

Chapter 15

Double Pipe Heat Exchangers

INTRODUCTION

Of the various types of heat exchangers that are employed in industry, perhaps the two most fundamental are the *double pipe* and the *shell and tube*. Despite the fact that shell and tube heat exchangers (see next chapter) generally provide greater surface area for heat transfer with a more compact design, greater ease of cleaning, and less possibility of leakage, the double pipe heat exchanger still finds use in practice.

The double pipe unit consists of two concentric pipes. Each of the two fluids—hot and cold—flow either through the inside of the inner pipe or through the annulus formed between the outside of the inner pipe and the inside of the outer pipe. Generally, it is more economical (from a heat efficiency perspective) for the hot fluid to flow through the inner pipe and the cold fluid through the annulus, thereby reducing heat losses to the surroundings. In order to ensure sufficient contacting time, pipes longer than approximately 20 ft are extended by connecting them to *return bends*. The length of pipe is generally kept to a maximum of 20 ft because the weight of the piping may cause the pipe(s) to sag. Sagging may allow the inner pipe to touch the outer pipe, distorting the annulus flow region and disturbing proper operation. When two pipes are connected in a “U” configuration by a return bend, the bend is referred to as a *hairpin*. In some instances, several hairpins may be connected in series.

Double pipe heat exchangers have been used in the chemical process industry for over 85 years. The first patent on this unit appeared in 1923. The unique (at that time) design provided “the fluid to be heated or cooled and flow longitudinally and transversely around a tube containing the cooling or heating liquid, etc.” The original design has not changed significantly since that time.

Although this unit is not extensively employed in industry (the heat transfer area is small relative to other heat exchangers), it serves as an excellent starting point from an academic and/or training perspective in the treatment of all the various heat exchangers reviewed in this Part.

Topics covered in this chapter include:

Equipment Description

Describing Equations

Effectiveness Factor and Number of Transfer Units

Wilson's Method

EQUIPMENT DESCRIPTION

As discussed in the Introduction, the simplest kind of heat exchanger, which confines both the hot and cold fluids, consists of two concentric tubes, or pipes. When conditions are such that only a few tubes per pass are required, the simplest construction is the double pipe heat interchanger shown in Figure 15.1. This consists of special fittings that are attached to standard iron (typically) pipe so that one fluid (usually liquid) flows through the *inside* pipe and the second fluid (also usually a liquid) flows through the *annular* space between the two pipes. Such a heat interchanger will usually consist of a number of passes which are almost invariably arranged in a vertical stack. If more than one pipe per pass is required, the proper number of such stacks is connected in parallel. Although they do not provide a large surface area for heat transfer, double pipe heat exchangers are at times used in industrial settings. A schematic of this unit and flow classification(s) is provided in Figure 15.2.

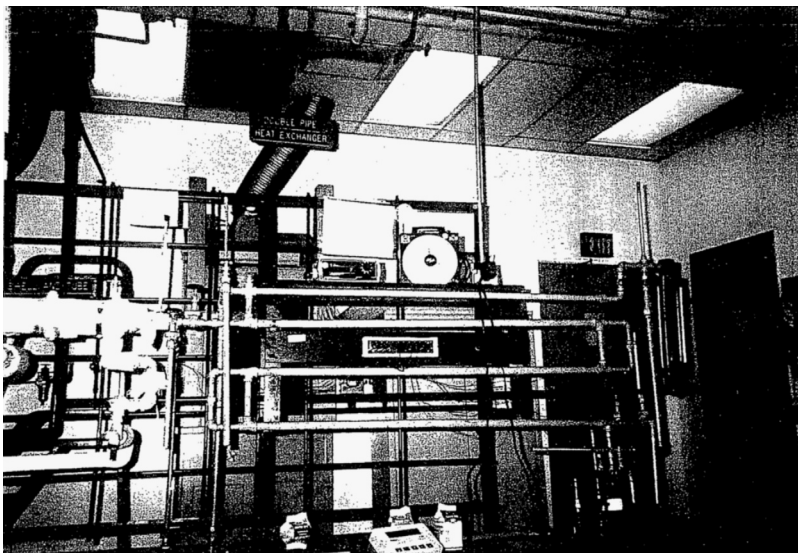


Figure 15.1 Double pipe heat exchanger (Manhattan College Unit Operations Laboratory).

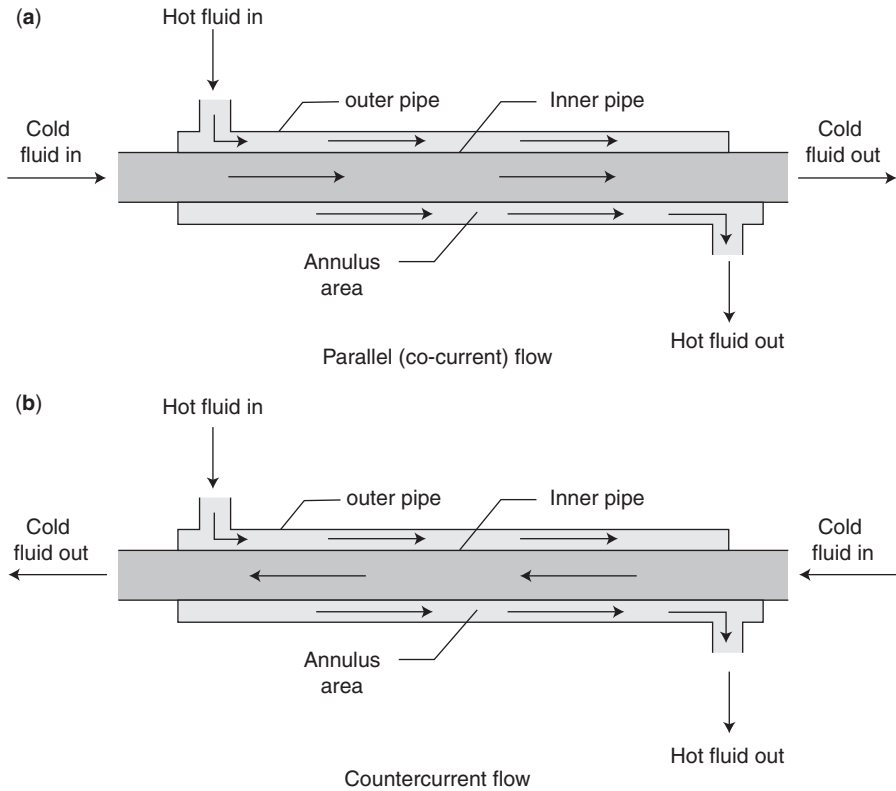


Figure 15.2 Double pipe schematic.

Recommended standard fittings for double pipe interchangers are provided in the literature.⁽¹⁾ Additional details are provided in Chapter 6 of Part One. Other types of tubes or pipes available include:

1. Plain tubes
2. Duplex tubes
3. Finned tubes

Plain tubes are used when the corrosion potential on either the tube or shell side (see next chapter) are approximately the same. Duplex tubes have walls of two different metals and are used when it is impossible to find a single metal that can adequately handle corrosion on both sides of the unit. Finned tubes (to be discussed in Chapter 17) are usually employed for heat transfer to gases where film coefficients are very low. Tube materials include: carbon steel, carbon alloy steels, stainless steels, brass and alloys, cupro-nickel, nickel, monel, glass, reinforced fiberglass plastic (RFP), etc.

As noted in Chapter 6, calculations for flow in the annular region referred to above require the use of a characteristic or *equivalent diameter*. By definition, this diameter,

D_{eq} , is given by 4 times the area available for flow divided by the “wetted” perimeter. Note that there are two wetted perimeters in an annular area. This reduces to

$$D_{\text{eq}} = 4\left(\frac{\pi}{4}\right)(D_{o,i}^2 - D_{i,o}^2)/\pi(D_{o,i} + D_{i,o}) \quad (15.1)$$

where $D_{o,i}$ is the inside diameter of the outer pipe and $D_{i,o}$ is the outside diameter of the inner pipe. This equation reduces to

$$D_{\text{eq}} = D_{o,i} - D_{i,o} \quad (15.2)$$

which is four times the hydraulic radius, i.e.,

$$D_{\text{eq}} = 4r_H \quad (15.3)$$

with

$$r_H = \frac{1}{4}(D_{o,i} - D_{i,o}) \quad (15.4)$$

The flow in a double pipe heat exchanger may be countercurrent or parallel (co-current). In countercurrent flow, the fluid in the pipe flows in a direction opposite to the fluid in the annulus. In parallel flow, the two fluids flow in the same direction. The variations of fluid temperature within the heat exchanger depend on whether the flow is parallel or countercurrent (see Figure 15.2).

The definitions below are employed in the development to follow (see also Figure 15.2):

T_1 = temperature of the hot fluid entering the inside pipe/tube

T_2 = temperature of the hot fluid exiting the inside pipe/tube

t_1 = temperature of the cold fluid entering the annulus

t_2 = temperature of the cold fluid exiting the annulus

The difference between the temperature of the tube side fluid and that of the annulus side is the *temperature difference driving force* (TDDF), ΔT .

As noted in the previous chapter, in a co-current flow heat exchanger, both hot and cold fluids enter on the same side and flow through the exchanger in the same direction. The *temperature approach* is defined as the temperature difference driving force at the heat exchanger entrance, ΔT_1 or $(T_1 - t_1)$. This driving force drops as the streams approach the exit of the exchanger. At the exit, the temperature difference driving force is ΔT_2 or $(T_2 - t_2)$. Thus, the heat exchanger is more effective at the entrance than at the exit. In a countercurrent flow exchanger, the two fluids exchange heat while flowing in opposite directions. The temperature approach at the tube entrance end ΔT_1 or $(T_1 - t_2)$ and at the annular entrance end, ΔT_2 or $(T_2 - t_1)$ is usually roughly the same. In addition, the thermal driving force is normally relatively constant over the length of the exchanger. The temperature profiles for both parallel and countercurrent systems are presented in Figure 15.3.

