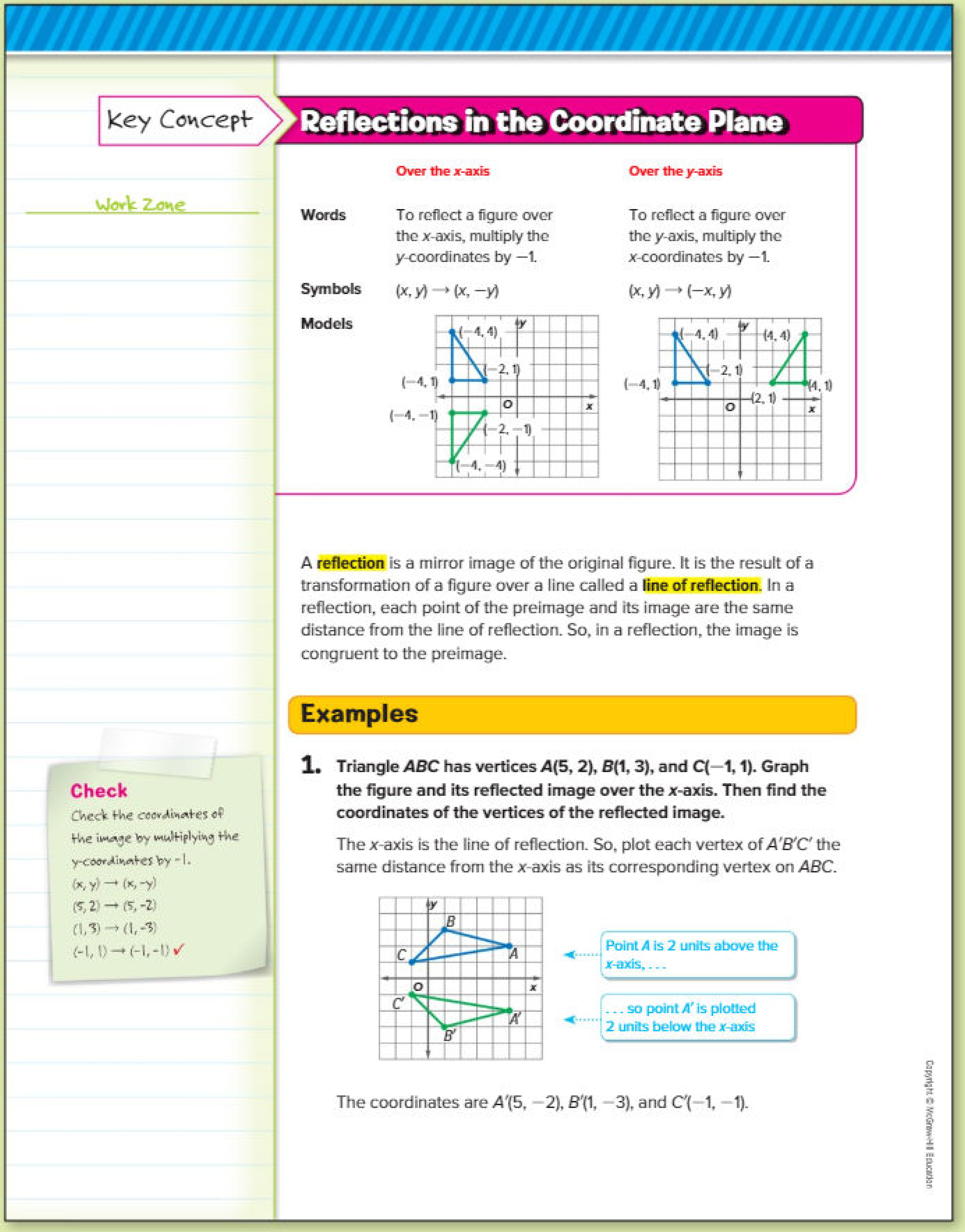
OL
- *If point A is 2 units above the x-axis, where will it be after the reflection?* 2 units below the *x*-axis
- *What algebraic notation explains the effect of this reflection?* $(x, y) \rightarrow (x, -y)$
- *What operation does the algebraic notation tell you to perform on the y-coordinates?* Multiply by -1.

BL
- *Is the orientation of the triangle preserved? Explain.* no; Sample answer: After the triangle is reflected, the orientation of the points are reversed from counter clockwise to clockwise.
- *Are the figures congruent?* yes

Need Another Example?

Quadrilateral *QRST* has vertices $Q(-1, 1)$, $R(0, 3)$, $S(3, 2)$, and $T(4, 0)$. Graph the figure and its reflected image over the *x*-axis. Then find the coordinates of the vertices of the reflected image. $Q'(-1, -1)$, $R'(0, -3)$, $S'(3, -2)$, $T'(4, 0)$





Key Concept

Reflections in the Coordinate Plane

	Over the *x*-axis	Over the *y*-axis
Words	To reflect a figure over the *x*-axis, multiply the *y*-coordinates by -1.	To reflect a figure over the *y*-axis, multiply the *x*-coordinates by -1.
Symbols	$(x, y) \rightarrow (x, -y)$	$(x, y) \rightarrow (-x, y)$
Models		

Work Zone

A **reflection** is a mirror image of the original figure. It is the result of a transformation of a figure over a line called a **line of reflection**. In a reflection, each point of the preimage and its image are the same distance from the line of reflection. So, in a reflection, the image is congruent to the preimage.

Examples

1. Triangle *ABC* has vertices $A(5, 2)$, $B(1, 3)$, and $C(-1, 1)$. Graph the figure and its reflected image over the *x*-axis. Then find the coordinates of the vertices of the reflected image.

The *x*-axis is the line of reflection. So, plot each vertex of *A′B′C′* the same distance from the *x*-axis as its corresponding vertex on *ABC*.

Point *A* is 2 units above the *x*-axis, . . .

. . . so point *A′* is plotted 2 units below the *x*-axis

The coordinates are $A'(5, -2)$, $B'(1, -3)$, and $C'(-1, -1)$.

Check

Check the coordinates of the image by multiplying the y-coordinates by -1.

(x, y) → (x, -y)

(5, 2) → (5, -2)

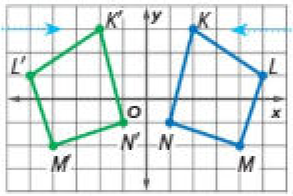
(1, 3) → (1, -3)

(-1, 1) → (-1, -1) ✓

2. **Quadrilateral *KLMN* has vertices *K*(2, 3), *L*(5, 1), *M*(4, −2), and *N*(1, −1). Graph the figure and its reflection over the *y*-axis. Then find the coordinates of the vertices of the reflected image.**

The *y*-axis is the line of reflection. So, plot each vertex of *K′L′M′N′* the same distance from the *y*-axis as its corresponding vertex on *KLMN*.

Point *K′* is 2 units to the left of the *y*-axis.

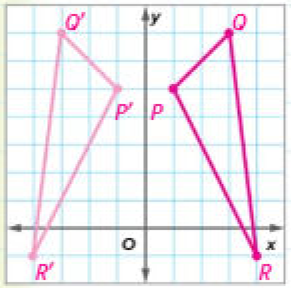


Point *K* is 2 units to the right of the *y*-axis.

The coordinates are *K′*(−2, 3), *L′*(−5, 1), *M′*(−4, −2), and *N′*(−1, −1).

Got it? **Do this problem to find out.**

a. Triangle *PQR* has vertices *P*(1, 5), *Q*(3, 7), and *R*(4, −1). Graph the figure and its reflection over the *y*-axis. Then find the coordinates of the reflected image.



Show your work.

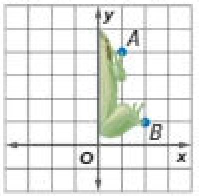
a. *P′*(−1, 5), *Q′*(−3, 7), *R′*(−4, −1)

Real-World Example

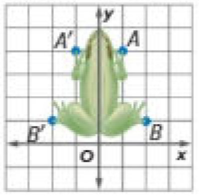
3. **The figure below is reflected over the *y*-axis. Find the coordinates of point *A′* and point *B′*. Then sketch the figure and its image on the coordinate plane.**

Point *A* is located at (1, 4). Point *B* is located at (2, 1). Since the figure is being reflected over the *y*-axis, multiply the *x*-coordinates by −1.

A(1, 4) → *A′*(−1, 4)



B(2, 1) → *B′*(−2, 1)



STOP and Reflect

Explain below how the x- and y-coordinates of an image relate to the x- and y-coordinates of the preimage after a reflection over the y-axis.

The *x*-coordinates are opposites and the *y*-coordinates are the same.

Examples

2. **Reflect a figure over the *y*-axis.**

AL
- *What line in the coordinate plane are we using as the mirror?* *y*-axis
- *When reflecting a point over the y-axis, which coordinate remains the same?* *y*-coordinate *Which coordinate changes?* *x*-coordinate

OL
- *If point K is 2 units to the right of the y-axis, where will it be after the reflection?* 2 units to the left of the *y*-axis
- *What algebraic notation explains the effect of this reflection?* (*x*, *y*) → (−*x*, *y*)

BL
- *What is another way to determine the coordinates of the reflected points?* Sample answer: Determine each point's location in respect to the origin. Point *K* is 2 units to the right of the origin and 3 units up. After the reflection, it will be 2 units to the left of the origin and 3 units up. Repeat this process for each point.

Need Another Example?

Triangle *XYZ* has vertices *X*(1, 2), *Y*(2, 1), and *Z*(1, −2). Graph the figure and its reflected image over the *y*-axis. Then find the coordinates of the vertices of the reflected image. See Answer Appendix.

3. **Use a reflection to sketch the figure.**

AL
- *Is the image of the frog symmetrical? Explain.* Yes, the left side is a reflection of the right side.

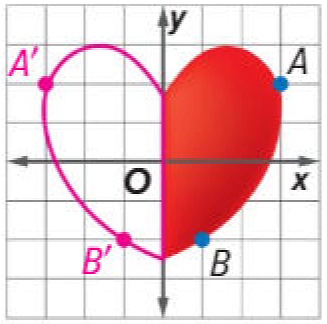
OL
- *How do you determine the new coordinates of a point that is reflected across the y-axis?* Multiply the *x*-coordinates by −1.

BL
- *What is another method you could use to plot A′?* *A* is 1 unit to the right and 4 units up from the origin. So, *A′* is 1 unit to the left and 4 units up from the origin.

Need Another Example?

The figure is reflected over the *y*-axis. Find the coordinates of point *A′* and point *B′*. Then sketch the figure and its image on the coordinate plane.

A′(−3, 2), *B′*(−1, −2)



Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL **LA** **Team-Pair-Solo** Have students work in a four-person team to complete Exercise 1, then with a partner to complete Exercise 2. Teams and partners should ensure that each person understands how to reflect a figure. Have students individually think through their response to Exercise 3, then discuss their response with their partner or group. **MP 1, 5, 6, 7**

BL **LA** **Trade-a-Problem** Have students draw their own figure in one quadrant of the coordinate grid and plot at least 4 points. Have students trade the grid and reflect each other's figure over the *x*- or *y*-axis. Then have them work together to write the algebraic notations that explain the effect of the reflections. **MP 1, 5, 6, 7**

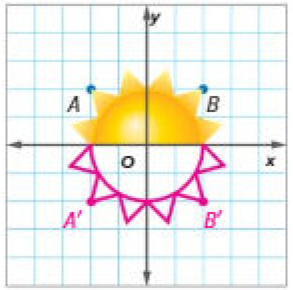
Watch Out!

Common Error In Exercise 1, students may reflect the image over the *y*-axis. Remind students that when reflecting over the *x*-axis, the *y*-coordinates are multiplied by −1, not the *x*-coordinates.

b. $A'(-2, -2)$, $B'(2, -2)$

Got it? **Do this problem to find out.**

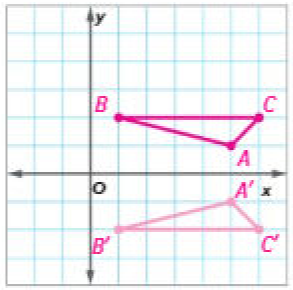
b. The figure at the right is reflected over the *x*-axis. Find the coordinates of point *A'* and point *B'*. Then sketch the image on the coordinate plane.



Guided Practice

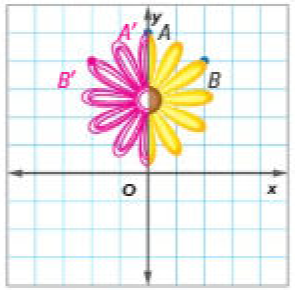
1. Graph △*ABC* with vertices *A*(5, 1), *B*(1, 2), and *C*(6, 2) and its reflection over the *x*-axis. Then find the coordinates of the image. (Examples 1 and 2)

$A'(5, -1)$, $B'(1, -2)$, $C'(6, -2)$



2. The figure is reflected over the *y*-axis. Find the coordinates of point *A'* and point *B'*. Then sketch the image on the coordinate plane. (Example 3)

$A'(0, 5)$, $B'(-2, 4)$



3. **Building on the Essential Question** How can you determine the coordinates of a figure after a reflection over either axis?

Sample answer: If you reflect over the *x*-axis, keep the *x*-coordinate and take the opposite of the *y*-coordinate. If you reflect over the *y*-axis, take the opposite of the *x*-coordinate and keep the *y*-coordinate.

Rate Yourself!

How well do you understand reflections? Circle the image that applies.

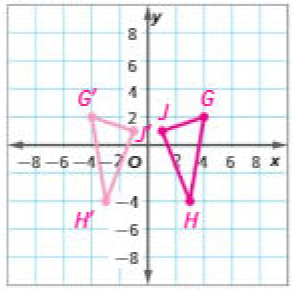
FOLDABLES Time to update your Foldable!

Name ______________________ My Homework ______________________

Independent Practice

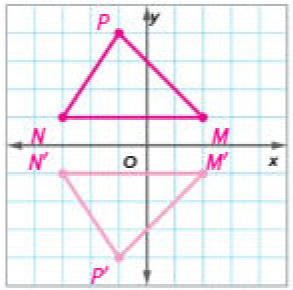
Graph each figure and its reflection over the indicated axis. Then find the coordinates of the reflected image. (Examples 1 and 2)

1. △*GHJ* with vertices *G*(4, 2), *H*(3, −4), and *J*(1, 1) over the *y*-axis



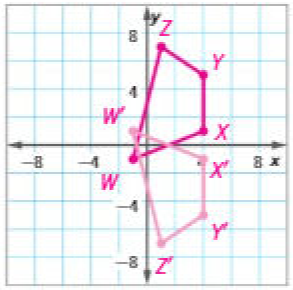
G′(−4, 2), *H*′(−3, −4), *J*′(−1, 1)

2. △*MNP* with vertices *M*(2, 1), *N*(−3, 1), and *P*(−1, 4) over the *x*-axis



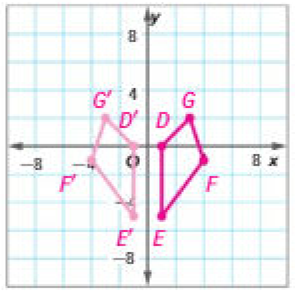
M′(2, −1), *N*′(−3, −1), *P*′(−1, −4)

3. quadrilateral *WXYZ* with vertices *W*(−1, −1), *X*(4, 1), *Y*(4, 5), and *Z*(1, 7) over the *x*-axis



W′(−1, 1), *X*′(4, −1), *Y*′(4, −5), *Z*′(1, −7)

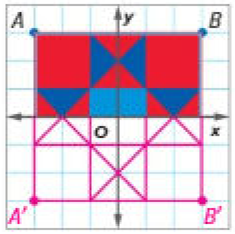
4. quadrilateral *DEFG* with vertices *D*(1, 0), *E*(1, −5), *F*(4, −1), and *G*(3, 2) over the *y*-axis



D′(−1, 0), *E*′(−1, −5), *F*′(−4, −1), *G*′(−3, 2)

5. The figure at the right is reflected over the *x*-axis. Find the coordinates of point *A*′ and point *B*′. Then sketch the image on the coordinate plane. (Example 3)

A′(−3, −3), *B*′(3, −3)



Identify Structure **The coordinates of a point and its image after a reflection are given. Describe the reflection as over the *x*-axis or *y*-axis.**

6. *A*(−3, 5) → *A*′(3, 5) *y*-axis

7. *M*(3, 3) → *M*′(3, −3) *x*-axis

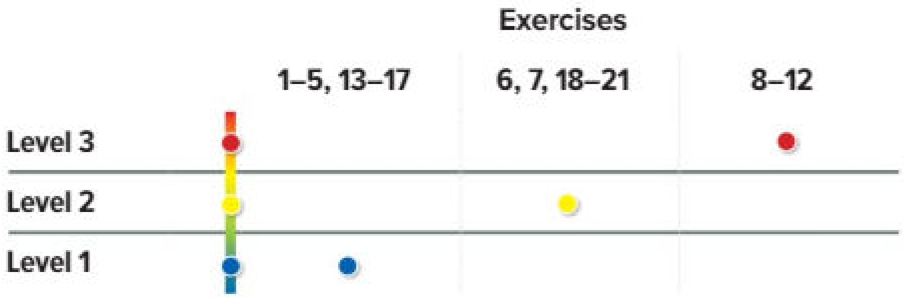
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7, 8, 10–12, 20, 21
OL	On Level	1–5 odd, 6–8, 10–12, 20, 21
BL	Beyond Level	6–12, 20, 21

MP MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	9
3 Construct viable arguments and critique the reasoning of others.	8, 10, 11, 12
7 Look for and make use of structure.	6, 7, 18, 19

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

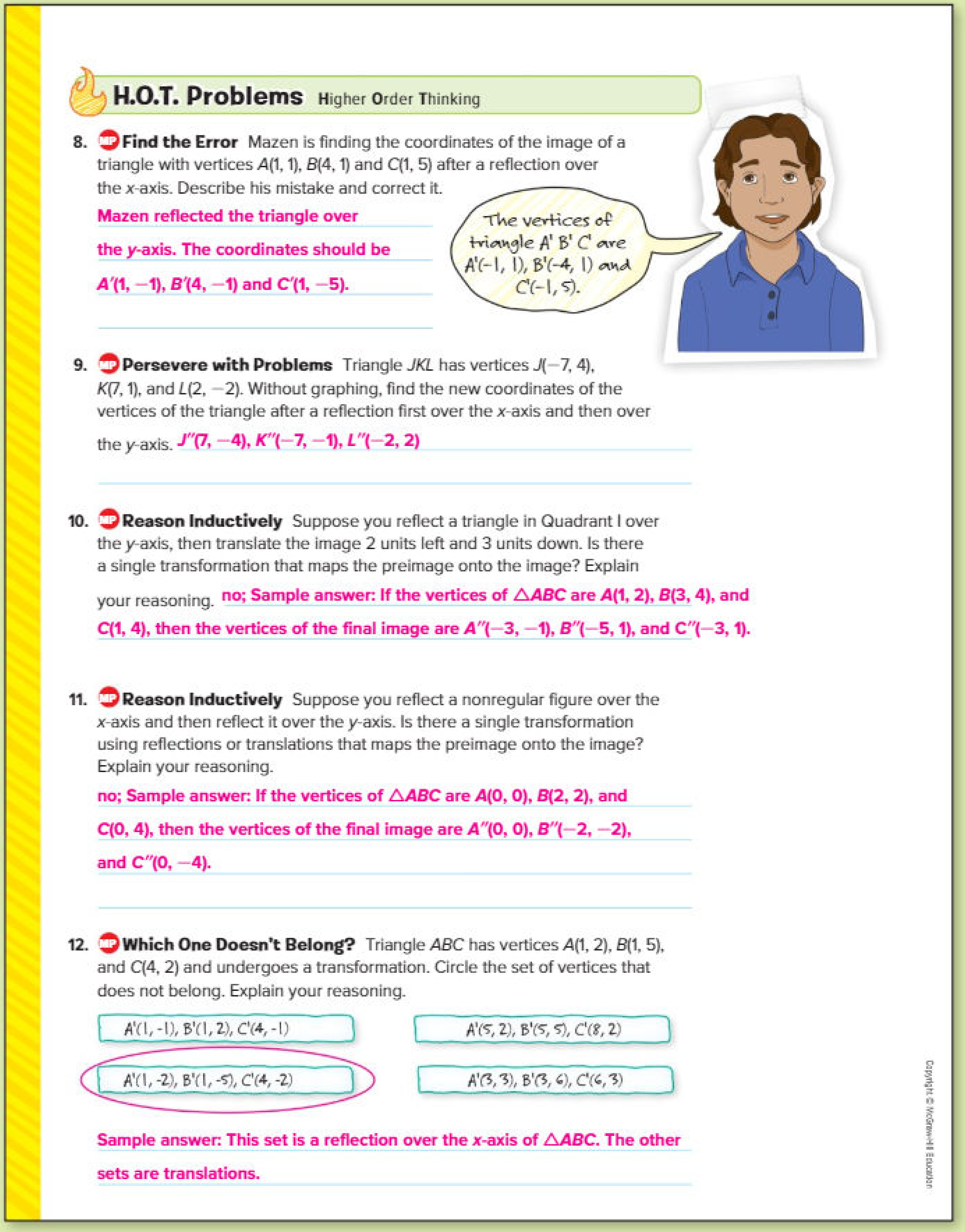
Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Ask students to compare translating a figure and reflecting a figure, and to write how previous lessons helped prepare them for concepts introduced in this lesson. Have them use the writing prompts below. See students' work.

- Reflecting a figure is similar to translating a figure because...
- Reflecting a figure is different from translating a figure because...
- What I learned about translations helped me to understand reflections in that...



H.O.T. Problems Higher Order Thinking

8\. MP **Find the Error** Mazen is finding the coordinates of the image of a triangle with vertices $A(1, 1)$, $B(4, 1)$ and $C(1, 5)$ after a reflection over the x-axis. Describe his mistake and correct it.

The vertices of triangle A' B' C' are A'(-1, 1), B'(-4, 1) and C'(-1, 5).

Mazen reflected the triangle over the y-axis. The coordinates should be $A'(1, -1)$, $B'(4, -1)$ and $C'(1, -5)$.

9\. MP **Persevere with Problems** Triangle JKL has vertices $J(-7, 4)$, $K(7, 1)$, and $L(2, -2)$. Without graphing, find the new coordinates of the vertices of the triangle after a reflection first over the x-axis and then over the y-axis. $J''(7, -4)$, $K''(-7, -1)$, $L''(-2, 2)$

10\. MP **Reason Inductively** Suppose you reflect a triangle in Quadrant I over the y-axis, then translate the image 2 units left and 3 units down. Is there a single transformation that maps the preimage onto the image? Explain your reasoning. no; Sample answer: If the vertices of $\triangle ABC$ are $A(1, 2)$, $B(3, 4)$, and $C(1, 4)$, then the vertices of the final image are $A''(-3, -1)$, $B''(-5, 1)$, and $C''(-3, 1)$.

11\. MP **Reason Inductively** Suppose you reflect a nonregular figure over the x-axis and then reflect it over the y-axis. Is there a single transformation using reflections or translations that maps the preimage onto the image? Explain your reasoning.

no; Sample answer: If the vertices of $\triangle ABC$ are $A(0, 0)$, $B(2, 2)$, and $C(0, 4)$, then the vertices of the final image are $A''(0, 0)$, $B''(-2, -2)$, and $C''(0, -4)$.

12\. MP **Which One Doesn't Belong?** Triangle ABC has vertices $A(1, 2)$, $B(1, 5)$, and $C(4, 2)$ and undergoes a transformation. Circle the set of vertices that does not belong. Explain your reasoning.

A'(1, -1), B'(1, 2), C'(4, -1)

A'(5, 2), B'(5, 5), C'(8, 2)

[✓ A'(1, -2), B'(1, -5), C'(4, -2)]

A'(3, 3), B'(3, 6), C'(6, 3)

Sample answer: This set is a reflection over the x-axis of $\triangle ABC$. The other sets are translations.

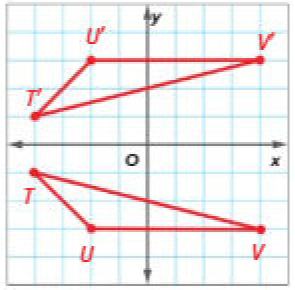
Name ______________________ My Homework ______________________

Extra Practice

Graph each figure and its reflection over the indicated axis. Then find the coordinates of the reflected image.

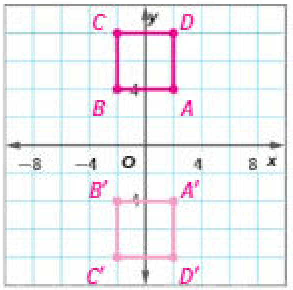
13. $\triangle TUV$ with vertices $T(-4, -1)$, $U(-2, -3)$, and $V(4, -3)$ over the x-axis

Show your work



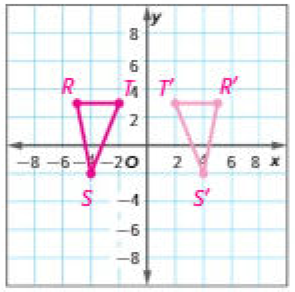
T'(-4, 1), U'(-2, 3), V'(4, 3)

14. square $ABCD$ with vertices $A(2, 4)$, $B(-2, 4)$, $C(-2, 8)$, and $D(2, 8)$ over the x-axis



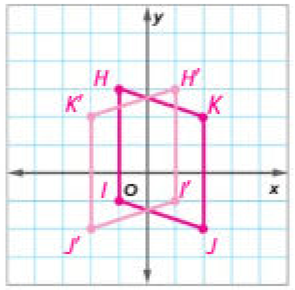
$A'(2, -4)$, $B'(-2, -4)$, $C'(-2, -8)$, $D'(2, -8)$

15. $\triangle RST$ with vertices $R(-5, 3)$, $S(-4, -2)$, and $T(-2, 3)$ over the y-axis



$R'(5, 3)$, $S'(4, -2)$, $T'(2, 3)$

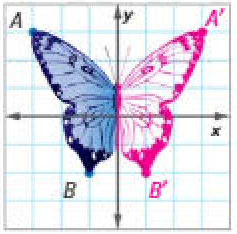
16. parallelogram $HIJK$ with vertices $H(-1, 3)$, $I(-1, -1)$, $J(2, -2)$, and $K(2, 2)$ over the y-axis



$H'(1, 3)$, $I'(1, -1)$, $J'(-2, -2)$, $K'(-2, 2)$

17. The figure at the right is reflected over the y-axis. Find the coordinates of point A' and point B'. Then sketch the image on the coordinate plane.

$A'(3, 3)$, $B'(1, -2)$



MP **Identify Structure** **The coordinates of a point and its image after a reflection are given. Describe the reflection as over the x-axis or y-axis.**

18. $X(-1, -4) \rightarrow X'(-1, 4)$ x-axis

19. $W(-4, 0) \rightarrow W'(4, 0)$ y-axis

Power Up! Test Practice

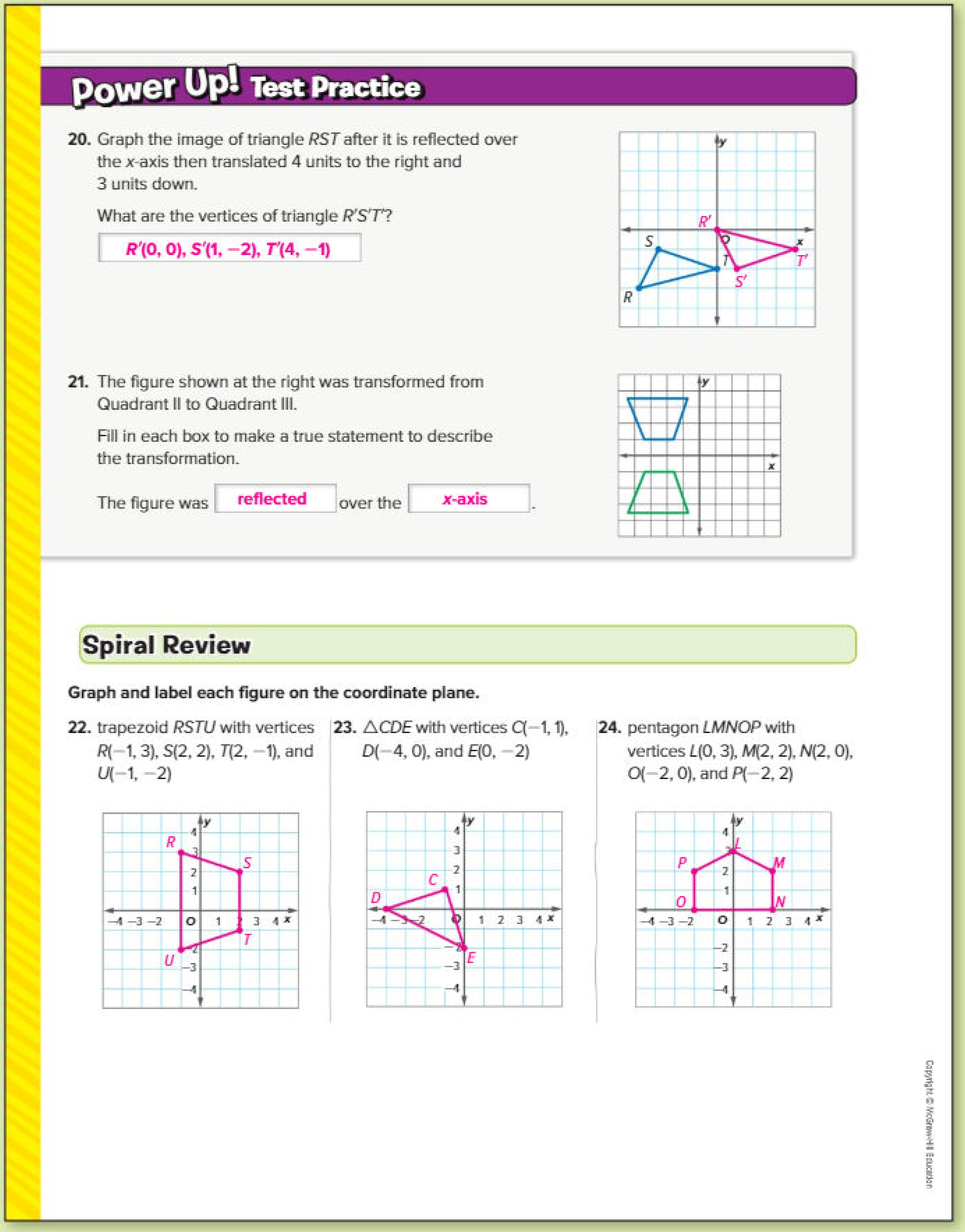
Exercises 20 and 21 prepare students for more rigorous thinking needed for assessment.

20.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly graph the figure and list the vertices.
1 point	Students correctly graph the figure OR list the vertices.

21.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1, MP5
Scoring Rubric	
1 point	Students correctly answer both parts of the question.



Power Up! Test Practice

20. Graph the image of triangle *RST* after it is reflected over the *x*-axis then translated 4 units to the right and 3 units down.

What are the vertices of triangle $R'S'T'$?

$R'(0, 0), S'(1, -2), T'(4, -1)$

21. The figure shown at the right was transformed from Quadrant II to Quadrant III.

Fill in each box to make a true statement to describe the transformation.

The figure was reflected over the *x*-axis.

