

Chapter 3

Process Variables

INTRODUCTION

The author originally considered the title “State, Physical, and Chemical Properties” for this chapter. However, since these three properties have been used interchangeably and have come to mean different things to different people, it was decided to employ the title “Process Variables.” The three aforementioned properties were therefore integrated into this all-purpose title and eliminated the need for differentiating between the three.

This third chapter provides a review of some basic concepts from physics, chemistry, and engineering in preparation for material that is covered in later chapters. All of these topics are important in some manner to heat transfer. Because many of these are unrelated to each other, this chapter admittedly lacks the cohesiveness that chapters covering a single topic might have. This is usually the case when basic material from such widely differing areas of knowledge as physics, chemistry, and engineering is surveyed. Though these topics are widely divergent and covered with varying degrees of thoroughness, all of them will find use later in this text. If additional information of these review topics is needed, the reader is directed to the literature in the reference section of this chapter.

Three additional sections complement the presentation for this chapter. They are:

Units and Dimensional Consistency

Key Terms and Definitions

Determination of Dimensionless Groups

ILLUSTRATIVE EXAMPLE 3.1

Discuss the traditional difference between chemical and physical properties.

SOLUTION: Every compound has a unique set of *properties* that allows one to recognize and distinguish it from other compounds. These properties can be grouped into two main

categories: physical and chemical. *Physical properties* are defined as those that can be measured without changing the identity and composition of the substance. Key physical properties include viscosity, density, surface tension, melting point, boiling point, and so on. *Chemical properties* are defined as those that may be altered via reaction to form other compounds or substances. Key chemical properties include upper and lower flammability limits, enthalpy of reaction, autoignition temperature, and so on.

These properties may be further divided into two categories—intensive and extensive. *Intensive properties* are not a function of the quantity of the substance, while *extensive properties* depend on the quantity of the substance. ■

UNITS AND DIMENSIONAL CONSISTENCY

Almost all process variables are dimensional (as opposed to dimensionless) and there are units associated with these terms. It is for this reason that a section on units and dimensional consistency has been included with this chapter. The units used in the text are consistent with those adopted by the engineering profession in the United States. One usually refers to them as the English or engineering units. Since engineers are often concerned with units and conversion of units, both the English and SI system of units are used throughout this book. All quantities, including physical and chemical properties, are expressed using either of these two systems.

Equations are generally dimensional and involve several terms. For the equality to hold, each term in the equation must have the same dimensions (i.e., the equation must be dimensionally homogeneous or consistent). This condition can be easily proved. Throughout the text, great care is exercised in maintaining the dimensional formulas of all terms and the dimensional consistency of each equation. The approach employed will often develop equations and terms in equations by first examining each in specific units (e.g., feet rather than length), primarily for the English system. Hopefully, this approach will aid the reader and will attach more physical significance to each term and equation.

Consider the example of calculating the perimeter, P , of a rectangle with length, L , and height, H . Mathematically, this may be expressed as $P = 2L + 2H$. This is about as simple as a mathematical equation can be. However, it only applies when P , L , and H are expressed in the same units.

A conversion constant/factor is a term that is used to obtain units in a more convenient form. All conversion constants have magnitude and units in the term, but can also be shown to be equal to 1.0 (unity) with *no* units. An often used conversion constant is

$$12 \text{ inches/foot}$$

This term is obtained from the following defining equation:

$$12 \text{ in} = 1.0 \text{ ft}$$

If both sides of this equation are divided by 1 ft, one obtains

$$12 \text{ in/ft} = 1.0$$

Note that this conversion constant, like all others, is also equal to unity without any units. Another defining equation (Newton's Law) is

$$1 \text{ lb}_f = 32.2 \frac{\text{lb} \cdot \text{ft}}{\text{s}^2}$$

If this equation is divided by lb_f , one obtains

$$1.0 = 32.2 \frac{\text{lb} \cdot \text{ft}}{\text{lb}_f \cdot \text{s}^2}$$

This serves to define the conversion constant g_c , which of necessity is also equal to unity with no units. Other conversion constants are given in Appendix A.6 and B.1.

ILLUSTRATIVE EXAMPLE 3.2

Convert the following:

1. 8.03 yr to seconds (s)
2. 150 miles/h to yards/h
3. 100.0 m/s^2 to ft/min^2
4. 0.03 g/cm^3 to lb/ft^3

SOLUTION:

The conversion factors needed include:

365 day/yr
 24 h/day
 60 min/h
 60 s/min

1. The following is obtained by arranging the conversion factors so that units cancel to leave only the desired units

$$(8.03 \text{ yr}) \left(\frac{365 \text{ day}}{\text{yr}} \right) \left(\frac{24 \text{ h}}{\text{day}} \right) \left(\frac{60 \text{ min}}{\text{h}} \right) \left(\frac{60 \text{ s}}{\text{min}} \right) = 2.53 \times 10^8 \text{ s}$$

2. In a similar fashion,

$$\left(\frac{150 \text{ miles}}{\text{h}} \right) \left(\frac{5280 \text{ ft}}{\text{mile}} \right) \left(\frac{\text{yd}}{3 \text{ ft}} \right) = 2.6 \times 10^5 \text{ yd/h}$$

$$3. (100.0 \text{ m/s}^2) \left(\frac{100 \text{ cm}}{\text{m}} \right) \left(\frac{\text{ft}}{30.48 \text{ cm}} \right) \left(\frac{60 \text{ s}}{\text{min}} \right)^2 = 1.181 \times 10^6 \text{ ft/min}^2$$

$$4. (0.03 \text{ g/cm}^3) \left(\frac{\text{lb}}{454 \text{ g}} \right) \left(\frac{30.48 \text{ cm}}{\text{ft}} \right)^3 = 2.0 \text{ lb/ft}^3$$



