Chapter 9

Forced Convection

INTRODUCTION

When a pot of water is heated on a stove, the portion of water adjacent to the bottom of the pot is the first to experience an increase in temperature. Eventually, the water at the top will also become hotter. Although some of the heat transfer from bottom to top is explainable by conduction through the water, most of the heat transfer is due to a second mechanism of heat transfer, *convection*. Agitation produced by a mixer, or the equivalent, adds to the convective effect. As the water at the bottom is heated, its density becomes lower. This results in convection currents as gravity causes the low density water to move upwards while being replaced by the higher density, cooler water from above. This macroscopic mixing is occasionally a far more effective mechanism for transferring heat energy through fluids than conduction. This process is called *natural* or *free* convection because no external forces, other than gravity, need be applied to move the energy in the form of heat. In most industrial applications, however, it is more economical to speed up the mixing action by artificially generating a current by the use of a pump, agitator, or some other mechanical device. This is referred to as *forced* convection and practicing engineers are primarily interested in this mode of heat transfer (i.e., most industrial applications involve heat transfer by convection).

Convective effects described above as *forced convection* are due to the bulk motion of the fluid. The bulk motion is caused by external forces, such as that provided by pumps, fans, compressors, etc., and is essentially independent of "thermal" effects. *Free* convection is the other effect that occasionally develops and was also briefly discussed above. This convective effect is attributed to buoyant forces that arise due to density differences within a system. It is treated analytically as another external force term in the momentum equation. The momentum (velocity) and energy (temperature) effects are therefore interdependent; consequently, both equations must be solved simultaneously. This treatment is beyond the scope of this text, but is available in the literature.(1) Also note that *both* forced and free convection may exist in some applications.

In order to circumvent the difficulties encountered in the analytical solution of microscopic heat-transfer problems, it is common practice in engineering to write

Heat Transfer Applications for the Practicing Engineer. Louis Theodore

the rate of heat transfer in terms of a heat transfer coefficient h, a topic that will receive extensive treatment in Part Three. If a surface temperature is T_S, and T_M represents the temperature of a fluid medium at some distance from the surface, one may write that

$$\dot{Q} = hA(T_S - T_M) \tag{9.1}$$

Since h and T_S are usually functions of the area A, the above equation may be rewritten in differential equation form

$$d\dot{Q} = h(T_S - T_M)\,dA \tag{9.2}$$

Integrating for the area gives

$$\int_0^{\dot{Q}} \frac{d\dot{Q}}{h(T_S - T_M)} = \int_0^A dA \tag{9.3}$$

This concept of a heat-transfer coefficient is an important concept in heat transfer, and is often referred to in the engineering literature as the *individual film* coefficient.

The expression in Equation (9.1) may be better understood by referring to heat transfer to a fluid flowing in a conduit. For example, if the resistance to heat transfer is thought of as existing only in a laminar film(2) adjacent to the wall of the conduit, the coefficient h may then be viewed as equivalent to $h/\Delta x_e$, where Δx_e is the equivalent thickness of a stationary film that offers the same resistance corresponding to the observed value of h. This is represented pictorially in Figure 9.1. In effect, this simply replaces the real resistance with a hypothetical one.

The reader should note that for flow past a surface, the velocity at the surface (s) is *zero* (no slip). The only mechanism for heat transfer at the surface is therefore conduction. One may therefore write

$$\dot{Q} = -kA\left.\frac{dT}{dx}\right|_s \tag{9.4}$$

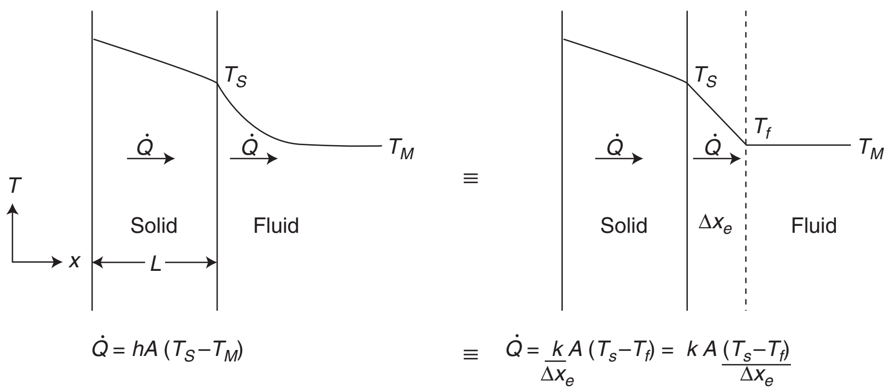

Figure 9.1 Convection temperature profile.

If the temperature profile in the fluid can be determined (i.e., $T = T(x)$), the gradient, dT/dx, can be evaluated at all points in the system including the surface, $dT/dx|_s$.

The heat transfer coefficient, h, was previously defined by

$$\dot{Q} = hA(T_S - T_M) \tag{9.1}$$

Therefore, combining Equations (9.1) and (9.4) leads to

$$-kA\left.\frac{dT}{dx}\right|_s = hA(T_S - T_M) \tag{9.5}$$

Since the thermal conductivity, k, of the fluid is usually known, information on h may be obtained. This information is provided later in this chapter.

To summarize, the transfer of energy by convection is governed by Equation (9.1) and is referred to as *Newton's Law of Cooling*,

$$\dot{Q} = hA(T_S - T_M) \tag{9.1}$$

where $\dot{Q}$ is the convective heat transfer rate (Btu/h); A, the surface area available for heat transfer (ft^2); T_S, the surface temperature (°F); T_M, the bulk fluid temperature (°F); and h is the aforementioned convection heat transfer coefficient, also termed the *film coefficient* or *film conductance* (Btu/h · ft^2 · °F or W/m^2 · K). Note that 1 Btu/h · ft^2 · °F = 5.6782 W/m^2 · K.

The magnitude of h depends on whether the transfer of heat between the surface and the fluid is by forced convection or by free convection, radiation, boiling, or condensation (to be discussed in later chapters). Typical values of h are given in Tables 9.1 and 9.2.

Topics covered in this chapter include:

Convective Resistances

Heat Transfer Coefficients: Qualitative Information

Heat Transfer Coefficients: Quantitative Information

Microscopic Approach

Table 9.1 Typical Film Coefficients

	h	
Mode	Btu/h · ft^2 · °F	W/m · K
Forced convection		
Gases	5–50	25–250
Liquids	10–4000	50–20,000
Free convection (see next chapter)		
Gases	1–5	5–25
Liquids	10–200	50–1000
Boiling/condensation	500–20,000	2500–100,000

Table 9.2 Film Coefficients in Pipes[a]

	h, Inside pipes	h, Outside pipes[b,c]
Gases	10–50	1–3 (n), 5–20 (f)
Water (liquid)	200–2000	20–200 (n), 100–1000 (f)
Boiling water[d]	500–5000	300–9000
Condensing steam[d]		1000–10,000
Nonviscous fluids	50–500	50–200 (f)
Boiling liquids[d]		200–2000
Condensing vapor[d]		200–400
Viscous fluids	10–100	20–50 (n), 10–100 (f)
Condensing vapor[d]		50–100

[a] $h = \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$.
[b] (n) = natural convection (see next chapter).
[c] (f) = forced convection.
[d] Additional details in Chapter 12.

