Chapter 16

Shell and Tube Heat Exchangers

INTRODUCTION

Shell and tube (also referred to as tube and bundle) heat exchangers provide a large heat transfer area economically and practically. The tubes are placed in a bundle and the ends of the tubes are mounted in tube sheets. The tube bundle is enclosed in a cylindrical shell, through which the second fluid flows. Most shell and tube exchangers used in practice are of welded construction. The shells are built as a piece of pipe with flanged ends and necessary branch connections. The shells are made of seamless pipe up to 24 inches in diameter; they are made of bent and welded steel plates, if above 24 inches. Channel sections are usually of built-up construction, with welding-neck forged-steel flanges, rolled-steel barrels and welded-in pass partitions. Shell covers are either welded directly to the shell, or are built-up constructions of flanged and dished heads and welding-neck forged-steel flanges. The tube sheets are usually nonferrous castings in which the holes for inserting the tubes have been drilled and reamed before assembly. Baffles can be employed to both control the flow of the fluid outside the tubes and provide turbulence.

There are vast industrial uses of shell and tube heat exchangers. These units are used to heat or cool process fluids, either through a single-phase heat exchanger or a two-phase heat exchanger. In single-phase exchangers, both the tube side and shell side fluids remain in the same phase that they enter. In two-phase exchangers (examples include condensers and boilers), the shell side fluid is usually condensed to a liquid or heated to a gas, while the tube-side fluid remains in the same phase.

Generally, shell and tube exchangers are employed when double pipe exchangers do not provide sufficient area for heat transfer. Shell and tube exchangers usually require less materials of construction and are consequently more economical when compared to double pipe and/or multiple double pipe heat exchangers in parallel.

Chapter topics include:

Equipment Description


Describing Equations

The "F" Factor

Effectiveness Factor and Number of Transfer Units

EQUIPMENT DESCRIPTION

There are more than 280 different types of shell and tube heat exchangers that have been defined by the Tubular Exchanger Manufacturers Association (TEMA). The simplest shell and tube heat exchanger has a single pass through the shell and a single pass through the tubes. This is termed a 1–1 shell and tube heat exchanger. A photograph of this unit is provided in Figure 16.1. A schematic (line diagram) of a 1–2 unit can be found in Figure 16.2. A side view of the tubes in a typical exchanger is shown in Figure 16.3. Fluids that flow through tubes at low velocity result in low heat transfer coefficients and low pressure drops. To increase the heat transfer rates, multipass operations may be used. As noted earlier, baffles are used to divert the fluid within the distribution header. An exchanger with one pass on the shell side and four tube passes is termed a 1–4 shell and tube heat exchanger. It is also possible to increase the number of passes on the shell side by using dividers. A 2–8 shell and tube heat exchanger has two passes on the shell side and 8 passes on the tube side.
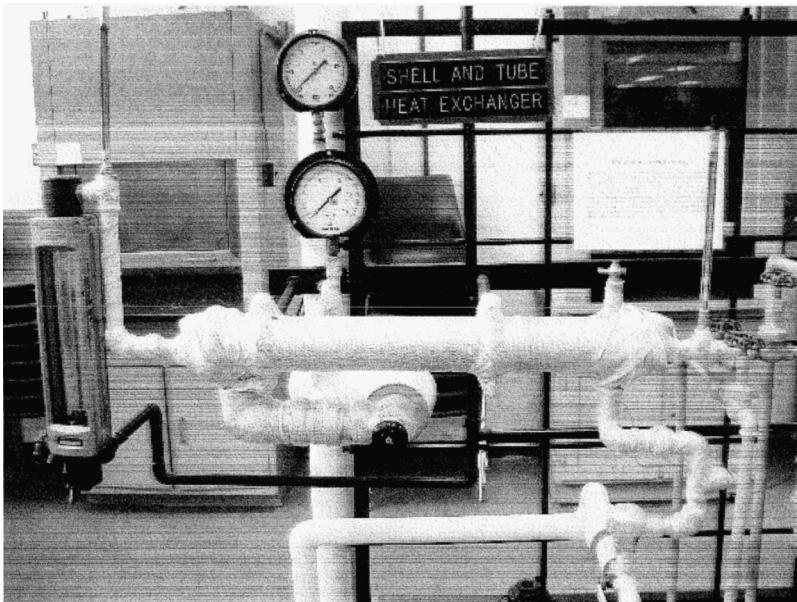
Figure 16.1   Shell and tube heat exchanger—Manhattan College Unit Operations Laboratory.
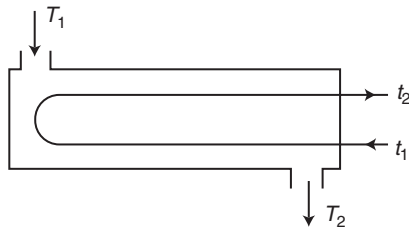
Figure 16.2   One shell pass, 2 tube passes schematic.
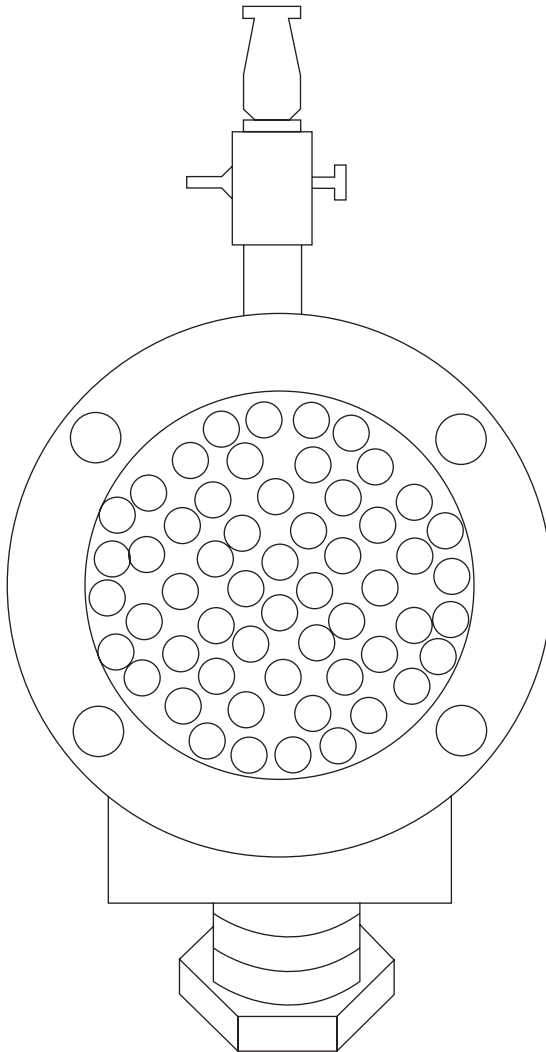
Figure 16.3   Side view of tube layout in the shell and tube heat exchanger.

ILLUSTRATIVE EXAMPLE 16.1

1. Determine the number of passes on the shell side and the number of passes on the tube side for a 4−6 shell and tube heat exchanger.
2. Provide a line diagram of a 3−6 shell and tube heat exchanger.

SOLUTION:

1. First consider how many passes there are on the shell side. There are four passes on the shell side. Finally, there are six passes on tube side. The number of shell passes depends, on the inner design of the shell while the number of tube passes depends on how many hairpins are formed by the bundle of tubes within the shell.
2. See Figure 16.4 for line diagram of a 3−6 shell and tube heat exchanger.   ∎

ILLUSTRATIVE EXAMPLE 16.2

What is the purpose of baffles? Select one answer.

a. To decrease the turbulence of the shell side fluid and therefore increase the contact between the shell side fluid and tube.
b. To decrease the turbulence of the tube side fluid and therefore increase the contact between the shell side fluid and tube.
c. To change the direction of flow from co-current to countercurrent and therefore increase the heat transfer coefficient.
d. To increase the turbulence of the tube side fluid and lower the effect of fouling.
e. To increase the turbulence of the shell side fluid and therefore provide better contact between the shell side fluid and tube.
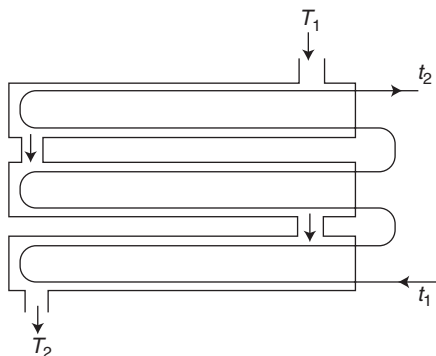
Figure 16.4   3−6 Exchanger.

SOLUTION: Baffles are included in heat exchangers to increase the turbulence of the shell side fluid and therefore increase contact between the shell side fluid and the tube. The more contact that takes place, the greater the heat transfer.

Therefore, the correct answer is (e).   ∎

ILLUSTRATIVE EXAMPLE 16.3

What is the difference between passes and crosses on the shell side? Select one answer.

   a. Passes are the number of times a fluid crosses the tube bundle while crosses are the number of times a fluid crosses an imaginary plane parallel to the tubes.

   b. Passes are the number of times a fluid crosses an imaginary plane parallel to the tubes while crosses are the number of times a fluid crosses the tube bundle.

   c. Passes are the number of times a fluid crosses an imaginary plane perpendicular to tubes while crosses are the number of times a fluid crosses the tube bundle.

   d. Passes are the number of times a fluid crosses the tube bundle while crosses are the number of times a fluid crosses an imaginary plane perpendicular to tubes.

   e. There is no difference; they are synonymous.

SOLUTION: Passes are the number of times the fluid crosses an imaginary plane perpendicular to the tubes. Crosses are the number of times the fluid crosses the tube bundle.

Therefore, the correct answer is (c).   ∎

When designing a shell and tube heat exchanger, the layout of the inner tube bundle is the main physical design criterion and includes:

   1. Tube diameter

   2. Tube wall thickness

   3. Tube length

   4. Tube layout

   5. Tube corrugation

   6. Baffle design.

Each are detailed below.

   1. *Tube Diameter*

   The fouling nature of the tube-side fluid determines the correct tube diameter. Smaller diameter tubes allow for compact and economical design; however, mechanical cleaning of smaller diameter tubes can be difficult. For cleaner fluids that do not foul, tubes may have an outside diameter as small as $\frac{1}{2}$ inch, while shell and tube heat exchangers that handle tars may have tubes that have outside diameters of 3 inches. Most shell and tube heat exchangers have tubes with an outside diameter that range from $\frac{3}{4}$ to 1 inch.[1]

2. *Tube Wall Thickness*

The tube wall thickness is governed with six different variables in mind. First and foremost, the tube must have enough thickness to handle the external pressure exerted by the shell-side fluid and the internal pressure exerted by the tube-side fluid. In most cases, pressure is not a factor. Other factors that the tube wall thickness must provide are listed below.

a. Allow enough margin for corrosion of the tube from use, or if any of the process fluids have corrosive properties.

b. Provide enough resistance against flow-induced vibrations.

c. Provide enough axial strength to reduce sagging or bending throughout the length of the tube.

d. Meet standards for ease in ordering replacement tubes.

e. Ordering thinner or thicker tubes must justify the associated extra costs.[1]

3. *Tube Length*

Generally, the most economical shell and tube heat exchangers have a small shell diameter and long tube length. Therefore, manufacturers strive to produce longer shell and tube heat exchangers. Long tubular bundles may prove difficult to replace. In practice, the maximum tube length for a fixed-bundle of tubes (individual tubes may not be withdrawn and replaced) may be 9 m with a total weight of about 20 tons. If a fixed tube sheet (individual tubes may be withdrawn and replaced) exchanger is used, then the tube length should be limited to about 15 m although tubes 22 m in length have been employed to meet process demands in industrial plants and are not uncommon.[1]

4. *Tube Layout*

When laying out a bundle of tubes, the tube pitch, i.e., the distance from the center of one tube to the next, should not exceed 1.25 times the outside diameter of the tubes. There are generally four types of tube bundle arrangements, as shown in Figure 16.5. The fouling properties of the shell-side fluid must also be considered when determining the tube pitch. A tube pitch angle of $30°$ or $60°$ can accommodate 15% more tubes than their $45°$ and $90°$ counterparts; however, $30°$ triangular pitch and $60°$ triangular pitch do not allow for external cleaning of the outer tubes. When the shell-side fluid is in the laminar region, then a $45°$ square pitch tube layout has better heat transfer rates as well as lower pressure drops across the exchanger. When the shell-side fluid is in the turbulent region, then the $90°$ square pitch provides for better heat transfer rates and a lower pressure drop across the exchanger.

