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General Stream

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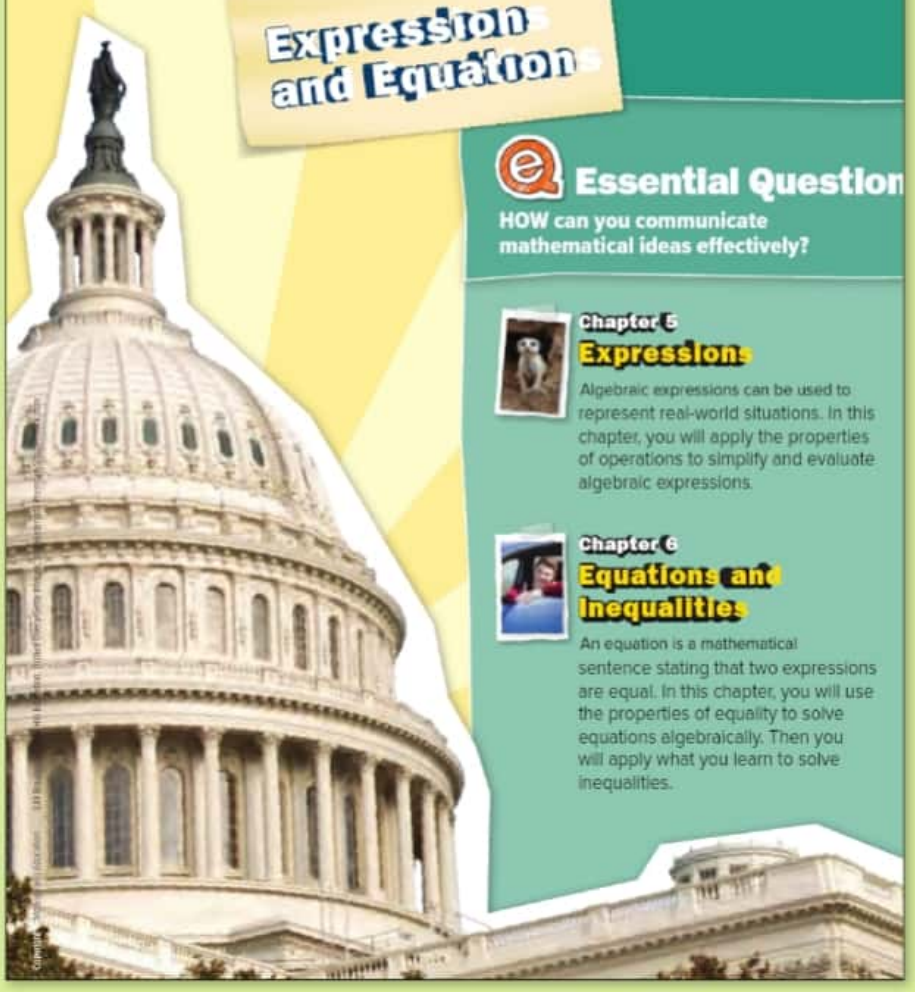
Expressions and Equations



Essential Question

HOW can you communicate mathematical ideas effectively?

Chapter 5

Expressions

Algebraic expressions can be used to represent real-world situations. In this chapter, you will apply the properties of operations to simplify and evaluate algebraic expressions.

Chapter 6

Equations and Inequalities

An equation is a mathematical sentence stating that two expressions are equal. In this chapter, you will use the properties of equality to solve equations algebraically. Then you will apply what you learn to solve inequalities.

Essential Question

At the end of this unit, students should be able to answer "How can you communicate mathematical ideas effectively?"

Each chapter explores a different essential question that assists students in answering the unit question. The lessons in each chapter include exercises that lead students to various aspects of the essential question.

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

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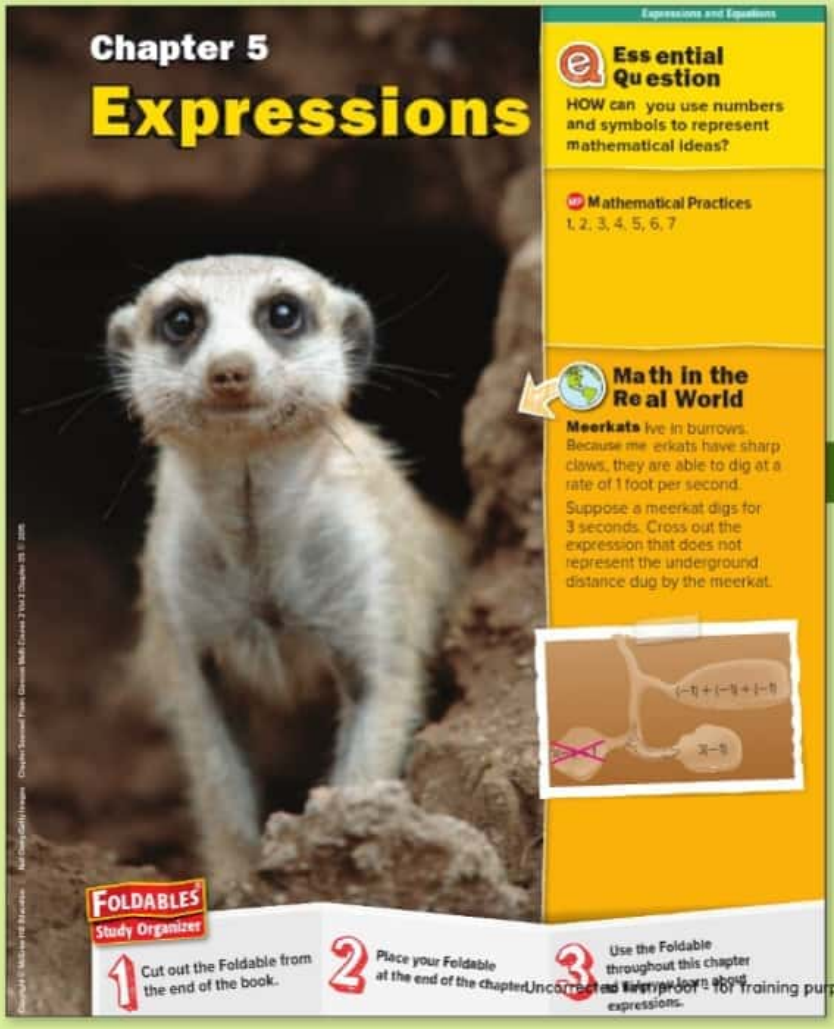


Focus narrowing the scope

This chapter focuses on **Expressions and Equations**.

Coherence connecting within and across grades

Previous	Now	Next
Students added, subtracted, multiplied, and divided integers and fractions.	Students use properties of operations to write and simplify expressions.	Students will apply the use of expressions to writing and solving equations and formulas.

Rigor pursuing concepts, fluency, and applications

The Levels of Complexity charts located throughout this chapter indicate how the exercises progress from conceptual understanding and procedural skills and fluency, to application and critical thinking.

Launch the Chapter

Math in the Real World

Meerkats Students may forget to represent the distance a meerkat digs below the surface as a negative integer. Remind students that the word *below* indicates use of a negative sign.

Are You Ready?

Use this page to determine if students have skills that are needed for the chapter.

Quick Review

Students with strong math backgrounds may opt to go directly to Quick Check.

Example(s)	Skill
1, 2	Evaluate and Write Powers
3, 4	Multiply Integers

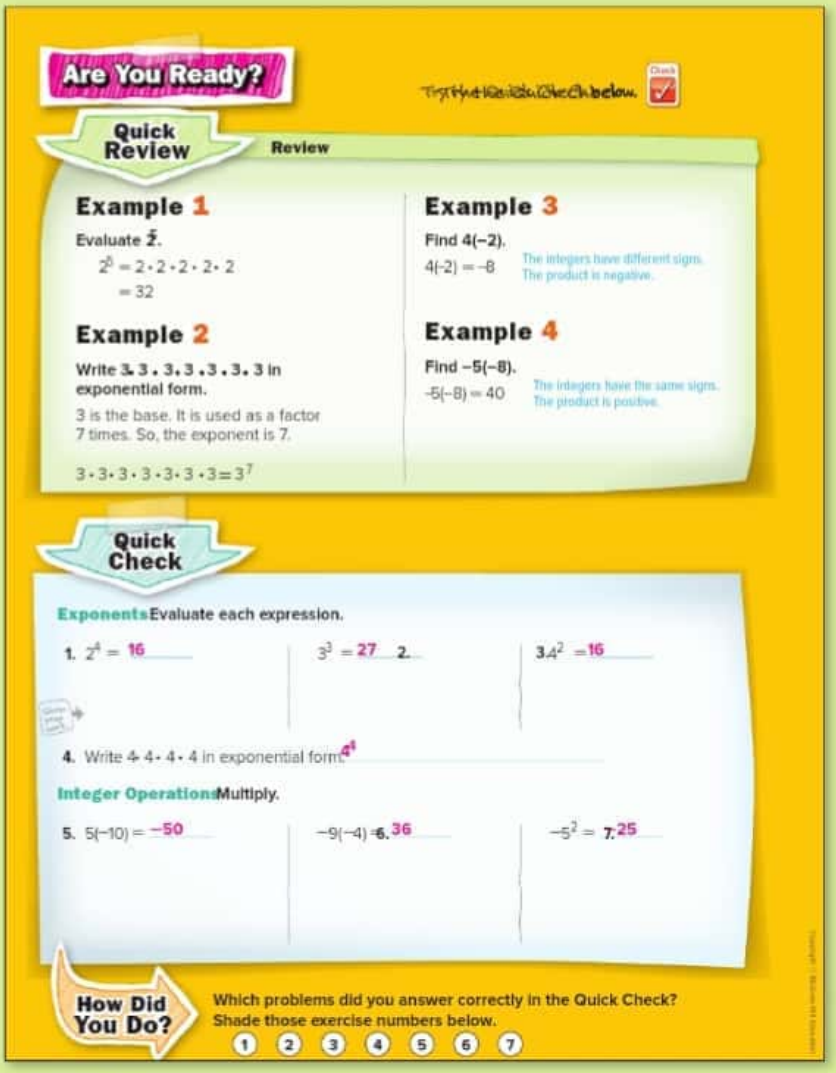
Quick Check

If students have difficulty with the exercises, present another example to clarify any misconceptions.

Exercises 1–3
Evaluate 7^4. 2,401

Exercise 4
Write $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$ in exponential form. 8^6

Exercises 5–7
Find $6(-4)$. -24



Are You Ready?

Quick Review

Review

Example 1

Evaluate 2^5.

$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$= 32$

Example 2

Write $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ in exponential form.

3 is the base. It is used as a factor 7 times. So, the exponent is 7.

$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^7$

Example 3

Find $4(-2)$.

$4(-2) = -8$ The integers have different signs. The product is negative.

Example 4

Find $-5(-8)$.

$-5(-8) = 40$ The integers have the same signs. The product is positive.

Quick Check

Exponents Evaluate each expression.

1. $2^4 = 16$
2. $3^3 = 27$
3. $4^2 = 16$
4. Write $4 \cdot 4 \cdot 4 \cdot 4$ in exponential form. 4^4

Integer Operations Multiply.

5. $5(-10) = -50$
6. $-9(-4) = 36$
7. $-5^2 = 25$

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4 5 6 7

Expressions and Equations

Lesson 1

Algebraic Expressions

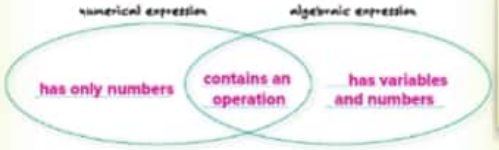
Vocabulary Start-Up

A **variable** is a symbol that represents an unknown quantity. An **algebraic expression** such as $n + 2$, is an expression that contains variables, numbers, and at least one operation.

Variable → $n + 2$

Write each of the following phrases in the correct section of the Venn diagram: *contains an operation, has variables and numbers, has only numbers.*

Characteristics of Expressions



Essential Question

HOW can you use numbers and symbols to represent mathematical ideas?

Vocabulary

variable
algebraic expression
algebra
coefficient
define a variable

Mathematical Practices
1, 2, 3, 4

Real-World Link

The expression $(F - 32) \times \frac{5}{9}$ can be used to convert a temperature from Fahrenheit to Celsius. In this algebraic expression, the variable F represents the temperature in degrees Fahrenheit.

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Evaluate simple algebraic expressions.

Coherence connecting within and across grades

Previous
Students used the order of operations to simplify numerical expressions.

Now
Students evaluate algebraic expressions given values for the variables.

Next
Students will add and subtract linear expressions.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 353.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL LA **Circle the Sage** Poll the class to find out who understands the difference between numerical and algebraic expressions. Those students, the sages, spread about the room. Divide the rest of the class into teams. Each student surrounds a sage while the sage explains what they know. The classmates listen and ask questions. Students return to their teams and discuss what they learned. MP 1, 2, 3

Alternate Strategy

AL Point out that the main difference between numerical and algebraic expressions is that an algebraic expression contains a variable and a numerical expression does not. MP 2, 7

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Evaluate an algebraic expression.

AL • *In what order do you perform operations on an expression?* parentheses, exponents, multiply and divide, add and subtract

• *What is the variable?* n

OL • *What is the value of n?* -4

• *What is the first step in evaluating the expression?* Replace n with -4.

BL • *Evaluate $2n + 6$ if $n = -4$. How does this answer compare to the answer in Example 1?* -2; It is the same.

Need Another Example?
Evaluate $3(t - 4)$ if $t = 6$. 6

2. Evaluate an algebraic expression.

AL • *What are the variables?* w and v

• *What is the value of w? of v?* 5; 3

OL • *What is the first step in evaluating the expression?* Replace w with 5 and v with 3.

• *What should you do next?* Multiply, then subtract.

BL • *Can you evaluate the expression $2b + 3c$? Explain.* no; you don't know the value of b and c, so you can't evaluate the expression.

Need Another Example?
Evaluate $6s - 3t$ if $s = 4$ and $t = -2$. 30

3. Evaluate an algebraic expression.

AL • *How do you find 3^3?* Multiply $3 \cdot 3 \cdot 3$.

OL • *What is the first step in evaluating the expression?* Replace y with 3.

BL • *Evaluate the expression if $y = \frac{3}{4^2}$* $\frac{5}{16}$

Need Another Example?
Evaluate $3m^2 - 4$ if $m = -3$. 23

Work Zone

Order of Operations
1. Evaluate the expressions inside grouping symbols.
2. Evaluate all powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

a. 5

b. 7

c. 9

d. 36

e. −69

f. 25

Evaluate an Algebraic Expression

The branch of mathematics that involves expressions with variables is called **algebra**. In algebra, the multiplication sign is often omitted.

$6d$ (6 times d) $\quad 9st$ (9 times s times t) $\quad mn$ (m times n) $\quad a^3$ (a times a times a)

The numerical factor of a multiplication expression that contains a variable is called **coefficient**. So, 6 is the coefficient of $6d$.

Expressions like $\frac{y}{2}$ can be written as $\frac{1}{2} \cdot 2$ or $y \times \frac{1}{2}$.

Examples

1. Evaluate $2(n + 3)$ if $n = -4$.

$2(n + 3) = 2(-4 + 3)$ Replace n with -4.
$= 2(-1)$ Evaluate inside the parentheses.
$= -2$ Multiply.

2. Evaluate $8w - 2v$ if $w = 5$ and $v = 3$.

$8w - 2v = 8(5) - 2(3)$ Replace w with 5 and v with 3.
$= 40 - 6$ Do all of the multiplication first.
$= 34$ Subtract 6 from 40.

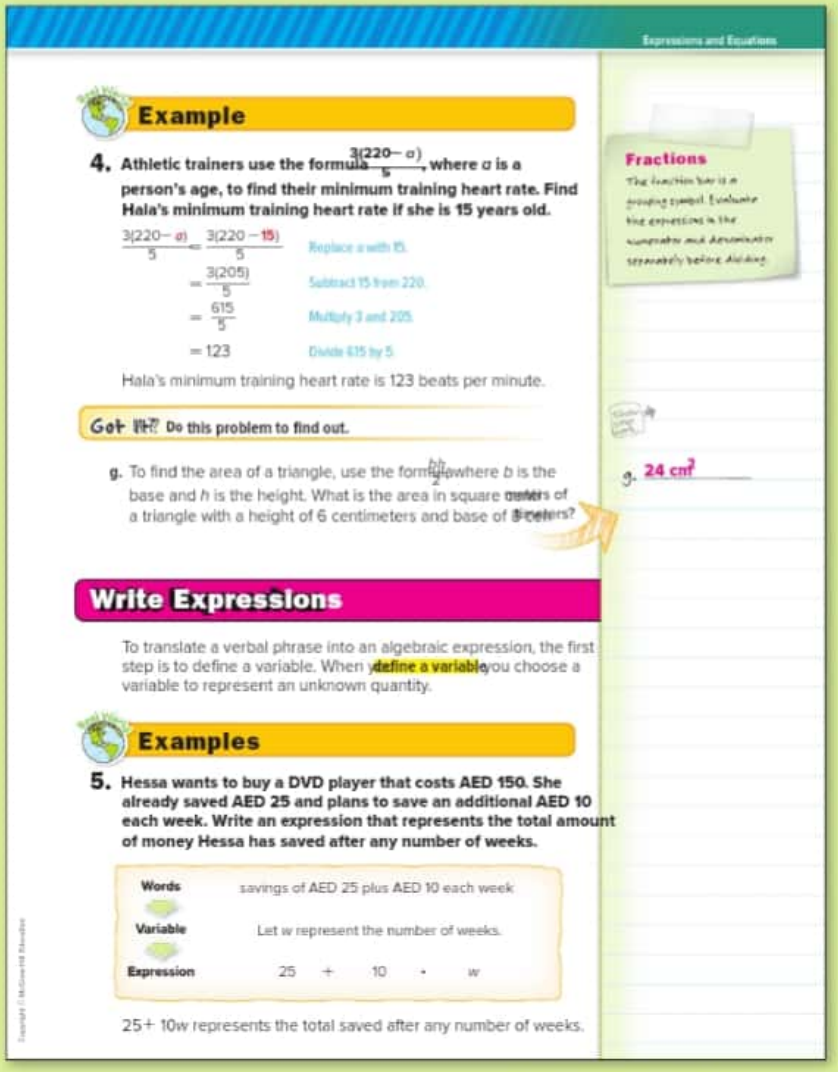
3. Evaluate $4y^3 + 2$ if $y = 3$.

$4y^3 + 2 = 4(3)^3 + 2$ Replace y with 3.
$= 4(27) + 2$ Evaluate the power.
$= 110$ Multiply, then add.

Got It? Do these problems to find out.

Evaluate each expression if $c = 8$ and $d = -5$.

a. $c - 3$ b. $15 - c$ c. $3(c + d)$
d. $2c - 4d$ e. $d - c^2$ f. $2d + 5d$



Expressions and Equations

Example

4. **Athletic trainers use the formula $\frac{3(220-a)}{5}$, where a is a person's age, to find their minimum training heart rate. Find Hala's minimum training heart rate if she is 15 years old.**

$\frac{3(220-a)}{5} = \frac{3(220-15)}{5}$ Replace a with 15.

$= \frac{3(205)}{5}$ Subtract 15 from 220.

$= \frac{615}{5}$ Multiply 3 and 205.

$= 123$ Divide 615 by 5.

Hala's minimum training heart rate is 123 beats per minute.

Got It? Do this problem to find out.

g. To find the area of a triangle, use the formula $\frac{bh}{2}$ where b is the base and h is the height. What is the area in square centimeters of a triangle with a height of 6 centimeters and base of 8 centimeters?

Fractions

The fraction bar is a grouping symbol. Evaluate the expressions in the numerator and denominator separately before dividing.

g. 24 cm²

Write Expressions

To translate a verbal phrase into an algebraic expression, the first step is to define a variable. When you define a variable, you choose a variable to represent an unknown quantity.

Examples

5. **Hessa wants to buy a DVD player that costs AED 150. She already saved AED 25 and plans to save an additional AED 10 each week. Write an expression that represents the total amount of money Hessa has saved after any number of weeks.**

Words	savings of AED 25 plus AED 10 each week
Variable	Let w represent the number of weeks.
Expression	$25 + 10 \cdot w$

$25 + 10w$ represents the total saved after any number of weeks.

Examples

4. **Evaluate an algebraic expression.**

AL • *What do you need to find?* Hala's minimum training heart rate

• *What expression will you use to find the heart rate?* $\frac{3(220-a)}{5}$

OL • *What is the value of a? Why?* 15; a represents the age and Hala's age is 15

BL • *How does the order of operations apply when evaluating this expression?* Use the order of operations to evaluate the numerator. Then divide by the denominator.

Need Another Example?

The formula for rewriting a Fahrenheit temperature as a Celsius temperature is $\frac{5(F-32)}{9}$, where F equals the temperature in degrees Fahrenheit. Find the Celsius equivalent of 77° Fahrenheit. 25°C

5. **Write an algebraic expression.**

AL • *What is the unknown value in the problem?* the number of weeks Hessa has saved money

• *What variable could you use to represent the number of weeks?* w

• *How much has she saved already?* AED 25

• *How much does she save each week?* AED 10

OL • *How can you find the total amount saved for any week?* Multiply 10 by the number of weeks and then add 25.

BL • *How many weeks do you think Hessa needs to save in order to buy the DVD player? Explain how you arrived at your answer.* 13 weeks; Sample answer: The DVD player is AED 150 and she already has AED 25, so she needs to save AED 125. At AED 10 per week, she would need to save for 13 weeks. If she only saved for 12 weeks, she would have saved AED 120, which is not enough.

Need Another Example?

Sumayya read 20 pages of a book. She plans to read 5 pages each day from now on. Write an expression that represents the total number of pages she will have read in d days. $20 + 5d$

Example

6. **Evaluate an algebraic expression.**

AL • *What do you need to find?* whether Hessa will have saved enough to buy the DVD player in 11 weeks

• *What value will you use for w? Explain.* 11; It represents the number of weeks she has saved.

OL • *Explain how you would use the order of operations to determine how much money she has saved.* In the order of operations, addition comes after multiplication, so multiply 10 • 11, then add 25.

• *What do you need to do after you evaluate the expression?* Compare the amount saved to the cost of the DVD player.

BL • *How much more does Hessa need to save?* AED 15

• *How many weeks will it take her to save that amount?* 2 weeks

Need Another Example?

Sumayya read 20 pages of a book. She plans to read 5 pages each day from now on. Use the expression from Need Another Example 5 to see if Sumayya will have read more than 60 pages in 7 days. Explain. No; 20 + 5 • 7 = 55 so, she will only have read 55 pages in 7 days.

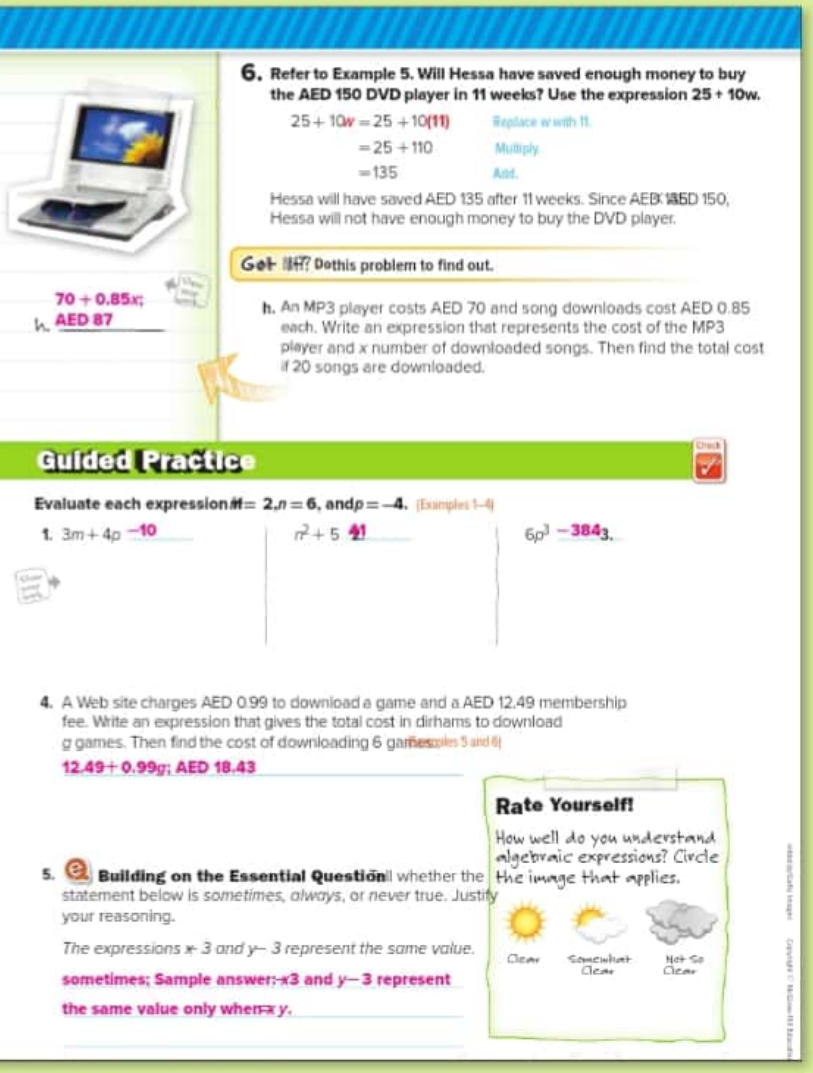
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Think-Pair-Solo** Divide students into teams of 4. Each team completes Exercise 1. Have students pair up to complete Exercise 2. The team regroups to discuss the solution. Have students complete Exercises 3 and 4 alone and regroup to share answers. Have them work together to find any errors. MP 1, 2, 4

BL LA **Find the Fib** Divide students into teams of 3–4. Have one student on each team create 3 problems similar to Exercises 1–3. Have them solve 2 correctly and 1 incorrectly. Have the team find the incorrect solution and solve it correctly. MP 3



6. **Refer to Example 5. Will Hessa have saved enough money to buy the AED 150 DVD player in 11 weeks? Use the expression 25 + 10w.**

$25 + 10w = 25 + 10(11)$ Replace w with 11.

$= 25 + 110$ Multiply.

$= 135$ Add.

Hessa will have saved AED 135 after 11 weeks. Since AED 135 < AED 150, Hessa will not have enough money to buy the DVD player.

Got it? Do this problem to find out.

h. An MP3 player costs AED 70 and song downloads cost AED 0.85 each. Write an expression that represents the cost of the MP3 player and x number of downloaded songs. Then find the total cost if 20 songs are downloaded.

70 + 0.85x; AED 87

Guided Practice

Evaluate each expression if $m = 2$, $n = 6$, and $p = -4$. (Examples 1–4)

1. $3m + 4p$ −10

$n^2 + 5$ 41

$6p^3$ −384

4. A Web site charges AED 0.99 to download a game and a AED 12.49 membership fee. Write an expression that gives the total cost in dirhams to download g games. Then find the cost of downloading 6 games. (Examples 5 and 6)

12.49 + 0.99g; AED 18.43

5. **Building on the Essential Question** Tell whether the statement below is *sometimes*, *always*, or *never* true. Justify your reasoning.

The expressions x − 3 and y − 3 represent the same value.

sometimes; Sample answer: x = 3 and y = 3 represent the same value only when x = y.

Rate Yourself!

How well do you understand algebraic expressions? Circle the image that applies.

Name _____ My Homework _____

Independent Practice

Evaluate each expression if $a = 8$, $e = 3$, $f = 4$, and $g = -4$. (Examples 1–3)

1. $2(d + 9)$ **34**
2. $\frac{d}{4}$ **2**
3. $\frac{ef}{4}$ **3**
4. $4f + d$ **24**
5. $\frac{5d - 25}{5}$ **3**
6. $d^2 + 7$ **71**
7. $\frac{d - 4}{2}$ **2**
8. $10(e + 7)$ **100**
9. $\frac{2g}{2}$ **−4**

10. The expression $5n + 2$ can be used to find the total cost in dirhams of bowling where n is the number of games bowled and 2 represents the cost of shoe rental. How much will it cost Omar to bowl 3 games? (Example 4)
AED 17

Reason Abstractly A car rental company's fees are shown. Suppose you rent a car using Option 2. Write an expression that gives the total cost in dirhams for driving k kilometers. Then find the cost for driving 150 kilometers. (Examples 5 and 6)
$50 + 2k$; AED 350

Car Rental Prices	
Option 1	Option 2
AED 20 per day	AED 50 fee
AED 2 per km	AED 2 per km

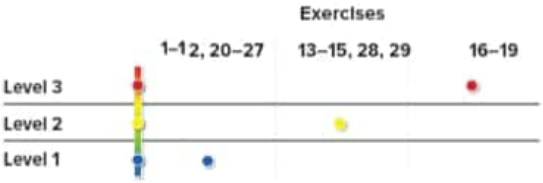
12. Refer to Exercise 11. Suppose you rent a car using Option 1. Write an expression that gives the total cost in dirhams to rent a car for d days and k kilometers. Then find the cost for renting a car for 2 days and driving 70 kilometers. (Examples 5 and 6)
$20d + 2k$, AED 180

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice
The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments
You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–13, 15–17, 28, 29
OL	On Level	1–11 odd, 13–17, 28, 29
BL	Beyond Level	13–19, 28, 29

Watch Out!
Common Error Some students may be confused as to the order of operations in expressions such as the one in Exercise 5. Remind students that the fraction bar acts as a grouping symbol. So, the expression above the bar must be evaluated before it is divided by the number below.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	18, 19
2 Reason abstractly and quantitatively.	11, 16
4 Model with mathematics.	17, 27

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a clos ing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students describe a word problem with a variable (or variables). Make sure they define the variables(s).

See students' work.

Evaluate each expression if $x = 3.2$, $y = 6.1$, and $z = 0.2$.

13. $x + y - z$ 9.1

14. $6 - (4 + y + z)$ 5.1

15. $z + y^2$ 37.85

H.O.T. Problems Higher Order Thinking

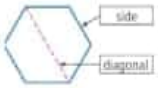
16. **Reason Abstractly** Write an algebraic expression with the variable x that has a value of 3 when evaluated.

Sample answer: $5x - 37$ if $x = 8$

17. **Model with Mathematics** Write a real-world problem that can be represented by the expression 5×10.

Sample answer: The fee to rent a bicycle is AED 10 plus AED 5 for each hour. The expression 5×10 represents the total cost for renting a bicycle for x hours.

18. **Persevere with Problems** To find the total number of diagonals for any given polygon, you can use the expression $\frac{n(n-3)}{2}$, where n is the number of sides of the polygon.



a. Determine the minimum value that n could be. 3

b. Make a table of four possible values of n. Then complete the table by evaluating the expression for each value of n.

c. Check by drawing the diagonals of a pentagon and counting the diagonals.

Sample answers given:

n	value
3	0
4	2
5	5
6	9

19. **Persevere with Problems** Jasser constructed the objects below using toothpicks.

Figure 1 Figure 2 Figure 3

Write two different rules that relate the figure number to the number of toothpicks in each figure.

Sample answer: $2n + 4$; $2(n + 2)$

Name _____ My Homework _____

Extra Practice

Evaluate each expression if $a = 8$, $e = 3$, $f = 4$, and $g = -4$.

20. $10 - e$ 7

$10 - e$

$10 - 3 = 7$

21. $\frac{16}{f}$ 4

$\frac{16}{2}$

$\frac{16}{4} = 4$

22. $4e^2$ 36

23. $8g - f$ −42

24. $\frac{(5+g)^2}{2}$ 8

25. $e^2 - 4$ 5

26. The expression $\frac{w}{11}$, where w is a person's weight in kilograms, is used to find the approximate number of liters of blood in the person's body. How many liters of blood does a 55-kilograms person have?

5 liters

27. **Model with Mathematics** Refer to the graphic novel frame below. Let n represent the number of text messages. Evaluate the expression $1.5(n - 250) + 50$ to find the cost of 275 text messages. AED 87.50

Power Up! Test Practice

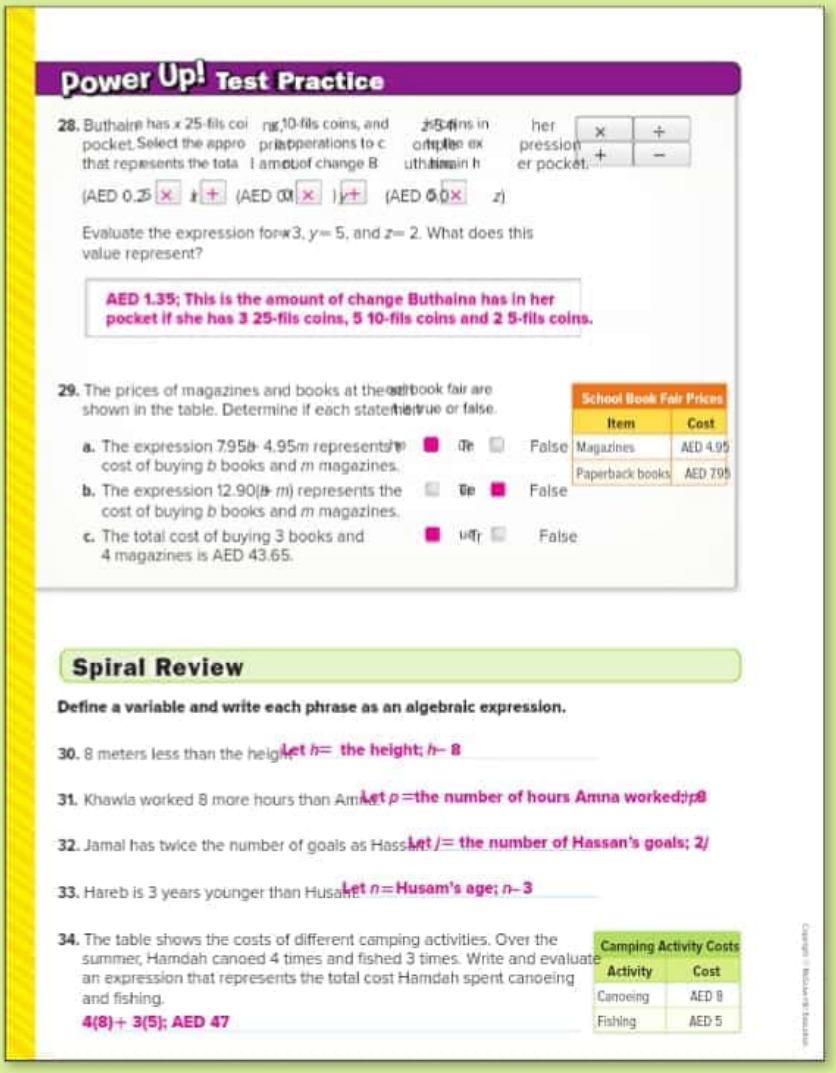
Exercises 28 and 29 prepare students for more rigorous thinking needed for the assessment.

28.

This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP3, MP4
Scoring Rubric	
2 points	Students correctly model, evaluate, and explain the expression.
1 point	Students correctly model and evaluate, but fail to explain the expression OR students evaluate and explain, but fail to model the expression.

29.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.



Power Up! Test Practice

28. Buthaina has x 25-fils coi ng 10-fils coins, and z 5-fils coins in pocket. Select the appro pri operations to c mplete the ex pression that represents the tota l amou of change B uthaina has in h er pocket.

× + + −

(AED 0.25 × x) + (AED 0.10 × y) + (AED 0.05 × z)

Evaluate the expression for $x = 3$, $y = 5$, and $z = 2$. What does this value represent?

AED 1.35; This is the amount of change Buthaina has in her pocket if she has 3 25-fils coins, 5 10-fils coins and 2 5-fils coins.

29. The prices of magazines and books at the school book fair are shown in the table. Determine if each statement is true or false.

School Book Fair Prices

Item	Cost
Magazines	AED 4.95
Paperback books	AED 7.95

a. The expression $7.95b + 4.95m$ represents the cost of buying b books and m magazines. ■ True □ False

b. The expression $12.90(b + m)$ represents the cost of buying b books and m magazines. □ True ■ False

c. The total cost of buying 3 books and 4 magazines is AED 43.65. ■ True □ False

Spiral Review

Define a variable and write each phrase as an algebraic expression.

30. 8 meters less than the height **Let h = the height; $h - 8$**

31. Khawla worked 8 more hours than Amna **Let p = the number of hours Amna worked; $p + 8$**

32. Jamal has twice the number of goals as Hassan **Let j = the number of Hassan's goals; $2j$**

33. Hareb is 3 years younger than Husam **Let n = Husam's age; $n - 3$**

34. The table shows the costs of different camping activities. Over the summer, Hamdah canoed 4 times and fished 3 times. Write and evaluate an expression that represents the total cost Hamdah spent canoeing and fishing.

Camping Activity Costs

Activity	Cost
Canoeing	AED 8
Fishing	AED 5

4(8) + 3(5); AED 47

Lesson 2

Sequences

Vocabulary Start-Up

A **sequence** is an ordered list of numbers. Each number in a sequence is called a **term**. In an **arithmetic sequence** each term is found by adding the same number to the previous term.

Complete the graphic organizer below.

arithmetic sequences

Numbers	Words
Continue each sequence.	Describe each sequence.
1, 3, 5, 7, 9, ...	Add 2 to the previous term.
1, 1.5, 2, 2.5, 3, 3.5, ...	Add 0.5 to the previous term.

Essential Question

HOW can you use numbers and symbols to represent mathematical ideas?

Vocabulary

sequence
term
arithmetic sequence

Mathematical Practices

1, 2, 3, 4

Real-World Link

Horseback Riding The number of students who went on each horseback riding trip is shown. Do the numbers represent the terms of an arithmetic sequence? Explain.

Trip	1	2	3	4	5
Number of Students	15	16	18	21	25

no; The same number is not added to the previous term. The numbers increase by 1, 2, 3, and 4.

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Describe the relationships and extend terms in arithmetic sequences.

Coherence connecting within and across grades

Previous Students used the order of operations to evaluate expressions.

Now Students evaluate expressions involving absolute values.

Next Students will add integers with the same sign and add integers with different signs.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 361.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

AL LA **Think-Pair-Share** Have students work in pairs. Give students 3–5 minutes to complete the graphic organizer. Have them share their responses with their partner, then together complete the Real-World Link. Then call on one student to share their response within a small group or large group discussion MP 5, 6

Alternate Strategy

BL Have students work in pairs to research *geometric sequences*. Ask them to compare and contrast them with arithmetic sequences by using a graphic organizer. Each pair can report their findings to a small group or to the entire class. MP 5, 6

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Describe and extend sequences.**

AL
- *What number do you need to add to 8 to get 13?* 5
- *What number do you need to add to 13 to get 18?* 5
- *What number would you add to each term to find the next term?* 5

OL
- *To find the fifth term, what should you do?* Add 5 to 23.

BL
- *What is the tenth term?* 53

Need Another Example?
Describe the relationship between the terms in the arithmetic sequence 7, 11, 15, 19, ... Then write the next three terms in the sequence. 4 is added to each term; 23, 27, 31

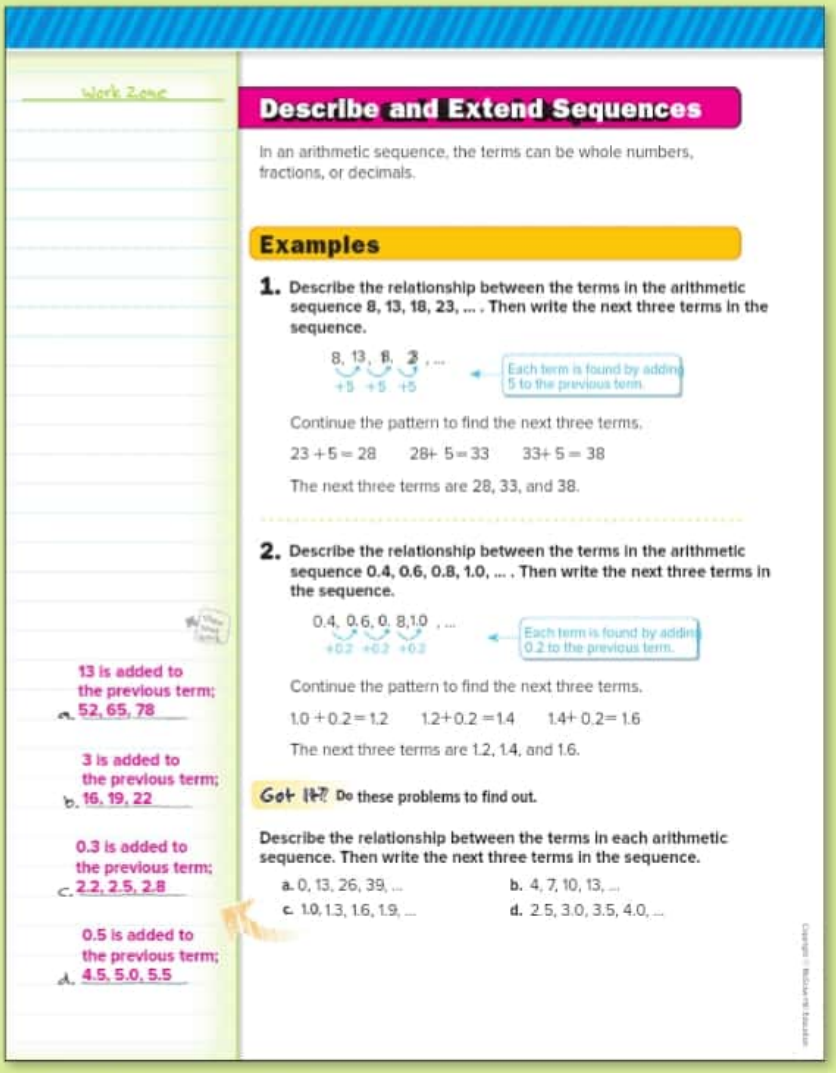
2. **Describe and extend sequences.**

AL
- *What number do you need to add to 0.4 to get 0.6?* 0.2
- *What number do you need to add to 0.6 to get 0.8?* 0.2
- *What number would you add to each term to find the next term?* 0.2

OL
- *To find the fifth term, what should you do?* Add 0.2 to 1.0.
- *What should you do next?* Continue adding 0.2 to the previous term.

BL
- *What is the tenth term? Describe how to find it.* 2.2; Sample answer: since you know the 7th term, you need to add 0.2 three more times to 1.6; 1.6 + 3(0.2) = 2.2.

Need Another Example?
Describe the relationship between the terms in the arithmetic sequence 0.1, 0.5, 0.9, 1.3, ... Then write the next three terms in the sequence. Each term is 0.4 greater than the previous term; 1.7, 2.1, 2.5.



Work Zone

13 is added to the previous term;
a. 52, 65, 78

3 is added to the previous term;
b. 16, 19, 22

0.3 is added to the previous term;
c. 2.2, 2.5, 2.8

0.5 is added to the previous term;
d. 4.5, 5.0, 5.5

Describe and Extend Sequences

In an arithmetic sequence, the terms can be whole numbers, fractions, or decimals.

Examples

1. **Describe the relationship between the terms in the arithmetic sequence 8, 13, 18, 23, ... Then write the next three terms in the sequence.**

8, 13, 18, 23, ...
+5 +5 +5
Each term is found by adding 5 to the previous term.

Continue the pattern to find the next three terms.

23 + 5 = 28 28 + 5 = 33 33 + 5 = 38

The next three terms are 28, 33, and 38.

2. **Describe the relationship between the terms in the arithmetic sequence 0.4, 0.6, 0.8, 1.0, ... Then write the next three terms in the sequence.**

0.4, 0.6, 0.8, 1.0, ...
+0.2 +0.2 +0.2
Each term is found by adding 0.2 to the previous term.

Continue the pattern to find the next three terms.

1.0 + 0.2 = 1.2 1.2 + 0.2 = 1.4 1.4 + 0.2 = 1.6

The next three terms are 1.2, 1.4, and 1.6.

Got It? **Do these problems to find out.**

Describe the relationship between the terms in each arithmetic sequence. Then write the next three terms in the sequence.

a. 0, 13, 26, 39, ... b. 4, 7, 10, 13, ...
c. 1.0, 1.3, 1.6, 1.9, ... d. 2.5, 3.0, 3.5, 4.0, ...

Write an Algebraic Expression

In a sequence, each term has a specific position within the sequence. Consider the sequence 2, 4, 6, 8,...

Notice that as the position number increases by 1, the value of the term increases by 2.

Position	Operation	Value of Term
1	1 • 2 = 2	2
2	2 • 2 = 4	4
3	3 • 2 = 6	6
4	4 • 2 = 8	8

+1, +1, +1 (between positions) +2, +2, +2 (between values)

You can also write an algebraic expression to represent the relationship between any term in a sequence and its position in the sequence. In this case, if n represents the position in the sequence, the value of the term is $2n$.

Arithmetic Sequences

When looking for a pattern between the position number and each term in the sequence, it is often helpful to make a table.

Example

3. **The greeting cards that Badria makes are sold in boxes at a gift store. The first week, the store sold 5 boxes. Each week, the store sells five more boxes. The pattern continues. What algebraic expression can be used to find the total number of boxes sold at the end of the 100th week? What is the total?**

Position	Operation	Value of Term
1	1 • 5	5
2	2 • 5	10
3	3 • 5	15
n	n • 5	$5n$

Each term is 5 times its position. So, the expression is $5n$.

$5n$ Write the expression.

$5(100) = 500$ Replace n with 100.

At the end of 100 weeks, 500 boxes will have been sold.

Example

3. **Write and evaluate an algebraic expression.**

AL
- *What do you need to find?* The number of boxes sold at the end of the 100th week
- *After Week 1, how many boxes of cards are sold?* 5
- *After Week 2, how many total boxes of cards are sold? Week 3? Week 4?* 10; 15; 20

OL
- *What is the relationship between the number of weeks and the number of boxes sold?* The number of boxes sold is equal to 5 times the number of weeks.
- *How can you represent this situation with an expression?* Sample answer: Let n represent the number of weeks and b represent the total number of boxes sold.

BL
- *Suppose the first week, the store sold 8 boxes, then each week after that, the store sells 5 more boxes. What expression could you write to describe the situation? Explain.* $5n + 3$; Sample answer: The terms are 8, 13, 18, 23, ... The value of each term is equal to 5 times the term position plus 3.

Need Another Example?