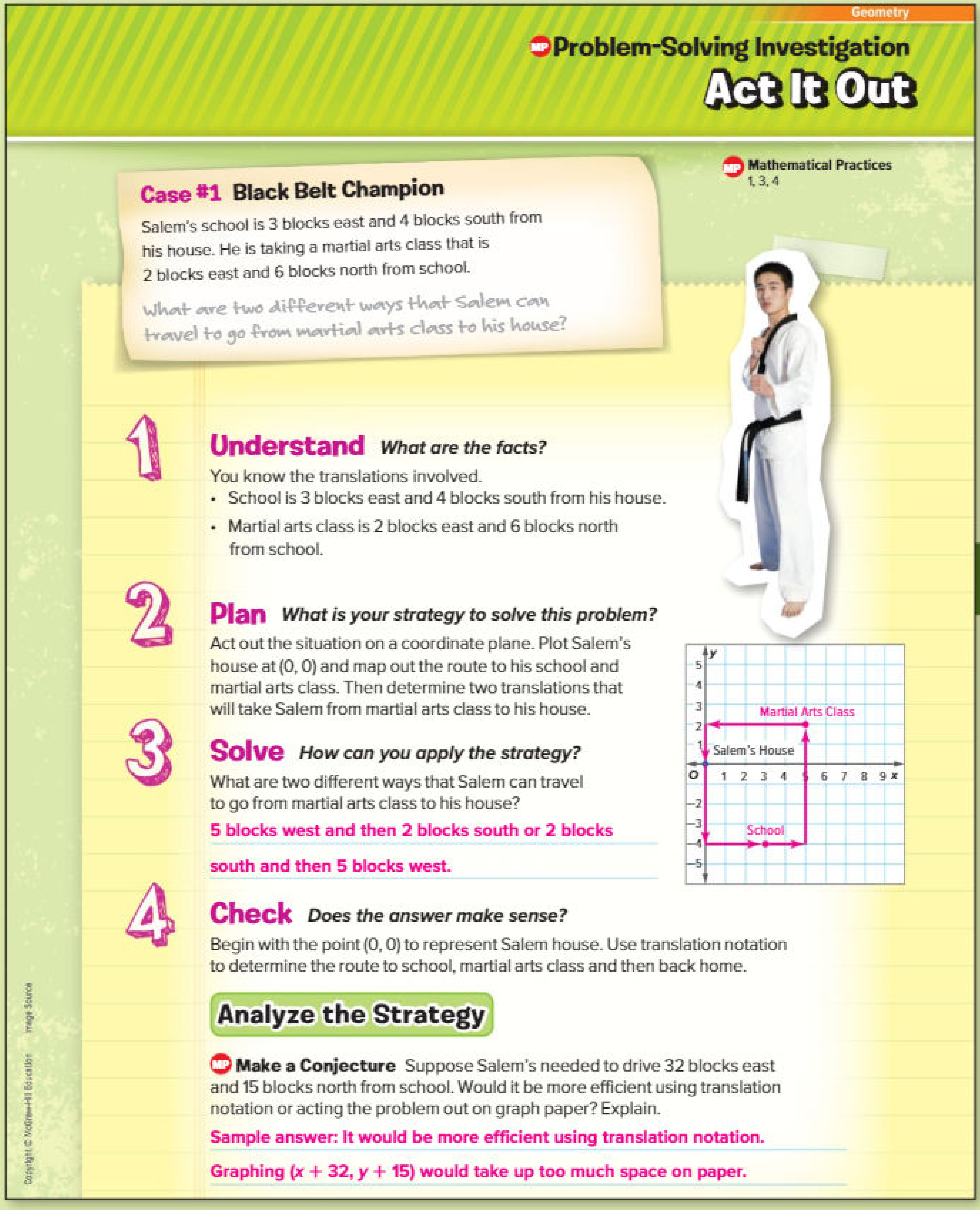
Spiral Review

Graph and label each figure on the coordinate plane.

22. trapezoid *RSTU* with vertices $R(-1, 3)$, $S(2, 2)$, $T(2, -1)$, and $U(-1, -2)$

23. $\triangle CDE$ with vertices $C(-1, 1)$, $D(-4, 0)$, and $E(0, -2)$

24. pentagon *LMNOP* with vertices $L(0, 3)$, $M(2, 2)$, $N(2, 0)$, $O(-2, 0)$, and $P(-2, 2)$



Problem-Solving Investigation

Act It Out

Mathematical Practices 1, 3, 4

Case #1 Black Belt Champion

Salem's school is 3 blocks east and 4 blocks south from his house. He is taking a martial arts class that is 2 blocks east and 6 blocks north from school.

What are two different ways that Salem can travel to go from martial arts class to his house?

1 Understand *What are the facts?*

You know the translations involved.

- School is 3 blocks east and 4 blocks south from his house.
- Martial arts class is 2 blocks east and 6 blocks north from school.

2 Plan *What is your strategy to solve this problem?*

Act out the situation on a coordinate plane. Plot Salem's house at (0, 0) and map out the route to his school and martial arts class. Then determine two translations that will take Salem from martial arts class to his house.

3 Solve *How can you apply the strategy?*

What are two different ways that Salem can travel to go from martial arts class to his house?

5 blocks west and then 2 blocks south or 2 blocks south and then 5 blocks west.

4 Check *Does the answer make sense?*

Begin with the point (0, 0) to represent Salem house. Use translation notation to determine the route to school, martial arts class and then back home.

Analyze the Strategy

Make a Conjecture Suppose Salem's needed to drive 32 blocks east and 15 blocks north from school. Would it be more efficient using translation notation or acting the problem out on graph paper? Explain.

Sample answer: It would be more efficient using translation notation. Graphing $(x + 32, y + 15)$ would take up too much space on paper.

Focus narrowing the scope

Objective Solve problems by using the *act it out* strategy. This lesson emphasizes **Mathematical Practice 4** Model with Mathematics.

Act It Out The strategy is especially helpful for students who are kinesthetic learners. For example, they model problems by arranging students or using counters.

Coherence connecting within and across grades

Now	**Next**
Students apply the content standards to solve non-routine problems.	Students will apply the act it out strategy to model transformation problems.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 471.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

The problems on pages 469 and 470 are for whole-group discussion on how to solve non-routine problems and provide scaffolded guidance. The problem on page 469 walks students through the solution, while the problem on page 470 asks students to come up with their own solutions.

Case #1 Black Belt Champion

BL Extend the problem using the question below.

Ask:

- *There is a grocery store 3 blocks east and 1 block south of Suhaila's house. If Suhaila needs to buy bread before going home, would it be better for her to stop at the store on her way to Martial Arts class, or after Martial Arts class on her way home? Explain.* Sample answer: It would better for her to go to the store on her way to Martial Arts class. She still has to travel 8 blocks to get to Martial Arts class. If she waits until after Martial Arts, she will add an additional 2 blocks to her trip.

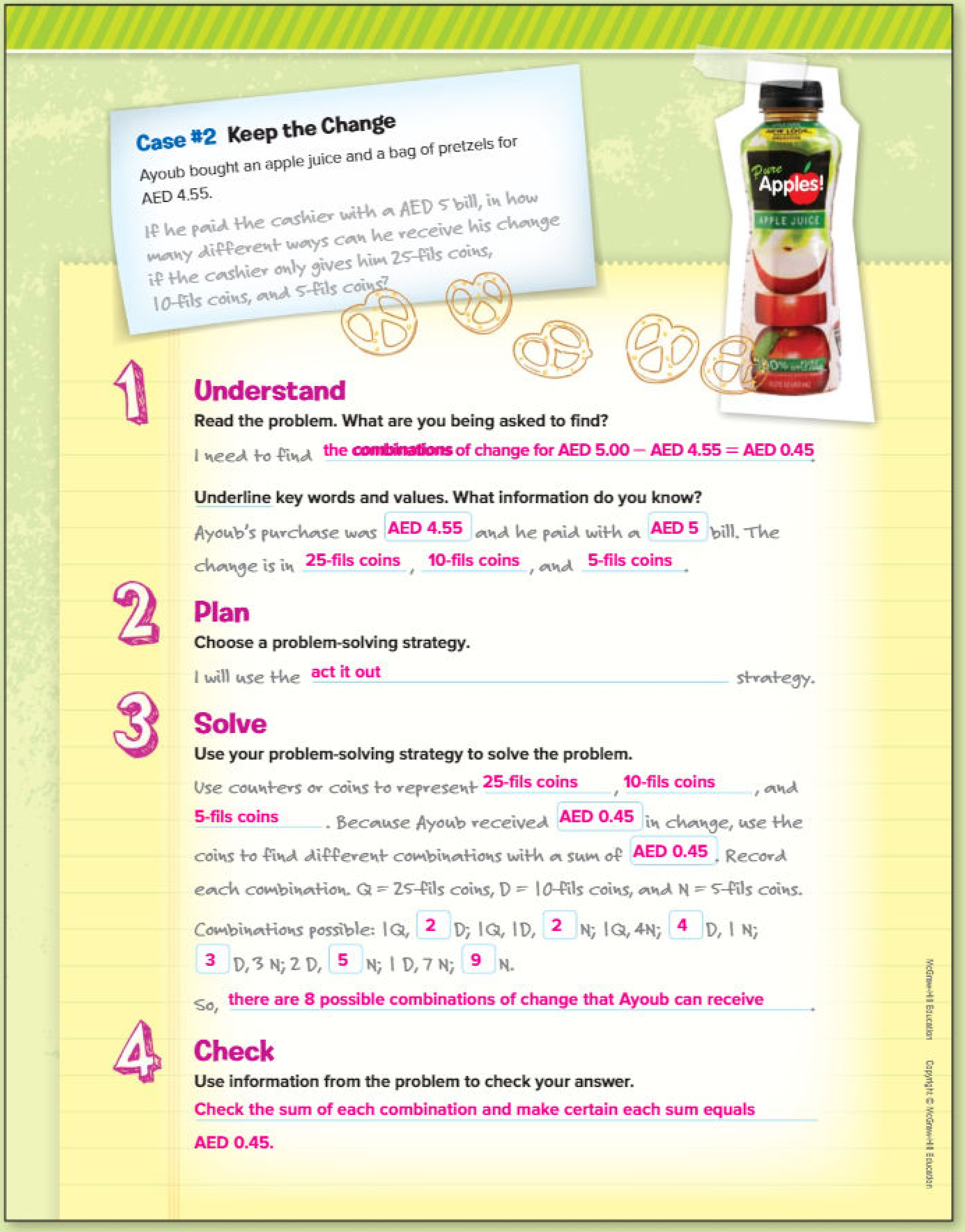
Case #2 Keep the Change

AL **LA** **Team Project** Have students work in 3- to 4-person learning teams. Give each team counters and have them act out the problem presented in the text. Then have each team share their answers with another team, listening carefully as each solution is explained and asking for clarification if necessary. MP 1, 2, 3, 4, 5, 6, 7, 8

BL **LA** **Trade-a-Problem** Have students create their own problem using the *act it out* strategy. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. MP 1, 2, 3, 4, 5, 6, 7, 8

Need Another Example?

How many ways are there to arrange five French club members for a yearbook photo if the president and vice president must be seated in front with the other three members behind them? 12



Case #2 Keep the Change

Ayoub bought an apple juice and a bag of pretzels for AED 4.55.

If he paid the cashier with a AED 5 bill, in how many different ways can he receive his change if the cashier only gives him 25-fils coins, 10-fils coins, and 5-fils coins?

1 Understand

Read the problem. What are you being asked to find?

I need to find the combinations of change for AED 5.00 – AED 4.55 = AED 0.45.

Underline key words and values. What information do you know?

Ayoub's purchase was AED 4.55 and he paid with a AED 5 bill. The change is in 25-fils coins, 10-fils coins, and 5-fils coins.

2 Plan

Choose a problem-solving strategy.

I will use the act it out strategy.

3 Solve

Use your problem-solving strategy to solve the problem.

Use counters or coins to represent 25-fils coins, 10-fils coins, and 5-fils coins. Because Ayoub received AED 0.45 in change, use the coins to find different combinations with a sum of AED 0.45. Record each combination. Q = 25-fils coins, D = 10-fils coins, and N = 5-fils coins.

Combinations possible: 1Q, 2 D; 1Q, 1D, 2 N; 1Q, 4N; 4 D, 1 N; 3 D, 3 N; 2 D, 5 N; 1 D, 7 N; 9 N.

So, there are 8 possible combinations of change that Ayoub can receive.

4 Check

Use information from the problem to check your answer.

Check the sum of each combination and make certain each sum equals AED 0.45.

Work with a small group to solve the following cases. Show your work on a separate piece of paper.

Case #3 Picture Exchanges

The French Club took a field trip to an exhibit of French art at the museum. Five of the club members held a picture exchange to share their pictures. Saeed brought more pictures than Yousif. Nasser brought more pictures than Mansour, but fewer than Yousif. Mahmoud brought more pictures than Nasser, but not as many as Yousif.

List the picture exchange participants in order from the most pictures to the fewest.

Saeed, Yousif, Mahmoud, Nasser, Mansour

Case #4 Fitness

The length of a basketball court is 25.2 meters. Kareem starts at one end of the court and runs 6 meters forward and then 2.40 meters back.

How many more times will he have to do this until he reaches the end of the basketball court? What equation represents this relationship?

6 times; Sample answer: $(6 - 2.4)x = 25.2$, where x = the number of times

Use any strategy!

Case #5 Dinner Parties

Wafa sent a text message to three friends inviting each of them to a dinner party. Each of those friends sent the message to three more friends. Then each of these friends sent the message to three more friends.

If two thirds of the friends receiving the text message attended the dinner, how many friends attended the dinner?

26 friends

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** EVALUATE

2 Collaborate

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	3, 4	5	6
Level 3			●
Level 2		●	
Level 1	●		

AL **LA** **Circle the Sage** Poll the class to see which students have some knowledge of translations. Those students (the sages) spread out around the room. Assign the rest of the students to teams. Have the teams split up with each team member going to a different sage, if possible. Have the sages lead work for Case 4. When the case is complete, students go back to their teams and compare solutions. Students discuss how the sages may have explained the steps differently.
MP 1, 2, 4, 5, 6, 7, 8

BL **LA** **Pairs Discussion** Have students work in pairs to answer the following extension question relating to Case 6.
MP 1, 2, 3, 4, 5, 6, 7, 8

Ask:

- *Wafa sent a text message to 2 friends inviting them to a party and they each sent a message to 2 more friends. Then each of these friends sent the message to 2 more friends. About two-thirds of the friends receiving the text message attended the party. How is the number of friends who attended this party different from the party where Wafa started by sending a message to 3 friends?* **Sample answer: When Wafa started with 2 friends, the total who attended the party was 9. This is a lot fewer people than the 26 who attended when she started with 3 friends.**

Mid-Chapter Check

If students have trouble with Exercises 1–5, they may need help with the following concepts.

Concept	Exercise(s)
translations (Lesson 1)	1, 3, 5
reflections (Lesson 2)	2, 4, 5

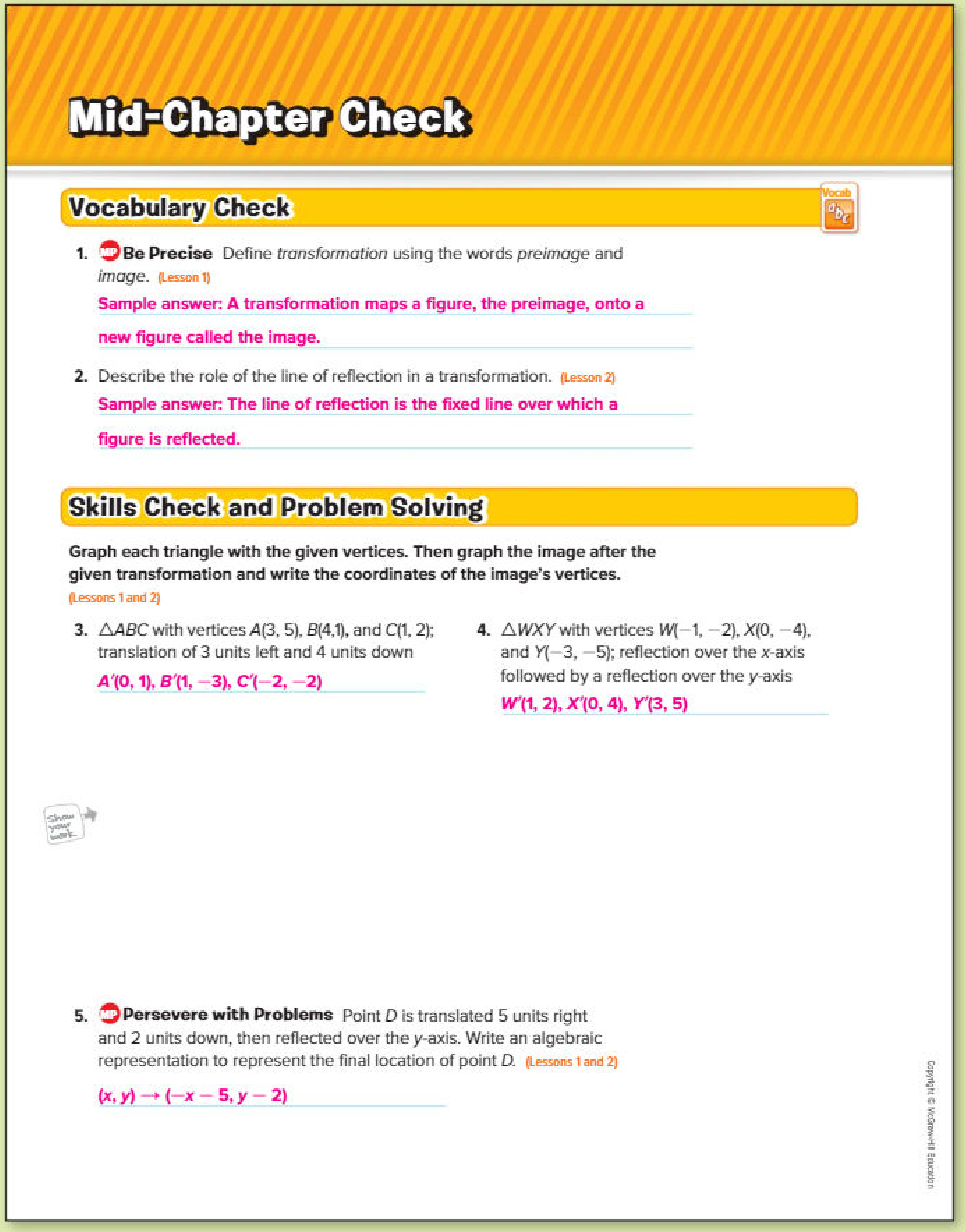
Vocabulary Activity

EL **Think-Pair-Share** Have students work in pairs to complete Exercise 1. Give them about one minute to individually think through their response. Then have them share their responses with a partner. Call on one set of pairs to share their responses with the class. MP 1, 6

Alternate Strategies

AL Provide students with the coordinates of the vertices of a triangle. Have students work with a partner to perform a reflection of the triangle over the *x*- or *y*-axis. Then have students discuss the role of the line of reflection in their transformation.

BL Have students create a graphic organizer of their own choosing to show the similarities and differences of translations and reflections.



Mid-Chapter Check

Vocabulary Check

1. MP **Be Precise** Define *transformation* using the words *preimage* and *image*. (Lesson 1)

   Sample answer: A transformation maps a figure, the preimage, onto a new figure called the image.

2. Describe the role of the line of reflection in a transformation. (Lesson 2)

   Sample answer: The line of reflection is the fixed line over which a figure is reflected.

Skills Check and Problem Solving

Graph each triangle with the given vertices. Then graph the image after the given transformation and write the coordinates of the image's vertices. (Lessons 1 and 2)

3. $\triangle ABC$ with vertices $A(3, 5)$, $B(4,1)$, and $C(1, 2)$; translation of 3 units left and 4 units down

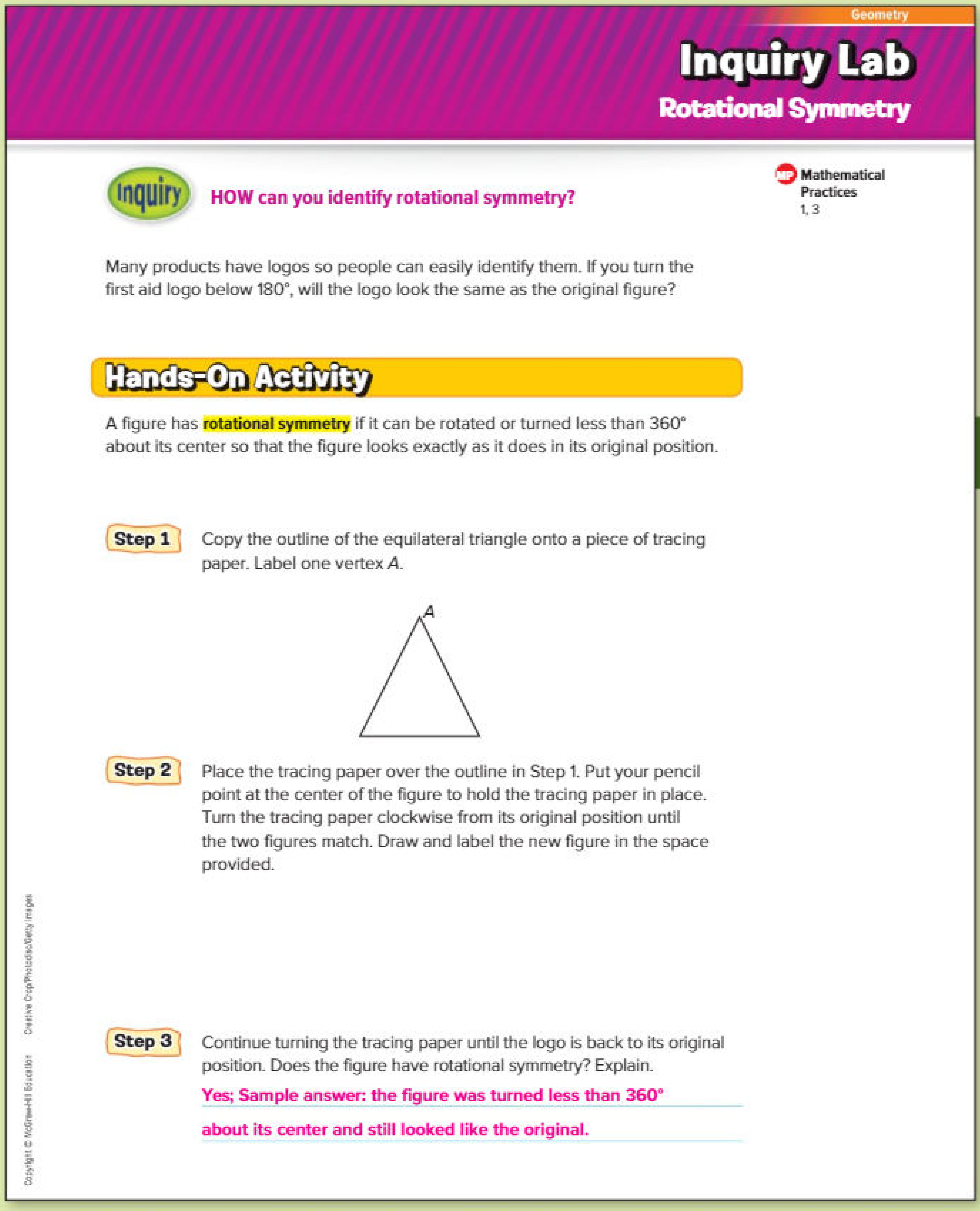
   $A'(0, 1)$, $B'(1, -3)$, $C'(-2, -2)$

4. $\triangle WXY$ with vertices $W(-1, -2)$, $X(0, -4)$, and $Y(-3, -5)$; reflection over the *x*-axis followed by a reflection over the *y*-axis

   $W'(1, 2)$, $X'(0, 4)$, $Y'(3, 5)$

5. MP **Persevere with Problems** Point *D* is translated 5 units right and 2 units down, then reflected over the *y*-axis. Write an algebraic representation to represent the final location of point *D*. (Lessons 1 and 2)

   $(x, y) \rightarrow (-x - 5, y - 2)$



Inquiry Lab
Rotational Symmetry

Inquiry **HOW can you identify rotational symmetry?**

Mathematical Practices
1, 3

Many products have logos so people can easily identify them. If you turn the first aid logo below 180°, will the logo look the same as the original figure?

Hands-On Activity

A figure has **rotational symmetry** if it can be rotated or turned less than 360° about its center so that the figure looks exactly as it does in its original position.

Step 1 Copy the outline of the equilateral triangle onto a piece of tracing paper. Label one vertex *A*.

Step 2 Place the tracing paper over the outline in Step 1. Put your pencil point at the center of the figure to hold the tracing paper in place. Turn the tracing paper clockwise from its original position until the two figures match. Draw and label the new figure in the space provided.

Step 3 Continue turning the tracing paper until the logo is back to its original position. Does the figure have rotational symmetry? Explain.

Yes; Sample answer: the figure was turned less than 360° about its center and still looked like the original.

Focus narrowing the scope

Objective Identify rotational symmetry.

Coherence connecting within and across grades

Now
Students will identify properties of rotational symmetry.

Next
Students will graph rotations on the coordinate plane.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 474.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Hands-On Activity

AL LA **Teammates Consult** Have students work in small teams. Give each team tracing paper. Ask a volunteer to lead the activity, showing how to rotate the paper without letting it slide. Then have them discuss the question in Step 3. Have one team share their responses with the class to initiate a class discussion about what it means for a figure to have rotational symmetry. MP 1, 3, 5, 6, 7

BL LA **Pairs Consult** After completing the activity, have students work with a partner to draw and color a pattern inside the logo that will change its rotational symmetry to be 180°. MP 1, 5, 6, 7

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises 1–3	 4–6	 7, 8
Level 3			●
Level 2		●	
Level 1	●		

Investigate

AL **LA** **Team-Pair-Solo** Give students tracing paper and have them trace the outline of each figure. As they did in the activity, have them use a pencil to hold the paper in place and rotate the paper until it matches with the original picture. Have students work in small teams to complete Exercise 1. Then have them work with a partner to complete Exercise 2, and have them work on their own to complete Exercise 3. Have them rejoin their original teams to discuss and compare responses. MP 1, 5, 6, 7

Create

BL Have students design and color their own personal logo with a degree of rotational symmetry. Pass their designs around the room and have students determine the degree of rotational symmetry for each other's work. MP 1, 5, 6, 7

Inquiry Students should be able to answer "HOW can you identify rotational symmetry?" Check for student understanding and provide guidance, if needed.

Investigate

Work with a partner. Determine whether the figure has rotational symmetry. Write *yes* or *no*.

1\.

yes

2\.

no

3\.

yes

Analyze and Reflect

4\. MP **Reason Inductively** The degree measure of an angle through which the figure is rotated is called the **angle of rotation**. Find the first angle of rotation of the equilateral triangle by dividing 360° by the total number of times the figures matched. 90°

5\. List the other angles of rotation of the equilateral triangle by adding the measure of the first angle of rotation to the previous angle measure. Stop when you reach 360°. 180°, 270°

6\. What is the angle of rotation of each figure in Exercises 1–3? Write *no* if there is no rotational symmetry.

Exercise 1 180° Exercise 2 no Exercise 3 60°

Create

7\. MP **Model with Mathematics** Draw two figures, one that has rotational symmetry and one that does not. **See students' work.**

Show your work.

8\. Inquiry HOW can you identify rotational symmetry?

Sample answer: You can identify rotational symmetry by turning the figure less than 360° and determining if the figure looks the same as the original.

Lesson 3

Rotations

Real-World Link

Prizes Majed is spinning the prize wheel shown below.

1. A spin can be *clockwise* or *counterclockwise*. Define these two words in your own words.

   clockwise **rotating to the right**

   counterclockwise **rotating to the left**

2. If the section labeled 8 on the left part of the wheel spins 90° clockwise, where will it land? **at the top**

3. If one of the sections labeled 4 makes three complete turns counterclockwise, how many degrees will it have traveled? **1,080** °

4. Are there any points on the wheel that stay fixed, or do not move, when the wheel spins? If so, what are the points?

   yes; the center

5. Does the center of the wheel change if the wheel is spun counterclockwise as opposed to clockwise? **no**

6. Does the distance from the center to the edge change as it spins? Explain.

   no; Sample answer: The distance from the center to the edge is the radius of the circle. The size of the circle does not change as it spins so the radius does not change.

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Essential Question

HOW can we best show or describe the change in position of a figure?

Vocabulary

rotation
center of rotation

Math Symbols

$(x, y) \rightarrow (y, -x)$
$(x, y) \rightarrow (-x, -y)$
$(x, y) \rightarrow (-y, x)$

Mathematical Practices
1, 3, 4, 7

Focus narrowing the scope

Objective Graph rotations on the coordinate plane.

Coherence connecting within and across grades

Previous	Now	Next
Students identified the properties of rotational symmetry.	Students will graph rotations on the coordinate plane.	Students will graph dilations on the coordinate plane.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 479.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Cooperative Play** Show a similar spinner at the front of the class or use an online spinner. Give students time to play with the spinner so that they can visualize the movements made in the activity. Then have them complete the activity with a partner. MP 1, 4, 5, 7

Alternate Strategy

AL LA Have students use a clock face to visualize the meanings of *clockwise* and *counterclockwise*. MP 1, 5

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

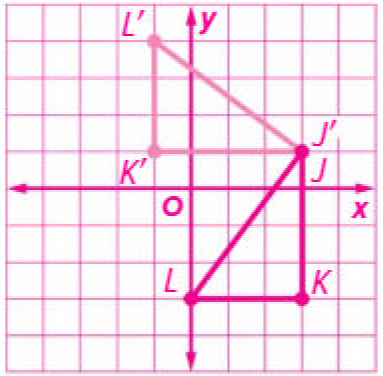
Example

1. Rotate a figure about a point.

- AL • *About what point are we rotating?* **about vertex *L***
- OL • *Describe vertex M in comparison to vertex L.* **Vertex *M* is 3 units up from vertex *L*.**
- • *After the rotation, how far away from vertex L will vertex M′ be?* **Vertex *M′* will be 3 units down from vertex *L*.**
- • *Describe vertex N in comparison to vertex L.* **Vertex *N* is 3 units up and 3 units to the right from vertex *L*.**
- • *After the rotation, how far away from vertex L will vertex N′ be?* **Vertex *N′* will be 3 units down and 3 units to the left from vertex *L*.**
- BL • *After the rotation, where will vertex L′ be in comparison to vertex L? Explain.* **Vertex *L′* will be in the same location as vertex *L*. Vertex *L* is the center of rotation.**
- • *Are the two figures congruent?* **yes**

Need Another Example?

Triangle *JKL* has vertices *J*(3, 1), *K*(3, −3), and *L*(0, −3). Graph the figure and its image after a clockwise rotation of 90° about vertex *J*. Then give the coordinates of the vertices for △*J′K′L′*.
***J′*(3, 1), *K′*(−1, 1), *L′*(−1, 4)**



Work Zone

Rotations

Rotations can be described in degrees and direction. For example, 90° clockwise or 270° counterclockwise.

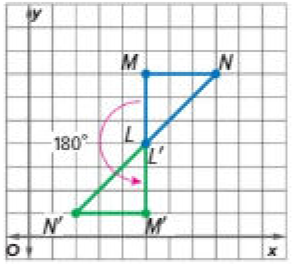
Rotate a Figure About a Point

A **rotation** is a transformation in which a figure is rotated, or turned, about a fixed point. The **center of rotation** is the fixed point. A rotation does not change the size or shape of the figure. So, the preimage and the image are congruent.

Real-World Example

1. **Triangle *LMN* with vertices *L*(5, 4), *M*(5, 7), and *N*(8, 7) represents a desk in Ibrahim's bedroom. He wants to rotate the desk counterclockwise 180° about vertex *L*. Graph the figure and its image. Then give the coordinates of the vertices for △*L′M′N′*.**

Step 1 Graph the original triangle.



Step 2 Graph the rotated image. Use a protractor to measure an angle of 180° with *M* as one point on the ray and *L* as the vertex. Mark off a point the same length as $\overline{ML}$. Label this point *M′* as shown.

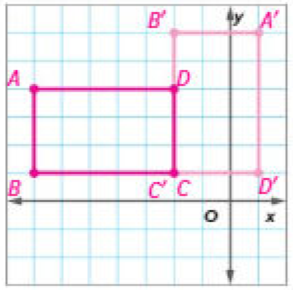
Step 3 Repeat Step 2 for point *N*. Since *L* is the point at which △*LMN* is rotated, *L′* will be in the same position as *L*.

So, the coordinates of the vertices of △*L′M′N′* are *L′*(5, 4), *M′*(5, 1), and *N′*(2, 1).

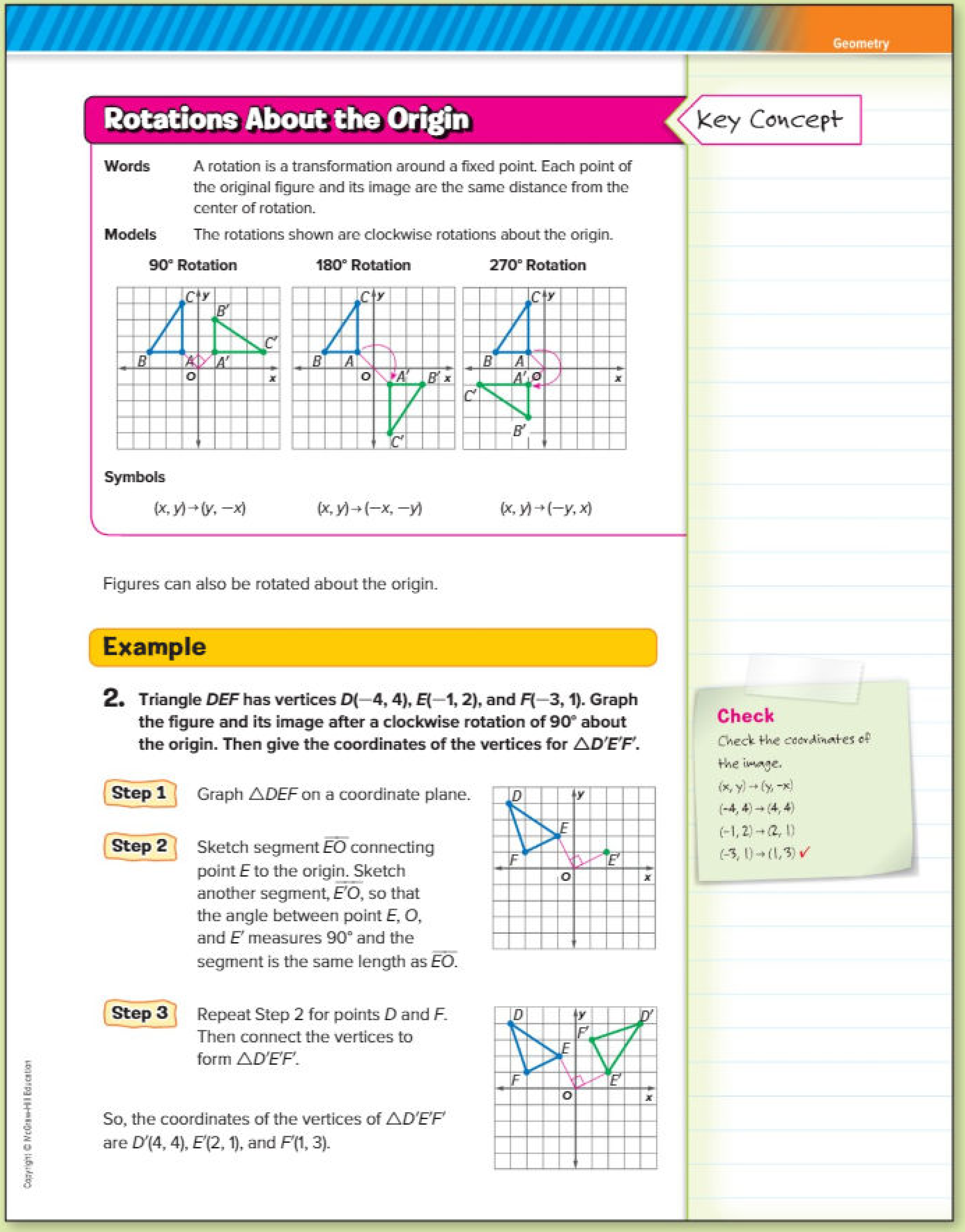
Show your work.