Summarizing, the double pipe heat exchanger employed in practice consists of two pipes: an inner and an outer pipe. Hot fluid normally flows in the inner pipe and the cold fluid flows in the annulus between the outer diameter of the inner pipe

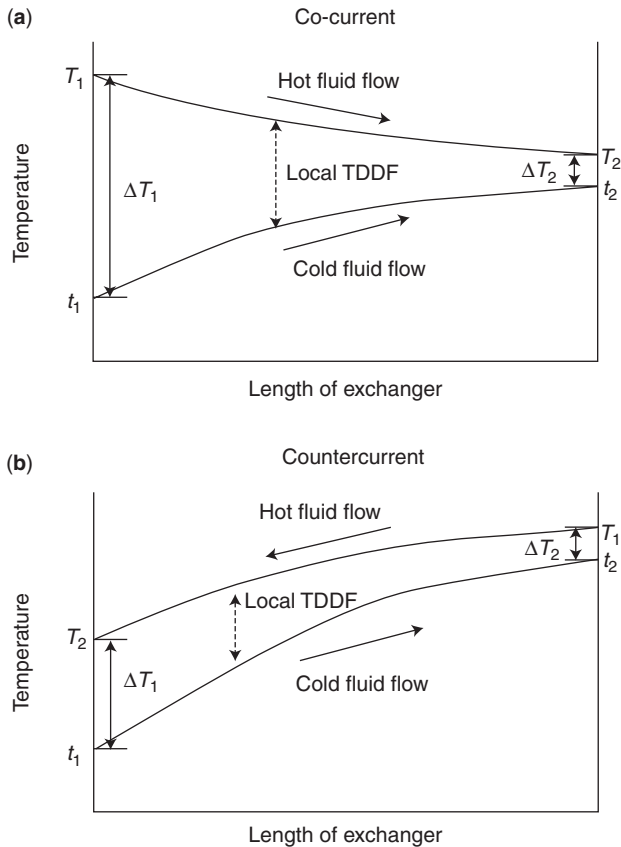


Figure 15.3 Co-current and countercurrent flow in a heat exchanger.

and inner diameter of the outer pipe. Heat transfer occurs from the inner pipe to the outer pipe as the fluids flow through the piping system. By recording both inner and outer fluid temperatures at various points along the length of the exchanger, it is possible to calculate heat exchanger duties and heat transfer coefficients (to be discussed shortly).

ILLUSTRATIVE EXAMPLE 15.1

Explain why the cold fluid flows in the annular region in nearly all double pipe heat exchanger applications.

SOLUTION: As discussed above, heat losses to the environment are lower (except in cryogenic applications) with the hot fluid flowing in the inside pipe. Any losses are thus transferred to the cold fluid and not the surroundings. ■

DESCRIBING EQUATIONS

In designing a double pipe heat exchanger, mass balances, heat balances, and the applicable heat transfer equation(s) are used. The steady-state heat balance equation is

$$\begin{aligned}\dot{Q} &= \Delta \dot{h}_H = \Delta \dot{h}_C \\ &= \dot{m}_H c_{pH}(T_1 - T_2) = \dot{m}_C c_{pC}(t_2 - t_1)\end{aligned}\quad (15.5)$$

This equation assumes steady state, no heat loss, no viscous dissipation, and no heat generation.⁽¹⁾ The rate equation used to design an exchanger is the design equation, which includes the previously developed log mean temperature difference (ΔT_{lm} or LMTD) and overall heat transfer coefficient, U

$$\dot{Q} = UA\Delta T_{\text{lm}} \quad (15.6)$$

where \dot{Q} is the heat load, U is the overall heat transfer coefficient, A is the heat transfer area, and ΔT_{lm} is the log mean (or global) temperature difference driving force. If the temperature difference driving forces are ΔT_1 and ΔT_2 at the entrance and exit of the heat exchanger, respectively, then

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} \quad (15.7)$$

If $\Delta T_1 = \Delta T_2$, then $\Delta T_{\text{lm}} = \Delta T_1 = \Delta T_2$. The overall heat transfer coefficient, U , is usually based on the inside area of the tube; the A term in Equation (15.6) should then be based on the inside surface area. Typical values of U for new, clean exchangers are available in the literature and, as noted earlier, are often assigned the symbol, U_{clean} . See also Equation (14.32).

If the heat duties are known (or have been calculated), values for the overall heat transfer coefficient, U , can be calculated as follows:

$$U_o = \frac{\dot{Q}_C}{A_o \Delta T_{\text{lm}}} \quad (15.8)$$

$$U_i = \frac{\dot{Q}_H}{A_i \Delta T_{\text{lm}}} \quad (15.9)$$

where A = surface area for heat transfer (outside or inside), ft²
 \dot{Q} = average heat duty (cold or hot), Btu/h
 U_o = overall heat transfer coefficient based on the outside area,
 Btu/ft² · h · °F
 U_i = overall heat transfer coefficient based on the inside area,
 Btu/ft² · h · °F

In addition, one may write

$$\frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} \quad (15.10)$$

As described in the previous chapter, the overall heat transfer coefficient, U , for flow in a tube is related to the individual coefficients by the following equation:

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o} \quad (15.11)$$

where h_i = inside heat transfer coefficient, $\text{Btu}/\text{ft}^2 \cdot \text{h} \cdot ^\circ\text{F}$
 h_o = outside heat transfer coefficient, $\text{Btu}/\text{ft}^2 \cdot \text{h} \cdot ^\circ\text{F}$
 $R_{f,i}$ = fouling factor based on inner tube surfaces
 $R_{f,o}$ = fouling factor based on outer tube surfaces
 D_i/D_o = diameter of pipe (inside or outside), ft
 k = pipe thermal conductivity, $\text{Btu}/\text{ft} \cdot \text{h} \cdot ^\circ\text{F}$
 L = tube length, ft

If the fouling factors, $R_{f,i}$ and $R_{f,o}$, and the tube wall resistance (middle term in right-hand side of Equation (15.11)) are negligible, then the relationship between the overall heat transfer coefficient and the individual coefficients simplifies as follows:

$$\frac{1}{U} = \frac{D_o}{h_i D_i} + \frac{1}{h_o} \quad (15.12)$$

or

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{D_i}{h_o D_o}$$

Individual coefficients, h_i and h_o , can be calculated by using empirical equations provided in Part Two, some of which is repeated below. Except for the viscosity term at the wall temperature, all of the physical properties in the equations that follow are evaluated at bulk temperatures. For the hot stream in the inner tube, the bulk temperature is $T_{H,\text{bulk}} = (T_{H,i} + T_{H,o})/2$. For the cold stream in the annulus, the bulk temperature is $t_{C,\text{bulk}} = (t_{C,i} + t_{C,o})/2$. Employing viscosity values at the wrong temperature can lead to substantial errors; however, the density and thermal conductivity of liquids do not vary significantly with temperature.

The Reynolds number for both the cold and hot process streams must be found in order to determine whether the flow rate for each stream is in the laminar, turbulent, or transition region. In all of the equations that follow, calculations for the annulus require that the aforementioned equivalent or hydraulic diameter, $D_{\text{eq}} = (D_{o,i} - D_{i,o})$, replace the tubular diameter, D . Thus, the Reynolds numbers, Re_i and Re_o , are defined as follows:

$$\text{Inner pipe: } \text{Re}_i = \frac{4\dot{m}_i}{\pi D_{i,i} \mu_i} \quad (15.13)$$

$$\text{Annulus between pipes: } \text{Re}_o = \frac{4\dot{m}_o}{\pi(D_{o,i} + D_{i,o})\mu_o} = \frac{D_{\text{eq}}\dot{m}_o}{\mu_o S} \quad (15.14)$$

Table 15.1 Reynolds Number Values vs. Type of Flow

Reynolds number, Re	Flow region
$Re < 2100$	Laminar
$2100 < Re < 10,000$	Transitional
$Re > 10,000$	Turbulent

where $D_{i,i}$ = inside diameter of inner pipe, ft
 $D_{i,o}$ = outside diameter of inner pipe, ft
 $D_{o,i}$ = inside diameter of outer pipe, ft
 D_{eq} = equivalent diameter, ft
 S = cross-sectional annular area, ft²
 μ = viscosity of hot or cold fluid at bulk temperature, lb/ft · h

Flow regimes for various Reynolds numbers appear in Table 15.1.

