Chapter 17

Fins and Extended Surfaces

INTRODUCTION

Consider the case of a heat exchanger that is heating air outside the tubes by means of steam inside the tubes. The steam-side coefficient will be very high and the air-side coefficient will be extremely low. Therefore, the overall heat transfer coefficient will approximate that of the air side (i.e., the air is the controlling resistance). One method of increasing the heat transfer rate is to increase the surface area of the heat exchanger. Therefore, if the surface of the metal on the air side could be increased, it would increase the area term without putting more tubes in the exchanger. This can be accomplished by mounting metal fins on a tube in such a way that there is good metallic contact between the base of the fin and the wall of the tube. If this contact is secured, the temperature throughout the fins will approximate that of the temperature of the heating (cooling) medium because of the high thermal conductivity of most metals used in practice. Consequently, the surface will be increased without more tubes.(1)

Additional metal is often added to the outside of ordinary heat transfer surfaces such as pipes, tubes, walls, etc. These extended surfaces, usually referred to as fins, increase the surface available for heat flow and result in an increase of the total transmission of heat. Examples of fin usage includes auto radiators, air conditioning, cooling of electronic components, and heat exchangers. They are primarily employed for heat transfer to gases where film coefficients are very low.

The remaining sections of this chapter include:

Fin Types

Describing Equations

Fin Effectiveness and Performance

Fin Considerations

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

FIN TYPES

Extended surfaces, or fins, are classified into longitudinal fins, transverse fins, and spine fins. *Longitudinal fins* (also termed straight fins) are attached continuously along the length of the surface (see Figure 17.1). They are employed in cases involving gases or viscous liquids. As one might suppose, they are primarily employed with double pipe heat exchangers. *Transverse* or *circumferential fins* are positioned approximately perpendicular to the pipe or tube axis and are usually used in the cooling of gases (see Figure 17.2). These fins find their major application with shell and tube exchangers. Transverse fins may be continuous or discontinuous (segmented). *Annular fins* are examples of continuous transverse fins. *Spine* or *peg fins* employ cones or cylinders, which extend from the heat transfer surface, and are used for either longitudinal flow or cross flow.

Fins are constructed of highly conductive materials. The optimum fin design is one that gives the highest heat transfer for the minimum amount of metal. The metal used in their manufacture has a strong influence on fin efficiency. Table 17.1 compares the volume and mass of three different metals required to give the same amount of heat transfer for fins with identical shapes. The values in the volume and mass columns are relative to the volume and mass of a copper fin.

ILLUSTRATIVE EXAMPLE 17.1

Consider the longitudinal (rectangular) fin pictured in Figure 17.1. Estimate the fin face area, neglecting the (top) area contribution associated with the fin thickness if $w = 1$ ft and $L =$ 1.5 in. The fin thickness is 0.1 in. Also calculate the total area of the fin.

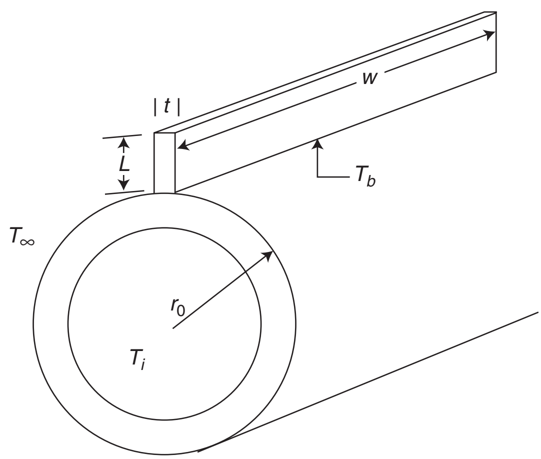

Figure 17.1 Longitudinal fins.

Table 17.1 Fin Metal Data

Metal	Thermal conductivity	Specific gravity	Relative volume	Relative mass
Copper	400	8.9	1.00	1.00
Aluminum	210	2.7	1.83	0.556
Steel	55	7.8	7.33	0.43

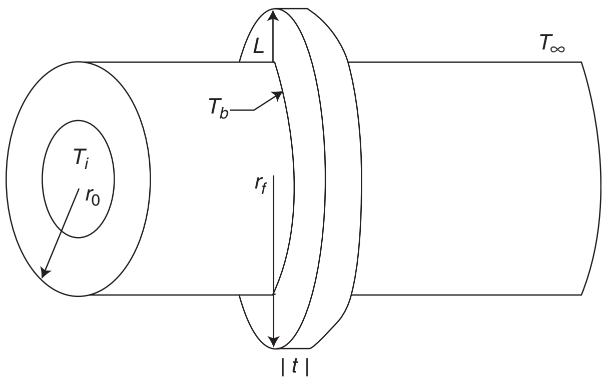

Figure 17.2 Transverse or circumferential fins.

***SOLUTION*:** The face area of the fin is given as

$$A_f = 2wL = 2(1.5)(12) = 36\,\text{in}^2 = 0.25\ \text{ft}^2$$

The total area is given by

$$A_{f,t} = 2wL + Lw = 36 + 12(0.1) = 36 + 1.2 = 37.2\,\text{in}^2 = 0.258\ \text{ft}^2$$ ■

ILLUSTRATIVE EXAMPLE 17.2

Comment on the results of the previous example.

***SOLUTION*:** Note that the external "rim" area, *wt*, does not contribute significantly to the total area and is normally neglected in fin calculations. ■

ILLUSTRATIVE EXAMPLE 17.3

Refer to Figure 17.2. Estimate the fin area, neglecting the area contribution associated with the fin thickness if $r_0 = 4.0$ in, $r_f = 6.0$ in, and $t = 0.1$ in.

SOLUTION: This is an example of a circumferential (annular) fin. For a circumferential fin, the face area is given by

$$A_f = 2\pi(r_f^2 - r_0^2) = 2\pi[(6)^2 - (4)^2] = 125.7\,\text{in}^2 = 0.873\,\text{ft}^2$$

The total area is given by

$$A_{f,t} = A_f + 2\pi r_f(t) = 125.7 + 2\pi(6)(0.1) = 125.7 + 3.8 = 129.5\,\text{in}^2 = 0.899\ \text{ft}^2$$

Once again, the external "rim" area does not contribute significantly to the total area. ■

DESCRIBING EQUATIONS

There are two problems in calculating the heat transfer coefficient for smooth finned tubes:

1. The mean surface temperature of the fin is lower than the surface temperature of a smooth tube under the same conditions (due to the flow of heat through the metal of the fin).
2. There is a question as to whether or not the flow of the fluid outside the tube is as great at the bottom of the space between the fins as in the unobstructed space. Both these factors depend on the size and thickness of the fins, their spacing, and the conditions of flow.(1)

To analyze the heat transfer in extended surfaces, the following assumptions are usually made:(2)