5. *Tube Corrugation*

Roped, fluted, and finned (see next chapter) tubes can be used in the design of a shell and tube heat exchangers to either increase the area available for heat transfer or ensure that the shell-side fluid remains in the turbulent regime. Finned tubes increase the area available for heat transfer in a shell and tube exchanger, while fluted and roped tubes induce turbulent flow in the shell-side fluid.[1]
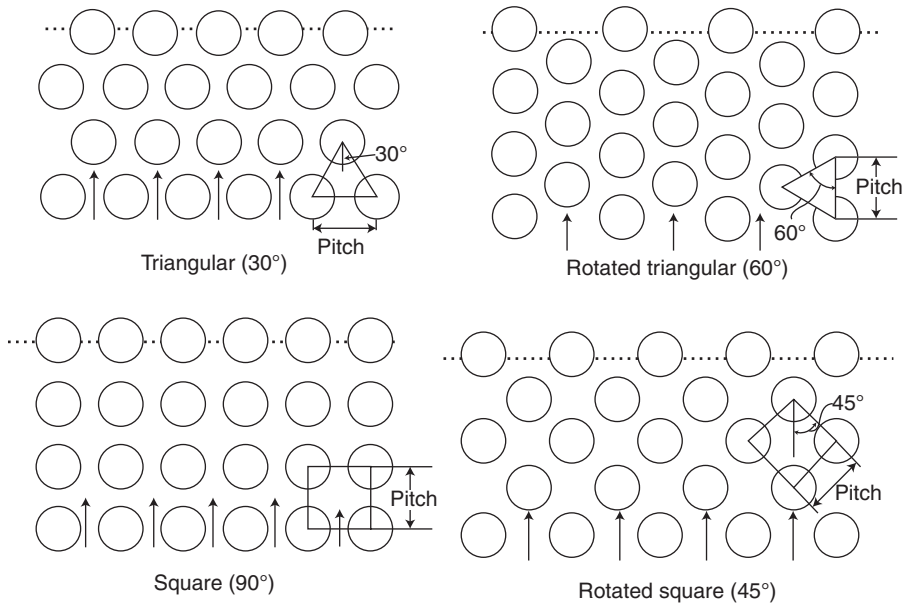
Figure 16.5   Types of tube pitches used in shell and tube heat exchangers.[1]

6. *Baffle Design*

Baffles in a shell and tube heat exchanger serve two purposes. First, they allow the shell-side fluid to flow across all of the tubes, preventing dead spaces occurring within the heat exchanger. Secondly, they provide support for longer tubes to prevent damage from sagging or flow-induced vibrations. Baffles can either be designed in a single-, double-, or triple-segmental pattern, as shown in order from top to bottom in Figure 16.6.[1] Single segmental baffles usually have a total length of 15–40% of the shell's inner diameter while the total length of double segmental baffles is 20–30% of the shell's inner diameter. Baffle spacing affects pressure drop; therefore, calculations of a pressure drop with certain baffle spacing must be performed to ensure that the pressure drop is reasonable. A general rule of thumb in industry is that baffles should not be spaced closer than one-fifth of the shell's inner diameter or two inches apart, whichever is greater.[1]

Finally, the choice of which fluid should flow in the tubes is an important design decision. If one of the fluids is particularly corrosive, it is typically introduced on the tube side since more expensive resistant tubes can be purchased. If one of the fluids is more likely to form scale and/or deposits, it should be the tube side fluid since the inside of the tubes are much easier to clean than the outside of the tubes. In addition, viscous fluids normally flow on the shell side in order to help induce turbulence. The impact on the overall heat transfer coefficient and pressure drop should also be considered when determining the flow locations of the fluids.
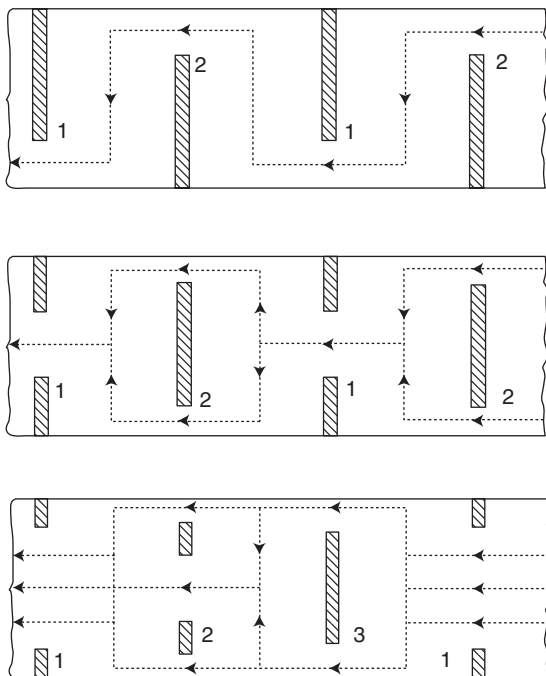
Figure 16.6 Baffle arrangements in shell and tube heat exchangers.[1]

ILLUSTRATIVE EXAMPLE 16.4

Discuss the effect of pressure drop in the design of shell and tube exchangers.

SOLUTION: Shell and tube heat exchangers are often designed to meet upper limit specifications on shell and tube side pressure drops. Many exchangers are designed for pressure drops less than 5 psi. For flow of low viscosity fluids (1–35 cP) through tubes, a good trial velocity is 6 fps. Equations are available in the literature[2,3] that estimate pressure drops for both the tube and shell side. ∎

In summary, shell and tube heat exchangers are among the most widely used exchangers due to the large surface area available for heat transfer. Countercurrent operation is the norm in industry but co-current operation is sometimes desired.

DESCRIBING EQUATIONS

When two process fluids at different temperatures pass one another as in a shell and tube exchanger, heat transfer occurs due to the temperature difference between the two streams. The energy required to accomplish this heat transfer, i.e., the heat

exchanger duty, can be determined for both process streams. For a well-insulated exchanger, the two duties should equal one another. The describing equations for the two duties are similar to those presented for the double pipe heat exchanger (see previous chapter). However, the following equation gives the heat exchanger duty of the hot process stream when saturated steam is introduced on the shell side (enthalpy balances between steam and condensate may also be used):

$$\dot{Q}_h = \dot{m}_s\lambda + \dot{m}_s c_{p,hw}(T_{s,i} - T_{s,o}) = \dot{m}_s\lambda + \dot{m}_s c_{p,hw}(T_1 - T_2) \tag{16.1}$$

where $\dot{Q}_h = \dot{Q}_H$ = rate of energy lost by steam, Btu/h

$\dot{m}_s$ = total mass flow rate of steam condensate, lb/h

$h_{vap} = \lambda$ = heat of vaporization, Btu/lb

$c_{p,hw}$ = average heat capacity of condensate (hot water), Btu/lb · °F

$T_{s,i} = T_1$ = steam inlet temperature, °F

$T_{s,o} = T_2$ = condensate outlet temperature, °F

The cold process stream duty is calculated similarly:

$$\dot{Q}_c = \dot{m}_c c_{p,cw}(t_{c,o} - t_{c,i}) = \dot{m}_c c_{p,cw}(t_2 - t_1) \tag{16.2}$$

where $\dot{Q}_c = \dot{Q}_C$ = rate of energy gained by cooling water, Btu/h

$\dot{m}_c$ = total mass flow rate of the cold water (*all* tubes), lb/h

$c_{p,cw}$ = average heat capacity of the cold process stream, Btu/lb · R

$t_{c,o} = t_2$ = cold water outlet temperature, °F

$t_{c,i} = t_1$ = cold water inlet temperature, °F

The reader should note that much of the material that follows was presented in the last chapter. However, as each chapter is written on a stand-alone basis, the applicable equations for shell and tube exchangers receive treatment.

If the heat exchanger duty is known, the overall heat transfer coefficient is calculated from either of the two equations provided below:

$$U_o = \frac{\dot{Q}_h}{A_o \Delta T_{lm}} \tag{16.3}$$

$$U_i = \frac{\dot{Q}_c}{A_i \Delta T_{lm}} \tag{16.4}$$

where U_o = overall heat transfer coefficient based on hot (h) process stream, Btu/ft² · h · °F

U_i = overall heat transfer coefficient based on cold (c) process stream, Btu/ft² · h · °F

A_o = exchanger transfer area based on outside tube surfaces, ft²

A_i = exchanger transfer area based on inside tube surface, ft^2

ΔT_{lm} = log-mean temperature difference, °F

Typical values for the *overall* heat transfer coefficient in steam condensers with water in the tubes range from $1000–6000 \, W/m^2 \cdot K$.[2] *Perry's Chemical Engineers' Handbook*[2] reports coefficients ranging from $400–1000 \, Btu/h \cdot ft^2 \cdot °R$.[3] These values are approximately the same after conversion of units.

For tubular heat exchangers, as with double pipe heat exchangers, the overall heat transfer coefficient is related to the individual coefficients by the following equation:

$$\frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} \tag{16.5}$$

$$= \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o} \tag{16.6}$$

where U = overall heat transfer coefficient, $Btu/h \cdot ft^2 \cdot °F$

A_i, A_o = inside and outside surface areas ($\pi D_i L$ or $\pi D_o L$), ft^2

A = average of the two areas, ft^2

U_i, U_o = coefficients based on inner and outer tube surfaces, $Btu/h \cdot ft^2 \cdot °F$

h_i, h_o = inside and outside heat transfer coefficients, $Btu/h \cdot ft^2 \cdot °F$

$R_{f,i}, R_{f,o}$ = fouling factors based on inner and outer tube surfaces

D_i, D_o = inside and outside pipe diameters, ft

k = thermal conductivity, $Btu/h \cdot ft \cdot °F$

L = tube length, ft

If the fouling factors, $R_{f,i}$ and $R_{f,o}$, and the tube wall resistance, i.e., the middle terms on the right-hand side of Equation (16.6) are negligible, then the relationship between the overall heat transfer coefficient and the individual coefficients simplifies once again to:

$$\frac{1}{U_o} = \frac{D_o}{h_i D_i} + \frac{1}{h_o} \tag{16.7}$$

The overall heat transfer coefficient, U, is an average value based on the average duty or $\dot{Q} = (\dot{Q}_h + \dot{Q}_c)/2$, and an average heat transfer area or $A = (A_i + A_o)/2$.

The individual coefficients above, h_i and h_o, are almost always calculated using empirical equations. First, the Reynolds number, Re, for the cold process fluid must be found in order to determine whether the flow is in the laminar, turbulent, or transition region. Refer to Table 15.1 for additional details. Except for the viscosity

term at the wall temperature, all of the physical properties of the tube-side fluid are evaluated at the average bulk temperature, $t_{c,\text{bulk}} = (t_{c,i} + t_{c,o})/2$.

$$\text{Re} = \frac{4\dot{m}_c}{\pi D_i \mu_c} \tag{16.8}$$

where $\dot{m}_c$ = mass flow rate of tube-side fluid through *one* tube, lb/h

 D_i = inside diameter of *one* tube, ft

 μ_c = viscosity of tube-side fluid at bulk temperature, lb/ft · h

 Re = Reynolds number

The Nusselt number, Nu, can be determined from empirical equations. For *laminar flow* in a circular tube (Re < 2100), the Nusselt number equals 4.36 for uniform surface heat flux (Q/A) or 3.66 for constant surface temperature.[2,4] This value should only be used for Graetz numbers, Gz, less than 10.[4] For laminar flow with Graetz numbers from 10 to 1000, the following equation applies:[4]

$$\text{Nu} = 2\,\text{Gz}^{1/3}\left(\frac{\mu_c}{\mu_{c,\text{wall}}}\right)^{0.14}; \qquad \text{Nu} = \frac{h_i D_i}{k_c}, \qquad \text{Gz} = \frac{\dot{m}_c c_{p,c}}{k_c L} \tag{16.9}$$

where $\mu_{c,\text{wall}}$ = viscosity of tube-side fluid at wall temperature, lb/ft · h

 k_c = thermal conductivity of tube-side fluid at bulk temperature, Btu/ft · h · °F

 $c_{p,c}$ = heat capacity of tube-side fluid at bulk temperature, Btu/lb · °F

 L = length of tubes, ft

 Nu = Nusselt number

 Gz = Graetz number

For *turbulent flow* (Re > 10,000), the Nusselt number may be calculated from the Dittus–Boelter equation if $0.7 \leq \text{Pr} \leq 160$ or the Sieder–Tate equation if $0.7 \leq \text{Pr} \leq 6700$, where Pr is the Prandtl number. Both equations are valid for L/D greater than 10.[2,4]

$$\text{Dittus–Boelter equation: Nu} = 0.023\,\text{Re}^{4/5}\,\text{Pr}^n \tag{16.10}$$

$$\text{Pr} = \frac{c_{p,c}\mu_c}{k_c}, \quad n = 0.4 \text{ for heating or } 0.3 \text{ for cooling}$$

$$\text{Sieder–Tate equation:} \quad \text{Nu} = 0.023\,\text{Re}^{4/5}\,\text{Pr}^{1/3}\left(\frac{\mu_c}{\mu_{c,\text{wall}}}\right)^{0.14} \tag{16.11}$$

The Dittus–Boelter equation should only be used for small to moderate temperature differences. The Sieder–Tate equation applies for larger temperature differences.[2-4] Errors as large as 25% are associated with both equations. Other

empirical equations with more complicated formulas and less error are available in the literature.[2-4]

The above empirical equations do not apply for flows in the *transition region* (between 2100 and 10,000). For the transition region, one should review the literature to determine the inside heat transfer coefficient. The reader is cautioned to use the correct mass flow rate; i.e., flow through a *single* tube rather than the total flow rate.