Similarly, Nusselt numbers, Nu_i and Nu_o , are defined by the following equations:

$$\text{Inner pipe: } Nu_i = \frac{h_i D_{i,i}}{k_i} \quad (15.15)$$

$$\text{Annulus between pipes: } Nu_o = \frac{h_o (D_{o,i} - D_{i,o})}{k_o} \quad (15.16)$$

where k = thermal conductivity at bulk temperature of hot or cold fluid,
 Btu/h · ft · °F
 Nu = Nusselt number, inside or outside

For *laminar flow*, the Nusselt number equals 4.36 for uniform surface heat flux (\dot{Q}/A) or 3.66 for constant surface temperature.^(2,3) This value should only be used for Graetz numbers, Gz , less than 10. For laminar flow with Graetz numbers from 10 to 1000, the following equation applies:⁽³⁾

$$Nu = 2.0 Gz^{1/3} \left(\frac{\mu}{\mu_{wall}} \right)^{0.14}; \quad Gz = \frac{\dot{m} c_p}{kL} \quad (15.17)$$

where μ_{wall} = viscosity at wall temperature, lb/ft · h
 L = total length of tubular exchanger, ft
 Gz = Graetz number

For *turbulent flow* ($Re > 10,000$), the Nusselt number may be calculated from the Dittus–Boelter equation if $0.7 \leq Pr \leq 160$ or the Sieder–Tate equation if $0.7 \leq Pr \leq 16,700$ where Pr is the Prandtl number. Both equations are valid for L/D greater than 10.⁽³⁾

Dittus–Boelter equation:

$$\begin{aligned} Nu &= 0.023 \operatorname{Re}^{0.8} \operatorname{Pr}^n \\ St &= 0.023 \operatorname{Re}(\operatorname{Re})^{-0.2} (\operatorname{Pr})^{-0.667} \\ \operatorname{Pr} &= \frac{c_p \mu}{k} \\ n &= 0.4 \text{ for heating or } 0.3 \text{ for cooling} \end{aligned} \quad (15.18)$$

Sieder–Tate equation:

$$Nu = 0.023 \operatorname{Re}^{0.8} \operatorname{Pr}^{1/3} \left(\frac{\mu}{\mu_{\text{wall}}} \right)^{0.14} \quad (15.19)$$

Note that Equation (15.18) is often written in terms of the Stanton number, St (occasionally referred to as the modified Nusselt number), where $St = Nu / \operatorname{RePr} = h / \rho V c_p$.

The Dittus–Boelter equation should only be used for small to moderate temperature differences. The Sieder–Tate equation applies for larger temperature differences.^(2,3) Errors as large as 25% are associated with both equations.⁽²⁾ Other empirical equations with more complicated formulas and less error are available in the literature.^(2–6)

Either Equation (15.17) or (15.19) requires a trial-and-error solution between the hot and cold streams because these equations include a viscosity term that is evaluated at the wall temperature, T_{wall} . An educated guess can be made for the wall temperature, or it can be estimated to be the average of the four known temperatures:

$$T_{\text{wall}} = \frac{t_{C,i} + t_{C,o} + T_{H,i} + T_{H,o}}{4}. \quad (15.20)$$

After the individual heat transfer coefficients are calculated, a new wall temperature can be calculated with the following equation:⁽³⁾

$$\Delta T_i = \frac{1/h_i}{(1/h_i) + (D_{i,i}/D_{i,o})h_o} (T_{H,\text{bulk}} - t_{C,\text{bulk}}) \quad (15.21)$$

with

$$T_{\text{wall}} = T_{H,\text{bulk}} - \Delta T_i \quad (15.22)$$

The entire calculation is repeated several times until T_{wall} converges. As previously stated, all of the terms in the operation above may be evaluated at the bulk temperatures of the hot and cold streams, except for μ_{wall} which is estimated at the wall temperature. Note that this correction factor is included to account for the distortion to the velocity profile that arises because of the viscosity variation with temperature.

Table 15.2 Representative Fouling Factors

Fluid	$R_f, \text{m}^2 \cdot \text{K}/\text{W}$
Seawater and treated boiler feedwater (below 50°C)	0.0001
Seawater and treated boiler feedwater (above 50°C)	0.0002
River water (below 50°C)	0.0002–0.0001
Fuel oil	0.0009
Quenching oil	0.0007
Refrigerating liquid	0.0002
Steam (not oil bearing)	0.0001

After several months of use, the tubes in an exchanger can become fouled by scale or dirt. This scale adds an extra resistance to heat transfer and causes a decrease in the overall heat transfer coefficient from U_{clean} to U_{dirty} . The relation between these two values of U —see previous chapter, Equation (14.32)—is given by

$$\frac{1}{U_{\text{dirty}}} = \frac{1}{U_{\text{clean}}} + R_f \quad (15.23)$$

where R_f is the fouling or dirt factor in typical units of $\text{ft}^2 \cdot \text{h} \cdot ^\circ\text{F}/\text{Btu}$ or $\text{m}^2 \cdot \text{K}/\text{W}$. Typical values of these fouling factors are given in Table 15.2. Information is also available in the previous chapter.

In designing a double pipe heat exchanger, it is often desirable to estimate the minimum required pipe length. This is accomplished by using the pipe conduction resistance as the only resistance to heat transfer and taking the larger of the two temperature differences at the ends of the exchanger, ΔT_{max} (i.e., either $T_1 - t_2$ or $T_2 - t_1$ for countercurrent flow and $T_1 - t_1$ for co-current flow), to be the driving force. For this condition,

$$\dot{Q} = \frac{2\pi k L_{\text{min}}}{\ln(r_o/r_i)} \Delta T_{\text{max}} \quad (15.24)$$

This finds limited use in some real world applications.

ILLUSTRATIVE EXAMPLE 15.2

The following data is provided for a double pipe heat exchanger

$$\begin{aligned} \dot{Q} &= 12,000 \text{ Btu/h} \\ U &= 48.0 \text{ Btu}/\text{ft}^2 \cdot \text{h} \cdot ^\circ\text{F} \\ \Delta T_{\text{lm}} &= 50^\circ\text{F} \end{aligned}$$

Calculate the area of the exchanger.

SOLUTION: Apply Equation (15.6):

$$\dot{Q} = UA\Delta T_{\text{lm}}$$

Substitute the data provided and solve for A .

$$\begin{aligned} A &= \frac{\dot{Q}}{U\Delta T_{\text{lm}}} \\ &= \frac{12,000}{(48.0)(50)} \\ &= 5.0 \text{ ft}^2 \end{aligned}$$

ILLUSTRATIVE EXAMPLE 15.3

Calculate the inlet cold water temperature flowing at a rate of 14.6 lb/min in a double pipe heat exchanger given the following data. Assume co-current operation.

$$\begin{aligned} t_1 &= t_{c,i} = 63^\circ\text{F} \\ T_1 &= T_{H,i} = 164^\circ\text{F} \\ T_2 &= T_{H,o} = 99^\circ\text{F} \\ U &= 35.35 \text{ Btu/ft} \cdot \text{h} \cdot ^\circ\text{F} \\ \dot{Q} &= 56,760 \text{ Btu/h} \\ A &= 32.1 \text{ ft}^2 \end{aligned}$$

SOLUTION: Apply Equation (15.6) once again and solve for ΔT_{lm} .

$$\begin{aligned} \dot{Q} &= UA\Delta T_{\text{lm}} \\ \Delta T_{\text{lm}} &= \frac{\dot{Q}}{UA} = \frac{56,760}{(35.35)(32.1)} \\ &= 50^\circ\text{F} \end{aligned}$$

Apply Equation (15.7).

$$\begin{aligned} \Delta T_{\text{lm}} &= \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} \\ \Delta T_1 &= 164 - 63 = 101 \\ \Delta T_2 &= 99 - t_{c,o} = 99 - t_2 \end{aligned}$$

Rearrange and solve for $t_{c,o}$.

$$t_{c,o} = t_2 = 79^\circ\text{F}$$

ILLUSTRATIVE EXAMPLE 15.4

As Shakespeare once said: “Something is rotten in the state of . . .” Comment on whether the information provided in the previous example is “consistent.”

SOLUTION: In addition to satisfying the heat transfer rate equation, the information provided must also satisfy the energy transfer (or conservation) equation. For this example,

$$\begin{aligned}\dot{Q} &= \dot{m}c_p C(t_{C,o} - t_{C,i}) = \dot{m}c_p C(t_2 - t_1) \\ &= [(14.6)(1.0)(79 - 63)] \\ &= 233.6 \text{ Btu/min} \\ &= 14,016 \text{ Btu/h}\end{aligned}$$

This result does not agree with the \dot{Q} provided in the problem statement. Shakespeare is right, something is indeed rotten. ■

ILLUSTRATIVE EXAMPLE 15.5

A heavy hydrocarbon oil with a heat capacity, $c_p = 0.55 \text{ Btu/lb} \cdot ^\circ\text{F}$, is being cooled in a double pipe heat exchanger from $T_1 = 210^\circ\text{F}$ to $T_2 = 170^\circ\text{F}$. The oil flows inside a tube at a rate of 8000 lb/h and the tube surface temperature is maintained at 60°F . The overall heat transfer coefficient $U = 63 \text{ Btu} \cdot \text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$. Calculate the required heat transfer area, A , in ft^2 .

SOLUTION: First calculate the LMTD:

$$\begin{aligned}\Delta T_1 &= 210 - 60 = 150^\circ\text{F} \\ \Delta T_2 &= 170 - 60 = 110^\circ\text{F}\end{aligned}$$

Apply Equation (15.7).