Humaid started a new exercise routine. The first day, he did 2 sit-ups. Each day after that, he did 2 more sit-ups than the previous day. If he continues this pattern, what algebraic expression can be used to help find the number of sit-ups on the nth day? Use the expression to find the number of sit-ups on the tenth day. $2n$; 20 sit-ups

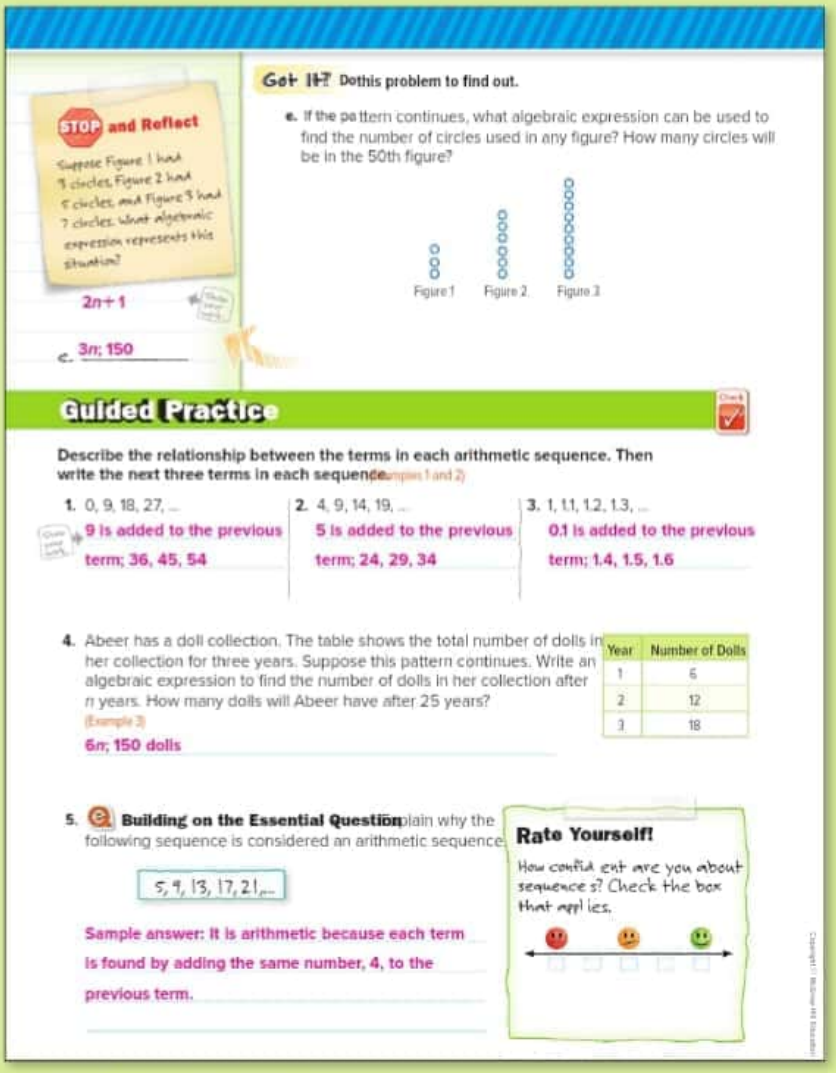
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA Pairs Discussion Have students work in pairs to complete Exercises 1–3. If they are having trouble finding what needs to be added to each term, give them centimeter cubes to build figures represented by each term in the sequence. Then, each pair trades their solutions with another pair of students and discusses any differences. MP 1, 4, 5, 7

BL LA Trade-a-Problem Have students create their own arithmetic sequences, similar to Exercises 1–3. Challenge them to use fractions and mixed numbers in their sequences. Also challenge them to have a different first term than what they will use for their common difference. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. MP 1, 7



STOP and Reflect

Suppose Figure 1 had 3 circles, Figure 2 had 5 circles, and Figure 3 had 7 circles. What algebraic expression represents this situation?

2n + 1

c. 3n; 150

Got it? Do this problem to find out.

e. If the pattern continues, what algebraic expression can be used to find the number of circles used in any figure? How many circles will be in the 50th figure?

Figure 1 Figure 2 Figure 3

Guided Practice

Describe the relationship between the terms in each arithmetic sequence. Then write the next three terms in each sequence. (Examples 1 and 2)

1. 0, 9, 18, 27, ...
9 is added to the previous term; 36, 45, 54

2. 4, 9, 14, 19, ...
5 is added to the previous term; 24, 29, 34

3. 1, 1.1, 1.2, 1.3, ...
0.1 is added to the previous term; 1.4, 1.5, 1.6

4. Abeer has a doll collection. The table shows the total number of dolls in her collection for three years. Suppose this pattern continues. Write an algebraic expression to find the number of dolls in her collection after n years. How many dolls will Abeer have after 25 years? (Example 3)

Year	Number of Dolls
1	6
2	12
3	18

6n; 150 dolls

5. **Building on the Essential Question** Explain why the following sequence is considered an arithmetic sequence.

5, 9, 13, 17, 21, ...

Sample answer: It is arithmetic because each term is found by adding the same number, 4, to the previous term.

Rate Yourself!

How confident are you about sequences? Check the box that applies.

Name _____ My Homework _____

Independent Practice

Describe the relationship between the terms in each arithmetic sequence. Then write the next three terms in each sequence. (Examples 1 and 2)

1. 0, 7, 14, 21, ...
   7 is added to the previous term; 28, 35, 42
2. 1, 7, 13, 19, ...
   6 is added to the previous term; 25, 31, 37
3. 26, 34, 42, 50, ...
   8 is added to the previous term; 58, 66, 74
4. 0.1, 0.4, 0.7, 1.0, ...
   0.3 is added to the previous term; 1.3, 1.6, 1.9
5. 2.4, 3.2, 4.0, 4.8, ...
   0.8 is added to the previous term; 5.6, 6.4, 7.2
6. 2.0, 3.1, 4.2, 5.3, ...
   1.1 is added to the previous term; 6.4, 7.5, 8.6
7. Refer to the table shown. If the pattern continues, what algebraic expression can be used to find the plant's height for any month? What will be the plant's height at 12 months? (Example 3)
   $3n$; 36 cm

Month	Height (cm)
1	3
2	6
3	9

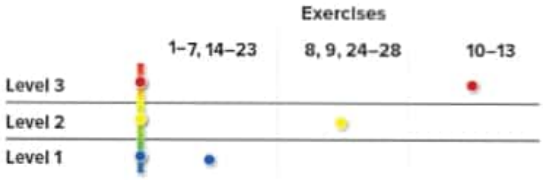
8. **Model with Mathematics** Explain how the number of text messages Badr sent and the cost form an arithmetic sequence. Then write an expression to find Badr's text messaging bill if he sends n text messages over 250.
   Sample answer: For each text message over 250, the cost increases by AED 1.50; $1.5n + 50$.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice
The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments
You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 10, 27, 28
OL	On Level	1–7 odd, 8–10, 27, 28
BL	Beyond Level	8–13, 27, 28

Watch Out!

Common Error If the expression for finding the nth term of a sequence is incorrectly determined, the terms that are found using that expression will also be incorrect. Have students check their expressions using all of the numbers that were given to them in the sequence.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	11–13
3 Construct viable arguments and critique the reasoning of others.	23
4 Model with mathematics.	9, 10
5 Use appropriate tools strategically.	8

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a clos ing formative assessment before dismissing students from your class.

TICKET Out the Door

Write the sequence 8, 16 , 24, 32, ... on the board. Then have students find the 9th term of the sequence. 72

Watch Out!

In Exercise 9, remind students that the coordinate plane can be used to plot ordered pairs. The horizontal axis is the *x*-axis and the vertical axis is the *y*-axis. To plot an ordered pair (x, y), the *x*-coordinate corresponds to the number along the *x*-axis and the *y*-coordinate corresponds to the number along the *y*-axis.

9. **Multiple Representations** Shaya is stacking boxes of tissues for a store display. She stacks 3 boxes in the first minute, 6 boxes by the end of the second minute, and 9 boxes by the end of the third minute. Suppose the pattern continues for parts **a–d**.

a. **Table** Make a table of values for 1, 2, 3, 4, and 5 minutes.

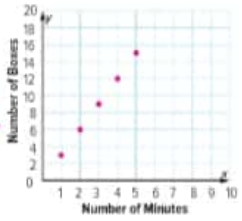
x	1	2	3	4	5
y	3	6	9	12	15

b. **Symbols** Write an expression to find the *n*th term in the sequence.

$3n$

c. **Graph** Graph the table of values from part **a** on the coordinate plane. Let *x* represent the number of minutes and *y* represent the number of boxes. Then describe the graph.

Sample answer: The number of boxes increases by 3 each minute. The points appear to fall in a straight line passing through the origin.



d. **Numbers** How many boxes will be displayed after 45 minutes?

135 boxes

H.O.T. Problems Higher Order Thinking

10. **Justify Conclusions** Write five terms of an arithmetic sequence and describe the rule for finding the terms.

Sample answer: 5, 6, 7, 8, 9, ...; $n + 4$, where *n* represents the position of the term

Persevere with Problems Not all sequences are arithmetic. But, there is still a pattern. Describe the relationship between the terms in each sequence. Then write the next three terms in the sequence.

11. 1, 2, 4, 7, 11, ...

+1, + 2, + 3, + 4, ...; 16, 22, 29

12. 0, 2, 6, 12, 20, ...

+ 2, + 4, + 6, + 8, ...; 30, 42, 56

13. **Persevere with Problems** Use an arithmetic sequence to find the number of multiples of 6 between 41 and 523. Justify your reasoning.

81; Sample answer: The multiples of 6 from 41 to 523 can be represented by the sequence 42, 48, 54, ... 522. The expression $6n + 36$ represents this sequence. When $n = 81$, the value of the expression is 522. So, the 81st term of the sequence is 522. There are 81 multiples of 6 between 41 and 523.

Name _____ My Homework _____

Extra Practice

Describe the relationship between the terms in each arithmetic sequence. Then write the next three terms in each sequence.

14. 19, 31, 43, 55, ...
12 is added to the previous term; 67, 79, 91

15. 6, 16, 26, 36, ...
10 is added to the previous term; 46, 56, 66

16. 33, 38, 43, 48, ...
5 is added to the previous term; 53, 58, 63

17. 4.5, 6.0, 7.5, 9.0, ...
1.5 is added to the previous term; 10.5, 12.0, 13.5

18. 1.2, 3.2, 5.2, 7.2, ...
2 is added to the previous term; 9.2, 11.2, 13.2

19. 4.6, 8.6, 12.6, 16.6, ...
4 is added to the previous term; 20.6, 24.6, 28.6

20. 18, 33, 48, 63, ...
15 is added to the previous term; 78, 93, 108

21. 20, 45, 70, 95, ...
25 is added to the previous term; 120, 145, 170

22. 38, 61, 84, 107, ...
23 is added to the previous term; 130, 153, 176

23. **Reason Abstractly** Refer to the figures for parts **a** and **b**.

Figure 1 Figure 2 Figure 3

a. Describe the relationship between the figures and the number of rectangles shown. Each figure is 8 less than the previous figure.

b. If the pattern continues, how many rectangles will be in the next 2 figures? 40, 32

The terms of an arithmetic sequence can be related by subtraction. Write the next three terms of each sequence.

24. 32, 27, 22, 17, ...
12, 7, 2

25. 45, 42, 39, 36, ...
33, 30, 27

26. 10.5, 10, 9.5, 9, ...
8.5, 8, 7.5

Power Up! Test Practice

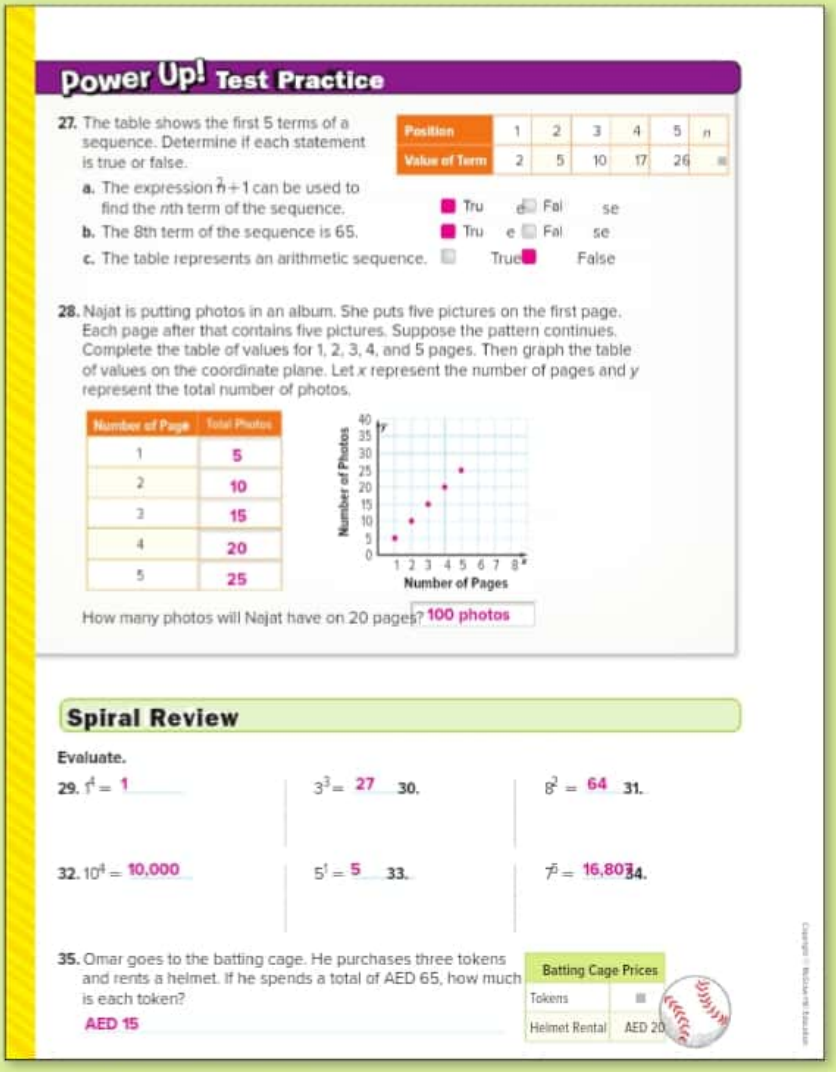
Exercises 27 and 28 prepare students for more rigorous thinking needed for the assessment.

27. This test item requires student to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

28. This test item requires student to analyze and solve complex real-world problems through the use of mathematical tools and models.

Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly complete the table, plot the points, and find the number of photos.
1 point	Students correctly complete the table and find the number of photos but fail to plot the points OR students complete the table and plot the points but fail to find the number of photos OR students incorrectly complete the table but plot the points and find the number of photos based on the error in the table.



Power Up! Test Practice

27. The table shows the first 5 terms of a sequence. Determine if each statement is true or false.

Position	1	2	3	4	5	n
Value of Term	2	5	10	17	26	■

a. The expression $n^2 + 1$ can be used to find the nth term of the sequence. ■ True ☐ False

b. The 8th term of the sequence is 65. ■ True ☐ False

c. The table represents an arithmetic sequence. ☐ True ■ False

28. Najat is putting photos in an album. She puts five pictures on the first page. Each page after that contains five pictures. Suppose the pattern continues. Complete the table of values for 1, 2, 3, 4, and 5 pages. Then graph the table of values on the coordinate plane. Let x represent the number of pages and y represent the total number of photos.

Number of Page	Total Photos
1	5
2	10
3	15
4	20
5	25

How many photos will Najat have on 20 pages? 100 photos

Spiral Review

Evaluate.

29. $1^4 = 1$ 30. $3^3 = 27$ 31. $8^2 = 64$

32. $10^4 = 10{,}000$ 33. $5^1 = 5$ 34. $7^5 = 16{,}807$

35. Omar goes to the batting cage. He purchases three tokens and rents a helmet. If he spends a total of AED 65, how much is each token?

AED 15

Batting Cage Prices	
Tokens	■
Helmet Rental	AED 20

Expressions and Equations

Inquiry Lab
Sequences

inquiry **HOW can geometric figures be used to model numerical patterns?**

MP Mathematical Practices 1, 3

A fencing company uses 4 planks of wood for one section of fencing, 7 planks for two sections, and 10 planks of wood for three sections. The fence sections are represented using the toothpicks shown. Determine how many planks would be used to create 5 sections of fencing.

1 Section 2 Sections 3 Sections

Hands-On Activity

Step 1 Find a pattern in the table. Then fill in the number of planks that would be in 4 and 5 sections of fencing.

Number of Sections	Number of Planks
1	4
2	7
3	10
4	13
5	16

Step 2 Check your work by using toothpicks to show 5 fence sections. Draw the result in the space below.

So, there will be 16 planks in 5 sections of fencing.

Focus narrowing the scope

Objective Explore patterns in sequences of geometric figures.

Coherence moving from concrete to abstract

Now Students examine sequences formed from geometric figures.

Next Students create algebraic representations for geometric sequences.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 366.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Hands-On Activity

AL LA Tell students that number of toothpicks represents the number of wood planks needed for each section.

Ask:

- *What number is added to the number of sections to find the next term?* 1
- *What number is added to the number of planks to find the next term?* 3

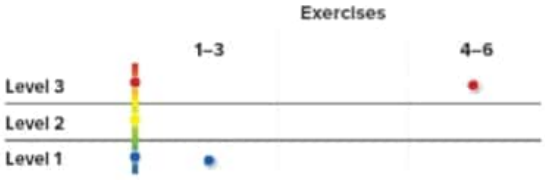
BL LA Have students work in pairs to alter the activity (and fence specifications) so that 5 planks are needed for 1 section, 9 planks are needed for 2 sectons, 13 planks are needed for 3 sections, and so on. Then have them complete a table similar to the one in the activity and use toothpicks to show 5 fence sections. There should be 21 toothpicks for 5 fence sections.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** section is intended to be used as a small-group investigation. The **Create** section is intended to be used as independent exercises

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Pairs Discussion** Have students work in pairs to complete Exercises 1–3. Have them trade their solutions with another pair of students and discuss any differences. MP 3, 7

Create

BL LA **Trade-a-Problem** Have students create their own pattern with toothpicks. Students trade their patterns with another student and write an expression for each other's pattern. MP 1, 7

Inquiry Students should be able to answer "HOW can geometric figures be used to model numerical patterns?" Check for student understanding and provide guidance, if needed.

Investigate

Work with a partner. Complete the table. You can use toothpicks to continue each pattern if needed.

1.

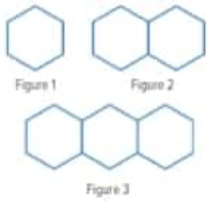


Figure Number	Number of Toothpicks
1	6
2	11
3	16
4	21
5	26

2. Refer to Exercise 1. Write an expression that could be used to find the number of toothpicks that would be needed for any figure.
$5n + 1$

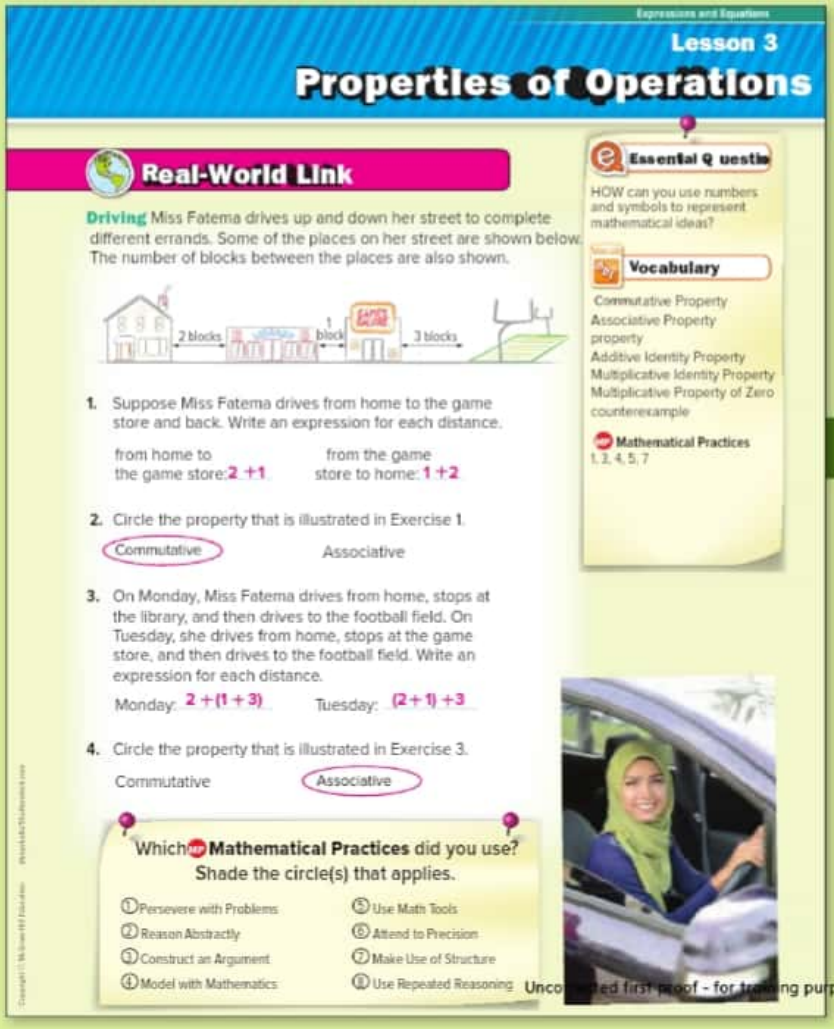
3. Use your expression from Exercise 2 to find the number of toothpicks that would be needed to create Figure 10. Explain.
51 toothpicks; Evaluate $5n + 1$ for $n = 10$; $5(10) + 1 = 51$.

Create

4. MP **Reason Abstractly** Refer to the activity. Write an expression that could be used to find the number of planks in any number of sections.
$3n + 1$

5. MP **Justify Conclusions** Use the expression in Exercise 4 to find the number of planks that would be needed to create 10 sections of fencing. Explain.
31 planks; Evaluate $3n + 1$ for $n = 10$; $3(10) + 1 = 31$.

6. Inquiry HOW can geometric figures be used to model numerical patterns?
Sample answer: Using a geometric figure to represent each number in a pattern helps to visualize the pattern.



Expressions and Equations

Lesson 3

Properties of Operations

Real-World Link

Driving Miss Fatema drives up and down her street to complete different errands. Some of the places on her street are shown below. The number of blocks between the places are also shown.

1. Suppose Miss Fatema drives from home to the game store and back. Write an expression for each distance.

   from home to the game store: 2 + 1 from the game store to home: 1 + 2

2. Circle the property that is illustrated in Exercise 1.

   (Commutative) Associative

3. On Monday, Miss Fatema drives from home, stops at the library, and then drives to the football field. On Tuesday, she drives from home, stops at the game store, and then drives to the football field. Write an expression for each distance.

   Monday: 2 + (1 + 3) Tuesday: (2 + 1) + 3

4. Circle the property that is illustrated in Exercise 3.

   Commutative (Associative)

Essential Question

HOW can you use numbers and symbols to represent mathematical ideas?

Vocabulary

Commutative Property
Associative Property
property
Additive Identity Property
Multiplicative Identity Property
Multiplicative Property of Zero
counterexample

Mathematical Practices
1, 3, 4, 5, 7

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems ⑤ Use Math Tools
② Reason Abstractly ⑥ Attend to Precision
③ Construct an Argument ⑦ Make Use of Structure
④ Model with Mathematics ⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Identify and use mathematical properties to simplify algebraic expressions.

Coherence connecting within and across grades

Previous Students wrote and evaluated algebraic expressions given a value for the variable.

Now Students use mathematical properties to simplify algebraic expressions.

Next Students will add and subtract linear expressions.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 371.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL LA **Think-Pair-Write** Have students work in pairs. Give them two minutes to think about the verbs *commute* and *associate* in everyday use. Have them discuss their ideas with their partner. Then have students write how the everyday uses of these verbs apply to the Commutative Properties and the Associative Properties. MP 1, 6

Alternate Strategy

AL Have students generate a list of words that are similar to *Commutative* and *Associative*, such as *commute*, *commuter*, *associates*, *associated*, and *associatio*. MP 1, 6

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Identify properties.

AL
- *What is the order of the numbers and variables on the left side of the equation?* 2, 5, *n*
- *What is the order of the numbers and variables on the right side of the equation?* 2, 5, *n*
- *Did the order change?* no
- *What changed?* the grouping

OL
- *What do the grouping symbols indicate?* The parentheses indicate the operations that are to be performed first.
- *What does each side of the equation simplify to?* 10?
- *What property does this statement illustrate?* Associative Property of Multiplication

BL
- *What properties are shown by the statement* $5 + (0 + x) = x + 5$*?* Commutative Property of Addition and the Additive Identity Property

Need Another Example?
Name the property shown by the statement $(3 \cdot m) \cdot 2 = 2 \cdot (3 \cdot m)$. Commutative Property of Multiplication

Key Concept **Properties of Operations**

Work Zone

Words The **Commutative Property** states that the order in which numbers are added or multiplied does not change the sum or product.

	Addition	Multiplication
Symbols	$a + b = b + a$	$a \cdot b = b \cdot a$
Examples	$6 + 1 = 1 + 6$	$7 \cdot 3 = 3 \cdot 7$

Words The **Associative Property** states that the way in which numbers are grouped when they are added or multiplied does not change the sum or product.

	Addition	Multiplication
Symbols	$a + (b + c) = (a + b) + c$	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$
Examples	$2 + (3 + 8) = (2 + 3) + 8$	$3 \cdot (4 \cdot 5) = (3 \cdot 4) \cdot 5$

A **property** is a statement that is true for any number. The following properties are also true for any numbers.

Property	Words	Symbols	Examples
Additive Identity	When 0 is added to any number, the sum is the number.	$a + 0 = a$ $0 + a = a$	$9 + 0 = 9$ $0 + 9 = 9$
Multiplicative Identity	When any number is multiplied by 1, the product is the number.	$a \cdot 1 = a$ $1 \cdot a = a$	$5 \cdot 1 = 5$ $1 \cdot 5 = 5$
Multiplicative Property of Zero	When any number is multiplied by 0, the product is 0.	$a \cdot 0 = 0$ $0 \cdot a = 0$	$8 \cdot 0 = 0$ $0 \cdot 8 = 0$

Example

1. Name the property shown by the statement $2 \cdot (5 \cdot n) = (2 \cdot 5) \cdot n$.

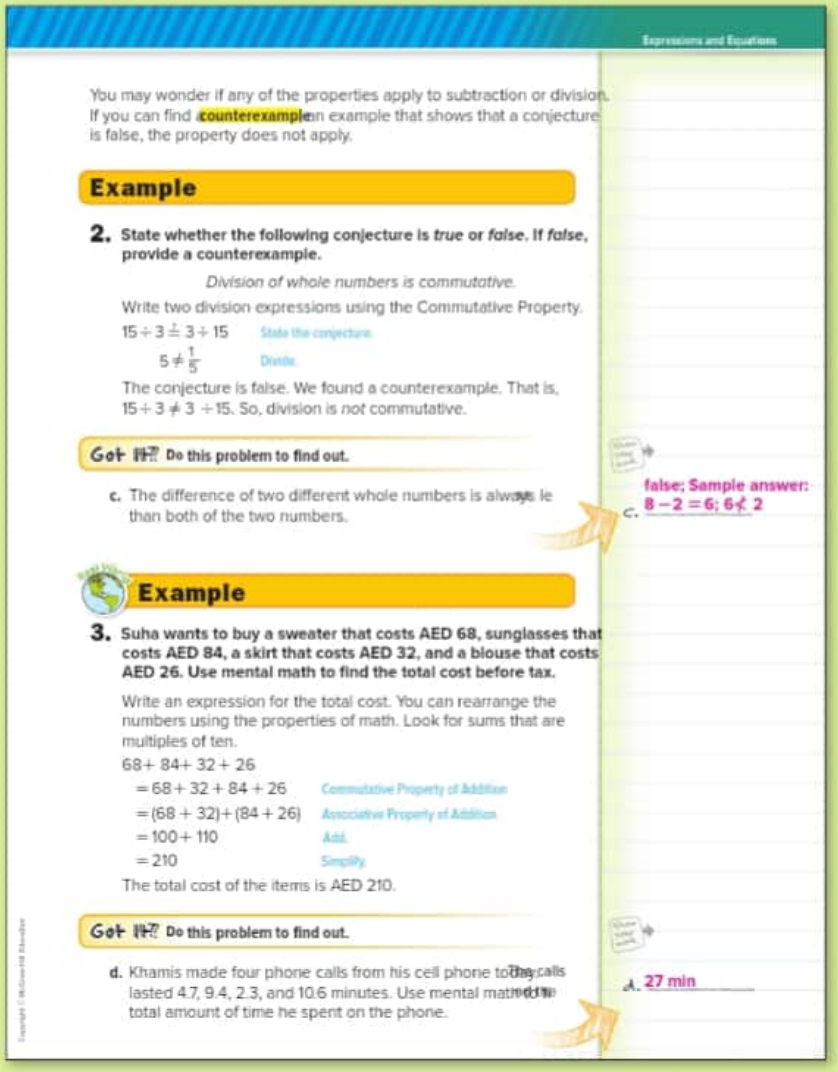
The order of the numbers and variable did not change, but their grouping did. This is the Associative Property of Multiplication.

Got It? Do these problems to find out.

a. Commutative (+)

b. Identity (+)

a. $42 + x + y = 42 + y + x$ b. $3x + 0 = 3x$



Expressions and Equations

You may wonder if any of the properties apply to subtraction or division. If you can find a **counterexample**, an example that shows that a conjecture is false, the property does not apply.

Example

2. **State whether the following conjecture is *true* or *false*. If *false*, provide a counterexample.**

Division of whole numbers is commutative.

Write two division expressions using the Commutative Property.

$15 \div 3 \stackrel{?}{=} 3 \div 15$ State the conjecture.

$5 \neq \frac{1}{5}$ Divide.

The conjecture is false. We found a counterexample. That is, $15 \div 3 \neq 3 \div 15$. So, division is *not* commutative.

Got It? **Do this problem to find out.**

c. The difference of two different whole numbers is always le than both of the two numbers.

c. false; Sample answer: $8 - 2 = 6$; $6 \not< 2$

Example

3. **Suha wants to buy a sweater that costs AED 68, sunglasses that costs AED 84, a skirt that costs AED 32, and a blouse that costs AED 26. Use mental math to find the total cost before tax.**

Write an expression for the total cost. You can rearrange the numbers using the properties of math. Look for sums that are multiples of ten.

$68 + 84 + 32 + 26$

$= 68 + 32 + 84 + 26$ Commutative Property of Addition

$= (68 + 32) + (84 + 26)$ Associative Property of Addition

$= 100 + 110$ Add.

$= 210$ Simplify.

The total cost of the items is AED 210.

Got It? **Do this problem to find out.**

d. Khamis made four phone calls from his cell phone today. The calls lasted 4.7, 9.4, 2.3, and 10.6 minutes. Use mental math to find the total amount of time he spent on the phone.

d. 27 min

Examples

2. **Find a counterexample.**

AL
- *What is a conjecture?* Sample answer: A statement that has not been proven.
- *What is a counterexample?* an example that shows a conjecture is false

OL
- *Why does the Commutative Property not apply to division?* Sample answer: The order in which you divide matters.

BL
- *Write an equation, using the inverse operation of $15 \div 3$, that illustrates the Commutative Property of Multiplication.* $15 \cdot \frac{1}{3} = \frac{1}{3} \cdot 15$

Need Another Example?

State whether the following conjecture is *true* or *false*. If false, provide a counterexample: *Subtraction of whole numbers is associative.* false; Sample answer: $(12 - 5) - 3 \neq 12 - (5 - 3)$

3. **Use mental math.**

AL
- *What are compatible numbers in addition?* numbers that are easy to add mentally
- *In the example, what number is compatible with 38? Why?* 22; 38 and 22 have a sum of 60, which ends in 0
- *In the example, what number is compatible with 14? Why?* 16; 14 and 16 have a sum of 30, which ends in 0

OL
- *Look at the cost of each item. Which pairs of numbers can you add mentally? Explain.* 38 and 22, 14 and 16; Sample answer: The ones digits in each pair make a ten ($8 + 2 = 10$ and $4 + 6 = 10$).
- *What property will allow you to reorder the addition expression?* Commutative Property of Addition
- *What property will allow you to regroup the numbers in the expression?* Associative Property of Addition

BL
- *Give an example of a problem where you would want to change the order of the problem to find a product mentally.* Sample answer: $4 \cdot 18 \cdot 25 = 4 \cdot 25 \cdot 18$

Need Another Example?

In a garden, a decorative pool in the shape of a box is 2 meters deep, 17 meters long, and 5 meters wide. Use mental math to find the volume of water in the pool. 170 m^3

Examples

4–5. Simplify expressions.

AL • *Refer to Example 4. What other term is like 7? Why?* They are known values.

• *Refer to Example 4. Can you add 7 and g? Explain.* You don't know the value of g.

OL • *Refer to Example 4. How can you rewrite (7 + g) + 5 so that you can add 7 and 5?* Switch the order of 7 and g, then regroup so that 7 and 5 are added first.

BL • *Refer to Example 5. What does the exponent indicate?* The variable m is multiplied by itself.

Need Another Example?

Simplify 6 + (d + 8). Justify each step.

$6 + (d + 8) = 6 + (8 + d)$ Commutative (+)

$= (6 + 8) + d$ Associative (+)

$= 14 + d$ Simplify.

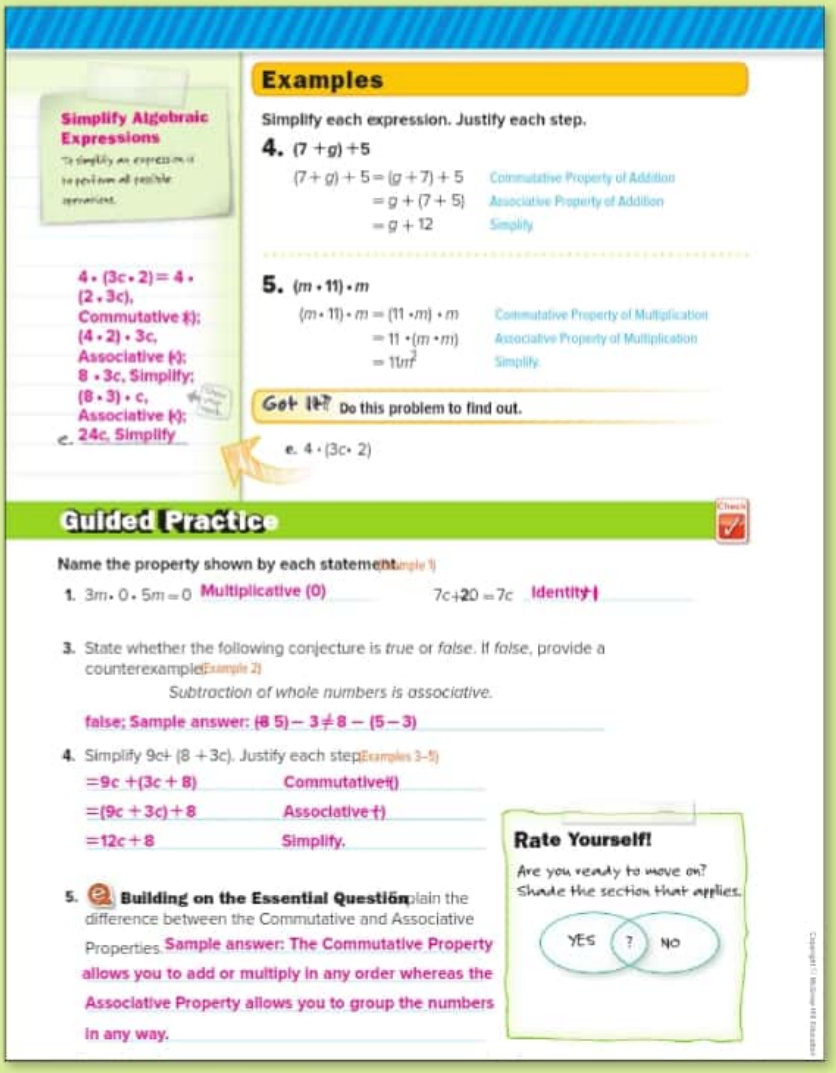
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Numbered Heads Together** Assign students to 3- or 4-person learning teams. Each member is assigned a number from 1 to 4. Each team completes Exercises 1–4, making sure that every member understands. If students are having difficulty coming up with the steps in Exercise 4, supply the steps and ask students to give the reason. Call on a specific number from one team to present the team's solution to the class. MP 1, 6

BL LA **Think-Pair-Share** Have students work in pairs. Give students three minutes to write five conjectures, some true and some false, about properties of whole numbers. Have them share their conjectures with their partner. Each partner should identify the conjectures as true or false. If false, students must give a counterexample. Call on one student to share a conjecture within a small group or large group discussion. MP 1, 3, 7



Simplify Algebraic Expressions

To simplify an expression is to perform all possible operations.

Examples

Simplify each expression. Justify each step.

4. $(7 + g) + 5$

$(7 + g) + 5 = (g + 7) + 5$ Commutative Property of Addition

$= g + (7 + 5)$ Associative Property of Addition

$= g + 12$ Simplify

5. $(m \cdot 11) \cdot m$

$(m \cdot 11) \cdot m = (11 \cdot m) \cdot m$ Commutative Property of Multiplication

$= 11 \cdot (m \cdot m)$ Associative Property of Multiplication

$= 11m^2$ Simplify

Got it? Do this problem to find out.

e. $4 \cdot (3c \cdot 2)$

$4 \cdot (3c \cdot 2) = 4 \cdot (2 \cdot 3c)$, Commutative (·); $(4 \cdot 2) \cdot 3c$, Associative (·); $8 \cdot 3c$, Simplify; $(8 \cdot 3) \cdot c$, Associative (·); $24c$, Simplify

Guided Practice

Name the property shown by each statement. Example 1

1. $3m \cdot 0 \cdot 5m = 0$ Multiplicative (0)

$7c + 0 = 7c$ Identity (+)

3. State whether the following conjecture is *true* or *false*. If *false*, provide a counterexample. Example 2

Subtraction of whole numbers is associative.

false; Sample answer: $(8 - 5) - 3 \neq 8 - (5 - 3)$

4. Simplify $9c + (8 + 3c)$. Justify each step. Examples 3–5

$= 9c + (3c + 8)$ Commutative (+)

$= (9c + 3c) + 8$ Associative (+)

$= 12c + 8$ Simplify.

5. **Building on the Essential Question** Explain the difference between the Commutative and Associative Properties. Sample answer: The Commutative Property allows you to add or multiply in any order whereas the Associative Property allows you to group the numbers in any way.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

Name _____ My Homework _____

Independent Practice

Name the property shown by each statement. (Example 1)

1. $a + (b + 12) = (b + 12) + a$
   Commutative (+)

2. $(5 + x) + 0 = 5 + x$
   Identity (+)

3. $16 + (c + 17) = (16 + c) + 17$
   Associative (+)

4. $d \cdot e \cdot 0 = 0$
   Multiplicative (0)

5. **Use a Counterexample** Indicate whether the conjecture is *true* or *false*. If *false*, provide a counterexample. (Example 2)
   Division of whole numbers is associative.
   false; Sample answer: $(24 \div 4) \div 2 \neq 24 \div (4 \div 2)$

6. Huda ordered a soda for AED 2.75, a sandwich for AED 8.50, and a dessert for AED 3.85. Sales tax was AED 1.15. Use mental math to find the total amount of the bill. Explain. (Example 3)
   AED 16.25;
   Sample answer: AED 3.85 → AED 1.15 = AED 5 and AED 2.75 → AED 8.50 = AED 2.50 + AED 0.25 + AED 8.50 = AED 11.25, AED 11.25 → AED 5 = AED 16.25

Simplify each expression. Justify each step. (Examples 4 and 5)

7. $15 + (12 + 8a)$

$= (15 + 12) + 8a$	Associative (+)
$= 27 + 8a$	Simplify.

8. $(5n \cdot 9) \cdot 2n$

$= (9 \cdot 5n) \cdot 2n$	Commutative (×)
$= 9 \cdot (5n \cdot 2n)$	Associative (×)
$= 9 \cdot 10n^2$	Simplify.
$= (9 \cdot 10) \cdot n^2$	Associative (×)
$= 90n^2$	Simplify.

9. $3x \cdot (7 \cdot x)$

$= 3x \cdot (x \cdot 7)$	Commutative (×)
$= (3x \cdot x) \cdot 7$	Associative (×)
$= 3x^2 \cdot 7$	Simplify.
$= 3 \cdot 7 \cdot x^2$	Commutative (×)
$= (3 \cdot 7) \cdot x^2$	Associative (×)
$= 21x^2$	Simplify.

10. $(4m \cdot 2) \cdot 5m$

$= (2 \cdot 4m) \cdot 5m$	Commutative (×)
$= 2 \cdot (4m \cdot 5m)$	Associative (×)
$= 2 \cdot 20m^2$	Simplify.
$= (2 \cdot 20)m^2$	Associative (×)
$= 40m^2$	Simplify.

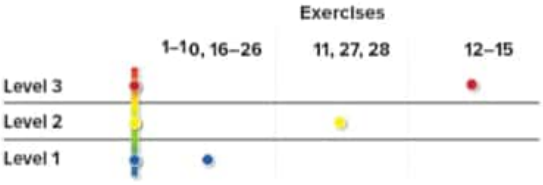
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–14, 27, 28
OL	On Level	1–9 odd, 11–14, 27, 28
BL	Beyond Level	11–15, 27, 28

Watch Out!

Common Error Some students may think that Exercise 1 represents the Associative Property of Addition because the statement uses parentheses. Explain to students that the variables and numbers grouped together inside the parentheses would have to change for the Associative Property to apply, such as $a + (b + 12) = (a + b) + 12$.

Name _____ My Homework _____

Independent Practice

Name the property shown by each statement. Example 1

1. $a + (b + 12) = (b + 12) + a$
   Commutative (+)

2. $(5 + x) + 0 = 5 + x$
   Identity (+)

3. $16 + (c + 17) = (16 + c) + 17$
   Associative (+)

4. $d \cdot e \cdot 0 = 0$
   Multiplicative (0)

5. **Use a Counterexample** Indicate whether the conjecture is *true* or *false*. If *false*, provide a counterexample. Example 2
   Division of whole numbers is associative.
   false; Sample answer: $(24 \div 4) \div 2 \neq 24 \div (4 \div 2)$

6. Huda ordered a soda for AED 2.75, a sandwich for AED 8.50, and a dessert for AED 3.85. Sales tax was AED 1.15. Use mental math to find the total amount of the bill. Explain. Example 3
   AED 16.25;
   Sample answer: AED 3.85 + AED 1.15 = AED 5 and AED 2.75 + AED 8.50 = AED 2.50 + AED 0.25 + AED 8.50 = AED 11.25, AED 11.25 + AED 5 = AED 16.25

Simplify each expression. Justify each step. Examples 4 and 5

7. $15 + (12 + 8a)$

$= (15 + 12) + 8a$	Associative (+)
$= 27 + 8a$	Simplify.

8. $(5n \cdot 9) \cdot 2n$

$= (9 \cdot 5n) \cdot 2n$	Commutative ( )
$= 9 \cdot (5n \cdot 2n)$	Associative ( )
$= 9 \cdot 10n^2$	Simplify.
$= (9 \cdot 10) \cdot n^2$	Associative ( )
$= 90n^2$	Simplify.

9. $3x \cdot (7 \cdot x)$

$= 3x \cdot (x \cdot 7)$	Commutative ( )
$= (3x \cdot x) \cdot 7$	Associative ( )
$= 3x^2 \cdot 7$	Simplify.
$= 3 \cdot 7 \cdot x^2$	Commutative ( )
$= (3 \cdot 7) \cdot x^2$	Associative ( )
$= 21x^2$	Simplify.

10. $(4m \cdot 2) \cdot 5m$

$= (2 \cdot 4m) \cdot 5m$	Commutative ( )
$= 2 \cdot (4m \cdot 5m)$	Associative ( )
$= 2 \cdot 20m^2$	Simplify.
$= (2 \cdot 20)m^2$	Associative ( )
$= 40m^2$	Simplify.

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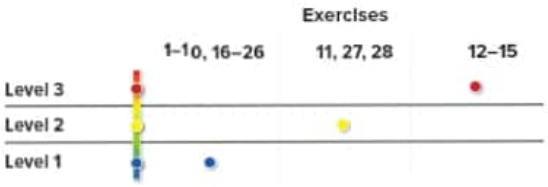

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MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	15
3 Construct viable arguments and critique the reasoning of others.	5, 13
4 Model with mathematics.	12
5 Use appropriate tools strategically.	21
7 Look for and make use of structure.	14

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a clos ing formative assessment before dismissing students from your class.

TICKET Out the Door

Ask students to give exa mples of the Associative Property and the Commutative P roperty. See students' work.

Watch Out!

Find the Error Students may need to be reminded in Exercise 13 that *associative* properties are related to *grouping*, while *commutative* properties are related to *order*.

11. Simplify the expression $(7 + 47 + 3)[5 \cdot (2 \cdot 3)]$. Use properties to justify each step.

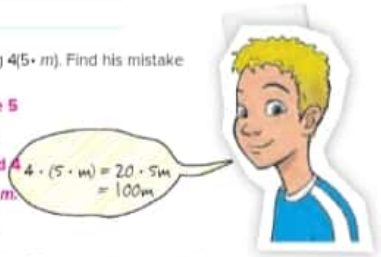
$[7 + (47 + 3)][5 \cdot (2 \cdot 3)]$, Associative (+); $(7 + 50)[5 \cdot (2 \cdot 3)]$, Simplify; $57[5 \cdot (2 \cdot 3)]$, Simplify; $57[(5 \cdot 2) \cdot 3]$, Associative (·); $57 \cdot 10 \cdot 3$, Simplify; $(57 \cdot 10) \cdot 3$, Associative (·); $570 \cdot 3$, Simplify; 1,710

H.O.T. Problems Higher Order Thinking

12. **Model with Mathematics** Write about something you do every day that is commutative. Then write about another situation that is not commutative.

See students' work.

13. **Find the Error** Mansour is simplifying $4(5 \cdot m)$. Find his mistake and correct it.



Mansour incorrectly multiplied both the 5 and m by 4. He should have used the Associative Property to group the 5 and 4 together, simplify and then multiply by m.

$4 \cdot (5 \cdot m) = 20m$

14. **Identify Structure** Does the Associative Property *always*, *sometimes*, or *never* hold for subtraction? Explain your reasoning using examples and counterexamples.

sometimes; Sample answer: It works for the expression $5 - (6 - 0)$. It does not work for the expression $(6 - 6)$.

15. **Persevere with Problems** If you take any two whole numbers and add them together, the sum is always a whole number. This is the Closure Property for Addition. The set of whole numbers is *closed* under addition.

a. Is the set of whole numbers closed under subtraction? If not, give a counterexample.

no; Sample answer: $2 - 3 = -1$ and -1 is not a whole number

b. Suppose you had a very small set of numbers that contained only 0 and 1. Would this set be closed under addition? If not, give a counterexample.

no; Sample answer: $1 + 1 = 2$ and 2 is not a member of the set.

Name _____ My Homework _____

Extra Practice

Name the property shown by each statement.

16. $9(ab) = (9a)b$

Associative (×)

17. $y \cdot 7 = 7y$

Commutative (×)

18. $1 \times c = c$

Identity (×)

19. $5 + (a + 8) = (5 + a) + 8$

Associative (+)

20. State whether the conjecture is *true* or *false*. If *false*, provide a counterexample.

Subtraction of whole numbers is commutative.

false; Sample answer: 10 4 ≠ 4 – 10

21. **Use Math Tools** The times for each leg of a relay for four runners are shown. Use mental math to find the total time for the relay team. Explain.

48 s; Sample answer: 12.4 + 12.6 = 25 and 11.8 + 11.2 = 23, 25 + 23 = 48

Runner	Time (s)
Jamal	12.4
Eissa	11.8
Khalifa	11.2
Majed	12.6

Simplify each expression. Justify each step.

22. $(22 + 19b) + 7$

$= (19b + 22) + 7$ Commutative (+)

$= 19b + (22 + 7)$ Associative (+)

$= 19b + 29$ Simplify.

23. $18 + (5 + 6m)$

$= (18 + 5) + 6m$ Associative (+)

$= 23 + 6m$ Simplify.

24. $11s(4)$

$= 11 \cdot 4 \cdot s$ Commutative (×)

$= (11 \cdot 4) \cdot s$ Associative (×)

$= 44s$ Simplify.

25. $10y(7)$

$= 10 \cdot 7 \cdot y$ Commutative (×)

$= (10 \cdot 7) \cdot y$ Associative (×)

$= 70y$ Simplify.