CONVECTIVE RESISTANCES

Consider heat transfer across a flat plate, as pictured in Figure 9.2. The total resistance (R_t) may be divided into three contributions: the inside film (R_i), the wall (R_w), and the outside film (R_o),

$$\sum R = R_t = R_i + R_w + R_o \tag{9.6}$$

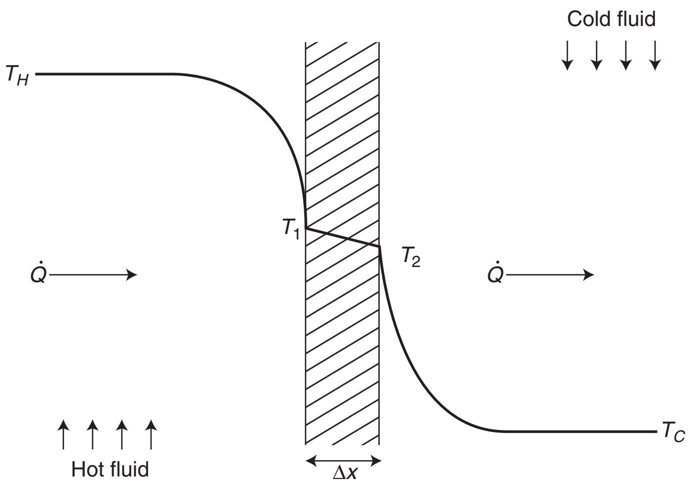

Figure 9.2 Flow past a flat plate.

where the individual resistances are defined by

$$R_t = \frac{1}{h_i A} + \frac{\Delta x}{kA} + \frac{1}{h_o A} \tag{9.7}$$

The term h_i is the *inside* film coefficient (Btu/h · ft^2 · °F); h_o, the *outside* film coefficient (Btu/h · ft^2 · °F); A, the surface area (ft^2); Δx, the thickness (ft); and, k, the thermal conductivity (Btu/h · ft · °F).

ILLUSTRATIVE EXAMPLE 9.1

Consider a closed cylindrical reactor vessel of diameter $D = 1$ ft, and length $L = 1.5$ ft. The surface temperature of the vessel, T_1, and the surrounding temperature, T_2, are 390°F and 50°F, respectively. The convective heat transfer coefficient, h, between the vessel wall and surrounding fluid is 4.0 Btu/h · ft · °F. Calculate the thermal resistance in °F · h/Btu.

***SOLUTION*:** Write the heat transfer rate equation:

$$\dot{Q} = hA(T_1 - T_2) \tag{9.1}$$

Since $D = 1$ ft and $L = 1.5$ ft, the total heat transfer area may be determined.

$$A = \pi(1)(1.5) + \frac{(2)\pi(1)^2}{4} = 6.28 \text{ ft}^2$$

Calculate the rate of heat transfer.

$$\dot{Q} = (4.0)(6.28)(390 - 50) = 8545 \text{ Btu/h}$$

Finally, calculate the thermal resistance associated with the film coefficient (see Equation 9.7).

$$R = \frac{1}{hA} = \frac{1}{(4.0)(6.28)} = 0.0398°\text{F} \cdot \text{h/Btu}$$

■

ILLUSTRATIVE EXAMPLE 9.2

Referring to the previous example, convert the resistance to K/W and °C/W.

***SOLUTION*:** First note that

$$1 \text{ W} = 3.412 \text{ Btu/h}$$

and a 1°C change corresponds to a 1.8°F change. Therefore,

$$\begin{aligned} R &= (0.0398)(3.412)/1.8 \\ &= 0.075°\text{C/W} \\ &= 0.075 \text{ K/W} \end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 9.3

Hot gas at 530°F flows over a flat plate of dimensions 2 ft by 1.5 ft. The convection heat transfer coefficient between the plate and the gas is 48 Btu/ft^2 · h · °F. Determine the heat transfer rate in Btu/h and kW from the air to one side of the plate when the plate is maintained at 105°F.

***SOLUTION*:** Assume steady-state conditions and constant properties. Write Newton's law of cooling to evaluate the heat transfer rate. Note that the gas is hotter than the plate. Therefore, $\dot{Q}$ will be transferred from the gas to the plate,

$$\dot{Q} = hA(T_S - T_M) \tag{9.1}$$

Substituting

$$\begin{aligned}\dot{Q} &= (48)(3)(530 - 105) = 61{,}200\,\text{Btu/h}\\ &= 61{,}200/3.4123 = 17{,}935\,\text{W}\\ &= 17.94\,\text{kW}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 9.4

The glass window shown in Figure 9.3 of area 3.0 m^2 has a temperature at the outer surface of 10°C. The glass has conductivity of 1.4 W/m · K. The convection coefficient (heat transfer coefficient) of the air is 100 W/m^2 · K. The heat transfer is 3.0 kW. Calculate the bulk temperature of the fluid.

***SOLUTION*:** Write the equation for heat convection. Equation (9.1) is given by

$$\dot{Q} = hA(T_S - T_M)$$

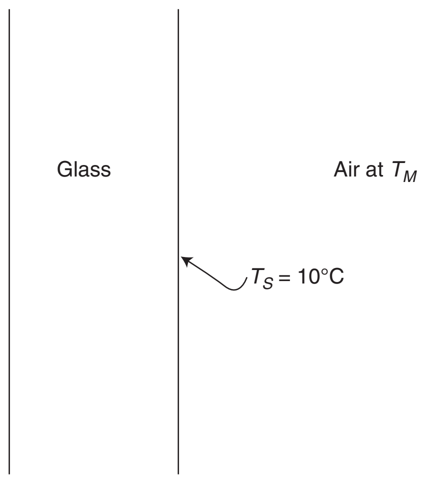

Figure 9.3 Convective glass.

where T_S = surface temperature at the wall and T_M = temperature of the bulk fluid. Solving for the unknown, the air temperature T_M, and substituting values yields

$$T_M = T_S - \frac{\dot{Q}}{hA}$$

$$T_M = (273 + 10) - \frac{3000}{(100)(3)} = 273\,\text{K} = 0^\circ\text{C}$$

■

ILLUSTRATIVE EXAMPLE 9.5

Refer to Illustrative Example 9.1. If the convective heat transfer coefficient, h, between the vessel wall and the surrounding fluid is constant at 4.0 Btu/h · ft^2 · °F and the plant operates 24 h/day, 350 days/yr, calculate the steady-state energy loss in Btu/yr.

***SOLUTION*:** The operating hours per year, N, are

$$N = (24)(350) = 8400\,\text{h/yr}$$

The yearly energy loss, $\dot{Q}$, is therefore

$$\dot{Q} = (8545)(8400) = 7.18 \times 10^7\,\text{Btu/yr}$$

■

Returning to Newton's law, the above development is now expanded and applied to the system pictured in Figure 9.2. Based on Newton's law of cooling,

$$\dot{Q}_1 = h_1 A(T_H - T_1) \tag{9.8}$$

$$\dot{Q}_2 = \frac{kA}{\Delta x}(T_1 - T_2) \tag{9.9}$$

$$\dot{Q}_3 = h_2 A(T_2 - T_C) \tag{9.10}$$

Assuming steady-state ($\dot{Q} = \dot{Q}_1 = \dot{Q}_2 = \dot{Q}_3$), and combining the above in a manner similar to the development provided in Chapter 7 yields:

$$\dot{Q} = \frac{T_H - T_C}{(1/h_1 A) + (\Delta x/kA) + (1/h_2 A)} \tag{9.11}$$

$$= \frac{T_H - T_C}{R_i + R_w + R_o} = \frac{T_H - T_C}{R_t} \tag{9.12}$$

HEAT TRANSFER COEFFICIENTS: QUALITATIVE INFORMATION

The heat transfer coefficient h is a function of the properties of the flowing fluid, the geometry and roughness of the surface, and the flow pattern of the fluid. Several methods are available for evaluating h: analytical methods, integral methods, and dimensional analyses. Only the results of these methods are provided in the next section. These may take the form of local and/or average values. Their derivation is not included. As will be demonstrated in the next Part, it is convenient to know these values for design purposes.