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{(150) - (110)}{\ln(150/110)} = 129^\circ\text{F}$$

Calculate the heat transferred or the duty. Apply Equation (15.5):

$$\dot{Q} = \dot{m}c_p \Delta T = 8000(0.55)(210 - 170) = 176,000 \text{ Btu/h}$$

Write the describing equation for the area and substitute:

$$\dot{Q} = UA\Delta T_{\text{lm}} \tag{15.6}$$

$$A = \frac{\dot{Q}}{U\Delta T_{\text{lm}}} = \frac{176,000}{63(129)} = 21.65 \text{ ft}^2 \quad \blacksquare$$

ILLUSTRATIVE EXAMPLE 15.6

Two variations of the design of a heat exchanger have been proposed. The unit is to cool a hot water stream from 140°F to 110°F through indirect contact with a cold water stream that is being heated from 60°F to 90°F. The water flow rate is 100 lb/min and the overall heat transfer coefficient may be assumed equal to 750 Btu/h · ft² · °F. Calculate the area requirements for the following two exchanger conditions:

1. double pipe co-current flow
2. double pipe countercurrent flow

SOLUTION:

1. For co-current flow,

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{(140 - 60) - (110 - 90)}{\ln(80/20)} = 43.3^\circ\text{F}$$

The rate of heat transfer is

$$\dot{Q} = \dot{m}c_p\Delta T = (100)(1)(30) = 3000 \text{ Btu/min} = 180,000 \text{ Btu/h}$$

The area for parallel flow is therefore

$$A_p = \frac{\dot{Q}}{U\Delta T_{\text{lm}}} = \frac{180,000}{750(43.3)} = 5.55 \text{ ft}^2$$

2. For countercurrent flow,

$$\Delta T_{\text{lm}} = 50^\circ\text{F (a constant)}$$

and

$$A_c = \frac{\dot{Q}}{U\Delta T_{\text{lm}}} = \frac{180,000}{750(50)} = 4.80 \text{ ft}^2$$

■

ILLUSTRATIVE EXAMPLE 15.7

Compare and comment on the area requirements for the two flow conditions in the previous example.

SOLUTION: As expected, the countercurrent exchanger yields the smaller (more compact) design due to the higher driving force. ■

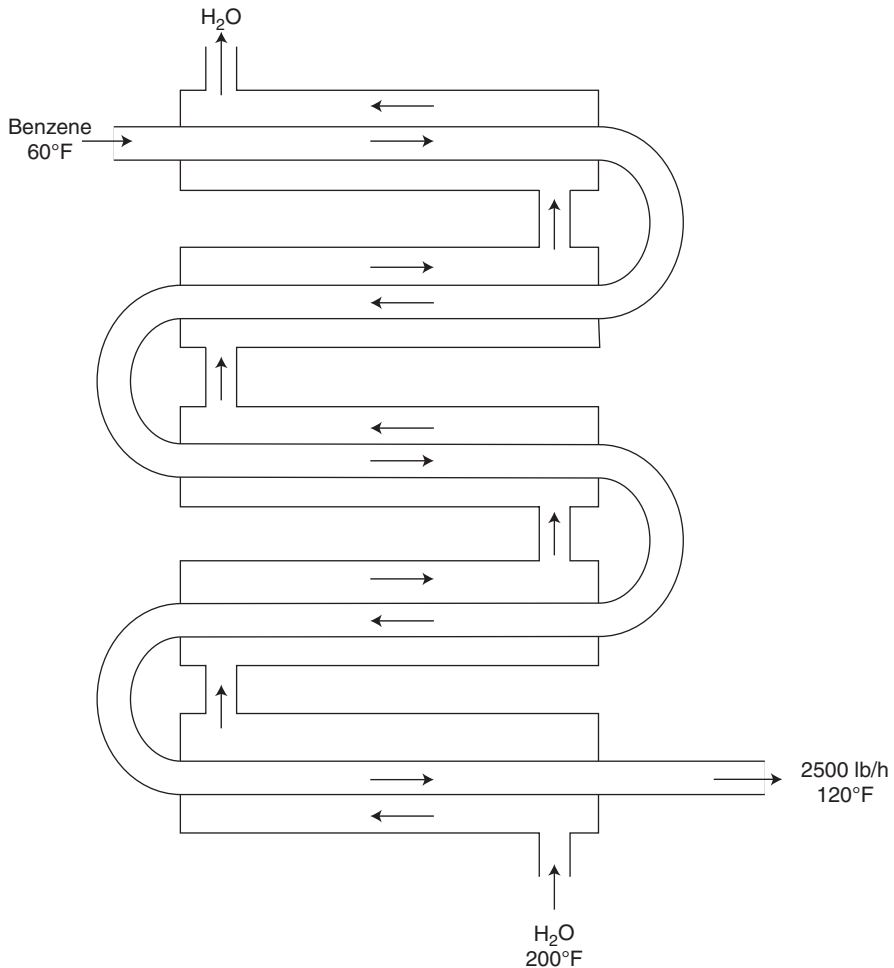


Figure 15.4 Trombone heat exchanger.

ILLUSTRATIVE EXAMPLE 15.8⁽⁷⁾

The double pipe heat exchanger shown in Figure 15.4 is used for heating 2500 lb/h benzene from 60°F to 120°F. Hot water at 200°F from a boiler is available for heating purposes in amounts up to 4000 lb/h. Use schedule 40 brass pipe and an integral number of 15 foot long sections are to be employed in the design of the exchanger.

The design is to be based on achieving a Reynolds number of 13,000 (turbulent flow) in both the inner pipe and annular region. As an additional constrain because of pressure drop consideration, the width of the annular region must be at least equal to one fourth the outside diameter of the inner pipe.

Note that one can obtain \dot{Q} by calculating the duty of the benzene stream, and a heat balance will yield the exit temperature of the water, T_w . However, the area or the overall heat transfer coefficient is not known. Obtaining U becomes more complicated since information on flow variables or a pipe diameter is not given.

Data is provided below for this problem, but can be estimated or obtained from Tables in the Appendix:

Benzene at 90°F:

$$\mu = 0.55 \text{ cP} = 3.70 \times 10^{-4} \text{ lb/ft} \cdot \text{s}$$

$$\text{SG} = 0.879 \text{ (at } 68.8^\circ\text{F)}$$

$$\rho = 0.879(62.4) = 54.8 \text{ lb/ft}^3$$

$$c_p = 0.415 \text{ Btu/lb} \cdot ^\circ\text{F}$$

$$k = 0.092 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F (at } 86^\circ\text{F)}$$

Water at 200°F:

$$\mu = 0.305 \text{ cP} = 2.05 \times 10^{-4} \text{ lb/ft} \cdot \text{s}$$

$$\rho = 60.13 \text{ lb/ft}^3$$

$$c_p = 1.0 \text{ Btu/lb} \cdot ^\circ\text{F}$$

$$k = 0.392 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F}$$

SOLUTION: For flow inside tubes (pipes)

$$\text{Re} = \frac{4\dot{m}}{\pi D_i \mu} \quad (15.13)$$

Substituting,

$$D_i = \frac{4\dot{m}}{\text{Re} \pi \mu} = \frac{4(2500/3600)}{(13,000)\pi(3.7 \times 10^{-4})} = 0.184 \text{ ft} = 2.20 \text{ in}$$

The closest standard pipe with $D_i = 2.20$ in is 2-inch schedule 40 pipe (see also Chapter 6). Thus,

$$D_i = 2.067 \text{ in}$$

This results in

$$\text{Re} = 13,874$$

For 2-inch schedule 40 pipe

$$A_i = 0.541 \text{ ft}^2/\text{ft}$$

$$A_o = 0.622 \text{ ft}^2/\text{ft}$$

$$D_i = 2.067 \text{ in}$$

$$D_o = 2.375 \text{ in}$$

$$S_i = 0.0233 \text{ ft}^2 \text{ (inside area)}$$

$$\Delta x_w = 0.154 \text{ in} = 0.0128 \text{ ft}$$

For flow in the annular region

$$\text{Re} = \frac{D_{eq} V_\rho}{\mu} = \frac{D_{eq} \dot{m}}{\mu S} \quad (15.14)$$

with

$$D_{eq} = 4r_H = \frac{4S}{L_p} = \frac{4S}{\pi(D_{o,i} + D_{i,o})}$$

Therefore,

$$\text{Re} = \frac{4\dot{m}}{\pi(D_{o,i} + D_{i,o})\mu}$$

However, note the constraint:

$$0.5(D_{i,o} - D_{o,i}) \geq 0.25D_{o,i}$$

or

$$D_{i,o} \geq 1.5D_{o,i} = 3.562 \text{ in}$$

The smallest pipe, which satisfies this constraint, is 4-inch schedule 40 pipe ($D_{i,o} = 4.026 \text{ in}$).

The outer pipe is therefore 4-inch schedule 40 pipe (see Chapter 6 once again).

For $\text{Re} = 13,000$, the water flow is:

$$\begin{aligned} \dot{m} &= \frac{\text{Re} \pi \mu (D_{o,i} + D_{i,o})}{4} \\ &= \frac{(13,000)\pi(2.05 \times 10^{-4})(3600)[(2.375/12) + (4.026/12)]}{4} \\ &= 4019 \text{ lb/h} \end{aligned}$$

This is close enough to 4000 lb/h.