Got it? **Do this problem to find out.**

a. Rectangle *ABCD* with vertices *A*(−7, 4), *B*(−7, 1), *C*(−2, 1), and *D*(−2, 4) represents the bed in Ibrahim's room. Graph the figure and its image after a clockwise rotation of 90° about vertex *C*. Then give the coordinates of the vertices for rectangle *A′B′C′D′*.



a. ***A′*(1, 6), *B′*(−2, 6), *C′*(−2, 1), *D′*(1, 1)**



Rotations About the Origin

Key Concept

Words A rotation is a transformation around a fixed point. Each point of the original figure and its image are the same distance from the center of rotation.

Models The rotations shown are clockwise rotations about the origin.

90° Rotation	180° Rotation	270° Rotation
$(x, y) \rightarrow (y, -x)$	$(x, y) \rightarrow (-x, -y)$	$(x, y) \rightarrow (-y, x)$

Symbols

Figures can also be rotated about the origin.

Example

2. **Triangle *DEF* has vertices *D*(−4, 4), *E*(−1, 2), and *F*(−3, 1). Graph the figure and its image after a clockwise rotation of 90° about the origin. Then give the coordinates of the vertices for △*D*′*E*′*F*′.**

Step 1 Graph △*DEF* on a coordinate plane.

Step 2 Sketch segment $\overline{EO}$ connecting point *E* to the origin. Sketch another segment, $\overline{E'O}$, so that the angle between point *E*, *O*, and *E*′ measures 90° and the segment is the same length as $\overline{EO}$.

Step 3 Repeat Step 2 for points *D* and *F*. Then connect the vertices to form △*D*′*E*′*F*′.

So, the coordinates of the vertices of △*D*′*E*′*F*′ are *D*′(4, 4), *E*′(2, 1), and *F*′(1, 3).

Check

Check the coordinates of the image.

(x, y) → (y, −x)

(−4, 4) → (4, 4)

(−1, 2) → (2, 1)

(−3, 1) → (1, 3) ✓

Example

2 **Rotate a figure about the origin.**

AL • *About what point are we rotating?* about the origin

OL • *Describe the location of point E in reference to the origin.* It is 1 unit to the left of and 2 units above the origin.

• *Describe the location of point E′ in reference to the origin.* It is 2 units to the right of and 1 unit above the origin.

• *Using this as a guide, what will be the location of point F′ in reference to the origin? point D′?* Point *F*′ will be 1 unit to the right and 3 units above the origin. Point *D*′ will be 4 units to the right and 4 units above the origin.

BL • *Are the two figures congruent?* yes

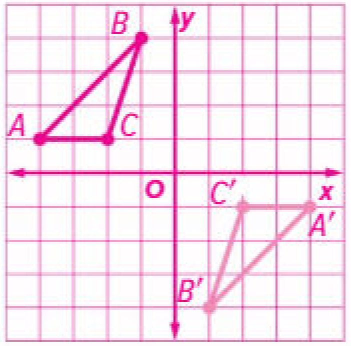
• *After a 180° rotation clockwise about the origin, what does the point (x, y) become?* (x, y) → (−x, −y)

• *After a 270° rotation clockwise about the origin, what does the point (x, y) become?* (x, y) → (−y, x)

• *After a 360° rotation clockwise about the origin, what does the point (x, y) become?* (x, y) → (x, y)

Need Another Example?

Triangle *ABC* has vertices *A*(−4, 1), *B*(−1, 4), and *C*(−2, 1). Graph the figure and its image after a counterclockwise rotation of 180° about the origin. Then give the coordinates of the vertices for △*A*′*B*′*C*′. ***A*′(4, −1), *B*′(1, −4), *C*′(2, −1)**



Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL **LA** **Groups-Pairs-Solo** If students are having trouble understanding how to rotate images, you may wish to complete Exercise 1 together as a whole group. You may also wish to give the students tracing paper to help them see how the image is rotated. Have them trace the original triangle and then press their pencil down on the point of rotation (vertex *X* for Exercise 1, the origin for Exercise 2) to rotate the paper. Then have students work with a partner to complete Exercise 2. Have them work individually to complete Exercise 3. Then have them rejoin the whole group to discuss answers and compare solutions. MP 1, 5, 7

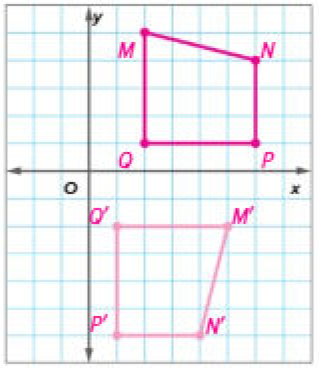
BL **LA** **Pairs Discussion** For Exercises 1 and 2, have students predict the coordinates of the image after the rotation without graphing. Then have them compare the coordinates after graphing to see if their prediction was correct. MP 1, 3, 5, 6, 7

Show your work.

b. *M*′(5, −2), *N*′(4, −6), *P*′(1, −6), *Q*′(1, −2)

Got it? **Do this problem to find out.**

b. Quadrilateral *MNPQ* has vertices *M*(2, 5), *N*(6, 4), *P*(6, 1), and *Q*(2, 1). Graph the figure and its image after a counterclockwise rotation of 270° about the origin. Then give the coordinates of the vertices for quadrilateral *M*′*N*′*P*′*Q*′.



Guided Practice

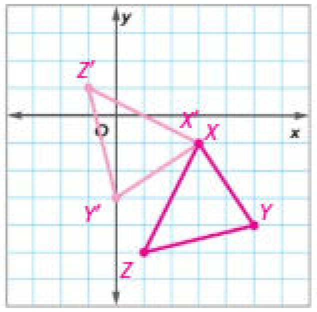
Check ✓

Triangle *XYZ* has vertices *X*(3, −1), *Y*(5, −4), and *Z*(1, −5). Graph △*XYZ* and its image after each rotation. Then give the coordinates of the vertices for △*X*′*Y*′*Z*′. (Examples 1 and 2)

1. 270° counterclockwise about vertex *X*

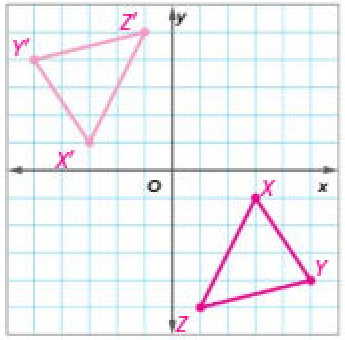
X′(3, −1), *Y*′(0, −3), *Z*′(−1, 1)

Show your work.



2. 180° clockwise about the origin

X′(−3, 1), *Y*′(−5, 4), *Z*′(−1, 5)



3. **Building on the Essential Question** What is the difference between rotating a figure about a given point that is a vertex and rotating the same figure about the origin if the rotation is less than 360°?

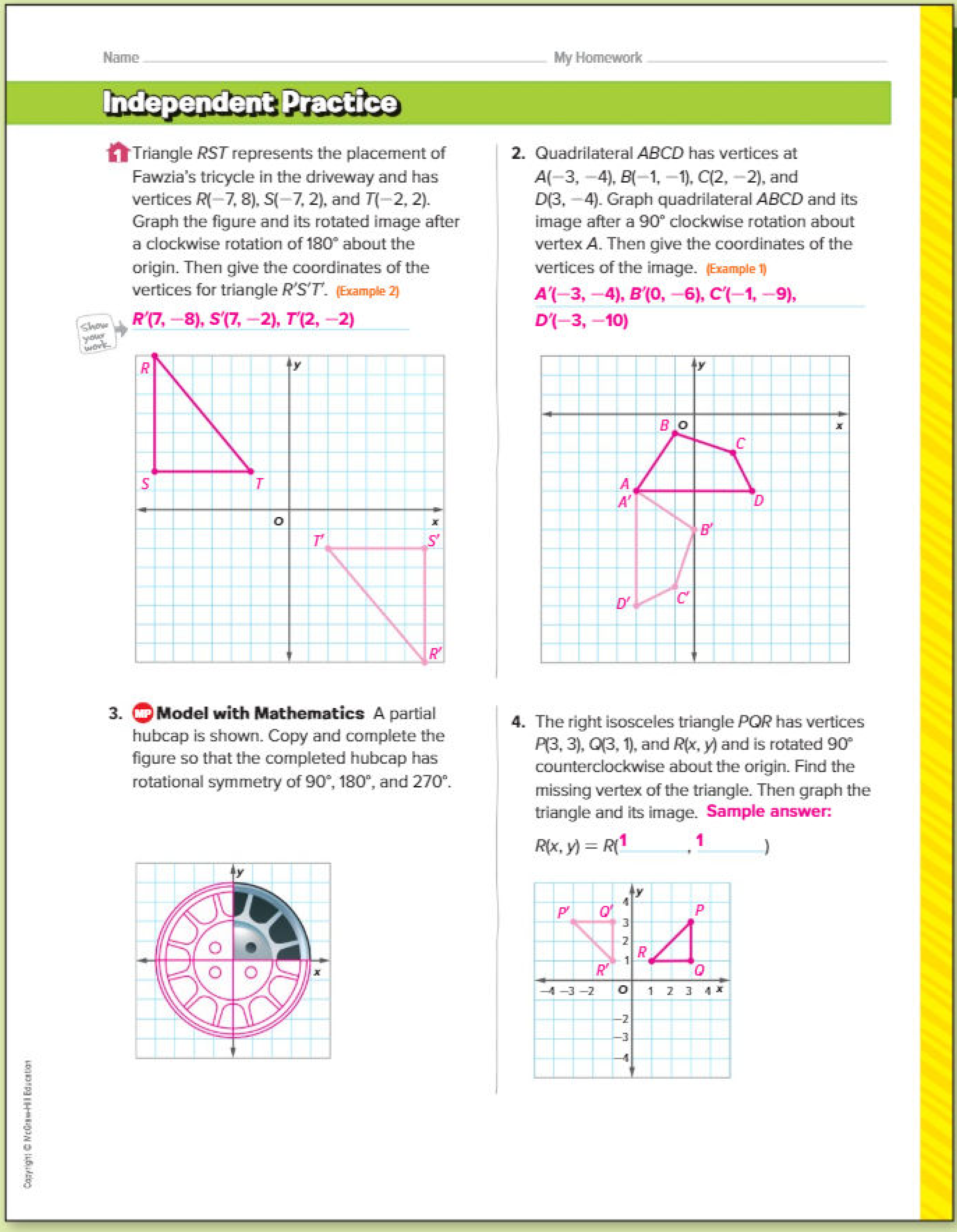
Sample answer: If you rotate the figure about one of the vertices, that point stays the same. If you rotate the same figure about the origin, all of the points are different unless one of the vertices is the origin.

Rate Yourself!

How confident are you about rotations? Check the box that applies.

☐ ☐ ☐ ☐ ☐

FOLDABLES Time to update your Foldable!



Name ______________________ My Homework ______________________

Independent Practice

1. Triangle *RST* represents the placement of Fawzia's tricycle in the driveway and has vertices *R*(−7, 8), *S*(−7, 2), and *T*(−2, 2). Graph the figure and its rotated image after a clockwise rotation of 180° about the origin. Then give the coordinates of the vertices for triangle *R′S′T′*. (Example 2)

Show your work.

R′(7, −8), S′(7, −2), T′(2, −2)

2. Quadrilateral *ABCD* has vertices at *A*(−3, −4), *B*(−1, −1), *C*(2, −2), and *D*(3, −4). Graph quadrilateral *ABCD* and its image after a 90° clockwise rotation about vertex *A*. Then give the coordinates of the vertices of the image. (Example 1)

A′(−3, −4), B′(0, −6), C′(−1, −9), D′(−3, −10)

3. **Model with Mathematics** A partial hubcap is shown. Copy and complete the figure so that the completed hubcap has rotational symmetry of 90°, 180°, and 270°.

4. The right isosceles triangle *PQR* has vertices *P*(3, 3), *Q*(3, 1), and *R*(*x*, *y*) and is rotated 90° counterclockwise about the origin. Find the missing vertex of the triangle. Then graph the triangle and its image. **Sample answer:**

R(*x*, *y*) = *R*(**1**, **1**)

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

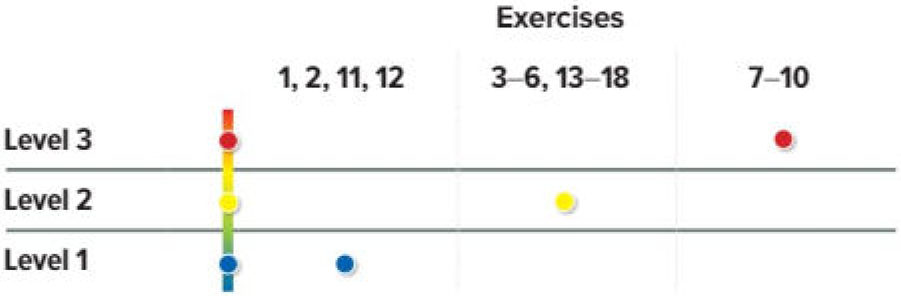
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–3, 5, 9, 10, 17, 18
OL	On Level	1, 3–6, 9, 10, 17, 18
BL	Beyond Level	3–10, 17, 18

Watch Out!

Common Error Watch for students who rotate figures about a vertex rather than about the origin. Remind students to first determine the center of rotation.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	7, 8
2 Reason abstractly and quantitatively.	10
4 Model with mathematics.	3, 6, 9
7 Look for and make use of structure.	13

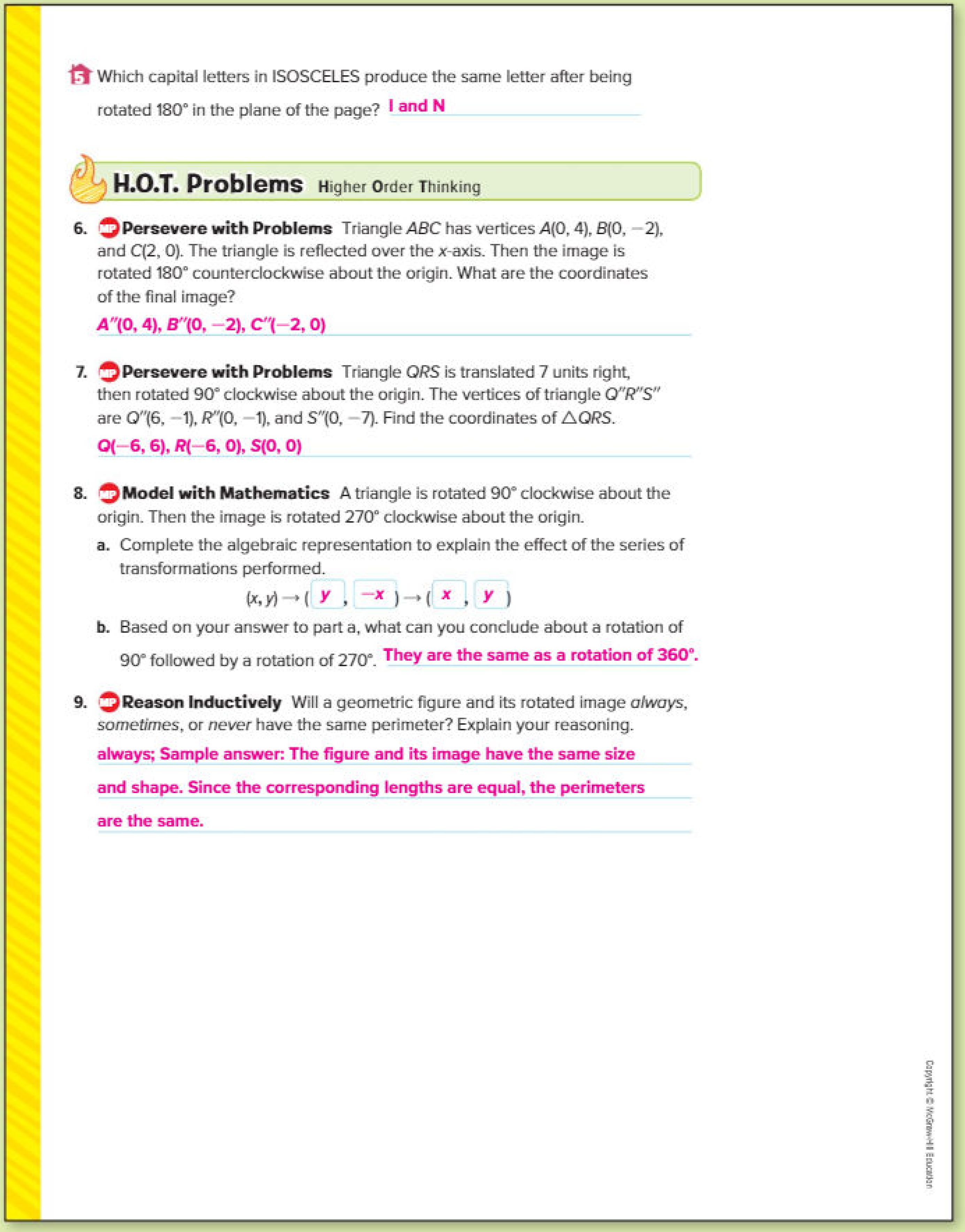
Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students respond to the following question: If point *T*(4, −3) is rotated 90° counterclockwise about the origin, what are the coordinates of *T*′? **(3, 4)**



5. Which capital letters in ISOSCELES produce the same letter after being rotated 180° in the plane of the page? **I and N**

H.O.T. Problems Higher Order Thinking

6. **Persevere with Problems** Triangle *ABC* has vertices *A*(0, 4), *B*(0, −2), and *C*(2, 0). The triangle is reflected over the *x*-axis. Then the image is rotated 180° counterclockwise about the origin. What are the coordinates of the final image?

A″(0, 4), B″(0, −2), C″(−2, 0)

7. **Persevere with Problems** Triangle *QRS* is translated 7 units right, then rotated 90° clockwise about the origin. The vertices of triangle *Q″R″S″* are *Q″*(6, −1), *R″*(0, −1), and *S″*(0, −7). Find the coordinates of △*QRS*.

Q(−6, 6), R(−6, 0), S(0, 0)

8. **Model with Mathematics** A triangle is rotated 90° clockwise about the origin. Then the image is rotated 270° clockwise about the origin.

   a. Complete the algebraic representation to explain the effect of the series of transformations performed.

   $$(x, y) \rightarrow (\boxed{y}, \boxed{-x}) \rightarrow (\boxed{x}, \boxed{y})$$

   b. Based on your answer to part a, what can you conclude about a rotation of 90° followed by a rotation of 270°. **They are the same as a rotation of 360°.**

9. **Reason Inductively** Will a geometric figure and its rotated image *always*, *sometimes*, or *never* have the same perimeter? Explain your reasoning.

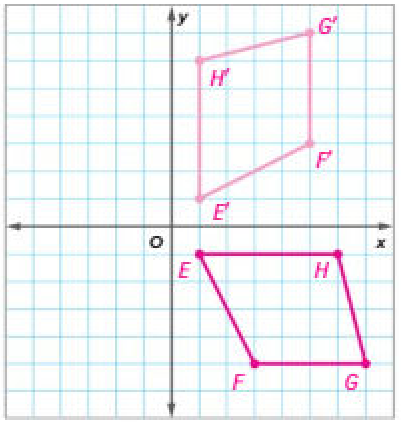
always; Sample answer: The figure and its image have the same size and shape. Since the corresponding lengths are equal, the perimeters are the same.

Name ______________________ My Homework ______________________

Extra Practice

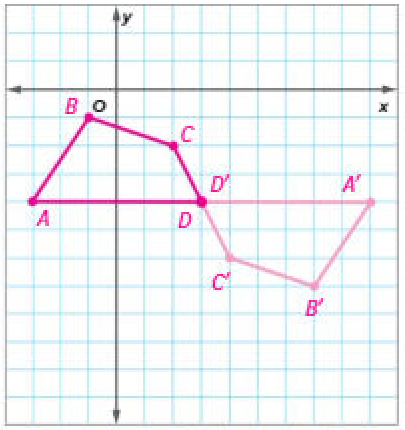
10. Quadrilateral *EFGH* has vertices *E*(1, −1), *F*(3, −5), *G*(7, −5), and *H*(6, −1). Graph the figure and its rotated image after a counterclockwise rotation of 90° about the origin. Then give the coordinates of the vertices for quadrilateral *E′F′G′H′*.

***E′*(1, 1), *F′*(5, 3), *G′*(5, 7), *H′*(1, 6)**



11. Quadrilateral *ABCD* has vertices at *A*(−3, −4), *B*(−1, −1), *C*(2, −2), and *D*(3, −4). Graph quadrilateral *ABCD* and its image after a 180° counterclockwise rotation about vertex *D*. Then give the coordinates of the vertices of the image.

***A′*(9, −4), *B′*(7, −7), *C′*(4, −6), *D′*(3, −4)**



12. **Identify Structure** Identify each transformation as a *translation*, *reflection*, or *rotation*.

a.

reflection

b.

translation

c.

rotation

Copy and Solve **Triangle *MNP* has vertices *M*(1, 4), *N*(3, 1), and *P*(5, 3). Find the vertices of *M′N′P′* after each rotation about the origin. Show your work on a separate piece of paper.**

13. 90° clockwise
***M′*(4, −1),**
***N′*(1, −3),**
***P′*(3, −5)**

14. 180° clockwise
***M′*(−1, −4),**
***N′*(−3, −1),**
***P′*(−5, −3)**

15. 90° counterclockwise
***M′*(−4, 1),**
***N′*(−1, 3),**
***P′*(−3, 5)**

Maciej Maksymowicz/123RF, Ingram Publishing/AGE Fotostock, Ivan Kuzmin/Shutterstock.com

Power Up! Test Practice

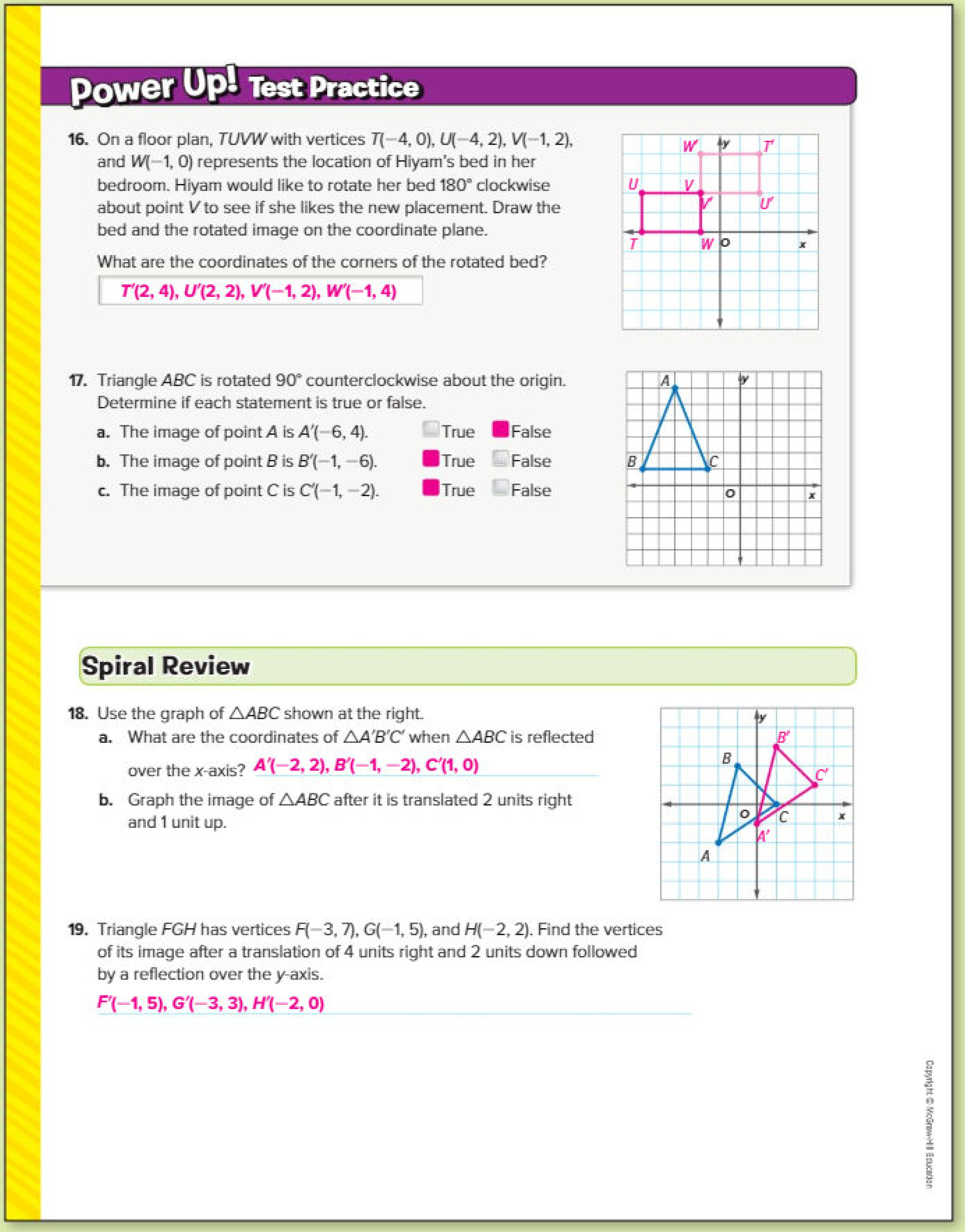
Exercises 17 and 18 prepare students for more rigorous thinking needed for assessment.

17.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

18.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK3
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly draw the figure and its rotation and list the coordinates.
1 point	Students correctly draw the figure and its rotation but fail to list the coordinates OR students correctly draw one figure and list the coordinates OR students correctly list the coordinates but fail to draw the figures.



Power Up! Test Practice

16. On a floor plan, *TUVW* with vertices *T*(−4, 0), *U*(−4, 2), *V*(−1, 2), and *W*(−1, 0) represents the location of Hiyam's bed in her bedroom. Hiyam would like to rotate her bed 180° clockwise about point *V* to see if she likes the new placement. Draw the bed and the rotated image on the coordinate plane.

What are the coordinates of the corners of the rotated bed?

T′(2, 4), U′(2, 2), V′(−1, 2), W′(−1, 4)

17. Triangle *ABC* is rotated 90° counterclockwise about the origin. Determine if each statement is true or false.

a. The image of point *A* is *A*′(−6, 4). ☐ True ☑ False

b. The image of point *B* is *B*′(−1, −6). ☑ True ☐ False

c. The image of point *C* is *C*′(−1, −2). ☑ True ☐ False

Spiral Review

18. Use the graph of △*ABC* shown at the right.

a. What are the coordinates of △*A′B′C′* when △*ABC* is reflected over the *x*-axis? **A′(−2, 2), B′(−1, −2), C′(1, 0)**

b. Graph the image of △*ABC* after it is translated 2 units right and 1 unit up.

19. Triangle *FGH* has vertices *F*(−3, 7), *G*(−1, 5), and *H*(−2, 2). Find the vertices of its image after a translation of 4 units right and 2 units down followed by a reflection over the *y*-axis.

F′(−1, 5), G′(−3, 3), H′(−2, 0)

Inquiry Lab

Dilations

Inquiry WHAT are the results of a dilation of a triangle?

MP Mathematical Practices 1, 3, 5

One way to create murals on a wall is to use a drawing grid method. Artists draw a grid on the artwork to be copied and draw a similar grid on the wall. By transferring sections of the artwork, the mural is the same shape as the artwork, but a different size.

Hands-On Activity 1

In this Activity, you will enlarge $\triangle ABC$ by a *scale factor* of 2 using grid paper. Point A will be the center point for the enlargement.

Step 1 On the grid shown below, $\overrightarrow{AB}$ is drawn to the edge of the grid. Draw $\overrightarrow{AC}$ in the same way.

Step 2 Draw point B' on $\overrightarrow{AB}$ so that $AB' = 2(AB)$. Draw point C' on $\overrightarrow{AC}$ so that $AC' = 2(AC)$.

Step 3 Draw $\overline{B'C'}$ to complete $\triangle AB'C'$.

What is the ratio of the length of $\overline{AB}$ to the length of $\overline{AB'}$? $\frac{5}{10}$ or $\frac{1}{2}$

What is the ratio of the length of $\overline{AC}$ to the length of $\overline{AC'}$? $\frac{6}{12}$ or $\frac{1}{2}$

What is the ratio of the length of $\overline{BC}$ to the length of $\overline{BC'}$? $\frac{1}{2}$

What do you notice about the ratios of corresponding sides?
Is $\triangle ABC$ similar to $\triangle A'B'C'$? They are equal; yes.

Focus narrowing the scope

Objective Identify dilations.

Coherence connecting within and across grades

Now
Students will measure angles and sides to generalize the properties of a dilation.

Next
Students will graph dilations on the coordinate plane.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 485.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activity 2.

Hands-On Activity 1

AL BL LA **Circle the Sage** Poll the class to see which students have some knowledge of dilations. Those students (the sages) spread out around the room. Assign the rest of the students to teams. Have the teams split up with each team member going to a different sage, if possible. Have the sages lead work for Activity 1. When the activity is complete, students go back to their teams and compare solutions. Students discuss how the sages may have explained the steps differently. MP 1, 5, 6, 7

Ask:

- *How does triangle ABC compare to triangle AB'C'?* Sample answer: They are the same shape but different sizes.