As one might suppose, heat transfer coefficients are higher for turbulent flow than with laminar flow, and heat transfer equipment are usually designed to take advantage of this fact. The heat transferred in most turbulent streams occurs primarily by the movement of numerous microscopic elements of fluid (usually referred to as eddies) in the system. Although one cannot theoretically predict the behavior of these eddies quantitatively with time, empirical equations are available for the practicing engineer.

Surface roughness can also have an effect on the heat transfer coefficient. However, the effect in laminar flow is, as one might expect, negligible. Except for a slight increase in the heat transfer area, the roughness has little to no effect on heat transfer with laminar flow. When the flow is turbulent, the roughness may have a significant effect. In general, the coefficient will not be affected if the rough "elements" do not protrude through the laminar sub-layer.

The heat transfer problems most frequently encountered have to do with the heating and cooling of fluids in pipes. Although the entrance effects can significantly affect overall performance in short pipes and conduits, this is normally neglected.

Before proceeding to a quantitative review of the equations that can be employed to calculate heat transfer film coefficients, it should be noted that a serious error can arise if the viscosity of the fluid is strongly dependent on temperature. The temperature gradient is normally greatest near the surface wall of the conduit and it is in this region that the velocity gradient is also greatest. The effect of temperature on the viscosity of the fluid at or near the wall may therefore have a pronounced effect on both the viscosity and temperature profiles. This effect is reflected in the heat transfer coefficient equations. If μ and μ_s represent the fluid viscosity at the average bulk fluid temperature and the viscosity at the wall temperature, respectively, the dimensionless term μ/μ_s is an empirical correction factor for the distortion of the velocity profile that results from the effect of temperature on viscosity.

A summary of the describing equations employed to predict convective heat transfer coefficients is presented in the next section. Most of the correlations contain dimensionless numbers, many of which were introduced in Chapter 3; some are redefined and reviewed again in this section. A host of illustrative examples complement the presentation.

HEAT TRANSFER COEFFICIENTS: QUANTITATIVE INFORMATION

Many heat transfer film coefficients have been determined experimentally. Typical ranges of film coefficients are provided in Tables 9.3–9.5. Explanatory details are provided in the subsections that follow. In addition to those provided in Tables 9.3–9.5, many empirical correlations can be found in the literature for a wide variety of fluids and flow geometries.[1]

Table 9.3 Summary of Forced Convection Heat Transfer Correlations for External Flow

Geometry	Correlation	Conditions
Flat plate: laminar	Hydrodynamic boundary layer, $\delta/x = 5\,\mathrm{Re}_x^{-0.5}$	Laminar flow, properties at the fluid film temperature, T_f
	Local friction factor, $f_x = 0.664\,\mathrm{Re}_x^{-0.5}$	Laminar, local, T_f
	Local Nusselt number $\mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{0.333}$	Laminar, local, T_f, $0.6 \le \mathrm{Pr} \le 50$
	Thermal boundary layer thickness, $\delta_t = \delta\,\mathrm{Pr}^{-0.333}$	Laminar, T_f
	Average friction factor, $\bar{f} = 1.328\,\mathrm{Re}_x^{-0.5}$	Laminar, average, T_f
	Average Nusselt number, $\overline{\mathrm{Nu}}_x = 0.644\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{0.333}$	Laminar, average, T_f, $0.6 \le \mathrm{Pr} \le 50$
Flat plate: liquid metal	Liquid metal, $\mathrm{Nu}_x = 0.565\,\mathrm{Pr}_x^{0.5}$	Laminar, local, T_f, $\mathrm{Pr} \le 0.05$
Flat plate: turbulent flow	Local friction factor, $f_x = 0.0592\,\mathrm{Re}_x^{-0.2}$	Turbulent, local, T_f, $\mathrm{Re}_x \le 10^8$
	Hydrodynamic boundary layer, $\delta/x = 0.37\,\mathrm{Re}_x^{0.2}$	Turbulent, local, T_f, $\mathrm{Re}_x \le 10^8$
	Local Nusselt number, $\mathrm{Nu}_x = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{0.333}$	Turbulent, local, T_f, $\mathrm{Re}_x \le 10^8$, $0.6 \lesssim \mathrm{Pr} \lesssim 60$
	Average Nusselt number, $\overline{\mathrm{Nu}} = 0.037\,\mathrm{Re}_L^{4/5}\,\mathrm{Pr}^{1/3}$	Turbulent, average, T_f, $\mathrm{Re} > \mathrm{Re}_{x,c} = 5 \times 10^5$, $0.6 < \mathrm{Pr} < 60$, $L \gg x_c$
	Average friction factor, $\bar{f} = 2\,j_H = 0.064\,\mathrm{Re}_x^{-0.2}$	Turbulent, average, T_f
Flat plate: mixed flow	Average friction factor, $\bar{f} = 0.074\,\mathrm{Re}_L^{-0.2} - 1472\,\mathrm{Re}_L^{-1}$	Mixed average, T_f, $\mathrm{Re}_{x,c} = 5 \times 10^5$, $\mathrm{Re}_L \le 10^8$
	Average Nusselt number, $\overline{\mathrm{Nu}} = (0.037\,\mathrm{Re}_L^{4/5} - 871)\,\mathrm{Pr}^{1/3}$	Mixed average, T_f, $\mathrm{Re}_{x,c} = 5 \times 10^5$, $\mathrm{Re}_L \le 10^8$, $0.6 < \mathrm{Pr} < 60$
Cylinder	Kundsen and Katz equation of average Nusselt number $\overline{\mathrm{Nu}}_D = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^{1/3}$ (Table 9.4 provides C and m information)	Average, T_f, $0.4 < \mathrm{Re}_d < 4 \times 10^5$, $\mathrm{Pr} \gtrsim 0.7$

(*Continued*)

Table 9.3 *Continued*

Geometry	Correlation	Conditions
Sphere	Whitaker equation for the average Nusselt number, $\overline{\mathrm{Nu}}_D = 2 + (0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3})\,\mathrm{Pr}^{0.4}(\mu/\mu_s)^{1/4}$	Average, properties at T_∞, $3.5 < \mathrm{Re}_D < 7.6 \times 10^4$, $0.71 < \mathrm{Pr} < 380$, $1.0 < (\mu/\mu_s) < 3.2$
Falling drop	$\overline{\mathrm{Nu}} = 2 + 0.6\,\mathrm{Re}_D^{1/2}\mathrm{Pr}^{1/3}[25(x/D)^{-0.7}]$	Average, T_∞
Packed bed of spheres	$\varepsilon\,\bar{j}_H = \varepsilon\,\bar{j}_m = 2.06\,\mathrm{Re}_D^{-0.575}$	Average, $\overline{T}$, $90 \le \mathrm{Re}_D \le 4000$, $\mathrm{Pr} \approx 0.7$

Notes: These correlations assume isothermal surfaces.

$\bar{j}_H$ = Colburn j_H factor = $\mathrm{St}\,\mathrm{Pr}^{2/3} = \mathrm{Nu}\,\mathrm{Pr}^{-1/3}$

ε = void fraction or porosity

For packed beds, properties are evaluated at the average fluid temperature, $\overline{T} = (T_i + T_o)/2$, or the average film temperature, $\overline{T}_f = (T_s + \overline{T})/2$.

Table 9.4 Constants of Knudsen and Katz for Heat Transfer to Cylinders in Cross Flow

Re	C	m
0.04–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

One of the more critical steps in solving a problem involving convection heat transfer is the estimation of the convective heat transfer coefficient. Cases considered in this section involve forced convection, while natural convection is considered in the next chapter. During forced convection heat transfer, the fluid flow may be either external to the surface (e.g., flow over a flat plate, flow across a cylindrical tube, or across a spherical object) or inside a closed surface (e.g., flow inside a circular pipe or a duct).