The outlet water temperature is obtained from a heat balance:

$$\begin{aligned} \dot{Q} &= \dot{m}_C c_{pC} \Delta t_C = \dot{m}_H c_{pH} \Delta T_H \\ &= (2500)(0.415)(60) \\ &= 62,250 \text{ Btu/h} \end{aligned}$$

From a water heat balance, with $c_{pH} = 1.0$,

$$\begin{aligned} \Delta T_H &= 62,250/4000 = 15.6^\circ\text{F} \\ T_{H,\text{out}} &= 200 - 15.6 = 184.4^\circ\text{F} \\ T_{H,\text{av}} &= (200 + 184.4)/2 = 192.2^\circ\text{F} \end{aligned}$$

Properties of water at 192°F are estimated from the data at 200°F. At 200°F,

$$\begin{aligned}\mu &= 0.322 \text{ cP} = 2.16 \times 10^{-4} \text{ lb/ft} \cdot \text{s} \\ \rho &= 60.13 \text{ lb/ft}^3 \\ c_p &= 1.0 \text{ Btu/lb} \cdot ^\circ\text{F} \\ k &= 0.390 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F}\end{aligned}$$

The Dittus–Boelter equation is employed to determine the heat transfer coefficients:

$$\text{St} = 0.023(\text{Re})^{-0.2}(\text{Pr})^{-0.667}$$

For the inside pipe (benzene)

$$\begin{aligned}\text{Re} &= 13,000 \\ \text{Pr} &= \frac{c_p \mu}{k} = \frac{0.415(3.70 \times 10^{-4})}{0.092/3600} = 6.01 \\ \text{St} &= 0.023(13,000)^{-0.2}(6.01)^{-0.667} = 0.00105 \\ h_i &= \frac{(\text{St})c_p \dot{m}}{S} = \frac{(0.00105)(0.415)(2500)}{0.0233} = 46.7 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}\end{aligned}$$

For the annular region (water)

$$\begin{aligned}\text{Re} &= \frac{4\dot{m}}{\pi\mu(D_{o,i} + D_{i,o})} = \frac{4(4000/3600)}{\pi(2.16 \times 10^{-4})[(2.375/12) + (4.026/12)]} = 12,279 \\ \text{Pr} &= \frac{c_p \mu}{k} = \frac{1(2.16 \times 10^{-4})}{0.390/(3600)} = 1.99 \\ \text{St} &= 0.023(12,278)^{-0.2}(1.99)^{-0.667} = 0.00221 \\ S_{\text{annulus}} &= \frac{\pi}{4}(D_{i,o}^2 - D_{o,i}^2) = \frac{\pi}{4}[(4.026)^2 - (2.375)^2]/144 = 0.0576 \text{ ft}^2 \\ h_o &= \frac{0.00221c_p \dot{m}}{S_{\text{annulus}}} = \frac{0.00221(1)(4000)}{0.0576} = 153 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}\end{aligned}$$

Neglecting fouling,

$$\frac{1}{U_o} = \frac{D_o}{D_i h_i} + \frac{\Delta x_w D_o}{k_w D_{\text{lm}}} + \frac{1}{h_o} \quad (15.11)$$

For the pipe,

$$\begin{aligned}D_{\text{lm}} &= \frac{2.375 - 2.067}{\ln(2.375/2.067)} = 2.217 \text{ in} \\ \frac{1}{U_o} &= \frac{2.375}{(2.067)(46.7)} + \frac{(0.0128)(2.375)}{(26)(2.217)} + \frac{1}{153} \\ &= 0.0246 + 0.0005274 + 0.00654 \\ &= 0.0316\end{aligned}$$

Therefore

$$U_o = 31.6 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$$

Finally, to obtain the length, use

$$\begin{aligned} \dot{Q} &= U_o A_o \Delta T_{\text{lm}} \\ \Delta T_{\text{lm}} &= \frac{124.4 - 80}{\ln(24.4/80)} = 100.6^\circ\text{F} \end{aligned}$$

In addition,

$$A_o = 0.622L$$

The heat exchanger equation is again employed to calculate the length

$$\begin{aligned} 62,250 &= (31.6)(0.622)(L)(100.6) \\ L &= 31.5 \text{ ft} \end{aligned} \quad \blacksquare$$

ILLUSTRATIVE EXAMPLE 15.9

With reference to the previous example, how many sections should be required in the design.

SOLUTION: Two sections would do the job. However, three sections might be recommended; the unit would then be somewhat over-designed. ■

CALCULATION OF EXIT TEMPERATURES⁽⁷⁾

The equations describing exit temperatures for parallel flow heat exchangers follows. These equations are based on three assumptions:

1. Streams do not experience phase change from inlet to exit.
2. Heat capacities of both streams are constant.
3. A single average U applies for the entire exchanger.

For parallel (p) flow, one notes (with capital and lowercase letters representing the hot and cold fluid, respectively, as per Famularo's⁽⁷⁾ notation)

$$\begin{aligned} \Delta T_1 &= T_1 - t_1 \\ \Delta T_2 &= T_2 - t_2 \end{aligned} \quad (15.25)$$

In addition,

$$\dot{Q}_p = \dot{M}C(T_1 - T_2) = \dot{m}c(t_2 - t_1) \quad (15.26)$$

or

$$\frac{T_1 - T_2}{B} = \frac{t_2 - t_1}{b}$$

with

$$B = \frac{1}{\dot{M}C}$$

$$b = \frac{1}{\dot{m}c}$$

Combining the above equation with the heat exchanger equation ($\dot{Q} = UA\Delta T_{lm}$) leads to

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -(B + b)UA \quad (15.27)$$

Combining Equations (15.26) and (15.27) gives

$$T_2 = \frac{[(b/B) + e^{-UA(B+b)}]T_1 + [1 - e^{-UA(B+b)}]t_1}{1 + (b/B)} \quad (15.28)$$

The cold fluid outlet temperature may be calculated by replacing T_2 , T_1 , and t_1 in Equation (15.28) by t_2 , t_1 , and T_1 , respectively.⁽⁷⁾

For countercurrent (c) flow, one notes

$$\Delta T_1 = T_1 - t_2$$

$$\Delta T_2 = T_2 - t_1 \quad (15.29)$$

Once again,

$$\dot{Q}_c = \dot{M}C(T_1 - T_2) = \dot{m}c(t_2 - t_1) \quad (15.30)$$

or

$$\frac{T_1 - T_2}{B} = \frac{t_2 - t_1}{b}$$

Combining the above equations with the heat transfer equation leads to

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -(B - b)UA \quad (15.31)$$

Combining Equation (15.30) and (15.31) gives

$$T_2 = \frac{[(B/b) - 1]T_1 + [(B/b)e^{UA(B-b)} - (B/b)]t_1}{(B/b)e^{UA(B-b)} - 1} \quad (15.32)$$

The cold fluid outlet temperature may be calculated by replacing T_2 , T_1 , and t_1 in Equation (15.32) by t_2 , t_1 , and T_1 , respectively.⁽⁷⁾

Another application involves a simplified case in which the fluid temperature on one side of the dividing wall (tube/pipe) is constant. If one can assume that

$$\dot{M}C \gg \dot{m}c$$

or

$$\frac{1}{\dot{M}C} \lll \frac{1}{\dot{m}c} \quad (15.33)$$

and $T_2 = T_1 = T$ and $B \lll b$, the co-current case reduces to

$$\ln\left(\frac{T - t_2}{T - t_1}\right) = -bUA \quad (15.34)$$

In addition, the countercurrent case reduces to

$$\ln\left(\frac{T - t_1}{T - t_2}\right) = bUA \quad (15.35)$$

If the cold stream is flowing inside the inner pipe, then

$$UA = U_i A_i \quad (15.36)$$

and

$$t_2 = T - (T - t_1)e^{-U_i A_i / \dot{m}c} \quad (15.37)$$

This equation applies to both the co-current and countercurrent case. There are two other situations in which the fluid temperature is constant, and the above equation applies:

1. Condensation of a vapor at T_s without subcooling. Set $T_s = T$ in Equation (15.37).
2. Outside flow at T_∞ normal to a tube. Set $T_\infty = T$ in Equation (15.37).

ILLUSTRATIVE EXAMPLE 15.10

Calculate T_2 and t_2 in (consistent units)

$$MC = \dot{m}_H c_{pH} = 2000$$

$$mc = \dot{m}_C c_{pC} = 1000$$

$$U = 2000$$

$$A = 10$$

$$T_1 = 300$$

$$t_1 = 60$$

Calculate the exit temperatures if the flow is *countercurrent* by employing Equation (15.32).⁽⁷⁾

SOLUTION: For this system:

$$B = 0.001, \quad b = 0.0005, \quad \text{and} \quad B/b = 2.0$$

In addition,

$$UA(B - b) = 2000(0.001 - 0.0005) = 1.0$$

so that

$$e^{1.0} = 2.7183$$

Substituting into Equation (15.32) gives

$$\begin{aligned} T_2 &= \frac{(2 - 1)300 + 2(2.7183 - 1)60}{2(2.7183) - 1} \\ &= \frac{506.2}{4.4366} = 114.1^\circ\text{F} = 114^\circ\text{F} \end{aligned}$$

Employ a revised Equation (15.32) or an overall energy balance to generate t_2 ,

$$\begin{aligned} t_2 &= t_1 + \frac{T_1 - T_2}{(B/b)} = 60 + \frac{300 - 114.1}{2} \\ &= 152.95^\circ\text{F} = 153^\circ\text{F} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 15.11

The following information is provided for a *co-current* flow double pipe heat exchanger. The pipe consists of 200 ft of 2-inch schedule 40 pipe, $k = 25 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F}$. The hot and cold film coefficients are 1200 and $1175 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$, respectively. In addition,

$$\begin{aligned} MC &= \dot{m}_H c_{pH} \\ mc &= \dot{m}_C c_{pC} = 22,300 \text{ Btu/h} \cdot ^\circ\text{F} \\ T_1 &= 300^\circ\text{F} \\ t_1 &= 60^\circ\text{F} \end{aligned}$$

Calculate T_2 and t_2 .

