Chapter 19

Insulation and Refractory

INTRODUCTION

The development presented in earlier chapters may be expanded to include insulation plus refractory and their effects. Industrial thermal insulation usually consists of materials of low thermal conductivity combined in a way to achieve a higher overall resistance to heat flow. Webster[1] defines insulation as: "to separate or cover with a non-conducting material in order to prevent the passage or leakage of ... heat ... etc." Insulation is defined in Perry's[2] in the following manner: "Materials or combinations of materials which have air- or gas-filled pockets or void spaces that retard the transfer of heat with reasonable effectiveness are thermal insulators. Such materials may be particulate and/or fibrous, with or without binders, or may be assembled, such as multiple heat-reflecting surfaces that incorporate air- or gas-filled void spaces." Refractory materials also serve the chemical process industries. In addition to withstanding heat, refractory also provides resistance to corrosion, erosion, abrasion, and/or deformation.

This chapter contains three remaining sections:

Describing Equations
Insulation
Refractory

However, the bulk of the material keys on insulation since it has found more applications than refractories, particularly in its ability to reduce heat losses. The reader should note that there is significant overlap of common theory, equations, and applications with Chapters 7 and 8, both of which are concerned with heat conduction.

DESCRIBING EQUATIONS

When insulation is added to a surface, the heat transfer between the wall surface and the surroundings will take place by a two-step steady-state process (see Figure 19.1 for flow past a base flat plate and Figure 19.2 for flow past an insulated plate): conduction from the wall surface at T_0 through the wall to T_1 and through the insulation from T_1 to

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

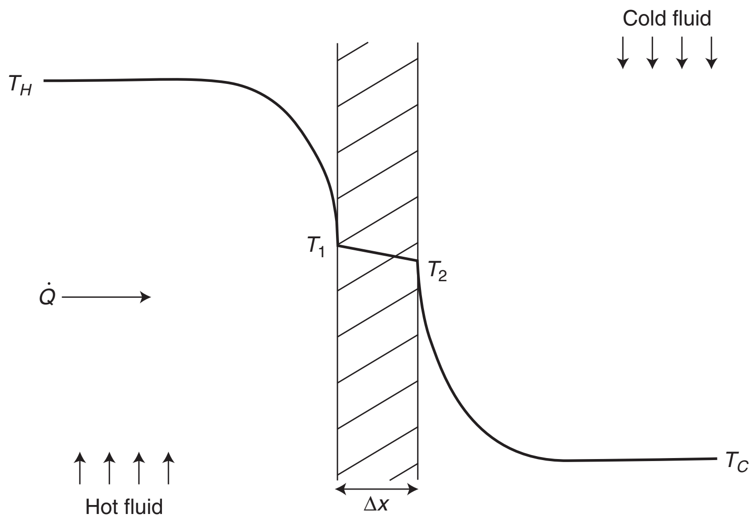

Figure 19.1 Flow past a flat plate.

T_2, and convection from the insulation surface at T_2 to the surrounding fluid at T_3. The temperature drop across each part of the heat flow path in Figure 19.2 is given below. The temperature drop across the wall and insulation is

$$(T_0 - T_1) = \dot{Q}R_0 \tag{19.1}$$

$$(T_1 - T_2) = \dot{Q}R_1 \tag{19.2}$$

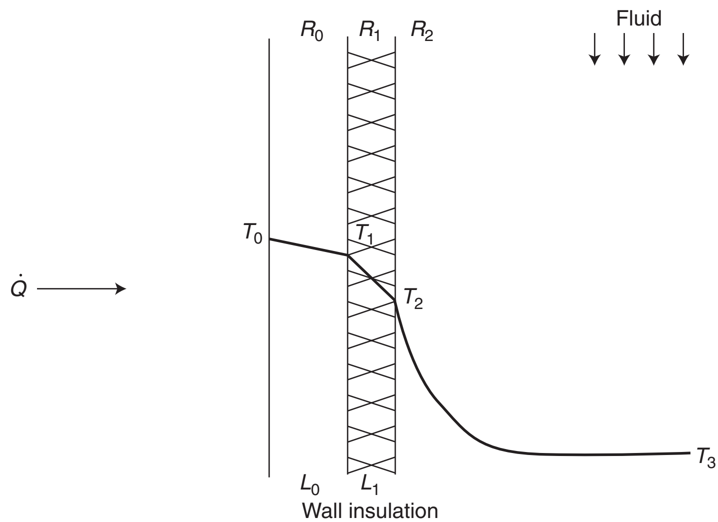

Figure 19.2 Flow past a flat insulated plate.

The temperature drop across the fluid film is

$$(T_2 - T_3) = \dot{Q}R_2 \tag{19.3}$$

where R_0 is the thermal resistance due to the conduction through the pipe wall (L_0/k_0A_0); R_1, the thermal resistance due to the conduction through the insulation (L_1/k_1A_1); and R_2, the thermal resistance due to the convection through the fluid ($1/h_2A_2$).

As with earlier analyses, the same heat rate, $\dot{Q}$, flows through each thermal resistance. Therefore, Equations (19.1) through (19.3) can be combined to give

$$(T_0 - T_3) = \dot{Q}R_t \tag{19.4}$$

The heat transfer is then

$$\dot{Q} = \frac{(T_0 - T_3)}{R_t} = \frac{\text{total thermal driving force}}{\text{total thermal resistance}} \tag{19.5}$$

Note that for the case of plane walls, the areas A_0, A_1, and A_2 are the same (see Figure 19.2). Dividing Equation (19.2) by Equation (19.3) yields

$$\frac{(T_1 - T_2)}{(T_2 - T_3)} = \frac{R_1}{R_2} = \frac{h_2L_1}{k_1}\frac{\text{conduction insulation resistance}}{\text{convection resistance}} \tag{19.6}$$

The group h_2L_1/k_1 is a dimensionless number termed earlier as the *Biot number*, Bi (refer to Table 9.1 in Part II),

$$\text{Bi} = \frac{\text{(fluid convection coefficient)(characteristic length)}}{\text{(thermal conductivity of insulation surface)}} = \frac{hL}{k} \tag{19.7}$$

ILLUSTRATIVE EXAMPLE 19.1

The following data is provided: a rectangular plane room wall, 2.5 m high and 4 m wide, has an outside surface temperature, $T_1 = 24°\text{C}$; the outside air temperature is $T_3 = -15°\text{C}$. Calculate the heat transfer rate. The convective heat transfer coefficient between the outside surface and the air is $11\ \text{W/m}^2 \cdot \text{K}$.

If loosely packed wool with $k = 0.04\ \text{W/m} \cdot \text{K}$ and a thickness of 7.62 mm (3 in.) is used for insulation on the outer wall, calculate a revised heat transfer rate.

***SOLUTION*:** Heat is transferred by convection from the outer wall surface at T_1 to the surrounding air at T_3. Therefore, Equation (19.3) may be applied with R replaced by $1/hA$:

$$\dot{Q} = hA(T_1 - T_3)$$

The heat transfer area is

$$A = (2.5)(4) = 10\,\text{m}^2$$

First, calculate $\dot{Q}_b$ for the base pipe without the insulation:

$$\begin{aligned}\dot{Q}_b &= hA(T_1 - T_3)\\ &= (11)(10)[24 - (-15)]\\ &= 4290\,\text{W} = 4.29\,\text{kW} = 14{,}639\,\text{Btu/h}\end{aligned}$$

Now consider the addition of insulation. Calculate the two thermal resistances (i.e., those due to conduction and convection). Since the insulation thickness, L_I, is 0.00762 m, the insulation resistance, R_I, is

$$R_I = \frac{L_I}{kA} = \frac{0.00762}{(0.04)(10)} = 0.0191\,\text{K/W}$$

In addition, the convective resistance, R_c, is

$$R_c = \frac{1}{hA} = \frac{1}{(11)(10)} = 0.0091\,\text{K/W}$$

The total resistance is

$$R_t = R_I + R_c = 0.0191 + 0.0091 = 0.0282\,\text{K/W}$$

The heat transfer rate is therefore

$$\begin{aligned}\dot{Q}_I &= \frac{T_1 - T_3}{R_t}\\ &= \frac{[24 - (-15)]}{0.0282}\\ &= 1383\,\text{W} = 1.38\,\text{kW} = 4583\,\text{Btu/h}\end{aligned}$$

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ILLUSTRATIVE EXAMPLE 19.2

Refer to Illustrative Example 19.1. Calculate the temperature at the wall-insulation surface.