The outside heat transfer coefficient, h_o, is calculated using empirical correlations for condensation of a saturated vapor on a cold surface if steam is employed in the shell. Under normal conditions, a continuous flow of liquid is formed over the surface (film condensation) and condensate flows downward due to gravity. In most cases, the motion of the condensate is laminar and heat is transferred from the vapor–liquid interface to the surface by conduction through the film. Heat transfer coefficients for laminar film condensation on a single horizontal tube or on a vertical tier of N horizontal tubes can be calculated using appropriate equations from the literature[2-4] or by reviewing Chapter 12.

As noted earlier, the shell and tube heat exchanger usually consists of a series of vertical tiers of horizontal tubes. In a vertical tier of N horizontal tubes, the average heat transfer coefficient for the stack of tubes is less than for a single tube. The equation for the average heat transfer coefficient for steam condensing on a vertical stack of horizontal tubes (laminar film condensation) follows:

$$h_o = 0.725 \left[\frac{k_f^3 \rho_f^2 g \lambda}{N(T_s - T_w)D_o \mu_f} \right]^{1/4} \tag{16.12}$$

where k_f = thermal conductivity of condensate at reference temperature, $\text{Btu}/\text{ft} \cdot \text{h} \cdot \text{°F}$

ρ_f = density of condensate at reference temperature, lb/ft^3

N = average number of tubes in a vertical tier or stack

T_s = temperature of condensing vapor, °F

T_w = temperature of outside surface of wall, °F

D_o = outside tube diameter, ft

μ_f = viscosity of condensate at reference temperature, $\text{lb}/\text{ft} \cdot \text{h}$

T_f = reference temperature, °F

Figure 16.3 is a drawing of the header of the shell and tube heat exchanger consisting of 56 tubes, N_{total}. The average number of tubes in a vertical tier (stack) can be determined by drawing a series of vertical lines down through the diagram of the header, then counting the tubes in each column and averaging the results. This procedure results in 11 vertical tiers of horizontal tubes with $N \approx 5$ tubes per tier. The correct value or $N \approx 5$ must be used in Equation (16.12).

Equation (16.12) is limited to cases where the steam-side Reynolds number, Re_s, is less than 2100 (laminar flow of shell-side fluid).[2] Other correlations are provided in the literature for condensation heat transfer in the turbulent region.[2-4]

Re_s is calculated as follows:

$$Re_s = \frac{4\Gamma_b}{\mu_L}; \qquad \Gamma_b = \frac{\dot{m}_s/N_{total}}{L} \qquad (16.13)$$

where Γ_b = condensate loading per unit length of tube, lb/h · ft

N_{total} = total number of tubes, 56 tubes in the discussion above

L = length of tubes in Equation (16.12)

The physical properties are evaluated at a reference temperature, T_f. A reasonable value for the reference temperature in the range between $T_{c,i}$ and $T_{s,i}$ can be assumed, or an average of the four known temperatures, $(t_{c,i} + t_{c,o} + T_{s,i} + T_{s,o})/4$, can be used for a first estimate. Note that $t_{c,i}$ and $t_{c,o}$ represent the inlet and outlet cold liquid temperatures, respectively.

A trial-and-error calculation is required because the wall temperature and the reference temperature are both unknown and the wall temperature, T_w, depends on h_o and h_i. Details are provided in the previous chapter.

There are additional calculations for shell and tube exchangers. The following notation (consistent units) is employed in the shell and tube development to follow:[2]

A = heat transfer area

D_t = inside diameter of tube

L = tube length

$\dot{m}_{1,tube}$ = mass flow rate of tube-side fluid per tube

$\dot{m}_1$ = mass flow rate of tube-side fluid

N_{tpp} = number of tubes per pass on tube side

N_p = number of passes on the tube side

N_t = total number of tubes

U = overall heat transfer coefficient

V = average velocity of fluid in the tube

The mass flow rate per tube is given by

$$\dot{m}_{1,tube} = \frac{\rho V \pi D_t^2}{4} \qquad (16.14)$$

The number of tubes per pass is obtained from the total mass flow of the fluid on the tube side.

$$N_{tpp} = \frac{\dot{m}_1}{\dot{m}_{1,tube}} \qquad (16.15)$$

The total number of tubes and heat transfer area in the heat exchanger are:

$$N_t = N_{tpp}N_p \qquad (16.16)$$

with

$$A = N_t \pi D_t L \tag{16.17}$$

Illustrative examples involving the equations presented in this section are pro-vided in the two sections that follow.

THE "F" FACTOR

Some heat exchangers use true countercurrent flow. However, these heat exchangers are not as economical as multipass and crossflow units. In multipass exchangers, the flow alternates between co-current and countercurrent between the different sections of the exchanger. As a result, the driving force is not the same as a true countercurrent or a true co-current exchanger. The aforementioned ΔT_{lm} (log mean temperature difference driving force or LMTD) for these exchangers is almost always less than that of countercurrent flow and greater than that of co-current flow.

The analysis of these units requires including an F factor in the describing equation (i.e., the heat transfer equation). For those multipass and crossflow exchan-gers, the log mean temperature difference (LMTD) method is applicable, i.e.,

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)} \tag{16.18}$$

or

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\dfrac{\Delta T_2}{\Delta T_1}\right)} \tag{16.19}$$

For the special case of

$$\Delta T_1 = \Delta T_2 = \Delta T \tag{16.20}$$

One may employ

$$\Delta T_{lm} = \Delta T \tag{16.21}$$

The heat transfer rate is still given by

$$\dot{Q} = UA\Delta T_{lm} \tag{16.22}$$

except that ΔT_{lm} must be corrected by a geometry factor, F. This correction factor accounts for portions of multipass heat exchangers where the flow is not countercur-rent (e.g., hairpin turns). Equation (16.22) is now written as

$$\dot{Q} = UAF\Delta T_{lm,cp} \qquad \Delta T_{lm} = F\Delta T_{lm,cp} \tag{16.23}$$

where $\Delta T_{lm,cp}$ is the LMTD for ΔTs based on "ideal" countercurrent operation and F is the geometry factor, or correction factor, applied to a different flow arrangement with the same hot and cold fluid temperature. Bowman et al.[5] coordinated the results of
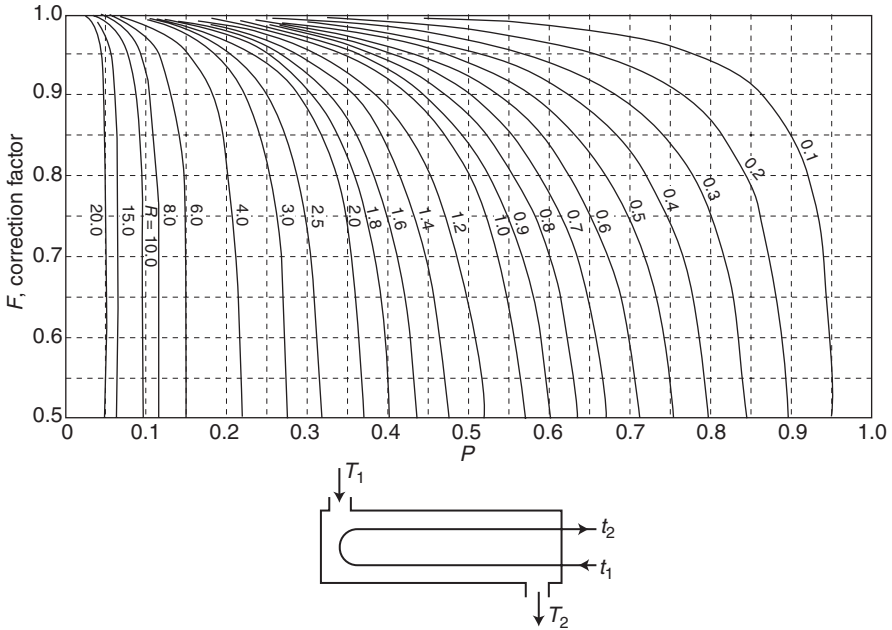
Figure 16.7   One shell pass; 2 or more tube passes.

earlier studies in his day regarding the ΔT_{lm} for exchangers that experience neither co-current or countercurrent flow to provide as complete a picture as possible of the various arrangements of surface conditions and flow directions. Shell and tube heat exchangers that experience any number of passes on the shell and tube side were covered in his work as well as crossflow exchangers with different pass arrangements. The results for the various classifications follow.

1. *One Pass Shell Side; Two Passes Tube Side*:
    The correction factor, F, for multipass heat exchangers having one shell pass and two tube-side passes is shown in Figure 16.7 for the $1-2$ pass exchanger versus P and R, where P and R are both dimensionless ratios given by:

$$P = \frac{t_2 - t_1}{T_1 - t_1} \tag{16.24}$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} \tag{16.25}$$

This graphical data is based on the integrated equations obtained by Underwood.[6] The integrated equation for ΔT_{lm} is:

$$\Delta T_{lm} = \frac{\sqrt{(T_1 - T_2)^2 + (t_2 - t_1)^2}}{\log\left\{ \dfrac{T_1 + T_2 - t_1 - t_2 + \sqrt{(T_1 - T_2)^2 + (t_2 - t_1)^2}}{T_1 + T_2 - t_1 - t_2 - \sqrt{(T_1 - T_2)^2 + (t_2 - t_1)^2}} \right\}} \tag{16.26}$$

The above equation was transposed into the following equation, in which the
F factor for a 1–2 exchanger is expressed in terms of the aforementioned
dimensionless ratios of P and R:

$$F_{1,2} = \frac{\sqrt{R^2+1}}{R-1}\log\left(\frac{1-P}{1-PR}\right) \Big/ \log\left[\frac{(2/P)-1-R+\sqrt{R^2+1}}{(2/P)-1-R-\sqrt{R^2+1}}\right] \quad (16.27)$$

Note that the term in the expression above

$$\frac{1}{R-1}\log\left(\frac{1-P}{1-PR}\right) \quad (16.28)$$

becomes indeterminate when $R = 1$. However, the usual treatment for such
indeterminates reduces Equation (16.28) to:

$$\frac{P}{2.3(1-P)} \quad (16.29)$$

2. *One Pass Shell Side; Four Passes Tube Side*
    The correction factor for 1–4 heat exchangers is slightly less than that for
1–2 exchangers. The correction factor is given by:

$$F_{1,4} = \frac{\sqrt{4R^2+1}}{2(R-1)}\log\left(\frac{1-P}{1-PR}\right) \Big/ \log\left[\frac{1+V(\sqrt{4R^2+1}-2R)}{1-V(\sqrt{4R^2+1}+2R)}\right] \quad (16.30)$$

where

$$V = \frac{t_2 - t_1}{4T_1 - (t_1 + 2t_i + t_2)} \quad (16.31)$$

and t_i is the intermediate temperature of the fluid where it leaves the second
tube-side pass and enters the third tube-side pass. The value of t_i may be
obtained using a trial-and-error solution of the following equation:

$$\left(\frac{t_i - t_1}{t_2 - t_i}\right)^{\sqrt{4R^2+1}} = \frac{1+V(\sqrt{4R^2+1}-2R)}{1-V(\sqrt{4R^2+1}+2R)} \quad (16.32)$$