SOLUTION: First calculate the overall inside heat transfer coefficient. For the pipe

$$\begin{aligned} \text{ID} &= 2.067 \text{ in} \\ \text{OD} &= 2.375 \text{ in} \\ \Delta x &= 0.154 \text{ in} \\ A'_i &= 0.541 \text{ ft}^2/\text{ft} \\ k &= 25 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F} \end{aligned}$$

Thus,

$$U_i = \frac{1}{\frac{1}{1200} + \frac{0.154}{(25)(12)} + \frac{1}{1175(2.375/2.067)}}$$

$$= 482 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$$

In addition,

$$A_i = (A'_i)(L)$$

$$= (0.541)(200)$$

$$= 108.2 \text{ ft}^2$$

There are two key equations—one based on an energy balance and one based on rate considerations. These two equations are:

$$\dot{Q}_H = \dot{Q} = MC(300 - T_2)$$

$$= (30,000)(300 - T_2) \quad (1a)$$

$$\dot{Q}_C = 22,300(t_2 - 60) \quad (1b)$$

with

$$\dot{Q}_C = \dot{Q}_H$$

and

$$\dot{Q} = U_i A_i \Delta T_{lm} \quad (2)$$

$$= (482)(108.2) \left\{ \frac{(300 - 60) - (T_2 - t_2)}{\ln[(300 - 60)/(T_2 - t_2)]} \right\}$$

There are also the two unknowns— T_2 and t_2 . These can be solved by any suitable numerical method. By trial-and-error, one obtains

$$T_2 = 200^\circ\text{F}$$

$$t_2 = 195^\circ\text{F}$$

with

$$\dot{Q} = 3.0 \times 10^6 \text{ Btu/h} \quad \blacksquare$$

ILLUSTRATIVE EXAMPLE 15.12

Solve the previous example employing the equations above provided by Famularo.⁽⁷⁾ Also calculate the discharge temperature, the LMTD, and \dot{Q} for the *countercurrent* case.

SOLUTION: Equation (5.28) applies for the cocurrent case

$$T_2 = \frac{[(b/B) + e^{-UA(B+b)}]T_1 + [1 - e^{-UA(B+b)}]t_1}{1 + (b/B)}$$

For $B = 3.33 \times 10^{-5}$, $b = 4.48 \times 10^{-5}$, $A = 108.2$ and $U = 482$, one obtains

$$T_2 = 199.4 \approx 200^\circ\text{F}$$

Apply an energy balance to both fluids.

$$30,000(300 - T_2) = 22,300(t_2 - 60)$$

For $T_2 = 200^\circ\text{F}$, one obtains

$$t_2 = 195^\circ\text{F}$$

The agreement between both methods is excellent. Note that temperature t_2 may also be calculated directly from Equation (15.28) with the temperature(s) appropriately reversed.

Substitute into Equation (15.32) to obtain T_2 for the countercurrent case:

$$T_2 = \frac{(B/b)[e^{UA(B-b)} - 1]t_1 + [(B/b) - 1]T_1}{(B/b)e^{UA(B-b)} - 1}$$

Once again, for $B = 3.33 \times 10^{-5}$, $b = 4.48 \times 10^{-5}$, $A = 108.2$ and $U = 482$, one obtains

$$T_2 = 164.20 = 164^\circ\text{F}$$

Substitute into Equation (15.32) with the temperature reversed to generate t_2 :

$$t_2 = \frac{(b/B)[e^{UA(b-B)} - 1]T_1 + [(b/B) - 1]t_1}{(b/B)e^{UA(b-B)} - 1}$$

Substituting gives

$$t_2 = 242.85 = 243^\circ\text{F}$$

For the LMTD,

$$\Delta T_{\text{lm}} = \frac{(T_2 - t_1) - (T_1 - t_2)}{\ln\left(\frac{T_2 - t_1}{T_1 - t_2}\right)}$$

Substituting

$$T_2 - t_1 = 164 - 60 = 104$$

$$T_1 - t_2 = 300 - 243 = 57$$

$$\begin{aligned} \Delta T_{\text{lm}} &= \frac{104 - 57}{\ln(104/57)} \\ &= 78.2^\circ\text{F} \end{aligned}$$

Apply the standard heat transfer equation to calculate \dot{Q} :

$$\begin{aligned} \dot{Q} &= UA\Delta T_{\text{lm}} & (15.6) \\ &= (482 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F})(108.2 \text{ ft}^2)(78.2^\circ\text{F}) \\ &= 4,078,000 \text{ Btu/h} \end{aligned}$$

This result may be checked by employing the heat load equation:

$$\begin{aligned}\dot{Q} &= MC(T_1 - T_2) \\ &= (30,000 \text{ Btu/h} \cdot ^\circ\text{F})(300 - 164.20) \\ &= 4,074,000 \text{ Btu/h}\end{aligned}$$

The two results are once again in reasonable agreement. ■

EFFECTIVENESS FACTOR AND NUMBER OF TRANSFER UNITS

The effectiveness factor (or effectiveness) and number of transfer units are two approaches that have been employed in heat exchanger design and analysis. The effectiveness, ε , of a heat exchanger is defined as

$$\varepsilon = \frac{\text{actual heat transfer rate}}{\text{maximum heat transfer rate}} = \frac{\dot{Q}}{\dot{Q}_{\max}} \quad (15.38)$$

A maximum rate, \dot{Q}_{\max} , is calculated for the hot side and the cold side (tube and annulus side, respectively) for the double pipe heat exchanger. For example, assuming countercurrent flow,

$$\dot{Q}_{\max, \text{tube}} = (\dot{m}c_p)_{\text{tube}}|T_1 - t_1| = C_{\text{tube}}|T_1 - t_1| \quad (15.39)$$

$$\dot{Q}_{\max, \text{annulus}} = (\dot{m}c_p)_{\text{annulus}}|T_1 - t_1| = C_{\text{annulus}}|T_1 - t_1| \quad (15.40)$$

The C term is defined as the thermal capacitance rate and is simply $\dot{m}c_p$. The lower \dot{Q}_{\max} value is used in evaluating the effectiveness.

The effectiveness factor, ε , is applied in simulation (predictive) studies rather than design. It enables one to calculate exit temperatures and heat transfer rates for an existing unit. The effectiveness factor is provided below for both systems (parallel, countercurrent) and for both hot and cold side. Note the development of these equations assumes that the hot fluid is flowing in the inside tube.

Cocurrent flow:

$$\begin{aligned}\text{tube: } \varepsilon_H &= \frac{T_1 - T_2}{T_1 - t_1} \\ \text{annulus: } \varepsilon_C &= \frac{t_2 - t_1}{T_1 - t_1}\end{aligned} \quad (15.41)$$

Countercurrent flow:

$$\begin{aligned} \text{tube: } \varepsilon_H &= \frac{T_1 - T_2}{T_1 - t_1} \\ \text{annulus: } \varepsilon_C &= \frac{t_2 - t_1}{T_1 - t_1} \end{aligned} \quad (15.42)$$

Note that subscripts 1 and 2 refer to inlet and outlet conditions, respectively.

Thus, if the effectiveness factor is known, one could calculate the exit temperature T_2 or t_2 (and the corresponding \dot{Q}). The question that remains is whether to use ε_H or ε_C . The fluid to employ, hot or cold, is that which undergoes the maximum temperature change (or difference) or the corresponding minimum value of C :

$$\dot{Q} = (\dot{m}c_p)\Delta T = (\dot{m}c_p)_H(T_1 - T_2) = (\dot{m}c_p)_C(t_2 - t_1) \quad (15.43)$$

If the temperatures are unknown and the flow rates are known, the minimum value of $\dot{m}c_p$ is obtained from flow rate data. The $\dot{m}c_p$ value for the fluid in question thus requires the use of the appropriate ε_H or ε_C , and this equation calculates the exit temperature. This general topic will be reviewed again in the next chapter, particularly as it applies to shell and tube heat exchangers.

Another term employed in heat exchanger studies is the number of transfer units, NTU, of a heat exchanger. It is defined as

$$\text{NTU} = \frac{UA}{(\dot{m}c_p)_{\min}} = \frac{UA}{C_{\min}} \quad (15.44)$$

where C_{\min} is the minimum thermal capacitance rate. NTU is determined by calculating C for each fluid and choosing the lower value.

ILLUSTRATIVE EXAMPLE 15.13

Is an absolute maximum value of 1.0 for ε achievable?

SOLUTION: The answer is no since ΔT would go to zero and therefore the area required would be infinite; this would be a violation of the Second Law of Thermodynamics.⁽⁸⁾ ■

ILLUSTRATIVE EXAMPLE 15.14

A double pipe heat exchanger is employed to heat 228 kg/h of water from 25°C to 73°C. Water is on the tube side and the flow is countercurrent. The heating fluid is saturated steam at 1 atm, with 55 kg/h of steam condensing on the annular side. No subcooling of condensed water takes place. The inner pipe of the double pipe heat exchanger has an inner radius ($r_{i,i}$) of 1.3 cm and an outer radius ($r_{o,i}$) of 1.9 cm. The inner radius of the outer tube ($r_{i,o}$) is 3.2 cm. Both tubes are 0.5% carbon steel. The overall heat transfer coefficient for the new heat

exchanger, U_{clean} , was originally determined to be $3994 \text{ W/m}^2 \cdot \text{K}$, based on the inside area of the inner pipe. After a period of use, the fouling factor, R_f , was estimated to be $0.0002 \text{ m}^2 \cdot \text{K/W}$. Determine:

1. the temperature profile of the water and steam along the length of the exchanger,
2. whether the steam has enough energy to heat the water,
3. the minimum length of the heat exchanger, and
4. the dirty overall heat transfer coefficient.