When a fluid flows over a flat plate maintained at a different temperature, heat transmission takes place by forced convection. The nature of the flow (laminar or turbulent) influences the forced convection heat transfer rate. The correlations used to calculate the heat transfer coefficient between the plate surface and the fluid are usually presented in terms of the average Nusselt number, $\overline{\mathrm{Nu}}_x$, the average Stanton number, $\overline{\mathrm{St}}$, (or, equivalently, the Colburn j_H factor), the local Reynolds number, Re_x, and the Prandtl number, Pr. In some cases, the Peclet number, Pe, is also used.

Correlation details as presented in Tables 9.3 and 9.4, apply in this section for:

1. Convection from a plane surface
2. Convection in circular pipes
   a. laminar flow
   b. turbulent flow
3. Convection in non-circular conduits
4. Convection normal to a cylinder
5. Convection normal to a number of circular tubes
6. Convection for spheres
7. Convection between a fluid and a packed bed

Topics 1 and 2 receives the bulk of the treatment. In addition, heat transfer to liquid metals is also discussed.

Flow Past a Flat Plate

For external flow, the fluid is assumed to approach the surface with a uniform constant velocity, V, and a uniform temperature, T_∞. The stagnant surface has a constant

Table 9.5 Summary of Forced Convection Correlations for Internal Flow

Correlation		Conditions
Darcy friction factor:[2]		
$f = 64/\mathrm{Re} = 64/\mathrm{Re}$	(1)	Laminar, fully developed
$\mathrm{Nu} = 4.364$	(2)	Laminar, fully developed, constant Q', UHF, $\mathrm{Pr} \geq 0.6$
$\mathrm{Nu}_D = 3.658$	(3)	Laminar, fully developed, constant T_s, UWT, $\mathrm{Pr} \geq 0.6$
Seider and Tate equation:		Laminar, combined entry length, properties at mean bulk temperature of the fluid
$\overline{\mathrm{Nu}} = 1.86\left(\mathrm{Re}_D \,\mathrm{Pr}\frac{D}{L}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$ $= 1.86\ \mathrm{Gz}^{0.333}\left(\frac{\mu}{\mu_s}\right)^{0.14}$	(4)	$\left[\left(\mathrm{Re}\,\mathrm{Pr}\frac{L}{D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}\right] \geq 2$ Constant wall temperature, T_s, $0.48 < \mathrm{Pr} < 16{,}700$, $0.0044 < (\mu/\mu_s) < 9.75$
$f = 0.316\,\mathrm{Re}_D^{-0.25}$	(5)	Turbulent, fully developed, smooth tubes, $\mathrm{Re} \leq 2 \times 10^4$
$f = 0.184\,\mathrm{Re}_D^{-0.2}$	(6)	Turbulent, fully developed, smooth tubes, $\mathrm{Re} \geq 2 \times 10^4$
Dittus–Boelter equation:		Turbulent, fully developed, $0.6 \leq \mathrm{Pr} \leq 160$, $\mathrm{Re} \geq 10{,}000$, $L/D \geq 10$
$\mathrm{Nu} = 0.023\,\mathrm{Re}_D^{0.8}\,\mathrm{Pr}^n$	(7)	$n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$

(Continued)

Table 9.5 *Continued*

Correlation		Conditions
Seider and Tate turbulent equation: $\mathrm{Nu} = 0.027\,\mathrm{Re}_D^{0.8}\,\mathrm{Pr}^{0.333}\left(\frac{\mu}{\mu_s}\right)^{0.14}$	(8)	Turbulent, fully developed, $0.7 \le \mathrm{Pr} \le 16{,}700$, $\mathrm{Re}_D \ge 10{,}000$, $L/D \gtrsim 10$, large change in fluid properties between the wall of the tube and the bulk flow
Nusselt turbulent entrance region equation: $\mathrm{Nu}_D = 0.036\,\mathrm{Re}_D^{0.8}\,\mathrm{Pr}^{0.333}\left(\frac{D}{L}\right)^{0.055}\left(\frac{\mu}{\mu_s}\right)^{0.14}$	(9)	Turbulent, entrance region, $10 < L/D < 400$, $0.7 \le \mathrm{Pr} \le 16{,}700$
Chilton–Colburn analogy: $j_H = \mathrm{St}\,\mathrm{Pr}^{2/3} = \mathrm{Nu}\,\mathrm{Re}^{-1}\,\mathrm{Pr}^{-1/3} = f/8$	(10)	Rough tubes, turbulent flow, St is based on the fluid bulk temperature, while Pr and f are based on the film temperature
Skupinski equation for liquid metals: $\mathrm{Nu}_D = 4.82 + 0.0185(\mathrm{Re}_D\,\mathrm{Pr})^{0.827}$ $= 4.82 + 0.0185\,\mathrm{Pe}_D^{0.827}$	(11)	Liquid metals, $0.003 < \mathrm{Pr} < 0.05$, turbulent, fully developed, constant $\dot{Q}_S$, $3.6 \times 10^3 < \mathrm{Re}_D < 9.05 \times 10^5$, $10^2 < \mathrm{Pe}_D$ (Peclet number based on diameter) $< 10^4$
Seban and Shimazaki equation for liquid metals: $\mathrm{Nu}_D = 5.0 + 0.025(\mathrm{Re}_D\,\mathrm{Pr})^{0.8}$ $= 5.0 + 0.025\,\mathrm{Pe}_D^{0.8}$	(12)	Liquid metals, turbulent, fully developed, constant T_S, $\mathrm{Pe}_D > 100$, properties at the average bulk temperature

Notes: Properties in Equations 2, 3, 7, 8, 11 and 12 are based on the mean fluid temperature T_m; properties in Equations 1, 5, and 6 are based on the average film temperature, $T_f \equiv (T_s + T_m)/2$; properties in Equation 4 are based on the average of the mean temperature of fluids entering and leaving $\overline{T} \equiv (T_{m,i} + T_{m,o})/2$.

$\mathrm{Re}_D \equiv D_h V/\nu$; D_h = hydraulic diameter $\equiv 4A/P_w$; $V \equiv \dot{m}/\rho A$.

UHF ≡ uniform heat flow; UWT ≡ uniform wall temperature.

temperature of T_S. Since a fluid is a material that cannot slip, the velocity of the fluid in contact with the surface is zero. A retardation layer forms near the stagnant surface. This layer is termed the "boundary-layer", or "hydrodynamic boundary layer". Its thickness, δ, increases as the fluid moves down the surface.

The dimensionless Reynolds number for flow over a flat plate depends on the distance, x, from the leading edge of the plate. It is termed the local Reynolds number, Re_x, and is defined by

$$\mathrm{Re}_x = xV/\nu = xV\rho/\mu \tag{9.13}$$

A value of Re_x less than 500,000 indicates that the boundary layer is in laminar flow. Critical Reynolds numbers above 500,000 indicate that most of the boundary layer is in turbulent flow. (Note that, even when the boundary layer is turbulent, the thin layer adjacent to the plate must still be in laminar flow.)