***SOLUTION*:** The temperature at the outer surface of the insulation, T_2, is given by

$$T_2 = T_1 - \dot{Q}R_I \tag{19.3}$$

Substitution gives

$$\begin{aligned}T_2 &= 24.0 - 1383(0.0191)\\ &= 24.0 - 26.4\\ &= -2.4^\circ\text{F}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 19.3

Calculate the Biot number in the previous example and comment on the results.

***SOLUTION*:** First calculate the Biot number. See Equations (19.6) and (19.7).

$$\mathrm{Bi} = hL/k$$

Substituting

$$\begin{aligned}\mathrm{Bi} &= \frac{(11)(0.00762)}{0.04} \\ &= 2.1\end{aligned}$$

This indicates that the conduction resistance is more than twice the convective resistance. ■

ILLUSTRATIVE EXAMPLE 19.4

One wall of an oven has a 3-inch insulation cover. The temperature on the inside of the wall is at 400°F; the temperature on the outside is at 25°C. What is the heat flux (heat flow rate per unit area) across the wall if the insulation is made of glass wool ($k = 0.022$ Btu/h · ft · °F)?

***SOLUTION*:** Once again, the thermal resistance associated with conduction is defined as

$$R = L/kA$$

where R = thermal resistance
k = thermal conductivity
A = area across which heat is conducted
L = length across which heat is conducted

The rate of heat transfer, $\dot{Q}$, is then

$$\dot{Q} = \Delta T/R \tag{19.4}$$

Thus,

$$\dot{Q} = kA\Delta T/L$$

Since 25°C is approximately 77°F and L is 0.25 ft,

$$\begin{aligned}\frac{\dot{Q}}{A} &= \frac{0.022(400 - 77)}{0.25} \\ &= 28.4\ \mathrm{Btu/h \cdot ft^2}\end{aligned}$$

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ILLUSTRATIVE EXAMPLE 19.5

A cold-storage room has a plane rectangular wall 8 m wide (w) and 3 m high (H). The temperature of the outside surface of the wall T_1 is -18°C. The surrounding air temperature T_3 is 26°C.

The convective heat transfer coefficient between the air and the surface is 21 $\mathrm{W/m^2 \cdot K}$. A layer of cork board insulation (thermal conductivity, $k = 0.0433\ \mathrm{W/\ m \cdot K}$) is to be attached to the outside wall to reduce the cooling load by 80%.

1. Calculate the rate of heat flow through the rectangular wall without insulation. Express the answer in tons of refrigeration (1 ton of refrigeration = 12,000 Btu/h). Which direction is the heat flowing?
2. Determine the required thickness of the insulation board.

***SOLUTION*:** 1. Calculate the heat transfer area, A:

$$A = (w)(H) = (8)(3) = 24\,\mathrm{m}^2$$

Calculate the rate of heat flow in the absence of insulation. Heat is transferred by convection from the wall surface to the surroundings. Applying Newton's law of cooling,

$$\begin{aligned}\dot{Q} &= hA(T_1 - T_3)\\ &= (21)(24)(-18 - 26) = -22{,}176\,\mathrm{W} = -22.18\ \mathrm{kW}\\ &= (-22{,}176)(3.4123) = -75{,}671\,\mathrm{Btu/h}\\ &= -75{,}671/12{,}000 = -6.3\ \text{ton of refrigeration}\end{aligned}$$

The negative sign indicates heat flow from the surrounding air into the cold room.

2. Calculate the heat rate with insulation. Since the insulation is to reduce $\dot{Q}$ by 80%, then

$$\dot{Q} = (0.2)(-22{,}176) = -4435.2\,\mathrm{W}$$

Calculate the total thermal resistance:

$$\begin{aligned}R_{\text{tot}} &= (T_1 - T_3)/\dot{Q} = (-18 - 26)/(-4435.2) = 0.00992^\circ\mathrm{C/W}\\ &= R_1 + R_2\end{aligned}$$

Calculate the convection thermal resistance, R_2:

$$R_2 = 1/hA = 1/[(21)(24)] = 0.00198^\circ\mathrm{C/W}$$

Also calculate the insulation conduction resistance, R_1:

$$R_1 = R_{\text{tot}} - R_2 = 0.00992 - 0.00198 = 0.00794^\circ\mathrm{C/W}$$

The required insulation thickness, L_I, is given by

$$R_I = L_I/k_I A$$

Substituting

$$\begin{aligned}L_I &= R_I k_I A = (0.00794)(0.0433)(24)\\ &= 0.00825\ \mathrm{m} = 8.25\,\mathrm{mm}\end{aligned}$$

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ILLUSTRATIVE EXAMPLE 19.6

Refer to the previous example.

1. Calculate the temperature at the interface between the cork board and the air.
2. Calculate the Biot number, Bi.
3. What can one conclude?

***SOLUTION*:** Calculate the interface temperature, T_2:

$$T_2 - T_3 = \dot{Q}R_2 \tag{19.3}$$

$$\begin{aligned} T_2 &= T_3 + \dot{Q}R_2 = 26 + (-4435.2)(0.00198) \\ &= 26 - 8.78 = 17.22^\circ\text{C} \end{aligned}$$

Also, calculate the Biot number:

$$\begin{aligned} \text{Bi} &= hL_1/k_1 \\ &= (21)(0.00825)/(0.0433) = 4.00 \end{aligned} \tag{19.7}$$

The result that Bi $\approx$ 4 indicates that the conduction resistance is four times as large as the convection resistance.

As will be demonstrated later in this chapter, if this dimensionless number is applied to insulation around a given pipe of diameter D_o, there is critical thickness (of outer diameter D_c or outer radius r_c) of insulation for which any increase in this value will result in a decrease in $\dot{Q}$, the heat loss. This critical length can be determined from Equation (19.8).

$$Bi = 2.0 = \frac{hD_c}{k}$$

or

$$Bi = 1.0 = \frac{hr_c}{k} \tag{19.8}$$

In effect, if the outer radius of the insulation is greater than the value obtained from Equation (19.8), any further increase in the insulation thickness will result in a corresponding decrease in heat transfer. Interestingly, for "small" diameter pipes, i.e., $D_o < D_c$, an increase in insulation initially produces an increase in heat loss until the insulation diameter achieves a value equal to D_c. Further increases in insulation produces a decrease in heat loss. ■

ILLUSTRATIVE EXAMPLE 19.7

A hypodermic needle with an external diameter D_1 of 0.50 mm is to be used to transfer a reactant preheated to 95°C in a laboratory reactor. To reduce the heat loss from the transfer line, the hypodermic needle is threaded through the center of a solid rubber insulating tube (thermal conductivity, $k_2 = 0.2$ W/m $\cdot$ K) with a diameter of D_2 of 2 mm.

1. Calculate the rate of the heat loss from the hypodermic needle with and without the rubber insulation.
2. Calculate the Biot and Nusselt numbers for the uninsulated needle taking the characteristic dimension to be the diameter of the needle.

The stainless steel needle has a thermal conductivity, k_1, of 16 $\mathrm{W/m \cdot K}$. The ambient air temperature is 20°C. The thermal conductivity of the air, k_3, is 0.0242 $\mathrm{W/m \cdot K}$. The heat transfer coefficient, h_3, from the outside surface of the transfer line to the surrounding air is primarily due to natural convection and is approximately equal to 12 $\mathrm{W/m^2 \cdot K}$; it may also be assumed independent of the radius or the temperature.

***SOLUTION*:** Neglect the resistance of the metal needle. Determine the value of R_{tot} for the case of no insulation. In the absence of insulation,

$$r_2 = r_1 = 2.5 \times 10^{-3}\ \text{m} \quad \text{and} \quad R_I = 0$$

Therefore,

$$\begin{aligned} R_{\text{tot}} &= R_3 \\ &= 1/[h_3(2\pi r_2 L)] \\ &= 1/[12(2\pi)(0.25 \times 10^{-3})(1)] \\ &= 53.05^\circ\text{C/W} \end{aligned}$$

Calculate $\dot{Q}$ without insulation:

$$\begin{aligned} \dot{Q} &= (T_1 - T_3)/R_{\text{tot}} \\ &= (95 - 20)/53.05 \\ &= 1.41\ \text{W} \end{aligned}$$

Calculate the Biot number:

$$\begin{aligned} \text{Bi} &= h_3 D_2/k_{\text{needle}};\ k_{\text{needle}} = k_1 \\ &= (12)(0.5 \times 10^{-3})/16 \\ &= 0.000375 \end{aligned}$$

Calculate the Nusselt number of the air:

$$\begin{aligned} \text{Nu} &= h_3 D_2/k_{\text{air}} \\ &= (12)(0.5 \times 10^{-3})/0.0242 \\ &= 0.248 \end{aligned}$$