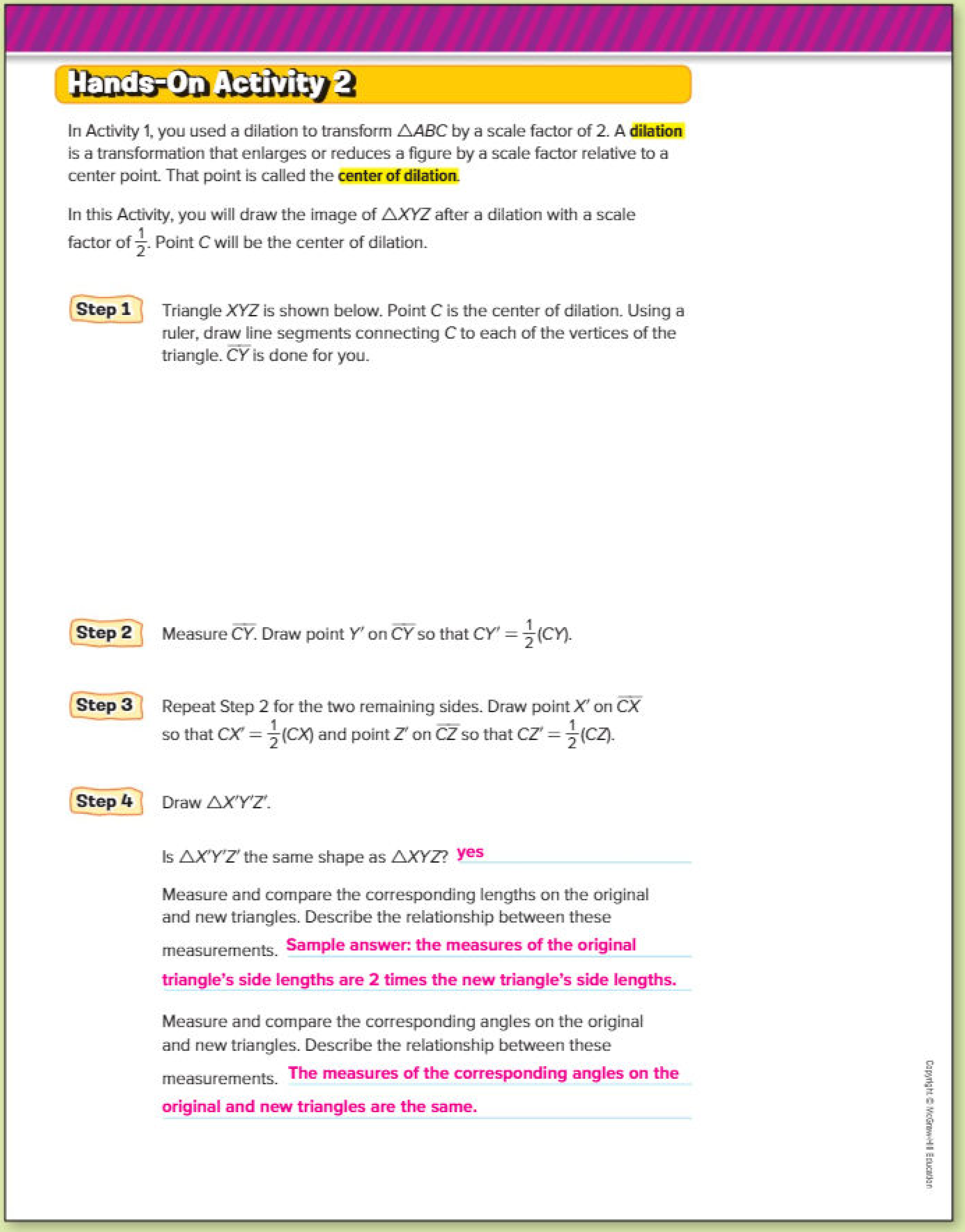
Hands-On Activity 2

AL LA **Circle the Sage** Select new sages to lead the activity based on their understanding of Activity 1. Repeat the same process, assigning new teams. MP 1, 3, 5, 6, 7

Ask:

- *Compare the scale factor in Activity 2 to the scale factor in Activity 1. How does the change in scale factor affect the dilation?* **Sample answer: The scale factor in Activity 1 was 2. The scale factor in Activity 2 is $\frac{1}{2}$. In Activity 1, the dilation was larger than the original triangle. In Activity 2, the dilation is smaller than the original triangle.**

BL LA **Rally Table** Assign students to pairs. Students take turns completing the tasks in Activity 2. Have them discuss their responses to the questions in Step 4, with each student listening carefully to the other's reasoning. Have them ask for clarification or assistance, if needed. MP 1, 5, 6, 7



Hands-On Activity 2

In Activity 1, you used a dilation to transform $\triangle ABC$ by a scale factor of 2. A **dilation** is a transformation that enlarges or reduces a figure by a scale factor relative to a center point. That point is called the **center of dilation**.

In this Activity, you will draw the image of $\triangle XYZ$ after a dilation with a scale factor of $\frac{1}{2}$. Point C will be the center of dilation.

Step 1 Triangle XYZ is shown below. Point C is the center of dilation. Using a ruler, draw line segments connecting C to each of the vertices of the triangle. $\overline{CY}$ is done for you.

Step 2 Measure $\overline{CY}$. Draw point Y' on $\overline{CY}$ so that $CY' = \frac{1}{2}(CY)$.

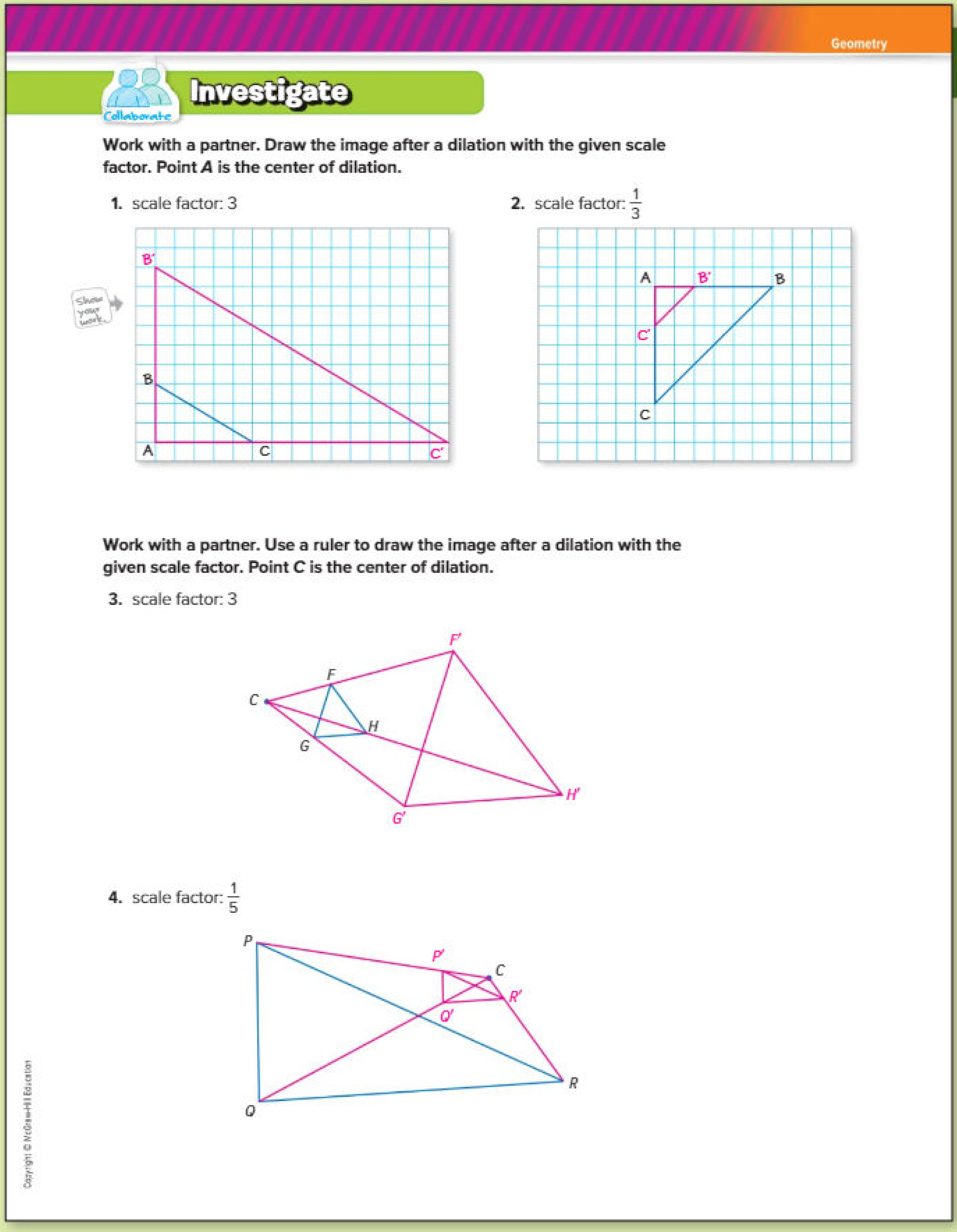
Step 3 Repeat Step 2 for the two remaining sides. Draw point X' on $\overline{CX}$ so that $CX' = \frac{1}{2}(CX)$ and point Z' on $\overline{CZ}$ so that $CZ' = \frac{1}{2}(CZ)$.

Step 4 Draw $\triangle X'Y'Z'$.

Is $\triangle X'Y'Z'$ the same shape as $\triangle XYZ$? **yes**

Measure and compare the corresponding lengths on the original and new triangles. Describe the relationship between these measurements. **Sample answer: the measures of the original triangle's side lengths are 2 times the new triangle's side lengths.**

Measure and compare the corresponding angles on the original and new triangles. Describe the relationship between these measurements. **The measures of the corresponding angles on the original and new triangles are the same.**



Investigate

Work with a partner. Draw the image after a dilation with the given scale factor. Point *A* is the center of dilation.

1. scale factor: 3

2. scale factor: $\frac{1}{3}$

Work with a partner. Use a ruler to draw the image after a dilation with the given scale factor. Point *C* is the center of dilation.

3. scale factor: 3

4. scale factor: $\frac{1}{5}$

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

	Exercises		
	1–4	5–10	11, 12
Level 3			●
Level 2		●	
Level 1	●		

Investigate

AL **LA** **Rally Coach** Have students work in pairs. While Student A works through Exercise 1, Student B watches, listens, coaches, and praises. Then partners trade roles for Exercise 2. Continue for Exercises 3 and 4. MP 1, 5, 6, 7

Ask:

- *What happens to the image when it is dilated by a scale factor that is a whole number greater than 1?* Sample answer: The new image is bigger.
- *What happens to the image when it is dilated by a scale factor that is a fraction between 0 and 1?* Sample answer: The new image is smaller.

BL **LA** **Trade-a-Problem** Students draw an image with 4 or 5 sides on a grid paper and choose a scale factor for the dilation. Students trade drawings and perform the dilation on their partner's drawing. Have students compare drawings and discuss any differences in solutions. MP 1, 5, 6, 7

Analyze and Reflect

Collaborate

AL **LA** **Think-Pair-Share** Give students time to complete Exercises 5–10 on their own. Then have them share answers with a partner and resolve any differences. MP 1, 5, 6, 7

Ask:

- *How are ratios written?* in fraction form

BL **LA** **Pairs Discussion** Give students time to complete Exercises 5–10 in pairs. Then have them trade answers with another pair of students and resolve any differences. MP 1, 5, 6, 7

Ask:

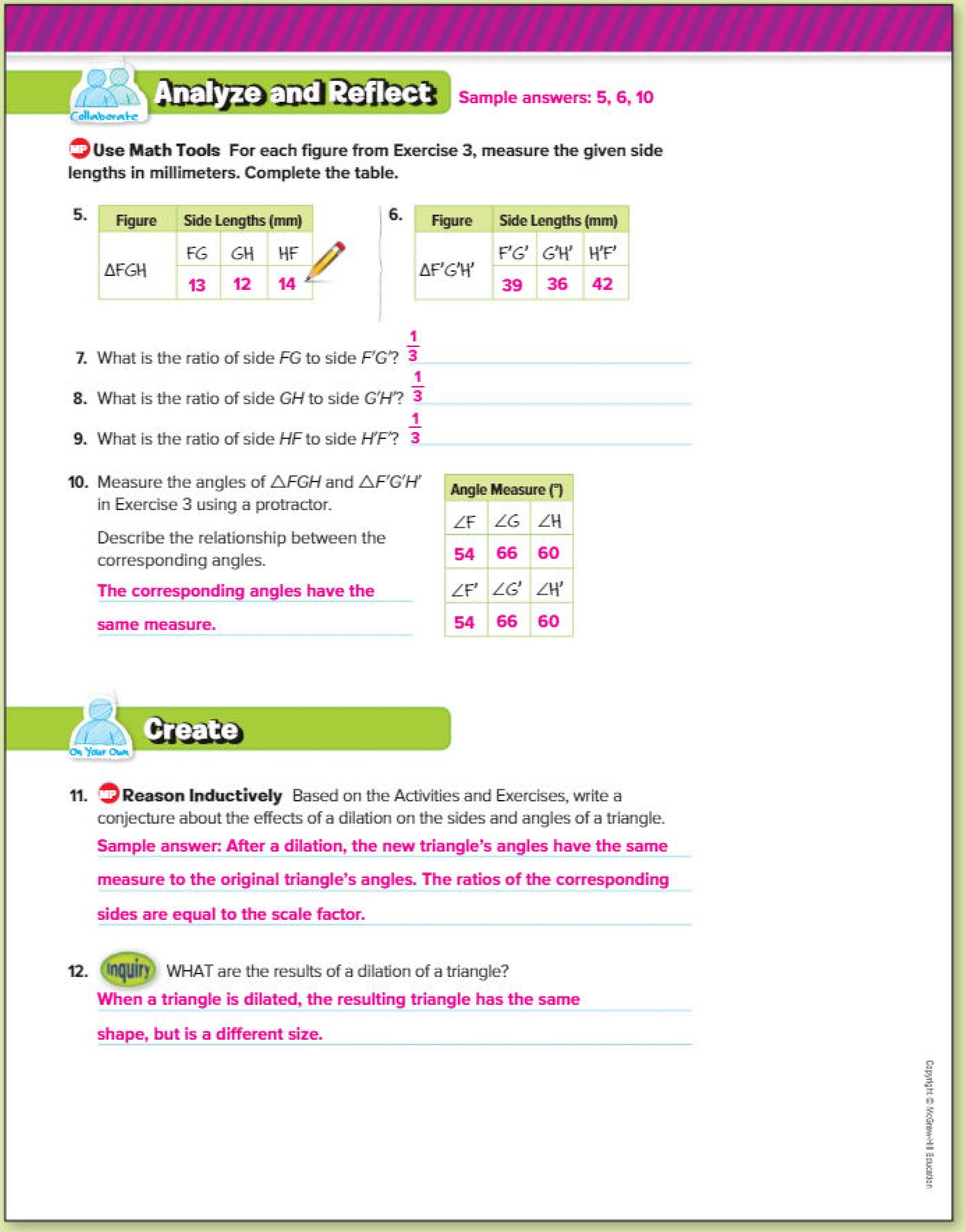
- *For Exercise 6, explain how you can complete the table without measuring.* Sample answer: Because the dilation had a scale factor of 3, I can multiply my answers in Exercise 5 by 3 to find the new side lengths.

Create

On Your Own

BL Discuss in a small group how to answer Exercises 11 and 12. Assign one group member as the leader. The leader facilitates the discussion and makes sure that every group member understands. MP 1, 3, 5, 6, 7

Inquiry Students should be able to answer "WHAT are the results of a dilation of a triangle?" Check for student understanding and provide guidance, if needed.



Analyze and Reflect

Collaborate

Sample answers: 5, 6, 10

MP **Use Math Tools** For each figure from Exercise 3, measure the given side lengths in millimeters. Complete the table.

5.

Figure	Side Lengths (mm)		
ΔFGH	FG	GH	HF
	13	12	14

6.

Figure	Side Lengths (mm)		
ΔF'G'H'	F'G'	G'H'	H'F'
	39	36	42

7. What is the ratio of side *FG* to side *F'G'*? $\frac{1}{3}$

8. What is the ratio of side *GH* to side *G'H'*? $\frac{1}{3}$

9. What is the ratio of side *HF* to side *H'F'*? $\frac{1}{3}$

10. Measure the angles of △*FGH* and △*F'G'H'* in Exercise 3 using a protractor.

Describe the relationship between the corresponding angles.

The corresponding angles have the same measure.

Angle Measure (°)		
∠F	∠G	∠H
54	66	60
∠F'	∠G'	∠H'
54	66	60

Create

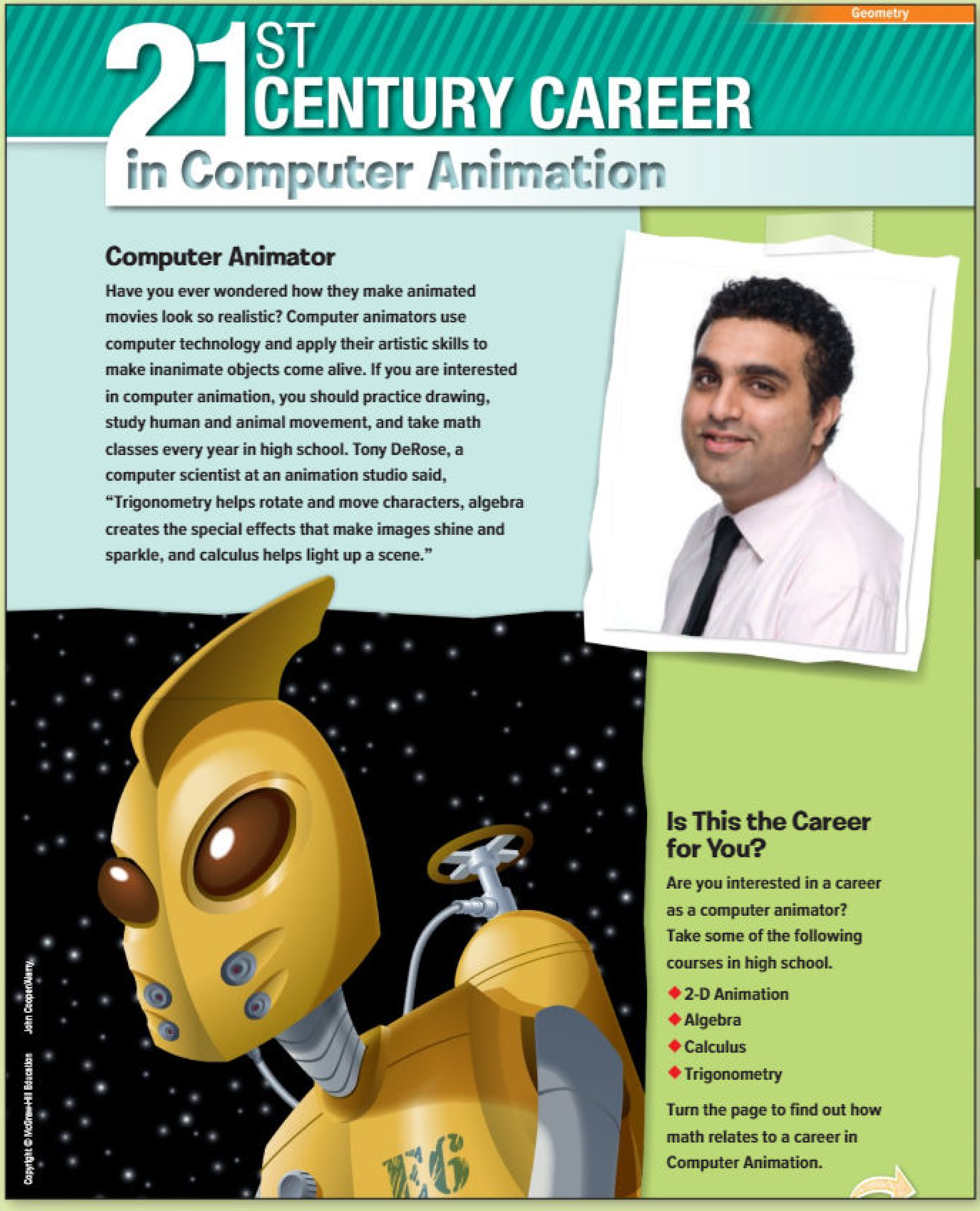
On Your Own

11. MP **Reason Inductively** Based on the Activities and Exercises, write a conjecture about the effects of a dilation on the sides and angles of a triangle.

Sample answer: After a dilation, the new triangle's angles have the same measure to the original triangle's angles. The ratios of the corresponding sides are equal to the scale factor.

12. Inquiry WHAT are the results of a dilation of a triangle?

When a triangle is dilated, the resulting triangle has the same shape, but is a different size.



21ST CENTURY CAREER in Computer Animation

Computer Animator

Have you ever wondered how they make animated movies look so realistic? Computer animators use computer technology and apply their artistic skills to make inanimate objects come alive. If you are interested in computer animation, you should practice drawing, study human and animal movement, and take math classes every year in high school. Tony DeRose, a computer scientist at an animation studio said, "Trigonometry helps rotate and move characters, algebra creates the special effects that make images shine and sparkle, and calculus helps light up a scene."

Is This the Career for You?

Are you interested in a career as a computer animator? Take some of the following courses in high school.

- ◆ 2-D Animation
- ◆ Algebra
- ◆ Calculus
- ◆ Trigonometry

Turn the page to find out how math relates to a career in Computer Animation.

Focus narrowing the scope

Objective Apply mathematics to problems arising in the workplace.

This lesson emphasizes **MP Mathematical Practice 4** Model with Mathematics.

Coherence connecting within and across grades

Previous	**Now**
Students described tranformations of figures.	Students apply the content standard to solve problems in the workplace.

Rigor pursuing concepts, fluency, and applications

See the Career Project on page 496.

ENGAGE **EXPLORE** EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ask students to read the information on the student page about computer animation and answer the following questions.

Ask:

- *What kinds of classes should you take to be a computer animator?* 2D Animation, Algebra, Calculus, Trigonometry
- *What does a computer animator do?* A computer animator uses computer technology and artistic skills to make inanimate objects come alive onscreen.
- *What should a person who wants to be a computer animator do to prepare?* practice drawing, study human and animal movement, and study the math needed to create the effects they are trying to achieve in their animation

Help students connect what they do today to their futures.

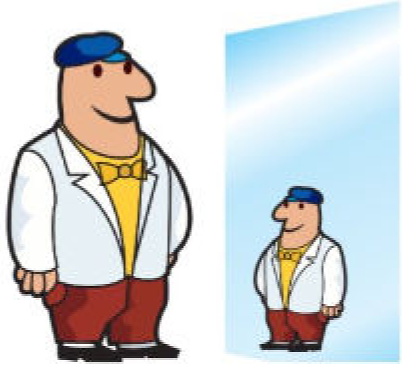
2 Collaborate

AL LA **Rally Coach** Have students work in pairs to complete Exercises 1–3. Student 1 completes the first exercise while Student 2 listens, coaches, and praises. Then, Student 2 completes the next exercise while Student 1 listens, coaches, and praises. Partners take turns until the exercises are completed. MP 1, 5, 6, 7

BL LA **Pairs Discussion** Have students work in pairs to extend the activity by answering the following question. MP 1, 3, 5, 6, 7

Ask:

- *Describe how more than one transformation was used to create the characters below.* The character on the right is a reflection and dilation of the character on the left.

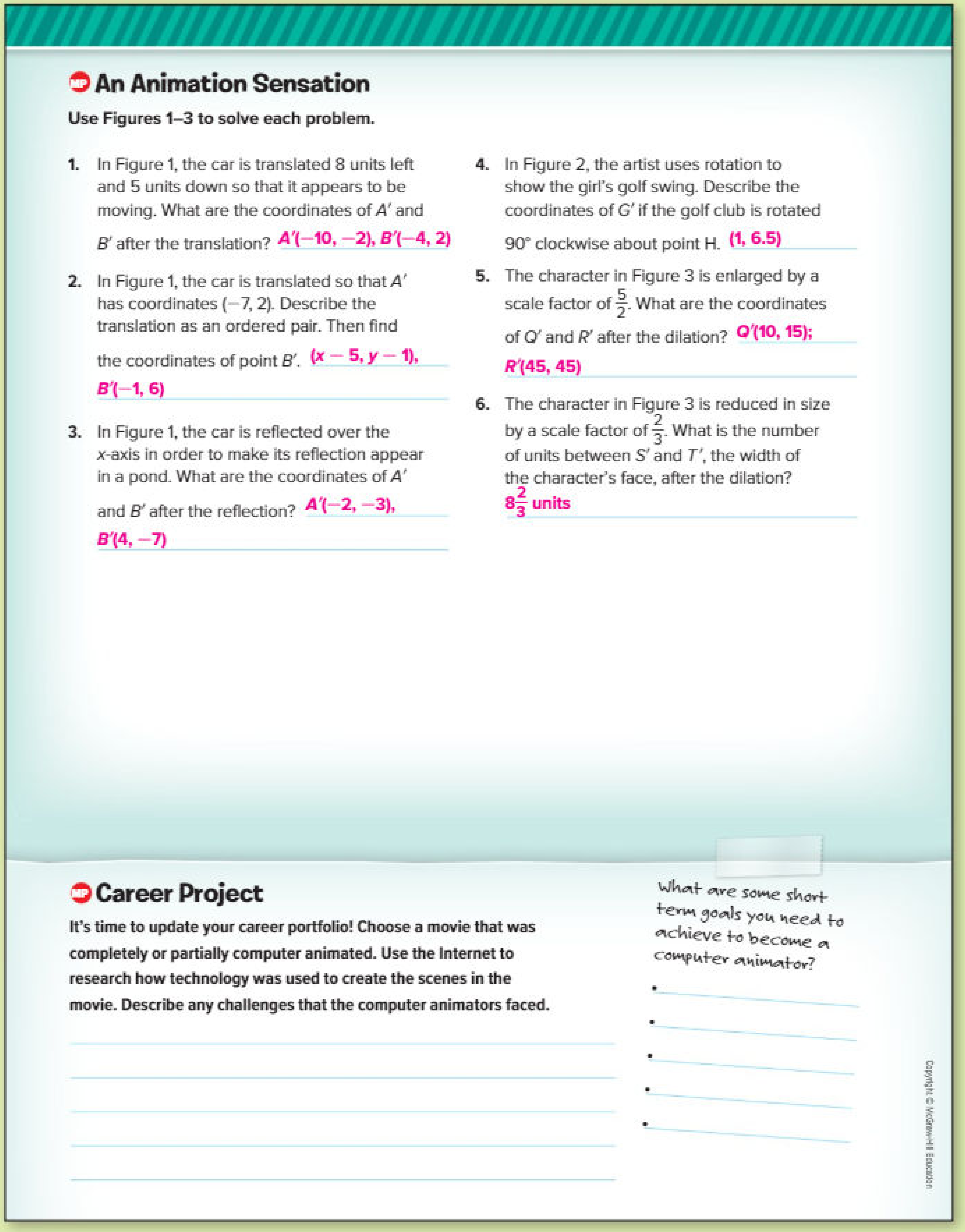


Career Portfolio

When students complete this page, have them add it to their Career Portfolio.

Career Facts

As mathematics and computer techniques become more advanced, computer animation becomes more lifelike. Computer animators are able to use advanced mathematics and computer techniques to create realistic water, waves, and splash effects.



MP An Animation Sensation

Use Figures 1–3 to solve each problem.

1. In Figure 1, the car is translated 8 units left and 5 units down so that it appears to be moving. What are the coordinates of A' and B' after the translation? **$A'(-10, -2)$, $B'(-4, 2)$**

2. In Figure 1, the car is translated so that A' has coordinates $(-7, 2)$. Describe the translation as an ordered pair. Then find the coordinates of point B'. **$(x - 5, y - 1)$, $B'(-1, 6)$**

3. In Figure 1, the car is reflected over the x-axis in order to make its reflection appear in a pond. What are the coordinates of A' and B' after the reflection? **$A'(-2, -3)$, $B'(4, -7)$**

4. In Figure 2, the artist uses rotation to show the girl's golf swing. Describe the coordinates of G' if the golf club is rotated 90° clockwise about point H. **(1, 6.5)**

5. The character in Figure 3 is enlarged by a scale factor of $\frac{5}{2}$. What are the coordinates of Q' and R' after the dilation? **$Q'(10, 15)$; $R'(45, 45)$**

6. The character in Figure 3 is reduced in size by a scale factor of $\frac{2}{3}$. What is the number of units between S' and T', the width of the character's face, after the dilation? **$8\frac{2}{3}$ units**

MP Career Project

It's time to update your career portfolio! Choose a movie that was completely or partially computer animated. Use the Internet to research how technology was used to create the scenes in the movie. Describe any challenges that the computer animators faced.

What are some short term goals you need to achieve to become a computer animator?

-
-
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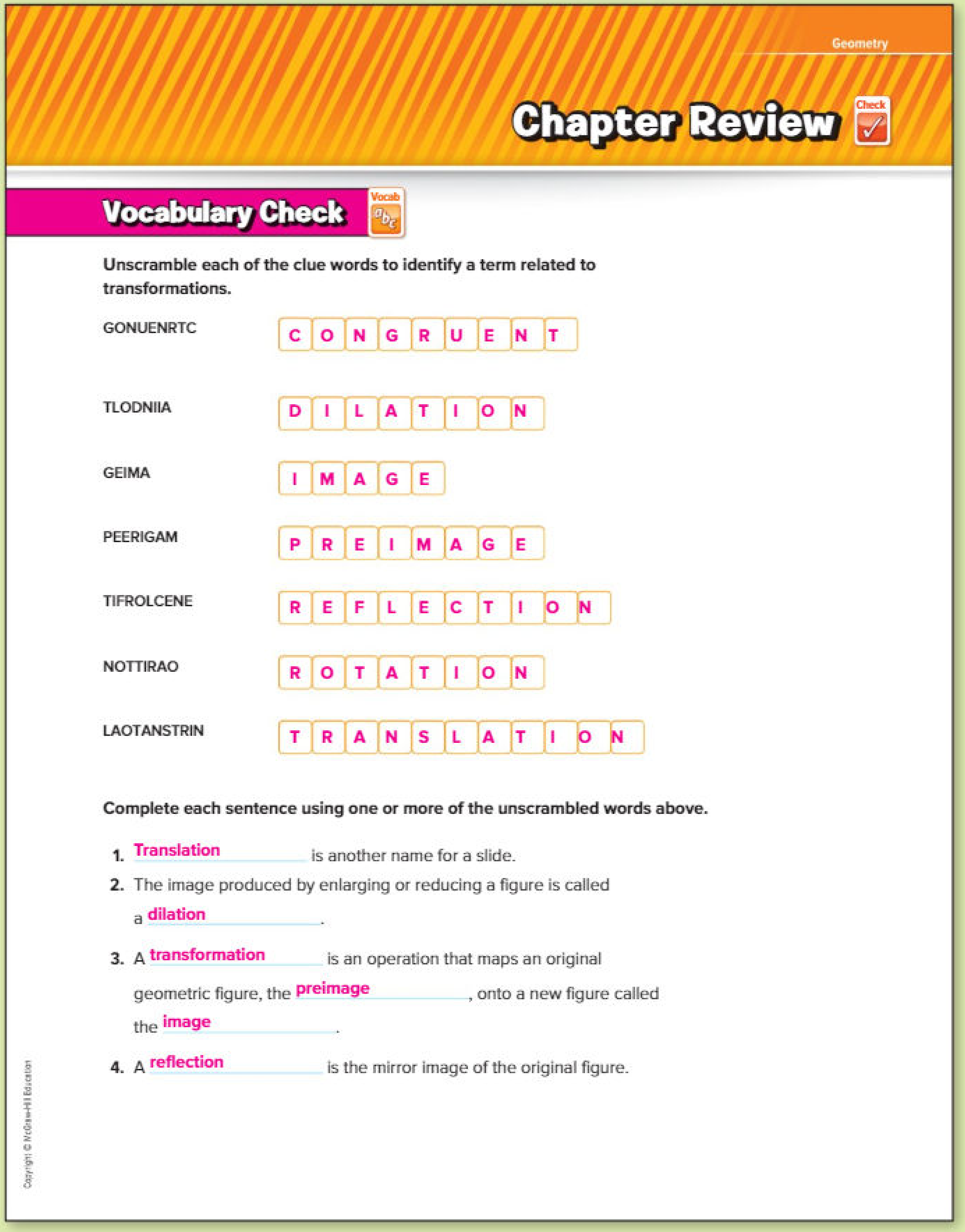


Chapter Review

Vocabulary Check

Unscramble each of the clue words to identify a term related to transformations.

GONUENRTC	C O N G R U E N T
TLODNIIA	D I L A T I O N
GEIMA	I M A G E
PEERIGAM	P R E I M A G E
TIFROLCENE	R E F L E C T I O N
NOTTIRAO	R O T A T I O N
LAOTANSTRIN	T R A N S L A T I O N

Complete each sentence using one or more of the unscrambled words above.

1. Translation is another name for a slide.
2. The image produced by enlarging or reducing a figure is called a dilation.
3. A transformation is an operation that maps an original geometric figure, the preimage, onto a new figure called the image.
4. A reflection is the mirror image of the original figure.

Vocabulary Check

LA **Find the Fib** Have students work in groups and write down two facts and one fib using the words in the Vocabulary Check. For example, one fact could be that a translation is another name for a slide. One fib could be that the original geometric figure in a transformation is called the image. Each team member shares their facts and fib by speaking aloud to the group. The job of the group is to listen carefully, discuss, and come to a consensus to identify the fib. MP 1, 3, 6

Alternate Strategy

AL LA To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include the following terms.

- congruent (Lesson 1)
- dilation (Lesson 4)
- image (Lesson 1)
- preimage (Lesson 1)
- reflection (Lesson 2)
- rotation (Lesson 3)
- translation (Lesson 1)

Key Concept Check

FOLDABLES LA A completed Foldable for this chapter should include translations, reflections, rotations, and dilations.

If you choose not to use this Foldable, have students write a brief review of the Key Concepts found throughout the chapter and give an example of each.