For laminar flow, the thickness of the laminar boundary layer, δ, is given by the Blasius equation,

$$\frac{\delta}{x} = \frac{5}{\sqrt{\mathrm{Re}_x}}, \quad \text{or} \quad \delta = 5\sqrt{\frac{\nu x}{V}} \tag{9.14}$$

where

δ = boundary layer thickness
ν = kinematic viscosity of the fluid = μ/ρ
μ = dynamic or absolute viscosity of the fluid
ρ = fluid density
V = free stream velocity of the fluid (outside the boundary layer)

Similar correlations for other types of external flow are listed in Table 9.3 with the coefficients for the Knudsen and Katz equation given in Table 9.4. When the fluid is a gas, the correlations are based on the assumption of incompressible flow, an assumption that is valid for Mach numbers less than 0.3. The Mach number is defined by the equation,

$$\mathrm{Ma} = V/c \tag{9.15}$$

where

c = speed of sound in the gas = $\sqrt{kRT/(MW)}$
k = ratio of gas heat capacities = c_p/c_v
c_p = constant pressure heat capacity
c_v = constant volume heat capacity
R = universal (ideal) gas constant
MW = molecular weight of the gas

For air, $k = 1.4$, $MW = 28.9$, $R = 8314.4\ \mathrm{m^2/s^2 \cdot K}$; the speed of sound in air from Equation (9.15) is therefore,

$$\begin{aligned} c_{\text{air}} &= 20\sqrt{T \text{ (in K)}};\ \text{m/s} \\ &= 49\sqrt{T \text{ (in °R)}};\ \text{ft/s} \end{aligned} \tag{9.16}$$

The heat transfer between the surface and the fluid is by convection. The heat transfer coefficient, h, will vary with position and is termed the local heat transfer coefficient, h_x. At any distance, x, on the plate surface, the heat flux, $\dot{Q}'$ (i.e., the heat flow rate through a unit cross-section, $\dot{Q}/A$) is

$$\dot{Q}' = \frac{\dot{Q}}{A} = h_x(T_S - T_\infty) \tag{9.17}$$

Since the fluid is stagnant at the plate surface, then (from a conduction heat transfer point-of-view)

$$\dot{Q}' = -k\left(\frac{dT}{dy}\right)_S \tag{9.18}$$

where k is the thermal conductivity of the fluid and y is the coordinate perpendicular to the direction of flow.

For a small area of the surface, dA, the heat transfer rate, $d\dot{Q}$, is

$$d\dot{Q} = h_x(T_S - T_\infty)\, dA \tag{9.19}$$

Integration of Equation (9.19) yields

$$\dot{Q} = (T_S - T_\infty)\int h_x\, dA = \bar{h}A(T_S - T_\infty) \tag{9.20}$$

where $\bar{h}$ is now the *average* heat transfer coefficient.

For flow over a flat plate of width, b, with the distance x measured from the leading edge of the plate, the surface area for heat transfer, A, is $A = bx$ so that $dA = b\, dx$. Combining this result and noting that the plate length, L, is

$$L = \int_0^L dx \tag{9.21}$$

yields the following relation for the average heat transfer coefficient over the whole length, L, of the plate:

$$\bar{h} = \frac{1}{L}\int_0^L h_x\, dx \tag{9.22}$$

For the entire plate, then,

$$\dot{Q}' = \frac{\dot{Q}}{A} = \bar{h}(T_S - T_\infty) \tag{9.23}$$

The local coefficient h_x for laminar flow is

$$\mathrm{Nu}_x = \frac{h_x x}{k} = (0.332)\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3} \tag{9.24}$$

The average coefficient over the interval $0 < x < L$ can be shown to be

$$\bar{h} = 2h_x|_{x=L}$$

so that

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}L}{k} = (0.664)\mathrm{Re}_L^{1/2}\mathrm{Pr}^{1/3} \tag{9.25}$$

Flow in a Circular Tube

Many industrial applications involve the flow of a fluid in a conduit (e.g., fluid flow in a circular tube). When the Reynolds number exceeds 2100, the flow is turbulent. Three cases are considered below. For commercial (rough) pipes, the friction factor is a function of the Reynolds number and the relative roughness of the pipe, ε/D.[(2)] In the region of complete turbulence (high Re and/or large ε/D), the friction factor depends mainly on the relative roughness.[(2)] Typical values of the roughness, ε, for various kinds of new commercial piping are available in the literature.[(2)]

1. *Fully Developed Turbulent Flow in a Smooth Pipe*
When the temperature difference is moderate, the Dittus and Boelter[(3)] equation may be used

$$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^n \tag{9.26}$$

where Nu = Nusselt number $= hD/k$, $\mathrm{Re} = DV/\nu = DV\rho/\mu$, and $\mathrm{Pr} = c_p\mu/k$. The properties in this equation are evaluated at the average (or mean) fluid bulk temperature, T_m, and the exponent n is 0.4 for heating (i.e., $T_1 < T_{s1}$, $T_2 < T_{s2}$) and 0.3 for cooling (i.e., $T_1 > T_{s1}$, $T_s > T_{s2}$). The equation is valid for fluids with Prandtl numbers, Pr, ranging from about 0.6 to 100.

If wide temperature differences are present in the flow, there may be an appreciable change in the fluid properties between the wall of the tube and the central flow. To take into account the property variations, the Seider and

Tate[1,4] equation should be used:

$$\mathrm{Nu} = 0.027\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}(\mu/\mu_s)^{0.14} \tag{9.27}$$

All properties are evaluated at bulk temperature conditions, except μ_s which is evaluated at the wall surface temperature, T_S.

2. *Turbulent Flow in Rough Pipes*
The recommended equation is the Chilton–Colburn j_H analogy between heat transfer and fluid flow. It relates the j_H factor ($=\mathrm{St}\,\mathrm{Pr}^{2/3}$) to the Darcy friction factor, f:

$$j_H = \mathrm{St}\,\mathrm{Pr}^{2/3} = \mathrm{Nu}\,\mathrm{Re}^{-1}\,\mathrm{Pr}^{-1/3} = f/8$$

or

$$\mathrm{Nu} = (f/8)\,\mathrm{Re}\,\mathrm{Pr}^{1/3} \tag{9.28}$$

where f is obtained from the Moody chart.[1–4] The average Stanton number (St) is based on the fluid bulk temperature, T_m, while Pr and f are evaluated at the film temperature, i.e., $(T_S + T_{\mathrm{av}})/2$.

Internal flow film coefficients are provided in Table 9.5. Special conditions that apply are provided in the note at the bottom of the Table.

Liquid Metal Flow in a Circular Tube

Liquid metals have small Prandtl numbers because they possess large thermal conductivities. This makes it possible for liquid metals to remove larger heat fluxes than other fluids. These fluids remain liquid at much higher temperatures than water or other substances. On the down side, liquid metals are difficult to handle, corrosive, and react violently with water. Despite their disadvantages, liquid metals are used in high heat flux systems. For example, they are commonly used to cool the cores of nuclear reactors.

For liquid metals, the Nusselt number has been found to depend on the product of the Reynolds and Prandtl numbers. This product is termed the aforementioned Peclet number:

$$(\mathrm{Re})(\mathrm{Pr}) = \mathrm{Pe} = DV/\alpha = DV\rho\,c_p/k \tag{9.29}$$

Table 9.5 lists the equations to use for predicting the heat transfer in liquid metals in smooth pipes. The Seban and Shimazaki equation is used for uniform wall temperature (UWT), or constant surface temperature, T_s, fully developed flow ($L/D > 60$) and a Peclet number, $\mathrm{Pe} > 100$.

$$\mathrm{Nu} = hD/k = 5.0 + 0.025\,\mathrm{Pe}^{0.8} \tag{9.30}$$

All properties are evaluated at the the average bulk temperature. The correlation of Srupinski is used for a uniform heat flux (UHF), Q_s', turbulent flow, fully developed flow, $0.003 < \mathrm{Pr} < 0.05$, $3.6 \times 10^3 < \mathrm{Re} < 9.05 \times 10^5$, and $100 < \mathrm{Pe} < 10{,}000$.

$$\mathrm{Nu} = 4.82 + 0.0185(\mathrm{Re\ Pr})^{0.827} = 4.82 + 0.0185\,\mathrm{Pe}^{0.827} \tag{9.31}$$

Convection Across Cylinders

Forced air coolers and heaters, forced air condensers, and cross-flow heat exchangers (to be discussed in Part Three) are examples of equipment that transfer heat primarily by the forced convection of a fluid flowing across a cylinder. For engineering calculations, the average heat transfer coefficient between a cylinder (at temperature T_s) and a fluid flowing across the cylinder (at a temperature T_∞) is calculated from the Knudsen and Katz equation (see Table 9.4)

$$\overline{\mathrm{Nu}}_D = \bar{h}D/k = C\,\mathrm{Re}^m\,\mathrm{Pr}^{1/3} \tag{9.32}$$

All the fluid physical properties are evaluated at the *mean film temperature* (the arithmetic average temperature of the cylinder surface temperature and the bulk fluid temperature). The constants C and m depend on the Reynolds number of the flow and are given in Table 9.4.