Terms in equations must also be constructed from a “magnitude” viewpoint. Differential terms cannot be equated with finite or integral terms. Care should also be exercised in solving differential equations. In order to solve differential equations to obtain a description of the pressure, temperature, composition, etc., of a system, it is necessary to specify boundary and/or initial conditions for the system. This information arises from a description of the problem or the physical situation. The number of boundary conditions (BC) that must be specified is the sum of the highest-order derivative for each independent differential position term. A value of the solution on the boundary of the system is one type of boundary condition. The number of initial conditions (IC) that must be specified is the highest-order time derivative appearing in the differential equation. The value for the solution at time equal to zero constitutes an initial condition. For example, the equation

$$\frac{d^2T}{dz^2} = 0 \quad (3.1)$$

requires 2 BCs (in terms of z). The equation

$$\frac{dT}{dt} = 0; \quad t = \text{time} \quad (3.2)$$

requires 1 IC. And finally, the equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2}; \quad \alpha = \text{thermal diffusivity} \quad (3.3)$$

requires 1 IC and 2 BCs (in terms of y).

Problems are frequently encountered in heat transfer and other engineering work that involve several variables. Engineers are generally interested in developing functional relationships (equations) between these variables. When these variables can be grouped together in such a manner that they can be used to predict the performance of similar pieces of equipment, independent of the scale or size of the operations, something very valuable has been accomplished. More details on this topic are provided in the last section.

Consider, for example, the problem of establishing a method of calculating the power requirements for heating liquids in open tanks. The obvious variables would be the depth of liquid in the tank, the density and viscosity of the liquid, the speed of the agitator, the geometry of the agitator, and the diameter of the tank. There are therefore six variables that affect the power, or a total of seven terms that must be considered. To generate a general equation to describe power variation with these variables, a series of tanks having different diameters would have to be set up in order to gather data for various values of each variable. Assuming that ten different values of each of the six variables were imposed on the process, 10^6 runs would be required. Obviously, a mathematical method for handling several variables that

requires considerably less than one million runs to establish a *design method* must be available. In fact, such a method is available and it is defined as *dimensional analysis*.⁽¹⁾

Dimensional analysis is a powerful tool that is employed in planning experiments, presenting data compactly, and making practical predictions from models without detailed mathematical analysis. The first step in an analysis of this nature is to write down the units of each variable. The end result of a dimensional analysis is a list of pertinent dimensionless numbers,⁽¹⁾ details of which are presented in the last section.

Dimensional analysis is a relatively “compact” technique for reducing the number and the complexity of the variables affecting a given phenomenon, process or calculation. It can help obtain not only the most out of experimental data but also scale-up data from a model to a prototype. To do this, one must achieve similarity between the prototype and the model. This similarity may be achieved through dimensional analysis by determining the important aforementioned dimensionless numbers, and then designing the model and prototype such that the important dimensionless numbers are the same in both.⁽¹⁾

KEY TERMS AND DEFINITIONS

This section is concerned with key terms and definitions in heat transfer. Since heat transfer is an important subject that finds wide application in engineering, the understanding of heat transfer jargon is therefore important to the practicing engineer. It should also be noted that the same substance in its different phases may have various properties that have different orders of magnitude. As an example, heat capacity values are low for solids, high for liquids, and usually intermediate for gases.

Fluids

For the purpose of this text, a fluid may be defined as a substance that does not permanently resist distortion. An attempt to change the shape of a mass of fluid will result in layers of fluid sliding over one another until a new shape is attained. During the change in shape, shear stresses (forces parallel to a surface) will result, the magnitude of which depends upon the viscosity (to be discussed shortly) of the fluid and the rate of sliding. However, when a final shape is reached, all shear stresses will have disappeared. Thus, a fluid at equilibrium is free from shear stresses. This definition applies for both liquids and gases.

Temperature

Whether in a gaseous, liquid, or solid state, all molecules possess some degree of kinetic energy, i.e., they are in constant motion—vibrating, rotating, or translating. The kinetic energies of individual molecules cannot be measured, but the combined

effect of these energies in a very large number of molecules can. This measurable quantity is known as *temperature*; it is a macroscopic concept only and as such does not exist at the molecular level.

Temperature can be measured in many ways; the most traditional method makes use of the expansion of mercury (usually encased inside a glass capillary tube) with increasing temperature. (However, thermocouples or thermistors are more commonly employed in industry.) The two most commonly used temperature scales are the Celsius (or Centigrade) and Fahrenheit scales. The Celsius scale is based on the boiling and freezing points of water at 1-atm pressure; to the former, a value of 100°C is assigned, and to the latter, a value of 0°C . On the older Fahrenheit scale, these temperatures correspond to 212°F and 32°F , respectively. Equations (3.4) and (3.5) illustrate the conversion from one scale to the other:

$$^{\circ}\text{F} = 1.8(^{\circ}\text{C}) + 32 \quad (3.4)$$

$$^{\circ}\text{C} = (^{\circ}\text{F} - 32)/1.8 \quad (3.5)$$

where $^{\circ}\text{F}$ = a temperature on the Fahrenheit scale and $^{\circ}\text{C}$ = a temperature on the Celsius scale.

Experiments with gases at low-to-moderate pressures (up to a few atmospheres) have shown that, if the pressure is kept constant, the volume of a gas and its temperature are linearly related (see Charles' law in Chapter 5) and that a decrease of 0.3663% or $(1/273)$ of the initial volume is experienced for every temperature drop of 1°C . These experiments were not extended to very low temperatures, but if the linear relationship were extrapolated, the volume of the gas would *theoretically* be zero at a temperature of approximately -273°C or -460°F . This temperature has become known as *absolute zero* and is the basis for the definition of two *absolute* temperature scales. (An *absolute* scale is one that does not allow negative quantities.) These absolute temperature scales are the Kelvin (K) and Rankine ($^{\circ}\text{R}$) scales; the former is defined by shifting the Celsius scale by 273°C so that 0 K is equal to -273°C . The Rankine scale is defined by shifting the Fahrenheit scale by 460° . Equation (3.6) shows this relationship for both absolute temperatures:

$$\begin{aligned} \text{K} &= ^{\circ}\text{C} + 273 \\ ^{\circ}\text{R} &= ^{\circ}\text{F} + 460 \end{aligned} \quad (3.6)$$

Pressure

There are a number of different methods used to express a pressure term or measurement. Some of them are based on a force per unit area, e.g., pound-force per square inch, dyne, psi, etc., and others are based on fluid height, e.g., inches of water, millimeters of mercury, etc. Pressure units based on fluid height are convenient when the pressure is indicated by a difference between two levels of a liquid. Standard barometric (or atmospheric) pressure is 1 atm and is equivalent to 14.7 psi, or 33.91 ft of water, or 29.92 in of mercury.