Water properties at the average bulk temperature $(25 + 73)/2 = 49^\circ\text{C}$ are: heat capacity, $c_p = 4174 \text{ J/kg} \cdot \text{K}$; density, $\rho = 993 \text{ kg/m}^3$; absolute viscosity, $\mu = 6.82 \times 10^{-4} \text{ kg/m} \cdot \text{s}$; kinematic viscosity, $\nu = 6.87 \times 10^{-7} \text{ m}^2/\text{s}$, and the Prandtl number, $\text{Pr} = 4.53$. The properties of water at the “average” film temperature $(100 + 49)/2 = 74.9^\circ\text{C}$ are: heat capacity, $c_p = 4190 \text{ J/kg} \cdot \text{K}$; density, $\rho = 975 \text{ kg/m}^3$; absolute viscosity, $\mu = 3.8 \times 10^{-4} \text{ kg/m} \cdot \text{s}$; kinematic viscosity, $\nu = 3.89 \times 10^{-7} \text{ m}^2/\text{s}$, and the Prandtl number, $\text{Pr} = 2.53$.

For saturated steam at 100°C : latent heat of condensation $= 2.26 \times 10^6 \text{ J/kg}$, vapor density $= 0.596 \text{ kg/m}^3$. The thermal conductivity for 0.5% carbon steel at an average temperature of $(100 + 25)/2 = 62.5^\circ\text{C}$ is $k = 54 \text{ W/m} \cdot \text{K}$.

Assumptions include steady-state operation, incompressible fluids, Newtonian fluids, constant properties, homogeneous isotropic materials, and there are negligible heat losses. Notations 1 and 2 denote the water inlet and outlet, respectively, and s the stream.

SOLUTION: Organize the given information by aligning the data in tabular form as shown in Table 15.3. The temperature profile in the heat exchanger is provided in Figure 15.5.

Calculate the temperature driving forces at both ends of the heat exchanger and ΔT_{lm} .

$$\Delta T_1 = (T_s - t_1) = 100 - 25 = 75^\circ\text{C} = 75 \text{ K}$$

$$\Delta T_2 = (T_s - t_2) = 100 - 73 = 27^\circ\text{C} = 27 \text{ K}$$

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{75 - 27}{\ln(75/27)} = 47^\circ\text{C}$$

Table 15.3 Key Parameters for Illustrative Example 15.14

Parameter	Stream			
	1	2	s	s
Type	Water		Steam	
Side	Tube		Annular	
ρ , kg/m^3	993		0.596	
c_p , $\text{J/kg} \cdot \text{K}$	4174		N/A	
μ , $\text{kg/m} \cdot \text{s}$	6.82×10^{-4}		N/A	
k , $\text{W/m} \cdot \text{K}$	0.63		N/A	
h_{vap} , J/kg	N/A		2.26×10^6	
t , T_s , $^\circ\text{C}$	25	73	100	100
\dot{m} , kg/s	0.0633		0.01528	

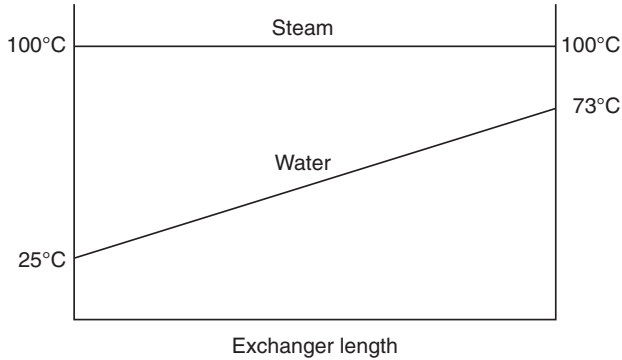


Figure 15.5 Temperature profile; Illustrative Example 15.14.

Calculate both the capacitance rate ($\dot{m}c_p$) of water and the heat load:

$$C_w = \dot{m}c_p = (228)(4174)(1 \text{ h}/3600 \text{ s}) = 264 \text{ W/K}$$

$$\dot{Q} = \dot{m}c_p(t_2 - t_1) = C_w(t_2 - t_1) = (264)(73 - 25) = 12,700 \text{ W} = 4568 \text{ kJ/h}$$

Determine the maximum heat term from the water stream:

$$\dot{Q}_{\max,w} = C_w(T_s - t_1) \quad (15.39)$$

Substituting

$$\begin{aligned} \dot{Q}_{\max,w} &= (264)(100 - 25) \\ &= 19,800 \text{ W} = C_w(T_s - t_1) \end{aligned}$$

Also, determine the maximum heat term for the steam assuming that all the steam condenses to liquid:

$$\dot{Q}_{\max,st} = \dot{m}_{\text{steam}}h_{\text{vap}} = (55)(2.26 \times 10^6)(1 \text{ h}/3600 \text{ s}) = 34,500 \text{ W}$$

Select the appropriate \dot{Q}_{\max} for calculating the effectiveness. Since $\dot{Q}_{\max,w}$ is the smaller value, $\dot{Q}_{\max} = \dot{Q}_{\max,w} = 19,800 \text{ W}$.

The effectiveness, ε , from Equation (15.38) is then

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{12,700}{19,800} = 0.641$$

The minimum required length of the heat exchanger is (see Equation (15.24)):

$$\begin{aligned} \dot{Q} &= \frac{2\pi k L_{\min}(T_s - t_1)}{\ln(r_{o,i}/r_{i,i})} \\ L_{\min} &= \frac{\dot{Q} \ln(r_{o,i}/r_{i,i})}{2\pi k(T_s - T_1)} = \frac{(12,700) \ln(1.9/1.3)}{2\pi(54)(100 - 25)} = 0.189 \text{ m} \end{aligned}$$

Determine the effects of fouling. Employ Equation (15.23):

$$\begin{aligned}
 R_f &= 0.0002 \text{ m}^2 \cdot \text{K/W} \\
 \frac{1}{U_{\text{dirty}}} &= \frac{1}{U_{\text{clean}}} + R_f = \frac{1}{3994} + 0.0002 = 0.00025 + 0.0002 \\
 &= 0.00045
 \end{aligned}$$

Inverting,

$$U_{\text{dirty}} = 2220 \text{ W/m}^2 \cdot \text{K} \quad \blacksquare$$

ILLUSTRATIVE EXAMPLE 15.15

Refer to the previous example. Calculate the area and the length of the heat exchanger.

SOLUTION: The heat transfer area may be calculated directly from Equation (15.6) employing U_{dirty} .

$$A = \frac{\dot{Q}}{U_{\text{dirty}} \Delta T_{\text{lm}}} = \frac{12,700}{2220(47)} = 0.1217 \text{ m}^2 = 1.31 \text{ ft}^2$$

The tube length (based on the inner radius) is then

$$L = \frac{A}{2\pi r_{i,i}} = \frac{0.1217}{2\pi(0.013)} = 1.49 \text{ m} \quad \blacksquare$$

ILLUSTRATIVE EXAMPLE 15.16

With reference to Illustrative Example 15.14, calculate the number of transfer units (NTU) of the exchanger.

SOLUTION: The effectiveness was calculated in the previous example. The number of transfer units may also be calculated from Equation (15.44):

$$\text{NTU} = \frac{U_{\text{dirty}} A}{C_w} = \frac{2220(0.1217)}{264} = 1.02 \quad \blacksquare$$

ILLUSTRATIVE EXAMPLE 15.17

How would the answers differ if the flow were co-current in Illustrative Example 15.14?

SOLUTION: Since T_s is constant, $\Delta T_1 = \Delta T_2 = \Delta T_{\text{lm}}$. Thus, the answers would be the same for a co-current heat exchanger. \blacksquare

WILSON'S METHOD

There is a procedure for evaluating the *outside* film coefficient for a double pipe unit. Wilson's method⁽⁹⁾ is a graphical technique for evaluating this coefficient. The inside coefficient is a function of the Reynolds and Prandtl numbers via the Dittus–Boelter equation presented in Equation (15.18), i.e.,

$$h_i = f(\text{Re}^{0.8} \text{Pr}^{0.3}) \quad (15.45)$$

A series of experiments can be carried out on a double pipe exchanger where all conditions are held relatively constant except for the velocity (V) of the cooling (in this case) inner stream. Therefore, for the proposed experiment:

$$h_i = f(\text{Re}^{0.8}) = f(V^{0.8}) = f(\dot{m}); \text{Re} = DV\rho/\mu \quad (15.46)$$

or, in equation form,

$$h_i = aV^{0.8} \quad (15.47)$$

where a is a constant. Equation (15.47) can be substituted into the overall coefficient equation

$$\frac{1}{U_o A_o} = R_o + R_w + R_i$$

so that

$$= \frac{1}{h_o A_o} + \frac{\Delta x}{k A_{lm}} + \frac{1}{a V^{0.8} A_i} \quad (15.48)$$

Data can be taken at varying velocities. By plotting $1/U_o A_o$ versus $1/V^{0.8}$, a straight line should be obtained since the first two terms of Equation (15.48) are constants. The intercept of this line corresponds to an infinite velocity and an inside resistance of zero. Thus, the above equation may be rewritten as

$$\left(\frac{1}{U_o A_o} \right)_{\text{intercept}} = \frac{1}{h_o A_o} + \frac{\Delta x}{k A_{lm}} \quad (15.49)$$

The second term on the right-hand side is known and/or can be calculated and h_o can then be evaluated from the intercept (details on this calculational scheme are provided later in the illustrative examples). Fouling coefficients, f , can be estimated by the Wilson method if the outside fluid coefficient, h_o , can be predicted or is negligible. Note that the fouling resistance is normally included in the intercept value.

