UNITED ARAB EMIRATES
MINISTRY OF EDUCATION

YEAR OF
ZAYED

MATH

7

McGraw-Hill Education

Integrated Math

United Arab Emirates Edition

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United Arab Emirates
Ministry of Education

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VISION
2021

McGraw-Hill Education

Integrated Math

United Arab Emirates Edition

GRADE **7** • VOLUME 3

Mc Graw Hill Education

Project: McGraw-Hill Education United Arab Emirates Edition Grade 07 Integrated Math Vol.3
FM, Glencoe Math Course 2 Vol 1 © 2015
8. Measure Figures, from Glencoe Math Course 2 Vol 2 Chapter 08 © 2015
9. Probabilty, from Glencoe Math Course 2 Vol 2 Chapter 09 © 2015
10. Statistics, from Glencoe Math Course 2 Vol 2 Chapter 10 © 2015

COVER: dyim/Shutterstock.com

mheducation.com/prek-12

Printed in the UAE.

ISBN: 978-1-52-682510-0 *(Student Edition)*
MHID: 1-52-682510-4 *(Student Edition)*
ISBN: 978-1-52-683225-2 *(Teacher Edition)*
MHID: 1-52-683225-9 *(Teacher Edition)*

1 2 3 4 5 6 7 8 9 XXX 22 21 20 19 18 17

ePub Edition

ISBN: 978-1-52-682801-9 *(Student Edition)*
MHID: 1-52-682801-4 *(Student Edition)*
ISBN: 978-1-52-683480-5 *(Teacher Edition)*
MHID: 1-52-683480-4 *(Teacher Edition)*

"Extensive knowledge and modern science must be acquired. The educational process we see today is in an ongoing and escalating challenge which requires hard work.
We succeeded in entering the third millennium, while we are more confident in ourselves."

H.H. Sheikh Khalifa Bin Zayed Al Nahyan
President of the United Arab Emirates

CONTENTS IN BRIEF

Units organized by domain

This book is organized into units based on groups called domains. The MP Mathematical Practices are embedded throughout the course.

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MP Mathematical Practices

Mathematical Practices Handbook

Tom Wang/Alamy

Ratios and Proportional Relationships

Chapter 1 **Ratios and Proportional Reasoning**

Chapter 2 **Percents**

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The Number System

Chapter 3 **Integers**

Chapter 4 **Rational Numbers**

Jill Braaten/McGraw-Hill Education

Expressions and Equations

Chapter 5 **Expressions**

Chapter 6 **Equations and Inequalities**

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Geometry

Chapter 7 **Geometric Figures**

Chapter 8 **Measure Figures**

1Apix/Alamy

Statistics and Probability

Chapter 9 **Probability**

Chapter 10 **Statistics**

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Our lead authors ensure that the McGraw-Hill mathematics programs are truly vertically aligned by beginning with the end in mind - success in Algebra 1 and beyond. By "backmapping" the content from the high school programs, all of our mathematics programs are well articulated in their scope and sequence.

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Dr. Carol Malloy was a fervent supporter of mathematics education. She was a Professor at the University of North Carolina, Chapel Hill, NCTM Board of Directors member, President of the Benjamin Banneker Association (BBA), and 2013 BBA Lifetime Achievement Award for Mathematics winner. She joined McGraw-Hill in 1996. Her influence significantly improved our programs' focus on real-world problem solving and equity. We will miss her inspiration and passion for education.

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UNIT 1 Ratios and Proportional Relationships

UNIT PROJECT PREVIEW
page 2

Chapter 1 Ratios and Proportional Reasoning

Essential Question

HOW can you show that two objects are proportional?

Chapter 2
Percents

Essential Question

HOW can percent help you understand situations involving money?

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Essential Question

WHAT happens when you add, subtract, multiply, and divide integers?

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Explore the Ocean Depths

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Equations and Inequalities

Essential Question

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HOW do measurements help you describe real-world objects?

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UNIT 5 Statistics and Probability

UNIT PROJECT PREVIEW

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Essential Question

HOW can you predict the outcome of future events?

Chapter 10 Statistics

Essential Question

HOW do you know which type of graph to use when displaying data?

UNIT PROJECT

Math Genes

This book focuses on four critical areas: (1) developing understanding of and applying proportional relationships; (2) operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings, geometric constructions, and surface area, and volume; and (4) drawing inferences about populations.

Content

Ratios and Proportional Relationships

- Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Expressions and Equations

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry

- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Statistics and Probability

- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices Handbook

Essential Question

WHAT practices help me explore and explain mathematics?

Mathematical Practices

The standards for mathematical practice will help you become a successful problem solver and to use math effectively in your daily life.

What You'll Learn

MP Throughout this handbook, you will learn about each of these mathematical practices and how they are integrated in the chapters and lessons of this book.

① **Focus on Mathematical Practice**
Persevere with Problems

② **Focus on Mathematical Practice**
Reason Abstractly and Quantitatively

③ **Focus on Mathematical Practice**
Construct an Argument

④ **Focus on Mathematical Practice**
Model with Mathematics

⑤ **Focus on Mathematical Practice**
Use Math Tools

⑥ **Focus on Mathematical Practice**
Attend to Precision

⑦ **Focus on Mathematical Practice**
Make Use of Structure

⑧ **Focus on Mathematical Practice**
Use Repeated Reasoning

Place a checkmark below the face that expresses how much you know about each Mathematical Practice. Then explain in your own words what it means to you.

I have no clue. I've heard of it. I know it!

Mathematical Practices				
Mathematical Practice				What it means to me
①				
②				
③				
④				
⑤				
⑥				
⑦				
⑧				

Persevere with Problems

What does it mean to persevere in solving problems?

Mathematical Practice 1

Make sense of problems and persevere in solving them.

Look up the word "persevere" in a dictionary. You might see "be persistent" or "follow something through to the end." When you persevere in solving math problems, you don't always stop at the first answer you get. You check if your solution is accurate, if it answers the problem, and if it makes sense!

Jared wants to paint his room. The dimensions of the room are 12 feet by 15 feet, and the walls are 9 feet tall. There are two windows, each with dimensions 6 feet by 5 feet. There are two doors, each with dimensions 30 inches by 6 feet. If a gallon of paint covers about 350 square feet, how many gallons of paint will he need to put two coats of paint on the wall?

1. **Understand** That's a lot of information! Go back and read the problem again. This time, circle the information given and underline what you are trying to find.

2. **Plan** Before you do ANY calculations, make a plan to solve the problem. List the steps you need to take.

3. **Solve** Apply your plan to solve the problem.

   Jared will need ☐ gallons of paint.

4. **Check** Is your solution accurate? Does it make sense? Explain.

5. Did you feel like giving up at any point while solving the problem? Explain.

It's Your Turn!

Solve each problem by using the four-step problem-solving model.

6. There are about 48,000 farms in Nebraska using approximately 45 million acres of land. This farmland covers about $\frac{9}{10}$ of the state. About how many acres are not made up of farmland?

Understand Circle the information you know and underline what you are trying to determine. Is there any information you will not use?

Plan What strategy will you use to solve this problem?

Solve Solve the problem. What is the solution?

Check Does your answer make sense? Can you solve the problem another way to check your work?

7. You and a friend went to the movies. You bought a student ticket and a drink. You split the cost of popcorn and a candy. You have AED4.75 left. How much did you take with you? Show your steps below. Check your solution.

Student	AED9	Popcorn	AED6.50
Adult	AED12	Candy	AED5
Senior	AED10	Drink	AED4.50

Find it in Your Book!

MP Persevere with Problems

Look in Chapter 1. Provide an example of where Mathematical Practice 1 is used. Explain why your example represents this practice.

Reason Abstractly and Quantitatively

I need to double this recipe. How much flour do I need?

Mathematical Practice 2

Reason abstractly and quantitatively.

Suppose you want to double the ingredients from the recipe below. If you write an expression or an equation to figure out what you need, you are reasoning quantitatively. When you simplify the expression or solve the equation algebraically, you are reasoning abstractly.

Granny's Pancakes (serves 4)

$\frac{3}{4}$ cup flour	$1\frac{3}{4}$ tsp baking powder
$\frac{1}{4}$ tsp salt	$1\frac{1}{2}$ tsp sugar
$\frac{1}{2}$ c and 2 T milk	$\frac{1}{8}$ c egg substitute
1 tsp vanilla	2 T butter, melted

1. What skill(s) will you use to see how much of each ingredient you would use if you were to double the recipe?

2. You planned for eight people to come for a pancake breakfast, but just found out that 10 people are coming! The recipe serves 4. Defne a variable and write an expression to determine the amount of each ingredient to serve 10 people.

3. Use the expression from Exercise 2 to complete the recipe card so that it serves 10 people. Is it appropriate to round any of the ingredients? Explain.

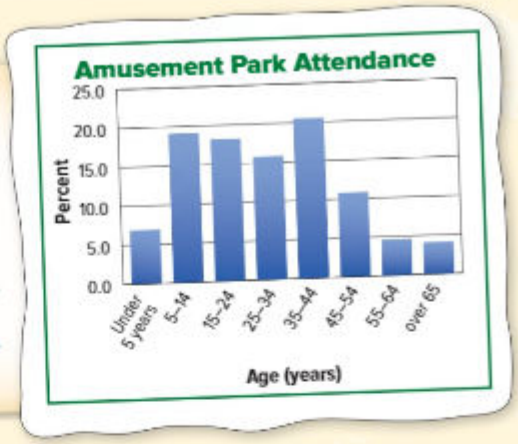
Granny's Pancakes (serves 10)

☐ cup flour	☐ tsp baking powder
☐ tsp salt	☐ tsp sugar
☐ c and ☐ T milk	☐ egg substitute
☐ tsp vanilla	☐ T butter, melted

It's Your Turn!

Reason abstractly or quantitatively to find a solution.

4. The graph shows the percent of people in different age groups that recently attended an amusement park. A total of 1.045 million people attended. How many of them were less than 25 years of age?



5. Cave exploration or spelunking is a very popular activity. Your family signs up for tours at a state park. On one of the tours, your brother is lowered 160 feet below the surface by rope. Then he continues another 70 feet below the surface to a room. You take the tree top tour where you climb to an adventure course that is 60 feet above ground. What is the difference between the elevations?

6. You and your family are traveling to a football game. You and your mother leave at 8:00 A.M. Your dad needs to wait for your sister to get home from aerobic practice, so he leaves at 9:30 A.M. If your mother drives at an average rate of 50 miles per hour, and your dad drives at an average rate of 65 miles per hour, when will he pass her? Suppose the game is 205 miles away. Who will get there first?

Find it in Your Book!

MP Reason Abstractly

Look in Chapter 1. Provide an example of where Mathematical Practice 2 is used. Explain why your example represents this practice.

Construct an Argument

Have you ever questioned something that someone else said?

Mathematical Practice 3

Construct viable arguments and critique the reasoning of others.

If your friend told you that his dog could run 45 miles per hour, would you believe him? What would your friend need to do to justify his comment? You might want to see the dog run and use a stopwatch to time him. In math, we often need to justify our conclusions as well. We can use *inductive* or *deductive* reasoning.

1. Use the Internet or another source to look up the meanings of the terms *inductive reasoning* and *deductive reasoning*. Write the meanings in your own words.

2. Label each example below as either using inductive or deductive reasoning.

_____ Reasoning

Every dog that Saeed met has fleas, so he believes that all dogs have fleas.

_____ Reasoning

Equilateral triangles have 3 congruent sides. Lamiaa has a triangle with 3 congruent sides, so she has an equilateral triangle.

Throughout this text, you may be asked to evaluate an argument that someone else made. If you determine that the argument is false, you may be asked to provide a counterexample. A *counterexample* is just one example that shows a statement is not true.

3. Determine if the following statement is true. If it is not true, provide a counterexample:

All prime numbers are odd.

It's Your Turn!

Complete each step in the solution shown. Use the Properties of Equality (Addition, Subtraction, Multiplication, or Division).

4. $a - 15 = 36$ Write the equation.

$+15 = +15$ _____

$a = 51$ Simplify.

5. $5p = 35$ Write the equation.

$\frac{5p}{5} = \frac{35}{5}$ _____

$p = 7$ Simplify.

For each of the following statements, determine if the statement is *true* or *false*. If false, provide a counterexample.

6. All four-legged pieces of furniture are tables.

_____

7. All rectangles have 4 right angles.

_____

8. The population of Pennsylvania is about 4% of the total population of the United States. Daniel claims that since the population of the United States is about 312 million, the population of Pennsylvania must be around 17.5 million. Is his claim reasonable? Explain.

_____

_____

_____

Find it in Your Book!

MP Construct an Argument

Look in Chapter 1. Provide an example of where Mathematical Practice 3 is used. Explain why your example represents this practice.

_____

_____

_____

_____

Model with Mathematics

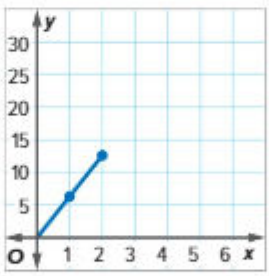
Are you a visual person or do you prefer to use words?

Mathematical Practice 4

Model with mathematics.

You might prefer to use diagrams or drawings when explaining ideas. Or you might prefer to use words. In math, we also use different ways to model the same idea. We can use words, graphs, tables, numbers, symbols, or diagrams.

1. Suppose you are selling T-shirts as a fundraiser for Key Club. The club makes a AED6.30 profit for every T-shirt sold. Complete each model shown.

<table>
<tr><th>Words</th><th>Numbers</th></tr>
<tr><td>_____ per T-shirt</td><td>
<table>
<tr><th>Profit (AED)</th><th>Number of shirts</th></tr>
<tr><td>6.30</td><td>1</td></tr>
<tr><td>12.60</td><td></td></tr>
<tr><td>18.90</td><td></td></tr>
</table>
</td></tr>
<tr><th>Symbols</th><th>Graph</th></tr>
<tr><td>Let p = profit
t = number of T-shirts sold
$p =$ ☐ t</td><td> 
</td></tr>
</table>

All of these model the same relationship between profit and number of T-shirts sold, just in different ways.

2. Which relationship would you prefer to use to determine the profit if 100 T-shirts were sold? Explain.

_____

_____

_____

It's Your Turn!

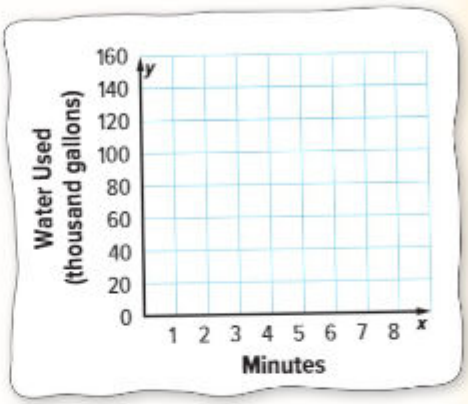
Use the models shown to solve each problem.

3. A waterpark cycles about 24,000 gallons per minute through the local river.

a. Tables Complete the table to show the number of gallons used in 1, 2, 3, 4, and 5 minutes.

Time, x (minutes)	Gallons, y (thousand gallons)

b. Graph Graph the ordered pairs on the coordinate plane.



c. Symbols Write an equation to show the number of gallons of water y used in x minutes.

_____

4. Kitra is creating a treasure hunt for the school carnival. The scale on the map is 0.5 inch = 0.25 mile.

a. Tables Complete the table to determine the actual distance for 0.5, 1, 1.5, 2, and 2.5 inches on the map.

b. Symbols Write an equation to determine the actual distance d for m inches on the map. _____

Map Length m (in.)	Distance d (mi)

Find it in Your Book!

MP Model with Mathematics

Look in Chapter 6. Provide an example of where Mathematical Practice 4 is used. Explain why your example represents this practice.

_____

_____

_____

_____

Use Math Tools

Which tools would you use to finish this piece of artwork?

Mathematical Practice 5

Use appropriate tools strategically.

You might need paints, a brush, or maybe charcoal or colored pencils. You might also need some art training! So, let's investigate how to choose and use the proper tools and strategies to solve math problems.

1. Math tools are objects like paper and pencil, calculators, algebra tiles, and rulers. List three more math tools that are helpful in solving problems.

Math Tools

2. Some math strategies are estimation, drawing a diagram, and using mental math. List three more math strategies that are helpful in solving problems.

Math Strategies

3. Describe a situation in which you would use a protractor.

It's Your Turn!

List the tools or strategies you would use to solve each problem. Then solve the problem.

4. You need to make a scale model of your room for your art class. The scale is $\frac{3}{4}$ inch represents 1 foot. What are the dimensions of your model?

_____

_____

5. Your family wants to go to a Green Bay Packers football game. About how much would it cost a family of four to go to a game, park, buy 2 programs, and each have a soda and a sandwich?

_____

_____

_____

Green Bay Packers Football	
Average Ticket Price	AED78.84
Parking	AED40.00
Soda	AED4.25
Sandwich	AED5.50
Program	AED6

6. Your group spent AED679.35 on props, hall space, and programs for a recital. The ticket prices for the recital are shown in the table. If you sold a total of 46 adult tickets and 59 student tickets, how much did the group make after paying for the supplies?

_____

_____

Recital Tickets	
Adult	AED15.00
Student	AED8.00

Find it in Your Book!

MP Use Math Tools

Look in Chapter 1. Provide an example of where Mathematical Practice 5 is used. Explain why your example represents this practice.

_____

_____

_____

_____

Attend to Precision

What does it mean to communicate precisely?

Mathematical Practice 6

Attend to precision.

Communicating precisely is not just about giving the right answer. It also includes using terms, units, symbols, ideas, and procedures appropriately when discussing or solving problems.

Mansour drives his scooter to soccer practice every day. Each week, his scooter uses a quarter of a tank of gas. The practice field is 3 miles from his house and the gas tank holds 2.4 gallons of gas. He wants to find the unit rate per gallon of gas. Pair up with a classmate to discuss and answer the following.

1. In your own words, write the definitions for *ratio, equivalent ratio, bar diagram,* and *unit rate*.

2. How do the words from Exercise 1 relate to the problem?

3. Discuss with your partner the steps you will use to solve this problem. Summarize your discussion, and then solve the problem.

4. What units of measure will describe the unit rate per gallon of gas?

5. What is the unit rate per gallon for Marlon's scooter?

It's Your Turn!

Solve each problem.

The state of Colorado is shaped like a rectangle as shown in the map. The rate $\frac{1\text{ cm}}{100\text{ km}}$ can be used to find actual distances.

6. Use equivalent rates to find the actual distance x.

a. $\frac{4.5\text{ cm}}{x}$ **b.** $\frac{6.1\text{ cm}}{x}$

7. What is the perimeter of the state on the map? the actual perimeter?

8. Sumaia is in charge of the 7th grade picnic and needs to order the food for the 90 students attending. She surveys a sample population of 18 students. Ten students chose beefburgers, 7 chose cheeseburgers, and 1 chose a veggie burger

a. Make a conjecture about how many students at the picnic will choose each type of food.

b. Discuss with a partner if these numbers are exact or estimates. Then determine what problems Sumaia may have by using those numbers.

Find it in Your Book!

MP Attend to Precision

Look in Chapter 1. Provide an example of where Mathematical Practice 6 is used. Explain why your example represents this practice.

It's Your Turn!

Describe the method you would use to solve each of the following. Then solve.

6. Yousef's scores on his science tests were 76%, 93%, 87%, 91%, and 83%. Haney wants a 90% test average for the term. If all tests are weighted the same, is it possible for him to get a 90% test average if there is only one more test? Explain.

7. You need to make a model of your bedroom for your art class. The scale is $\frac{3}{4}$-inch $= 1$ foot. What are the dimensions of your model?

8. A rectangle has a length of 4 centimeters and a width of 3 centimeters. The length and width are each multiplied by a factor of 3. Is the ratio $\frac{\text{area of new rectangle}}{\text{area of original rectangle}}$ equivalent to the ratio $\frac{\text{side length of new rectangle}}{\text{side length of original rectangle}}$? If not, explain how they are related.

Find it in Your Book!

MP Make Use of Structure

Look in Chapter 1. Provide an example of where Mathematical Practice 7 is used. Explain why your example represents this practice.

Use Repeated Reasoning

How can repeated reasoning help me in math?

Sometimes, if you find repeated reasoning or patterns in mathematics, you can actually create shortcuts that help you in calculations.

Mathematical Practice 8

Look for and express regularity in repeated reasoning.

1. Complete the table by choosing a set of three consecutive numbers. Then compare the product of the two outer numbers to the middle number squared. The first one is started for you.

Consecutive Numbers	Product of Two Outer Numbers	Middle Number Squared
4, 5, 6	$4 \times 6 =$	$5 \times 5 =$

2. What is the relationship between the product of the two outer numbers and the middle number squared?

3. Suppose you want to find 22×24. Write an expression using the middle number that can help you find the product.

4. Use this reasoning to astound your family and friends by using mental math to find 49×51!

5. Do you think this process works for three digit numbers? How can you test your conjecture?

It's Your Turn!

Solve.

6. Amany is creating a treasure hunt for the school carnival. The scale on the map is 0.5 inch = 0.25 mile.

   a. **Table** Complete the table to determine the actual distance for 0.5 inch, 1 inch, 1.5 inches, 2 inches, and 2.5 inches on the map.

   b. **Symbols** Write an equation to find the actual distance d for m inches on the map.

   _____

Map Length (in.) m	Distance (mi) d

A famous number pattern is Pascal's Triangle shown below.
Use Pascal's Triangle to complete Exercises 7 and 8.

7. Complete the triangle below. What relationship exists among the numbers in each row compared to the numbers in the previous row?

   _____

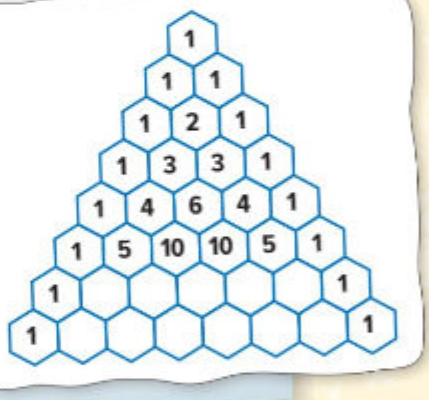
   _____

8. Determine the sum of the numbers in each row. Analyze the relationship between the sum of the numbers in each row compared to the sum of the numbers in the previous row.

   _____

   _____

   _____



Find it in Your Book!

MP Use Repeated Reasoning

Look in Chapter 5. Provide an example of where Mathematical Practice 8 is used. Explain why your example represents this practice.

_____

_____

_____

_____

Use the Mathematical Practices

Solve.

The courtyard at Eastmoor Middle School is shaped like a rectangle that is 40.3 feet long. The width of the courtyard is 14.6 feet less than the length.

a. Draw and label a diagram of the school courtyard. What is the perimeter of the courtyard? _____

b. Student council wants to plant 14 trees so they are equally spaced around the courtyard. Draw a diagram showing where the trees should be planted. About how far apart are the trees? _____

Determine which mathematical practices you used to determine the solution. Shade the circles that apply.

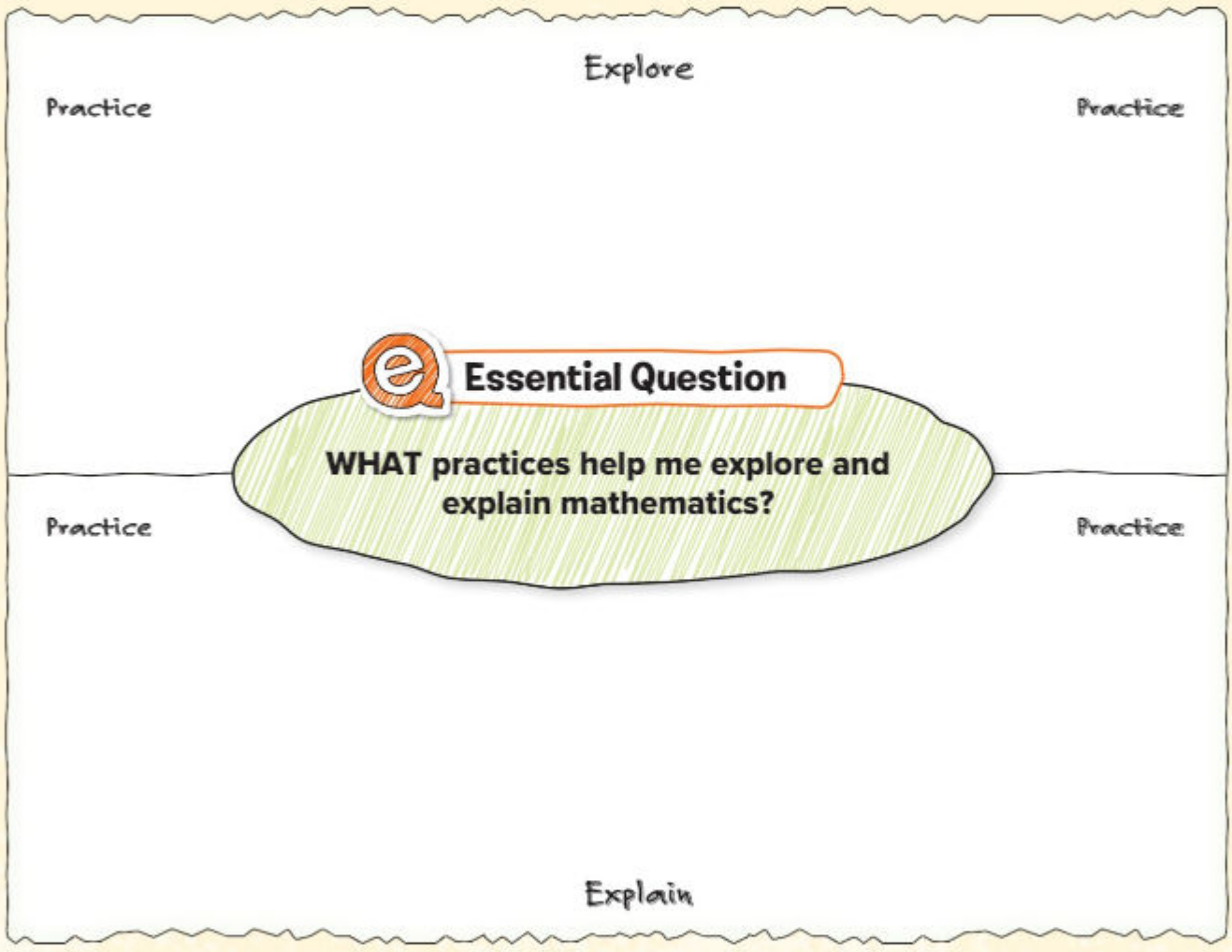
Which MP **Mathematical Practices** did you use?
Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Reflect

Answering the Essential Question

Use what you learned about the mathematical practices to complete the graphic organizer. Write two different practices that you use for each category. Then describe how each practice helps you explore and explain mathematics.



Answer the Essential Question. WHAT practices help me explore and explain mathematics?

Chapter 8

Measure Figures

Essential Question

How do measurements help you describe real-world objects?

MP Mathematical Practices

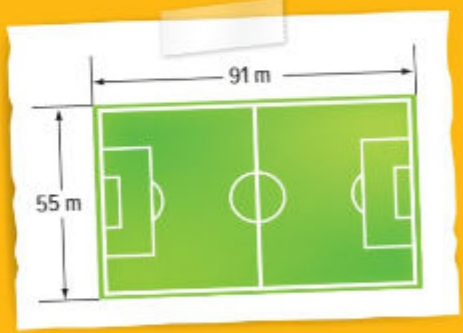
1, 2, 3, 4, 5, 6, 8

Math in the Real World

Soccer is a sport that is played on a rectangular field. The dimensions of a regulation size soccer field are 91 meters long and 55 meters wide.

What is the area of the soccer field shown?

$A =$ ____ square meters



FOLDABLES® Study Organizer

1. Cut out the Foldable on page FL9 of this book.
2. Place your Foldable on page 700.
3. Use the Foldable throughout this chapter to help you learn about measuring figures.

What Tools Do You Need?

Vocabulary

center	lateral face	semicircle
circle	lateral surface area	slant height
circumference	pi	surface area
composite figure	radius	volume
diameter	regular pyramid	

Study Skill: Studying Math

Power Notes *Power notes* are similar to lesson outlines, but they are simpler to organize. Power notes use the numbers 1, 2, 3, and so on. You can have more than one detail under each power. You can even add drawings or examples to your power notes.

Power 1: This is the main idea.
Power 2: This provides details about the main idea.
Power 3: This provides details about Power 2.
and so on...

Complete the following sample of power notes for this chapter.

1: Circles

2: Circumference

3: _____

3: _____

2: Area

3: _____

What Do You Already Know?

Place a checkmark below the face that expresses how much you know about each concept. Then scan the chapter to find a definition or example of it.

I have no clue. I've heard of it. I know it!

Integers				
Concept				Definition or Example
area of a circle				
area of composite figures				
lateral surface area				
pi (π)				
total surface area				
volume				

When Will You Use This?

Here is an example of how volume and surface area are used in the real world.

Activity 1 When wrapping a present, how do you determine how much paper to use? Describe a method that you could use to make sure that you cut the right size piece of wrapping paper.

Are You Ready?

Try the Quick Check below.

Quick Review

Review

Example 1

Find the area of the rectangle.

$A = \ell w$ Area of a rectangle

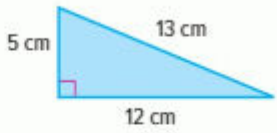
$A = (10)(4)$ Replace ℓ with 10 and w with 4.

$A = 40$ Simplify.

The area of the rectangle is 40 square meters.

Example 2

Find the area of the triangle.



$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}(12)(5)$ Replace b with 12 and h with 5.

$A = \frac{1}{2}(60)$ Multiply.

$A = 30$ Simplify.

The area of the triangle is 30 square centimeters.

Quick Check

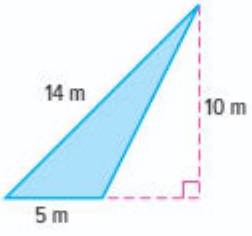
Area **Find the area of each figure.**

1.

Show your work.

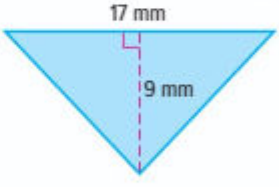
$A =$ _____

2.



$A =$ _____

3.



$A =$ _____

4. Hana's yard is in the shape of a triangle. It has a height of 10 meters and a base of 15 meters. What is the area of the yard?

_____

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4

Inquiry Lab
Circumference

Inquiry

HOW is the circumference of a circle related to its diameter?

MP Mathematical Practices 1, 3, 6

The distance around a flying disc, or its *circumference*, is 37.7 centimeters. The distance across the disc through its center, or its *diameter*, is 12 centimeters. How is the circumference of a circular object, such as a flying disc, related to its diameter?

Hands-On Activity

Step 1 Cut a piece of string the length of the circumference of a circular object such as a jar lid. Use a centimeter ruler to measure the length of the string to the nearest tenth of a centimeter. Record this measurement in the table below.

Object	Circumference (C)	Diameter (d)	$\frac{C}{d}$
Disc	37.7 cm	12 cm	

Step 2 Measure and record the diameter of the lid.

Step 3 Use a calculator to find the ratio of the circumference of the flying disc to its diameter. Repeat for the circular object you measured in Steps 1 and 2. Round answers to the nearest hundredth.

Step 4 Repeat Steps 1 through 3 for other circular objects.

Describe the ratios $\frac{C}{d}$ you found. Identify the number that is closest to the value of each ratio. _____

_____

Write a rule in the form $\frac{C}{d} \approx \blacksquare$, where $\blacksquare$ is the number you identified in the question above. _____

Investigate

Make a Prediction Work with a partner. Measure the diameter of two different circular objects. Predict each circumference. Check your predictions by measuring. Then find each ratio $\frac{C}{d}$. Record your values to the nearest hundredth in the table below.

	Object	Diameter (d)	Predicted Circumference	Measured Circumference (C)	Ratio $\frac{C}{d}$
1.					
2.					

Analyze and Reflect

3. **Reason Inductively** How do the ratios $\frac{C}{d}$ in the table compare to the ones in the Activity? Identify the number that is closest to the value of all of the ratios.

Create

4. **Reason Abstractly** Write a formula in the form $\frac{C}{d} = \blacksquare$, that gives the approximate ratio of the circumference C of a circle to its diameter d, where $\blacksquare$ is the number you identified in Exercise 3.

5. **Reason Abstractly** Multiply both sides of your formula by d to write an equivalent formula in the form $C = \blacksquare \times d$ *that gives the approximate circumference* C if you know the diameter d of a circle.

6. **Reason Abstractly** The radius of a circle is one half of its diameter. Write a formula that relates the circumference C of a circle to its radius r.

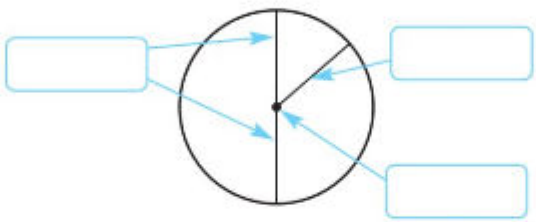
7. Inquiry HOW is the circumference of a circle related to its diameter?

Lesson 1 Circumference

Vocabulary Start-Up

A **circle** is the set of all points in a plane that are the same distance from a point, called the **center**. The **circumference** is the distance around a circle. The **diameter** is the distance across a circle through its center. The **radius** is the distance from the center to any point on the circle.

Fill in each box with one of the following terms: *center*, *diameter*, and *radius*.



Real-World Link

1. The table shows the approximate measurements of two sizes of hula hoops.

Size	Radius (cm)	Diameter (cm)	Circumference (cm)
child	14	28	88
adult	20	40	126

a. Describe the relationship between the diameter and radius of each hula hoop. _____

_____

b. Describe the relationship between the circumference and diameter of each hula hoop. _____

_____

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Essential Question

HOW do measurements help you describe real-world objects?

Vocabulary

circle
center
circumference
diameter
radius
pi π

MP Mathematical Practices
1, 3, 4, 6, 8

Key Concept

Radius and Diameter

Words The diameter d of a circle is twice its radius r. The radius r of a circle is half of its diameter d.

Symbols $d = 2r$ $r = \frac{d}{2}$

Work Zone

Examples

1. **The diameter of a circle is 14 centimeters. Find the radius.**

$r = \frac{d}{2}$ Radius of circle

$r = \frac{14}{2}$ Replace d with 14.

$r = 7$ Divide.

The radius is 7 centimeters.

2. **The radius of a circle is 8 meters. Find the diameter.**

$d = 2r$ Diameter of circle

$d = 2 \cdot 8$ Replace r with 8.

$d = 16$ Multiply.

The diameter is 16 meters.

Got it? **Do these problems to find out.**

Find the radius or diameter of each circle with the given dimension.

a. $d = 23$ cm **b.** $r = 3$ cm.

c. $d = 16$ m **d.** $r = 5.2$

STOP and Reflect

The diameter of a circle is 36 centimeters. Circle the radius.

72 cm 18 cm

Show your work.

a. _____

b. _____

c. _____

d. _____

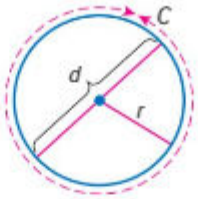
Circumference

Key Concept

Words The circumference of a circle is equal to π times its diameter or π times twice its radius.

Symbols $C = \pi d$ or $C = 2\pi r$

Model



In the Inquiry Lab, you learned that $\frac{C}{d} \approx 3$. The exact ratio is represented by the Greek letter **π (pi)**. The value of π is 3.1415926... . The decimal never ends, but it is often approximated as 3.14.

Another approximation for π is $\frac{22}{7}$. Use this value when the radius or diameter is a multiple of 7 or has a multiple of 7 in its numerator if the radius is a fraction.

Estimation
To estimate the circumference of a circle, you can use 3 for π since $\pi \approx 3$.

Example

3. **Find the circumference of a circle with a radius of 21 centimeters.**

Since 21 is a multiple of 7, use $\frac{22}{7}$ for π.

$C = 2\pi r$ Circumference of a circle

$C \approx 2 \cdot \frac{22}{7} \cdot 21$ Replace π with $\frac{22}{7}$ and r with 21.

$C \approx 2 \cdot \frac{22}{\cancel{7}_{1}} \cdot \frac{\cancel{21}^{3}}{1}$ Divide by the GCF, 7.

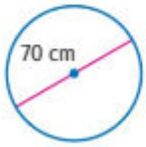
$C \approx 132$ Simplify.

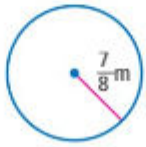
The circumference of the circle is about 132 centimeters.

Got it? **Do these problems to find out.**

Show your work.

Find the circumference of each circle. Use $\frac{22}{7}$ for π.

e. 

f. 

e. _____

f. _____

Show your work.

g. _____

Example

4. **Big Ben is a famous clock tower in London, England. The diameter of the clock face is 7 meters. Find the circumference of the clock face. Round to the nearest meter.**

$C = \pi d$ Circumference of a circle

$C \approx 3.14(7)$ Replace π with 3.14 and d with 23.

$C \approx 22$ Multiply.

So, the distance around the clock is about 22 meters.

Got it? **Do this problem to find out.**

g. A circular fence is being placed to surround a tree. The diameter of the fence is 4 meters. How much fencing is used? Use 3.14 for π. Round to the nearest tenth if necessary.

Guided Practice

Check

Find the radius or diameter of each circle with the given dimension.

(Examples 1 and 2)

Show your work.

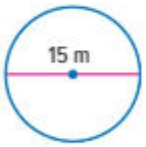
1. $d = 3$ m _____

2. $r = 14$ m _____

3. $d = 20$ cm. _____

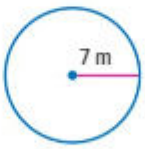
Find the circumference of each circle. Use 3.14 or $\frac{22}{7}$ for π. Round to the nearest tenth if necessary. (Examples 3 and 4)

4.



_____

5.



_____

6. **Building on the Essential Question** A circle has a circumference of about 16.3 meters and a diameter of about 5.2 meters. What is the relationship between the circumference and diameter of this circle?

_____

_____

_____

Rate Yourself!

How confident are you about finding the circumference? Check the box that applies.

Name _____ My Homework _____

Independent Practice

Find the radius or diameter of each circle with the given dimensions.
(Examples 1 and 2)

1. $d = 5$ mm _____

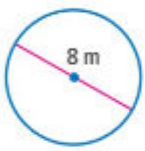
2. $d = 24$ m _____

3. $r = 17$ cm _____

Show your work.

Find the circumference of each circle. Use 3.14 or $\frac{22}{7}$ for π. Round to the nearest tenth if necessary. (Example 3)

4.



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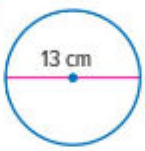


_____

6.

_____

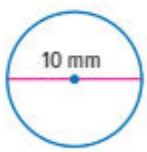
7 The largest tree in the world by volume is in Sequoia National Park. The diameter at the base is 11 meters. If a person with outstretched arms can reach 1.8 meters, how many people would it take to reach around the base of the tree? (Example 4)

_____

8. The Belknap shield volcano is located in the Cascade Range in Oregon. The volcano is circular and has a diameter of 8 kilometers. What is the circumference of this volcano. Round your answer to the nearest tenth? (Example 4)

_____

9. MP **Be Precise** Refer to the circle at the right.



a. Find the circumference of the circle. Use 3 as the estimate of π.

_____

b. Find the circumference of the circle using 3.14 for π.

_____

c. Another estimate of π is 3.14159. Find the circumference using this estimate.

_____

d. What do you notice about the estimate used for π and the circumference of the circle?

_____

_____

Copy and Solve **For Exercises 10–14, show your work on a separate piece of paper.**

Find the diameter given each circumference. Use 3.14 for π.

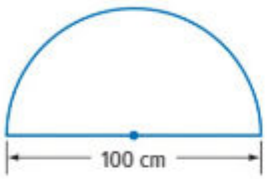
10. a satellite dish with a circumference of 957.7 meters

11. a basketball hoop with a circumference of 141.3 centimeters

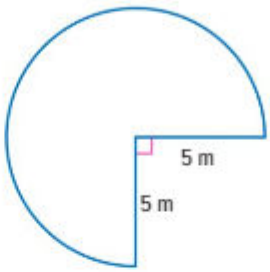
12. a nickel with a circumference of about 65.94 millimeters

Find the distance around each figure. Use 3.14 for π.

13.



14.



H.O.T. Problems Higher Order Thinking

15. MP **Justify Conclusions** Determine if the circumference of a circle with a radius of 4 meters will be greater or less than 24 meters. Explain.

16. MP **Model with Mathematics** Draw and label a circle that has a diameter more than 5 centimeters, but less than 10 centimeters. Estimate its circumference and then find its circumference using a calculator. Compare your results.

17. MP **Persevere with Problems** Analyze how the circumference of a circle would change if the diameter was doubled. Provide an example to support your explanation.

18. MP **Justify Conclusions** Determine whether the relationship between the circumference of a circle and its diameter is a direct variation. If so, identify the constant of proportionality. Justify your response.

Name _____ My Homework _____

Extra Practice

Find the radius or diameter of each circle with the given dimensions.

19. $d = 7$ cm. 3.5 cm.

Homework Help

$r = \frac{d}{2}$

$r = \frac{7}{2}$ or 3.5

20. $d = 30$ m _____

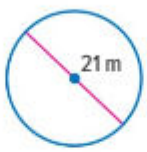
21. $r = 36$ m _____

Find the circumference of each circle. Use 3.14 or $\frac{22}{7}$ for π.

22.

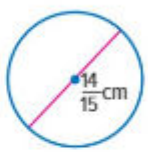
_____

23.



_____

24.



_____

25. a button with a radius of 21 millimeters

_____

26. a dunk tank with a radius of 90 centimeters

_____

27. The diameter of a music CD is 12 centimeters. Find the circumference of a CD to the nearest tenth. _____

28. At a local park, Suhaila can choose between two circular paths to walk. One path has a diameter of 120 meters, and the other has a radius of 45 meters. How much farther can Suhaila walk on the longer path than the shorter path if she walks around the path once? _____

29. MP **Identify Repeated Reasoning** The diagram at the right is made up of circles with the same center. The innermost circle has a diameter of 1 unit. Each circle moving outward has a diameter one more unit than the previous. Without calculating, how much longer is the circumference of each circle? _____

_____

Power Up! Test Practice

30. A bicycle tire has a radius of 31.25 centimeters. Select values to complete the equation below to find the circumference of the wheel. Use 3.14 for π.

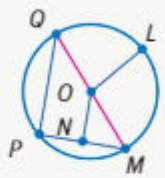
0.5	4
2	31.25
3.14	62.5

$C \approx$ [ ] × [ ] × [ ]

How far does the tire roll in one complete revolution?

[ ]

31. A circle with center at point *O* is shown. Determine if each statement is true or false.



a. $\overline{ON}$ is a radius of the circle. ☐ True ☐ False

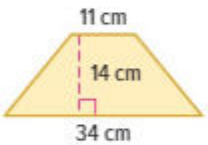
b. $\overline{QM}$ is a diameter of the circle. ☐ True ☐ False

c. To find the circumference, multiply the length of $\overline{OL}$ by π. ☐ True ☐ False

Spiral Review

Find the area of each trapezoid. 6.G.1

32.



_____

33.

_____

34.

_____

35. Find the area of glass used on the side of the parallelogram-shaped building shown. 6.G.1

_____

36. Find the area of a triangle with a base of 25 centimeters and a height of 30 centimeters. 6.G.1

_____

Inquiry Lab
Area of Circles

Inquiry **HOW are the circumference and area of a circle related?**

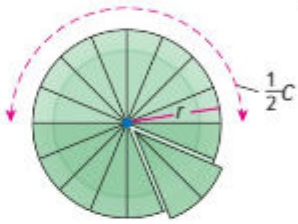
MP Mathematical Practices 1, 3, 6

Mrs. Huda wants to create a family message center on a wall in her house. There are 4 family members, including Mrs. Huda. She decides to paint 1 circle for each family member using magnetic paint. Each circle will have a 30-centimeter radius. How do you find the area of a circle?

Hands-On Activity

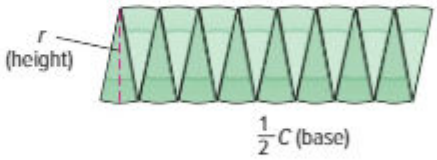
Let's develop a formula for finding the area of a circle.

Step 1 Fold a paper plate in half four times to divide it into 16 equal sections.



Step 2 Label the radius r as shown. Let C represent the circumference of the circle.

Step 3 Cut out each section. Reassemble the sections to form a parallelogram-shaped figure.



What expressions represent the measurements of the base and the height?

Base: _____ Height: _____

Substitute these values into the formula for the area of a parallelogram, $A = b \times h$. Write the new formula. _____

Replace C with the expression for the circumference of a circle, $2\pi r$. Simplify the equation and describe what it represents.

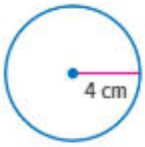
_____

Investigate

Collaborate

Work with a partner. Use the circle to draw and label a parallelogram that would result from cutting and reassembling the circle. Use 3.14 for π.

1.

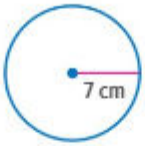


Base: _____

Height: _____

Area of Parallelogram: _____

2.



Base: _____

Height: _____

Area of Parallelogram: _____

Analyze and Reflect

Collaborate

3. MP **Reason Inductively** Use the formula you wrote on the previous page to find the area of the circles in Exercises 1 and 2 above. Use 3.14 for π.

Area of circle in Exercise 1: _____

Area of circle in Exercise 2: _____

4. Compare the area of the circles you found in Exercise 3 to the area of the parallelograms in Exercises 1 and 2. What do you notice? Explain.

_____

_____

Create

On Your Own

5. MP **Model with Mathematics** Find a real-world example of a circle. Measure the radius of the circle. Draw a resulting parallelogram from reassembling the circle. Then calculate the circle's area. _____

6. Inquiry HOW are the circumference and area of a circle related?

_____

_____

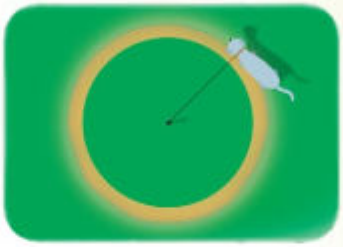
Lesson 2

Area of Circles

Real-World Link

Pets Hidaya bought an 20-centimeter leash for her cat.

1. Hidaya wants to find the distance the cat runs when it runs one circle with the leash fully extended. Should she calculate the circumference or area? Explain.



2. Suppose she wants to find the amount of running room the cat has with the leash fully extended. Should she calculate the circumference or area? Explain.

3. Describe a real-world situation that would involve finding the area of a circle.

4. Describe a real-world situation that would involve finding the circumference of a circle.

Essential Question

HOW do measurements help you describe real-world objects?

Vocabulary

semicircle

Mathematical Practices
1, 3, 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

(1) Persevere with Problems	(5) Use Math Tools
(2) Reason Abstractly	(6) Attend to Precision
(3) Construct an Argument	(7) Make Use of Structure
(4) Model with Mathematics	(8) Use Repeated Reasoning

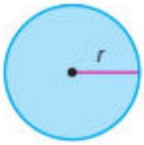
Key Concept

Find the Area of a Circle

Words The area A of a circle equals the product of π and the square of its radius r.

Symbols $A = \pi r^2$

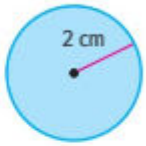
Model



Work Zone

Examples

1. **Find the area of the circle. Use 3.14 for π.**



Estimate $3 \times 2 \times 2 = 12$

$A = \pi r^2$ Area of a circle

$A \approx 3.14 \cdot 2^2$ Replace r with 2.

$A \approx 3.14 \cdot 4$ $2^2 = 2 \cdot 2$ or 4

$A \approx 12.56$ Multiply.

Check for Reasonableness $12.56 \approx 12$ ✓

The area of the circle is approximately 12.56 square centimeters.

2. **Find the area of a circle with a radius of 14 centimeters. Use $\frac{22}{7}$ for π.**

Estimate $3 \times 14 \times 14 = 588$

$A = \pi r^2$ Area of a circle

$A \approx \frac{22}{7} \cdot 14^2$ Replace π with $\frac{22}{7}$ and r with 14.

$A \approx \frac{22}{7} \cdot 196$ $14^2 = 14 \cdot 14$ or 196

$A \approx \frac{22}{\cancel{7}_{1}} \cdot \cancel{196}^{28}$ Divide by the GCF, 7.

$A \approx 616$ Multiply.

Check for Reasonableness $616 \approx 588$ ✓

The area of the circle is approximately 616 square centimeters.

STOP and Reflect

Cross out the formula that is not used for finding the area of a circle.

$A = \pi r^2$ $A = 3.14r^2$

$A = \frac{22}{7}r^2$ $A = \frac{1}{2}bh$

Got it? **Do this problem to find out.**

Show your work

a. Find the area of a circle with a radius of 3.2 centimeters. Round to the nearest tenth.

a. _____

Example

Tutor

3. **Find the area of the face of the UAE 25 fils coin with a diameter of 24 millimeters. Use 3.14 for π. Round to the nearest tenth if necessary.**

The radius is $\frac{1}{2}(24)$ or 12 millimeters.

$A = \pi r^2$ Area of a circle

$A \approx 3.14 \cdot 12^2$ Replace r with 12.

$A \approx 452.16$ Multiply.

The area is approximately 452.2 square millimeters.

Got it? **Do this problem to find out.**

b. The bottom of a circular swimming pool with a diameter of 9 meters is painted blue. How many square meters are blue?

b. _____

Calculating with π

When evaluating expressions involving π, using the π key on a calculator will result in a different approximation.

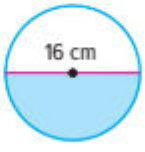
Show your work.

Area of Semicircles

A **semicircle** is half of a circle. The formula for the area of a semicircle is $A = \frac{1}{2}\pi r^2$.

Example

4. **Find the area of the semicircle. Use 3.14 for π. Round to the nearest tenth.**



$A = \frac{1}{2}\pi r^2$ Area of a semicircle

$A \approx \frac{1}{2}(3.14)8^2$ Replace r with 8.

$A \approx 0.5(3.14)(64)$ $8^2 = 8 \cdot 8$ or 64

$A \approx 100.5$ Simplify.

The area of the semicircle is approximately 100.5 square centimeters.

Got it? **Do this problem to find out.**

c. Find the approximate area of a semicircle with a radius of 6 centimeters.

c. _____

Example

5. On a basketball court, there is a semicircle above the free-throw line that has a radius of 2 meters. Find the area of the semicircle. Use 3.14 for π. Round to the nearest tenth.

$A = \frac{1}{2}\pi r^2$ Area of a semicircle

$A \approx 0.5(3.14)(2^2)$ Replace π with 3.14 and r with 2.

$A \approx 0.5(3.14)(4)$ $2^2 = 2 \cdot 2$ or 4

$A \approx 6.28$ Multiply.

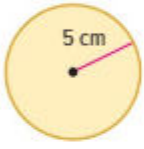
So, the area of the semicircle is approximately 6.28 square meters.

Guided Practice

Check

Find the area of each circle. Round to the nearest tenth. Use 3.14 or $\frac{22}{7}$ for π. (Examples 1–3)

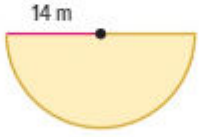
1. Show your work.



2.

3. diameter = 16 m

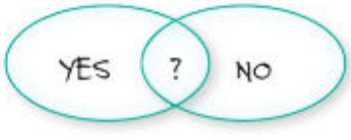
4. Ayoub draws the semicircle shown at the right. What is the area of the semicircle? Use 3.14 for π. (Examples 4 and 5)



5. **Building on the Essential Question** Name one way the circumference and area of a circle are the same and one way they are different.

Rate Yourself!

Are you ready to move on? Shade the section that applies.



Name _____ My Homework _____

Independent Practice

Find the area of each circle. Round to the nearest tenth. Use 3.14 or $\frac{22}{7}$ for π. (Examples 1–3)

1.

Show your work.

2.

3

4. diameter = 10.5 cm

5. radius = 6.3 mm

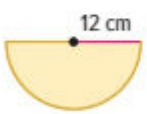
6. radius = $3\frac{1}{4}$ m

7. Refer to the pets problem at the beginning of this lesson. Find the area, to the nearest tenth, of grass that Hidaya's cat may run in if the leash is 2.7 meters long. (Example 3)

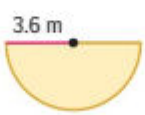
8. A rotating sprinkler that sprays water at a radius of 3.3 meters is used to water a lawn. Find the area of the lawn that is watered. Use 3.14 for π. (Example 3)

Find the area of each semicircle. Round to the nearest tenth. Use 3.14 for π. (Example 4)

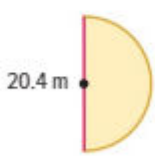
9



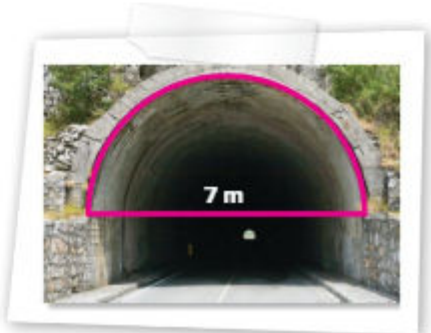
10.



11.



12. The tunnel opening shown is a semicircle. Find the area, to the nearest tenth, of the opening of the tunnel enclosed by the semicircle. (Example 5)



13. **Justify Conclusions** Harry's Pizzeria is having a sale on medium and large pizzas. Medium pizzas are 10 inch and cost AED7.99. Large pizzas are 16 inch and cost AED14.99. Which size pizza is the better deal? Explain. (*Hint*: Find the cost per square centimeter of each pizza.)

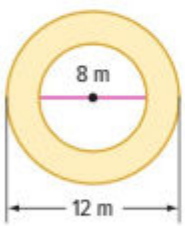
H.O.T. Problems Higher Order Thinking

14. **Model with Mathematics** Write a real-world problem that involves finding the area of two circles. Then solve your problem.

15. **Reason Inductively** If the length of the radius of a circle is doubled, how does that affect the circumference and area? Explain.

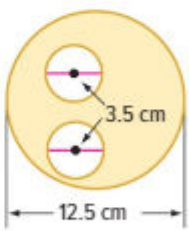
Persevere with Problems Find the area of the shaded region in each figure. Round to the nearest tenth.

16.



17.

18.



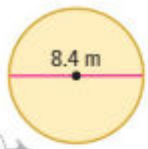
19. **Persevere with Problems** Explain how you could find the area of the quarter circle shown at the right. Then write a formula that could be used to find the area of a quarter circle and use the formula to find the area to the nearest tenth.

Name _____ My Homework _____

Extra Practice

Find the area of each circle. Round to the nearest tenth. Use 3.14 or $\frac{22}{7}$ for π.

20.



$A = \pi r^2$

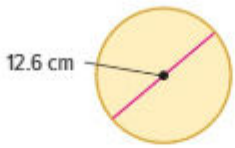
$A = 3.14 \cdot 4.2^2$

$A = 55.4 \text{ m}^2$

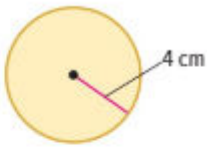
Homework Help

$3.14 \times 4.2 \times 4.2 = 55.4 \text{ m}^2$

21.



22.



23. diameter = 10.8 m

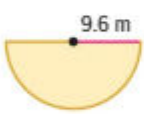
24. radius = $3\frac{4}{5}$ m

25. radius = 9.3 mm

26. Find the area of the Girl Scout patch shown if the diameter is 3 centimeters. Round to the nearest tenth.

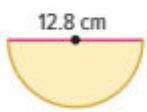
Find the area of each semicircle. Round to the nearest tenth. Use 3.14 for π.

27.



28.

29.



30. A window that is in the shape of a semicircle has a diameter of 70 centimeters. Find the area of the window. Round to the nearest tenth.

31. MP **Justify Conclusions** Which has a greater area, a triangle with a base of 100 meters and a height of 100 meters or a circle with diameter of 100 meters? Justify your selection.

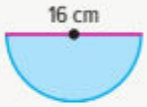
32. A radio station sends a signal in a circular area with an 80-mile radius. Find the approximate area in square kilometers that receives the signal. (*Hint*: 1 square mile ≈ 2.6 square kilometers)

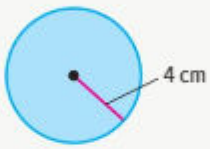
Power Up! Test Practice

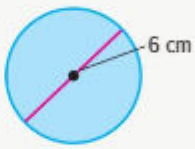
33. A large pizza at a restaurant has the dimensions shown. Find the area of the pizza. Use $\frac{22}{7}$ for π.

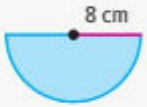
Why does it make sense to use $\frac{22}{7}$ as the estimate for π? Explain your reasoning.

34. Refer to the figures shown below. Which figures have the same area? Select all that apply.

☐ 

☐ 

☐ 

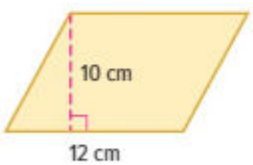
☐ 

Spiral Review

35. A frame for a collage of pictures is in the shape of a trapezoid. The two bases are 38 centimeters and 50 centimeters. The height of the trapezoid is 30 centimeters. What is the area enclosed by the frame? _____

Find the area of each parallelogram. Round to the nearest tenth if necessary.

36.



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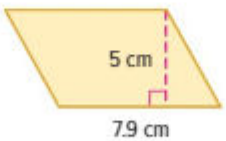

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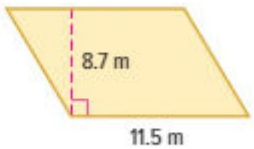


Lesson 3

Area of Composite Figures

Real-World Link

Stained Glass Windows An image of a stained glass window is shown below.

1. Identify two of the shapes that make up the window.

_____

_____

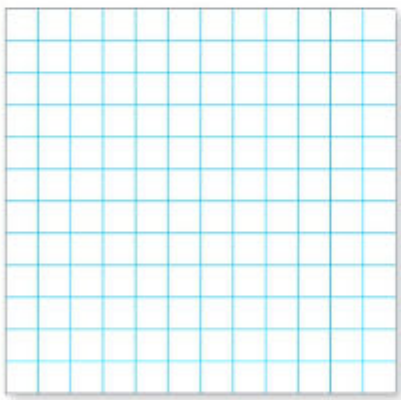
2. How could you find the area of the entire window except for the shapes you identified in Exercise 1?

_____

_____

_____

3. Draw a figure that is made up of a triangle and a rectangle on the grid below. Then find the area of your figure by counting square units.



Area: _____ square units

Essential Question

HOW do measurements help you describe real-world objects?

Vocabulary

composite figure

Mathematical Practices

1, 2, 3, 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

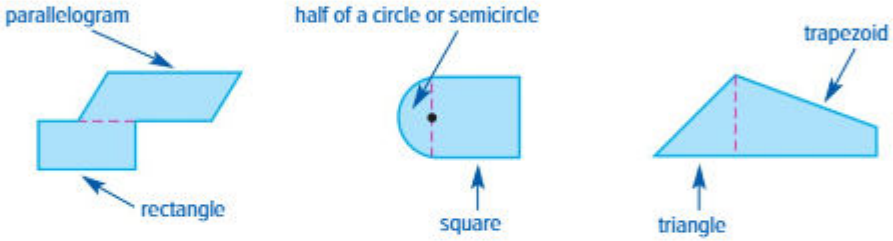
Work Zone

Find the Area of a Composite Figure

A **composite figure** is made up of two or more shapes.

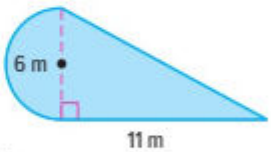
To find the area of a composite figure, decompose the figure into shapes with areas you know. Then find the sum of these areas.

Shape	Words	Formula
Parallelogram	The area A of a parallelogram is the product of any base b and its height h.	$A = bh$
Triangle	The area A of a triangle is half the product of any base b and its height h.	$A = \frac{1}{2}bh$
Trapezoid	The area A of a trapezoid is half the product of the height h and the sum of the bases, b_1 and b_2.	$A = \frac{1}{2}h(b_1 + b_2)$
Circle	The area A of a circle is equal to π times the square of the radius r.	$A = \pi r^2$



Example

1. **Find the area of the composite figure.**



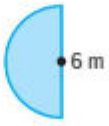
The figure can be separated into a semicircle and a triangle.

Area of semicircle

$A = \frac{1}{2}\pi r^2$

$A \approx \frac{1}{2} \cdot 3.14 \cdot 3^2$

$A \approx 14.1$

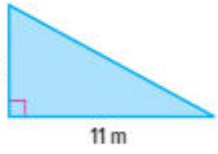


Area of triangle

$A = \frac{1}{2}bh$

$A = \frac{1}{2} \cdot 11 \cdot 6$

$A = 33$



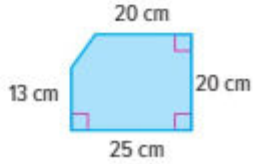
The area of the figure is about 14.1 + 33 or 47.1 square meters.

Got it? **Do this problem to find out.**

Show your work.

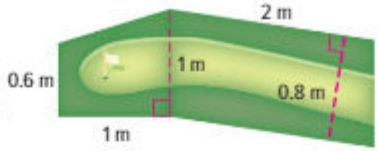
a. Find the area of the figure. Round to the nearest tenth if necessary.

a. _____



Example

2. A miniature golf hole is composed of a trapezoid and a parallelogram. How many square meters of turf does the hole cover?

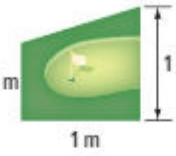


Area of trapezoid

$A = \frac{1}{2}h(b_1 + b_2)$

$A = \frac{1}{2}(1)(0.6 + 1)$

$A = 0.8$

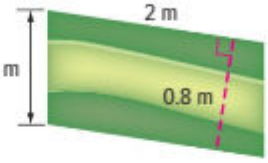


Area of parallelogram

$A = bh$

$A = 2 \cdot 0.8$

$A = 1.6$

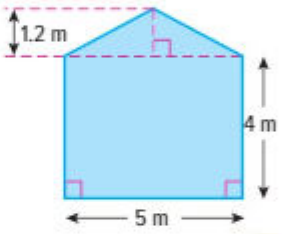


So, 0.8 + 1.6 or 2.4 square meters of turf will be needed.

Got it? Do this problem to find out.

Show your work.

b. Saeed's father is building a shed. How many square meters of wood are needed to build the back of the shed shown at the right?



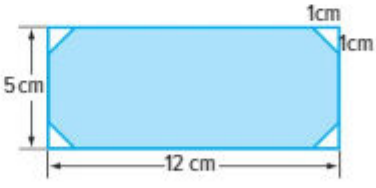
b. _____

Find the Area of a Shaded Region

Use the areas you know to find the area of a shaded region.

Examples

3. Find the area of the shaded region.



Find the area of the rectangle and subtract the area of the four congruent triangles.

Area of rectangle

$A = \ell w$

$A = 12 \cdot 5$ $\ell = 12, w = 5$

$A = 60$ Simplify.

Area of triangles

$A = 4 \cdot \left(\frac{1}{2}bh\right)$

$A = 4 \cdot \frac{1}{2} \cdot 1 \cdot 1$ $b = 1, h = 1$

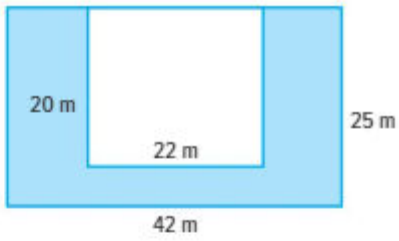
$A = 2$ Simplify.

The area of the shaded region is 60 − 2 or 58 square centimeters.

Congruent Triangles

Congruent triangles have corresponding sides and angles that are congruent.

4. **The blueprint for a hotel swimming area is represented by the figure shown. The shaded area represents the pool. Find the area of the pool.**



Find the area of the entire rectangle and subtract the section that is not shaded.

Area of the entire rectangle

$A = \ell w$

$A = 42 \cdot 25$ or 1,050

Area not shaded

$A = \ell w$

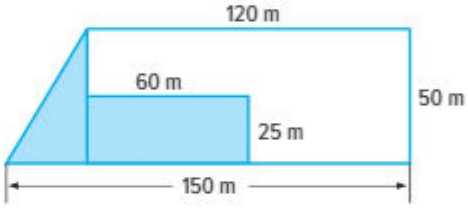
$A = 22 \cdot 20$ or 440

The area of the shaded region is 1,050 − 440 or 610 square meters.

Show your work.