- **a.** Steady-state operation
- **b.** Constant properties
- **c.** Constant surrounding air temperature of T_∞
- **d.** Homogeneous isotropic material, with thermal conductivity, k
- **e.** One-dimensional heat transfer by conduction in the radial direction
- **f.** No internal heat generation
- **g.** Heat transfer coefficient, h, is uniform along the fin surface
- **h.** Negligible thermal radiation
- **i.** Fin perimeter at any cross section is P
- **j.** The fin cross-sectional area is A_f
- **k.** The temperature of the heat transfer surface (exposed and unexposed) at the base of the fin is constant, T_b
- **l.** The maximum temperature driving force for convection is $T_b - T_\infty$

The maximum rate of heat transfer, $\dot{Q}_{f,\max}$, from a fin will occur when the entire fin surface is isothermal at $T = T_b$. In the case of a fin of total surface area A_f, $\dot{Q}_{f,\max}$

is written as

$$\dot{Q}_{f,\max} = hA_f(T_b - T_\infty) = hA_f\theta_b \tag{17.1}$$

where θ, termed the excess temperature, is defined as $(T - T_\infty)$. Thus, θ_b is the excess temperature at the base of the fin, and T_∞ is the fluid temperature.[2]

Since the fin has a finite thermal conductivity, a temperature gradient will exist along the fin. The actual heat transfer rate from the fin to the outside fluid, $\dot{Q}_f$, will be less than $\dot{Q}_{f,\max}$. The *fin efficiency*, η_f, is a measure of how close $\dot{Q}_f$ comes to $\dot{Q}_{f,\max}$ and is defined as:

$$\eta_f = \frac{\dot{Q}_f}{\dot{Q}_{f,\max}} = \frac{\dot{Q}_f}{hA_f\theta_b} \tag{17.2}$$

From Equations (17.1) and (17.2),

$$\dot{Q}_f = \eta_f hA_f\theta_b = \frac{T_b - T_\infty}{(1/\eta_f hA_f)} = \frac{T_b - T_\infty}{R_{t,f}} \tag{17.3}$$

where

$$R_{t,f} = \text{fin thermal resistance} = \frac{1}{\eta_f hA_f} \tag{17.4}$$

Figure 17.3 is a plot of the efficiency of straight (longitudinal) fins ($\eta\%$) versus the following dimensionless group,

$$L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} \tag{17.5}$$

where L_c is the corrected fin length and A_p is the profile area of the fin. Table 17.2 provides expressions to calculate the corrected length, L_c, the projected or profile area, A_p, and the surface area, A_f, in terms of the fin length, L, fin thickness at the base, t, and the fin width, w, for various fin types.

The fin effficiency figures are valid for fin Biot number ≤ 0.25, i.e.,

$$\text{Bi}_f = \frac{h(t/2)}{k} \leq 0.25 \tag{17.6}$$

Barkwill et al.[3] recently converted the graphical results presented in Figure 17.3 into equation form. His results are provided in Table 17.3.

Figure 17.4 shows the fin efficiency ($\eta_f\%$) of annular fins of rectangular profile. The variables L_c and A_p used in the abscissa of the graph are related to the fin height, L,

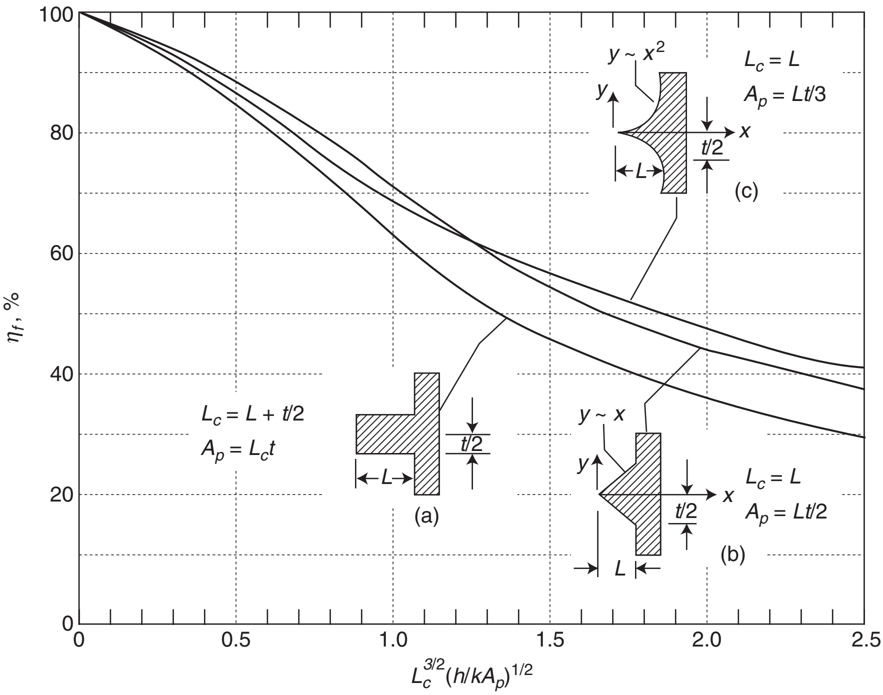

Figure 17.3 Efficiency of straight rectangular fins (a), triangular fins (b), and parabolic profile fins (c). (Adapted from Incropera and De Witt, *Fundamentals of Heat and Mass Transfer*, John Wiley & Sons, 1981, Figure 3.17.)

pipe radius, r_o, and fin outside radius, r_f, i.e.,

$$L = \text{ fin height} = r_f - r_o$$
$$r_{2c} = \text{corrected outside radius} = r_f + (t/2)$$
$$L_c = \text{corrected height} = L + t/2$$
$$A_p = \text{ profile (cross-sectional) area} = L_c t$$
$$A_f = \text{ fin surface area} = 2\pi(r_{2,c}^2 - r_o^2)$$

Table 17.2 Fin Data

Variable	Rectangular fin	Triangular fin	Parabolic fin
L_c = corrected height	$L + t/2$	L	L
A_p = profile area	$L_c t$	$Lt/2$	$Lt/3$
A_f = fin surface area of length w	$2wL_c$	$2w\sqrt{L^2 - (t/2)^2}$	$2.05w\sqrt{L^2 - (t/2)^2}$

Table 17.3 Efficiency of Straight Rectangular Fins, Triangular Fins, and Parabolic Profile Fins

For rectangular fins:

$$Y = (4.5128 \times X^3 - 10.079 \times X^2 - 31.413 \times X + 101.47) \quad \text{(A)}$$

For triangular and parabolic profile fins:

$$Y = (3.1453 \times X^3 - 7.5664 \times X^2 - 25.536 \times X + 101.18) \quad \text{(B)}$$