Gauge pressure is the pressure relative to the surrounding (or atmospheric) pressure and it is related to the absolute pressure by the following equation:

$$P = P_a + P_g \quad (3.7)$$

where P is the absolute pressure (psia), P_a is the atmospheric pressure (psi) and P_g is the gauge pressure (psig). The absolute pressure scale is absolute in the same sense that the absolute temperature scale is absolute (i.e., a pressure of zero psia is the lowest possible pressure theoretically achievable—a perfect vacuum).

In stationary fluids subjected to a gravitational field, the *hydrostatic pressure difference* between two locations A and B is defined as

$$P_A - P_B = - \int_{z_A}^{z_B} \rho g dz \quad (3.8)$$

where z is positive in the vertical upward direction, g is the gravitational acceleration, and ρ is the fluid density. This equation will be revisited in Chapter 10.

Expressed in various units, the standard atmosphere is equal to 1.00 atmosphere (atm), 33.91 feet of water (ft H₂O), 14.7 pound-force per square inch absolute (psia), 2116 pound-force per square foot (psfa), 29.92 inches of mercury (in Hg), 760.0 millimeters of mercury (mm Hg), and 1.013×10^5 Newtons per square meter (N/m²). The pressure term will be reviewed again in several later chapters.

Vapor pressure, usually denoted p' , is an important property of liquids and, to a much lesser extent, of solids. If a liquid is allowed to evaporate in a confined space, the pressure in the vapor space increases as the amount of vapor increases. If there is sufficient liquid present, a point is eventually reached at which the pressure in the vapor space is exactly equal to the pressure exerted by the liquid at its own surface. At this point, a dynamic equilibrium exists in which vaporization and condensation take place at equal rates and the pressure in the vapor space remains constant.⁽²⁾ The pressure exerted at equilibrium is called the vapor pressure of the liquid. The magnitude of this pressure for a given liquid depends on the temperature, but not on the amount of liquid present. Solids, like liquids, also exert a vapor pressure. Evaporation of solids (called *sublimation*) is noticeable only for those with appreciable vapor pressures.

ILLUSTRATIVE EXAMPLE 3.3

Consider the following pressure calculations.

1. A liquid weighing 100 lb held in a cylindrical column with a base area of 3 in² exerts how much pressure at the base in lb_f/ft²?
2. If a pressure reading is 35 psig (pounds per square inch gauge), what is the absolute pressure?

SOLUTION:

1. See an earlier section in this chapter.

$$\begin{aligned} F &= mg/g_c = 100\text{lb}(1\text{ lb}_f/\text{lb}) \\ &= 100\text{ lb}_f \end{aligned}$$

Note: As already discussed, g_c is a conversion factor equal to $32.2\text{ lb} \cdot \text{ft}/\text{lb}_f \cdot \text{s}^2$; g is the gravitational acceleration, which is equal, or close to, $32.2\text{ ft}/\text{s}^2$ on Earth's surface.

Therefore,

$$\begin{aligned} P &= F/\text{area} = 100\text{ lb}_f/3\text{ in}^2 \\ &= 33.33\text{ lb}_f/\text{in}^2 \\ &= 4800\text{ lb}_f/\text{ft}^2 \end{aligned}$$

2. $P = P_g + P_a = 35 + 14.7$
 $= 49.7\text{ psia}$

This assumes the surrounding pressure to be atmospheric. ■

Moles and Molecular Weights

An atom consists of protons and neutrons in a nucleus surrounded by electrons. An electron has such a small mass relative to that of the proton and neutron that the weight of the atom (called the *atomic weight*) is approximately equal to the sum of the weights of the particles in its nucleus. Atomic weight may be expressed in *atomic mass units (amu) per atom* or *in grams per gram · atom*. One gram · atom contains 6.02×10^{23} atoms (Avogadro's number). The atomic weights of the elements are available in the literature.⁽³⁾

The *molecular weight* (MW) of a compound is the sum of the atomic weights of the atoms that make up the molecule. Atomic mass units per molecule (amu/molecule) or grams per gram · mole (g/gmol) are used for molecular weight. One gram · mole (gmol) contains an Avogadro number of molecules. For the English system, a pound · mole (lbmol) contains $454 \times 6.023 \times 10^{23}$ molecules.

Molal units are used extensively in heat transfer calculations as they greatly simplify material balances where chemical (including combustion) reactions are occurring. For mixtures of substances (gases, liquids, or solids), it is also convenient to express compositions in mole fractions or mole percentages instead of mass fractions. The mole fraction is the ratio of the number of moles of one component to the total number of moles in the mixture. Equations (3.9)–(3.12) express these relationships:

$$\begin{aligned} \text{moles of A} &= \frac{\text{mass A}}{\text{molecular weight of A}} \\ n_A &= \frac{m_A}{(\text{MW})_A} \end{aligned} \tag{3.9}$$

$$\begin{aligned} \text{mole fraction A} &= \frac{\text{moles A}}{\text{total moles}} \\ y_A &= \frac{n_A}{n} \end{aligned} \quad (3.10)$$

$$\begin{aligned} \text{mass fraction A} &= \frac{\text{mass A}}{\text{total mass}} \\ w_A &= \frac{m_A}{m} \end{aligned} \quad (3.11)$$

$$\begin{aligned} \text{volume fraction A} &= \frac{\text{volume A}}{\text{total volume}} \\ v_A &= \frac{V_A}{V} \end{aligned} \quad (3.12)$$

The reader should note that, in general, mass fraction (or percent) is not equal to mole fraction (or percent).