Now consider the insulated needle. Calculate the thermal resistances:

$$D_3 = 2\ \text{mm} \qquad r_3 = 1\ \text{mm} = 0.001\ \text{m}$$

Note that cylindrical coordinates apply (see Chapters 8 and 9):

$$\begin{aligned}
R_I = R_2 &= \frac{\ln(r_2/r_1)}{2\pi k_2 L} \\
&= \frac{\ln(0.001/0.00025)}{2\pi(0.2)(1)} \\
&= 1.103^\circ\text{C/W} \\
R_3 &= 1/[h_3(2\pi r_2 L)] \\
&= 1/[(12)(2\pi)(0.001)(1)] \\
&= 13.26^\circ\text{C/W} \\
R_{\text{tot}} &= R_2 + R_3 \\
&= 14.37^\circ\text{C/W}
\end{aligned}$$

Calculate the rate of heat loss:

$$\begin{aligned}
\dot{Q} &= (T_1 - T_3)/R_{\text{tot}} \\
&= (95 - 20)/(14.37) \\
&= 5.22\,\text{W}
\end{aligned}$$

Check on the critical radius of insulation. Since

$$\text{Bi}_c = 2.0 = h\,D_c/k \tag{19.7}$$

or

$$\begin{aligned}
r_c &= k_2/h_3 \\
&= 0.2/12 = 0.0166\text{ m} = 16.6\,\text{mm} \\
D_c &= 33.3\,\text{mm}
\end{aligned}$$

Repeat the calculations for other insulation diameters and present the results in tabular form (see Table 19.1). The table indicates that the addition of insulation increases the rate of heat loss

Table 19.1 Heat Rates for Different Insulators

Outside pipe insulation diameter, D_3, mm	Insulation resistance, R_2, °C/W	Convection resistance R_3, °C/W	Total resistance $R_{\text{tot}} = R_2 + R_3$, °C/W	Heat rate $\dot{Q}$, W
0.5	0	53.05	53.05	1.41
2.0	1.103	13.26	14.37	5.22
4.0	1.654	6.63	8.29	9.05
6.0	1.977	4.42	6.39	11.72
8.0	2.206	3.32	5.52	13.58
10.0	2.384	2.65	5.036	14.89
12.0	2.529	2.21	4.739	15.82
14.0	2.651	1.895	4.546	16.49
$33.3 = D_c$	3.342	0.796	4.138	18.13
50.0	3.3667	0.531	4.198	17.87

because the increase in the conduction resistance is proportionally less than the decrease in the convection resistance. The total resistance therefore decreases for this "small" diameter pipe but increases when $D > D_c$. ■

ILLUSTRATIVE EXAMPLE 19.8

Comment further on the results of the previous example.

***SOLUTION*:** As shown in Table 19.1, the maximum value of the heat loss (at the critical radius) is 18.13 W. The initial increase in heat loss may be viewed as occurring because of what the author refers to as a "radius of curvature" effect for small radii. In effect, the area increases at a faster rate than the resistance decreases. More details follow in the next section. The reader may choose to verify that $\dot{Q} \to 0$ in the limit as $D \to \infty$. ■

ILLUSTRATIVE EXAMPLE 19.9

The surface temperature of a circular conducting rod is maintained at 200°C (T_1) by the passage of an electric current. The rod diameter is 10 mm, the length is 2.5 m, the thermal conductivity is 60 W/m · K, the density is 7850 kg/m^3, and the heat capacity is 434 J/kg · K. A bakelite coating (thermal conductivity = 1.4 W/m · K) is applied to the rod. The rod is in a fluid at 25°C (T_3), and the convection heat transfer coefficient is 140 W/m^2 · K. The thermal conductivity of the fluid is 0.6 W/m · K.

1. Calcualte the rate of heat transfer for the bare rod?
2. What is the critical radius associated with the bakelite coating? What is the heat transfer rate at the critical radius?
3. If the bakelite insulation thickness is 55 mm, determine the fractional reduction in heat transfer rate relative to the case of a bare rod.

***SOLUTION*:**

1. Calculate $\dot{Q}$ for the bare rod:

$$\begin{aligned}\dot{Q}_{\text{bare}} &= h(\pi D_1 L)(T_1 - T_3) = (140)(\pi)(0.01)(2.5)(200 - 25) \\ &= 1924\,\text{W}\end{aligned}$$

2. Calculate the critical bakelite (subscript 2) radius:

$$\begin{aligned}\text{Bi}_i &= \text{critical Biot number} = h_3 D_{2,c}/k_2 = 2.0 \\ D_{2,c} &= 2k_2/h_3 = 2(1.4)/140 = 0.02\ \text{m} = 20\,\text{mm} \\ r_{2,c} &= D_{2,c}/2 = 0.01\ \text{m} = 10\ \text{mm}\end{aligned}$$

Since $r_1 < r_{2,c}$, the addition of bakelite will increase $\dot{Q}$ until $r_1 = r_{2,c}$.

$$R_1 = \frac{\ln(r_2/r_1)}{2\pi k_2 L} = \frac{\ln(0.01/0.005)}{(2\pi)(1.4)(2.5)} = 0.0315^\circ\text{C/W}$$

$$R_2 = \text{convection resistance} = \frac{1}{h_3(2\pi r_2 L)} = \frac{1}{(140)(2\pi)(0.01)(2.5)} = 0.0455^\circ\text{C/W}$$

$$R_{\text{tot}} = R_1 + R_2 = 0.077^\circ\text{C/W}$$

$$\dot{Q}_{\text{crit}} = (T_1 - T_3)/R_{\text{tot}} = (200 - 25)/0.077 = 2273\ \text{W}$$

3. For insulation of thickness = 55 mm = 0.055 m, calculate $\dot{Q}_{\text{insul}}$:

$$r_2 = r_1 + 0.055 = 0.06\ \text{m}$$

$$R_1 = \text{conduction (bakelite) resistance} = \frac{\ln(0.06/0.005)}{2\pi(1.4)(2.5)} = 0.113^\circ\text{C/W}$$

$$R_2 = \text{convection resistance} = 1/[(140)(2\pi)(0.06)(2.5)] = 0.0076^\circ\text{C/W}$$

$$R_{\text{tot}} = 0.12^\circ\text{C/W}$$

$$\dot{Q} = \dot{Q}_{\text{insul}} = (T_1 - T_3)/R_{\text{tot}} = (200 - 25)/0.12 = 1451.3\ \text{W}$$

Since $r_2 > r_{2,c}$, $\dot{Q}_{\text{insul}}$ decreases. Therefore, the percent reduction in $\dot{Q}$ relative to the bare case is:

$$\begin{aligned}\text{Reduction} &= \left(\frac{\dot{Q}_{\text{bare}} - \dot{Q}_{\text{insul}}}{\dot{Q}_{\text{bare}}}\right)100\% \\ &= \left(\frac{1924 - 1451.3}{1924}\right)100\% \\ &= 24.6\%\end{aligned}$$

The results are in agreement with the discussion presented earlier for a small diameter pipe, i.e.,

$$Q_{\text{crit}} > Q_{\text{bare}}$$

and

$$Q_{\text{insul}} < Q_{\text{crit}} \text{ if } r > r_c$$

■

ILLUSTRATIVE EXAMPLE 19.10

A stainless steel tube carries hot ethylene glycol at 124°C (T_1). The surrounding air outside the tube is at 2°C (T_5). To reduce the heat losses from the ethylene glycol, the tube is surrounded by asbestos insulation. For a 1 m length of the tube, calculate:

1. the rate of heat transfer without insulation,
2. the rate of heat transfer with insulation, and

3. the overall heat transfer coefficient based on the inside area of the tube,
4. the overall heat transfer coefficient based on the outside area of the insulation,
5. the temperature, T_3, at the steel–insulation interface,
6. the inside and outside Biot numbers, and the outside Nusselt number, and
7. the log mean radius of insulation.

The following data is provided.

Stainless steel pipe: inside radius, $r_1 = 1.1$ cm; outside radius, $r_2 = 1.3$ cm; thermal conductivity, $k_2 = 19\ \mathrm{W/m \cdot K}$; heat transfer coefficient from ethylene glycol to the stainless steel pipe, $h_1 = 190\ \mathrm{W/m^2 \cdot K}$.

Asbestos insulation: inside radius, $r_2 = 1.3$ cm; outside radius, $r_3 = 3.8$ cm; thermal conductivity, $k_3 = 0.2\ \mathrm{W/m \cdot K}$.