Note: $Y = \eta_f$, %

$$X = L_c^{3/2}(h/kA_p)^{1/2} = \sqrt{\frac{L_c^3 h}{kA_p}}$$

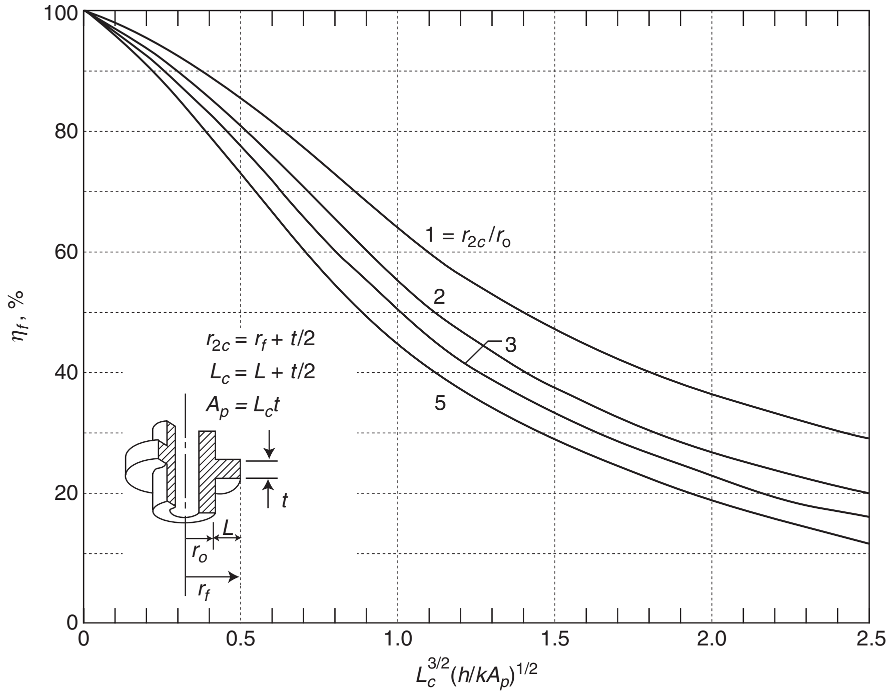

Figure 17.4 Efficiency of annular fins of rectangular profiles. (Adapted from Incroprera and De Witt, *Fundamentals of Heat and Mass Transfer*, John Wiley & Sons, 1981, Figure 3.18.)

Table 17.4 Efficiency of annular fins of rectangular profiles

$$Y = \left[(-5.1459)(X^4) + (30.478)(X^3)\right] - \left[(51.613)(X^2) - (9.3683)(X) + (100.08)\right] - \left\{\ln\left[(-20X^3) + (95)(X^2) + (77)(X) + (0.5)(Z)\right]\ln(Z)^2\right\}$$

Note:

$$Y = \eta_f, \%$$

$$X = L_c^{3/2}(h/kA_p)^{1/2} = \sqrt{\frac{L_c^3 h}{kA_p}}$$

$$Z = r_{2c}/r_o$$

The parameter of the curves is the ratio, r_{2c}/r_o. Barkwill et al.[3] also converted the results of Figure 17.4 into equation form. His results are provided in Table 17.4.

For a straight fin (one of uniform cross-section as opposed to one that, for example, tapers down to a point), the heat transfer from the fin may be represented mathematically by

$$\dot{Q} = \sqrt{hPkA_c}\,\theta_c \tanh(mL_c) \tag{17.7}$$

where P is the fin cross-section perimeter, A_c is the (cross-sectional) area of the fin, and $m = \sqrt{hP/kA_c}$.

ILLUSTRATIVE EXAMPLE 17.4

The following information is provided for a straight rectangular fin: $h = 15\ \mathrm{W/m^2\cdot K}$, $k = 300\ \mathrm{W/m\cdot K}$, $L = 3$ in, and $t = 1$ in. Estimate the fin efficiency.

***SOLUTION*:** Refer to Figure 17.3. For a rectangular fin:

$$L_c = L + t/2 = 3 + (1/2) = 3.5\ \mathrm{in} = 0.0889\ \mathrm{m}$$

and

$$A_p = L_c t = 3.5(1) = 3.5\ \mathrm{in}^2 = 0.00226\ \mathrm{m}^2$$

Generate the x-coordinate of Figure 17.3:

$$\sqrt{\frac{L_c^3 h}{kA_p}} = L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} = (0.0899)^{3/2}\left(\frac{15}{300(0.00226)}\right)^{1/2}$$

$$= (0.0265)(22.124)^{1/2}$$

$$= 0.1246$$

Reading off Figure 17.3,

$$\eta_f \approx 98\%$$

■

ILLUSTRATIVE EXAMPLE 17.5

Estimate the fin efficiency in the previous example using the equation developed by Barkwill et al.[3]

***SOLUTION*:** Apply Equation (A) from Table 17.3:

$$Y = 4.5128X^3 - 10.079X^2 - 31.413X + 101.47$$

with

$$X = 0.1246$$

and

$$Y = \eta_f$$

Substituting,

$$\begin{aligned}\eta_f &= 4.5128(0.1246)^3 - 10.079(0.1246)^2 - 31.413(0.1246) + 101.47\\ &= 0.008730 - 0.15648 - 3.91406 + 101.47\\ &= 0.974\\ &= 97.4\%\end{aligned}$$

The two results are in agreement with each other. ■

ILLUSTRATIVE EXAMPLE 17.6

A set of micro-fins is designed to cool an electronic circuit. Each micro-fin has a square cross-section of 0.2 cm by 0.2 cm and a length of 1 cm. The conductivity of the fin material is 400 $W/m \cdot K$ and the air heat transfer coefficient is 16 $W/m^2 \cdot K$. The circuit temperature is 100°C and the air temperature is 25°C. Calculate the heat transfer from each micro-fin in W.

***SOLUTION*:** Write the equation for the heat transfer application given above:

$$\dot{Q} = \sqrt{hPkA_c}\,\theta_b \tanh(mL_c) \tag{17.7}$$

Substitute known values and compute the heat transfer:

$$P = (4)(0.2) = 0.8\,\text{cm}$$
$$L_c = L + A_c/P = 1.0 + (0.2)(0.2)/(4.0)(0.2) = 1.05\,\text{cm} = 0.0105\,\text{m}$$
$$m = [hP/kA_c]^{1/2} = [(16)(4)(0.002)/(400)(0.002)(0.002)]^{1/2} = 8.95\,\text{m}^{-1}$$
$$\dot{Q} = \sqrt{(16)(4)(0.002)(400)(0.002)(0.002)}(100 - 25)\tanh[(8.95)(0.0105)]$$
$$= 0.10\,\text{W}$$

■

ILLUSTRATIVE EXAMPLE 17.7

Calculate the total heat transfer from the set of micro-fins in the previous example.

***SOLUTION*:** Although the heat transfer from one micro-fin is known, the total number of fins in the set is *not* known. Therefore, the total heat transfer cannot be calculated. ■

ILLUSTRATIVE EXAMPLE 17.8

Air and water are separated by a 1.5 mm plane wall made of steel ($k = 38\,\text{W/m}\cdot\text{K}$; density, $\rho = 7753\,\text{kg/m}^3$; heat capacity, $c_p = 486\,\text{J/kg}\cdot\text{K}$). The air temperature, T_1, is 19°C, and the water temperature, T_4, is 83°C. Denote the temperature at the air–wall interface T_2 and let T_3 be the temperature at the wall–water interface. The air-side heat transfer coefficient, h_1, is $13\,\text{W/m}^2\cdot\text{K}$ and the water side heat transfer coefficient, h_3, is $260\,\text{W/m}^2\cdot\text{K}$. Assume an area of the wall that is 1 m high and 1 m wide as a basis.