ILLUSTRATIVE EXAMPLE 3.4

If a 55-gal tank contains 20.0 lb of water,

1. How many pound · moles of water does it contain?
2. How many gram · moles does it contain?
3. How many molecules does it contain?

SOLUTION: The molecular weight of the water (H_2O) is

$$\begin{aligned} \text{MW} &= (2)(1.008) + (15.999) = 18.015 \text{ g/gmol} \\ &= 18.015 \text{ lb/lbmol} \end{aligned}$$

Therefore,

1. $(20.0 \text{ lb}) \left(\frac{1 \text{ lbmol}}{18.015 \text{ lb}} \right) = 1.11 \text{ lbmol water}$
2. $(20.0 \text{ lb}) \left(\frac{454 \text{ g}}{1 \text{ lb}} \right) \left(\frac{1 \text{ gmol}}{18.015 \text{ g}} \right) = 504 \text{ gmol water}$
3. $(504 \text{ gmol}) \left(\frac{6.023 \times 10^{23} \text{ molecules}}{1 \text{ gmol}} \right) = 3.036 \times 10^{26} \text{ molecules}$

Note that the volume of the tank does not impact the calculations. ■

Mass and Volume

The *density* (ρ) of a substance is the ratio of its mass to its volume and may be expressed in units of pounds per cubic foot (lb/ft^3), kilograms per cubic meter

(kg/m^3), and so on. For solids, density can be easily determined by placing a known mass of the substance in a liquid and determining the displaced volume. The density of a liquid can be measured by weighing a known volume of the liquid in a volumetric flask. For gases, the ideal gas law, to be discussed in Chapter 5, can be used to calculate the density from the pressure, temperature, and molecular weight of the gas.

Densities of pure solids and liquids are relatively independent of temperature and pressure and can be found in standard reference books.^(3,4) The *specific volume* (v) of a substance is its volume per unit mass (ft^3/lb , m^3/kg , etc.) and is, therefore, the inverse of its density.

The *specific gravity* (SG) is the ratio of the density of a substance to the density of a reference substance at a specific condition:

$$\text{SG} = \rho/\rho_{\text{ref}} \quad (3.13)$$

The reference most commonly used for *solids* and *liquids* is water at its maximum density, which occurs at 4°C ; this reference density is $1.000 \text{ g}/\text{cm}^3$, $1000 \text{ kg}/\text{m}^3$, or $62.43 \text{ lb}/\text{ft}^3$. Note that, since the specific gravity is a ratio of two densities, it is dimensionless. Therefore, any set of units may be employed for the two densities as long as they are consistent. The specific gravity of *gases* is used only rarely; when it is, air at the same conditions of temperature and pressure as the gas is usually employed as the reference substance.

Another dimensionless quantity related to density is the API (American Petroleum Institute) gravity, which is often used to indicate densities of fuel oils. The relationship between the API scale and specific gravity is

$$\text{degrees API} = {}^\circ\text{API} = \frac{141.5}{\text{SG}(60/60^\circ\text{F})} - 131.5 \quad (3.14)$$

where $\text{SG}(60/60^\circ\text{F})$ = specific gravity of the liquid at 60°F using water at 60°F as the reference.

Petroleum refining is a major industry. Petroleum products serve as an important fuel for the power industry, and petroleum derivatives are the starting point for many syntheses in the chemical industry. Petroleum is also a mixture of a large number of chemical compounds. A list of the common petroleum fractions derived from crude oil and ${}^\circ\text{API}$ is given in Table 3.1.

Table 3.1 API Values for Crude Oil Fractions

Fractions from crude oil	Approximate ${}^\circ\text{API}$
Light ends and gases	114
Gasoline	75
Naphtha	60
Kerosene	45
Distillate	35
Gas oil	28
Lube oil	18–30
Fuel oil (residue)	25–35

ILLUSTRATIVE EXAMPLE 3.5

The following information is given:

Specific gravity of liquid (methanol) = 0.92 (at 60°F)

Density of reference substance (water) = 62.4 lb/ft³ (at 60°F)

Determine the density of methanol in lb/ft³.

SOLUTION: Calculate the density of methanol in English units by multiplying the specific gravity by the density of water [see Equation (3.13)]:

$$\begin{aligned}\text{Density of methanol} &= (\text{specific gravity})(\text{density of water}) \\ &= (0.92)(62.4) \\ &= 57.4 \text{ lb/ft}^3\end{aligned}$$

The procedure is reversed in order to calculate specific gravity from density data. As noted above, the notation for density is usually, but not always, ρ . The notation ρ_V and ρ_G are also occasionally employed for gases while ρ_L may be employed for liquids. ■

Viscosity

Viscosity is a property associated with a fluid's resistance to flow. More precisely, this property accounts for the energy losses that result from the shear stresses that occur between different portions of a fluid moving at different velocities. The *absolute* or *dynamic viscosity* (μ) has units of mass per length · time; the fundamental unit is the *poise* (P), which is defined as 1 g/cm · s. This unit is inconveniently large for many practical purposes, and viscosities are frequently given in *centipoises* (0.01 poise), which is abbreviated cP. The viscosity of pure water at 68.6°F is 1.00 cP. In English units, absolute viscosity is expressed either as pounds (mass) per foot · second (lb/ft · s) or pounds per foot · hour (lb/ft · h). The absolute viscosity depends primarily on temperature and to a lesser degree on pressure. The *kinematic viscosity* (ν) is the absolute viscosity divided by the density of the fluid and is useful in certain fluid flow and heat transfer problems; the units for this quantity are length squared per time (e.g., square foot per second (ft²/s) or square meters per hour (m²/h)). A kinematic viscosity of 1 cm²/s is called a *stoke*. For pure water at 70°F, $\nu = 0.983$ cS (centistokes). Unlike the absolute viscosity, the kinematic viscosity of gases is a strong function of pressure due to its effect on the density. Because fluid viscosity changes rapidly with temperature, a numerical value of viscosity has no significance unless the temperature is specified.