ILLUSTRATIVE EXAMPLE 15.18

In 1975, Arthur Andrews⁽¹⁰⁾—a chemical engineering senior at Manhattan College—conducted a double pipe heat exchanger experiment in the Unit Operations Laboratory. Some of the

Table 15.4 Wilson Method Information: Illustrative Example 15.18

Run	$1/U_o A_o, \text{ } ^\circ\text{F} \cdot \text{h}/\text{Btu}$	$T, \text{ } ^\circ\text{F}$ Average temperature	$1/V, \text{ h}/\text{ft}$
1	1.2176×10^{-3}	124.5	2.29×10^{-4}
2	0.9454×10^{-3}	125	1.65×10^{-4}
3	0.9366×10^{-3}	129	1.29×10^{-4}
4	0.8618×10^{-3}	121.5	1.117×10^{-4}
5	0.7966×10^{-3}	122	0.962×10^{-4}

co-current flow data and calculations submitted are presented in Table 15.4. Employing Andrew's data and Wilson's method, estimate h_o for the exchanger fluid. For this lab unit (see Figure 15.1), $A_o = 1.85 \text{ ft}^2$.

SOLUTION: A "best" straight line representation plot of $1/U_o A_o$ versus $1/V^{0.8}$ is provided in Figure 15.6. The intercept ($1/h_o A_o$) is approximately 0.560×10^{-3} . This value may now be used to estimate h_o :

$$h_o = 1/(0.560 \times 10^{-3})(1.85) = 965 \text{ Btu}/\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$$

ILLUSTRATIVE EXAMPLE 15.19

Another experiment⁽¹¹⁾ on the double pipe heat exchanger in the Unit Operation Laboratory at Manhattan College produced the following data and calculated results for U (overall heat transfer coefficient) and \dot{m}_C (inside cold water flow rate); see Table 15.5.

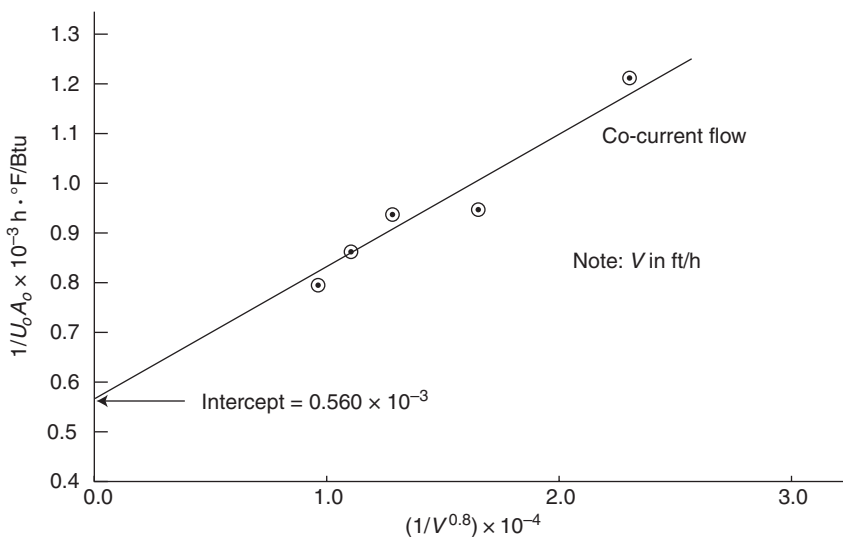
**Figure 15.6** Wilson method plot: co-current flow.

Table 15.5 Wilson Experiment Data and Results

U , Btu/h · ft ² · °F	\dot{m}_C , lb/h	$\dot{m}_C^{-0.8}$
687.98	500.97	144.49
740.74	1001.94	251.58
748.10	1502.90	347.97
757.08	2003.87	438.02
756.31	2204.26	472.73

Using the Wilson method, estimate the outside fluid film coefficient. Once again, neglect the effects of scale and pipe wall.

SOLUTION⁽¹²⁾: Noting once again that \dot{m}_C is linearly related to V , regress the data in columns (2) and (5) of Table 15.6 to the equation

$$\frac{1}{U} = a + b\dot{m}_C^{-0.8}$$

For this data (see Figure 15.7)

$$a = 0.00126$$

$$b = 0.0276$$

The term a represents the inverse of the h_o so that

$$h_o = 1.0/a = 1.0/0.00126$$

$$= 794 \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{°F}$$

■

ILLUSTRATIVE EXAMPLE 15.20

Refer to the previous illustrative example. Perform the same calculation but include the resistance associated with the tube (Admiralty metal) if the inside and outside diameters are 1.00 and 0.902 in, respectively; k for the tube is 60 Btu/h · ft² · °F.

Table 15.6 Wilson Experiment Data and Results

U , Btu/h · ft ² · °F	$1/U$, h · ft ² · °F	\dot{m}_C , lb/h	$\dot{m}_C^{0.8}$	$\dot{m}_C^{-0.8}$
687.98	0.0014535	500.97	144.49	0.00692071
740.74	0.00135	1001.94	251.58	0.003974904
748.10	0.0013367	1502.90	347.97	0.002873796
757.08	0.0013209	2003.87	438.02	0.002282992
756.31	0.0013222	2204.26	472.73	0.002115387

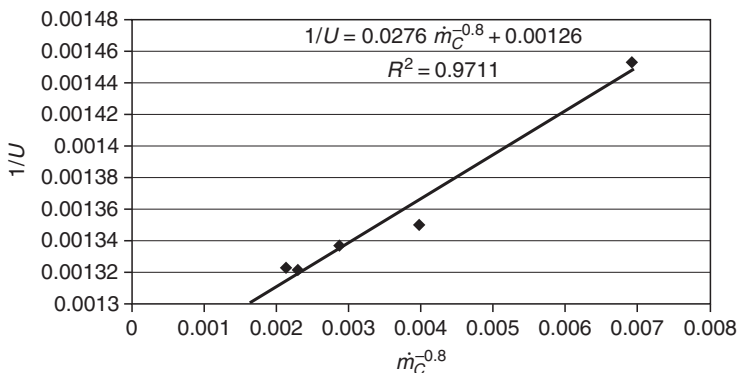


Figure 15.7 Wilson Plot for Illustrative Example 15.19.

SOLUTION: From the concept of resistances in series, it is clear that the total resistance is equal to the sum of the individual resistances (i.e., $1/U = R_o + R_w + R_s + R_i$). If the scale is neglected, the a term in the previous example represents the outside and wall resistances.

From Equation (15.49), the resistance of the wall is (neglecting any area correction) is

$$\begin{aligned} R_w &= \frac{\Delta r}{k} = \frac{(1.00 - 0.902)/2}{(12)(60)} \\ &= 0.681 \times 10^{-4} \text{ Btu/h} \cdot ^\circ\text{F} \end{aligned}$$

Therefore, the contribution of the wall resistance to the term a is 0.681×10^{-4} . The revised h_o is then

$$\begin{aligned} h_o &= \frac{1.0}{0.00126 - 0.0000681} \\ &= 839 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F} \end{aligned}$$

ILLUSTRATIVE EXAMPLE 15.21

Data from experiments based on Wilson's method produced the following results:

$$\begin{aligned} \text{Clean tube: } \frac{1}{U_{\text{clean}}} &= 0.00044 + \frac{0.0050}{(\dot{m})^{0.8}} \\ \text{Tube with scale: } \frac{1}{U_{\text{dirty}}} &= 0.00089 + \frac{0.0050}{(\dot{m})^{0.8}} \end{aligned}$$

Calculate the scale film coefficient if the wall resistance can be neglected.

SOLUTION: The overall resistance has been subdivided into component resistances. By comparing the two equations, one obtains the resistance associated with the scale to be:

$$R_{\text{dirty}} = R_s = 0.00089 - 0.00044 \\ = 0.00045$$

Thus

$$h_s = \frac{1}{0.00045} = 2222 \text{ Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F} \quad \blacksquare$$

ILLUSTRATIVE EXAMPLE 15.22

With reference to the Wilson method, discuss why the resistance of the wall is often neglected.

SOLUTION: The resistance of the wall is usually small (due to the high conductivity of the metal) in comparison to the resistances of the two flowing fluids. ■

REFERENCES

1. W. BADGER and J. BANCHERO, *Introduction to Chemical Engineering*, McGraw-Hill, New York City, NY, 1955.
2. F. INCROPERA and D. DE WITT, *Fundamentals of Heat and Mass Transfer*, 4th edition, Chapters 8, 11. John Wiley & Sons, Hoboken, NJ, 1996.
3. W. McCABE, J. SMITH, and P. HARRIOTT, *Unit Operations of Chemical Engineering*, 5th edition, Chapters 11–12. McGraw-Hill, New York City, NY, 1993.
4. W. McADAMS, *Heat Transmission*, 3rd edition, Chapters 9–10, McGraw-Hill, New York City, NY, 1954.
5. R. PERRY, D. GREEN, and J. O. MALONEY (editors), *Perry's Chemical Engineers' Handbook*, 7th edition, 10-67 to 10-137, McGraw-Hill, New York City, NY, 1997.
6. O. ZIMMERMAN and I. LAVINE, *Chemical Engineering Laboratory Equipment*, 2nd edition, 79–145. Industrial Research Service, Inc., Dover, NH, 1955.
7. J. FAMULARO: private communication to L. THEODORE, 1980.
8. L. THEODORE, F. RICCI, and T. VAN VLIET, *Thermodynamics for the Practicing Engineer*, John Wiley & Sons, Hoboken, NJ, 2009.
9. E. WILSON, *Trans. ASME*, 34, 47, New York City, NY, 1915.
10. A. ANDREWS, report submitted to L. THEODORE, Manhattan College, Bronx, NY, date unknown.
11. Unknown source, unknown date.
12. C. MOCKLER, submitted to L. THEODORE, Manhattan College, Bronx, NY, 2009.