1. Show whether the conduction resistance may be neglected.
2. What is the rate of heat transfer from water to air?

To increase the rate of heat transfer, it is proposed to add steel fins to the wall. These straight rectangular steel fins will be 2.5 cm long, 1.3 mm thick, and will be spaced such that the fin pitch, S, is 1.3 cm between centers.

3. Calculate the percent increase in steady-state heat transfer rate that can be realized by adding fins to the air side of the plane wall.
4. Calculate the percent increase in steady-state heat transfer rate that can be realized by adding fins to the water side of the plane wall.

***SOLUTION*:** The base wall area, A, is $1.0\,\text{m}^2$.

1. R_1 = air resistance = $1/(h_1A) = 1/13 = 0.0769°\text{C/W}$
   R_2 = conduction resistance = $L_2/(k_1A) = 0.0015/38 = 3.95 \times 10^{-5}°\text{C/W}$
   R_3 = water resistance = $1/(h_4A) = 1/260 = 0.00385°\text{C/W}$
   Clearly R_2 (conduction) $\ll R_1$ or R_3 and may be neglected.
2. $R_{\text{tot}} = R_1 + R_3 = 0.0769 + 0.00385 = 0.0807°\text{C/W}$
   Therefore,

$$\dot{Q} = (T_4 - T_1)/R_{\text{tot}} = (83 - 19)/0.0807 = 793.1\,\text{W}$$

This represents the total heat transfer from the base pipe without fins, i.e. $\dot{Q}_{t,w/o,f}$.

3. For the case where fins have been added on the air side, write an expression for the total heat transfer rate, $\dot{Q}_t$:

$$\begin{aligned}\dot{Q}_t &= \dot{Q}_{be} + \dot{Q}_{ft} = \dot{Q}_{be} + N_f\dot{Q}_f \\ &= \dot{Q}_{be} + N_f\eta_f\dot{Q}_{f,\,\max} \\ &= h\theta_b(A_{be} + N_f\eta_f A_f)\end{aligned}$$

(See also Equations 17.10–17.13 in the next section for notation/nomenclature.) For a unit area, 1 m in length and 1 m in width, calculate the number of fins, N_f, and the exposed base surface area, A_{be}:

$$\begin{aligned}N_f &= \text{number of fins} = 1/0.013 = 77\,\text{fins};\; S = 1.3\,\text{cm} \\ L_{be} &= \text{unfinned exposed base surface} = w - N_f t;\; t = 1.3\,\text{mm} \\ &= 1 - (77)(0.0013) = 1 - 0.1 = 0.9\,\text{m} \\ A_{be} &= wL_{be} = (1)(0.9) = 0.9\,\text{m}^2\end{aligned}$$

For the rectangular fin, calculate the corrected length, L_c, the profile area, A_p, and the fin surface area, A_f:

$$\begin{aligned}L_c &= \text{corrected length} = L + t/2 = 0.025 + 0.0013/2 = 0.02565\,\text{m} \\ A_p &= \text{profile area} = L_c t = (0.02565)(0.0013) = 3.334 \times 10^{-5}\,\text{m}^2 \\ A_f &= \text{fin surface area} = 2wL_c = 2(1)(0.02565) = 0.0513\,\text{m}^2\end{aligned}$$

(See also next section for additional details.)
Check the Biot number to verify that the use of the fin efficiency figure is valid:

$$\text{Bi}_f = \frac{h(t/2)}{k_f} = \frac{(13)(0.001312/2)}{38} = 2.2 \times 10^{-4}$$

Since $\text{Bi}_f < 0.25$, the use of the figure is valid.

Calculate the abscissa of the fin efficiency diagram and obtain the fin efficiency:

$$\begin{aligned}\sqrt{\frac{L_c^3 h}{kA_p}} &= L_c^{3/2}(h/kA_p)^{1/2} \\ &= \sqrt{\frac{(0.02565)^3(13)}{(13)(3.334 \times 10^{-5})}} = \sqrt{0.1731} = 0.416\end{aligned}$$

From Figure 17.3, $\eta_f \simeq 0.88$.

Calculate the air thermal resistances:

$$R_{\text{base}} = \frac{1}{hA_{be}} = \frac{1}{(13)(0.9)} = 0.0855^\circ\text{C/W}$$

$$\begin{aligned}R_{\text{fins}} &= \frac{1}{hN_f A_f \eta_f} = \frac{1}{(13)(77)(0.0513)(0.88)} \\ &= 0.0221^\circ\text{C/W}\end{aligned}$$

The total resistance of the fin array is therefore given by

$$\frac{1}{R_{\text{tot}}} = \frac{1}{R_{\text{base}}} + \frac{1}{R_{\text{fins}}} = \frac{1}{0.0855} + \frac{1}{0.0221}$$
$$= 11.69 + 45.2 = 56.9\,\text{W}/^{\circ}\text{C}$$
$$R_{\text{tot}} = 0.0176^{\circ}\text{C/W (due to finned surface)}$$

This is the *outside* resistance; the water side resistance remains the same.

Determine the total resistance to heat transfer and the heat transfer rate:

$$R_{\text{tot}} = 0.0176 + 0.00385 = 0.0214^{\circ}\text{C/W}$$
$$\dot{Q}_t = (T_1 - T_4)/R_{\text{tot}} = (83 - 19)/0.0214 = 2867\,\text{W}$$

Calculate the percent increase in $\dot{Q}$ due to the fins on the air side:

$$\%\ \text{increase} = 100\left(\frac{2867}{793.1} - 1\right) = 261.5\%$$

Therefore, $\dot{Q}$ will increase by 261.5% when fins are added on the air side.

4. For the case where fins have been added on the water side, determine the new fin efficiency:

$$\text{Abscissa} = L_c^{3/2}(h/kA_p)^{1/2}\sqrt{\frac{L_c^3 h}{kA_p}}$$
$$= \sqrt{\frac{(0.02565)^3(260)}{(38)(3.334 \times 10^{-5})}} = \sqrt{3.46} = 1.86$$

From Figure 17.3, η_f is about 38%.