Liquid viscosity is usually measured by the amount of time it takes for a given volume of liquid to flow through an orifice. The *Saybolt universal viscometer* is the most widely used device in the United States for the determination of the viscosity of fuel oils and liquids. It should be stressed that Saybolt viscosities, which are expressed in *Saybolt seconds* (SSU), are not even approximately proportional to

absolute viscosities except in the range above 200 SSU; hence, converting units from Saybolt seconds to other units requires the use of special conversion tables. As the time of flow decreases, the deviation becomes more marked. In any event, viscosity is an important property because of potential flow problems that are associated with viscous liquids and/or petroleum products.

The viscosities of air at atmospheric pressure and water are presented in Tables 3.2 and 3.3, respectively, as functions of temperature. Viscosities of other substances are available in the literature.^(3,4) Finally, the notation for the viscosity of a gas is usually μ (but occasionally μ_V or μ_G); μ_L is often employed for liquids.

Table 3.2 Viscosity of Air at 1 Atmosphere

T ($^{\circ}\text{C}$)	Viscosity, micropoise (μP)
0	170.8
18	182.7
40	190.4
54	195.8
74	210.2
229	263.8

1P = 100 cP = 10^6 μP ; 1 cP = 6.72×10^{-4} lb/ft · s.

Table 3.3 Viscosity of Water

T ($^{\circ}\text{C}$)	Viscosity, centipoise (cP)
0	1.792
5	1.519
10	1.308
15	1.140
20	1.000
25	0.894
30	0.801
35	0.723
40	0.656
50	0.594
60	0.469
70	0.406
80	0.357
90	0.317
100	0.284

ILLUSTRATIVE EXAMPLE 3.6

What is the kinematic viscosity of a gas, if the specific gravity and absolute viscosity are 0.8 and 0.02 cP, respectively?

SOLUTION: Convert to engineering units.

$$\mu = \left(\frac{0.02 \text{ cP}}{1} \right) \left(\frac{6.720 \times 10^{-4} \text{ lb/ft} \cdot \text{s}}{1 \text{ cP}} \right) = 1.344 \times 10^{-5} \text{ lb/ft} \cdot \text{s}$$

$$\rho = (\text{SG})(\rho_{\text{ref}}) = (0.8)(62.43 \text{ lb/ft}^3) = 49.94 \text{ lb/ft}^3$$

Substituting,

$$\begin{aligned} \nu &= \mu/\rho = (1.344 \times 10^{-5} \text{ lb/ft} \cdot \text{s})/(49.94 \text{ lb/ft}^3) \\ &= 2.691 \times 10^{-7} \text{ ft}^2/\text{s} \end{aligned}$$

ILLUSTRATIVE EXAMPLE 3.7

To illustrate the use of a monogram, calculate the absolute viscosity of a 98% sulfuric acid solution at 45°C.

SOLUTION: Referring to Figure C.1 (see Appendix C), the coordinates of 98% H₂SO₄ are given as $X = 7.0$ and $Y = 24.8$ (number 97). Locate these coordinates on the grid and call it point A. From 45°C, draw a straight line through point A and extend it to cut the viscosity axis. The intersection occurs at approximately 12 centipoise (cP). Therefore,

$$\mu = 12 \text{ cP} = 0.12 \text{ P} = 0.12 \text{ g/cm} \cdot \text{s}$$

Heat Capacity

The *heat capacity* of a substance is defined as the quantity of heat required to raise the temperature of that substance by one degree on a unit mass (or mole) basis. The term *specific heat* is frequently used in place of *heat capacity*. This is not strictly correct, because specific heat has been traditionally defined as the ratio of the heat capacity of a substance to the heat capacity of water. However, since the heat capacity of water is approximately 1 cal/g · °C or 1 Btu/lb · °F, the term *specific heat* has come to imply heat capacity.

For gases, the addition of heat to cause a 1° temperature rise may be accomplished either at constant pressure or at constant volume. Since the amounts of heat necessary are different for the two cases, subscripts are used to identify which heat capacity is being used— c_p for constant pressure and c_v for constant volume. For liquids and solids, this distinction does not have to be made since there is little difference between the two. Values of heat capacity are available in the literature.⁽²⁻⁵⁾

Heat capacities are often used on a *molar* basis instead of a *mass* basis, in which case the units become $\text{cal/gmol} \cdot ^\circ\text{C}$ or $\text{Btu/lbmol} \cdot ^\circ\text{F}$. To distinguish between the two bases, uppercase letters (C_P, C_V) are used in this text to represent the molar-based heat capacities, and lowercase letters (c_p, c_v) will be used for the mass-based heat capacities or specific heats.

Heat capacities are functions of both the temperature and pressure, although the effect of pressure is generally small and is neglected in almost all engineering calculations. The effect of temperature on C_P can be described by

$$C_P = \alpha + \beta T + \gamma T^2 \quad (3.15)$$

or

$$C_P = a + bT + cT^{-2} \quad (3.16)$$

Values for α, β, γ , and a, b, c , as well as average heat capacity information are provided in tabular form by Theodore et al.⁽²⁾ *Average* or *mean* heat capacity data over specific temperature ranges are also available. It should be noted that heat capacities for solids are a weak function of both temperature and pressure. Liquids are less dependent on pressure than solids, but are slightly influenced by temperature. Gases exhibit a strong temperature dependence while the effect of pressure is small, except near the critical state where the pressure dependence diminishes with increasing temperature.

ILLUSTRATIVE EXAMPLE 3.8

The following is given:

$$\text{Heat capacity of methanol} = 0.61 \text{ cal/g} \cdot ^\circ\text{C} \text{ (at } 60^\circ\text{F)}$$

Convert the heat capacities to English units.