The outside heat transfer coefficient from the air to the surface of the insulation (or of the pipe, in the case of no insulation), $h_3 = 14\ \mathrm{W/m^2 \cdot K}$. The thermal conductivity, k_4, of the air is $0.0242\ \mathrm{W/m \cdot K}$.

***SOLUTION*:** Calculate the inside and outside areas of heat transfer. Let A_1 and A_2 represent the inside and outside surface areas of the stainless steel tube, respectively, and A_3 represent the outside surface area of the insulation:

$$\begin{aligned} A_1 &= 2\pi r_1 L = (2\pi)(0.011)(1) \\ &= 0.0691\ \mathrm{m^2} \\ A_2 &= 2\pi r_2 L = (2\pi)(0.013)(1) \\ &= 0.0817\ \mathrm{m^2} \\ A_3 &= 2\pi r_3 L = (2\pi)(0.038)(1) \\ &= 0.239\ \mathrm{m^2} \end{aligned}$$

1. Calculate the inside convection resistance:

$$\begin{aligned} R_1 &= 1/[(190)(0.0691)] \\ &= 0.0762^\circ\mathrm{C/W} \end{aligned}$$

Calculate the conduction resistance through the tube (see Chapters 7 and 14):

$$\begin{aligned} R_2 &= \frac{\ln(r_2/r_1)}{2\pi k_2 L} \\ &= \frac{\ln(0.013/0.011)}{(2\pi)(19)(1)} \\ &= 0.0014^\circ\mathrm{C/W} \end{aligned}$$

For the case of no insulation, calculate the outside convection resistance:

$$\begin{aligned} R_3 &= 1/h_4 A_2 \\ &= 1/[(14)(0.0817)] \\ &= 0.874^\circ\mathrm{C/W} \end{aligned}$$

Calculate the heat transfer rate:

$$\begin{aligned} R_{\text{tot}} &= 0.0762 + 0.0014 + 0.874 \\ &= 0.952^{\circ}\text{C/W} \end{aligned}$$

$$\begin{aligned} \dot{Q} &= (T_1 - T_5)/R_{\text{tot}} \\ &= (124 - 2)/0.952 \\ &= 128.2\,\text{W} \end{aligned}$$

2. For the case of insulation, calculate the conduction resistance associated with the insulation:

$$\begin{aligned} R_3 &= \frac{\ln(r_3/r_2)}{2\pi k_3 L} \\ &= \frac{\ln(0.038/0.013)}{2\pi(0.2)(1)} \\ &= 0.854^{\circ}\text{C/W} \end{aligned}$$

Calculate the outside convection resistance:

$$\begin{aligned} R_4 &= 1/h_4 A_3 \\ &= 1/[(14)(0.239)] \\ &= 0.3^{\circ}\text{C/W} \end{aligned}$$

Calculate the total resistance:

$$\begin{aligned} R_{\text{tot}} &= 0.0762 + 0.0014 + 0.854 + 0.3 \\ &= 1.23^{\circ}\text{C/W} \end{aligned}$$

Calculate the heat transfer rate:

$$\begin{aligned} \dot{Q} &= (T_1 - T_5)/R_{\text{tot}} \\ &= (124 - 2)/(1.23) \\ &= 99.2\,\text{W} \end{aligned}$$

3. Calculate the overall heat transfer coefficient, U, based on the inside area:

$$\begin{aligned} U_1 &= 1/R_{\text{tot}} A_1 \\ &= 1/[(1.23)(0.0691)] \\ &= 11.76\,\text{W/m}^2 \cdot \text{K} \end{aligned}$$

4. Calculate U based on the outside area:

$$\begin{aligned} U_3 &= 1/R_{\text{tot}} A_3 \\ &= 1/[(1.23)(0.239)] \\ &= 3.4\,\text{W/m}^2 \cdot \text{K} \end{aligned}$$

5. Calculate the temperature T_3:

$$\begin{aligned} T_3 &= T_1 - (R_1 + R_2)\dot{Q} \\ &= 124 - (0.076 + 0.0014)(99.2) \\ &= 116.3^\circ\text{C} \end{aligned}$$

6. Calculate the outside Biot number:

$$\begin{aligned} \text{Bi}_o &= h_4(2r_3)/k_3 \\ &= (14)(2)(0.038)/0.2 \\ &= 5.32 \end{aligned}$$

Calculate the inside Biot number:

$$\begin{aligned} \text{Bi}_i &= h_1(2r_1)/k_3 \\ &= (190)(2)(0.011)/19 \\ &= 0.22 \end{aligned}$$

Calculate the Nusselt number of the air:

$$\begin{aligned} \text{Nu} &= \frac{h_{\text{air}} D_3}{k_{\text{air}}} \\ &= \frac{h_4 D_3}{k_4} \\ &= \frac{(190)(2)(0.011)}{0.0242} \\ &= 172.7 \end{aligned}$$

7. Calculate the log mean radius of the insulation:

$$\begin{aligned} r_{\text{lm}} &= \frac{r_3 - r_2}{\ln(r_3/r_2)} \\ &= \frac{0.038 - 0.013}{\ln(0.038/0.013)} \\ &= 0.0233\ \text{m} \\ &= 2.33\ \text{m} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 19.11

A recently developed synthetic oil is stored in a vertical tank 10 feet in diameter and 30 feet high which is insulated with a 2-inch layer of insulation ($k = 0.039$ Btu/h · ft · °F). To prevent freezing of the oil, it is maintained at a temperature of 115°F by a heating coil consisting of an 18-gauge, $\frac{3}{4}$-inch copper tube ($k = 224$ Btu/h · ft · °F) containing saturated steam at 10 psig.

Assuming that the minimum outdoor temperature is 5°F and that the oil temperature is uniform, calculate of copper tubing required in feet to maintain the tank at 120°F in the coldest weather. Neglect upper and lower tank surface heat losses. The following additional information is provided:

$$\text{Steam condensing inside coil, } h = 800\,\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

$$\text{Oil outside coil, } h = 40\,\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

$$\text{Oil inside tank wall, } h = 40\,\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

$$\text{Outer tank wall to ambient air, } h = 2.0\,\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

***SOLUTION*:** First consider the tank (t). Since wall thermal conductivity data is not provided, assume the wall resistance is negligible, and any area corrections can be neglected because of the large tank diameter (i.e., $A_i = A_o = A_{\text{lm}} = A_t$). Thus,

$$\begin{aligned} U_o &= \frac{1.0}{\dfrac{1}{40} + \dfrac{2.0}{(12)(0.039)} + \dfrac{1}{2.0}} \\ &= 0.21\,\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F} \end{aligned}$$

The heat transfer rate lost from the tank can now be calculated:

$$\begin{aligned} \dot{Q}_{\max} = \dot{Q} &= U_o A_t \Delta T_t \\ &= (0.21)(\pi)[10 + (4/12)](30)(120 - 5) \\ &= 23{,}000\,\text{Btu/h} \end{aligned}$$

For the coil, also neglect any area correction factors, i.e.,

$$A_i = A_o = A_{\text{lm}} = A_c$$

From Table 6.3, $\Delta x_c = 0.049$ in. Thus,

$$\begin{aligned} U_o &= \frac{1}{\dfrac{1}{40} + \dfrac{0.049}{(12)(224)} + \dfrac{1}{800}} \\ &= 38.0\,\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F} \end{aligned}$$

For 10 psia (24.7 psia) steam, $T_{\text{st}} = 240°$F. Therefore,

$$\begin{aligned} \dot{Q} &= U_o A_c \Delta T_c \\ 23{,}000 &= (38.0)(A_c)(240 - 120); \; c = \text{copper tubing} \\ A_c &= 5.04\,\text{ft}^2 \end{aligned}$$

The area of the copper tube is $0.1963\,\text{ft}^2/\text{ft}$. The length of tube, L, is then

$$\begin{aligned} L &= \frac{5.04}{0.1963} \\ &= 25.7\,\text{ft} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 19.12

Ricci and Theodore (R&T) Consultants have been assigned the job of selecting insulation for all the plant piping at the local power plant. Included in the plant piping are 8000 ft of 1-inch schedule 40 steel (1% C) pipe carrying steam at 240°F. It is estimated that the heat transfer coefficient for condensing steam on the inside of the pipe is 2000 Btu/h · ft^2 · °F. The air temperature outside of the pipe can drop to 20°F, and with wind motion the outside heat transfer coefficient can be as high as 100 Btu/h · ft · °F.