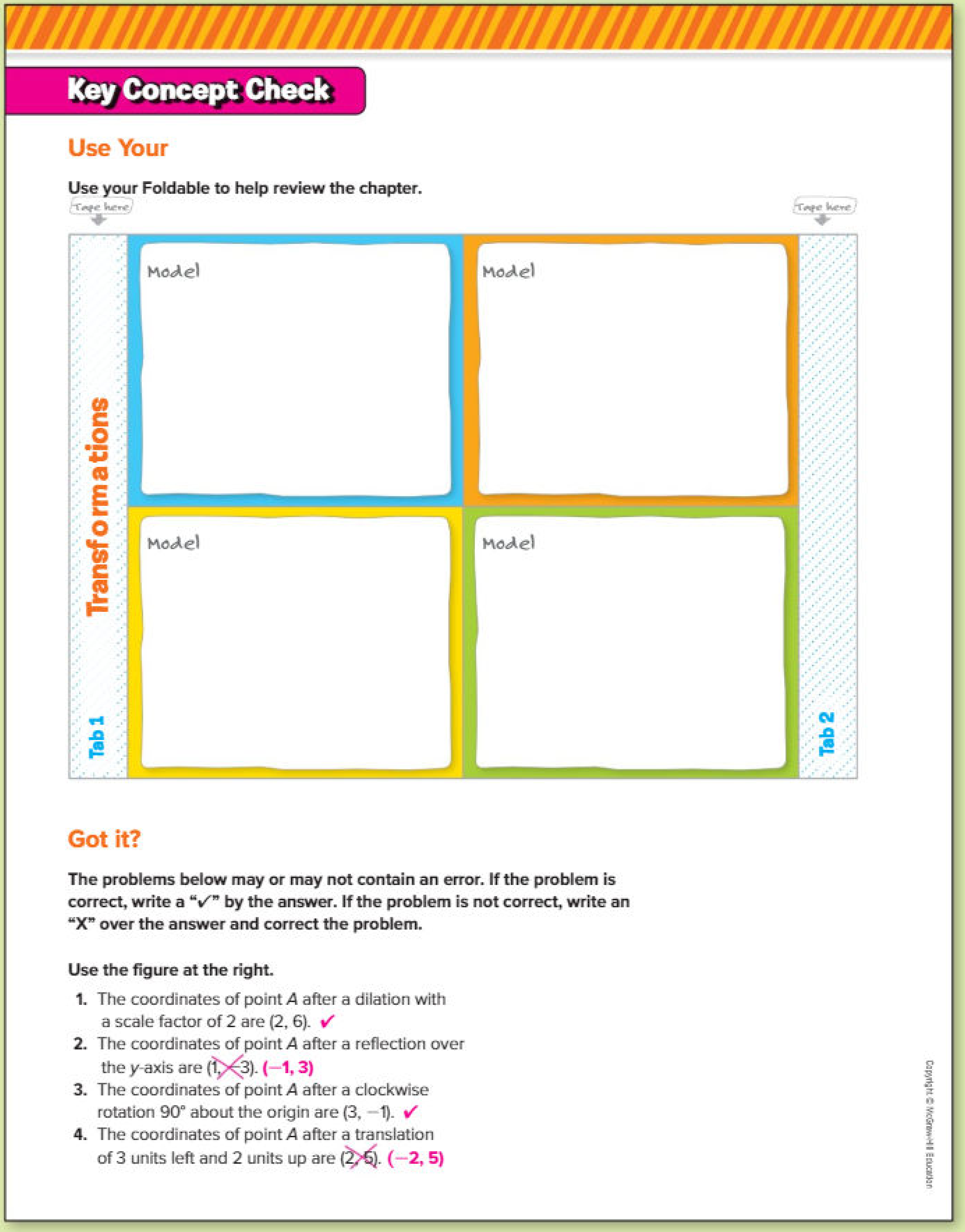
Ideas for Use

LA **Gallery Walk** Have students work with a partner to share their completed Foldables. Then have each student add or adjust anything in their Foldable based on the discussion with their partner. Display all of the Foldables around the room and have students walk around the room studying each Foldable. Have them determine if they should add anything to their Foldable based upon what they saw in others' Foldables.
MP 1, 5, 6, 7

Got It?

If students have trouble with Exercises 1–4, they may need help with the following concept(s).

Concept	Exercise(s)
dilations (Lesson 4)	1
reflections (Lesson 2)	2
rotations (Lesson 3)	3
translations (Lesson 1)	4



Key Concept Check

Use Your

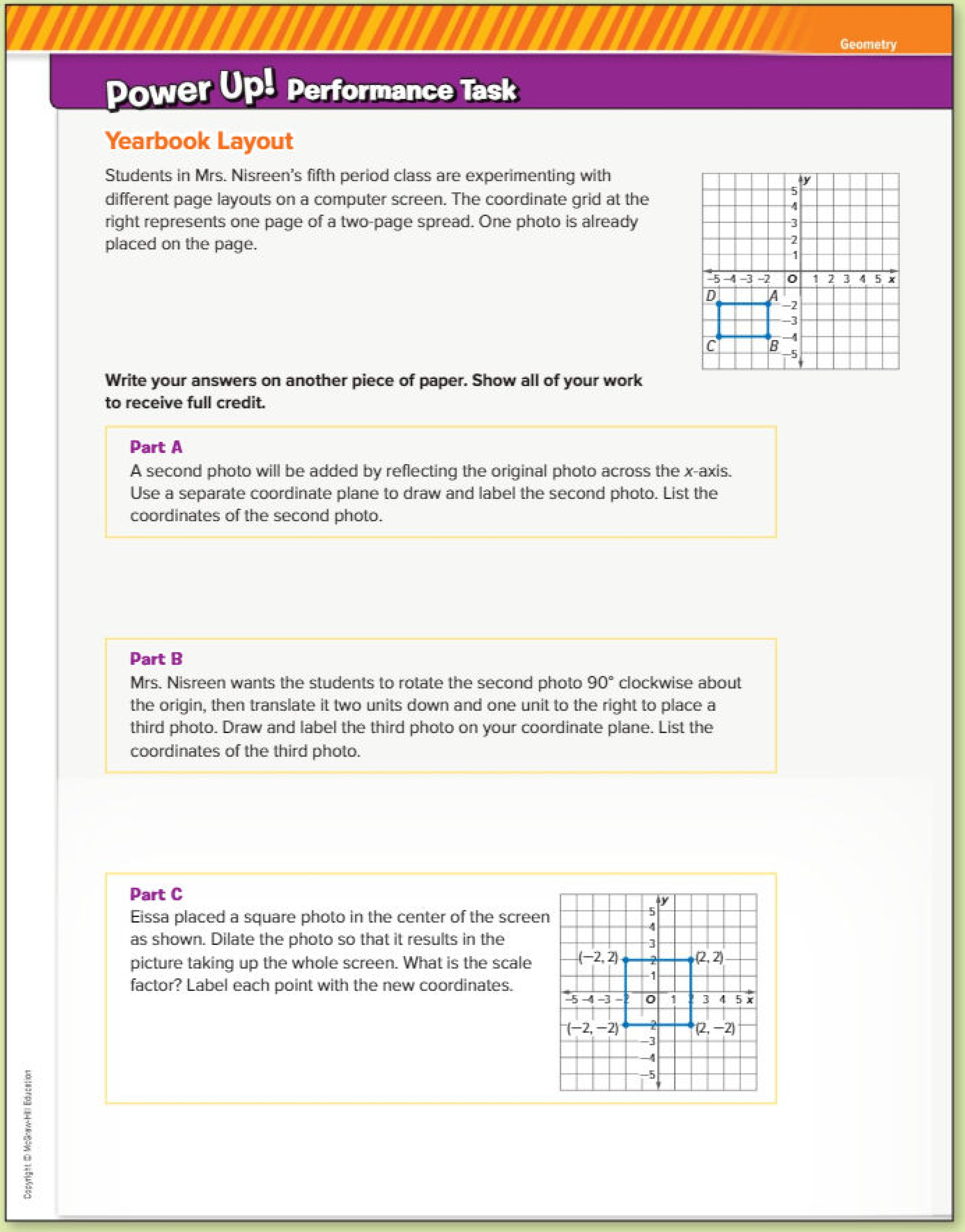
Use your Foldable to help review the chapter.

Got it?

The problems below may or may not contain an error. If the problem is correct, write a "✓" by the answer. If the problem is not correct, write an "X" over the answer and correct the problem.

Use the figure at the right.

1. The coordinates of point *A* after a dilation with a scale factor of 2 are (2, 6). ✓
2. The coordinates of point *A* after a reflection over the *y*-axis are ~~(1, 3)~~. **(−1, 3)**
3. The coordinates of point *A* after a clockwise rotation 90° about the origin are (3, −1). ✓
4. The coordinates of point *A* after a translation of 3 units left and 2 units up are ~~(2, 5)~~. **(−2, 5)**



Power Up! Performance Task

Yearbook Layout

Students in Mrs. Nisreen's fifth period class are experimenting with different page layouts on a computer screen. The coordinate grid at the right represents one page of a two-page spread. One photo is already placed on the page.

Write your answers on another piece of paper. Show all of your work to receive full credit.

Part A

A second photo will be added by reflecting the original photo across the *x*-axis. Use a separate coordinate plane to draw and label the second photo. List the coordinates of the second photo.

Part B

Mrs. Nisreen wants the students to rotate the second photo 90° clockwise about the origin, then translate it two units down and one unit to the right to place a third photo. Draw and label the third photo on your coordinate plane. List the coordinates of the third photo.

Part C

Eissa placed a square photo in the center of the screen as shown. Dilate the photo so that it results in the picture taking up the whole screen. What is the scale factor? Label each point with the new coordinates.

Power Up! Performance Task

This Performance-Based Assessment requires students to solve multi-step problems through abstract reasoning, precision, and perseverance. This practice scenario can be used to help students prepare for the thinking skills that will be used on the CCSS Assessment.

A complete scoring rubric with answers to the Exercises can be found on page PT2.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the **Building on the Essential Question** exercises found in each lesson of the chapter.

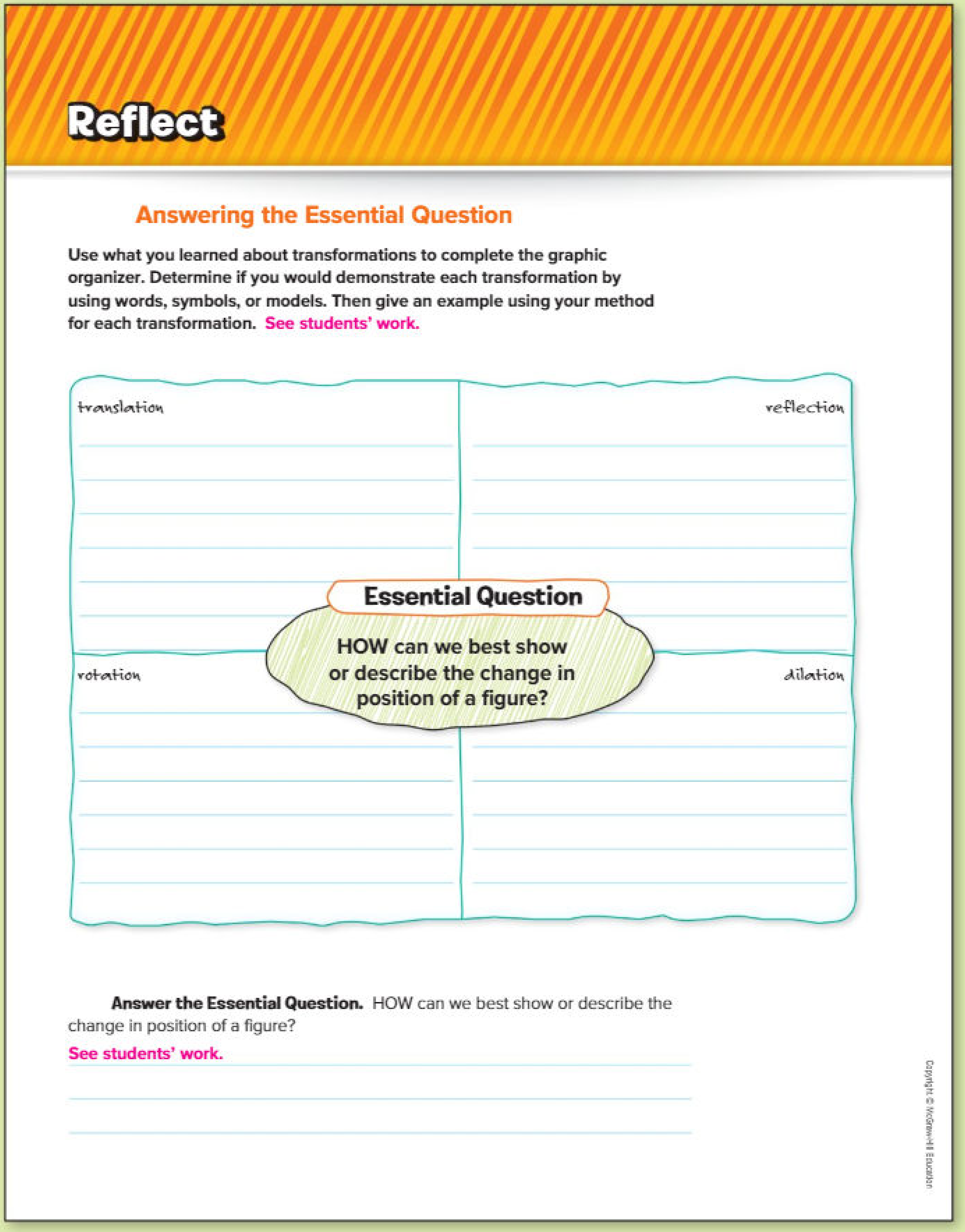
- How are figures translated on the coordinate plane? (p. 456)
- How can you determine the coordinates of a figure after a reflection over either axis? (p. 464)
- What is the difference between rotating a figure about a given point that is a vertex and rotating the same figure about the origin if the rotation is less than 360°? (p. 478)
- How are dilations similar to scale drawings? (p. 490)

Ideas for Use

LA **Think-Pair-Share** Have students work in pairs. Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer. Then have them share their responses with their partner before they complete the graphic organizer. MP 1, 5, 6, 7

Track Your Progress

Return to the beginning of the chapter to review the objectives that it addressed. Students should see that their knowledge of the key ideas has increased now that they have completed this chapter.



Reflect

Answering the Essential Question

Use what you learned about transformations to complete the graphic organizer. Determine if you would demonstrate each transformation by using words, symbols, or models. Then give an example using your method for each transformation. **See students' work.**

translation

reflection

Essential Question

HOW can we best show or describe the change in position of a figure?

rotation

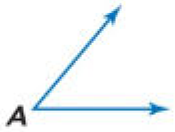
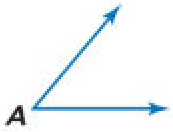
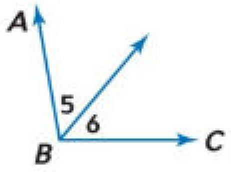
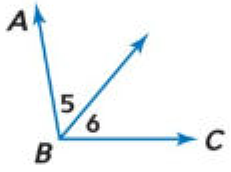
dilation

Answer the Essential Question. HOW can we best show or describe the change in position of a figure?

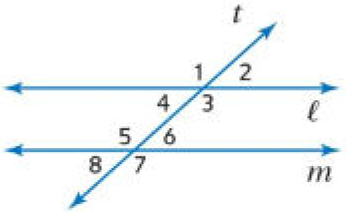
See students' work.

قاموس المصطلحات

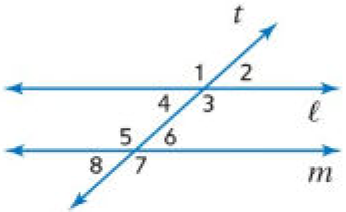
A

English	العربية
accuracy The degree of closeness of a measurement to the true value.	**دقة القياس** درجة قرب القياس إلى القيمة الحقيقية.
acute angle An angle whose measure is less than 90°.	**الزاوية الحادة** زاوية قياسها أصغر من 90 درجة.
	
acute triangle A triangle with all acute angles.	**المثلث حاد الزوايا** مثلث كل زواياه حادة.
Addition Property of Equality If you add the same number to each side of an equation, the two sides remain equal.	**الجمع في المعادلة** إذا أضفت العدد نفسه إلى كل طرف في المعادلة، يبقى الطرفان متساويين.
adjacent angles Angles that share a common vertex, a common side, and do not overlap. In the figure, the adjacent angles are $\angle 5$ and $\angle 6$.	**الزوايا المتجاورة** الزوايا التي لها رأس مشترك، وضلع مشترك، ولا تتداخل. في الشكل التالي، الزوايا المتجاورة هي $\angle 5$ و $\angle 6$.
	
algebra A branch of mathematics that involves expressions with variables.	**الجبر** فرع من الرياضيات يحتوي على عبارات تتضمن متغيرات.
algebraic expression A combination of variables, numbers, and at least one operation.	**التعبير الجبري** مجموعة من المتغيرات والأعداد وعملية واحدة على الأقل.

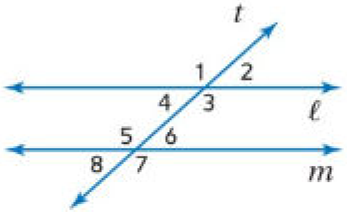
alternate exterior angles Exterior angles that lie on opposite sides of the transversal. In the figure, transversal t intersects lines l and m. $\angle 1$ and $\angle 7$, and $\angle 2$ and $\angle 8$ are alternate exterior angles. If line l and m are parallel, then these pairs of angles are congruent.



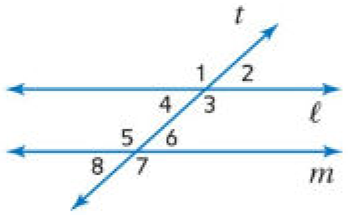
الزوايا الخارجية المتبادلة زوايا خارجية توجد على جانبين متقابلين من القاطع. في الشكل التالي، يتداخل القاطع t مع المستقيمين ℓ و m. $\angle 1$ و $\angle 7$، $\angle 2$ و $\angle 8$ هي زوايا خارجية متبادلة. إذا كان المستقيمان ℓ و m متوازيين، فإن أزواج هذه الزوايا تكون متطابقة.



alternate interior angles Interior angles that lie on opposite sides of the transversal. In the figure below, transversal t intersects lines l and m. $\angle 3$ and $\angle 5$, and $\angle 4$ and $\angle 6$ are alternate interior angles. If lines l and m are parallel, then these pairs of angles are congruent.



الزوايا الداخلية البديلة زوايا داخلية توجد على ضلعين متقابلين من القاطع. في الشكل أدناه، يتداخل القاطع t مع المستقيمين ℓ و m. $\angle 3$ و $\angle 5$، $\angle 4$ و $\angle 6$ هي زوايا داخلية متبادلة. إذا كان المستقيمان ℓ و m متوازيين، فإن أزواج الزوايا هذه متطابقة.



angle of rotation The degree measure of the angle through which a figure is rotated.

زاوية الدوران المحوري مقياس بالدرجات للزاوية التي يستدير من خلالها الشكل.

arc One of two parts of a circle separated by a central angle.

القوس أحد جزأي دائرة تفصله زاوية مركزية.

Associative Property The way in which three numbers are grouped when they are added or multiplied does not change their sum or product.

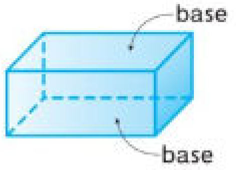
خاصية التجميع الطريقة التي يتم فيها تجميع ثلاثة أعداد عند إضافتها أو ضربها بحيث لا يتغير المجموع أو ناتج الضرب.

B

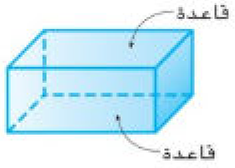
base In a power, the number that is the common factor. In 10^3, the base is 10. That is, $10^3 = 10 \times 10 \times 10$.

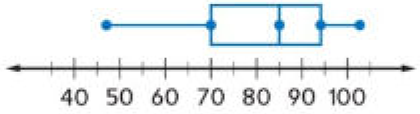
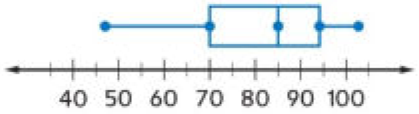
الأساس عملية القوى، هو العدد الذي يتكرّر في عملية الضرب. في المثال 10^3، يكون الأساس هو العدد 10. بمعنى أن، $10^3 = 10 \times 10 \times 10$.

base One of the two parallel congruent faces of a prism.

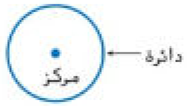


قاعدة المنشور أحد الوجهين المتطابقين المتوازيين في المنشور.



biased sample A sample drawn in such a way that one or more parts of the population are favored over others.	**العينة المُنحازة** عينة تُجمع بطريقة يُفضلها جزء أو أكثر من المجتمع الإحصائي مقارنةً بالمجتمعات الأخرى.
bivariate data Data with two variables, or pairs of numerical observations.	**البيانات ذات المتغيرين** بيانات تحتوي على متغيرين (أو زوجين من المشاهدات العددية).
box plot A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.  	**مخطط الصندوق ذي العارضين** طريقة لعرض توزيع قيم البيانات بصريًا باستخدام الوسيط والرُبعيّات وأطراف مجموعة البيانات. ويظهر الصندوق الوسيط بنسبة 50% من البيانات. 

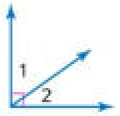
C

center The given point from which all points on a circle are the same distance.	**مركز الدائرة** النقطة المعلومة التي تبعد عنها كل النقاط على الدائرة المسافة نفسها.
center of dilation The center point from which dilations are performed.	**مركز تغيير الأبعاد** نقطة المركز الذي يتم تغيير الأبعاد منها.
center of rotation A fixed point around which shapes move in a circular motion to a new position.	**مركز الدوران المحوري** نقطة ثابتة تتحرك حولها الأشكال بحركة دائرية إلى موقع جديد.
central angle An angle that intersects a circle in two points and has its vertex at the center of the circle.	**الزاوية المركزية** زاوية رأسها مركز وضلعاها أنصاف أقطار في الدائرة.
circle The set of all points in a plane that are the same distance from a given point called the center.  	**الدائرة** مجموعة النقاط في المستوى التي لها البُعد نفسه عن نقطة معلومة ثابتة تُسمى المركز. 
circumference The distance around a circle.  	**محيط الدائرة** المسافة حول الدائرة. 
chord A segment with endpoints that are on a circle.	**الوتر** قطعة طرفاها على محيط دائرة.
coefficient The numerical factor of a term that contains a variable.	**المعامل** عامل عددي للحد الذي يحتوي على متغير.
common difference The difference between any two consecutive terms in an arithmetic sequence.	**الفرق المشترك** الفرق بين حدين متتاليين في المتتالية الحسابية. ويُسمّى أساس المتتالية الحسابية d.

Commutative Property The order in which two numbers are added or multiplied does not change their sum or product.

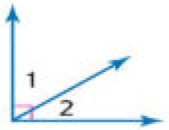
خاصية التبديل الترتيب الذي يتم به إضافة أو ضرب عددين بحيث لا يتغير مجموعهما أو ناتج ضربهما.

complementary angles Two angles are complementary if the sum of their measures is 90°.



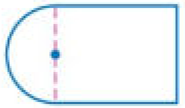
∠1 and ∠2 are complementary angles.

الزاويتان المتتامتان تكون الزاويتان متتامتين إذا كان مجموع قياسهما يساوي 90°.

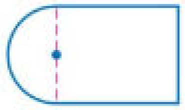


∠1 و ∠2 زاويتان متتامتان.

composite figure A figure that is made up of two or more shapes.



الشكل المركب الشكل المكوّن من شكلين أو أكثر.



composite solid An object made up of more than one type of solid.

المجسم المركب جسم يتكون من أكثر من نوع واحد من المجسمات.

composition of transformations The resulting transformation when a transformation is applied to a figure and then another transformation is applied to its image.

تركيب التحويلات التحويل الناتج عند تطبيق تحويل على شكل ما ثم تطبيق تحويل آخر على صورة هذا الشكل.

compound event An event that consists of two or more simple events.

الحدث المركب حدث مكوّن من حدثين بسيطين أو أكثر.

compound interest Interest paid on the initial principal and on interest earned in the past.

الفائدة المركبة فائدة تدفع على رأس المال الأولي وعلى الفائدة المكتسبة سابقًا.

cone A three-dimensional figure with one circular base connected by a curved surface to a single vertex.

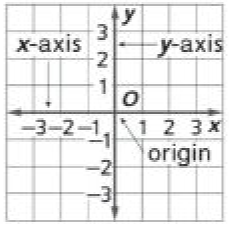
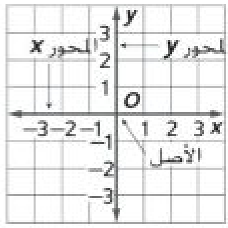
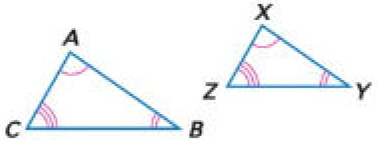
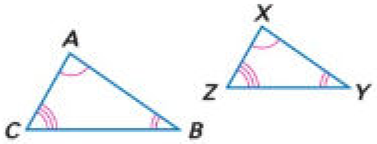
المخروط شكل ثلاثي الأبعاد قاعدته دائرية الشكل ومتصلة بسطح منحنٍ ورأس واحدة فقط.

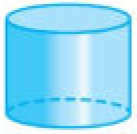
congruent Having the same measure; if one image can be obtained by another by a sequence of rotations, reflections, or translations.

المتطابق زاوية لها القياس نفسه؛ أو شكل مماثل تمامًا لشكل آخر وينطبق عليه عن طريق سلسلة من الدورانات المحورية أو الانعكاسات أو التفسيرات.

constant A term without a variable.

الثابت حد لا يحتوي على متغير.

constant of proportionality The constant ratio in a proportional linear relationship.	**ثابت التناسب** نسبة ثابتة في علاقة التناسب الخطية.
constant of variation A constant ratio in a direct variation.	**ثابت التغير** نسبة ثابتة في حالة التغير الطردي.
constant rate of change The rate of change between any two points in a linear relationship is the same or constant.	**معدل التغير الثابت** تماثل أو ثبات نسبة التغير بين أي نقطتين في علاقة خطية.
continuous data Data that can take on any value. There is no space between data values for a given domain. Graphs are represented by solid lines.	**البيانات المتصلة** بيانات قد تأخذ أي قيمة. ولا توجد مساحة بين قيم البيانات الخاصة بمجال معين. وتمثل بيانيًا عن طريق خطوط المجسم.
convenience sample A sample which includes members of the population that are easily accessed.	**العينة الملائمة** عينة تتضمن أفرادًا من المجتمع الإحصائي يسهل الوصول إليهم.
converse The converse of a theorem is formed when the parts of the theorem are reversed. The converse of the Pythagorean Theorem can be used to test whether a triangle is a right triangle. If the sides of the triangle have lengths a, b, and c, such that $c^2 = a^2 + b^2$, then the triangle is a right triangle.	**العكس** يتشكل عكس نظرية عندما يتم عكس أجزاء من النظرية. يمكن استخدام عكس نظرية فيثاغورس لمعرفة إذا كان المثلث قائم الزاوية.إذا كانت أطوال أضلاع المثلث هي a، وb، و c. بحيث إن $c^2 = a^2 + b^2$. إذا هذا المثلث قائم الزاوية..
coordinate plane A coordinate system in which a horizontal number line and a vertical number line intersect at their zero points.	**المستوى الإحداثي** نظام إحداثي يكون فيه خط الأعداد الأفقي وخط الأعداد الرأسي متقاطعين في النقاط الصفرية.
	
coplanar Lines that lie in the same plane.	**مستقيمات مستوية** المستقيمات التي تقع في المستوى نفسه.
corresponding angles Angles that are in the same position on two parallel lines in relation to a transversal.	**الزوايا المتناظرة** زوايا تقع في الجهة والموقع أنفسهما من قاطع يقطع مستقيمين متوازيين.
corresponding parts Parts of congruent or similar figures that match.	**الأجزاء المتناظرة** أجزاء من أشكال متطابقة أو متشابهة تتوافق مع بعضها.
	

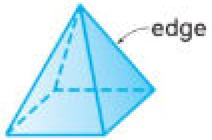
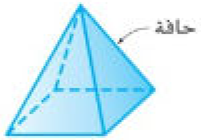
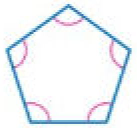
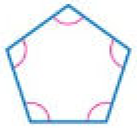
counterexample A statement or example that shows a conjecture is false.	**المثال المضاد** عبارة أو مثال يدل على فرضية غير صحيحة.
cross section The intersection of a solid and a plane.	**المقطع** تقاطع مجسم ومستوى.
cube root One of three equal factors of a number. If $a^3 = b$, then a is the cube root of b. The cube root of 64 is 4 since $4^3 = 64$.	**الجذر التكعيبي** أحد العوامل الثلاثة المتساوية في العدد. إذا كان $a^3 = b$. فإن a هو الجذر التكعيبي لــ b. الجذر التكعيبي للعدد 125 هو 5. لأن $64 - 4^3$.
cylinder A three-dimensional figure with two parallel congruent circular bases connected by a curved surface.	**الأسطوانة** شكل ثلاثي الأبعاد يحتوي على قاعدتين دائريتين متطابقتين ومتوازيتين وتتصلان ببعضهما عن طريق سطح منحنٍ.
	

D

deductive reasoning A system of reasoning that uses facts, rules, definitions, or properties to reach logical conclusions.	**الاستدلال الاستنتاجي** أسلوب التفكير الذي يستخدم الحقائق أو القواعد أو التعريفات أو الخصائص للتوصل إلى استنتاجات صحيحة.
defining a variable Choosing a variable and a quantity for the variable to represent in an expression or equation.	**تعيين متغير** اختيار متغير وكمية لهذا المتغير لتمثيلها في عبارة أو معادلة.
degree A unit used to measure angles.	**الدرجة** وحدة قياس الزوايا.
degree A unit used to measure temperature.	**الدرجة** وحدة قياس درجة الحرارة.
dependent events Two or more events in which the outcome of one event does affect the outcome of the other event or events.	**الأحداث غير المستقلة** حدثان أو أكثر تؤثر نتيجة إحداهما في نتيجة الحدث الآخر أو الأحداث الأخرى.
dependent variable The variable in a relation with a value that depends on the value of the independent variable.	**المتغير التابع** المتغير الذي تعتمد قيمته في علاقة على قيمة المتغير المستقل.
derived unit A unit that is derived from a measurement system base unit, such as length, mass, or time.	**الوحدة المشتقة** وحدة مشتقة من وحدة أساسية لأحد أنظمة القياس، مثل الطول أو الكتلة أو الزمن.
diagonal A line segment whose endpoints are vertices that are neither adjacent nor on the same face.	**قطر ثلاثي الأبعاد** قطعة مستقيمة ثلاثية الأبعاد نقطتا نهايتها عبارة عن رأسين ليسا متجاورين ولا على الوجه نفسه.

diameter The distance across a circle through its center.  	**قطر الدائرة** المسافة المارة بالمركز داخل الدائرة، وهو وتر مار بمركز الدائرة. 
dilation A transformation that enlarges or reduces a figure by a scale factor.	**تغيير الأبعاد بمقياس** تحويل هندسي لتصغير أو لتكبير شكل عن طريق معامل المقياس.
dimensional analysis The process of including units of measurement when you compute.	**التحليل البُعدي** عملية تضمين وحدات قياس أثناء إجراء العمليات الحسابية.
direct variation A relationship between two variable quantities with a constant ratio.	**التغيّر الطردي** العلاقة بين كميتي متغير ذات نسبة ثابتة.
discount The amount by which a regular price is reduced.	**الخصم** مقدار تخفيض السعر المعتاد.
discrete data Data with space between possible data values. Graphs are represented by dots.	**البيانات المتقطعة** بيانات بها مساحة بين قيم البيانات المحتملة. وتمثل بيانيًا عن طريق نقاط.
disjoint events Events that cannot happen at the same time.	**الأحداث المنفصلة** الأحداث التي لا يمكن أن تحدث في الوقت نفسه.
Distance Formula The distance d between two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$	**قانون المسافة بين نقطتين** المسافة d بين نقطتين في المستوى الإحداثي (x_1, y_1) و (x_2, y_2) يُمكن الحصول عليها من خلال القانون: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$
distribution A way to show the arrangement of data values.	**التوزيع** طريقة توضح ترتيب قيم البيانات.
Distributive Property To multiply a sum by a number, multiply each addend by the number outside the parentheses. $5(x + 3) = 5x + 15$	**خاصية التوزيع** ضرب الإجمالي في أي عدد، وضرب كل طرف جمع في العدد الموجود خارج الأقواس. $5(x + 3) = 5x + 15$
Division Property of Equality If you divide each side of an equation by the same nonzero number, the two sides remain equal.	**القسمة في المعادلة** في حالة قسمة طرفي معادلة على العدد نفسه غير الصفري، يبقى الطرفان متساويين.
domain The set of x-coordinates in a relation.	**المجال** مجموعة من إحداثيات x.
double box plot Two box plots graphed on the same number line.	**مخطط الصندوق ذو العارضين المزدوج** مخطط لصندوقين يتم تمثيلهما بالرسم البياني على خط الأعداد نفسه.