Got it? **Do this problem to find out.**

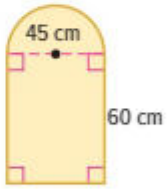
c. A diagram for a park is shown. The shaded area represents the picnic sections. Find the area of the picnic sections.



c. _____

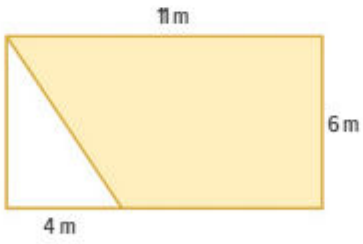
Guided Practice

1. Yousif installed the window shown. How many square centimeter is the window? Round to the nearest tenth. Use 3.14 for π.



Show your work. (Examples 1 and 2) _____

2. A triangle is cut from a rectangle. Find the area of the shaded region.



(Examples 3 and 4) _____

3. **Building on the Essential Question** Is your answer to Exercise 1 an exact or approximate answer? Explain.

_____

_____

_____

Rate Yourself!

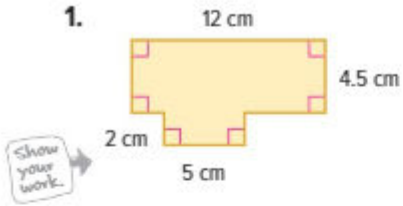
How confident are you about finding the area of composite figures? Check the box that applies.

Name _____ My Homework _____

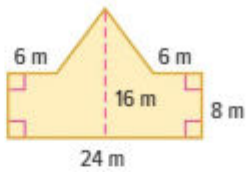
Independent Practice

Find the area of each figure. Round to the nearest tenth if necessary. (Example 1)

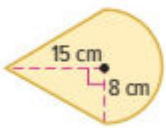
1.



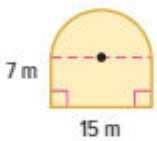
2.



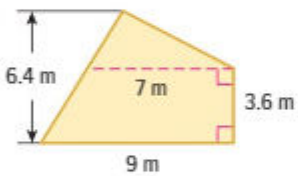
3.



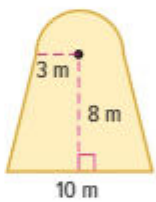
4.



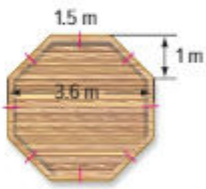
5



6.

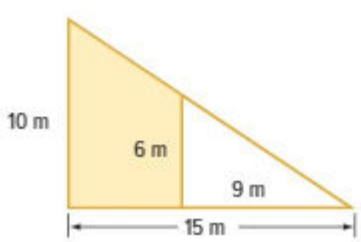


7 Nasser is constructing a deck like the one shown. What is the area of the deck? (Example 2)

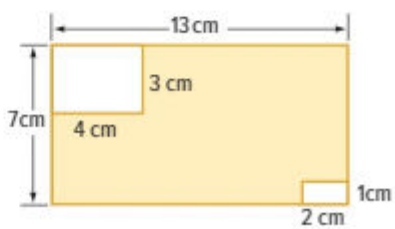


Find the area of the shaded region. Round to the nearest tenth if necessary. (Examples 3 and 4)

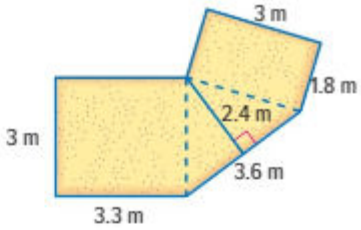
8.



9.



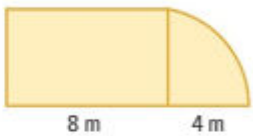
10. **Persevere with Problems** Noura's mom is carpeting her bedroom and needs to know the amount of floor space. How many square meters of carpeting are needed for the room? If she is also installing baseboards on the bottom of all the walls, how many meters of baseboards are needed? _____



H.O.T. Problems Higher Order Thinking

11. **Persevere with Problems** The composite figure shown is made from a rectangle and part of a circle. Find the approximate area and perimeter of the entire figure. Round to the nearest tenth.

_____

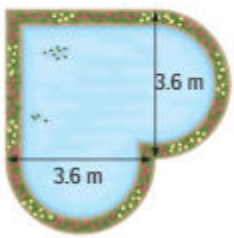


12. **Reason Abstractly** The side length of the square in the figure at the right is x units. Write expressions that represent the perimeter and area of the figure.

_____

13. **Persevere with Problems** In the diagram shown at the right, a 0.6-meter-wide flower border surrounds the heart-shaped pond. What is the area of the border?

_____



14. **Model with Mathematics** Find a real-world object that is a composite figure. Measure the dimensions of the figure. Draw a model of the figure with appropriate labels. Then find the area of the composite figure. _____

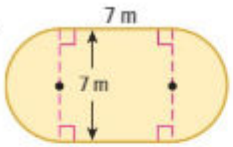
Show your work.

Name _____ My Homework _____

Extra Practice

Find the area of each figure. Round to the nearest tenth if necessary.

15.



87.5 m^2

Homework Help

Area of circle

$A = \pi r^2$

$A = 3.14 \cdot 3.5^2$ or 38.5

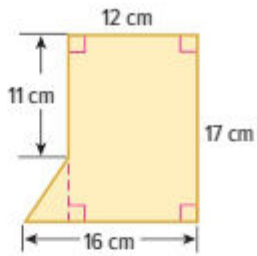
Area of square

$A = lw$

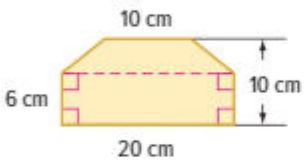
$A = 7 \cdot 7$ or 49

$38.5 + 49 = 87.5$

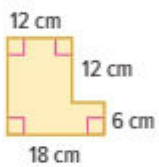
16.



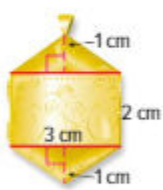
17.



18.

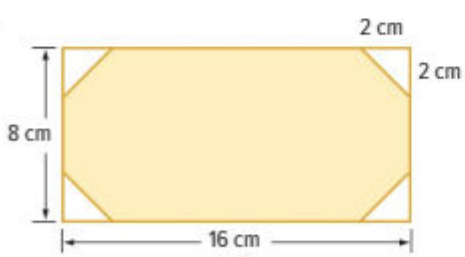


19. A necklace comes with a gold pendant. What is the area of the pendant in square centimeters? _____

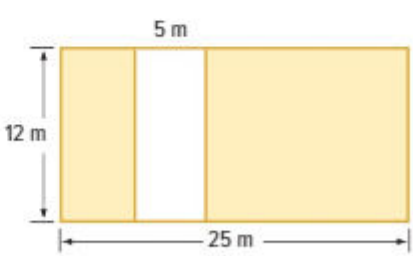


Find the area of the shaded region. Round to the nearest tenth if necessary.

20.

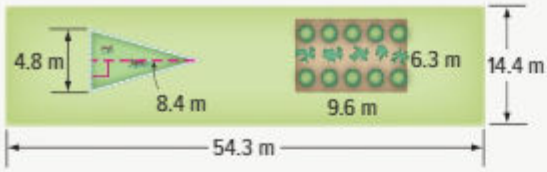


21.



Power Up! Test Practice

22. Mansour's backyard has a rectangular vegetable garden and a triangular cat exercise area.



Match each part of the yard with the correct area.

Cat Exercise Area: _____ m^2

Vegetable Garden Area: _____ m^2

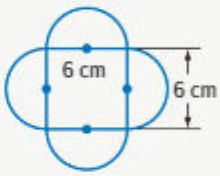
Total Backyard Area: _____ m^2

9.5	60.4
20.2	98.3
41.2	676.8
49	781.9

How much of the backyard is not being used for the vegetable garden or pet exercise area?

_____

23. The figure is made up of a square and four semicircles. Fill in each box to complete each statement. Round to the nearest hundredth.



a. The area of the square is _____ cm^2.

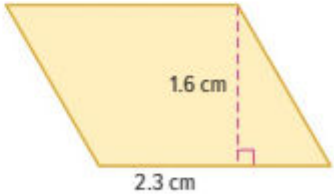
b. The area of each semicircle is about _____ cm^2.

c. The total area of the figure is about _____ cm^2.

Spiral Review

24. What is the area of a triangle with a base of 52 meters and a height of 38 meters? 6.G.1 _____

25. Find the area of the parallelogram at the right. Round to the nearest tenth. 6.G.1 _____



26. Find the height of a parallelogram with an area of 104 square meters and a base of 8 meters. 6.G.1

_____

27. Find the base of a parallelogram with a height of 3.2 meters and an area of 15.04 square meters. 6.G.1

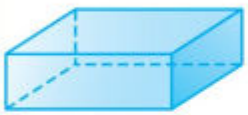
_____

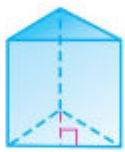
Lesson 4
Volume of Prisms

Vocabulary Start-Up

Recall that a prism is a polyhedron with two parallel, congruent bases. The bases of a *rectangular prism* are rectangles, and the bases of a *triangular prism* are triangles.

Write *rectangular prism* or *triangular prism* on the line below each figure.





_____ _____

Essential Question

HOW do measurements help you describe real-world objects?

Vocabulary

volume

Mathematical Practices

1, 2, 3, 4

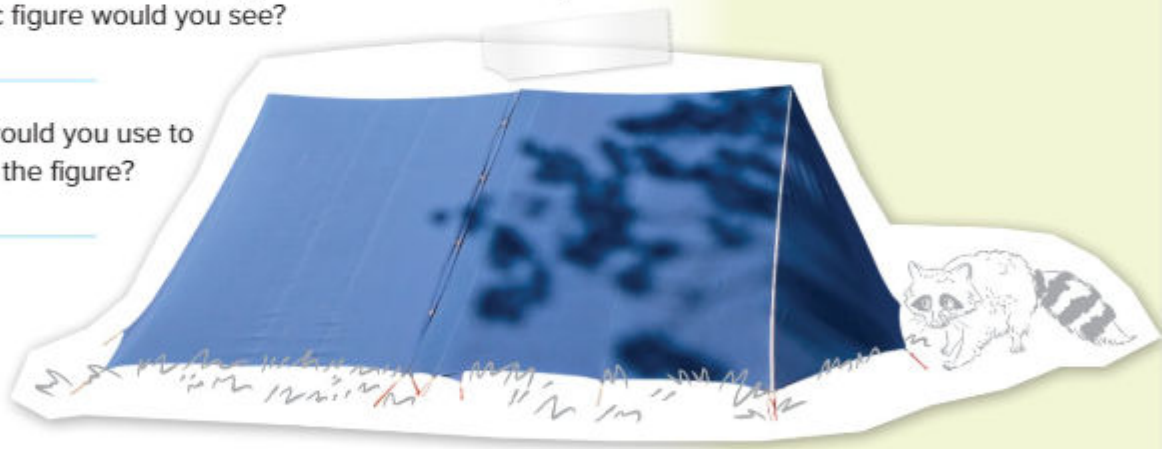
Real-World Link

1. Suppose you observed the camping tent shown from directly above. What geometric figure would you see?

_____

2. What formula would you use to find the area of the figure?

_____



Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

(1) Persevere with Problems	(5) Use Math Tools
(2) Reason Abstractly	(6) Attend to Precision
(3) Construct an Argument	(7) Make Use of Structure
(4) Model with Mathematics	(8) Use Repeated Reasoning

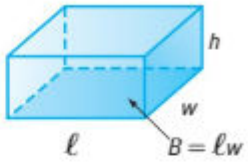
Key Concept

Volume of a Rectangular Prism

Words The volume V of a rectangular prism is the product of the length ℓ, the width w, and the height h. It is also the area of the base B times the height h.

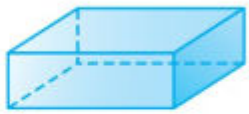
Symbols $V = \ell wh$ or $V = Bh$

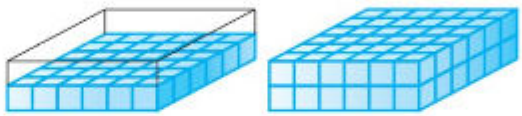
Model



Work Zone

The **volume** of a three-dimensional figure is the measure of space it occupies. It is measured in cubic units such as cubic centimeters (cm^3) or cubic inches (in^3).





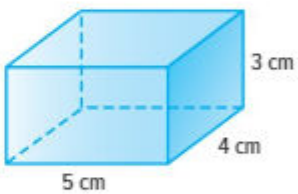
It takes 2 layers of 36 cubes to fill the box. So, the volume of the box is 72 cubic centimeters.

Decomposing Figures

Think of the volume of the prism as consisting of three congruent slices. Each slice contains the base area, 20 square centimeters, and a height of 1 centimeter.

Example

1. **Find the volume of the rectangular prism.**



$V = \ell wh$ Volume of a prism

$V = 5 \cdot 4 \cdot 3$ $\ell = 5$, $w = 4$, and $h = 3$

$V = 60$ Multiply.

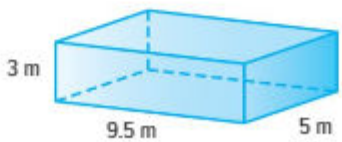
The volume is 60 cubic centimeters or 60 cm^3.

Got it? **Do this problem to find out.**

Show your work.

a. Find the volume of the rectangular prism shown below.

a. _____



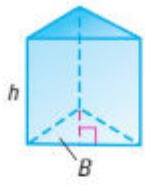
Volume of a Triangular Prism

Key Concept

Words The volume V of a triangular prism is the area of the base B times the height h.

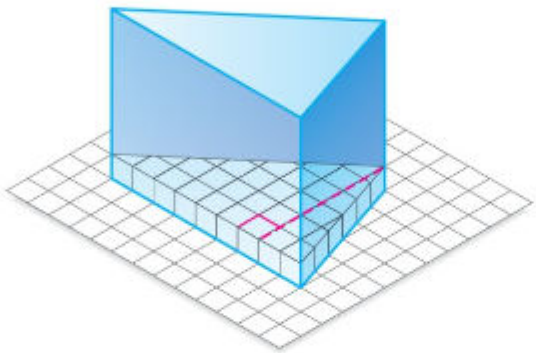
Symbols $V = Bh$, where B is the area of the base.

Model



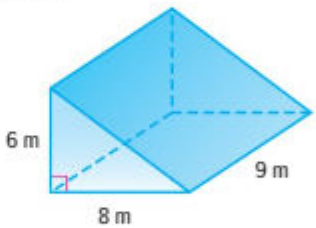
Height
Do not confuse the height of the triangular base with the height of the prism.

The diagram below shows that the volume of a triangular prism is also the product of the area of the base B and the height h of the prism.



Example

2. **Find the volume of the triangular prism shown.**



The area of the triangle is $\frac{1}{2} \cdot 6 \cdot 8$, so replace B with $\frac{1}{2} \cdot 6 \cdot 8$.

$V = Bh$ Volume of a prism

$V = \left(\frac{1}{2} \cdot 6 \cdot 8\right)h$ Replace B with $\frac{1}{2} \cdot 6 \cdot 8$.

$V = \left(\frac{1}{2} \cdot 6 \cdot 8\right)9$ The height of the prism is 9.

$V = 216$ Multiply.

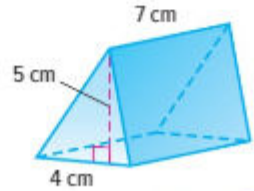
The volume is 216 cubic meters or 216 m^3.

Before finding the volume of a prism, identify the base. In Example 2, the base is a triangle, so you replace B with $\frac{1}{2}bh$.

Got it? **Do this problem to find out.**

Show your work.

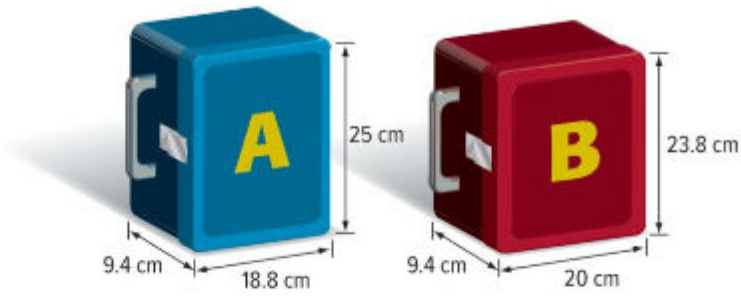
b. Find the volume of the triangular prism.



b. _____

Example

3. **Which lunch box holds more food?**



Find the volume of each lunch box. Then compare.

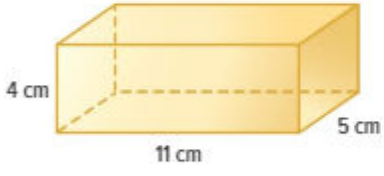
Lunch Box A	Lunch Box B
$V = \ell wh$	$V = \ell wh$
$V = 18.8 \cdot 9.4 \cdot 25$	$V = 20 \cdot 9.4 \cdot 23.8$
$V = 4{,}418 \text{ cm}^3$	$V = 4{,}474.4 \text{ cm}^3$

Since $4{,}474.4 \text{ cm}^3 > 4{,}418 \text{ cm}^3$, Lunch Box B holds more food.

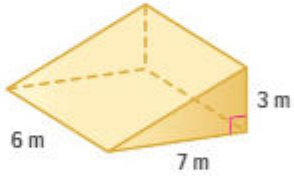
Guided Practice

Find the volume of each prism. Round to the nearest tenth if necessary.
(Examples 1–2)

1.



2.



Show your work.

3. One room measures 3 meters by 2.5 meters by 5 meters. A second measures 4 meters by 3.5 meters by 4.5 meters. Which volume is greater? Explain. (Example 3)

4. **Building on the Essential Question** Compare and contrast finding the volume of a rectangular prism and a triangular prism.

Rate Yourself!

How confident are you about finding volume for prisms? Check the box that applies.

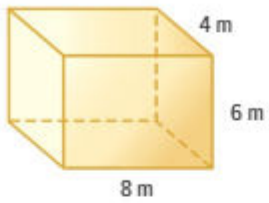
FOLDABLES Time to update your Foldable!

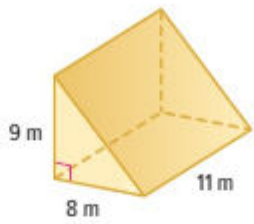
Name _____ My Homework _____

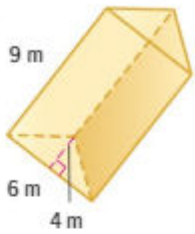
Independent Practice

Find the volume of each prism. Round to the nearest tenth if necessary.

(Examples 1–2)

1. 

2. 

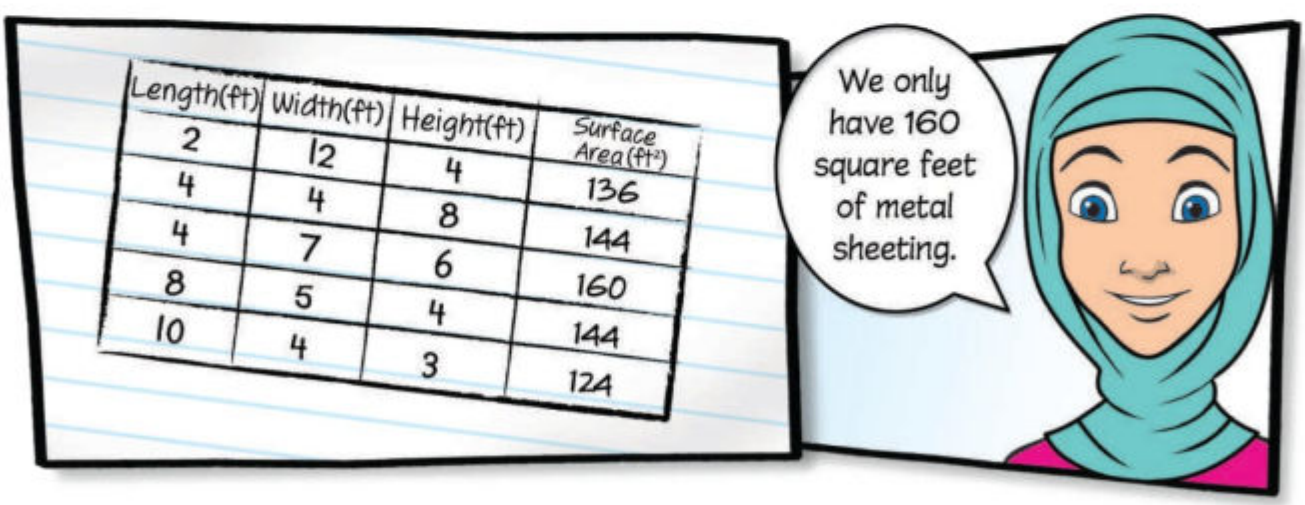
3. 

Show your work.

4. Which container holds more detergent? Justify your answer. (Example 3)

5. **Model with Mathematics** Refer to the graphic novel frame below. The table shows possible dimensions for the dunk tank.

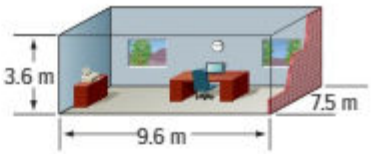
Length(ft)	Width(ft)	Height(ft)	Surface Area(ft^2)
2	12	4	136
4	4	8	144
4	7	6	160
8	5	4	144
10	4	3	124



a. Find the volume of each given dunk tank.

b. Which dimensions are reasonable for a dunk tank? Explain.

6. The diagram shows the dimensions of an office. It costs about AED 3.88 per year to air condition one cubic meter of space. On average, how much does it cost to air condition the office for one month? _____



H.O.T. Problems Higher Order Thinking

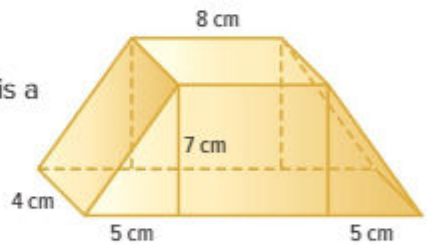
7. MP **Reason Inductively** A rectangular prism is shown.

   a. Suppose the length of the prism is doubled. How does the volume change? Explain your reasoning. _____

   b. Suppose the length, width, and height are each doubled. How does the volume change? _____

   c. Which will have a greater effect on the volume of the prism: doubling the height or doubling the width? Explain your reasoning.

8. MP **Persevere with Problems** The prism shown has a base that is a trapezoid. Find the volume of the prism. _____



9. MP **Model with Mathematics** Find the volume of a real-world object that is in the shape of a rectangular or triangular prism using appropriate units. Draw a model of the prism including the dimensions. _____

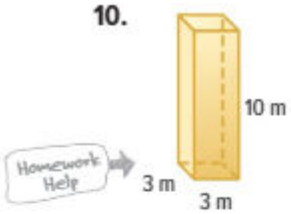
Show your work.

Name _____ My Homework _____

Extra Practice

Find the volume of each prism. Round to the nearest tenth if necessary.

10.



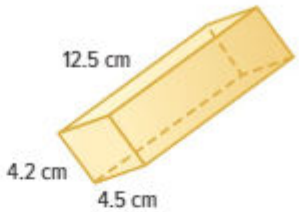
$90\ m^3$

$V = lwh$

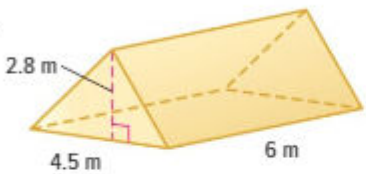
$V = 3 \cdot 3 \cdot 10$

$V = 90$

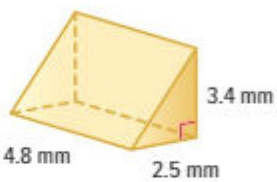
11.



12.



13.



14. A toy company makes rectangular sandboxes that measure 1.8 meters by 1.5 meters by 0.36 meter. A customer buys a sandbox and 1 cubic meter of sand. Did the customer buy too much or too little sand? Justify your answer.

_____

_____

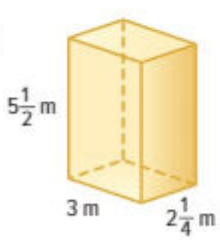
15. The base of a rectangular prism has an area of 19.4 square meters and the prism has a volume of 306.52 cubic meters. Write an equation that can be used to find the height h of the prism. Then find the height of the prism.

_____

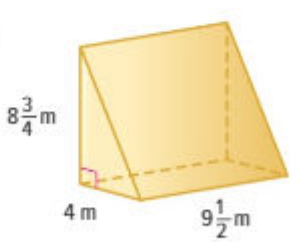
_____

Find the volume of each prism.

16.



17.



18. **Reason Abstractly** Write a formula for finding the volume of a cube. Use an exponent and the variable s to represent the side lengths. Then use the formula to find the volume of a cube with side lengths of 7 centimeters.

_____

Power Up! Test Practice

19. The volume of a box is 1.5 cubic meters. Which of the following are possible dimensions of the box? Select all that apply.

- ☐ 2 m. by 1.5 m. by 0.5 m.
- ☐ 2 m. by 1 m. by 1 m.
- ☐ 3 m. by 0.5 m. by 1.5 m.
- ☐ 3 m. by 1 m. by 0.5 m.

20. The table shows the dimensions of 4 containers. Sort the containers from least to greatest volume.

Container	ℓ (m)	w (m)	h (m)
A	2	2	2
B	1	3	3
C	3	4	0.5
D	3	2	0.5

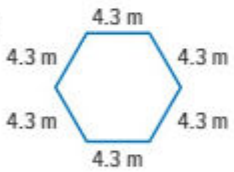
	Container	Volume (m^3)
Least		
Greatest		

Which container has the greatest volume? _____

Spiral Review

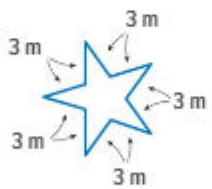
Find the perimeter of each figure. 4.MD.3

21.



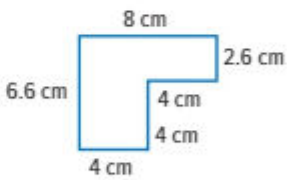
_____

22.



_____

23.



_____

24. Write a formula for finding the perimeter of a square. Use your formula to find the perimeter of a square with side length of 0.5 centimeter. 6.G.3

_____

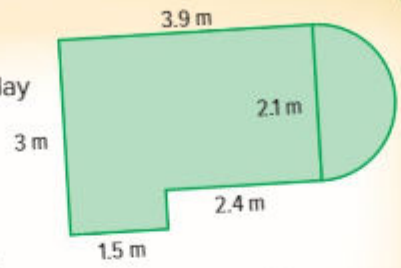
MP Problem-Solving Investigation

Solve a Simpler Problem

MP Mathematical Practices 1, 3

Case #1 Playgrounds

Mahmoud is helping to mulch the play area at the community center. The diagram shows the dimensions of the play area.



What is the area of the play area to be mulched? Round to the nearest tenth if necessary.

1 Understand *What are the facts?*

You know the shape and dimensions of the play area.

2 Plan *What is your strategy to solve this problem?*

Find the area of the two rectangles and the semi-circle, and then add.

3 Solve *How can you apply the strategy?*

Area of Rectangle 1	Area of Rectangle 2	Area of Semi-Circle
$A = \ell w$	$A = \ell w$	$A = \frac{\pi r^2}{2}$
$A = 1.5 \cdot 3$	$A = 2.4 \cdot 2.1$	$A = \frac{3.14 \cdot (1.05)^2}{2}$
$A =$ ☐	$A =$ ☐	$A =$ ☐

The total area is ☐ + ☐ + ☐ or ☐ square meters.

4 Check *Does the answer make sense?*

The play area is about 3.9 • 3 or 11.7 square meters. So, an answer of ☐ square meters is reasonable.

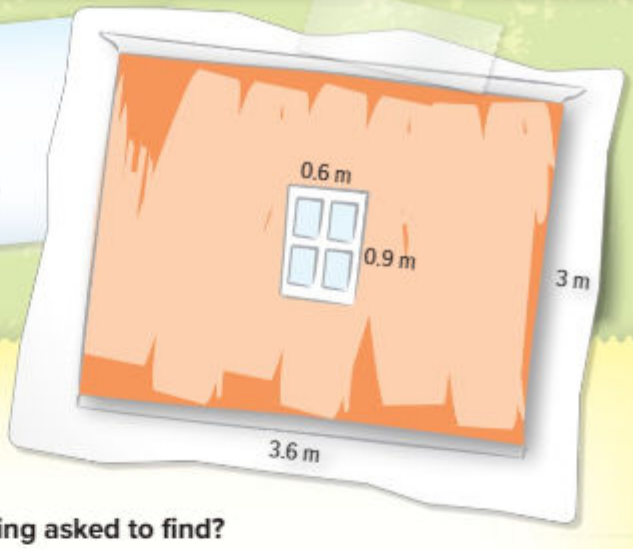
Analyze the Strategy

MP **Reason Inductively** Why is breaking this problem into simpler parts a good strategy to solve it?

Case #2 Wallpaper

Nahla is painting a wall in her house.

What is the area that will be painted?



1 Understand

Read the problem. What are you being asked to find?

I need to find _____.

What information do you know?

The picture shows the wall is _____ long and _____ high.

There is a window that is _____ by _____.

2 Plan

Choose a problem-solving strategy.

I will use the _____ strategy.

3 Solve

Use your problem-solving strategy to solve the problem.

Find the area of the wall. Then subtract the area of the window.

The dimensions of the wall are ☐ meters by ☐ meters.

So, the area of the wall is ☐ × ☐ = ☐ m^2.

The dimensions of the window are ☐ meters by ☐ meters.

So, the area of the window is ☐ × ☐ = ☐ m^2.

☐ − ☐ = ☐

So, _____.

4 Check

Use information from the problem to check your answer.

Use estimation to check the reasonableness of your answer. The area of the wall is approximately 3 × 3.6 = ☐ m^2. The answer is reasonable.

Collaborate

Work with a small group to solve the following cases.
Show your work on a separate piece of paper.

Case #3 Woodworking

Two workers can make two chairs in two days.

How many chairs can 8 workers working at the same rate make in 20 days?

Case #4 Tips

Reham wants to leave an 18% tip for a AED19.5 restaurant bill. The tax is 5%, which is added to the bill before the tip.

How much money does Ebony spend at the restaurant? Explain.

Case #5 Continents

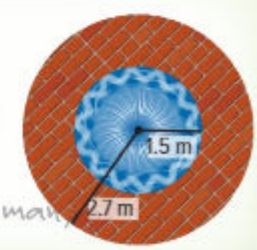
The land area of Earth is 147,214,613 square kilometers.

To the nearest tenth, how much larger is the land area of Asia than North America? Explain.

Continent	Percent of Earth's Land
Asia	30
Africa	20.2
North America	16.5

Case #6 Fountains

Mr. Ali has a circular fountain with a radius of 1.5 meters. He plans on installing a brick path around the fountain.



If each brick covers 0.18 square meter, how many bricks will he need to buy?.

Mid-Chapter Check

Vocabulary Check

1. **Be Precise** Define *circumference*. Explain how to find the circumference of a circle. (Lesson 1)

2. Fill in the blank in the sentence below with the correct term. (Lesson 3)

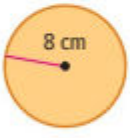
   A _____ is made up of two or more shapes.

Skills Check and Problem Solving

Find the circumference and area of each circle. Use 3.14 for π. Round to the nearest tenth if necessary. (Lessons 1 and 2)

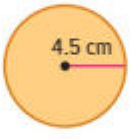
3. circumference = _____

   area = _____



4. circumference = _____

   area = _____

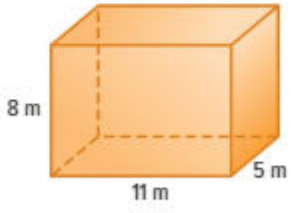


5. circumference = _____

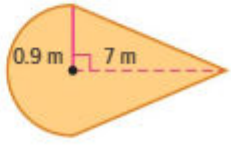
   area = _____

Show your work.

6. The dimensions of a box are shown in the figure at the right. What is the volume of the box? (Lesson 4)



7. **Persevere with Problems** The figure at the right represents the design for a new hole for a miniature golf course. The new turf to cover the hole costs AED16.2 per square meter. How much will it cost to cover the entire area? (Lesson 3)



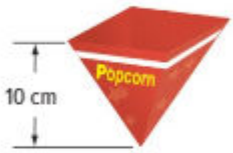
Inquiry Lab
Volume of Pyramids

Inquiry

WHAT is the relationship between the volume of a prism and the volume of a pyramid with the same base area and height?

MP Mathematical Practices 1, 3, 5

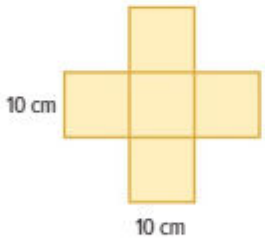
A movie theater offers two different containers of popcorn: a square prism and a square pyramid. Both containers are 10 centimeters tall and have a base area of 100 square centimeters. Determine the container that holds more popcorn.

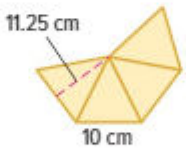


Hands-On Activity

Nets are two-dimensional patterns of three-dimensional figures.

Step 1 Draw the nets of the popcorn containers shown below onto card stock. Cut out and tape each net to form its shape. The prism and pyramid will be open. The pyramid is composed of ☐ congruent isosceles triangles with bases of 10 centimeters and heights of 11.25 centimeters.





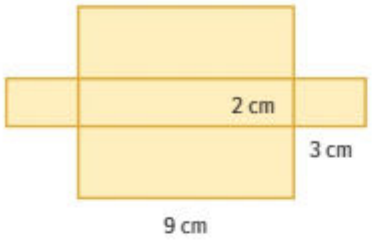
Step 2 Fill the pyramid with rice. Pour the rice from the pyramid into the prism and repeat until the prism is full. Slide a ruler across the top to level the amount.

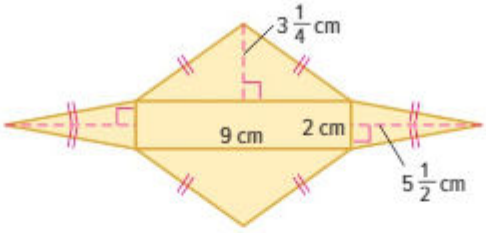
It took ☐ pyramids of rice to fill the prism.

So, the square _____ container holds more popcorn than the square _____ container.

Investigate

Use Math Tools Work with a partner to repeat the Activity with the rectangular prism and the rectangular pyramid shown.





1. How many pyramids of rice did it take to fill the prism?

2. What is true about the bases of your rectangular prism and rectangular pyramid? the heights?

3. Refer to the Activity. What is true about the bases of the square prism and square pyramid? the heights?

Analyze and Reflect

4. What fraction of the volume of the rectangular prism is the volume of the rectangular pyramid? _____

5. Refer to the Activity. What fraction of the volume of the square prism is the volume of the square pyramid? _____

Create

6. **Reason Inductively** How can you find volume of a pyramid given a prism with the same base area and height? Write a formula for the volume of a pyramid based on the formula for the volume of a prism.

7. Inquiry WHAT is the relationship between the volume of a prism and the volume of a pyramid with the same base area and height?

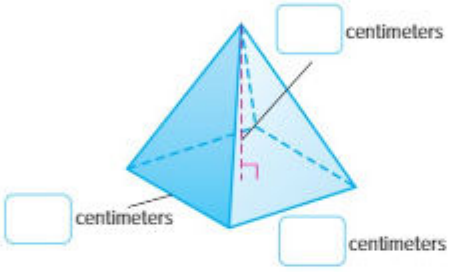
Lesson 5

Volume of Pyramids

Real-World Link

Sand Sculpture Mohammad is helping his mother build a sand sculpture at the beach in the shape of a pyramid. The square pyramid has a base with a length and width of 30 centimeters each and a height of 35 centimeters.

1. Label the dimensions of the sand sculpture on the square pyramid below.



2. What is the area of the base of the pyramid?

_____

3. What is the volume of a square prism with the same dimensions as the pyramid?

_____

Essential Question

HOW do measurements help you describe real-world objects?

Vocabulary

lateral face

Mathematical Practices

1, 3, 4, 6

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

(1) Persevere with Problems	(5) Use Math Tools
(2) Reason Abstractly	(6) Attend to Precision
(3) Construct an Argument	(7) Make Use of Structure
(4) Model with Mathematics	(8) Use Repeated Reasoning

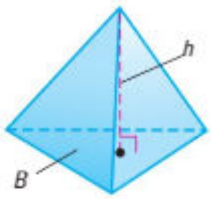
Key Concept

Volume of a Pyramid

Words The volume V of a pyramid is one third the area of the base B times the height of the pyramid h.

Symbols $V = \frac{1}{3}Bh$

Model



Work Zone

In a polyhedron, any face that is not a base is called a **lateral face**. The lateral faces of a pyramid meet at a common vertex. The height of a pyramid is the distance from the vertex perpendicular to the base.

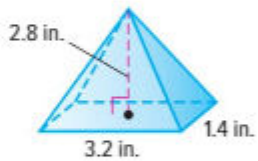
Examples

1. **Find the volume of the pyramid. Round to the nearest tenth.**

$V = \frac{1}{3}Bh$ Volume of a pyramid

$V = \frac{1}{3}(3.2 \cdot 1.4)2.8$ $B = 3.2 \cdot 1.4$, $h = 2.8$

$V \approx 4.2$ Simplify.



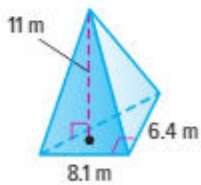
The volume is about 4.2 cubic centimeters.

2. **Find the volume of the pyramid. Round to the nearest tenth.**

$V = \frac{1}{3}Bh$ Volume of a pyramid

$V = \frac{1}{3}\left(\frac{1}{2} \cdot 8.1 \cdot 6.4\right)11$ $B = \frac{1}{2} \cdot 8.1 \cdot 6.4$, $h = 11$

$V = 95.04$ Simplify.



The volume is about 95.0 cubic meters.

Show your work

Got it? **Do this problem to find out.**

a. Find the volume of a pyramid that has a height of 9 centimeters and a rectangular base with a length of 7 centimeters and a width of 3 centimeters.

a. _____

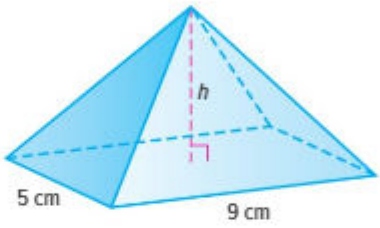
Find the Height of a Pyramid

You can also use the formula for the volume of a pyramid to find a missing height.

Examples

3. **The rectangular pyramid shown has a volume of 90 cubic centimeters. Find the height of the pyramid.**

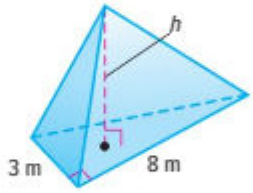
$V = \frac{1}{3}Bh$	Volume of a pyramid
$90 = \frac{1}{3}(9 \cdot 5)h$	$V = 90, B = 9 \cdot 5$
$90 = 15h$	Multiply.
$\frac{90}{15} = \frac{15h}{15}$	Divide by 15.
$6 = h$	Simplify.



The height of the pyramid is 6 centimeters.

4. **A triangular pyramid has a volume of 44 cubic meters. It has an 8-meter base and a 3-meter height. Find the height of the pyramid.**

$V = \frac{1}{3}Bh$	Volume of a pyramid
$44 = \frac{1}{3}\left(\frac{1}{2} \cdot 8 \cdot 3\right)h$	$V = 44, B = \frac{1}{2} \cdot 8 \cdot 3$
$44 = 4h$	Multiply.
$\frac{44}{4} = \frac{4h}{4}$	Divide by 4.
$11 = h$	Simplify.



The height of the pyramid is 11 meters.

Got it? **Do these problems to find out.**

b. A triangular pyramid has a volume of 840 cubic centimeters. The triangular base has a base length of 20 centimeters and a height of 21 centimeters. Find the height of the pyramid.

c. A rectangular pyramid has a volume of 525 cubic meters. It has a base of 25 meters by 18 meters. Find the height of the pyramid.

Multiplying Fractions

To find $\frac{1}{3} \cdot \frac{1}{2} \cdot 8 \cdot 3$, multiply $\frac{1}{3} \cdot \frac{1}{2}$ and $8 \cdot 3$ to get $\frac{1}{6}$ and 24, then find $\frac{1}{6}$ of 24.

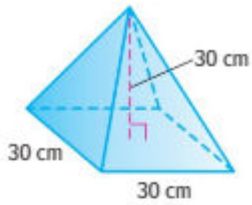
Show your work.

b. _____

c. _____

Example

5. **Kamilah is making a model of the Food Guide Pyramid for a class project. Find the volume of the square pyramid.**



$V = \frac{1}{3}Bh$ Volume of a pyramid

$V = \frac{1}{3}(30 \cdot 30)30$ $B = 30 \cdot 30, h = 30$

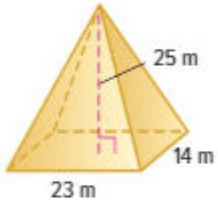
$V = 9{,}000$ Multiply.

The volume is 9,000 cubic centimeters.

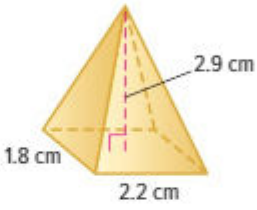
Guided Practice

Find the volume of each pyramid. Round to the nearest tenth if necessary. (Examples 1 and 2)

1.



2.



Find the height of each pyramid. (Examples 3 and 4)

3. square pyramid: volume 1,024 cm^3; base edge 16 cm _____

4. triangular pyramid: volume 48 cm^3; base edge 9 cm.; base height 4 cm. _____

Show your work.

5. The Transamerica Pyramid is a skyscraper in San Francisco. The rectangular base has a length of 52.5 meters and a width of 36 meters. The height is 256 meters. Find the volume of the building. (Example 5) _____

6. **Building on the Essential Question** When you are finding the volume of a pyramid, why is it important to know the shape of the base of the pyramid?

_____

_____

Rate Yourself!

How well do you understand volume of pyramids? Circle the image that applies.

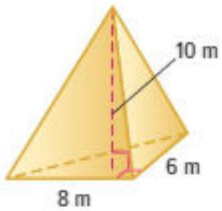
FOLDABLES Time to update your Foldable!

Independent Practice

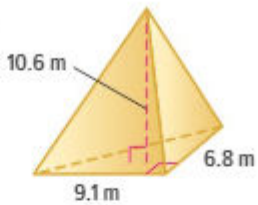
Find the volume of each pyramid. Round to the nearest tenth if necessary. (Examples 1 and 2)

1

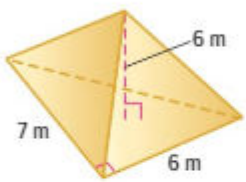
Show your work.



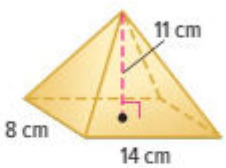
2.



3.



4.



Find the height of each pyramid. (Examples 3 and 4)

5. rectangular pyramid: volume 448 cm^3; base edge 12 cm.; base length 8 cm.

6. triangular pyramid: volume 270 cm^3; base edge 15 cm; height of base 4 cm

7 A glass pyramid has a height of 10 centimeters. Its rectangular base has a length of 7.5 centimeters and a width of 6.25 centimeters. Find the volume of glass used to create the pyramid. (Example 5)

8. The Pyramid Arena in Memphis, Tennessee, is a square pyramid that is 96.3 meters tall. The base has 180-meter sides. Find the volume of the pyramid. (Example 5)

9. **Reason Inductively** A rectangular pyramid has a length of 14 centimeters, a width of 9 centimeters, and a height of 10 centimeters. Explain the effect on the volume if each dimension were doubled.

10. Find the height of a square pyramid that has a volume of $25\frac{3}{5}$ meters and a base with 4 meter sides.

H.O.T. Problems Higher Order Thinking

11. **Be Precise** A rectangular pyramid has a volume of 160 cubic meters. Find two possible sets of measurements for the base area and height of the pyramid.

12. **Persevere with Problems** A square pyramid and a cube have the same bases and volumes. How are their heights related? Explain.

13. **Reason Inductively** The two figures shown have congruent bases. How does the volume of the two square pyramids in Figure B compare to the volume of the square pyramid in Figure A?

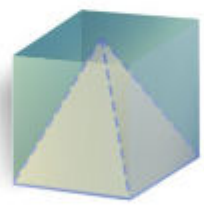


Figure A

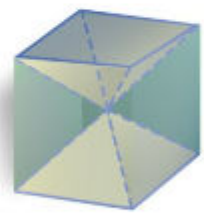


Figure B

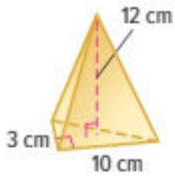
14. **Reason Inductively** Determine whether the following statement is *true* or *false*. Explain your reasoning.

The volumes of a rectangular-based pyramid and a triangular-based pyramid with congruent heights and equal base areas are equal.

Extra Practice

Find the volume of each pyramid. Round to the nearest tenth if necessary.

15.



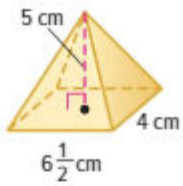
Homework Help

$V = \frac{1}{3}Bh$

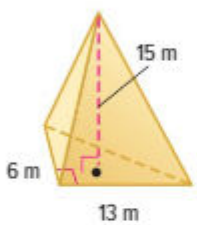
$V = \frac{1}{3}\left(\frac{1}{2} \cdot 10 \cdot 3\right)12$

$V = 60$

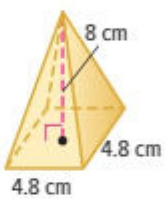
16.



17.



18.



Find the height of each pyramid.

19. square pyramid: volume 297 m^3; area of the base 81 m^2

20. hexagonal pyramid: volume 1,320 m^3; area of the base 120 m^2

21. square pyramid: volume 550 cm^3; area of the base 75 cm^2

22. rectangular pyramid: volume 3,800 m^3; area of the base 300 m^2

23. An ancient stone pyramid has a height of 13.6 meters. The edges of the square base are 16.5 meters. Find the volume of the stone pyramid.

Power Up! Test Practice

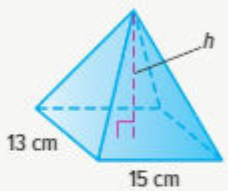
24. The table shows the base dimensions and heights of 4 rectangular pyramids. Sort the pyramids from least to greatest volume.

Pyramid	ℓ (m)	w (m)	h (m)
A	4	9	5
B	6	6	7
C	5	5	9
D	3	6	12

	Pyramid	Volume (m^3)
Least		
Greatest		

Which pyramid has the greatest volume? _____

25. The rectangular pyramid shown has a volume of 1,560 cubic centimeters. What is the height of the pyramid? Explain how you found your answer.



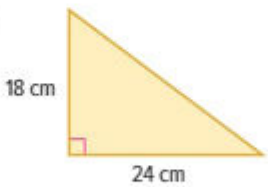
Spiral Review

Find the area of each figure.

26.

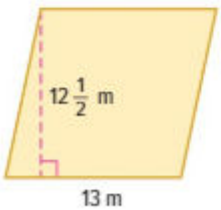
_____

27.



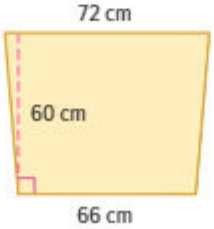
_____

28.



_____

29.



_____

Inquiry Lab

Nets of Three-Dimensional Figures

Inquiry **HOW can models and nets help you find the surface area of prisms?**

MP Mathematical Practices 1, 3, 6

Nets are used to design and manufacture items such as boxes and labels. Find the shapes that make up the net of a cereal box.

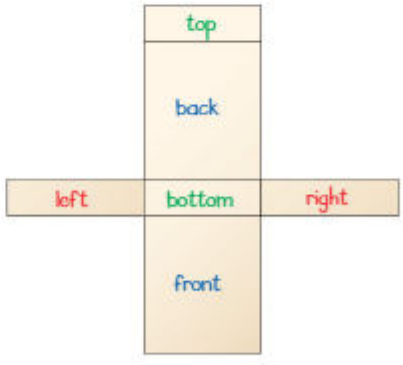
Hands-On Activity 1

Make a net from a rectangular prism.

Step 1 Use an empty cereal box. Cut off one of the two top flaps. The remaining top flap is the top face.

Step 2 Label the top and bottom faces using a green marker. Label the front and back faces using a blue marker. Label the left and right faces using a red marker.

Step 3 Carefully cut along the three edges of the top face. Then cut down each vertical edge.



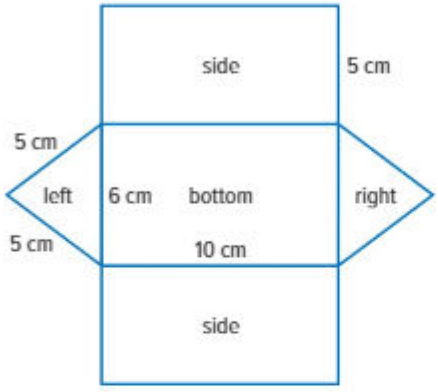
The net of a cereal box is made up of a total of ☐ rectangles.

What do you notice about the top and bottom faces, the left and right faces, and the front and back faces?

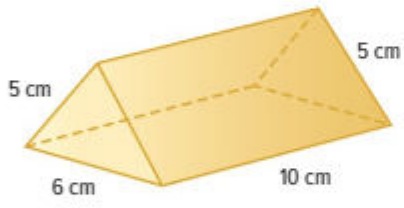
Hands-On Activity 2

Make a triangular prism from a net.

Step 1 Draw a net on a piece of card stock with the dimensions shown below.



Step 2 Fold the net into a triangular prism. Tape together adjacent edges.



The triangular prism is made up of ☐ triangles and ☐ rectangles.

What is true about the triangular bases?

How is the side of one of the rectangles related to the base of one of the triangles?

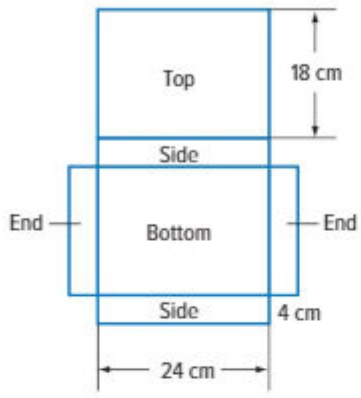
Explain one way to find the total surface area of a triangular prism.

Collaborate

Investigate

Work with a partner to solve each problem.

1. A net of a rectangular prism that is 24 centimeters by 18 centimeters by 4 centimeters is shown. The net of the prism is labeled with *top*, *bottom*, *side*, and *end*. Fill in the boxes to find the total area of the rectangular prism.



Area of Top and Bottom → ☐ cm^2

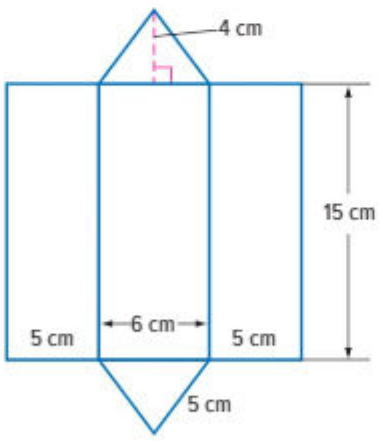
Area of Both Sides → ☐ cm^2

Area of Both Ends → + ☐ cm^2

Total Area → ☐ cm^2

2. Describe in words how you could find the total surface area of a rectangular prism.

3. A net of a triangular prism is shown. Fill in the boxes to find the total area of the triangular prism.



Area of Center Rectangle → ☐ cm^2

Area of Left and Right Rectangles → ☐ cm^2

Area of Triangles → + ☐ cm^2

Total Area → ☐ cm^2

4. Describe in words how you could find the total surface area of a triangular prism.

Analyze and Reflect

Collaborate

Work with a partner.

5. **Reason Inductively** Suppose Majed wants to wrap a present in a container that is a rectangular prism. How can he determine the amount of wrapping paper that he will need? _____

Circle each correct surface area. Draw and label the net for each figure if needed. The first one is done for you.

	Prism	Measures	Surface Area		
	Rectangular	Length: 10 cm Width: 8 cm Height: 5 cm	170 cm^2	340 cm^2	400 cm^2
6.	Rectangular	Length: 3 m Width: 2 m Height: 5 m	30 m^2	31 m^2	62 m^2
7.	Rectangular	Length: 2 m Width: 1 m Height: 1.5 m	3 m^2	6.5 m^2	13 m^2
8.	Triangular	Area of Top and Bottom Triangles: 3 mm^2 Area of Center Rectangle: 12 mm^2 Area of Left and Right Rectangles: 10 mm^2	25 mm^2	28 mm^2	38 mm^2
9.	Triangular	Area of Top and Bottom Triangles: 6 cm^2 Area of Center Rectangle: 50.4 cm^2 Area of Left and Right Rectangles: 56 cm^2	174.4 cm^2	118.4 cm^2	112.4 cm^2

Create

On Your Own

10. **Be Precise** *Surface area* is the sum of the areas of all the surfaces of a three-dimensional figure. Write the formula for the total surface area of a rectangular prism.

11. Inquiry HOW can models and nets help you find the surface area of prisms? _____

Lesson 6

Surface Area of Prisms

Real-World Link

Message Board Members of a local recreation center are permitted to post messages on 21.25-centimeter by 27.5-centimeter paper on the board. Assume the signs are posted vertically and do not overlap, as shown below.

1. Suppose 6 messages fit across the board widthwise.

   What is the width of the board in centimeters? ____ centimeters

2. Suppose 3 messages fit down the board lengthwise.

   What is the length of the board in centimeters? ____ centimeters

3. What is the area in square centimeters of the message board?

   _____

4. Messages can also be posted on the other side of the board. What is the total area of the front and back of the board in square centimeters?

   _____

Essential Question

HOW do measurements help you describe real-world objects?

Vocabulary

surface area

Mathematical Practices

1, 3, 4, 6

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

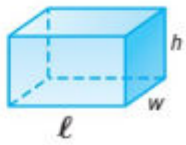
① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Key Concept

Surface Area of a Rectangular Prism

Words The surface area *S.A.* of a rectangular prism with base ℓ, width w, and height h is the sum of the areas of its faces.

Model



Symbols $S.A. = 2\ell h + 2\ell w + 2hw$

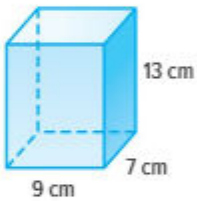
Work Zone

The sum of the areas of all the surfaces, or faces, of a three-dimensional figure is the **surface area**. In the previous Inquiry Lab, you used a net to find the surface area of a rectangular prism. You can also use a formula to find surface area.

When you find the surface area of a three-dimensional figure, the units are square units, not cubic units.

Example

1. **Find the surface area of the rectangular prism shown at the right.**



Replace ℓ with 9, w with 7, and h with 13.

$$\begin{aligned} \text{surface area} &= 2\ell h + 2\ell w + 2hw \\ &= 2 \cdot 9 \cdot 13 + 2 \cdot 9 \cdot 7 + 2 \cdot 13 \cdot 7 \\ &= 234 + 126 + 182 \quad \text{Multiply first. Then add.} \\ &= 542 \end{aligned}$$

The surface area of the prism is 542 square centimeters.

Show your work.

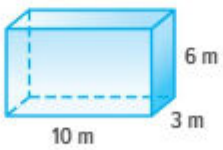
Got it? **Do these problems to find out.**

Find the surface area of each rectangular prism.

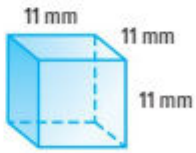
a. _____

b. _____

a.



b.



Example

2. **Fahd built a toy box 150 centimeters long, 60 centimeters wide, and 90 centimeters high. He has 1 liter of paint that covers about 8 square meters of surface. Does he have enough to paint the outside of the toy box? Justify your answer.**

Step 1 Find the surface area of the toy box.

Replace ℓ with 150, w with 60, and h with 90.

$$\begin{aligned}\text{surface area} &= 2\ell h + 2\ell w + 2hw \\ &= 2 \cdot 150 \cdot 90 + 2 \cdot 150 \cdot 60 + 2 \cdot 90 \cdot 60 \\ &= 55{,}800 \text{ cm}^2\end{aligned}$$

Step 2 Find the number of square centimeters the paint will cover.

$$\begin{aligned}1 \text{ m}^2 &= 1 \text{ m} \times 1 \text{ m} && \text{Replace 1 m with 100 cm.} \\ &= 100 \text{ cm.} \times 100 \text{ cm.} && \text{Multiply.} \\ &= 10{,}000 \text{ cm}^2\end{aligned}$$

So, 8 square meters is equal to 8 × 10,000 or 80,000 square centimeters.

Since 80,000 > 55,800, Fahd has enough paint.

Consistent Units

Since the surface area of the toy box is expressed in centimeters, convert 8 m^2 to square inches so that all measurements are expressed using the same units.

Got it? **Do this problem to find out.**

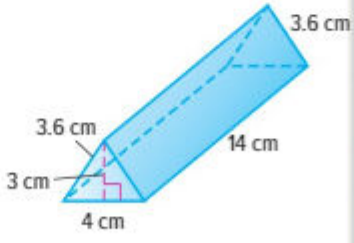
Show your work.

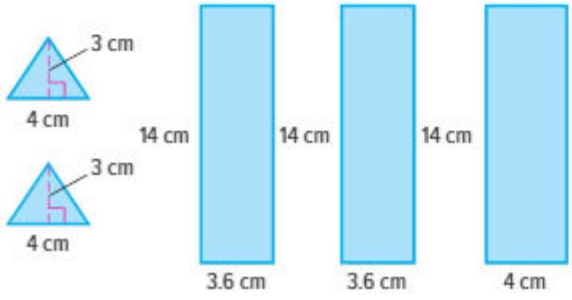
c. The largest corrugated cardboard box ever constructed measured about 6.9 meters long, 2.7 meters high, and 2.4 meters wide. Would 85.5 square meters of paper be enough to cover the box? Justify your answer.

c. _____

Surface Area of Triangular Prisms

To find the surface area of a triangular prism, it is more efficient to find the area of each face and calculate the sum of all of the faces rather than using a formula.





Example

3. **Faleh is mailing his aunt the package shown. How much cardboard is used to create the shipping container?**

Find the area of each face and add.

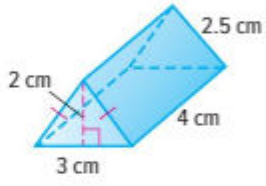
The area of each triangle is $\frac{1}{2} \cdot 10 \cdot 7.5$ or 37.5.

The area of two of the rectangles is 35 · **9 or 315. The area** of the third rectangle is 35 · 10 or 350.

The sum of the areas of the faces is 37.5 + 37.5 + 315 + 315 + 350 or 1,055 square centimeters.

Got it? **Do this problem to find out.**

d. Find the surface area of the triangular prism.



Show your work.

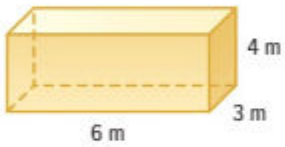
d. _____

Guided Practice

Check

Find the surface area of each prism. (Examples 1–3)

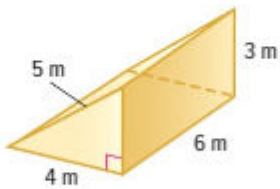
1.



Show your work.

_____

2.



_____

3. **Building on the Essential Question** Why is the surface area of a three-dimensional figure measured in square units rather than in cubic units?

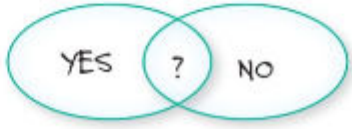
_____

_____

_____

Rate Yourself!

Are you ready to move on? Shade the section that applies.

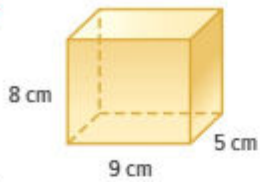


FOLDABLES Time to update your Foldable!

Independent Practice

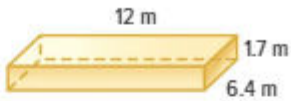
Find the surface area of each rectangular prism. Round to the nearest tenth if necessary. (Example 1)

1.



Show your work.

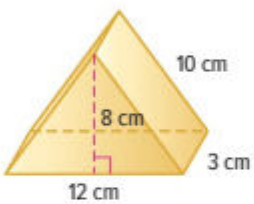
2.



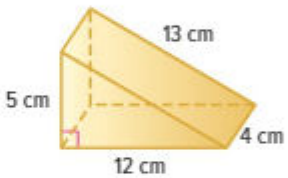
3. When making a book cover, Faris adds an additional 50 square centimeters to the surface area to allow for overlap. How many square centimeters of paper will Faris use to make a book cover for a book 27.5 centimeters long, 20 centimeters wide, and 2.5 centimeters high? (Example 2) _____

Find the surface area of each triangular prism. (Example 3)

4.



5.



6. **Model with Mathematics** Refer to the graphic novel frame below. What whole number dimensions would allow the students to maximize the volume while keeping the surface area at most 14.4 square meters? Explain. _____

7. Write a formula for the surface area *S.A.* of a cube in which each side measures x units.

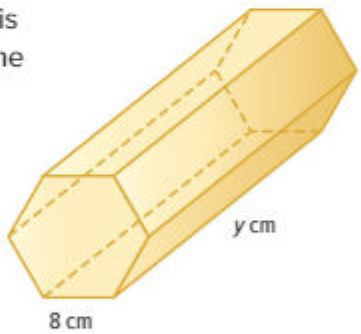
8. A company will make a cereal box with whole number dimensions and a volume of 100 cubic centimeters. If cardboard costs AED0.05 per 100 square centimeters, what is the least cost to make 100 boxes?

H.O.T. Problems Higher Order Thinking

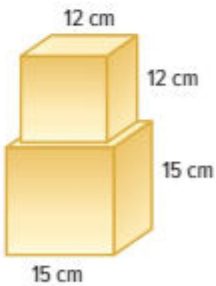
9. MP **Reason Inductively** Determine if the following statement is *true* or *false*. Explain your reasoning.

If you double one of the dimensions of a rectangular prism, the surface area will double.

10. MP **Reason Inductively** A prism with a base that is a regular hexagon is shown. How would you find the surface area of the hexagonal prism if the area of the base of the prism is x square centimeters?



11. MP **Persevere with Problems** The figure at the right is made by placing a cube with 12-centimeter sides on top of another cube with 15-centimeter sides. Find the surface area of the figure.



12. MP **Model with Mathematics** Draw and label a rectangular prism that has a total surface area between 100 and 200 square units. Then find the surface area of your prism.

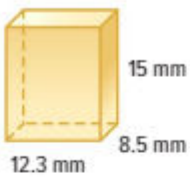
Show your work.

Name _____ My Homework _____

Extra Practice

Find the surface area of each prism. Round to the nearest tenth if necessary.

13.



833.1 mm^2

Homework Help

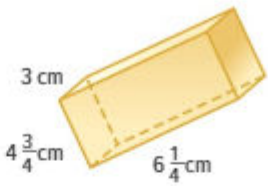
S.A. $= 2lh + 2lw + 2hw$

$= 2 \cdot 12.3 \cdot 15 + 2 \cdot 12.3 \cdot 8.5 + 2 \cdot 15 \cdot 8.5$

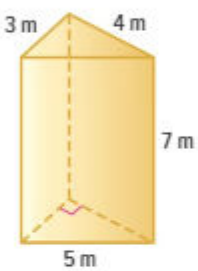
$= 369 + 209.1 + 255$

$= 833.1$

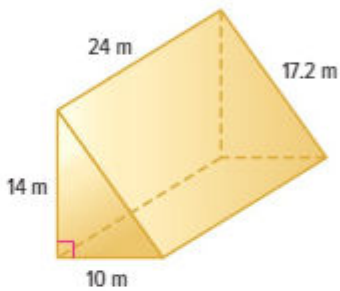
14.



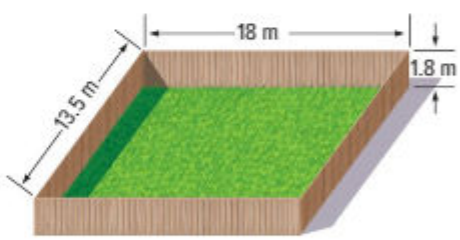
15.



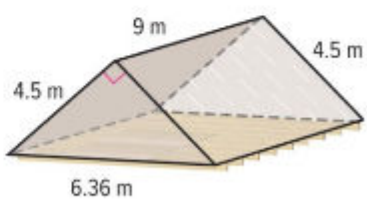
16.



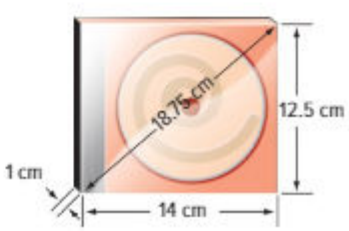
17. If one liter of paint covers 31.5 square meters, will 8 liters of paint be enough to paint the inside and outside of the fence shown once? Explain.