SOLUTION: Note that $1.0 \text{ Btu/lb} \cdot ^\circ\text{F}$ is equivalent to $1.0 \text{ cal/g} \cdot ^\circ\text{C}$. This also applies on a mole basis, that is

$$1 \text{ Btu/lbmol} \cdot ^\circ\text{F} = 1 \text{ cal/gmol} \cdot ^\circ\text{C}$$

The heat capacity can also be converted from units of $\text{cal/g} \cdot ^\circ\text{C}$ to $\text{Btu/lb} \cdot ^\circ\text{F}$ using appropriate conversion factors:

$$\left(\frac{0.61 \text{ cal}}{\text{g} \cdot ^\circ\text{C}}\right) \left(\frac{454 \text{ g}}{\text{lb}}\right) \left(\frac{\text{Btu}}{252 \text{ cal}}\right) \left(\frac{^\circ\text{C}}{1.8^\circ\text{F}}\right) = 0.61 \text{ Btu/lb} \cdot ^\circ\text{F} \quad \blacksquare$$

Thermal Conductivity

Experience has shown that when a temperature difference exists across a solid body, energy in the form of heat will transfer from the high-temperature region to the low-temperature region until thermal equilibrium (same temperature) is reached. This

Table 3.4 Typical Thermal Conductivity Values

Material	k , $W/m \cdot ^\circ C$
Solid metals	15–400
Liquids	0.1–10
Gases	0.01–0.2

mode of heat transfer, where vibrating molecules pass along kinetic energy through the solid, is called *conduction*. Liquids and gases may also transport heat in this fashion. The property of *thermal conductivity*, k , provides a measure of how fast (or how easily) heat flows through a substance. It is defined as the amount of heat that flows per unit time through a unit surface area of unit thickness as a result of a unit difference in temperature. It is also a property of the material. Typical units for conductivity are $Btu \cdot ft/h \cdot ft^2 \cdot ^\circ F$ or $Btu/h \cdot ft \cdot ^\circ F$ or $W/m \cdot ^\circ C$. Typical values of k are given in Table 3.4.

Thermal conductivities of solids are primarily dependent on temperature. In general, thermal conductivities for a pure metal decrease with temperature; alloying elements tend to reverse this trend. Liquids are for the most part temperature dependent but insensitive to pressure. The thermal conductivities of most liquids decrease with increasing temperature. It should be noted that water has the highest thermal conductivity of all liquids except the so-called liquid metals. The thermal conductivity of a gas increases with increasing temperature; it is a weak function of pressure for pressures close to atmospheric. However, the effect of pressure is significant for high pressures. Also note that the thermal conductivity of steam exhibits a strong pressure dependence.

With regard to heat transfer applications, this particular property finds extensive application in designing heat exchangers (see Chapters 7 and 8, and Part III).

ILLUSTRATIVE EXAMPLE 3.9

The following data is given:

$$\text{Thermal conductivity of methanol} = 0.0512 \text{ cal/m} \cdot \text{s} \cdot ^\circ \text{C (at } 60^\circ \text{F)}$$

Convert the thermal conductivity to English units.

SOLUTION: The factor for converting $\text{cal/m} \cdot \text{s} \cdot ^\circ \text{C}$ to $\text{Btu/ft} \cdot \text{h} \cdot ^\circ \text{F}$ can be shown to be 2.419. The thermal conductivity of methanol can therefore be converted to $\text{Btu/ft} \cdot \text{h} \cdot ^\circ \text{F}$ from $\text{cal/m} \cdot \text{s} \cdot ^\circ \text{C}$ as follows:

$$\begin{aligned} k &= \left(\frac{0.0512 \text{ cal}}{\text{m} \cdot \text{s} \cdot ^\circ \text{C}} \right) \left(\frac{\text{Btu}}{252 \text{ cal}} \right) \left(\frac{0.3048 \text{ m}}{\text{ft}} \right) \left(\frac{3600 \text{ s}}{\text{h}} \right) \left(\frac{^\circ \text{C}}{1.8^\circ \text{F}} \right) \\ &= (0.0512)(2.419) \\ &= 0.124 \text{ Btu/ft} \cdot \text{h} \cdot ^\circ \text{F} \end{aligned}$$

Note that the usual engineering notation for thermal conductivity is k and that $1 \text{ Btu/h} \cdot \text{ft} \cdot ^\circ\text{F} = 1.7307 \text{ W/m} \cdot \text{K}$. ■

ILLUSTRATIVE EXAMPLE 3.10

Estimate the thermal conductivity ($\text{W/m} \cdot \text{K}$) of air at ambient conditions.

SOLUTION: Refer to Table D.7 in the Appendix. The thermal conductivity is approximately $0.025 \text{ W/m} \cdot \text{K}$. ■

Thermal Diffusivity

A useful combination of terms already considered is the *thermal diffusivity*, α ; it is defined by

$$\alpha = \frac{k}{\rho c_p} \quad (3.17)$$

As with the kinematic viscosity, the units of α are ft^2/h or m^2/s . As one might expect, thermal energy diffuses rapidly through substances with high α and slowly through those with low α . There is a strong dependence of α for gases for both pressure and temperature.

Reynolds Number

The Reynolds number, Re , is a dimensionless number that indicates whether a moving fluid is flowing in the laminar or turbulent mode. *Laminar* flow is characteristic of fluids flowing slowly enough so that there are no eddies (whirlpools) or macroscopic mixing of different portions of the fluid. (*Note:* In any fluid, there is always *molecular* mixing due to the thermal activity of the molecules; this is distinct from *macroscopic* mixing due to the swirling motion of different portions of the fluid.) In laminar flow, a fluid can be imagined to flow like a deck of cards, with adjacent layers sliding past one another. *Turbulent* flow is characterized by eddies and macroscopic currents. In practice, moving gases are generally in the turbulent region. For flow in a pipe, a Reynolds number above 2100 is an indication of turbulent flow.