E

edge The line segment where two faces of a polyhedron intersect.	**الحافة** القطعة المستقيمة التي يتقابل فيها وجهان من شكل متعدد الأوجه.
	
equation A mathematical sentence stating that two quantities are equal.	**المعادلة** عبارة رياضية توضح وجود كميتين متساويتين.
equiangular A polygon in which all angles are congruent.	**متطابق الزوايا** مضلع كل زواياه متطابقة.
	
equilateral triangle A triangle with three congruent sides.	**المثلث متساوي الأضلاع** مثلث يحتوي على ثلاثة أضلاع متطابقة.
equivalent expressions Expressions that have the same value regardless of the value(s) of the variable(s).	**التعابير المتكافئة** تعابير لها القيمة نفسها بغض النظر عن قيمة المتغير (قيم المتغيرات).
event An outcome is a possible result.	**الحدث** نتيجة ممكنة.
experimental probability An estimated probability based on the relative frequency of positive outcomes occurring during an experiment.	**الاحتمال التجريبي** احتمال مقدر قائم على التكرار النسبي للنتائج الإيجابية التي تقع أثناء إجراء التجربة العشوائية.
exponent In a power, the number of times the base is used as a factor. In 10^3, the exponent is 3.	**الأس** في عملية القوى، هو عدد المرات التي يتم فيها استخدام الأساس كعامل. وفي 10^3، يكون الأس 3.
exponential function A nonlinear function in which the base is a constant and the exponent is an independent variable.	**الدالة الأسية** دالة غير خطية أساسها عبارة عن ثابت وأُسها عبارة عن متغير مستقل.
exterior angles The four outer angles formed by two lines cut by a transversal.	**الزوايا الخارجية** أربع زوايا خارجية تتكوّن من خطين يقطعهما قاطع.

F

face A flat surface of a polyhedron.	**الوجه** سطح مستو لشكل متعدد الأوجه.
	

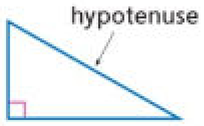
fair game A game where each player has an equally likely chance of winning.	**اللعبة العادلة** لعبة يكون لدى كل لاعب فيها فرصة متساوية لاحتمالية الفوز.
five-number summary A way of characterizing a set of data that includes the minimum, first quartile, median, third quartile, and the maximum.	**ملخص الخمسة مقاييس** طريقة تصنيف مجموعة من البيانات تتضمن الحد الأدنى، والرُبيّع الأول، والوسيط، والرُبيّع الثالث، والحد الأقصى.
formal proof A two-column proof containing statements and reasons.	**البرهان الشكلي** برهان مكون من عمودين يحتويان على العبارات والاستدلالات.
function A relation in which each member of the domain (input value) is paired with exactly one member of the range (output value).	**الدالة** علاقة يقترن فيها كل عضو في المجال (قيمة المدخل) بعضو واحد آخر في المدى (قيمة المخرج).
function table A table organizing the domain, rule, and range of a function.	**جدول الدالة** جدول ينظم المجال والقاعدة والمدى للدالة.
Fundamental Counting Principle Uses multiplication of the number of ways each event in an experiment can occur to find the number of possible outcomes in a sample space.	**مبدأ العد الأساسي** استعمال عملية الضرب لإيجاد عدد الطرق الممكنة لوقوع كل حدث في التجربة من أجل التوصل إلى عدد من النتائج المحتملة في فضاء عيني ما.

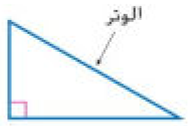
G

geometric sequence A sequence in which each term after the first is found by multiplying the previous term by a constant.	**المتتالية الهندسية** متتالية يتم فيها إيجاد كل حد بعد الحد الأول عن طريق ضرب الحد السابق في ثابت.

H

half-plane The part of the coordinate plane on one side of the boundary.	**نصف المستوى الإحداثي** جزء المستوى الإحداثي الواقع على جانب واحد من الحد الفاصل بين الجزأين.
hemisphere One of two congruent halves of a sphere.	**نصف الكرة** أحد النصفين المتطابقين للكرة.
hypotenuse The side opposite the right angle in a right triangle.	**وتر المثلث القائم الزاوية** الضلع المقابل للزاوية القائمة في المثلث القائم الزاوية.

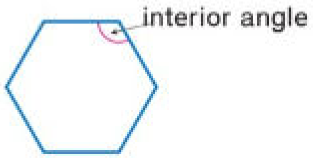


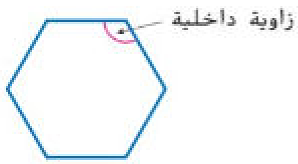


I

identity An equation that is true for every value for the variable.	**متطابقة** عبارة رياضية صحيحة لكل قيمة من قيم المتغير.

image The resulting figure after a transformation.	**الصورة** الشكل الناتج بعد التحويل.
independent events Two or more events in which the outcome of one event does not affect the outcome of the other event(s).	**الأحداث المستقلة** حدثان أو أكثر لا تؤثر نتيجة إحداهما في نتيجة الحدث الآخر أو الأحداث الأخرى.
independent variable The variable in a function with a value that is subject to choice.	**المتغير المستقل** متغير في الدالة تخضع قيمته للاختبار.
indirect measurement A technique using properties of similar polygons to find distances or lengths that are difficult to measure directly.	**القياس غير المباشر** تقنية تستخدم خصائص المضلعات المتماثلة للعثور على مسافات أو أطوال يصعب قياسها مباشرة.
inductive reasoning Reasoning that uses a number of specific examples to arrive at a plausible generalization or prediction. Conclusions arrived at by inductive reasoning lack the logical certainty of those arrived at by deductive reasoning.	**الاستدلال الاستقرائي** الاستدلال الذي يستخدم عددًا من الأمثلة المحددة للوصول إلى تعميم منطقي أو توقع. تفتقر النتائج التي نتوصل إليها عن طريق الاستدلال الاستقرائي إلى اليقين المنطقي المتوفر في النتائج التي نتوصل إليها من خلال الاستدلال الاستنتاجي.
inequality A mathematical sentence that contains <, >, ≠, ≤, or ≥.	**المتباينة** عبارة رياضية تحتوي على < أو > أو ≠ أو ≥ أو ≤.
inscribed angle An angle that has its vertex on the circle. Its sides contain chords of the circle.	**الزاوية المحيطية** زاوية رأسها على محيط الدائرة. وضلعاها وتران في الدائرة.
informal proof A paragraph proof.	**البرهان غير الشكلي** برهان الفقرة.
interest The amount of money paid or earned for the use of money.	**الفائدة** مقدار المال المدفوع أو المكتسب نظير استخدام المال.
interior angle An angle inside a polygon.	**الزاوية الداخلية** زاوية داخل مضلع.





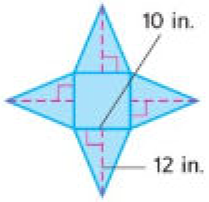
interior angles The four inside angles formed by two lines cut by a transversal.	**الزوايا الداخلية** أربع زوايا داخلية تتكون من مستقيمين يقطعهما قاطع.
interquartile range A measure of variation in a set of numerical data. It is the difference between the first quartile and the third quartile.	**المدى الرُبيعي** مقياس تشتت مجموعة من البيانات الرقمية. ويقصد به الفرق بين الرُبَيْع الأول والرُبَيْع الثالث.
inverse operations Pairs of operations that undo each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.	**العمليات العكسية** أزواج من العمليات التي تلغي عمل بعضها البعض. يُعد الجمع والطرح عمليتين متعاكستين. كما يُعد الضرب والقسمة أيضًا عمليتين متعاكستين.
irrational number A number that cannot be expressed as the quotient $\frac{a}{b}$, where a and b are integers and b ≠ 0.	**العدد غير النسبي** العدد الذي لا يمكن التعبير عنه كناتج القسمة $\frac{a}{b}$، حيث إن a و b أعداد صحيحة و $b \neq 0$.

isosceles triangle A triangle with at least two congruent sides.

المثلث متساوي الساقين مثلث يحتوي على ضلعين متطابقين على الأقل.

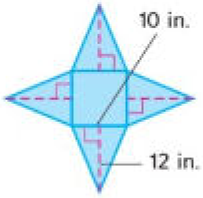
L

lateral area The sum of the areas of the lateral faces of a solid.



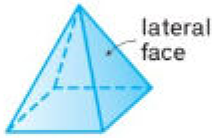
lateral area = $4\left(\frac{1}{2} \times 10 \times 12\right) = 240$ square inch

مساحة جانبية مجموع مساحات الأوجه الجانبية لأحد المجسمات.

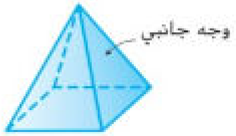


مساحة جانبية $= 4\left(\frac{1}{2} \times 10 \times 12\right) = 240$ بوصة مربعة

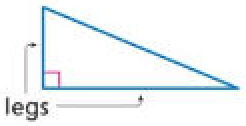
lateral face Any flat surface that is not a base.



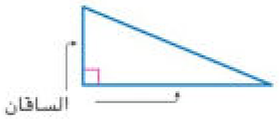
الوجه الجانبي أي وجه غير قاعدة المُجسّم.



legs The two sides of a right triangle that form the right angle.



ضلعا القائمة ضلعا المثلث القائم الزاوية اللذان يشكلان الزاوية القائمة.



like fractions Fractions that have the same denominators.

الكسور المتشابهة الكسور التي لها المقامات نفسها.

like terms Terms that contain the same variable(s) to the same powers.

الحدود المتشابهة حدود تتكون من نفس (المتغير) المتغيرات ومرفوعة إلى الأسس نفسها.

linear To fall in a straight line.

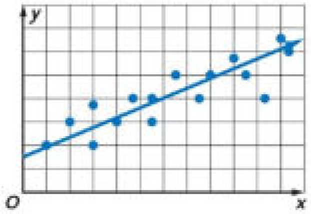
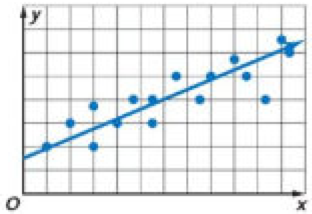
خطّي أن يكون في خط مستقيم.

linear equation An equation with a graph that is a straight line.

المعادلة الخطية معادلة يتم تمثيلها بيانيًا بخط مستقيم.

linear function A function in which the graph of the solutions forms a line.

الدالة الخطية دالة يمثل الرسم البياني للحلول فيها خطًا مستقيمًا.

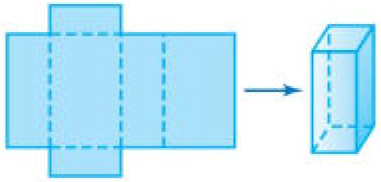
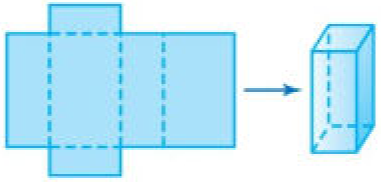
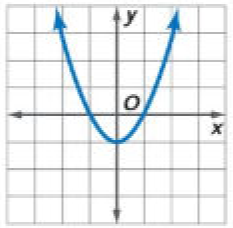
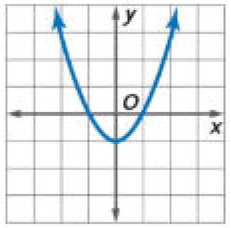
linear relationship A relationship that has a straight-line graph.	**العلاقة الخطية** علاقة يكون التمثيل البياني فيها عبارة عن خط مستقيم.
line of best fit A line that is very close to most of the data points in a scatter plot.  	**المستقيم الأفضل تمثيلاً (خط الانتشار)** خط قريب للغاية من معظم نقاط البيانات بالرسم البياني المتفرق. 
line of reflection The line over which a figure is reflected.	**خط الانعكاس** الخط الذي تتكون صورة الشكل بالانعكاس عليه.
line of symmetry Each half of a figure is a mirror image of the other half when a line of symmetry is drawn.	**خط التناظر** يعتبر كل نصف من أي شكل بمثابة صورة مطابقة للنصف الآخر عند رسم خط التناظر.
line symmetry A figure has line symmetry if a line can be drawn so that one half of the figure is a mirror image of the other half.	**التناظر المحوري** يكون للشكل تناظر محوري إذا كان يُمكن رسم خط مستقيم على الشكل بحيث يكون نصف الشكل صورة معكوسة للنصف الآخر.
literal equation An equation or formula that has more than one variable.	**المعادلة متعددة المتغيرات** معادلة أو صيغة بها أكثر من متغير.

M

markup The amount the price of an item is increased above the price the store paid for the item.	**هامش الربح** مقدار زيادة سعر عنصر عن السعر الذي دفعه المتجر مقابل هذا العنصر.
mean The sum of the data divided by the number of items in the set.	**المتوسط الحسابي** مجموع البيانات مقسومًا على عدد البيانات.
mean absolute deviation The average of the absolute values of differences between the mean and each value in a data set.	**الانحراف المتوسط** متوسط القيم المطلقة للفروق بين المتوسط الحسابي وكل قيمة في مجموعة البيانات.
measures of center Numbers that are used to describe the center of a set of data.These measures include the mean, median, and mode.	**مقاييس النزعة المركزية** أرقام تُستخدم لوصف تمركز مجموعة من البيانات، وتتضمن هذه القياسات المتوسط الحسابي والوسيط، والمتوال.
measures of variation Numbers used to describe the distribution or spread of a set of data.	**مقاييس التباين** أرقام تستخدم لوصف توزيع أو نشر مجموعة من البيانات.

median A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values.	**الوسيط** مقياس من مقاييس النزعة المركزية في مجموعة من البيانات العددية. وسيط مجموعة من القيم هو القيمة التي تظهر في مركز القيم العددي بها أو المتوسط الحسابي للقيمتين المركزيتين. إذا كانت القائمة تحتوي على عدد زوجي من القيم.
mode The number(s) or item(s) that appear most often in a set of data.	**المنوال** العدد (الأعداد) أو العنصر (العناصر) الأكثر تكرارًا في مجموعة من البيانات.
monomial A number, a variable, or a product of a number and one or more variables.	**أحادي الحد** عدد أو متغير أو ناتج ضرب عدد ومتغير واحد أو أكثر.
Multiplication Property of Equality If you multiply each side of an equation by the same number, the two sides remain equal.	**الضرب في المعادلة** في حالة ضرب طرفي أية معادلة في نفس العدد، يبقى الطرفان متساويين.
multiplicative inverses Two numbers with a product of 1. The multiplicative inverse of $\frac{2}{3}$ is $\frac{3}{2}$.	**النظير الضربي** عدد ناتج ضربه في العدد يساوي 1. النظير الضربي للعدد $\frac{2}{3}$ هو $\frac{3}{2}$، أو هو مقلوب العدد.

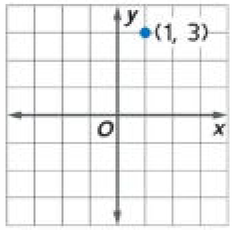
N

net A two-dimensional pattern of a three-dimensional figure.	**الشبكة** مخطط مستوي لشكل ثلاثي الأبعاد.
	
nonlinear function A function whose rate of change is not constant. The graph of a nonlinear function is not a straight line.	**الدالة غير الخطية** دالة يكون فيها معدل التغير ليس ثابتًا. ويكون الرسم البياني لمعادلة غير خطية عبارة عن خط غير مستقيم.
	
null set The empty set.	**المجموعة الخالية** المجموعة الفارغة.

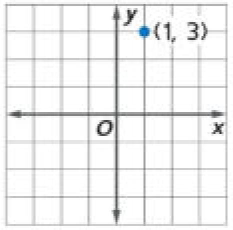
O

obtuse angle An angle whose measure is between 90° and 180°.	**الزاوية المنفرجة** زاوية قياسها بين 90° و 180°.
obtuse triangle A triangle with one obtuse angle.	**المثلث منفرج الزاوية** مثلث إحدى زواياه منفرجة.

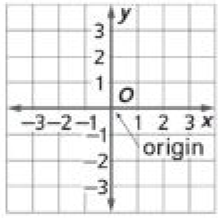
ordered pair A pair of numbers used to locate a point in the coordinate plane. The ordered pair is written in this form: (x-coordinate, y-coordinate).



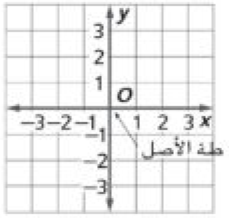
الزوج المرتب زوج من الأعداد يُستخدم لتحديد نقطة في المستوى الإحداثي، ويكتب الزوج المرتب على هذا الشكل: (x, y).



origin The point of intersection of the x-axis and y-axis in a coordinate plane.



نقطة الأصل نقطة تقاطع المحور x مع المحور y في مستوى إحداثي.



outcome One possible result of a probability event. For example, 4 is an outcome when a number cube is rolled.

نتيجة غير عشوائية ناتج ممكن لحدث محتمل.
مثال: يكون العدد 4 نتيجة في حالة دحرجة مكعب أعداد.

outlier Data that are more than 1.5 times the interquartile range from the first or third quartiles.

القيمة المتطرفة بيانات تكون أكبر بمقدار مرة ونصف من المدى من الرُبيعي الأول أو الثالث.

P

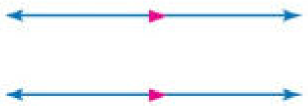
paragraph proof A paragraph that explains why a statement or conjecture is true.

برهان الفقرة تفسير صحة عبارة أو فرضية.

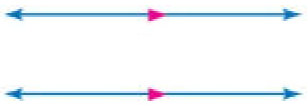
parallel Lines that never intersect no matter how far they extend.

المتوازي مستقيمات لا تتقاطع أبدًا مهما طالت المسافة بها.

parallel lines Lines in the same plane that never intersect or cross. The symbol ∥ means parallel.



المستقيمات المتوازية مستقيمات في نفس المستوى ولا تتقاطع أو تتلاقى أبدًا. يعني الرمز ∥ توازيًا.



parallelogram A quadrilateral with both pairs of opposite sides parallel and congruent.

متوازي الأضلاع شكل رباعي الأضلاع له زوجان من الأضلاع المتقابلة المتوازية والمتطابقة.

percent equation An equivalent form of a percent proportion in which the percent is written as a decimal.

part = percent · whole

percent of change A ratio that compares the change in quantity to the original amount.

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original amount}}$$

percent of decrease When the percent of change is negative.

percent of increase When the percent of change is positive.

percent proportion Compares part of a quantity to the whole quantity using a percent.

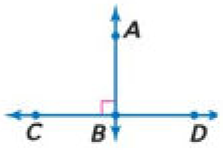
$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

perfect cube A rational number whose cube root is a whole number. 27 is a perfect cube because its cube root is 3.

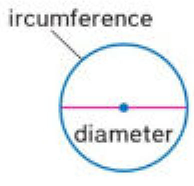
perfect square A rational number whose square root is a whole number. 25 is a perfect square because its square root is 5.

permutation An arrangement or listing in which order is important.

perpendicular lines Two lines that intersect to form right angles.



pi The ratio of the circumference of a circle to its diameter. The Greek letter π represents this number. The value of pi is always 3.1415926... .



$$\pi = \frac{c}{d}$$

point-slope form An equation of the form $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a given point on a nonvertical line.

المعادلة المئوية صيغة مكافئة للتناسب المئوي، يتم التعبير فيها عن النسبة المئوية على صورة كسر عشري.

الجزء = النسبة المئوية · الكل

النسبة المئوية للتغير نسبة تقارن مقدار تغير كمية بالنسبة إلى الكم الأصلي.

$$\text{النسبة المئوية للتغير} = \frac{\text{مقدار التغير}}{\text{المقدار الأصلي}}$$

النسبة المئوية للتناقص عندما تكون النسبة المئوية للتغير سالبة.

النسبة المئوية للتزايد عندما تكون النسبة المئوية للتغير موجبة.

مقدار النسبة المئوية هو المقارنة بين جزء من كمية وإجمالي الكمية باستخدام نسبة مئوية.

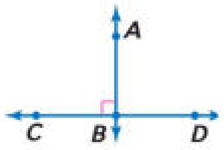
$$\frac{\text{الجزء}}{\text{الكل}} = \frac{\text{النسبة المئوية}}{100}$$

المكعب الكامل عدد نسبي جذره التكعيبي عبارة عن عدد صحيح. ويعد العدد 27 مكعبًا كاملاً؛ لأن جذره التكعيبي يساوي 3.

المربع الكامل عدد نسبي جذره التربيعي عبارة عن عدد صحيح. ويُعد العدد 25 مربعًا كاملاً لأن الجذر التربيعي للعدد 25 هو 5.

التبديل تنسيق أو وضع قائمة يلعب الترتيب فيها دورًا مهماً.

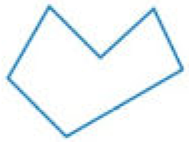
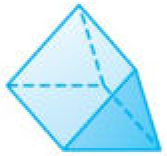
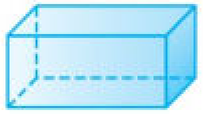
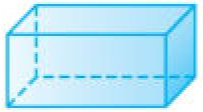
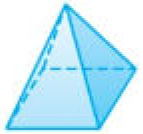
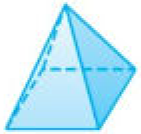
المستقيمان المتعامدان مستقيمان يتقاطعان لتكوين زوايا قائمة.



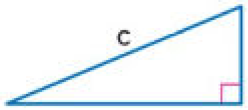
(باي) نسبة محيط الدائرة إلى قطرها. ويمثل الحرف اليوناني π هذا العدد. ودائمًا ما تكون قيمة (باي) تساوي ...3.1415926.

$$\pi = \frac{c}{d}$$

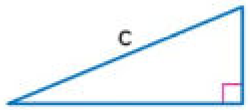
معادلة المستقيم بدلالة ميله ونقطة عليه معادلة الشكل $y - y_1 = m(x - x_1)$، حيث إن m هو الميل و(x_1, y_1) هي نقطة معينة على ضلع غير متعامد.

polygon A simple, closed figure formed by three or more line segments.	**المضلع** شكل مستوٍ مغلق مكون من ثلاث قطع مستقيمة أو أكثر.
	
polyhedron A three-dimensional figure with faces that are polygons.	**متعدد الوجوه** هو شكل ثلاثي الأبعاد له وجوه عبارة عن مضلعات.
	
power A product of repeated factors using an exponent and a base. The power 7^3 is read seven to the third power, or seven cubed.	**القوى** ناتج ضرب عوامل متكررة باستخدام الأس والأساس. فعملية القوى7^3 تُقرأ القوة الثالثة للعدد 7 أو سبعة تكعيب.
precision The ability of a measurement to be consistently reproduced.	**الدقة** إمكانية الحصول على نفس القياس على الدوام.
preimage The original figure before a transformation.	**الصورة الأصلية** الشكل الأصلي قبل التحويل.
principal The amount of money invested or borrowed.	**رأس المال** مقدار المال المستثمر أو المقترض.
prism A polyhedron with two parallel congruent faces called bases.	**المنشور** مُجسّم متعدد الوجوه له وجهان متوازيان ومتطابقان يطلق عليهما القاعدتان.
	
probability The chance that some event will happen. It is the ratio of the number of ways a certain event can occur to the number of possible outcomes.	**الاحتمال** فرصة حدوث بعض الأحداث. وهو نسبة عدد الطرق الممكنة لوقوع الحدث إلى عدد النتائج الكليّة.
proof A logical argument in which each statement that is made is supported by a statement that is accepted as true.	**البرهان** البرهانِ المنطقي الذي يكون فيه كل عبارة تقدمها مدعومة بعبارة مسلّم بأنها صحيحة.
property A statement that is true for any numbers.	**الخاصّية** عبارة حقيقية بالنسبة لأي أعداد.
pyramid A polyhedron with one base that is a polygon and three or more triangular faces that meet at a common vertex.	**الهرم** شكل متعدد الوجوه له قاعدة واحدة على شكل مضلع، وثلاثة أوجه أو أكثر على شكل مثلث تلتقي في قمة مشتركة.
	

Pythagorean Theorem In a right triangle, the square of the length of the hypotenuse c is equal to the sum of the squares of the lengths of the legs a and b. $a^2 + b^2 = c^2$

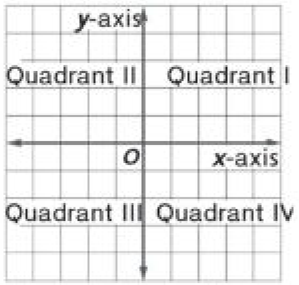


نظرية فيثاغورث في المثلث قائم الزاوية، مربع طول الوتر c يساوي مجموع مربعي طولي الساقين a و b. $a^2 + b^2 = c^2$

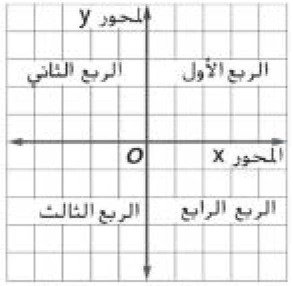


Q

quadrants The four sections of the coordinate plane.



رُبعيّات الأقسام الأربعة التي تشكل المستوى الإحداثي.



quadratic function A function in which the greatest power of the variable is 2.

الدالة التربيعية الدالة التي تكون قوة المتغير فيها تساوي 2.

quadrilateral A closed figure with four sides and four angles.

رباعي الأضلاع شكل مستوٍ مغلق مكون من أربعة أضلاع وأربع زوايا.

qualitative graph A graph used to represent situations that do not necessarily have numerical values.

تمثيل نوعي رسم بياني يُستخدم في تمثيل حالات لا تحتوي بالضرورة على قيم عددية.

quantitative data Data that can be given a numerical value.

بيانات كمية بيانات يمكن إعطاؤها قيمة عددية.

quartiles Values that divide a set of data into four equal parts.

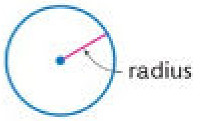
أرباع قيم تقسم مجموعة البيانات إلى أربعة أجزاء متساوية.

R

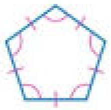
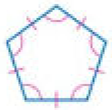
radical sign The symbol used to indicate a positive square root, $\sqrt{}$.

رمز الجذر الرمز المستخدم للدلالة على جذر تربيعي موجب $\sqrt{}$.

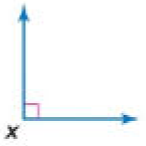
radius The distance from the center of a circle to any point on the circle.



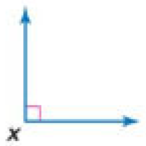
نصف القطر المسافة بين مركز الدائرة وأي نقطة على الدائرة.

random Outcomes occur at random if each outcome is equally likely to occur.	**عشوائي** تحدث النتائج بشكل عشوائي إذا تساوت في احتمال حدوثها.
range The set of y-coordinates in a relation.	**مدى العلاقة** مجموعة إحداثيات y في علاقة ما.
range The difference between the greatest number (maximum) and the least number (minimum) in a set of data.	**مدى** الفرق بين أكبر عدد (الأقصى) وأقل عدد (الأدنى) في مجموعة من البيانات.
rational number Numbers that can be written as the ratio of two integers in which the denominator is not zero. All integers, fractions, mixed numbers, and percents are rational numbers.	**عدد نسبي** العدد الذي يمكن كتابته في صورة كسر بسطه ومقامه عددان صحيحان، ولا يكون مقامه صفرًا. وتشمل الأعداد النسبية جميع الأعداد الصحيحة، والكسور، والكسور المعتلة، والنسب المئوية.
real numbers The set of rational numbers together with the set of irrational numbers.	**أعداد حقيقية** أعداد مكونة من مجموعة الأعداد النسبية ومجموعة الأعداد غير النسبية.
reciprocals The multiplicative inverse of a number. The product of reciprocals is 1.	**نظير ضربي** هو المقلوب الضربي للعدد. وناتج ضرب العدد في نظيره الضربي يساوي 1.
reflection A transformation where a figure is flipped over a line. Also called a flip.	**انعكاس** تحويل هندسي يُقلب فيه الشكل على خط مستقيم. ويُطلق عليه أيضًا «القلب».
regular polygon A polygon that is equilateral and equiangular. 	**مضلع منتظم** مضلع يكون متساوي الأضلاع ومتساوي الزوايا. 
regular pyramid A pyramid whose base is a regular polygon.	**هرم منتظم** هرم تكون قاعدته مضلعًا منتظمًا.
relation Any set of ordered pairs.	**علاقة** أي مجموعة من الأزواج المرتبة.
relative frequency The ratio of the number of experimental successes to the total number of experimental attempts.	**تكرار نسبي** نسبة وقوع الحدث إلى إجمالي عدد مرات إجراء التجربة.
remote interior angles The angles of a triangle that are not adjacent to a given exterior angle.	**زوايا داخلية غير مجاورة** زوايا المثلث التي تكون غير مجاورة للزاوية الخارجية المعطاة.
repeating decimal Decimal form of a rational number.	**كسر عشري دوري** الصيغة العشرية من بعض الأعداد النسبية.
rhombus A parallelogram with four congruent sides.	**معين** متوازي أضلاع له أربعة أضلاع متطابقة.

right angle An angle whose measure is exactly 90°.



زاوية قائمة زاوية قياسها 90° بالضبط.



right triangle A triangle with one right angle.

مثلث قائم الزاوية مثلث إحدى زواياه قائمة.

rise The vertical change between any two points on a line.

ارتفاع التغير الرأسي بين أي نقطتين على الخط.

rotation A transformation in which a figure is turned about a fixed point.

دوران محوري تحويل هندسي يتم فيه تدوير الشكل حول نقطة ثابتة.

rotational symmetry A type of symmetry a figure has if it can be rotated less than 360° about its center and still look like the original.

تناظر دوراني نوع من أنواع التناظر الذي يتصف به الشكل إذا أمكن تدويره بزاوية أقل من 360° حول مركزه مع الاحتفاظ بشكله الأصلي.

run The horizontal change between any two points on a line.

امتداد أفقي التغير الأفقي بين أي نقطتين على المحور X.

S

sales tax An additional amount of money charged on certain goods and services.

ضريبة المبيعات مبلغ إضافي من المال يُفرض على سلع وخدمات معينة.

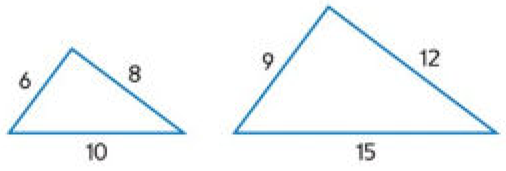
sample A randomly-selected group chosen for the purpose of collecting data.

عينة مجموعة مختارة بشكل عشوائي بهدف جمع البيانات.

sample space The set of all possible outcomes of a probability experiment.

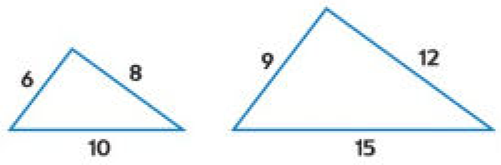
الفضاء العيني (Ω) مجموعة النتائج المحتملة للتجربة الاحتمالية. ويُرمز له بالرمز أوميجا (Ω).

scale factor The ratio of the lengths of two corresponding sides of two similar polygons.



scale factor = $\frac{3}{2}$

معامل المقياس نسبة طولي ضلعين متناظرين لمضلعين متشابهين.

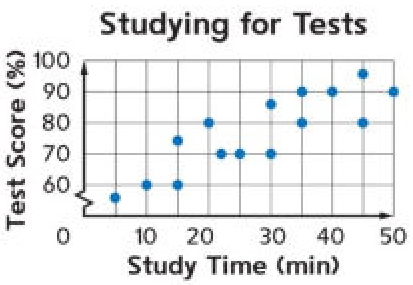


معامل المقياس = $\frac{3}{2}$

scalene triangle A triangle with no congruent sides.

مثلث مختلف الأضلاع مثلث أضلاعه غير متطابقة.

scatter plot A graph that shows the relationship between a data set with two variables graphed as ordered pairs on a coordinate plane.



scientific notation A compact way of writing numbers with absolute values that are very large or very small. In scientific notation, 5,500 is 5.5×10^3.

selling price The amount the customer pays for an item.

semicircle An arc measuring 180°.

sequence An ordered list of numbers, such as 0, 1, 2, 3 or 2, 4, 6, 8.

similar If one image can be obtained from another by a sequence of transformations and dilations.

similar polygons Polygons that have the same shape.

similar solids Solids that have exactly the same shape, but not necessarily the same size.

simple interest Interest paid only on the initial principal of a savings account or loan.

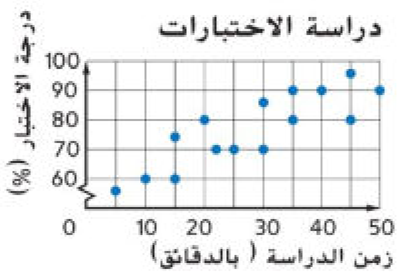
simple random sample A sample where each item or person in the population is as likely to be chosen as any other.

simplest form An algebraic expression that has no like terms and no parentheses.

simplify To perform all possible operations in an expression.

simulation An experiment that is designed to model the action in a given situation.

مخطط الانتشار رسم بياني يعرض العلاقة بين مجموعة بيانات ذات متغيرين، ويمثلها في صورة أزواج مرتبة على المستوى الإحداثي.



ترميز علمي طريقة موجزة لكتابة الأعداد ذات القيم المطلقة الكبيرة للغاية أو الصغيرة للغاية. وفي الترميز العلمي، 5500 يساوي 5.5×10^3.