26. $(9 + 31 + 5)[(7 \cdot 5) \cdot 4]$

[(9 + 31) + 5][(7 · 5) · 4], Associative (×); (40 + 5)(7 · 5) · 4], Simplify; 45[(7 · 5) · 4], Simplify; 45[7 · (5 · 4)], Associative (×); 45 · 7 · 20, Simplify; 45 · 20 · 7, Commutative (×); (45 · 20) · 7, Associative (×); 900 · 7, Simplify; 6,300, Simplify

Power Up! Test Practice

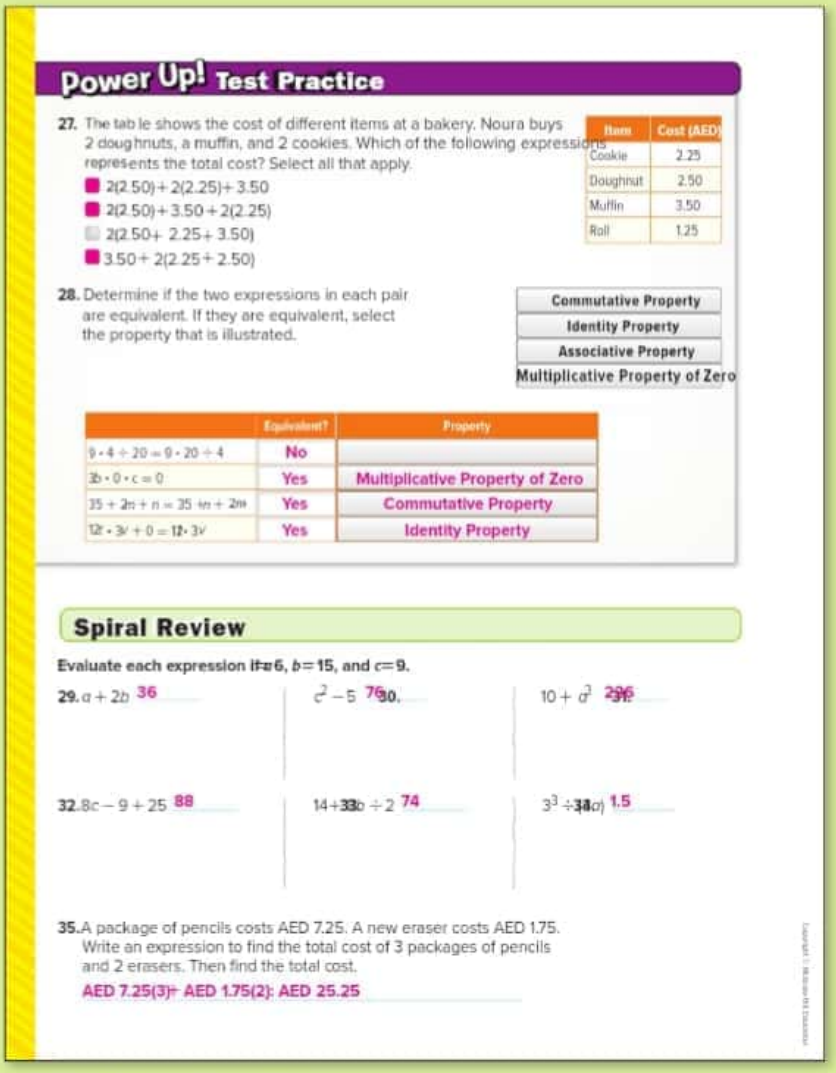
Exercises 27 and 28 prepare students for more rigorous thinking needed for the assessment.

27. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

28. This test item requires students to reason abstractly and quantitatively when problem solving.

Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP7
Scoring Rubric	
2 points	Students correctly place all seven values.
1 point	Students correctly place five or six of the seven values.



Power Up! Test Practice

27. The table shows the cost of different items at a bakery. Noura buys 2 doughnuts, a muffin, and 2 cookies. Which of the following expressions represents the total cost? Select all that apply.

Item	Cost (AED)
Cookie	2.25
Doughnut	2.50
Muffin	3.50
Roll	1.25

- [x] 2(2.50) + 2(2.25) + 3.50
- [x] 2(2.50) + 3.50 + 2(2.25)
- [ ] 2(2.50 + 2.25 + 3.50)
- [x] 3.50 + 2(2.25 + 2.50)

28. Determine if the two expressions in each pair are equivalent. If they are equivalent, select the property that is illustrated.

Commutative Property
Identity Property
Associative Property
Multiplicative Property of Zero

	Equivalent?	Property
$9 \cdot 4 \div 20 = 9 \cdot 20 \div 4$	No	
$3b \cdot 0 \cdot c = 0$	Yes	Multiplicative Property of Zero
$35 + 2n + n = 35 + n + 2n$	Yes	Commutative Property
$12r \cdot 3v + 0 = 12 \cdot 3v$	Yes	Identity Property

Spiral Review

Evaluate each expression if $a = 6$, $b = 15$, and $c = 9$.

29. $a + 2b$ 36

$c^2 - 5$ 76 **30.**

$10 + a^2$ 236 **31.**

32. $8c - 9 + 25$ 88

$14 + 3b \div 2$ 74

$3^3 \div 3a$ 1.5

35. A package of pencils costs AED 7.25. A new eraser costs AED 1.75. Write an expression to find the total cost of 3 packages of pencils and 2 erasers. Then find the total cost.

AED 7.25(3) + AED 1.75(2); AED 25.25

Expressions and Equations

Lesson 4

The Distributive Property

Real-World Link

School Supplies Mazen buys three notebooks that cost AED 5 each. He also buys three packages of pens at AED 6 each.

1. Write an expression that shows the cost of three notebooks added to the cost of three packages of pens.

   3 × 5 + 3 × 6

2. Write an expression that shows three times the cost of one notebook and one package of pens.

   3 (5 + 6)

3. Evaluate both expressions. What do you notice?

   AED 33; AED 33; The expressions have the same value.

4. Suppose Mazen buys five notebooks at AED 3 each and five packages of pens that cost AED 7 each. Circle the expressions that represent Mazen's purchases.

   5 × 3 + 5 × 7 (circled) 5 × 3 × 5 × 7 5(3 + 7) (circled)

5. Suppose Mazen buys two rulers that cost AED 1 each and two folders that cost AED 1.50 each. Circle the expressions that represent Mazen's purchases.

   2 + 1 + 2 + 1.50 2(1 + 1.50) (circled) 2 × 1 + 2 × 1.50 (circled)

Essential Question

HOW can you use numbers and symbols to represent mathematical ideas?

Vocabulary

Distributive Property

equivalent expressions

Mathematical Practices

1, 3, 4, 5, 7

Which Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems ② Reason Abstractly ③ Construct an Argument ④ Model with Mathematics ⑤ Use Math Tools ⑥ Attend to Precision ⑦ Make Use of Structure ⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Apply the Distributive Property to rewrite algebraic expressions.

Coherence connecting within and across grades

Previous Students used properties to simplify algebraic expressions.

Now Students use the Distributive Property to simplify algebraic expressions.

Next Students will use the Distributive Property to factor linear expressions.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 379.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL LA **Pairs Discussion** Have students work in pairs to complete Exercises 1–5. Then ask them to consider how they could apply this concept to mentally multiply 54 by 22. Have them discuss their findings with another pair of students. MP 1, 7

Alternate Strategy

AL Give students two-sided counters. One side represents AED 5 for each notebook, and the other side represents AED 6 for each package of pens. Have students model the two different situations in Exercises 1 and 2. They can then assign different values to the counters to model Exercises 4 and 5. MP 1, 4, 7

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Evaluate expressions.

AL
- *What number is outside the parentheses?* 8
- *What does it mean that 8 is outside the parentheses?* Sample answer: 8 is being multiplied by both −9 and 4.
- *How can you use the Distributive Property in the Example?* Multiply each term by 8, then add.

OL
- *Do you need to multiply the first term, the second term, or both terms by 8?* both
- *After you distribute the multiplication, what should you do?* Add the two products.

BL
- *What does the word distribute mean in everyday use?* Sample answer: to spread something out
- *How does the everyday meaning of the word distribute relate to the Distributive Property?* Sample answer: When you use the Distributive Property, you spread the multiplication out over the addition or subtraction.

Need Another Example?
Use the Distributive Property to evaluate 7(12).
7 • 3 + 7 • 12; 105

Key Concept **Use the Distributive Property**

Work Zone

Words	The Distributive Property states that to multiply a difference by a number, multiply each term inside the parentheses by the number outside the parentheses.	
Symbols	$a(b + c) = ab + ac$	$a(b - c) = ab - ac$
Examples	$4(6 + 2) = 4 \cdot 6 + 4 \cdot 2$	$3(7 - 5) = 3 \cdot 7 - 3 \cdot 5$

You can model the Distributive Property with algebraic expressions using algebra tiles. The expression 2(x + 2) is modeled below.

Model x + 2 using algebra tiles.

Double the amount of tiles to represent 2(x + 2).

Rearrange the tiles by grouping together the ones with the same shapes.

$2(x + 2) = 2(x) + 2(2)$ Distributive Property
$= 2x + 4$ Multiply

The expressions 2(x + 2) and 2x + 4 are **equivalent expressions**. No matter what x is, these expressions have the same value.

Example

1. Use the Distributive Property to evaluate 8(−9 + 4).

$8(-9 + 4) = 8(-9) + 8(4)$ Expand using the Distributive Property.
$= -72 + 32$ or -40 Multiply. Then add.

Got It? Do these problems to find out.

a. 5(−9 + 11) b. 7(10 − 5) c. (12 − 8)9

a. 10
b. 35
c. 36

Expressions and Equations

Examples

Use the Distributive Property to rewrite each expression.

2. $4(x+7)$

$4(x+7) = 4(x) + 4(7)$ Expand using the Distributive Property.
$= 4x + 28$ Simplify.

3. $6(p-5)$

$6(p-5) = 6[p+(-5)]$ Rewrite $p-5$ as $p+(-5)$.
$= 6(p) + 6(-5)$ Expand using the Distributive Property.
$= 6p + (-30)$ Simplify.
$= 6p - 30$ Definition of subtraction

4. $-2(x-8)$

$-2(x-8) = -2[x+(-8)]$ Rewrite $x-8$ as $x+(-8)$.
$= -2(x) + -2(-8)$ Expand using the Distributive Property.
$= -2x + 16$ Simplify.

5. $5(-3x+7y)$

$5(-3x+7y) = 5(-3x) + 5(7y)$ Expand using the Distributive Property.
$= -15x + 35y$ Simplify.

6. $\frac{1}{3}(x-6)$

$\frac{1}{3}(x-6) = \frac{1}{3}[x+(-6)]$ Rewrite $x-6$ as $x+(-6)$.
$= \frac{1}{3}(x) + \left(\frac{1}{3}(-6)\right)$ Expand using the Distributive Property.
$= \frac{1}{3}x + (-2)$ Simplify.
$= \frac{1}{3}x - 2$ Definition of subtraction

Got It? Do these problems to find out.

d. $6(a+4)$ e. $(m+3n)8$
f. $-3(y-10)$ g. $\frac{1}{2}(w-4)$

d. $6a + 24$
e. $8m + 24n$
f. $-3y + 30$
g. $\frac{1}{2}w - 2$

Examples

2–3. Write equivalent expressions.

AL • *How are Examples 2 and 3 different?* Example 2 is the product of a number and a sum. Example 3 is the product of a number and a difference.

• *In Example 3, why is $p-5$ changed to $p+(-5)$?* It helps us to remember to multiply 6 by -5, not 5. Subtracting a positive integer is the same as adding a negative integer.

OL • *In Example 3, why is $6p-(-30)$ changed to $6p-30$?* Since they are equivalent expressions, it is simplified when written as $6p-30$.

• *In Example 3, can you subtract 30 from 6p? Explain.* No; they are not like terms.

BL • *Refer to Example 3. What is another way you can rewrite the expression, without first changing the subtraction to addition?* Sample answer: You can distribute the multiplication over the subtraction and keep the subtraction sign; $6(p-5) = 6p - 30$

Need Other Examples?
Use the Distributive Property to rewrite each expression.
a. $11(x+5)$ $11x+55$ b. $5(p-8)$ $5p-40$

4–6. Write equivalent expressions.

AL • *In Example 4, do you distribute just the 2, just the negative sign, or the entire number 2 to both terms?* -2

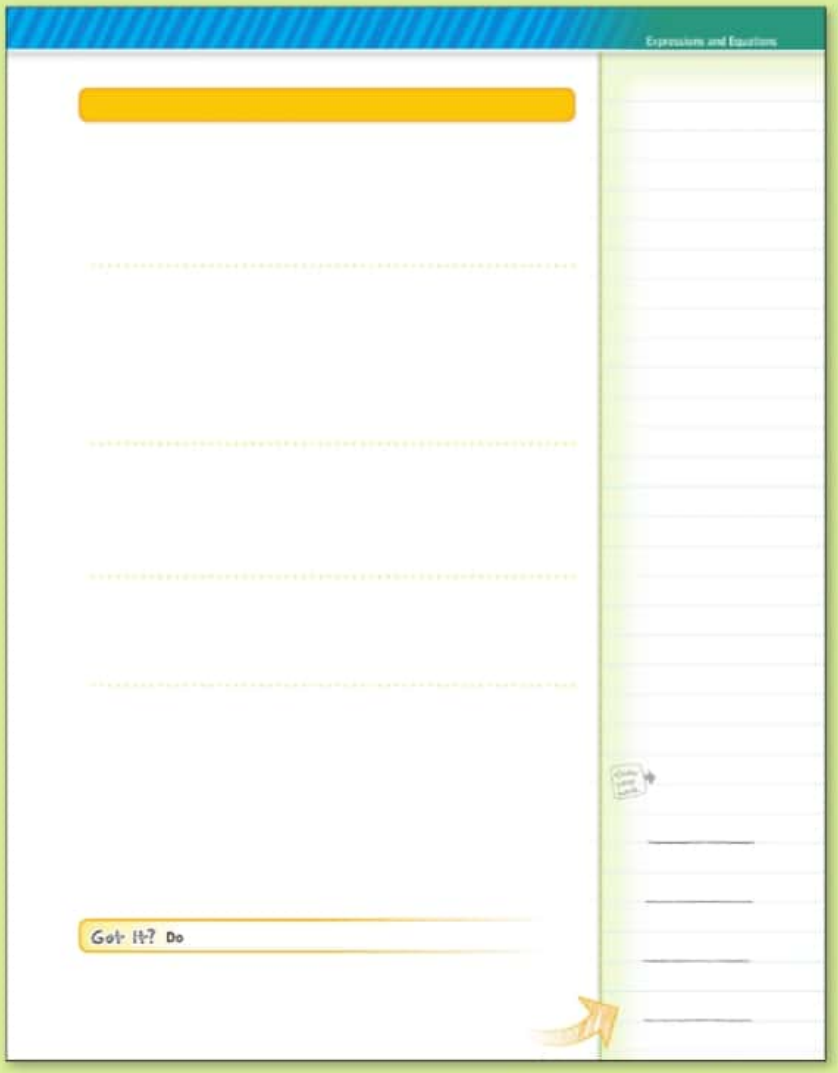
• *In Example 4, what is $-2 \cdot (-8)$?* 16

• *In Example 6, what is $\frac{1}{3} \cdot x$?* $\frac{1}{3}x$

OL • *In Example 5, can you add $15x$ and $35y$? Explain.* no; Sample answer: They are not like terms since the variables are different.

BL • *In Example 4, why was it beneficial to change subtraction to addition before distributing?* Sample answer: It helped to avoid making a sign mistake by remembering to multiply -2 by -8, and not by 8.

Need Other Examples?
Use the Distributive Property to rewrite each expression.
a. $-3(x-2)$ $-3x+6$ b. $\frac{1}{4}(x-16)$ $\frac{1}{4}x-4$



Examples

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BL • *Refer to Example 3. What is another way you can rewrite the expression, without first changing the subtraction to addition?* Sample answer: You can distribute the multiplication over the subtraction and keep the subtraction sign; $6(-5) = 6 - 30$

Need Other Examples?

Use the Distributive Property to rewrite each expression.

a. $11(x + 5)$ $11 + 55$ **b.** $5(p - 8)$ $5 - 40$

4–6. Write equivalent expressions.

AL • *In Example 4, do you distribute just the 2, just the negative sign, or the entire number −2 to both terms?* -2

• *In Example 4, what is* $-2 \cdot (-8)$*?* 16

• *In Example 6, what is* $\frac{1}{3} \cdot x$*?* $\frac{1}{3}$

OL • *In Example 5, can you add 15x and 35y? Explain.* no; Sample answer: They are not like terms since the variables are different.

BL • *In Example 4, why was it beneficial to change subtraction to addition before distributing?* Sample answer: It helped to avoid making a sign mistake by remembering to multiply −2 by −8, and not by 8.

Need Other Examples?

Use the Distributive Property to rewrite each expression.

a. $-3(x - 2)$ $-3 + 6$ **b.** $\frac{1}{4}(x - 16)$ $\frac{1}{4} - 4$

Example

7. Simplify expressions.

AL • *What do you need to find?* the total cost of 9 helmets

• *Why rename AED 19.95 as AED 20.00 − AED 0.05?* to be able to use the Distributive Property to mentally multiply 9 by AED 20.00 and 9 by AED 0.05

OL • *How does the Distributive Property allow you to use mental math in this example?* Sample answer: I can find 9 × AED 20 and 9 × AED 0.05 quicker than I can find 9 × AED 19.95.

BL • *Is there another way to solve this problem using the Distributive Property? If so, write the expression you could use.* yes; Sample answer: 19.95(10 − 1)

Need Another Example?
Fifteen students are buying T-shirts that cost AED 10.60 each. Use the Distributive Property to find the total cost of the shirts.
AED 159; 15(AED 10 + AED 0.60) = 15 • 10 + 15 • 0.6 = 150 + 9, or 159

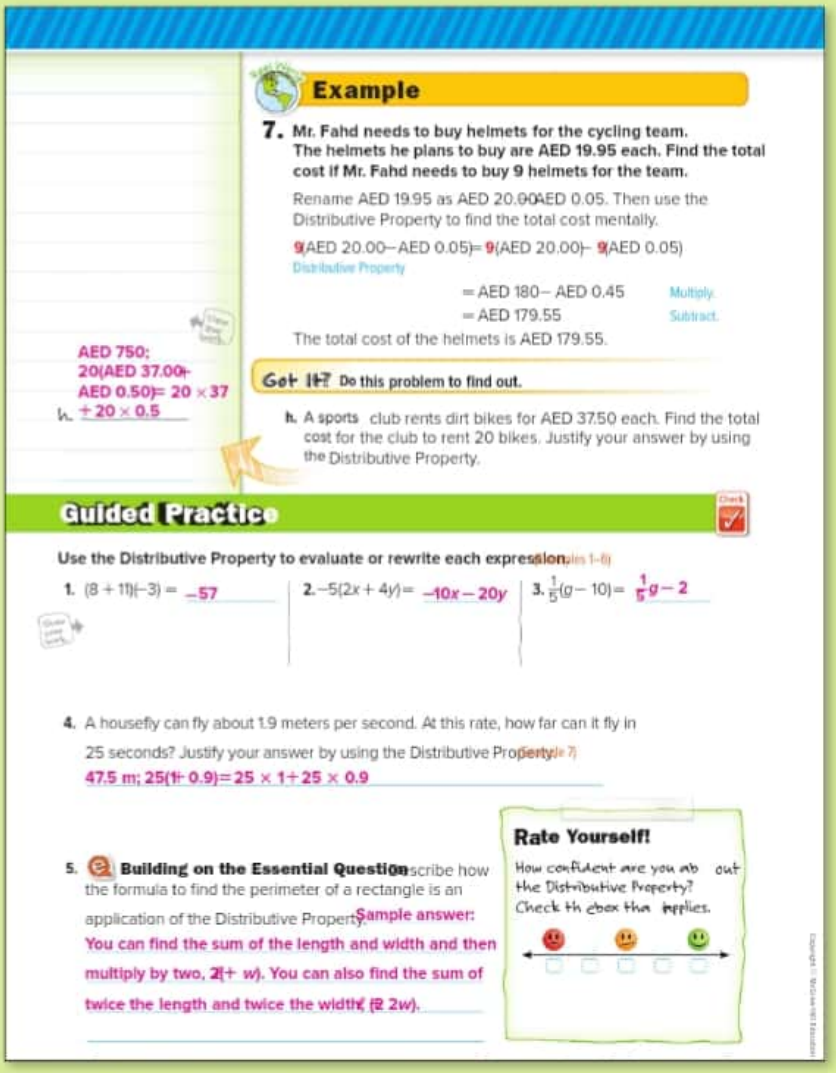
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Team Pair Solo** Divide students into teams of four. Each team completes Exercises 1 and 2. Then students pair up to complete Exercise 3. The team regroups to discuss the solution. Finally, students work alone to complete Exercise 4, return to the team, and share answers. If the solutions do not agree, students work together to find the error. MP 1, 7

BL LA **Pairs Discussion** Have students work in pairs. Tell students the "official" names of the Distributive Property: the Distributive Property of Multiplication over Addition, and the Distributive Property of Multiplication over Subtraction. Ask students to consider the expression 5(3) and determine whether there is a Distributive Property of Multiplication over Multiplication. Then call on one student to share and justify their response within a small group or large group discussion. MP 1, 6, 7



Example

7. Mr. Fahd needs to buy helmets for the cycling team. The helmets he plans to buy are AED 19.95 each. Find the total cost if Mr. Fahd needs to buy 9 helmets for the team.

Rename AED 19.95 as AED 20.00 − AED 0.05. Then use the Distributive Property to find the total cost mentally.

9(AED 20.00 − AED 0.05) = 9(AED 20.00) − 9(AED 0.05) Distributive Property

= AED 180 − AED 0.45 Multiply.

= AED 179.55 Subtract.

The total cost of the helmets is AED 179.55.

Got It? Do this problem to find out.

h. A sports club rents dirt bikes for AED 37.50 each. Find the total cost for the club to rent 20 bikes. Justify your answer by using the Distributive Property.

AED 750; 20(AED 37.00 + AED 0.50) = 20 × 37 + 20 × 0.5

Guided Practice

Use the Distributive Property to evaluate or rewrite each expression. Examples 1–6

1. (8 + 11)(−3) = −57
2. −5(2x + 4y) = −10x − 20y
3. $\frac{1}{5}(g - 10) = \frac{1}{5}g - 2$
4. A housefly can fly about 1.9 meters per second. At this rate, how far can it fly in 25 seconds? Justify your answer by using the Distributive Property. Example 7
47.5 m; 25(1 + 0.9) = 25 × 1 + 25 × 0.9
5. **Building on the Essential Question** Describe how the formula to find the perimeter of a rectangle is an application of the Distributive Property. Sample answer: You can find the sum of the length and width and then multiply by two, 2(ℓ + w). You can also find the sum of twice the length and twice the width, (2ℓ + 2w).

Rate Yourself!
How confident are you about the Distributive Property? Check the box that applies.

Name _____ My Homework _____

Independent Practice

Use the Distributive Property to evaluate each expression. (Example 1)

1. $3(5+6) = 33$
2. $(6+4)(-12) = -120$
3. $-6(9-4) = -30$
4. $5(-6+4) = -10$
5. $4(8-7) = 4$
6. $(5-7)(-3) = 6$

Identify Structure Use the Distributive Property to rewrite each expression. (Examples 2–4)

7. $3(-4x+8) = -12x+24$
8. $4(x-6y) = 4x-24y$
9. $6(5-q) = 30-6q$
10. $\frac{1}{2}(c-8) = \frac{1}{2}c-4$
11. $-3(5-b) = -15+3b$
12. $(d-2)(-7) = -7d-14$

13. Eiman bought roast beef for AED 59.85 per kilogram. Find the total cost if Eiman bought 4 kilograms of roast beef. Justify your answer by using the Distributive Property. (Example 7)

AED 239.40; $4(\text{AED } 60.00 - \text{AED } 0.15) = 4 \times 60 - 4 \times 0.15$

14. The table shows the different prices of items at a movie theater.
    a. Suppose Wafa and two of her friends go to the movies. Write an expression that could be used to find the total cost for them to go to the movies and buy one of each item.

    $3(2.25 + 3.25 + 4.50 + 19.50)$

    b. What is the total cost for all three people?

    AED 88.50

Movie Theater Prices	
Item	Cost (AED)
box of candy	2.25
drink	3.25
popcorn	4.50
ticket	19.50

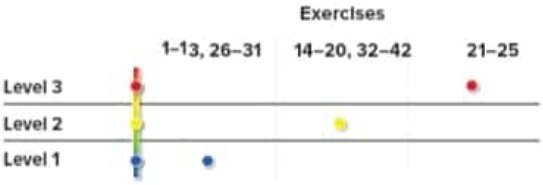
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–13, 15–19 odd, 21, 22, 24, 41, 42
OL	On Level	1–13 odd, 14–22, 24, 41, 42
BL	Beyond Level	15–25, 41, 42

Watch Out!

Common Error Some students may make errors simplifying expressions such as $(x+7)5$ because of the order. Have students use the Commutative Property of Multiplication to write the equivalent expression $5(x+7)$.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	23, 25
2 Reason abstractly and quantitatively.	21
3 Construct viable arguments and critique the reasoning of others.	24, 32
5 Use appropriate tools strategically.	15–20
7 Look for and make use of structure.	7–12, 22

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students explain h ow they think today's lesson on the Distributive Property wi ll help them with simplifying algebraic expressions. See students' work.

Use Math Tools Find each product mentally. Justify your answer.

15. 9 • 35 = 315
9(30 + 5) = 9(30) + 9(5)
= 270 + 45

16. 8 • 28 = 224
8(20 + 8) = 8(20) + 8(8)
= 160 + 64

17. 112 • 6 = 672
(100 + 12)6 = 100(6) + 12(6)
= 600 + 72

18. 85 • 8 = 680
(80 + 5)8 = 80(8) + 5(8)
= 640 + 40

19. 4 • 122 = 488
4(120 + 2) = 4(120) + 4(2)
= 480 + 8

20. 12 • 64 = 768
12(60 + 4) = 12(60) + 12(4)
= 720 + 48

H.O.T. Problems Higher Order Thinking

21. **Reason Abstractly** Write an expression that when using the Distributive Property can be simplified to 12*b* − 6c.
Sample answer: 6(2*b* − *c*)

22. **Identify Structure** Use the Distributive Property to rewrite the expression 7*bx* + 7*by* as an equivalent expression.
7*b*(*x* + *y*)

23. **Persevere with Problems** Use the Distributive Property to write an equivalent expression for the expression (*a* + *b*)(2 + *y*).
2*a* + *ay* + 2*b* + *by*

24. **Find the Error** Hana is using the Distributive Property to simplify 3(*x* + 2). Find her mistake and correct it.

$3(x + 2) = 3x + 2$

Sample answer: Hana did not distribute the 3 to the second term inside the parentheses; the correct answer is 3*x* + 6.

25. **Persevere with Problems** 3 + (*x* × *y*) = (3 + *x*) × (3 + *y*) a true statement? If so, explain your reasoning. If not, give a counterexample.
No; 3 + (4 × 5) = 23 but (3 + 4) × (3 + 5) = 56

Name _____ My Homework _____

Extra Practice

Use the Distributive Property to evaluate each expression.

26. $(3+6)(-8) =$ -72

$3 \cdot (-8) + 6 \cdot (-8) =$

$-24 + (-48) = -72$

27. $4(11-5) =$ 24

28. $(12-4)(-5)$ -40

Use the Distributive Property to rewrite each expression.

29. $-8(a+b) =$ $-8a-8b$

30. $(2b+8)5 =$ $10b+40$

31. $(p+7)(-2) =$ $-2p-14$

32. Justify Conclusions Wafa is planning on making a fleece blanket for her nephew. She learns that the fabric she wants to use is AED 7.99 per meter. Find the total cost of 4 meters of fabric. Justify your answer by using the Distributive Property.

AED 31.96; 4(AED 8.00 AED 0.01)=

$4 \times 8 - 4 \times 0.01$

33. You are ordering T-shirts with your school's mascot printed on them. Each T-shirt costs AED 24.75. The printer charges a setup fee of AED 30 and AED 2.50 to print each shirt. Write two expressions to represent the total cost of printing n T-shirts.

$n(24.75+2.50)+30$; $27.25n+30$

Use the Distributive Property to rewrite each expression.

34. $0.5x(y-z)$

$= 0.5xy - 0.5xz$

35. $-6a(2b+5c)$

$= -12ab - 30ac$

36. $-4m(3n-6p)$

$= -12mn + 24mp$

37. $3(2y+4z)$

$= 6y + 12z$

38. $-2(3a-2b)$

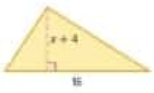
$= -6a + 4b$

39. $-6(12p-8n)$

$= -72p + 48n$

40. Write two equivalent expressions for the area of the figure.

$8(x+4)$; $8x+32$



Power Up! Test Practice

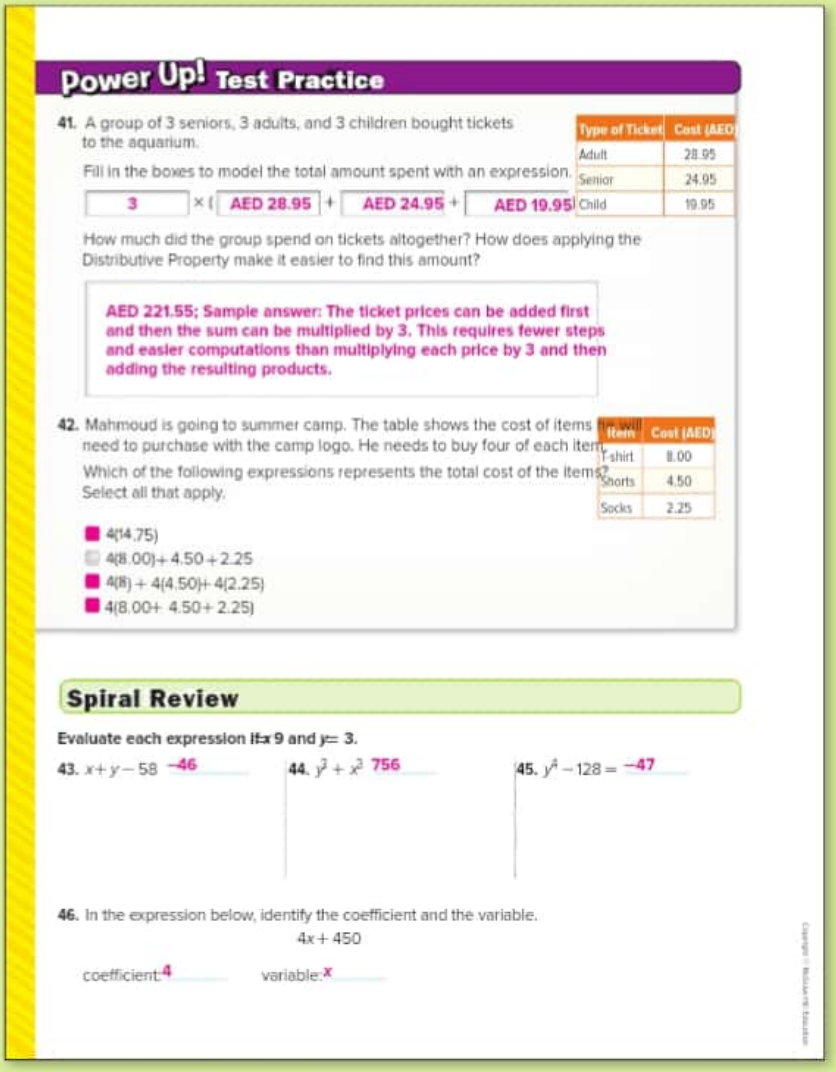
Exercises 41 and 42 prepare students for more rigorous thinking needed for the assessment.

41.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP3, MP4
Scoring Rubric	
2 points	Students correctly model the situation and answer the question.
1 point	Students correctly model the situation OR students correctly answer the question.

42.

This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Knowledge	DOK3
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.



Power Up! Test Practice

41. A group of 3 seniors, 3 adults, and 3 children bought tickets to the aquarium.

Type of Ticket	Cost (AED)
Adult	28.95
Senior	24.95
Child	19.95

Fill in the boxes to model the total amount spent with an expression.

3 × (AED 28.95 + AED 24.95 + AED 19.95)

How much did the group spend on tickets altogether? How does applying the Distributive Property make it easier to find this amount?

AED 221.55; Sample answer: The ticket prices can be added first and then the sum can be multiplied by 3. This requires fewer steps and easier computations than multiplying each price by 3 and then adding the resulting products.

42. Mahmoud is going to summer camp. The table shows the cost of items he will need to purchase with the camp logo. He needs to buy four of each item.

Item	Cost (AED)
T-shirt	8.00
Shorts	4.50
Socks	2.25

Which of the following expressions represents the total cost of the items? Select all that apply.

- [x] 4(14.75)
- [ ] 4(8.00)+ 4.50 + 2.25
- [x] 4(8) + 4(4.50)+ 4(2.25)
- [x] 4(8.00+ 4.50+ 2.25)

Spiral Review

Evaluate each expression if $x = 9$ and $y = 3$.

43. $x + y - 58$ −46

44. $y^3 + x^3$ 756

45. $y^4 - 128 =$ −47

46. In the expression below, identify the coefficient and the variable.

$4x + 450$

coefficient: 4 variable: x

Problem-Solving Investigation
Make a Table

MP Mathematical Practices 1, 3, 4

Case #1 Mountain Biking

Ali wants to purchase a membership to a bike park. The cost depends on the number of people on the membership. It costs AED 55 for 5 people, AED 65 for 6 people, and AED 75 for 7 people.

Find the cost of a membership that includes 8 people.

1 Understand *What are the facts?*

The cost of a membership depends on the number of people included on the membership.

2 Plan *What is your strategy to solve this problem?*

Make a table that shows the number of people and the cost.

3 Solve *How can you apply the strategy?*

Make a table. Find the cost for 8 people.

Number of People (p)	Cost
5	AED 55
6	AED 65
7	AED 75
8	AED 85

+1a +1a +1a

So, the cost for 8 people is AED 85.

4 Check *Does the answer make sense?*

The expression 10p + 5 can be used to represent the situation.

Since AED 10(8) + AED 5 = AED 85, the solution is reasonable.

Analyze the Strategy

Justify Conclusions Ali wants to purchase a membership for four people. Explain how the table would change and then solve.

Sample answer: The table would be set up in descending order. To solve for 4 people, subtract AED 10 from AED 55; AED 45

Focus narrowing the scope

Objective Solve problems by making a table. This lesson emphasizes MP **Mathematical Practice 4** Using tables, students model real-world situations. Then they justify conclusions with written explanations and computations.

Make a Table Making a table is a useful strategy for solving many kinds of problems. In this lesson, students will work with tables involving numbers, objects, and geometric figures.

Coherence connecting within and across grades

Now Students solve non-routine problems.

Next Students will apply the make a table strategy to find patterns.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 385.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

The problems on pages 383 and 384 are intended to be used as a whole-group discussion on how to solve non-routine problems and are designed to provide scaffolded guidance. The problem on page 383 walks students through the solution, while the problem on page 384 asks students to come up with their own solutions.

Case #1 Mountain Biking

BL Have students extend the problem by having them answer the questions below.

Ask:

- *Find the average cost per person using the cost for one person and the cost for four people.* The cost for one person is AED 15. The average cost per person for four people is AED 11.25.

Case #2 Financial Literacy

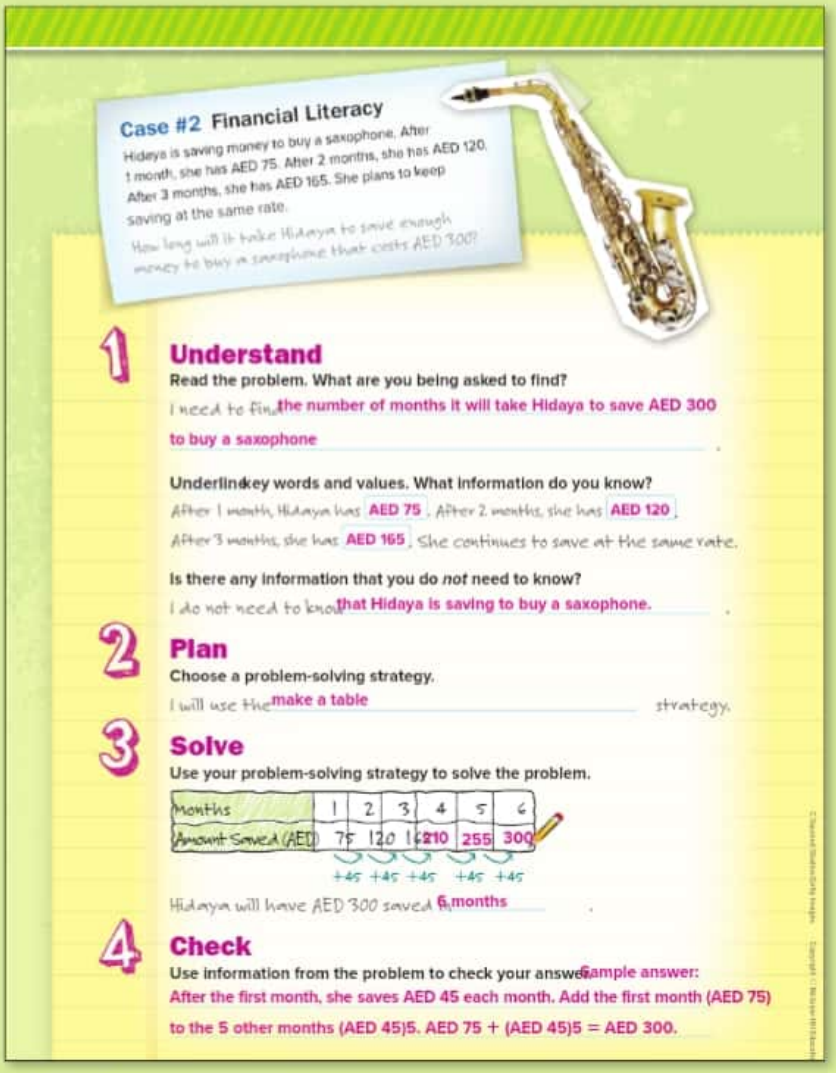
AL LA **Circle the Sage** Have students work in teams of 3–5 students to read through the problem. Poll the class to see who feels that they have a solid understanding on how to solve the problem. Those students (the sages) spread out around the room. Have the teams split up with each team member going to a different sage, if possible. Have the sages explain how they completed the exercises while the classmates listen, ask questions, and take notes. MP 1, 3,

BL LA **One Stray** Have students complete the problem in teams of 3 or 4. Call one or more students to stand. The standing student(s) move to a team with an empty seat. The student(s) who moved compare their team's responses with the new team's responses. As the student(s) return to their original team, give them time to change any of their responses, if necessary. MP 1

Need Another Example?
In a pyramid of balloons, there is one balloon at the top, four balloons in the second layer, and seven balloons in the third layer. The pattern continues for a total of six layers. How many balloons are in the sixth layer?

Layer	1	2	3	4	5	6
Number of Balloons	1	4	7	10	13	16

There are 16 balloons in the sixth layer.



Work with a small group to solve the following cases. Show your work on a separate piece of paper.

Case #3 Carnivals

For a carnival game, containers are arranged in a triangular display. The top row has 1 container. The second row has 2 containers. The third row has 3 containers. The pattern continues until the bottom row, which has 10 containers.

A contestant knocks down 29 containers on their first throw. How many containers remain?

26 containers

Case #4 Budget

Yasmin earns AED 2,050 each month. She spends 65% of the amount she earns. The rest of the money is equally divided and deposited into two separate accounts.

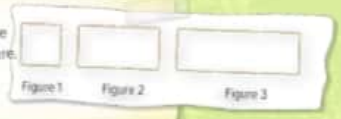
How many months until Yasmin has deposited more than AED 2,500 in one of her accounts?

7 months

Case #5 Toothpicks

Write an expression that can be used to find the number of toothpicks needed to make any figure. Then find the number of toothpicks needed to make the eighth figure.

$2n + 2$; 18 toothpicks



Use any strategy!

Case #6 Diving

A diver descends to −4.5 meters after 1 minute, −9 meters 2 minutes, and −13.5 meters after 3 minutes.

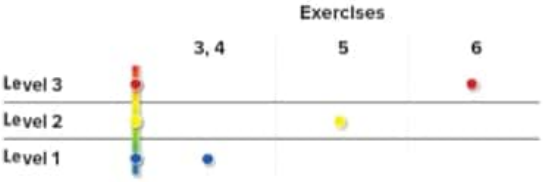
If the diver keeps descending at this rate, what is their position after 12 minutes?

−54 meters

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** EVALUATE

2 Collaborate

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



AL LA **Rally Coach** Have students work in pairs to complete Cases 1–4. Student 1 completes Case 1, talking through their solution process, while Student 2 watches, listens, coaches, and praises. Partners alternate roles until all exercises have been completed. MP 1, 6

BL LA **Team Discussion** In teams of 3–4, have students discuss how making a table can help them solve a problem involving a linear relationship. Have them study their tables in Cases 1–4 to determine if each relationship is linear. Have them justify their responses. MP 1, 6, 7

Ask:

- *How does making a table help you solve the problem?* Sample answer: Making a table helps to organize the information to discover a pattern or see if there is a constant rate of change.
- *How do you know if a relationship is linear, by studying the table?* Sample answer: If the table shows a constant rate of change, the relationships is linear.

Mid-Chapter Check

If students have trouble with Exercises 1–10, they may need help with the following concepts.

Concept	Exercise(s)
sequences (Lesson 2)	2–5
algebraic expressions (Lesson 1)	1, 10
Distributive Property (Lesson 4)	6–8
mathematical properties (Lesson 3)	9

Vocabu lary Activity

LA **Pairs Discussion** Have students work in pairs to complete Exercise 2. One student makes a list of similarities between an arithmetic sequence and a geometric sequence. The other student makes a list of differences. Have them swap lists and discuss, modifying their lists if necessary.

MP 1, 6, 7

Mid-Chapter Check

Vocabulary Check

1. Fill in the blank in the sentence below with the correct term. (Lesson 1)
   A **variable** is a symbol that represents an unknown quantity.

2. Define *arithmetic sequence*. Then provide an example. (Lesson 2)
   In an arithmetic sequence each term is found by adding the same number to the previous term. Sample answer: 5, 7, 9, 11, ...

Skills Check and Problem Solving

Describe the relationship between the terms in each arithmetic sequence. Then write the next three terms in each sequence. (Lesson 2)

3. 5, 8, 11, 14, ...
   Add 3 to the previous term; 17, 20, 23

4. 4, 11, 18, 25, ...
   Add 7 to the previous term; 32, 39, 46

5. 5.8, 10.8, 15.8, 20.8, ...
   Add 5 to the previous term; 25.8, 30.8, 35.8

Use the Distributive Property to rewrite each expression. (Lesson 4)

6. $4(x + 9) =$ **$4x + 36$**

7. $2(x + 5) =$ **$2x + 10$**

8. $3(-2x - 4) =$ **$-6x + 12$**

9. **Identify Structure** What property is shown by the statement $8x + 0 = 8x$? (Lesson 3)
   Identity (+)

10. **Persevere with Problems** A coach bought some baseball bats and five baseball gloves. Let b represent the number of bats. Write an expression that can be used to find the total cost of the bats and gloves. Then find the total cost if he bought three bats. (Lesson 1)
    $5(48) + 35b$; AED 345

Lesson 5

Simplify Algebraic Expressions

Real-World Link

Music Store Salem, Khalaf, and Ayoub work at a music store. Each week, Salem works three more than twice the number of hours that Khalaf works. Ayoub works 2 less hours than Khalaf.

1. Let x represent the number of hours that Khalaf works each week. The number of hours that Khalaf, Salem and Ayoub work can be modeled as shown below. Write an expression that represents each person's number of hours.

Expression: x Expression: $2x+3$ Expression: $x-2$

2. Model the total number of hours that Salem and Ayoub work. Draw the result below. Then write an expression for the drawing.

Expression: $2x+3+x+(-2)$

3. Like tiles are tiles that have the same shape. Group like tiles together and remove the zero pairs. Draw the result below. Then write an expression for your drawing.

Expression: $3x+1$

Essential Question

HOW can you use numbers and symbols to represent mathematical ideas?

Vocabulary

term
like terms
constant
simplest form

Mathematical Practices
1, 2, 3, 4, 6

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Simplify algebraic expressions.

Coherence connecting within and across grades

Previous
Students used properties to simplify algebraic expressions.

Now
Students simplify algebraic expressions with more than two terms by combining like terms.

Next
Students will combine like terms when adding and subtracting linear expressions

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 391.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

AL LA **Paired Heads Together** Have students work in pairs to complete Exercises 1–3, making sure each member understands how to arrive at the answer. Call on one member to present the team's solution in a small group or whole class discussion. MP 1, 4

Alternate Strategy

BL Tell students that terms with the same variable can be combined because of the Distributive Property. For example, $5x + 3x$ is the same as $(5+3)x$ or $8x$. They can combine those terms by performing the addition or subtraction indicated by the coefficients. MP 1, 7

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Identify parts of an expression.

AL
- *What are like terms?* terms that contain the same variable to the same power
- *What is a coefficient?* the numerical part of a term
- *What is a constant?* a term without a variable

OL
- *What terms are like terms in* $6n$, $-7n$, and n?
- *What is the coefficient of* n?
- *How will you add the like terms?* add the coefficients

BL
- *Why do you think the term that is called constant got its name?* Since there is no variable, the value of the term does not change, it stays constant.

Need Another Example?

Identify the terms, like terms, coefficients, and constants in the expression $3x - 5 + 2x - x$. terms: $3x$, -5, $2x$, and $-x$; like terms: $3x$, $2x$, and $-x$; coefficients: 3, 2, and -1; constant: -5

Work Zone

STOP and Reflect

Circle the term below that is a like term with $-4x^2$.

$-4x^2$ x^2 -4

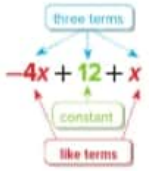
a. terms: $9y$, -4, $-11y$, 7; like terms: $9y$ and $-11y$, -4 and 7; coefficients: 9, -11; constants: -4, 7

b. terms: $3x$, 2, -10, $-3x$; like terms: $3x$ and $-3x$, 2 and -10; coefficients: 3, -3; constants: 2, -10

Identify Parts of an Expression

When addition or subtraction signs separate an algebraic expression into parts, each part is called a **term**. Recall that the numerical factor of a term that contains a variable is called the coefficient of the variable.

Like terms contain the same variables to the same powers. For example, $3x^2$ and $-7x^2$ are like terms. So are $8xy^2$ and $12xy^2$. But $10x^2z$ and $22xz^2$ are *not* like terms. A term without a variable is called a **constant**. Constant terms are also like terms.



Example

1. **Identify the terms, like terms, coefficients, and constants in the expression $6n - 7n - 4 + n$.**

$6n - 7n - 4 + n = 6n + (-7n) + (-4) + 1n$ Rewrite the expression.

- Terms: $6n$, $-7n$, -4, n
- Like terms: $6n$, $-7n$, n All of these terms have the same variable.
- Coefficients: 6, -7, 1
- Constants: -4 This is the only term without a variable.