R&T have decided to use a fiberglass insulation having a thermal conductivity of 0.01 Btu/h · ft · °F. It is available in 6 ft lengths in the four thicknesses listed below:

$\frac{3}{8}$-inch thick	\$1.51/6 ft length
$\frac{1}{2}$-inch thick	\$3.54/6 ft length
$\frac{3}{4}$-inch thick	\$5.54/6 ft length
1-inch thick	\$8.36/6 ft length

Calculate the energy saved per dollar of insulation investment in going from $\frac{3}{8}$-inch to $\frac{1}{2}$-inch thick insulation. Repeat this calculation in going from $\frac{1}{2}$-inch to $\frac{3}{4}$-inch and $\frac{3}{4}$-inch to 1-inch. Express the results in units of Btu/h per dollar.

***SOLUTION*:** Employ cylindrical coordinates. Calculate the outside log-mean diameter of the pipe. For the 1-inch pipe schedule 40:

$$D_i = 1.049\ \text{inch} = 0.0874\ \text{ft}$$
$$D_o = 1.315\ \text{inch} = 0.1096\ \text{ft}$$

The log-mean diameter of the pipe is therefore

$$\begin{aligned}\bar{D} &= \frac{D_o - D_i}{\ln(D_o/D_i)}\\ &= \frac{0.1096 - 0.0874}{\ln(0.1096/0.0874)} = 0.09808\ \text{ft}\end{aligned}$$

Provide an expression for the insulation outside diameter in terms of Δx_I, the insulation thickness in inches:

$$D_I = \frac{1.315 + 2(\Delta x_I)}{12\ \text{in/ft}}$$

Thus,

Thickness, inch	D_I, ft
$\frac{3}{8}$	0.172
$\frac{1}{2}$	0.193
$\frac{3}{4}$	0.235
1	0.276

Obtain the relationship between the insulation thickness and the insulation log-mean diameter of the pipe:

$$\bar{D}_I = \frac{D_I - 0.1096}{\ln(D_I/0.1096)}$$

Therefore,

Thickness, inch	$\overline{D}_I$, ft
$\frac{3}{8}$	0.139
$\frac{1}{2}$	0.147
$\frac{3}{4}$	0.164
1	0.180

The pipe wall resistance and pipe thickness are:

$$R_w = \frac{\Delta x_w}{k\pi \bar{D}_w L}$$

$$\Delta x_w = \frac{0.1096\,\text{ft} - 0.0874\,\text{ft}}{2} = 0.0111\,\text{ft}$$

For steel:

$$k = 24.8\,\text{Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}$$

Therefore, for an 8000 ft pipe, the wall resistance is:

$$R_w = \frac{0.0111\,\text{ft}}{(24.8\,\text{Btu/hr} \cdot\ \text{ft} \cdot {}^\circ\text{F})\pi(0.09808\,\text{ft})(8000\,\text{ft})}$$
$$= 1.816 \times 10^{-7}(\text{Btu/h} \cdot {}^\circ\text{F})^{-1}$$

The inside steam convection resistance is:

$$R_i = \frac{1}{h_i \pi D_i L}$$
$$= \frac{1}{(2000\,\text{Btu/h} \cdot\ \text{ft}^2 \cdot {}^\circ\text{F})\pi(0.0874\,\text{ft})(8000\,\text{ft})}$$
$$= 2.28 \times 10^{-7}(\text{Btu/h} \cdot {}^\circ\text{F})^{-1}$$

Express the insulation resistance in terms of Δx_I:

$$R_I = \frac{\Delta x_I}{k_I \pi \bar{D}_I L}$$

Thickness, inch	R_I, $(\text{Btu/h} \cdot {}^\circ\text{F})^{-1}$
$\frac{3}{8}$	8.978×10^{-4}
$\frac{1}{2}$	1.125×10^{-3}
$\frac{3}{4}$	1.514×10^{-3}
1	1.839×10^{-3}

Also express the outside air convection resistance in terms of the thickness, Δx_I:

$$R_o = \frac{1}{h_o \pi D_I L}$$

Thickness, inch	R_o, $(\text{Btu/h} \cdot {}^\circ\text{F})^{-1}$
$\frac{3}{8}$	2.312×10^{-6}
$\frac{1}{2}$	2.062×10^{-6}
$\frac{3}{4}$	1.696×10^{-6}
1	1.440×10^{-6}

The total resistance, R, is

$$R = R_i + R_w + R_I + R_o$$

The total resistance for each thickness is therefore:

Thickness, inch	R, $(\text{Btu/h} \cdot {}^\circ\text{F})^{-1}$
$\frac{3}{8}$	9.005×10^{-4}
$\frac{1}{2}$	1.128×10^{-3}
$\frac{3}{4}$	1.156×10^{-3}
1	1.841×10^{-3}

The overall outside heat transfer coefficient, U_o, is given by

$$U_o = \frac{1}{R \pi D_I L}$$

so that

Thickness, inch	U_o, $(\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F})$
$\frac{3}{8}$	0.257
$\frac{1}{2}$	0.183
$\frac{3}{4}$	0.112
1	0.078

The inside overall heat transfer coefficient, U_i, is

$$U_i = \frac{1}{R\pi D_i L}$$

so that

Thickness, inch	U_i, (Btu/h · ft^2 · °F)
$\frac{3}{8}$	0.506
$\frac{1}{2}$	0.404
$\frac{3}{4}$	0.300
1	0.247

The energy loss is

$$\dot{Q} = \frac{\Delta T}{R} = UA\Delta T$$

with

$$A_i = \pi D_i L = \pi(0.0874)(8000) = 2195\,\text{ft}^2$$
$$\Delta T = 240 - 20 = 220$$

Therefore,

Thickness, inch	$\dot{Q}$, Btu/h
$\frac{3}{8}$	2.443×10^5
$\frac{1}{2}$	1.951×10^5
$\frac{3}{4}$	1.451×10^5
1	1.195×10^5

Finally, calculate the energy saved per dollar of insulation investment going from $\frac{3}{8}$-inch to $\frac{1}{2}$-inch, from $\frac{1}{2}$-inch to $\frac{3}{4}$-inch, and from $\frac{3}{4}$-inch to 1-inch.

$$\frac{\text{Energy}}{\$} = \frac{\Delta\dot{Q}}{(8000)(\Delta\text{Unit cost per 6 ft})}$$

From $\frac{3}{8}$-inch to $\frac{1}{2}$-inch:

$$\frac{\text{Energy}}{\$} = \frac{(2.443 \times 10^5\,\text{Btu/h} - 1.951 \times 10^5\,\text{Btu/h})}{(8000\,\text{ft})(\$3.54 - \$1.51)/(6\,\text{ft})} = 18.1\,\text{Btu/h} \cdot \$$$

From $\frac{1}{2}$-inch to $\frac{3}{4}$-inch:

$$\frac{\text{Energy}}{\$} = \frac{(1.951 \times 10^5\,\text{Btu/h} - 1.451 \times 10^5\,\text{Btu/h})}{(8000\,\text{ft})(\$5.54 - \$3.54)/(6\,\text{ft})} = 18.7\,\text{Btu/h} \cdot \$$$

From $\frac{3}{4}$-inch to 1-inch:

$$\frac{\text{Energy}}{\$} = \frac{(1.451 \times 10^5\,\text{Btu/h} - 1.195 \times 10^5\,\text{Btu/h})}{(8000\,\text{ft})(\$8.36 - \$5.54)/(6\,\text{ft})} = 6.8\,\text{Btu/h} \cdot \$$$

■

ILLUSTRATIVE EXAMPLE 19.13

Comment on the results of the previous example.

***SOLUTION*:** The energy recovery quotient is the highest when going from $\frac{1}{2}$-inch pipe to $\frac{3}{4}$-inch pipe. Obviously, the insulation should be in this range based on the calculations. Additional calculations may be necessary employing different insulation thicknesses since either of these pipes may not be the choice for the most cost effective system. In order to decide which system is the best, the total annual capital and operating costs should also be calculated. The total annual capital cost also depends upon the plant life and the rate of return (see also Chapter 27). In addition to cost minimization, a safety factor must be taken into consideration. The insulation must be adequate to prevent exposed surfaces from exceeding the temperature that would be safe for the person working nearby. Some of these factors are considered in the next (and last) Illustrative Example in this chapter.

Additional economic and finance illustrative examples are provided in Chapter 27 ■

INSULATION

Fiber, *powder*, and *flake-type* insulation consist of finely dispersed solids throughout an air space. The ratio of the air space to the insulator volume is called the *porosity* or *void fraction*, ε. In *cellular* insulation, a material with a rigid matrix contains entrapped air pockets. An example of such rigid insulation is *foamed insulation* which is made from *plastic* and *glass* material. Another type of insulation consists of multi-layered thin sheets of *foil* of high reflectivity. The spacing between the foil sheets is intended to restrict the motion of air. This type of insulation is referred to as *reflective* insulation.