ثمن البيع مبلغ يدفعه الزبون للحصول على منتج ما.

نصف دائرة قوس قياسه 180°.

نمط قائمة مرتبة من الأعداد، مثل 0, 1, 2, 3 أو 2, 4, 6, 8.

متماثل إمكانية الحصول على صورة من صورة أخرى عن طريق سلسلة من التحويلات وتغييرات الأبعاد.

مضلعات متماثلة مضلعات لها الشكل نفسه.

مجسمات متشابهة مجسمات لها نفس الشكل بالضبط ولكنها ليست بالضرورة بالحجم ذاته.

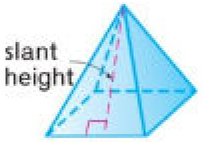
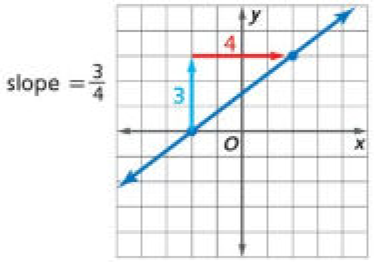
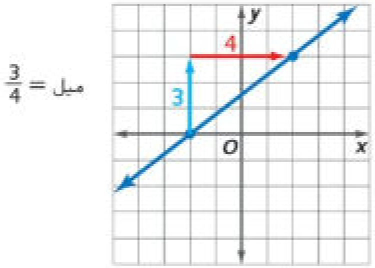
فائدة بسيطة فائدة تُدفع فقط على رأس المال الأولي لحساب الادخار أو القرض.

عينة عشوائية بسيطة عينة يكون فيها احتمال اختيار كل عنصر أو شخص في التعداد مماثلاً بالنسبة إلى جميع العناصر أو الأشخاص.

أبسط صورة تعبير جبري ليس فيه حدود متشابهة وأقواس.

تبسيط إجراء جميع العمليات المحتملة في تعبير ما.

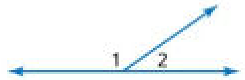
محاكاة تجربة مصممة لصياغة نموذج من إجراء ما في حالة معينة.

slant height The altitude or height of each lateral face of a pyramid.	**ارتفاع جانبي** الارتفاع أو العلو لكل وجه جانبي للهرم.
	
slope The rate of change between any two points on a line. The ratio of the rise, or vertical change, to the run, or horizontal change.	**ميل** معدل التغير بين أي نقطتين على الخط. نسبة الارتفاع (أو التغير الرأسي) إلى الامتداد (أو التغير الأفقي).
	
slope-intercept form An equation written in the form $y = mx + b$, where m is the slope and b is the y-intercept.	**معادلة المستقيم بدلالة الميل والجزء المقطوع من محور y** معادلة مكتوبة بالصيغة $y = mx + b$.حيث m هو الميل و b هو الجزء المقطوع من محور y.
solid A three-dimensional figure formed by intersecting planes.	**مجسم** شكل ثلاثي الأبعاد يتشكل بتقاطع المستويات.
	
sphere The set of all points in space that are a given distance from a given point called the center.	**كرة** مجموعة النقاط في الفراغ التي لها بعد محدد عن نقطة معلومة تُسمى المركز.
square root One of the two equal factors of a number. If $a^2 = b$, then a is the square root of b. A square root of 144 is 12 since $12^2 = 144$.	**جذر تربيعي** أحد العاملين المتساويين للعدد. إذا كان $a^2= b$. إذن a هو الجذر التربيعي لـــb. الجذر التربيعي للعدد 144 هو 12 لأن $12^2 - 144$.
standard deviation A measure of variation that describes how the data deviates from the mean of the data.	**انحراف معياري** مقياس الانحراف الذي يصف مدى انحراف البيانات عن المتوسط الحسابي للبيانات.
standard form An equation written in the form $Ax + By = C$.	**الصورة القياسية لمعادلة المستقيم** معادلة تُكتب بالشكل $Ax + By = C$.
straight angle An angle whose measure is exactly 180°.	**زاوية مستقيمة** زاوية قياسها 180° بالضبط.
	
substitution An algebraic model that can be used to find the exact solution of a system of equations.	**تعويض** نموذج جبري يمكن استخدامه لإيجاد حل دقيق لنظام من المعادلات.

Subtraction Property of Equality If you subtract the same number from each side of an equation, the two sides remain equal.

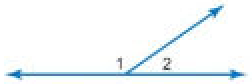
الطرح في المعادلة إذا طرحت العدد نفسه من كلا طرفي المعادلة، يبقى الطرفان متساويين.

supplementary angles Two angles are supplementary if the sum of their measures is 180°.



∠1 and ∠2 are supplementary angles.

زاويتان متكاملتان تكون الزاويتان متكاملتين إذا كان مجموع قياسهما يساوي 180°.



∠1 و ∠2 هما زاويتان متكاملتان

symmetric A description of the shape of a distribution in which the left side of the distribution looks like the right side.

متماثل وصف لأحد أشكال التوزيع الذي يكون فيه الجانب الأيسر من التوزيع مشابهًا للجانب الأيمن.

system of equations A set of two or more equations with the same variables.

نظام المعادلات مجموعة المعادلات المكونة من معادلتين فأكثر والتي تحتوي على نفس المتغيرات.

T

term A number, a variable, or a product of numbers and variables.

حد العدد أو المتغير أو ناتج ضرب أو قسمة أعداد ومتغيرات.

term Each part of an algebraic expression separated by an addition or subtraction sign.

حد كل جزء من التعبير الجبري تفصله علامة الجمع أو الطرح.

terminating decimal A repeating decimal where the repeating digit is zero.

كسر عشري مُتناه كسر عشري ذو منازل عشرية محددة.

theorem A statement or conjecture that can be proven.

نظرية عبارة أو فرضية يمكن إثباتها.

theoretical probability Probability based on known characteristics or facts.

احتمال نظري احتمال مبني على خصائص أو حقائق معلومة.

third quartile For a data set with median M, the third quartile is the median of the data values greater than M.

الرُبيّع الثالث بالنسبة لمجموعة البيانات ذات الوسيط M. يكون الربع الثالث هو وسيط قيم البيانات التي تكون أكبر من قيمة M.

three-dimensional figure A figure with length, width, and height.

شكل ثلاثي الأبعاد شكل له طول وعرض وارتفاع، أو شكل يشغل حيزًا من الفراغ.

total surface area The sum of the areas of the surfaces of a solid.

مساحة السطح الكُليّة مجموع مساحات أسطح المجسم.

transformation An operation that maps a geometric figure, preimage, onto a new figure, image.

تحويل هندسي عملية صياغة الشكل الهندسي (أو الصورة الأصلية) إلى شكل جديد، صورة الشكل.

translation A transformation that slides a figure from one position to another without turning.	**إزاحة** تحويل هندسي يتم فيه تحريك الشكل من أحد مواضعه إلى آخر دون تدويره.
transversal A line that intersects two or more other lines.	**قاطع مستعرض** مستقيم يقطع مستقيمين فأكثر.
trapezoid A quadrilateral with exactly one pair of parallel sides.	**شبه المنحرف** شكل رباعي الأضلاع مكون من زوج واحد من الأضلاع المتوازية.
tree diagram A diagram used to show the total number of possible outcomes in a probability experiment.	**مخطط الشجرة** مخطط مستخدم في عرض الفضاء العيني في التجربة الاحتمالية.
triangle A figure formed by three line segments that intersect only at their endpoints.	**مثلث** مضلع مكون من ثلاث قطع مستقيمة تتقاطع فقط عند نقاط نهايتها.
two-column proof A formal proof that contains statements and reasons organized in two columns. Each step is called a statement, and the properties that justify each step are called reasons.	**برهان ذو عمودين** برهان شكلي يحتوي على عبارات ومبررات منظمة في عمودين. ويُطلق على كل خطوة عبارة، والخصائص التي تبرر كل خطوة يُطلق عليها مبررات.
two-step equation An equation that contains two operations.	**معادلة ذات خطوتين** معادلة تحتوي على عمليتين.
two-step inequality An inequality that contains two operations.	**متباينة ذات خطوتين** متباينة تحتوي على عمليتين.
two-way table A table that shows data that pertain to two different categories.	**جدول ثنائي الاتجاه** جدول يعرض بيانات خاصة بفئتين مختلفتين.

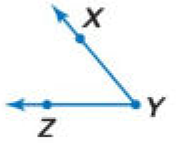
U

unbiased sample A sample that is selected so that it is representative of the entire population.	**عينة محايدة** عينة مختارة لتمثيل التعداد بالكامل.
unit rate/ratio A rate or ratio with a denominator of 1.	**كسر/نسبة الوحدة** نسبة أو كسر مقامه 1.
univariate data Data with one variable.	**بيانات أحادية المتغير** بيانات ذات متغير واحد.
unlike fractions Fractions whose denominators are different.	**كسور غير متشابهة** كسور تختلف في المقام.

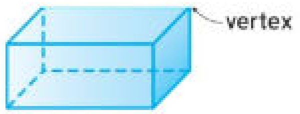
V

variable A symbol, usually a letter, used to represent a number in mathematical expressions or sentences.	**متغير** رمز عادةً ما يكون حرفًا، يُستخدم في تمثيل عدد في التعابير أو العبارات الرياضية.

vertex The point where the sides of an angle meet.

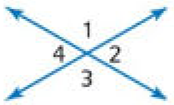


vertex The point where three or more faces of a polyhedron intersect.

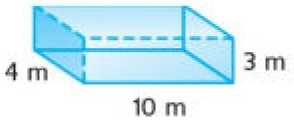


vertex The point at the tip of a cone.

vertical angles Opposite angles formed by the intersection of two lines. Vertical angles are congruent. In the figure, the vertical angles are $\angle 1$ and $\angle 3$, and $\angle 2$ and $\angle 4$.



volume The measure of the space occupied by a solid. Standard measures are cubic units such as in^3 or ft^3.

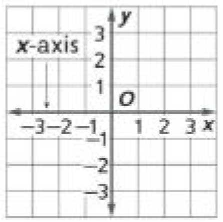


$V = 10 \times 4 \times 3 = 120$ cubic meters

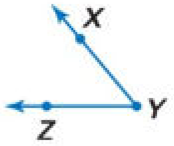
voluntary response sample A sample which involves only those who want to participate in the sampling.

X

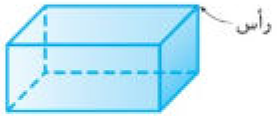
x-axis The horizontal number line that helps to form the coordinate plane.



رأس الزاوية نقطة التقاء ضلعي الزاوية.

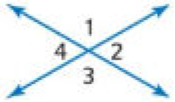


رأس نقطة تقاطع ثلاثة وجوه فقط للشكل متعدد الوجوه.

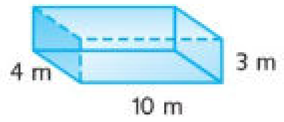


نقطة الرأس النقطة عند نهاية الشكل المخروطي.

زوايا متقابلة بالرأس زوايا متقابلة تتشكل بتقاطع خطين مستقيمين. والزوايا المتقابلة بالرأس تعد زوايا متطابقة. وتتمثل الزاويتان المتقابلتان بالرأس بهذا الشكل في $\angle 1$ و $\angle 3$ و $\angle 2$ و $\angle 4$.



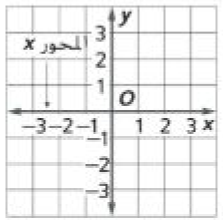
الحجم مقدار الفراغ الذي يشغله المجسم. ويكون القياس المعياري بالوحدات المكعبة مثل in^3 أو ft^3.



$V = 10 \times 4 \times 3 = 120$ متر مكعب

عينة الاستجابة الطوعية عينة تضم فقط الأشخاص الراغبين في المشاركة في العينة المأخوذة.

المحور الأفقي x خط الأعداد الأفقي في تشكيل المستوى الإحداثي.

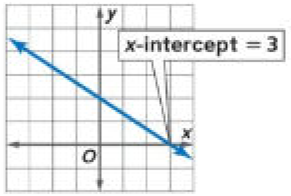


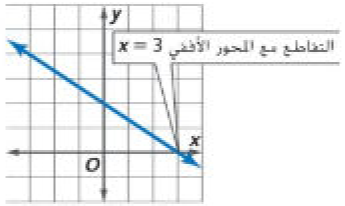
x-coordinate The first number of an ordered pair.

الإحداثي x العدد الأول في الزوج المرتب.

x-intercept The x-coordinate of the point where the line crosses the x-axis.

التقاطع مع المحور x الإحداثي x للنقطة حيث يقطع المستقيم المحور x.

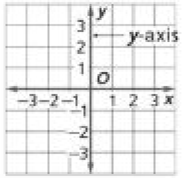


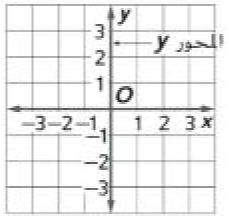


Z

y-axis The vertical number line that helps to form the coordinate plane.

المحور الرأسي y خط الأعداد الرأسي في المستوى الإحداثي.



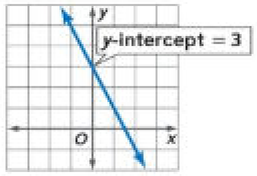


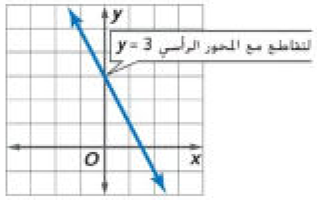
y-coordinate The second number of an ordered pair.

الإحداثي y العدد الثاني في الزوج المرتب.

y-intercept The y-coordinate of the point where the line crosses the y-axis.

التقاطع مع المحور y الإحداثي y للنقطة حيث يقطع المستقيم المحور y.





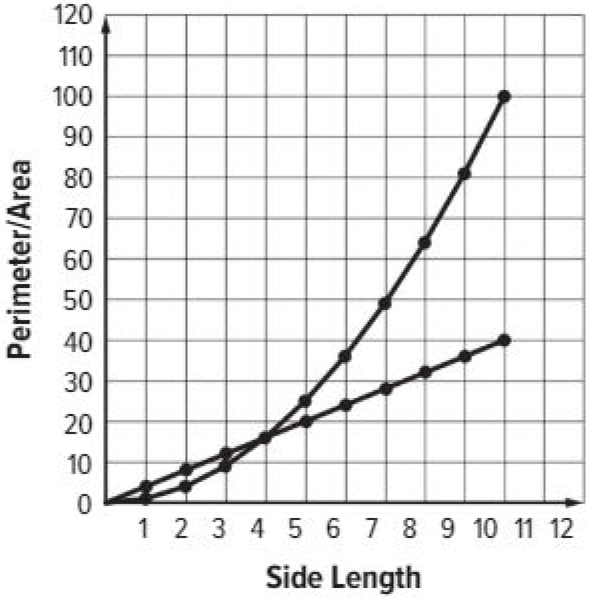
Chapter 1 Real Numbers

Lesson 1-2 Extra Practice

27a.

Side Length (in.)	Perimeter (in.)	Area (in²)
1	4	1
2	8	4
3	12	9
4	16	16
5	20	25
6	24	36
7	28	49
8	32	64
9	36	81
10	40	100

27b.

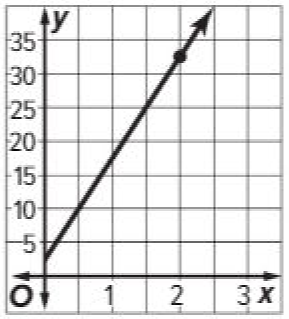


27c. Sample answer: The graph representing perimeter of a square is linear because each side length is multiplied by 4. The graph representing area of a square is nonlinear because each side length is squared and does not increase at a constant rate.

Chapter 3 Equations in Two Variables

Lesson 3-4 Need Another Example?

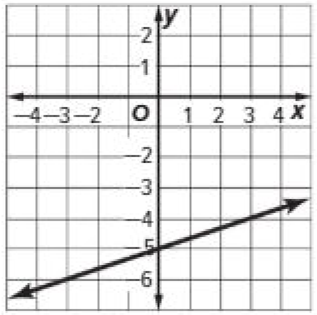
4–5.



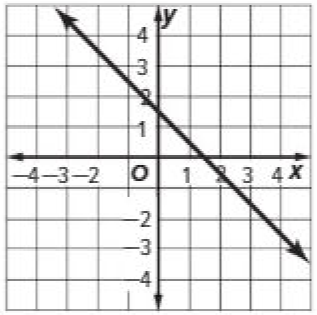
The slope 15 represents the rate of change or cost per hour. The *y*-intercept 2.5 is the charge for instruction.

Lesson 3-4 Independent Practice

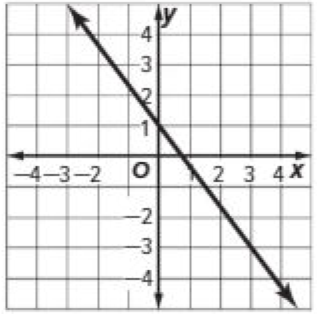
9.



10.

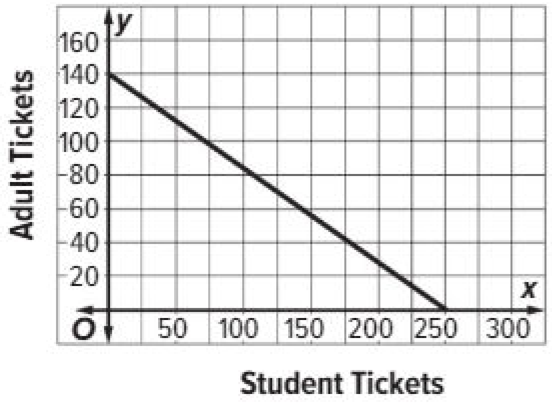


11.



Lesson 3-5 Need Another Example?

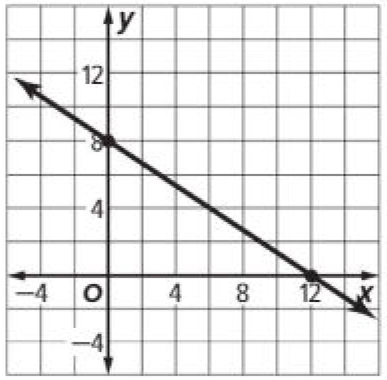
2–3.



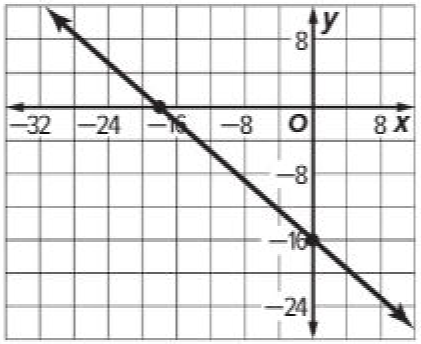
The *x*-intercept of 252 means that if 252 student tickets and 0 adult tickets were sold, the total sales would be $1,260. The *y*-intercept of 140 means that if 0 student tickets and 140 adult tickets were sold, the total sales would be $1,260.

Lesson 3-5 Extra Practice

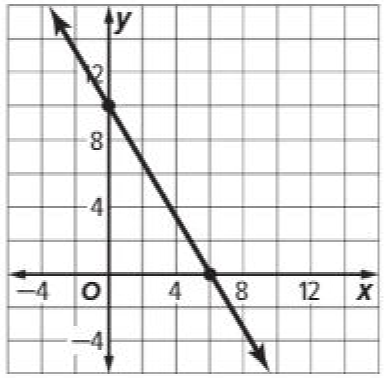
11. (12, 0), (0, 8);



12. (−18, 0), (0, −16);

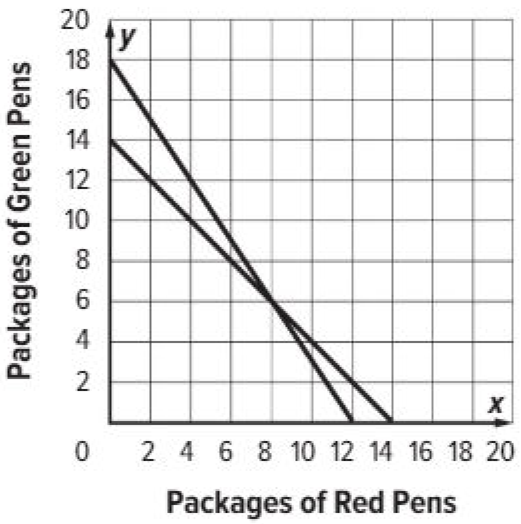


13. (6, 0), (0, 10);



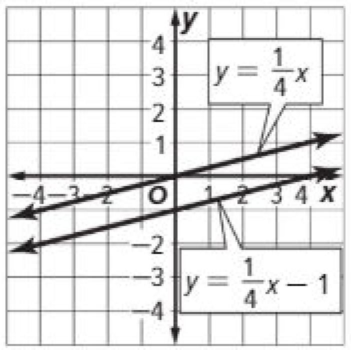
Lesson 3-7 Need Another Example?

2–3. $x + y = 14$; $6x + 4y = 72$



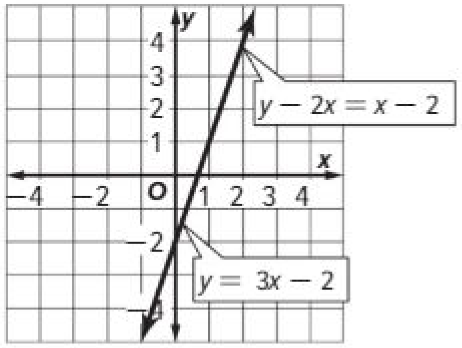
(8, 6); Ms. Baker bought 8 packages of red pens and 6 packages of green pens.

4.



no solution

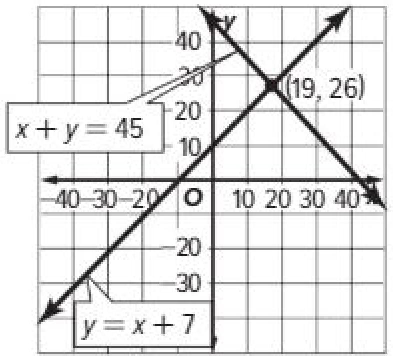
5.



an infinite number of solutions

Lesson 3-7 Independent Practice

7. Sample answer: Let $x =$ the number of birds and $y =$ the number of cats; $x + y = 45$, $y = x + 7$; There are 19 birds and 26 cats.



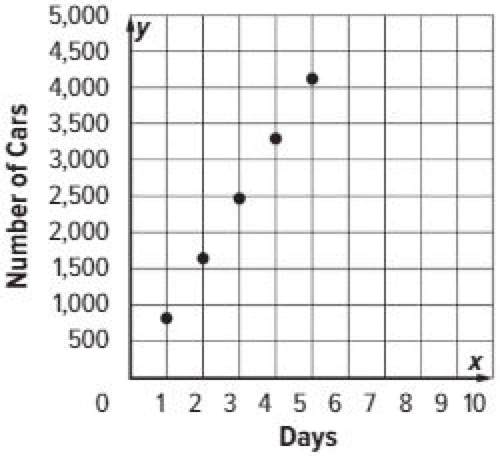
Chapter 4 Functions

Lesson 4-2 Independent Practice

3.

x	825*x*	*y*
1	825(1)	825
2	825(2)	1,650
3	825(3)	2,475
4	825(4)	3,300
5	825(5)	4,125

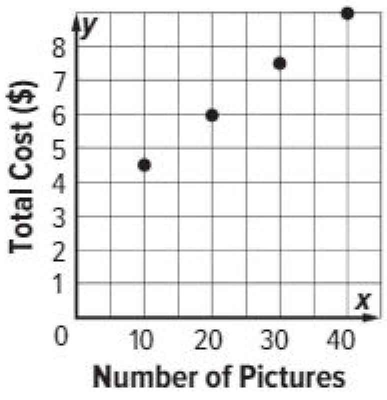
4.



6a.

x	$0.15x + 2.99$	y
10	0.15(10) + 2.99	4.49
20	0.15(20) + 2.99	5.99
30	0.15(30) + 2.99	7.49
40	0.15(40) + 2.99	8.99

6b.



6c. 75 pictures: 0.15(75) + 2.99 = $14.24
100 pictures: 0.15(100) + 2.99 = $17.99

Lesson 4-3 Need Another Example?

2. Sample answer:

Domain	Rule	Range
x	$f(x) = 4x - 1$	$f(x)$
−2	4(−2) − 1	−9
−1	4(−1) − 1	−5
0	4(0) − 1	−1
1	4(1) − 1	3

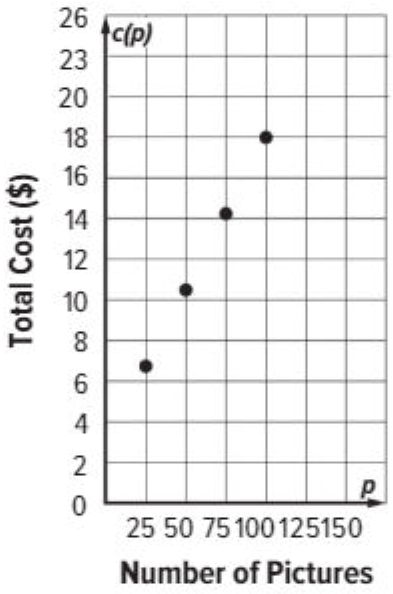
D: {−2, −1, 0, 1}; R: {−9, −5, −1, 3}

Lesson 4-3 Independent Practice

8b.

p	$c(p) = 0.15p + 2.99$	$c(p)$
25	$c(p) = 0.15(25) + 2.99$	6.74
50	$c(p) = 0.15(50) + 2.99$	10.49
75	$c(p) = 0.15(75) + 2.99$	14.24
100	$c(p) = 0.15(100) + 2.99$	17.99

8c.



about 145 pictures

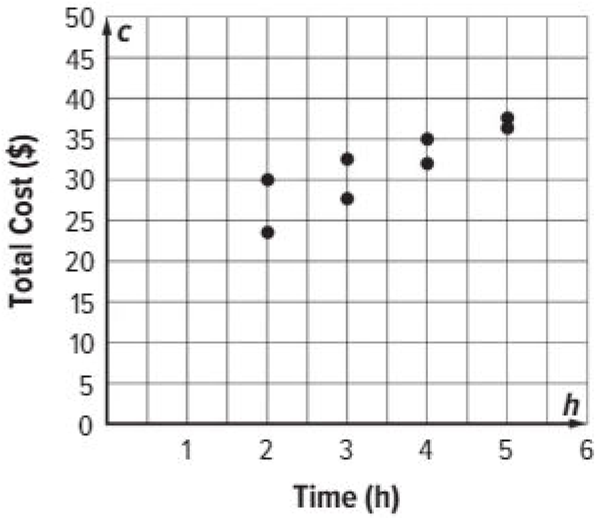
Lesson 4-4 Independent Practice

5b.

Mountain Bike Rental		
h	$15 + 4.25h$	c
2	15 + 4.25(2)	23.50
3	15 + 4.25(3)	27.75
4	15 + 4.25(4)	32.00
5	15 + 4.25(5)	36.25

Scooter Rental		
h	$25 + 2.5h$	c
2	25 + 2.5(2)	30.00
3	25 + 2.5(3)	32.50
4	25 + 2.5(4)	35.00
5	25 + 2.5(5)	37.50

5c.



Lesson 4-5 Need Another Example?

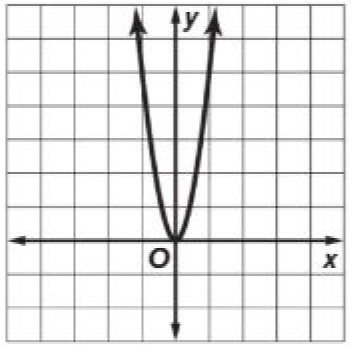
1. The water garden has a flow rate of 52 gallons per minute and the koi pond has flow rate of 60 gallons per minute. The water in the koi pond has a greater rate of change.

2a. The function for Package A has a *y*-intercept of 5 and the function for Package B has a *y*-intercept of 0. Package A costs $7 per person and Package B costs $9 per person. The rate of change for Package B is greater.

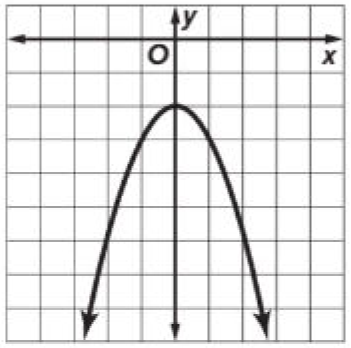
3a. The function for the first company has a *y*-intercept of 5 and a rate of change of 1.5. The function for the second company has a *y*-intercept of 4 and a rate of change of 2. The *y*-intercept for the first company is greater than the *y*-intercept for the second company but the rate of change for the first company is less than the rate of change for the second company.

Lesson 4-8 Need Another Example?

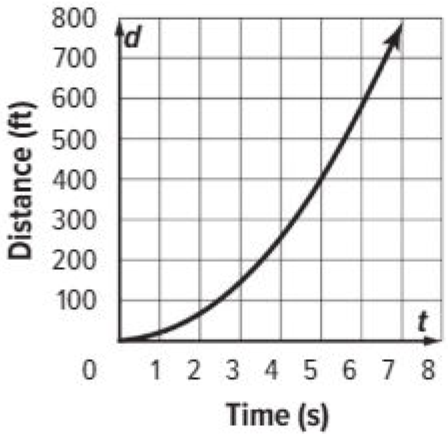
1.



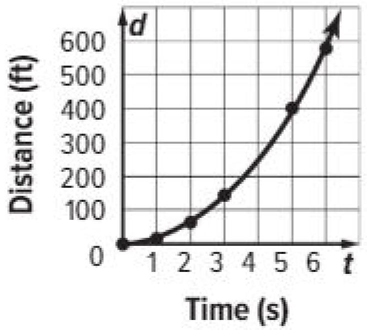
2.



3.

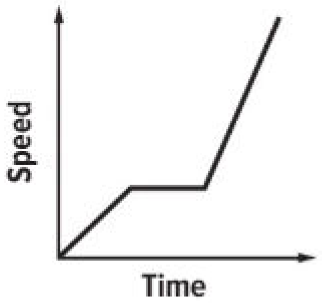


4.

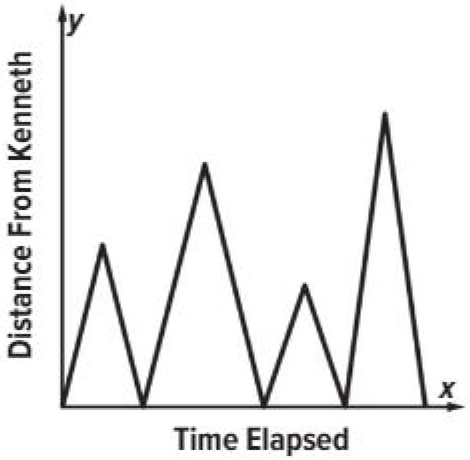


Lesson 4-9 Need Another Example?

2. Sample answer:



3. Sample answer:



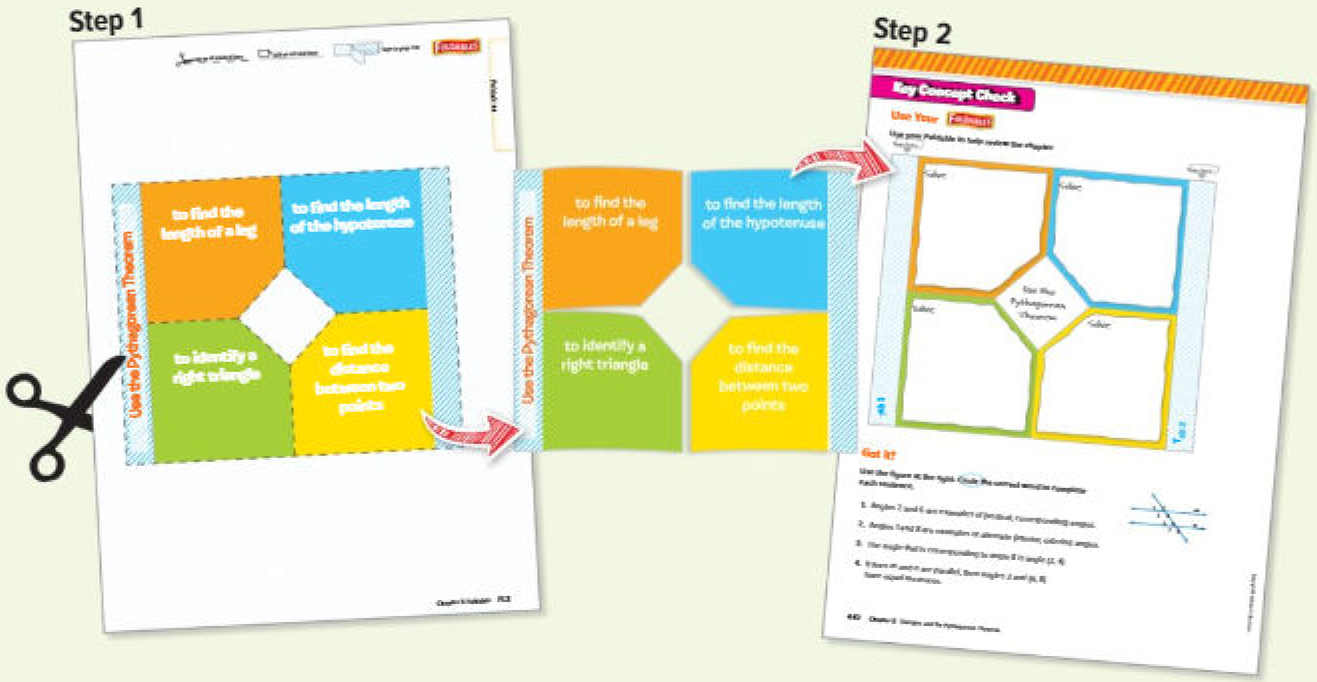
FOLDABLES® Study Organizers

What Are Foldables and How Do I Create Them?