The Reynolds number is dependent on the fluid velocity, density, viscosity, and some *length* characteristic of the system or conduit; for pipes, this characteristic length is the inside diameter:

$$Re = DV\rho/\mu = DV/\nu \quad (3.18)$$

where Re = Reynolds number
 D = inside diameter of the pipe (ft)
 V = fluid velocity (ft/s)
 ρ = fluid density (lb/ft^3)
 μ = fluid viscosity ($\text{lb}/\text{ft} \cdot \text{s}$)
 ν = fluid kinematic viscosity (ft^2/s)

Any consistent set of units may be used with Equation (3.18). In addition, the reader should note that both v and V are employed to represent velocity while volume is solely designated V (see next chapter).

ILLUSTRATIVE EXAMPLE 3.11

Calculate the Reynolds number for a fluid flowing through a 5-inch diameter pipe at 10 fps (feet per second) with a density of 50 lb/ft^3 and a viscosity of 0.65 cP . Is the flow turbulent or laminar?

SOLUTION: By definition

$$\text{Re} = DV\rho/\mu \quad (3.18)$$

Substitution yields

$$\begin{aligned} \text{Re} &= \left(\frac{50 \text{ lb}}{\text{ft}^3}\right) \left(\frac{10 \text{ ft}}{\text{s}}\right) \left(\frac{5 \text{ in}}{1}\right) \left(\frac{1 \text{ ft}}{12 \text{ in}}\right) \left(\frac{1}{0.65 \text{ cP}}\right) \left(\frac{1 \text{ cP}}{6.720 \times 10^{-4} \text{ lb/ft} \cdot \text{s}}\right) \\ &= (50 \text{ lb/ft}^3)(10 \text{ ft/s})[(5/12)\text{ft}]/(0.65 \times 6.72 \times 10^{-4} \text{ lb/ft} \cdot \text{s}) \\ &= 477,000 \end{aligned}$$

The Reynolds number is >2100 ; therefore, the flow is turbulent. ■

Kinetic Energy

Consider a body of mass, m , that is acted upon by a force, F . If the mass is displaced a distance, dL , during a differential interval of time, dt , the energy expended is given by

$$dE_{KE} = m\left(\frac{a}{g_c}\right)dL \quad (3.19)$$

Since the acceleration is given by $a = dV/dt$,

$$dE_{KE} = \left(\frac{m}{g_c}\right)\left(\frac{dV}{dt}\right)dL = \left(\frac{m}{g_c}\right)\left(\frac{dL}{dt}\right)dV \quad (3.20)$$

Noting that $V = dL/dt$, the above expression becomes:

$$dE_{KE} = m\left(\frac{V}{g_c}\right)dV \quad (3.21)$$

If this equation is integrated from V_1 to V_2 , the change in energy is

$$\Delta E_{KE} = \frac{m}{g_c} \int_{V_1}^{V_2} V dV = \frac{m}{g_c} \left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) \quad (3.22)$$

or

$$\Delta E_{KE} = \left(\frac{mV_2^2}{2g_c} - \frac{mV_1^2}{2g_c} \right) = \Delta \left(\frac{mV^2}{2g_c} \right) \quad (3.23)$$

The term above is defined as the change in kinetic energy.

The reader should note that for flow through pipes and tubes, the above kinetic energy term can be retained as written if the velocity profile is uniform i.e., the local velocities at all points in the cross-section are the same. Ordinarily, there is a velocity gradient across the passage; this introduces an error in the above calculation, the magnitude of which depends on the nature of the velocity profile and the shape of the cross section. For the usual case where the velocity is approximately uniform (e.g., turbulent flow), the error is not serious and, since the error tends to cancel because of the appearance of kinetic terms on each side of any energy balance equation, it is customary to ignore the effect of velocity gradients. When the error cannot be ignored, the introduction of a correction factor is needed.⁽¹⁾

Potential Energy

A body of mass m is raised vertically from an initial position z_1 to z_2 . For this condition, an upward force at least equal to the weight of the body must be exerted on it, and this force must move through the distance $z_2 - z_1$. Since the weight of the body is the force of gravity on it, the minimum force required is given by Newton's law:

$$F = \frac{ma}{g_c} = m \left(\frac{g}{g_c} \right) \quad (3.24)$$

where g is the local acceleration of gravity. The minimum work required to raise the body is the product of this force and the change in vertical displacement, that is,

$$\Delta E_{PE} = F(z_2 - z_1) = m \left(\frac{g}{g_c} \right) (z_2 - z_1) = \Delta \left(m \frac{g}{g_c} z \right) \quad (3.25)$$

The term above is defined as the change in the potential energy of the mass.

ILLUSTRATIVE EXAMPLE 3.12

As part of a heat transfer course, a young environmental engineering major has been requested to determine the potential energy of water before it flows over a waterfall 10 meters in height above ground level conditions.

SOLUTION: The potential energy of water depends on two considerations:

1. the quantity of water, and
2. a reference height.

For the problem at hand, take as a basis 1 kilogram of water and assume the potential energy to be zero at ground level conditions. Apply Equation (3.25). Based on the problem statement, set $z_1 = 0$ m and $z_2 = 10$ m, so that

$$\Delta z = 10 \text{ m}$$

At ground level conditions,

$$PE_1 = 0$$

Therefore

$$\begin{aligned} \Delta(PE) &= PE_2 - PE_1 = PE_2 \\ PE_2 &= E_{PE_2} = m(g/g_c)z_2 \\ &= (1 \text{ kg})(9.8 \text{ m/s}^2)(10 \text{ m}) \\ &= 98 \text{ kg} \cdot \text{m}^2/\text{s}^2 \\ &= 98 \text{ J} \end{aligned}$$

■

DETERMINATION OF DIMENSIONLESS GROUPS

To scale-up (or scale-down) a process, it is necessary to establish geometric and dynamic similarities between the model and the prototype. These two similarities are discussed below.