3. *One Pass Shell Side; Six Passes Tube Side*
    Nagle[7] found that the correction factors for 1–6 exchangers agree rather
closely with those of 1–2 exchangers operating with the same terminal temp-
eratures. Therefore, the curves of the 1–2 exchanger graph may be used to
determine the correction factor for 1–6 exchangers in this specific case.
Therefore, from this discussion above, one can conclude that the correction
factor, F, for heat exchangers with one shell-side pass and two, four, six
and infinite tube-side passes are approximately the same.
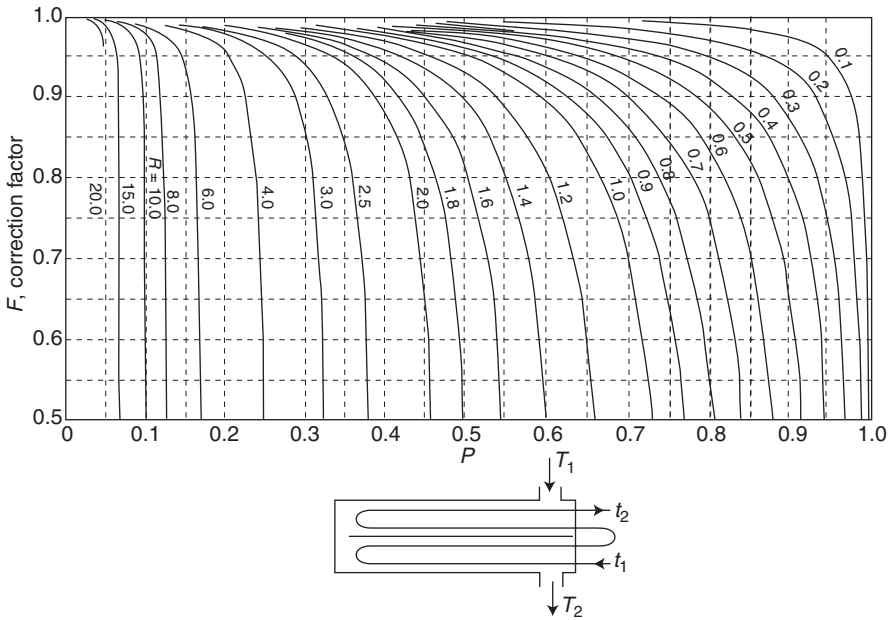
Figure 16.8   Two shell passes; four or more tube passes.

4. *Two Passes Shell Side; Four Passes Tube Side*

The correction factor for 2–4 exchangers designed to approach countercurrent flow is given in Figure 16.8. Underwood's algebraic equation for 2–4 heat exchangers is given by

$$F_{2,4} = \frac{\sqrt{R^2 + 1}}{2(R - 1)} \log\left(\frac{1 - P}{1 - PR}\right) \Bigg/$$

$$\log\left[\frac{(2/P) - 1 - R + (2/P)\sqrt{(1 - P)(1 - PR)} + \sqrt{R^2 + 1}}{(2/P) - 1 - R + (2/P)\sqrt{(1 - P)(1 - PR)} - \sqrt{R^2 + 1}}\right]$$

(16.33)

5. *Three and More Shell-Side Passes*

A general method was developed by Bowman et al.[5,8] to calculate the correction factor, F, for the 3–6, 4–8, and 6–12, etc., exchangers. This method uses the correction factors from the 1–2 exchangers to help estimate the correction factor for three or more shell-side passes. At any given values of F and R, the value of P for an exchanger having N shell-side and $2N$ tube-side passes is related to P for a 1–2 exchanger by the equation:

$$P_{N,2N} = \frac{1 - \left(\frac{1 - P_{1,2}R}{1 - P_{1,2}}\right)^N}{R - \left(\frac{1 - P_{1,2}R}{1 - P_{1,2}}\right)^N}$$

(16.34)

For the special case of $R = 1$, Equation (16.34) becomes indeterminate and reduces to:

$$P_{N,2N} = \frac{P_{1,2}N}{P_{1,2}N - P_{1,2} + 1} \tag{16.35}$$

Graphical results for four and six shell passes are provided in Figures 16.9 and 16.10, respectively.

6. *Fluids Mixed*

Graphical results for single pass, crossflow exchangers with both fluids unmixed (compartmentalized), and with one fluid mixed and the other unmixed are provided in Figures 16.11 and 16.12, respectively.

From the above analysis, the correction factor approaches unity as the number of shell-side passes is increased for any value of P and R. This is to be expected since a multipass exchanger with several shell-side passes approaches the ideal countercurrent heat exchanger more closely than one shell-side and two or more tube passes. In addition, one notes that

$$F_{1-2} \approx F_{1-4} \approx F_{1-6} \tag{16.36}$$

$$F_{1-3} \approx F_{1-6} \approx F_{1-9} \tag{16.37}$$

$$F_{2-4} \approx F_{2-8} \approx F_{2-12} \tag{16.38}$$

$$F_{3-6} \approx F_{3-12} \approx F_{3-18} \tag{16.39}$$
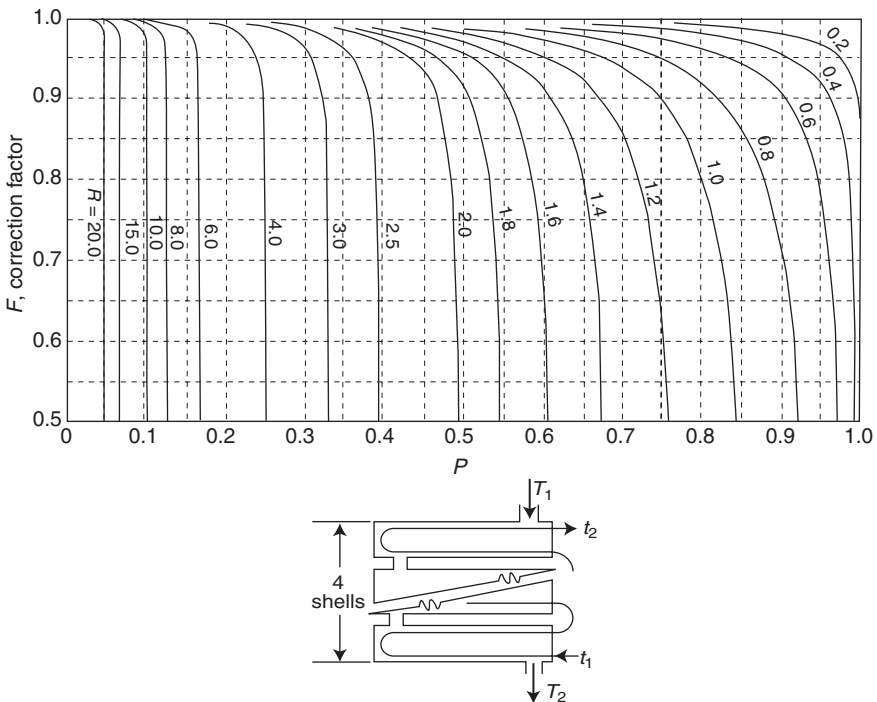
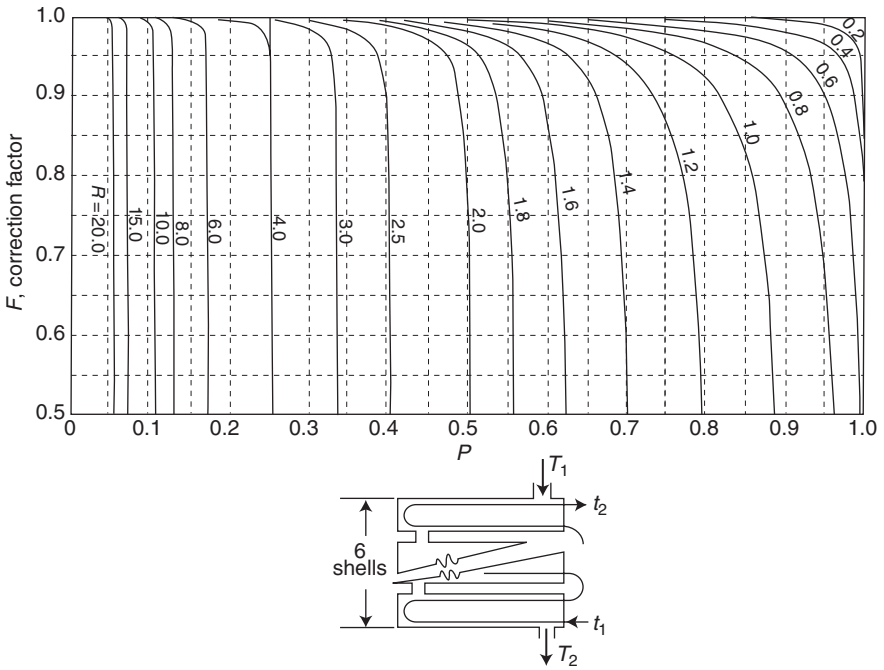**Figure 16.9**   Four shell passes; eight or more tube passes.

Figure 16.10   Six shell passes; 12 or more tube passes.
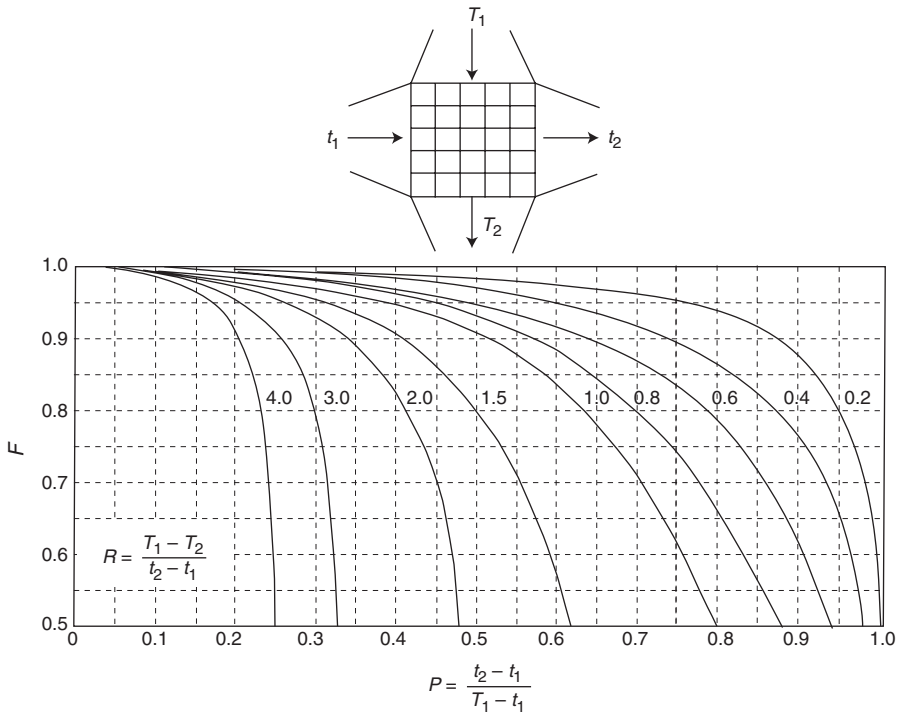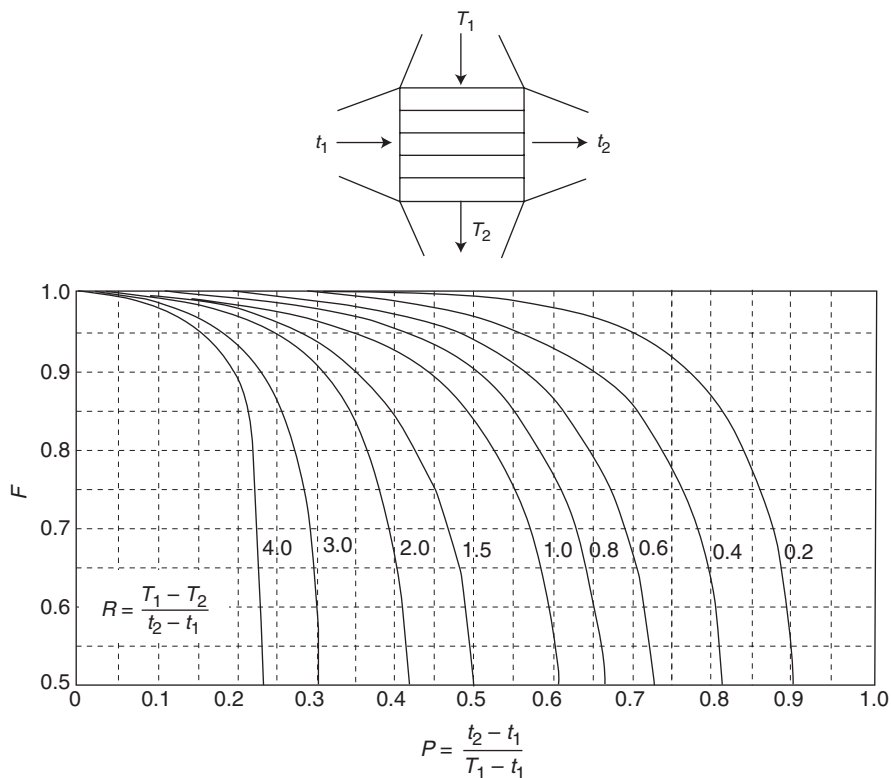

Figure 16.11   Correction factor for a single pass, cross-flow heat exchanger with both fluids unmixed.