18. The attic shown is a triangular prism. Insulation will be placed inside all walls, not including the floor. Find the surface area that will be covered with insulation.

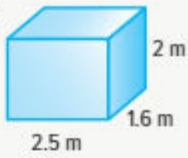


19. **Be Precise** To the nearest tenth, find the approximate amount of plastic covering the outside of the CD case.



Power Up! Test Practice

20. A cardboard box has the dimensions shown. Select the correct values to complete the formula to find the surface area of the box.

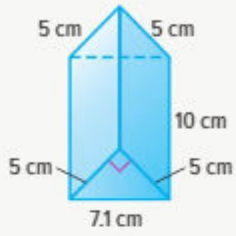


1.6
2
2.5

$SA =$ ☐ · ☐ · ☐ + ☐ · ☐ · ☐ + ☐ · ☐ · ☐ · ☐

How much cardboard is needed to make the box? ☐

21. A triangular prism has the dimensions shown. Fill in each box to complete each statement.



a. The area of each triangular base is ☐ square centimeters.

b. The area of each of the two congruent rectangular faces is ☐ square centimeters.

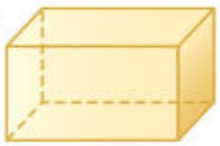
c. The area of the third rectangular face is ☐ square centimeters.

d. The total surface area of the prism is ☐ square centimeters.

Spiral Review

Describe the shape resulting from a vertical, horizontal, and angled cross section for each figure.

22.

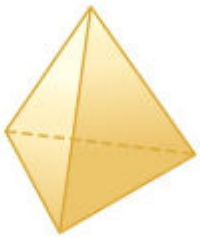


vertical: _____

horizontal: _____

angled: _____

23.



vertical: _____

horizontal: _____

angled: _____

24.

vertical: _____

horizontal: _____

angled: _____

25.

vertical: _____

horizontal: _____

angled: _____

Inquiry Lab

Relate Surface Area and Volume

Inquiry **HOW does the shape of a rectangular prism affect its volume and surface area?**

MP Mathematical Practices 1, 3, 4

You can arrange blocks in many ways. How can you arrange 8 blocks to create the least possible surface area?

What do you know? _____

_____

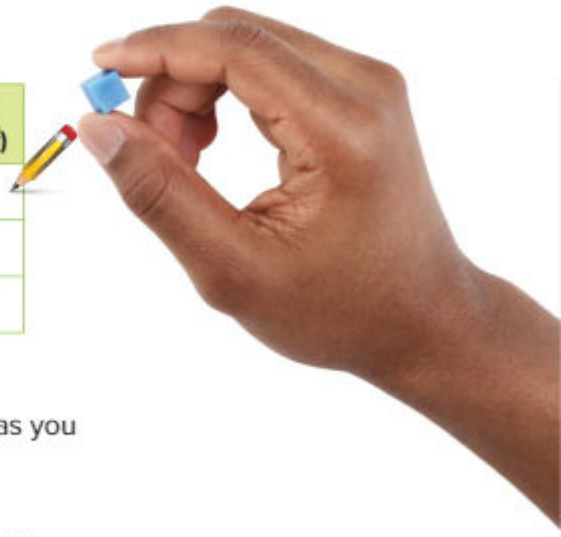
What do you need to find? _____

_____

Hands-On Activity 1

Tools

Step 1 Create a rectangular prism using 8 centimeter cubes. Record the dimensions in the table below. Find and record the volume and surface area of the prism.



Rectangular Prism	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
1	2	2			
2					
3					

Step 2 Repeat Step 1 for as many different rectangular prisms as you can create with 8 cubes.

Does the volume change when the prism changes? Explain.

_____

_____

The rectangular prism measuring ☐ × ☐ × ☐ has the least surface area.

Hands-On Activity 2

Suppose you make structures in the shape of the ones shown below. What is the volume of each structure? Which structure has the lesser surface area? Draw a net if necessary.

Figure 1

Figure 2

Step 1 Use centimeter cubes to create the rectangular prism shown in Figure 1. Write its dimensions, volume, and surface area in the table below.

Rectangular Prism	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
Figure 1	3				
Figure 2					

Step 2 Use centimeter cubes to create the rectangular prism shown in Figure 2. Write its dimensions, volume, and surface area in the table.

Step 3 Compare the volume and surface areas of Figure 1 and Figure 2.

What do you notice about the volume of Figure 1 and Figure 2?

The surface area of Figure 1 is ☐ square centimeters.

The surface area of Figure 2 is ☐ square centimeters.

Compare the surface areas using an inequality.

☐ square centimeters < ☐ square centimeters

So, Figure ☐ has the lesser surface area.

Investigate

Collaborate

Work with a partner. Compare the two figures that have the same volume. Then determine which figure has a greater surface area. Draw a net if necessary.

1.

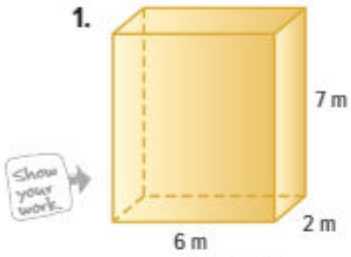


Figure 1

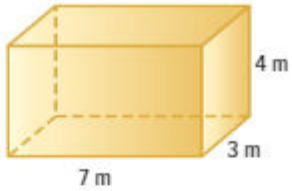


Figure 2

Surface Area: _____

Surface Area: _____

2.

Figure 1

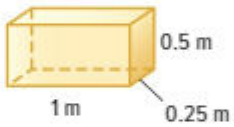


Figure 2

Surface Area: _____

Surface Area: _____

3.

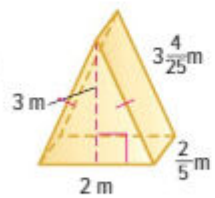


Figure 1

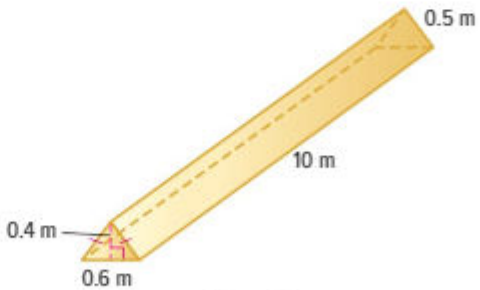


Figure 2

Surface Area: _____

Surface Area: _____

Analyze and Reflect

Collaborate

Work with a partner to solve the following problems. Draw a net if necessary.

4. Najat sews together pieces of fabric to make rectangular gift boxes. She only uses whole numbers. What are the dimensions of a box with a volume of 50 cubic centimeters that has the greatest amount of surface area?

5. Eissa is creating a decorative container to fill with colored sand. He uses only whole numbers. The top of the container is open. What are the dimensions of the rectangular prism that holds 100 cubic centimeters with the least amount of surface area?

6. MP **Construct an Argument** Omar needs to melt a stick of butter that measures 5 centimeters by 1 centimeter by 1 centimeter. He is going to put the butter in a pan on top of the stove. Explain why cutting the butter into smaller pieces will help the butter melt faster.

Create

On Your Own

7. MP **Model with Mathematics** Draw a sketch of a triangular prism with a volume of 120 cubic units and a surface area of 184 square units.

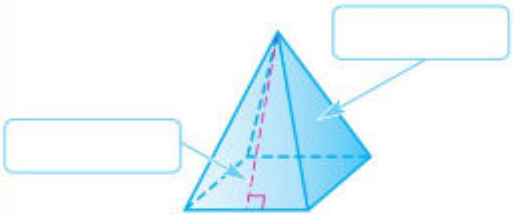
8. Inquiry HOW does the shape of a rectangular prism affect its volume and surface area?

Lesson 7
Surface Area of Pyramids

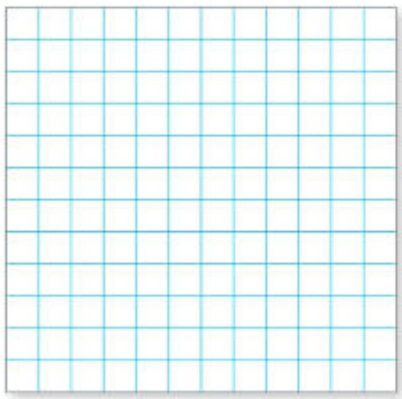
Vocabulary Start-Up

Pyramids Ancient Egyptians built pyramids, such as the one shown in the photo below. A right square pyramid has a square base and four isosceles triangles that make up the lateral faces. The **lateral surface area** is the sum of the areas of all its lateral faces. The height of each lateral face is called **slant height**.

1. Fill in the blanks on the diagram below with the terms *slant height* and *lateral face*.



2. Draw a net of a square pyramid.



Essential Question

HOW do measurements help you describe real-world objects?

Vocabulary

lateral surface area
slant height
regular pyramid

Mathematical Practices
1, 3, 4, 5

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Key Concept

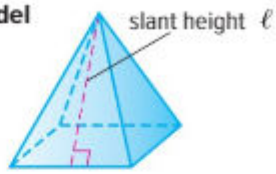
Surface Area of a Pyramid

Work Zone

Lateral Area

Words The lateral surface area *L.A.* of a regular pyramid is half the perimeter *P* of the base times the slant height ℓ.

Model



Symbols $L.A. = \frac{1}{2}P\ell$

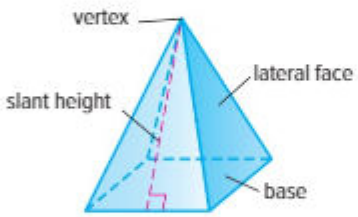
Total Surface Area

Words The total surface area *S.A.* of a regular pyramid is the lateral area *L.A.* plus the area of the base *B*.

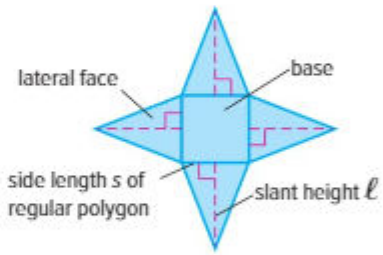
Symbols $S.A. = B + L.A.$ or $S.A. = B + \frac{1}{2}P\ell$

A **regular pyramid** is a pyramid with a base that is a regular polygon.

Model of Regular Square Pyramid



Net of Regular Square Pyramid



To find the lateral area *L.A.* of a regular pyramid, refer to the net. The lateral area is the sum of the areas of the triangles.

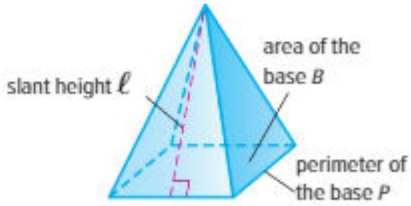
$L.A. = 4\left(\frac{1}{2}s\ell\right)$ Area of the lateral faces

$L.A. = \frac{1}{2}(4s)\ell$ Commutative Property of Multiplication

$L.A. = \frac{1}{2}P\ell$ The perimeter of the base *P* is $4s$.

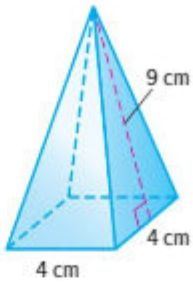
The total surface area of a regular pyramid is the lateral surface area *L.A.* plus the area of the base *B*.

$$S.A. = B + \frac{1}{2}P\ell$$



Examples

1. **Find the total surface area of the pyramid. Round to the nearest tenth.**



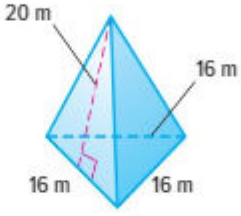
$S.A. = B + \frac{1}{2}P\ell$ Surface area of a pyramid

$S.A. = 16 + \frac{1}{2}(16 \cdot 9)$ $B = 4 \cdot 4$, $P = 4 \cdot 4$ or 16, $\ell = 9$

$S.A. = 88$ Simplify.

The surface area is 88 square centimeters.

2. **Find the total surface area of the pyramid with a base area of 111 square meters.**



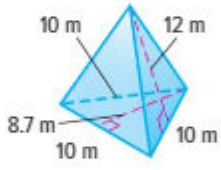
$S.A. = B + \frac{1}{2}P\ell$ Surface area of a pyramid

$S.A. = 111 + \frac{1}{2}(48 \cdot 20)$ $B = 111$, $P = 16 + 16 + 16$ or 48, $\ell = 20$

$S.A. = 591$ Simplify.

The surface area of the pyramid is 591 square meters.

3. **Find the total surface area of the pyramid.**



$S.A. = B + \frac{1}{2}P\ell$ Surface area of a pyramid

$S.A. = 43.5 + \frac{1}{2}P\ell$ $B = \frac{1}{2} \cdot 10 \cdot 8.7$ or 43.5

$S.A. = 43.5 + \frac{1}{2}(30 \cdot 12)$ $P = 10 + 10 + 10$ or 30, $\ell = 12$

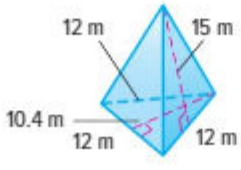
$S.A. = 223.5$ Simplify.

The surface area is 223.5 square meters.

Got it? **Do these problems to find out.**

a. Find the surface area of a square pyramid that has a slant height of 8 centimeters and a base length of 5 centimeters.

b. Find the total surface area of the pyramid shown.



Perimeter of a Square

The formula for the perimeter of a square is $P = 4s$.

Show your work.

a. _____

b. _____

Example

4. **Ali is wrapping gift boxes that are square pyramids for party favors. They have a slant height of 3 centimeters and base edges 2.5 centimeters long. How many square centimeters of card stock are used to make one gift box?**

$S.A. = B + \frac{1}{2}P\ell$ Surface area of a pyramid

$S.A. = 6.25 + \frac{1}{2}(10 \cdot 3)$ $B = 2.5^2$ or 6.25, $P = 4(2.5)$ or 10, $\ell = 3$

$S.A. = 21.25$ Simplify.

So, 21.25 square centimeters of card stock are used to make one gift box.

Got it? **Do this problem to find out.**

Show your work.

c. Adnan purchased a bottle of perfume that is in the shape of a square pyramid. The slant height of the bottle is 11.25 centimeters and the base is 5 centimeters. Find the surface area.

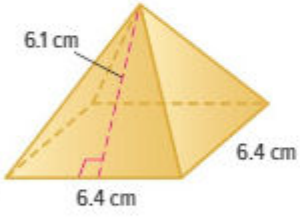
c. _____

Guided Practice

Find the total surface area of each pyramid. Round to the nearest tenth.

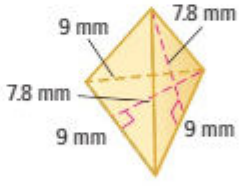
(Examples 1–3)

1.



Show your work.

2.



3. The Washington Monument is an obelisk with a square **pyramid top**. The slant height of the pyramid is 16.65 meters, and the square base has sides of 10.35 meters. Find the lateral area of the pyramid. (Example 4) _____

4. **Building on the Essential Question** Justify the formula for the surface area of a pyramid.

Rate Yourself!

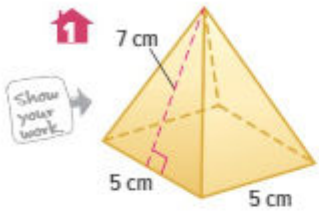
ow confident are you about finding the surface area of pyramids? Check the box that applies.

FOLDABLES Time to update your Foldable!

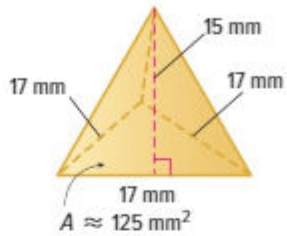
Independent Practice

Find the total surface area of each pyramid. Round to the nearest tenth. (Examples 1–3)

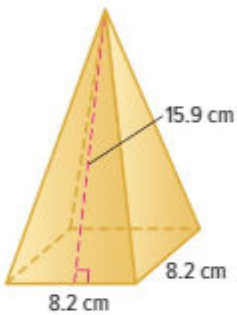
1



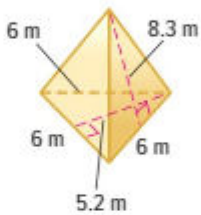
2.



3.



4.



5. A triangular pyramid has a slant height of 0.75 meter. The equilateral triangular base has a perimeter of 1.2 meters and an area of about 0.07 square meter. Find the approximate surface area. (Example 4)

_____

6. The gemstone shown is a square pyramid that has a base with sides 3.4 centimeters long. The slant height of the pyramid is 3.8 centimeters. Find the surface area of the gemstone. (Example 4)

_____

7 Mazen is building a birdhouse for a class project. The birdhouse is a regular hexagonal pyramid. The base has side lengths of 7.5 centimeters and an area of about 150 square centimeters. The slant height is 15 centimeters. Find the approximate surface area of the birdhouse. (Example 4)

_____

8. **Persevere with Problems** A square pyramid has a surface area of 175 square centimeters. The square base has side lengths of 5 centimeters. Find the slant height of the pyramid.

9. A square pyramid has a lateral area of 107.25 square centimeters and a slant height of 8.25 centimeters. Find the length of each side of its base.

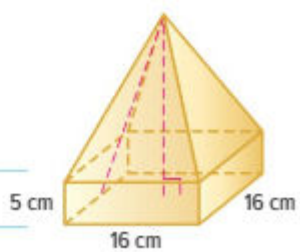
H.O.T. Problems Higher Order Thinking

10. **Justify Conclusions** Suppose you could climb to the top of the Great Pyramid of Giza in Egypt. Which path would be shorter, climbing a lateral edge or the slant height? Justify your response.

11. **Model with Mathematics** Draw a rectangular pyramid and a square pyramid. Explain the differences between the two.

Rectangular Pyramid **Square Pyramid**

12. **Persevere with Problems** The total height of the figure shown is 20 centimeters and the slant height is 17 centimeters. Which has a greater surface area, the prism or the pyramid? Explain your reasoning.

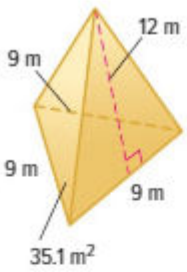


Name _____ My Homework _____

Extra Practice

Find the total surface area of each pyramid. Round to the nearest tenth.

13.



197.1 m^2

$S.A. = B + \frac{1}{2}P\ell$

$S.A. = 35.1 + \frac{1}{2}(27 \cdot 12)$

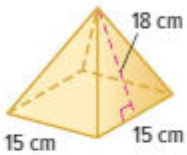
$S.A. = 197.1$

Homework Help

14.

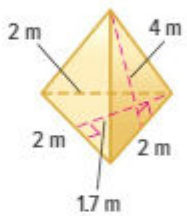
_____

15.



_____

16.

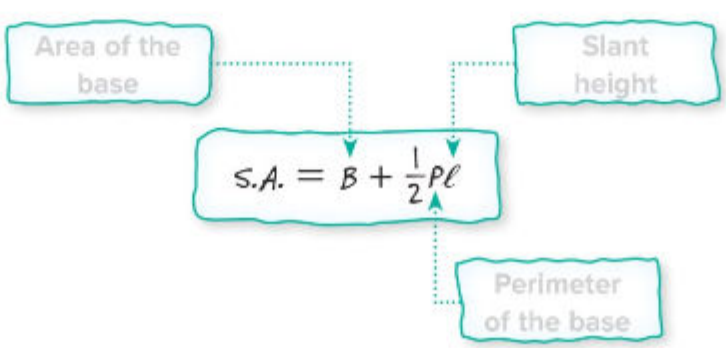


_____

17. A square pyramid has a slant height of $4\frac{2}{3}$ meters. The base has side lengths of $2\frac{1}{4}$ meters. Find the surface area. _____

18. A building in San Francisco is shaped like a square pyramid. It has a slant height of 256.8 meters and each side of its base is 43.5 meters. Find the lateral area of the building. _____

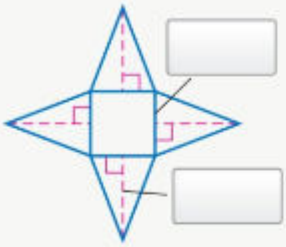
19. MP **Use Math Tools** Complete the organizer below to help you remember what each part of the formula for the surface area of a pyramid represents.



Power Up! Test Practice

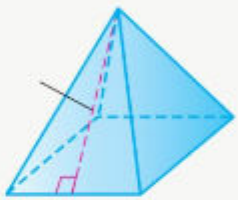
20. The base of a square pyramid has a perimeter of 28 centimeters. The height of the pyramid is 2.1 centimeters longer than the side length of the base. Label the net of the pyramid with the correct dimensions.

2.1 cm	7 cm
4 cm	9.1 cm
4.9 cm	28 cm



What is the total surface area of the pyramid? _____

21. An entertainment company is constructing a tent in the shape of a square pyramid, without a floor, to be used at a dish party. Determine if each statement is true or false.



a. The area of the ground covered by the tent is 56.25 square meters. ☐ True ☐ False

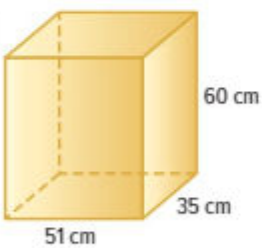
b. The area of each triangular face of the tent is 38.25 square meters. ☐ True ☐ False

c. The amount of material needed to make the tent is 209.25 square meters. ☐ True ☐ False

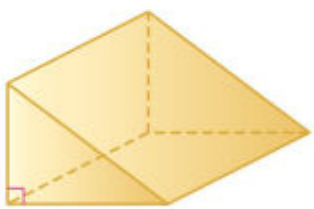
Spiral Review

Find the surface area of each prism. 7.G.6

22. _____



23. _____



24. The volume of the prism shown below is 140 cubic meters. Find the height of the prism. 7.G.6 _____

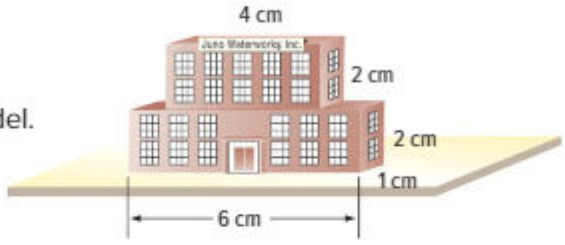
25. The volume of the prism shown below is 10,360 cubic meters. Find the width of the prism. 7.G.6 _____

Inquiry Lab
Composite Figures

Inquiry **HOW can you find the volume and surface area of a composite figure?**

MP Mathematical Practices 1, 3, 4

A company made a model of a new office building. The building is composed of rectangular prisms. You can use centimeter cubes to find the volume of the building model.



Hands-On Activity 1

Tools

The model is a *composite figure* because it is made from two rectangular prisms.

Step 1 Model the top and bottom rectangular prisms using cubes.


Bottom

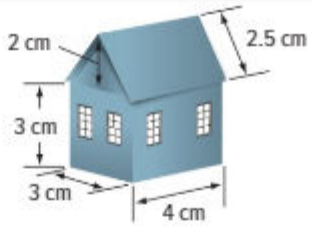

Top

Step 2 Count the cubes to find the dimensions. Write the dimensions in the table below. Then use the cube models to find the volume of both prisms. Write these measures in the table below step 3.

Step 3 Use the table to find the volume of the entire building model. Write these measures in the composite row of the table.

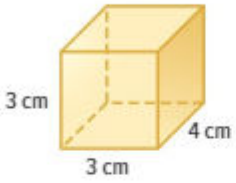
Model	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
Bottom	6	1		
Top				
Composite				

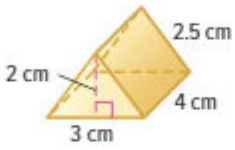
Mr. Obaid's class made a model of a house. The model was composed of a rectangular prism and a triangular prism. Determine the volume and surface area of the model house.



Hands-On Activity 2

Step 1 Use a rectangular prism to model the bottom of the house. Use a triangular prism to model the top of the house.





Step 2 Complete the tables below using the models from Step 1.

Prism	Length (cm)	Width (cm)	Height (cm)
Rectangular	4	3	

Prism	Length (cm)	Base (cm)	Height (cm)
Triangular	4	3	

Step 3 Use the information from the tables and the models to find the total volume of the model house.

$\square$ cm^3 (Volume of Rectangular Prism) + $\square$ cm^3 (Volume of Triangular Prism) = $\square$ cm^3 (Total Volume)

Step 4 Use the information from the tables and the models to find the total surface area of the model house.

$\square$ cm^2 (Surface Area of Rectangular Prism) + $\square$ cm^2 (Surface Area of Triangular Prism) − $\square$ cm^2 (Areas where Prisms Connect) = $\square$ cm^2 (Total Surface Area)

The total volume of the model house is $\square$ cubic centimeters. The total surface area is $\square$ square centimeters.

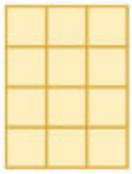
Investigate

Collaborate

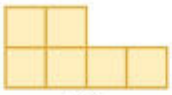
Work with a partner.

1. **Model with Mathematics** Use the top, side, and front views to build a figure using centimeter cubes.

Show your work.



top



side

front

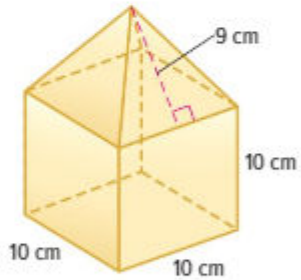
   a. Make a sketch of the figure you built.

Show your work.

   b. Find the volume and surface area of the figure.

   Volume: _____ Surface Area: _____

Refer to the figure at the right for Exercises 2–4.



2. The figure is comprised of a _____ and a square _____.

3. Complete the following to find the volume of the figure.

   a. The volume of the cube is ☐ cubic centimeters.

   b. The volume of the square pyramid is 250 cubic centimeters.

   c. So, the volume of the composite figure is ☐ cubic centimeters.

4. Complete the following to find the surface area of the figure.

   a. The surface area of the cube is ☐ square centimeters.

   b. The surface area of the square pyramid is ☐ square centimeters.

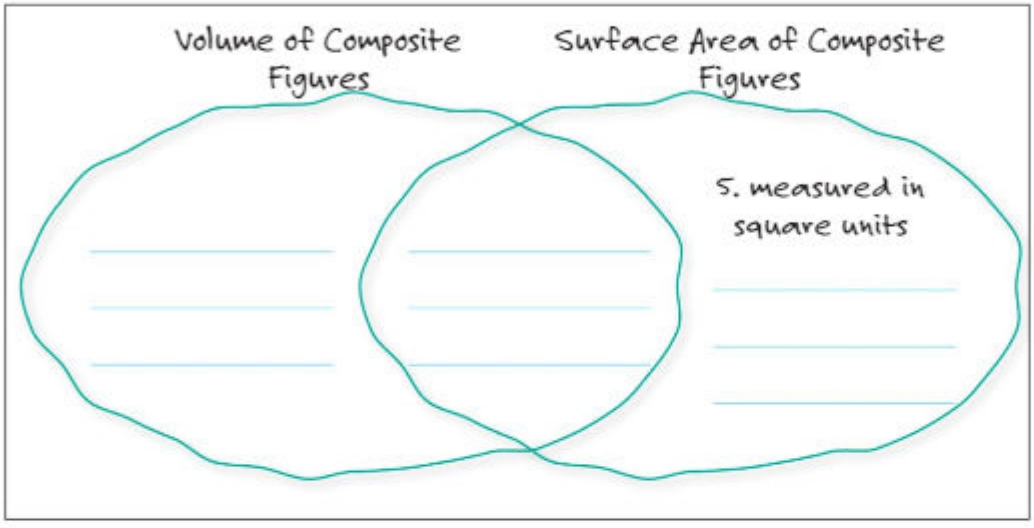
   c. The area where the figures overlap is ☐ square centimeters.

   d. The surface area of the composite figure is ☐ square centimeters.

Analyze and Reflect

Reason Inductively **Work with a partner. Write each of the following statements in the correct location. One statement is done for you.**

5. *measured in square units*
6. *measured in cubic units*
7. *involves adding measures of each figure*
8. *involves subtracting where figures overlap*



Create

9. **Model with Mathematics** Describe a real-world situation where it might be necessary to use a model or a drawing to find the volume or surface area.

10. Inquiry HOW can you find the volume and surface area of a composite figure?

Lesson 8

Volume and Surface Area of Composite Figures

Real-World Link

Abdalla and his sister Maysoun are making a bat house for their backyard like the one shown. They need to determine the surface area to find how much wood they will need.

1. Look at the largest bat house. What three-dimensional figures make up the bat house?

2. What method could you use to find the surface area of the bat house?

3. Suppose you wanted to find the volume of the bat house. What method could you use?

Essential Question

HOW do measurements help you describe real-world objects?

MP Mathematical Practices

1, 3, 4

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Work Zone

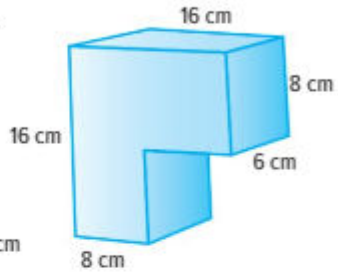
Volume of a Composite Figure

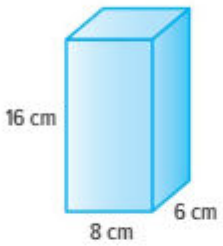
The volume of a composite figure can be found by decomposing the figure into solids whose volumes you know how to find.

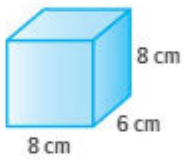
Examples

1. **Find the volume of the composite figure.**

Find the volume of each prism.







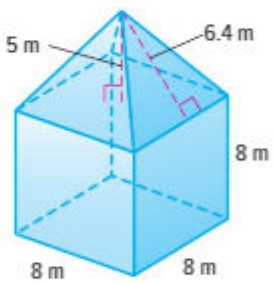
$V = \ell wh$ $V = \ell wh$

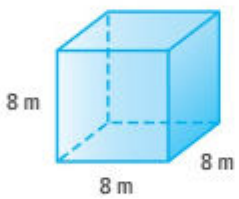
$V = 8 \cdot 6 \cdot 16$ or 768 $V = 8 \cdot 6 \cdot 8$ or 384

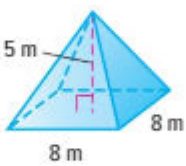
The volume is 768 + 384 or 1,152 cubic centimeters.

2. **Find the volume of the composite figure.**

Find the volume of the cube and the pyramid. Round to the nearest tenth.







$V = \ell wh$ $V = \frac{1}{3}Bh$

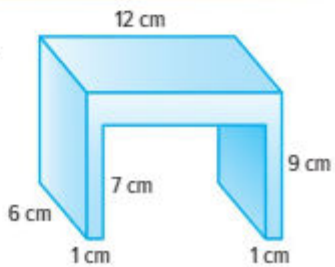
$V = 8 \cdot 8 \cdot 8$ or 512 $V = \frac{1}{3}(8 \cdot 8)5$ or 106.7

The volume is 512 + 106.7 or 618.7 cubic meters.

Show your work.

Got it? **Do this problem to find out.**

a. Find the volume of the composite figure.



a. _____

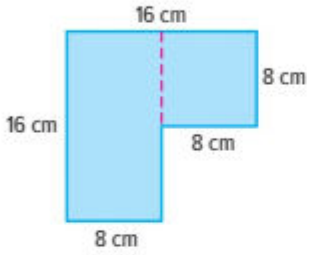
Surface Area of a Composite Figure

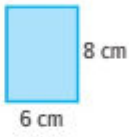
You can also find the surface area of composite figures by finding the areas of the faces that make up the composite figure.

Examples

3. **Find the surface area of the figure in Example 1.**

The surface is made up of three different polygons.





$A = \ell w + \ell w$
$A = (8 \cdot 16) + (8 \cdot 8)$
$A = 128 + 64$ or 192

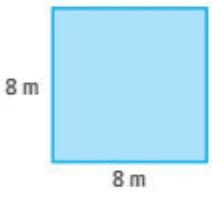
$A = \ell w$
$A = 6 \cdot 16$
$A = 96$

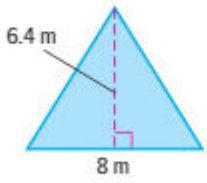
$A = \ell w$
$A = 6 \cdot 8$
$A = 48$

The total surface area is $2(192) + 2(96) + 4(48)$ or 768 square centimeters.

4. **Find the surface area of the composite figure in Example 2.**

The figure is made up of two different polygons.





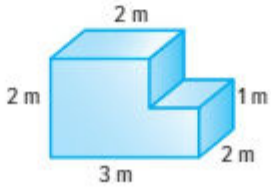
$A = \ell w$
$A = 8 \cdot 8$ or 64

$A = \frac{1}{2}bh$
$A = \frac{1}{2} \cdot 8 \cdot 6.4$ or 25.6

The total surface area is $5(64) + 4(25.6)$ or 422.4 square meters.

Got it? **Do this problem to find out.**

b. Find the surface area of the steps that are represented by the composite figure shown.



Surface Area

To make it easier to see each face, sketch the faces and label the dimensions of each.

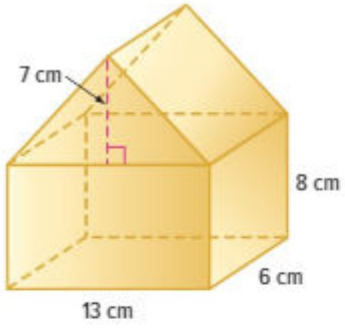
Show your work.

b. _____

Guided Practice

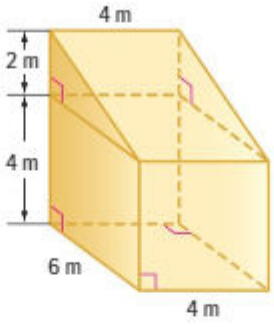
Find the volume of each composite figure. Round to the nearest tenth if necessary. (Examples 1 and 2)

1.



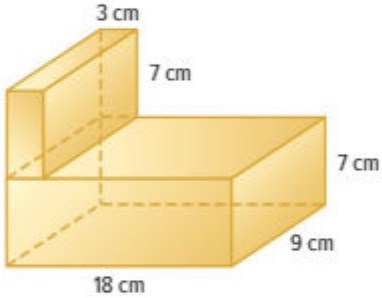
Show your work.

2.

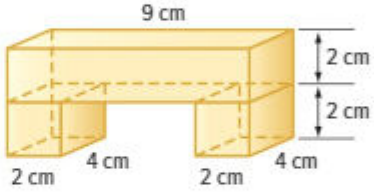


Find the surface area of each composite figure. Round to the nearest tenth if necessary. (Examples 3 and 4)

3.



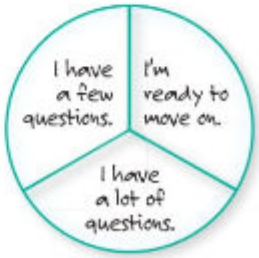
4.



5. **Building on the Essential Question** How do the previous lessons in this chapter help you find the surface area and volume of a composite figure?

Rate Yourself!

Are you ready to move on? Shade the section that applies.

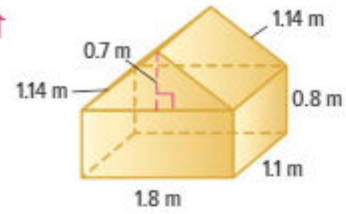


Name _____ My Homework _____

Independent Practice

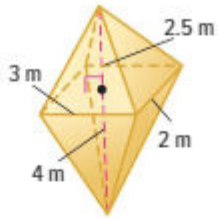
Find the volume of each composite figure. Round to the nearest tenth if necessary. (Examples 1 and 2)

1.



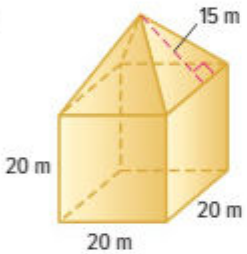
Show your work.

2.

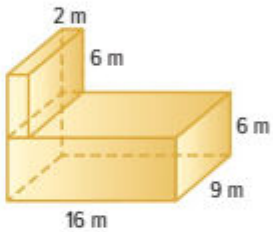


Find the surface area of each composite figure. Round to the nearest tenth if necessary. (Examples 3 and 4)

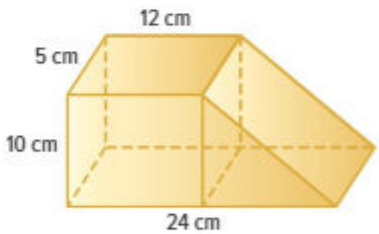
3.



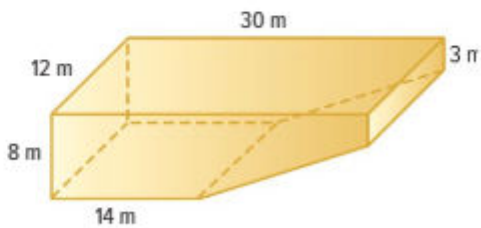
4.



5. Find the volume of the figure at the right in cubic meters. Round to the nearest tenth. (Examples 1 and 2)



6. **Reason Inductively** The swimming pool at the right is being filled with water. Find the number of cubic meters that it will take to fill the swimming pool. (*Hint:* The area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$.) (Examples 1 and 2)

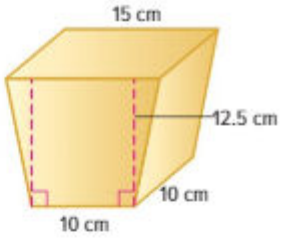


Copy and Solve **For Exercises 7–8, show your work on a separate piece of paper. Round to the nearest tenth.** (Examples 1–4)

7. Find the surface area of the figure in Exercise 1.

8. Find the volume of the figure in Exercise 4.

9. A carryout container is shown. The bottom base is a 10-centimeters square and the top base is a 10-centimeters by 15-centimeters rectangle. The height of the container is 12.5 centimeters. Find the volume of food that it holds. _____



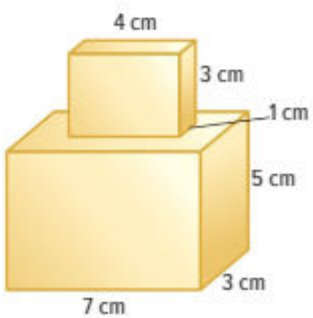
10. Refer to the house shown. Find the surface area and volume of the house. Do not include the bottom of the house when calculating the surface area. _____

H.O.T. Problems Higher Order Thinking

11. MP **Model with Mathematics** Draw a composite figure that is made up of a cube and a square pyramid. Label its dimensions and find the volume of the figure. _____

12. MP **Persevere with Problems** Draw an example of a composite figure that has a volume between 250 and 300 cubic units.

13. MP **Construct an Argument** Will the surface area of the figure at the right be greater than or less than 180 square centimeters? Explain your reasoning. _____

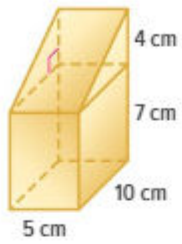


Name _____ My Homework _____

Extra Practice

Find the volume of each composite figure. Round to the nearest tenth if necessary.

14.



450 cm^3

Homework Help

Rectangular Prism

$V = lwh$

$V = 5 \cdot 10 \cdot 7$

$V = 350$

Triangular Prism

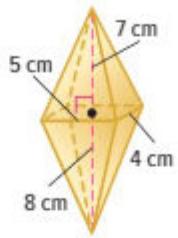
$V = Bh$

$V = \frac{1}{2} \cdot 10 \cdot 4 \cdot 5$

$V = 100$

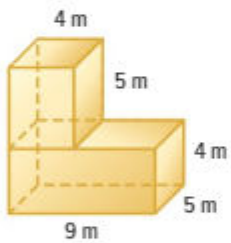
Total Volume = 350 + 100 or 450 cm^3

15.

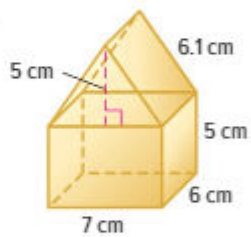


Find the surface area of each composite figure. Round to the nearest tenth if necessary.

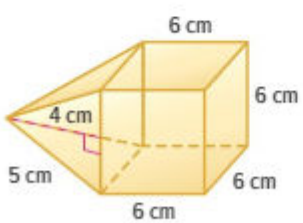
16.

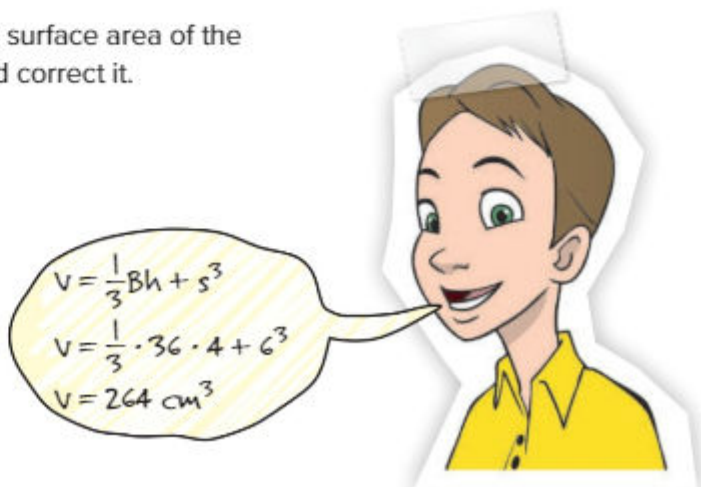


17.



18. MP **Find the Error** Abdulkarim is finding the surface area of the composite figure shown. Find his mistake and correct it.



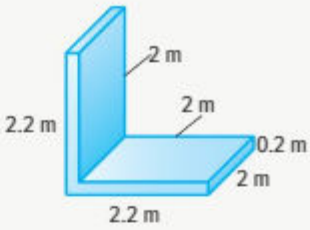


Power Up! Test Practice

19. Refer to the composite figure with the dimensions shown. Fill in the boxes to complete each statement.

a. The volume of the composite figure is ☐.

b. The total surface area of the composite figure is ☐.



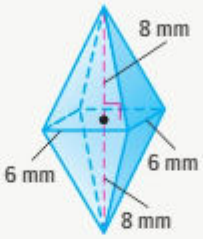
20. Refer to the composite figure with the dimensions shown.

Select the correct values to complete the expression to find the volume of the figure.

$V = \square \cdot \frac{1}{3} \cdot \square \cdot \square \cdot \square$

What is the volume of the composite figure?

☐



2
3
4
6
8

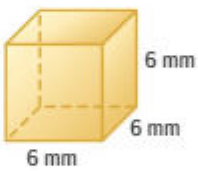
Spiral Review

Draw a net for each figure. 6.G.4

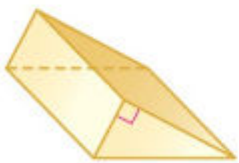
21.

Show your work.

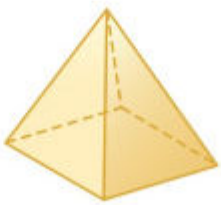
22.



23.



24.



21ST CENTURY CAREER in Landscape Architecture

Landscape Architect

Do you have an artistic side, and do you enjoy being outdoors? If so, a career in landscape design might be a perfect fit for you. Landscape architects design outside areas such as yards, parks, playgrounds, campuses, shopping centers, and golf courses. Their designed areas are not only meant to be beautiful, but also functional and compatible with the natural environment. A landscape architect must be proficient in mathematics, science, and the use of computer-aided design.

College & Career READINESS

Is This the Career for You?

Are you interested in a career as a landscape architect? Take some of the following courses in high school.

- Algebra
- Architectural Design
- Botany
- Drafting/Illustrative Design Technology
- Geometry

Find out how math relates to a career in Landscape Architecture.

Planting in Circles

For each problem, use the information in the designs.

1. In Design 2, what is the radius of the larger grassy area? _____

2. The small circular fountain in Design 1 is surrounded by a stone wall. Find the circumference of the wall. Use $\frac{22}{7}$ for π.

   _____

3. Find the circumference of the smaller grassy area in Design 2. Use 3.14 for π.

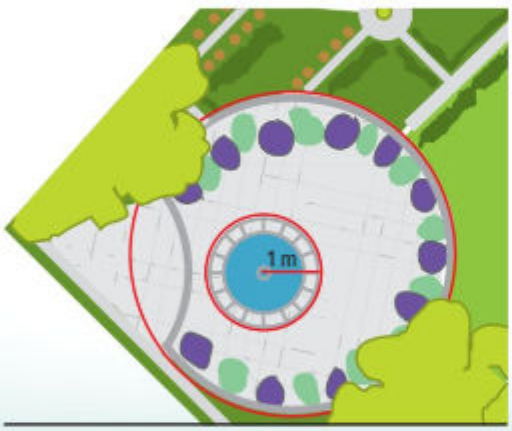
   _____

4. In Design 2, how much greater is the lawn area in the larger circle than in the smaller circle? Use 3.14 for π. _____

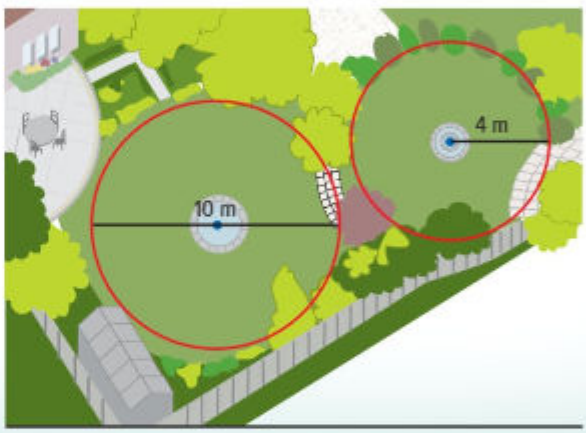
5. In Design 2, the smaller circle is surrounded by a path 1 meter wide. What is the outer circumference of the path? Use the π key on a calculator and round to the nearest tenth.

   _____

6. In Design 1, the area of the large circular patio is about 18.1 square meters. What is the radius of the patio? Round to the nearest tenth of a meter. _____



Design 1



Design 2

Career Project

It's time to update your career portfolio! Download free landscaping software from the Internet and use it to create your own landscape design. Include a list of all the plants, materials, and hard elements used in your design. Also, provide an estimate of the total cost of the landscaping project.

_____

_____

_____

_____

What is something you really want to do in the next ten years?

- _____
- _____
- _____
- _____
- _____

Chapter Review

Vocabulary Check

Complete each sentence using the vocabulary list at the beginning of the chapter. Then circle the word that completes the sentence in the word search.

1. The distance across a circle through its center is called the _____.
2. The _____ is the distance from the center to any point on the circle.
3. A _____ is the set of all points in a plane that are the same distance from a point.
4. The point in a circle from which all other points are equidistant is called the _____.
5. The distance around a circle is the _____.
6. The ratio of circumference to diameter is called _____.
7. A _____ is half of a circle.
8. A _____ figure is made up of two or more shapes.
9. The _____ of a three-dimensional figure is the measure of the space it occupies.
10. The sum of the areas of all the faces of a three-dimensional figure is the _____ area.
11. The triangular faces of a pyramid that are not bases are _____ faces.
12. The height of each lateral face of a pyramid is called the _____ height.

G	U	H	M	I	R	H	X	P	E	S	F	S	S	Q	W	C	P	A	C	O	Q	
P	I	B	A	E	V	O	L	U	M	E	E	V	W	Z	A	X	S	D	O	O	C	
P	G	J	T	V	C	T	Y	A	G	E	R	M	R	Q	A	Q	I	E	M	I	O	
C	N	N	C	D	E	D	Z	M	M	L	R	L	I	I	N	A	D	P	P	I	D	
M	E	Z	C	K	G	H	N	L	O	C	P	I	V	C	M	C	W	Z	O	C	Z	
C	H	C	I	R	C	U	M	F	E	R	E	N	C	E	I	P	M	C	S	N	P	
X	E	O	V	G	B	L	H	V	Z	I	B	B	T	P	T	R	S	O	I	C	N	
S	Y	V	T	D	K	L	A	B	W	C	O	E	E	T	J	L	C	V	T	D	L	
S	P	M	T	Y	S	U	Z	R	I	T	R	J	Q	O	R	Y	B	L	E	Y	Q	
Y	P	V	S	Z	B	E	Z	O	E	H	N	W	G	J	C	T	T	Q	E	H	S	
P	R	D	B	T	T	L	D	W	T	T	N	W	Y	X	Z	S	V	J	X	L	J	
E	K	A	J	N	J	G	W	C	M	P	A	C	D	M	K	L	M	O	S	L	P	
A	I	Y	D	A	Z	X	F	Q	F	C	X	L	Y	M	Y	A	W	X	F	Z	F	
V	X	Y	M	I	N	H	P	I	W	D	U	Z	L	A	T	N	Q	L	O	Q	J	
Z	U	O	Y	M	U	U	K	K	K	L	I	C	E	W	H	T	A	J	M	U	H	
E	C	A	F	R	U	S	G	Z	E	X	K	W	S	T	R	X	J	X	K	I	K	

Key Concept Check

Use Your FOLDABLES

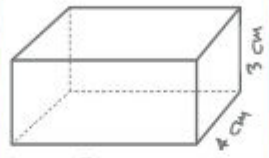
Use your Foldable to help review the chapter.

Tape here

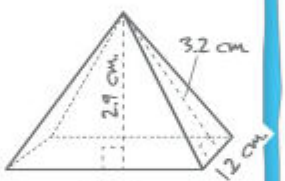
Tape here

Volume

Volume =



Volume =



Tab 1

Surface Area

Surface area =

Surface area =

Tab 2

Got it?

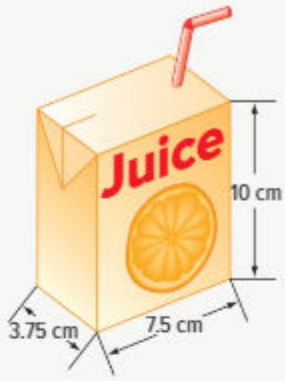
Circle the correct term or number to complete each sentence.

1. The diameter of a circle is (twice, three times) its radius.
2. The area of a circle equals the product of pi and the square of its (radius, diameter).
3. The volume of a rectangular prism can be found by multiplying the area of the base times the (length, height).
4. To find the surface area of a triangular prism, find the area of each face and calculate the (sum, product) of all the faces.

Power Up! Performance Task

Juice Box Packaging

Supreme Packaging Company manufactures juice boxes for juice companies. They are examining different ways to make the juice boxes using various lengths, widths, and heights. The measurements of one juice box are shown.



Write your answers on another piece of paper. Show all of your to receive full credit.

Part A

What is the volume of the juice box shown? The company received an order to make a jumbo juice box that has twice the volume as the one shown. Could you double the current dimensions to make the jumbo juice box at the suggested volume? Explain.

Part B

Draw and label a net to find the surface area of the original juice box. It costs Supreme Packaging AED0.02 per square centimeter to create one juice box. The company groups eight juice boxes together as one package. How much does it cost to create one package?

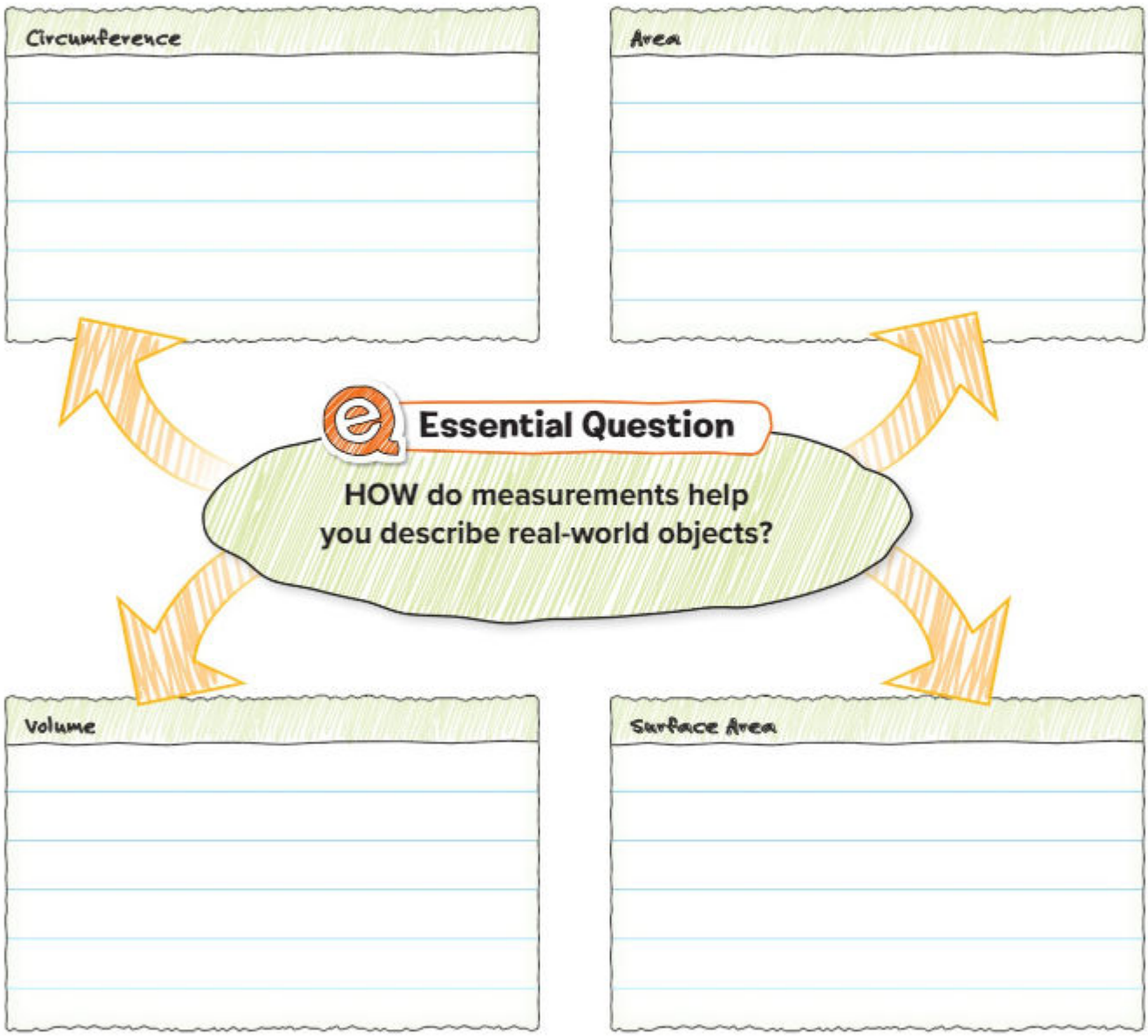
Part C

An artist created the picture of the citrus fruit on the label. The picture is a circle and has an area of 78.5 square centimeters. Will the artist's picture fit on the juice box label? Explain. Use 3.14 for π.

Reflect

Answering the Essential Question

Use what you learned about measuring figures to complete the graphic organizer.



Answer the Essential Question. HOW do measurements help you describe real-world objects?

UNIT PROJECT

Turn Over a New Leaf The flatness of leaves serves an important purpose. In this project you will:

- **Collaborate** with your classmates as you research the primary function of leaves.
- **Share** the results of your research in a creative way.
- **Reflect** on how you use different measurements to solve real-life problems.

Collaborate

Work with your group to research and complete each activity. You will use your results in the Share section on the following page.

1. Suppose you have a cube that is 10 centimeters on each side. Find the volume, surface area, and surface area to volume ratio.

2. Disassemble the cube from Exercise 1 into centimeter cubes. Arrange the cubes in a 50-by-20-by-1 prism. Find the volume, surface area, and surface area to volume ratio.

3. Compare and contrast the volume, surface area, and surface area to volume ratio from Exercises 1 and 2.

4. Trace the outline of a leaf onto centimeter grid paper. Estimate the volume of the leaf. (Assume the height of your leaf is 0.1 centimeter.) Estimate the surface area. (You can ignore the edge of the leaf.) Find the surface area to volume ratio.

5. Do research to find the primary function of a leaf. Explain how the surface area to volume ratio of a leaf aids in its function.

6. Find examples from nature or man-made objects that have a small surface area to volume ratio. Explain the benefits.

Share

Collaborate

With your group, decide on a way to share what you have learned about the surface area to volume ratio of leaves. Some suggestions are listed below, but you could also think of other creative ways to present your information. Remember to show how you used mathematics to complete each of the activities in this project!

- Create a digital presentation that compares two types of leaves. Use what you learned about surface area to volume ratios in your presentation.
- Imagine you discovered a new type of leaf. Create an annotated diagram of your leaf. The annotations should include the type of information you learned in this project.

Check out the note on the right to connect this project with other subjects.

Connect with Science

Environmental Literacy Write a paragraph detailing facts about the leaves you researched. Some questions to consider are:

- What are the names of the trees that dropped these leaves?
- Are these types of trees common in your state?

Reflect

On Your Own

6. **Answer the Essential Question** How can you use different measurements to solve real-life problems?

   a. How did what you learned about geometric figures help you use different measurements to solve real-life problems in this project?

   b. How did what you learned about measuring figures to help you use different measurements to solve real-life problems in this project?

UNIT 5
Statistics and Probability

Essential Question

WHY is learning mathematics important?

Chapter 9
Probability

Probability describes the likelihood of an event occurring. In this chapter, you will develop probability models and find probabilities of simple and compound events.

Chapter 10
Statistics

Statistics can be used to draw conclusions about a population. In this chapter, you will use random samples to make predictions and compare populations.

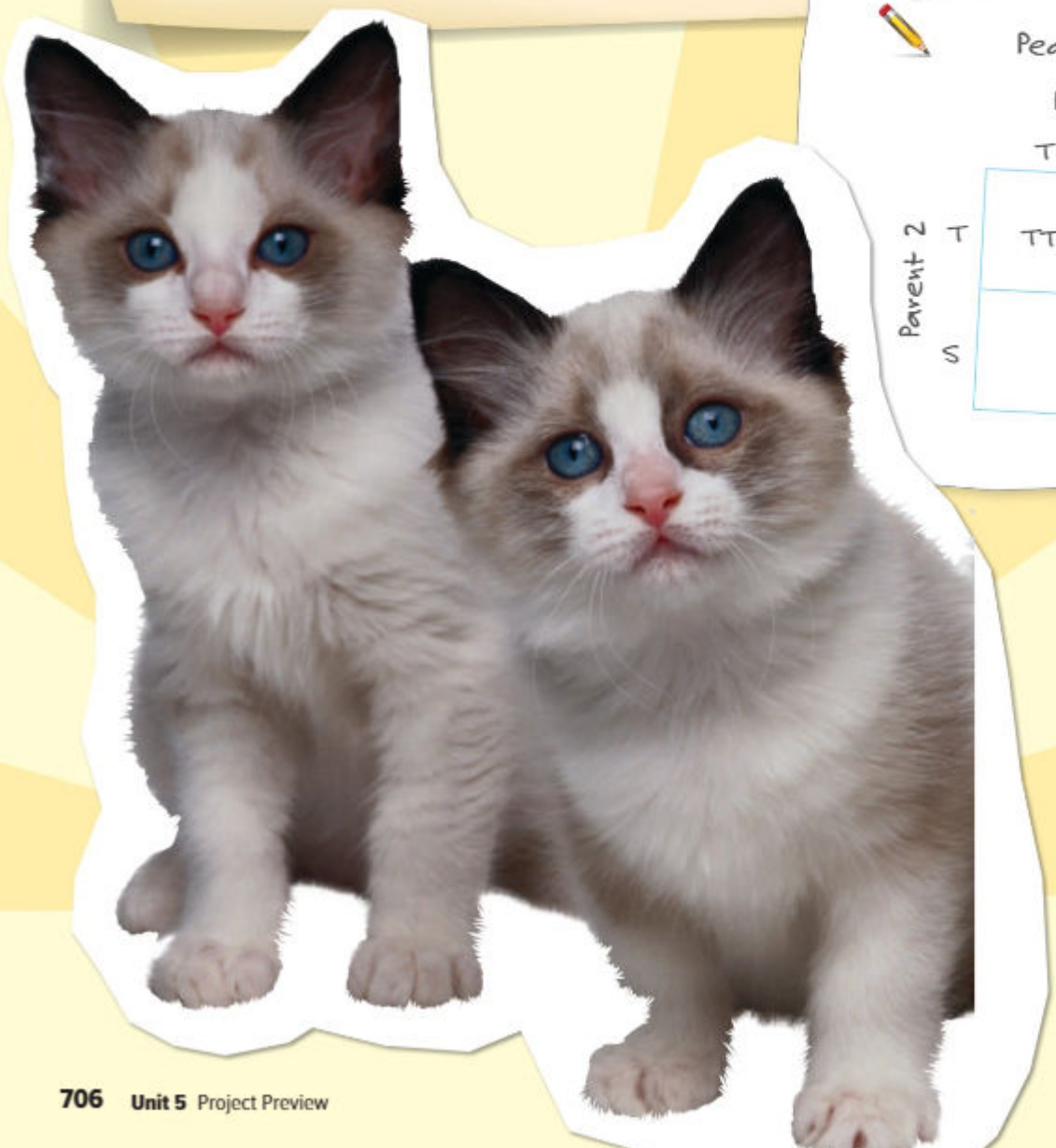
Collaborate

Unit Project Preview

Math Genes A *Punnett Square* is a diagram that is used to predict the genetic traits of offspring.

A pea plant can be tall (described by TT, TS, or ST) or short (described by SS). Complete the Punnett Square below. What percent of the outcomes indicate that the offspring will be short? _____

At the end of Chapter 10, you'll complete a project in which you use pets' traits to make predictions about their offspring. Put on your lab coat and grab your math tool kit to begin this adventure!



Pea Plants

Parent 2 \ Parent 1	T	S
T	TT	
S		

Chapter 9
Probability

Essential Question

HOW can you predict the outcome of future events?

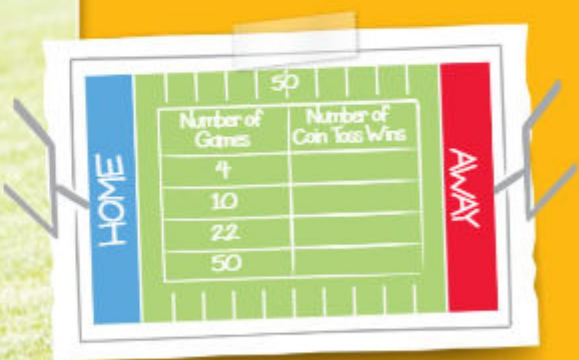
MP Mathematical Practices

1, 3, 4, 5

Math in the Real World

Probability is the likelihood or chance of an event occurring.

At the beginning of a football game, a coin is tossed to determine which team receives the ball first. Fill in the table below to indicate the number of times a team would expect to win the coin toss based on the number of games played.



FOLDABLES® Study Organizer

1. Cut out the Foldable on page FL11 of this book.
2. Place your Foldable on page 786.
3. Use the Foldable throughout this chapter to help you learn about probability.

What Tools Do You Need?

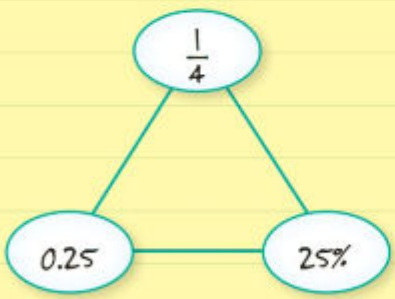
Vocabulary

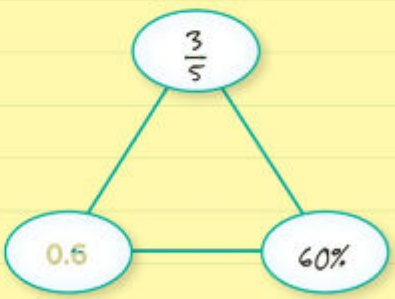
complementary events	outcome	simple event
compound event	permutation	simulation
dependent events	probability	theoretical probability
experimental probability	random	tree diagram
fair	relative frequency	uniform probability model
Fundamental Counting Principle	sample space	unfair
independent events		

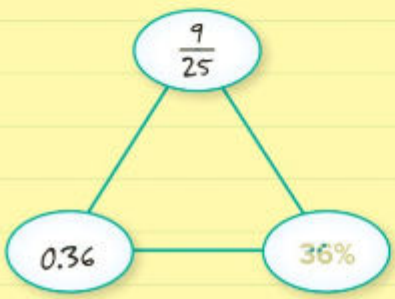
Review Vocabulary

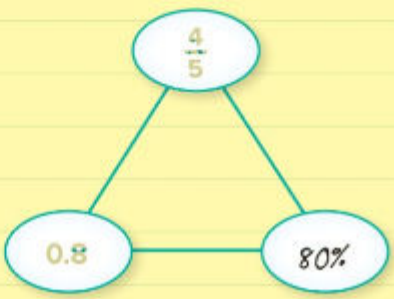
Fractions, Decimals, and Percents Equivalent rational numbers are numbers that have the same value. For example, three-fourths is equivalent to 0.75 or 75%.