Calculate the thermal resistance of the base, fins, and the total resistance of the finned surface:

$$R_{\text{base}} = \frac{1}{hA_{be}} = \frac{1}{(260)(0.9)} = 0.00427^{\circ}\text{C/W}$$
$$R_{\text{fins}} = \frac{1}{hN_f A_f \eta_f} = \frac{1}{(260)(77)(0.0513)(0.38)} = 0.00256^{\circ}\text{C/W}$$
$$R_t\ \text{(finned surface)} = \frac{1}{\dfrac{1}{0.00427} + \dfrac{1}{0.00256}} = \frac{1}{624.6} = 0.0016^{\circ}\text{C/W}$$

This resistance is on the water side; the air resistance remains the same (i.e., as it was before fins were added to the air side):

$$R_{tot} = 0.0769 + 0.0016 = 0.0785°C/W$$

$$\dot{Q}_t = (83 - 19)/(0.0785) = 815.3\,W$$

Finally, calculate the percent increase in $\dot{Q}$ due to water-side fins:

$$\%\ \text{increase} = 100\left(\frac{815.3}{793.1} - 1\right) = 2.8\%$$ ■

ILLUSTRATIVE EXAMPLE 17.9

Comment on the results of the previous example.

***SOLUTION*:** It is concluded that fins should be added on the air side; the fins on the water side are practically useless. ■

ILLUSTRATIVE EXAMPLE 17.10

A circular tube has an outside diameter of 2.5 cm and a surface tempeature, T_b, of 170°C. An annular aluminum fin of rectangular profile is attached to the tube. The fin has an outside radius, r_f, of 2.75 cm, a thickness, t, of 1 mm, and a thermal conductivity, k, of 200 W/m · K. The surrounding fluid is at a temperature $T_\infty = 25°C$ and the associated heat transfer coefficient, h, is 130 W/m^2 · K. Calculate the heat transfer rate without the fin, $\dot{Q}_{w/o,f}$, the corrected length, L_c, the outer radius, r_{2c}, the maximum heat transfer rate from the fin, $\dot{Q}_{f,max}$, the fin efficiency, η_f, the fin heat transfer rate, q_f, and the fin thermal resistance, $R_{t,f}$.

***SOLUTION*:** Determine the area of the base of the fin:

$$r_o = D_o/2 = 0.025/2 = 0.0125\,m$$

$$A_b = 2\pi r_o t = (2\pi)(0.0125)(0.001) = 7.854 \times 10^{-5}\,m^2$$

Calculate the excess temperature at the base of the fin.

$$\theta_b = T_b - T = 170 - 25 = 145\,K$$

The total heat transfer rate without the fin, $\dot{Q}_{w/o,f}$ is then (see Equation 17.8)

$$\dot{Q}_{t,w/o,f} = \dot{Q}_{w/o,f} = hA_b\theta_b = (130)(7.854 \times 10^{-5})(145) = 1.48\,W = 5.0\,Btu/h$$

Calculate the Biot number. Determine if it is valid to use the fin efficiency figures provided in this section since the Biot number must be less than 0.25:

$$\mathrm{Bi}_f = \frac{h(t/2)}{k_f} = \frac{130(0.001/2)}{200} = 3.25 \times 10^{-4} < 0.25$$

The fin efficiency figures may be used.

Calculate the fin height, L. Since

$$r_f = 0.0275\,\mathrm{m}$$
$$L = r_f - r_o = 0.0275 - 0.0125 = 0.015\,\mathrm{m}$$

Calculate the corrected radius and height:

$$r_{f,c} = r_f + t/2 = 0.0275 + 0.001/2 = 0.028\,\mathrm{m}$$
$$L_c = L + t/2 = 0.015 + 0.001/2 = 0.0155\,\mathrm{m}$$

Calculate the profile area and the fin surface area:

$$A_p = L_c t = (0.0155)(0.001) = 1.55 \times 10^{-5}\,\mathrm{m}^2$$
$$A_f = 2\pi(r_{f,c}^2 - r_0^2) = (2\pi)(0.028^2 - 0.0125^2) = 3.94 \times 10^{-3}\,\mathrm{m}^2$$

The maximum fin heat transfer rate, $\dot{Q}_{f,\max}$, is therefore:

$$\dot{Q}_{f,\max} = hA_f\theta_b = (130)(3.94 \times 10^{-3})(145) = 74.35\,\mathrm{W}$$

Calculate the abscissa and the curve parameter for the fin efficiency figure in Figure 17.3:

$$\text{Abscissa} = L_c^{3/2}(h/kA_p)^{1/2} = \left(\frac{L_c^3 h}{kA_p}\right)^{1/2} = \sqrt{\frac{(0.0155)^3(130)}{(200)(1.55 \times 10^{-5})}} = 0.40$$
$$\text{Curve parameter} = 0.028/0.0125 = 2.24$$

From Figure 17.4 (interpolating),

$$\eta_f = 86\% = 0.86$$

Therefore, the fin heat transfer rate, $\dot{Q}_f$, is given by Equation (17.2),

$$\dot{Q}_f = \eta_f \dot{Q}_{f,\max} = (0.86)(74.35) = 64\,\mathrm{W} = 218\,\mathrm{Btu/h}$$

The corresponding fin resistance, from Equations (17.2)–(17.4), is

$$R_{t,f} = \frac{\theta_b}{\dot{Q}_f}; \quad \theta_b = T - T_\infty$$

Substituting,

$$R_{t,f} = \frac{145}{64} = 2.27\,\text{K/W} \qquad \blacksquare$$

FIN EFFECTIVENESS AND PERFORMANCE

Another dimensionless quantity used to assess the benefit of adding fins is the *fin effectiveness*, ε_f, or *fin performance coefficient*, FPC. It is defined as

$$\varepsilon_f = \text{FPC} = \frac{\dot{Q}_f}{\dot{Q}_{\text{w/o},f}} \tag{17.8}$$

where $\dot{Q}_{\text{w/o},f}$ is the rate of heat transfer without fins, i.e.,

$$\dot{Q}_{\text{w/o},f} = hA_b\theta_b \tag{17.9}$$

Usually, fins are not justified unless ε_f or FPC ≥ 2.

The fin efficiency, η_f, and the effectiveness, ε_f, characterize the performance of a single fin. As indicated above, arrays of fins are attached to the base surface in many applications. The distance from the center of one fin to the next one along the same tube surface is termed the *fin pitch*, S. In this case, the total heat transfer area, A_t, includes contributions due to the fin surfaces and the exposed (unfinned) base surface, that is,

$$A_t = A_{be} + N_f A_f \tag{17.10}$$

The total heat transfer area without fins, $A_{t,\text{w/o},f}$, is

$$A_{t,\text{w/o},f} = A_{be} + N_f A_b \tag{17.11}$$

where $N_f =$ number of fins, $A_f =$ surface area per fin, $A_{be} =$ total exposed (unfinned) base area of the surface, and $A_b =$ the base area of one fin. Equation (17.11) is often used to determine A_{be} from knowledge of the surface geometry, fin base area, and number of fins.

The total heat transfer rate, without fins, $\dot{Q}_{t,\text{w/o},f}$ is

$$\dot{Q}_{t,\text{w/o},f} = hA_{t,\text{w/o},f}\theta_b = h(A_{be} + N_f A_b)\theta_b \tag{17.12}$$

The total heat transfer rate from the finned surface, $\dot{Q}_t$, is

$$\begin{aligned}
\dot{Q}_t &= \dot{Q}_{be} + \dot{Q}_{ft} \\
&= \dot{Q}_{be} + N_f\dot{Q}_f \\
&= hA_{be}\theta_b + N_f hA_f\eta_f\theta_b \\
&= h(A_{be} + N_f A_f\eta_f)\theta_b
\end{aligned} \tag{17.13}$$

where $\dot{Q}_{ft}$ is the heat transfer rate due to all (total) fins and $\dot{Q}_f$ is that due to a single fin.