Geometric similarity implies using the same geometry of equipment. A circular pipe prototype should be modeled by a tube in the model. Geometric similarity establishes the scale of the model/prototype design. A 1/10th scale model means that the characteristic dimension of the model is 1/10th that of the prototype.

Dynamic similarity implies that the important dimensionless numbers must be the same in the model and the prototype. For a flowing fluid, which is being heated in a tube of an exchanger, it has been shown that the friction factor, f , is a function of the dimensionless Reynolds number (defined earlier). By selecting the operating conditions such that Re in the model equals the Re in the prototype, then the friction factor in the prototype will equal the friction factor in the model.⁽¹⁾

It should now be apparent that dimensionless numbers, as well as dimensionless groups, play an important role in the engineering analysis of heat transfer phenomena. The Buckingham π (pi) theorem may be used to determine the number of independent dimensionless groups required to obtain a relation describing a physical phenomenon. The theorem requires that equations relating variables must be dimensionally homogenous (consistent). The approach can be applied to the problem of correlating experimental heat transfer data for a fluid flowing across (or through) a heated tube. This is defined as a convective process that requires expressing a heat transfer coefficient, a term and topic to be defined and treated in Chapter 9, in terms of its dependent variables. The units of this coefficient are energy/time \cdot area \cdot temperature. For this process, it is reasonable to expect that the physical quantities listed below are pertinent to the description of this system. These have been expressed in terms of the four primary dimensions—length (L), mass (M), time (t), and temperature (T).

The aforementioned heat transfer coefficient is assumed a function of the following variables

$$h = \psi(D, k, V, \rho, \mu, C) \tag{3.26}$$

Each variable is expressed below in terms of the primary dimensions

Tube diameter	D	L
Thermal conductivity	k	$ML/t^2 \cdot T$
Fluid velocity	V	L/t
Fluid density	ρ	M/L^3
Fluid viscosity	μ	$M/L \cdot t$
Heat capacity	C	$L^2/t^2 \cdot T$
Heat transfer coefficient	h	$M/t^3 \cdot T$

Note that energy can be expressed as $M \cdot L^2/t^2$, heat capacity is represented with a capital C , and velocity with a capital V .

The following procedure is employed for the Buckingham π theorem:

$$\pi = D^a k^b V^c \rho^d \mu^e C^f h^g \tag{3.27}$$

Substituting primary units gives

$$\pi = (L)^a (M \cdot L/t^2 \cdot T)^b (L/t)^c (M/L^3)^d (M/L \cdot t)^e (L^2/t^2 \cdot T)^f (M/t^3 \cdot T)^g \tag{3.28}$$

For π to be dimensionless, the exponents of each primary dimension must separately add up to zero, i.e.,

$$\begin{aligned} M: & \quad b + d + e + g = 0 \\ L: & \quad a + b + c - 3d - e + 2f = 0 \\ t: & \quad -3b - c - e - 2f - 3g = 0 \\ T: & \quad -b - f - g = 0 \end{aligned}$$

There are seven unknown but only four equations at this point. Since h is the dependent variable, set $g = 1$. A trial-and-error procedure, based to some extent on experience is now applied. Set $c = d = 0$.

For this condition (there are now four equations and four unknowns), resulting in

$$a = 1; \quad b = -1; \quad e = f = 0 \tag{3.29}$$

and the first dimensionless group, π_1 , is

$$\pi_1 = \frac{hD}{k} = \text{Nusselt number} = \text{Nu} \tag{3.30}$$

For π_2 , once again set $g = 0$, and $a = 1, f = 0$. The solution now becomes

$$b = 0; \quad c = d = 1; \quad e = -1 \tag{3.31}$$

with

$$\pi_2 = \frac{DV\rho}{\mu} = \text{Reynolds number} = \text{Re}$$

For π_3 , set $e = 1$, and $c = g = 0$. The solution now becomes

$$\pi_3 = \frac{C\mu}{k} = \text{Prandtl number} = \text{Pr} \quad (3.32)$$

Thus, the function relationship can be written as

$$\text{Nu} = f(\text{Re}, \text{Pr}) \quad (3.33)$$

Another dimensionless number that appears in combined natural and forced convection systems as well as just natural convection (Chapter 10) systems is the Graetz number. There are several defining equations for this number:

$$\text{Gz} = (\text{Re})(\text{Pr})\left(\frac{D}{L}\right); \quad L = \text{characteristic length} \quad (3.34)$$

Table 3.5 Dimensionless Numbers

Biot number (Bi)	$\frac{hL}{k}$
Graetz number (Gz)	$\frac{\dot{m}c_p}{kL}$
Grashof number (Gr)	$\frac{L^3 \rho^2 g \beta \Delta T}{\mu^2}$
Liquid Jacob number (Ja)	$\frac{c_p \Delta T_e}{h_{\text{vap}}}$
Nusselt number (Nu)	$\frac{hD}{k}$
Peclet number (Pe)	$\frac{DV\rho c_p}{k}$
Prandtl number (Pr)	$\frac{c_p \mu}{k}$
Reynolds number (Re)	$\frac{DV\rho}{\mu}, \frac{DG}{\mu}$
Stanton number (St)	$\frac{h}{c_p V\rho}, \frac{h}{c_p G}$

$$\text{Gz} = \frac{4 \dot{m} c_p}{\pi kL}; \quad \dot{m} = \text{mass flow rate} \quad (3.35)$$

However, it should be noted that some define the Graetz number by

$$\text{Gz} = \frac{\dot{m} c_p}{kL} \quad (3.36)$$

These dimensionless numbers, as well as others, will appear with regular consistency in both this and the next two Parts. Extensive reference to these terms is made in Chapter 9.

There are several dimensionless numbers encountered in heat transfer applications. Some of the more important and most commonly used numbers are provided in alphabetical order in Table 3.5, with some of the terms to be defined later. With reference to Table 3.5, also note that

$$\text{Nu} = (\text{St})(\text{Re})(\text{Pr}) \quad (3.37)$$

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