ILLUSTRATIVE EXAMPLE 9.6

Identify the following three dimensionless groups:

1. $h_f L/k_f$ (subscript f refers to fluid)
2. $h_f L/k_s$ (subscript s refers to solid surface)
3. (Reynolds number)(Prandtl number), i.e., (Re)(Pr)

***SOLUTION*:** As noted above,

1. $h_f L/k_f$ = Nusselt number = Nu
2. $h_f L/k_s$ = Biot number = *Bi* (see also Table 3.5 and Equation 9.35 later in Chapter)
3. $(\mathrm{Re})(\mathrm{Pr}) = \left(\frac{LV\rho}{\mu}\right)\left(\frac{c_p\mu}{k}\right) = \frac{LV\rho\, c_p}{k}$ = Peclet number = Pe ■

ILLUSTRATIVE EXAMPLE 9.7

For a flow of air over a horizontal flat plate, the local heat transfer coefficient, h_x, is given by the equation

$$h_x = \frac{25}{x^{0.4}},\ \mathrm{W/m^2 \cdot K}$$

where h_x is the local heat transfer coefficient at a distance x from the leading edge of the plate and x is the distance in meters. The critical Reynolds number, Re_{cr}, which is the Reynolds number at which the flow is no longer laminar, is 500,000.

Consider the flow of air at $T_\infty = 21°C$ ($c_p = 1004.8\ J/kg \cdot K$, $\nu = 1.5 \times 10^{-5}\ m^2/s$, $k = 0.025\ W/m \cdot K$, $Pr = 0.7$), at a velocity of 3 m/s, over a flat plate. The plate has a thermal conductivity $k = 33\ W/m \cdot K$, surface temperature, $T_S = 58°C$, width, $b = 1$ m, and length, $L = 1.2$ m. Calculate

1. the heat flux at 0.3 m from the leading edge of the plate
2. the local heat transfer coefficient at the end of the plate
3. the ratio $\bar{h}/h_x$ at the end of the plate

***SOLUTION*:**

1. Calculate the local heat transfer coefficient, h_x, at $x = 0.3$ m:

$$h_x = 25/x^{0.4} = 25/(0.3)^{0.4}$$
$$= 40.5\ W/m^2 \cdot K$$

Calculate the heat flux, $\dot{Q}'$, at $x = 0.3$ m:

$$\dot{Q}' = h_{x=0.3}(T_S - T_\infty)$$
$$= 40.5(58 - 21)$$
$$= 1497\ W/m^2$$

2. Calculate h_x at the end of the plate. Since $x = L$,

$$h_L = 25/(1.2)^{0.4}$$
$$= 23.2\ W/m^2 \cdot K$$

3. Calculate the average heat transfer coefficient, $\bar{h}$. Apply Equation (9.22).

$$\bar{h} = (1/L)\int_0^L h_x\,dx$$
$$= (1/L)\int_0^L (25/x^{0.4})\,dx$$
$$= (25/L)(1/0.6)(x^{0.6})|_0^L$$
$$= 41.67L^{0.6}/L = 41.67/L^{0.4} = 41.67/(1.2)^{0.4}$$
$$= 38.7\ W/m^2 \cdot K$$

Calculate the ratio $\bar{h}/h_x$ at the end of the plate:

$$\bar{h}/h_L = 38.7/23.2 = 1.668$$

■

ILLUSTRATIVE EXAMPLE 9.8

Refer to the previous example. Calculate the rate of heat transfer over the whole length of the plate.

***SOLUTION*:** Calculate the area for heat transfer for the entire plate

$$A = bL = (1)(1.2) = 1.2\,\text{m}^2$$

Apply Equation (9.23).

$$\begin{aligned}\dot{Q} &= \bar{h}A(T_s - T_\infty)\\ &= (38.7)(1.2)(58 - 21)\\ &= 1720\,\text{W}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 9.9

Air with a mass rate of 0.075 kg/s flows through a tube of diameter $D = 0.225$ m. The air enters at 100°C and, after a distance of $L = 5$ m, cools to 70°C. Determine the heat transfer coefficient of the air. The properties of air at 85°C are approximately, $c_p = 1010\,\text{J/kg}\cdot\text{K}$, $k = 0.030$ $\text{W/m}\cdot\text{K}$, $\mu = 208 \times 10^{-7}\,\text{N}\cdot\text{s/m}^2$, and Pr = 0.71.

***SOLUTION*:** The Reynolds number is

$$\text{Re} = \frac{4\dot{m}}{\pi D\mu} = \frac{(4)(0.075)}{(\pi)(0.225)(208 \times 10^{-7}\,\text{N})} = 20{,}400$$

Apply either Equation (7) in Tables 9.5 and/or Equation (9.26), with $n = 0.3$ for heating,

$$\text{Nu} = hD/k = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023(20{,}400)^{0.8}(0.71)^{0.3} = 58.0$$

Thus

$$\begin{aligned}h &= \left(\frac{k}{D}\right)\text{Nu} = (0.03/0.225)58.0\\ &= 7.73\,\text{W/m}^2\cdot\text{K}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 9.10

Calculate the average film heat transfer coefficient ($\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$) on the water side of a single pass steam condenser. The tubes are 0.902 inch inside diameter, and the cooling water enters at 60°F and leaves at 70°F. Employ the Dittus–Boelter equation and assume the average water velocity is 7 ft/s. Pertinent physical properties of water at an average temperature of 65°F are:

$$\rho = 62.3\,\text{lb/ft}^3$$
$$\mu = 2.51\,\text{lb/ft}\cdot\text{h}$$

$c_p = 1.0\,\text{Btu/lb}\cdot{}^\circ\text{F}$
$k = 0.340\,\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}$

***SOLUTION*:** For heating, Equation (9.26) applies:

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4}$$

or

$$h = \frac{k}{D}0.023\left(\frac{DVe}{\mu}\right)^{0.8}\left(\frac{c_p\mu}{k}\right)^{0.4}$$

The terms $\text{Re}^{0.8}$ and $\text{Pr}^{0.4}$ are given by

$$\begin{aligned}\text{Re}^{0.8} &= [(0.902/12)(7)(62.4)/(2.51/3600)]^{0.8}\\ &= (47{,}091)^{0.8} = 5475\\ \text{Pr}^{0.4} &= [(1.0)(2.51)/(0.340)]^{0.4}\\ &= (7.38)^{0.4}\\ &= 2.224\end{aligned}$$

Therefore,

$$\begin{aligned}h &= \left(\frac{0.340}{0.0752}\right)(0.023)(5475)(2.224)\\ &= 1266\,\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 9.11

Air at 1 atm and 300°C is cooled as it flows at a velocity 5.0 m/s through a tube with a diameter of 2.54 cm. Calculate the heat transfer coefficient if a constant heat flux condition is maintained at the wall and the wall temperature is 20°C above the temperature along the entire length of the tube.