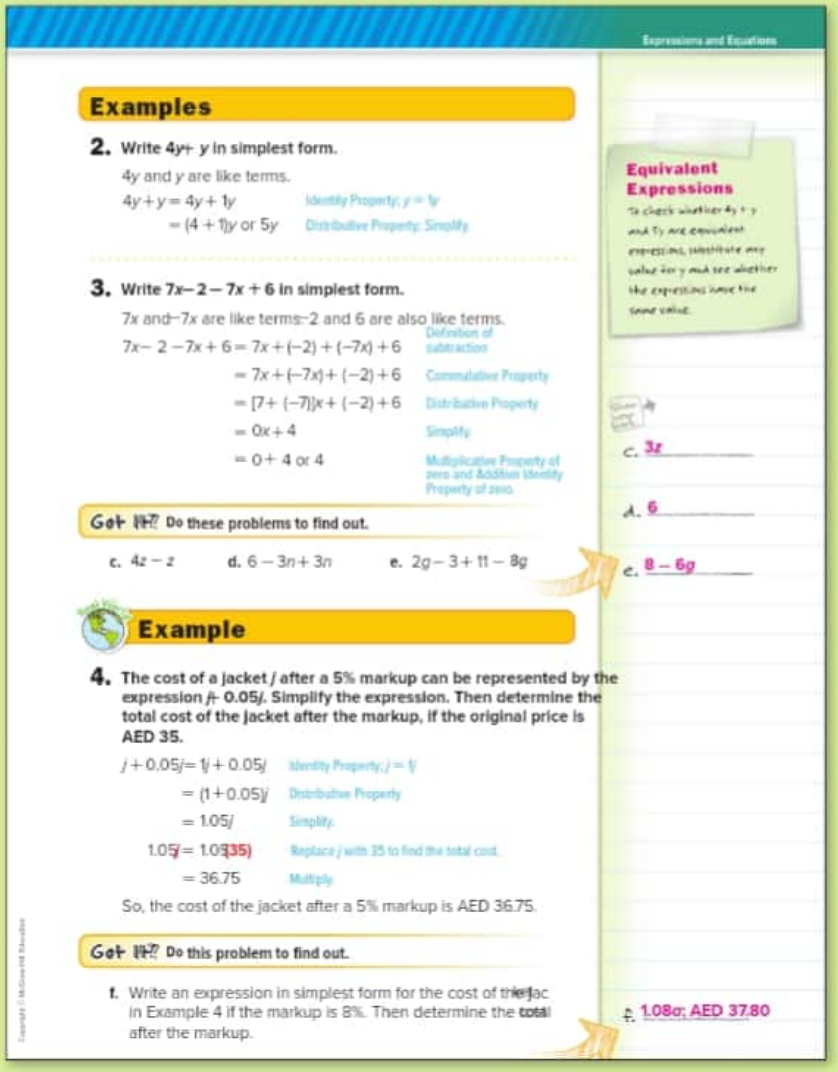
Got It? Do these problems to find out.

Identify the terms, like terms, coefficients, and constants in each expression.

a. $9y - 4 - 11y + 7$ b. $3x + 2 - 10 - 3x$

Simplify Algebraic Expressions

An algebraic expression is in **simplest form** if it has no like terms and no parentheses. Use the Distributive Property to combine like terms.



Expressions and Equations

Examples

2. Write $4y + y$ in simplest form.

$4y$ and y are like terms.

$4y + y = 4y + 1y$ Identity Property: $y = 1y$

$= (4 + 1)y$ or $5y$ Distributive Property; Simplify

3. Write $7x - 2 - 7x + 6$ in simplest form.

$7x$ and $-7x$ are like terms; -2 and 6 are also like terms.

$7x - 2 - 7x + 6 = 7x + (-2) + (-7x) + 6$ Definition of subtraction

$= 7x + (-7x) + (-2) + 6$ Commutative Property

$= [7 + (-7)]x + (-2) + 6$ Distributive Property

$= 0x + 4$ Simplify

$= 0 + 4$ or 4 Multiplicative Property of zero and Additive Identity Property of zero

Got It? Do these problems to find out.

c. $4z - z$ d. $6 - 3n + 3n$ e. $2g - 3 + 11 - 8g$

Equivalent Expressions

To check whether $4y + y$ and $5y$ are equivalent expressions, substitute any value for y and see whether the expressions have the same value.

c. $3z$

d. 6

e. $8 - 6g$

Example

4. The cost of a jacket j after a 5% markup can be represented by the expression $j + 0.05j$. Simplify the expression. Then determine the total cost of the jacket after the markup, if the original price is AED 35.

$j + 0.05j = 1j + 0.05j$ Identity Property: $j = 1j$

$= (1 + 0.05)j$ Distributive Property

$= 1.05j$ Simplify.

$1.05j = 1.05(35)$ Replace j with 35 to find the total cost.

$= 36.75$ Multiply.

So, the cost of the jacket after a 5% markup is AED 36.75.

Got It? Do this problem to find out.

f. Write an expression in simplest form for the cost of the jacket in Example 4 if the markup is 8%. Then determine the total cost after the markup.

f. $1.08c$; AED 37.80

Examples

2–3. Simplify expressions.

AL
- *In Example 2, what are the like terms?* $4y$ and y
- *In Example 2, what are the coefficients of the like terms?* 4 and 1

OL
- *What are the like terms in Example 3?* $7x$ and $-7x$; -2 and 6
- *In Example 3, how would you combine the like terms?* For the x terms, add 7 and -7, for the constants, add -2 and 6.

BL
- *In Example 2, why is $4y + y = 5y$ and not $5y^2$?* Sample answer: You are adding the terms, so you add the coefficients 4 and 1 and keep the variable y. You do not multiply y by y.

Need Another Example?

Write $8z + z - 5 - 9z + 2$ in simplest form. -3

4. Simplify expressions.

AL
- *What are you trying to find?* the total cost of the jacket after the 5% markup
- *Why is the expression $j + 0.05j$ and not $j + 5j$?* The 5% markup should be expressed as a decimal, 0.05.
- *What are the coefficients of j in the terms j and $0.05j$?* 1 and 0.05

OL
- *Does simplifying $j + 0.05j$ as $1.05j$ answer the question presented in the example? Explain.* Yes; Sample answer: The question asks for the total cost of the jacket after the markup. We need to replace j with 35 to find the total cost.

BL
- *Is it easier for you to simplify the expression first, then evaluate it for $j = 35$, or evaluate it for $j = 35$ first, then simplify? Explain.* See students' preferences.
- *Write an expression, in simplest form, that would represent the total cost of the jacket after a 15% markup. Then find the total cost.* $1.15j$; AED 40.25

Need Another Example?

The cost of a set of DVDs after a 25% markup can be represented by the expression $c + 0.25c$. Simplify the expression. Then determine the total cost of the DVDs after the markup if the original price is AED 40. $1.25c$; AED 50

Example

5. Simplify expressions.

AL • *What do you need to find?* an expression for the total cost of some T-shirts and CDs

• *What do you know about the number of T-shirts and CDs purchased?* The number is the same.

• *How can you represent the number of T-shirts and CDs purchased?* with a variable

OL • *What is the cost of one T-shirt? one CD?* AED 12; AED 7.50

• *Why can you combine 12x and 7.5x?* They are like terms.

BL • *Suppose you bought 5 of each. How much did you spend?* AED 97.50

Need Another Example?
Ayoub buys some boxes of cereal for AED 4.85 each and the same number of bags of pretzels for AED 2.90 each. Write an expression in simplest form that represents the total amount spent. 7.75x

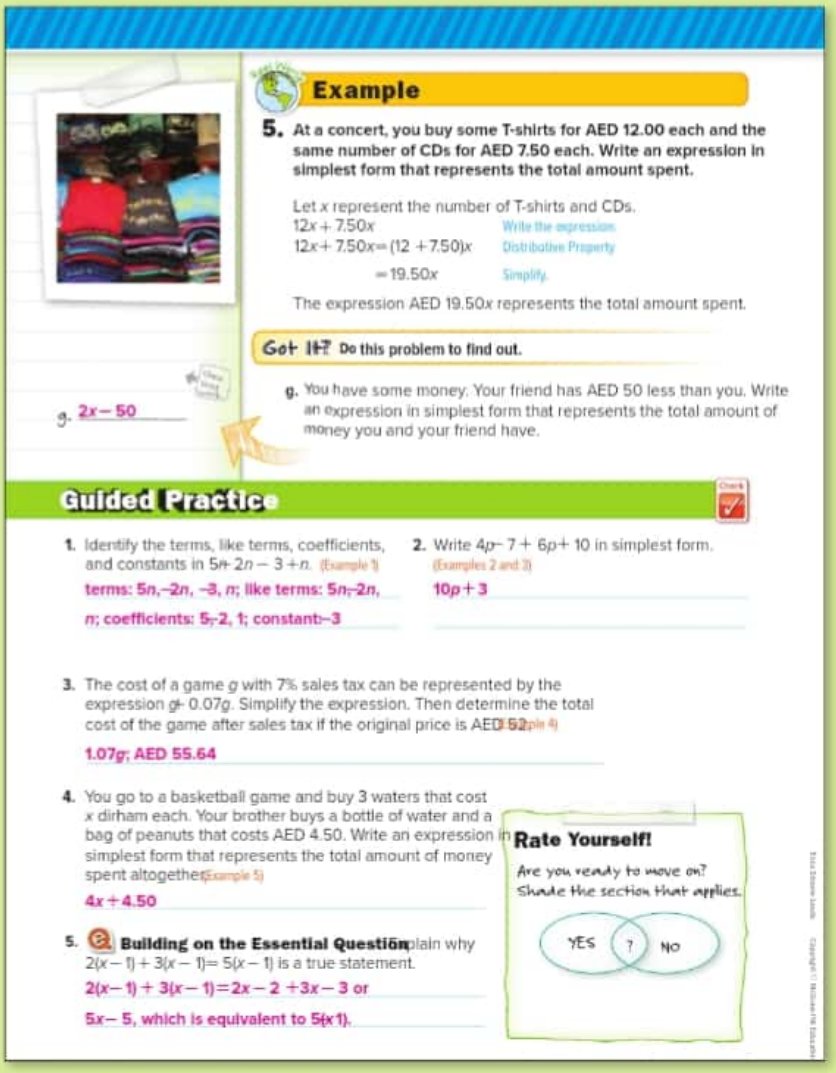
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Discussion** Have students work in pairs to complete Exercises 1–5. Have them trade their solutions with another pair of students and discuss any differences. MP 3, 2, 3

BL LA **Trade-a-Problem** Have students create their own problem, similar to Exercise 4. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. Challenge students to use more than one variable or variables raised to a power. MP 1, 2, 6



Example

5. **At a concert, you buy some T-shirts for AED 12.00 each and the same number of CDs for AED 7.50 each. Write an expression in simplest form that represents the total amount spent.**

Let x represent the number of T-shirts and CDs.

$12x + 7.50x$ Write the expression

$12x + 7.50x = (12 + 7.50)x$ Distributive Property

$= 19.50x$ Simplify.

The expression AED 19.50x represents the total amount spent.

Got It? Do this problem to find out.

g. You have some money. Your friend has AED 50 less than you. Write an expression in simplest form that represents the total amount of money you and your friend have.

g. $2x - 50$

Guided Practice

1. Identify the terms, like terms, coefficients, and constants in $5n - 2n - 3 + n$. (Example 1)
terms: $5n, -2n, -3, n$; like terms: $5n, -2n, n$; coefficients: $5, -2, 1$; constant: -3

2. Write $4p - 7 + 6p + 10$ in simplest form. (Examples 2 and 3)
$10p + 3$

3. The cost of a game g with 7% sales tax can be represented by the expression $g + 0.07g$. Simplify the expression. Then determine the total cost of the game after sales tax if the original price is AED 52. (Example 4)
$1.07g$; AED 55.64

4. You go to a basketball game and buy 3 waters that cost x dirham each. Your brother buys a bottle of water and a bag of peanuts that costs AED 4.50. Write an expression in simplest form that represents the total amount of money spent altogether. (Example 5)
$4x + 4.50$

5. **Building on the Essential Question** Explain why $2(x - 1) + 3(x - 1) = 5(x - 1)$ is a true statement.
$2(x - 1) + 3(x - 1) = 2x - 2 + 3x - 3$ or $5x - 5$, which is equivalent to $5(x - 1)$.

Rate Yourself!
Are you ready to move on? Shade the section that applies.

Name _____ My Homework _____

Independent Practice

Identify the terms, like terms, coefficients, and constants in each expression. (Example 1)

1. $2 + 3a + 9a$
   terms: 2, 3a, 9a; like terms: 3a, 9a; coefficients: 3, 9; constant: 2

2. $7 - 5x + 1$
   terms: 7, –5x, 1; like terms: 7, 1; coefficients: –5; constants: 7, 1

3. $9 - z + 3 - 2z$
   terms: 9, –z, 3, –2z; like terms: 9 and 3, –z and –2z; coefficients: –1, –2; constants: 9, 3

Write each expression in simplest form. (Examples 2 and 3)

4. $n + 5n =$ 6n

5. $12c - c =$ 11c

6. $-4j - 16 - 4j + 6 =$ –8j + 5

7. The cost of a ticket t to a concert with a 3% sales tax can be represented by the expression $t + 0.03t$. Simplify the expression. Then determine the total cost after the sales tax if the original price is AED 72. (Example 4)
   1.03t; AED 74.16

Write an expression in simplest form that represents the total amount in each situation. (Example 5)

8. You rent x pairs of shoes for AED 20 each. You buy the same number of drinks for AED 9.50 each. You also pay AED 19 for a bowling lane.
   29.50x + 19

9. You watch x minutes of television on Monday, the same amount on Wednesday, and 30 minutes on Friday.
   2x + 30

10. In a municipality committee, there were 119 more members in the auditing department than in the public relations and media department. If there were m members in the public relations and media department, write an expression to represent the total members in the committee.
    2m + 119

11. Nasser and his friends paid a total of AED 27 for tickets to the school football game. While at the game, they bought 5 sandwiches at x dirhams each, 4 boxes of popcorn at y dirhams each, and 2 pretzels at z dirhams each.
    a. Write an expression to show the total cost of admission and the snacks.
       27 + 5x + 4y + 2z
    b. Sandwiches cost AED 4, popcorn cost AED 3, and pretzels cost AED 2. What was the total cost for admission and snacks?
       AED 63

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

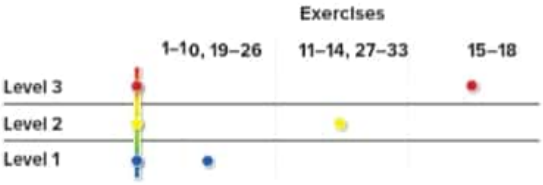
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–11, 13, 15, 16, 18, 32, 33
OL	On Level	1–9 odd, 11–16, 18, 32, 33
BL	Beyond Level	11–18, 32, 33

Watch Out!

Common Error Some students may overlook the coefficient of a variable if the coefficient is 1 or –1. Remind students that a coefficient of 1 is *not* usually written in an algebraic expression. For example, $8n + n$ means $8n + 1n$.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	17
2 Reason abstractly and quantitatively.	12–14, 28
3 Construct viable arguments and critique the reasoning of others.	16, 18
6 Attend to precision.	15

Mathematical Practices 1, 3, 4, and 5 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a clos ing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students explain h ow to simplify the expression $5n + n$. Add the coeffici ents 5 and 1 to get 6

Reason Abstractly Write an expression in simplest form for the perimeter of each figure.

12. $11x + 5y$

13. $16a + 8b + 4$

14. $4h + 16j$

H.O.T. Problems Higher Order Thinking

15. **Be Precise** Write an expression that has three terms and simplifies to $4x - 7$. Identify the coefficient(s) and constant(s) in your expression.

Sample answer: $3x + x - 7$; coefficients: 3, 1; constant: 7

16. **Which One Doesn't Belong?** Identify the expression that is not equivalent to the other three. Explain your reasoning.

$x - 2 + 3x$ | $4(x - 2)$ | $-2 + 7x - 3x$ | $4x - 2$

$4(x - 2)$; Sample answer: $4(x - 2)$ is equivalent to $4x - 8$, while the other three expressions are equivalent to $4x - 2$.

17. **Persevere with Problems** Simplify the expression $8x + 2x + 12x - 3$. Show that your answer is true for $x = 2$.

$18x - 3$; $18x - 3 = 18(2) - 3 = 33$ and

$8x - 2x + 12x - 3 = 8(2) - 2(2) + 12(2) - 3 = 33$

18. **Justify Conclusions** Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

When using the Distributive Property, if the term outside the parentheses is negative, then the sign of each term inside the parentheses will change.

always; Sample answer: If the term outside the parentheses is negative and is multiplied by a term with a positive coefficient, the product will be negative. If the coefficient of the term in the parentheses is negative, then the product will be positive.

Name _____ My Homework _____

Extra Practice

Identify the terms, like terms, coefficients, and constants in each expression.

19. $4 + 5y - 6y + y$

terms: 4, 5, −6, y

like terms: 5, −6, y

coefficients: 5, −6, 1

constant: 4

20. $n + 4n - 7n - 1$

terms: n, $4n$, $-7n$, -1; like terms: n, $4n$, $-7n$; coefficients: 1, 4, −7; constants: −1

21. $-3d + 8 - d - 2$

terms: $-3d$, 8, $-d$, -2; like terms: $-3d$ and $-d$, 8 and -2; coefficients: −3, −1; constants: 8, −2

Write each expression in simplest form.

22. $5x + 4 + 9x$

$= 14x + 4$

23. $2 + 3d + d$

$= 2 + 4d$

24. $-3r + 7 - 3r - 12$

$= -6r - 5$

Write an expression in simplest form that represents the total amount in each situation.

25. You subscribe to m different magazines. Your friend subscribes to 2 fewer than you.

$m - 2$

26. Your friend is y years old. Her brother is 5 years younger.

$y - 5$

27. You spent m minutes studying on Sunday. On Monday, you studied 15 more minutes than you did on Sunday. Tuesday, you studied 30 minutes less than you did on Monday. You studied twice as long on Wednesday as you did on Sunday. On Thursday, you studied 20 minutes less than you did on Wednesday. Write an expression in simplest form to represent the number of minutes you studied in all.

$7m - 20$

28. **Reason Abstractly** Write a real-world situation for $7.50y + 9$.

Sample answer: You buy y movie tickets that cost AED 7.50 each and spend AED 9 at the concession stand.

Simplify each expression.

29. $3(4x - 5) + 4(2x + 6)$

$= 20x + 9$

30. $-8(2a - 3b) - 5(6b - 4a)$

$= 4a - 6b$

31. $10(5g + 2h - 3) - 4(3g - 4h + 2)$

$= 38g + 36h - 38$

Power Up! Test Practice

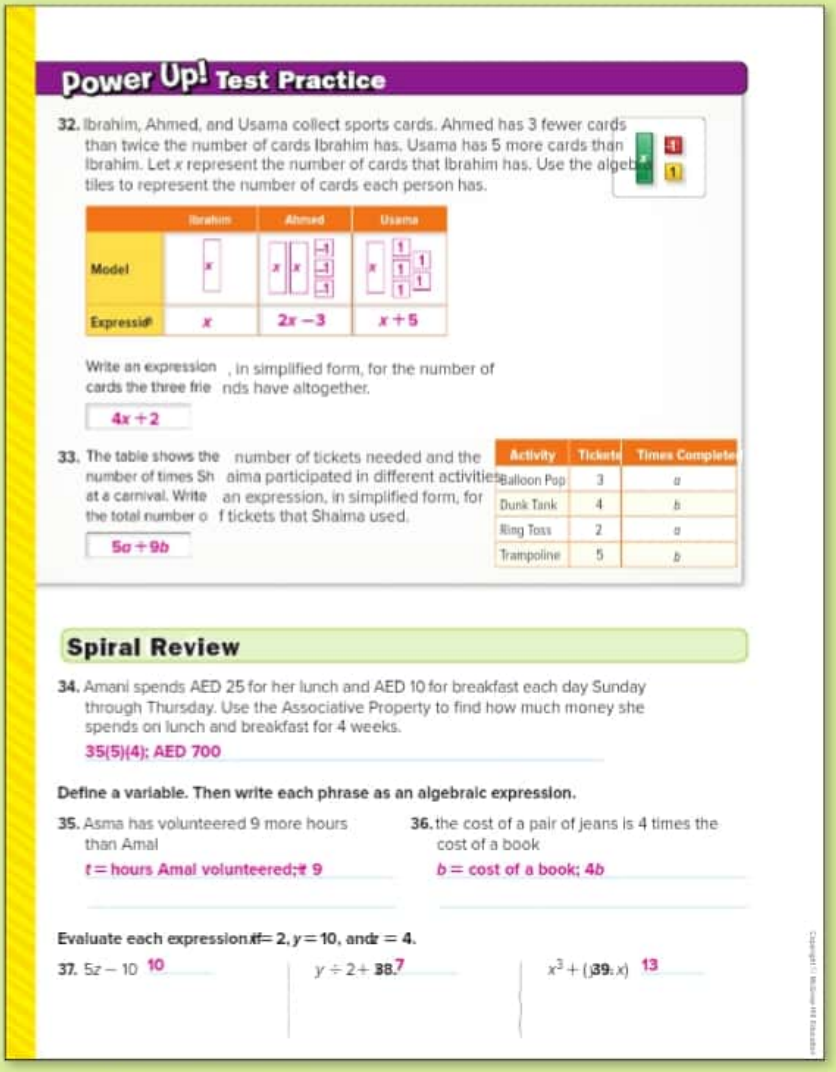
Exercises 32 and 33 prepare students for more rigorous thinking needed for the assessment.

32. This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.

Depth of Knowledge	DOK3
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly model the amount of cards each person has and write an expression to represent each amount.
1 point	Students correctly model all three but fail to write the expressions OR students write correct expressions for each but fail to model OR students correctly model and write the corresponding expressions for two of the people.

33. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.



Power Up! Test Practice

32. Ibrahim, Ahmed, and Usama collect sports cards. Ahmed has 3 fewer cards than twice the number of cards Ibrahim has. Usama has 5 more cards than Ibrahim. Let x represent the number of cards that Ibrahim has. Use the algebra tiles to represent the number of cards each person has.

	Ibrahim	Ahmed	Usama
Model	x	x x -1 -1 -1	x 1 1 1 1 1
Expression	x	$2x - 3$	$x + 5$

Write an expression , in simplified form, for the number of cards the three frie nds have altogether.

$4x + 2$

33. The table shows the number of tickets needed and the number of times Sh aima participated in different activities at a carnival. Write an expression, in simplified form, for the total number o f tickets that Shaima used.

$5a + 9b$

Activity	Tickets	Times Complete
Balloon Pop	3	a
Dunk Tank	4	b
Ring Toss	2	a
Trampoline	5	b

Spiral Review

34. Amani spends AED 25 for her lunch and AED 10 for breakfast each day Sunday through Thursday. Use the Associative Property to find how much money she spends on lunch and breakfast for 4 weeks.

35(5)(4); AED 700

Define a variable. Then write each phrase as an algebraic expression.

35. Asma has volunteered 9 more hours than Amal

t = hours Amal volunteered; $t + 9$

36. the cost of a pair of jeans is 4 times the cost of a book

b = cost of a book; $4b$

Evaluate each expression if $x = 2$, $y = 10$, and $z = 4$.

37. $5z - 10$ 10

$y \div 2 + 3$ 8.7

$x^3 + (3 \cdot x)$ 13

Expressions and Equations

Lesson 6

Add Linear Expressions

Real-World Link

Homework Badr has 20 math problems and 11 science questions for homework. Saeed has 23 math problems and 10 science questions for homework.

1. The expression below represents the types of exercises that Badr has for homework.

   20 math problems + 11 science questions

   Complete the expression that represents the types of exercises that Saeed has for homework.

   23 math problems + 10 science questions

2. Write an expression for the total number of math problems and science questions for both boys.

   43 math problems + 21 science questions

3. Suppose Badr has x math problems and 5 science questions for homework and Saeed has x math problems and 6 science questions. The algebra tiles below represent the total number of math problems and science questions for both boys. Write an expression in simplest form that represents the algebra tiles.

Expression: $2x + 11$

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems ② Reason Abstractly ③ Construct an Argument ④ Model with Mathematics ⑤ Use Math Tools ⑥ Attend to Precision ⑦ Make Use of Structure ⑧ Use Repeated Reasoning

Essential Question: HOW can you use numbers and symbols to represent mathematical ideas?

Vocabulary: linear expression

MP Mathematical Practices 1, 2, 3, 4

Focus narrowing the scope

Objective Add linear expressions.

Coherence connecting within and across grades

Previous Students simplified algebraic expressions by combining like terms.

Now Students add linear expressions with more than two terms by combining like terms.

Next Students will combine like terms when subtracting linear expressions.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 399.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

AL LA **Trade-a-Problem** Ask each student to write an expression that can be simplified with six terms on a piece of paper. Then have them exchange papers with a neighbor and simplify the given expression. Repeat as time allows. If the solutions do not agree, students work together to find the error. MP 1, 7

Alternate Strategy

BL Ask students to study the lesson to research the phrase *linear expression*. Ask them to identify if each of the following *is* or *is not* a linear expression, and to explain their reasoning. MP 1, 6

$3x^2 + x$ $5y - 3$ $6p + 7p + 4$ $15st$

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Add linear expressions.

AL • *What are the like terms?* 2x and x, 3 and 4

• *How does using algebra tiles help you add linear expressions?* Sample answer: I can visually see the like terms and determine the total number of tiles and the total number of 1-tiles.

OL • *What is 2x + x?* 3x

• *Can you simplify 3x + 7? Explain.* No; 3x and 7 are not like terms.

BL • *What properties would you use to add the two expressions?* Commutative Property of Addition and Associative Property of Addition

Need Another Example?
Find $(6x + 2) + (x + 3)$. $7x + 5$

2. Add linear expressions.

AL • *What are the like terms?* 2x and x, −1 and −5

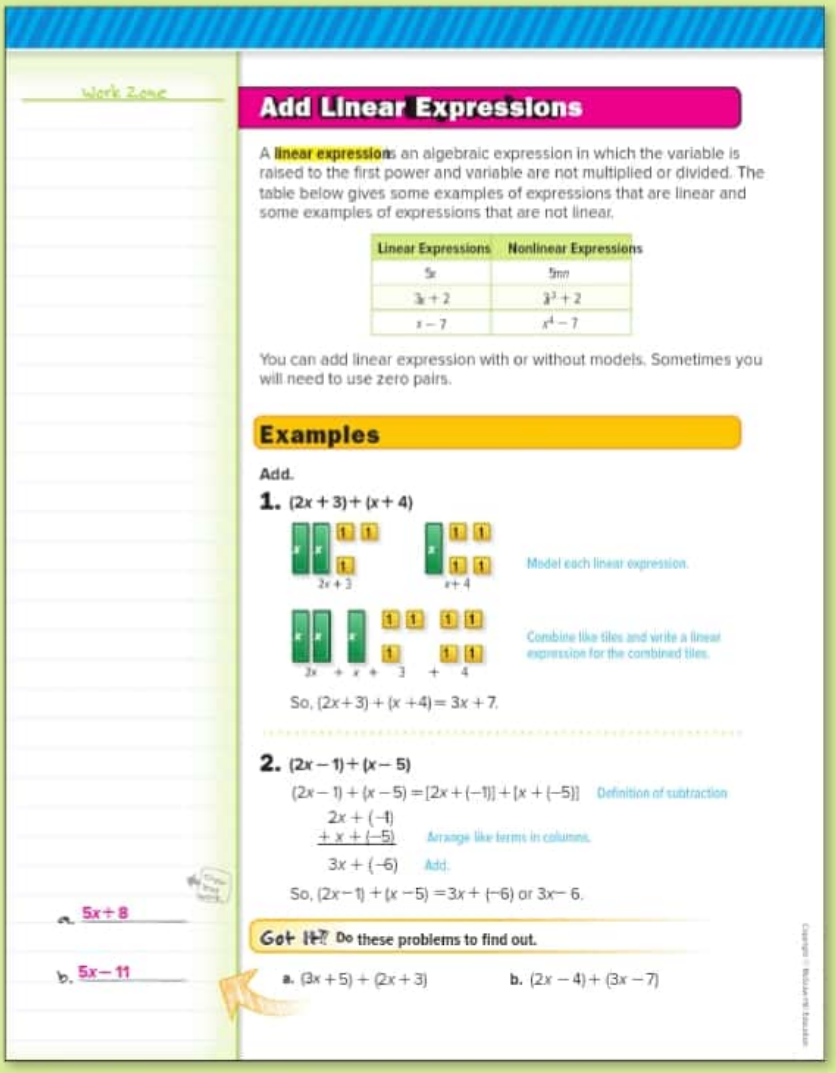
• *How would you use algebra tiles to model this expression?*
Use 2x-tiles and one −1-tile to model $2x - 1$, then add one x-tile and five −1-tiles to model $x - 5$.

OL • *What are the two coefficients of 2?* 2 and 1

• *What are the two constants?* −1 and −5

BL • *Can you simplify this expression another way? Explain.* Sample answer: Rewrite the expression so that like terms are together, then add $2x + x + (-1) + (-5)$

Need Another Example?
Find $(4x - 2) + (x - 4)$. $5x - 6$



Work Zone

a. 5x + 8

b. 5x − 11

Add Linear Expressions

A **linear expression** is an algebraic expression in which the variable is raised to the first power and variable are not multiplied or divided. The table below gives some examples of expressions that are linear and some examples of expressions that are not linear.

Linear Expressions	Nonlinear Expressions
$5x$	$5m$
$3x + 2$	$3^2 + 2$
$x - 7$	$x^4 - 7$

You can add linear expression with or without models. Sometimes you will need to use zero pairs.

Examples

Add.

1. $(2x + 3) + (x + 4)$

So, $(2x + 3) + (x + 4) = 3x + 7$.

2. $(2x - 1) + (x - 5)$

$(2x - 1) + (x - 5) = [2x + (-1)] + [x + (-5)]$ Definition of subtraction

$2x + (-1)$
$+ x + (-5)$ Arrange like terms in columns.

$3x + (-6)$ Add.

So, $(2x - 1) + (x - 5) = 3x + (-6)$ or $3x - 6$.

Got It? Do these problems to find out.

a. $(3x + 5) + (2x + 3)$ b. $(2x - 4) + (3x - 7)$

Example

6. Simplify expressions.

AL • *What do you need to find?* the perimeter of the triangle

• *What do you know?* the lengths of the sides as algebraic expressions and the value of x

OL • *Look at the lengths of the sides of the triangle. What are the like terms?* $3x$, $2x$, and $5x$; -3, and 9

• *What does the expression $10x + 6$ represent?* the perimeter as an algebraic expression

BL • *Is there another way to solve this problem? Explain.* Sample answer: Replace x with 5 before you write an expression for the perimeter and simplify to find each side length. Then add the lengths of the sides.

Need Another Example?

The side length of a square is $(5x)$ centimeters. Write a linear expression in simplest form to represent the perimeter of the square. Then find the perimeter if x equals 2. $20x + 4$; 84 cm.

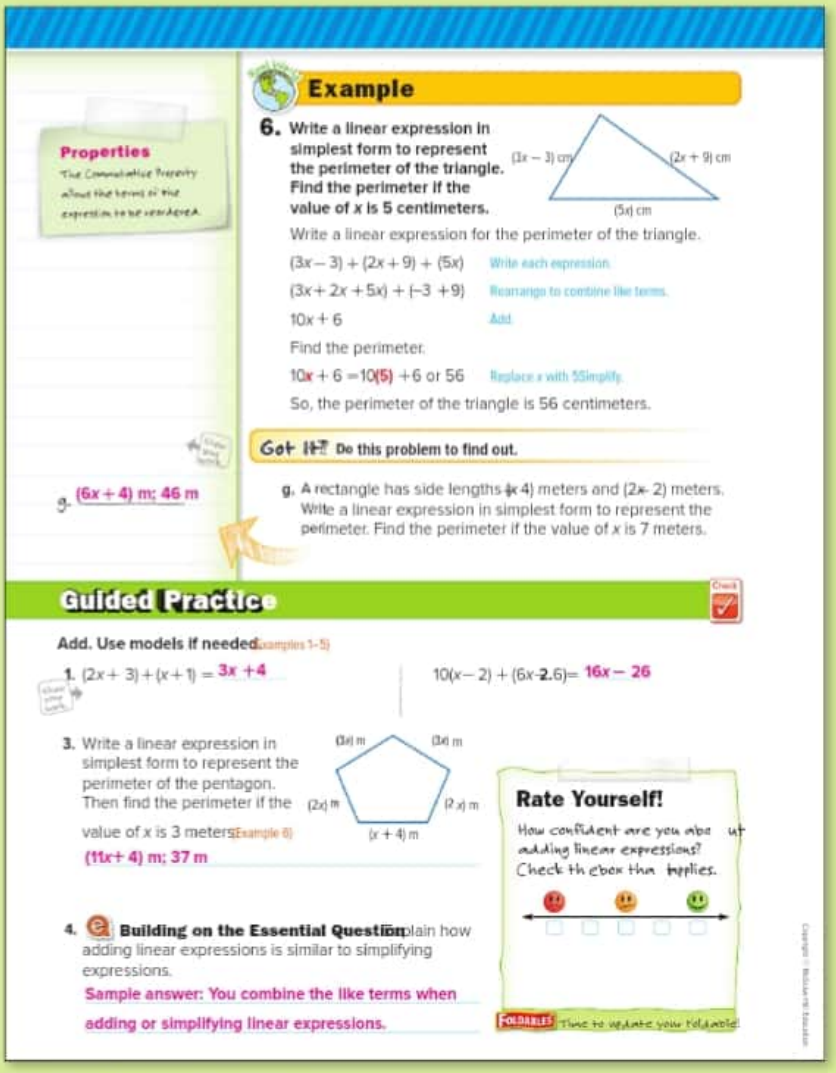
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Think-Pair-Share** Have students work in pairs. Give them one minute to complete Exercise 1. One student should use algebra tiles, while the other uses pencil and paper. Have them share their responses with their partner and discuss any differences. Then for Exercise 2, have them switch roles. Call on one student to share the team's answers in a small group or large group discussion. MP 1, 4, 5

BL LA **Trade-a-Problem** Have students create their own problem, similar to Exercise 3, but involving finding the perimeter of a composite figure. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. Challenge students to use more than one variable or to include fractions in their expressions. MP 1, 2



Properties

The Commutative Property allows the terms of the expression to be rearranged.

Example

6. Write a linear expression in simplest form to represent the perimeter of the triangle. Find the perimeter if the value of x is 5 centimeters.

Write a linear expression for the perimeter of the triangle.

$(3x - 3) + (2x + 9) + (5x)$ Write each expression.

$(3x + 2x + 5x) + (-3 + 9)$ Rearrange to combine like terms.

$10x + 6$ Add.

Find the perimeter.

$10x + 6 = 10(5) + 6$ or 56 Replace x with 5. Simplify.

So, the perimeter of the triangle is 56 centimeters.

Got it? Do this problem to find out.

g. A rectangle has side lengths $(x + 4)$ meters and $(2x - 2)$ meters. Write a linear expression in simplest form to represent the perimeter. Find the perimeter if the value of x is 7 meters.

g. $(6x + 4)$ m; 46 m

Guided Practice

Add. Use models if needed. Examples 1–5

1. $(2x + 3) + (x + 1) = 3x + 4$

$10(x - 2) + (6x - 2.6) = 16x - 26$

3. Write a linear expression in simplest form to represent the perimeter of the pentagon. Then find the perimeter if the value of x is 3 meters. Example 6

(11x + 4) m; 37 m

4. **Building on the Essential Question** Explain how adding linear expressions is similar to simplifying expressions.

Sample answer: You combine the like terms when adding or simplifying linear expressions.

Rate Yourself!

How confident are you about adding linear expressions? Check the box that applies.

FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

Add. Use models if needed. Examples 1–5

1. $(4x + 8) + (7x + 3) =$ **11x + 11**

$(-3x + 7) + (-6x + 9) =$ **−9x + 16**

$(x - 10) + (3x - 6) =$ **4x − 16**

$(-3x - 7) + (4x + 7) =$ **x**

5. $2(x + 14) + (2x - 14) =$ **4x + 14**

$(11x - 8) + (7x - 1) =$ **18x − 15**

7. Write a linear expression in simplest form to represent the perimeter of the triangle at the right. Then find the perimeter if the value of x is 10 millimeters. Example 6

(22x + 10); 230 mm

8. A rectangle has side lengths $(2x + 5)$ meters and $(2x + 6)$ meters. Write a linear expression in simplest form to represent the perimeter. Find the perimeter if the value of x is 12 meters. Example 6

(8x + 2) m; 98 m

Find the sum of $(x + 5)$, $(-4x - 2)$, and $(2x - 1)$.

−x + 2

Add.

10. $(-3.5x + 1.7) + (9.1x - 0.3) =$ **5.6x + 1.4**

$(0.5x + 15) + (8.2x - 16.6) =$ **8.7x − 1.6**

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

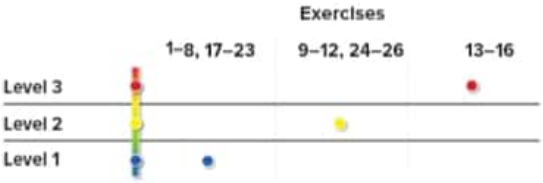
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 11, 13, 14, 16, 25, 26
OL	On Level	1–7 odd, 9–14, 16, 25, 26
BL	Beyond Level	9–16, 25, 26

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	15, 17
2 Reason abstractly and quantitatively.	12, 24
3 Construct viable arguments and critique the reasoning of others.	13, 14, 16

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write an explanation of how to add two linear expressions. See students' work.

12. **Reason Abstractly** The table shows the breakdown of the points scored in last week's basketball game.

	1st Quarter Points	2nd Quarter Points	3rd Quarter Points	4th Quarter Points	Total Free Throw Points
Panthers	$x-6$	$x+2$	x	$x-6$	9

a. Write a linear expression in simplest form to represent the total points scored in the first two quarters.
$3x - 4$

b. Write a linear expression in simplest form to represent the total points scored in the game.
$6x - 1$

H.O.T. Problems Higher Order Thinking

13. **Reason Inductively** Write two linear expressions with a sum of $-5x + 4$.
Sample answer: $(10x + 2)$ and $(-15x + 2)$

14. **Construct an Argument** Will the sum of two linear expressions, each with an x-term, *always*, *sometimes*, or *never* have an x-term? Explain your reasoning.
Sometimes; sample answer: If the coefficients of the x-term are not opposites, then the statement is always true. If they are opposites, then the statement is false.

15. **Persevere with Problems** An integer can be represented by x. The next integer can then be represented as $x + 1$. Write a linear expression that represents the sum of any two consecutive integers. Show that the sum of any two consecutive integers is always odd.
$2x + 1$; The expression $2x + 1$ will always be odd when x is an integer because when an integer is doubled, the result is always even. Adding one to the result will give an odd number.

16. **Reason Inductively** Explain how algebra tiles represent like terms and zero pairs.
Sample answer: Algebra tiles that represent like terms have the same size and shape. When adding algebraic expressions, a red tile and a tile of any other color with the same size and shape form a zero pair and may be removed. The result is the sum of the algebraic expressions.

Name _____ My Homework _____

Extra Practice

Add. Use models if needed.

17. $(-x + 10) + (-3x + 6) =$ $-4x + 16$

18. $(-4x + 13) + (-2x + 8) =$ $-6x + 11$

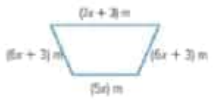
19. $(-6x + 5) + (4x - 7) =$ $-2x - 2$

20. $(-4x + 5) + (15x - 3) =$ $11x + 2$

21. $(-5x + 4) + -1(x - 1) =$ $-6x + 5$

22. $17(2x - 2) + (-x + 4) =$ $33x - 81$

23. Write a linear expression in simplest form to represent the perimeter of the trapezoid at the right. Then find the perimeter if the value of x is 7 meters. $(24x + 9)$ m; 177 m



24. **Reason Abstractly** The table shows the points earned by a contestant in four rounds on a game show.

Round 1	Round 2	Round 3	Round 4
$2x + 40$	$5x + 12$	100	$8 - 10$

a. Write a linear expression in simplest form to represent the total points earned by the contestant in rounds 1 and 2.
$7x + 52$

b. Write a linear expression in simplest form to represent the total points earned in all four rounds.
$13x + 142$

c. If the value of x is 8, what is the total points earned in all four rounds?
246 points

Power Up! Test Practice

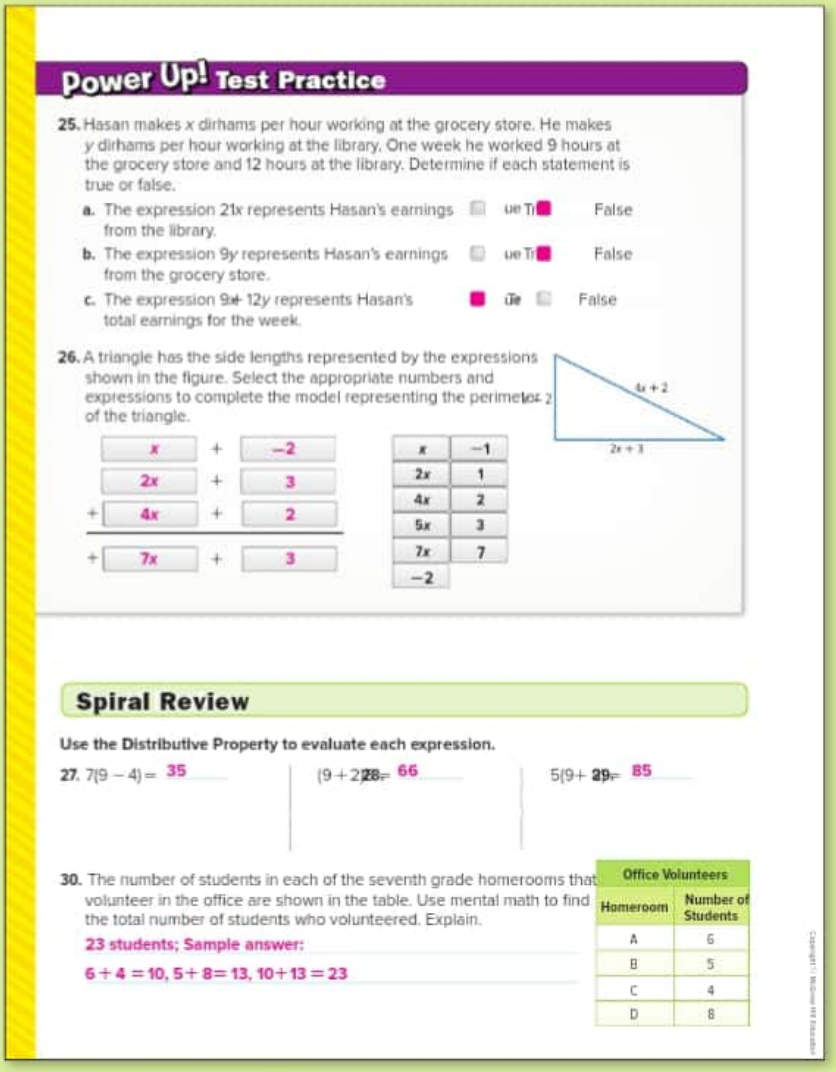
Exercises 25 and 26 prepare students for more rigorous thinking needed for the assessment.

25.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

26.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly place all eight values.
1 point	Students correctly represent two of the sides, but fail to correctly represent the third side and complete the answer accordingly OR students correctly represent all three sides, but fail to represent the perimeter.



Power Up! Test Practice

25. Hasan makes x dirhams per hour working at the grocery store. He makes y dirhams per hour working at the library. One week he worked 9 hours at the grocery store and 12 hours at the library. Determine if each statement is true or false.

a. The expression $21x$ represents Hasan's earnings from the library. — use Ti — False

b. The expression $9y$ represents Hasan's earnings from the grocery store. — use Ti — False

c. The expression $9x + 12y$ represents Hasan's total earnings for the week. — True — use Ti

26. A triangle has the side lengths represented by the expressions shown in the figure. Select the appropriate numbers and expressions to complete the model representing the perimeter of the triangle.

$4x + 2$

$2x + 3$

x	+	-2	
$2x$	+	3	
+ $4x$	+	2	
+ $7x$	+	3	

x	-1
$2x$	1
$4x$	2
$5x$	3
$7x$	7
-2	

Spiral Review

Use the Distributive Property to evaluate each expression.

27. $7(9-4) =$ 35

$(9+2$ 28 $=$ 66

$5(9+$ 29 $=$ 85

30. The number of students in each of the seventh grade homerooms that volunteer in the office are shown in the table. Use mental math to find the total number of students who volunteered. Explain.

23 students; Sample answer:

$6+4=10$, $5+8=13$, $10+13=23$

Office Volunteers	
Homeroom	Number of Students
A	6
B	5
C	4
D	8

Lesson 7

Subtract Linear Expressions

Real-World Link

Dog Sledding The Iditarod is a dog sledding race over 1,840 kilometers across Alaska. The table shows two winning times.

Iditarod	Days	Hours	Minutes	Seconds
Race 1	9	11	46	48
Race 2	9	5	8	41

Essential Question

HOW can you use numbers and symbols to represent mathematical ideas?

Mathematical Practices
1, 2, 3, 4

1. What is the difference in hours, minutes, and seconds between the two races?

6 h 38 min 7 s

2. Explain how you could find the difference in times between any two races, given the days, hours, minutes, and seconds.

Subtract like units; keep the labels.

3. Describe another situation in which finding the difference involves subtracting like units.

Sample answer: To compare the ingredient amounts in a recipe you would compare cups of flours and teaspoons of vanilla in one recipe to cups of flours and teaspoons of vanilla in another recipe.

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Subtract linear expressions.

Coherence connecting within and across grades

Previous Students added linear expressions by combining like terms.

Now Students subtract linear expressions with more than two terms by combining like terms.

Next Students will combine like terms when solving linear equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 407.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

AL LA **Pairs Discussion** Have students complete the Real-World Link in their texts, ensuring that each student understands that only like units, such as hours, can be combined. Call on one set of pairs to share their responses with the class. MP 1, 6

Alternate Strategy

BL Have students rewrite the rows as linear expressions. For example, the row for Race 1 could be written as $9d + 11h + 46m + 48s$. Then have them write and simplify a subtraction expression that models the situation. Have them compare their answer to the answer they determined in Exercise 1. MP 1, 7

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Subtract linear expressions.

AL
- *When using algebra tiles, how do you show subtraction?* by removing tiles
- *Using algebra tiles, how would you model subtracting 2x from 6x?* Remove two x-tiles from the six x-tiles.
- *Using algebra tiles, how would you model subtracting 2 from 3?* Remove two 1-tiles from the three 1-tiles.

OL
- *What is 6x − 2x?* $4x$
- *What is 3 − 2?* 1
- *What is (6x + 3) − (2x + 2)?* $4x + 1$

BL
- *Can you simplify 4x + 1? Explain.* No; $4x$ and 1 are not like terms.

Need Another Example?
Find (4x + 5) − (3x + 1). Use models if needed. $x + 4$

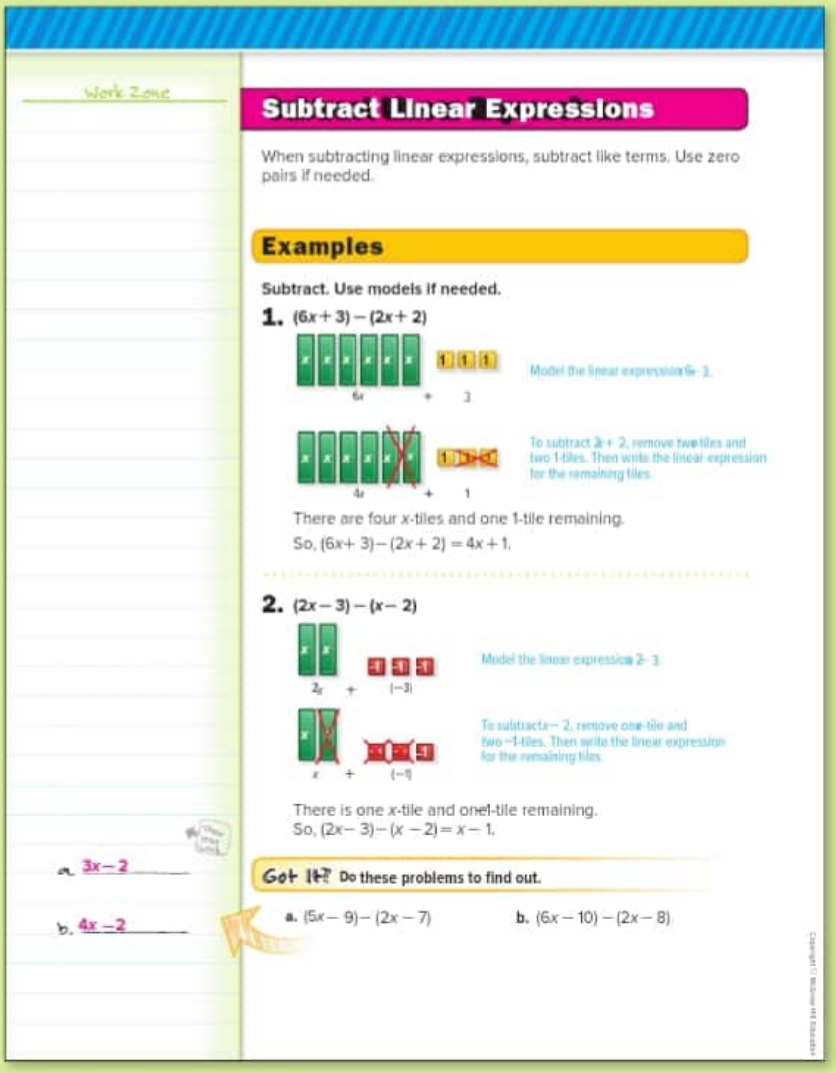
2. Subtract linear expressions.