A probability can be expressed as a fraction, decimal, or percent. For each rational number, write the missing equivalent values. Write fractions in simplest form.









What Do You Already Know?

List three things you already know about probability in the first section. Then list three things you would like to learn about probability in the second section.

Probability	
What I know	What I want to find out

When Will You Use This?

Here is an example of how probability is used in the real world.

Activity 1 Have you ever read something like "The chances of winning are 75%." or "30% of the people surveyed said that they prefer vanilla ice cream."? Find an example like the ones given. Describe your example and what it means to you.

Are You Ready?

Try the Quick Check below. Check

Quick Review

Example 1

Write $\frac{21}{28}$ in simplest form.

$\frac{21}{28} \overset{\div 7}{=} \frac{3}{4}$

Divide the numerator and denominator by the GCF, 7.

Example 2

Find $7 \cdot 6 \cdot 5 \cdot 4$.

$7 \cdot 6 \cdot 5 \cdot 4 = 42 \cdot 5 \cdot 4$
$= 210 \cdot 4$
$= 840$

Multiply from left to right.

Quick Check

Fractions Write each fraction in simplest form.

1. $\frac{5}{15} =$ _____

2. $\frac{3}{18} =$ _____

3. $\frac{8}{12} =$ _____

4. $\frac{12}{20} =$ _____

Show your work.

Products Find each product.

5. $6 \cdot 5 =$ _____

6. $10 \cdot 9 \cdot 8 =$ _____

7. $4 \cdot 3 \cdot 2 \cdot 1 =$ _____

8. Suppose you listen to 9 songs each hour for 5 hours every day this week. How many songs will you have listened to this week?

_____

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4 5 6 7 8

Lesson 1

Probability of Simple Events

Vocabulary Start-Up

Probability is the chance that some event will occur. A **simple event** is one outcome or a collection of outcomes. What is an **outcome**?

Math Definition		Real-World Definition
A possible result in a probability experiment.	Outcome	

Real-World Link

For a sledding trip, you randomly select one of the four hats shown. Complete the table to show the possible outcomes.

Hat Selection Outcomes			
Outcome 1	green hat	**Outcome 3**	
Outcome 2		**Outcome 4**	

1. Write a ratio that compares the number of blue hats to the total number of hats. _____

2. Describe a hat display in which you would have a better chance of selecting a red hat.

_____

_____

Essential Question

HOW can you predict the outcome of future events?

Vocabulary

probability
outcome
simple event
random
complementary events

Mathematical Practices
1, 3, 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

(1) Persevere with Problems	(5) Use Math Tools
(2) Reason Abstractly	(6) Attend to Precision
(3) Construct an Argument	(7) Make Use of Structure
(4) Model with Mathematics	(8) Use Repeated Reasoning

Key Concept

Probability

Words The probability of an event is a ratio that compares the number of favorable outcomes to the number of possible outcomes.

Symbols $P(event) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

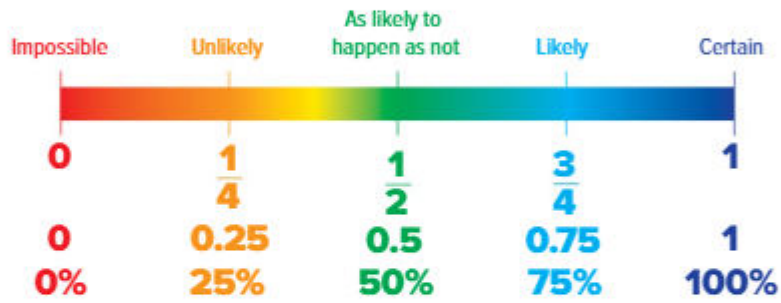
Work Zone

STOP and Reflect

In the space below, describe an example of a simple event that is certain to occur.

The probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Greater numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

Probability can be written as a fraction, decimal, or percent.



Outcomes occur at **random** if each outcome is equally likely to occur.

Example

Tools

There are six equally likely outcomes if a number cube with sides labeled 1 through 6 is rolled.

1. **Find *P*(6) or the probability of rolling a 6.**

There is only one 6 on the number cube.

$P(6) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

$= \frac{1}{6}$

The probability of rolling a 6 is $\frac{1}{6}$, or about 17%, or about 0.17.

Show your work.

Got it? **Do this problem to find out.**

a. A coin is tossed. Find the probability of the coin landing on heads. Write your answer as a fraction, percent, and decimal.

a. _____

Example

2. **Find the probability of rolling a 2, 3, or 4 on the number cube.**

The word *or* indicates that the number of favorable outcomes needs to include the numbers 2, 3, and 4.

$$P(2, 3, \text{or } 4) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$= \frac{3}{6} \text{ or } \frac{1}{2} \quad \text{Simplify.}$$

The probability of rolling a 2, 3, or 4 is $\frac{1}{2}$, 50%, or 0.5.

Got it? **Do these problems to find out.**

Show your work

The spinner at the right is spun once. Find the probability of each event. Write each answer as a fraction, percent, and decimal.

b. $P(\text{F})$ **c.** $P(\text{D or G})$ **d.** $P(\text{vowel})$

b. _____

c. _____

d. _____

Find Probability of the Complement

Complementary events are two events in which either one or the other must happen, but they cannot happen at the same time. For example, a coin can either land on heads or *not* land on heads. The sum of the probability of an event and its complement is 1 or 100%.

Complement

In everyday language complement means the quantity required to make something complete. This is similar to the math meaning.

Example

3. **Find the probability of *not* rolling a 6 in Example 1.**

The probability of *not* rolling a 6 and the probability of rolling a 6 are complementary. So, the sum of the probabilities is 1.

$P(6) + P(\textit{not } 6) = 1$ $P(6)$ and $P(\textit{not } 6)$ are complements.

$\frac{1}{6} + P(\textit{not } 6) = 1$ Replace $P(6)$ with $\frac{1}{6}$.

$\frac{1}{6} + \frac{5}{6} = 1$ **THINK** $\frac{1}{6}$ plus what number equals 1?

The probability of *not* rolling a 6 is $\frac{5}{6}$, or about 83% or 0.83.

Got it? **Do this problem to find out.**

e. A bag contains 5 blue, 8 red, and 7 green marbles. A marble is selected at random. Find the probability the marble is *not* red.

e. _____

Example

4. **Mr. Nasser surveyed his class and discovered that 30% of his students have blue eyes. Identify the complement of this event. Then find its probability.**

The complement of having blue eyes is *not* having blue eyes. The sum of the probabilities is 100%.

$P(\text{blue eyes}) + P(\textit{not}\ \text{blue eyes}) = 100\%$ — *P*(blue eyes) and *P*(*not* blue eyes) are complements.

$30\% + P(\textit{not}\ \text{blue eyes}) = 100\%$ — Replace *P*(blue eyes) with 30%.

$30\% + 70\% = 100\%$ — **THINK** 30% plus what number equals 100%?

So, the probability that a student does *not* have blue eyes is 70%, 0.7, or $\frac{7}{10}$.

Guided Practice

Check

A letter tile is chosen randomly. Find the probability of each event. Write each answer as a fraction, percent, and decimal. (Examples 1–3)

1. *P*(D) _____

2. *P*(S, V, or L) _____

3. *P*(*not* D) _____

4. The probability of choosing a "Go Back 1 Space" card in a board game is 25%. Describe the complement of this event and find its probability. (Example 4) _____

5. **Building on the Essential Question** Explain the relationship between the probability of an event and its complement. Give an example.

Rate Yourself!

How confident are you about finding the probability of simple events? Shade the ring on the target.

FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

The spinner shown is spun once. Find the probability of each event. Write each answer as a fraction, percent, and decimal. (Examples 1–3)

1. *P*(blue)

2. *P*(red or yellow)

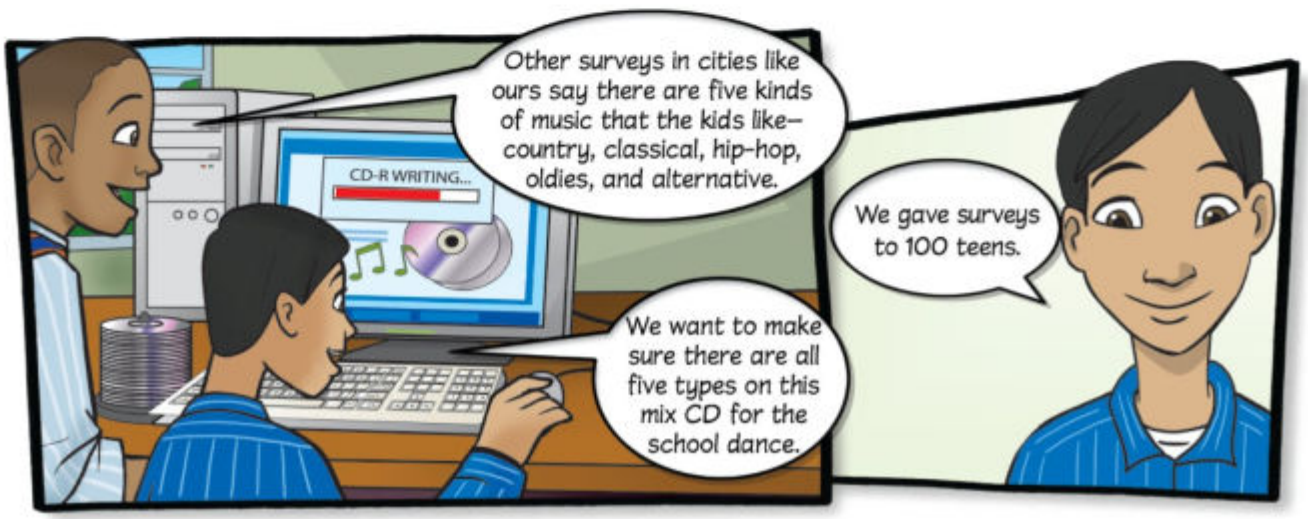
3. *P*(*not* brown)

4. *P*(*not* green)

5. Refer to the table on air travel at selected airports. Suppose a flight that arrived at El Centro is selected at random. What is the probability that the flight did *not* arrive on time? Write the answer as a fraction, decimal, and percent. Explain your reasoning. (Example 4)

Air Travel	
Airport	**Arrivals (Percent on-time)**
El Centro (CA)	80
Baltimore (MD)	82

6. **Model with Mathematics** Refer to the graphic novel frame below. Mazen and Ali decide to create a music mix and include an equal number of songs from each genre. What is the probability that any given song would be from the hip-hop genre? _____



One jelly bean is picked, without looking, from the dish. Write a sentence that explains how likely it is for each event to happen.

7. black

8. purple, red, or yellow

H.O.T. Problems Higher Order Thinking

9. MP **Persevere with Problems** The probability of landing in a certain section on a spinner can be found by considering the size of the angle formed by that section. On spinner shown, the angle formed by the yellow section is one-fourth of the angle formed by the entire circle. So, $P(\text{yellow}) = \frac{1}{4}$, 0.25, or 25%.

a. Determine $P(\text{green})$ and $P(\text{orange})$ for the spinner. Write the probabilities as fractions, decimals, and percents.

b. Determine $P(\textit{not}\ \text{yellow})$.

10. MP **Persevere with Problems** A bag contains 6 red, 4 blue, and 8 green marbles. How many marbles of each color should be added so that the total number of marbles is 27, but the probability of randomly selecting one marble of each color remains unchanged?

11. MP **Which One Doesn't Belong?** Circle the pair of probabilities that does not belong with the other three. Explain your reasoning.

$0.625, \frac{3}{8}$ $0.38, 62\%$ $\frac{7}{8}, 0.125$ $70\%, \frac{1}{3}$

Name _____ My Homework _____

Extra Practice

Ten cards numbered 1 through 10 are mixed together and then one card is drawn. Find the probability of each event. Write each answer as a fraction, percent, and decimal.

12. $P(8)$

$\frac{1}{10}$, 10%, or 0.1

Only 1 card has an 8. So, $P(8)$ is $\frac{1}{10}$, 10%, or 0.1.

Homework Help

13. $P(7 \text{ or } 9)$

$\frac{1}{5}$, 20%, or 0.2

There is 1 card with a 7 and 1 card with a 9. So, $P(7 \text{ or } 9)$ is $\frac{1}{5}$, or 20%, or 0.2.

14. $P(\text{less than } 5)$

15. $P(\text{greater than } 3)$

16. $P(\text{odd})$

17. $P(\text{even})$

18. $P(\textit{not}$ a multiple of 4)

19. $P(\textit{not}$ 5, 6, 7, or 8)

20. $P(\text{divisible by } 3)$

21. Of the students at Grant Middle School, 63% are girls. The school newspaper is randomly selecting a student to be interviewed. Describe the complement of selecting a girl and find the probability of the complement. Write the answer as a fraction, decimal, and percent.

22. The table shows the number of rabbits and cats at a groomer. If a pet is selected at random to be groomed, find the probability that Patches the cat will be selected. Then find the probability that a cat will be selected.

Pets at the Groomer	
Cats	Rabbits
12	16

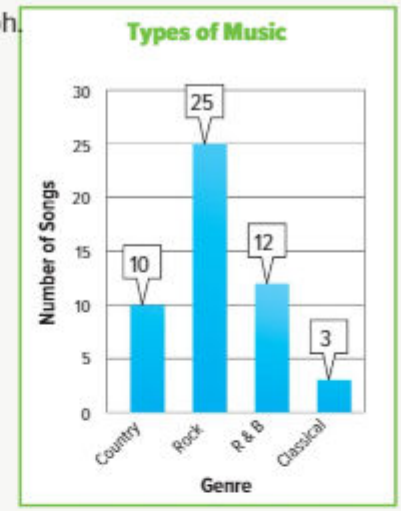
23. MP **Persevere with Problems** For a certain game, the probability of choosing a card with the number 13 is $\frac{8}{1{,}000}$. Find the probability of *not* choosing a card with the number 13. Then describe the likelihood of the event occurring.

Power Up! Test Practice

24. The types of songs on Ahmed's MP3 player are shown on the graph. Max will play one of the songs at random. Complete the model below to find *P*(country or R&B).

P(country or R&B) =

$$\frac{\square + \square}{\square + \square + \square + \square} = \square$$



25. Amani has a bowl containing the numbers of colored candies shown in the table. Which of the following probabilities are correct? Select all that apply.

Color	Number
Red	5
Orange	3
Yellow	1
Green	6

- ☐ $P(\text{red}) = \frac{1}{4}$
- ☐ $P(\text{orange}) = \frac{1}{5}$
- ☐ $P(\text{yellow}) = \frac{1}{10}$
- ☐ $P(\text{green}) = \frac{2}{5}$

Spiral Review

Compare each decimal using <, >, or =.

26. 0.2 ◯ 0.3

27. 0.75 ◯ 0.7

28. 5.89 ◯ 5.899

29. Yousif misses 12% of his foul shots and Ayoub misses 0.2 of his foul shots. Write 12% and 0.2 as fractions in simplest form. Then compare the fractions to determine who misses more foul shots.

Inquiry Lab
Relative Frequency

Mathematical Practices 1, 3

Inquiry **HOW is probability related to relative frequency?**

In a board game, you get an extra turn if you roll doubles or two of the same number.

You can conduct an experiment to find the relative frequency of rolling doubles using two number cubes. **Relative frequency** is the ratio of the number of experimental successes to the number of experimental attempts.

Hands-On Activity

Tools

Step 1 Complete the table to show all of the possible outcomes for rolling two number cubes. Shade all of the possible outcomes that are doubles.

(1, 1)	(2, 1)				
(1, 2)	(2, 2)				
(1, 3)	(2, 3)				
(1, 4)					
(1, 5)					
(1, 6)					

The probability of rolling doubles is _____.

How many times would you expect doubles to be rolled if you roll the number cubes 50 times? Explain. _____

Step 2 Roll two number cubes and record the number of doubles in the table. Repeat the experiment 50 times.

Number of Rolls	50
Number of Doubles	

Step 3 Find the relative frequency of rolling doubles. Use the ratio $\frac{\text{number of times doubles were rolled}}{\text{number of rolls}}$. _____

Compare the ratios in Steps 1 and 3. What do you notice? Explain.

Suppose the number cubes are rolled 100 times. Would you expect the results to be the same? Explain why or why not.

Investigate

Work with a partner.

1. Place a paperclip around the tip of a pencil. Then place the tip on the center of the spinner. Spin the paperclip 40 times. Record the results in the table below.

Section	A	B	C	D
Frequency				
Relative Frequency				

The spinner above is spun once. Find the probability of each event.

2. $P(A)$ ____

3. $P(B)$ ____

4. $P(C)$ ____

5. $P(D)$ ____

Analyze and Reflect

6. Based on your results from the spinner experiment, are the outcomes of A, B, C, or D equally likely? ____

7. **Reason Inductively** What would you expect to happen to the long-run relative frequency of spinning an A as you increase the number of spins from 40 to 1,000? ____

Create

8. **Justify Conclusions** If you rolled a number cube 600 times, approximate the relative frequency of rolling a 3 or 6. Explain your reasoning to a classmate. ____

9. Inquiry HOW is probability related to relative frequency?

Lesson 2

Theoretical and Experimental Probability

Real-World Link

Carnival Games The prize wheels for a carnival game are shown. You receive a less expensive prize if you spin and win on wheel A. You receive a more expensive prize if you spin and win on wheel B.

Wheel A

Wheel B

In a **uniform probability model**, each outcome has an equal probability of happening.

1. Which wheel has uniform probability? ________

2. Use a paperclip and the tip of your pencil to spin each wheel 4 times. Record your results.

Spin	Wheel A	Wheel B
1		
2		
3		
4		

3. Why do you think winners on wheel A receive a less expensive prize than winners on wheel B?

____

____

Which **MP Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Essential Question

HOW can you predict the outcome of future events?

Vocabulary

uniform probability model
theoretical probability
experimental probability

MP Mathematical Practices
1, 3, 4

Work Zone

Experimental and Theoretical Probability

Theoretical probability is based on uniform probability — what *should* happen when conducting a probability experiment. **Experimental probability** is based on relative frequency — what *actually* occurrs during such an experiment.

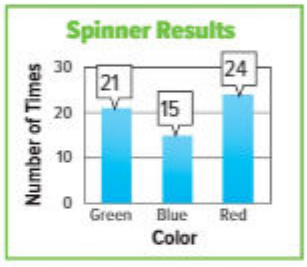
The theoretical probability and the experimental probability of an event may or may not be the same. As the number of attempts increases, the theoretical probability and the experimental probability should become closer in value.

Trials
A trial is one experiment in a series of successive experiments.

Examples

1. **The graph shows the results of an experiment in which a spinner with 3 equal sections is spun sixty times. Find the experimental probability of spinning red for this experiment.**

The graph indicates that the spinner landed on red 24 times, blue 15 times, and green 21 times.



$$P(\text{red}) = \frac{\text{number of times red occurs}}{\text{total number of spins}}$$
$$= \frac{24}{60} \text{ or } \frac{2}{5}$$

The experimental probability of spinning red is $\frac{2}{5}$.

2. **Compare the experimental probability you found in Example 1 to its theoretical probability.**

The spinner has three equal sections: red, blue, and green. So, the theoretical probability of spinning red is $\frac{1}{3}$. Since $\frac{2}{5} \approx \frac{1}{3}$, the experimental probability is close to the theoretical probability.

Show your work.

Got it? **Do these problems to find out.**

a. _____

b. _____

a. Refer to Example 1. If the spinner was spun 3 more times and landed on green each time, find the experimental probability of spinning green for this experiment.

b. Compare the experimental probability you found in Exercise **a** to its theoretical probability.

Examples

3. **Two number cubes are rolled together 20 times. A sum of 9 is rolled 8 times. What is the experimental probability of rolling a sum of 9?**

$P(9) =$ number of times a sum of 9 occurs ___ total number of rolls

$= \frac{8}{20}$ or $\frac{2}{5}$

The experimental probability of rolling a sum of 9 is $\frac{2}{5}$.

4. **Compare the experimental probability you found in Example 3 to its theoretical probability. If the probabilities are not close, explain a possible reason for the discrepancy.**

When rolling two number cubes, there are 36 possible outcomes.

The theoretical probability of rolling a sum of 9 is $\frac{4}{36}$ or $\frac{1}{9}$.

Rolls with Sum of 9	
First Cube	Second Cube
3	6
4	5
5	4
6	3

Since $\frac{1}{9}$ is not close to $\frac{2}{5}$, the experimental probability is *not* close to the theoretical probability. One possible explanation is that there were not enough trials.

Got it? **Do these problems to find out.**

Show your work.

c. _____

d. _____

e. _____

c. In Example 3, what is the experimental probability of rolling a sum that is *not* 9?

d. Two coins are tossed 10 times. Both coins land on picture 6 times. Compare the experimental probability to the theoretical probability. If the probabilities are not close, explain a possible reason for the discrepancy.

e. Suppose three coins are tossed 10 times. All three coins land on picture 1 time. Compare the experimental probability to the theoretical probability. If the probabilities are not close, explain a possible reason for the discrepancy.

Predict Future Events

Theoretical and experimental probability can be used to make predictions about future events.

Example

Solving Proportions
The cross products of any proportion are equal.
$\frac{29}{100} = \frac{x}{5{,}000}$

5. **Last year, a DVD store sold 670 action DVDs, 580 comedy DVDs, 450 drama DVDs, and 300 horror DVDs. A media buyer expects to sell 5,000 DVDs this year. Based on these results, how many comedy DVDs should she buy? Explain.**

2,000 DVDs were sold and 580 were comedy. So, the probability is $\frac{580}{2{,}000}$ or $\frac{29}{100}$.

$\frac{29}{100} = \frac{x}{5{,}000}$ Write a proportion.

$29 \cdot 5{,}000 = 100 \cdot x$ Find the cross products.

$145{,}000 = 100x$ Multiply.

$1{,}450 = x$ Divide each side by 100.

She should buy about 1,450 comedy DVDs.

Guided Practice

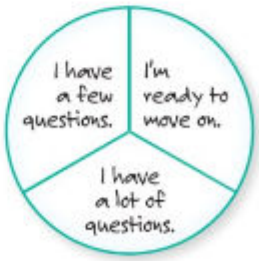
1. A coin is tossed 50 times, and it lands on picture 28 times. Find the experimental probability and the theoretical probability of the coin landing on picture. Then, compare the experimental and theoretical probabilities. (Examples 1–4)

2. Yesterday, 50 bakery customers bought muffins and 11 of those customers bought banana muffins. If 100 customers buy muffins tomorrow, how many would you expect to buy a banana muffin? (Example 5)

3. **Building on the Essential Question** How are experimental probability and theoretical probability alike?

Rate Yourself!

Are you ready to move on? Shade the section that applies.



FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

1 A number cube is rolled 20 times and lands on 1 two times and on 5 four times. Find each experimental probability. Then compare the experimental probability to the theoretical probability. (Examples 1–4)

a. landing on 5

b. *not* landing on 1

2. The spinner at the right is spun 12 times. It lands on blue 1 time. (Examples 1–4)

a. What is the experimental probability of the spinner landing on blue?

b. Compare the experimental and theoretical probabilities of the spinner landing on blue. If the probabilities are not close, explain a possible reason for the discrepancy.

3. The frequency table shows the results of a survey of 70 zoo visitors who were asked to name their favorite animal exhibit. (Example 5)

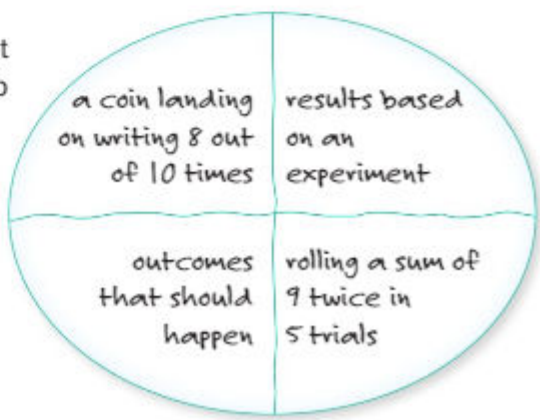
What is your Favorite Animal Exhibit?

Exhibit	Tally	Frequency
Bears	𝍸 |	6
Elephants	𝍸 𝍸 𝍸 ||	17
Monkeys	𝍸 𝍸 𝍸 𝍸 |	21
Penguins	𝍸 𝍸 |||	13
Snakes	𝍸 𝍸 |||	13

a. Suppose 540 people visit the zoo. Predict how many people will choose the monkey exhibit as their favorite. _____

b. Suppose 720 people visit the zoo. Predict how many people will choose the penguin exhibit as their favorite. _____

4. MP **Make a Conjecture** Cross out the part of the concept circle that does *not* belong. Then describe the relationship among the remaining parts.



5 MP **Multiple Representations** A spinner with three equal-sized sections marked A, B, and C is spun 100 times.

a. Numbers What is the theoretical probability of landing on A?

b. Numbers The results of the experiment are shown in the table. What is the experimental probability of landing on A? on C?

Section	Frequency
A	24
B	50
C	26

c. Models Make a drawing of what the spinner might look like based on its experimental probabilities. Explain.

H.O.T. Problems Higher Order Thinking

6. MP **Persevere with Problems** The experimental probability of a coin landing on picture is $\frac{7}{12}$. If the coin landed on writing 30 times, find the number of tosses.

7. MP **Reason Inductively** Twenty sharpened pencils are placed in a box containing an unknown number of unsharpened pencils. Suppose 15 pencils are removed at random and five of the removed pencils are sharpened. Based on this, is it reasonable to assume that the number of unsharpened pencils was 40? Explain your reasoning.

8. MP **Reason Inductively** The results of spinning a spinner with six equal sections are shown. Determine the minimum number of additional spins needed and their frequency of landing on each color so that the experimental probabilities will be equal to the theoretical probabilities. Explain your reasoning.

Color	Frequency
Blue	8
Green	6
Orange	12
Purple	10
Red	8
Yellow	4

Extra Practice

For Exercises 9 and 10, find each experimental probability. Then compare the experimental probability to its theoretical probability. If the probabilities are not close, explain a possible reason for the discrepancy.

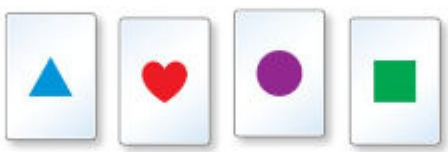
9. A coin is tossed 20 times. It lands on picture 9 times.

Homework Help

$P(\text{heads}) = \frac{\text{number of times picture occurs}}{\text{total number of coin tosses}} = \frac{9}{20}$

The experimental probability of $\frac{9}{20}$ is close to the theoretical probability of $\frac{1}{2}$.

10. A heart is randomly chosen 7 out of 12 times from the cards shown.



Solve.

11. Last month, customers at a gift shop bought 40 baby shower cards, 19 congratulations cards, 20 holiday cards, and 21 thank you cards. Suppose 125 customers buy greeting cards next month. How many would you expect to buy a baby shower card?

12. Use the graph at the right.

a. What is the probability that a mother received a gift of flowers or plants? Write the probability as a fraction in simplest form.

b. Suppose 400 mothers will receive a gift. Predict how many will receive flowers or plants.

Power Up! Test Practice

13. Omar tossed a coin 100 times. Fill in the boxes to complete each statement.

Based on Omar's results, the _____ probability of tossing picture is _____ %. This is _____ than the theoretical probability of tossing picture with a coin.

14. Determine if each situation represents experimental or theoretical probability.

a. Suha flips a coin 20 times and determines that the probability of flipping picture is 0.55. ☐ experimental ☐ theoretical

b. Muna has made 16 out of 25 free throws. The probability that she will make her next free throw is 64%. ☐ experimental ☐ theoretical

c. There are 4 1-fils coins, 2 5-fils coins, 5 10-fils coins, and 5 25-fils coins in a jar. The probability that a randomly selected coin is a 1-fils coin is $\frac{1}{4}$. ☐ experimental ☐ theoretical

Spiral Review

For Exercises 15 and 16, circle the greater probability.

15. The spinner at the right is spun.

P(red) *P*(not red)

16. A number cube is rolled.

P(multiple of 3) *P*(prime number)

17. A restaurant offers three flavors of ice cream on its dessert menu: vanilla, chocolate, and strawberry. Dessert options are sundaes or ice cream cones. List all of the possible desserts. Then determine if it is likely, unlikely, or equally likely of randomly choosing a sundae.

Inquiry Lab
Fair and Unfair Games

Inquiry **HOW can you determine if a game is fair?**

MP Mathematical Practices 1, 3

In a counter-toss game, players toss three two-color counters. The winner of each game is determined by how many counters land with either the red or yellow side facing up. Find out if this game is fair or unfair.

Mathematically speaking, a two-player game is **fair** if each player has an equal chance of winning. A game is **unfair** if there is not such a chance.

Hands-On Activity 1

Tools

Work in pairs to play the game described above.

Step 1 Player 1 tosses the counters. If 2 or 3 counters land red-side up, Player 1 wins. If 2 or 3 counters land yellow-side up, Player 2 wins. Record the results in the table below. Place a check in the winner's column for each game.

Game	Player 1	Player 2	Game	Player 1	Player 2
1			6		
2			7		
3			8		
4			9		
5			10		

Step 2 Player 2 then tosses the counters and the results are recorded.

Step 3 Continue alternating turns until the counters have been tossed 10 times.

Based on your results, do you think the game is fair or unfair? Circle your response below.

Fair Unfair

Investigate

Collaborate

Work with a partner.

1. Complete the organized list of all the possible outcomes resulting from one toss of the three counters described in Activity 1.

Counter 1	Counter 2	Counter 3	Outcome
red	red	red	red, red, red

2. In the outcome column of the table above, draw a circle around the outcomes that are a win for Player 1. Draw a box around the outcomes that are a win for Player 2.

3. Calculate the theoretical probability of each player winning. Write each probability as a fraction and as a percent. Is the game fair or unfair?

4. Use your results from Activity 1 to calculate the experimental probability of each player winning.

Analyze and Reflect

Collaborate

5. MP **Justify Conclusions** Compare the probabilities you found in Exercises 3 and 4. Explain any discrepancies.

6. MP **Reason Inductively** Predict the number of times Player 1 would win if the game were played 100 times. Explain your reasoning.

Ahmed and Salem made up a game using a plastic cup. A cup is tossed. If it lands right-side up or open-end down, Ahmed wins. If it lands on its side, Salem wins. Is this game fair?

Hands-On Activity 2

Work in pairs to play the game and determine if Ahmed and Salem created a fair game.

Step 1 Player 1 tosses the cup. If it lands right-side up or open-end down, Player 1 gets a point. If the cup lands on its side, Player 2 gets a point. Record your results in the table below.

Toss	Player 1	Player 2	Toss	Player 1	Player 2
1			6		
2			7		
3			8		
4			9		
5			10		

Step 2 Player 2 then tosses the cup and the results are recorded.

Step 3 Continue alternating turns until there is a total of 10 tosses.

Based on your results, do you think the game Ahmed and Salem created is fair or unfair? Circle your response below.

Fair Unfair

There are three possible outcomes when tossing the cup and Ahmed wins if two of those outcomes happen. It may appear that Ahmed has a better chance of winning, however this is not necessarily true.

Explain why Salem actually has a better chance at winning the game.

What was the experimental probability for the cup landing right-side up or open-end down?

Investigate

Work with a partner.

7. A game involves rolling two number cubes. Player 1 wins the game if the total of the numbers rolled is 5 or if a 5 is shown on one or both number cubes. Otherwise, Player 2 wins. Fill in the table for all of the possible outcomes of rolling two number cubes.

	1	2	3	4	5	6
1	1 + 1 = 2	1 + 2 = 3	1 + 3 = 4	1 + 4 = 5	1 + 5 = 6	1 + 6 = 7
2	2 + 1 = 3					
3						
4						
5						
6						

8. Shade in the cells of the table in which Player 1 is a winner.

Analyze and Reflect

9. For the number cube game, calculate the theoretical probability of each player winning. Write each probability as a fraction and as a percent.

10. MP **Justify Conclusions** Is the number cube game fair? Explain.

Create

11. MP **Model with Mathematics** Design and describe a game in which the outcome is not fair. Then explain how you could change the game to make it fair.

12. Inquiry HOW can you determine if a game is fair?

Lesson 3

Probability of Compound Events

Real-World Link

Travel Suhaila wants to pack enough items to create 6 different outfits. She packs 1 jacket, 3 shirts, and 2 pairs of jeans. Can Suhaila create 6 different outfits from her clothing items?

1. Complete the table below.

Outfit	Clothing Items
1	jacket, shirt 1, jeans 1
2	jacket, shirt 1, jeans 2
3	jacket, shirt 2, jeans 1
4	jacket, shirt 2,
5	jacket, shirt 3,
6	jacket,

2. The table is an example of an organized list. What is another way to show the different outfits that Suhaila can create?

3. Describe another situation for which you might want to list all of the possible outcomes.

Essential Question

HOW can you predict the outcome of future events?

Vocabulary

sample space
tree diagram
compound event

Mathematical Practices
1, 3, 4, 5

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Find a Sample Space

The set of all of the possible outcomes in a probability experiment is called the **sample space**. Organized lists, tables, and **tree diagrams** can be used to represent the sample space.

Examples

1. **The three students chosen to represent Mr. Fahd's class in a school assembly are shown. All three of them need to sit in a row on the stage. Use a list to find the sample space for the different ways they can sit in a row.**

Students
Amna
Hassan
Khalid

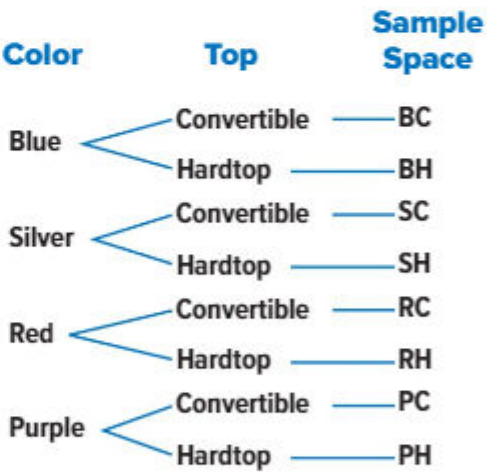
Use A for Amna, H for Hassan, and K for Khalid. Use each letter exactly once.

AHK AKH HAK HKA KAH KHA

So, the sample space consists of 6 outcomes.

2. **A car can be purchased in blue, silver, red, or purple. It also comes as a convertible or hardtop. Use a table or a tree diagram to find the sample space for the different styles in which the car can be purchased.**

Color	Top
blue	convertible
blue	hardtop
silver	convertible
silver	hardtop
red	convertible
red	hardtop
purple	convertible
purple	hardtop



Using either method, the sample space consists of 8 outcomes.

Show your work.

Got it? **Do this problem to find out.**

a. _____

a. The table shows the sandwich choices for a picnic. Find the sample space using a list, table, or tree diagram for a sandwich consisting one type of meat and one type of bread.

Meat	Bread
chicken turkey	rye sourdough white

Find Probability

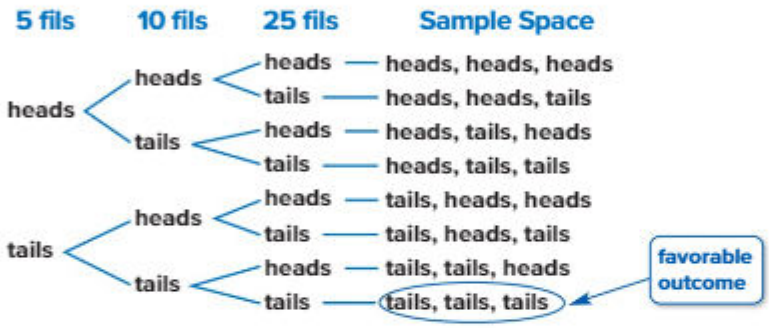
A **compound event** consists of two or more simple events. The probability of a compound event, just as with simple events, is the fraction of outcomes in the sample space for which the compound event occurs.

Example

Tools

3. **Suppose you toss a 25-fils coin, a 10-fils coin, and a 5-fils coin. Find the sample space. What is the probability of getting three tails?**

Make a tree diagram to show the sample space.



$P(3 \text{ tails}) = \frac{1}{8}$ ← number of favorable outcomes
← number of possible outcomes

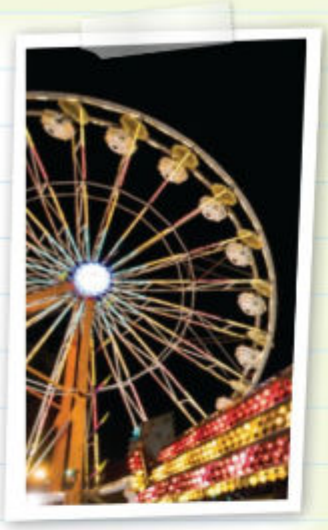
So, the probability of getting three Tails is $\frac{1}{8}$.

Got it? **Do this problem to find out.**

b. The animal shelter has both male and female cats in yellow, brown, or black. There is an equal number of each kind. What is the probability of choosing a female yellow cats? Show your work in the space below.

Random

When choosing an outcome, assume that each outcome is chosen randomly.



Example

4. **To win a carnival prize, you need to choose one of 3 doors labeled 1 through 3. Then you need to choose a red, yellow, or blue box behind each door. What is the probability that the prize is in the blue or yellow box behind door 2?**

The table shows that there are 9 total outcomes. Two of the outcomes are favorable.

So, the probability that the prize is in a blue or yellow box behind door 2 is $\frac{2}{9}$.

Outcomes	
door 1	red box
door 1	yellow box
door 1	blue box
door 2	red box
door 2	yellow box
door 2	blue box
door 3	red box
door 3	yellow box
door 3	blue box

Guided Practice

For each situation, find the sample space. (Examples 1–2)

1. A coin is tossed twice.

2. A pair of brown or black sandals are available in sizes 7, 8, or 9.

3. Khalaf spins a spinner with four equal sections, labeled A, B, C, and D, twice. If letter A is spun at least once, Khalaf wins. Otherwise, Khalifa wins. Use a list to find the sample space. Then find the probability that Khalifa wins. (Examples 3–4)

4. **Bulding on the Essential Question** How do tree diagrams, tables, and lists help you find the probability of a compound event?

Rate Yourself!

- ☐ I understand how to show a sample space.

Great! You're ready to move on!

- ☐ I still have questions about showing a sample space.

Name _____ My Homework _____

Independent Practice

For each situation, find the sample space. (Examples 1–2)

1. tossing a coin and spinning the spinner at the right

2. picking a number from 1 to 5 and choosing the color red, white, or blue

3 choosing a purple, green, black, or silver bike having 10, 18, 21, or 24 speeds

4. choosing a letter from the word SPACE and choosing a consonant from the word MATH

For each game, find the sample space. Then find the indicated probability. (Examples 3–4)

5. Hessa tosses 2 number cubes. She wins if she rolls double sixes.

Find *P*(Hessa wins). _____

6. Jamal rolls a number cube, tosses a coin, and chooses a card from two cards marked A and B. If an even number and heads appears, Jamal wins, no matter which card is chosen. Otherwise Ismail wins.

Find *P*(Jamal wins). _____

7 MP **Persevere with Problems** The following is a game for two players.

- Three counters are labeled according to the table at the right.
- Toss the three counters.
- If exactly 2 counters match, Player 1 scores a point. Otherwise, Player 2 scores a point.

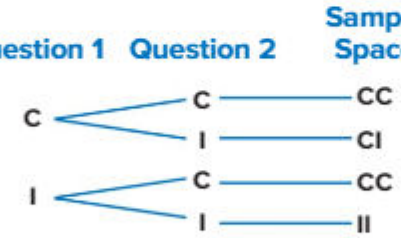
Counters	Side 1	Side 2
Counter 1	red	blue
Counter 2	red	yellow
Counter 3	blue	yellow

Find the probability that each player scores a point.

H.O.T. Problems Higher Order Thinking

8. MP **Persevere with Problems** Refer to Exercise 7. Do the two players both have an equal chance of winning? Explain.

9. MP **Find the Error** Asma wants to determine the probability of guessing correctly on two true-false questions on her history test. She draws the tree diagram below using C for correct and I for incorrect. Find her mistake and correct it.

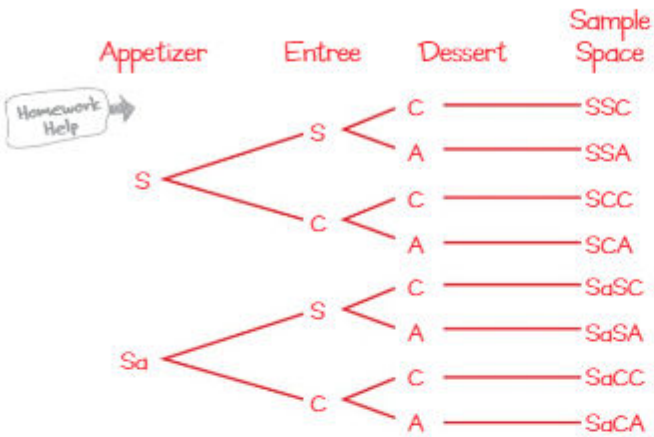


10. MP **Model with Mathematics** Write a real-world problem in which the probability of a compound event occurring is 0.25.

Extra Practice

11. Three-course dinners can be made from the menu shown. Find the sample space for a dinner consisting of an appetizer, entrée, and dessert.

Appetizers	Entrees	Desserts
Soup	Steak	Carrot cake
Salad	Chicken	Apple pie



12. Mr. and Mrs. Faris are expecting triplets. Suppose the chance of each child being a boy is 50% and of being a girl is 50%. Find the probability of each event.

a. *P*(all three children will be boys) _____

b. *P*(at least one boy and one girl) _____

c. *P*(two boys and one girl) _____

d. *P*(at least two girls) _____

Copy and Solve **For Exercises 13 and 14, show your work on a separate piece of paper.**

13. The University of Oregon's football team has many different uniforms. The coach can choose from four colors of jerseys and pants: green, yellow, white, and black. There are three helmet options: green, white, and yellow. Also, there are the same four colors of socks and two colors of shoes, black and yellow.

a. How many jersey/pant combinations are there?

b. If the coach picks a jersey/pant combination at random, what is the probability he will pick a yellow jersey with green pants?

c. Use a tree diagram to find all of the possible shoe and sock combinations.

14. MP **Use Math Tools** Use the Internet or another source to find the top five best-selling animated movies. Then create a list of the possibilities for choosing a movie and choosing a wide-screen or full-screen version.

Power Up! Test Practice

15. Mr. Usama will choose one student from each of the two groups to present their history report to the class. Which of the following represent possible outcomes? Select all that apply.

Group 1	Group 2
Ayman	Husam
Jassim	Hamad
Khalid	

- ☐ (Ayman, Husam)
- ☐ (Khalid, Hamad)
- ☐ (Jassim, Khalid)
- ☐ (Jassim, Husam)

16. Campers choose one activity from each of the morning, afternoon, and evening activities shown below.

Morning	Afternoon	Evening
Hiking (H)	Archery (A)	Horseback Riding (R)
Canoeing (C)	Bird Watching (B)	Campfire Building (F)
		Navigating (N)

Make a list to show the sample space for the possible morning, afternoon, and evening activities.

What is the probability that a randomly selected camper will be horseback riding in the evening?

Spiral Review

Eight cards numbered 1–8 are shuffled together. A card is drawn at random. Find the probability of each event.

17. $P(8)$ _____

18. $P(\text{greater than } 5)$ _____

19. $P(\text{even})$ _____

20. $P(3 \text{ or } 7)$ _____

21. What is the probability of rolling a number greater than 4 on a number cube? Explain.

_____

_____

Lesson 4

Simulations

Real-World Link

Music Downloads A new electronics store is opening at the mall. One out of six new customers will receive a free music download. The winners are chosen at random. On Monday, the store had 50 customers. You can act out or *simulate* 50 random customers by using the random number generator on a graphing calculator.

Type in the following keystrokes to set 1 as the lower bound and 6 as the upper bound for 50 trials.

Keystrokes: MATH ◀ 5 1 , 6 , 50) ENTER

The screen should look similar to the screen shown below.

A set of 50 numbers ranging from 1 to 6 appears. Use the right arrow key to see the next number in the set.

1. Let the number 3 represent a customer who wins a free download. Write the experimental probability of winning a download.

2. Compare the experimental probabilities found in Exercise 1 to the theoretical probability of winning a download.

Essential Question

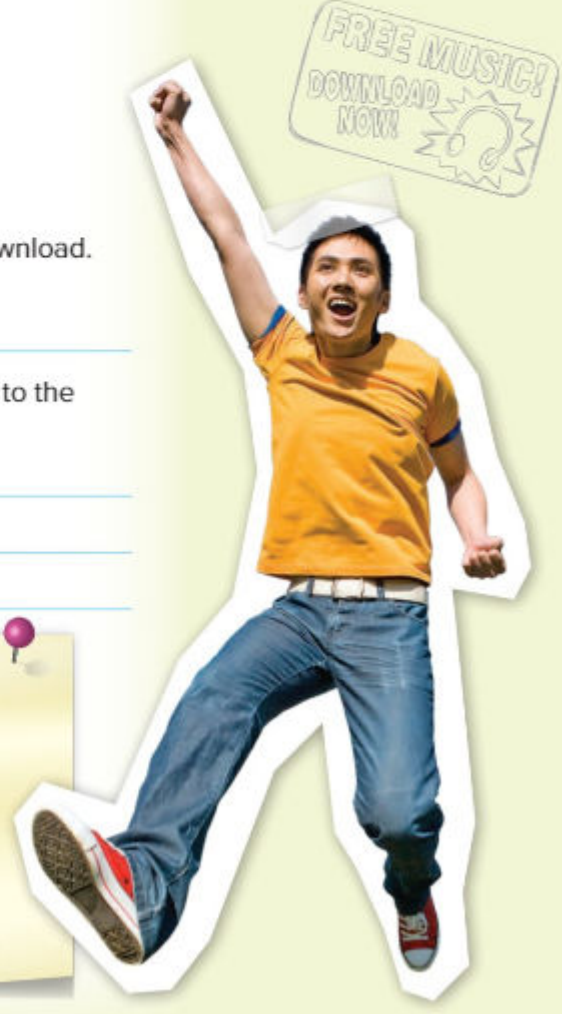
HOW can you predict the outcome of future events?

Vocabulary

simulation

Mathematical Practices

1, 3, 4



Lane Oatey/Blue Jean Images RF/Getty Images

Which Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Work Zone

Model Equally Likely Outcomes

A **simulation** is an experiment that is designed to model the action in a given situation. For example, you used a random number generator to simulate rolling a number cube. Simulations often use models to act out an event that would be impractical to perform.

Example

1. **A cereal company is placing one of eight different trading cards in its boxes of cereal. If each card is equally likely to appear in a box of cereal, describe a model that could be used to simulate the cards you would find in 15 boxes of cereal.**

Choose a method that has 8 possible outcomes, such as tossing 3 coins. Let each outcome represent a different card.

For example, the outcome of all three coins landing heads up could simulate finding card 1.

Toss 3 coins to simulate the cards that might be in 15 boxes of cereal. Repeat 15 times.

Coin Toss Simulation			
Outcome	Card	Outcome	Card
HHH	1	TTT	5
HHT	2	TTH	6
HTH	3	THT	7
HTT	4	THH	8

Show your work.

Got it? **Do this problem to find out.**

a. A restaurant is giving away 1 of 5 different toys with its children's meals. If the toys are given out randomly, describe a model that could be used to simulate which toys would be given with 6 children's meals.

a. _____

Example

2. **Every student who volunteers at the concession stand during basketball games will receive a free school T-shirt. The T-shirts come in 3 different designs.**

Design a simulation that could be used to model this situation. Use your simulation to find how many times a student must volunteer in order to get all 3 T-shirts.

Use a spinner divided into 3 equal sections. Assign each section one of the T-shirts. Spin the spinner until you land on each section.

first spin

second spin

third spin

fourth spin

Based on this simulation, a student should volunteer 4 times in order to get all 3 T-shirts.

Got it? **Do this problem to find out.**

Show your work.

b. Mr. Hamdan must wear a dress shirt and a tie to work. Each day he picks one of his 6 ties at random. Design a simulation that could be used to model this situation. Use your simulation to find how many days Mr. Hamdan must work in order to wear all of his ties.

b. _____

Model Unequally Likely Outcomes

Simulations can also be used to model events in which the outcomes are not equally likely.

STOP and Reflect

How could you simulate a 20% chance? Write your answer below.

Example

3. **There is a 60% chance of rain for each of the next two days. Describe a method you could use to find the experimental probability of having rain on both of the next two days.**

Place 3 red and 2 blue marbles in a bag. Let 60% or $\frac{3}{5}$ of them represent rain. Let 40% or $\frac{2}{5}$ of them represent no rain.

Randomly pick one marble to simulate the first day. Replace the marble and pick again to simulate the second day. Find the probability of rain on both days.

Show your work.

Got it? **Do this problem to find out.**

c. During the regular season, Bilal made 80% of his free throws. Describe an experiment to find the experimental probability of Bilal making his next two free throws.

c. _____

Guided Practice

Check

1. An ice cream store offers waffle cones or sugar cones. Each is equally likely to be chosen. Describe a model that could be used to simulate this situation. Based on your simulation, how many people must order an ice cream cone in order to sell all possible combinations? (Examples 1 and 2)

2. An electronics store has determined that 45% of its customers buy a wide-screen television. Describe a model that you could use to find the experimental probability that the next three television-buying customers will buy a wide-screen television. (Example 3)

3. **Building on the Essential Question** Explain how using a simulation is related to experimental probability.

Rate Yourself!

How well do you understand simulations? Circle the image that applies.

Clear | Somewhat Clear | Not So Clear

Name _____ My Homework _____

Independent Practice

1. The questions on a multiple-choice test each have 4 answer choices. Describe a **model that you could use to simulate the** outcome of guessing the correct answers to a 50-question test. (Example 1)

2. A game requires drawing balls numbered 0 through 9 for each of four digits to determine the winning number. Describe a model that could be used to simulate the selection of the number. (Example 1)

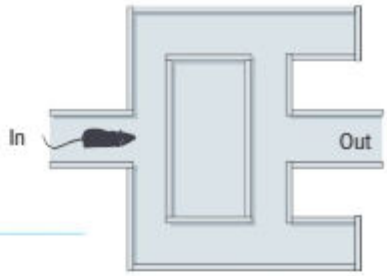
MP Model with Mathematics Describe a model you could use to simulate each event.

3. A jar of cookies contains 18 different types of cookies. Each type is equally likely to be chosen. Based on your simulation, how many times **must a cookie be chosen in order to get each type**? (Example 2)

4. A cooler contains 5 bottles of lemonade, 4 bottles of water, and 3 bottles of juice. Each type is equally likely to be chosen. Based on your simulation, how many times must a drink be chosen in order to get each type? (Example 3)

5. Players at a carnival game win about 30% of the time. Based on your simulation, what is the experimental probability that the next four players will win. (Example 3)

6. **Model with Mathematics** Suppose a mouse is placed in the maze at the right. If each decision about direction is made at random, create a simulation to determine the probability that the mouse will find its way out before coming to a dead end or going out the In opening.



H.O.T. Problems Higher Order Thinking

7. **Model with Mathematics** Describe a situation that could be represented by a simulation. What objects could be used in your simulation?

8. **Persevere with Problems** A simulation uses cards numbered 0 through 9 to generate five 2-digit numbers. A card is selected for the tens digit and not replaced. Then a card for the ones digit is drawn and not replaced. The process is repeated until all the cards are used. If the simulation is performed 10 times, about how many times could you expect a 2-digit number to begin with a 5? Explain.

9. **Justify Conclusions** Determine whether the following statement is *sometimes*, *always*, or *never* true. Justify your answer.

A spinner can be used to model equally likely outcomes.

10. **Justify Conclusions** Humaid believes that the coin his teacher uses for an experiment gives an advantage to one team of students. His teacher has students toss the coin 50 times each and record their results. Based on the results in the table, do you think the coin is fair? Explain.

Student	heads	Tails
1	17	33
2	22	28
3	28	22
4	21	29
5	13	37
6	20	30

Name _____ My Homework _____

Extra Practice

11. A store employee randomly gives scratch-off discount cards to the first 50 customers. The cards offer discounts of 10%, 20%, 25%, 30%, or 40%. There is an equal chance of receiving any of the 5 cards. Describe a model that could be used to simulate the discount received by 4 customers.

Homework Help

Use a spinner with 5 equal sections to represent the 5 different discounts. Spin 4 times to simulate 4 customers receiving cards.

12. On average, 75% of the days in Henderson county are sunny, with little or no cloud cover. Describe a model that you could use to find the experimental probability of sunny days each day for a week in Henderson county.

MP **Model with Mathematics** **Describe a model you could use to simulate each event.**

13. Every student who participated in field day activities received a water bottle. The water bottles came in 2 different colors. Based on your simulation, how many students had to receive a water bottle in order to distribute water bottles in both colors?

14. A field hockey team wins 80% of its games. Based on your simulation, what is the experimental probability of the team winning its next 3 games?

15. There are 4 different magazines on Rana's nightstand. Each evening, she randomly selects one magazine to read. Based on your simulation, how many days must she select a magazine in order to read all 4 magazines?

Power Up! Test Practice

16. The table shows the chance of rain this weekend. Select values to fill in the boxes in the model below to describe a method you could use to find the experimental probability of having rain on both days.

Day	Saturday	Sunday
Chance of Rain	30%	30%

2	4	6	7	red
10	30	60	70	blue

Place 3 red and ☐ blue marbles in a bag. Let the ☐ marbles represent rain since ☐ % of the marbles are this color. Let the ☐ marbles represent no rain since ☐ % of the marbles are this color.

Randomly pick one marble to simulate the first day. Replace the marble and pick again to simulate the second day. Find the experimental probability of rain on both days. Do you think it matters how many trials of the simulation you conduct? Will conducting more trials result in a better prediction? Explain your reasoning.

17. At a restaurant, 1 out of every 6 kids' meals wins a prize. Determine which probability models could be used to simulate winning a prize. Select all that apply.

- ☐ Toss a coin. Let tossing heads represent winning a prize and let tossing tails represent not winning a prize.
- ☐ Spin a spinner with equal size spaces labeled A, B, C, D, E, and F. Let spinning A represent winning a prize and let spinning other letters represent not winning a prize.
- ☐ Roll a number cube. Let rolling a 1 represent winning a prize and let rolling a 2, 3, 4, 5, or 6 represent not winning a prize.

Spiral Review

18. A local video store has advertised that one out of every four customers will receive a free box of popcorn with their video rental. So far, 15 out of 75 customers have received popcorn. Compare the experimental and theoretical probabilities of receiving popcorn.

Inquiry Lab

Simulate Compound Events

Inquiry **HOW do simulations help you understand the probability of events happening?**

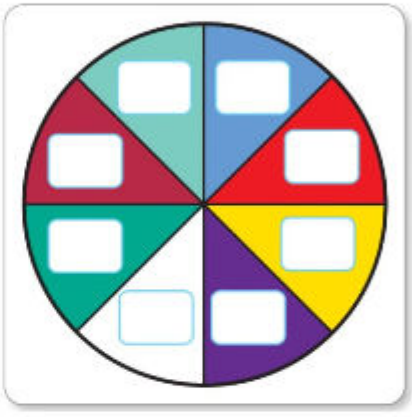
MP Mathematical Practices 1, 3

A local shop randomly gives coupons to 3 out of every 8 customers. Use a spinner to determine the probability that a customer will receive a coupon two days in a row.

Hands-On Activity 1

Tools

Step 1 A spinner with eight equal sections can be used to simulate the situation. Label three of the sections with the letter C to represent the people that receive a coupon. Label five of the sections with the letter D to represent the people that do not receive a coupon.



Step 2 Every two spins of the spinner represents one trial. Use a paperclip and the tip of your pencil to spin the spinner twice and record the results in the table. Perform a total of 15 trials.

Trial	Spin 1	Spin 2	Trial	Spin 1	Spin 2	Trial	Spin 1	Spin 2
1			6			11		
2			7			12		
3			8			13		
4			9			14		
5			10			15		

Based on your results, what is the experimental probability that a customer will receive a coupon two days in a row?

You can also use a random number table to simulate a compound event.

There is a 10% chance of rain for a city on Sunday and a 20% chance of rain on Monday. Use a random number table to find the probability that it will rain on both days.

Hands-On Activity 2

Step 1 A random number table has random digits in rows that can be grouped in different combinations as needed. These digits are arranged in groups of 5, but the grouping often does not matter. Since the situation we want to represent involves two days, continue drawing lines to separate the numbers into two-digit numbers.

48587	49460	89640	30270
19507	87835	99812	52353
11364	35645	90087	64254
87045	39769	77995	14316
69913	93449	68497	31270
81827	32901	82033	43714
33386	99637	25725	31900
41575	86692	40882	44123
77351	12790	62795	77307

Step 2 Using the digits 0 through 9, assign one digit in the tens place for rain on Sunday and assign two different digits in the ones place for rain on Monday. For example, the digit 1 in the tens place can represent rain occurring on Sunday and the digits 1 and 2 in the ones place can represent rain occurring on Monday.

Step 3 Find the numbers in the table that have a 1 in the tens place and either a 1 or 2 in the ones place. Those numbers are 11 and 12. Circle those numbers in the table.

Step 4 Find the probability using the numbers found in Step 3.

There were ☐ instances of the random numbers 11 and 12 occurring out of 90 random numbers.

So, the probability that it will rain on both days is $\frac{\square}{90}$ or $3\frac{1}{3}\%$.

Investigate

Collaborate

Work with a partner.

1. Ibrahim plays goalie on his soccer team. He usually stops 2 out of every 6 penalty kicks. Label the sections of the spinner at the right. Then use the spinner to determine the experimental probability that Ibrahim stops 2 penalty kicks in a row.

Trial	Spin 1	Spin 2	Trial	Spin 1	Spin 2	Trial	Spin 1	Spin 2
1			6			11		
2			7			12		
3			8			13		
4			9			14		
5			10			15		

The experimental probability is _____.

2. Suppose 40% of customers who enter a pet shelter own a cat. What is the probability that it will take at least 4 customers before a cat owner enters the shelter? Use a random number table to simulate this compound event.

In the table below, separate the numbers into groups of 4. Then use the digits 0, 1, 2, and 3 to represent people who own cats. You are looking for groups of 4 numbers that do *not* contain a 0, 1, 2, or 3. Circle those groups.

1877 1	47374	36541	83454
97907	40978	34947	78482
2607 1	12644	94567	35467
02459	78467	06 161	85897
44480	71716	13 166	44096
72769	18974	24 186	50866
35842	78478	45468	1 544 1
58438	37487	16 1 87	89892
8371 1	54631	19846	08483

In this case, the probability is $\frac{\square}{45}$ or 15.6%.

So, the experimental probability that it takes at least 4 customers before a cat owner enters the store is 15.6%.

Analyze and Reflect

Collaborate

3. In Exercise 1, what does spinning a Stop on your first spin, and spinning a Goal on your second spin represent in this situation?

4. **Justify Conclusions** Explain how your results might change for Exercise 1 if you simulated 100 penalty kicks.

5. In Exercise 2, why were the numbers from the random number table separated into groups of four?

6. In Exercise 2, you could have used any 4 numbers to represent cat owners. Complete the simulation four more times using the numbers in the table to represent the cat owners.

Numbers that Represent Cat Owners	Experimental Probability
4, 5, 6, 7	$\frac{\square}{45}$
0, 1, 8, 9	$\frac{\square}{45}$
3, 4, 5, 6	$\frac{\square}{45}$

Create

On Your Own

7. **Model with Mathematics** Design a simulation that could be used to predict the probability of taking a four question multiple-choice test with four answer choices and getting all four questions correct by guessing. Conduct 50 trials of the experiment. Then calculate the experimental probability of getting all four questions correct by guessing.

8. Inquiry HOW do simulations help you understand the probability of events happening?

MP Problem-Solving Investigation
Act It Out

MP Mathematical Practices 1, 3, 4

Case #1 Winning Serves

Sally has been practicing her volleyball serve every day after school. She hits a good serve an average of 3 out of 4 times.

What is the probability that Sally will hit two good serves in a row?

1 Understand *What are the facts?*

You know that Sally hits a good serve an average of 3 out of 4 times. Act it out with a spinner.

2 Plan *What is your strategy?*

Spin a spinner, numbered 1 to 4, two times. If the spinner lands on 1, 2, or 3, she hits a good serve. If the spinner lands on 4, she doesn't. Repeat the experiment 10 times.

3 Solve *How can you apply the strategy?*

Here are some possible results. Circle the columns that show two good serves. The first two are done for you.

Trials	1	2	3	4	5	6	7	8	9	10
First Spin	4	1	4	3	1	2	2	1	3	2
Second Spin	2	3	3	2	1	4	1	4	3	3

The circled columns show that six out of 10 trials resulted in two good serves in a row. So, the probability is _____%.

4 Check *Does the answer make sense?*

Repeat the experiment several times to see whether the results agree.

Analyze the Strategy

MP **Reason Inductively** Describe an advantage of using the *act it out* strategy?

_____

_____

Case #2 Tests

Tools

Rashid uses a spinner with four equal sections to answer a five-question multiple-choice quiz. Each question has choices A, B, C, and D.

Is this a good way to answer the quiz questions?

1 Understand

- **Read the problem. What are you being asked to find?**

I need to find _____

_____.

- **What information do you know?**

The spinner has 4 equal parts. There are 5 multiple-choice questions.

The answer choices are A, B, C, and D.

2 Plan

- **Choose a problem-solving strategy.**

I will use the _____ strategy.

3 Solve

Use your problem-solving strategy to solve the problem.
Spin a spinner with four equal parts labeled A, B, C, and D five times. Repeat the experiment two times. Make a table of the results.

Question	1	2	3	4	5
Trial 1					
Trial 2					

With each spin there is an equal chance of landing on any section. Since the probability of an answer being A, B, C, or D is _____ likely, any answer choice is possible.

Is using a spinner to answer a multiple-choice question a good idea? _____

4 Check

Use information from the problem to check your answer.
Repeat the experiment several times to see if the results agree.

Collaborate

Work with a small group to solve the following cases.
Show your work on a separate piece of paper.

Case #3 Chess

A chess tournament will be held and 32 students will participate. If a player loses one match, he or she will be eliminated.

How many total games will be played in the tournament?

Case #4 Running

Six runners are entered in a race. Assume there are no ties.

In how many ways can first and second places be awarded?

Case #5 Fair Games

Alia and Bilal are playing a game with number cubes. Each number cube is numbered 1 to 6. They roll both number cubes. If the product is a multiple of 3, Bilal wins. If the product is a multiple of 4, Alia wins.

Is this game fair or unfair? Justify your response.

Use any strategy!

Case #6 Algebra

The figure shown at the right is known as Pascal's Triangle. Make a conjecture for the numbers in the 6th and 7th rows.

Mid-Chapter Check

Vocabulary Check

1. Define *probability*. Give an example of the probability of a simple event. (Lesson 1)

2. Fill in the blank in the sentence below with the correct term. (Lesson 4)

   A(n) _____ is an experiment that is designed to act out a given situation.

Skills Check and Problem Solving

The table shows the number of science fiction, action, and comedy movies Bilal has in his collection. Suppose one movie is selected at random. Find each probability. Write as a fraction in simplest form. (Lesson 1)

Type of Movie	
Science Fiction	10
Action	7
Comedy	3

3. *P*(science fiction) _____

4. *P*(*not* action) _____

5. A coin is tossed 20 times. It lands picture 4 times. Compare the experimental probability to its theoretical probability. If the probabilities are not close, explain a possible reason for the discrepancy. (Lesson 2)

6. A weather forecaster predicts a 30% chance of rain for each of the next three days. Describe a way to simulate the chance that it will rain the next three days. (Lesson 4)

7. **Persevere with Problems** Without looking, Ghaya took a handful of multi-colored candies from a bag and found that 20% of the candies were yellow and 15% were green. Suppose there were 480 candies in the bag. Based on Ghaya's results, how many more yellow candies would you expect there to be than green candies? (Lesson 1)

Lesson 5

Fundamental Counting Principle

Real-World Link

Classes Khamis wants to take a class at the community center. The table shows the class options he is considering. All of the classes are offered only on Monday and Tuesday.