The maximum heat transfer rate, $\dot{Q}_{t,\max}$, of the surface occurs when $n_f = 1.0$, i.e., when the temperature of the base and all the fins is T_b, and is given by

$$\dot{Q}_{t,\max} = h\theta_b(A_{be} + N_f A_f) = hA_t\theta_b \tag{17.14}$$

The *overall fin efficiency*, $\eta_{o,f}$, is defined as

$$\eta_{o,f} = \frac{\dot{Q}_t}{\dot{Q}_{t,\max}} = \frac{A_{be} + N_f A_f \eta_f}{A_t} \tag{17.15}$$

Substituting from Equation (17.10) into (17.15) yields

$$\eta_{o,f} = 1 - \left(\frac{N_f A_f}{A_t}\right)(1 - \eta_f) \tag{17.16}$$

Finally, the *overall surface effectiveness*, $\varepsilon_{o,f}$, is defined as

$$\varepsilon_{o,f} = \frac{\dot{Q}_t}{\dot{Q}_{t,\mathrm{w/o},f}} \tag{17.17}$$

ILLUSTRATIVE EXAMPLE 17.11

Refer to Illustrative Example 17.10. Calculate the fin effectiveness, ε_f, and whether the use of the fin is justified.

***SOLUTION*:** Calculate the fin effectiveness or performance coefficient using Equation (17.8):

$$\varepsilon_f = \frac{\dot{Q}_f}{\dot{Q}_{\mathrm{w/o},f}} = \frac{64}{1.48} = 43.2$$

Since,

$$\varepsilon_f = 43.2 \gg 2.0$$

the use of the fin is justified. ■

ILLUSTRATIVE EXAMPLE 17.12

If the tube described in Illustrative Example 17.10 had a length of one meter and fin pitch of 10 mm, what would be the total surface area for heat transfer, the exposed tube base total heat transfer rate, $\dot{Q}_t$, the overall efficiency of the surface, and the overall surface effectiveness?

Note: This problem is an extension of Illustrative Example 17.10. The information is obtained from it and its solution is used in the solution of this problem.

***SOLUTION*:** Calculate the number of fins in the tube length:

$$N_f = w/S = 1/0.01 = 100\,\text{fins};\ S = 1.0\,\text{mm} = 0.01\,\text{cm}$$

Calculate the unfinned base area:

$$w_{be} = \text{unfinned (exposed) base length} = w - N_f t = 1 - (100)(0.001) = 0.9\,\text{m}$$

$$A_{be} = \text{unfinned base area} = 2\pi r_o w_{be} = (2\pi)(0.0125)(0.9) = 0.0707\,\text{m}^2$$

The total transfer surface area, A_t, may now be calculated:

$$A_{t,\text{w/o},f} = A_t = A_{be} + N_f A_f = 0.0707 + (100)(3.94\times10^{-3}) = 0.465\,\text{m}^2$$

Equation (17.12) is used to obtain the total heat rate without fins:

$$\dot{Q}_{t,\,\text{w/o},f} = h(2\pi r_o w)\theta_b = (130)(2\pi)(0.0125)(1)(145) = 1480\,\text{W}$$

Calculate the heat flow rate from the exposed tube base:

$$\begin{aligned}\dot{Q}_{be} &= hA_{be}\theta_b \\ &= (130)(0.0707)(145) = 1332.7\,\text{W} = 4548\,\text{Btu/h}\end{aligned} \tag{17.13}$$

Calculate the heat flow rate from all the fins:

$$\begin{aligned}\dot{Q}_{ft} &= N_f \dot{Q}_f \\ &= (100)(64) = 6400\,\text{W}\end{aligned} \tag{17.13}$$

The total heat flow rate may now be calculated:

$$\begin{aligned}\dot{Q}_t &= \dot{Q}_{be} + \dot{Q}_{ft} \\ &= 1332.7 + 6400 = 7732.7\,\text{W}\end{aligned} \tag{17.13}$$

Calculate the maximum heat transfer rate from Equation (17.14):

$$\dot{Q}_{t,\max} = hA_t\theta_b = (130)(0.465)(145) = 8765.3\,\text{W}$$

By definition, the overall fin efficiency is given in Equation (17.15):

$$\eta_{o,f} = \frac{\dot{Q}_t}{\dot{Q}_{t,\,\max}} = \frac{7732.7}{8765.3} = 0.882$$

The corresponding overall effectiveness is therefore given by Equation (17.17):

$$\varepsilon_{o,f} = \frac{\dot{Q}_t}{Q_{t,\,\text{w/o},f}} = \frac{7732.7}{1480} = 5.22$$

Finally, calculate the thermal resistance:

$$R_{\text{base}} = \frac{1}{hA_{be}} = \frac{1}{(130)(0.0707)} = 0.109\,\text{K/W}$$

$$R_{\text{fins}} = \frac{1}{hN_f A_f \eta_f} = \frac{1}{(130)(100)(3.94\times10^{-3})(0.82)} = 0.0238\,\text{K/W}$$

■

ILLUSTRATIVE EXAMPLE 17.13

Consider the case of aluminum fins of triangular profile that are attached to a plane wall with a surface temperature is 250°C. The fin base thickness is 2 mm and its length is 6 mm. The system is in ambient air at a temperature of 20°C and the surface convection coefficient is 40 $W/m^2 \cdot K$. Consider a 1 m width of a single fin. Determine:

1. the heat transfer rate without the fin,
2. the maximum heat transfer rate from the fin, and
3. the fin efficiency, thermal resistance, and effectiveness.

Properties of the aluminum may be evaluated at the average temperature, $(T_b + T_\infty)/2 = (250 + 20)/2 = 135°C = 408$ K, where $k \simeq 240$ W/m · K.

***SOLUTION*:** Determine the base area of the fin:

$$A_b = (t)(w) = (0.002)(1) = 0.002\,\text{m}^2$$

Calculate the excess temperature at the base of the fin:

$$\theta_b = T_b - T_\infty = 250 - 20 = 230°\text{C} = 230\,\text{K}$$

Calculate the heat transfer rate without a fin, $\dot{Q}_{\text{w/o},f}$, employing Equation (17.9).

$$\dot{Q}_{\text{w/o},f} = hA_b\theta_b = (40)(0.002)(230) = 18.4\,\text{W}$$

Also, calculate the maximum heat transfer rate, $\dot{Q}_{f,\max}$:

$$\dot{Q}_{f,\max} = hA_f\theta_b = (40)(0.012)(230) = 110\,\text{W};\ A_f = 0.0118 \simeq 0.012\,\text{m}^2$$

Check the Biot number criterion to determine if the use of the fin efficiency figure is valid:

$$\text{Bi}_f = \frac{h(t/2)}{k} = \frac{(40)(0.002/2)}{240} = 1.67 \times 10^{-4} < 0.25$$

Since $\text{Bi}_f < 0.25$, the use of the figure is permitted.