AL
- *How do you represent −3 using algebra tiles?* Use three red −1-tiles
- *How would you use algebra tiles to model this expression?* Use two x-tiles and three −1-tiles, then remove one x-tile and two −1-tiles.

OL
- *What is 2x − x?* x
- *What is −3 − (−2)?* −1
- *What is (2x − 3) − (x − 2)?* $x - 1$

BL
- *Can you solve this problem another way? Explain.* Sample answer: Yes, subtract each set of like terms.

Need Another Example?
Find (4x − 6) − (2x − 4). Use models if needed. $2x - 2$



Work Zone

Subtract Linear Expressions

When subtracting linear expressions, subtract like terms. Use zero pairs if needed.

Examples

Subtract. Use models if needed.

1. $(6x + 3) - (2x + 2)$

Model the linear expression $6x + 3$.

To subtract $2x + 2$, remove two x-tiles and two 1-tiles. Then write the linear expression for the remaining tiles.

There are four x-tiles and one 1-tile remaining.
So, $(6x + 3) - (2x + 2) = 4x + 1$.

2. $(2x - 3) - (x - 2)$

Model the linear expression $2x - 3$.

To subtract $x - 2$, remove one x-tile and two −1-tiles. Then write the linear expression for the remaining tiles.

There is one x-tile and one 1-tile remaining.
So, $(2x - 3) - (x - 2) = x - 1$.

a. $3x - 2$

b. $4x - 2$

Got It? Do these problems to find out.

a. $(5x - 9) - (2x - 7)$ b. $(6x - 10) - (2x - 8)$

Example

3. Find $(-2x - 4) - (2x)$. Use models if needed.

$-2x + (-4)$

Model the linear expression $-2x - 4$.

zero pairs

Since there are no positive x-tiles to remove, add two zero pairs of x-tiles. Remove two positive x-tiles.

So, $(-2x - 4) - (2x) = -4x - 4$.

Got It? Do these problems to find out.

c. $(3x - 2) - (5x - 4)$

d. $(4x - 4) - (-2x + 2)$

c. $-2x + 2$

d. $6x - 6$

Use the Additive Inverse to Subtract

When subtracting integers, you add the opposite, or the additive inverse. The same process is used when subtracting linear expressions.

Examples

4. Find $(6x + 5) - (3x + 1)$.

$$\begin{array}{r} 6x + 5 \\ (+)\,-3x - 1 \\ \hline 3x + 4 \end{array}$$

Arrange like terms in columns.

The additive inverse of $3x + 1$ is $(-3x - 1)$.

Additive Inverse

The additive inverse is found by multiplying the given expression by -1.

5. Find $(-4x - 7) - (-5x - 2)$.

$$\begin{array}{r} -4x - 7 \\ (+)\,5x + 2 \\ \hline x - 5 \end{array}$$

Arrange like terms in columns.

The additive inverse of $(-5x - 2)$ is $(5x + 2)$.

Got It? Do these problems to find out.

e. $(4x - 3) - (2x + 7)$

f. $(5x - 4) - (2x + 3)$

e. $2x - 10$

f. $3x - 7$

Examples

3. Subtract linear expressions.

AL • *Name one way in which the model in Example 3 is different from the models in the previous examples.* Sample answer: There are negative x-tiles in Example 3 and only positive x-tiles in Examples 1 and 2.

• *How do you model -4?* Use four red -1-tiles.

OL • *Why did you need to add two zero pairs of x-tiles to the model?* There were no positive x-tiles to take away, so add two zero pairs. Then remove two positive x-tiles.

OL • *Does adding a zero pair change the value of the expression?* No; a zero pair represents 0 which does not affect the value of the expression.

BL • *Rewrite $(-2x - 4) - (2x)$ as an addition expression. Then simplify.* $-2x + (-4) + (-2x) = -4x - 4$

Need Another Example?
Find $(-7x - 6) - (7x)$. $-14x - 6$

4. Use the additive inverse to subtract expressions.

AL • *How could you model this expression using algebra tiles?*
Set up six x-tiles and five 1-tiles, and then remove three x-tiles and one 1-tile.

• *What is $6x - 3x$? $5 - 1$?* $3x$; 4

OL • *What is the additive inverse of $3x + 1$?* $-3x - 1$

• *After finding the additive inverse of $3x + 1$, what are the like terms?* $6x$ and $-3x$, 5 and -1

BL • *How does the Distributive Property apply to finding the additive inverse of $3x + 1$?* You find the additive inverse of each term.

Need Another Example?
Find $(12x + 8) - (6x + 2)$. $6x + 6$

5. Use the additive inverse to subtract expressions.

AL • *What is the additive inverse of $5x$? of -2?* $5x$; 2

OL • *After you find the additive inverse, what are the like terms?* $-4x$ and $5x$; -7 and 2

BL • *Find $-2(x - 3) - 2(2x - 5)$.* $-6x + 16$

Need Another Example?
Find $(-5x - 9) - (-6x - 1)$. $x - 8$

Example

6. Use linear expressions.

AL • *What do you need to find?* how many more college hats were sold than professional hats

• *What do you know?* you know expressions for the number of each type of hat

• *What operation will you use?* subtraction

OL • *What is the additive inverse of* $5m - 2$*?* $-5m + 2$

• *After you have simplified the expression, how will you evaluate the expression?* Replace m with 10 and simplify.

BL • *Is there another way to solve this problem? Explain.* Sample answer: Replace m with 10, in the expressions $6m + 3$ and $5m - 2$. Simplify, then subtract.

Need Another Example?

A bakery wants to know how many more chocolate chip cookies than sugar cookies were sold last month. The number of chocolate chip cookies sold is represented by the expression $(7n + 6)$. The number of sugar cookies sold is represented by the expression $(6n + 3)$. Write an expression to show how many more chocolate cookies were sold last month. Then evaluate the expression if n equals 15. $n + 9$; 24

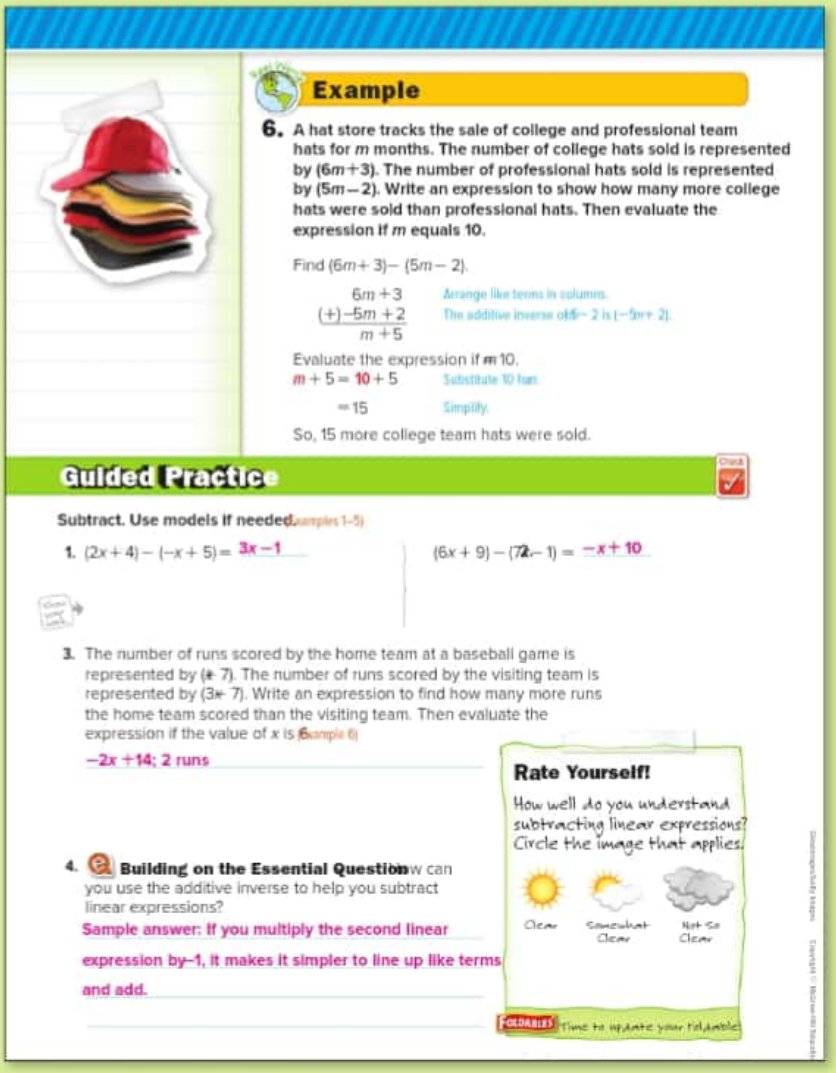
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Discussion** Have pairs complete Exercises 1 and 2. One student should use algebra tiles to model the expressions as the other student writes the problems vertically on their paper. Then, they discuss any differences in the answers. MP 1, 7

BL LA **Make a Game** Have students create a game to help practice adding and subtracting linear expressions. For example, they could write 20 linear expressions on index cards, and roll a die to determine if they need to add or subtract. The group at play determines if the player earns or loses a point. If the games are successful, the entire class can play. MP 1, 7



Example

6. A hat store tracks the sale of college and professional team hats for m months. The number of college hats sold is represented by $(6m + 3)$. The number of professional hats sold is represented by $(5m - 2)$. Write an expression to show how many more college hats were sold than professional hats. Then evaluate the expression if m equals 10.

Find $(6m + 3) - (5m - 2)$.

$$\begin{array}{rl} & 6m + 3 \quad \text{Arrange like terms in columns.} \\ (+) & -5m + 2 \quad \text{The additive inverse of } -5m + 2 \text{ is } (-5m + 2). \\ \hline & m + 5 \end{array}$$

Evaluate the expression if m is 10.

$m + 5 = 10 + 5$ Substitute 10 for m.

$= 15$ Simplify.

So, 15 more college team hats were sold.

Guided Practice

Check

Subtract. Use models if needed. Examples 1–5

1. $(2x + 4) - (-x + 5) =$ $3x - 1$

$(6x + 9) - (7x - 1) =$ $-x + 10$

3. The number of runs scored by the home team at a baseball game is represented by $(x + 7)$. The number of runs scored by the visiting team is represented by $(3x + 7)$. Write an expression to find how many more runs the home team scored than the visiting team. Then evaluate the expression if the value of x is 6. Example 6

$-2x + 14$; 2 runs

4. **Building on the Essential Question** How can you use the additive inverse to help you subtract linear expressions?

Sample answer: If you multiply the second linear expression by -1, it makes it simpler to line up like terms and add.

Rate Yourself!

How well do you understand subtracting linear expressions? Circle the image that applies.

Clear | Somewhat Clear | Not So Clear

FOLDABLES Time to update your foldable!

Name _____ My Homework _____

Independent Practice

Subtract. Use models if needed. (Examples 1–5)

1. $(9x + 5) - (4x + 3) =$ $5x + 2$

2. $(-x + 3) - (x - 5) =$ $-2x + 8$

3. $(3x + 4) - (x + 2) =$ $2x + 2$

4. $(7x + 5) - (3x - 2) =$ $4x + 3$

5. $(9x - 8) - (x + 4) =$ $8x - 12$

6. $(9x - 12) - (5x - 7) =$ $4x - 5$

7. **Reason Abstractly** The number of customers in a store on the first day is represented by $(6x + 3)$. The number of customers on the second day is represented by $(x + 1)$. Write an expression to find how many more customers visited the store on the first day. Then evaluate the expression if x is equal to 50. (Example 6)

   $5x + 2$; 248 customers

8. The perimeter of the garden shown is $(6x + 2)$ units. Find the length of the missing side.

   $x + 1$

9. The cost for shipping a package that weighs x kilograms from Dubai to Sharjah is shown at the right. How much more does Shipping Central charge than Globe Delivery?

   $5x + 0.51$

Company	Cost (AED)
Shipping Central	$3.5 + 3.50$
Globe Delivery	$2.5 + 2.99$

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

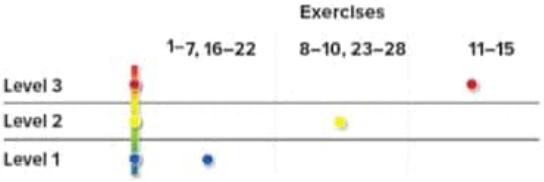
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 11, 12, 15, 27, 28
OL	On Level	1–7 odd, 8–12, 15, 27, 28
BL	Beyond Level	8–15, 27, 28

Watch Out!

Common Error When subtracting linear expressions, students often forget to add the opposite of every term. Encourage them to show all of their work including sign changes in the subtrahend and to keep their work organized.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	13, 14
2 Reason abstractly and quantitatively.	7, 22
3 Construct viable arguments and critique the reasoning of others.	11, 12, 15

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a clos ing formative assessment before dismissing students from your class.

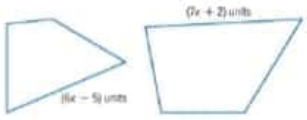
TICKET Out the Door

Have students write an explanation of how to subtract two linear expressions. See student's work.

Watch Out!

Find the Error In Exercise 11, Theresa forgot to add the opposite of 1 in the second polynomial. Suggest that students highlight or circle each term in an expression that requires a change in sign when adding the opposite.

10. Find the difference in the given lengths of the polygons. $(x + 7)$ units

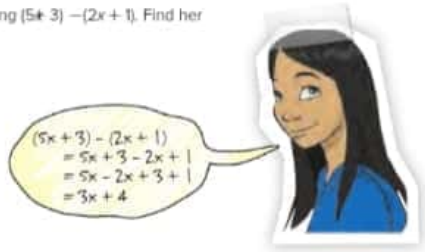


H.O.T. Problems Higher Order Thinking

11. **Find the Error** Suhaila is finding $(5x + 3) - (2x + 1)$. Find her mistake and correct it.

Sample answer: The additive inverse of $(2x + 1)$ is $(-2x - 1)$.

$(5x + 3) - (2x + 1)$
$= (5x + 3) + (-2x - 1)$
$= 5x + 3 + (-2x) + (-1)$
$= 5x + (-2x) + 3 + (-1)$
$= 3x + 2$



12. **Reason Inductively** Name two linear expressions whose difference is $5x - 4$.

Sample answer: $(7x + 1) - (2x + 3) = 7x - 1 - 2x - 3 = 5x - 4$

13. **Persevere with Problems** One linear expression is subtracted from a second linear expression and the difference is $5x$. What is the difference when the second linear expression is subtracted from the first? $-x + 5$

14. **Persevere with Problems** Suppose A and B represent linear expressions. If $A + B = 2x - 2$ and $A - B = 4x - 8$, find A and B.

$A = 3x - 5$; $B = -x + 3$

15. **Reason Inductively** Explain how you can apply the rule for subtracting integers to linear expressions.

Sample answer: The rule is to add the inverse when subtracting integers, and is applied to each term in the linear expression that is subtracted.

Name _____ My Homework _____

Extra Practice

Subtract. Use models if needed.

16. $(-3x-2)-(7x+9)=$ $-10x-11$

$-3x-2$
$(+)\ -7x-9$
$-10x-11$

17. $(-2x-7)-(x-7)=$ $-3x+6$

18. $(9x+5)-(6x-8)=$ $3x+13$

19. $(-8x+1)-(8x-1)=$ $-16x+2$

20. $(4x+10)-(-3x+5)=$ $7x+5$

21. $(-6x+11)-(-2x-4)=$ $-4x-7$

22. Reason Abstractly The number of questions on a math test is represented $(3x-1)$. The number of questions on a spelling test is represented by $(x-12)$. Write an expression to find how many more questions were on the math test. Then evaluate the expression if the value of x is 8.

$2x-11$; 5 questions

Subtract.

23. $(5.7x-0.8)-(4.9x-1.4)=$ $0.8x+0.6$

24. $\left(-\frac{5}{6}x+5\frac{1}{2}\right)-\left(\frac{2}{3}x+4\right)=$ $-1\frac{1}{2}x+1\frac{1}{2}$

25. $2(x+1)-3x=$ $-x+2$

26. $5(x-3)-x=$ $4x-15$

Power Up! Test Practice

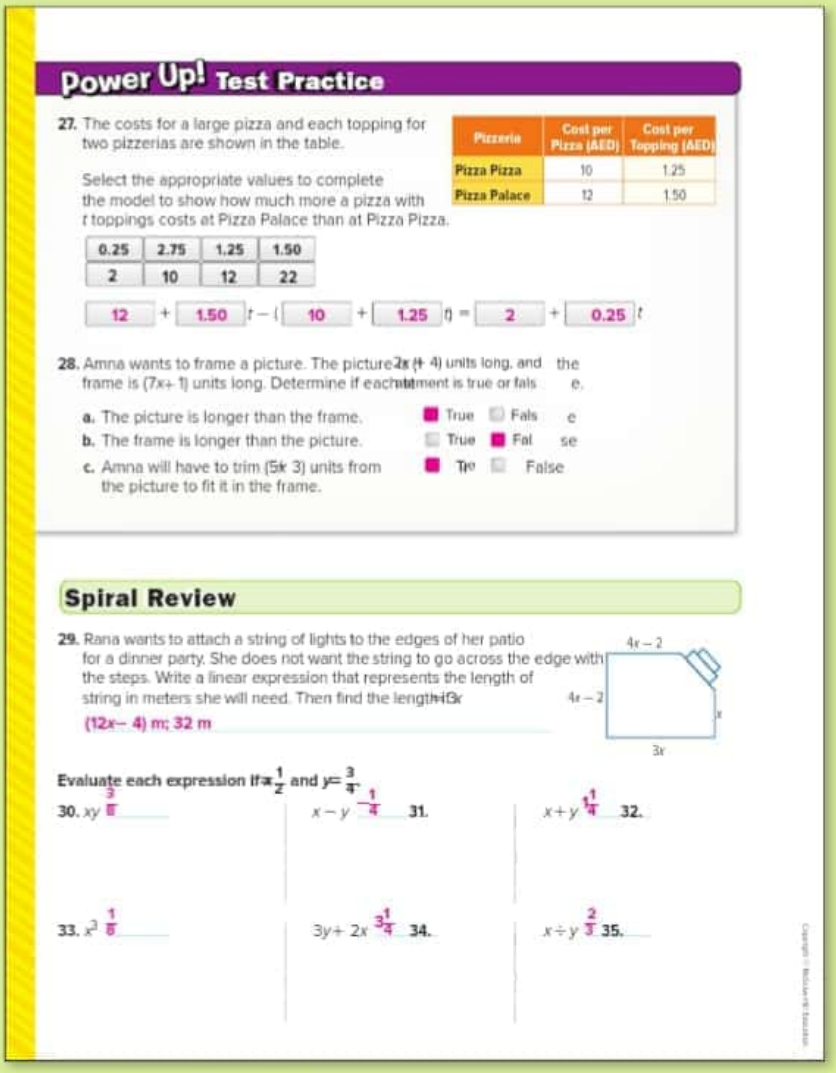
Exercises 27 and 28 prepare students for more rigorous thinking needed for the assessment.

27.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly model the equation.
1 point	Students correctly place four or five of the six values.

28.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.



Power Up! Test Practice

27. The costs for a large pizza and each topping for two pizzerias are shown in the table.

Pizzeria	Cost per Pizza (AED)	Cost per Topping (AED)
Pizza Pizza	10	1.25
Pizza Palace	12	1.50

Select the appropriate values to complete the model to show how much more a pizza with t toppings costs at Pizza Palace than at Pizza Pizza.

0.25	2.75	1.25	1.50
2	10	12	22

$12 + 1.50t - (10 + 1.25t) = 2 + 0.25t$

28. Amna wants to frame a picture. The picture is $2x(+ 4)$ units long, and the frame is $(7x+ 1)$ units long. Determine if each statement is true or fals e.

a. The picture is longer than the frame. ■ True □ Fals e

b. The frame is longer than the picture. □ True ■ Fal se

c. Amna will have to trim $(5x 3)$ units from the picture to fit it in the frame. ■ Tre □ False

Spiral Review

29. Rana wants to attach a string of lights to the edges of her patio for a dinner party. She does not want the string to go across the edge with the steps. Write a linear expression that represents the length of string in meters she will need. Then find the length if x is 3.

$(12x- 4)$ m; 32 m

Evaluate each expression if $x = \frac{1}{2}$ and $y = \frac{3}{4}$.

30. xy $\frac{3}{8}$

$x - y$ $-\frac{1}{4}$ 31.

$x + y$ $1\frac{1}{4}$ 32.

33. x^3 $\frac{1}{8}$

$3y + 2x$ $3\frac{1}{4}$ 34.

$x \div y$ $\frac{2}{3}$ 35.

Inquiry Lab
Factor Linear Expressions

inquiry HOW do models help you factor linear expressions?

MP Mathematical Practices 1, 3

Hamad has enough 1 centimeter square glass tiles to create a rectangular piece of mosaic art that has an area of 24 square centimeters. Some of the possible dimensions of the rectangle are listed in the table. Write the two missing possible dimensions.

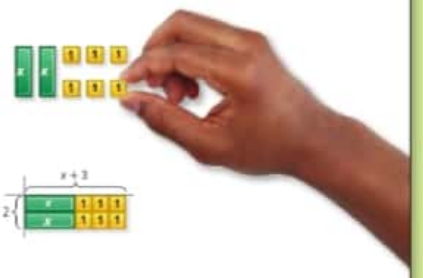
Length (cm)	Width (cm)
24	1
3	8
12	2
6	4

Each of the dimensions listed are factors of 24. Sometimes, you know the product and are asked to find the factors. This process is called *factoring*.

Hands-On Activity 1

Use algebra tiles to factor $2x + 6$.

Step 1 Model the expression $2x + 6$.



Step 2 Arrange the tiles into a rectangle with equal rows and columns. The total area of the tiles represents the product. Its length and width represent the factors.

The rectangle has a width of two 1-tiles and a length of one x-tile and three 1-tiles.

So, $2x + 6 = 2(x + \boxed{3})$.

Focus narrowing the scope

Objective Factor linear expressions using models.

Coherence moving from concrete co abstract

Now
Students use algebra tiles to model factoring linear expressions.

Next
Students will combine like terms when solving linear equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 413.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1–3 are intended to be used as whole-group activities. Activities 1 and 2 are designed to provide more guidance than Activity 3.

Hands-On Activity 1

AL LA Remind students that when using algebra tiles, a green x-tile is used to represent x, a yellow tile is used to represent 1, and a red tile is used to represent -1. Have students practice modeling integers like -4, 7, and 3 before moving on to the Activity. MP 1, 4, 5

Ask:

- *To model the expression, how many x-tiles do you need?* 2
- *To model the expression, how many 1-tiles do you need?* 6
- *In Step 2, what do the length and width of the rectangle represent?* the factors

BL Have students alter the model so that the factors are 2 and $x + 4$. Then have them determine the expression of the product that is modeled, $2x +$ MP 1, 4, 5

Hands-On Activity 2

AL LA If students have trouble understanding why they use negative 1-tiles in Activity 2, remind them that $2x - 8$ can be rewritten as $2x + (-8)$. This should help them see the connection between the tiles and the expressi MP 1, 7

Ask:

- *To model the expression, how many x-tiles do you need?* 2
- *To model the expression, how many negative 1-tiles do you need?* 8
- *In Step 2, what do the length and width of the rectangle represent?* the factors

BL Have students alter the model so that the factors are 3 and $x - 4$. Then have them determine the expression of the product that is modeled, $3x - 1$ MP 1, 4, 5

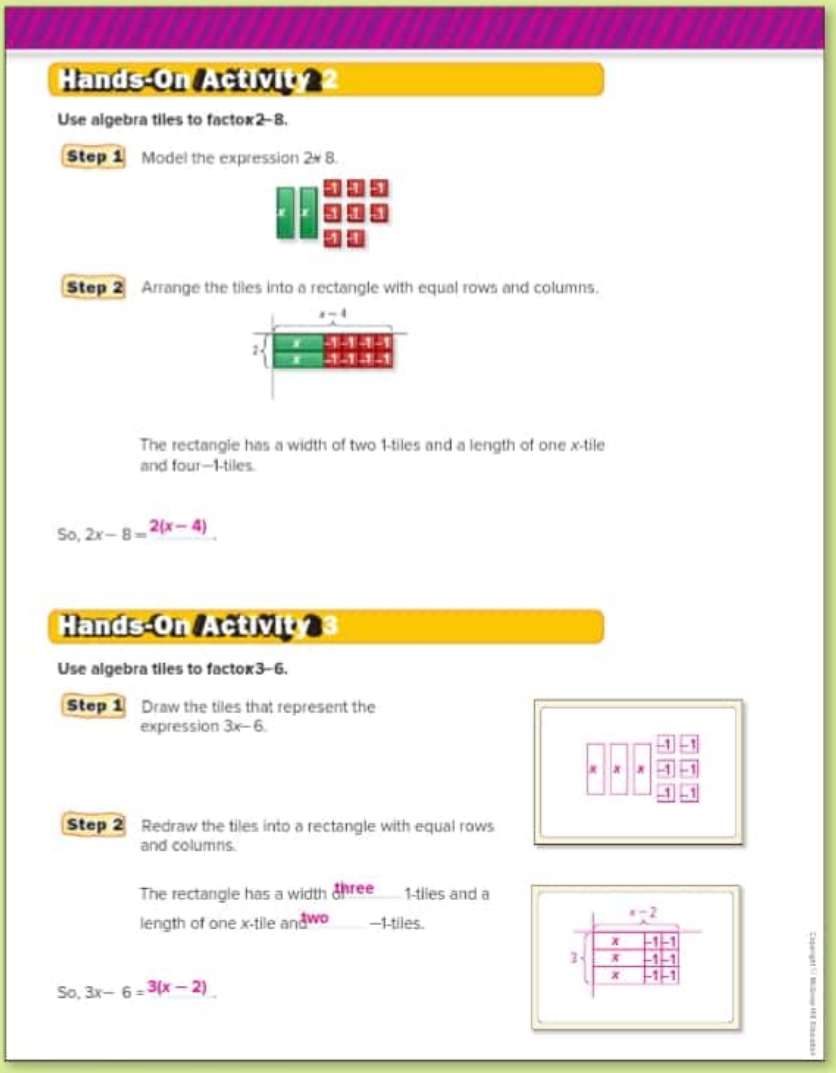
Hands-On Activity 3

AL LA Make sure students understand that the algebra tiles need to form a rectangle. If not, they cannot produce the correct *length* × *width* expressio MP 1

Ask:

- *To model the expression, how many x-tiles do you need?* 3
- *To model the expression, how many negative 1-tiles do you need?* 6
- *In Step 2, what do the length and width of the rectangle represent?* the factors

BL Have students alter the model so that the factors are 4 and $x - 4$. Then have them determine the expression of the product that is modeled, $4x - 16$. Then have them determine the factors of the expression $5x - 20$ without using a model. MP 1, 4, 5



Hands-On Activity 2

Use algebra tiles to factor $2x - 8$.

Step 1 Model the expression $2x - 8$.

Step 2 Arrange the tiles into a rectangle with equal rows and columns.

The rectangle has a width of two 1-tiles and a length of one x-tile and four -1-tiles.

So, $2x - 8 =$ $2(x - 4)$

Hands-On Activity 3

Use algebra tiles to factor $3x - 6$.

Step 1 Draw the tiles that represent the expression $3x - 6$.

Step 2 Redraw the tiles into a rectangle with equal rows and columns.

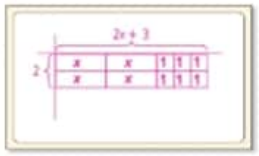
The rectangle has a width of three 1-tiles and a length of one x-tile and two -1-tiles.

So, $3x - 6 =$ $3(x - 2)$

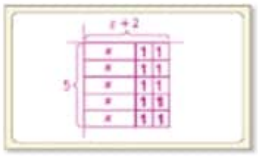
Investigate

Work with a partner. Factor each expression by arranging the appropriate algebra tiles into equal rows and columns. Draw the finished product.

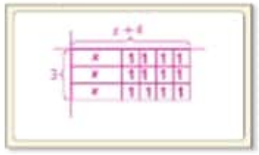
1. $4x + 6 =$ **2(2x + 3)**



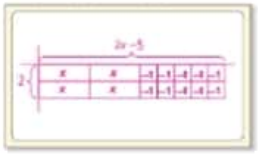
2. $5x + 10 =$ **5(x + 2)**



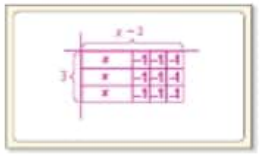
3. $3x + 12 =$ **3(x + 4)**



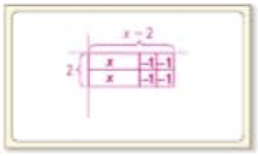
4. $4x - 10 =$ **2(2x − 5)**



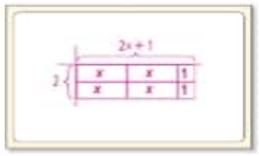
5. $3x - 9 =$ **3(x − 3)**



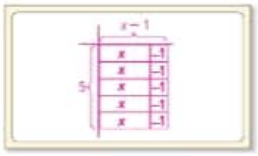
6. $2x - 4 =$ **2(x − 2)**



7. $4x + 2 =$ **2(2x + 1)**



8. $5x - 5 =$ **5(x − 1)**



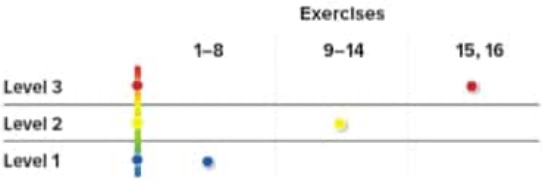
ENGAGE **EXPLORE** **EXPLAIN** **ELABORATE** **EVALUATE**

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Pairs Discussion** Have students work in pairs to complete Exercises 1–8. Have them trade their solutions with another pair of students and discuss any differen MP 1, 4, 5

BL LA **Trade-a-Problem** Have students create their own problem, similar to Exercises 1–8. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. MP 1, 4, 5

Analyze and Reflect

AL LA **Rally Coach** For Exercises 9–14, have students work in pairs. Student 1 should complete Exercise 1 while Student 2 watches, listens, coaches, and praises. Student 2 completes the next exercise while Student 1 watches, listens, coaches, and praises. Students take turns until the exercises are completed. MP 1, 7

BL LA After completing Exercises 9–14, ask students to give a n example of an expression that cannot be factored. MP 1, 7

Create

BL LA **Trade-a-Problem** Have students create a word problem that could be solved by factoring a linear expression. Then have them trade their word problems, solve each other's problem using a model or diagram, and compare solutions. Ask them if there are any other ways to solve the problem. MP 1, 7

Inquiry Students should be able to answer "HOW do models help you factor linear expressions?" Check for student understanding and provide guidance, if needed.

Analyze and Reflec

Work with a partner to complete the table. Use algebra tiles if needed.

	Original Expression	Factored Expression	Distributive Property
	$2x + 8$	$2(x + 4)$	$2(x) + 2(4) = 2x + 8$
9.	$4x - 8$	$4(x - 2)$	$4(x) - 4(2) = 4x - 8$
10.	$6x + 2$	$2(3x + 1)$	$2(3x) + 2(1) = 6x + 2$
11.	$2x - 10$	$2(x - 5)$	$2(x) - 2(5) = 2x - 10$
12.	$8x + 6$	$2(4x + 3)$	$2(4x) + 2(3) = 8x + 6$

13. MP **Reason Inductively** How is factoring related to using the Distributive Property?
Sample answer: Factoring and using the Distributive Property are inverse processes. Check the solution of one by performing the other.

14. MP **Construct an Argument** Is the expression $2x - 2$ equivalent to the expression $2(x - 2)$? Explain.
They are not equivalent. Sample answer: $2(2)$ is equivalent to $2x - 4$.

Create

15. MP **Justify Conclusions** Explain how you could use algebra tiles to factor $5x + 15$.
Sample answer: Arrange 5 rows of one x-tile and three 1-tiles. The resulting factors would then be 5 and $(x + 3)$.

16. Inquiry HOW do models help you factor linear expressions?
Sample answer: Models help you to visualize the factors as if they were the area of a rectangle.

Lesson 8

Factor Linear Expressions

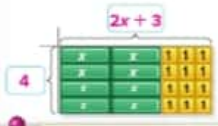
Real-World Link

Yard Sale A rectangular yard is being separated into four equal-size sections for different items at a yard sale. The area of the yard is $(8x + 12)$ square meters.

1. How can you find the area of each section of the yard sale?
Since there are 4 equal sections, divide $8x + 12$ by 4.

2. What is the area of each section? Explain your answer.
$(2x + 3)$ m^2; Sample answer: Four multiplied by some number is $8x + 12$. Using the Distributive Property, $4(2x + 3) = 8x + 12$. So, the area of each section is $(2x + 3)$ square meters.

3. The algebra tiles represent the area of the entire yard sale. Fill in the length and width. Write an expression that represents the area in terms of the length and width of the model. $4(2x + 3)$



Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Essential Question
HOW can you use numbers and symbols to represent mathematical ideas?

Vocabulary
monomial
factor
factored form

Mathematical Practices
1, 2, 3, 4

Focus narrowing the scope

Objective Read and write integers, and find the absolute value of an integer.

Coherence connecting within and across grades

Previous
Students used the Distributive Property to multiply a sum or difference by a number.

Now
Students use the Distributive Property to factor the GCF out of a sum or difference.

Next
Students will combine like terms when solving linear equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 419.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL LA **Think-Write-Pair** Give students one minute to think about how the Distributive Property can be used to multiply a number and an algebraic expression, such as writing $5(3x + 1)$ as $15x + 5$. Ask them to write a rule that would use the Distributive Property backwards to write $15x + 5$ as $5(3x + 1)$. Have them share their rules with a partner. MP 1, 7

Alternate Strategy

AL Have students work in pairs. Give each pair a set of algebra tiles. Have them use the algebra tiles to equally separate $2x + 8$ into two groups, $3x + 6$ into three groups, and finally $4x + 12$ into four groups. MP 1, 4, 5

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Find the GCF.

AL
- *What is the Greatest Common Factor of two numbers?* the greatest number that will divide evenly into both numbers
- *How would you find the GCF of 16 and 20?* Write the prime factorization of each number, circle the common factors, and then multiply the common factors.
- *What is the GCF of 16 and 20?* 4

OL
- *What is the prime factorization of 4x?* $2 \cdot 2 \cdot x$
- *What is the prime factorization of 12x?* $2 \cdot 2 \cdot 3 \cdot x$
- *What are the common factors?* 2, 2, and x

BL
- *What is the GCF of $15x^2$ and $25x$?* $5x$

Need Another Example?
Find the GCF of $48x$ and $28x$. $4x$

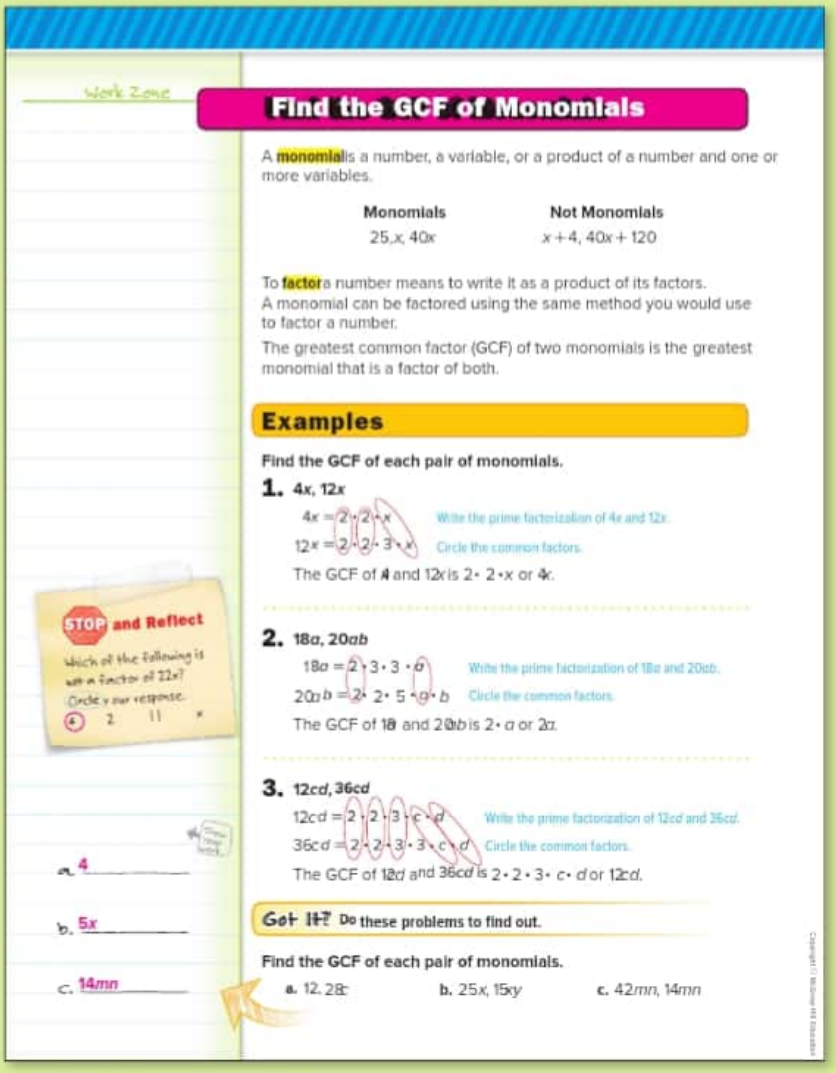
2–3. Find the GCF.

AL
- *In Example 2, what is the prime factorization of 18a? 20ab?* $2 \cdot 3 \cdot 3 \cdot a$; $2 \cdot 2 \cdot 5 \cdot a \cdot b$
- *What factors are common to 18a and 20ab?* 2 and a
- *What is the GCF of 18a and 20ab?* $2a$

OL
- *In Example 3, what is the prime factorization of 12cd? of 36cd?* $2 \cdot 2 \cdot 3 \cdot c \cdot d$; $2 \cdot 2 \cdot 3 \cdot 3 \cdot c \cdot d$
- *What factors are common to 12cd and 36cd?* 2, 2, 3, c and d
- *What is the GCF of 12cd and 36cd?* $12cd$

BL
- *Why is it helpful to know the GCF when using the Distributive Property in reverse?* Sample answer: The GCF is the number that will appear outside of the parentheses.

Need Another Example?
Find the GCF of $14ab$ and $28a$. $14ab$



Work Zone

Find the GCF of Monomials

A **monomial** is a number, a variable, or a product of a number and one or more variables.

Monomials	Not Monomials
$25, x, 40x$	$x + 4, 40x + 120$

To **factor** a number means to write it as a product of its factors. A monomial can be factored using the same method you would use to factor a number.

The greatest common factor (GCF) of two monomials is the greatest monomial that is a factor of both.

Examples

Find the GCF of each pair of monomials.

1. $4x$, $12x$

$4x = 2 \cdot 2 \cdot x$
$12x = 2 \cdot 2 \cdot 3 \cdot x$
Write the prime factorization of $4x$ and $12x$. Circle the common factors.

The GCF of $4x$ and $12x$ is $2 \cdot 2 \cdot x$ or $4x$.

2. $18a$, $20ab$

$18a = 2 \cdot 3 \cdot 3 \cdot a$
$20ab = 2 \cdot 2 \cdot 5 \cdot a \cdot b$
Write the prime factorization of $18a$ and $20ab$. Circle the common factors.

The GCF of $18a$ and $20ab$ is $2 \cdot a$ or $2a$.

3. $12cd$, $36cd$

$12cd = 2 \cdot 2 \cdot 3 \cdot c \cdot d$
$36cd = 2 \cdot 2 \cdot 3 \cdot 3 \cdot c \cdot d$
Write the prime factorization of $12cd$ and $36cd$. Circle the common factors.

The GCF of $12cd$ and $36cd$ is $2 \cdot 2 \cdot 3 \cdot c \cdot d$ or $12cd$.

Got It? Do these problems to find out.

Find the GCF of each pair of monomials.

a. $12, 28c$ **b.** $25x, 15xy$ **c.** $42mn, 14mn$

STOP and Reflect

Which of the following is not a factor of $12x$? Circle your response.

4 2 11 x

a. 4

b. $5x$

c. $14mn$

Factor Linear Expressions

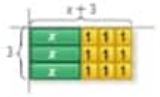
You can use the Distributive Property and the work backward strategy to express a linear expression as a product of its factors. A linear expression is in **factored form** when it is expressed as the product of its factors.

$8x + 4y = 4(2x) + 4(y)$ The GCF of $8x$ and $4y$ is 4.
$= 4(2x + y)$ Distributive Property

Examples

4. Factor $3x + 9$.

Method 1 Use a model.



Arrange three x tiles and nine 1 tiles into equal rows and columns. The rectangle has a width of three 1-tiles, or 3, and a length of one x tile and three 1-tiles, or $x + 3$.

Method 2 Use the GCF.

$3x = 3 \cdot x$ Write the prime factorization of $3x$ and 9.
$9 = 3 \cdot 3$ Circle the common factors.

The GCF of 3 and 9 is 3. Write each term as a product of the GCF and its remaining factors.

$3x + 9 = 3(x) + 3(3)$
$= 3(x + 3)$ Distributive Property

So, $3x + 9 = 3(x + 3)$.

5. Factor $12x + 7y$.

Find the GCF of $12x$ and $7y$.

$12x = 2 \cdot 2 \cdot 3 \cdot x$
$7y = 1 \cdot 7 \cdot y$

There are no common factors, so $12x + 7y$ *cannot be factored*.

Got It? Do these problems to find out.

Factor each expression. If the expression cannot be factored, write *cannot be factored*. Use algebra tiles if needed.

d. $4x - 28$ **e.** $3x + 33y$ **f.** $4x + 35$

Factoring Expressions
To check your factored answers, multiply your factors out. You should get your original expression as a result.

d. $4(x - 7)$

e. $3(x + 11y)$

f. cannot be factored

Examples

4. Factor algebraic expressions.

AL
- *What does it mean to factor an expression?* If possible, write it as a product of its factors.
- *Using a model, what do the length and width of the rectangle represent?* the factors

OL
- *Using Method 2, what is the GCF of $3x$ and 9?* 3
- *What is the factored form of $3x + 9$?* $3(x + 3)$
- *How can you check to see if the factored form is correct?* Use the Distributive Property to multiply. The answer should be the original expression.

BL
- *Do you prefer Method 1 or Method 2 when factoring an expression? Explain.* See students' preferences.

Need Another Example?
Factor $12x + 48$. $12(x + 4)$

5. Factor algebraic expressions.

AL
- *What is the prime factorization of $12x$?* $2 \cdot 2 \cdot 3 \cdot x$
- *What the prime factorization of $7y$?* $7 \cdot y$ or $7 \cdot y$
- *Are there any common factors?* There are no common factors.

OL
- *Why can't $12x + 7y$ be factored?* There are no common factors.

BL
- *When an expression cannot be factored, it is considered prime. Is $12x + 7y$ prime?* yes
- *Compare and contrast prime expressions and prime numbers.* Sample answer: The only factors of each one are 1 and itself. A prime expression may have individual parts that are not prime numbers, but the entire expression is prime.

Need Another Example?
Factor $3x + 11$. cannot be factored

Example

6. Use linear expressions.

AL
- *What do you need to find?* the dimensions of the garden
- *What do you know?* the total area of the garden
- *How will you find possible dimensions?* factor $15x + 18$

OL
- *What is a possible width of the garden?* 3 m
- *What the corresponding possible length of the garden?* $(5x + 6)$ m

BL
- *Is there another possible solution to the problem? If so, name one.* yes; Sample answer: 2 m by $(7.5x - 9)$ m

Need Another Example?
Lily's favorite picture has an area of $(8x + 12)$ square centimeters. Find possible dimensions of the picture. 3 cm by $(x + 4)$ cm

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

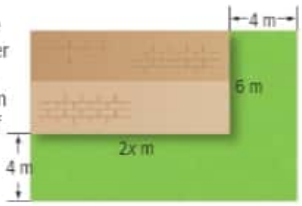
If some of your students are not ready for assignments, use the differentiated activities below.

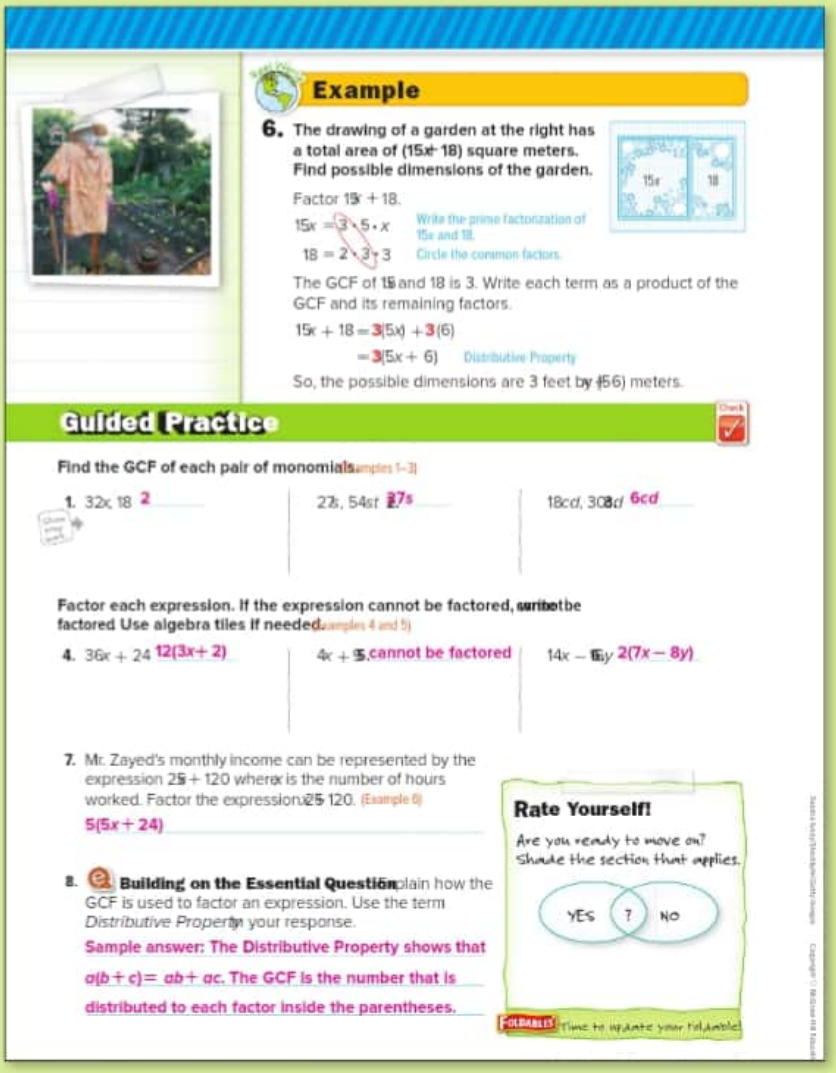
AL LA **Pairs Check** Have students work in pairs to completes Exercises 1–6. One partner completes the problem while the other partner coaches. Students switch roles for the next problem. After each section, the pairs check their answers with another pair and discuss any differences in the answers.