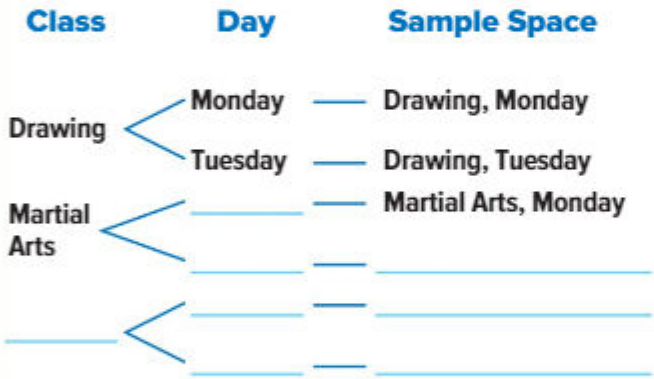
Class	Day
Drawing Martial Arts Dance	Monday Tuesday

1. According to the table, how many classes is he considering? _____

2. How many days are the classes offered?

_____

3. Complete the tree diagram to find the number of different class and day outcomes.



4. Find the product of the two numbers you found in Exercises 1 and 2. How does the number of outcomes compare to the product?

_____

Essential Question

HOW can you predict the outcome of future events?

Vocabulary

Fundamental Counting Principle

Mathematical Practices

1, 3, 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Key Concept

Fundamental Counting Principle

If event M has m possible outcomes and event N has n possible outcomes, then event M followed by event N has $m \times n$ possible outcomes.

Work Zone

You can use multiplication instead of making a tree diagram to find the number of possible outcomes in a sample space. This is called the **Fundamental Counting Principle**.

Example

1. **Find the total number of outcomes when a coin is tossed and a number cube is rolled.**

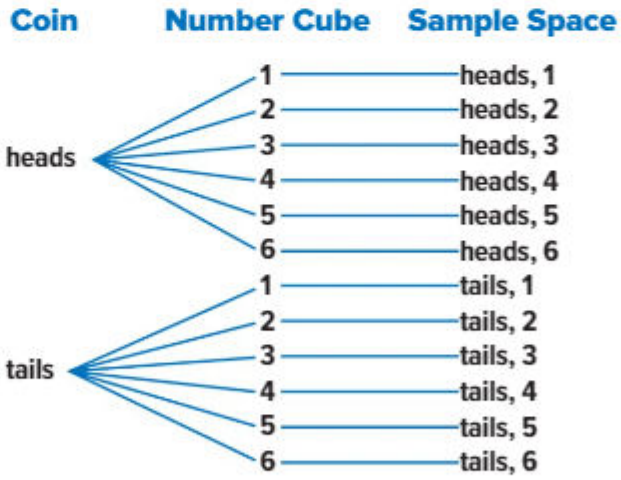
A coin has 2 possible outcomes. A number cube has 6 possible outcomes. Multiply the possible outcomes of each event.

coin · number cube = total

$2 \cdot 6 = 12$ Fundamental Counting Principle

There are 12 different outcomes.

Check Draw a tree diagram to show the sample space.



The tree diagram also shows that there are 12 outcomes. ✓

Got it? **Do this problem to find out.**

Show your work.

a. Find the total number of outcomes when choosing from bike helmets that come in three colors and two styles.

a. _____

Find Probability

You can use the Fundamental Counting Principle to help find the probability of events.

Examples

2. **Find the total number of outcomes from rolling a number cube with sides labeled 1–6 and choosing a letter from the word NUMBERS. Then find the probability of rolling a 6 and choosing an M.**

number cube · letters = total

$6 \cdot 7 = 42$ Fundamental Counting Principle

There are 42 different outcomes.

There is only one favorable outcome. So, the probability of rolling a 6 and choosing an M is $\frac{1}{42}$ or about 2%.

3. **Find the number of different jeans available at The Jeans Shop. Then find the probability of randomly selecting a size 32 × 34 slim fit. Is it likely or unlikely that the jeans would be chosen?**

The Jeans Shop

Waist Size	Length (in.)	Style
30	30	slim fit
32	32	bootcut
34	34	loose fit
36		
38		

size · length · style = total

$5 \cdot 3 \cdot 3 = 45$ Fundamental Counting Principle

There are 45 different types of jeans to choose. Out of the 45 possible outcomes, only one is favorable. So, the probability of randomly selecting a 32 × 34 slim fit is $\frac{1}{45}$ or about 2%.

It is very unlikely that the size would be chosen at random.

Jean Size

In men's jeans, the size is labeled waist x length. So, a 32 x 34 is a 32-inch waist with a 34-inch length.

Got it? **Do this problem to find out.**

b. Two number cubes are rolled. What is the probability that the sum of the numbers on the cubes is 12? How likely is it that the sum would be 12?

Show your work.

b. _____

Example

4. **A box of toy cars contains blue, orange, yellow, red, and black cars. A separate box contains a male and a female action figure. What is the probability of randomly choosing an orange car and a female action figure? Is it likely or unlikely that this combination is chosen?**

First, find the number of possible outcomes.

There are 5 choices for the car and 2 choices for the action figure.

$5 \cdot 2 = 10$ Fundamental Counting Principal

There are 10 possible outcomes. There is one way to choose an orange car and a female action figure. It is very unlikely that this combination is chosen at random.

$P(\text{orange car, female action figure}) = \frac{1}{10}$ or 10%.

Guided Practice

1. Use the Fundamental Counting Principle to find the number of outcomes from tossing a 25-fils coin, a 10-fils coin, and a 5-fils coin. (Example 1)

2. How many outcomes are possible when rolling a number cube and picking a cube from 4 different colored cubes? (Example 1)

3. Find the number of different outfits that can be made from 3 sweaters, 4 blouses, and 6 skirts. Then find the probability of randomly selecting a particular sweater-blouse-skirt outfit. Is the probability of this event likely or unlikely? (Examples 2–4)

4. **Building on the Essential Question** Compare and contrast tree diagrams and the Fundamental Counting Principle.

Rate Yourself!

How confident are you about using the Fundamental Counting Principle? Shade the ring on the target.

Name _____ My Homework _____

Independent Practice

Use the Fundamental Counting Principle to find the total number of outcomes for each situation. (Example 1)

1. choosing a bagel with one type of cream cheese from the list shown in the table

Bagels	Cream Cheese
Plain Blueberry Cinnamon raisin Garlic	Plain Chive Sun-dried tomato

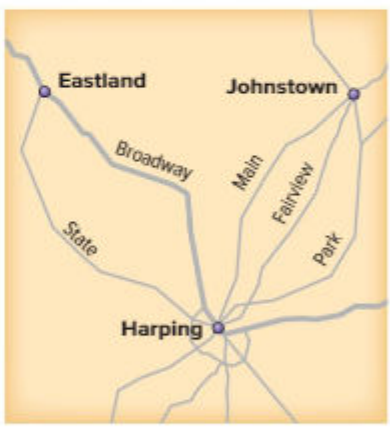
2. choosing a sandwich and a side from the list shown in the table

Sandwiches	Sides
chicken Turkey Roast Beef Tuna Salad Vegetarian	Pasta Salad Fruit Cup Potato Chips Side Salad

3. picking a month of the year and a day of the week _____

4. choosing from a comedy, horror, or action movie each shown in four different theaters

5. Find the number of possible routes from Eastland to Johnstown that pass through Harping. Then find the probability that State and Fairview will be used if a route is selected at random. State the probability as a fraction and percent. (Examples 2–3)



6. Find the number of possible choices for a 2-digit number that is greater than 19. Then find the number of possible choices for a 4-digit Personal Identification Number (PIN) if the digits cannot be repeated. (Example 1)

7. An electronics company makes educational apps for 5 subjects, including math. The app has 10 versions, with a different avatar in each version. One version has an avatar that looks similar to a lion. The company is randomly giving free apps to its customers. Find the probability of randomly receiving a math app with a lion avatar. How likely is the probability of receiving this app at random? (Examples 2–4)

8. A sandwich shop offers 4 different meats and 2 different cheeses. Suppose the sandwich shop offers 24 different meat-cheese sandwiches. How many different breads does the sandwich shop use?

9 MP **Justify Conclusions** A store offers 32 different T-shirt designs and 11 choices of color. Is the store's advertisement true? Explain.

H.O.T. Problems Higher Order Thinking

10. MP **Persevere with Problems** Determine the number of possible outcomes when tossing one coin, two coins, and three coins. Then determine the number of possible outcomes for tossing *n* coins. Describe the strategy you used.

11. MP **Which One Doesn't Belong?** Identify the choices for events *M* and *N* that do not result in the same number of outcomes as the other two. Explain your reasoning.

9 drinks, 8 desserts	18 shirts, 4 pants	10 groups, 8 activities

12. MP **Justify Conclusions** Zayed has a choice of a white, gray, or black shirt to wear with a choice of tan, black, brown, or denim pants. Without calculating the number of possible outcomes, how many more outfits can he make if he buys a green shirt? Explain your reasoning to a classmate.

13. MP **Persevere with Problems** Write an algebraic expression to find the number of outcomes if a number cube is rolled *x* times.

Name _____ My Homework _____

Extra Practice

Use the Fundamental Counting Principle to find the total number of outcomes for each situation.

14. Rolling a number cube and spinning a spinner with eight equal sections 48

$6 \cdot 8 = 48$

Homework Help

15. Tossing a coin and selecting one letter from the word MATH _____

16. Selecting one sweatshirt from a choice of five sweatshirts and one pair of pants from a choice of four pairs of pants _____

17. Selecting one entrée from a choice of nine entrées and one dessert from a choice of three desserts _____

18. Rolling a number cube and tossing two coins _____

19. Choosing tea in regular, raspberry, lemon, or peach; sweetened or unsweetened; and in a glass or bottle _____

20. A cafeteria offers oranges, apples, or bananas as its fruit option. It offers peas, green beans, or carrots as the vegetable option. Find the number of fruit and vegetable options. If the fruit and the vegetable are chosen at random, what is the probability of getting an orange and carrots? Is it likely or unlikely that a customer would get an orange and carrots?

_____

21. MP **Justify Conclusions** The table shows cell phone options offered by a wireless phone company. If a phone with one payment plan and one accessory is given away at random, predict the probability that it will be Brand B and have a headset. Explain your reasoning.

Phone Brands	Payment Plans	Accessories
Brand A Brand B Brand C	Individual Family Business Government	Leather case Car mount Headset Travel charger

_____

_____

_____

_____

Power Up! Test Practice

22. A restaurant has 24 different lunch combinations. Which of the following could describe the lunch options? Select all that apply.

- ☐ 3 drink sizes, 4 main dishes, 2 side dishes
- ☐ 2 appetizers, 6 main dishes, 3 desserts
- ☐ 3 kinds of bread, 8 kinds of sandwiches
- ☐ 2 drink sizes, 7 appetizers, 2 main dishes

23. Hat Shack sells 9 different styles of hats in several different colors for 2 different sports teams. The company makes 108 kinds of hats in all. Select the correct values to complete the formula below to find the number of different color hats the Hat Shack makes.

Hat Shack		
Styles	**Colors**	**Teams**
9	?	2

☐ = ☐ × ☐ × ☐

2
9
108
c

How many different colors does the company use for hats?

☐

Spiral Review

Find each probability.

24. A coin is tossed and a spinner with 4 equal sections labeled W–Z is spun. Find *P*(heads and Z).

25. A pizza shop offers a single item pizza with choice of beef pepperoni, green peppers, pineapple, tuna, or mushroom toppings. The pizza can be thick crust or thin crust. Find *P*(thick crust).

Describe a model that could be used to simulate each situation.

26. There is a fifty percent chance of rain on Monday.

27. A restaurant randomly gives away 1 of 6 toys. Determine the number of times a child needs to visit the restaurant to receive all 6 toys.

Lesson 6

Permutations

Real-World Link

Scheduling Amer is planning his Saturday. He wants to mow the grass, go swimming, and do his homework. How many different ways are there to arrange what he wants to do?

Fill in the blanks of the organized list below to find all of the possible arrangements of the activities.

1: Mowing	2: Swimming	3: Homework
1: Mowing	2: Homework	3: ______
1: Swimming	2: Mowing	3: Homework
1: Swimming	2: Homework	3: ______
1: Homework	2: ______	3: ______
1: ______	2: ______	3: ______

1. How many choices does Amer have for his first activity?

______

2. Once the first activity is selected, how many choices does Amer have for the second activity?

______

3. Once the first and second activities are selected, how many choices does Amer have for the third activity?

______

Which MP **Mathematical Practices** did you use?
Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Essential Question

HOW can you predict the outcome of future events?

Vocabulary

permutation

MP Mathematical Practices
1, 3, 4

Work Zone

Find a Permutation

A **permutation** is an arrangement, or listing, of objects in which order is important.

You can use the Fundamental Counting Principle to find the number of permutations.

Examples

1. **Lamya is scheduling her first three classes. Her choices are math, science, and language arts. Use the Fundamental Counting Principle to find the number of different ways Lamya can schedule her first three classes.**

There are **3** choices for the first class.

There are **2** choices that remain for the second class.

There is **1** choice that remains for the third class.

$3 \cdot 2 \cdot 1 = 6$ ← the number of permutations of 3 classes

There are 6 possible arrangements, or permutations, of the 3 classes.

2. **An ice cream shop has 31 flavors. Hassan wants to buy a three-scoop cone with three different flavors. How many cones could he buy if the order of the flavors is important?**

There are 31 choices for the first scoop, 30 choices for the second scoop, and 29 choices for the third scoop.

Use the Fundamental Counting Principle.

$31 \cdot 30 \cdot 29 = 26{,}970$

Hassan could buy 26,970 different cones.

Show your work.

Got it? **Do these problems to find out.**

a. _____

b. _____

a. In how many ways can the starting six players of a volleyball team stand in a row for a picture?

b. In a race with 7 runners, in how many ways can the runners end up in first, second, and third place?

The symbol $P(31, 3)$ represents the number of permutations of 31 things taken 3 at a time.

Start with 31.

$$P(31, 3) = 31 \cdot 30 \cdot 29$$

Use three factors.

Example

3. **Find $P(8, 3)$.**

$P(8, 3) = 8 \cdot 7 \cdot 6$ or 336 — 8 things taken 3 at a time

Got it? **Do these problems to find out.**

c. $P(12, 2)$ **d.** $P(4, 4)$ **e.** $P(10, 5)$

c. _____

d. _____

Show your work.

e. _____

Find Probability

Permutations can be used when finding probabilities of real-world situations.

Examples

4. **Lamis MP3 player has a setting that allows the songs to play in a random order. She has a playlist that contains 10 songs. What is the probability that the MP3 player will randomly play the first three songs in order?**

First find the permutation of ten things taken three at a time or $P(10, 3)$.

10 songs — Choose 3

$P(10, 3) = 10 \cdot 9 \cdot 8$ ← 10 choices for the 1st song; 9 choices for the 2nd song; 8 choices for the 3rd song

$= 720$

So, there are 720 different ways to play the first 3 songs. Since you want the first three songs in order, there is only 1 out of the 720 ways to do this.

So, the probability that the first 3 songs will play in order is $\frac{1}{720}$.

Notation

In Example 4, the notation $P(10, 3)$ indicates a permutation while the notation $P(\text{playing the first three songs in order})$ indicates probability.

Reasonable Answers

A possible probability of $\frac{1}{56}$ indicates that it is very unlikely that Yumii will finish first and Paquita will finish second.

5. **A swimming event features 8 swimmers. If each swimmer has an equally likely chance of finishing in the top two, what is the probability that Fatheya will be in first place and Shaima in second place?**

Swimmers	
Abeer	Fawzia
Laila	Shaima
Fatema	Ayesha
Fatheya	Maha

First find the permutation of 8 things taken two at a time or $P(8, 2)$.

$$P(8, 2) = 8 \cdot 7$$
$$= 56$$

There are 56 possible arrangements, or permutations, of the two places. Since there is only one way of having Fatheya come in first and Shaima second, the probability of this event is $\frac{1}{56}$.

Show your work.

Got it? **Do this problem to find out.**

f. Two different letters are randomly selected from the letters in the word *math*. What is the probability that the first letter selected is *m* and the second letter is *h*?

f. _____

Guided Practice

Check

1. In how many ways can a president, vice president, and secretary be randomly selected from a class of 25 students? (Examples 1 and 2)

2. Find the value of $P(5, 3)$. (Example 3)

3. Manal, Najla, and two of their friends will sit in a row at a baseball game. If each friend is equally likely to sit in any seat, what is the probability that Manal will sit in the first seat and Najat will sit in the second seat? (Examples 4 and 5)

4. **Building on the Essential Question** HOW can you find the number of permutations of a set of objects?

Rate Yourself!

☐ I understand how to find permutations.

Great! You're ready to move on!

☐ I still have questions about finding permutations.

Independent Practice

1. In the Battle of the Bands contest, in how many ways can the four participating bands perform? (Examples 1 and 2)

_____

2. A garage door code has 5 digits. If no digit is repeated, how many codes are possible?

_____

Find each value. Use a calculator if needed. (Example 3)

3. $P(7, 4)$ _____

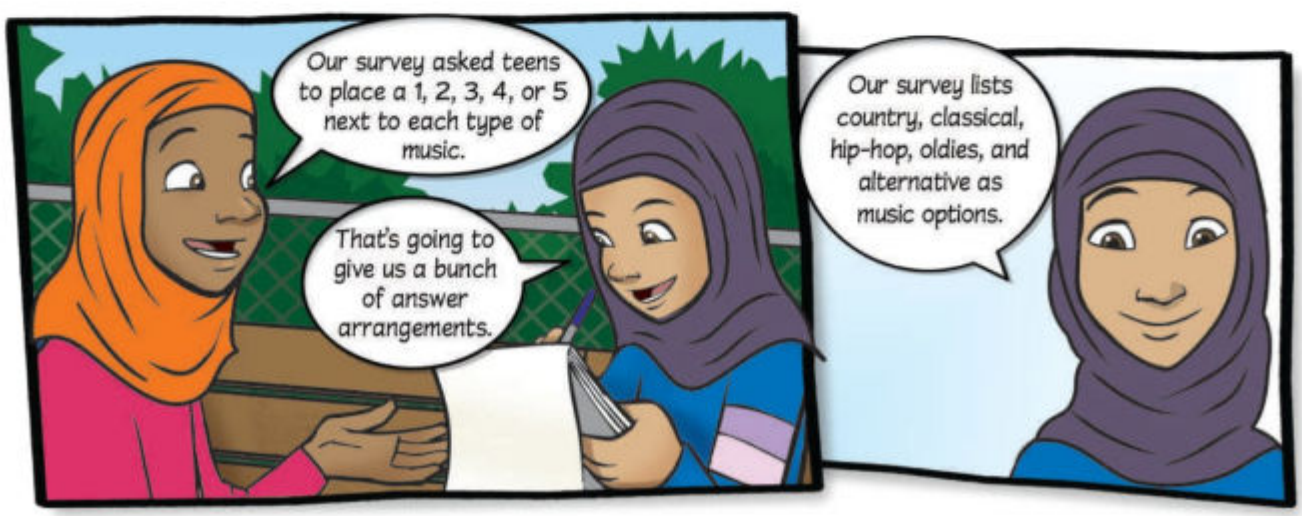
4. $P(12, 5)$ _____

5. $P(8, 8)$ _____

6. You have five seasons of your favorite TV show on DVD. If you randomly select two of them from a shelf, what is the probability that you will select season one first and season two second? (Examples 4 and 5)

_____

7. **Model with Mathematics** The graphic novel frame below explains how the survey has students rank their favorite kinds of music. In how many ways can the survey be answered? _____



8. A certain number of friends are waiting in line to board a new roller coaster. They can board the ride in 5,040 different ways. How many friends are in line?

_____

9. The Obaid family discovered they can stand in a row for their family portrait in 720 different ways. How many members are in the Obaid family? _____

10. Howland Middle School assigns a four-digit identification number to each student. The number is made from the digits 1, 2, 3, and 4, and no digit is repeated. If assigned randomly, what is the probability that an ID number will end with a 3? _____

H.O.T. Problems Higher Order Thinking

11. MP **Model with Mathematics** Describe a real-world situation that has 6 permutations.

_____

_____

12. MP **Persevere with Problems** There are 1,320 ways for three students to win first, second, and third place during a debate match. How many students are there on the debate team? Explain your reasoning.

_____

_____

13. MP **Persevere with Problems** A *combination* is an arrangement where order is *not* important. You can find the number of combinations of items by dividing the number of permutations by the number of ways the smaller set can be arranged. The combination at the right shows the number of combinations if you choose 2 flavors of ice cream out of 5 flavors. Use this method to find each value.

From 5 flavors take 2 at a time.

$$C(5, 2) = \frac{5 \cdot 4}{2 \cdot 1} \text{ or } 10$$

There are $2 \cdot 1$ ways to order 2 flavors.

a. $C(6, 4)$ _____ b. $C(10, 3)$ _____

c. $C(5, 3)$ _____ d. $C(8, 6)$ _____

Name _____ My Homework _____

Extra Practice

14. How many permutations are possible of the letters in the word FRIEND? 720

Homework Help → $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

15. How many different 3-digit numbers can be formed using the digits 9, 3, 4, 7, and 6? Assume no number can be used more than once. _____

Find each value. Use a calculator if needed.

16. $P(9, 2)$ _____

17. $P(5, 5)$ _____

18. $P(7, 7)$ _____

19. The members of the Evergreen Junior High Quiz Bowl team are listed in the table. If a captain and an assistant captain are chosen at random, what is the probability that Saleh is selected as captain and Abdulrahman as co-captain? _____

Evergreen Junior High Quiz Bowl Team	
Adnan	Tarek
Hareb	Abdulrahman
Humaid	Abdulraheem
Sultan	Abdulaziz
Saleh	Abdulkarim

20. Tarek, Eissa, Faleh, and Majed are playing a video game. If they each have an equally likely chance of getting the highest score, what is the probability that Majed will get the highest score and Tarek the second highest? _____

21. A child has wooden blocks with the letters shown. Find the probability that the child randomly arranges the letters in the order TIGER. _____

Power Up!

22. The schools listed in the table are finalists in a science competition. First through third places will win a prize. Each school is equally likely to win the competition. Select values to complete the model below to find the probability that Lincoln wins first place, River Valley wins second place, and Glenwood wins third place.

Finalists
Chester Middle School
Glenwood Middle School
Lincoln Middle School
River Valley Middle School
South Middle School

Find the number of ways the schools can finish in first, second, and third place:

$P(\square, \square) = \square$

1	2	3	4	5
10	20	30	60	90

The number of ways that Lincoln can finish first, River Valley second, and Glenwood third is equal to $\square$.

$P(\text{Lincoln first, River Valley second, Glenwood third}) = \frac{\square}{\square}$

23. The five finalists in a writing contest are Nahla, Nabila, Noura, Nisreen, and Maysa. There will be a first place award and a second place award. Each finalist is equally likely to win an award. Determine if each statement is true or false.

a. There are 10 permutations of 5 finalists taken 2 at a time. ☐ True ☐ False

b. There is only 1 way that Nabila can earn first place and Noura can earn second place. ☐ True ☐ False

c. The probability that Nabila earns first place and Noura earns second place is 0.05. ☐ True ☐ False

Spiral Review

A card is pulled from a stack of 30 cards labeled 1–30. Find each probability. Write as a fraction in simplest form.

24. *P*(greater than 5) _____

25. *P*(*not* 1) _____

26. *P*(an even number) _____

27. A cross country athlete has a white, a red, and a gray sweatshirt. She has black and gray running pants. Make a list to show the possible combinations of training outfits. _____

Inquiry Lab

Independent and Dependent Events

Inquiry **HOW can one event impact a second event in a probability experiment?**

MP Mathematical Practices 1, 3

Maysoun wants to go to the movies and Najla wants to go skating. They decide by doing a simulation. They place two red counters in a bag to represent going to the movies and two white counters to represent going skating. If they draw or remove two red counters, they will go to the movies. If they draw two white counters they will go skating. If they draw a red and a white counter, they will stay home.

You can simulate this activity using counters.

Hands-On Activity

Step 1 Place two red counters and two white counters in a paper bag.

Step 2 Without looking, draw a counter from the bag and record its color in the table below. Place the counter back in the bag.

Step 3 Without looking, draw a second counter and record its color in the table. The two colors are one trial. Place the counter back in the bag.

Step 4 Repeat until you have 18 trials.

Trial	1st Color	2nd Color	Trial	1st Color	2nd Color	Trial	1st Color	2nd Color
1			7			13		
2			8			14		
3			9			15		
4			10			16		
5			11			17		
6			12			18		

What is the experimental probability that the girls will go to the movies?

Investigate

Work with a partner.

1. Complete the same experiment from the Activity. Except do not replace the counter after the first draw for each trial. Record your results.

Trial	1st Color	2nd Color	Trial	1st Color	2nd Color	Trial	1st Color	2nd Color
1			7			13		
2			8			14		
3			9			15		
4			10			16		
5			11			17		
6			12			18		

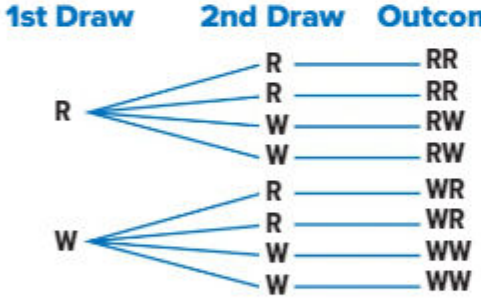
What is the experimental probability that the girls will go to the movies?

_____

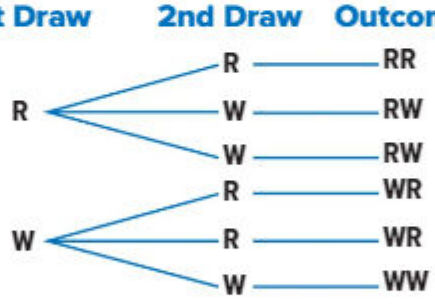
Analyze and Reflect

The tree diagrams below represent the possible outcomes for the Activity and for Exercise 1. Use the diagrams to answer Exercises 2–3.

Investigation



Exercise 1



2. What is the theoretical probability of drawing two reds in the Investigation? In Exercise 1? _____

3. **Reason Inductively** Is there a better chance that the girls will go to the movies if the counters are replaced after the first draw? Explain.

_____

Create

4. Inquiry HOW can one event impact a second event in a probability experiment?

_____

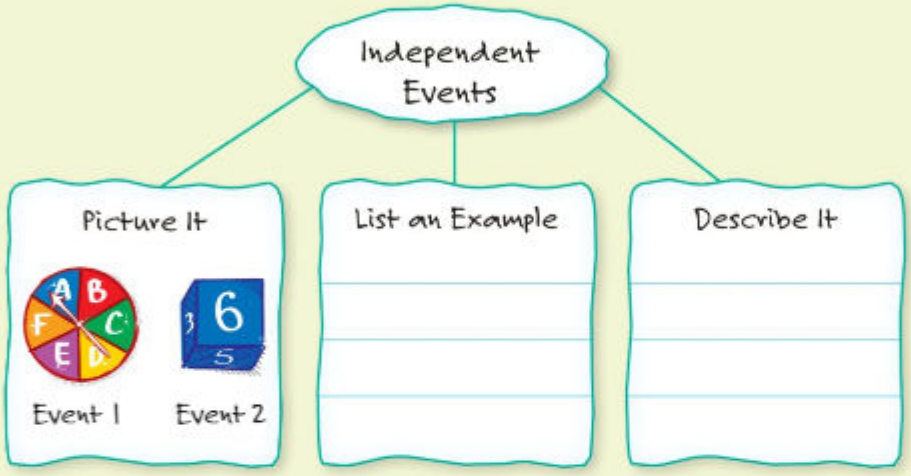
_____

Lesson 7

Independent and Dependent Events

Vocabulary Start-Up

When one event does not affect the outcome of the other event, the events are **independent events**. For example, if you toss a coin twice, the first toss has no affect on the second toss. Complete the graphic organizer below.



Essential Question

HOW can you predict the outcome of future events?

Vocabulary

independent events
dependent events

Mathematical Practices
1, 3, 4

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Key Concept

Probability of Independent Events

Work Zone

Words The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event.

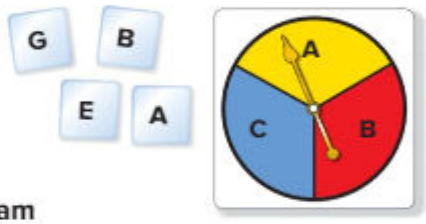
Symbols $P(A \text{ and } B) = P(A) \cdot P(B)$

You can use organized lists, tables, tree diagrams, or multiplication to find the probability of compound events.

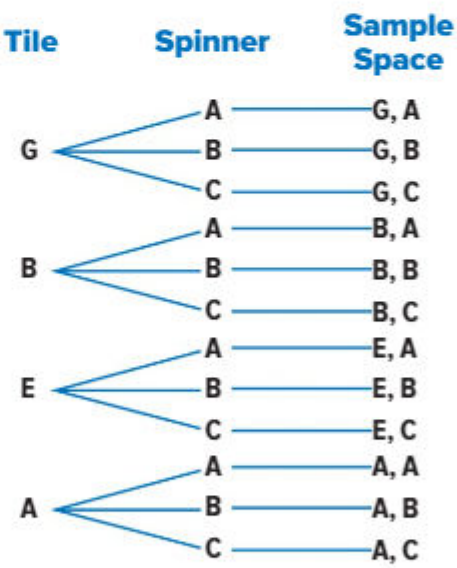
Examples

Tools

1. **One letter tile is selected and the spinner is spun. What is the probability that both will be a vowel?**



Method 1 **Make a Tree Diagram**



There are 12 outcomes. Two outcomes contain only vowels. The probability that both will be a vowel is $\frac{2}{12}$ or $\frac{1}{6}$.

Method 2 **Use Multiplication**

$P(\text{selecting a vowel}) = \frac{2}{4}$ or $\frac{1}{2}$. $P(\text{spinning a vowel}) = \frac{1}{3}$.

$P(\text{both vowels}) = \frac{1}{2} \cdot \frac{1}{3}$ or $\frac{1}{6}$. Multiply the probabilities.

So, using either method the probability is $\frac{1}{6}$.

2. **The spinner and number cube shown are used in a game. What is the probability of a player *not* spinning blue and then rolling a 3 or 4?**

You are asked to find the probability of the spinner *not* landing on blue and rolling a 3 or 4 on a number cube. The events are independent because spinning the spinner does not affect the outcome of rolling a number cube.

First, find the probability of each event.

$P(\text{not blue}) = \frac{4}{5}$ ← $\frac{\text{number of ways not to spin blue}}{\text{number of possible outcomes}}$

$P(3 \text{ or } 4) = \frac{2}{6} \text{ or } \frac{1}{3}$ ← $\frac{\text{number of ways to roll 3 or 4}}{\text{number of possible outcomes}}$

Then, find the probability of both events occurring.

$P(\text{not blue and 3 or 4}) = \frac{4}{5} \cdot \frac{1}{3}$ $P(A \text{ and } B) = P(A) \cdot P(B)$

$= \frac{4}{15}$ Multiply.

The probability is $\frac{4}{15}$.

Check Make an organized list, table, or a tree diagram to show the sample space.

Got it? **Do this problem to find out.**

Show your work.

a. A game requires players to roll two number cubes to move the game pieces. The faces of the cubes are labeled 1 through 6. What is the probability of rolling a 2 or 4 on the first number cube and then rolling a 5 on the second?

a. _____

Probability of Dependent Events

Key Concept

Words	If two events A and B are dependent, then the probability of both events occurring is the product of the probability of A and the probability of B after A occurs.
Symbols	$P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

If the outcome of one event affects the outcome of another event, the events are called **dependent events**. For example, you have a bag with blue and green marbles. You pick one marble, do not replace it, and pick another one.

Example

3. **There are 4 oranges, 7 bananas, and 5 apples in a fruit basket. Mansour selects a piece of fruit at random and then Mahmoud selects a piece of fruit at random. Find the probability that two apples are chosen.**

Since the first piece of fruit is not replaced, the first event affects the second event. These are dependent events.

$P(\text{first piece is an apple}) = \frac{5}{16}$ ← number of apples / ← total pieces of fruit

$P(\text{second piece is an apple}) = \frac{4}{15}$ ← number of apples left / ← total pieces of fruit left

$P(\text{two apples}) = \frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{16}}} \cdot \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{15}}}$ or $\frac{1}{12}$.

The probability that two apples are chosen is $\frac{1}{12}$.

Show your work.

b. _____

c. _____

Got it? **Do these problems to find out.**

Refer to the situation above. Find each probability.

b. *P*(two bananas)

c. *P*(orange then apple)

Guided Practice

Check

A coin is tossed and a number cube is rolled. Find each probability. (Examples 1–2)

1. *P*(tails and 3) _____

2. *P*(heads and odd) _____

3. Cards labeled 5, 6, 7, 8, and 9 are in a stack. A card is drawn and not replaced. Then, a second card is drawn at random. Find the probability of drawing two even numbers.

(Example 3) _____

4. **Building on the Essential Question** Explain the difference between independent events and dependent events.

_____

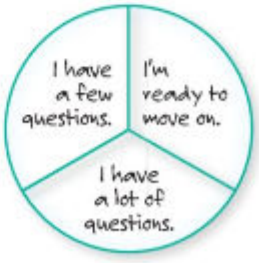
_____

_____

_____

Rate Yourself!

Are you ready to move on? Shade the section that applies.



Name _____ My Homework _____

Independent Practice

A number cube is rolled and a marble is selected at random from the bag at the right. Find each probability. Show your work. (Example 1)

1. *P*(1 and red) _____

Show your work.

2. *P*(3 and purple) _____

3 *P*(even and yellow) _____

4. *P*(odd and *not* green) _____

5. A carnival game wheel has 12 equal sections. One of the sections contains a star. To win a prize, players must land on the section with the star on two consecutive spins. What is the probability of a player winning? (Example 2) _____

6. A standard set of dominoes contains 28 tiles, with each tile having two sides of dots from 0 to 6. Of these tiles, 7 have the same number of dots on each side. If four players each randomly choose a tile, without replacement, what is the probability that each chooses a tile with the same number of dots on each side? (Example 3) _____

Mrs. Huda class has 5 students with blue eyes, 7 with brown eyes, 4 with hazel eyes, and 4 with green eyes. Two students are selected at random. Find each probability. (Example 3)

7 *P*(green then brown) _____

8. *P*(two blue) _____

9. *P*(hazel then blue) _____

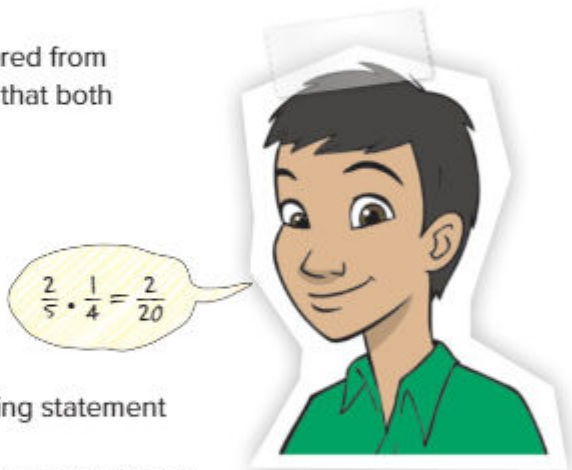
10. *P*(brown then blue) _____

11. **Reason Inductively** You and a friend plan to see 2 movies over the weekend. You can choose from 6 comedy, 2 drama, 4 romance, 1 science fiction, or 3 action movies. You write the movie titles on pieces of paper, place them in a bag, and each randomly select a movie. What is the probability that neither of you selects a comedy? Is this a dependent or independent event? Explain.

H.O.T. Problems Higher Order Thinking

12. **Model with Mathematics** There are 9 marbles representing 3 different colors. Write a problem where 2 marbles are selected at random without replacement and the probability is $\frac{1}{6}$.

13. **Find the Error** A spinner with equal sections numbered from 1 to 5 is spun twice. Mohammad is finding the probability that both spins will result in an even number. Find his mistake and correct it.



14. **Justify Conclusions** Determine whether the following statement is *true* or *false*. If false, provide a counterexample.
If two events are independent, then the probability of both events is less than 1.

15. **Persevere with Problems** A company has determined that 2% of the pudding cups it produces are defective in some way. The pudding cups are sold in packages of two.

a. What is the probability that both pudding cups in a package are defective?

b. The company produces 1,000,000 packages each year. Predict the number of packages in which both cups are defective.

Name _____ My Homework _____

Extra Practice

A number cube is rolled and a letter is selected from the word AMERICA. Find each probability. Show your work.

16. *P*(less than 4 and vowel) _____

Homework Help

$P(\text{less than 4}) = \frac{1}{2}$

$P(\text{vowel}) = \frac{4}{7}$

$\frac{1}{2} \cdot \frac{4}{7} = \frac{4}{14}$ or $\frac{2}{7}$

17. *P*(greater than 1 and a consonant) _____

18. A number cube is rolled and a coin is tossed. What is the probability of the cube landing on 5 or 6 and the coin landing on heads?

19. A laundry basket contains 18 blue socks and 24 black socks. What is the probability of randomly picking 2 black socks, without replacement, from the basket?

20. MP **Persevere with Problems** Fahd is playing a board game that requires rolling two number cubes to move a game piece. He needs to roll a sum of 6 on his first turn and then a sum of 10 on his second turn to land on the next two bonus spaces. What is the probability that Fahd will roll a sum of 6 and then a sum of 10 on his next two turns? _____

Copy and Solve **Solve Exercises 21–28 on a separate sheet of paper.**

A card is pulled from a stack of 15 cards labeled 1–15 and the spinner shown is spun. Find each probability.

21. *P*(less than 10 and red)

22. *P*(odd and red or blue)

23. *P*(even and blue)

24. *P*(prime number and blue)

Hala is packing for a trip. In her closet, there are 3 red, 4 black, 2 green, and 2 yellow blouses. She randomly selects 2 blouses. Find each probability.

25. *P*(red and red)

26. *P*(black and yellow)

27. *P*(red and black)

28. *P*(green and green)

Power Up!

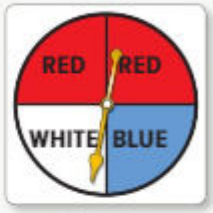
29. A bag contains letter tiles. There are 6 vowels in the bag and 14 consonants. On Hasan next turn, he will select a letter tile at random from the bag. Without replacing the first tile, he will then select a second letter tile. Determine if each of the following probabilities is true or false.

a. $P(\text{vowel then vowel}) = \frac{3}{38}$ ☐ True ☐ False

b. $P(\text{vowel then consonant}) = \frac{21}{95}$ ☐ True ☐ False

c. $P(\text{consonant then consonant}) = \frac{49}{100}$ ☐ True ☐ False

30. The spinners are each spun once.



Do the two spins represent independent or dependent events? ☐

Select the correct values to complete the model below to find $P(2 \text{ and white})$.

$$P(2 \text{ and white}) = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$$

1	6
2	8
3	12
4	16

Spiral Review

Solve each proportion.

31. $\frac{1}{4} = \frac{x}{72}$ $x =$ _____

32. $\frac{8}{n} = \frac{0.5}{0.9}$ $n =$ _____

33. $\frac{1}{3} = \frac{m}{153}$ $m =$ _____

34. $\frac{0.2}{a} = \frac{1.8}{18}$ $a =$ _____

35. 9 is 15% of what number? Write an equation. Then solve.

_____

36. A school librarian surveyed students about their favorite type of novel. The results are shown in the table at the right. What percent of students chose science fiction as their favorite type of novel? Round to the nearest whole percent.

_____

Type of Novel	Number of Students
mystery	18
romance	10
science fiction	26
other	4

21ST CENTURY CAREER in Medicine

Pediatricians

Do you have compassion, a sense of humor, and the ability to analyze data? You might want to consider a career in medicine. Pediatricians care for the health of infants, children, and teenagers. They diagnose illnesses, interpret diagnostic tests, and prescribe and administer treatment.

College & Career READINESS

Is This the Career for You?

Are you interested in a career as a pediatrician? Take some of the following courses in high school.

- Algebra
- Biology
- Calculus
- Chemistry
- Psychology

Find out how math relates to a career in Medicine.

On Call for Kids

Use the information in the table below to solve each problem. Write each answer as a percent rounded to the nearest whole number.

1. What is the probability that one of the patients tested has strep throat? _____
2. If a patient has strep throat, what is the probability that they have a positive test? _____
3. What is the probability that a patient with the disease has a negative test? _____
4. If a patient does not have the disease, what is the probability that they have a positive test? _____
5. What is the probability that a patient that does *not* have strep throat tested negative for the disease? _____
6. The *positive predictive value*, or *PPV*, is the probability that a patient with a positive test result will have the disease. What is the PPV? _____
7. The *negative predictive value*, or *NPV*, is the probability that a patient with a negative test result will not have the disease. What is the NPV? _____

200 Patients Tested for Strep Throat		
	Patients Have Strep Throat	**Patients do *Not* Have Strep Throat**
Test is Positive	*True Positive (TP)* 90	*False Positive (FP)* 17
Test is Negative	*False Negative (FN)* 8	*True Negative (TN)* 85

Career Project

It's time to update your career portfolio! Interview your pediatrician. Be sure to ask what he or she enjoys most about being a pediatrician and what is most challenging. Include all the interview questions and answers in your portfolio.

What are some short term goals you need to achieve to become a pediatrician?

- _____
- _____
- _____
- _____
- _____

Chapter Review

Vocabulary Check

Unscramble each of the clue words. After unscrambling all of the terms, use the numbered letters to find a sentence assoicated with probability.

HELATCORTEI

PORTUNMETAI

LEAPMS ECPAS

COAPELMERTMYN

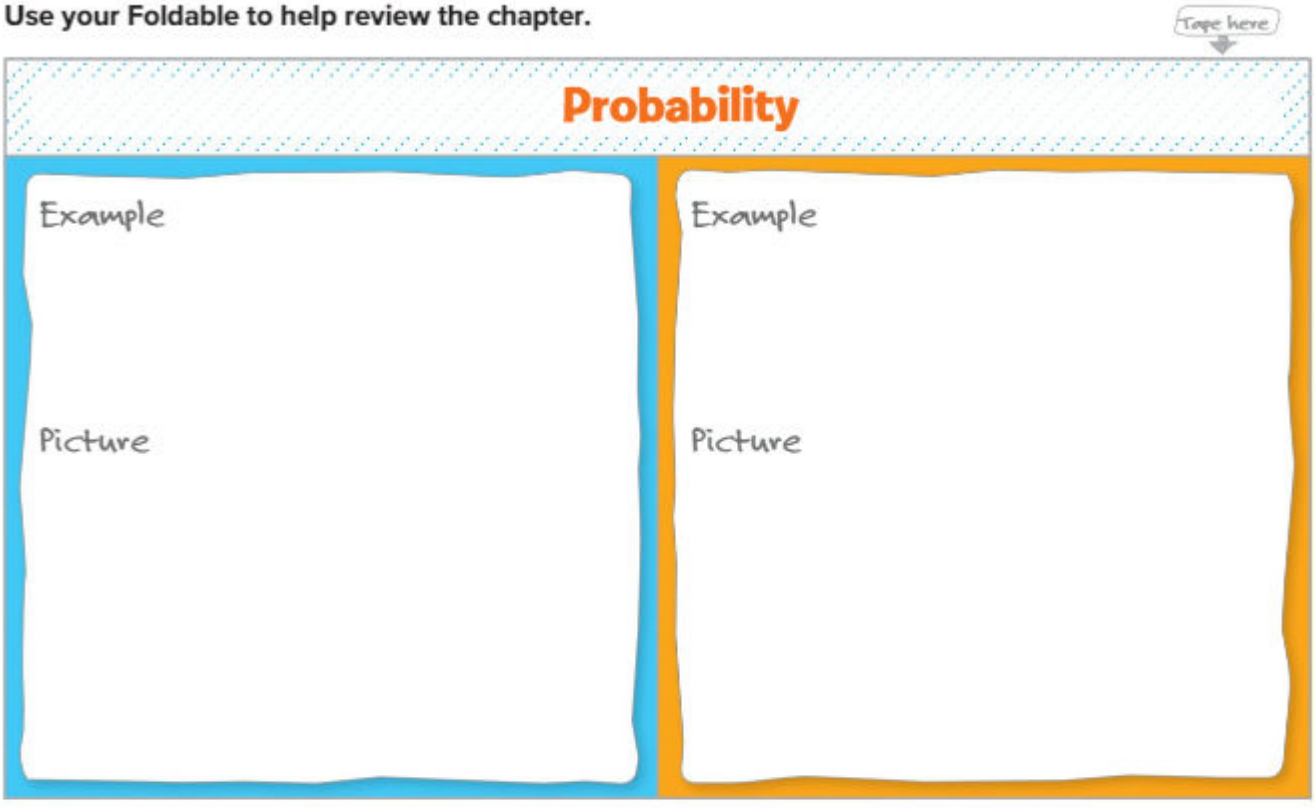
Complete each sentence using one of the unscrambled words above.

1. The _____ is the set of all of the possible outcomes of a probability experiment.
2. A _____ is an arrangement, or listing, of objects in which order is important.
3. The _____ probability is based on what should happen when conducting a probability experiment.
4. Two events in which one or the other must happen, but they cannot happen at the same time are _____.

Key Concept Check

Use Your FOLDABLES

Use your Foldable to help review the chapter.



Got it?

Match each term or phrase on the left with the words on the right.

1. Based on what actually occurred in a probability experiment
2. The outcome of one event affects the outcome of a separate event
3. Consists of two or more simple events
4. Can be used to find the sample space

a. compound event

b. experimental probability

c. Fundamental Counting Principle

d. dependent event

e. tree diagrams

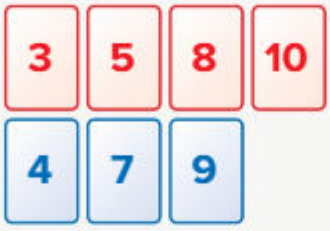
f. organized lists

Power Up! Performance Task

Carnival Prizes

Sumayya is in charge of a game booth at the school carnival. The game has two simple rules.

- Randomly pick one blue card and one red card.
- If the product of the two numbers is greater than or equal to 45, you win a prize.



Write your answers on another piece of paper. Show all of your work to receive full credit.

Part A

Create a sample space and find the product of each combination. What is the probability that a person wins the game? Express your answer as a fraction in lowest terms and as a percent rounded to the nearest whole number.

Part B

The sponsor of the booth determines that they are giving away too many prizes. Recommend a minimum winning number that lowers the chance of winning to 25%. Explain your reasoning.

Part C

Participants achieving a winning score of 70 or higher in four consecutive attempts will receive a large stuffed animal. What is the probability of this occurring?

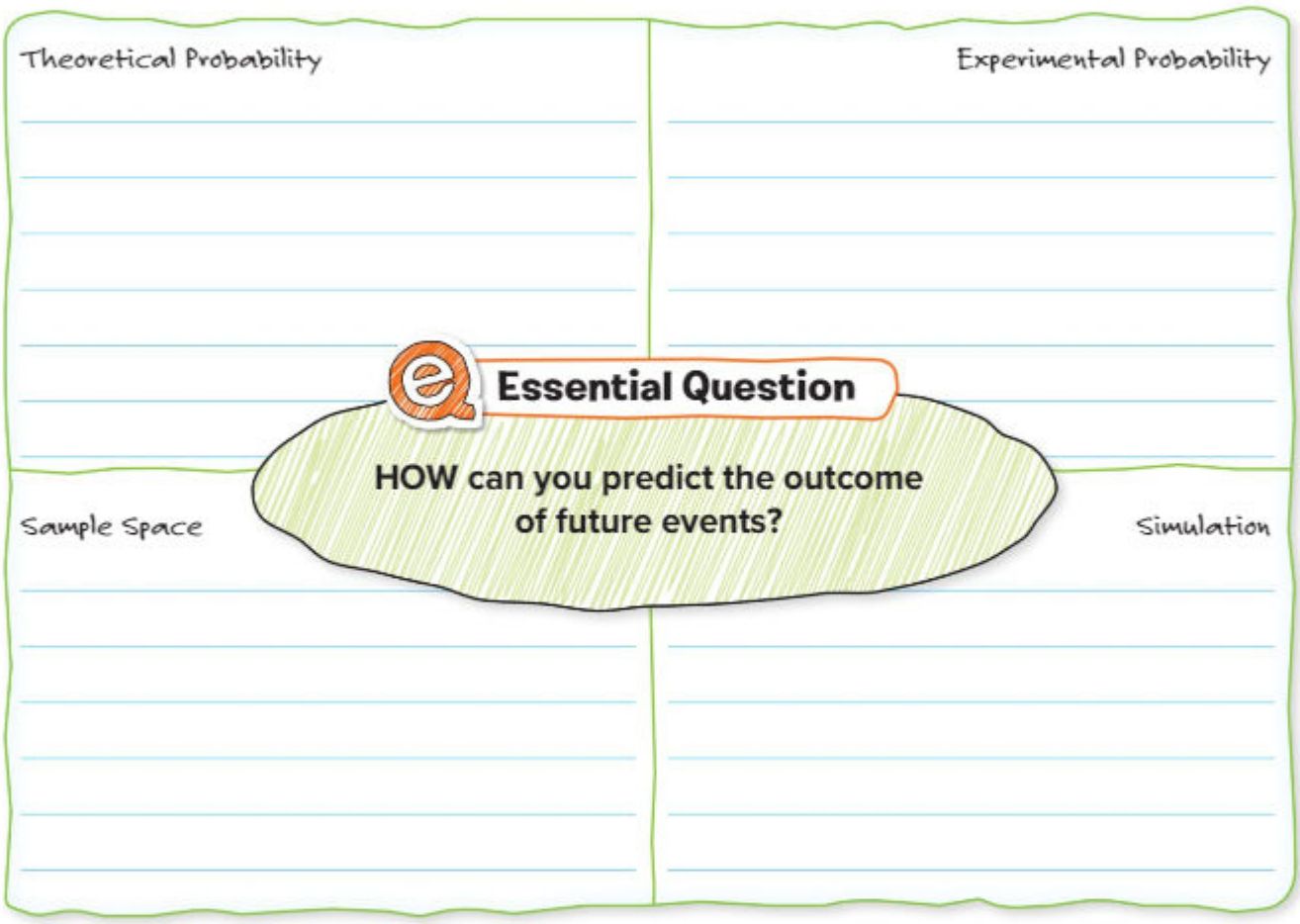
Part D

After changing the game rules, patrons and onlookers are disappointed when the first five games yield products of 12, 21, 32, 35, and 12. Recommend a statement that the sponsor can use to reassure customers that the game is fair.

Reflect

Answering the Essential Question

Use what you learned about probability to complete the graphic organizer.



Answer the Essential Question. HOW can you predict the outcome of future events?

Chapter 10
Statistics

Essential Question

HOW do you know which type of graph to use when displaying data?

MP Mathematical Practices

1, 3, 4, 5, 6,

Math in the Real World

Surveys are used to collect information. Survey results can be shown in graphs.

The results of a survey of 50 middle school students are shown in the table. On the circle graph, write the percent of students who preferred each activity.

Activity	Number of Students
Gaming	22
Social Networking	18
Viewing Movies	6
Other	4

FOLDABLES® Study Organizer

1. Cut out the Foldable on page FL13 of this book.
2. Place your Foldable on page 850.
3. Use the Foldable throughout this chapter to help you learn about statistics.

What Tools Do You Need?

Vocabulary

biased sample	population	survey
convenience sample	sample	systematic random sample
double box plot	simple random sample	unbiased sample
double dot plot	statistics	voluntary response sample

Study Skill: Writing Math

Describe Data When you *describe* something, you represent it in words.

The table shows the prices for takeout orders at Lombardo's Restaurant.

Takeout	Price (AED)
Main Dish	8.00
Side Dish	2.50
Dessert	4.00

Use the table to complete the following statements.

1. The price of a dessert is _____.
2. The price of a main dish is twice the price of _____.
3. A _____ is the least expensive item.

Write two other statements that describe the data.

4. _____
5. _____

What Do You Already Know?

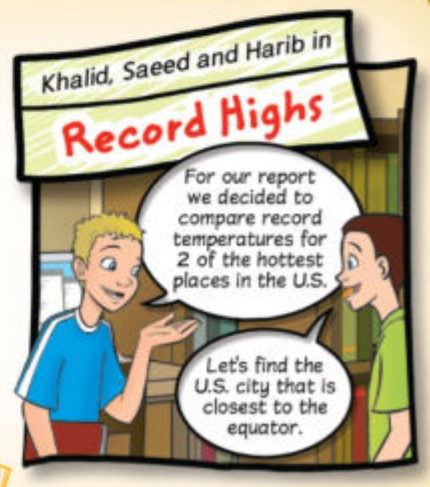
Read each statement. Decide whether you agree (A) or disagree (D). Place a checkmark in the appropriate column and then justify your reasoning.

Statistics			
Statement	A	D	Why?
Statistics deal with collecting, organizing, and interpreting data.			
A sample is the same thing as the entire population.			
A biased sample accurately represents the entire population.			
Graphs are sometimes made to influence conclusions by misrepresenting the data.			
A double box plot consists of two box plots that are drawn on the same number line.			
Any type of display can be used to represent data.			

When Will You Use This?

Here is an example of how statistics are used in the real world.

Activity 1 Find the average monthly high and low temperatures for the city where you live. Then find the average monthly high and low temperatures for another city. How do these temperatures compare to the temperatures for your city?



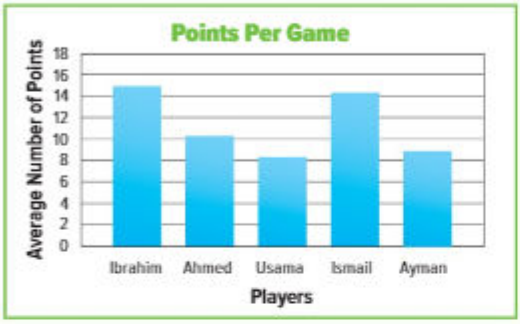
Are You Ready?

Try the Quick Check below.

Quick Review

Example 1

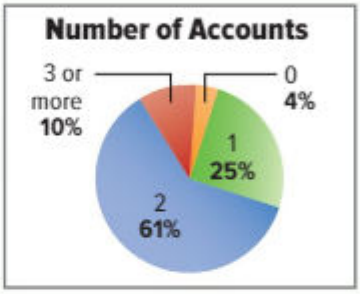
Which players averaged more than 10 points per game?



Ibrahim, Ahmed, and Ismail averaged more than 10 points per game.

Example 2

Use the circle graph. Suppose 300 people were surveyed. How many people have two accounts?



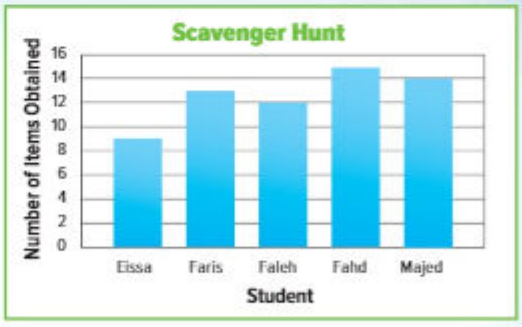
Find 61% of 300.

$61\% \text{ of } 300 = 61\% \times 300$
$= 0.61 \times 300 \text{ or } 183$

So, 183 people have two accounts.

Quick Check

Graphs **The bar graph at the right shows the number of items each student obtained during a scavenger hunt.**



1. Who obtained the most items?

2. Who obtained the least items?

3. Refer to the circle graph in Example 2. Suppose 300 people were surveyed. How many people have 1 account?

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

(1) (2) (3)

Lesson 1

Make Predictions

Vocabulary Start-Up

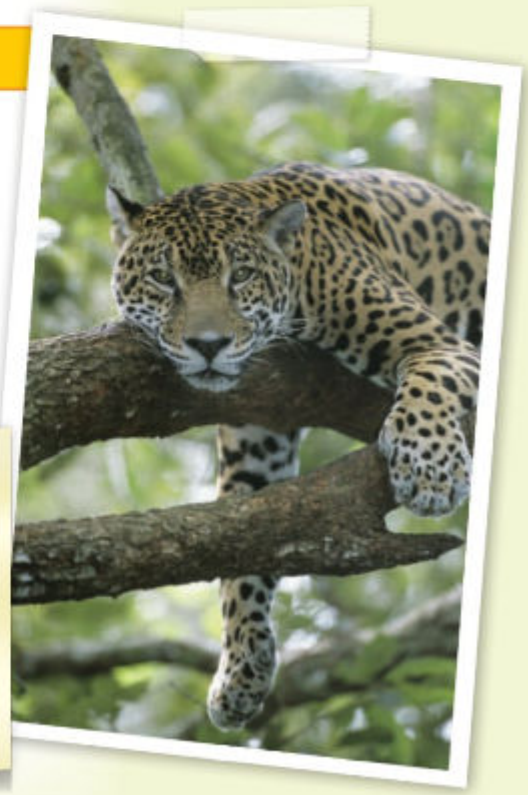
Statistics deal with collecting, organizing, and interpreting data. A **survey** is a method of collecting information. The group being studied is the **population**. Sometimes the population is very large. To save time and money, part of the group, called a **sample**, is surveyed.

For each survey topic, determine which set represents the population and which represents a sample of the population. Write *population* or *sample*.

	Survey Topic	Set A	Set B
1.	dress code changes	the students in a middle school	the seventh graders in the middle school
2.	favorite flavors of ice cream	the customers at an ice cream shop in the town	the residents of a town

Real-World Link

Badr wants to survey students in his school about their favorite and least favorite zoo exhibit. Describe a possible sample Badr could survey instead of surveying the entire school.



Essential Question

HOW do you know which type of graph to use when displaying data?

Vocabulary

statistics
survey
population
sample

Mathematical Practices

1, 3, 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Make Predictions Using Ratios

You can use the results of a survey or past actions to predict the actions of a larger group. Since the ratios of the responses of a good sample are often the same as the ratios of the responses of the population.

Examples

The students in Mr. Zayed's class brought photos from their summer break. The table shows how many students brought each type of photo.

Summer Break Photos	
Location	**Students**
beach	6
campground	4
home	7
theme park	11

1. What is the probability that a student brought a photo taken at a theme park?

$$P(\text{theme park}) = \frac{\text{number of theme park photos}}{\text{number of students with a photo}} = \frac{11}{28}$$

So, the probability of a theme park photo is $\frac{11}{28}$.

2. There are 560 students at the school where Mr. Zayed teaches. Predict how many students would bring in a photo taken at a theme park.

Let s represent the number of theme park photos.

$\frac{11}{28} = \frac{s}{560}$ Write an equivalent ratio.

$\frac{11}{28} \overset{\times 20}{=} \frac{s}{560}$ Since $28 \times 20 = 560$, multiply 11 by 20 to find s.

$\frac{11}{28} = \frac{220}{560}$ $s = 220$

Of the 560 students, you can expect about 220 to bring a photo from a theme park.

Got it? Do these problems to find out.

A survey found that 6 out of every 10 students have a blog.

a. What is the probability that a student at the school has a blog?

b. Suppose there are about 250 students at the school. About how many have a blog?

Make Predictions Using Equations

You can also use the percent equation to make predictions.

Examples

3. **A survey found that 85% of people use emoticons on their instant messengers. Predict how many of the 2,450 students at Washington Middle School use emoticons.**

Words	What number of students is 85% of 2,450 students?				
Variable	Let n represent the number of students.				
Equation	n	$=$	0.85	$\cdot$	2,450

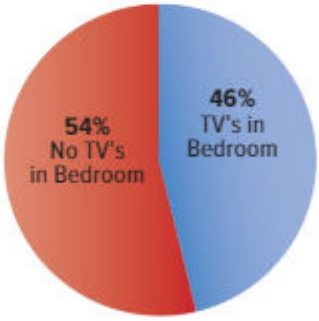
$n = 0.85 \cdot 2{,}450$ Write the percent equation.

$n = 2{,}082.5$ Multiply.

About 2,083 of the students use emoticons.

4. **The circle graph shows the results of a survey in which children ages 8 to 12 were asked whether they have a television in their bedroom. Predict how many out of 1,725 students would not have a television in their bedroom.**

TV's in the Bedroom



You can use the percent equation and the survey results to predict what part p of the 1,725 students have no TV in their bedroom.

part = percent · whole

$p = 0.54 \cdot 1{,}725$ Survey results: 54%

$p = 931.5$ Multiply.

About 932 students do not have a television in their bedroom.

STOP and Reflect

What proportion could you use to solve Example 4? Write your answer below.

Got it? **Do this problem to find out.**

c. Refer to Example 4. Predict how many out of 1,370 students have a television in their bedroom.

Show your work.

c. _____

Guided Practice

The table shows the results of a survey of Hamilton Middle School seventh graders. Use the table to find the following probabilities. (Examples 1 and 2)

Career Field	Students
Entertainment	17
Education	14
Medicine	11
Public service	6
Sports	2

1. the probability of choosing a career in public service

Show your work.

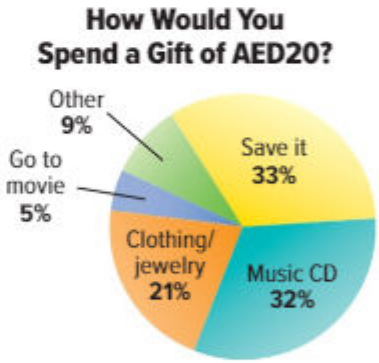
2. the probability of choosing a career in education

3. the probability of choosing a career in sports

4. Predict how many students out of 400 will enter the education field.

5. Predict how many students out of 500 will enter the medical field.

6. Use the circle graph that shows the results of a poll to which 60,000 teens responded. Predict how many of the approximately 28 million teens in the United States would buy a music CD if they were given AED20. (Examples 3 and 4)



7. **Building on the Essential Question** When can statistics be used to gain information about a population from a sample?

Rate Yourself!

How confident are you about making predictions? Check the box that applies.

Independent Practice

The table shows the results of a survey of 150 students. Use the table to find the probability of a student participating in each sport. (Example 1)

Sport	Students
Baseball/softball	36
Basketball	30
Football	45
Gymnastics	12
Tennis	18
Volleyball	9

Show your work.

1. football

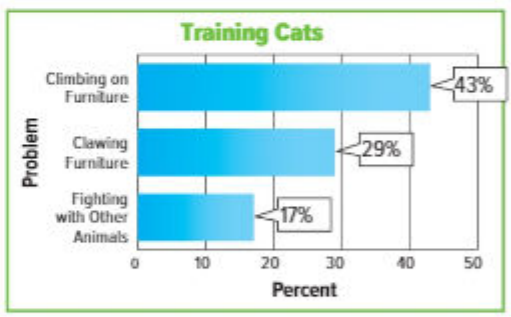
2. tennis

3. gymnastics

4. volleyball

5. Three out of every 10 students ages 6–14 have a magazine subscription. Suppose there are 30 students in Eiman's class. About how many will have a magazine subscription? (Example 2)

6. Use the graph that shows the percent of cat owners who train their cats in each category. (Examples 3 and 4)



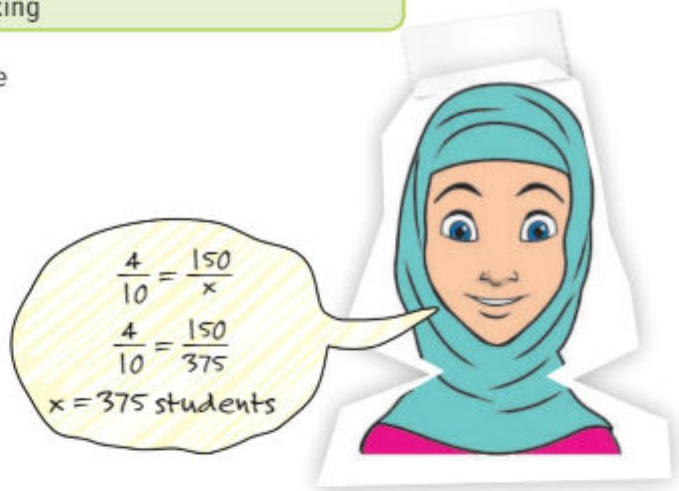
   a. Out of 255 cat owners, predict how many owners trained their cat not to climb on furniture.

   b. Out of 316 cat owners, predict how many cat owners trained their cat not to claw on furniture.

7. **Make a Prediction** The school librarian recorded the types of books students checked out on a typical day. Suppose there are 605 students enrolled at the school. Predict the number of students that prefer humor books. Compare this to the number of students at the school who prefer nonfiction.

H.O.T. Problems Higher Order Thinking

8. **Find the Error** A survey of a seventh-grade class showed that 4 out of every 10 students are taking a trip during spring break. There are 150 students in the seventh grade. Maysoun is trying to determine how many of the seventh-grade students can be expected to take a trip during spring break. Find her mistake and correct it.



9. **Persevere with Problems** One letter tile is drawn from the bag and replaced 300 times. Predict how many times a consonant will *not* be picked.

10. **Persevere with Problems** A survey found that 80% of teens enjoy going to the movies in their free time. Out of 5,200 teens, predict how many said that they do not enjoy going to the movies in their free time.