Calculate the fin L_c, A_p, and P, noting that this is a triangular fin:

$$L_c = \text{corrected length} = L = 0.006\,\text{m}$$
$$A_p = \text{profile area} = Lt/2 = (0.006)(0.002)/2 = 6 \times 10^{-6}\,\text{m}^2$$
$$A_f = \text{fin surface area} = 2w\sqrt{L^2 - (t/2)^2} \approx 0.012\ \text{m}^2$$

Determine the abscissa for the fin efficiency figure:

$$\text{Abscissa} = \sqrt{\frac{L_c^3 h}{kA_p}} = \sqrt{\frac{(0.006)^3(40)}{(240)(6 \times 10^{-6})}} \simeq 0.0775$$

Obtain the fin efficiency from Figure 17.4 for triangular profile straight fins:

$$\eta_f \approx 0.99$$

Calculate the fin heat transfer rate, $\dot{Q}_f$ employing Equation (17.15).

$$\dot{Q}_f = \eta_f \dot{Q}_{f,\max} = (0.99)(110) = 108.9\,\text{W}$$

The fin thermal resistance, $R_{t,f}$, is therefore

$$\begin{aligned} R_{t,f} &= \theta_b/\dot{Q}_f \qquad (17.3) \\ &= 230/108.9 = 2.1^\circ\text{C/W} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 17.14

Determine whether the use of the fin is justified in the previous example.

***SOLUTION*:** Calculate the fin effectiveness, ε_f, or performance coefficient, FPC, employing Equation (17.8).

$$\varepsilon_f = \text{FPC} = \dot{Q}_f/\dot{Q}_{\text{w/o},f} = 108.9/18.4 = 5.92$$

Is the use of the fin justified? Since $\varepsilon_f (= \text{FPC}) = 5.94 > 2$, the use of the fin is justified. Note that the triangular fin is known to provide the maximum heat transfer per unit mass. ■

ILLUSTRATIVE EXAMPLE 17.15

Annular aluminum fins of rectangular profile are attached to a circular tube. The outside diameter of the tube is 50 mm and the temperature of its outer surface is 200°C. The fins are 4 mm thick and have a length of 15 mm. The system is in ambient air at a temperature of 20°C and the surface convection coefficient is $40\,\text{W/m}^2\cdot\text{K}$. The thermal conductivity of aluminum is $240\,\text{W/m}\cdot\text{K}$. What are the efficiency, thermal resistance, effectiveness, and heat transfer rate of a single fin? Is the use of the fin justified?

***SOLUTION*:** Determine the base area of the fin:

$$A_b = 2\pi r_o t = (2\pi)(0.025)(0.004) = 6.283 \times 10^{-4}\,\text{m}^2$$

Calculate the excess temperature at the fin base:

$$\theta_b = T_b - T_\infty = 200 - 20 = 180^\circ\text{C} = 180\,\text{K} = 324^\circ\text{R} = 324^\circ\text{F}$$

Calculate the heat transfer rate without a fin, $\dot{Q}_{\text{w/o},f}$, using Equation (17.9).

$$\dot{Q}_{\text{w/o},f} = hA_b\theta_b = (40)(6.283 \times 10^{-4})(180) = 4.52\,\text{W}$$

Check the Biot number criterion to determine if the use of the fin efficiency figure is valid:

$$\text{Bi}_f = \frac{h(t/2)}{k} = \frac{(40)(0.004/2)}{240} = 3.33 \times 10^{-4} < 0.25; \quad \text{OK}$$

Calculate the fin corrected radius and length, profile area, and surface area:

$$r_f = r_0 + L = 0.025 + 0.015 = 0.04\,\text{m}$$
$$r_{f,c} = r_f + t/2 = 0.04 + 0.002 = 0.042\,\text{m}$$
$$L_c = L + t/2 = 0.015 + 0.002 = 0.017\,\text{m}$$
$$A_p = L_c t = (0.017)(0.004) = 6.8 \times 10^{-5}\,\text{m}^2$$
$$A_f = 2\pi(r_{2,f}^2 - r_0^2) = 2\pi((0.042)^2 - (0.025)^2) = 7.157 \times 10^{-3}\,\text{m}^2$$

Calculate the maximum heat transfer rate from a *single* fin using Equation (17.14).

$$\dot{Q}_{f,\max} = hA_f\theta_b = (40)(7.157 \times 10^{-3})(180) = 51.53\,\text{W}$$

Determine the abscissa and curve parameter for the efficiency figure (for annular rectangular fins):

$$r_{f,c}/r_0 = 0.042/0.025 = 1.68$$
$$\text{Abscissa} = \sqrt{\frac{L_c^3 h}{kA_p}} = \sqrt{\frac{(0.0017)^3(40)}{(240)(6.8 \times 10^{-5})}} = 0.11$$

Read the fin efficiency from the figure for annular rectangular straight fins:

$$\eta_f = 0.97$$

The fin heat transfer rate, $\dot{Q}_f$, is therefore

$$\dot{Q}_f = \eta_f \dot{Q}_{f,\max} \tag{17.2}$$

Substituting,

$$\dot{Q}_f = (0.97)(51.53)$$
$$= 50\,\text{W}$$

Calculate the fin resistance and effectiveness:

$$R_{t,f} = \theta_b/\dot{Q}_f = 180/50$$
$$= 3.6^\circ\text{C/W}$$
$$\varepsilon_f = \dot{Q}_f/\dot{Q}_{\text{w/o},f} = 50/4.52$$
$$= 11.06$$

Since $\varepsilon_f = \text{FCP} > 2$, the use of the fin is justified. ■

ILLUSTRATIVE EXAMPLE 17.16

What is the rate of heat transfer per unit length of tube in the previous illustrative example, if there are 125 such fins per meter of tube length? Also calculate the total efficiency and effectiveness.