***SOLUTION*:** The density is first calculated using the ideal gas law in order to obtain the Reynolds number. Apply the appropriate value of *R* for air on a *mass* basis.

$$\rho = \frac{P}{RT} = \frac{1(1.0132\times10^5)}{(287)(573)} = 0.6161\,\text{kg/m}^3$$

The following data is obtained from the Appendix assuming air to have the properties of nitrogen:

$$\text{Pr} = 0.713$$

$$\mu = 1.784 \times 10^{-5}\,\text{kg/m}\cdot\text{s}$$
$$k = 0.0262\,\text{W/m}\cdot\text{K}$$
$$c_p = 1.041\,\text{kJ/kg}\cdot\text{K}$$

Thus,

$$\text{Re} = \frac{DV\rho}{\mu} = \frac{(0.0254)(5)(0.6161)}{1.784 \times 10^{-5}} = 4386$$

Since the flow is turbulent, Equation (7) in Table 9.5 and/or Equation (9.26) applies:

$$\text{Nu} = \frac{\text{hD}}{\text{k}} = 0.023\,\text{Re}^{0.8}\text{Pr}^{0.3} = 0.023(4386)^{0.8}(0.713)^{0.3} = 17.03$$

Thus,

$$h = \left(\frac{k}{D}\right)\text{Nu} = \left(\frac{0.0262}{0.0254}\right)(17.03) = 17.57\,\text{W/m}^2\cdot\text{K}$$

■

ILLUSTRATIVE EXAMPLE 9.12

Water flows with an average velocity of 0.355 m/s through a long copper tube (inside diameter = 2.2 cm) in a heat exchanger. The water is heated by steam condensing at 150°C on the outside of the tube. Water enters at 15°C and leaves at 60°C. Determine the heat transfer coefficient, h, for the water. (Adapted from Griskey.[5])

***SOLUTION*:** First evaluate the average bulk temperature of the water which is

$$(15 + 60)/2 = 37.5^\circ\text{C}$$

Evaluating water properties (from the Appendix) at this temperature yields,

$$\rho = 993\,\text{kg/m}^3$$
$$\mu = 0.000683\,\text{kg/m}\cdot\text{s}$$
$$c_p = 4.17 \times 10^3\,\text{J/kg}\cdot\text{K}$$
$$k = 0.630\,\text{W/m}\cdot\text{K}$$

Calculate the Reynolds and Prandtl numbers for the flowing water:

$$\text{Re} = \frac{DV\rho}{\mu} = \frac{(0.022)(0.355)(993)}{6.83 \times 10^{-4}} = 11{,}350$$

$$\text{Pr} = \frac{c_p\mu}{k} = \frac{(4170)(6.83 \times 10^{-4})}{0.630} = 4.53$$

The flow is turbulent and, since the tube is a long one (i.e., no L/D effect), one may use the Seider and Tate turbulent relation [Equation (8) in Table 9.5 and/or Equation (9.25)] for internal flow:

$$\frac{hD}{k} = 0.027\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.33}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

All of the quantities in the above equation are known except μ_w. To obtain this value, the fluids' average wall temperature must be determined. This temperature is between the fluid's bulk average temperature of 37.5°C and the outside wall temperature of 150°C. Once again, use a value of 93.75°C (the average of the two). At this temperature (see Appendix)

$$\mu_w = 0.000306\,\mathrm{kg/m\cdot s}$$

Then

$$h = \frac{0.630}{0.022}(0.027)(11{,}350)^{0.8}(4.53)^{0.33}\left(\frac{0.000683}{0.000306}\right)^{0.14} = 2498.1\,\mathrm{W/m^2\cdot K}$$

■

An important dimensionless number that arises in some heat transfer studies and calculations is the Biot number, *Bi*. It plays a role in conduction calculations that also involve convection effects, and provides a measure of the temperature change within a solid relative to the temperature change between the surface of the solid and a fluid.

Consider once again the heat transfer process described in Figure 9.1. Under steady-state conditions, one may express the heat transfer rate as

$$\dot{Q} = \frac{kA}{L}(T - T_S) = hA(T_S - T_M) \tag{9.33}$$

The above equation may be rearranged to give

$$\frac{T - T_S}{T_S - T_M} = \frac{hA}{kA/L} = \frac{L/kA}{1/hA} = \frac{R_{\mathrm{cond}}}{R_{\mathrm{conv}}} = Bi = \frac{hL}{k} \tag{9.34}$$

The above equation indicates that, for small values of Bi, one may assume that the temperature across the solid is (relatively speaking) constant. If $Bi = 1.0$, the resistances are approximately equal. If $Bi \ll 1.0$, the resistance to conduction within the solid is much less than the resistance to convection across the fluid boundary layer. The reverse applies if $Bi \gg 1.0$. Generally, if $Bi < 2.0$, one may assume that for the scenario provided in Figure 9.1, the temperature profile in the solid is constant.

ILLUSTRATIVE EXAMPLE 9.13

Refer to Illustrative Example 9.7. Calculate the average Biot number.

***SOLUTION*:** Since the thickness of the flat plate is specified, the Biot number can be calculated.

$$Bi = \frac{hL}{k} = \frac{(38.7)(1.2)}{0.025} = 1858$$

■

ILLUSTRATIVE EXAMPLE 9.14

The surface temperature T_s of a circular conducting rod is maintained at 250°C by the passage of an electric current. The rod diameter is 10 mm, the length is 2.5 m, the thermal conductivity is 60 W/m · K, the density is 7850 kg/m^3, and the heat capacity is 434 J/kg · K. The rod is in a fluid at temperature $T_f = 25$°C, and the convection heat transfer coefficient is 140 W/m^2 · K. The thermal conductivity of the fluid is 0.6 W/m · K.

1. What is the thermal diffusivity of the bare rod?
2. What is the Nusselt number of the fluid in contact with the bare rod?
3. What is the Biot number of the bare rod?
4. Calculate the heat transferred from the rod to the fluid.

***SOLUTION*:**

1. The thermal diffusivity, α, of the bare rod is

$$\alpha = k/(\rho c_p) = 60/[(7850)(434)] = 1.76 \times 10^{-5}\ \mathrm{m^2/s}$$

2. The Nusselt number of the fluid is

$$\mathrm{Nu} = hD/k_f = (140)(0.01)/0.6 = 2.33$$

3. The Biot number of the bare rod is

$$Bi = hD/k_f = (140)(0.01)/60 = 0.0233$$

Finally, calculate $\dot{Q}$ for the bare rod:

$$\begin{aligned}\dot{Q}_{\mathrm{bare}} &= h(\pi DL)(T_s - T_f) = (140)(\pi)(0.01)(2.5)(250 - 25)\\ &= 2474\ \mathrm{W}\end{aligned}$$

■

ILLUSTRATIVE EXAMPLE 9.15

Comment on result (3) in the previous Illustrative example.

***SOLUTION*:** Since $Bi = 0.0233$, which is $\ll 2.0$, one may assume that the temperature profile in the electric current conducting rod is relatively constant. ■

MICROSCOPIC APPROACH

The microscopic (transport) equations employed to describe heat transfer in a flowing fluid are presented in Table 9.6.[6,7] The equations describe the temperature profile in a moving fluid. Note that this table is essentially an extension of the microscopic material presented in the previous two chapters. The six illustrative examples that follow, drawn from the work of Theodore,[7] involve the application of this table.