11. **Model with Mathematics** Explain how to use a sample to predict what a group of people prefer. Then give an example of a situation in which it makes sense to use a sample.

12. **Model with Mathematics** Design a survey to give to your classmates. Construct a bar graph in the space below to represent your data. Then write and solve a problem that involves a prediction based on the data you collected.

Show your work.

Name _____ My Homework _____

Extra Practice

Solve.

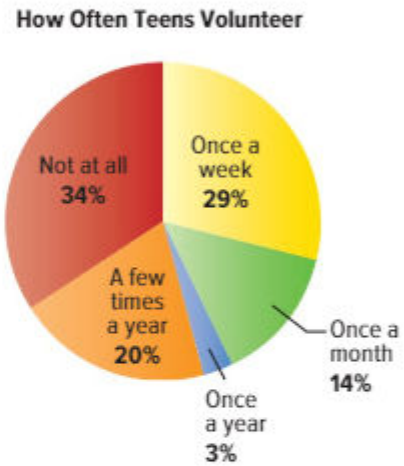
13. Bilal won 12 of the last 20 video games he played. Find the probability of Bilal winning the next game he plays. $\frac{3}{5}$, 0.6, or 60%

Homework Help

$$P(\text{winning}) = \frac{\text{number of games won}}{\text{number of games played}} = \frac{12}{20} \text{ or } \frac{3}{5}$$

14. Refer to Exercise 13. Suppose Bilal plays a total of 60 games with his friends over the next month. Predict how many of these games Bilal will win. _____

15. Use the graph that shows the number of times teens volunteer.



a. About 300,000 teens ages 12–14 live in Virginia. Predict the number of teens in this age group who volunteer a few times a year.

_____

b. Tennessee has about 250,000 teens ages 12–14. Predict the number of teens in this age group who volunteer once a week.

_____

c. About 240,000 teens ages 12–14 live in Missouri. Predict the number of teens in this age group who volunteer once a year.

_____

16. MP **Make a Prediction** The probability of Jamal making a free throw is 15%. Predict the number of free throws that he can expect to make if he attempts 40 free throws.

_____

Draw a line to match each situation with the appropriate equation or proportion.

17. 27 MP3s is what percent of 238 MP3s?

18. 238% of 27 is what number?

19. 27% of MP3 owners download music weekly. Predict how many MP3 owners out of 238 owners download music weekly.

a. $n = 27 \cdot 2.38$

b. $\frac{27}{100} = \frac{p}{238}$

c. $\frac{27}{238} = \frac{n}{100}$

Power Up! Test Practice

20. There were 515 students surveyed on how they spend time with their families. Which of the following estimates are accurate? Select all that apply.

- ☐ About 175 students spend time with their family eating dinner.
- ☐ About 72 students spend time with their family playing sports.
- ☐ About 50 students spend time with their family watching TV.
- ☐ About 38 students spend time with their family taking walks.

How Students Spend Time with Family	
Eating Dinner	34%
Watching TV	20%
Talking	14%
Playing Sports	14%
Talking Walks	4%
Other	14%

21. Yesterday a bakery baked 54 loaves of bread in 20 minutes. Today the bakery needs to bake 405 loaves of bread at the same rate. Select values to complete the model below to predict how long it will take to bake the bread today.

$$\frac{\square}{\square} = \frac{\square}{\square}$$

20
54
405
x

How long will it take to bake the bread today?

Spiral Review

22. A magazine rack contains 5 sports magazines, 7 news magazines, and 10 fashion magazines. After a magazine is chosen, it is *not* replaced. Find the probability of randomly choosing two fashion magazines.

23. Each week, Hareb's mother has him randomly choose a chore that he must complete from the list shown. The first week, he chose washing the dishes. What is the probability that Hareb will choose washing the dishes two more weeks in a row?

Weekly Chores
Collecting the trash
Folding the laundry
Cleaning the house
Washing the dishes
Cutting the grass

24. In how many different orders can a person watch 3 different movies? Use a list to show the sample space.

Lesson 2

Unbiased and Biased Samples

Real-World Link

Entertainment A T.V. programming manager wants to conduct a survey to determine which reality television show is the favorite of viewers in a certain viewing area. He is considering the three samples shown. Draw an X through the two samples that would not fairly represent all of the people in the viewing area.

Sample 1
100 people that are trying out for a reality show

Sample 2
100 students at your middle school

Sample 3
Every 100th person at a shopping mall

Explain why the two samples that you crossed out do *not* fairly represent all of the people in the viewing area? Explain.

Essential Question

HOW do you know which type of graph to use when displaying data?

Vocabulary

unbiased sample
simple random sample
systematic random sample
biased sample
convenience sample
voluntary response sample

Mathematical Practices
1, 3, 4, 5

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Work Zone

Biased and Unbiased Samples

To get valid results, a sample must be chosen very carefully. An **unbiased sample** is selected so that it accurately represents the entire population. Two ways to pick an unbiased sample are listed below.

Unbiased Samples		
Type	**Description**	**Example**
Simple Random Sample	Each item or person in the population is as likely to be chosen as any other.	Each student's name is written on a piece of paper. The names are placed in a bowl, and names are picked without looking.
Systematic Random Sample	The items or people are selected according to a specific time or item interval.	Every 20th person is chosen from an alphabetical list of all students attending a school.

In a **biased sample**, one or more parts of the population are favored over others. Two ways to pick a biased sample are listed below.

Biased Samples		
Type	**Description**	**Example**
Convenience Sample	A convenience sample consists of members of a population that are easily accessed.	To represent all the students attending a school, the principal surveys the students in one math class.
Voluntary Response Sample	A voluntary response sample involves only those who want to participate in the sampling.	Students at a school who wish to express their opinions complete an online survey.

Everyday Use
Bias is a tendency or prejudice

Math Use
Bias is an error introduced by selecting or encouraging a specific outcome

Examples

Determine whether the conclusion is valid. Justify your answer.

1. **A random sample of students at a middle school shows that 10 students prefer listening to rock, 15 students prefer listening to hip hop, and 25 students prefer no music while they exercise. It can be concluded that half the students prefer no music while they exercise.**

This is a simple random sample. So, the sample is unbiased and the conclusion is valid.

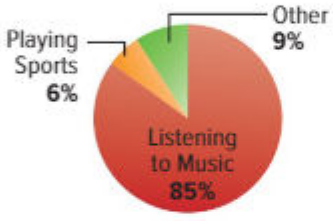
Determine whether each conclusion is valid. Justify your answer.

2. **Every tenth person who walks into a department store is surveyed to determine his or her music preference. Out of 150 customers, 70 stated that they prefer rock music. The manager concludes that about half of all customers prefer rock music.**

Since the population is every tenth customer of a department store, the sample is an unbiased, systematic random sample. The conclusion is valid.

3. **The customers of a music store are surveyed to determine their favorite leisure time activity. The results are shown in the graph. The store manager concludes that most people prefer to listen to music in their leisure time.**

Leisure Time Activities



The customers of a music store probably like to listen to music in their leisure time. The sample is a biased, convenience sample since all of the people surveyed are in one specific location. The conclusion is not valid.

Got it? **Do this problem to find out.**

Show your work.

a. A radio station asks its listeners to indicate their preference for one of two candidates in an upcoming election. Seventy-two percent of the listeners who responded preferred candidate A, so the radio station announced that candidate A would win the election. Is the conclusion valid? Justify your answer.

a. _____

Use Sampling to Predict

A valid sampling method uses unbiased samples. If a sampling method is valid, you can make generalizations about the population.

Example

4. **A store sells 3 types of pants: jeans, capris, and cargos. The store workers survey 50 customers at random about their favorite type of pants. The survey responses are indicated at the right. If 450 pairs of pants are ordered, how many should be jeans?**

Type	Number
Jeans	25
Capris	15
Cargos	10

First, determine whether the sample method is valid. The sample is a simple random sample since customers were randomly selected. Thus, the sample method is valid.

$\frac{25}{50}$ or 50% of the customers prefer jeans. So, find 50% of 450.

$0.5 \times 450 = 225$, so about 225 pairs of jeans should be ordered.

Guided Practice

1. Husam is trying to decide which of three golf courses is the best. He randomly surveyed people at a sports store and recorded the results in the table. Is the sample method valid? If so, suppose Husam surveyed 150 more people. How many people would be expected to vote for Rolling Meadows? (Example 4)

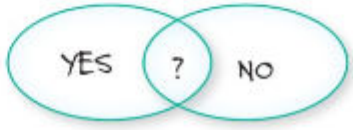
Course	Number
Whispering Trail	10
Tall Pines	8
Rolling Meadows	7

2. To find how much money the average American family spends to cool their home, 100 Alaskan families are surveyed at random. Of the families, 85 said that they spend less than AED75 per month on cooling. The researcher concluded that the average American family spends less than AED75 on cooling per month. Is the conclusion valid? Explain. (Examples 1–3)

3. **Building on the Essential Question** How is using a survey one way to determine experimental probability?

Rate Yourself!

Are you ready to move on? Shade the section that applies.



FOLDABLES Time to update your Foldable!

Independent Practice

Determine whether each conclusion is valid. Justify your answer.

(Examples 1–3)

1 To evaluate the quality of their product, a manufacturer of cell phones checks every 50th phone off the assembly line. Out of 200 phones tested, 4 are defective. The manager concludes that about 2% of the cell phones produced will be defective.

Show your work.

_____

_____

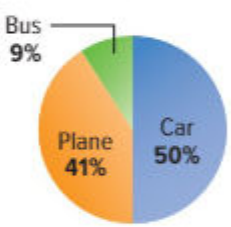
2. To determine whether the students will attend an arts festival at the school, Hassan surveys his friends in the art club. All of Hassan's friends plan to attend. So, Hassan assumes that all the students at his school will also attend.

_____

_____

3 A random sample of people at a mall shows that 22 prefer to take a family trip by car, 18 prefer to travel by plane, and 4 prefer to travel by bus. Is the sample method valid? If so, how many people out of 500 would you expect to say they prefer to travel by plane? (Example 4)

Preferred Ways to Travel



_____

_____

4. MP **Use Math Tools** Use the organizer to determine whether the conclusion is valid.

Step 1: Read the situation.	Hasan wants to predict the next student council president. He polls every fourth person from each grade level as they exit the cafeteria. In his poll, 65% chose Jassim. So, Hasan predicts Jassim will win the election.
Step 2: Determine the type of sample taken.	_____ _____
Step 3: Determine if the conclusion is valid.	_____ _____

H.O.T. Problems Higher Order Thinking

5. **Persevere with Problems** How could the wording of a question or the tone of voice of the interviewer affect a survey? Provide an example.

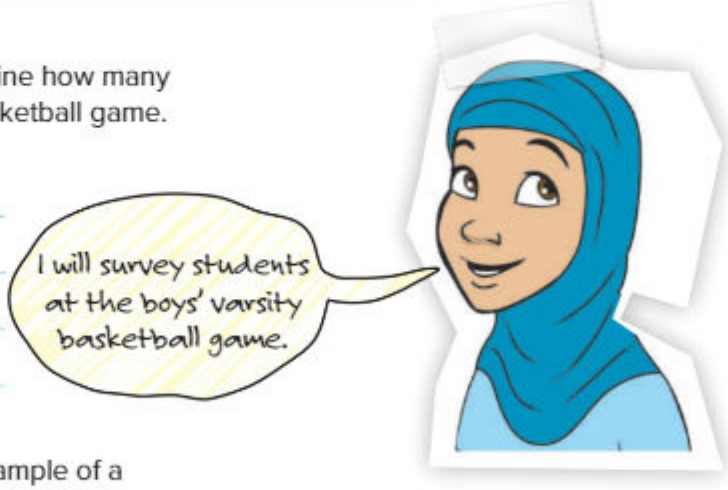
Justify Conclusions Determine whether each statement is *sometimes*, *always*, or *never* true. Explain your reasoning to a classmate.

6. A biased sample is valid.

7. A simple random sample is valid.

8. A voluntary response sample is valid.

9. **Find the Error** Hessa wants to determine how many students plan to attend the girls' varsity basketball game. Find her mistake and correct it.



10. **Model with Mathematics** Give an example of a data set from a random sample. Then make an inference about the population represented by the sample.

Name _____ My Homework _____

Extra Practice

Determine whether each conclusion is valid. Justify your answer.

11. To determine what people in California think about a proposed law, 5,000 people from the state are randomly surveyed. Of the people surveyed, 58% are against the law. The legislature concludes that the law should not be passed.

This is an unbiased, simple random sample because randomly selected Californians were surveyed. So, the conclusion is valid.

Homework Help

12. A magazine asks its readers to complete and return a questionnaire about popular scientists. The majority of those who replied liked one scientist the most, so the magazine decides to write more articles about that scientist.

13. The Student Council advisor asked every tenth student in the lunch line how they preferred to be contacted with school news. The results are shown in the table. Is this a random sample? If yes, suppose there are 684 students at the school. How many can be expected to prefer E-mail?

Method	Number
E-mail	16
Newsletter	12
Announcement	5
Telephone	3

MP **Justify Conclusions** **Each of the following surveys results in a biased sample. For each situation, explain why the survey is biased. Then explain how you would change the survey to obtain an unbiased sample.**

14. A store manager sends an E-mail survey to customers who have registered at the store's Web site.

15. A school district surveys the family of every tenth student to determine if they would vote in favor of the construction of a new school building.

Power Up! Test Practice

16. Reham surveyed all the members of her softball team about their favorite sport. Based on these results, Reham concludes that softball is the favorite sport among all her classmates. Explain why Reham's conclusion might *not* be valid. How could she change the survey to achieve a more valid conclusion?

Sport	Number of Members
Softball	12
Basketball	5
Soccer	3
Volleyball	8

17. Ms. Khadija determined that 60% of the students in her classes brought an umbrella to school when the weather forecast predicted rain. She has a total of 150 students in her classes. Determine if each statement represents a valid or invalid conclusion.

a. On days when rain is forecasted, less than $\frac{2}{5}$ of her students bring an umbrella to school. ☐ Valid ☐ Invalid

b. On days when rain is forecasted, about 90 of her students bring an umbrella to school. ☐ Valid ☐ Invalid

c. On days when rain is forecasted, more than $\frac{1}{2}$ of her students bring an umbrella to school. ☐ Valid ☐ Invalid

Spiral Review

For Exercises 18 and 19, use the table that shows Khawla's first six math test scores.

Test	1	2	3	4	5	6
Score	88%	92%	70%	96%	84%	96%

18. Find the mean, median, and mode of Khawla's test scores. Round to the nearest tenth if necessary.

mean: _____ median: _____ mode: _____

19. Determine which measure of center best represents Khawla's performance. Justify your reasoning.

_____

_____

Inquiry Lab

Multiple Samples of Data

Inquiry **WHY is it important to analyze multiple samples of data before making predictions?**

MP Mathematical Practices 1, 3, 4, 5

A host at a restaurant randomly hands out crayons to young children. There are three different color crayons: green (G), red (R), and blue (B). The server gives out the green crayon 40% of the time, the red crayon 40% of the time, and the blue crayon 20% of the time.

Hands-On Activity 1

Tools

When you draw a conclusion about a population from a sample of data, you are making *inferences* about that population. Sometimes, making inferences about a population from only one sample is not as accurate as using multiple samples of data.

Use a spinner to simulate the situation above.

Step 1 Create a spinner with five equal sections. Label two sections G. Label another two sections R and label one section B.

Step 2 Each spin of the spinner represents a young child receiving a crayon. Spin the spinner 20 times. Record the number of times each color of crayon was received in the column labeled Sample 1 in the table below. Repeat two more times. Record the results in the columns labeled Sample 2 and Sample 3 in the table.

Color	Sample 1 Frequency	Sample 2 Frequency	Sample 3 Frequency
Green			
Red			
Blue			

Compare the results of the 3 samples. Do you notice any differences?

The most commonly used keyboard is the QWERTY keyboard. However, there is another type of keyboard called the Dvorak keyboard that is based on letter frequency. Complete the Activity below about letter frequencies.

Hands-On Activity 2

The table at the right contains fifteen randomly selected words from the English language dictionary.

Sample 1		
airport	juggle	sewer
blueberry	lemon	standard
costume	mileage	thread
doorstop	percentage	vacuum
instrument	print	whale

Step 1 Find the frequency of each letter. Record the frequencies in the Sample 1 rows of the tables below.

Letter	a	b	c	d	e	f	g	h	i	j	k	l	m
Sample 1 Frequency													
Sample 2 Frequency													
Sample 3 Frequency													

Letter	n	o	p	q	r	s	t	u	v	w	x	y	z
Sample 1 Frequency													
Sample 2 Frequency													
Sample 3 Frequency													

Step 2 Randomly select another 15 words from a dictionary. Record the frequency of the letters in the rows labeled Sample 2 in the tables above.

Step 3 Repeat Step 2. Record the frequency of the letters in the rows labeled Sample 3.

Investigate

Work with a partner to collect multiple samples based on the following situation.

Halima and Hamdah are making centerpieces for their school's fall ceremony. They randomly select a ribbon to use in each centerpiece. There are four different colors of ribbon to choose from: brown (B), green (G), orange (O), and yellow (Y).

1. **Model with Mathematics** Design a method to simulate how many times each ribbon will be selected. Describe your simulation.

2. Use the method you described in Exercise 1 to simulate the ribbon selection 20 times. Record the frequency of each color selection in the Sample 1 Frequency column of the table below.

Color	Sample 1 Frequency	Sample 2 Frequency	Sample 3 Frequency
Brown			
Green			
Orange			
Yellow			

3. Repeat the process described in Exercise 2 two more times. Record the frequencies of each color selection in the Sample 2 and Sample 3 columns.

4. Make an inference to determine which color was selected the most often in each sample.

5. The *relative frequency* of a color being selected is the ratio of the number of times the color was selected to the total number of selections. Find the relative frequency of an orange ribbon being selected for each sample.

   Sample 1: _____ Sample 2: _____ Sample 3: _____

6. Hamdah predicts that 5 out of 10 centerpieces will have an orange ribbon. How far off is Hamdah's prediction? Explain.

Analyze and Reflect

Work with a partner to answer the following questions. Refer to Activity 2.

7. What is the relative frequency for the letter *e* for each sample? Round to the nearest hundredth.

Sample 1: _____ Sample 2: _____ Sample 3: _____

8. What is the mean relative frequency of the letter *e* for the three samples? the median relative frequency? Round to the nearest tenth if necessary.

mean relative frequency: _____ median relative frequency: _____

9. MP **Use Math Tools** Research on the Internet to find the actual relative frequency of the letter *e* for words in the English language. How do your sample results compare to the actual relative frequency?

_____

_____

10. MP **Reason Inductively** Write a few sentences describing the inferences you can make about the frequency of letters in the words in the English language using your three samples.

_____

_____

_____

Create

11. MP **Justify Conclusions** Research on the Internet to find the relative frequency of other letters in words in the English language. Write how your sample results compare to the actual frequencies. Note any differences.

_____

_____

_____

12. Inquiry WHY is it important to analyze multiple samples of data before making predictions?

_____

_____

Lesson 3

Misleading Graphs and Statistics

Real-World Link

Hockey The Stanley Cup is awarded annually to the champion team in the National Hockey League. The graph shows the total number of points scored in Stanley Cup playoff games by three players during their careers.

1. According to the size of the players, how many times more points does Messier appear to have than Kurri?

2. Do you think this is representative of the players' number of points? Explain.

3. What reason could someone have for intentionally creating a misleading Stanley Cup graph?

Essential Question

HOW do you know which type of graph to use when displaying data?

Mathematical Practices
1, 3, 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

(1) Persevere with Problems	(5) Use Math Tools
(2) Reason Abstractly	(6) Attend to Precision
(3) Construct an Argument	(7) Make Use of Structure
(4) Model with Mathematics	(8) Use Repeated Reasoning

Work Zone

Identify a Misleading Graph

Graphs let readers analyze data easily, but are sometimes made to influence conclusions by misrepresenting the data.

Example

1. **Explain how the graphs differ.**

Graph A

Graph B

The graphs show the same data. However, the graphs differ in that Graph A uses an interval of 4, and Graph B uses an interval of 2.

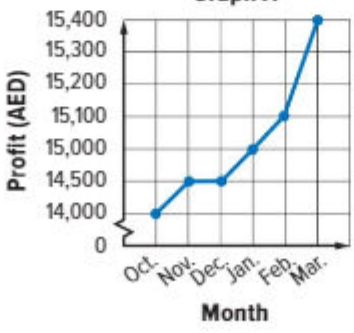
Which graph appears to show a sharper increase in price?
Graph B makes it appear that the prices increased more rapidly even though the price increase is the same.

Which graph might the Student Council use to show that while ticket prices have risen, the increase is not significant? Why?
They might use Graph A. The scale used on the vertical axis of this graph makes the increase appear less significant.

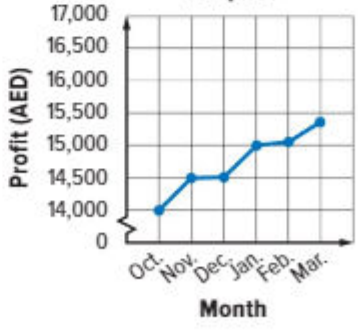
Got it? **Do this problem to find out.**

a. The line graphs show monthly profits of a company from October to March. Which graph suggests that the business is extremely profitable? Is this a valid conclusion? Explain.

Graph A



Graph B



Show your work.

a. _____

Changing Scales
To emphasize a change over time, reduce the scale interval on the vertical axis.

Misleading Statistics

Statistics can also be used to influence conclusions.

Example

2. **An amusement park boasts that the average height of their roller coasters is 51 meters Explain how this might be misleading.**

Park Roller Coaster Heights	
Coaster	**Height (m)**
Viper	32
Monster	40
Red Zip	35
Tornado	110
Riptide	38

Mean $\frac{32 + 40 + 35 + 110 + 38}{5} = \frac{255}{5}$

$= 170$

Median 32, 35, 38, 40, 110

Mode none

The average used by the park was the mean. This measure is much greater than most of the heights listed because of the coaster that is 110 meters. So, it is misleading to use this measure to attract visitors.

A more appropriate measure to describe the data is the median, 38 meters, which is closer to the height of most of the coasters.

Mode

The mode is the number or numbers that appear most often in a set of data.

Got it? **Do this problem to find out.**

b. Find the mean, median, and mode of the sofa prices shown in the table. Which measurement might be misleading in describing the average cost of a sofa? Explain.

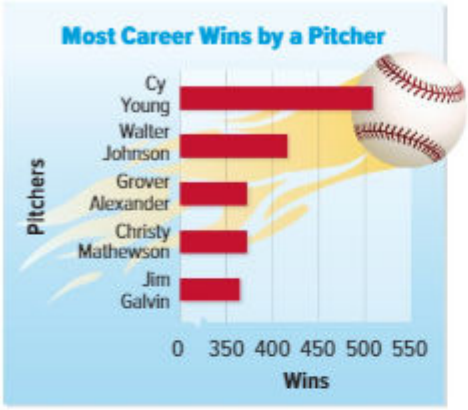
Sofa Prices	
Sofa Style	**Cost**
leather	AED1,700
reclining	AED1,400
DIY assembly	AED350
sectional	AED1,600
micro-fiber	AED1,400

Show your work.

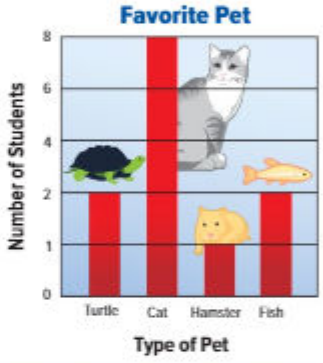
b. _____

Guided Practice

1. The graph suggests that Cy Young had three times as many wins as Jim Galvin. Is this a valid conclusion? Explain. (Example 1)



2. The graph at the right shows the results of a survey to determine students' favorite pets. Why is the graph misleading? (Example 1)



3. The table lists the five largest land vehicle tunnels in the United States. Write a convincing argument for which measure of center you would use to emphasize the average length of the tunnels. (Example 2)

U.S. Vehicle Tunnels	Length (ft)
Anton Anderson Memorial	3,990
E. Johnson Memorial	2,688
Eisenhower Memorial	2,682
Allegheny	1,822
Liberty Tubes	1,776

4. **Building on the Essential Question** Describe at least two ways in which the display of data can influence the conclusions reached.

Rate Yourself!

How well do you understand misleading graphs and statistics? Circle the image that applies.

Name _____ My Homework _____

Independent Practice

Go online for Step-by-Step Solutions eHelp

1 Which graph could be used to indicate a greater increase in monthly gas prices? Explain. (Example 1)

Show your work.

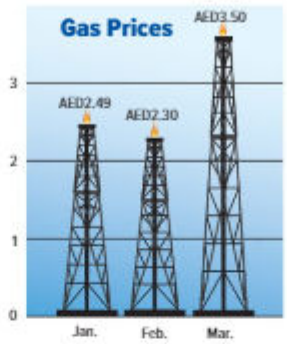
_____

_____

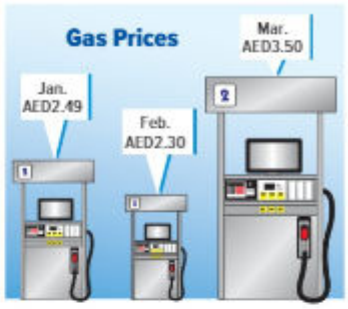
_____

_____

Graph A



Graph B



For Exercises 2 and 3, use the table. (Example 2)

2. Find the mean, median, and mode of the data. Which measure might be misleading in describing the average annual number of visitors who visit these sights? Explain.

_____

_____

_____

Annual Sight-Seeing Visitors	
Sight	**Visitors**
Cape Cod	4,600,000
Grand Canyon	4,500,000
Lincoln Memorial	4,000,000
Castle Clinton	4,600,000
Smoky Mountains	10,200,000

3 Which measure would be best if you wanted a value close to the most number of visitors? Explain.

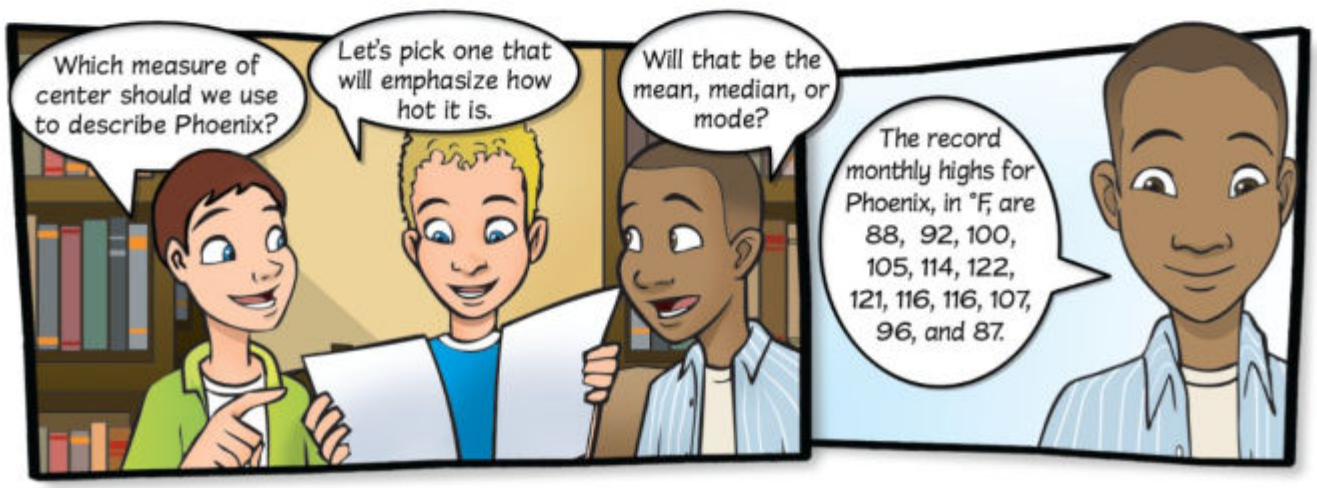
_____

_____

4. MP **Model with Mathematics** Refer to the graphic novel frame below. Which measure of center should the students use? _____

_____

_____



For Exercises 5 and 6, create a display that would support each argument. The monthly costs to rent an apartment for the last five years are AED500, AED525, AED560, AED585, and AED605.

5. Rent has remained fairly stable.

Show your work.

6. Rent has increased dramatically.

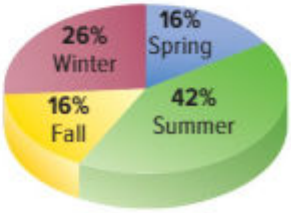
H.O.T. Problems Higher Order Thinking

7. MP **Reason Inductively** How could the graph you created in Exercise 5 help influence someone's decision to rent the apartment?

8. MP **Persevere with Problems** Does adding values that are much greater or much less than the other values in a set of data affect the median of the set? Give an example to support your answer.

9. MP **Reason Inductively** The circle graph shows the results of a survey. In what way is this graph misleading? Explain.

Favorite Time of Year

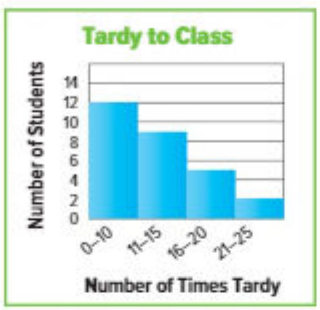


Name _____ My Homework _____

Extra Practice

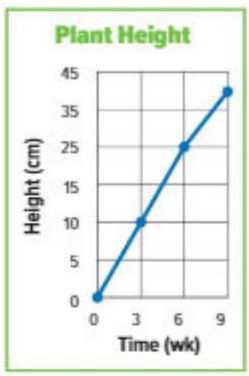
10. To determine how often his students are tardy, Mr. Rana considered Rana the attendance record for his first period class. Why is this graph misleading?

Homework Help



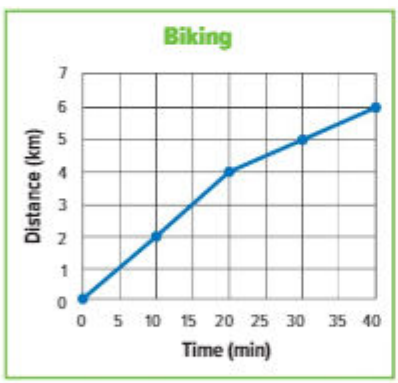
There are not equal intervals on the horizontal axis. So, the height of the bars is not representative of the sample.

11. The graph shows the height of a plant after 9 weeks of growth. Why is the graph misleading?

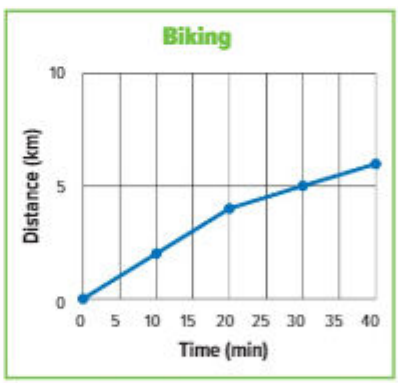


12. **Justify Conclusions** Each of the graphs below show the distance Khamis travels on his bike. Khamis wants to impress his friends with the distance he travels. Which graph should he show his friends? Explain.

Graph A



Graph B



13. The scores Ayesha received on her math tests were 80, 90, 85, 100, 100, and 84. Why might it be misleading for Ayesha to say that most of the time she receives a score of 100?

Power Up! Test Practice

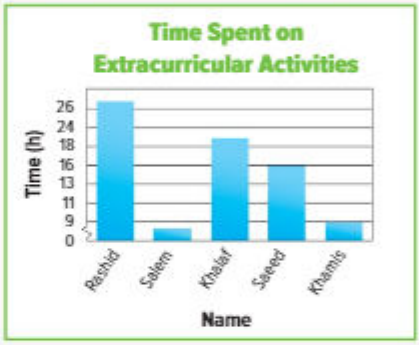
14. Phones For All uses the display shown at the right to compare the number of minutes that they offer per month versus their competitor.

a. How many more minutes per month does Phones For All offer than its competitor?

b. Why might the display be misleading?

15. The graph shows the average number of hours each week that certain students spend on extracurricular activities after school. Which of the following describe reasons why the graph may be misleading? Select all that apply.

- ☐ The graph does not show the number of hours each student spent on extracurricular activities.
- ☐ The intervals on the vertical scale are inconsistent.
- ☐ The graph's title is misleading.



Spiral Review

Draw a histogram to represent the set of data.

16.

Test Scores		
Percent	**Tally**	**Frequency**
50–59	|	1
60–69	||	2
70–79	||||	4
80–89	𝍸 𝍸 |	11
90–99	𝍸 |||	8

MP Problem-Solving Investigation
Use a Graph

MP Mathematical Practices 1, 3, 4

Case #1 Fishy Waters

Shaima recently purchased a saltwater aquarium. She needs to add 1 tablespoon of sea salt for every 5 liters of water.

Sea Salt Requirements						
Tablespoons of Sea Salt	1	2	3	4	5	6
Capacity of Tank (liters)	5	10	15	20	25	30

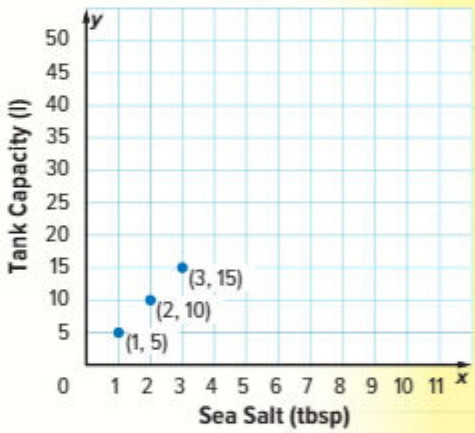
How can she use a graph to show the number of tablespoons of salt required for a 50-liter saltwater fish tank?

1 Understand *What are the facts?*

You know the number of liters of the tank. You need to show the number of tablespoons of sea salt.

2 Plan *What is your strategy to solve this problem?*

Organize the rest of the data in a graph so you can easily see any trends.



3 Solve *How can you apply the strategy?*

Continue the graph until you align horizontally with 50 liters. Graph a point. What value of sea salt corresponds with the point?

4 Check *Does the answer make sense?*

Find the unit rate of tablespoons of sea salt per liter of water. Multiply the unit rate by the number of liters to find the number of tablespoons of sea salt.

$$\frac{0.2 \text{ tbsp salt}}{1 \text{ l water}} \times \frac{50 \text{ l water}}{1} = \square \text{ tbsp salt} \quad ✓$$

Analyze the Strategy

MP **Make a Prediction** Suppose the tank holds 32 liters. Predict how much sea salt is required.

Case #2 Calories

The table shows the average number of Calories burned while sleeping for various numbers of hours. Assume the trend continues.

Calories Burned While Sleeping	
Hours	Calories
6	386
7	450
8	514
9	579

Make a graph to determine the approximate number of Calories that are burned by sleeping for 10 hours.

1 Understand

Read the problem. What are you being asked to find?

I need to find _____.

What information do you know?

There is an average of _____ Calories burned while sleeping for 6 hours and 514 Calories burned while sleeping for _____ hours.

2 Plan

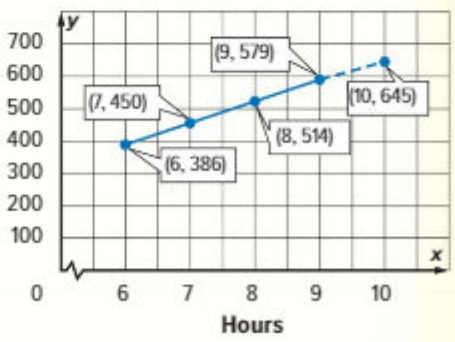
Choose a problem-solving strategy.

I will use the _____ strategy.

3 Solve

Use your problem-solving strategy to solve the problem.

Continue the graph until it is aligned vertically with 10 hours. Graph a point. Find what value of Calories corresponds with the point. So, about _____ Calories are burned while sleeping for 10 hours.



4 Check

Review the data in the table.

450 − 386 = 64; 514 − 450 = 64; 579 − 514 = 65. 645 − 579 = 66.

So, the answer seems reasonable.

Collaborate

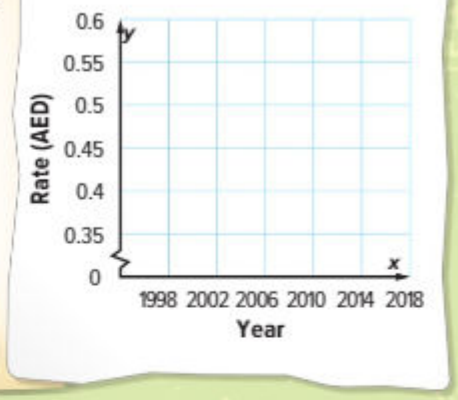
Work with a small group to solve the following cases. Show your work on a separate piece of paper.

Case #3 Postage

The table shows the postage stamp rate from 1999 to 2009.

Make a graph of the data. Predict the year the postage rate will reach AED0.52.

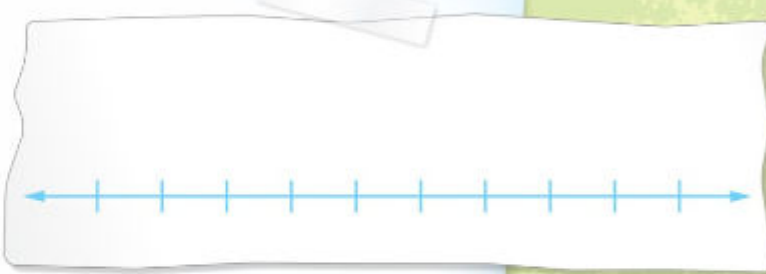
Postage Stamp Rates	
Year	Cost (AED)
1999	0.33
2001	0.34
2002	0.37
2006	0.39
2007	0.41
2008	0.42
2009	0.44



Case #4 Trains

The lengths of various train rides are 4, 1, 2, 3, 6, 2, 3, 2, 5, 8, and 4 hours.

Draw a box plot for the data set. What percent of the train rides are longer than 3 hours?



Case #5 Advertising

A local newspaper charges AED14.50 for every three lines of a classified ad plus a 5% sales tax.

What is the cost of a 7-line ad? Round to the nearest hundredth.

Use any strategy!

Case #6 Anatomy

Each human hand has 27 bones. There are 6 more bones in the fingers than in the wrist. There are 3 fewer bones in the palm than in the wrist.

How many bones are in each part of the hand?

Mid-Chapter Check

Vocabulary Check

1. **Be Precise** Define *sample*. Give an example of a sample of the students in a middle school. (Lesson 1)

2. Fill in the blank in the sentence below with the correct terms. (Lesson 2)

_____ and _____ are two types of unbiased samples.

Skills Check and Problem-Solving

3. A travel agent surveyed her customers to determine their favorite vacation locations. Use the table to find the probability of choosing a beach vacation. (Lesson 1)

Show your work.

Vacation Locations	
Location	**Customers**
amusement park	2
beach	11
campground	8
national park	4

4. Refer to the table. Suppose 120 customers are planning vacations. Predict how many will plan a national park vacation. (Lesson 1)

5. The number of points Sultan scored in 5 basketball games is 10, 8, 9, 8, and 30. Why might it be misleading for Sultan to say that he averages 13 points per game? (Lesson 3)

6. **Persevere with Problems** An online gaming site conducted a survey to determine the types of games people play online. The results are shown in the circle graph. If 1,500 people participated in the study, how many more would play card games than arcade games?

(Lesson 1)

Games People Play Online

7. An owner of a restaurant wants to conduct a survey about possible menu changes. Give an example of a sampling method that would produce a valid sample. (Lesson 2)

Inquiry Lab
Collect Data

Inquiry **HOW can you use the measures of center and the range to compare two populations?**

MP Mathematical Practices 1, 3, 4

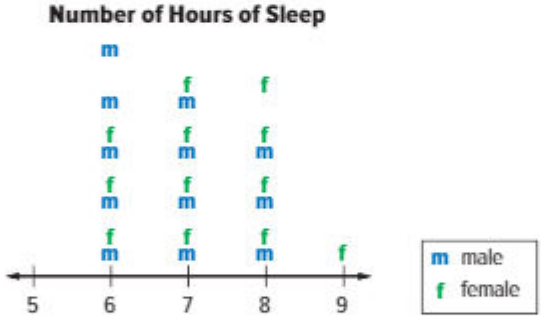
Studies show that teens need around 9 hours of sleep each night to stay healthy.

Hands-On Activity

Step 1 The results of a survey that asked 24 teens how many hours they slept last night are shown below. The teens were split into two populations, male and female.

Males	7	7	6	8	6	8	7	6	7	6	8	6
Females	8	8	7	6	8	7	6	6	7	8	9	7

Step 2 Graph the data for each population on a single line plot.



Step 3 Find the measures of center and range for each population.

	Mean	Median	Mode	Range
Males	$6.8\overline{3}$			
Females				

Are the data for males more or less varied than females?

Which measure most accurately represents the data of the whole class?

Explain.

Investigate

Collaborate

1. Work with a partner to design your own survey that meets the following guidelines.

- Create a survey question that involves two populations. For example, you might want to know about how many hours of sleep per night male students get in your school versus female students. Write your survey question below.

- Survey a random sample that is representative of your school's population. Survey at least 25 people. Collect the data and record your results in a table on a separate piece of paper.
- Create a display of your data. Be sure that the display shows the two populations.

Analyze and Reflect

Collaborate

Work with a partner to complete the exercises below based on the data you collected above.

2. Determine the measures of center (mean, median, and mode) and the range for each population's set of data.

3. MP **Reason Inductively** Compare the two populations. Are the data for one population more or less varied than the data for the other population? Justify your response.

4. Describe any other comparative inferences, or conclusions, you can make about differences in the two populations.

Create

On Your Own

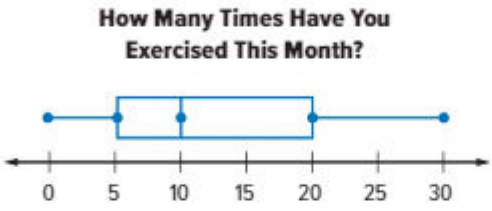
5. Inquiry HOW can you use the measures of center and the range to compare two populations?

Lesson 4

Compare Populations

Real-World Link

Exercise Mr. Amer surveyed the students in his first period gym class to find out how many times they exercised this month. The box plot below shows the results.



1. Find the following values.

Minimum: ☐ First Quartile: ☐

Maximum: ☐ Third Quartile: ☐

Range: ☐ Interquartile Range: ☐

2. What is the median? What does the median represent?

3. Write a conclusion that you can make from the box plot.

Essential Question

HOW do you know which type of graph to use when displaying data?

Vocabulary

double box plot

double dot plot

Mathematical Practices

1, 3, 4, 6

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

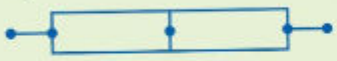
(1) Persevere with Problems	(5) Use Math Tools
(2) Reason Abstractly	(6) Attend to Precision
(3) Construct an Argument	(7) Make Use of Structure
(4) Model with Mathematics	(8) Use Repeated Reasoning

Work Zone

Box plots

A box plot is symmetric if the data are balanced at the center.

Symmetric



Not Symmetric

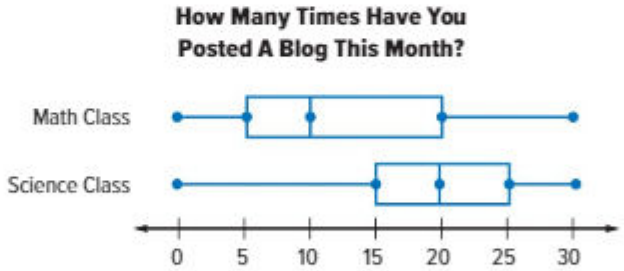
Compare Two Populations

A **double box plot** consists of two box plots graphed on the same number line. A **double dot plot** consists of two dot plots that are drawn on the same number line. You can draw inferences about two populations in a double box plot or double dot plot by comparing their centers and variations. The centers and variations to use are shown.

Most Appropriate Measures			
	Both sets of data are symmetric.	**Neither set of data is symmetric.**	**Only one set of data is symmetric.**
Measure of Center	mean	median	median
Measure of Variation	mean absolute deviation	interquartile range	interquartile range

Example

1. **Lamya surveyed a different group of students in her science and math classes. The double box plot shows the results for both classes. Compare their centers and variations. Write an inference you can draw about the two populations.**



Neither box plot is symmetric. Use the median to compare the centers and the interquartile range to compare the variations.

	Math Class	**Science Class**
Median	10	20
Interquartile Range	20 − 5, or 15	25 − 15, or 10

Overall, the science students posted more blogs than the math students. The median for the science class is twice the median for the math class. There is a greater spread of data around the median for the math class than the science class.

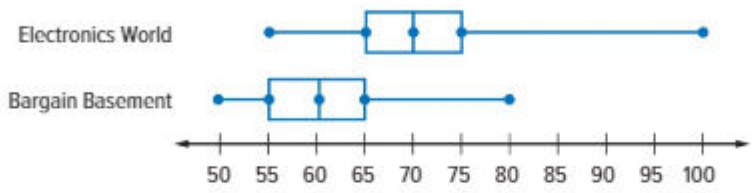
Got it? Do this problem to find out.

Show your work.

a. The double box plot shows the costs of MP3 players at two different stores. Compare the centers and variations of the two populations. Write an inference you can draw about the two populations.

a. _____

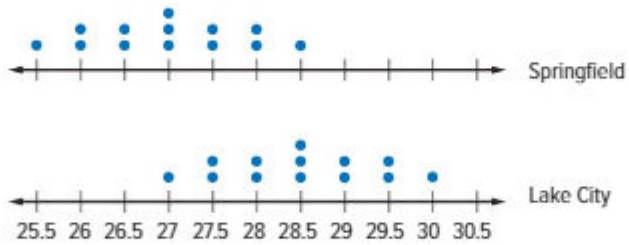
Cost of MP3 Players (AED)



Example

2. **The double dot plot below shows the daily high temperatures for two cities for thirteen days. Compare the centers and variations of the two populations. Write an inference you can draw about the two populations.**

Daily High Temperatures (°C)



Both dot plots are symmetric. Use the mean to compare the centers and use the mean absolute deviation, rounded to the nearest tenth, to compare the variations.

	Springfield	Lake City
Mean	27	28.5
Mean Absolute Deviation	0.8	0.8

While both cities have the same variation, or spread of data about each of their means, Lake City has a greater mean temperature than Springfield.

Mean Absolute Deviation

To find the mean absolute deviation, find the absolute values of the differences between each value and the mean. Then find the average of those differences.

Show your work.

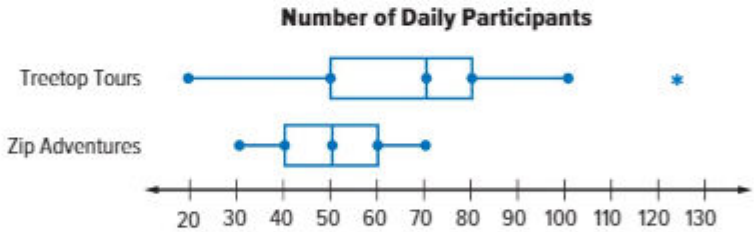
b. _____

Got it? **Do this problem to find out.**

b. The double dot plot shows the number of new E-mails in each of Abdulrahman's and Tarek's inboxes for sixteen days. Compare the centers and variations of the two populations. Write an inference you can draw about the two populations.

Examples

3. **The double box plot shows the daily participants for two zip line companies for one month. Compare the centers and variations of the two populations. Which company has the greater number of daily participants?**



The distribution for Zip Adventures is symmetric, while the distribution for Treetop Tours is not symmetric. Use the median and the interquartile range to compare the populations.

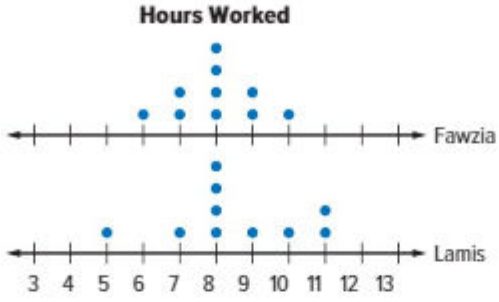
	Treetop Tours	Zip Adventures
Median	70	50
Interquartile Range	30	20

Overall, Treetop Tours has a greater number of daily participants. However, Treetop Tours also has a greater variation, so it is more difficult to predict how many participants they may have each day. Zip Adventures has a greater consistency in their distribution.

STOP and Reflect

What can you tell about the set of data for Zip Adventures by looking at its box plot? Write your answer in the space below.

4. **The double dot plot shows Fawzia's and Lamis's number of hours worked in two weeks at their part-time jobs. Compare the centers and variations of the two populations. Who typically works the greater number of hours in a week?**



The distribution for Fawzia's number of hours is symmetric, while the distribution for Lamis's number of hours is not symmetric. Use the median and interquartile range to compare the populations.

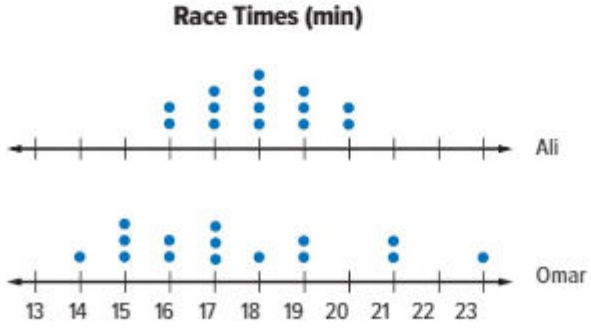
	Fawzia	Lamis
Median	8	8
Interquartile Range	2	2

The median and interquartile range for both sets of data are the same. However, the interquartile range for Lamis's number of hours worked is the difference of 10 and 8, while the interquartile range for Fawzia's number of hours is the difference of 9 and 7. So, Lamis typically works more per week.

Got it? **Do this problem to find out.**

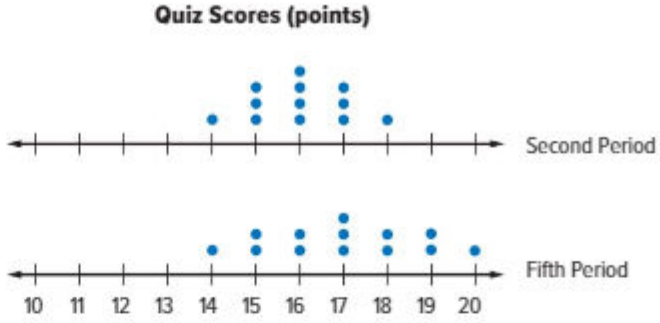
c. The double dot plot shows Ali's and Omar's race times for a five-kilometer race. Compare the centers and variations of the two populations. Which runner is more likely to run a faster race?

c. _____



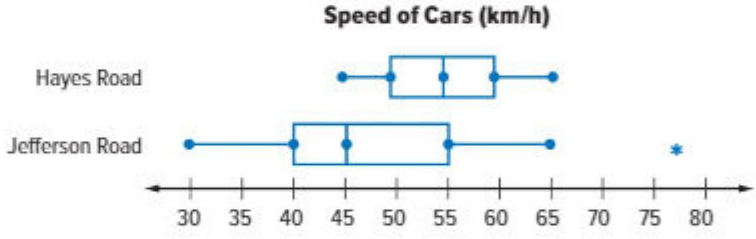
Guided Practice

1. The double dot plot at the right shows the quiz scores out of 20 points for two different class periods. Compare the centers and variations of the two populations. Round to the nearest tenth. Write an inference you can draw about the two populations. (Examples 1 and 2)



Show your work.

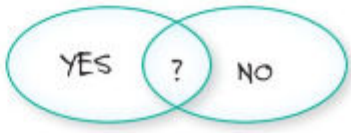
2. The double box plot shows the speeds of cars recorded on two different roads in Hamilton County. Compare the centers and variations of the two populations. On which road are the speeds greater? (Examples 3 and 4)



3. **Building on the Essential Question** Muna recorded the daily temperatures for two cities for 30 days. The two populations have similar centers, but City A has a greater variation than City B. For which city can you more accurately predict the daily temperature? Explain.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

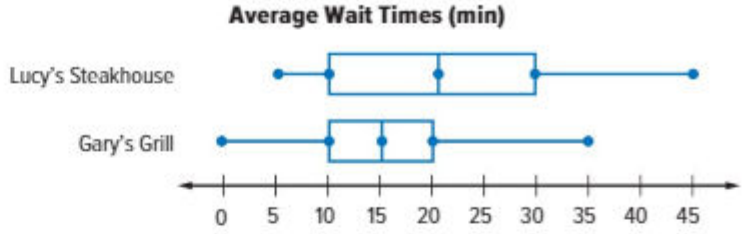


Name _____ My Homework _____

Independent Practice

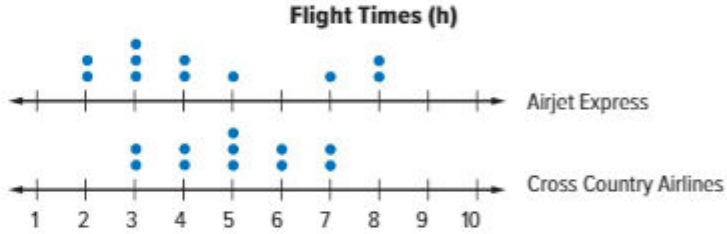
Go online for Step-by-Step Solutions eHelp

1 Obaid randomly asked customers at two different restaurants how long they waited for a table before they were seated. The double box plot shows the results. Compare their centers and variations. Write an inference you can draw about the two populations. (Examples 1 and 2)



Show your work.

2. The double dot plot shows the times, in hours, for flights of two different airlines flying out of the same airport. Compare the centers and variations of the two populations. Which airline's flights had shorter flight times? (Examples 3 and 4)



Copy and Solve **Write your answers for Exercise 3 on a separate piece of paper.**

3. MP **Multiple Representations** For a science project, Maha is measuring the growth of two plants.

Weekly Plant Growth (cm)								
	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8
Plant A	2	3	2	2.5	3.4	3	2.5	3
Plant B	3	2.5	3	3.4	3.2	3.8	3.5	2.5

a. **Numbers** Find the median and interquartile range for both plants.

b. **Graphs** Graph the data using a double box plot.

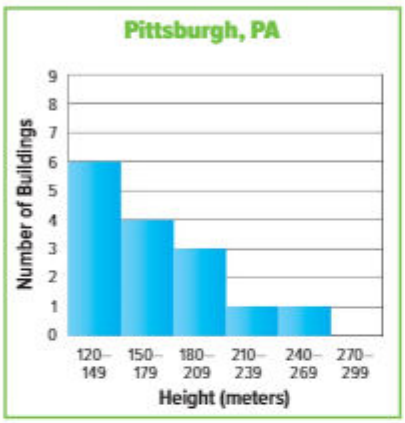
c. **Words** Write an inference you can draw about the two populations.

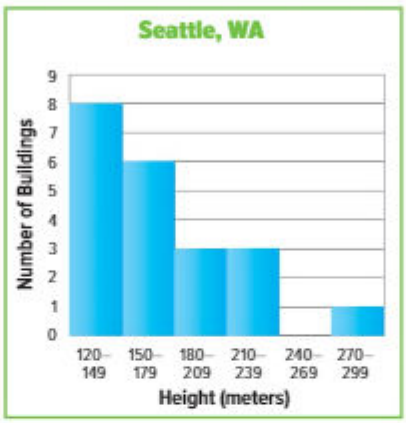
4. The median and interquartile range of a set of data is shown. Write a set of data consisting of seven values for the pair of measures.

Median: 6 Interquartile Range: 5

H.O.T. Problems Higher Order Thinking

5. **Persevere with Problems** The histograms below show the number of tall buildings for two cities. Explain why you cannot describe the specific location of the centers and spreads of the histograms.





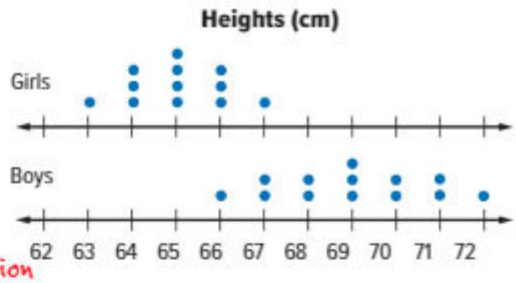
6. **Model with Mathematics** Refer to Exercise 1. What is a specific question you could ask about the two populations?

7. **Model with Mathematics** Two hockey teams, the Warriors and the Bulldogs, played 15 games each during a month. Both scored a minimum of 0 goals and a maximum of 8 goals. The Bulldogs generally scored fewer goals than the Warriors. Draw a double box plot that could represent the situation.

Show your work.

Extra Practice

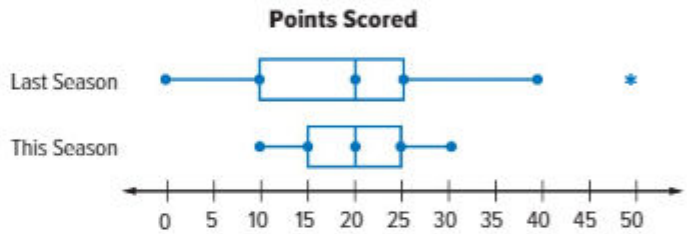
8. The double dot plot shows the heights in centimeters for the girls and boys in Adnan's math class. Compare the centers and variations of the two populations. Round to the nearest tenth. Write an inference you can draw about the two populations.



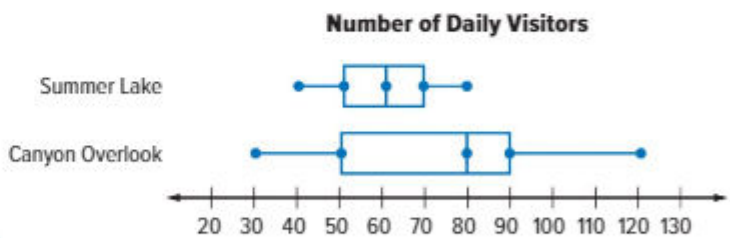
Homework Help

Both plots are symmetric. The girls' heights have a mean of 162.5 centimeters with a mean absolute deviation of about 2 centimeters. The boys' heights have a mean of 172.5 centimeters with a mean absolute deviation of about 3.5 centimeters. Overall, the girls' heights are lower than the boys' heights and are also more consistently grouped together.

9. The double box plot shows the number of points scored by the football team for two seasons. Compare the centers and variations of the two populations. During which season was the team's performance more consistent?



10. The double box plot shows the number of daily visitors to two different parks. Compare the centers and variations of the two populations. In general, which park has more daily visitors?

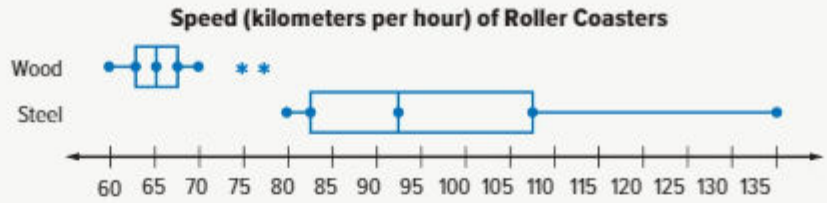


11. **Be Precise** The median and interquartile range of a set of data is shown. Write a set of data consisting of seven values for the pair of measures.

Median: 5 Interquartile Range: 5

Power Up! Test Practice

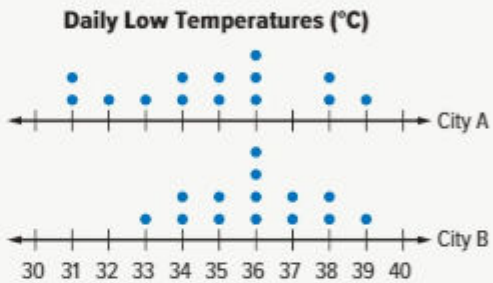
12. The double box plot shows the top speeds reached by wood and steel roller coasters.



Which of the following is not true about the double box plot? Select all that apply.

- ☐ The data for steel coasters is symmetric.
- ☐ The data for wood coasters is symmetric.
- ☐ The top speed of the fastest steel coaster is 135 kilometers per hour.
- ☐ The top speed of the slowest wooden coaster is 60 kilometers per hour.

13. The double dot plot shows the daily low temperatures of two cities in January over a two week period. Determine if each statement is true or false



a. The medians are the same. ☐ True ☐ False

b. The interquartile ranges are the same. ☐ True ☐ False

c. The temperatures for City B are more consistent. ☐ True ☐ False

Spiral Review

Find the mean absolute deviation of each set of data. Round to the nearest hundredth if necessary.

14. _____

Maximum Speeds of Boats (km/h)			
40	48	58	60
66	72	80	88

15. _____

Populations of Largest U.S. Cities (millions)			
1.3	3.8	1.5	8.4
0.9	1.4	2.3	1.3

16. Refer to the graph in Exercise 2. Describe the shape of the distribution of the data for Airjet Express.

17. Refer to the graph in Exercise 10. Describe the shape of the distribution of the data for Canyon Overlook.

Inquiry Lab

Visual Overlap of Data Distributions

Inquiry **WHAT does the ratio $\frac{\text{difference in means}}{\text{mean absolute deviation}}$ tell you about how much visual overlap there is between two distributions with similar variation?**

MP Mathematical Practices 1, 3

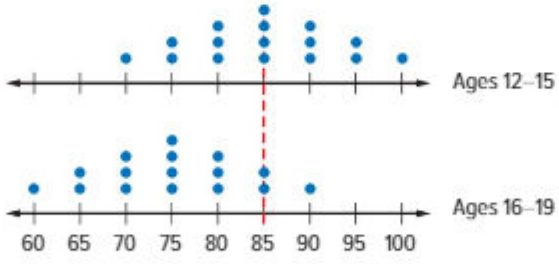
A survey was done. The tables below show the number of text messages sent and received daily for two different age groups.

Hands-On Activity

You can compare two numerical data sets by comparing the shape of their distributions. The **visual overlap** of two distributions with similar variation is a visual demonstration that compares their centers to their variation, or spread.

Step 1 Use a double dot plot to display the data in each table.

Text Messages Sent and Received



Step 2 Find the mean number of text messages for each age group.

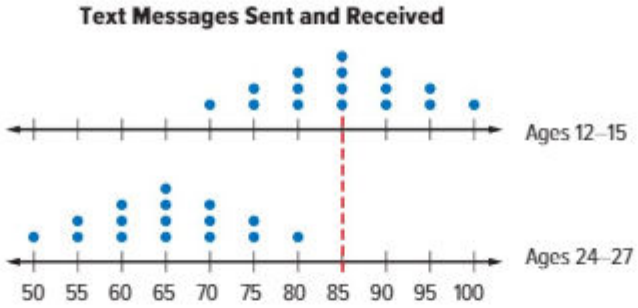
Ages 12–15 mean = ☐ **Ages 16–19 mean =** ☐

Step 3 A red dotted line has been drawn through both dot plots that corresponds to the mean for the age group, 12–15 years. Draw a vertical dotted line through both dot plots that corresponds to the mean for the age group, 16–19 years. The dotted lines show the visual overlap between the centers.

Investigate

Collaborate

Work with a partner.
The double dot plot compares the number of text messages sent and received by a third age group to the age group, 12–15 years.



1. What is the mean number of texts for the age group, 24–27 years?

2. In the graph above, draw a vertical dotted line through both dot plots that corresponds to the mean for the age group, 24–27 years.

Analyze and Reflect

Collaborate

Work with a partner.

3. What is the difference between the means of the distributions for the Activity? for Exercise 1?

4. The mean absolute deviation of each distribution is 6.25 texts. For the Activity and Exercise 1, write the difference between the means and the mean absolute deviation as a ratio. Express the ratio as a decimal.

5. MP **Reason Inductively** Compare the ratios you wrote in Exercise 4.

Create

On Your Own

6. Inquiry What does the ratio $\frac{\text{difference in means}}{\text{mean absolute deviation}}$ tell you about how much visual overlap there is between two distributions with similar variation?

Lesson 5

Select an Appropriate Display

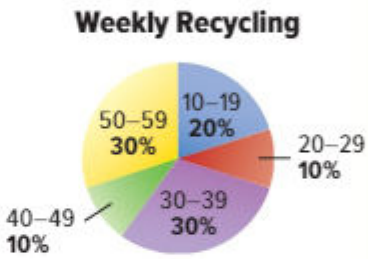
Real-World Link

There are many different types of graphs that are used to display all kinds of statistical data. List all of the types of graphs you can think of below.

_____

_____

The graphs below display the total number of kilograms of plastic recycled each week during a ten-week period in different ways.



_____ _____

1. On the line below each graph, write the type of graph used.
2. Which display more easily shows the number of weeks the class collected between 30 and 39 kilograms of plastic? _____
3. Which display more easily shows the percent of time that 40 to 49 kilograms of plastic was recycled? _____

Essential Question

HOW do you know which type of graph to use when displaying data?