Most thermal insulation systems consist of the insulation *and* a so-called "finish." The finish provides protection against water or other liquid entry, mechanical damage, and ultraviolet degradation; it can also provide fire protection. The finish usually consists of any form of coating (e.g., polymeric paint material, etc.), a membrane (e.g., felt, plastic laminate, foil, etc.), or a sheet material (e.g., fabric, plastic, etc.). Naturally, the finish must be able to withstand any potential temperature excursion in its immediate vicinity.

Critical Insulation Thickness

It would normally seem that the thicker the insulation, the less the heat loss, i.e., increasing the insulation should reduce the heat loss to the surroundings. But, and as discussed in the previous section, this is not always the case. There is a "critical insulation thickness" below which the system will experience a greater heat loss due to an increase in insulation. This situation arises for "small" diameter pipes when the increase in area increases more rapidly then the resistance opposed by the thicker insulation.

Consider the system shown in Figure 19.3. As noted in Chapters 7 and 14, the area terms for the heat transfer equations in rectangular coordinates are no longer equal in cylindrical coordinates (e.g., for the inside surface, the heat transfer area is given by $2\pi r_i$L). Applying the above development to a pipe/cylinder system leads to

$$\dot{Q} = \frac{T_i - T_o}{\dfrac{1}{2\pi r_i L}\left(\dfrac{1}{h_i}\right) + \dfrac{\Delta x_w}{k_w 2\pi L r_{\text{lm},w}} + \dfrac{\Delta x_i}{k_i 2\pi L r_{\text{lm},i}} + \dfrac{1}{2\pi r L}\left(\dfrac{1}{h_o}\right)}$$

$$= \frac{2\pi L(T_i - T_o)}{\dfrac{1}{r_i h_i} + \dfrac{\ln(r_o/r_i)}{k_w} + \dfrac{\ln(r/r_o)}{k_i} + \dfrac{1}{r h_o}} = \frac{2\pi L(T_i - T_o)}{f(r)} \qquad (19.9)$$

where

$$f(r) = \frac{1}{r_i h_i} + \frac{\ln(r_o/r_i)}{k_w} + \frac{\ln(r/r_o)}{k_i} + \frac{1}{r h_o} \qquad (19.10)$$

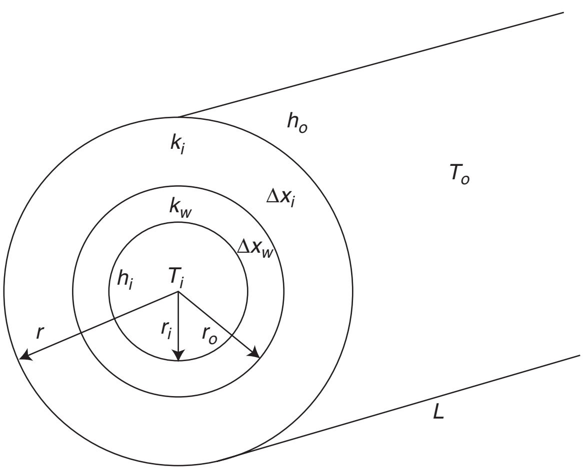

Figure 19.3 Critical insulation thickness for a pipe.

Assuming that $\dot{Q}$ goes through a maximum or minimum as r is varied, l'Hôpital's rule can be applied to Equation (19.10):

$$\frac{d\dot{Q}}{dr} = 2\pi L(T_i - T_o)\left[\frac{-\dfrac{df(r)}{dr}}{f(r)^2}\right] = 0 \tag{19.11}$$

with

$$-\frac{df(r)}{dr} = -\frac{1}{rk_i} + \frac{1}{r^2 h_0}$$

For $d\dot{Q}/dr = 0$, one may therefore write

$$\frac{d\dot{Q}}{dr} = -\frac{d(fr)}{dr} = \frac{1}{rk_i} + \frac{1}{r^2 h_o} = 0 \tag{19.12}$$

For this maximum/minimum condition, set $r = r_c$ and solve for r_c.

$$r_c = \frac{k_i}{h_o} \tag{19.8}$$

The second derivative of $d\dot{Q}_r/dr$ of Equation (19.11) provides information as to whether $\dot{Q}$ experiences a maximum or minimum at r_c.

$$\frac{d}{dr}\left(\frac{d\dot{Q}_r}{dr}\right) + \frac{1}{r^2 k_i} - \frac{2}{r^3 h_o} = \frac{h_o^2}{k^3} - \frac{2h_o^2}{k^3} = \frac{h_o^2}{k^3}(1-2) \tag{19.13}$$

Clearly, the second derivative is a negative number; $\dot{Q}$ is therefore a *maximum* at $r = r_c$. $\dot{Q}$ then decreases monotonously as r is increased beyond r_c. However, one should exercise care in interpreting the implications of the above development. This result applies *only* if r_o is *less* than r_c, i.e., it generally applies to "small" diameter pipes/tubes. Thus, r_c represents the outer radius (not the thickness) of the insulation that will maximize the heat loss and at which point any further increase in insulation thickness will result in a decrease in heat loss.

A graphical plot of the resistance R versus r is provided in Figure 19.4. (The curve is inverted for the plot of $\dot{Q}$ or r.) In other words, the maximum heat loss from a pipe occurs when the critical radius equals the ratio of the thermal conductivity of the insulation to the surface coefficient of heat transfer. This ratio has the dimension of length (e.g., ft). Note that once again the equation for r_i can be rewritten in terms of a dimensionless number defined earlier:

$$\frac{k_i}{h_o r_c} = \mathrm{Bi}^{-1} \tag{19.7}$$

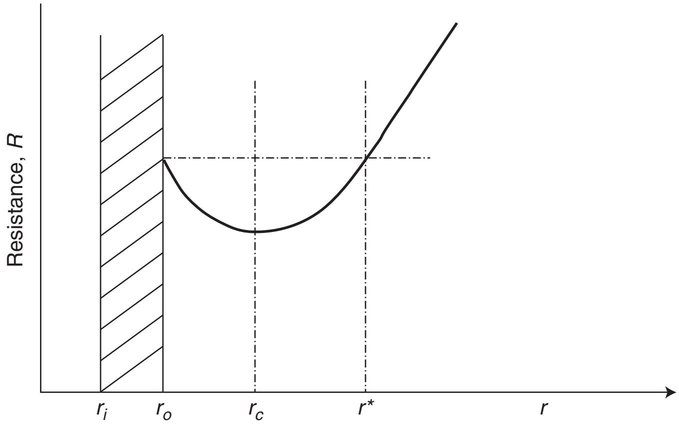

Figure 19.4 Resistance associated with the critical insulation thickness for a bare surface.

To reduce $\dot{Q}$ below that for a bare wall ($r = r_0$), r must be greater than r_c, i.e., $r > r_c$. The radius at which this occurs is denoted as r^*. The term r^* may be obtained by solving the equation

$$\dot{Q}_{\text{bare}} = \dot{Q}_{I,r^*} \tag{9.14}$$

so that

$$\frac{2\pi L(T_i - T_o)}{\dfrac{1}{r_i h_i} + \dfrac{\ln(r_o/r_i)}{k_w} + \dfrac{1}{r_o h_o}} = \frac{2\pi L(T_i - T_o)}{\dfrac{1}{r_i h_i} + \dfrac{\ln(r_o/r_i)}{k_w} + \dfrac{\ln(r^*/r_o)}{k_i} + \dfrac{1}{r^* h_o}} \tag{19.15}$$

One may now solve for r^* using a suitable trial-and-error procedure.

The above development applies when r is less than r_c. If r_o is larger than r_c, the above analysis again applies, but only to the results presented for $r > r_c$, i.e., $\dot{Q}$ will decrease indefinitely as r increases. Note that there is *no* maximum/minimum (inflection) for this case since values of $\ln(r/r_o)$ are indeterminate for $r < r_o$. Once again, $\dot{Q}$ approaches zero in the limit as r approaches infinity.

ILLUSTRATIVE EXAMPLE 19.14

Explain why it is important to determine the critical radius.

***SOLUTION*:** As noted above, as the thickness of the insulation is increased, the cost associated with heat lost decreases but the insulation cost increases. The optimum thickness is determined by the minimum of the total costs. Thus, as the thickness of the insulation is increased, the heat loss reaches a maximum value and then decreases with further increases in insulation. Reducing this effect can be accomplished by using an insulation of low conductivity. ■

ILLUSTRATIVE EXAMPLE 19.15

It has come to the attention of a young engineer that there is a bare pipe that is releasing a significant amount of heat into the atmosphere. List factors that should be considered in selecting the optimum insulation diameter for the pipe.