ILLUSTRATIVE EXAMPLE 9.16

An incompressible fluid enters the reaction zone of an insulated tubular (cylindrical) reactor at temperature T_0. The chemical reaction occurring in the zone causes a rate of energy per unit volume to be liberated. This rate is proportional to the temperature and is given by

$$(S)(T)$$

where S is a constant. Obtain the steady-state equations describing the temperature in the reactor zone if the flow is laminar. Neglect axial diffusion.

***SOLUTION*:** This problem is solved in cylindrical coordinates. The pertinent equation is "extracted" from Table 9.6. Based on the problem statement, the temperature is a function of *both* z and r, and the only velocity component is v_z:

$$v_z \frac{\partial T}{\partial z} = \alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right] + \frac{A}{\rho c_p}$$

Table 9.6 Energy-Transfer Equation for Incompressible Fluids[7]

Rectangular coordinates:

$$\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z} = \alpha\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{A}{\rho c_p} \tag{1}$$

Cylindrical coordinates:

$$\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\phi}{r}\frac{\partial T}{\partial \phi} + v_z\frac{\partial T}{\partial z} = \alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{A}{\rho c_p} \tag{2}$$

Spherical coordinates:

$$\begin{aligned}&\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\\ &= \alpha\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right] + \frac{A}{\rho c_p}\end{aligned} \tag{3}$$

Note. Lowercase v employed for velocity.

If one neglects axial diffusion

$$\frac{\partial^2 T}{\partial z^2} = 0$$

The source term A is given by

$$A = (S)(T)$$

The resulting equation is

$$v_z \frac{\partial T}{\partial z} = \alpha \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \right] + \left(\frac{S}{\rho c_p} \right) T \qquad (1)$$

If the flow is laminar[2],

$$v_z = V \left[1 - \left(\frac{r}{a} \right)^2 \right]$$

where $a =$ radius of cylinder and $V =$ maximum velocity, located at $r = 0$. Equation (1) now takes the form

$$V \left[1 - \left(\frac{r}{a} \right)^2 \right] \frac{\partial T}{\partial z} = \alpha \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \right] + \left(\frac{S}{\rho c_p} \right) T$$

■

ILLUSTRATIVE EXAMPLE 9.17

Refer to the previous example. Obtain the describing equation if the flow is plug.

***SOLUTION*:** If the flow is plug, v_z is constant[2] across the area of the reactor. Based on physical grounds, the radial-diffusion term is zero and T is solely a function of z. Equation (1) then becomes

$$v_z \frac{dT}{dz} = \left(\frac{S}{\rho c_p} \right) T$$

■

ILLUSTRATIVE EXAMPLE 9.18

Obtain the temperature profile in the reactor of Illustrative Example 9.17 if the flow is plug.

***SOLUTION*:** The describing DE is obtained from Illustrative Example 9.17,

$$v_z \frac{dT}{dz} = \left(\frac{S}{\rho c_p} \right) T$$

The equation is rewritten as

$$\frac{dT}{T} = \left(\frac{S}{\rho c_p v_z} \right) dz$$

Integrating the above equation gives

$$\ln T = \left(\frac{S}{\rho c_p v_z}\right)z + B$$

The BC is

$$T = T_0 \quad \text{at } z = 0$$

so that

$$B = \ln T_0$$

Therefore

$$\ln\left(\frac{T}{T_0}\right) = \left(\frac{S}{\rho c_p v_z}\right)z$$

or

$$T = T_0 e^{(S/\rho c_p v_z)z}$$

If energy is absorbed due to chemical reaction, the above solution becomes

$$T = T_0 e^{-(S/\rho c_p v_z)z}$$

■

ILLUSTRATIVE EXAMPLE 9.19

A fluid is flowing (in the y direction) through the reaction zone of a rectangular parallelepiped reactor. Chemical reaction is occurring in the zone and energy is liberated at a constant rate. Obtain the equation(s) describing the steady-state temperature in the reaction zone if the flow is laminar. Assume the height (in the z direction) of the reactor h to be very small compared to its width w. Do not neglect diffusion effects.

***SOLUTION*:** For laminar flow,

$$T = T(y, z)$$
$$v_y = v_y(z)$$

From Table 9.6, Equation (1),

$$v_y \frac{\partial T}{\partial y} = \alpha\left[\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{A}{\rho c_p}$$

In addition, Theodore[(7)] has shown that for this application,

$$v_y = \frac{6q}{wh^3}(zh - z^2)$$

where q = volumetric flow rate.

■

ILLUSTRATIVE EXAMPLE 9.20

Refer to Illustrative Example 9.19. Obtain the describing equation if the flow is plug.

***SOLUTION*:** For plug flow,

$$T = T(y)$$
$$v_y = \text{constant}$$

From Table 9.6, Equation (1),

$$v_y \frac{dT}{dy} = \alpha \frac{d^2T}{dy^2} + \frac{A}{\rho c_p}$$

■

ILLUSTRATIVE EXAMPLE 9.21

Consider the motion of a fluid past a thin flat plate as pictured in Figure 9.4. The boundary-layer thickness for energy transfer δ_T is arbitrarily defined at the points where $T = 0.99T_0$. The temperature of the fluid upstream from the plate is T_0. The temperature at every point on the plate is maintained at zero. Obtain the steady-state equations describing the temperature in and outside the boundary layer.

***SOLUTION*:** The describing DE is obtained from Equation (1) in Table 9.6. Since the flow is two-dimensional,

$$v_x = 0$$
$$v_y \neq 0$$
$$v_z \neq 0$$

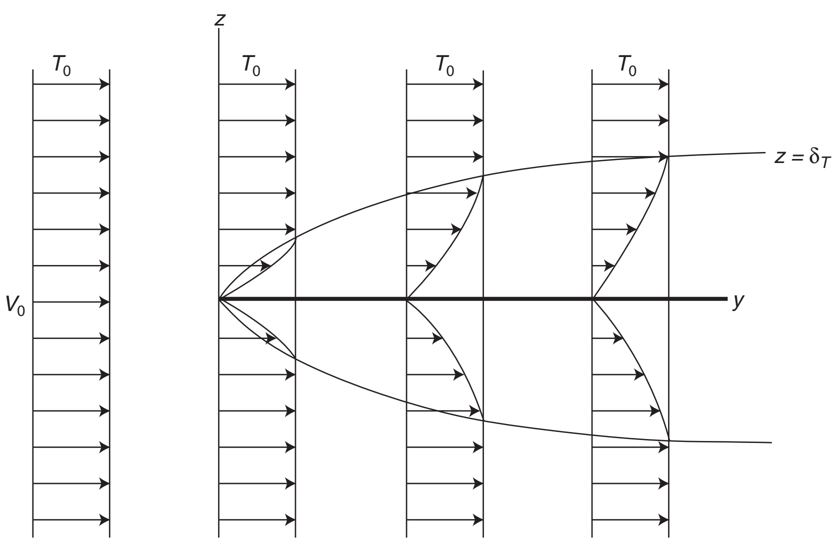

Figure 9.4 Boundary layers.

Based on the problem statement

$$T = T(y, z)$$

Since the flow is "primarily" in the y-direction, neglect thermal diffusion effects in that direction, i.e.,

$$\frac{\delta^2 T}{\delta y^2} = 0$$

The following equation results from Table 9.6:

$$v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} = \alpha \frac{\partial^2 T}{\partial z^2} \tag{1}$$

The equation of continuity in rectangular coordinates for this application is[2,7]

$$\frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \tag{2}$$

The BC are

$$\begin{aligned} T = 0\ , \quad & v_y = 0 \quad \text{at } z = 0 \\ T = T_0, \quad & v_y = V_0 \quad \text{at } z = \infty \end{aligned}$$

Equations (1) and (2) can now be solved to give the temperature profile in the system. The complete solution is not presented. However, the profile outside the boundary layer is (naturally)

$$T = T_0$$

■

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