MP Mathematical Practices
1, 3, 4

Which **MP Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Key Concept

Select an Appropriate Display

Type of Display	Best Used to...
Bar Graph	show the number of items in specific categories
Box Plot	show measures of variation for a set of data; also useful for very large sets of data
Circle Graph	compare parts of the data to the whole
Double Bar Graph	compare two sets of categorical data
Histogram	show frequency of data divided into equal intervals
Line Graph	show change over a period of time
Line Plot	show frequency of data with a number line

Work Zone

Data Displays

Many situations have more than one appropriate display.

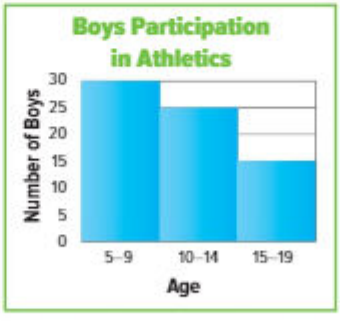
When deciding what type of display to use, ask these questions.

- What type of information is given?
- What do you want the display to show?
- How will the display be analyzed?

Example

1. **Select an appropriate display to show the number of boys of different age ranges that participate in athletics.**

Since the display will show an interval, a histogram like the one below would be an appropriate display to represent this data.



Got it? **Do this problem to find out.**

Show your work.

a. Select an appropriate display for the percent of students in each grade at a middle school.

a. _____

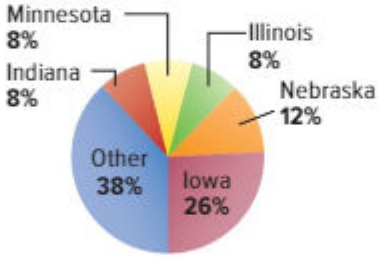
Example

2. **Select an appropriate type of display to compare the percent of ethanol production by state. Justify your reasoning. Then construct the display. What can you conclude from your display?**

Ethanol Production by State Per Year						
State	Iowa	Nebraska	Illinois	Minnesota	Indiana	Other
Liters (millions)	3,534	1,665	1,135	1,102	1,074	5,098

You are asked to compare parts to a whole. A circle graph would be an appropriate display.

Ethanol Production by State



Indiana, Minnesota, and Illinois produce about the same amount of ethanol.

Got it? **Do this problem to find out.**

b. The table lists the ticket prices for school musicals during recent years. Select an appropriate display to predict the price of a ticket in 2013. Justify your reasoning. Then construct the display. What can you conclude from your display?

Ticket Prices	
Year	**Price (AED)**
2009	5.00
2010	5.50
2011	6.50
2012	7.00

Show your work.

Guided Practice

Select an appropriate display for each situation. Justify your reasoning.
(Example 1)

1. the number of people who have different kinds of pets

2. the percent of different ways electricity is generated

3. The prices of sandwiches at a restaurant are AED4.50, AED5.59, AED3.99, AED2.50, AED4.99, AED3.75, AED2.99, AED3.29, and AED4.19. Select an appropriate display to determine how many sandwiches range from AED3.00 to AED3.99. Justify your reasoning. Then construct the display. What can you conclude from your display? (Example 2)

4. A survey asked teens which subject they felt was most difficult. Of those who responded, 25 said English, 39 said social studies, 17 said science, and 19 said other. Construct an appropriate display of the data. Justify your reasoning. Then name one thing you can conclude from the display. (Example 2)

5. **Building on the Essential Question** What are some of the factors to consider when selecting an appropriate display for a set of data?

Rate Yourself!

How confident are you about selecting an appropriate display? Shade the ring on the target.

Name _____ My Homework _____

Independent Practice

Select an appropriate display for each situation. Justify your reasoning.

(Example 1)

1. the median age of members in a community band

Show your work.

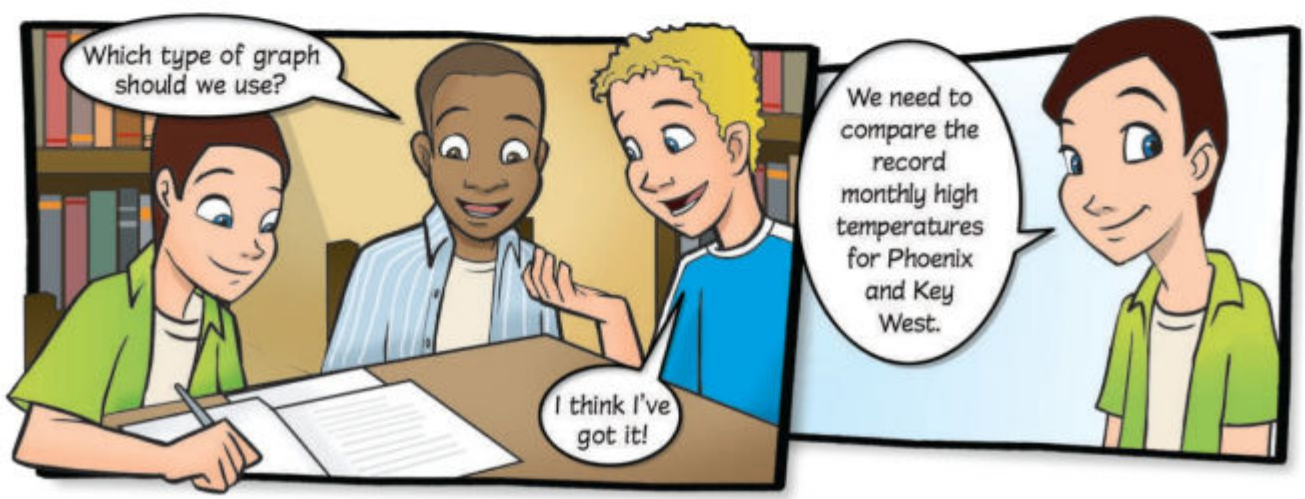
2. the number of students that favor chocolate or vanilla as a frosting

3. Select an appropriate display for the data. Justify your reasoning. Then construct the display. What can you conclude from your display? (Example 2)

Show your work.

Number of Push-ups			
45	35	42	37
44	40	36	42
45	40	42	39
44	43	36	39

4. MP **Model with Mathematics** Refer to the graphic novel frame below. What is the best type of display to use for this data? Explain.



5. Refer to the situations described below.

Situation A	Situation B
the number of customers ages 12–19 compared to all age groups	the number of customers ages 12, 13, 14, 15, and 16 who made a purchase

a. Which situation involves data that is best displayed in a bar graph?

Explain your reasoning. _____

b. Refer to the situation you selected in part **a.** Could you display the data using another type of display? If so, which display? Explain.

H.O.T. Problems Higher Order Thinking

6. MP **Model with Mathematics** Give an example of a data set that would be best represented in a line graph. _____

7. MP **Reason Inductively** Determine if the following statement is *always, sometimes,* or *never* true. Justify your response.

A circle graph can be used to display data from a bar graph.

8. MP **Persevere with Problems** Determine if the following statement is *true* or *false*. Explain your reasoning.

A line plot can be used to display data from a histogram.

9. MP **Reason Inductively** Compare and contrast bar graphs and histograms. Explain when it is appropriate to use a histogram rather than a bar graph.

Name _____ My Homework _____

Extra Practice

Justify Conclusions **Select an appropriate display for each situation. Justify your reasoning.**

10. the resale value of a person's car over time

Homework Help

line graph; A line graph compares change over time.

11. the percent of people that drink 0, 1, 2, 3, or more than 3 glasses of water a day

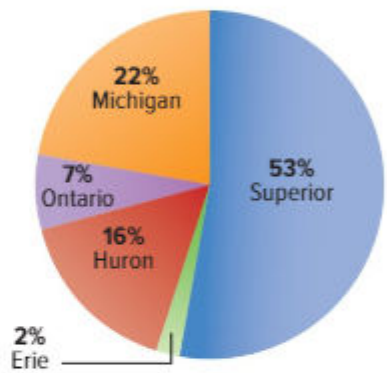
12. the number of different colored cars at a car dealership

13. The circle graph shows the approximate percent of the total volume of each Great Lake.

a. Display the data using another type of display.

Show your work.

Volume of the Great Lakes



b. Write a convincing argument telling which display is more appropriate.

Copy and Solve **Select an appropriate display for each situation. Then justify your reasoning and construct the display on a separate sheet of paper. What can you conclude from your display?**

14.

Favorite Movies	
Type of Movie	Number of People
Comedy	48
Action	17
Drama	5
Horror	2

15.

Age Group	Number of Texts per Day
11–15	25
16–20	23
21–25	17
26–30	10

Power Up! Test Practice

16. The number of home runs hit by each player of a professional baseball team is shown in the table.

Home Runs					
10	15	5	10	12	5
12	12	4	5	10	7

Determine if each statement is true or false.

a. A line plot would be most appropriate for showing the frequency of the data on a number line. ☐ True ☐ False

b. A histogram would be most appropriate for showing the frequency of the data in equal intervals. ☐ True ☐ False

c. A circle graph would be most appropriate for showing how the number of home runs changes over time. ☐ True ☐ False

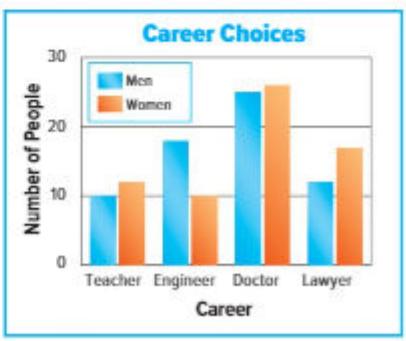
17. Select the most appropriate type of display for each situation.

bar graph	histogram	line plot
circle graph	line graph	

Situation	Type of Graph
Mr. Omar measured the amount of rain that had fallen every 15 minutes during a storm. He wants to show how the amount of rain fallen changed over time during the storm.	
Nabila recorded how many hours of her free time she spent playing sports, watching TV, talking with her friends, or playing video games. She wants to compare the percentages of her free time that she spends on each activity.	
Maysoun collected data on how many of her classmates ride the bus, get a ride, or walk to school. She wants to compare how many students are in each category.	

Spiral Review

Use the graph to answer Exercises 18–20. The graph shows the number of male and the number of female students that chose certain occupations to research.



18. About how many people are represented in the graph? _____

19. About how many men and how many women are represented in the graph? _____

20. How many more women chose to research law? _____

21ST CENTURY CAREER in Market Research

Market Research Analyst

Do you think that gathering and analyzing information about people's opinions, tastes, likes, and dislikes sounds interesting? If so, then you should consider a career in market research. Market research analysts help companies understand what types of products and services consumers want. They design Internet, telephone, or mail response surveys and then analyze the data, identify trends, and present their conclusions and recommendations. Market research analysts must be analytical, creative problem-solvers, have strong backgrounds in mathematics, and have good written and verbal communication skills.

College & Career READINESS

Is This the Career for You?

Are you interested in a career as a market research analyst? Take some of the following courses in high school.

- Algebra
- Calculus
- Computer Science
- English
- Statistics

Find out how math relates to a career in Market Research.

Keeping Your Eye on the Target Market!

Use the results of the survey in the table below to solve each problem.

1. At Hastings Middle School, 560 of the students use social networking sites. Predict how many of them use the sites to make plans with friends. _____

2. Suppose 17.9 million teens use online social networks. Predict how many will be using the sites to make new friends.

   _____

3. According to the survey, what percent of a teen's networking site friends are people they regularly see? _____

4. Majed randomly selects a friend from his social networking site. What is the probability that it is someone he never sees in person? Write as a percent. _____

5. Najla wants to leave a message on 8 of her friends' social networking sites. In how many ways can she leave a message on her friends' sites? _____

Survey Results: Teens and Social Networking	
Reason to Use Social Networks	**Percent of Respondents**
Stay in touch with friends	91%
Make plans with friends	72%
Make new friends	49%
Friends on Social Networking Sites	**Average Number**
People who are regularly seen	43
People who are occasionally seen	23
People who are never seen in person	33
Total	99

Career Project

It's time to update your career portfolio! Use the Internet or another source to research a career as a market research analyst. Write a paragraph that summarizes your findings.

_____

_____

_____

_____

_____

_____

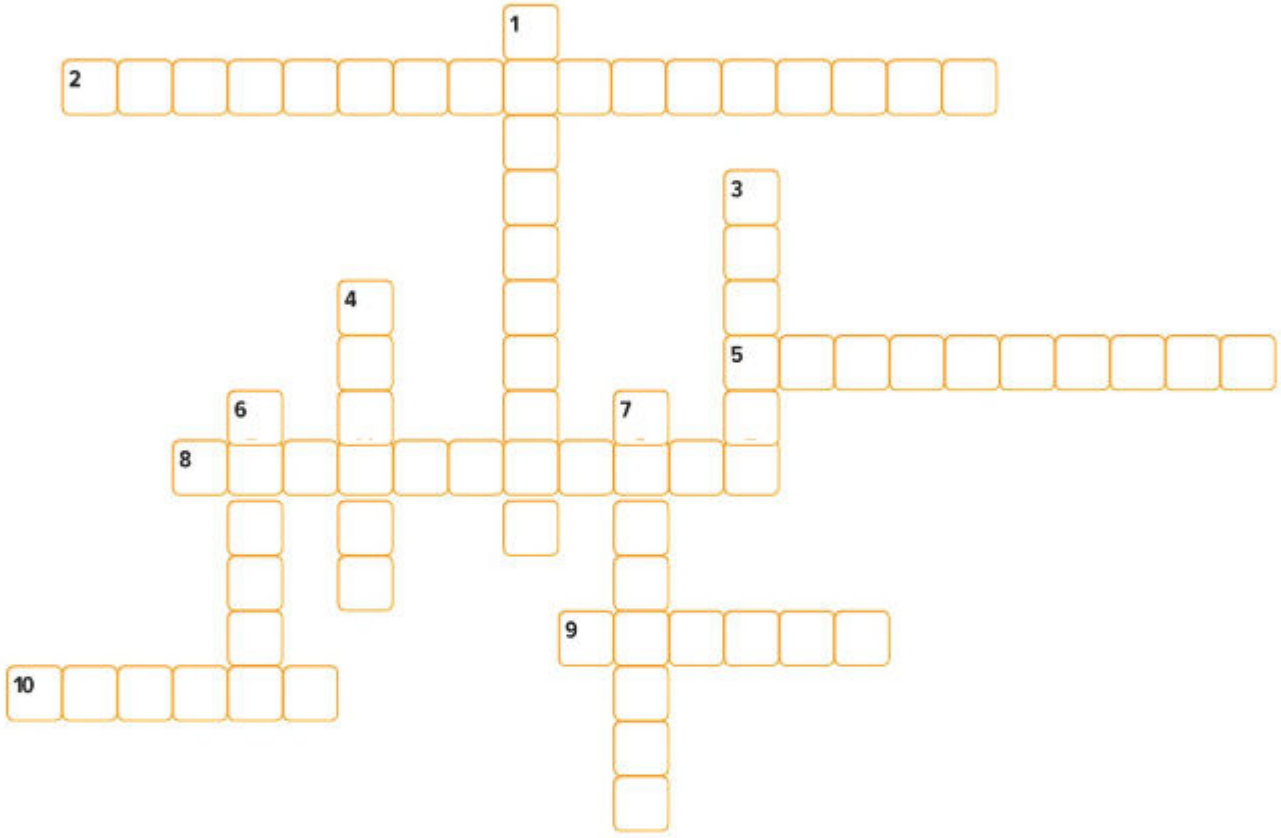
What skills would you need to improve to succeed in this career?

- _____
- _____
- _____
- _____
- _____

Chapter Review

Vocabulary Check

Complete the crossword puzzle using the vocabulary list at the beginning of the chapter.



Across

2. sample involving only those who want to participate (two words)

5. the group being studied

8. sample in which members of a population are easily accessed

9. part of a group

10. sample in which one or more parts of the population are favored over other parts

Down

1. random sample in which items are selected according to a specific time or interval

3. random sample in which each item is as likely to be chosen as any other item

4. a method of collecting information

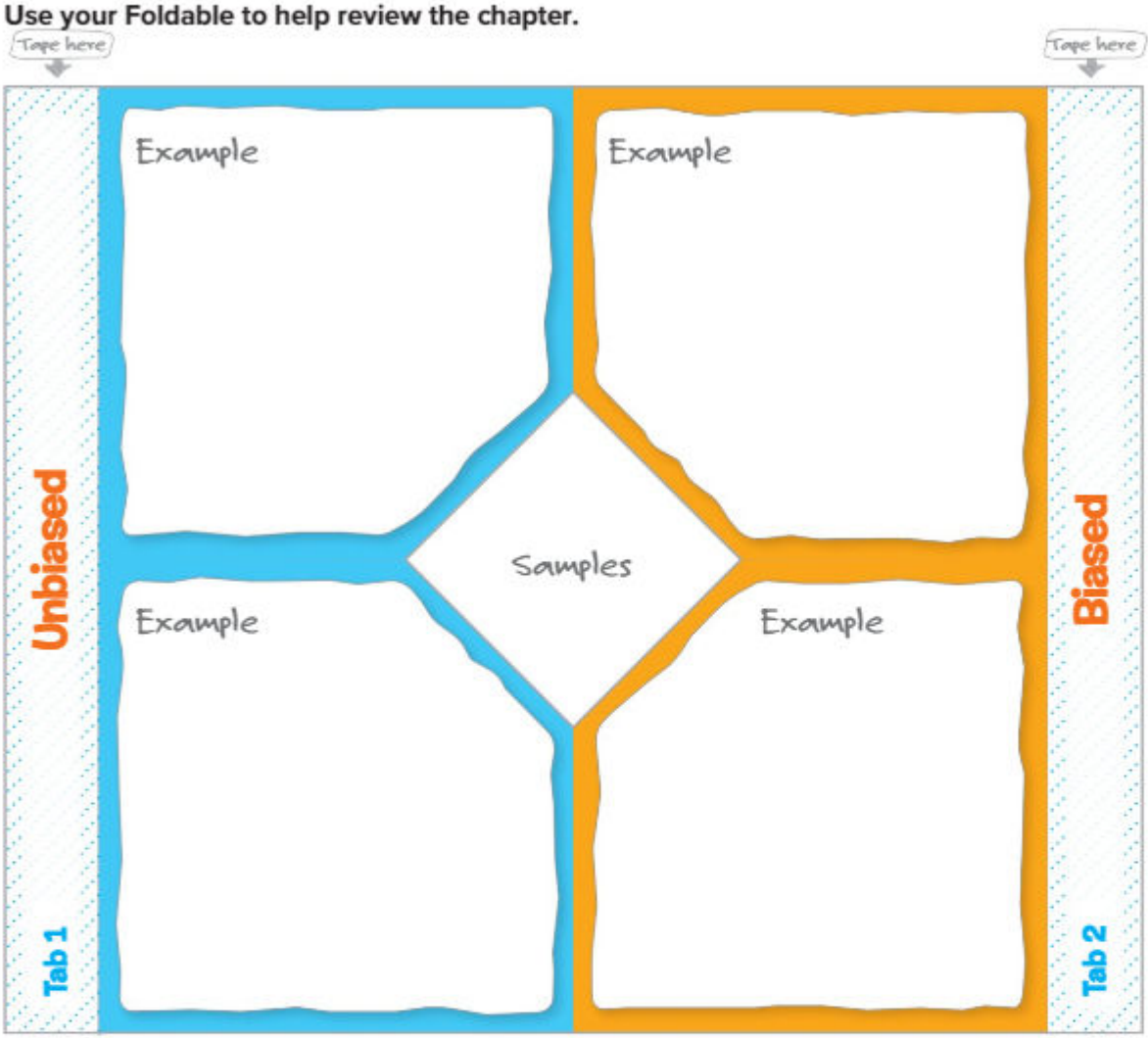
6. two box plots on the same number line

7. sample that represents the entire population

Key Concept Check

Use Your FOLDABLES

Use your Foldable to help review the chapter.



Got it?

Match each phrase with the correct term.

1. a method of collecting information
2. the group being studied
3. when one or more parts of the population is favored
4. a sample that involves only those who want to participate

a. voluntary response sample

b. biased sample

c. survey

d. population

e. convenience sample

Power Up! Performance Task

Class Evaluation

Mr. Faleh is analyzing his student's grades over the last three years. He has had approximately 65 students each year. To simplify his analysis, he has decided to use a random sample of data from only ten students from 2012 and 2013. He had no records of the grades from 2014, so he asked students to bring in transcripts. He used the first few transcripts he received for his 2014 data.

	Student Grades									
2012	?	58	86	78	82	79	84	83	82	72
2013	83	85	85	85	87	87	88	90	91	91
2014	79	83	84	88	88	90	93	93	94	95

Write your answers on another piece of paper. Show all of your work to receive full credit.

Part A

Is the above information likely to be a legitimate representation of all the students of each respective year? Is the 2012 data biased or unbiased? Explain your answers.

Part B

In the year 2012, one of the grades is missing. If the mean was 80.1, what is the missing grade?

Part C

Considering the years 2013 and 2014, in which year were the grades the most consistent? Which of the two years had the better scores? What type of display would best show the data? Justify your responses.

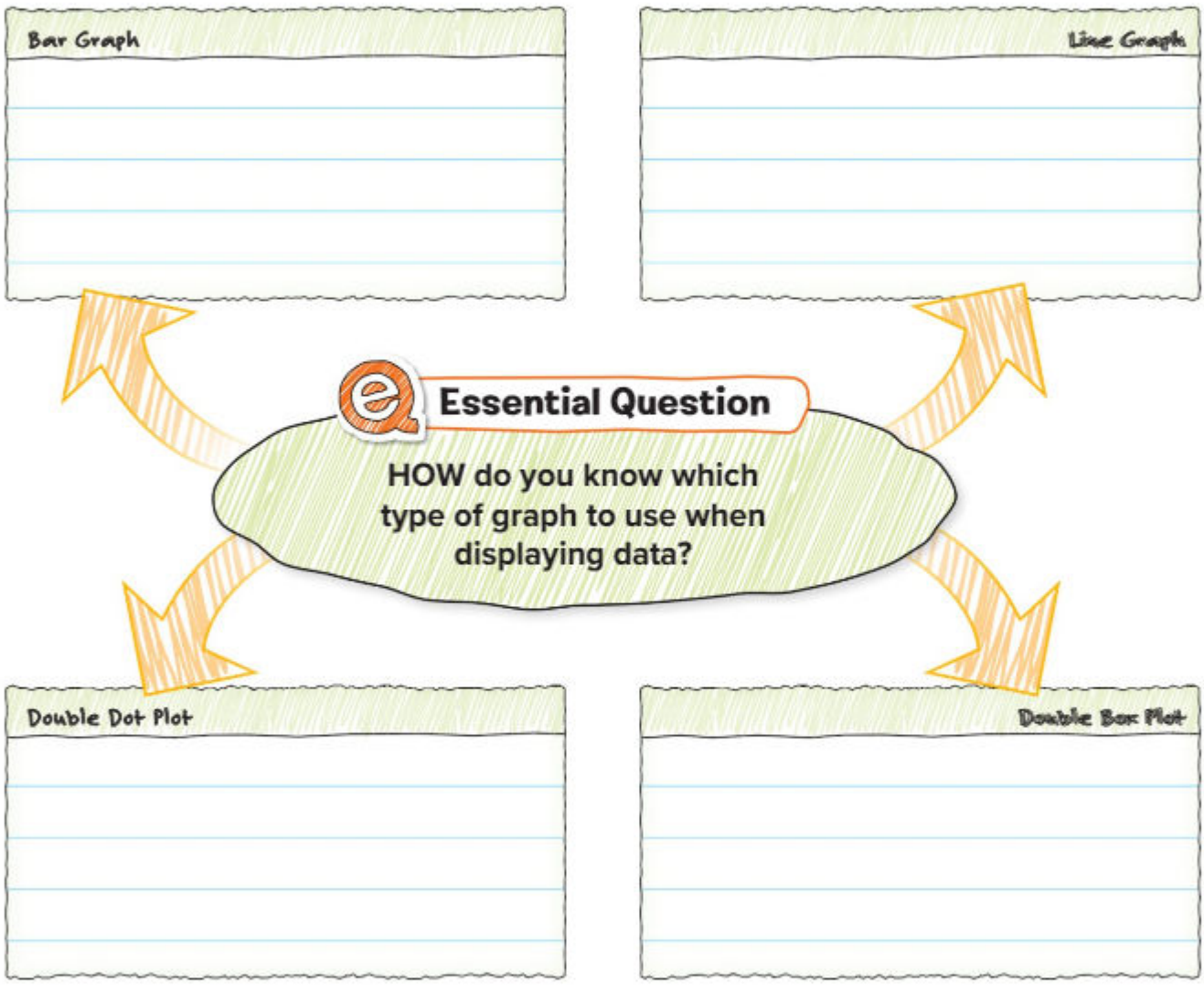
Part D

Mr. Faleh wants to submit the data from one year for a Teacher of the Year award. Choose a year. Could the data be seen as misleading? Explain.

Reflect

Answering the Essential Question

Use what you learned about statistics to complete the graphic organizer.



Answer the Essential Question. HOW do you know which type of graph to use when displaying data?

UNIT PROJECT

Math Genes A Punnett Square is a graphical way to predict the genetic traits of offspring. In this project you will:

- **Collaborate** with your classmates as you research genetics and the Punnet Square.
- **Share** the results of your research in a creative way.
- **Reflect** on why learning mathematics is important.

Complete the activities below and discover the fun you can have with genetics.

Collaborate

Go Online Work with your group to research and complete each activity. You will use your results in the Share section on the following page.

1. Use the Internet to research Punnett Squares and their role in genetics. Write a paragraph describing your findings.

2. Create sample genes for pet traits. Then create a Punnett Square using those traits. Describe what each outcome represents. Include a graph with your explanation.

3. Refer to Exercise 2. How many different genetic outcomes are possible according to your Punnett Square? What is the probability of each outcome occurring?

4. Create three word problems that involve using probability and the Punnett Squares to help answer the questions.

5. Collect two or more genetic-related information samples about students in your class. For example, you can collect data on attached/unattached earlobes. Analyze the data and make a prediction about the genetics of the entire school. Draw an appropriate graph of your results.

Share

Collaborate

With your group, decide on a way to share what you have learned about genetics and Punnett Squares. Some suggestions are listed below, but you can also think of other creative ways to your present your information. Remember to show how you used mathematics to complete each of the activities in this project.

- Create a digital presentation of the facts you learned about genetics.
- Act as a genetic scientist. Write a journal entry that explains your current research on predicting traits passed down from generations.

Check out the note on the right to connect this project with other subjects.

Connect with Health

Health Literacy Select a health condition or disease and research how genetics may play a part in the disease. Write 1–2 paragraphs explaining how genetics may influence someone's risk of getting the disease and steps that can be taken to reduce the risk factors.

Reflect

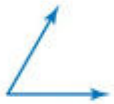
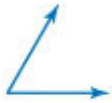
On Your Own

6. **Answer the Essential Question** Why is learning mathematics important?

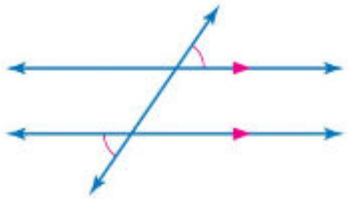
   a. How did what you learned about probability help you to understand why learning mathematics is important?

   b. How did what you learned about statistics help you to understand why learning mathematics is important?

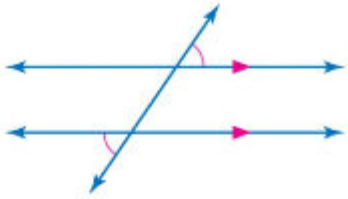
Glossary/القاموس

English	العربية
A	
absolute value The distance the number is from zero on a number line.	**القيمة المطلقة** هي المسافة بين العدد والصفر على خط الأعداد.
acute angle An angle with a measure greater than 0° and less than 90°.	**الزاوية الحادة** هي زاوية بقياس أكبر من الدرجة 0 وأصغر من الدرجة 90.
	
acute triangle A triangle having three acute angles.	**المثلث الحاد** هو مثلث يحتوي على ثلا ث زوايا حادة.
	
Addition Property of Equality If you add the same number to each side of an equation, the two sides remain equal.	**خاصية الجمع في المعادلات** إذا أضفت العدد نفسه إلى كل طرف في معادلة، يظل الطرفان متساويين.
Addition Property of Inequality If you add the same number to each side of an inequality, the inequality remains true.	**خاصية الجمع في المتباينة** إذا أضفت العدد نفسه لكل طرف في متباينة، تظل المتباينة صحيحة.
Additive Identity Property The sum of any number and zero is the number.	**خاصية المحايد الجمعي** مجموع أي عدد وصفر يكون العدد نفسه.
additive inverse Two integers that are opposites. The sum of an integer and its additive inverse is zero.	**المعكوسان الجمعيان** هما عددان صحيحان معاكسان لبعضهما. ويكون حاصل جمع أي عدد صحيح ومعكوسه الجمعي صفرًا.
adjacent angles Angles that have the same vertex, share a common side, and do not overlap.	**الزوايا المجاورة** هي زوايا لها الرأس نفسه وتتقاسم ضلعًا مشتركًا ولا تتداخل.
algebra A branch of mathematics that involves expressions with variables.	**الجبر** هو فرع من الرياضيات ينطوي على عبارات تتضمن متغيرات.
algebraic expression A combination of variables, numbers, and at least one operation.	**العبارة الجبرية** هي عبارة عن مجموعة من المتغيرات والأعداد وعملية واحدة على الأقل.

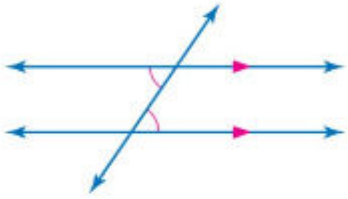
alternate exterior angles Angles that are on opposite sides of the transversal and outside the parallel lines.



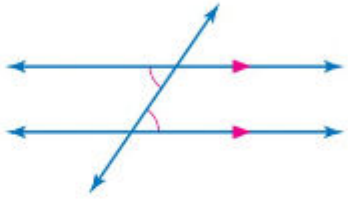
الزوايا الخارجية البديلة هي زوايا تكون على الأضلاع المقابلة للقاطع المستعرض وخارج المستقيمات المتوازية.



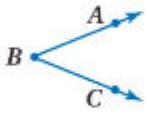
alternate interior angles Angles that are on opposite sides of the transversal and inside the parallel lines.



الزوايا الداخلية البديلة هي زوايا تكون على الأضلاع المقابلة للقاطع المستعرض وداخل المستقيمات المتوازية.

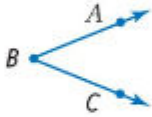


angle Two rays with a common endpoint form an angle. The rays and vertex are used to name the angle.



$\angle ABC$, $\angle CBA$, or $\angle B$

الزاوية هي شعاعان لهما نقطة مشتركة تشكل زاوية. ويتم استخدام الشعاعين والرأس في تسمية الزاوية.



$\angle B$ أو $\angle CBA$ ،$\angle ABC$

arithmetic sequence A sequence in which the difference between any two consecutive terms is the same.

المتتالية الحسابية هي متتالية يكون فيها الفرق بين أي حدين متتاليين متماثلاً.

Associative Property The way in which numbers are grouped does not change their sum or product.

خاصية التجميع هي الطريقة التي يتم فيها تجميع الأعداد بحيث لا يتغير مجموعها أو ناتجها.

B

bar notation In repeating decimals, the line or bar placed over the digits that repeat. For example, $2.\overline{63}$ indicates that the digits 63 repeat.

رمز العدد الدوري في الكسر العشري الدوري، هو الخط أو الشريط الذي يوضع أعلى الأرقام المتكررة. على سبيل المثال، 2.63 تشير إلى تكرار الرقمين 63.

base In a power, the number used as a factor. In 10^3, the base is 10. That is, $10^3 = 10 \times 10 \times 10$.

الأساس في عملية الرفع، هو العدد الذي يتم استخدامه كعامل. في المثال 10^3، يكون الأساس هو العدد 10. بمعنى أن، $10^3 = 10 \times 10 \times 10$.

base One of the two parallel congruent faces of a prism.

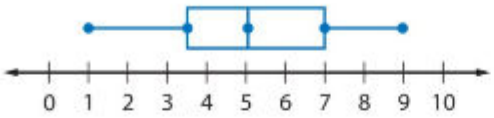
القاعدة هي أحد الوجهين المتطابقين المتوازيين في المنشور.

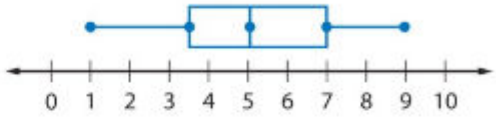
biased sample A sample drawn in such a way that one or more parts of the population are favored over others.

العينة المتحيزة هي عينةٌ تجمع بطريقة يفضلها جزء أو أكثر من المجتمع الإحصائي مقارنة بالمجتمعات الأخرى.

box plot A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.

مخطط الصندوق ذي العارضين هو طريقة لعرض توزيع قيم البيانات بصريًا باستخدام المتوسط الحسابي والربيعيات وأطراف مجموعة البيانات. ويظهر الصندوق الوسط الحسابي بنسبة 50% من البيانات.





C

center The point from which all points on circle are the same distance.

المركز هو النقطة التي تبعد عنها جميع النقاط على الدائرة المسافة نفسها.

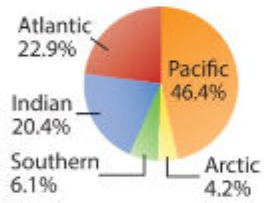
circle The set of all points in a plane that are the same distance from a given point called the center.

الدائرة هي مجموعة النقاط في المستوى التي لها البعد نفسه عن نقطة معلومة تُسمى المركز.

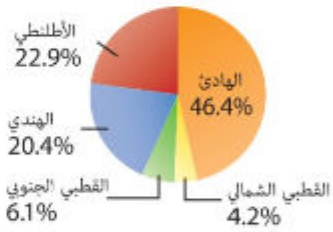
circle graph A graph that shows data as parts of a whole. In a circle graph, the percents add up to 100.

الرسم البياني الدائري هو رسم بياني يوضح البيانات كأجزاء من الكل. في الرسم البياني الدائري، يكون مجموع النسب المئوية 100.

Area of Oceans

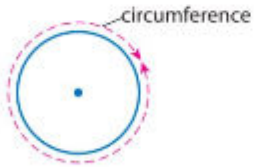


مناطق من المحيطات



circumference The distance around a circle.

محيط الدائرة هو المسافة حول الدائرة.



coefficient The numerical factor of a term that contains a variable.

المعامل هو عامل عددي للحد الذي يحتوي على متغير.

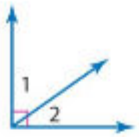
common denominator A common multiple of the denominators of two or more fractions. 24 is a common denominator for $\frac{1}{3}$, $\frac{5}{8}$, and $\frac{3}{4}$ because 24 is the LCM of 3, 8, and 4.

المقام المشترك هو مضاعف مشترك في مقامي كسرين أو أكثر. يعتبر العدد 24 مقامًا مشتركًا للأعداد $\frac{1}{3}$، و $\frac{5}{8}$، و $\frac{3}{4}$ لأن العدد 24 يعتبر المضاعف المشترك الأصغر للعدد 3 و8 و4.

Commutative Property The order in which two numbers are added or multiplied does not change their sum or product.

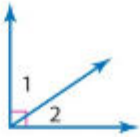
خاصية التبديل هي الترتيب الذي يتم به إضافة أو ضرب عددين بحيث لا يتغير مجموعهما أو ناتجهما.

complementary angles Two angles are complementary if the sum of their measures is 90°.



∠1 and ∠2 are complementary angles.

الزاويتان المتتامتان تكون الزاويتان متتامتين إذا كان مجموع قياسهما يساوي 90 درجة.



∠1 و ∠2 زاويتان متتامتان.

complementary events The events of one outcome happening and that outcome not happening. The sum of the probabilities of an event and its complement is 1 or 100%. In symbols, P(A) + P(not A) = 1.

الأحداث المتممة هي أحداث خاصة بنتيجة ما، غير أن هذه الأحداث قيد التنفيذ، بينما تُعد النتيجة غير واقعة. ويكون مجموع الاحتمالات لحدث ما ومتممه 1 أو 100%. وللتوضيح بالرموز P(A) + P(not A) = 1.

complex fraction A fraction $\frac{A}{B}$ where A or B are fractions and B does not equal zero.

الكسر المجمع هو كسر $\frac{A}{B}$ حيث إن A أو B كسرين، و B لا يساوي الصفر.

composite figure A figure that is made up of two or more three-dimensional figures.

الشكل المركب شكل مكون من شكلين ثلاثيي الأبعاد أو أكثر.

compound event An event consisting of two or more simple events.

الحدث المركب هو حدث مكّون من حدثين بسيطين أو أكثر.

cone A three-dimensional figure with one circular base connected by a curved surface to a single vertex.

المخروط هو شكل ثلاثي الأبعاد قاعدته دائرية الشكل ومتصلة بسطح منحنٍ ورأس واحد فقط.

congruent Having the same measure.

التطابق هو وجود القياس نفسه.

congruent angles Angles that have the same measure.

∠1 and ∠2 are congruent angles.

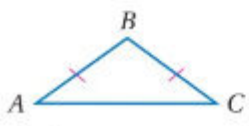
الزوايا المتطابقة هي الزوايا التي لها نفس القياس.

∠1 و ∠2 هما زاويتان متطابقتان.

congruent figures Figures that have the same size and same shape and corresponding sides and angles with equal measure.

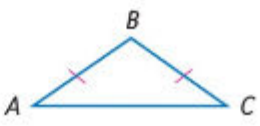
الأشكال المتطابقة هي الأشكال التي لها القياس نفسه والشكل نفسه؛ ويكون قياس أضلاعها وزواياها المتناظرة متساويًا.

congruent segments Sides with the same length.



Side $\overline{AB}$ is congruent to side $\overline{BC}$.

القطع المتطابقة هي الأضلاع التي لها الطول نفسه.



الضلع $\overline{AB}$ متطابق مع الضلع $\overline{BC}$.

constant A term that does not contain a variable.

الثابت هو الحد الذي لا يحتوي على متغير.

constant of proportionality A constant ratio or unit rate of two variable quantities. It is also called the constant of variation.

ثابت التناسب هو نسبة ثابتة أو معدل وحدة خاص بكميتين متغيرتين. ويسمى أيضًا بثابت التغير.

constant of variation The constant ratio in a direct variation. It is also called the constant of proportionality.

ثابت التغير هو نسبة ثابتة في متغير مباشر. وهو يسمى أيضًا بثابت التناسب.

constant rate of change The rate of change in a linear relationship.

معدل التغير الثابت هو معدل التغير في علاقة خطية.

continuous data Data that take on any real number value. It can be determined by considering what numbers are reasonable as part of the domain.

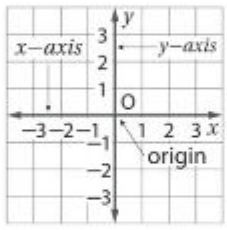
البيانات المتصلة هي البيانات التي تأخذ أي قيمة عددية حقيقية. ويمكن تحديدها من خلا ل النظر في الأرقام التي من المعقول وجودها كجزء من المجال.

convenience sample A sample which consists of members of a population that are easily accessed.

العينة الملائمة هي عينة تتكون من أفراد المجتمع الإحصائي الذين يسهل الوصول إليهم.

coordinate plane A plane in which a horizontal number line and a vertical number line intersect at their zero points. Also called a coordinate grid.

المستوى الإحداثي هو مستوى يكون فيه خط الأعداد الأفقي وخط الأ عداد الرأسي متقاطعين في النقاط الصفرية. ويطلق عليه أيضًا الشبكة الإحداثية.



coplanar Lines or points that lie in the same plane.

متحد المستوى يطلق على الخطوط أو النقاط متحدة المستوى إذا كانت تقع في المستوى نفسه.

corresponding angles Angles in the same position on parallel lines in relation to a transversal.

الزاوية المتناظرة هي الزاوية التي تقع في الموقع نفسه من مستقيمات متوازية يقطعها مستقيم مستعرض.

corresponding sides The sides of similar figures that are in the same relative position.

الأضلاع المتناظرة هي أضلاع الأ شكال المتماثلة في موقع نسبي مماثل.

counterexample A specific case which proves a statement false.

مثال مضاد هو حالة محددة تثبت عدم صحة عبارة ما.

cross product The product of the numerator of one ratio and the denominator of the other ratio. The cross products of any proportion are equal.

الضرب التقاطعي هو ضرب بسط إحدى النسب في مقام النسبة الأخرى. ويكون حاصل الضرب التقاطعي لأي نسبة متساويًا.

cross section The cross section of a solid and a plane.

المقطع العرضي هو المقطع العرضي لجسم مستوٍ.

cube root One of three equal factors of a number. If $a^3 = b$, then a is the cube root of b. The cube root of 125 is 5 since $5^3 = 125$.

الجذر التكعيبي هو أحد العوامل الثلا ثة المتساوية في العدد. إذا كان $a^3 = b$ إذًا a هو الجذر التكعيبي لـ b. الجذر التكعيبي للعدد 125 هو 5، وذلك لأن $5^3 = 125$.

cubed The product in which a number is a factor three times. Two cubed is 8 because $2 \times 2 \times 2 = 8$.

التكعيب هو حاصل ضرب يحل فيه العدد محل العامل ثلاث مرات. 2 تكعيب يساوي 8 لأن $2 \times 2 \times 2 = 8$.

cylinder A three-dimensional figure with two parallel congruent circular bases connected by a curved surface.

الأسطوانة هي شكل ثلاثي الأبعاد يحتوي على قاعدتين دائريتين متطابقتين ومتوازيتين ويتصلان بيعضهما عن طريق سطح منحن.

D

decagon A polygon having ten sides.

عشاري الأضلاع هو مضلع له عشرة أضلاع.

defining a variable Choosing a variable and a quantity for the variable to represent in an expression or equation.

تعيين متغير هو اختيار متغير وكمية لهذا المتغير لتمثيلها في عبارة أو معادلة.

degrees The most common unit of measure for angles. If a circle were divided into 360 equal-sized parts, each part would have an angle measure of 1 degree.

الدرجات هي الوحدة الأكثر شيوعًا لقياس الزوايا. إذا تم تقسيم دائرة إلى 360 جزءًا متساويًا في القياس، فسيكون كل جزء عبارة عن زاوية قياسها درجة واحدة.

dependent events Two or more events in which the outcome of one event affects the outcome of the other event(s).

الأحداث غير المستقلة تكون الأحداث غير مستقلة إذا كان هناك حدثان أو أكثر تؤثر نتيجة حدث منها على نتيجة (الحدث) الأحداث الأخرى.

dependent variable The variable in a relation with a value that depends on the value of the independent variable.

المتغير التابع هو المتغير الذي تعتمد قيمته في علاقة على قيمة المتغير المستقل.

derived unit A unit that is derived from a measurement system base unit, such as length, mass, or time.

الوحدة المشتقة هي وحدة مشتقة من وحدة قاعدة نظام قياس، مثل الطول أو الكتلة أو الزمن.

diagonal A line segment that connects two nonconsecutive vertices.

القطر هو قطعة مستقيمة تربط بين رأسين غير متتاليين.

diameter The distance across a circle through its center.

قطر الدائرة هو المسافة المارة بالمركز داخل الدائرة.

dimensional analysis The process of including units of measurement when you compute.

التحليل البعدي هو عملية تضمين وحدات قياس أثناء إجراء العمليات الحسابية.

direct variation The relationship between two variable quantities that have a constant ratio.

التغيّر المباشر هو العلا قة بين كميتي متغير ذواتي نسبة ثابتة.

discount The amount by which the regular price of an item is reduced.

الخصم هو مقدار تخفيض السعر المعتاد لأحد العناصر.

discrete data When solutions of a function are only integer values. It can be determined by considering what numbers are reasonable as part of the domain.

البيانات المتقطعة عندما تكون حلول دالة عبارة عن قيم أعداد صحيحة فقط. ويمكن تحديدها من خلا ل النظر في الأرقام التي من المعقول وجودها كجزء من المجال.

disjoint events Events that cannot happen at the same time.

الأحداث المنفصلة هي الأحداث التي لا يمكن حدوثها في الوقت نفسه.

Distributive Property To multiply a sum by a number, multiply each addend of the sum by the number outside the parentheses. For any numbers a, b, and c, $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.

Example: $2(5 + 3) = (2 \times 5) + (2 \times 3)$ and $2(5 - 3) = (2 \times 5) - (2 \times 3)$

خاصية التوزيع هي ضرب عملية جمع في عدد ما، أو ضرب كل حد جمعي في عملية جمع في العدد الموجود خارج الأقواس. بالنسبة إلى الأ عداد a ، b، c، تكون $a(b + c) = ab + ac$ و $a(b - c) = ab - ac$.

مثال: $2(5 + 3) = (2 \times 5) + (2 \times 3)$ و $2(5 - 3) = (2 \times 5) - (2 \times 3)$

Division Property of Equality If you divide each side of an equation by the same nonzero number, the two sides remain equal.

خاصية القسمة في المعادلة في حالة قسمة حديّ أي معادلة على العدد نفسه غير الصفري، يظل الحدان متساويين.

Division Property of Inequality When you divide each side of an inequality by a negative number, the inequality symbol must be reversed for the inequality to remain true.

خاصية القسمة في المتباينة عند قسمة حديّ المتباينة على عدد سالب، يجب عكس رمز المتباينة لتظل المتباينة صحيحة.

domain The set of input values for a function.

المجال هو مجموعة القيم المدخلة على دالة.

double box plot Two box plots graphed on the same number line.

مخطط الصندوق المزدوج هو مخطط لصندوقين ممثلين بالرسم البياني على خط الأعداد ذاته.

double dot plot A method of visually displaying a distribution of two sets of data values where each value is shown as a dot above a number line.

الرسم البياني بالنقاط المزدوج هو طريقة لعرض توزيع مجموعتين من قيم البيانات بصرًيا حيث تظهر كل قيمة بمثابة نقطة فوق خط الأعداد.

E

edge The line segment where two faces of a polyhedron intersect.

الحافة هي القطعة المستقيمة التي يتقابل فيها وجهان من شكل متعدد الأوجه.

enlargement An image larger than the original.

التكبير هو جعل صورة أكبر من الصورة الأصلية.

equation A mathematical sentence that contains an equals sign, =, stating that two quantities are equal.

المعادلة هي جملة رياضية تضم علامة يساوي (=)، لتوضيح أن هناك كميتين متساويتين.

equiangular In a polygon, all of the angles are congruent.

متساوي الزوايا ُيطلق هذا المصطلح على المضلع، إذا كانت جميع زواياه متطابقة.

equilateral In a polygon, all of the sides are congruent.

متساوي الأضلاع يُطلق هذا المصطلح على المضلع، إذا كانت جميع أضلاعه متطابقة.

equilateral triangle A triangle having three congruent sides.

المثلث متساوي الأضلاع هو مثلث يحتوي على ثلاثة أضلاع متطابقة.

equivalent equations Two or more equations with the same solution.

المعادلات المتكافئة هي معادلتان أو أكثر لها الحل نفسه.

equivalent expressions Expressions that have the same value.

التعبيرات المتكافئة هي التعبيرات التي لها القيمة نفسها.

equivalent ratios Two ratios that have the same value.

النسب المتكافئة هي النسب التي لها القيمة نفسها.

evaluate To find the value of an expression.

التقييم هو إيجاد قيمة تعبير ما.

experimental probability An estimated probability based on the relative frequency of positive outcomes occurring during an experiment. It is based on what *actually* occurred during such an experiment.

الاحتمال التجريبي هو احتمال مقدر قائم على التكرار النسبي لحدوث النتائج الإيجابية التي تحدث أثناء التجربة. فهو قائم على ما يحدث فعلياً خلال هذه التجربة.

exponent In a power, the number that tells how many times the base is used as a factor. In 5^3, the exponent is 3. That is, $5^3 = 5 \times 5 \times 5$.

الأس في عملية الرفع، هو العدد الذي يوضح عدد المرات التي يتم فيها استخدام الأساس كعامل. وفي 5^3، يكون الأس 3. بمعنى أن $5^3 = 5 \times 5 \times 5$.

exponential form Numbers written with exponents.

الصيغة الأسية هي الأعداد المكتوبة مع الأسس.

F

face A flat surface of a polyhedron.

الوجه هو سطح مستو لشكل متعدد الأوجه.

factor To write a number as a product of its factors.

العامل هو كتابة عدد في شكل حاصل ضرب عوامله.

factored form An expression expressed as the product of its factors.

على شكل عوامل هو توضيح التعبير في شكل حاصل ضرب عوامله.

factors Two or more numbers that are multiplied together to form a product.

العوامل هي عددان أو أكثر يتم ضربهما معًا لتكوين حاصل ضرب.

fair game A game where each player has an equally likely chance of winning.

اللعبة العادلة هي لعبة يكون لدى كل لاعب فرصة متساوية لاحتمالية الفوز.

first quartile For a data set with median M, the first quartile is the median of the data values less than M.

الربيع الأول بالنسبة إلى مجموعة البيانات ذات الوسيط M، يكون الربيع الأول عبارة عن وسيط قيم البيانات الأقل من قيمة M.

formula An equation that shows the relationship among certain quantities.

الصيغة هي معادلة تظهر العلاقة بين كميات معينة.

function A relationship which assigns exactly one output value for each input value.

الدالة هي علاقة تحدد بالضبط قيمة مُخرج واحد بالنسبة إلى كل قيمة مدخل.

function rule The operation performed on the input of a function.

قاعدة الدالة هي العملية التي تم إجراؤها على مدخل الدالة.

function table A table used to organize the input numbers, output numbers, and the function rule.

جدول الدالة هو جدول يستخدم لتنظيم أرقام المُدخل وأرقام المُخرج وقاعدة الدالة.

Fundamental Counting Principle Uses multiplication of the number of ways each event in an experiment can occur to find the number of possible outcomes in a sample space.

مبدأ العد الأساسي هو مبدأ يستخدم الضرب بعدد من الطرق التي يمكن أن تحدث في كل تجربة للتوصل إلى عدد من النتائج المحتملة في مجال عينة.

G

gram A unit of mass in the metric system equivalent to 0.001 kilogram. The amount of matter an object can hold.

الجرام هو وحدة كتلة في النظام المتري ويعادل 0.001 كيلوجرامًا. ويعبر عن مقدار المادة الذي قد يحويه الجسم.

graph The process of placing a point on a number line or on a coordinate plane at its proper location.

الرسم البياني هو عملية وضع نقطة على خط أعداد أو على مستوى إحداثي في موقعها الصحيح.

gratuity Also known as a tip. It is a small amount of money in return for a service.

الإكرامية تعرف أيضًا باسم العطية. وهي مبلغ صغير يُقدم مقابل خدمة.

H

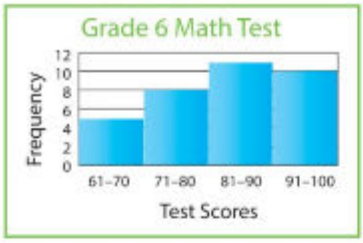
heptagon A polygon having seven sides.

سباعي الأضلاع هو مضلع له سبعة أضلاع.

hexagon A polygon having six sides.

سدادي الأضلاع هو مضلع له ستة أضلاع.

histogram A type of bar graph used to display numerical data that have been organized into equal intervals.



المدرج الإحصائي هو نوع من التمثيل البياني بالأعمدة تُستخدم فيه الأ عمدة لعرض بيانات عددية منظمة على فواصل متساوية.

I

Identity Property of Zero The sum of an addend and zero is the addend. Example: 5 + 0 = 5

خاصية المحايد للصفر (الجمعي) هي حاصل جمع حد جمعي والصفر الذي يعمل كالحد الجمعي الآخر. مثال: 5 = 0 + 5

independent events Two or more events in which the outcome of one event does not affect the outcome of the other event(s).

الأحداث المستقلة هي حادثتان أو أكثر لا تؤثر نتيجة إحداها في نتيجة (الحدث الآخر) الحوادث الأخرى.

independent variable The variable in a function with a value that is subject to choice.

المتغير المستقل هو متغير في الدالة تخضع قيمته للاختيار.

indirect measurement Finding a measurement using similar figures to find the length, width, or height of objects that are too difficult to measure directly.

القياس غير المباشر هو إيجاد قياس باستخدام أشكال مماثلة لإ يجاد طول أو عرض أو ارتفاع الأ جسام التي يصعب للغاية قياسها بشكل مباشر.

inequality An open sentence that uses <, >, ≠, ≤, or ≥ to compare two quantities.

المتباينة هي عبارة مفتوحة تستخدم <، أو >، أو ≠، أو ≤، أو≥ للمقارنة بين كميتين.

integer Any number from the set {..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...}, where ... means continues without end.

العدد الصحيح هو أي عدد ضمن المجموعة (..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...) حيث يعني «...» استمرار المجموعة دون نهاية.

interquartile range A measure of variation in a set of numerical data. It is the distance between first and third quartiles of the data set.

المدى الرُبيعي مقياس التباين في مجموعة من البيانات الرقمية. وهو المسافة بين الربيعين الأ ول والثالث لمجموعة من البيانات.

inverse variation A relationship where the product of x and y is a constant k. As x increases in value, y decreases in value, or as y decreases in value, x increases in value.

التغير العكسي هو علا قة تجعل حاصل ضرب x في y ثابتًا وهو k. وكلما زادت قيمة x ، نقصت قيمة y ؛ أو كلما نقصت قيمة y زادت قيمة x.

irrational number A number that cannot be expressed as the ratio of two integers.

العدد غير النسبي هو عدد لا يمكن التعبير عنه في صورة كسر من عددين صحيحين.

isosceles triangle A triangle having at least two congruent sides.

المثلث متساوي الساقين هو مثلث يحتوي على ضلعين متطابقين على الأقل.

K

kilogram The base unit of mass in the metric system. One kilogram equals 1,000 grams.

الكيلوجرام هو الوحدة الأساسية للكتلة في النظام المتري. الكيلوجرام الواحد يساوي 1000 جرام.

L

lateral face In a polyhedron, a face that is not a base.

الوجه الجانبي في مضلع، هو أي وجه غير القاعدة.

lateral surface area The sum of the areas of all of the lateral faces of a solid.

مساحة السطح الجانبي هي مجموع مساحات الأوجه الجانبية لأحد الأجسام.

least common denominator (LCD) The least common multiple of the denominators of two or more fractions. You can use the LCD to compare fractions.

المقام المشترك الأصغر (LCD) هو أصغر مضاعف مشترك في مقامي كسرين أو أكثر. ويمكنك استخدام المقام المشترك الأصغر في المقارنة بين الكسور.

like fractions Fractions that have the same denominators.

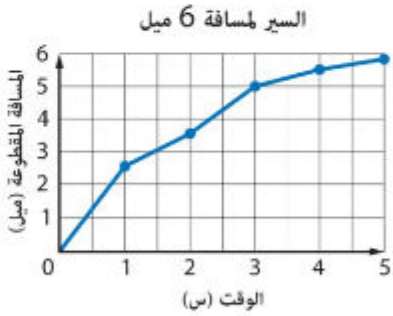
الكسور المتشابهة هي الكسور التي لها المقامات نفسها.

like terms Terms that contain the same variables raised to the same power. Example: 5x and 6x are like terms.

الحدود المتشابهة هي حدود تتكون من المتغيرات نفسها المرفوعة للأس نفسه. مثال: 5x و6x هي حدود متشابهة.

line graph A type of statistical graph using lines to show how values change over a period of time.

الرسم البياني الخطي هو نوع من الرسم البياني الإحصائي يستخدم خطوطًا لإظهار كيف تتغير القيم على مدى فترة زمنية.



linear expression An algebraic expression in which the variable is raised to the first power, and variables are not multiplied nor divided.

التعبير الخطي هو تعبير جبري يتم فيه رفع المتغير إلى الأس الأول، ولا يتضمن ضرب المتغيرات أو قسمتها.

linear function A function for which the graph is a straight line.

الدالة الخطية هي دالة يكون الرسم البياني فيها عبارة عن خط مستقيم.

linear relationship A relationship for which the graph is a straight line.

العلاقة الخطية هي علاقة يكون الرسم فيها البياني عبارة عن خط مستقيم.

liter The base unit of capacity in the metric system. The amount of dry or liquid material an object can hold.

اللتر هو الوحدة الأساسية للسعة في النظام المتري. وهو كمية المادة الجافة أو السائلة التي يستطيع أي جسم احتواءها.

M

markdown An amount by which the regular price of an item is reduced.

تخفيض الأسعار هو مقدار تخفيض السعر المعتاد لأحد العناصر.

markup The amount the price of an item is increased above the price the store paid for the item.

هامش الربح هو مقدار زيادة سعر عنصر عن السعر الذي دفعه المتجر مقابل هذا العناصر.

mean The sum of the data divided by the number of items in the data set.

المتوسط الحسابي هو مجموع البيانات مقسمة على عدد قيم البيانات.

mean absolute deviation A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values.

متوسط الانحراف المطلق هو قياس التباين في مجموعة من البيانات الرقمية التي يتم حسابها بإضافة مسافات بين كل قيمة بيانات والمتوسط الحسابي، ثم قسمة الناتج على عدد قيم البيانات.

measures of center Numbers that are used to describe the center of a set of data. These measures include the mean, median, and mode.

المقياس المركزي هو أرقام تستخدم لوصف مركز مجموعة من البيانات. ويشمل هذا المقياس الوسط الحسابي والوسيط، والمنوال.

measures of variation A measure used to describe the distribution of data.

مقاييس التغير هي عبارة عن وسيلة قياس لوصف توزيع البيانات الإحصائية.

median A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values.

الوسيط هو وسيلة قياس لمركز مجموعة من البيانات الرقمية. وسيط قائمة القيم هو القيمة التي تظهر في مركز النسخة المصنفة من القائمة أو المتوسط الحسابي للقيمتين المركزيتين، إذا كانت القائمة تحتوي على قيم بعدد زوجي.

meter The base unit of length in the metric system.

المتر هو وحدة الطول الأساسية في النظام المتري.

metric system A decimal system of measures. The prefixes commonly used in this system are kilo-, centi-, and milli-.

النظام المتري هو النظام العشري للقياسات. ووحدات القياس المستخدمة عادة في هذا النظام هي الكيلومتر والسنتيمتر والمليمتر.

mode The number or numbers that appear most often in a set of data. If there are two or more numbers that occur most often, all of them are modes.

المنوال هو العدد أو الأعداد الأكثر تكرارا في مجموعة من البيانات. وفي حالة وجود عددين أو أكثر هي الأكثر تكرارًا، تكون جميعها منوالا.

monomial A number, variable, or product of a number and one or more variables.

أحادي الحد هو عدد أو متغير أو حاصل ضرب لعدد ومتغير واحد أو أكثر.

Multiplication Property of Equality If you multiply each side of an equation by the same nonzero number, the two sides remain equal.

خاصية الضرب في المعادلة في حالة ضرب حدي أية معادلة في نفس العدد غير الصفري، فسوف يظل الحدان متساويين.

Multiplication Property of Inequality When you multiply each side of an inequality by a negative number, the inequality symbol must be reversed for the inequality to remain true.

خاصية الضرب في المتباينة عند ضرب حدي المتباينة في عدد سالب، يجب عكس رمز المتباينة لتظل المتباينة صحيحة.

Multiplicative Identity Property The product of any number and one is the number.

خاصية المحايد الضربي هي حاصل ضرب أي عدد في واحد يساوي نفس العدد.

Multiplicative Property of Zero The product of any number and zero is zero.

خاصّيّة الصّفر في الضّرب هي حاصل ضرب أي عدد في صفر يساوي صفرًا.

multiplicative inverse Two numbers with a product of 1. For example, the multiplicative inverse of $\frac{2}{3}$ is $\frac{3}{2}$.

المعكوس الضربي هو أي عددين حاصل ضربهما يساوي 1 على سبيل المثال، المعكوس الضربي للعدد $\frac{2}{3}$ هو $\frac{3}{2}$.

N

negative exponent Any nonzero number to the negative n power. It is the multiplicative inverse of its nth power.

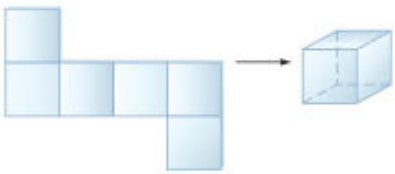
الأس السالب هو أي عدد غير الصفر مرفوع إلى الأس n السالب. ويكون عبارة عن المعكوس الضربي للأس n المرفوع إليه.

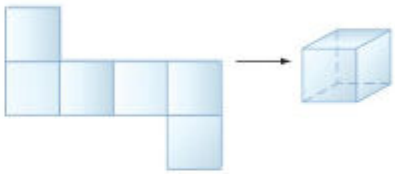
negative integer An integer that is less than zero. Negative integers are written with a - sign.

العدد الصحيح السالب هو أي عدد صحيح أقل من صفر. ويكتب بجوار العدد الصحيح علامة $-$.

net A two-dimensional figure that can be used to build a three-dimensional figure.

الشبكة هي شكل ثنائي الأبعاد يمكن استخدامه لبناء شكل ثلاثي الأبعاد.





nonagon A polygon having nine sides.

تساعي الأضلاع هو مضلع له تسعة أضلاع.

nonlinear function A function for which the graph is not a straight line.

الدالة غير الخطية هي دالة لا يكون الرسم البياني فيها خطًا مستقيمًا.

nonproportional The relationship between two ratios with a rate or ratio that is not constant.

غير متكافئ هو علاقة بين نسبتين بمعدل أو نسبة غير ثابتة.

numerical expression A combination of numbers and operations.

التعبير العددي هو مزيج من الأعداد والعمليات.

O

obtuse angle Any angle that measures greater than 90° but less than 180°.

الزاوية المنفرجة هي أي زاوية يكون قياسها أكبر من 90 درجة وأصغر من 180 درجة.

obtuse triangle A triangle having one obtuse angle.

المثلث منفرج الزاوية هو مثلث إحدى زواياه منفرجة.

octagon A polygon having eight sides.

ثماني الأضلاع هو مضلع له ثمانية أضلاع.

opposites Two integers are opposites if they are represented on the number line by points that are the same distance from zero, but on opposite sides of zero. The sum of two opposites is zero.

العددان المتقابلان هما عددان صحيحان يكونان معكوسين في حالة تمثليهما على خط الأ عداد من النقاط التي تكون على مسافة واحدة من الصفر، ولكنهما على ضلعين متقابلين من الصفر. ويكون مجموع العددين المتقابلين صفرًا.

order of operations The rules to follow when more than one operation is used in a numerical expression.

1. Evaluate the expressions inside grouping symbols.
2. Evaluate all powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

ترتيب العمليات هو القواعد الواجب اتباعها عند استخدام أكثر من عملية في تعبير عددي.

1. إيجاد قيم التعبيرات داخل رموز المجموعات.
2. إيجاد قيم جميع القوى.
3. الضرب والقسمة بالترتيب من اليسار إلى اليمين.
4. الإ ضافة والطرح بالترتيب من اليسار إلى اليمين.

ordered pair A pair of numbers used to locate a point in the coordinate plane. An ordered pair is written in the form (*x*-coordinate, *y*-coordinate).

الزوج المرتب هو زوج من الأ عداد يُستخدم لتحديد نقطة في المستوى الإحداثي. ويكتب الزوج المرتب على شكل (الإحداثي *y*، والإحداثي *x*).

origin The point at which the *x*-axis and the *y*-axis intersect in a coordinate plane. The origin is at (0, 0).

نقطة الأصل هي النقطة التي يتقاطع المحور الأفقي *x* مع المحور الرأسي *y* فيها في مستوى إحداثي. تكون نقطة الأصل هي (0، 0).

outcome Any one of the possible results of an action. For example, 4 is an outcome when a number cube is rolled.

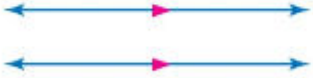
النتيجة هي إحدى النتائج الممكنة لحدث ما. مثال: يكون العدد 4 نتيجة في حالة دحرجة مكعب أعداد.

outlier A data value that is either much *greater* or much *less* than the median.

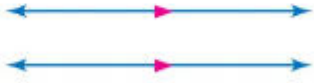
القيمة المتطرفة هي قيمة بيانات تكون إما أكبر بكثير أو أصغر بكثير من الوسيط الحسابي.

P

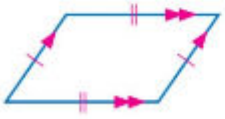
parallel lines Lines in a plane that never intersect.



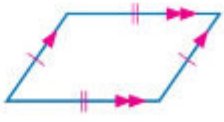
المستقيمات المتوازية هي مستقيمات في مستوى واحد ولا تتقاطع أبدًا.



parallelogram A quadrilateral with opposite sides parallel and opposite sides congruent.