Figure 16.12   Correction factor for a single pass, crossflow heat exchanger with one fluid mixed and the other unmixed (t_1, t_2).

ILLUSTRATIVE EXAMPLE 16.5

Consider the shell and tube heat exchanger in Figure 16.13. Calculate the log mean temperature driving force. Also perform the calculation for the exchanger described in Figure 16.14.

SOLUTION: The system is first treated as an ideal countercurrent system as shown in Figure 16.13. For this ideal countercurrent system,

$$\text{LMTD} = \Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} \tag{16.18}$$

$$= \frac{(150 - 80) - (100 - 50)}{\ln(70/50)} = 59.4°\text{F}$$

Figure 16.13   Ideal countercurrent heat exchanger; Illustrative Example 16.5.
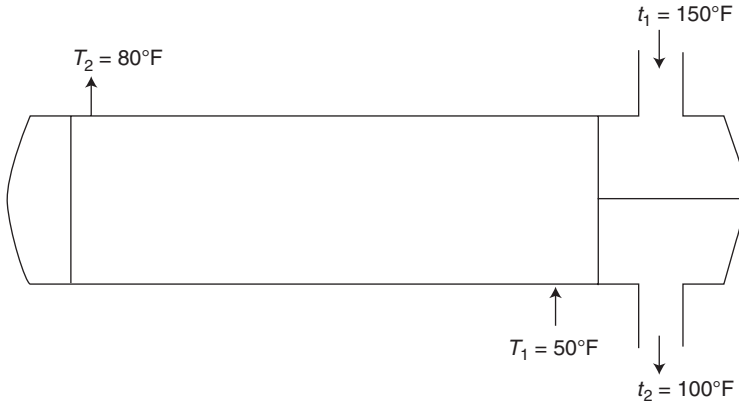
Figure 16.14   Shell and tube exchanger; Illustrative Example 16.5.

For the system in Figure 16.14,

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{80 - 50}{150 - 50} = 0.30$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{150 - 100}{80 - 50} = 1.67$$

From Figure 16.7, $F = 0.925$. Therefore,

$$\Delta T_{\text{lm}} = F\Delta T_{\text{lm},cp} = (0.925)(59.4) = 54.9°\text{F}$$

This somewhat lower value represents the LMTD for the actual (real) system shown in Figure 16.14.   ∎

ILLUSTRATIVE EXAMPLE 16.6

A shell and tube heat exchanger having two shell passes and four tube passes is being used for cooling. The shell-side fluid enters at 400°F and leaves at 250°F and the tube-side fluid enters at 100°F and leaves at 175°F. What is the log mean temperature difference between the hot fluid and the cold fluid?

SOLUTION: The log mean temperature difference (driving force) based ideal upon ideal (or true) countercurrent operation is first evaluated:

$$\text{LMTD} = \Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln\dfrac{\Delta T_1}{\Delta T_2}} \tag{16.18}$$

$$= \frac{(400 - 250) - (175 - 100)}{\ln\left(\dfrac{400 - 250}{175 - 100}\right)} = \frac{150 - 75}{\ln\dfrac{150}{75}} = 108°\text{F}$$

The *F* factor can be obtained from Figure 16.8,

$$P = \frac{75}{350} = 0.214$$

$$R = \frac{150}{75} = 2.0$$

From Figure 16.8,

$$F = 0.985$$

Thus,

$$\Delta T_{\text{lm}} = F\Delta T_{\text{lm,cp}} = (0.985)(108)$$
$$= 106.4°\text{F}$$

∎

ILLUSTRATIVE EXAMPLE 16.7

Recalculate the results of the two previous examples using the equation upon which the graphs are based.

SOLUTION: For Illustrative Example 16.5,

$$P = 0.30$$
$$R = 1.67$$

Applying Equation (16.27) gives

$$F = 0.92$$

Therefore,

$$\Delta T_{\text{lm}} = F\Delta T_{\text{lm,cp}} \tag{16.23}$$
$$= (0.92)(59.4)$$
$$= 54.65°\text{F}$$

For Illustrative Example (16.6),

$$P = 0.214$$
$$R = 2.0$$

Applying Equation (16.33) leads to

$$F \simeq 0.985$$

so that

$$\Delta T_{\text{lm}} = (0.985)(108)$$
$$= 106.4°\text{F}$$

The results are in excellent agreement with the two previous examples. ∎

ILLUSTRATIVE EXAMPLE 16.8

A shell and tube heat exchanger has one pass on the shell side and two passes on the tube side (i.e., a 1–2 shell and tube heat exchanger). It is being used for oil cooling. The oil flows in the tube side. It enters at 110°C and leaves at 75°C. The shell-side fluid is water at a flowrate of 1.133 kg/s, entering at 35°C and leaving at 75°C. The heat capacity of the water is 4180 J/kg · K. The overall heat-transfer coefficient for the heat exchanger is 350 W/m² · K. The geometry factor F has been previously estimated to be 0.965. Calculate the heat-transfer area requirement for this unit in square meters.[9]

SOLUTION: The heat load is

$$\dot{Q} = \dot{m}c_p(T_{\text{out}} - T_{\text{in}}) = \dot{m}c_p(t_2 - t_1)$$
$$= (1.133)(4180)(75 - 35)$$
$$= 189{,}400 \text{ W}$$

The countercurrent log-mean temperature difference is

$$\Delta T_1 = 110 - 75 = 35°\text{C}$$
$$\Delta T_2 = 75 - 35 = 40°\text{C}$$
$$\Delta T_{\text{lm,cp}} = \frac{35 - 40}{\ln(35/40)}$$
$$= 37.4°\text{C}$$
$$= 310.6 \text{ K}$$

The corrected log-mean temperature difference is calculated employing the correction factor, F:

$$\Delta T_{\text{lm}} = F\Delta T_{\text{lm,cp}}$$
$$= (0.965)(310.6)$$
$$= 299.7 \text{ K}$$

The required heat-transfer area is then

$$A = \frac{\dot{Q}}{U\Delta T_{\text{lm}}}$$

$$= \frac{189,400}{(350)(299.7)}$$

$$= 1.806 \text{ m}^2$$ ∎

ILLUSTRATIVE EXAMPLE 16.9

Discuss the problem that can arise in employing the "F" factor.

SOLUTION: The LMTD method is adequate when the terminal temperatures are known. If the heat-transfer area is given and the exit temperatures are unknown, the problem often requires solution by trial and error. This is discussed in the next section. ∎

ILLUSTRATIVE EXAMPLE 16.10

A shell and tube heat exchanger is to be designed for heating 10,000 kg/h of water from 16°C to 84°C by a new high heat capacity hot engine oil flowing through the shell. The oil makes a single shell pass, entering at 160°C and leaving at 94°C. The water flows through 11 brass tubes of 22.9 mm inside diameter and 25.4 mm outside diameter with each tube making four passes through the shell. The overall heat transfer coefficient (including the fouling resistance effect) is approximately 350 W/m² · °C and the thermal conductivity of brass is 137 W/m · °C. Calculate

1. the heat load, in MW,
2. the countercurrent flow log mean temperature difference, and
3. the F correction factor and the corrected log mean temperature difference.

SOLUTION: Calculate the average bulk temperatures of the water and oil:

$$T_{w,\text{av}} = (16 + 84)/2 = 50°C$$
$$T_{\text{oil,av}} = (160 + 94)/2 = 127°C$$

Obtain the properties of both fluids and organize this information in tabular form (see Table 16.1):

for water: $\rho = 987 \text{ kg/m}^3$, $c_p = 4176 \text{ J/kg} \cdot °C$

for oil: $\rho = 822 \text{ kg/m}^3$, $c_p = 4820 \text{ J/kg} \cdot °C$

Table 16.1 Data/Information for Illustrative Example 16.10

	Stream			
Property	Inlet	Outlet	Inlet	Outlet
Fluid	Water (being heated)		Hot oil	
Side/passes	Tube (four passes)		Shell (one pass)	
$t, T, °C$	16	84	160	94
Average $t, T, °C$	50		127	
ρ, kg/m^3	987		822	
c_p, J/kg · °C	4176		4820	
$\dot{m}$, kg/s	2.778		2.480	
Capacitance rate, $C = \dot{m}c_p$, W/°C	11,600		11,950	

Calculate the heat load, $\dot{Q}$, from the water-side information:

$$\dot{m}_1 = \dot{m}_w = 10{,}000/3600 = 2.778 \, \text{kg/s}$$

$$\dot{Q} = \dot{m}_1 c_{p,w}(t_2 - t_1) = (2.778)(4176)(84 - 16) = 788{,}800 \, \text{W} = 0.788 \, \text{MW}$$

Calculate the oil flow rate, $\dot{m}_3$:

$$\dot{Q} = 788{,}800 = (c_{p,\text{oil}})(\dot{m}_{\text{oil}})(T_1 - T_2) = (4820)(\dot{m}_{\text{oil}})(160 - 94)$$
$$\dot{m}_2 = \dot{m}_{\text{oil}} = 2.480 \, \text{kg/s}$$

Calculate the log mean temperature difference based on the countercurrent flow, $\Delta T_{\text{lm},cp}$:

$$\Delta T_1 = 94 - 16 = 78°C$$
$$\Delta T_2 = 160 - 84 = 76°C$$
$$\Delta T_{\text{lm},cp} = (78 - 76)/\ln(78/76) = 77°C$$

Calculate P and R:

$$t_1 = 16°C, \quad t_2 = 84°C, \quad T_1 = 160°C, \quad T_2 = 94°C$$
$$P = (84 - 16)/(160 - 16) = 0.472$$
$$R = (160 - 94)/(84 - 16) = 0.971$$

Using Figure 16.7, for a 1–4 shell and tube heat exchanger, $F \simeq 0.965$. Therefore,

$$\Delta T_{\text{lm}} = F\Delta T_{\text{lm},cp}$$
$$= (0.965)(77)$$
$$= 74.3°C \qquad\qquad ■$$

ILLUSTRATIVE EXAMPLE 16.11

Refer to the previous example. Calculate the area and the length of the tubes required for this heat exchanger.