MP 1

BL LA **Pairs Discussion** Have students work in pairs to complete the problem shown below. Have them trade their solutions with another pair and discuss any differences. MP 7

Nasser's cat shelter sits in the corner of a field as shown in the diagram. The walkway around the shelter is 4 meters wide. Write an expression in factored form that represents the area of the walkway around the shelter. $8(x + 5)$ m^2





Example

6. The drawing of a garden at the right has a total area of $(15x + 18)$ square meters. Find possible dimensions of the garden.

Factor $15x + 18$.

$15x = 3 \cdot 5 \cdot x$ Write the prime factorization of $15x$ and 18.

$18 = 2 \cdot 3 \cdot 3$ Circle the common factors.

The GCF of $15x$ and 18 is 3. Write each term as a product of the GCF and its remaining factors.

$15x + 18 = 3(5x) + 3(6)$

$= 3(5x + 6)$ Distributive Property

So, the possible dimensions are 3 feet by $(5x + 6)$ meters.

Guided Practice

Find the GCF of each pair of monomials. Examples 1–3

1. $32x$, 18 2
2. $27s$, $54st$ $27s$
3. $18cd$, $30cd$ $6cd$

Factor each expression. If the expression cannot be factored, write cannot be factored **Use algebra tiles if needed.** Examples 4 and 5

4. $36x + 24$ $12(3x + 2)$
5. $4x + 5$ cannot be factored
6. $14x - 16y$ $2(7x - 8y)$

7. Mr. Zayed's monthly income can be represented by the expression $25x + 120$ where x is the number of hours worked. Factor the expression $25x + 120$. Example 6
$5(5x + 24)$

8. **Building on the Essential Question** Explain how the GCF is used to factor an expression. Use the term *Distributive Property* in your response.
Sample answer: The Distributive Property shows that $a(b + c) = ab + ac$. The GCF is the number that is distributed to each factor inside the parentheses.

Rate Yourself!
Are you ready to move on? Shade the section that applies.

FOLDABLES Time to update your foldable!

Name _____ My Homework _____

Independent Practice

Find the GCF of each pair of monomials. Examples 1–3

1. $24, 48n$ **24**

$32a, 48b^2$ **16**

$36k, 144m$ **36k**

Factor each expression. If the expression cannot be factored, write *cannot be factored*. Use algebra tiles if needed. Examples 4 and 5

4. $3x + 6$ **3(x + 2)**

$2x - 15$ **cannot be factored**

$12x + 30y$ **6(2x + 5y)**

7. The area of a rectangular weight lifting area is $(4x - 48)$ square units. Factor $4x - 8$ to find possible dimensions of the weight lifting area. Example 6
4 units by (x − 2) units

8. The area of a rectangular porch is $(x^2 + 9x + 18)$ square units. Factor $x^2 + 9x + 18$ to find possible dimensions of the porch. Example 6
9 units by (x + 2) units

9. Six friends visited a museum to see the new holograms exhibit. The group paid for admission to the museum and AED 12 for parking. The total cost of the visit can be represented by the expression $\frac{x}{6} + 12$. What expression would represent the cost of the visit for one person?
(x + 2) dirhams

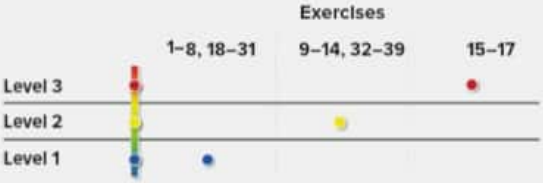
10. The diagram represents a flower border that is 3 meters wide surrounding a rectangular sitting area. Write an expression in factored form that represents the area of the flower border.
6(3x + 11) m²

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice
The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments
You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–8, 9–13 odd, 15, 16, 38, 39
OL	On Level	1–7 odd, 9–16, 38, 39
BL	Beyond Level	9–17, 38, 39

Watch Out!
Common Error Students may attempt to factor an expression that cannot be factored. Remind them that they should always check their answers by using the Distributive Property to see if the product of their factors is equivalent to the original expression.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	17
2 Reason abstractly and quantitatively.	11–14
3 Construct viable arguments and critique the reasoning of others.	15, 16

Mathematical Practices 1, 3, 4, and 5 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

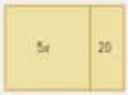
TICKET Out the Door

Have students explain how to factor 54x18 using the GCF. See students' work.

Watch Out!

Find the Error Point out that in Exercise 16, Jamar has the incorrect number of terms inside the parentheses. When factoring a simplified expression, there should be the same number of terms inside the parentheses as there were terms in the original expression.

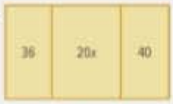
Reason Abstractly Write an expression in factored form to represent the total area of each rectangle.

11. 

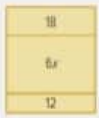
5(x + 4) units²

12. 

7(1 + 7x) units²

13. 

4(5x + 19) units²

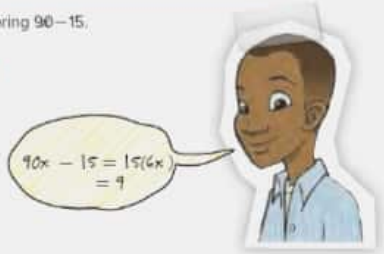
14. 

6(x + 5) units²

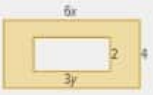
H.O.T. Problems Higher Order Thinking

15. **Reason Inductively** Write two monomials whose greatest common factor is 4m.
Sample answer: 20m and 12mn

16. **Find the Error** Jamar is factoring 90 − 15. Find his mistake and correct it.
Sample answer: The GCF is 15. When he factored out 15, he wrote 6x instead of 6x 1. The correct answer should be 15(6x − 1).



17. **Persevere with Problems** The area of a rectangle is found using the formula $A = \ell w$, where ℓ is the length and w is the width of the rectangle. Write an expression in factored form that represents the area of the shaded region at the right.
6(4x − y)



Name _____ My Homework _____

Extra Practice

Find the GCF of each pair of monomials.

18. $63p$, 84 21

$63p = 3 \cdot 3 \cdot 7 \cdot p$
$84 = 2 \cdot 2 \cdot 3 \cdot 7$
Homework Help
The GCF of $63p$ and 84 is $3 \cdot 7$ or 21.

$30rs$, $42rs$ $6rs$

$30rs = 2 \cdot 3 \cdot 5 \cdot r \cdot s$
$42rs = 2 \cdot 3 \cdot 7 \cdot r \cdot s$
The GCF of $30rs$ and $42rs$ is $2 \cdot 3 \cdot r \cdot s$ or $6rs$.

$60jk$, $45km$ $15jk$

21. $40x$, $60x$ $20x$

$54gh$, $72g$ $18g$

$100xy$, $25xyz$ $25xy$

Factor each expression. If the expression cannot be factored, write *cannot be factored*. Use algebra tiles if needed.

24. $5x + 5$ $5(x + 1)$

$18x + 6$ $6(3x + 1)$

$4x - 7$ cannot be factored

27. $10x - 35$ $5(2x - 7)$

$32x + 28y$ $8(4x + 3y)$

$30x - 40$ $10(3x - 4)$

30. Ahmed has AED 120 in his savings account and plans to save AED x each month for 6 months. The expression $6x + 120$ represents the total amount in the account after 6 months. Factor the expression $6x + 120$.

$6(x + 20)$

31. A square scrapbooking page has a perimeter of $(8x + 20)$ centimeters. What is the length of one side of the scrapbooking page?

$(2x + 5)$ cm

Copy and Solve Write an expression in factored form that is equivalent to the given expression. Show your work on a separate piece of paper. **32–37.** Sample answers given.

32. $\frac{1}{2}x + 4$ $\frac{1}{2}(x + 8)$

33. $\frac{2}{3}x + 6$ $\frac{2}{3}(x + 9)$

34. $\frac{3}{4}x - 24$ $\frac{3}{4}(x - 32)$

35. $\frac{5}{6}x - 30$ $\frac{5}{6}(x - 36)$

36. $\frac{2}{5}x + 16$ $\frac{2}{5}(x + 40)$

37. $\frac{3}{8}x + 18$ $\frac{3}{8}(x + 48)$

Power Up! Test Practice

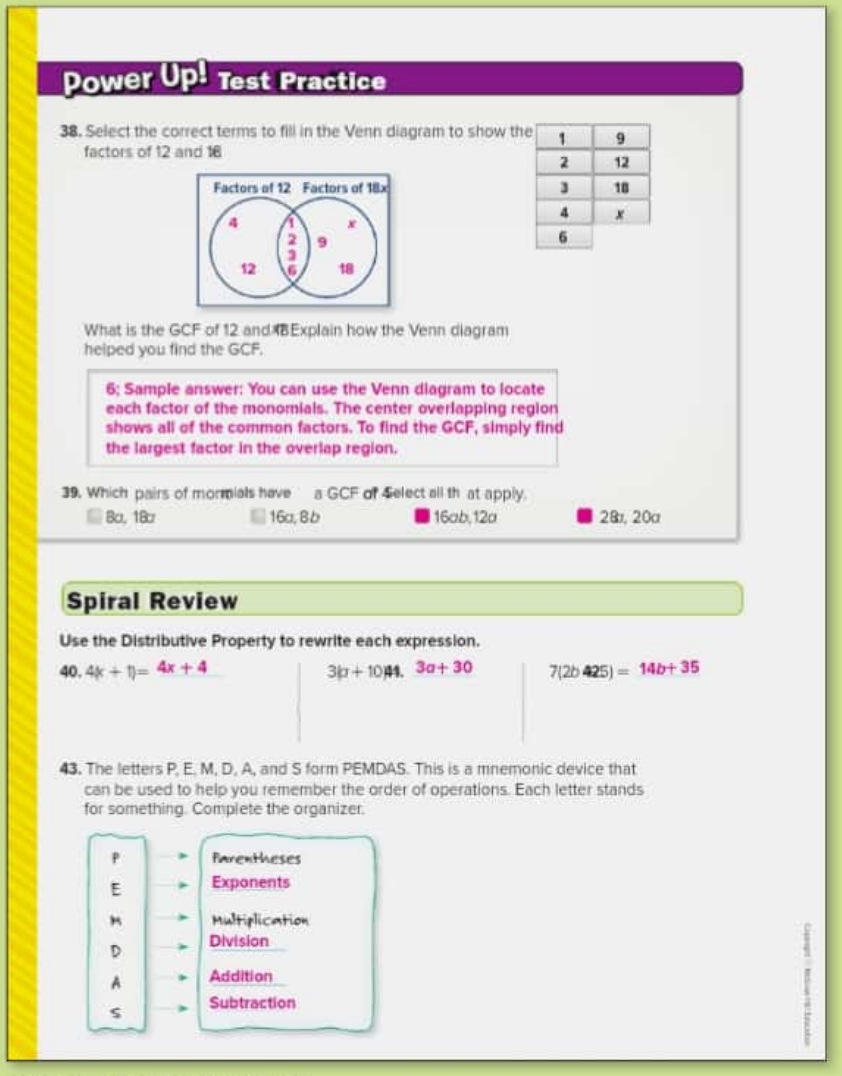
Exercises 38 and 39 prepare students for more rigorous thinking needed for the assessment.

38. This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.

Depth of Knowledge	DOK3
Mathematical Practices	MP1, MP3, MP4
Scoring Rubric	
2 points	Students correctly complete the Venn Diagram, find the GCF and explain.
1 point	Students correctly complete the Venn Diagram, find the GCF, but fail to explain OR students correctly place 6–9 of the values in the diagram and find the GCF OR students correctly place 6–9 of the values in the diagram, find the GCF, and explain OR students fail to find the GCF, place 6–9 values on the diagram, and explain.

39. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.



Power Up! Test Practice

38. Select the correct terms to fill in the Venn diagram to show the factors of 12 and 18.

1	9
2	12
3	18
4	x
6	

What is the GCF of 12 and 18? Explain how the Venn diagram helped you find the GCF.

6; Sample answer: You can use the Venn diagram to locate each factor of the monomials. The center overlapping region shows all of the common factors. To find the GCF, simply find the largest factor in the overlap region.

39. Which pairs of monomials have a GCF of 4? Select all that apply.

☐ $8a$, $18a$ ☐ $16a$, $8b$ ■ $16ab$, $12a$ ■ $28x$, $20a$

Spiral Review

Use the Distributive Property to rewrite each expression.

40. $4(x + 1) =$ $4x + 4$

41. $3(a + 10) =$ $3a + 30$

42. $7(2b + 5) =$ $14b + 35$

43. The letters P, E, M, D, A, and S form PEMDAS. This is a mnemonic device that can be used to help you remember the order of operations. Each letter stands for something. Complete the organizer.

P	Parentheses
E	Exponents
M	Multiplication
D	Division
A	Addition
S	Subtraction

21ST CENTURY CAREER in Animal Conservation

Shark Scientist

Are you fascinated by sharks, especially those that are found around the coasts of the United States? If so, you should consider a career as a shark scientist. Shark scientists use satellite-tracking devices, called tags, to study and track the movements of sharks. By analyzing the data transmitted by the tags, scientists are able to learn more about the biology and ecology of sharks. Their research is helpful in protecting shark populations around the world.

Is This the Career for You?

Are you interested in a career as a shark scientist? Take some of the following courses in high school.

- Algebra
- Calculus
- Physics
- Statistics

Find out how math relates to a career in Animal Conservation.

Focus narrowing the scope

Objective Apply mathematics to problems arising in the workplace.

This lesson emphasizes **Mathematical Practice** Model with Mathematics.

Coherence connecting within and across grades

Previous
Students use expressions to represent mathematical ideas.

Now
Students apply the content standard to solve problems in the workplace.

Rigor pursuing concepts, fluency, and applications

See the Career Project on page 424.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ask students to read the information on the student page about shark scientists and answer the following questions.

Ask:

- *What does a shark scientist do?* Sample answer: studies the biology and ecology of sharks
- *What courses should you take if you are interested in becoming a shark scientist?* Sample answers: Calculus, Algebra, Physics, and Statistics

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** EVALUATE

2 Collaborate

AL LA **Think-Pair-Share** Have students work in pairs to complete Exercises 1, 3, and 5 using the following scaffolded questions. Answers shown are for Exercise 1. MP 1, 2, 4

Ask:

- *What information from the note cards do you need to complete this exercise?* The total length of a hammerhead shark is about 1.3 times the fork length.
- *What variable is used to represent the fork length (or total length in Exercise 5)?* *m*
- *What operation is indicated by the note cards?* multiplication

BL LA **Trade-a-Problem** Have students alter Exercises 2, 4, and 6 using different given fork lengths for a hammerhead shark and sandbar shark, and total length of a white shark. Then have them trade their altered problems with another student. Each student solves the other's problems and compare solutions. If the solutions do not agree, students should work together to determine any errors. MP 1, 2, 3

Career Portfolio

When students complete this page, have them add it to their Career Portfolio.

Career Facts

Due to fisheries, unregulated catching of sharks, and habitat destruction, more than 50 percent of oceanic shark species are currently listed as *threatened with extinction*. Through their research projects, scientists help promote conservation of sharks worldwide.

Tag, You're It!

The *fork length* of a shark is the length from the tip of the snout to the fork of the tail. Use the information on the note cards to solve each problem.

1. Write an expression to represent the total length of a hammerhead shark that has a fork length of m meters. $1.3m$
2. Use the expression from Exercise 1 to find the total length of a hammerhead shark that has a fork length of 3.5 meters. 4.55 m
3. Write an expression to represent the average fork length of a tiger shark, given the average fork length s of a sandbar shark. $2s - 55$
4. Use the expression from Exercise 3 to find the average fork length of a tiger shark if the average fork length of a sandbar shark is 129 centimeters. 203 cm
5. Write an expression to find the average fork length of a white shark with a total length of t centimeters. $0.94t - 5.74$
6. The total length of a white shark is 204 centimeters. Use the expression in Exercise 5 to find the approximate fork length of the white shark. 186 cm

Tiger Shark
A study found that the average fork length of a tiger shark is 55 centimeters less than twice the average fork length of a sandbar shark.

White Shark
The fork length of a white shark is about 5.74 centimeters less than 0.94 times the total length.

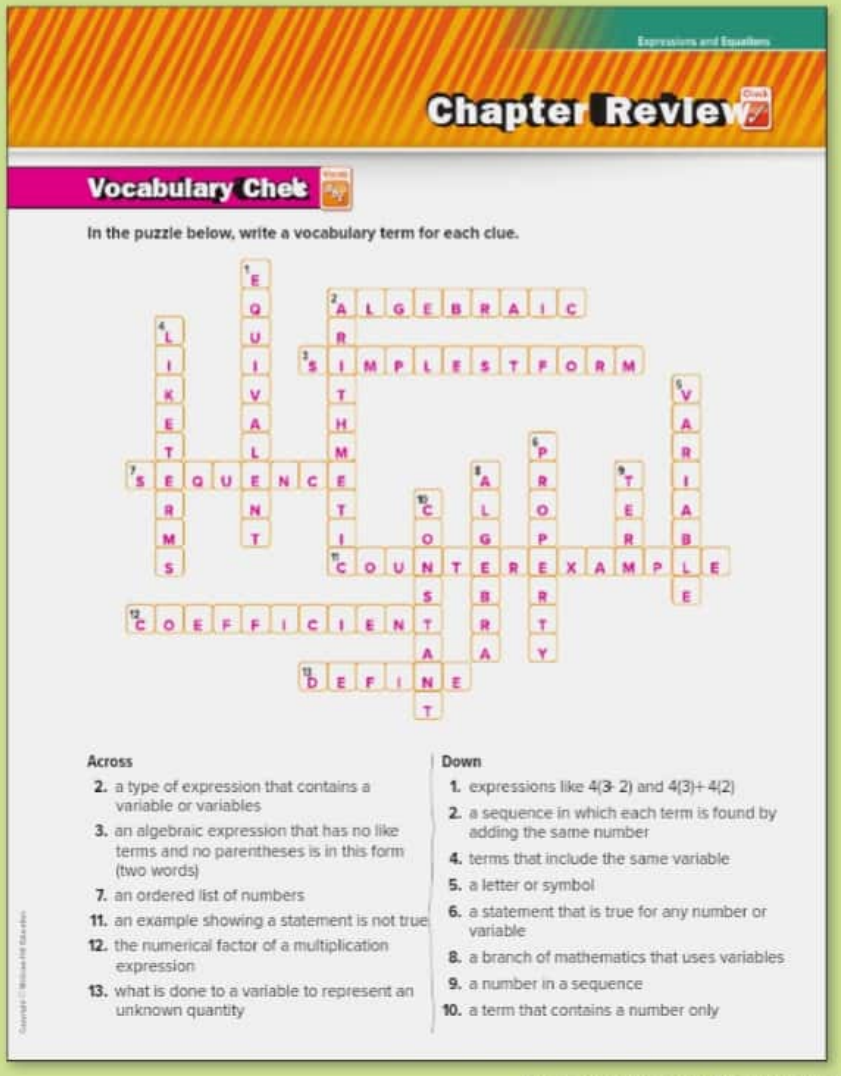
Hammerhead Shark
The total length of a hammerhead shark is about 1.3 times the fork length.

Career Project

It's time to update your career portfolio. Describe the skills that would be necessary for a shark scientist to possess. Determine whether this type of career would be a good fit for you.

List several challenges associated with this career.

-
-
-
-
-



Vocab ulary Check

LA **Numbered Heads Together** Assign students to 3- or 4-person learning teams. Each member is assigned a number from 1 to 4. Each team completes the Vocabulary Check, making sure every team member understands the terms and their definitions. Call on a specific number from one team to present the group's solution to the class. MP 1, 6

Alternate Strategy

AL LA To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include:

- algebra (Lesson 1)
- algebraic expression (Lesson 1)
- arithmetic sequence (Lesson 2)
- coefficient (Lesson 1)
- constant (Lesson 5)
- counterexample (Lesson 3)
- define a variable (Lesson 1)
- equivalent expression (Lesson 4)
- like terms (Lesson 5)
- property (Lesson 3)
- sequence (Lesson 2)
- simplest form (Lesson 5)
- term (Lessons 2 and 5)
- variable (Lesson 1)

Key Conc ept Check

FOLDABLES LA A completed Foldable for this chapter should include a review of adding, subtracting, and factoring linear expressions.

If you choose not to use the Foldable, have students write a brief review of the Key Concepts found throughout the chapter and give an example of each.

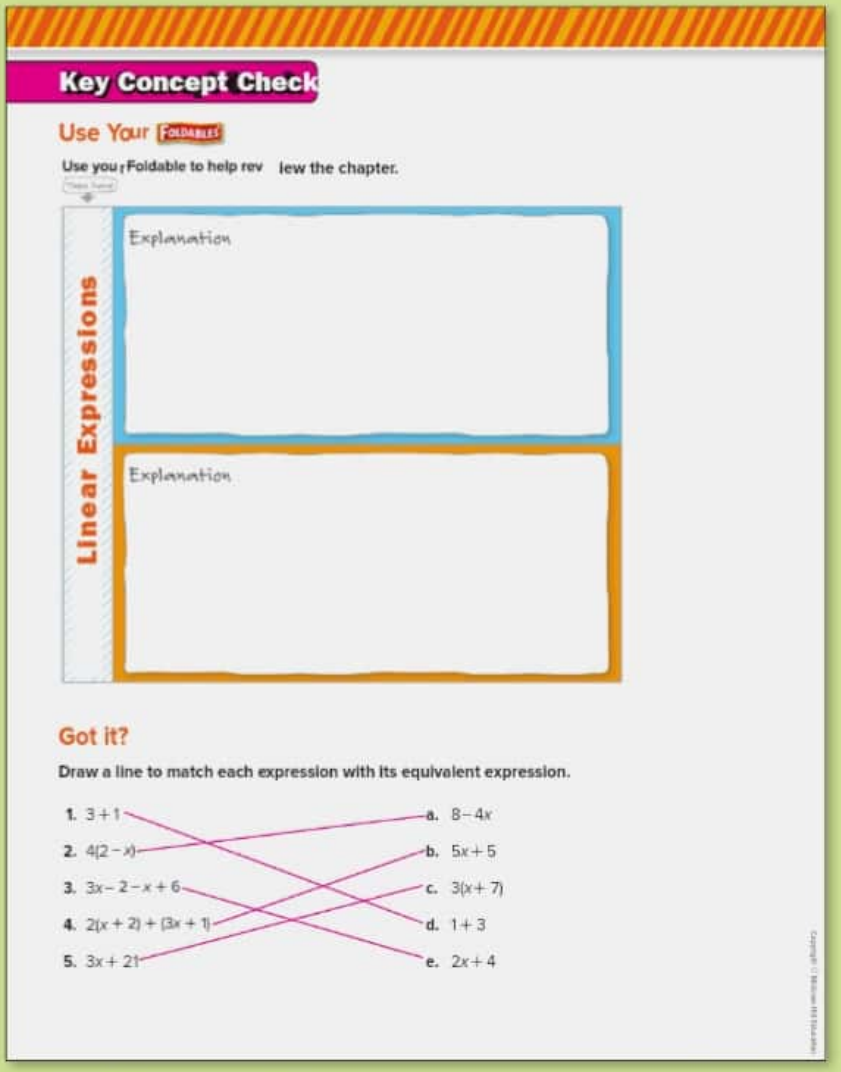
Ideas for Use

LA Have students work in pairs to discuss their Foldables. Have them practice speaking in a collaborative setting by sharing how they have completed their Foldable thus far and how they could finish it. Have each student complete their Foldable and trade with their partner to discuss any similarities and differences MP 1, 3, 5

Got It?

If students have trouble with Exercises 1–5, they may need help with the following concepts.

Concept	Exercise(s)
Commutative Property (Lesson 3)	1
Distributive Property (Lesson 4)	2, 4
simplifying expressions (Lesson 5)	3
factoring expressions (Lesson 8)	5



Key Concept Check

Use Your FOLDABLES

Use your Foldable to help rev lew the chapter.

Linear Expressions

Explanation

Explanation

Got it?

Draw a line to match each expression with its equivalent expression.

1. $3+1$
2. $4(2-x)$
3. $3x-2-x+6$
4. $2(x+2)+(3x+1)$
5. $3x+21$

a. $8-4x$
b. $5x+5$
c. $3(x+7)$
d. $1+3$
e. $2x+4$

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the **Building on the Essential Question** exercises found in each lesson of the chapter.

- Will the expressions $x + 3$ and $y - 3$ sometimes, always, or never represent the same value? (p. 352)
- Why is 5, 9, 13, 17, 21, ... considered an arithmetic sequence? (p. 360)
- What is the difference between the Commutative and Associative Properties? (p. 370)
- How is the formula for the perimeter of a rectangle an application of the Distributive Property? (p. 378)
- Why are the expressions $2(x + 1) + 3(x - 1)$ and $5(x - 1)$ equivalent? (p. 390)
- How is adding linear expressions similar to simplifying expressions? (p. 398)
- How can you use the additive inverse to help you subtract linear expressions? (p. 406)
- How is the GCF used to factor expressions? (p. 418)

Ideas for Use

LA **Think-Pair-Share** Have students work in pairs. Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer. Then have them share their responses with their classmate before they complete the graphic organizer.

MP 1, 3, 5

Reflect

Answering the Essential Question

Use what you learned about algebraic expressions to complete the graphic organizer. Then answer the chapter's Essential Question below.

What do you know about variables?

Sample answer: A variable is a letter or symbol that represents a number or set of numbers. Variables are used to represent an unknown quantity. Suppose a real-world problem states that each pizza costs AED 7, but the problem does not state how many pizzas are ordered. Let p represent the number of pizzas ordered and $7p$ represent the cost of the pizzas.

Essential Question

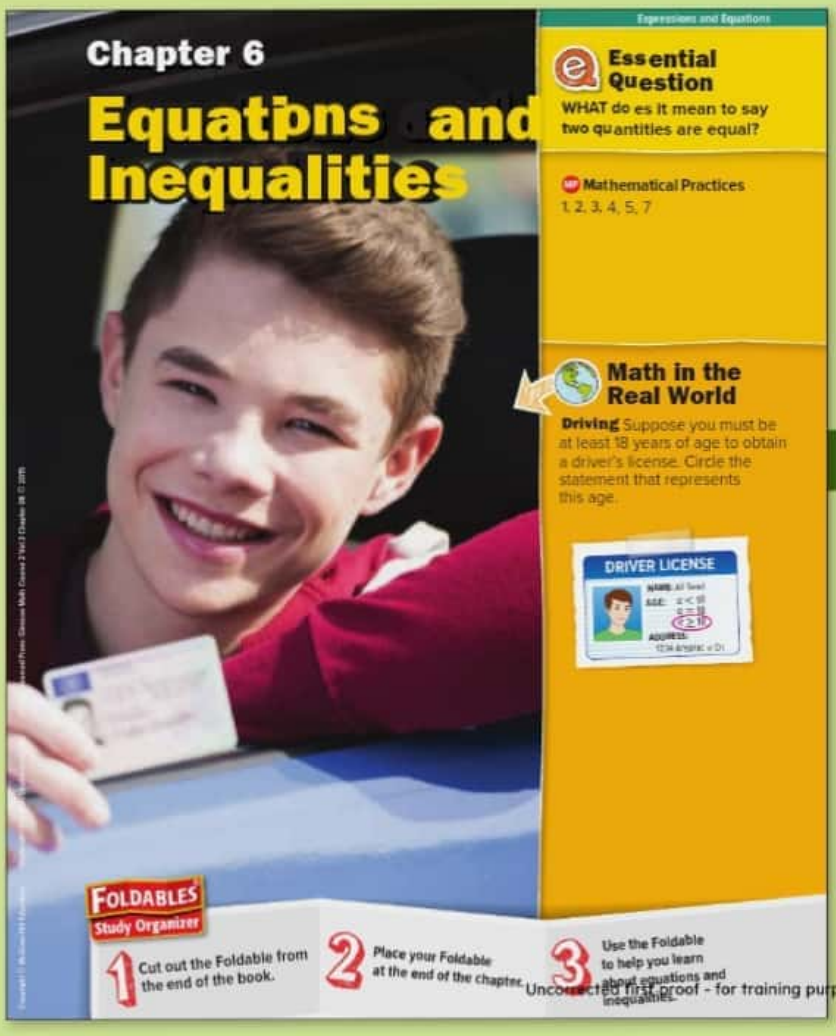
HOW can you use numbers and symbols to represent mathematical ideas?

How do you know which operation symbol to use?

Sample answer: Four operation symbols are for addition, − for subtraction, ×, •, or ( ) for multiplication, and ÷ for division. Read a real-world problem carefully to look for key words indicating which operation to use, such as *more than* or *increased by* for addition and *less than* or *decreased by* for subtraction. Then use the symbol for that operation.

Answer the Essential Question HOW can you use numbers and symbols to represent mathematical ideas?

See students' work.



Focus narrowing the scope

This chapter focuses on **Expressions and Equations**.

Coherence connecting within and across grades

Previous	Now	Next
Students used properties of operations to simplify expressions.	Students solve equations and inequalities.	Students will use expressions and equations to solve geometric relationships.

Rigor pursuing concepts, fluency, and applications

The Levels of Complexity charts located throughout this chapter indicate how the exercises progress from conceptual understanding and procedural skills and fluency, to application and critical thinking.

Launch the Chapter

Math in the Real World

Driving Remind students that the phrase *at least* indicates that you should use the ≥ symbol.

What Tools Do You Need

Vocabulary Activity

LA As you proceed through the chapter, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define: A coefficient is the numerical factor of a term that contains a variable.

Example: In the term $\frac{1}{2}b$, $\frac{1}{2}$ is the coefficient.

Ask:

- *What is the coefficient in the equation* $\frac{2}{5}n = \frac{12}{15}$? $\frac{2}{5}$

Reading Math

LA Students identify key information in word problems. This is important in the understanding and solving of word problems. Have students read the steps to identify key information in the Reading Math section.

Ask:

- *How does putting the important information in one sentence help with solving a word problem?* Sample answer: It helps to sort out what you know and what you need to find.
- *What are some key words or phrases in Exercise 2?* Sample answer: 3.1 million kilograms more than, how many

What Tools Do You Need

Vocabulary

Addition Property of Equality	equation	solution
Addition Property of Inequality	equivalent equation	Subtraction Property of Equality
coefficient	inequality	Subtraction Property of Inequality
Division Property of Equality	Multiplication Property of Equality	two-step equation
Division Property of Inequality	Multiplication Property of Inequality	two-step inequality

Study Skill: Reading Math

Identify Key Information Have you ever tried to solve a word problem and didn't know where to start. Start by looking for key words in the text and images. Then write the important information in one sentence.

1. Highlight or circle key words in the following real-world problem.

During a recent Super Bowl, millions of kilograms of potato chips and tortilla chips were consumed. The number of kilograms of potato chips consumed was 3.1 million kilograms more than the number of kilograms of tortilla chips. How many kilograms of tortilla chips were consumed?

2. Write a sentence that summarizes the information provided. Include information from the text and the image. Sample answer: 12.4 million kilograms of potato chips were 3.1 million more than the number of kilograms of tortilla chips.

Are You Ready?

Use this page to determine if students have skills that are needed for the chapter.

Quick Review

Students with strong math backgrounds may opt to go directly to the Quick Check.

Quick Check

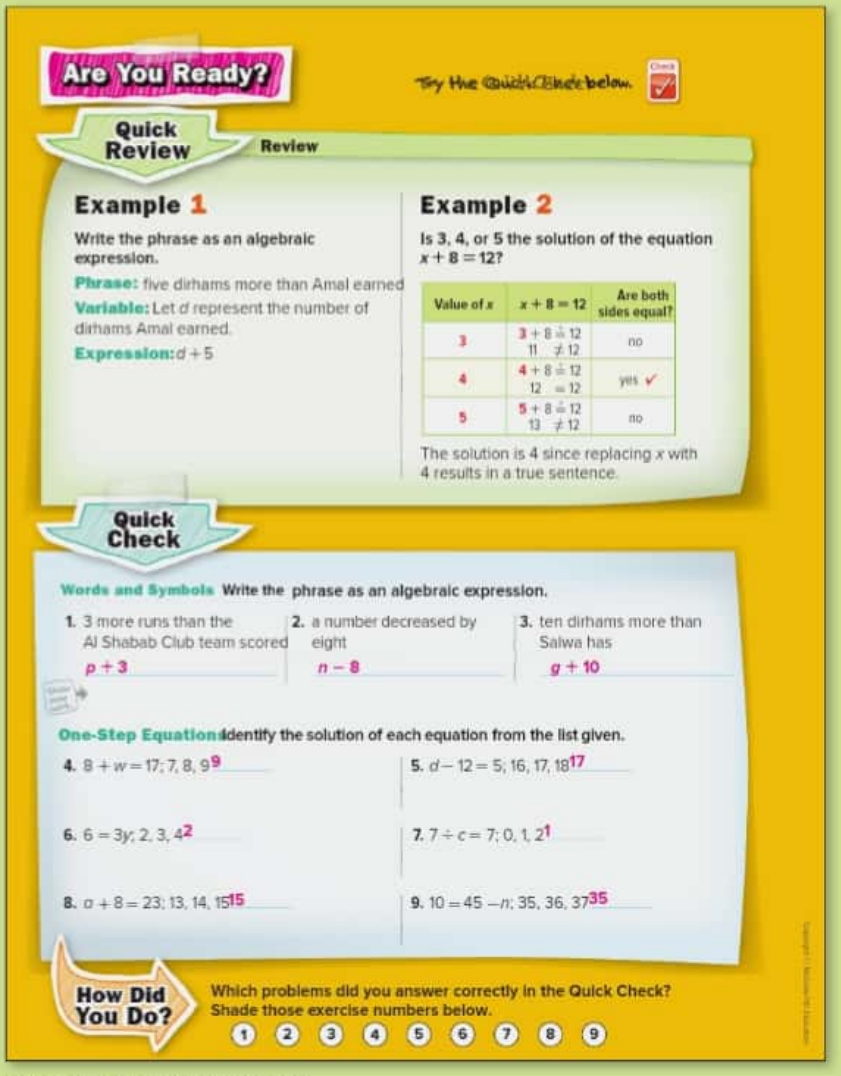
If students have difficulty with the exercises, present an additional example to clarify any misconceptions.

Exercises 1–3

Write the phrase *12 years older than his sister* as an algebraic expression. $s + 12$

Exercises 4–9

Is 28, 29, or 30 the solution of the equation $x - 25 = 54$? 29



Are You Ready?

Try the Quick Check below.

Quick Review

Review

Example 1

Write the phrase as an algebraic expression.

Phrase: five dirhams more than Amal earned

Variable: Let d represent the number of dirhams Amal earned.

Expression: $d + 5$

Example 2

Is 3, 4, or 5 the solution of the equation $x + 8 = 12$?

Value of x	$x + 8 = 12$	Are both sides equal?
3	$3 + 8 \stackrel{?}{=} 12$ $11 \neq 12$	no
4	$4 + 8 \stackrel{?}{=} 12$ $12 = 12$	yes ✓
5	$5 + 8 \stackrel{?}{=} 12$ $13 \neq 12$	no

The solution is 4 since replacing x with 4 results in a true sentence.

Quick Check

Words and Symbols Write the phrase as an algebraic expression.

1. 3 more runs than the Al Shabab Club team scored — $p + 3$
2. a number decreased by eight — $n - 8$
3. ten dirhams more than Salwa has — $g + 10$

One-Step Equations Identify the solution of each equation from the list given.

4. $8 + w = 17$; 7, 8, 9 — 9
5. $d - 12 = 5$; 16, 17, 18 — 17
6. $6 = 3y$; 2, 3, 4 — 2
7. $7 \div c = 7$; 0, 1, 2 — 1
8. $a + 8 = 23$; 13, 14, 15 — 15
9. $10 = 45 - n$; 35, 36, 37 — 35

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4 5 6 7 8 9

Inquiry Lab

Solve One-Step Addition and Subtraction Equations

inquiry **HOW can bar diagrams or algebra tiles help you solve an equation?**

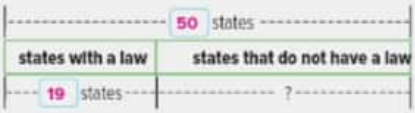
MP Mathematical Practices 1, 2, 3, 5

In a recent year, 19 of the 50 states had a law banning the use of handheld cell phones while driving a school bus. Determine how many states did *not* have this law.

Hands-On Activity 1

You can represent this situation with an equation.

Step 1 The bar diagram represents the total number of states and the number of states that have passed a cell phone law. Fill in the missing information.



Step 2 Write an equation from the bar diagram. Let x represent the states that do not have a cell phone law for school bus drivers.

$$19 + x = 50$$

Step 3 Use the *work backward* strategy to solve the equation. Since $19 + x = 50$, $x = 50 - 19$. So, $x =$ 31.

Check $19 +$ 31 $= 50$ ✓

So, 31 states did *not* have a law banning the use of cell phones by bus drivers.

Focus narrowing the scope

Objective Use models to write and solve addition and subtraction equations.

Coherence connecting within and across grades

Now Students use bar diagrams and algebra tiles to write and solve one-step addition and subtraction equations.

Next Students will use Properties of Equality to solve one-step equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 434.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activity 1 is intended to be used as a whole-group activity. Activity 1 is designed to provide more guidance than Activity 2

Hands-On Activity 1

AL LA Remind students that the bar diagram represents the sum of the states with a law and the states that do not have a law.

Ask:

- *How many states have this law?* 19 states
- *How can you use the bar diagram to write an equation?* The first section represents the 19 states with a law, the second section represents the states that do not. The sum of these is equal to 50.

BL Omit Activities 1 and 2 and proceed directly to the Investigate section. MP 1, 4, 5

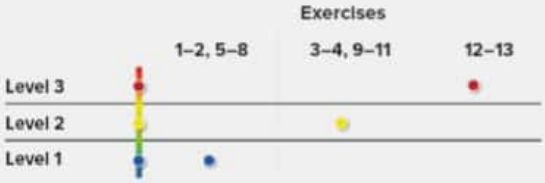
ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Pairs Discussion** Have students work in pairs to complete Exercises 1 and 2. Have them trade their solutions with another pair of students and discuss any differences. MP 1, 2, 4, 5

BL LA **Trade-a-Problem** Have students create their own problem, similar to Exercises 1 and 2. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. MP 1, 2, 4

Analyze and Reflect

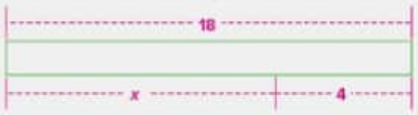
AL LA **Think-Pair-Share** Have students work in pairs. Give students one minute to think through their responses to Exercises 3 and 4. Have them share their responses with their partner. Then call on one student to share their response within a small group or large group discussion. MP 1, 2, 5

Investigate

Work with a partner to solve each problem.

1. Draw a bar diagram and write an addition equation to represent the following situation. Then solve the equation.

The sum of a number and four is equal to 18.



Equation: $x + 4 = 18$ Solution: $x = 14$

2. MP **Use Math Tools** Salem collects postage stamps. He sold 7 of his stamps and had 29 stamps left. Complete the bar diagram below. Then write and solve a subtraction equation to find the number of stamps Salem had at the beginning.

Equation: $n - 7 = 29$ Solution: $n = 36$

So, Salem had 36 stamps at the beginning.

Analyze and Reflect

3. Suppose Salem sold 15 stamps and had 21 stamps left. How would the bar diagram change?

Sample answer: Since Salem sold 15 stamps instead of 7, the stamps sold bar would be longer.

4. MP **Reason Abstractly** Suppose Salem had 40 stamps in the beginning and sold 7 of them. How would the bar diagram change? What equation could you write to represent the situation?

Sample answer: Write 40 stamps above the bar and use a variable for the number of stamps left; $n - 7 = 40$.

Expressions and Equations

Hands-On Activity 2

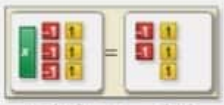
Solve $x - 3 = -2$ using algebra tiles.

Remember a 1-tile and 1 tile combine to make a *zero pair*. You can add or subtract zero pairs from either side of an equation without changing its value.

Step 1 Model the equation.

$x - 3 = -2$

Step 2 Add three 1-tiles to the left side of the mat and three 1-tiles to the right side of the mat to form zero pairs on each side of the mat.



$x - 3 + 3 = -2 + 3$

Step 3 Remove all of the zero pairs from each side. There are one 1-tile on the right side of the mat.

$x = 1$

Therefore, $x =$ 1.

Check 1 $-3 = -2$ ✓

Hands-On Activity 2

AL LA Explain to students that a 1-tile and ~~a~~ −1-tile make a zero pair because when 1 and ~~a~~ −1 are opposites. Zero pairs can be removed from or added to the equation mat without changing the value on that side of the equation.

Ask:

- *To isolate the variable, do you need to add 1-tiles or −1-tiles?* 1-tiles
- *How many 1-tiles need to be added to each side of the mat to isolate the variable?* 3
- *How many zero pairs are now on the left side of the mat?* 3
- *How many zero pairs are now on the right side of the mat?* 2
- *How many 1-tiles remain on the right side of the mat?* 1

Investigate

AL LA **Rally Coach** For Exercises 5–8, have students work in pairs. Partner A should answer the first Exercise while Partner B watches, listens, coaches, and praises. Partner B solves the next Exercise while Partner A watches, listens, coaches, and praises. Partners take turns until the Exercises are complete. MP 1, 2, 5

Analyze and Reflect

AL LA **Numbered Heads Together** Assign students to 3- or 4-person learning teams. Each member is assigned a number from 1 to 4. Each team completes Exercises 9–11 making sure that every member understands. Call on a specific number from one team to present the team's solution to the class. MP 1, 2

BL LA **Trade-a-Problem** Have students create their own problem, similar to Exercises 9–11. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the error. MP 1, 2

Create

BL LA **Pairs Discussion** Have students work in pairs to complete Exercises 12 and 13. Have them trade their solutions with another pair of students and discuss any differences. MP 1, 7

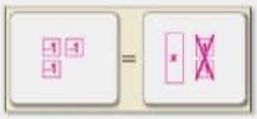
Inquiry Students should be able to answer "HOW can bar diagrams or algebra tiles help you solve an equation?" Check for student understanding and provide guidance, if needed.

Investigate

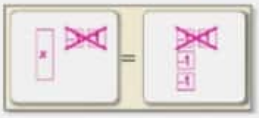
Use Math Tools Work with a partner to solve each equation. Use algebra tiles. Show your work using drawings.

5. $x + 4 = 4$ $x = 0$

6. $-2 = x + 1$ $x = -3$



7. $x - 1 = -3$ $x = -2$



8. $4 = x - 2$ $x = 6$

Analyze and Reflect

Work with a partner to complete the table. The first one is done for you.

	Equation	Related Equation
	$x + 3 = 4$	$x = 4 - 3$
9.	$6 + x = 10$	$x = 10 - 6$
10.	$x + 3 = -1$	$x = -1 - 3$
11.	$6 + x = -7$	$x = -7 - 6$

Create

12. MP **Construct an Argument** Write a rule that you can use to solve addition equations without using models or a drawing.

Sample answer: Write them as subtraction equations.

13. Inquiry HOW can bar diagrams or algebra tiles help you solve an equation?

Sample answer: A bar diagram or an algebra tile model provides a visual aid when deciding which operation to use.

Lesson 1

Solve One-Step Addition and Subtraction Equations

Vocabulary Start-Up

An **equation** is a sentence stating that two quantities are equal. The value of a variable that makes an equation true is called a **solution** of the equation.

$x + 2 = 6$
$-2 = -2$
$x = 4$

The equations $x + 2 = 6$ and $x = 4$ are **equivalent equations** because they have the same solution, 4.

Circle the equations below that are equivalent to $x = 3$. Use algebra tiles if needed.

$x + 3 = 6$ $x + 1 = 6$ $x + 6 = 8$
$x + 3 = 3$ $x + 1 = 4$ $x + 2 = 5$

Essential Question

WHAT does it mean to say two quantities are equal?

Vocabulary

equation
solution
equivalent equation
Subtraction Property of Equality
Addition Property of Equality

Mathematical Practices
1, 2, 3, 4, 5

Real-World Link

Video Games Amna had some video games, and then she bought 4 more games. Now she has 10 games. This scenario can be described using the equation $x + 4 = 10$.

1. What does x represent in the equation?
   The x represents the number of games Amna started with.
2. Write two different equations that are equivalent to $x + 4 = 10$.
   Sample answer: $x = 6$ and $2x = 12$.

Which Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Solve addition and subtraction equations.

Coherence connecting within and across grades

Previous Students wrote and evaluated algebraic expressions.

Now Students write and solve one-step linear equations.

Next Students will solve two-step linear equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 441.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA AL **Think-Pair-Share** Have students work in pairs to complete the Vocabulary Start-Up. When they get to the vocabulary word *equivalent equations*, give students one minute to think about the word equivalent, what it means, and where they have used it before. Have them share their responses with their partner. Then call on one student to share their response within a small group or large group discussion. MP 1, 6

Alternate Strategy

BL Have students work in pairs. Have one student give an example of an equation. Then have the other student write an equation that is equivalent to the first equation. Challenge students to write more complex equations. MP 1, 4

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Solve addition equations.**

AL • *What is the variable?* x
- *What number is on the same side of the equals sign as the variable?* 6
- *What operation is used between x and 6?* addition
- *How do you undo addition?* subtraction

OL • *What is the first step to solving the equation?* subtract 6 from each side
- *What is 4 − 6?* −2

BL • *What property allows you to subtract 6 from each side of the equation?* Subtraction Property of Equality

Need Another Example?
Solve $14 + y = 20$. Check your solution. 6

2. **Solve addition equations.**

AL • *What is the variable?* b
- *What number is on the same side of the equals sign as the variable?* 8
- *What operation is used between b and 8?* addition
- *How do you undo addition?* subtraction

OL • *What is the first step to solving the equation?* subtract 8 from each side
- *What is −5 − 8?* −13
- *What property allows you to subtract 8 from each side of the equation?* Subtraction Property of Equality

BL • *How can you check your solution?* Replace *b* with −13 in the original equation, simplify, and determine if the final statement is true.

Need Another Example?
Solve $-12 = 4 + c$. Check your solution. −16

Key Concept **Subtraction Property of Equality**

Work Zone

Words The **Subtraction Property of Equality** states that the two sides of an equation remain equal when you subtract the same number from each side.

Symbols If $a = b$, then $a - c = b - c$.

You can use bar diagrams and the *work backward* problem-solving strategy to solve equations arithmetically. Or, you can use the properties of equality to solve equations algebraically.

Examples

1. **Solve $x + 6 = 4$. Check your solution.**

$x + 6 = 4$ Write the equation.
$-6 = -6$ Subtraction Property of Equality
$x = -2$ Simplify.

Check $x + 6 = 4$ Write the original equation.
$-2 + 6 \stackrel{?}{=} 4$ Replace x with -2.
$4 = 4$ ✓ The sentence is true.

So, the solution is −2.

2. **Solve $-5 = b + 8$. Check your solution.**

$-5 = b + 8$ Write the equation.
$-8 = -8$ Subtraction Property of Equality
$-13 = b$ Simplify.

Check $-5 = b + 8$ Write the original equation.
$-5 \stackrel{?}{=} -13 + 8$ Replace b with -13.
$-5 = -5$ ✓ The sentence is true.

So, the solution is −13.

Got it? Do these problems to find out.