***SOLUTION*:** Two important factors include durability and maintainability. If it is determined that cost is the number one factor, and a cheaper insulation is chosen, it would be wise to investigate these two factors for that insulation. The end result might be that the cheaper insulation does not have a long life and might have to be maintained much more often than a more expensive one. This could cause the more expensive one to be more cost effective than the cheaper insulation.

Temperature difference will also play a role in determining the insulation diameter. It may be imperative to keep the temperature of the material in the pipe just above freezing, or it may be that the temperature needs to be 60° above freezing. These different situations call for different insulation diameters. Another factor that falls under the temperature category is the location of the pipe. It would be extremely different if a pipe is insulated in New York, Alaska, or Tahiti. All of these places have different climates and it is imperative that these be investigated in order to know how large or small the diameter of the insulation needs to be.

Finally, the other factors that need to be looked at, and may be as important as the first, are whether the materials that constitute the insulation are harmful. First and foremost, there is asbestos, and due to recent studies, insulation with no or very little asbestos should be used. Another insulation material is fiberglass. A good number of insulators are made with fiberglass. When the insulation is cut, fiberglass escapes into the air and workers should not breathe this harmful material. ■

ILLUSTRATIVE EXAMPLE 19.16

Calculate the outer critical radius of insulation on a 2.0-inch OD pipe. Assume the air flow coefficient to be 1.32 Btu/h · ft^2 · °F and the loosely-packed insulation's thermal conductivity to be 0.44 Btu/h · ft · °F. Comment on the effect of insulation on the heat rate lost from the pipe.

***SOLUTION*:** Employ Equation (19.8):

$$\begin{aligned} r_c &= \frac{k_i}{h_o} \\ &= \frac{0.44}{1.32} = 0.333\ \text{ft} \\ &= 4.0\ \text{in} \end{aligned}$$

The critical insulation thickness is therefore:

$$L_I = \Delta x_I = 4.0 - (2.0/2) = 3.0\ \text{in}$$

Since $r_o = 1.0$ in, $r_o < r_c$, so that the heat loss will increase as insulation is added, but start to decrease when the radius of the insulation increases above r_c, i.e., when $r > r_c$. ■

REFRACTORY

Webster[1] defines refractory as: "resistant to heat, hard to melt or work . . . not yielding to treatment . . . a heat resistant material used in lining furnaces, etc." Refractory materials must obviously be chemically and physically stable at high temperatures. Depending on the operating environment, they must also be resistant to thermal shock, chemically inert, and resistant to wear. Refractories normally require special warm-up periods to reduce the possibility of thermal shock and/or drying stresses.

The oxides of aluminum (alumina), silicon (silica), and magnesium (magnesia) are the most common materials used in the manufacture of refractories. Another oxide usually found in refractories is the oxide of calcium (lime). Fireclays are also widely used in the manufacture of refractories. Additional details are provided in Tables 19.2 and 19.3.

Refractories are selected based primarily on operating conditions. Some applications require special refractory materials. Zirconia is used when the material must withstand extremely high temperatures. Silicon carbide and carbon are two other refractory materials used in some very severe temperature conditions, but they cannot be used in contact with oxygen, as they will oxidize and burn.

Table 19.2 Properties of Non-metals

Structural and heat-resistant materials					
Substance	Temperature, °C	k, W/m · °C	ρ, kg/m^3	c_p, kJ/kg · °C	α, m^2/s × 10^7
Asphalt	20–55	0.74–0.76			
Brick					
Building brick, common	20	0.69	1600	0.84	5.2
face		1.32	2000		
Carborundum brick	600	18.5			
	1400	11.1			
Chrome brick	200	2.32	3000	0.84	9.2
	550	2.47			9.8
	900	1.99			7.9
Diatomaceous earth, molded and fired	200	0.24			
	870	0.31			
Fire clay, burnt 2426°F	500	1.04	2000	0.96	5.4
	800	1.07			
	1100	1.09			
burnt 2642°F	500	1.28	2300	0.96	5.8
	800	1.37			
	1100	1.40			
Fireclay brick	200	1.0	2645	0.96	3.9
	655	1.5			

(*Continued*)

Table 19.2 *Continued*

Substance	Temperature, °C	k, W/m · °C	ρ, kg/m^3	c_p, kJ/kg · °C	α, m^2/s × 10^7
Structural and heat-resistant materials					
	1205	1.8			
Missouri	200	1.00	2600	0.96	4.0
	600	1.47			
	1400	1.77			
Magnesite	200	3.81		1.13	
	650	2.77			
	1200	1.90			
Cement, Portland		0.29	1500		
Mortar	23	1.16			
Concrete, cinder	23	0.76			
Stone, 1-2-4 mix	20	1.37	1900–2300	0.88	8.2–6.8
Glass, window	20	0.78 (avg)	2700	0.84	3.4
Borosilicate	30–75	1.09	2200		
Plaster, gypsum	20	0.48	1440	0.84	4.0
Metal lath	20	0.47			
Wood lath	20	0.28			
Stone					
Granite		1.73–3.98	2640	0.82	8–18
Limestone	100–300	1.26–1.33	2500	0.90	5.6–5.9
Marble		2.07–2.94	2500–2700	0.80	10–13.6
Sandstone	40	1.83	2160–2300	0.71	11.2–11.9
Wood (across the grain)					
Balsa, 8.8 lb/ft^3	30	0.055	140		
Cypress	30	0.097	460		
Fir	23	0.11	420	2.72	0.96
Maple or oak	30	0.166	540	2.4	1.28
Yellow pine	23	0.147	640	2.8	0.82
White pine	30	0.112	430		
Asbestos					
Loosely packed	−45	0.149			
	0	0.154	470–570	0.816	3.3–4
	100	0.161			
	20	0.74			
Asbestos–cement boards	20	0.74			
Sheets	51	0.166			
Felt					
40 laminations/in	38	0.057			
	150	0.069			
	260	0.083			
20 laminations/in	38	0.078			

(*Continued*)

Table 19.2 *Continued*

Structural and heat-resistant materials					
Substance	Temperature, °C	k, W/m · °C	ρ, kg/m^3	c_p, kJ/kg · °C	α, m^2/s × 10^7
	150	0.095			
	260	0.112			
Corrugated, 4 piles/in	38	0.087			
	93	0.100			
	150	0.119			
Asbestos cement	–	2.08			
Balsam wood, 2.2 lb/ft^3	32	0.04	35		
Cardboard, corrugated	–	0.064			
Celotex	32	0.048			
Corkboard, 10 lb/ft^3	30	0.043	160		
Cork, regranulated	32	0.045	45–120	1.88	2–5.3
Ground	32	0.043	150		
Diatomaceous earth (Sil-o-cel)	0	0.061	320		
Felt, hair	30	0.036	130–200		
Wool	30	0.052	330		
Fiber, insulating board	20	0.048	240		
Glass wool, 1.5 lb/ft^3	23	0.038	24	0.7	22.6
Insulex, dry	32	0.064			
		0.144			
Kapok	30	0.035			
Magnesia, 85%	38	0.067	270		
	93	0.071			
	150	0.074			
	204	0.080			
Rock wool, 10 lb/ft^3	32	0.040	160		
Loosely packed	150	0.067	64		
	260	0.087			
Sawdust	23	0.059			
Silica aerogel	32	0.024	140		
Wood shavings	23	0.059			

There is no single design and selection procedure for refractories. Three general rules can be followed:

1. Design for compressive loading.
2. Allow for thermal expansion.
3. Take advantage of the full range of materials, forms, and shapes.

These apply to whether the design is essentially brickwork and masonry construction or whether the refractory is one that might have been made of speciality metal.