SOLUTION: The heat transfer area may now be calculated:

$$\dot{Q} = UA\Delta T_{lm} \tag{16.22}$$

$$A = \dot{Q}/(U\Delta T_{lm}) = 788{,}800/[(350)(74.3)] = 30.33 \text{ m}^2$$

Calculate the tube length, L:

N_t = total number of tubes

= (number of tubes per pass, N_{tpp})(number of passes, N_p)

$$= N_{tpp} N_P \tag{16.16}$$

= (11)(4)

= 44 tubes

Therefore,

$$A = \pi D_i L N_t$$

$$L = (30.34)/[(\pi)(0.0229)(44)] = 9.58 \text{ m} = 31.3 \text{ ft} \qquad \blacksquare$$

ILLUSTRATIVE EXAMPLE 16.12

Steam enters the shell-side of a shell and tube heat exchanger at 450°F and exits at a temperature of 300°F. An organic material that has fluid properties similar to water enters the tube-side of the same heat exchanger at 200°F and is heated by the steam to an exit temperature of 300°F. Draw the diagram of each of the following specifications and/or heat exchangers:

(a) Countercurrent flow
(b) Co-current flow
(c) One shell-side pass and two tube-side passes
(d) One shell-side pass and four tube side passes
(e) Two shell-side passes and four tube-side passes
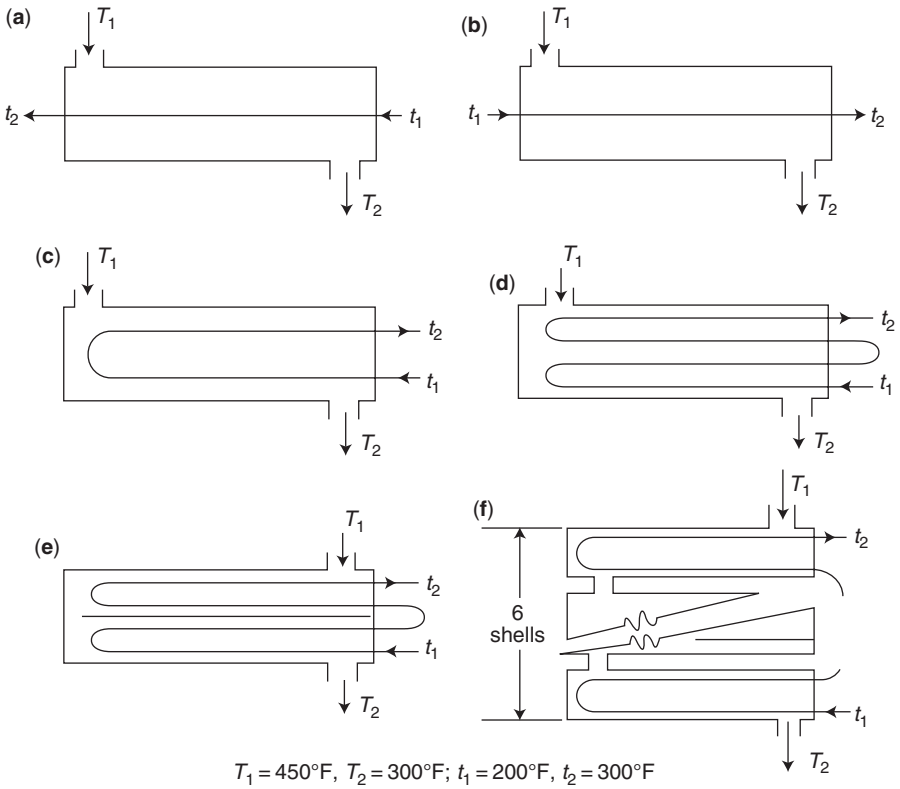**(f)** Six shell-side passes and twelve tube-side passes

SOLUTION: The appropriate labeling of one of the heat exchanger examples is as follows:

$$T_1 = 450°F \qquad T_2 = 300°F$$

and

$$t_1 = 200°F \qquad t_2 = 300°F$$

The diagrams of each specification and/or heat exchanger are shown in Figure 16.15a–f.   ∎

$T_1 = 450°F, \ T_2 = 300°F; \ t_1 = 200°F, \ t_2 = 300°F$

Figure 16.15   Diagrams of shell tube passes; Illustrative Example 16.12.

ILLUSTRATIVE EXAMPLE 16.13

Refer to the previous example. Calculate the correction factor, F, and the log mean temperature, ΔT_{lm}, for each specification employing the figures provided earlier.

SOLUTION: The F and ΔT_{lm} values obtained using the figures for the given exchanger specifications are:

		F	ΔT_{lm}
a	Countercurrent Flow	1.000	123.3
b	Co-current Flow	0	0
c	1–2 Multipass	0.810	99.9
d	1–4 Multipass	0.805	99.3
e	2–4 Multipass	0.960	118.4
f	6–12 Multipass	1.000	123.3

■

ILLUSTRATIVE EXAMPLE 16.14

Use the "Bowman" equations to solve the previous example.

SOLUTION: The F and ΔT_{lm} values generated using the equations for the given exchanger specifications are:

		F	ΔT_{lm}
a	Countercurrent Flow	1.000	123.3
b	Co-current Flow	0	0
c	1–2 Multipass	0.803	99.1
d	1–4 Multipass	0.799	98.5
e	2–4 Multipass	0.957	118.1
f	6–12 Multipass	1.000	123.3

■

ILLUSTRATIVE EXAMPLE 16.15

Comment on the results of the two previous examples.

SOLUTION: The agreement between the two approaches is excellent.   ■

ILLUSTRATIVE EXAMPLE 16.16

Provide some sample calculations for Illustrative Example 16.14.[10]

SOLUTION: These sample calculations show the different steps to calculating the correction factor, F, and log mean temperature, ΔT_{lm}, for the different specifications and/or heat exchangers.

The first log mean temperature, ΔT_{lm}, is calculated assuming ideal countercurrent flow. This log mean temperature, ΔT_{lm}, calculated will be used throughout the example with the correction factor, F, for each of the different heat exchangers examined.

$$
\begin{aligned}
\Delta T_{lm} &= \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\dfrac{T_1 - t_2}{T_2 - t_1}\right)} \\[2mm]
&= \frac{(450°F - 300°F) - (300°F - 200°F)}{\ln\left(\dfrac{450°F - 300°F}{300°F - 200°F}\right)} \\[2mm]
&= \frac{(150°F) - (100°F)}{\ln\left(\dfrac{150°F}{100°F}\right)} \\[2mm]
&= 123.3°F
\end{aligned}
$$

For ideal countercurrent flow, the correction factor F is assumed to be equal to one. Therefore, the log mean temperature of countercurrent flow is 123.3°F.

The calculation of the correction factor, F, for a 2–4 multipass exchanger follows. First calculate P and R.

$$P = \frac{100}{250} = 0.4$$

$$R = \frac{150}{100} = 1.5$$

In addition,

$$F_{2,4} = \frac{\sqrt{R^2 + 1}}{2(R-1)} \log\left(\frac{1-P}{1-PR}\right) \Bigg/$$

$$\log\left[\frac{(2/P) - 1 - R + (2/P)\sqrt{(1-P)(1-PR)} + \sqrt{R^2+1}}{(2/P) - 1 - R + (2/P)\sqrt{(1-P)(1-PR)} - \sqrt{R^2+1}}\right] \qquad (16.33)$$

Substituting,

$$F_{2,4} = \frac{\sqrt{1.5^2 + 1}}{2(1.5-1)} \log\left(\frac{1-0.4}{1-0.4 \times 1.5}\right)$$

$$\Bigg/ \log\left[\frac{(2/0.4) - 1 - 1.5 + (2/0.4)\sqrt{(1-0.4)(1-0.4 \times 1.5)} + \sqrt{1.5^2+1}}{(2/0.4) - 1 - 1.5 + (2/0.4)\sqrt{(1-0.4)(1-0.4 \times 1.5)} - \sqrt{1.5^2+1}}\right]$$

$$= 0.957$$

For this case,

$$\Delta T_{lm} = F\Delta T_{lm,cp} \qquad (16.23)$$

Substituting,

$$\Delta T_{lm,corrected} = (0.957)(123.3)$$
$$= 118.1°F$$

The calculation of the correction factor for a 1–4 multipass exchanger requires an iterative calculation of the intermediate temperature, t_i. In order to determine this value, the dimensionless constant, V, is calculated and used in the steps to follow. The goal seek function in Microsoft Excel can be used to perform the final calculation of the intermediate temperature.

Calculation of the dimensionless constant, V:

$$V = \frac{t_2 - t_1}{4T_1 - (t_1 + 2t_i + t_2)} \qquad (16.31)$$

Substituting,

$$V = \frac{300 - 200°F}{4 \times 450°F - (200°F + 2t_i + 300°F)}$$

Below is the (iteration) equation for the intermediate temperature, t_i:

$$\left(\frac{t_i - t_1}{t_2 - t_i}\right)^{\sqrt{4R^2+1}} = \frac{1 + V\left(\sqrt{4R^2 + 1} - 2R\right)}{1 - V\left(\sqrt{4R^2 + 1} + 2R\right)} \tag{16.32}$$

Substituting,

$$\left(\frac{t_i - 200°F}{300 - 200°F}\right)^{\sqrt{4R^2+1}} = \frac{1 + V\left(\sqrt{4 \times 1.5^2 + 1} - 2 \times 1.5\right)}{1 - V\left(\sqrt{4 \times 1.5^2 + 1} + 2 \times 1.5\right)}$$

$$0 = \frac{1 + \left(\dfrac{300 - 200°F}{4 \times 450°F - (200°F + 2t_i + 300°F)}\right)\left(\sqrt{4 \times 1.5^2 + 1} - 2 \times 1.5\right)}{1 - \left(\dfrac{300 - 200°F}{4 \times 450°F - (200°F + 2t_i + 300°F)}\right)\left(\sqrt{4 \times 1.5^2 + 1} + 2 \times 1.5\right)}$$
$$- \left(\frac{t_i - 200°F}{300 - 200°F}\right)^{\sqrt{4R^2+1}}$$

Solving,

$$t_i = 262.4°F$$

for which

$$V = 0.129$$

The calculation for the correction factor F follows:

$$F_{1,4} = \frac{\sqrt{4R^2 + 1}}{2(R - 1)} \log\left(\frac{1 - P}{1 - PR}\right) \Big/ \log\left[\frac{1 + V\left(\sqrt{4R^2 + 1} - 2R\right)}{1 - V\left(\sqrt{4R^2 + 1} + 2R\right)}\right] \tag{16.30}$$

Substituting,

$$F_{1,4} = \frac{\sqrt{4 \times 1.5^2 + 1}}{2(1.5 - 1)} \log\left(\frac{1 - 0.4}{1 - 0.4 \times 1.5}\right) \Big/ \log\left[\frac{1 + 0.129\left(\sqrt{4 \times 1.5^2 + 1} - 2 \times 1.5\right)}{1 - 0.129\left(\sqrt{4 \times 1.5^2 + 1} + 2 \times 1.5\right)}\right]$$

$$= 0.799$$

The corrected driving force may now be calculated

$$\Delta T_{lm} = F \Delta T_{lm,cp} \tag{16.23}$$

Substituting,

$$\Delta T_{lm} = (0.799)(123.3°F)$$
$$= 98.5°F \qquad \blacksquare$$

EFFECTIVENESS FACTOR AND NUMBER OF TRANSFER UNITS

The effectiveness factor number of transfer units (or ε-NTU) method described in the previous chapter for double pipe exchangers is again preferable to use when only inlet

temperatures to the shell and tube heat exchanger are known. The following notation (different from the previous chapter since the colder fluid is normally on the tube side) will be used for this shell and tube exchanger development:

$\dot{m}_1$ = mass flow rate of fluid entering or leaving the tube side

$\dot{m}_2$ = mass flow rate of fluid entering or leaving the shell side

T_1, T_2 = inlet and exit temperatures of the shell side fluid, respectively

t_1, t_2 = inlet and exit temperatures of the tube side fluid, respectively

$\dot{Q}$ = heat duty, heat load, or heat transfer rate

$\dot{Q}_{max}$ = maximum possible heat transfer rate

c_1, c_2 = heat capacity of fluids on the tube and shell sides, respectively

C_1, C_2 = capacitance rate of fluids on the tube and shell sides, respectively

As noted earlier, the heat duty of a shell and tube exchanger is calculated from the energy balance on the tube side or on the shell side:

$$\dot{Q} = \dot{m}_1 c_1 (T_1 - T_2) = \dot{m}_2 c_2 (T_1 - T_2) = C_1(t_2 - t_1) = C_2(T_1 - T_2) \qquad (16.40)$$

This assumes that the fluid on the tube side is being heated ($t_2 > t_1$) and the shell side fluid is being cooled ($T_1 > T_2$). To determine $\dot{Q}_{max}$, the maximum heat transfer rates for both the tube, $\dot{Q}_{max,t}$, and shell side fluids, $\dot{Q}_{max,s}$, are first determined and the lower of the two values is used. For the tube side, $\dot{Q}_{max,t}$ occurs when the exit fluid temperature, t_2, is the same as the temperature of the incoming fluid on the shell side (i.e., $t_2 = T_1$), so that

$$\dot{Q}_{max,t} = C_1(T_1 - t_1) \qquad (16.41)$$

Similarly, for the shell side fluid, $\dot{Q}_{max,s}$ occurs when $T_2 = t_1$:

$$\dot{Q}_{max,s} = C_2(T_1 - t_1) \qquad (16.42)$$

The lower of the two capacitance rates, C_1 and C_2, determines C_{min} and the permissible maximum heat load, $\dot{Q}_{max}$. If the capacitance rate $C_1 < C_2$, then $C_{min} = C_1$ and $C_{max} = C_2$. In this case,

$$\dot{Q}_{max} = C_1(T_1 - t_1) = C_{min}(T_1 - t_1) \qquad (16.43)$$

Using the earlier definition of the effectiveness, ε, and combining Equations (16.40) and (16.43) yields