Solve each equation. Check your solution.

a. $y + 6 = 9$ b. $x + 3 = 1$ c. $-3 = a + 4$

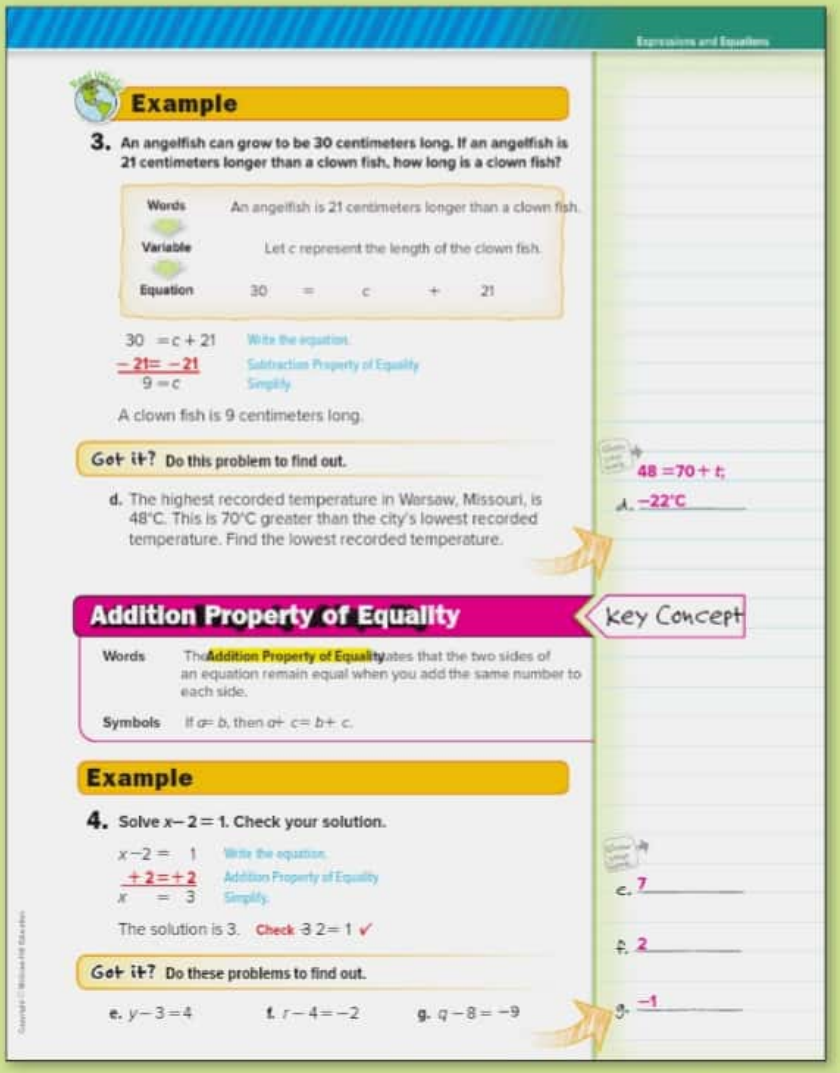
Solutions

Notice that your new equation, $x = -2$, has the same solution as the original equation, $x + 6 = 4$.

a. 3

b. −2

c. −7



Expressions and Equations

Example

3. An angelfish can grow to be 30 centimeters long. If an angelfish is 21 centimeters longer than a clown fish, how long is a clown fish?

Words An angelfish is 21 centimeters longer than a clown fish.

Variable Let c represent the length of the clown fish.

Equation $30 = c + 21$

$30 = c + 21$ Write the equation.

$-21 = -21$ Subtraction Property of Equality

$9 = c$ Simplify.

A clown fish is 9 centimeters long.

Got it? Do this problem to find out.

d. The highest recorded temperature in Warsaw, Missouri, is 48°C. This is 70°C greater than the city's lowest recorded temperature. Find the lowest recorded temperature.

48 = 70 + t;

d. −22°C

Addition Property of Equality

Key Concept

Words The **Addition Property of Equality** states that the two sides of an equation remain equal when you add the same number to each side.

Symbols If $a = b$, then $a + c = b + c$.

Example

4. Solve $x - 2 = 1$. Check your solution.

$x - 2 = 1$ Write the equation.

$+2 = +2$ Addition Property of Equality

$x = 3$ Simplify.

The solution is 3. **Check** $3 - 2 = 1$ ✓

Got it? Do these problems to find out.

e. $y - 3 = 4$ **f.** $r - 4 = -2$ **g.** $q - 8 = -9$

e. 7

f. 2

g. −1

Examples

3. Write and solve an addition equation.

AL • *What do you need to find?* the length of the clown fish

• *What do you know?* the length of an angelfish is 21 centimeters longer than the clown fish and an angelfish can grow to 30 centimeters

• *The length of the clown fish plus what is the equal to the length of the angelfish?* 21

OL • *What addition equation represents this situation?* $c + 21 = 30$

• *How would you solve this equation?* subtract 21 from each side of the equation

BL • *Suppose a certain betta fish is 3 centimeters longer than the clown fish. How long is the betta fish?* 12 centimeters

Need Another Example?
A grapefruit weighs 310 grams, which is 170 grams more than an apple. Write and solve an equation to find the weight of the apple. $a + 170 = 310$; 140 g

4. Solve a subtraction equation.

AL • *What is the variable?* x

• *What number is on the same side of the equals sign as the variable?* 2

• *What operation is used between x and 2?* subtraction

• *How do you undo addition?* addition

OL • *What is the first step to solving the equation?* Add 2 to each side.

• *What property allows you to add 2 to each side of the equation?* Addition Property of Equality

BL • *Solve the equation.* $-12 = p - (-20)$. −32

Need Another Example?
Solve $12 = z - 8$. Check your solution. 20

Example

5. Write and solve a subtraction equation.

AL • *What do you need to find?* the cost of the jeans

• *What do you know?* A pair of shoes that are AED 25 is AED 14 less than the cost of the jeans.

• *The cost of the jeans minus what is equal to the cost of the shoes?* 14

OL • *What subtraction equation represents this situation?* $j - 14 = 25$ or $25 = j - 14$

• *How would you solve this equation?* Add 14 to each side of the equation and simplify.

BL • *Are the two equations $j - 14 = 25$ and $25 = j - 14$ equivalent? Explain.* yes; Both equations have the same solution, $j = 39$.

Need Another Example?
Khadija practiced the piano for 32 minutes. She practiced 11 minutes less than her brother did. Write and solve an equation to determine how long her brother practiced the piano. $b - 11 = 32$; 43 minutes

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Discussion** Have students work in pairs to solve Exercises 1–4. Students can use algebra tiles if needed to model the equations. Have them trade their solutions with another pair of students and discuss any differences. MP 1, 2, 4

BL LA **Think-Pair-Share** Have students work in pairs. Give them a minute to think through their responses to the problems shown. Have them share their responses with their partner. Then call on one student to share their response within a small group or large group discussion. MP 1, 2

$-x + 5 = -7$ $-3 - x = -4$ $\frac{2}{3} + x = 2$ $x - 5\frac{1}{2} = -7$

Models

jeans, j

shoes	
AED 95	AED 14

$j - 14 = 125$

$\ell - 3 = 17$;
h. 20 years

Example

5. A pair of shoes costs AED 95. This is AED 14 less than the cost of a pair of jeans. Find the cost of the jeans.

Shoes are AED 14 less than jeans. Let j represent the cost of jeans.

$95 = j - 14$ Write the equation.
$+14 = +14$ Addition Property of Equality
$109 = j$ Simplify.

The jeans cost AED 109.

Got it? Do this problem to find out.

h. The average lifespan of a tiger is 17 years. This is 3 years less than the average lifespan of a lion. Write and solve an equation to find the average lifespan of a lion.

Guided Practice

Solve each equation. Check your solution. (Examples 1, 2, and 4)

1. $n + 6 = 8$ 2
2. $7 = y + 2$ 5
3. $-7 = c - 6$ −1

4. Orville and Wilbur Wright made the first airplane flights in 1903. Wilbur's flight was 111 meters. This was 36 meters longer than Orville's flight. Write an equation to represent the flights. Use a bar diagram if needed. Then solve to find the length of Orville's flight. (Examples 3 and 5)

$d + 36 = 111$; 75m

5. **Building on the Essential Question** What are two methods for solving a real-world problem that can be represented by an equation?

Sample answer: One method is to solve arithmetically using a bar diagram. Another method is to solve the equation algebraically using the properties of equality.

Rate Yourself!

☐ I understand how to solve one-step addition and subtraction equations.

Great! You're ready to move on!

☐ I still have some questions about solving equations.

Name _____ My Homework _____

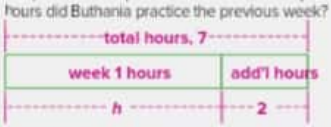
Independent Practice

Solve each equation. Check your solution. (Examples 1, 2, and 4)

1. $a + 3 = 10$ **7**
2. $y + 5 = -11$ **−16**
3. $s - 8 = 9$ **17**
4. $5 = x + 8$ **−3**
5. $-2 = p - 1$ **−1**
6. $14 = s + 7$ **7**

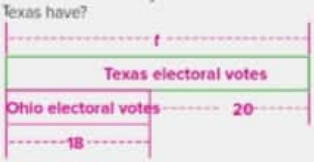
Use a bar diagram to solve arithmetically. Then use an equation to solve algebraically. (Examples 3 and 5)

7. Last week Buthania practiced her bassoon a total of 7 hours. This was 2 hours more than she practiced the previous week. How many hours did Buthania practice the previous week?



7 = h + 2; 5 h

8. In a recent presidential election, Ohio had 18 electoral votes. This is 20 votes less than Texas had. How many electoral votes did Texas have?



18 = t − 20; 38 votes

9. **Multiple Representations** Use the table to solve.

Tallest Wooden Roller Coasters	Height (meters)	Drop (meters)	Speed (km/h)
Colossos	h	48	109
T Express	55	45	104
El Toro	54	53	s
Voyage	49	d	107

a. **Symbols** The difference in speeds of El Toro and T Express is 8 kilometers per hour. If El Toro has the greater speed, write and solve a subtraction equation to find its speed.

s − 104 = 8; 112 kmph

b. **Diagram** Voyage has a drop that is 6.6 meters less than El Toro. Draw a bar diagram to the right and write an equation to find the height of Voyage.

d + 6.6 = 53; 46.4 m

c. **Words** Let h represent the height of the Colossos roller coaster. Explain why $h - 4 = 55$ and $h - 10 = 49$ are equivalent equations. Then explain the meaning of the solution.

The solution of each equation is 59; Colossos is 59 meters tall.

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 11–16, 35, 36
OL	On Level	1–7 odd, 9–16, 35, 36
BL	Beyond Level	9–16, 35, 36

Watch Out!

Common Error Students often forget whether to add or subtract to solve an addition or subtraction equation. Remind them to use the inverse of the operation in the equation. To solve an addition equation, use subtraction. To solve a subtraction equation, use addition.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
2 Reason abstractly and quantitatively.	14
3 Construct viable arguments and critique the reasoning of others.	9, 12, 13, 15, 16
5 Use appropriate tools strategically.	23, 24

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a clos ing formative assessment before dismissing students from your class.

TICKET Out the Door

Ask students to write th e steps explaining how to solve $x - 3 = 19$. See student s' work.

Watch Out!

Find the Error For Exercise 13, remind students to use inverse operation to solve equations. In this addition equation, they subtract in order to isolate the variable.

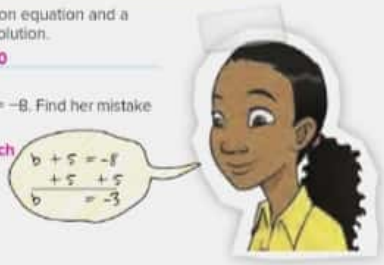
10. The sum of the measures of the angles of a triangle is 180°. Write and solve an equation to find the missing measure.
35 + 45 + x = 180; 100

11. The sum of the measures of a quadrilateral is 360°. Write and solve an equation to find the missing measure.
115 + 115 + 65 + x = 360; 65

H.O.T. Problems Higher Order Thinking

12. **Reason Inductively** Write an addition equation and a subtraction equation that have 10 as a solution.
Sample answer: 20 + x = 30; 30 − x = 20

13. **Find the Error** Aisha is finding $b + 5 = -8$. Find her mistake and correct it.
She should have subtracted 5 from each side; −13



14. **Reason Abstractly** Suppose $x + y = 11$ and the value of x increases by 2. If their sum remains the same, what must happen to the value of y? Justify your response
The value of y decreases by 2; Sample answer: (x + 2) + (y − 2) = 11 if x + y = 11, because if you add 2 and the sum remains the same, you must also subtract 2.

15. **Which One Doesn't Belong?** Identify the equati on that does not belong with the other three. Explain your reasoning.

$x + 4 = -2$ | $x + 5 = -1$ | $x + 2 = 8$ | $3 - x = 9$

x + 2 = 8; The solution for the other equations is −6.

16. **Reason Inductively** In the equation $x + y = 5$, the value for x is a whole number greater than 2 but less than 6. Find the possible solutions for y.
0, 1, 2

Name _____ My Homework _____

Extra Practice

Solve each equation. Check your solution.

17. $r + 6 = -3$

$r + 6 = -3$
$-6 \quad -6$
$r = -9$

18. $w - 7 = 11$ **18**

19. $k + 3 = -9$ **−12**

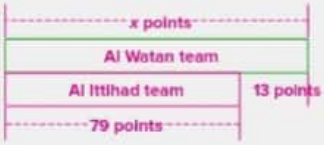
20. $-1 = q - 8$ **7**

21. $9 = r + 2$ **7**

22. $y + 15 = 11$ **−4**

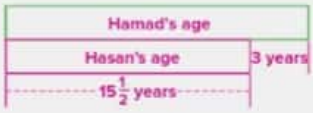
Use Math Tools **Use a bar diagram to solve arithmetically. Then use an equation to solve algebraically.**

23. The Al Ittihad team scored 79 points. This was 13 points less than the Al Watan team. How many points did the Al Watan team score?



$x - 13 = 79$; **92 points**

24. Hasan is $15\frac{1}{2}$ years old. This is 3 years younger than his brother Hamad. How old is Hamad?



$15\frac{1}{2} = y - 3$; $18\frac{1}{2}$ **years old**

25. The table shows a golfer's scores for four rounds of a recent Women's Open. Her total score was even with par. What was her score for the third round?

Round	Score
First	−1
Second	−3
Third	s
Fourth	+2

$-1 + (-3) + s + 2 = 0$; **+2**

Copy and Solve **Solve each equation. Check your solution. Show your work on a separate piece of paper.**

26. $a - 3.5 = 14.9$ **18.4**

27. $b + 2.25 = 1$ **−1.25**

28. $-\frac{1}{3} = r - \frac{3}{4}$ $\frac{5}{12}$

29. $x - 2.8 = 9.5$ **12.3**

30. $r - 8.5 = -2.1$ **6.4**

31. $z - 9.4 = -3.6$ **5.8**

32. $m + \frac{5}{6} = \frac{11}{12}$ $\frac{1}{12}$

33. $-\frac{5}{6} + c = -\frac{11}{12}$ $-\frac{1}{12}$

34. $s - \frac{1}{9} = \frac{5}{18}$ $\frac{7}{18}$

Power Up! Test Practice

Exercises 35 and 36 prepare students for more rigorous thinking needed for the assessment.

35. This test item requires students to reason abstractly and quantitatively when problem solving.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

36. This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.

Depth of Knowledge	DOK2
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly model and solve the equation.
1 point	Students correctly model OR solve the equation.

Power Up! Test Practice

35. The model represents the equation $x - 2 = 5$. Determine if each statement is true or false.

a. To solve the equation, add 2 positive counters to each side of the equation mat. ■ True ☐ False

b. To solve the equation, add 5 negative to each side of the equation mat. ☐ True ■ False

c. The value of x is 7. ■ True ☐ False

36. Badria practiced the piano a total of 7 hours this week. This is 3 hours less than she practiced last week. Select the correct labels to complete the bar diagram that is used to find the number of hours w Badria practiced last week.

this week | last week, w | 3 hours | 4 hours | 7 hours | 10 hours

How many hours did Badria practice the piano last week? 10 hours

Spiral Review

Multiply or divide.

37. $5(-4) =$ −20

38. $\frac{36}{-9} =$ −4

39. $(-10)(-6) =$ 60

40. $\frac{-42}{-7} =$ 6

41. $(-3)(12) =$ −36

42. $\frac{-54}{2} =$ −27

43. While playing a round of golf, Rana had a score of three under par after the first three holes. Write and solve an equation to find Rana's average score per hole h after three holes. $3h = -3; h = -1$

44. On Friday morning, the temperature dropped 2 degrees per hour for four hours. Write and solve an equation to find the total number of degrees d the temperature dropped on Friday morning. $-2 \times 4 = d; d = -8$

Inquiry Lab

Multiplication Equations with Bar Diagrams

inquiry **HOW do you know which operation to use when solving an equation?**

MP Mathematical Practices 1, 2, 3

Hessa tutors students to earn money to buy a new Blu-ray™ player that costs AED 63. She is able to tutor seven hours in a week. How much should she charge per hour to have enough money by the end of the week?

What do you know? the cost of the Blu-ray™ player, the number of hours Hessa can tutor.

What do you need to find? the amount per hour Hessa should charge

Hands-On Activity

Step 1 Draw a bar diagram that represents the money Hessa needs to earn and the number of hours she is available to tutor that week.

Step 2 Write an equation from the bar diagram. Let x represent the amount she should charge each hour.

$7x = 63$

Step 3 Use the *work backward* strategy to solve the equation. Since $7x = 63$, $x = 63 \div 7$. So, $x =$ 9.

Check $7 \times$ 9 $= 63$ ✓

So, Hessa should charge AED 9 per hour.

Focus narrowing the scope

Objective Use bar diagrams to write and solve problems involving ratios.

Coherence connecting within and across grades

Now Students used bar diagrams to write and solve one-step addition and subtraction equations.

Next Students will use Properties of Equality to solve one-step equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 446.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Hands-On Activity

AL LA Remind students that the bar diagram represents the total amount of money she needs to earn.

Ask:

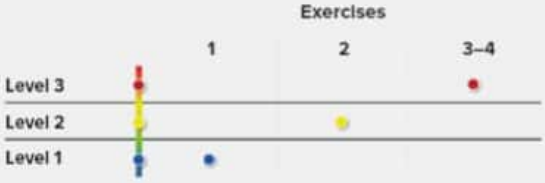
- *Into how many sections should the bar diagram be divided? Why?* 7 sections; This represents the 7 hours that she can tutor.
- *How much money does she need?* AED 63
- *What equation is represented by the bar diagram?* $7x = 63$

BL Omit the activity and proceed directly to the Investigate section. MP 1, 4

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Pairs Discussion** Have students work in pairs to complete Exercise 1. Have them trade their solution with another pair of students and discuss any differences. MP 1, 2, 5

BL LA **Trade-a-Problem** Have students create their own problem, similar to Exercise 1. Students trade their problems, solve, and compare solutions. MP 1, 2, 4

Analyze and Reflect

AL LA **Think-Pair-Share** Have students work in pairs. Give students one minute to think through their responses to Exercise 2. Have them share their responses with their partner. Then call on one student to share their responses. MP 3, 2, 5

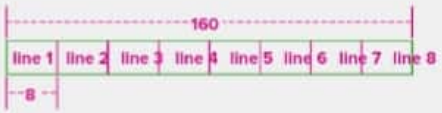
Create

inquiry Students should be able to answer "HOW do you know which operation to use when solving an equation?" Check for student understanding and provide guidance, if needed.

Investigate

Work with a partner to solve.

1. The screen on Hamdan's cell phone allows for 8 lines of text per message. The maximum number of characters for each message is 160. How many characters can each line hold? Complete the bar diagram below and write an equation. Then solve the equation.



$8x = 160$; 20 characters

Analyze and Reflect

Work with a partner to answer the following question.

2. MP **Make a Conjecture** Hamdan's refer to Exercise 1. Suppose Hamdan's cell phone allows 4 lines of text and a maximum of 80 characters for each text message. How would the bar diagram and equation change?

Sample answer: The bar would be divided into 4 sections instead of 8. The equation would be $4x = 80$.

Create

3. MP **Reason Abstractly** Halimah spent AED 70 for 4 hours of cooking classes. How much did she spend per hour of cooking class? Draw a bar diagram below and write an equation. Then solve the equation.

$4x = 70$; AED 17.50

4. inquiry HOW do you know which operation to use when solving an equation?

Sample answer: Use the inverse operation. Division is the inverse of multiplication. So, use division to solve a multiplication equation.

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Solve a multiplication equation.**

AL
- *What is the variable?* x
- *What operation is used between x and 4?* multiplication
- *How do you undo multiplication?* division

OL
- *What is the coefficient in the expression 4x?* 4
- *What is the first step to solving the equation?* Divide each side by 4

BL
- *What property allows you to divide each side of the equation by 4?* Division Property of Equality

Need Another Example?
Solve $39 = 3y$. Check your solution. 13

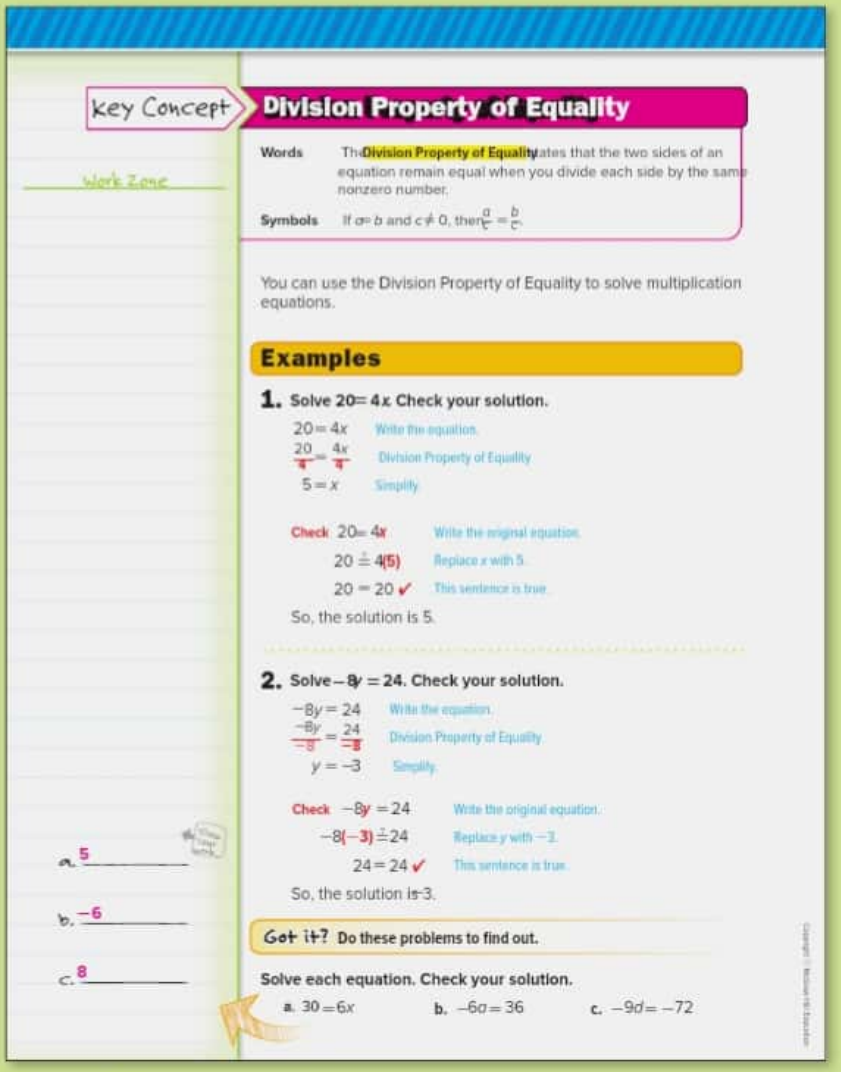
2. **Solve a multiplication equation.**

AL
- *What is the variable?* y
- *What is the coefficient in the expression −8y?* −8
- *What operation is used between y and −8?* multiplication
- *How do you undo multiplication?* division

OL
- *What is the coefficient in the expression −8y?* −8
- *What is the first step to solving the equation?* Divide each side by −8
- *What property allows you to divide each side of the equation by −8?* Division Property of Equality

BL
- *How can you check your solution?* Replace y with −3 in the original equation, simplify, and determine if the final statement is true.

Need Another Example?
Solve $-4z = 60$. Check your solution. −15



Key Concept **Division Property of Equality**

Work Zone

Words The Division Property of Equality states that the two sides of an equation remain equal when you divide each side by the same nonzero number.

Symbols If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

You can use the Division Property of Equality to solve multiplication equations.

Examples

1. **Solve $20 = 4x$. Check your solution.**

$20 = 4x$ Write the equation.
$\frac{20}{4} = \frac{4x}{4}$ Division Property of Equality
$5 = x$ Simplify

Check $20 = 4x$ Write the original equation.
$20 \stackrel{?}{=} 4(5)$ Replace x with 5.
$20 = 20$ ✓ This sentence is true.

So, the solution is 5.

2. **Solve $-8y = 24$. Check your solution.**

$-8y = 24$ Write the equation.
$\frac{-8y}{-8} = \frac{24}{-8}$ Division Property of Equality
$y = -3$ Simplify.

Check $-8y = 24$ Write the original equation.
$-8(-3) \stackrel{?}{=} 24$ Replace y with −3.
$24 = 24$ ✓ This sentence is true.

So, the solution is −3.

Got it? Do these problems to find out.

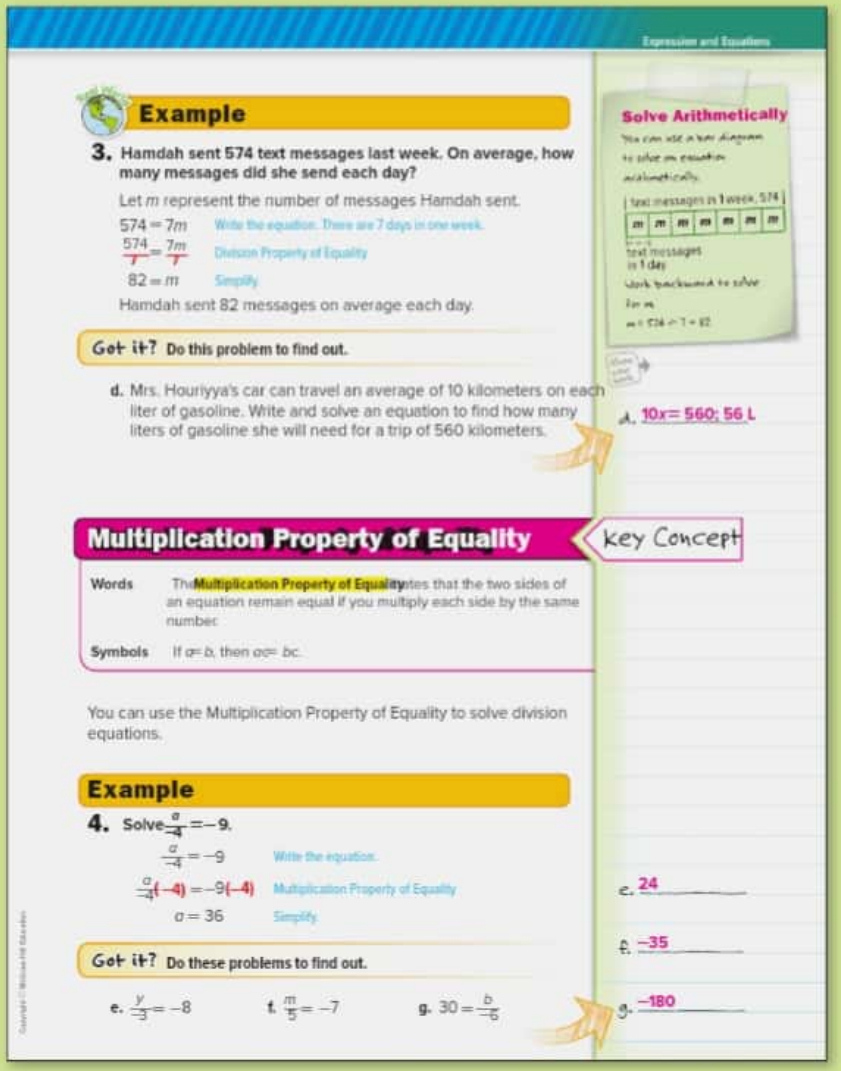
Solve each equation. Check your solution.

a. $30 = 6x$ b. $-6a = 36$ c. $-9d = -72$

a. 5

b. −6

c. 8



Expression and Equations

Example

3. **Hamdah sent 574 text messages last week. On average, how many messages did she send each day?**

Let m represent the number of messages Hamdah sent.

$574 = 7m$ Write the equation. There are 7 days in one week.

$\frac{574}{7} = \frac{7m}{7}$ Division Property of Equality

$82 = m$ Simplify

Hamdah sent 82 messages on average each day.

Got it? Do this problem to find out.

d. Mrs. Houriyya's car can travel an average of 10 kilometers on each liter of gasoline. Write and solve an equation to find how many liters of gasoline she will need for a trip of 560 kilometers.

d. 10x = 560; 56 L

Solve Arithmetically

You can use a bar diagram to solve an equation arithmetically.

text messages in 1 week, 574						
m	m	m	m	m	m	m

text messages in 1 day

Work backward to solve for m.

$m = 574 \div 7 = 82$

Multiplication Property of Equality

Key Concept

Words The **Multiplication Property of Equality** states that the two sides of an equation remain equal if you multiply each side by the same number.

Symbols If $a = b$, then $ac = bc$.

You can use the Multiplication Property of Equality to solve division equations.

Example

4. Solve $\frac{a}{-4} = -9$.

$\frac{a}{-4} = -9$ Write the equation.

$\frac{a}{-4}(-4) = -9(-4)$ Multiplication Property of Equality

$a = 36$ Simplify

Got it? Do these problems to find out.

e. $\frac{y}{-3} = -8$ **f.** $\frac{m}{5} = -7$ **g.** $30 = \frac{b}{-6}$

e. 24

f. −35

g. −180

Examples

3. **Write and solve a multiplication equation.**

AL
- *What do you need to find?* the average number of text message she sent per day
- *What do you know?* the number of messages she sent last week

OL
- *How many messages did she send altogether?* 574
- *How many days are in a week?* 7
- *What multiplication equation represents this situation?* $7m = 574$
- *How would you solve this equation?* divide each side of the equation by 7

BL
- *How many text messages would you expect Lelah to send in 30 days?* 2,460 text messages

Need Another Example?
Serena mailed 138 letters during the last 6 days. Write and solve an equation to find, on average, how many letters she mailed each day. $6x = 138$; 23 letters

4. **Solve a division equation.**

AL
- *How do you know this is a division equation?* The fraction $\frac{a}{-4}$ means a divided by -4.
- *How do you undo division?* multiplication

OL
- *What is the first step to solving the equation?* Multiply each side by -4.
- *What property allows you to multiply each side of the equation by -4?* Multiplication Property of Equality

BL
- *Rewrite this equation as a multiplication equation.* $-\frac{1}{4}a = -9$.
- *How would you solve this equation?* Multiply each side of the equation by -4.

Need Another Example?
Solve $\frac{m}{2} = -10$. Check your solution. -20

Example

5. Solve a division equation.

AL • *What is the variable?* d

• *What number is on the same side of the equals sign as the variable?* 3

• *What operation is used between d and 3?* division

• *How do you undo division?* multiplication

OL • *How would you solve this equation?* Multiply each side of the equation by 3 and simplify.

BL • *Use what you learned in this lesson to explain how to solve the equation d = rt for the variable r. Then solve for r.* Use the Division Property of Equality to divide each side by t; $r = \frac{d}{t}$

Need Another Example?

The distance *d* that Salem travels on his bike at a rate of 20 kilometers per hour for 4 hours is given by the equation $\frac{d}{4} = 20$. How far did he ride? 80 km

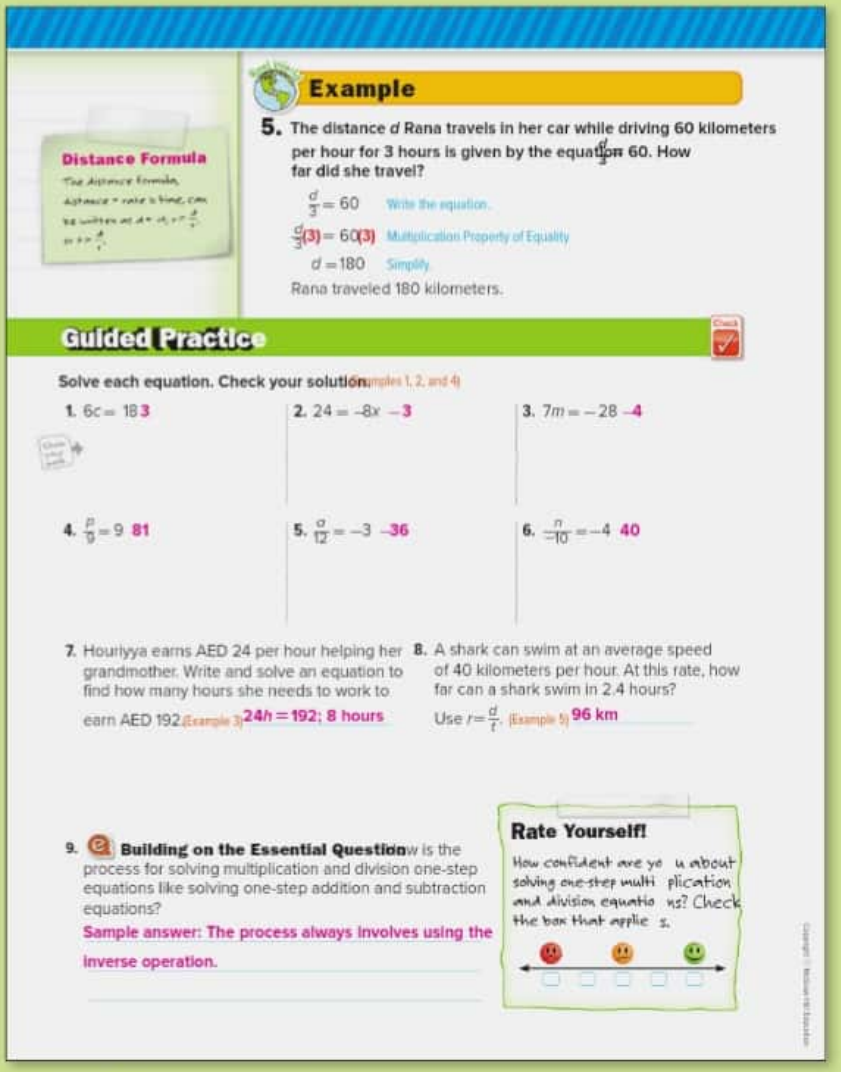
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Team-Pair-Solo** Have students work in teams of 4 to solve Exercise 1, then break up in pairs to solve Exercise 2, and finally work alone to solve Exercise 3. Have them repeat the process to solve Exercises 4–6. Have them trade their pair and solo solutions with the rest of the group and discuss any differences. MP 1, 2

BL LA **Trade-a-Problem** Have students create their own problem, similar to Exercise 7, using either multiplication or division. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the error. MP 1, 2, 4



Distance Formula

The distance formula, distance = rate × time, can be written as $d = rt$, $r = \frac{d}{t}$, or $t = \frac{d}{r}$.

Example

5. The distance *d* Rana travels in her car while driving 60 kilometers per hour for 3 hours is given by the equation $\frac{d}{3} = 60$. How far did she travel?

$\frac{d}{3} = 60$ Write the equation.

$\frac{d}{3}(3) = 60(3)$ Multiplication Property of Equality

$d = 180$ Simplify

Rana traveled 180 kilometers.

Guided Practice

Solve each equation. Check your solution. (Examples 1, 2, and 4)

1. $6c = 18$ 3
2. $24 = -8x$ −3
3. $7m = -28$ −4
4. $\frac{p}{9} = 9$ 81
5. $\frac{a}{12} = -3$ −36
6. $\frac{n}{-10} = -4$ 40
7. Hourlyya earns AED 24 per hour helping her grandmother. Write and solve an equation to find how many hours she needs to work to earn AED 192. (Example 3) $24h = 192$; 8 hours
8. A shark can swim at an average speed of 40 kilometers per hour. At this rate, how far can a shark swim in 2.4 hours? Use $r = \frac{d}{t}$. (Example 5) 96 km
9. **Building on the Essential Question** How is the process for solving multiplication and division one-step equations like solving one-step addition and subtraction equations?

Sample answer: The process always involves using the inverse operation.

Rate Yourself!

How confident are you about solving one-step multiplication and division equations? Check the box that applies.

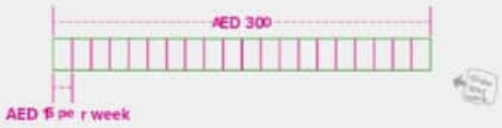
Name _____ My Homework

Independent Practice

Solve each equation. Check your solution. (Examples 1, 2, and 4)

1. $7a = 49$ **7**
2. $-6 = 2x$ **−3**
3. $-32 = -4b$ **8**
4. $\frac{u}{6} = 9$ **54**
5. $-8 = \frac{c}{-10}$ **80**
6. $54 = -9d$ **−6**
7. $-12y = 60$ **−5**
8. $\frac{r}{20} = -2$ **−40**
9. $\frac{g}{10} = -9$ **−90**

10. Najla wants to buy a digital camera that costs AED 300. Suppose she saves AED 15 each week. In how many weeks will she have enough money for the camera? Use a bar diagram to solve arithmetically. Then use an equation to solve algebraically. (Example 3) **15w = 300; 20 weeks**



11. A race car can travel at a rate of 205 kilometers per hour. At this rate, how far would it travel in 3 hours? Use $r = \frac{d}{t}$. Write an equation and then solve. (Example 3) **$205 = \frac{d}{3}$; 615 kilometers**

12. A certain hurricane travels at 20.88 kilometers per hour. The distance from Cuba to Key West is 145 kilometers. Write and solve a multiplication equation to find about how long it would take the hurricane to travel from Cuba to Key West. **$20.88h = 145$; $7h$**

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–11, 13–15, 27, 28
OL	On Level	1–11 odd, 12–15, 27, 28
BL	Beyond Level	12–17, 27, 28

Watch Out!

Common Error Students may have trouble solving equations when the variable is on the right side of the equal sign. Remind them that if $30 = \frac{x}{-5}$, then $= \frac{x}{-5} = 30$.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	16, 17
2 Reason abstractly and quantitatively.	14
3 Construct viable arguments and critique the reasoning of others.	13, 26
7 Look for and make use of structure.	15

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Write the distance/time/rate formula $=dt$ on the board. Have students write and solve their own rate problems.

See students' work.

13. **Multiple Representations** Hiyam saves AED 5.50 for each hour she works. She needs to save an additional AED 44 to buy an E-reader. How many more hours does Hiyam need to work to pay for the E-reader?

a. **Diagram** Draw a bar diagram that represents the situation.

b. **Algebra** Write an equation that represents the situation.

$5.5x = 44$

c. **Words** Describe the process you would use to solve your equation. Then solve.

Sample answer: Divide each side by 5.5. Then simplify $8x$

H.O.T. Problems Higher Order Thinking

14. **Reason Abstractly** Describe a real-world situation in which you would use a division equation to solve a problem. Write your equation and then solve your problem.

Situation: See students' work.

Equation: _____ Solution: _____

15. **Identify Structure** *True* or *false*. To solve the equation $5x = 20$ you can use the Multiplication Property of Equality. Explain your reasoning.

True; Sample answer: Multiply each side of the equation by $\frac{1}{5}$ instead of dividing each side by 5.

16. **Persevere with Problems** Solve $3|x| = 12$. Explain your reasoning.

The absolute value of a positive or negative number is always positive.

So, $x = 4$ or $x = -4$.

17. **Persevere with Problems** Explain how you would solve $\frac{-30}{x} = 6$. Then solve the equation.

Sample answer: Multiply both sides by x, then divide both sides of the equation by 6. $x = -5$.

Name _____ My Homework _____

Extra Practice

Solve each equation. Check your solution.

18. $-4j = 36$

$\frac{-4j}{-4} = \frac{36}{-4}$

$j = -9$

19. $-4s = -16$ **4**

20. $63 = -9d$ **−7**

21. $\frac{m}{10} = 7$

$\frac{m}{10} = 7$

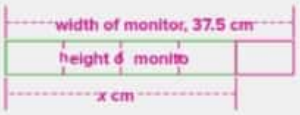
$\frac{m}{10}(10) = 7(10)$

$m = 70$

22. $\frac{h}{-3} = 12$ **−36**

23. $\frac{g}{12} = -10$ **−120**

24. The width of a computer monitor is 1.25 times its height. Find the height of the computer monitor at the right. Use a bar diagram to solve arithmetically. Then use an equation to solve algebraically. **1.25x = 30; 24 cm.**



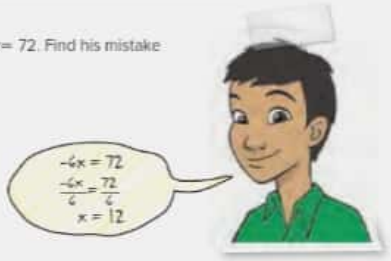
25. A dragonfly, the fastest insect, can fly a distance of 15 feet at a speed of 7.5 feet per second. Find the time in seconds. Write the equation in the form $d = rt$, then solve.

15 = 7.5t; 2 s

26. HOT **Find the Error** Amer is solving $-6x = 72$. Find his mistake and correct it.

Sample answer: Amer divided by 6 and should have divided by −6.;

x = −12



Watch Out!

Find the Error For Exercise 26, remind students that when solving this equation for x, they must divide each side by the coefficient. The coefficient ~~is~~6, so divide each side of the equation by −6.

Power Up! Test Practice

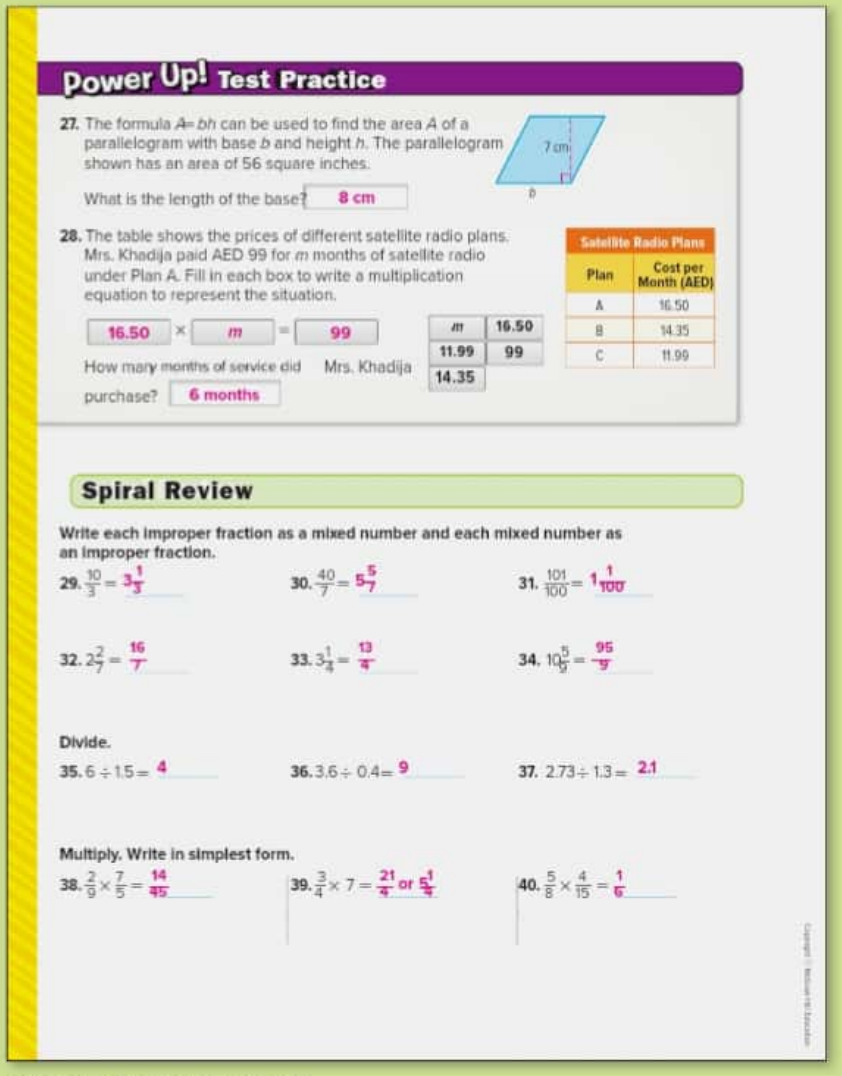
Exercises 27 and 28 prepare students for more rigorous thinking needed for the assessment.

27.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

28.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly model and solve the equation.
1 point	Students correctly model OR solve the equation.



Power Up! Test Practice

27. The formula $A = bh$ can be used to find the area A of a parallelogram with base b and height h. The parallelogram shown has an area of 56 square inches.

What is the length of the base? 8 cm

28. The table shows the prices of different satellite radio plans. Mrs. Khadija paid AED 99 for m months of satellite radio under Plan A. Fill in each box to write a multiplication equation to represent the situation.

Satellite Radio Plans	
Plan	Cost per Month (AED)
A	16.50
B	14.35
C	11.99

16.50 × m = 99

m	16.50
11.99	99
14.35	

How many months of service did Mrs. Khadija purchase? 6 months

Spiral Review

Write each improper fraction as a mixed number and each mixed number as an improper fraction.

29. $\frac{10}{3} = 3\frac{1}{3}$
30. $\frac{40}{7} = 5\frac{5}{7}$
31. $\frac{101}{100} = 1\frac{1}{100}$
32. $2\frac{2}{7} = \frac{16}{7}$
33. $3\frac{1}{4} = \frac{13}{4}$
34. $10\frac{5}{9} = \frac{95}{9}$

Divide.

35. $6 \div 1.5 = 4$
36. $3.6 \div 0.4 = 9$
37. $2.73 \div 1.3 = 2.1$

Multiply. Write in simplest form.

38. $\frac{2}{9} \times \frac{7}{5} = \frac{14}{45}$
39. $\frac{3}{4} \times 7 = \frac{21}{4}$ or $5\frac{1}{4}$
40. $\frac{5}{8} \times \frac{4}{15} = \frac{1}{6}$

Inquiry Lab

Solve Equations with Rational Coefficients

inquiry **HOW can you use bar diagrams to solve equations with rational coefficients?**

MP Mathematical Practices 1, 3

Two thirds of Ahmed's homeroom class plan to participate in the school talent show. If 16 students from the class plan to participate, how many students are in the homeroom class?

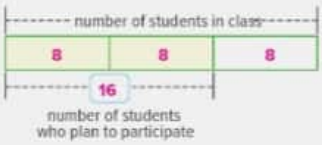
What do you know? the fraction and number of students participating in the talent show

What do you need to find? the number of students in Ahmed's homeroom class

Hands-On Activity

You can represent the situation above with an equation.

Step 1 Draw a bar diagram that represents the total number of students in the class and how many plan to participate.



Step 2 Write an equation from the bar diagram. Let c represent the total number of students in the class. $\frac{2}{3}c = 16$

Step 3 Find the number of students represented by the sections of the bar. Write that number in each section of the bar in Step 1.

Since each section represents 8 students, there are 3 or 24 students in the class.

Check $\frac{2}{3} \times 24 = \frac{2}{3} \times \frac{24}{1}$

$= \frac{48}{3}$ or 16 ✓

Focus narrowing the scope

Objective Use bar diagrams to model and solve equations with rational coefficients.

Coherence connecting within and across grades

Now Students will use models to solve equations with rational coefficients.