Table 19.3 Physical Properties of Some Common Low-Melting-Point Metals

Metal	Melting point, °C	Normal boiling point, °C	Temperature, °C	Density, $kg/m^3 \times 10^{-3}$	Viscosity, $kg/m \cdot s \times 10^3$	Heat capacity $kJ/kg \cdot K$	Thermal conductivity, $W/m \cdot K$	Prandtl number
Bismuth	271	1477	316	10.01	1.62	0.144	16.4	0.014
			760	9.47	0.79	0.165	15.6	0.0084
Lead	327	1737	371	10.5	2.40	0.159	16.1	0.024
			704	10.1	1.37	0.155	14.9	0.016
Lithium	179	1317	204	0.51	0.60	4.19	38.1	0.065
			982	0.44	0.42	4.19		
Mercury	−39	357	10	13.6	1.59	0.138	8.1	0.027
			316	12.8	0.86	0.134	14.0	0.0084
Potassium	63.8	760	149	0.81	0.37	0.793	45.0	0.0066
			704	0.67	0.14	0.754	33.1	0.0031
Sodium	97.8	883	93	0.93	0.67	1.38	86.2	0.011
			204	0.90	0.43	1.34	80.3	0.0072
			704	0.78	0.18	1.26	59.7	0.0038
Sodium potassium								
22% Na	19	826	93.3	0.848	0.49	0.946	24.4	0.019
			760	0.69	0.146	0.883		
56% Na	−11	784	93.3	0.89	0.58	1.13	25.6	0.026
			760	0.74	0.16	1.04	28.9	0.058
Lead bismuth								
44.5% Pb	125	1670	288	10.3	1.76	0.147	10.7	0.025
			649	98.4	1.15			

Example: For lead at 371°C, density = 10,500 kg/m^3, viscosity = 0.0024 $kg/m \cdot s$, heat capacity = 159 $J/kg \cdot K$.

ILLUSTRATIVE EXAMPLE 19.17

Provide melting point temperature ranges for various refractories.

***SOLUTION*:**

Refractory metals: 3500–6000°F
Refractory oxides: 4500–6000°F
Refractory ceramics: 4000–7000°F
Refractory cermets (ceramic–metals): 3500–5500°F ■

ILLUSTRATIVE EXAMPLE 19.18

A flat incinerator wall with a surface area of 480 ft^2 consists of 6 inches of firebrick with a thermal conductivity of 0.61 Btu/h · ft · °F and an 8-inch outer layer of rock wool insulation with a thermal conductivity of 0.023 Btu/h · ft · °F. If the temperature of the insulation of the inside face of the firebrick and the outside surface of the rock wool insulation are 1900 and 140°F, respectively, calculate the following:

1. The heat loss through the wall in Btu/h.
2. The temperature of the interface between the firebrick and the rock wool.

***SOLUTION*:** As noted earlier

$$\dot{Q} = \frac{\Delta T}{\sum R} \tag{19.4}$$

The individual resistances are:

$$\begin{aligned} R_{\text{firebrick}} &= \frac{L_f}{k_f A} \\ &= \frac{0.5}{(0.61)(480)} \\ &= 0.0017\,\text{h} \cdot {}^\circ\text{F/Btu} \\ R_{\text{rock wool}} &= \frac{L_{rw}}{k_{rw} A} \\ &= \frac{0.67}{(0.023)(480)} \\ &= 0.0604\,\text{h} \cdot {}^\circ\text{F/Btu} \end{aligned}$$

Thus,

$$\sum R = 0.0017 + 0.0604 = 0.0621\,\text{h} \cdot {}^\circ\text{F/Btu}$$

The heat loss through the wall is then

$$\dot{Q} = \frac{1900 - 140}{0.0621}$$
$$= 28{,}341 \text{ Btu/h} \qquad \blacksquare$$

ILLUSTRATIVE EXAMPLE 19.19

Heat is flowing from steam on one side of a 0.375-inch thick vertical steel sheet to air on the other side. The steam heat-transfer coefficient is 1700 $\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$ and that of the air is 2.0 $\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$. The total temperature difference is 120°F. How would the rate of heat transfer be affected if:

1. the wall was copper rather than steel,
2. by increasing the steam coefficient to 2500, and
3. by increasing the air coefficient to 12.0?

Note that the thermal conductivities, k, for steel and copper are 26 and 218 $\text{Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}$, respectively.

***SOLUTION*:** The describing equation is once again

$$\dot{Q} = \frac{\sum \Delta T}{\sum R} \tag{19.4}$$

For the existing application, assume a basis of 1.0 ft^2. Therefore,

$$R_{\text{steam}} = \frac{1}{hA} = \frac{1}{(1700)(1)} = \frac{1}{1700}$$
$$R_{\text{air}} = \frac{1}{hA} = \frac{1}{(2)(1)} = \frac{1}{2}$$
$$R_{\text{steel}} = \frac{\Delta k}{kA} = \frac{0.375/12}{(26)(1)}; \quad k_{\text{steel}} = 26 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}$$
$$\therefore \sum R = \frac{1}{1700} + \frac{1}{2} + \frac{0.375/12}{(26)}$$
$$= 0.502 \text{ h} \cdot {}^\circ\text{F/Btu}$$

1. If copper ($k = 218 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}$) is employed,

$$\sum R = \frac{1}{1700} + \frac{1}{2} + \frac{0.375/12}{(218)}$$
$$= 0.50 \text{ h} \cdot {}^\circ\text{F/Btu}$$

Thus, the rate of heat transfer is essentially unaffected.

2. If h_{steam} is 2500 Btu/h · ft² · °F,

$$\sum R = \frac{1}{2500} + \frac{1}{2} + \frac{0.375/12}{(26)}$$
$$= 0.50 \text{ h} \cdot {}^\circ\text{F/Btu}$$

The rate is again unaffected.

3. However, if h_{air} is 12 Btu/h · ft² · °F,

$$\sum R = \frac{1}{1700} + \frac{1}{12} + \frac{0.375/12}{(26)}$$
$$= 0.0852 \text{ h} \cdot {}^\circ\text{F/Btu}$$

The rate is affected for this case. Thus, it can be concluded that the air is the *controlling* resistance. ■

ILLUSTRATIVE EXAMPLE 19.20[3]

An incinerator is 30 ft long, has a 12-ft ID and is constructed of $\frac{3}{4}$-inch carbon steel. The inside of the steel shell is protected by 10 in. of firebrick ($k = 0.608$ Btu/h · ft · °F) and 5 inches of Sil-o-cel insulation ($k = 0.035$ Btu/h · ft · °F) covers the outside. The ambient air temperature is 85°F and the average inside temperature is 1800°F. The present heat loss through the furnace wall is 6% of the heat generated by combustion of a fuel. Calculate the thickness of Sil-o-cel insulation that must be added to cut the losses to 3%.

***SOLUTION*:** A diagram of this system is presented in Figure 19.5. Although the resistance of the steel can be neglected, the other two need to be considered. For cylindrical systems, the effect of the radius of curvature must once again be included in the resistance equations. These take the form presented below:

$$R_{\text{firebrick}} = \frac{\ln(r_{fo}/r_{fi})}{2\pi L k_f}$$
$$= \frac{\ln(6.000/5.167)}{2\pi(30)(0.608)}$$
$$= 1.304 \times 10^{-3} \text{h} \cdot {}^\circ\text{F/Btu}$$

$$R_{\text{Sil-o-cel}} = \frac{\ln(r_{so}/r_{si})}{2\pi L k_s}$$
$$= \frac{\ln(6.479/6.063)}{2\pi(30)(0.035)}$$
$$= 10.059 \times 10^{-3} \text{ h} \cdot {}^\circ\text{F/Btu}$$

Thus,

$$\sum R = 11.363 \times 10^{-3} \text{ h} \cdot {}^\circ\text{F/Btu}$$

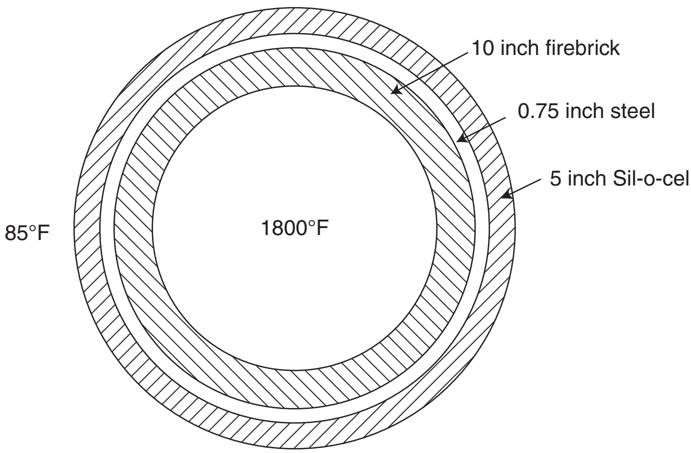

Figure 19.5 Diagram for Illustrative Example 19.20.

To cut the heat loss in half, R must be doubled. The additional Sil-o-cel resistance is therefore 11.363×10^{-3} h · °F/Btu. The new outside radius, r_o, is calculated from:

$$R_{\text{addedSil-o-cel}} = 11.363 \times 10^{-3} = \frac{\ln(r_o/6.479)}{2\pi(30)(0.035)}$$

$$r_o = 6.983\ \text{ft}$$

The extra thickness is $6.983 - 6.479 = 0.504$ ft $= 6.05$ in ■

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