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{t_2 - t_1}{T_1 - t_1} \qquad (16.44)$$

If the shell side fluid has the lower heat capacity rate (i.e., $C_2 < C_1$), then $C_{min} = C_2$, $C_{max} = C_1$, and

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{T_1 - T_2}{T_1 - t_1} \qquad (16.45)$$

For any heat exchanger, it can be shown that

$$\varepsilon = f\left(NTU, \frac{C_{min}}{C_{max}}\right); \qquad NTU = \frac{UA}{C_{min}} \qquad (16.46)$$

Several graphs in which ε is plotted against the NTU, with the C_{min}/C_{max} ratio C_r as the curve parameter, are presented below. Six graphs are available for six different heat exchanger geometries.[2] These are:

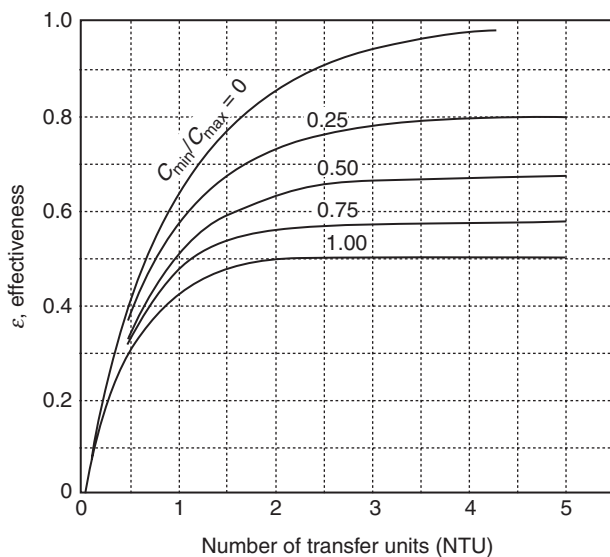Figure 16.16: for a parallel flow heat exchanger.

Figure 16.17: for a counterflow heat exchanger.

Figure 16.18: for a shell and tube heat exchanger with one shell pass and any even multiple of two tube passes (two, four, etc.).

Figure 16.19: for a shell and tube heat exchanger with two shell passes and any even multiple of four tube passes (four, eight, etc.).
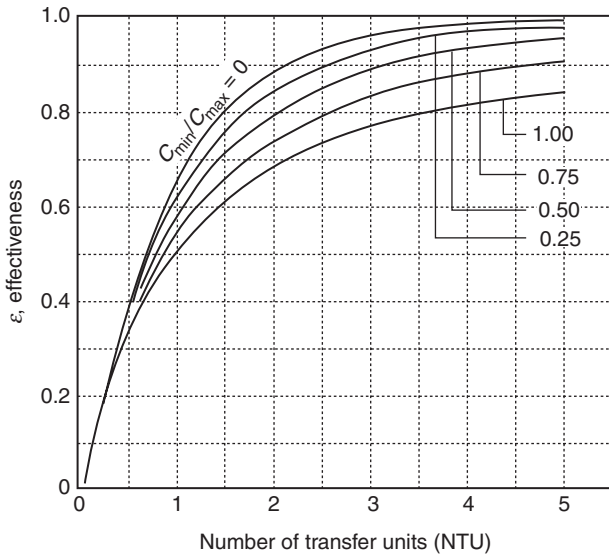
Figure 16.20: for a single pass, crossflow heat exchanger with both fluids unmixed.

Figure 16.21: for a single pass, crossflow heat exchanger with one fluid mixed and the other unmixed.
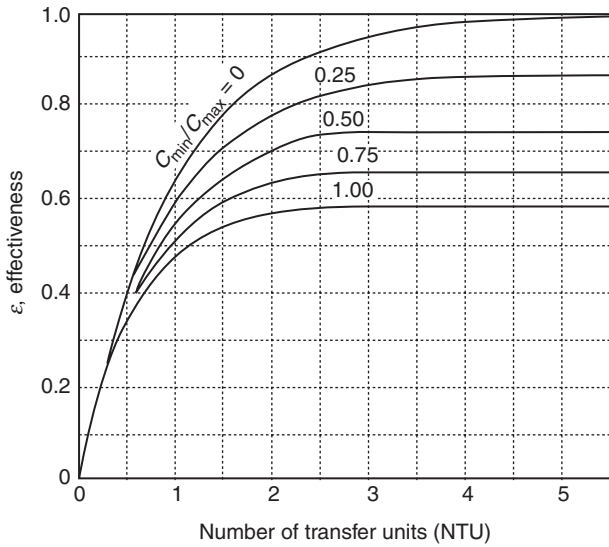

Figure 16.16   Effectiveness of transfer units for a parallel-flow heat exchanger.
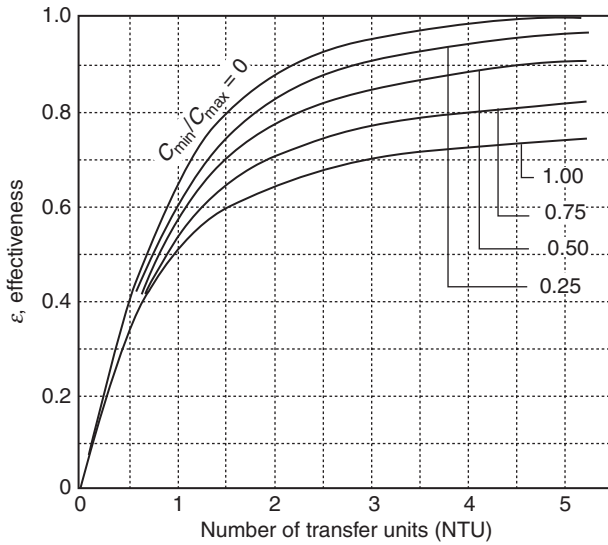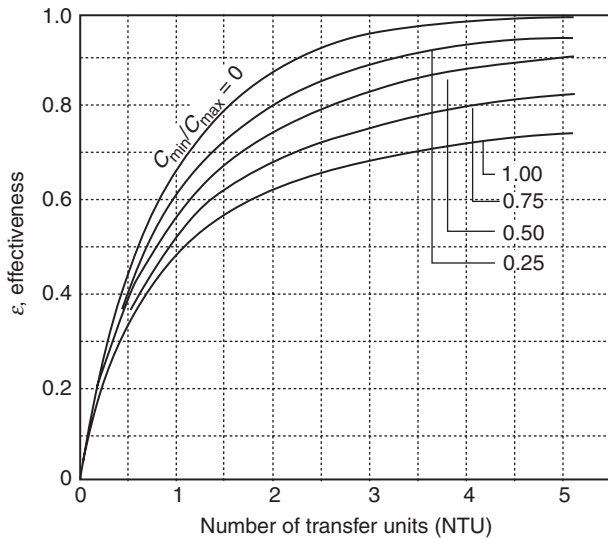
Figure 16.17   Effectiveness of transfer units for a counterflow heat exchanger.
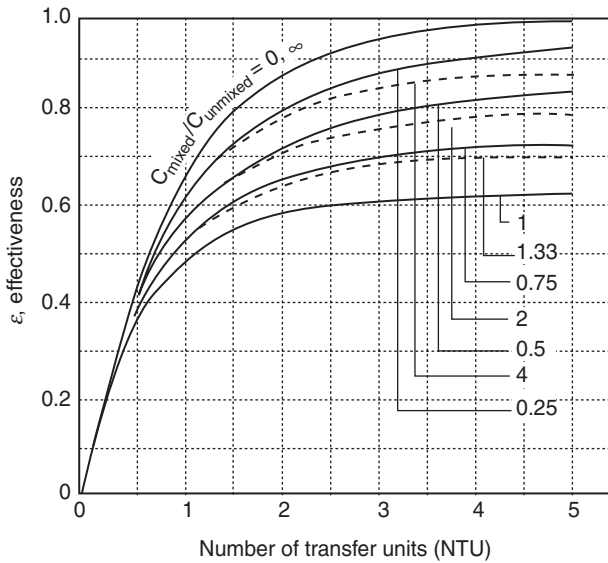

Figure 16.18   Effectiveness of transfer units for a shell and tube heat exchanger with one shell pass and any even multiple of two tube passes.

Figure 16.19 Effectiveness of transfer units for a shell and tube heat exchanger with two shell passes and any even multiple of four tube passes.

Figure 16.20 Effectiveness of transfer units for a single pass, crossflow heat exchanger with both fluids unmixed.

Figure 16.21   Effectiveness of transfer units for a single pass, crossflow heat exchanger with one fluid mixed and the other unmixed. The solid curves correspond to C_{min} mixed and C_{max} unmixed while the dashed curves correspond to C_{min} unmixed and C_{max} mixed.

The figures show that when the capacitance rate ratio $C_{min}/C_{max} = C_r = 0$, all heat exchangers have the same effectiveness. Moreover, if NTU < 0.25, all heat exchangers have the same effectiveness regardless of the value of C_r. More generally, for $C_r > 0$ and NTU > 0.25, the counterflow exchanger is the most effective.

ILLUSTRATIVE EXAMPLE 16.17

Describe the relationship between the maximum and minimum values of the effectiveness and C_r.

SOLUTION: Examining Figures (16.16–16.20) indicates that the maximum and minimum value of the effectiveness are associated with $C_r = 0$ and $C_r = 1$, respectively.

Griskey[11,12] has also provided effectiveness relationships in equation form. These are provided in Table 16.2. Barkwill[13] has also provided these relationships in equation form as can be seen in Table 16.3. However, the equations of Griskey, if applicable, are recommended.   ■

ILLUSTRATIVE EXAMPLE 16.18

Refer to Illustrative Example 16.10. Determine the effectiveness and number of transfer units.

Table 16.2   Heat Exchanger Effectiveness Relations[11,12]

System	Effectiveness relation
Double pipe	
Parallel flow (Figure 16.16)	$\varepsilon = \dfrac{1 - \exp[-N(1 + C)]}{1 + C}$
Countercurrent flow (Figure 16.17)	$\varepsilon = \dfrac{1 - \exp[-N(1 + C)]}{1 - C\exp[-N(1 - C)]}$
Cross-flow	
Both fluids unmixed (Figure 16.20)	$\varepsilon = 1 - \exp\{C/n[\exp(-NCn) - 1]\}$ where $n = N^{-0.22}$
C_{max} mixed, C_{min} unmixed (Figure 16.21)	$\varepsilon = (1/C)\{1 - \exp[C(1 - e^{-N})]\}$
C_{max} unmixed, C_{min} mixed (Figure 16.21)	$\varepsilon = 1 - \exp\{(1/C)[1 - \exp(-NC)]\}$
Shell and tube	
One shell pass; 2, 4, 6 tube passes (Figure 16.18)	$\varepsilon = 2\{1 + C + (1 + C^2)^{1/2}\,\dfrac{\left[1 + \exp[-N(1 + C^2)^{1/2}]\right]}{\left[1 - \exp[-N(1 + C^2)^{1/2}]\right]}\}^{-1}$

where N is NTU or $UA/(\dot{m}c_p)_{min}$ and $C = C_r = (\dot{m}c_p)_{min}/(\dot{m}c_p)_{max}$

Table 16.3   Regression Equations: Heat Exchanger Effectiveness Relations[13]

System	Effectiveness relation
Parallel flow (Figure 16.16)	$y = [-\exp(-x) + 1] - \left[(0.625)(z)\left(\dfrac{0.85 \times x^2}{x^2 + 1.5}\right)\right]$
Counterflow (Figure 16.17)	$y = [-\exp(-x) + 1] - \left[(0.22)(z)\left(\dfrac{x^2}{x^2 + 3.5}\right)\right]$
One shell pass + multiple of 2 tube passes (Figure 16.18)	$y = [-\exp(-x) + 1] - \left[(0.40)(z)\left(\dfrac{x^2}{x^2 + 1}\right)\right]$
Two shell pass + multiple of 4 tube passes (Figure 16.19)	$y = [-\exp(-x) + 1] - \left[(0.29)(z)\left(\dfrac{x^2}{x^2 + 2}\right)\right]$
Single pass, no mixed fluids (Figure 16.20)	$y = [-\exp(-x) + 1] - \left[(0.29)(z)\left(\dfrac{x^2}{x^2 + 3}\right)\right]$
Single pass mixed fluids, (Figure 16.21)	$y = [-\exp(-x) + 1] - \left[(0.40)(z)\left(\dfrac{x^2}{x^2 + 2}\right)\right]$

Note: y = effectiveness number, x = NTU, $z = C_{min}/C_{max}$.