***SOLUTION*:** For an array of 125 fins per meter ($N_f = 125$), calculate the unfinned base area:

$$w_{be} = \text{unfinned exposed base length} = w - N_f t = 1 - (125)(0.004) = 0.5\,\text{m}$$

$$A_{be} = (2\pi)(0.025)(0.5) = 0.0785\,\text{m}^2$$

Calculate the total heat transfer surface area, A_t:

$$A_t = A_{be} + N_f A_f = 0.0785 + (125)(7.157 \times 10^{-3}) = 0.973\,\text{m}^2$$

Calculate the heat rate without fins:

$$\dot{Q}_{\text{w/o},f} = h(2\pi r_o w)\Delta T_b = (40)(2\pi)(0.025)(1)(180) = 1131\,\text{W}$$

Calculate the heat rate from the base and fins, and the total heat rate:

$$\dot{Q}_{be} = hA_{be}\theta_b = (40)(0.0785)(180) = 565.2\,\text{W}$$

$$\dot{Q}_{ft} = N_f \dot{Q}_f = (125)(50) = 6250\,\text{W}$$

$$\begin{aligned}\dot{Q}_t &= 565.2 + 6250 = 6815.2\,\text{W}\\ &= 6.82\,\text{kW}\end{aligned}$$

Calculate the maximum heat transfer rate:

$$\dot{Q}_{\max} = hA_t\theta_b = (40)(0.973)(180) = 7005.6\,\text{W}$$

The overall fin efficiency is therefore

$$\begin{aligned}\eta_{o,f} &= \dot{Q}_t/\dot{Q}_{t,\max} \qquad (17.15)\\ &= 6815.2/7008.6 = 0.973\end{aligned}$$

The corresponding overall fin effectiveness is

$$\begin{aligned}\varepsilon_{o,f} &= \dot{Q}_t/\dot{Q}_{t,\text{w/o},f} \qquad (17.17)\\ &= 6815.2/1131 = 6.03\end{aligned}$$

Finally, the thermal resistances are

$$R_{\text{base}} = \frac{1}{hA_{be}} = \frac{1}{(40)(0.0785)} = 0.318^\circ\text{C/W}$$

$$\begin{aligned}R_f &= \frac{1}{hN_f A_f \eta_f}\\ &= \frac{1}{(40)(125)(7.157 \times 10^{-3})(0.97)} = 0.0288^\circ\text{C/W} \qquad (17.4)\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 17.17

A metal fin 1 inch high and $\frac{1}{8}$ inch thick has a thermal conductivity, k, of 25 Btu/h · ft · °F and a uniform base temperature of 250°F. It is exposed to an air stream at 60°F with a velocity past the fin such that the convection coefficient of heat transfer, h, is 15 Btu/h · ft · °F. Calculate the temperature at the tip of the fin and the heat transfer from the fin per foot of fin. Solve the problem analytically.

***SOLUTION*:** Set the fin height, thickness, and length equal to L, t, and w, respectively. Bennett and Meyers[4] have shown that the equation describing the temperature profile of this system is:

$$\frac{d^2T}{dx^2} - \frac{2h}{\mathrm{k}t}(T - 70) = 0; \; x = \text{height coordinate}$$

Let

$$\alpha^2 = \frac{2h}{(\mathrm{k})(t)} = (2)(15)/(25)(1/96) = 115$$

so that

$$\frac{d^2(T-70)}{dx^2} - \alpha^2(T - 70) = 0; \quad \alpha = 10.7$$

The solution to this ordinary differential equation is:

$$(T - 70) = c_1e^{\alpha x} + c_2e^{-\alpha x}$$

or

$$T = 70 + c_1e^{\alpha x} + c_2e^{-\alpha x}$$

In addition,

$$\frac{dT}{dx} = \alpha c_1e^{\alpha x} - \alpha c_2e^{-\alpha x}$$

and

$$\frac{d^2T}{dx^2} = \alpha^2c_1e^{\alpha x} + \alpha^2c_2e^{-\alpha x}$$

The reader is left the exercise of showing that this solution satisfies the above second order ordinary differential equation. The solution must also satisfy the two boundary conditions (BC) below that are employed to evaluate the two integration constants c_1 and c_2.

$$\text{BC(1):} \quad T = 250 \text{ at } x = 0 \text{ (fin base)}$$

$$\text{BC(2):} \quad -kA\frac{dT}{dx} = hA(T - 70); \quad A = (t)(w)$$

or

$$\frac{dT}{dx} = -\frac{h}{k}(T - 70) \quad \text{at } x = L = 1/12\,\text{ft}$$

From BC(1),

$$180 = c_1 + c_2$$

From BC(2),

$$-k(\alpha c_1 e^{\alpha x} - \alpha c_2 e^{-\alpha x}) = h(c_1 e^{\alpha x} + c_2 e^{-\alpha x})$$

Solving these two equations simultaneously gives

$$c_1 = 25.9 \quad c_2 = 154.1$$

The fin top $\left(x = L = \frac{1}{12}\text{ ft}\right)$ temperature is therefore

$$\begin{aligned}(T - 70) &= (25.9)(2.4392) + (154.1)(0.40997)\\ &= 63.2 + 63.2\\ &= 126.4\\ T &= 70 + 126.4 = 196.4^\circ\text{F}\end{aligned}$$

The heat transfer rate is:

$$\begin{aligned}\dot{Q} &= -k(A_{\text{base}})\frac{dT}{dx}\bigg|_{x=0}; \ \frac{dT}{dx}\bigg|_{\lambda=0} = \alpha(c_1 - c_2)\\ &= -(25)(1/8)(1/12)(w)(10.7)(25.9 - 154.1); \quad L = 1.0\\ &= 357.2\,\text{Btu/h}\cdot\text{ft of fin length}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 17.18

Estimate the fin efficiency in the previous example, if the efficiency is defined (as noted earlier) as the actual heat transfer divided by the heat rate of the entire fin with the same temperature as its base.

***SOLUTION*:** For the entire fin at $T = 250^\circ$F,

$$\begin{aligned}\dot{Q} &= hA(T - 70)\\ &= (15)\frac{[(1)(w) + (1)(w) + (1/8)w]}{12}(T - 70)\end{aligned}$$

For $w = 1$ ft

$$\begin{aligned}\dot{Q} &= (15)(2.125/12)(250 - 70)\\ &= 478\,\text{Btu/h}\cdot\ \text{ft of fin length}\end{aligned}$$

The fin efficiency from Equation (17.15) is therefore

$$\begin{aligned}\eta_f &= (357.2/478)100\\ &= 74.7\%\end{aligned}$$

■

FIN CONSIDERATIONS

The selection of suitable fin geometry requires an overall comparison of

1. economics,
2. mass of fin,
3. space (if available),
4. pressure drop (if applicable), and
5. the heat transfer characteristics of the fin.

Generally, fins are effective with gases; are less effective with liquids in forced (or natural) convection; are very poor with boiling liquids; and, are extremely poor with condensing vapors.

As discussed earlier, fins should be placed on the side of the heat exchanger surface where h is the lowest. Thin, closely spaced fins (subject to economic constraints) are generally superior to fewer thicker fins. Their thermal conductivity, k, should obviously be high.

In summary, the same basic heat exchanger equation applies for fins:

$$\dot{Q} = UA\Delta T \tag{17.18}$$

with

$$\frac{1}{U} = \frac{1}{h_o} + \frac{\Delta x}{k} + \frac{1}{h_i} \tag{17.19}$$

For many applications, $h_i \ggg h_o$ and $\Delta x/k \ggg h_o$ so that

$$\frac{1}{U} \simeq \frac{1}{h_o} \tag{17.20}$$

The outside coefficient is generally the controlling resistance and it may be large. One way to increase Q is to increase A. As noted earlier, this may be accomplished by the addition of fins to the appropriate heat transfer surface and these can be mounted longitudinally or circumferentially.

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