Foldables are three-dimensional graphic organizers that help you create study guides for each chapter in your book.

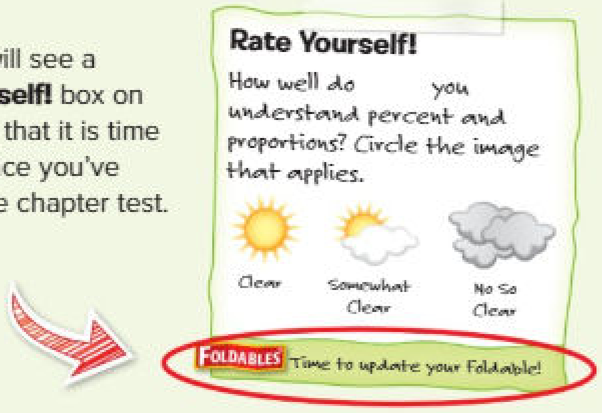
Step 1 Go to the back of your book to find the Foldable for the chapter you are currently studying. Follow the cutting and assembly instructions at the top of the page.

Step 2 Go to the Key Concept Check at the end of the chapter you are currently studying. Match up the tabs and attach your Foldable to this page. Dotted tabs show where to place your Foldable. Striped tabs indicate where to tape the Foldable.



How Will I Know When to Use My Foldable?

When it's time to work on your Foldable, you will see a Foldables logo at the bottom of the **Rate Yourself!** box on the Guided Practice pages. This lets you know that it is time to update it with concepts from that lesson. Once you've completed your Foldable, use it to study for the chapter test.



Rate Yourself!

How well do you understand percent and proportions? Circle the image that applies.

FOLDABLES Time to update your Foldable!

How Do I Complete My Foldable?

No two Foldables in your book will look alike. However, some will ask you to fill in similar information. Below are some of the instructions you'll see as you complete your Foldable. **HAVE FUN** learning math using Foldables!

Instructions and what they mean

Best Used to...	Complete the sentence explaining when the concept should be used.
Definition	Write a definition in your own words.
Description	Describe the concept using words.
Equation	Write an equation that uses the concept. You may use one already in the text or you can make up your own.
Example	Write an example about the concept. You may use one already in the text or you can make up your own.
Formulas	Write a formula that uses the concept. You may use one already in the text.
How do I ...?	Explain the steps involved in the concept.
Models	Draw a model to illustrate the concept.
Picture	Draw a picture to illustrate the concept.
Solve Algebraically	Write and solve an equation that uses the concept.
Symbols	Write or use the symbols that pertain to the concept.
Write About It	Write a definition or description in your own words.
Words	Write the words that pertain to the concept.

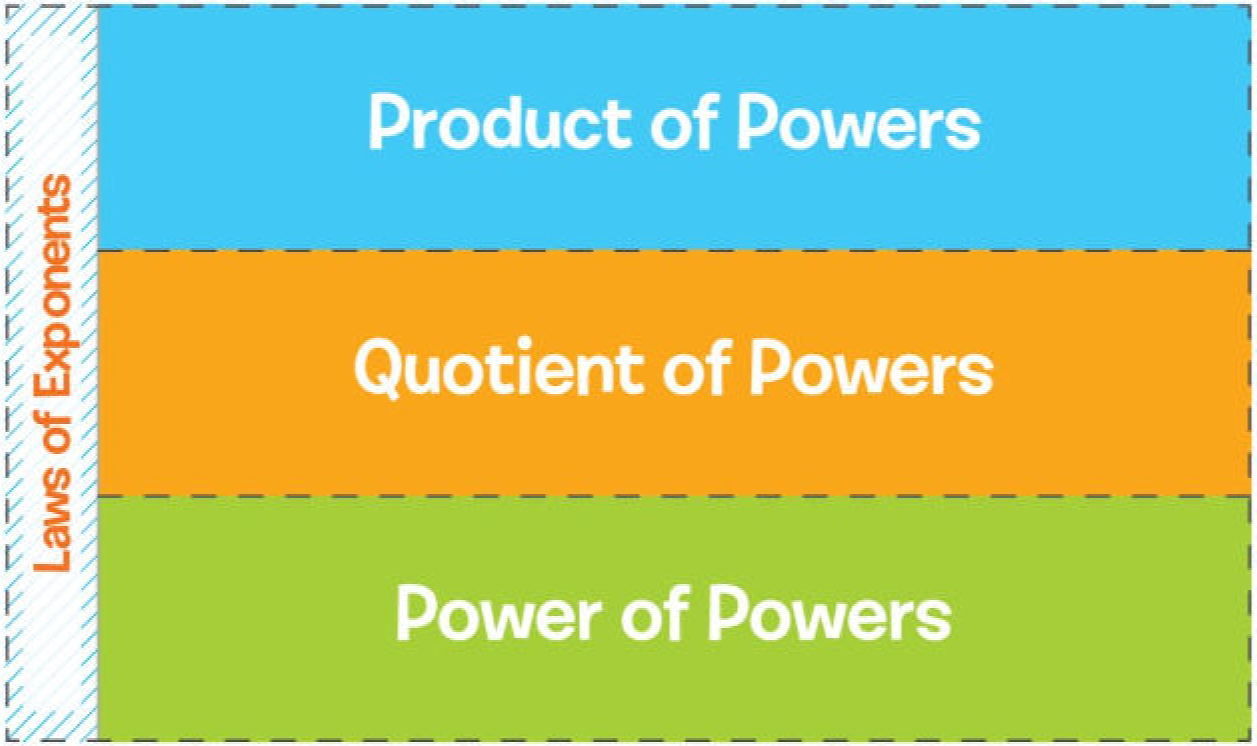
Meet Foldables Author Dinah Zike

Dinah Zike is known for designing hands-on manipulatives that are used nationally and internationally by teachers and parents. She is an explosion of energy and ideas. Her excitement and joy for learning inspires everyone she touches.

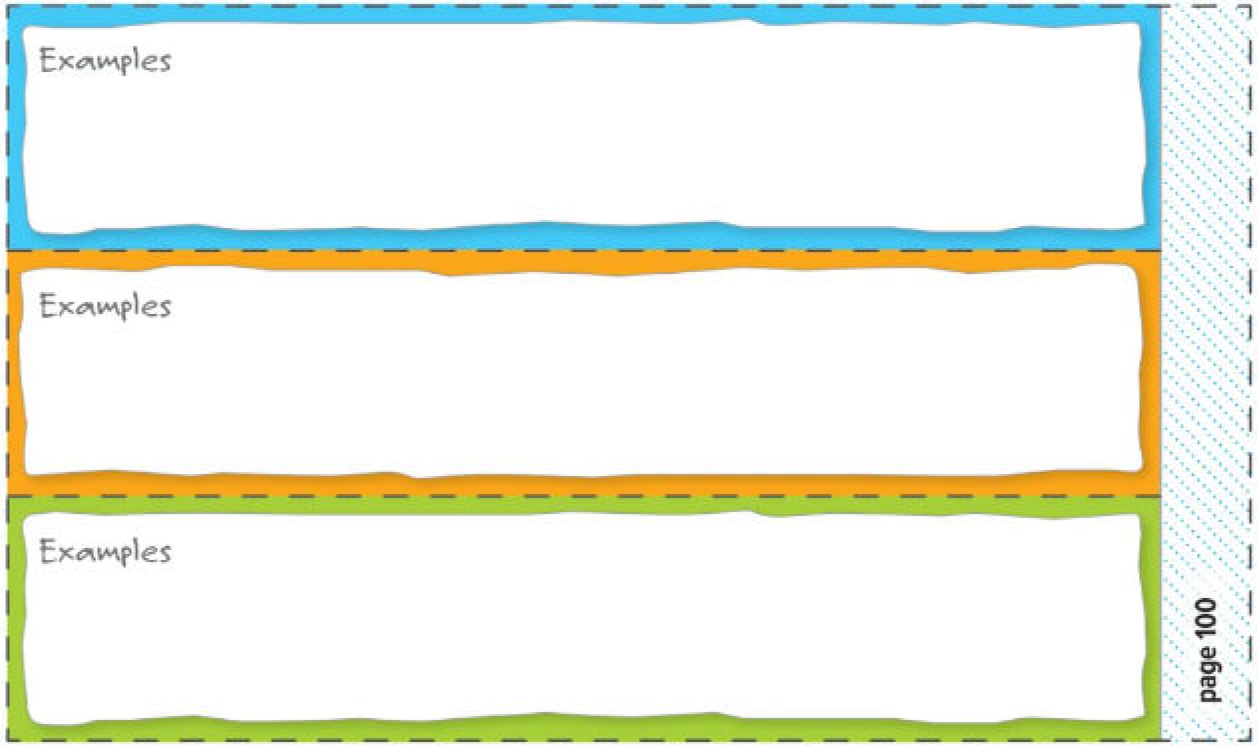
Courtesy of Dinah Zike

Use this Foldable with Chapter 1.

cut on all dashed lines fold on all solid lines tape to page 100 FOLDABLES



cut on all dashed lines fold on all solid lines tape to page 100 FOLDABLES



Use this Foldable with Chapter 1.

Use this Foldable with Chapter 2.

cut on all dashed lines

fold on all solid lines

tape to page 164

cut on all dashed lines

fold on all solid lines

tape to page 164

page 164

Step 1

Step 2

Step 3

Step 4

Tab 2

Distributive Property

Addition or Subtraction Property of Equality

Multiplication or Division Property of Equality

page 164

Tab 1

Use this Foldable with Chapter 2.

Use this Foldable with Chapter 3.

cut on all dashed lines

fold on all solid lines

tape to page 256

FOLDABLES

Solve Systems of Equations

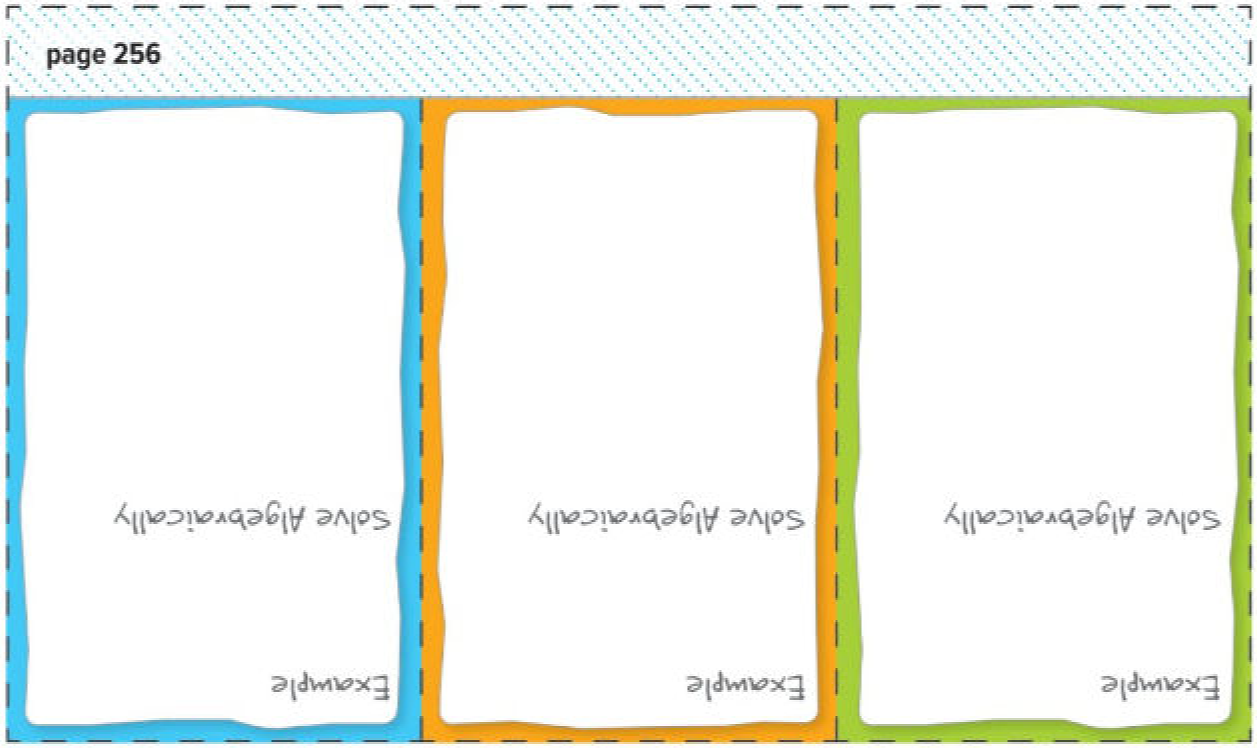
one solution	no solution	infinite number of solutions

cut on all dashed lines

fold on all solid lines

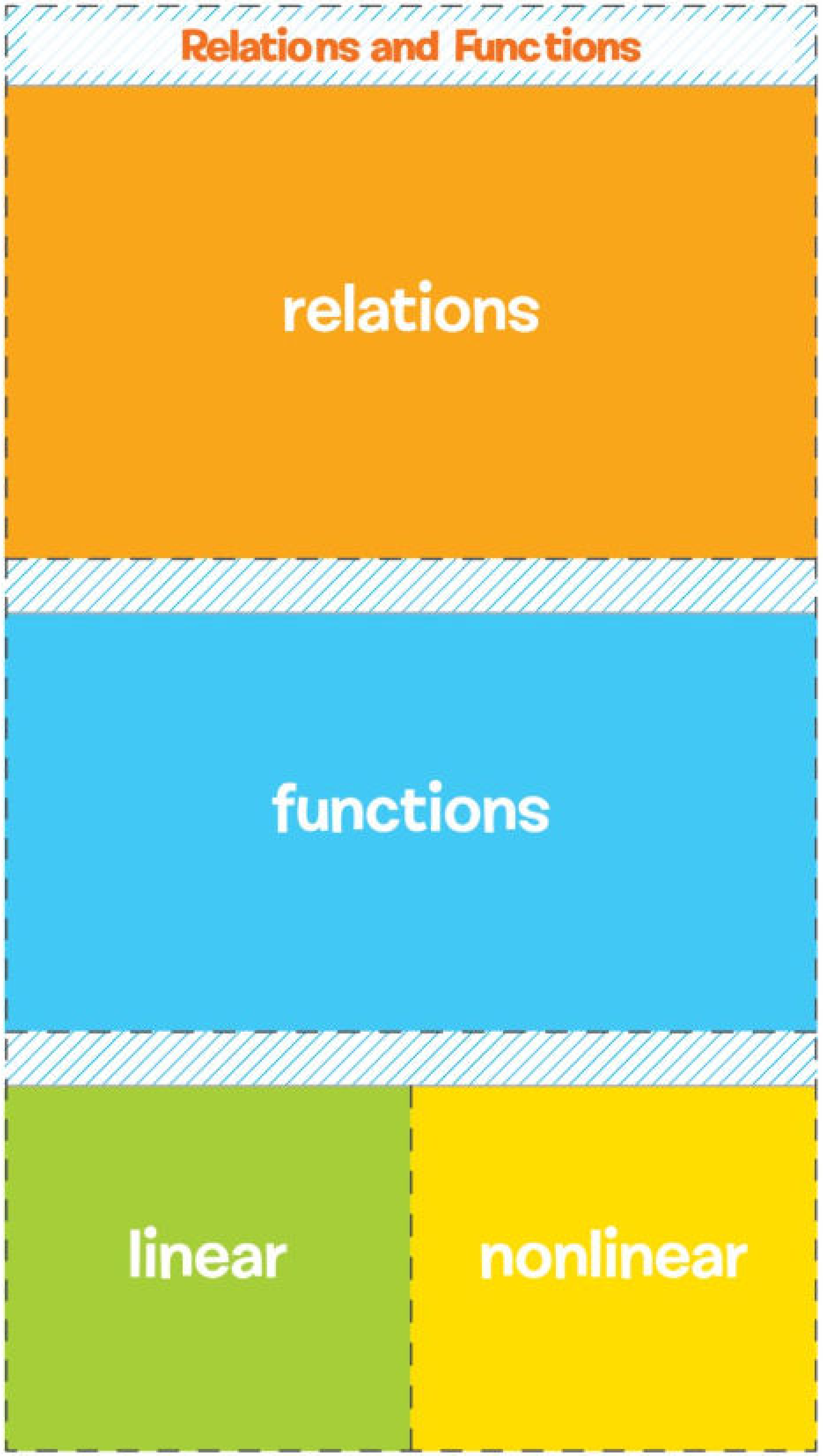
tape to page 256

Use this Foldable with Chapter 3.



Use this Foldable with Chapter 4.

cut on all dashed lines fold on all solid lines tape to page 358



cut on all dashed lines fold on all solid lines tape to page 358

Use this Foldable with Chapter 4.

page 358 **Tab 3**

Examples

Words

page 358 **Tab 2**

Examples

Words

page 358 **Tab 1**

Example

Words

Example

Words

Chapter 1 Real Numbers

Planetary Play

Mathematical Practices	MP1, MP2, MP3, MP4, MP6
Depth of Knowledge	DOK3

Part	Max Points	Scoring Rubric
A	2	Full Credit: $1.5 \times 10^8 - 1.1 \times 10^8 = 4 \times 10^7$ $4.5 \times 10^9 \div (5.8 \times 10^7) \approx 77.6$ times farther Earth is 4×10^7 kilometers farther from the Sun than it is from Venus. Neptune is 77.6 times farther from the Sun than Mercury is from the Sun. Partial Credit will be given for one correct answer. No credit will be given for an incorrect answer.
B	1	Full Credit: Standard form: 0.24 Fraction form: $\frac{24}{100}$ or $\frac{6}{25}$ No credit will be given for an incorrect answer.
C	1	Full Credit: $8.27 \times 10^{14} \div 766 \approx 1.08 \times 10^{12}$ The approximate volume of Earth is 1.08×10^{12}. No credit will be given for an incorrect answer.
D	3	Full Credit: Mercury: $\frac{1}{1 \times 10^8} = \frac{x}{5.8 \times 10^7} : x = 5.8 \times 10^{-1}$ or 0.58 cm Venus: $\frac{1}{1 \times 10^8} = \frac{x}{1.1 \times 10^8} : x = 1.1$ or 1.1 cm Earth: $\frac{1}{1 \times 10^8} = \frac{x}{1.5 \times 10^8} : x = 1.5$ or 1.5 cm Mars: $\frac{1}{1 \times 10^8} = \frac{x}{2.3 \times 10^8} : x = 2.3$ or 2.3 cm Jupiter: $\frac{1}{1 \times 10^8} = \frac{x}{7.8 \times 10^8} : x = 7.8$ or 7.8 cm Saturn: $\frac{1}{1 \times 10^8} = \frac{x}{1.4 \times 10^9} : x = 14$ or 14 cm Neptune: $\frac{1}{1 \times 10^8} = \frac{x}{4.5 \times 10^9} : x = 45$ or 45 cm Refer to the distances shown and see students' work for the model. Students will need to use two pieces of paper joined together or a large (11 × 17) piece of paper. Partial Credit: 2 points will be given if the student finds all of the proportions but has an error in modeling OR if the student has an error in finding between one and three of the distances and the error(s) is reflected in the model. 1 point will be given if the student finds all of the proportions but fails to model the distances. No credit will be given for an incorrect answer.
TOTAL	7	

Chapter 2 Equations in One Variable

Bowling Alley Fun

Mathematical Practices	MP1, MP2, MP3, MP4, MP6
Depth of Knowledge	DOK3

Part	Max Points	Scoring Rubric
A	2	Full Credit: The situation can be represented by the equation $5(5g + 3) + 25 = 115$. Each family member can bowl three games. The fees for each person will be modeled by the expression $5g + 3$. There are 5 family members so the expression for the entire family is $5(5g + 3)$. Add the gas and food expenses and the result is $5(5g + 3) + 25$. Set this expression equal to the total budget of \$115. $5(5g + 3) + 25 = 115$ $25g + 15 + 25 = 115$ $25g + 40 = 115$ $25g = 75$ $g = 3$ Partial Credit will be given for the correct equation. No credit will be given for an incorrect answer.
B	2	Full Credit: The situation can be represented by the equation $4(2g + 9) + 30 = 106$. Each family member can bowl five games. The fees for each person will be modeled by the expression $2g + 9$. There are 4 family members so the expression for the entire family is $4(2g + 9)$. Add the gas and food expenses and you get $4(2g + 9) + 30$. Set this expression equal to the total budget of \$106. $4(2g + 9) + 30 = 106$ $8g + 36 + 30 = 106$ $8g + 66 = 106$ $8g = 40$ $g = 5$ Partial Credit will be given for the correct equation. No credit will be given for an incorrect answer.
C	1	Full Credit: Dan's Alley: $5g + 3$ Bowling Palace: $2g + 9$ $5g + 3 = 2g + 9$ $3g + 3 = 9$ $3g = 6$ $g = 2$ Mr. Bradley would need to bowl 2 games to pay an equal amount at each bowling alley. No credit will be given for an incorrect answer.
D	2	Full Credit: A student will receive full credit for writing a reasonable scenario, solving the equation correctly, and telling what each number represents. Possible scenario: The Bradley family, plus one friend, goes bowling at a different bowling alley. The cost per game is \$8, shoe rental is \$2, and they set aside \$45 for gas and food. The total budget is \$153. $6 =$ the number of people that bowl $8g =$ \$8 per game $2 =$ \$2 shoe rental $45 =$ money for gas and food $153 =$ total budget $6(8g + 2) + 45 = 153$ $48g + 12 + 45 = 153$ $48g + 57 = 153$ $48g = 96$ $g = 2$ So they each bowled two games. Partial Credit will be given for a correct scenario. No credit will be given for an incorrect answer.
TOTAL	7	

Chapter 3 Equations in Two Variables

Play for a Prize

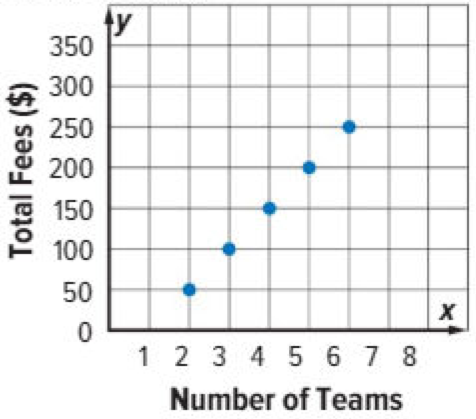
Mathematical Practices	MP1, MP2, MP4, MP5, MP6
Depth of Knowledge	DOK3

Part	Max Points	Scoring Rubric
A	3	Full Credit: Game #1: $y = \frac{1}{60}x + 2$ Game #2: $y = \frac{1}{40}x$ The slope $\frac{1}{60}$ and y-intercept of 2 for Game 1 indicate that 2 tickets are received when the game begins and additional ticket is earned after each 60 points. The slope $\frac{1}{40}$ and y-intercept of 0 for Game 2 indicate that no tickets are received when the game begins and 1 ticket is received after each 40 points. Partial Credit: 2 points will be given for a correct equation and the correct interpretation of slope and y-intercept for that equation. 1 point will be given for a correct equation with no other correct information. No credit will be given for an incorrect answer.
B	1	Full Credit: Yes; the relationship for Game 2 is proportional. Possible explanation: The function $y = \frac{1}{40}x$ has an x-intercept of zero and the graph of the function passes through the origin. No credit will be given for incorrect answer.
C	3	Full Credit will be given for a correct equation with work shown and solution of (240, 6), with the interpretation of the solution. $\frac{1}{60}x + 2 = \frac{1}{40}x$ $\frac{2}{120}x + 2 = \frac{3}{120}x$ $2 = \frac{3}{120}x - \frac{2}{120}x$ $2 = \frac{1}{120}x$ $(120)(2) = (120)\left(\frac{1}{120}x\right)$ $240 = x$ $y = \frac{1}{40}(240)$ $y = 6$ Possible interpretation: The solution tells what point level, 240 points, both games give an equal number of tickets, 6. Partial Credit: 2 points will be given for a correct algebraic solution with work shown and no interpretation of the solution OR a correct algebraic solution and interpretation lacking shown work. 1 point will be given for a correct solution. No credit will be given for incorrect answer.
D	1	Full Credit: Antoine should play game #2. Possible explanation: With game #2, Antoine will receive 10 tickets after 400 points. In order to get 10 tickets with game #1, he would need 480 points. No credit will be given for an incorrect answer.
TOTAL	8	

Chapter 4 Functions

Tournament Time

Mathematical Practices	MP1, MP2, MP3, MP4, MP5, MP6
Depth of Knowledge	DOK3

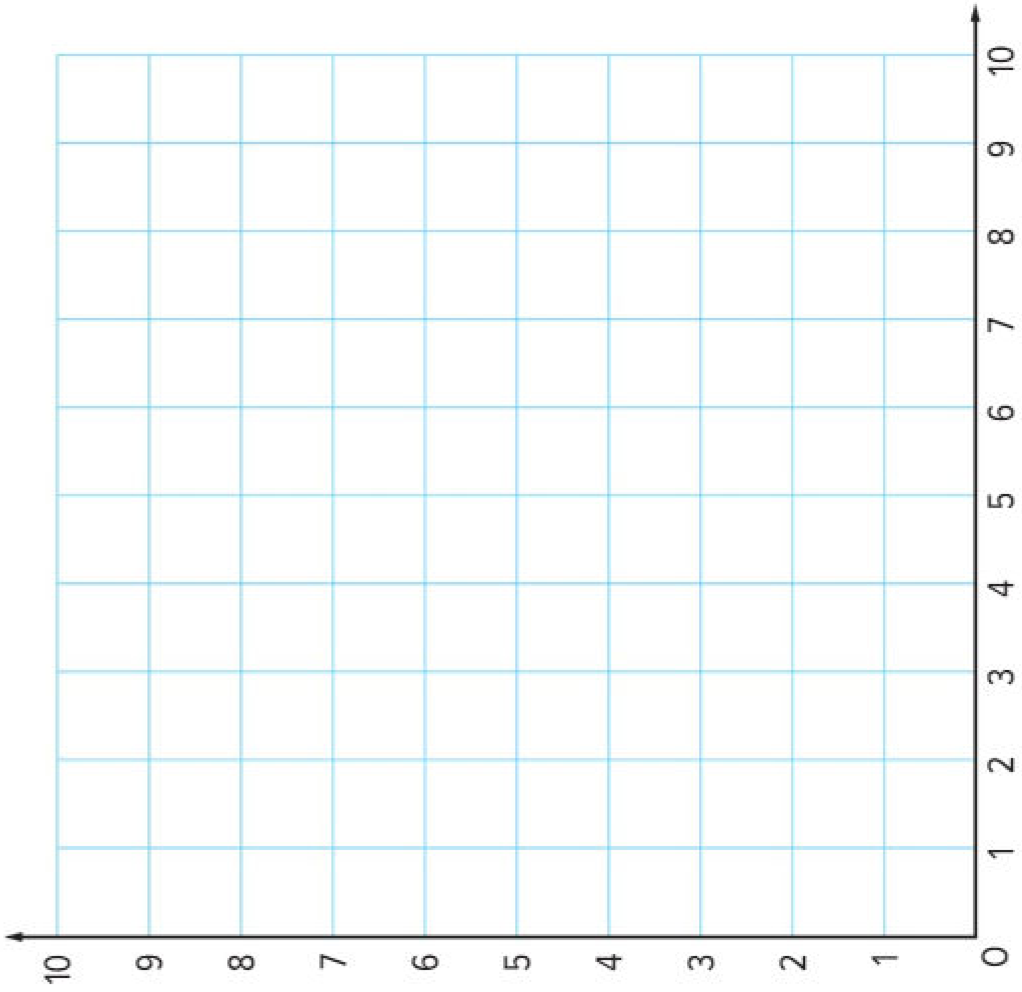
Part	Max Points	Scoring Rubric
A	1	Full Credit: The function $g(x) = x - 1$ relates the number of teams to the number of games played. No credit will be given for an incorrect answer.
B	2	Full Credit: $r(x) = 50(2) - 50;\ \$50$ $r(x) = 50(3) - 50;\ \$100$ $r(x) = 50(4) - 50;\ \$150$ $r(x) = 50(5) - 50;\ \$200$ $r(x) = 50(6) - 50;\ \$250$ Partial Credit will be given for three or four correct answers. No credit will be given for an incorrect answer.
C	3	Full Credit will be given for 5 correctly plotted points, for a correct answer of yes, and for a correct explanation. Possible Explanation: The points form a straight line, so the function is linear. Also accept that the function $r(x) = 50x - 50$ is linear because the x-value has an exponent of 1.  Partial Credit: 2 points will be given for 2–4 correctly plotted points and a correct answer of yes. 1 point will be given for 2–4 correctly plotted points OR an answer of yes with an appropriate explanation. No credit will be given for an incorrect answer.
D	1	Full Credit: $r(x) = 50(x - 1)$ $750 = 50(x - 1)$ $750 = 50x - 50$ $800 = 50x$ $16 = x$ Sixteen teams can enter the tournament. No credit will be given for an incorrect answer.
TOTAL	7	

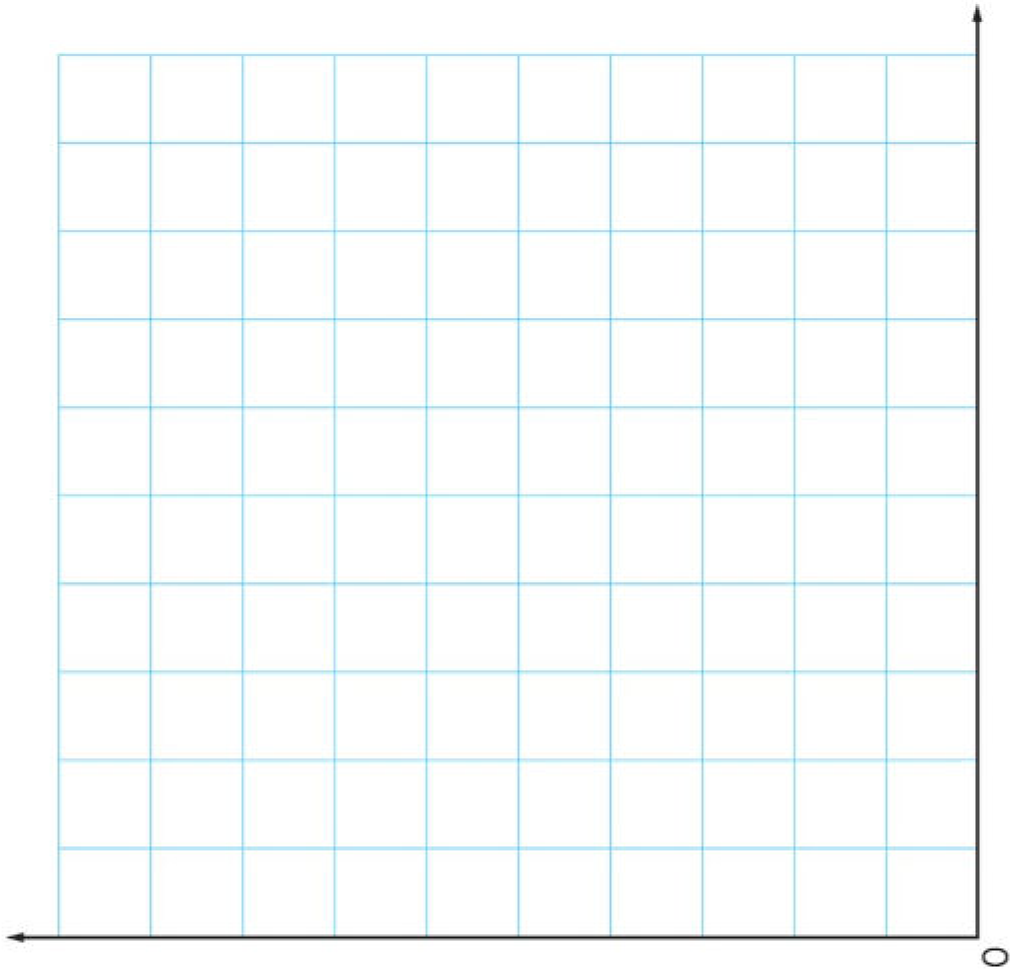
Name

=

Name ____________________

Name





Name