SOLUTION: $C_1 = C_w =$ capacitance rate of water $= \dot{m}_1 c_{p,w} = (2.778)(4176) = 11{,}600 \text{ W}/^\circ\text{C}$
and $C_2 = C_{\text{oil}} = \dot{m}_2 c_{p,\text{oil}}(2.480)(4820) = 11{,}950 \text{ W}/^\circ\text{C}$.
   To determine the effectiveness, ε, note that tube-side water has the minimum capacitance rate. Thus,

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\text{water,max}}} = \frac{C_w(84-16)}{C_w(160-16)} = \frac{84-16}{160-16}$$

$$= 0.472$$

   The number of transfer units, NTU is

$$\text{NTU} = \frac{UA}{C_{\min}} = \frac{(350)(30.33)}{11{,}600}$$

$$= 0.915$$

   Finally, calculate the capacitance rate ratio:

$$\frac{C_{\min}}{C_{\max}} = \frac{C_w}{C_{\text{oil}}} = \frac{11{,}600}{11{,}950} = 0.971$$ ∎

ILLUSTRATIVE EXAMPLE 16.19

The feedwater heater for a boiler supplies 10,000 kg/h of water at 65°C. The raw feedwater enters the tubes with an inlet temperature of 20°C and is to be heated in a single-shell, two-tube pass heat exchanger by condensing steam at 1.30 bars. Water is on the tube side and steam is on the shell side. The overall heat transfer coefficient is 2000 W/m² · K. Determine the required heat transfer area for the exchanger. Use both the LMTD method and the ε-NTU approach to obtain the results.

SOLUTION: Organize the information as presented in Table 16.4.

Table 16.4 Data and Information for Illustrative Example 16.19

	Water (1)		Steam (2)	
Properties	Inlet	Outlet	Inlet	Outlet
Side		Tube		Shell
P, bars		N.A.		1.3
t, T, °C	20	65	107.3	107.3
$\dot{m}$, kg/s		2.778		
c_p, J/kg · K		4179		Latent heat of condensation, $h_{\text{vap}} = 2.238 \times 10^6$ J/kg
Capacitance rate, C, W/°C		11,608.3		∞
Capacitance rate, C, classification		$C_{\min}$		$C_{\max}$

Calculate the heat load, $\dot{Q}$:

$$\dot{Q} = C_w(t_2 - t_1) = 11{,}608.3(65 - 20) = 5.224 \times 10^5 \text{ W}$$

Calculate the steam condensation rate:

$$\dot{Q} = \dot{m}_2 h_{vap}$$
$$\dot{m}_2 = (5.224 \times 10^5)/(2.238 \times 10^6) = 0.2334 \text{ kg/s}$$

Calculate the LMTD ($\Delta T_{lm,cp}$):

$$\Delta T_1 = 107.3 - 20 = 87.3°C = 87.3 \text{ K}$$
$$\Delta T_2 = 107.3 - 65 = 42.3°C = 42.3 \text{ K}$$
$$\Delta T_{lm,cp} = (87.3 - 42.3)/\ln(87.3/42.3) = 62.1°C = 62.1 \text{ K}$$

Calculate the correction factor, F. Since the shell side has a constant temperature, $F = 1$:

$$\Delta T_{lm} = F\Delta T_{lm,cp} = 62.1°C$$

Calculate the heat transfer area, A (i.e., by the LMTD method):

$$\dot{Q} = UA\Delta T_{lm}$$
$$A = (5.224 \times 10^5)/[(2000)(62.1)] = 4.21 \text{ m}^2 = 45.32 \text{ ft}^2$$

Calculate C_{min}/C_{max} and ε using the effectiveness-NTU (number of transfer units) method:

$$C_{min}/C_{max} = 0$$

Since water has the lower capacitance rate, the effectiveness is based on the temperature change of water:

$$\varepsilon = \frac{\text{actual temperature change of water}}{\text{maximum temperature change of water}} = \frac{65 - 20}{107.3 - 20} = 0.515$$

From the ε vs. NTU figure for a shell and tube heat exchanger with one pass on the shell side and two passes on the tube side (Figure 16.18), one estimates

$$\text{NTU} \simeq 0.7$$

Calculate the heat transfer area, A by the ε-NTU method:

$$\text{NTU} = UA/C_{min} \tag{16.46}$$
$$A = \frac{(\text{NTU})/(C_{min})}{U} = \frac{(0.7)(11{,}608.3)}{2000} = 4.1 \text{ m}^2 \qquad\blacksquare$$

ILLUSTRATIVE EXAMPLE 16.20

Compare the results obtained using the two approaches in the previous example.

SOLUTION: The NTU result is very close to the area result obtained from the LMTD method. This was to be expected. $\blacksquare$

ILLUSTRATIVE EXAMPLE 16.21

A shell and tube heat exchanger has one pass on the shell side and two passes on the tube side (i.e., 1–2 shell and tube heat exchanger). It is being used for oil cooling. The oil flows in the tube side. It enters at 110°C and leaves at 75°C. The shell-side fluid is water at a flow rate of 1.133 kg/s, entering at 35°C and leaving at 75°C. The oil flows in the tubes at an average velocity of 0.3 m/s. The tube inside diameter is $\frac{3}{4}$ inch or 0.01905 m. The overall heat transfer coefficient for a clean heat exchanger is 350 W/m² · K. The fouling factor, R_f, is 0.00027 m² · K/W. The oil properties are: $\rho_{oil} = 850$ kg/m³ and $c_{oil} = 1900$ J/kg · K. Calculate:

1. the mass flow rate of the oil
2. the minimum and maximum heat capacity rate
3. the heat load, $\dot{Q}$
4. the total number of tubes
5. the tube length.

SOLUTION: Organize the information in the problem statement as shown in Table 16.5. Calculate the water capacitance rate:

$$C_w = \dot{m}_w c_w;\ C_w = C_{p,w}$$
$$= (1.133)(4180)$$
$$= 4736\ \text{W/K}$$

Calculate the heat load:

$$\dot{Q} = \dot{m}_w c_w (t_2 - t_1) = C_w (t_2 - t_1)$$
$$= (4736)(75 - 35)$$
$$= 189{,}440\ \text{W}$$

Table 16.5   Data and Information for Illustrative Example 16.21

Parameter	Stream			
	Inlet	Outlet	Inlet	Outlet
Fluid		Oil		Water
Side		Tube		Shell
ρ, kg/m³		850		995
C_p, J/kg · K		1900		4180
$\dot{m}$, kg/s		*		1.133
Kinematic viscosity, ν, m²/s	0.0008		N/A	
Capacitance rate, $C = \dot{m}$, W/K		*		4736
T, t, °C	110	75	35	75
V, m/s		0.3		N/A

*To be determined later.

Calculate the oil capacitance rate and mass flow rate:

$$\dot{Q} = \dot{m}_{oil}c_{oil}(T_1 - T_2); \qquad c_{oil} = C_{p,oil}$$
$$= C_{oil}(T_1 - T_2)$$

$$C_{oil} = \dot{Q}/(T_1 - T_2)$$
$$= 189{,}440/(110 - 75)$$
$$= 5412.6 \text{ W/K}$$

$$\dot{m}_{oil} = 5412.6/1900$$
$$= 2.85 \text{ kg/s}$$

Determine the minimum and maximum capacitance rate:

$$C_{min} = C_w = 4736 \text{ W/K}$$
$$C_{max} = C_{oil} = 5412.6 \text{ W/K}$$

Calculate the oil flow rate per tube:

$$\dot{m}_{oil,tube} = \rho V(\pi/4)D^2$$
$$= (850)(0.3)(\pi/4)(0.01905)^2$$
$$= 0.0727 \text{ kg/s}$$

Determine the number of tubes:

$$N_{tpp} = \dot{m}_{oil,total}/\dot{m}_{oil,tube} \qquad\qquad (16.15)$$
$$= 2.85/0.0727$$
$$= 39.2$$
$$= 40 \text{ tubes per pass}$$
$$N_t = N_{tpp}(\text{number of passess, } N_p) \qquad\qquad (16.16)$$
$$= (40)(2)$$
$$= 80 \text{ tubes}$$

For countercurrent flow in a double-pipe heat exchanger, calculate the log mean temperature difference:

$$\Delta T_1 = 110 - 75 = 35°C$$
$$\Delta T_2 = 75 - 35 = 40°C$$

$$\Delta T_{lm} = \frac{35 - 40}{\ln(35/40)} = 37.44°C = 37.44 \text{ K}$$

Determine the parameters P and R needed to obtain the correction factor, F, from the appropriate F chart or equation:

$$P = (t_2 - t_1)/(T_1 - t_1) \qquad (16.24)$$
$$= (75 - 35)/(110 - 35)$$
$$= 0.533$$
$$R = (T_1 - T_2)/(t_2 - t_1) \qquad (16.25)$$
$$= (110 - 75)/(75 - 35)$$
$$= 0.875$$

From Figure 16.7, $F = 0.81$. The corrected log mean temperature difference is therefore

$$\Delta T_{lm} = F\Delta T_{lm,cp}$$
$$= (0.81)(37.44)$$
$$= 30.33°C$$

The "dirty" overall heat transfer coefficient is

$$\frac{1}{U_{dirty}} = \frac{1}{U_{clean}} + R_f$$
$$= \frac{1}{350} + 0.00027$$
$$U_{dirty} = 320 \text{ W/m}^2 \cdot \text{K}$$

Calculate the required heat transfer area:

$$A = \dot{Q}/[U_{dirty}\Delta T_{lm}]$$
$$= 189{,}440/[(320)(30.33)]$$
$$= 19.5 \text{ m}^2$$

Finally, calculate the tube length:

$$A = N_t \pi DL$$
$$L = A/(N_t \pi D)$$
$$= 19.5/[(80)(\pi)(0.01905)]$$
$$= 4.1 \text{ m}$$

∎

ILLUSTRATIVE EXAMPLE 16.22

Determine the effectiveness and NTU for the previous example.

SOLUTION: For the effectiveness of the exchanger, the actual water temperature change $=$ $75 - 35 = 40°F$. The maximum water temperature change $= 110 - 35 = 75°F$. Therefore

$$\varepsilon = \frac{40}{75} = 0.533$$

Calculate the NTU using earlier results:

$$NTU = UA/C_{min}; \; C_{min} = C_w \qquad (16.46)$$
$$= (320)(19.5)/4736$$
$$= 1.318$$

■

REFERENCES

1. E. SAUNDERS, *Heat Exchangers: Selection Design and Construction*, Longman Scientific and Technical, p. 46. John Wiley & Sons, Hoboken, NJ, 1988.
2. F. INCROPERA and D. DE WITT, *Fundamentals of Heat and Mass Transfer*, 4th edition, Chapters 8, 10, 11. John Wiley & Sons, Hoboken, NJ, 1996.
3. D. GREEN and R. PERRY (editors), *Perry's Chemical Engineers' Handbook*, 8th edition, McGraw-Hill, New York City, NY, 2008.
4. W. MCCABE, J. SMITH, and P. HARRIOTT, *Unit Operations of Chemical Engineering*, 5th edition, McGraw-Hill, New York City, NY, 1993.
5. R. BOWMAN, A. MUELLER, and W. NAGLE, *Trans. ASME*, 62, 283, New York City, NY, 1940.
6. A. UNDERWOOD, *J. Inst. Petroleum Technol.*, 20, 145–158, 1934.
7. W. NAGLE, *Ind. Eng. Chem.*, 25, 604–608, New York City, NY, 1933.
8. R. BOWMAN, A. MUELLER, and W. NAGLE, *Trans. ASME*, New York City, NY, 62, 283–294, 1940.
9. J. REYNOLDS, J. JERIS, and L. THEODORE, *Handbook of Chemical and Environmental Engineering Calculations*, John Wiley & Sons, Hoboken, NJ, 2004.
10. B. BARKWILL, M. SPINELLI, and K. VALENTINE, *The F Factor*, term project submitted to L. Theodore, 2009.
11. R. GRISKEY, *Transport Phenomena and Unit Operations*, John Wiley & Sons, Hoboken, NJ, 2002.
12. R. GRISKEY, *Chemical Engineering for Chemists*, American Chemical Society, Washington, DC, 1997.
13. B. BARKWILL, *The Effectiveness Number*, term project submitted to L. Theodore, 2009.