متوازي الأضلاع هو شكل رباعي الأضلا ع فيه كل ضلعين متقابلين متطابقان.



pentagon A polygon having five sides.

خماسي الأضلاع هو مضلع له خمسة أضلاع.

percent equation An equation that describes the relationship between the part, whole, and percent.

$$\text{part} = \text{percent} \cdot \text{whole}$$

المعادلة المئوية هي المعادلة التي تصف العلاقة بين الجزء، والكل والنسبة المئوية.

$$\text{الجزء} = \text{النسبة المئوية} \times \text{الكل}$$

percent error A ratio that compares the inaccuracy of an estimate (amount of error) to the actual amount.

النسبة المئوية للخطأ هي النسبة التي تقارن عدم دقة تقدير (كم الخطأ) بالكم الفعلي.

percent of change A ratio that compares the change in a quantity to the original amount.

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original amount}}$$

النسبة المئوية للتغيير هي نسبة تقارن مقدار تغير كمية بالنسبة إلى الكم الأصلي.

$$\text{النسبة المئوية للتغيير} = \frac{\text{كم التغيير}}{\text{الكم الأصلي}}$$

percent of decrease A negative percent of change.

النسبة المئوية للتناقص هي النسبة المئوية السالبة للتغيير.

percent of increase A positive percent of change.

النسبة المئوية للتزايد هي النسبة المئوية الموجبة للتغيير.

percent proportion One ratio or fraction that compares part of a quantity to the whole quantity. The other ratio is the equivalent percent written as a fraction with a denominator of 100.

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

مقدار النسبة المئوية هو نسبة أو كسر يقارن جزءًا من كمية بمجمل الكمية. وتمثل النسبة الأخرى النسبة المئوية المكافئة المكتوبة في شكل كسر مقامه 100.

$$\frac{\text{الجزء}}{\text{الكل}} = \frac{\text{النسبة المئوية}}{100}$$

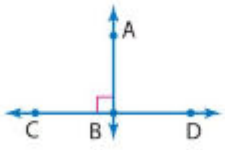
perfect squares Numbers with square roots that are whole numbers. 25 is a perfect square because the square root of 25 is 5.

المربعات الكاملة هي الأعداد التي لها جذور تربيعية وهي عبارة عن أعداد صحيحة. ويُعد العدد 25 مربعًا كاملاً لأن الجذر التربيعي للعدد 25 هو 5.

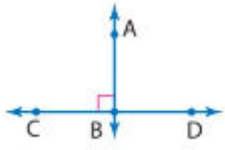
permutation An arrangement, or listing, of objects in which order is important.

التبديل هو التنسيق أو قائمة بمجموعة من الأشياء التي يعدّ الترتيب فيها مهمًا.

perpendicular lines Lines that meet or cross each other to form right angles.



المستقيمات المتعامدة هي المستقيمات التي تتقابل أو تتقاطع مع بعضها لتشكل زوايا قائمة.



pi The ratio of the circumference of a circle to its diameter. The Greek letter π represents this number. The value of pi is 3.1415926. . . . Approximations for pi are 3.14 and $\frac{22}{7}$.

باي π هي نسبة محيط الدائرة إلى قطرها. ويمثل الحرف اليوناني π هذا العدد. وقيمة π تساوي 3.1415926. . . . ويُمكن تقريب π إلى 3.14 و $\frac{22}{7}$.

plane A two-dimensional flat surface that extends in all directions.

المستوى هو سطح مسطح ثنائي الأبعاد يمتد في جميع الاتجاهات.

polygon A simple closed figure formed by three or more straight line segments.

المضلع هو شكل مغلق بسيط مكون من ثلا ث قطع مستقيمة أو أكثر.

polyhedron A three-dimensional figure with faces that are polygons.

متعدد الوجوه هو شكل ثلاثي الأ بعاد له وجوه عبارة عن مضلعات.

population The entire group of items or individuals from which the samples under consideration are taken.

المجتمع الإحصائي هو مجموعة كاملة من العناصر أو الأفراد التي يتم منها أخذ العينات قيد البحث.

positive integer An integer that is greater than zero. They are written with or without a + sign.

العدد الصحيح الموجب هو العدد الصحيح الأكبر من صفر. ويمكن كتابته بعلامة + أو بدونها.

powers Numbers expressed using exponents. The power 3^2 is read *three to the second power*, or *three squared*.

عمليات الرفع هي التعبير عن الأرقام باستخدام الأسس. فعملية الرفع 3^2 تقرأ ثلاثة أس اثنين أو ثلاثة تربيع.

precision The ability of a measurement to be consistently reproduced.

الدقة هي إمكانية الحصول على القياس نفسه على الدوام.

principal The amount of money deposited or borrowed.

رأس المال هو مقدار المال المودع أو المقترض.

prism A polyhedron with two parallel congruent faces called bases.

المنشور هو شكل متعدد الوجوه له وجهان متوازيان ومتطابقان يطلق عليهما القاعدتان.

probability The chance that some event will happen. It is the ratio of the number of favorable outcomes to the number of possible outcomes.

الاحتمال هو فرصة حدوث بعض الأحداث. ويُعبر عن نسبة عدد النتائج المفضلة إلى عدد النتائج المحتملة.

probability model A model used to assign probabilities to outcomes of a chance process by examining the nature of the process.

نموذج الاحتمال هو نموذج يستخدم لتعيين الاحتمالات إلى النتائج الخاصة بفرصة ما من خلا ل دراسة طبيعة العملية.

properties Statements that are true for any number or variable.

الخواص هي البيانات التي تعتبر حقيقية بالنسبة إلى أي عدد أو متغير.

proportion An equation stating that two ratios or rates are equivalent.

التناسب هو معادلة تنص على أن نسبتين أو معدلين متكافئان.

proportional The relationship between two ratios with a constant rate or ratio.

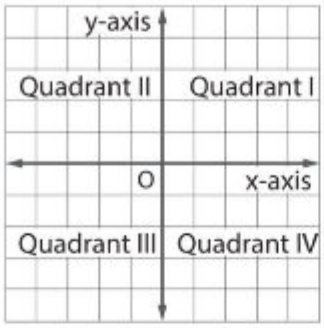
التناسب هو العلا قة بين نسبتين بمعدل أو نسبة ثابتة.

pyramid A polyhedron with one base that is a polygon and three or more triangular faces that meet at a common vertex.

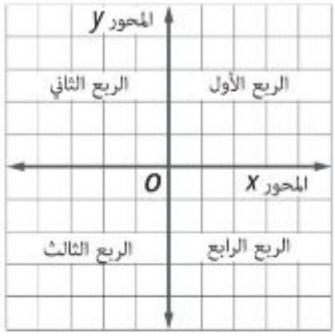
الهرم هو شكل متعدد الوجوه له قاعدة واحدة على شكل مضلع وثلا ثة أوجه مثلثة أو أكثر تلتقي في قمة مشتركة.

Q

quadrant One of the four regions into which the two perpendicular number lines of the coordinate plane separate the plane.



الربع هو أحد الأقسام الأربعة التي يفصلها خطا أعداد متعامدان على مستوى إحداثي داخل المستوى نفسه.



quadrilateral A closed figure having four sides and four angles.

رباعي الأضلاع هو شكل مغلق له أربعة أضلاع وأربع زوايا.

quartile A value that divides the data set into four equal parts.

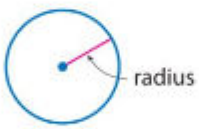
الربيع هو القيمة التي تقسم مجموعة من البيانات إلى أربعة أجزاء متساوية.

R

radical sign The symbol used to indicate a nonnegative square root, $\sqrt{\ }$.

علامة الجذر هي الرمز المستخدم للدلالة على الجذر التربيعي غير السالب، $\sqrt{\ }$.

radius The distance from the center of a circle to any point on the circle.



نصف القطر هو المسافة بين مركز الدائرة وأي نقطة على الدائرة.

random Outcomes occur at random if each outcome occurs by chance. For example, rolling a number on a number cube occurs at random.

العشوائية هي النتائج التي تحدث عشوائيا إذا حدثت كل النتائج عن طريق المصادفة. مثل، تدحرج عدد على مكعب عددي عشوائيًا.

range The set of output values for a function.

المدى هو مجموعة قيم المخرجات الخاصة بدالة.

range The difference between the greatest and least data value.

المدى هو الفرق بين قيمتي البيانات الكبرى والصغرى.

rate A ratio that compares two quantities with different kinds of units.

المعدل هو معدل يقارن بين كميتين في أنواع مختلفة من الوحدات.

rate of change A rate that describes how one quantity changes in relation to another. A rate of change is usually expressed as a unit rate.

معدل التغير هو معدل يصف كيف تتغير كمية بالنسبة إلى كمية أخرى. وعادة ما يتم التعبير عن معدل التغير بمعدل الوحدة.

rational numbers The set of numbers that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Examples: $1 = \frac{1}{1}$, $\frac{2}{9}$, $-2.3 = -2\frac{3}{10}$

الأعداد النسبية هي مجموعة من الأعداد التي يمكن كتابتها في شكل $\frac{a}{b}$ ، بحيث يكون a و b عددَين صحيحين و $b \neq 0$.

أمثلة: $1 = \frac{1}{1}$, $\frac{2}{9}$, $-2.3 = -2\frac{3}{10}$

real numbers A set made up of rational and irrational numbers.

الأعداد الحقيقية هي مجموعة مكونة من أعداد جذرية وغير جذرية.

reciprocal The multiplicative inverse of a number.

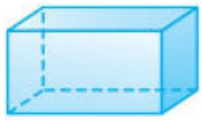
المعكوس الضربي هو المقلوب الضربي للعدد.

rectangle A parallelogram having four right angles.

المستطيل هو متوازي أضلا ع به أربع زوايا قائمة.

rectangular prism A prism that has two parallel congruent bases that are rectangles.

المنشور القائم هو منشور يحتوي على قاعدتين متطابقتين ومتوازيتين على شكل مستطيلين.

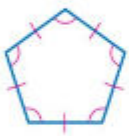


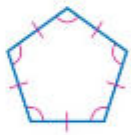
reduction An image smaller than the original.

الاختزال هو الصورة الأ صغر من الشكل الأصلي.

regular polygon A polygon that has all sides congruent and all angles congruent.

مضلع منتظم مضلع جميع أضلا عه وزواياه متطابقة.





regular pyramid A pyramid whose base is a regular polygon and in which the segment from the vertex to the center of the base is the altitude.

الهرم المنتظم هو هرم قاعدته عبارة عن مضلّع منتظم وارتفاعه هو القطعة المستقيمة الواصلة من رأسه إلى مركز قاعدته.

relation Any set of ordered pairs.

العلاقة هي أي مجموعة من الأزواج المرتبة.

relative frequency A ratio that compares the frequency of each category to the total.

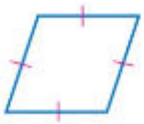
التكرار النسبيّ هو نسبة تقارن التكرار بين كل فئة والمجموع الكلي.

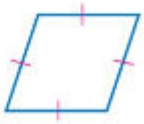
repeating decimal The decimal form of a rational number.

الكسر العشري الدوري هو الصيغة العشرية من العدد النسبي.

rhombus A parallelogram having four congruent sides.

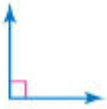
المعين هو متوازي أضلا ع مكون من أربعة أضلاع متطابقة.





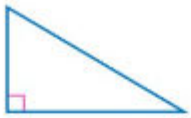
right angle An angle that measures exactly 90°.

الزاوية القائمة هي زاوية قياسها 90 درجة بالضبط.



right triangle A triangle having one right angle.

المثلث القائم الزاوية هو مثلث إحدى زواياه قائمة.



S

sales tax An additional amount of money charged on items that people buy.

الضريبة على المبيعات هي مبلغ إضافي من المال يُفرض على البنود التي يشتريها المواطنون.

sample A randomly selected group chosen for the purpose of collecting data.

العينة هي مجموعة مختارة بشكل عشوائي بهدف تجميع البيانات.

sample space The set of all possible outcomes of a probability experiment.

مجال العينة هو مجموعة النتائج المحتملة للتجربة الاحتمالية.

scale The scale that gives the ratio that compares the measurements of a drawing or model to the measurements of the real object.

المقياس يعطي المقياس نسبة تعمل على مقارنة مقاييس رسم أو نموذج لقياس الشيء الحقيقي.

scale drawing A drawing that is used to represent objects that are too large or too small to be drawn at actual size.

الرسم المقياسي هو الرسم المستخدم لتمثيل الأشياء الكبيرة للغاية أو الصغيرة للغاية التي يمكن رسمها بالحجم الفعلي.

scale factor A scale written as a ratio without units in simplest form.

عامل القياس تدريج يُكتب على هيئة نسبة في أبسط صورها دون استخدام الوحدات.

scale model A model used to represent objects that are too large or too small to be built at actual size.

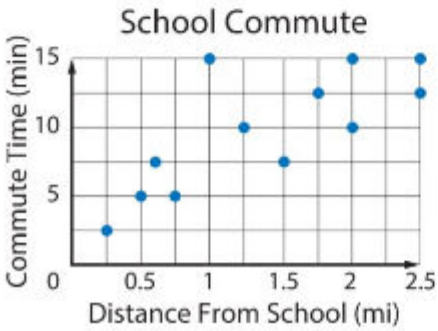
النموذج المقياسي هو نموذج يستخدم لتمثيل أشياء كبيرة للغاية أو صغيرة للغاية لدرجة أنه لا يمكن بناؤه بالحجم الفعلي.

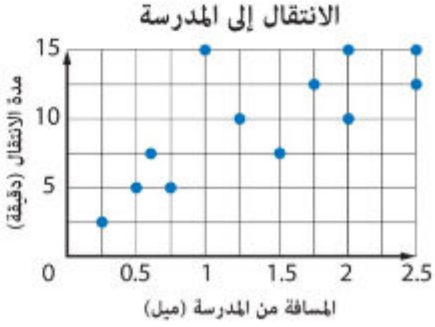
scalene triangle A triangle having no congruent sides.

المثلث مختلف الأضلاع هو مثلث أضلاعه غير متطابقة.

scatter plot In a scatter plot, two sets of related data are plotted as ordered pairs on the same graph.

مخطّط الانتشار في مخطّط الانتشار، يتم رسم مجموعتين من البيانات ذات الصلة على شكل زوج مرتب على الرسم البياني نفسه.





selling price The amount the customer pays for an item.

سعر البيع هو مبلغ يدفعه العميل للحصول على منتج ما.

semicircle Half of a circle. The formula for the area of a semicircle is $A = \frac{1}{2}\pi r^2$.

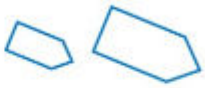
نصف الدائرة هو أحد الجزأين المتساويين في الدائرة. وصيغة مساحة نصف الدائرة $A = \frac{1}{2}\pi r^2$.

sequence An ordered list of numbers, such as 0, 1, 2, 3 or 2, 4, 6, 8.

المتتالية هي قائمة مرتبة من الأعداد، مثل 0، 1، 2، 3 أو 2، 4، 6، 8.

similar figures Figures that have the same shape but not necessarily the same size.

الأشكال المتشابهة هي أشكال لها الشكل نفسه ولكن ليس بالضرورة أن يكون لها نفس الحجم.



similar solids Solids with the same shape. Their corresponding linear measures are proportional.

المجسّمات متماثلة هي مجسّمات لها نفس الشكل. وتكون القياسات الخطية المتطابقة فيها نسبية.

simple event One outcome or a collection of outcomes.

الحدث البسيط هو نتيجة واحدة أو مجموعة نتائج.

simple interest The amount paid or earned for the use of money. The formula for simple interest is $I = prt$.

الفائدة البسيطة هي المبلغ المدفوع أو المكتسب نظير استخدام المال. صيغة الفائدة البسيطة هي $I = prt$.

simple random sample An unbiased sample where each item or person in the population is as likely to be chosen as any other.

العينة العشوائية البسيطة هي عينة محايدة يكون فيها احتمال اختيار كل عنصر أو شخص في المجتمع الإحصائي مماثلاً بالنسبة إلى جميع العناصر أو الأشخاص.

simplest form An expression is in simplest form when it is replaced by an equivalent expression having no like terms or parentheses.

أبسط صورة يكون التعبير في أبسط صورة عندما يتم استبداله بتعبير مكافئ ليس له حدود أو أقواس مشابهة.

simplify Write an expression in simplest form.

التبسيط هو كتابة تعبير في أبسط صورة.

simulation An experiment that is designed to model the action in a given situation.

المحاكاة هي تجربة مصممة لصياغة إجراء ما في موقف محدد.

skew lines Lines that do not intersect and are not coplanar.

المستقيمات المتخالفة هي المستقيمات التي لا تتقاطع وغير متحدة المستوى.

slant height The height of each lateral face.

الارتفاع الجانبي هو ارتفاع كل وجه جانبي.

slope The rate of change between any two points on a line. It is the ratio of vertical change to horizontal change. The slope tells how steep the line is.

الميل هو معدل التغير بين أي نقطتين على المستقيم. وهو نسبة التغير الرأسي إلى التغير الأفقي. ويوضح الميل مقدار انحدار المستقيم.

solution A replacement value for the variable in an open sentence. A value for the variable that makes an equation true. Example: The solution of $12 = x + 7$ is 5.

حل قيمة لحل المتغير في جملة مفتوحة. أو هو قيمة المتغير التي تجعل المعادلة صحيحة. مثال: حل المعادلة $12 = x + 7$ هو 5.

square The product of a number and itself. 36 is the square of 6.

المربع هو حاصل ضرب عدد في نفسه. 36 هو مربع العدد 6.

square A parallelogram having four right angles and four congruent sides.

المربع هو متوازي أضلاع له أربع زوايا قائمة وأربعة أضلاع متطابقة.

square root The factors multiplied to form perfect squares.

الجذر التربيعي هو عوامل مضاعفة لتشكيل مربعات كاملة.

squared The product of a number and itself. 36 is the square of 6.

التربيع هو حاصل ضرب عدد ونفسه. 36 هو مربع العدد 6.

standard form Numbers written without exponents.

الصيغة القياسية هي الأعداد المكتوبة بدون الأسس.

statistics The study of collecting, organizing, and interpreting data.

علم الإحصاء هو دراسة جمع البيانات وتنظيمها وتفسيرها.

straight angle An angle that measures exactly 180°.

الزاوية المستقيمة هي زاوية قياسها 180 درجة بالضبط.

Subtraction Property of Equality If you subtract the same number from each side of an equation, the two sides remain equal.

خاصية الطرح في المعادلة إذا طرحت العدد نفسه من كلا طرفي المعادلة، يظل الطرفان متساويين.

Subtraction Property of Inequality If you subtract the same number from each side of an inequality, the inequality remains true.

خاصية الطرح في المتباينة إذا طرحت العدد نفسه من كلا طرفي المتباينة، تظل المتباينة صحيحة.

supplementary angles Two angles are supplementary if the sum of their measures is 180°.

الزاويتان المتكاملتان تكون الزاويتان متكاملتين إذا كان مجموع قياسهما يساوي 180°.

∠1 and ∠2 are supplementary angles.

∠1 و ∠2 هما زاويتان متكاملتان.

surface area The sum of the areas of all the surfaces (faces) of a three-dimensional figure.

مساحة السطح هي مجموع مساحات أسطح (أوجه) الشكل ثلاثي الأبعاد.

survey A question or set of questions designed to collect data about a specific group of people, or population.

دراسة استقصائية هي عبارة عن سؤال أو مجموعة أسئلة تهدف إلى جمع البيانات حول مجموعة معينة من البشر أو مجتمع إحصائي.

systematic random sample A sample where the items or people are selected according to a specific time or item interval.

عينة عشوائية منتظمة هي عينة يتم فيها اختيار العناصر أو الأشخاص وفق فترة زمنية محددة أو عنصر محدد.

T

term Each number in a sequence.

الحد هو كل عدد في متتالية.

term A number, a variable, or a product or quotient of numbers and variables.

الحد هو العدد أو المتغير أو حاصل ضرب أو قسمة الأعداد والمتغيرات.

terminating decimal A repeating decimal which has a repeating digit of 0.

الكسر العشري المنتهي هو كسر عشري متكرر يوجد به رقم متكرر وهو الصفر.

theoretical probability The ratio of the number of ways an event can occur to the number of possible outcomes. It is based on what should happen when conducting a probability experiment.

احتمال نظري هو نسبة عدد طرق إمكانية حدوث الحدث إلى عدد النتائج المحتملة. ويقوم الاحتمال النظري على ما يجب حدوثه عند إجراء تجربة احتمالية.

three-dimensional figure A figure with length, width, and height.

الشكل ثلاثي الأبعاد هو شكل له طول وعرض وارتفاع.

third quartile For a data set with median M, the third quartile is the median of the data values greater than M.

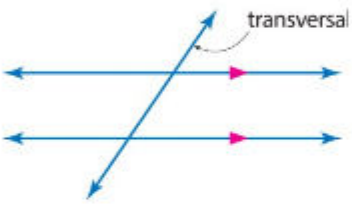
الربيع الثالث بالنسبة لمجموعة البيانات ذات الوسيط M، يكون الربيع الثالث هو وسيط قيم البيانات التي تكون أكبر من قيمة M.

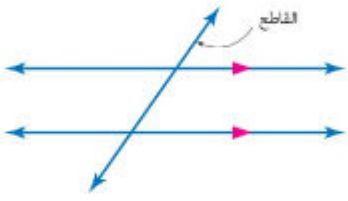
tip Also known as a gratuity, it is a small amount of money in return for a service.

العطية تُعرف أيضًا باسم الإكرامية، وهي مبلغ صغير يُقدم مقابل خدمة.

transversal The third line formed when two parallel lines are intersected.

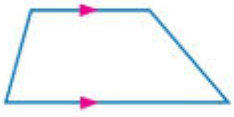
القاطع هو المستقيم الثالث الذي يتكوّن عندما يقطع مستقيمين متوازيين.

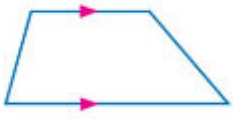




trapezoid A quadrilateral with one pair of parallel sides.

شبه المنحرف هو شكل رباعي الأضلاع مكون من زوج من الأضلاع المتوازية.





tree diagram A diagram used to show the sample space.

مخطط الشجرة هو مخطط يستخدم في عرض مجال عينة.

triangle A figure with three sides and three angles.

المثلث هو شكل له ثلاثة أضلاع وثلاث زوايا.

triangular prism A prism that has two parallel congruent bases that are triangles.

المنشور الثلاثي هو منشور يحتوي على قاعدتين متطابقتين ومتوازيتين على شكل مثلثين.

two-step equation An equation having two different operations.

معادلة من خطوتين هي معادلة تحتوي على عمليتين مختلفتين.

two-step inequality An inequality than contains two operations.

متباينة من خطوتين هي متباينة تحتوي على عمليتين.

U

unbiased sample A sample representative of the entire population.

العينة الحيادية هي عينة تمثل جميع السكان.

unfair game A game where there is not a chance of each player being equally likely to win.

اللعبة غير العادلة هي لعبة لا يوجد بها فرصة متساوية لكل لاعب في تحقيق الفوز.

uniform probability model A probability model which assigns equal probability to all outcomes.

نموذج الاحتمال المنتظم هو نموذج احتمال يخصص الاحتمالات المتكافئة لجميع النتائج.

unit rate A rate that is simplified so that it has a denominator of 1 unit.

معدل الوحدة هو معدل تم تبسيطه بحيث تصبح قيمة المقام وحدة واحدة.

unit ratio A unit rate where the denominator is one unit.

نسبة الوحدة هي معدل الوحدة عندما يكون المقام وحدة واحدة.

unlike fractions Fractions with different denominators.

الكسور غير المتشابهة هي كسور لها مقامات مختلفة.

V

variable A symbol, usually a letter, used to represent a number in mathematical expressions or sentences.

المتغير هو رمز عادةً ما يكون حرفًا، ويُستخدم في تمثيل عدد في التعبيرات أو العبارات الرياضية.

vertex A vertex of an angle is the common endpoint of the rays forming the angle.

الرأس هو رأس إحدى الزوايا التي لديها نقطة نهاية مشتركة للأشعة التي تشكل الزاوية.

vertex The point where three or more faces of a polyhedron intersect.

الرأس هو نقطة تقاطع ثلاثة وجوه فأكثر للشكل ثلاثي الأبعاد.

vertex The point at the tip of a cone.

الرأس هو نقطة نهاية الشكل المخروطي.

vertical angles Opposite angles formed by the intersection of two lines. Vertical angles are congruent.

∠1 and ∠2 are vertical angles.

الزوايا المتقابلة بالرأس هي الزوايا المتقابلة ونشأت عن تقاطع خطين مستقيمين. والزوايا المتقابلة بالرأس تعد زوايا متطابقة.

∠1 و ∠2 هما زاويتان متقابلتان بالرأس.

visual overlap A visual demonstration that compares the centers of two distributions with their variation, or spread.

التداخل البصري هو عرض بصري يقارن مراكز توزيعين اثنين من حيث التباين أو الانتشار.

volume The number of cubic units needed to fill the space occupied by a solid.

الحجم هو عدد الوحدات المكعبة اللازمة لملء مساحة يشغلها جسم صلب.

voluntary response sample A sample which involves only those who want to participate in the sampling.

عينة الاستجابة الطوعية هي عينة تضم فقط الأشخاص الراغبين في المشاركة في العينة.

X

***x*-axis** The horizontal number line in a coordinate plane.

المحور الأفقي *x* هو خط أعداد أفقي في مستوى إحداثي.

***x*-coordinate** The first number of an ordered pair. It corresponds to a number on the *x*-axis.

الإحداثي *x* هو العدد الأول في الزوج المرتب. ويطابق الإحداثي *x* العدد الموجود في المحور الأفقي *x*.

Y

y-axis The vertical number line in a coordinate plane.

المحور الرأسي *y* هو خط أعداد رأسي في مستوى إحداثي.

y-coordinate The second number of an ordered pair. It corresponds to a number on the *y*-axis.

الإحداثي *y* هو العدد الثاني في الزوج المرتب. ويطابق العدد الموجود في المحور الرأسي *y*.

Z

zero pair The result when one positive counter is paired with one negative counter. The value of a zero pair is 0.

الزوج الصفري هو النتيجة التي نحصل عليها عند اقتران عدد موجب مع عدد سالب. وتكون قيمة الزوج الصفري هي 0.

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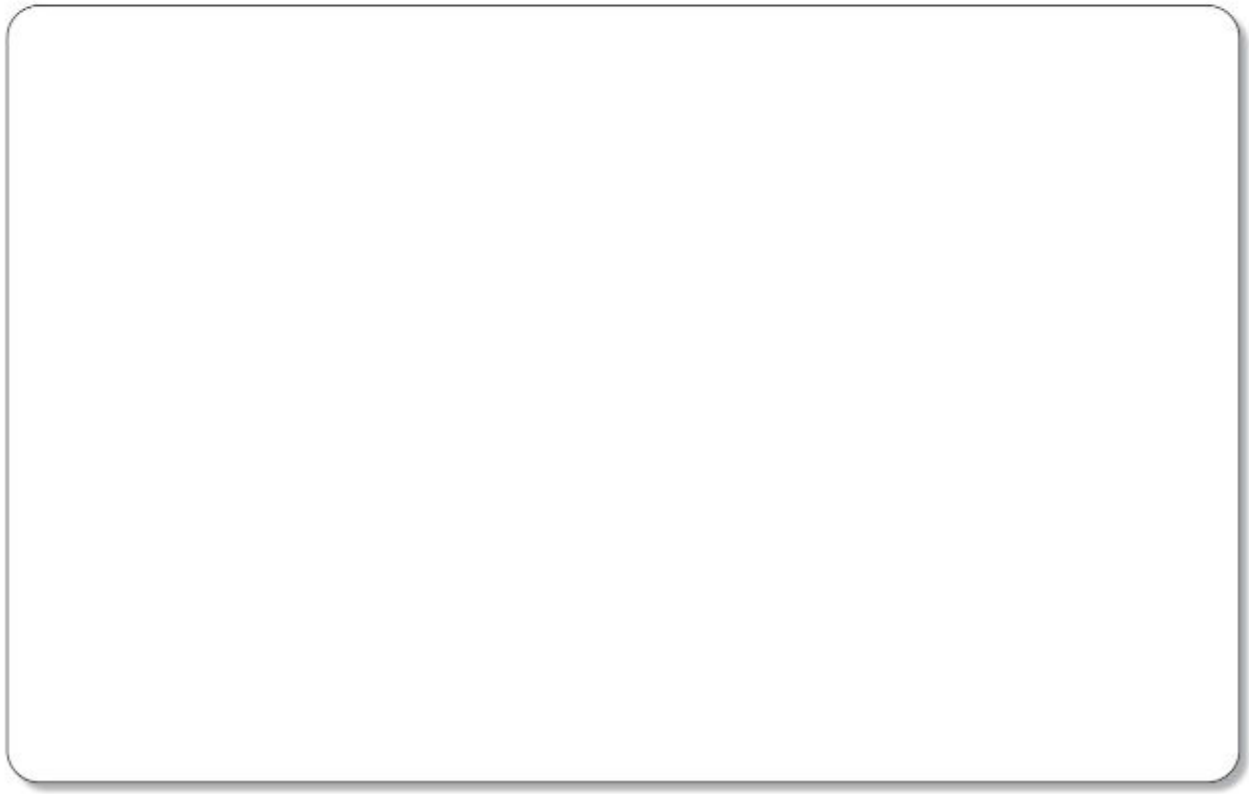

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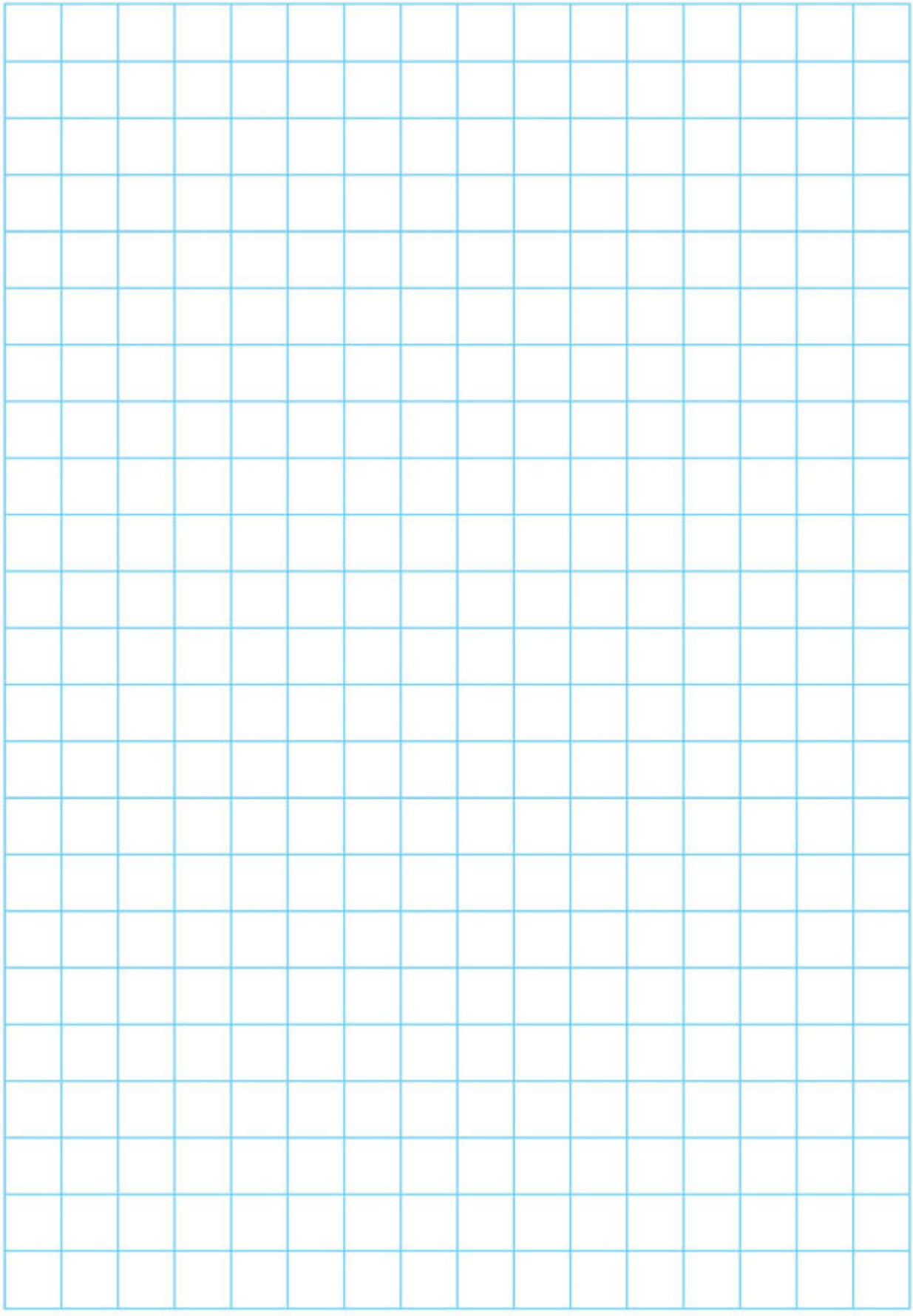

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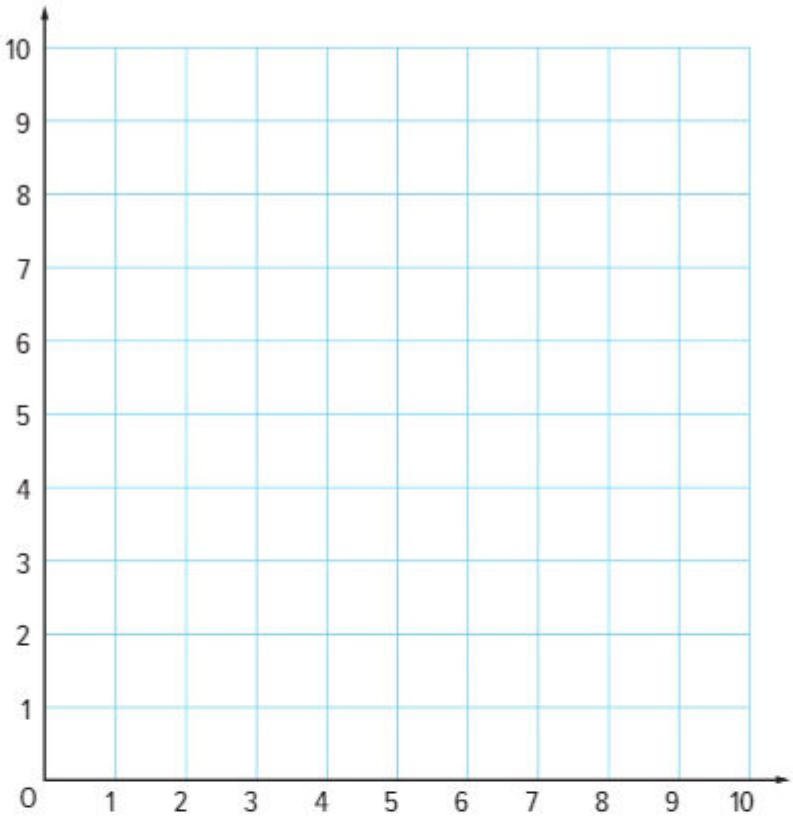

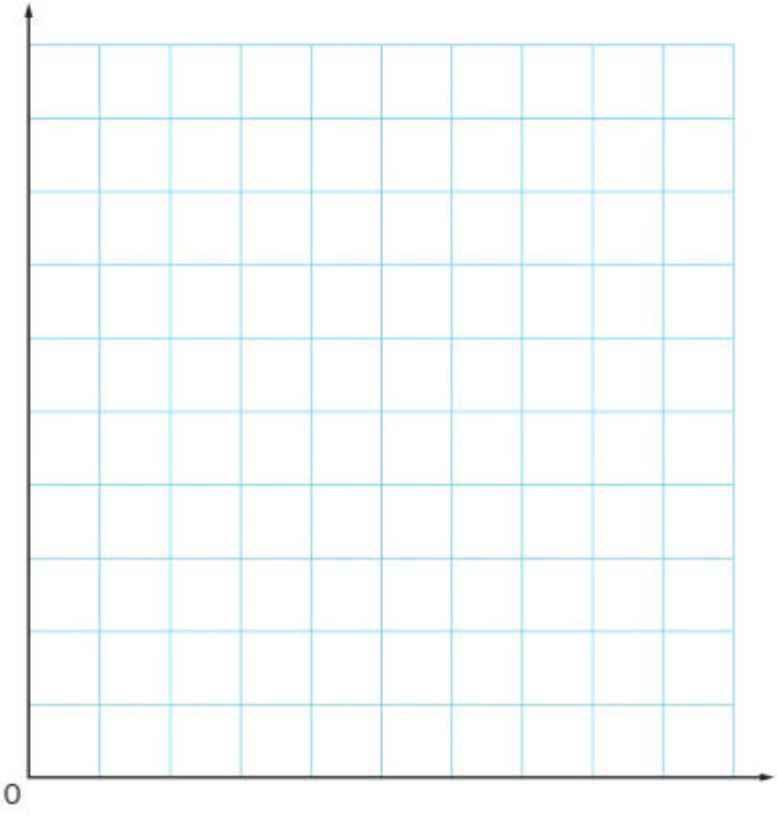

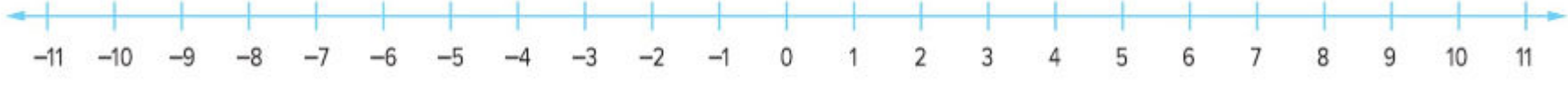

Name

Work Mats

WM4 Number Lines

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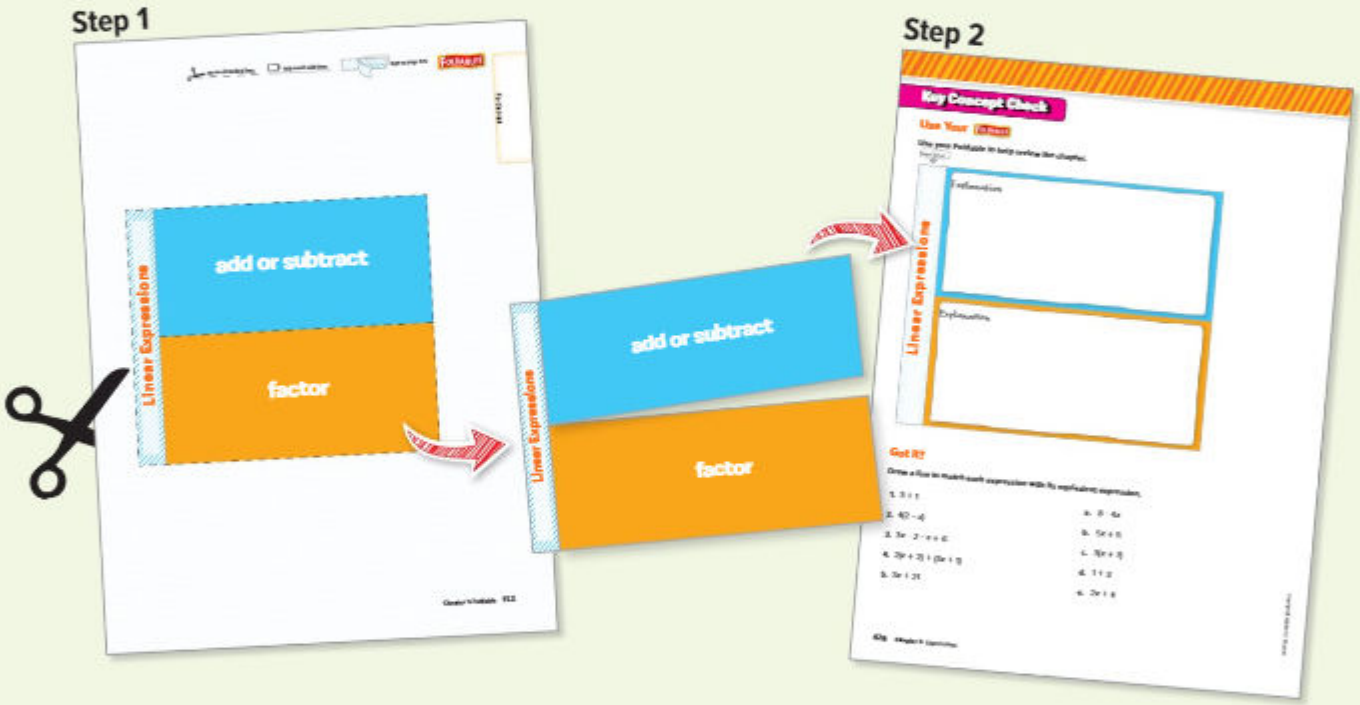
Foldables are three-dimensional graphic organizers that help you create study guides for each chapter in your book.

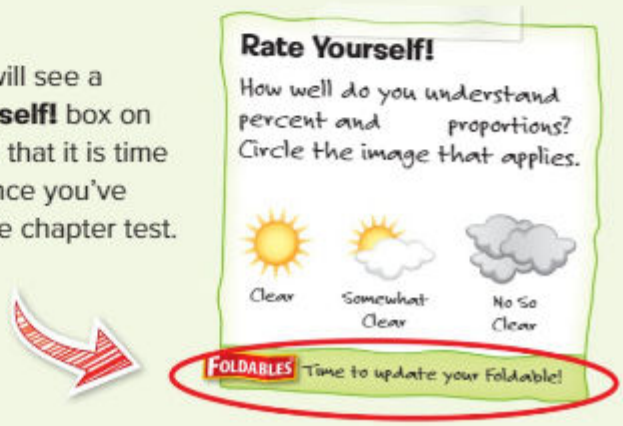
Step 1 Go to the back of your book to find the Foldable for the chapter you are currently studying. Follow the cutting and assembly instructions at the top of the page.

Step 2 Go to the Key Concept Check at the end of the chapter you are currently studying. Match up the tabs and attach your Foldable to this page. Dotted tabs show where to place your Foldable. Striped tabs indicate where to tape the Foldable.



How Will I Know When to Use My Foldable?

When it's time to work on your Foldable, you will see a Foldables logo at the bottom of the **Rate Yourself!** box on the Guided Practice pages. This lets you know that it is time to update it with concepts from that lesson. Once you've completed your Foldable, use it to study for the chapter test.



How Do I Complete My Foldable?

No two Foldables in your book will look alike. However, some will ask you to fill in similar information. Below are some of the instructions you'll see as you complete your Foldable. **HAVE FUN** learning math using Foldables!

Instructions and what they mean

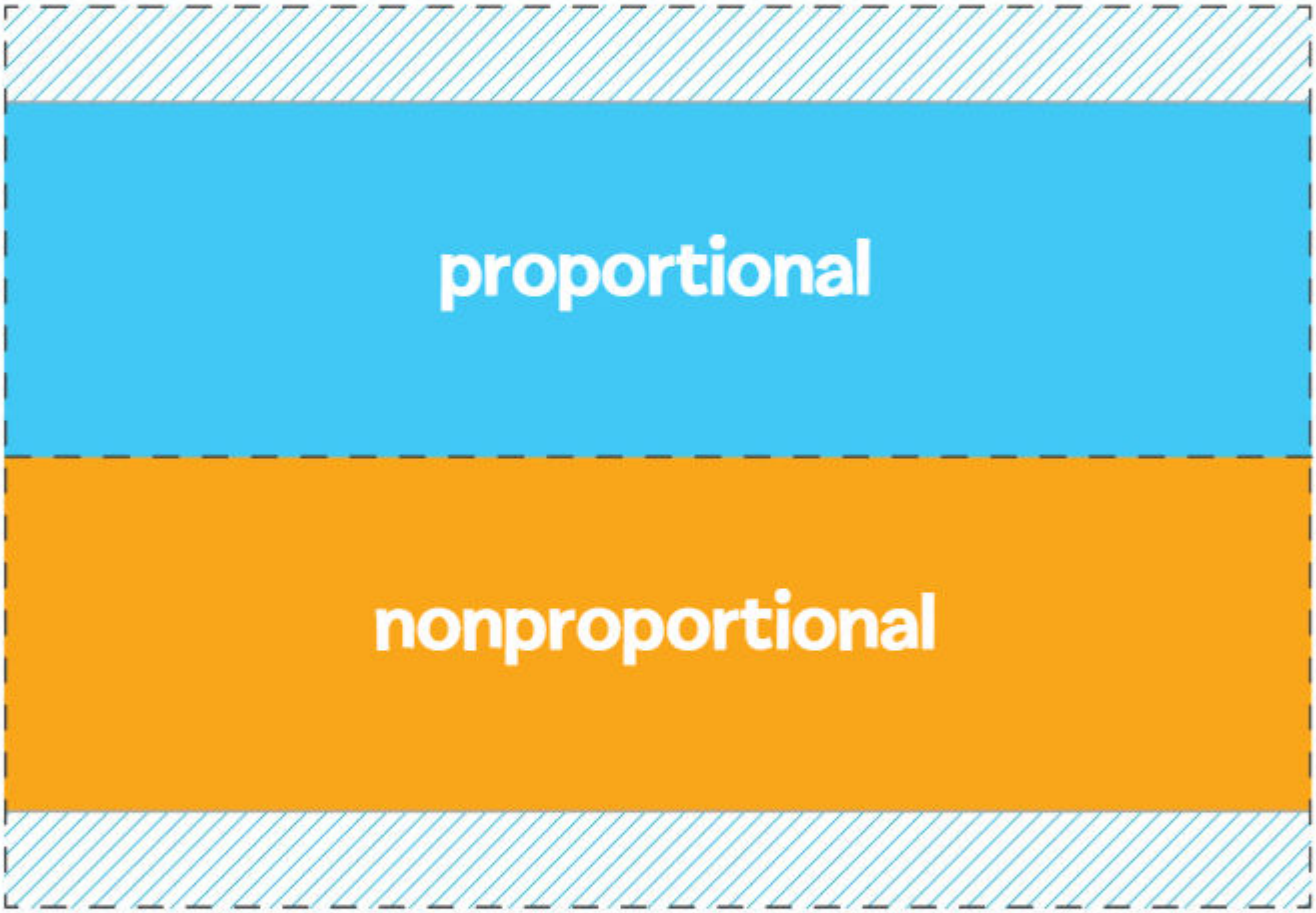
Best Used to...	Complete the sentence explaining when the concept should be used.
Definition	Write a definition in your own words.
Description	Describe the concept using words.
Equation	Write an equation that uses the concept. You may use one already in the text or you can make up your own.
Example	Write an example about the concept. You may use one already in the text or you can make up your own.
Formulas	Write a formula that uses the concept. You may use one already in the text.
How do I ...?	Explain the steps involved in the concept.
Models	Draw a model to illustrate the concept.
Picture	Draw a picture to illustrate the concept.
Solve Algebraically	Write and solve an equation that uses the concept.
Symbols	Write or use the symbols that pertain to the concept.
Write About It	Write a definition or description in your own words.
Words	Write the words that pertain to the concept.

Meet Foldables Author Dinah Zike

Dinah Zike is known for designing hands-on manipulatives that are used nationally and internationally by teachers and parents. Dinah is an explosion of energy and ideas. Her excitement and joy for learning inspires everyone she touches.

cut on all dashed lines fold on all solid lines tape to page 92 FOLDABLES

Foldables



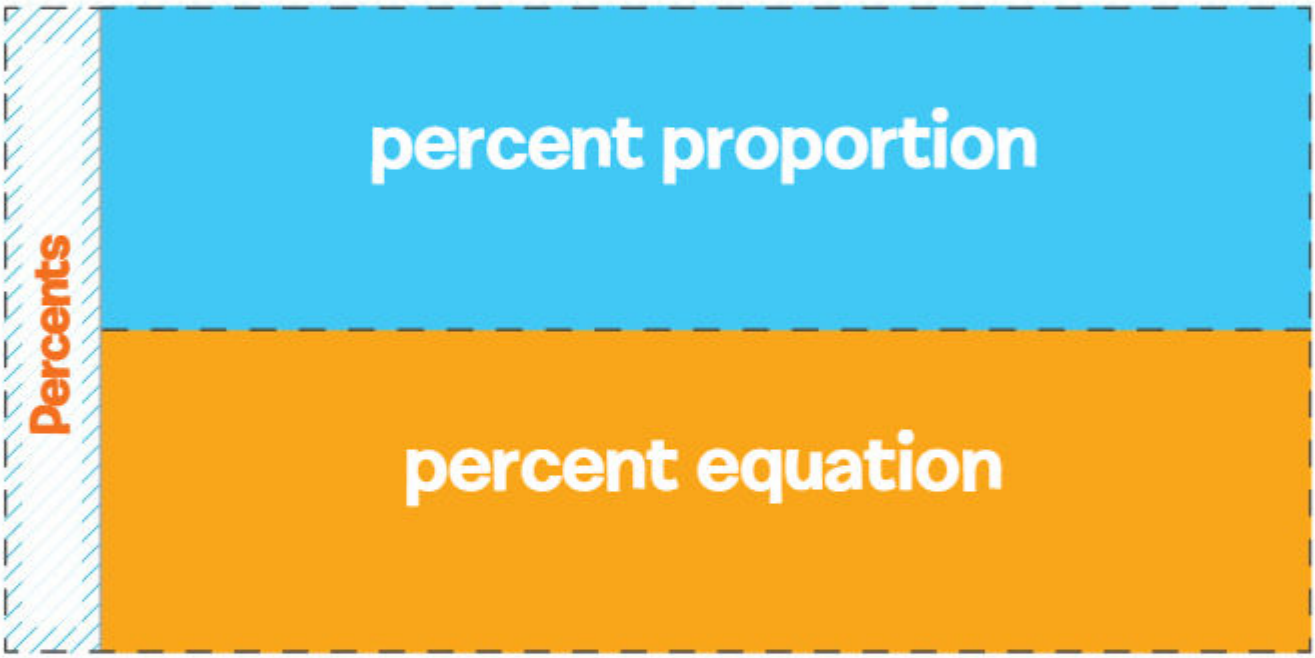
cut on all dashed lines fold on all solid lines tape to page 92 FOLDABLES

page 92 Tab 1

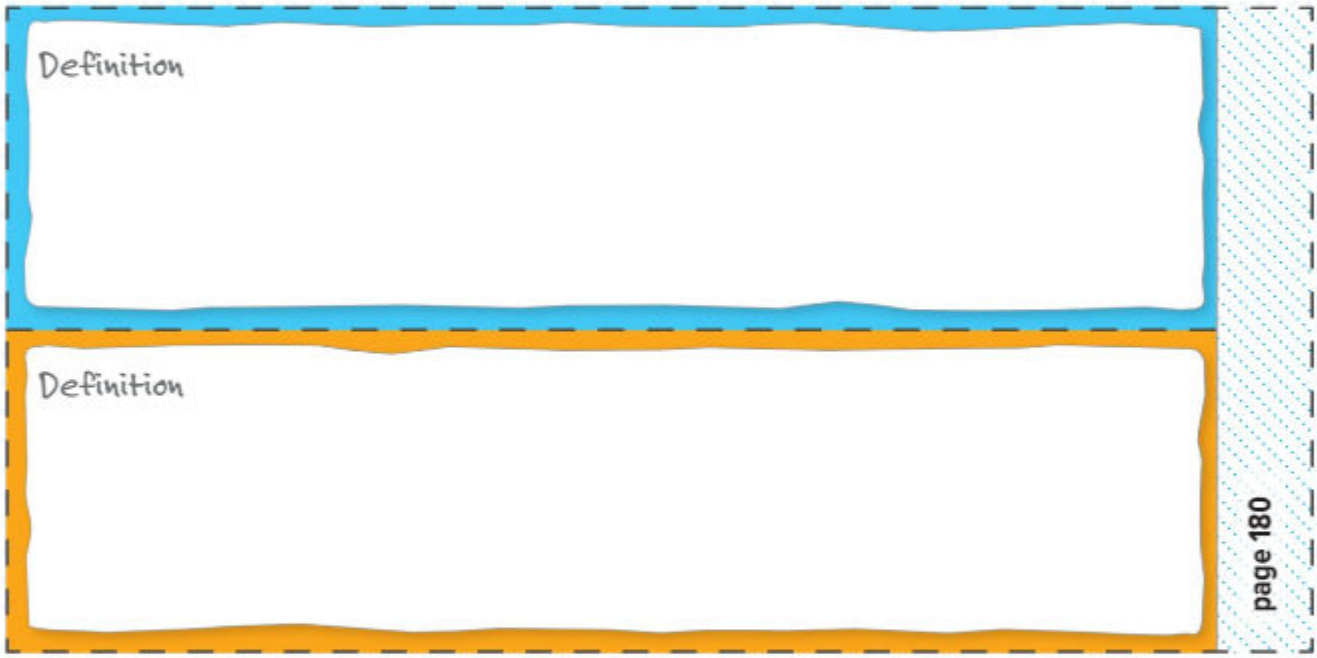
Write About It

Write About It

page 92 Tab 2

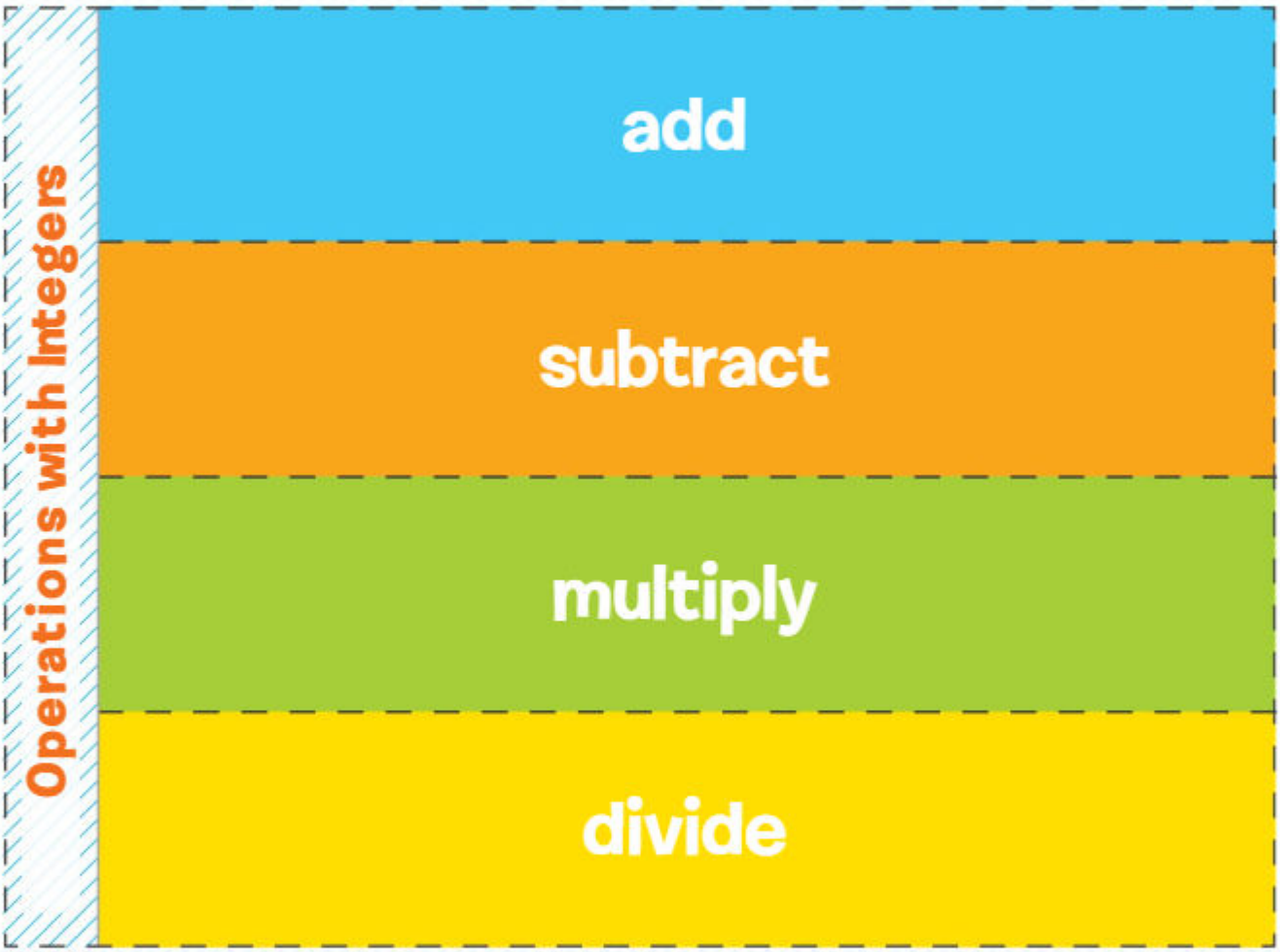


cut on all dashed lines fold on all solid lines tape to page 180 FOLDABLES

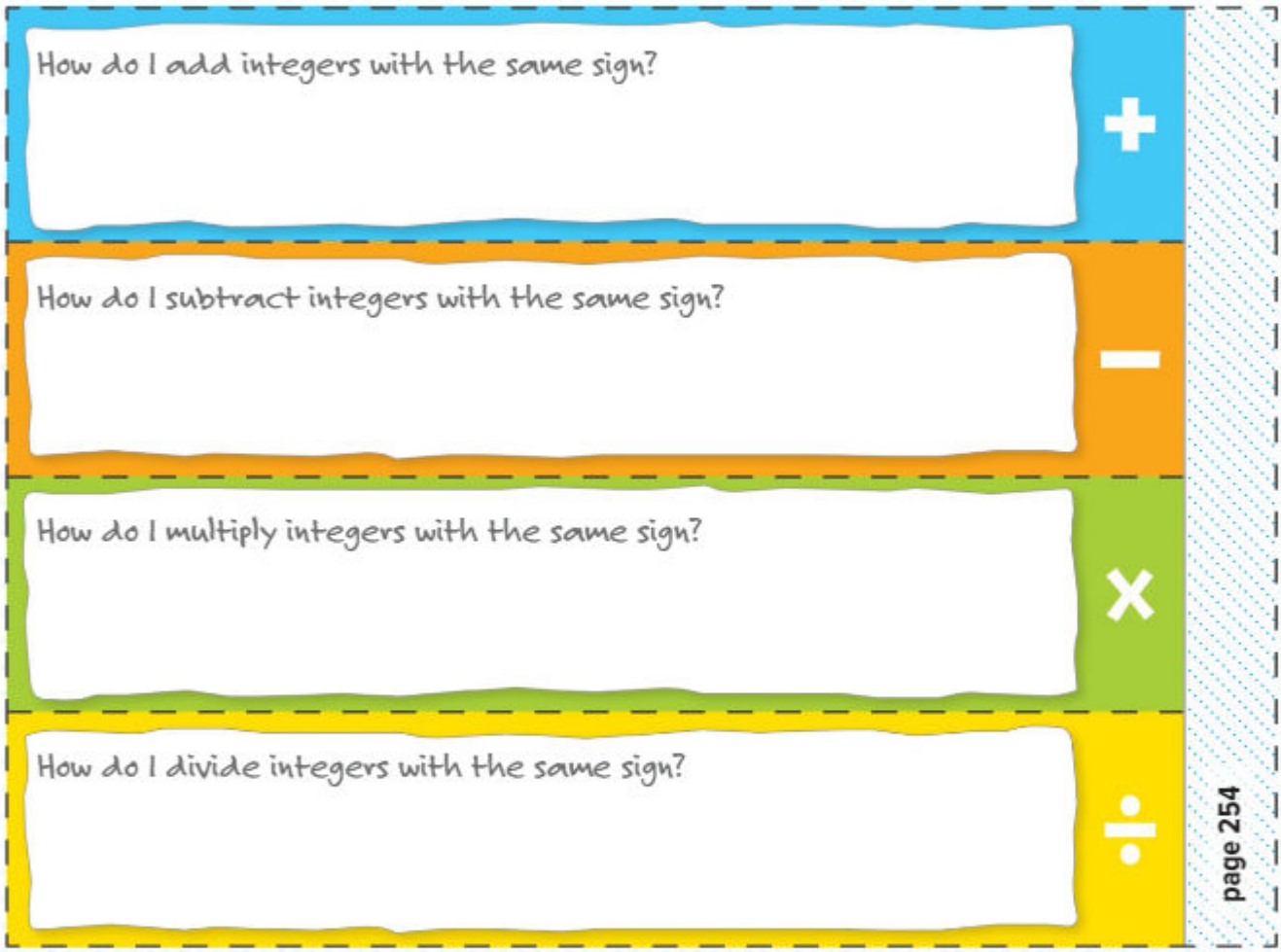


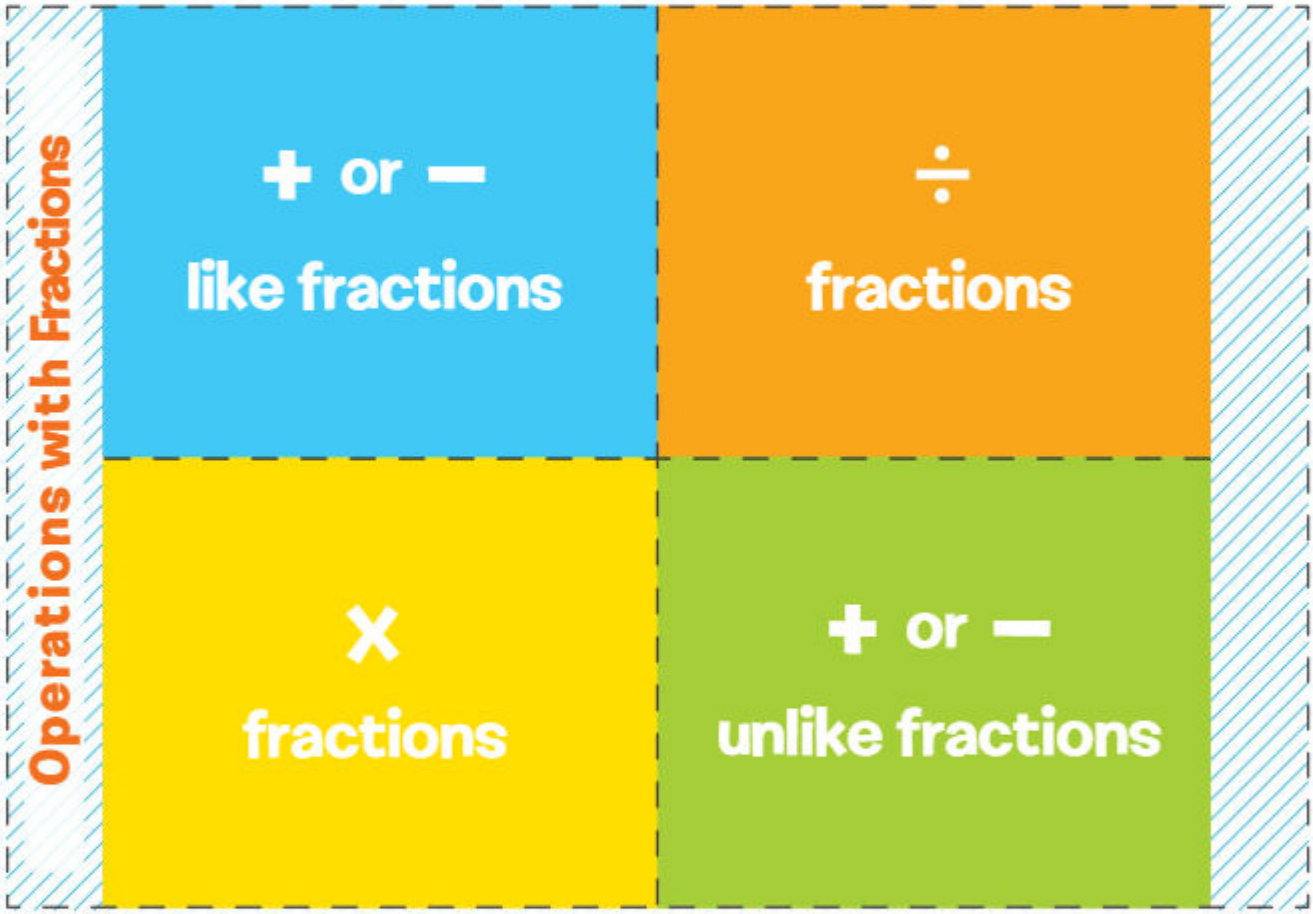
cut on all dashed lines fold on all solid lines tape to page 254 FOLDABLES

Foldables



cut on all dashed lines fold on all solid lines tape to page 254 FOLDABLES





cut on all dashed lines fold on all solid lines tape to page 254 FOLDABLES