Next Students will solve equations with rational coefficients.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 456.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Hands-On Activity

AL LA Paired Heads Together Have students work in pairs to complete the activity. Assign each student a number. Pairs should ensure that both students understand how to draw a bar diagram, write an equation, and use the bar diagram to solve the equation. Students are responsible to ask for help, if needed. Call on one numbered student to share their responses with the class. MP 1, 2, 4, 5

BL LA Pairs Discussion Have students work with a partner to determine a different method that can be used to solve the problem. Have them present their method to the class. MP 1, 2, 7

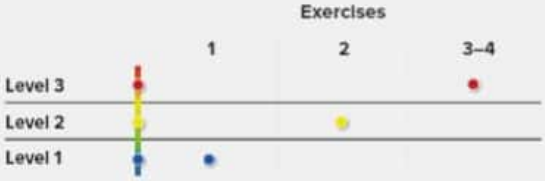
ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA Think-Pair-Write Give students one minute to think through how they would draw a bar diagram and write an equation to complete Exercise 1. Then have them work in pairs to complete the problem. Have students diagram the parts of the equation (coefficient, variable, constant) and write out in words what each part of the equation represents. For example, *three-fifths of Reham's monthly allowance is the amount she wants to spend, $24* MP 1, 2, 5

Analyze and Reflect

AL LA Think-Pair-Draw Give students time to think of a real-world problem that could be represented by an equation with a rational coefficient. Have them work in pairs to write equations that represent the problem. Have them draw bar diagrams that can be used to solve the equations. MP 1, 2, 4, 5

Inquiry Students should be able to answer "HOW can you use bar diagrams to solve equations with rational coefficients?" Check for student understanding. Provide guidance, if needed.

Investigate

Work with a partner to solve the following problem.

1. Reham is spending $\frac{3}{5}$ of her monthly allowance on a costume for the talent show. She plans to spend AED 24. Draw a bar diagram to represent the situation. Then write and solve an equation to find the amount of Reham's monthly allowance.

Equation: $\frac{3}{5}a = 24$ Solution: AED 40

Analyze and Reflect

Work with a partner to answer the following question.

2. MP **Make a Conjecture** Suppose Reham planned on spending $\frac{3}{4}$ of her monthly allowance on a costume. How would the diagram and equation be different?

Sample answer: The bar would be divided into four equal sections. The equation would be $\frac{3}{4}a = 24$.

Create

3. MP **Model with Mathematics** Write a real-world problem that could be represented by the equation $\frac{2}{3}x = 12$. Then solve the equation.

Sample answer: Amna sold $\frac{2}{3}$ of his video collection at a community yard sale. If he sold 12 videos, how many did he originally have to sell? 18 videos

4. Inquiry HOW can you use bar diagrams to solve equations with rational coefficients?

Sample answer: You find the value of one section by dividing, then you multiply by the number of sections.

Expressions and Equations

Lesson 3

Solve Equations with Rational Coefficients

Real-World Link

Social Networks Three-fourths of the students in Sally's class belong to a social network. There are 15 students in her class that belong to a social network.

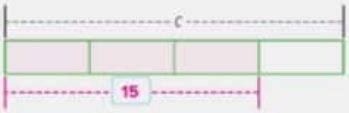
Essential Question

WHAT does it mean to say two quantities are equal?

Mathematical Practices

1, 2, 3, 4

1. Create a bar diagram and shade $\frac{3}{4}$ or 0.75, of it.



Label 15 along the bottom to show the amount of the bar that represents 15 students.

2. Based on the diagram, circle the equation that can be used to find c, the number of students in Sally's class.

$15c = \frac{3}{4}$ $0.75c = 15$ $4c = 15$

3. Based on what you know about solving equations, explain how you could solve the equation you circled in Exercise 2.

Divide each side of the equation by 0.75.

4. How many students are in Sally's class?

20 students

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Solve one-step equations with rational coefficients.

Coherence connecting within and across grades

Previous Students wrote and solved one-step equations with integer coefficients.

Now Students solve equations with rational coefficients.

Next Students will model and solve two-step equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 461.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Talking Chips** Have students work in small groups to discuss and complete Exercises 1–4. Provide each student with 5 chips. Students must place a chip in the center of the table as they contribute to the discussion. After they have used all of their chips, they may not contribute. All students must use all of their chips. MP 1, 2, 4, 5

Alternate Strategy

AL Have students explain what is different about the equations $3c = 15$ and $\frac{3}{4}c = 15$. Ask them what operation is being performed on the variable in each equation and how they should solve each equation. MP 1, 3

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Solve equations with decimal coefficients.**

AL • *What is a coefficient?* the number being multiplied by the variable

• *What is the coefficient in this example?* 0.25

OL • *What operation is indicated by the coefficient?* multiplication

• *What is the inverse operation of multiplying by 0.25?* dividing by 0.25

BL • *Describe a way to solve this equation mentally.* Sample answer: The coefficient is 0.25, which is one-fourth. One-fourth of a number is equal to 16, so that number must be 64.

Need Another Example?
Solve $10 = 0.5x$. Check your solution. 20

2. **Solve equations with decimal coefficients.**

AL • *What does the variable represent?* the number of cones the coach can buy

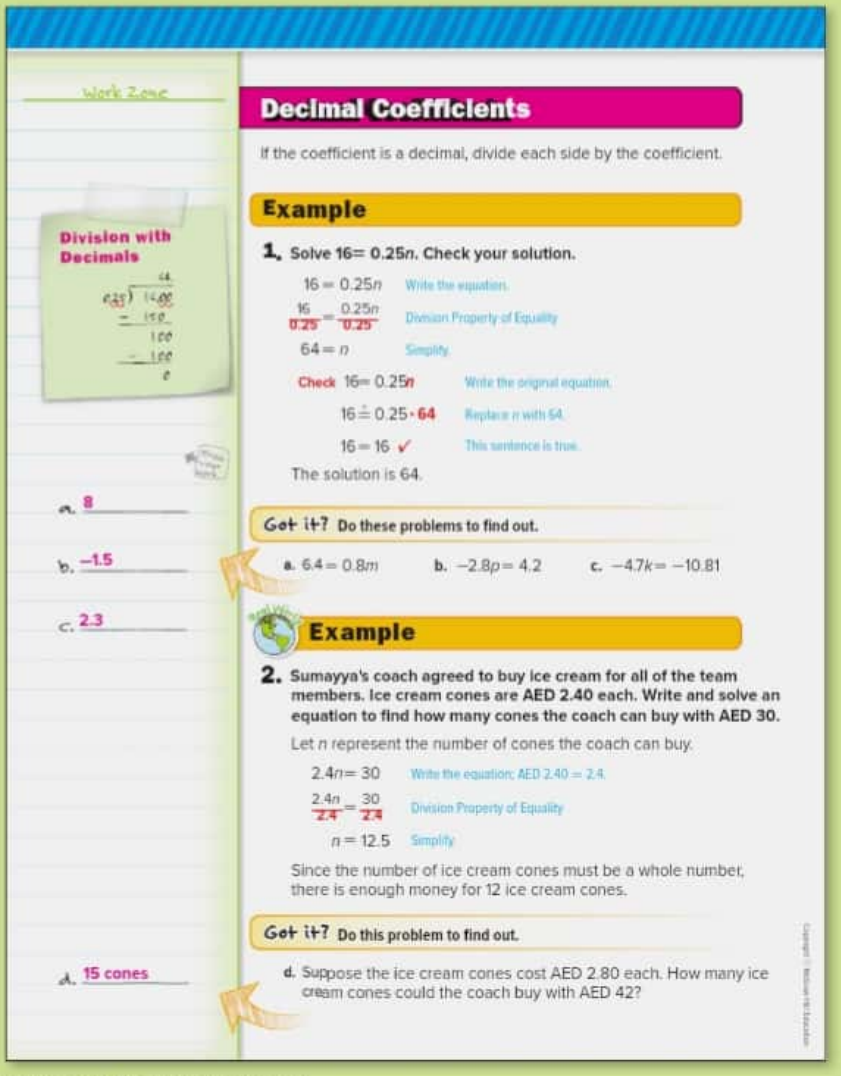
• *What is the cost of each cone?* AED 2.40

OL • *How do you know that the coefficient will be 2.4?* The cost, AED 2.40, will be multiplied by the number of cones, n.

• *How can you solve this equation?* Divide each side by 2.4.

BL • *How can you use estimation to determine if your answer is reasonable?* Sample answer: Round 2.4 to 2. Since $30 \div 2 = 15$ and 12.5 is close to 15, the answer is reasonable.

Need Another Example?
Nasser wants to cut pieces of siding that are each 3.5 meters long to fit between a window and the end of the house. If the original piece of siding is 21 meters long, write and solve an equation to determine the total number of 3.5-meter-long pieces he can cut. $3.5x = 21$; 6 pieces



Work Zone

Division with Decimals

a. 8

b. −1.5

c. 2.3

d. 15 cones

Decimal Coefficients

If the coefficient is a decimal, divide each side by the coefficient.

Example

1. Solve $16 = 0.25n$. Check your solution.

$16 = 0.25n$ Write the equation.

$\frac{16}{0.25} = \frac{0.25n}{0.25}$ Division Property of Equality

$64 = n$ Simplify.

Check $16 = 0.25n$ Write the original equation.

$16 \stackrel{?}{=} 0.25 \cdot 64$ Replace n with 64.

$16 = 16$ ✓ This sentence is true.

The solution is 64.

Got it? Do these problems to find out.

a. $6.4 = 0.8m$ b. $-2.8p = 4.2$ c. $-4.7k = -10.81$

Example

2. **Sumayya's coach agreed to buy ice cream for all of the team members. Ice cream cones are AED 2.40 each. Write and solve an equation to find how many cones the coach can buy with AED 30.**

Let n represent the number of cones the coach can buy.

$2.4n = 30$ Write the equation; AED 2.40 = 2.4.

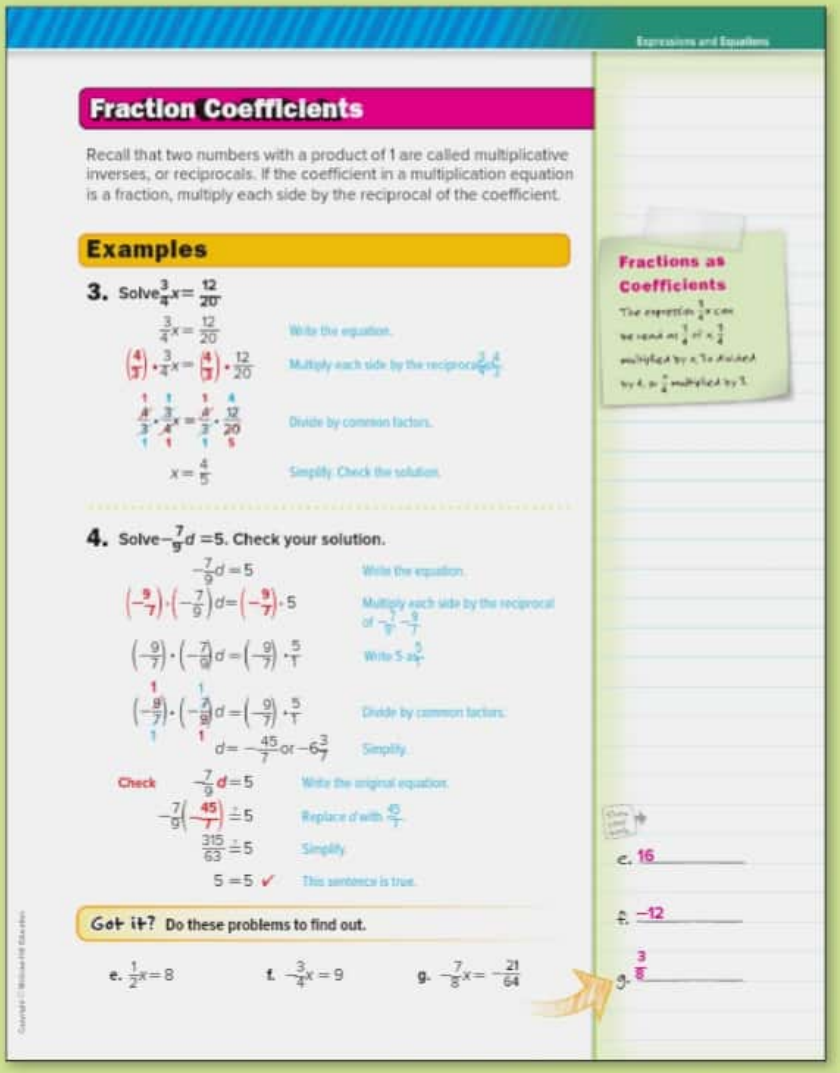
$\frac{2.4n}{2.4} = \frac{30}{2.4}$ Division Property of Equality

$n = 12.5$ Simplify.

Since the number of ice cream cones must be a whole number, there is enough money for 12 ice cream cones.

Got it? Do this problem to find out.

d. Suppose the ice cream cones cost AED 2.80 each. How many ice cream cones could the coach buy with AED 42?



Expressions and Equations

Fraction Coefficients

Recall that two numbers with a product of 1 are called multiplicative inverses, or reciprocals. If the coefficient in a multiplication equation is a fraction, multiply each side by the reciprocal of the coefficient.

Examples

3. Solve $\frac{3}{4}x = \frac{12}{20}$

$\frac{3}{4}x = \frac{12}{20}$ Write the equation.

$\left(\frac{4}{3}\right) \cdot \frac{3}{4}x = \left(\frac{4}{3}\right) \cdot \frac{12}{20}$ Multiply each side by the reciprocal $\frac{4}{3}$.

$\frac{1}{\cancel{4}_{1}} \cdot \frac{\cancel{3}^{1}}{\cancel{3}_{1}} x = \frac{\cancel{4}^{1}}{\cancel{3}_{1}} \cdot \frac{\cancel{12}^{3}}{\cancel{20}_{5}}$ Divide by common factors.

$x = \frac{4}{5}$ Simplify. Check the solution.

4. Solve $-\frac{7}{9}d = 5$. Check your solution.

$-\frac{7}{9}d = 5$ Write the equation.

$\left(-\frac{9}{7}\right) \cdot \left(-\frac{7}{9}\right)d = \left(-\frac{9}{7}\right) \cdot 5$ Multiply each side by the reciprocal of $-\frac{7}{9}$, $-\frac{9}{7}$.

$\left(-\frac{9}{7}\right) \cdot \left(-\frac{7}{9}\right)d = \left(-\frac{9}{7}\right) \cdot \frac{5}{1}$ Write 5 as $\frac{5}{1}$.

$\left(-\frac{\cancel{9}^{1}}{\cancel{7}_{1}}\right) \cdot \left(-\frac{\cancel{7}^{1}}{\cancel{9}_{1}}\right)d = \left(-\frac{9}{7}\right) \cdot \frac{5}{1}$ Divide by common factors.

$d = -\frac{45}{7}$ or $-6\frac{3}{7}$ Simplify.

Check $-\frac{7}{9}d = 5$ Write the original equation.

$-\frac{7}{9}\left(-\frac{45}{7}\right) \stackrel{?}{=} 5$ Replace d with $-\frac{45}{7}$.

$\frac{315}{63} \stackrel{?}{=} 5$ Simplify.

$5 = 5$ ✓ This sentence is true.

Got it? Do these problems to find out.

e. $\frac{1}{2}x = 8$ f. $-\frac{3}{4}x = 9$ g. $-\frac{7}{8}x = -\frac{21}{64}$

Fractions as Coefficients

The expression $\frac{1}{4}x$ can be read as $\frac{1}{4}$ of x, $\frac{1}{4}$ multiplied by x, x divided by 4, or $\frac{x}{4}$ multiplied by 1.

e. 16

f. −12

g. $\frac{3}{8}$

Examples

3. Solve equations with fractional coefficients.

AL
- *What is the coefficient?* $\frac{3}{4}$
- *What operation is indicated by the coefficient?* multiplication

OL
- *Why can you multiply both sides by the reciprocal of the coefficient?* When you multiply any value by its reciprocal, the product is 1, which isolates the variable.
- *What is the reciprocal of* $\frac{3}{4}$*?* $\frac{4}{3}$

BL
- *What is another way to solve the equation?* Sample answer: Divide both sides of the equation by $\frac{3}{4}$, which is equivalent to multiplying both sides of the equation by $\frac{4}{3}$.

Need Another Example?

Solve $\frac{2}{3}x = \frac{6}{15}$. Check your solution. $\frac{3}{5}$

4. Solve equations with fractional coefficients.

AL
- *What is the coefficient?* $-\frac{7}{9}$
- *What is the inverse operation of multiplying by* $\frac{7}{9}$*?* dividing by $-\frac{7}{9}$

OL
- *What is the reciprocal of* $\frac{7}{9}$*?* $-\frac{9}{7}$
- *Why is dividing by common factors helpful in solving this equation?* Sample answer: Dividing by common factors makes the multiplication easier.

BL
- *What is another way to solve the equation?* Sample answer: Divide both sides of the equation by $-\frac{7}{9}$, which is equivalent to multiplying both sides of the equation by $-\frac{9}{7}$.

Need Another Example?

Solve $\frac{2}{3}x = 12$. Check your solution. 18

Example

5. Write and solve an equation to solve a real-world problem.

AL • *What are you trying to find?* How many hats Hessa can make with 6 meters of fabric

• *How much fabric is needed for one hat?* $\frac{2}{3}$ m

OL • *What equation can be used to represent the problem?* $\frac{2}{3}n = 6$

• *What is the reciprocal of the coefficient?* $\frac{3}{2}$

BL • *How do you know if your answer is reasonable?* Sample answer: Hessa needs between $\frac{1}{2}$ meter and 1 meter to make one hat. So, to make 9 hats, she will need between $4\frac{1}{2}$ meters and 9 meters. 6 meters is between these two measurements, so the answer is reasonable.

Need Another Example?

Wafa answered $\frac{4}{5}$ of the questions on her science quiz correctly. If she answered 8 questions correctly, write and solve an equation to find the number of questions that were on the quiz. $\frac{4}{5}x = 8$; 10 questions

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

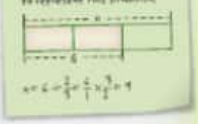
If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Three Stay, One Stray** Have students work in small groups to complete Exercises 1–3. Have one student stray to a new group to share responses and resolve any differences. Have the new group complete Exercises 4–6. Then have the students share responses in their original group. MP 1, 2

BL LA **Trade-a-Problem** Have students work in pairs to write a real-world problem that can be represented by a one-step equation involving a rational coefficient. Have them trade problems with a partner. Each partner solves the equation and checks each other's work. MP 1, 4

Bar Diagrams

A bar diagram can be used to represent this situation.



Example

5. Hessa needs $\frac{2}{3}$ meter of fabric to make each hat for the school play. Write and solve an equation to find how many hats she can make with 6 meters of fabric.

Write and solve a multiplication equation. Let n represent the number of hats.

$\frac{2}{3}n = 6$ Write the equation.

$\left(\frac{3}{2}\right) \cdot \frac{2}{3}n = \left(\frac{3}{2}\right) \cdot 6$ Multiply each side by $\frac{3}{2}$.

$n = 9$ Simplify.

So, Hessa can make 9 hats.

Guided Practice

Solve each equation. Check your solution. (Examples 1, 3, and 4)

1. $1.6k = 3.2$ 2
2. $-2.5b = 20.5$ −8.2
3. $-\frac{1}{2} = -\frac{5}{18}h$ $1\frac{4}{5}$

Write and solve an equation. (Examples 2 and 5)

4. The average growth of human hair is 1.25 centimeters per month. Find how long it takes a human to grow 7.5 centimeters of hair.

Equation: $1.25t = 7.5$ Solution: 6 months

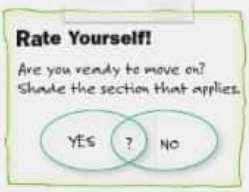
5. Three fourths of the fruit in a refrigerator are apples. There are 24 apples in the refrigerator. How many pieces of fruit are in the refrigerator?

Equation: $\frac{3}{4}n = 24$ Solution: 32 pieces of fruit

6. **Building on the Essential Question** What is the process for solving a multiplication equation with a rational coefficient? Sample answer: Divide by decimal coefficients. If the coefficient is a fraction, multiply by the reciprocal of the fraction.

Rate Yourself!

Are you ready to move on? Shade the section that applies.



Name _____ My Homework

Independent Practice

Solve each equation. Check your solution. (Examples 1, 3, and 4)

1. $1.2x = 6$ **5**

2. $14.4 = -2.4b$ **−6**

3. $-3.6h = -10.8$ **3**

4. $\frac{2}{5}t = \frac{12}{25}$ $\frac{6}{5}$ or $1\frac{1}{5}$

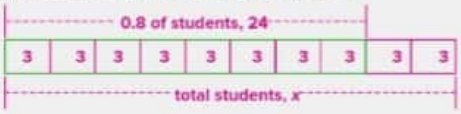
5. $-3\frac{1}{3} = -\frac{1}{2}g$ $\frac{20}{3}$ or $6\frac{2}{3}$

6. $-\frac{7}{9}m = \frac{11}{6}$ $-\frac{33}{14}$ or $-2\frac{5}{14}$

Financial Literacy Ibrahim deposited $\frac{3}{4}$ of his paycheck into the bank. The deposit slip shows how much he deposited. Write and solve an equation to find the amount of his paycheck. (Example 2)

Equation: $\frac{3}{4}p = 46.50$ Solution: AED 62

8. Twenty-four students brought their permission slips to attend the class field trip to the local art museum. If this represented eight tenths of the class, how many students are in the class? Use a bar diagram to solve arithmetically. Then use an equation to solve algebraically. (Example 5)



Equation: $0.8x = 24$; Solution: 30 students

9. **Justify Conclusions** Seventy-five percent, or 15, of the students in Rana's homeroom class are going on a field trip. Two thirds, or 12, of the students in Hessa's homeroom class are going on the field trip. Which class has more students? Justify your answer.

Rana's homeroom class;
Sample answer: Write and solve the equations $0.75s = 15$ and $\frac{2}{3}s = 12$;
$s = 20$ and $s = 18$; Since $20 > 18$, Rana's homeroom class has more students.

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

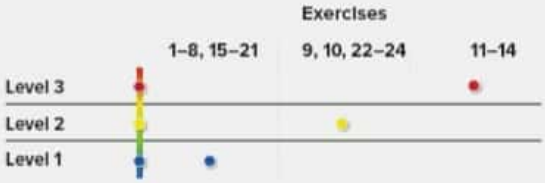
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 11, 12, 14, 23, 24
OL	On Level	1–7 odd, 9–12, 14, 23, 24
BL	Beyond Level	9–14, 23, 24

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	13
2 Reason abstractly and quantitatively.	10
3 Construct viable arguments and critique the reasoning of others.	9, 11, 12
4 Model with mathematics.	14, 22

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a clos ing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students explain in writing how what they learned about solving one-step equations will help them to solve two-step equations. Use the writing prompt below.

See students' work.

- Solving one-step equations will help me to solve two-step equations because...

Watch Out!

Common Error Students may have difficulty finding the reciprocal of whole numbers such as 5. Review that all whole numbers can be written as a fraction with a denominator of 1. For example, $5 = \frac{5}{1}$, so the reciprocal of $\frac{5}{1}$ is $\frac{1}{5}$.

10. **Reason Abstractly** Noura and Rana are making stuffed animals for a toy drive. The table shows the fabric purchased they made. Who purchased the more expensive fabric? Explain your reasoning. Rana; Sample answer: Write and solve the equations $\frac{2}{3}n = 4$ and $0.8r = 6$; $n = 6$ and $r = 7.5$; Since $7.5 > 6$, Rana purchased the more expensive fabric.

Purchaser	Amount Purchased (m)	Amount Paid (AED)
Noura	$\frac{2}{3}$	4
Rana	0.8	6

H.O.T. Problems Higher Order Thinking

11. **Reason Inductively** Complete the statement: If $8\frac{m}{4}$, then $m - 12 =$ ■. Explain. 20; Sample answer: Solve $8\frac{m}{4}$ to find that $m = 32$. So, replace m with 32 to find $32 - 12 = 20$.

12. **Which One Doesn't Belong?** Identify the pair of numbers that does not belong with the other three. Explain. $\frac{3}{5}$; The other pairs of numbers are reciprocals.

$\frac{4}{9}, \frac{6}{7}$ $\quad 4, \frac{1}{4}$ $\quad \frac{3}{5}, 5$ $\quad \frac{2}{7}, \frac{7}{2}$

13. **Persevere with Problems** The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where b_1 and b_2 are both bases and h is the height. Find the value of h in terms of A, b_1, and b_2. Justify your answer. Sample answer: Multiply each side by 2. Then divide each side by $(b_1 + b_2)$. So, $\frac{2A}{b_1 + b_2} = h$.

14. **Model with Mathematics** Write a real-world problem that can be represented by the equation $224 = 3.5r$. Then solve the problem and explain the solution. Sample answer: Alia drove 224 kilometers in 3.5 hours. Find the rate r at which Alia was traveling; 64 km/h; She drove at a rate of 64 kilometers per hour.

Name _____ My Homework _____

Extra Practice

Solve each equation. Check your solution.

15. $0.4d = 2.8$

$\frac{0.4d}{0.4} = \frac{2.8}{0.4}$

$d = 7$

16. $-5w = -24.5$ **4.9**

17. $-22.8 = 6n$ **-3.8**

18. $\frac{7}{8}k = \frac{5}{6}$

$\left(\frac{7}{7}\right) \cdot \frac{7}{8}k = \left(\frac{7}{7}\right) \cdot \frac{5}{6}$

$k = \frac{40}{42}$ or $\frac{20}{21}$

19. $-6\frac{1}{4} = \frac{3}{5}c$ $-\frac{125}{12}$ **or** $-10\frac{5}{12}$

20. $-\frac{4}{7}v = -8\frac{2}{3}$ $\frac{91}{6}$ **or** $15\frac{1}{6}$

21. The Mammoth Cave Discovery Tour includes an elevation change of 42 meters. This is $\frac{7}{15}$ of the elevation change on the Wild Cave Tour. What is the elevation change on the Wild Cave Tour? Use a bar diagram to solve arithmetically. Then use an equation to solve algebraically.

Equation: $42 = \frac{7}{15}x$ Solution: 900 meters

22. **Model with Mathematics** Refer to the graphic novel frame below. Write and solve an equation to find how many movies they have time to show.

Equation: $1.75m = 3.5$ Solution: 2 movies

Power Up! Test Practice

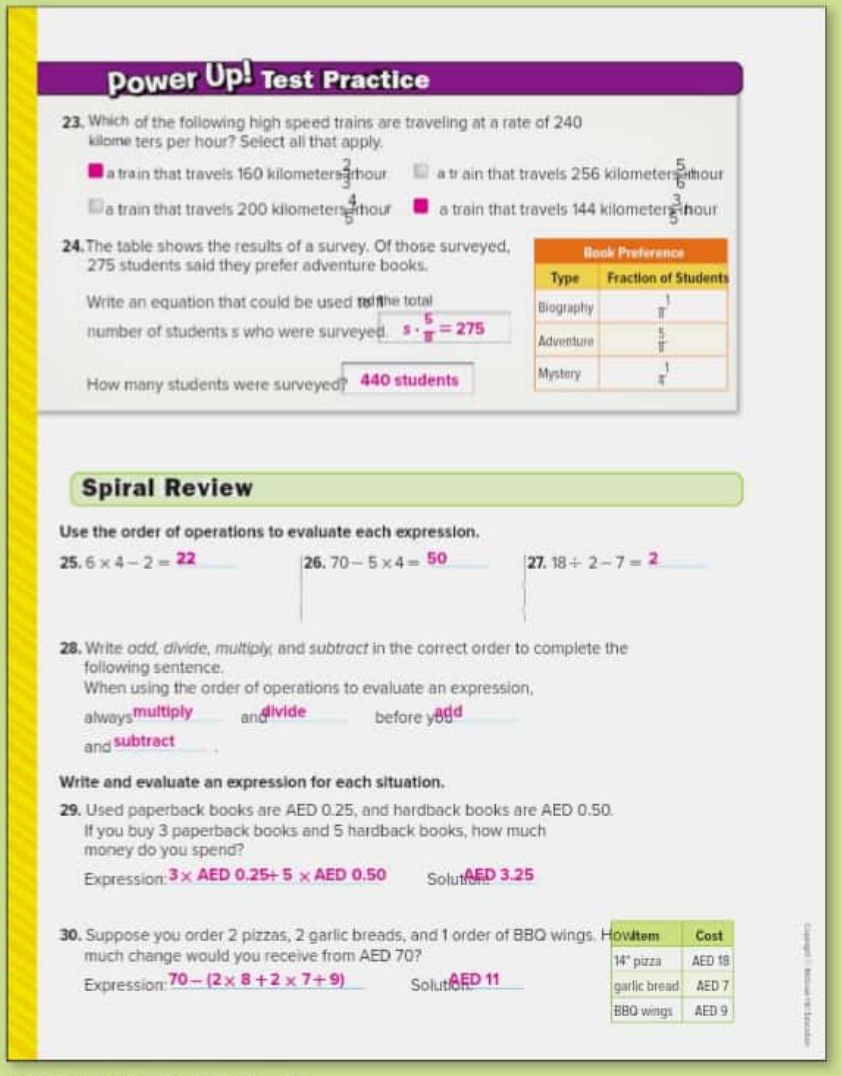
Exercises 23 and 24 prepare students for more rigorous thinking needed for the assessment.

23. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

24. This test item requires students to reason abstractly and quantitatively when problem solving.

Depth of Knowledge	DOK2
Mathematical Practice	MP1, MP7
Scoring Rubric	
2 points	Students correctly write and solve an equation.
1 point	Students correctly write OR solve an equation.



Power Up! Test Practice

23. Which of the following high speed trains are traveling at a rate of 240 kilometers per hour? Select all that apply.

- ■ a train that travels 160 kilometers in $\frac{2}{3}$ hour
- ☐ a train that travels 256 kilometers in $\frac{5}{6}$ hour
- ☐ a train that travels 200 kilometers in $\frac{4}{5}$ hour
- ■ a train that travels 144 kilometers in $\frac{3}{5}$ hour

24. The table shows the results of a survey. Of those surveyed, 275 students said they prefer adventure books.

Book Preference	
Type	**Fraction of Students**
Biography	$\frac{1}{11}$
Adventure	$\frac{5}{11}$
Mystery	$\frac{1}{4}$

Write an equation that could be used to find the total number of students s who were surveyed. $s \cdot \frac{5}{11} = 275$

How many students were surveyed? 440 students

Spiral Review

Use the order of operations to evaluate each expression.

25. $6 \times 4 - 2 =$ 22

26. $70 - 5 \times 4 =$ 50

27. $18 \div 2 - 7 =$ 2

28. Write *add*, *divide*, *multiply*, and *subtract* in the correct order to complete the following sentence.
When using the order of operations to evaluate an expression, always multiply and divide before you add and subtract.

Write and evaluate an expression for each situation.

29. Used paperback books are AED 0.25, and hardback books are AED 0.50. If you buy 3 paperback books and 5 hardback books, how much money do you spend?
Expression: $3 \times$ AED $0.25 + 5 \times$ AED 0.50 Solution: AED 3.25

30. Suppose you order 2 pizzas, 2 garlic breads, and 1 order of BBQ wings. How much change would you receive from AED 70?

Item	Cost
14" pizza	AED 18
garlic bread	AED 7
BBQ wings	AED 9

Expression: $70 - (2 \times 8 + 2 \times 7 + 9)$ Solution: AED 11

Inquiry Lab
Solve Two-Step Equations

inquiry **HOW can a bar diagram or algebra tiles help you solve a real-world problem?**

MP Mathematical Practices 1, 2, 3, 4

Reham plays basketball and tennis. She has two basketballs and three tennis balls that weigh a total of 1,360 grams. Each tennis ball weighs 60 grams. What is the weight of a basketball?

Hands-On Activity 1

You can use a bar diagram to represent the situation.

Step 1 Draw a bar diagram that represents the total weight.

Step 2 Write an equation that is modeled by the bar diagram. Let x represent the weight of a basketball.

Step 3 Use the bar diagram to solve the equation. Subtract the weight of the tennis balls, 180 grams, from the total weight, 1,360 grams.

The two basketballs together weigh 1,360 − 180 = 1,180 grams.

Divide the weight by 2 to find the weight of one basketball.

So, x = 590. The weight of one basketball is 1,180 ÷ 2 = 590 grams.

Check $2 \cdot 590 + 6 = 48$ ✓

The weight of one basketball is 590 grams.

Focus narrowing the scope

Objective Use bar diagrams to model and solve two-step equations.

Coherence connecting within and across grades

Now Students model and solve two-step equations.

Next Students will solve two-step equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 467.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activity 2.

Materials: algebra tiles

Hands-On Activity 1

AL LA **Pairs to Groups** Arrange students to work in pairs to complete Steps 1–3, ensuring that each student understands how the bar diagram models the equation. Have pairs of students join up to form groups of 4. The group should share their responses and resolve any differences. Call on each group to share results with the cla MP 1, 2, 4, 5

Hands-On Activity 2

AL LA **Circle the Sage** Designate several "sages" for Activity 2. Sages should be proficient in using algebra tiles, solving one-step equations, and communication skills. Have the sages stand in the front of the room. Have one student from each group report to a sage, with no two students from the same group reporting to the same sage, if possible. Sages lead the discussion for the steps in the activity. Then have students report back to their original groups and compare what they learned from each sage. Students may use manipulatives, such as a paper bag and counters, if algebra tiles are not readily available. MP 1, 2, 5

BL LA **Pairs Discussion** Have students work with a partner to determine how to solve the equation without using algebra tiles, or other manipulatives. MP 1, 2

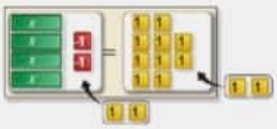
Hands-On Activity 2

You can use algebra tiles to model and solve the equation $4x = 10$.

Step 1 Model the equation.

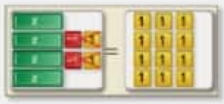
$4x - 2 = 10$

Step 2 Add 2 1-tiles to each side of the mat to form zero pairs on the left side.



$4x - 2 + 2 = 10 + 2$

Step 3 Remove both zero pairs from the left side so that the variable is by itself.



$4x = 12$

Step 4 Divide the remaining tiles into 4 equal groups.

$\frac{4x}{4} = \frac{12}{4}$

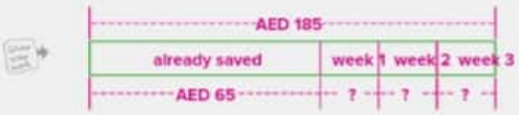
So, $x =$ 3.

Check $4 \cdot 3 - 2 = 10$ ✓

Investigate

Work with a partner to solve the following problem.

1. **Reason Abstractly** Ahmed is saving money to buy a skateboard that costs AED 185. He has already saved AED 65. He plans to save the same amount each week for three weeks. Draw a bar diagram. Then write an equation. How much should Ahmed save each week?



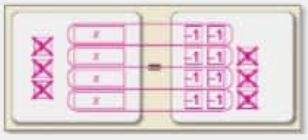
$3x + 65 = 185$; Ahmed should save AED 40 each week.

Work with a partner to solve each equation. Use algebra tiles. Show your work using drawings.

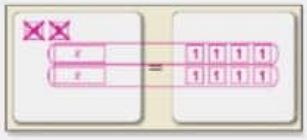
2. $2x + 1 = 5$ $x = 2$

3. $3x + 2 = 11$ $x = 3$

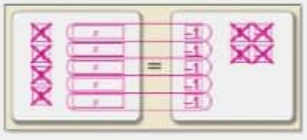
4. $4x + 3 = -5$ $x = -2$



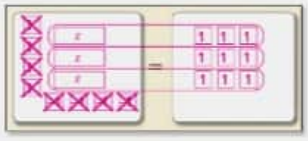
5. $2x - 1 = 7$ $x = 4$



6. $5x - 2 = -7$ $x = -1$



7. $3x - 4 = 5$ $x = 3$

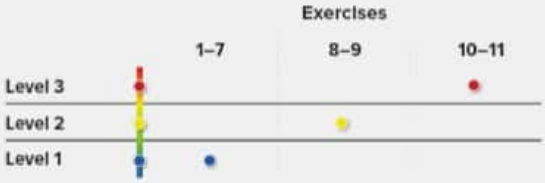


2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Rally Coach** Have students work in pairs to complete Exercises 1–7. Student 1 should read the problem aloud and talk through their steps to complete the problem, while Student 2 watches, listens, coaches, and praises. Students alternate roles for each exercise. MP 1, 2, 5

BL LA **Trade-a-Problem** Have students work in pairs to draw either a bar diagram or a diagram showing algebra tiles that model a two-step equation. Without including the equation, have students trade their diagrams with a partner. Each partner should determine the equation that is modeled and then solve the equation. The pair should discuss and resolve any differences. MP 1, 2, 5

Analyze and Reflect

AL LA **Pairs Discussion** Have students work with a partner to complete Exercises 8 and 9. Have Student 1 read the steps aloud, while Student 2 listens carefully and models the correct equation using algebra tiles. Students trade roles each exercise. Students may use paper bags and counters if algebra tiles are not readily available. MP 1, 2, 5, 7

Create

BL LA **Think-Pair-Write** Give students one minute to think about how they would write the real-world problem. Then have students work in pairs to write two different real-world problems that can be represented by the bar diagram. Challenge them to alter their real-world problem and explain how the bar diagrams, equations, and solutions would change. MP 1, 2, 4, 5

Inquiry Students should be able to answer "HOW can a bar diagram or algebra tiles help you solve a real-world problem?" Check for student understanding and provide guidance, if needed.

Analyze and Reflect

8. MP **Reason Inductively** Work with a partner. Read the steps to model and solve an equation using algebra tiles. Then circle each correct equation.

Steps to Solve	Choices of Equation		
• Add three 1-tiles to each side of the mat. • Divide tiles into two equal groups.	$x + 3 = 15$	$3x + 2 = 15$	$2x - 3 = 15$
• Add four 1-tiles to each side of the mat. • Divide tiles into three equal groups.	$3x - 4 = 11$	$3x + 4 = 11$	$4x - 3 = 11$
• Remove seven 1-tiles from each side of the mat. • Divide tiles into three equal groups.	$x + 3 = 10$	$3x + 7 = 10$	$3x - 7 = 10$
• Add two −1-tiles to each side of the mat. • Remove two zero pairs from the left side of the mat. • Divide tiles into five equal groups.	$5x - 2 = -8$	$5x + 2 = -8$	$2x + 5 = -8$

9. MP **Construct an Argument** What did you observe while choosing the correct equations in the table above?

Sample answer: The number of equal groups matches the coefficient of the variable in the equation.

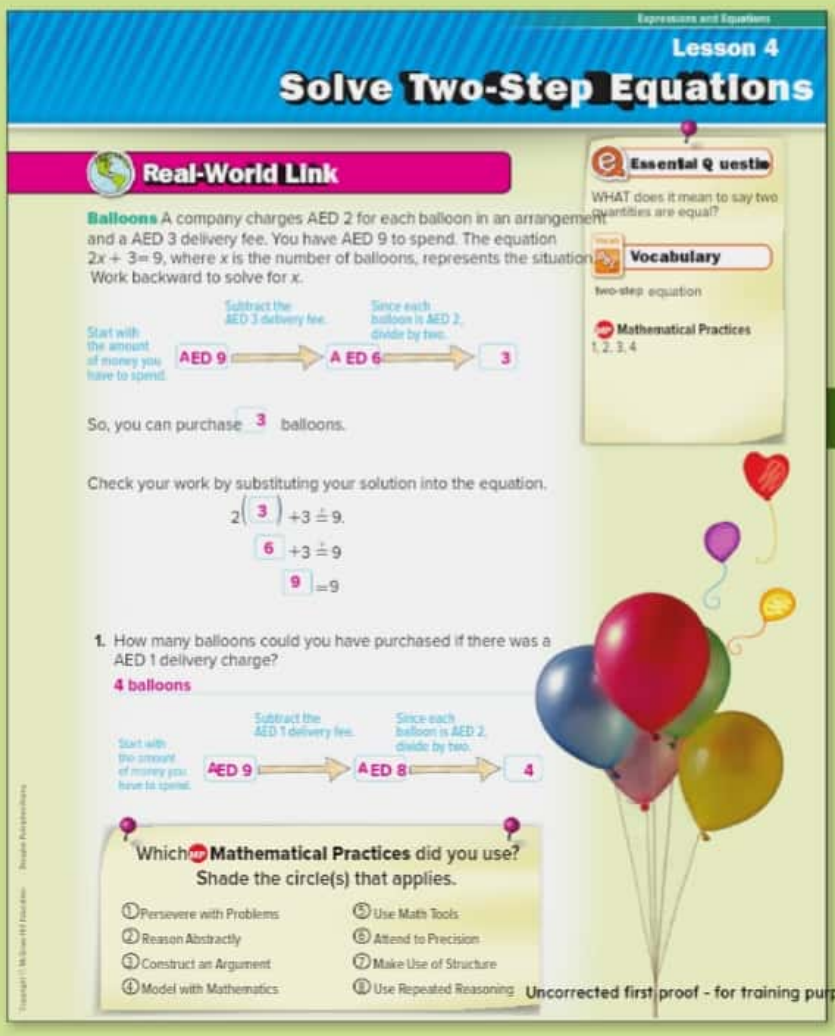
Create

10. MP **Model with Mathematics** Write a real-world problem and an equation that the bar diagram below could represent. Then solve your problem.

Sample answer: Rashid and two friends downloaded 540 songs. Rashid downloaded 200 songs. If the two friends downloaded an equal number of songs, how many songs did each download? $x + 200 = 540$; 170 songs

11. Explain HOW can a bar diagram or algebra tiles help you solve a real-world problem?

Sample answer: Bar diagrams and algebra tile models provide visual representations to write an equation for a real-world problem. They also help you decide the steps needed to solve the equation.



Expressions and Equations

Lesson 4

Solve Two-Step Equations

Real-World Link

Balloons A company charges AED 2 for each balloon in an arrangement and a AED 3 delivery fee. You have AED 9 to spend. The equation $2x + 3 = 9$, where x is the number of balloons, represents the situation. Work backward to solve for x.

Start with the amount of money you have to spend. AED 9 → Subtract the AED 3 delivery fee. A ED 6 → Since each balloon is AED 2, divide by two. 3

So, you can purchase 3 balloons.

Check your work by substituting your solution into the equation.

$2(3) + 3 \stackrel{?}{=} 9$

$6 + 3 \stackrel{?}{=} 9$

$9 = 9$

1. How many balloons could you have purchased if there was a AED 1 delivery charge?

4 balloons

Start with the amount of money you have to spend. AED 9 → Subtract the AED 1 delivery fee. A ED 8 → Since each balloon is AED 2, divide by two. 4

Essential Question

WHAT does it mean to say two quantities are equal?

Vocabulary

two-step equation

Mathematical Practices

1, 2, 3, 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Solve two-step equations.

Coherence connecting within and across grades

Previous	Now	Next
Students solved one-step equations with rational coefficients.	Students solve two-step equations.	Students will solve two-step equations in the form $p(x + q) = r$.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 473.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Think-Pair-Share** Give students one minute to individually think through their response to Exercises 1. Then have them complete the exercises with a partner, ensuring that each student understands how to set up the two-step equation. Call on one pair of students to share their response with the class. MP 1, 2

Alternate Strategy

AL Provide students with a partially completed bar diagram to model the problem. Have them highlight the number that represents the coefficient in the problem. MP 1, 4

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Solve a two-step equation.

AL
- *What two operations are being performed on the variable?* multiplication and addition
- *Which operation is being performed first on the variable?* multiplication

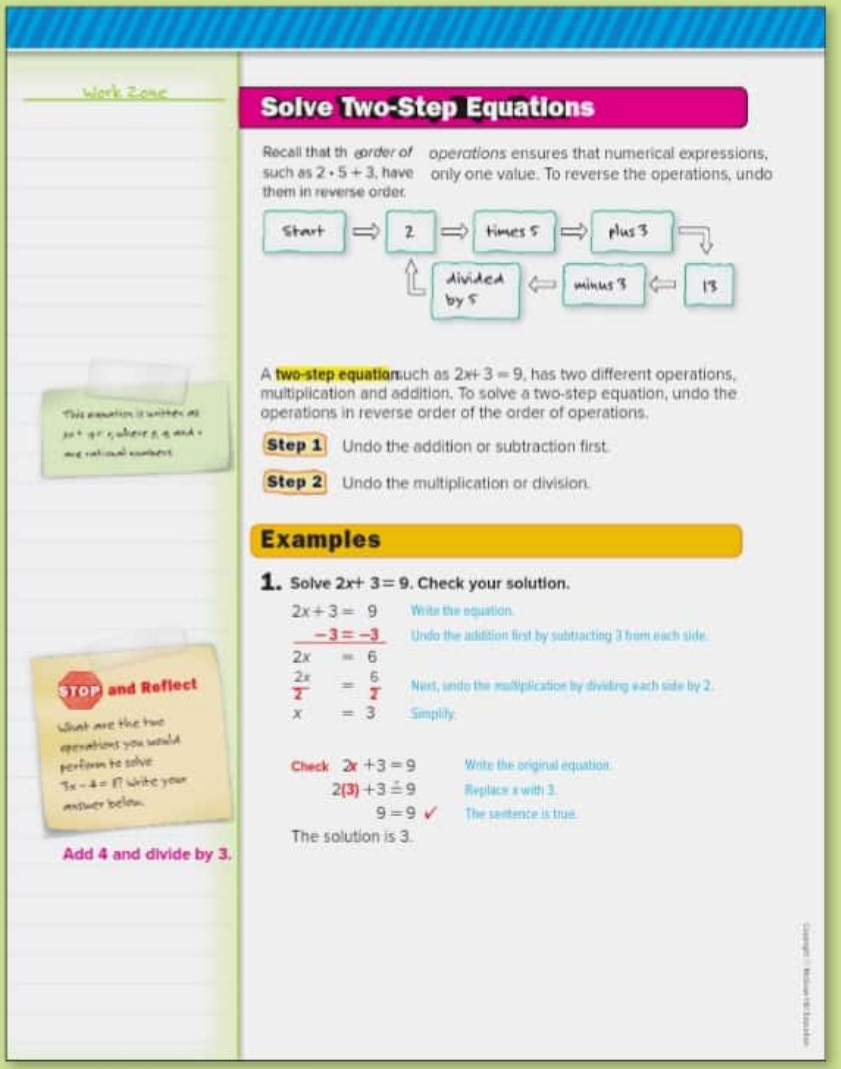
OL
- *What step will you do first? Why?* subtract 3 from both sides; Sample answer: Undo the operations in the reverse order of the order of operations.
- *After subtracting 3 from both sides, what is the next step?* divide both sides by 2
- *How can you use the number line to check your solution?* Sample answer: Start at 0. 2 groups of 3 is equal to 6. Then after adding 3, the result is 9, so the equation is true.

BL
- *Write a word problem that can be represented by the given equation.* See students' work.

Need Another Example?
Solve $4x + 3 = 19$. Check your solution. 4

Watch Out!

Common Error In exercises where the coefficient is negative, remind students to divide by the entire coefficient, including the negative sign for negative coefficients.



Work Zone

This equation is written as $px + qr = s$, where p, q, and s are rational numbers.

STOP and Reflect

What are the two operations you would perform to solve $7x - 4 = 17$? Write your answer below.

Add 4 and divide by 3.

Solve Two-Step Equations

Recall that th *order of operations* ensures that numerical expressions, such as $2 \cdot 5 + 3$, have only one value. To reverse the operations, undo them in reverse order.

Start ⇒ 2 ⇒ times 5 ⇒ plus 3 ⇒ 13 ⇐ minus 3 ⇐ divided by 5

A **two-step equation** such as $2x + 3 = 9$, has two different operations, multiplication and addition. To solve a two-step equation, undo the operations in reverse order of the order of operations.

Step 1 Undo the addition or subtraction first.

Step 2 Undo the multiplication or division.

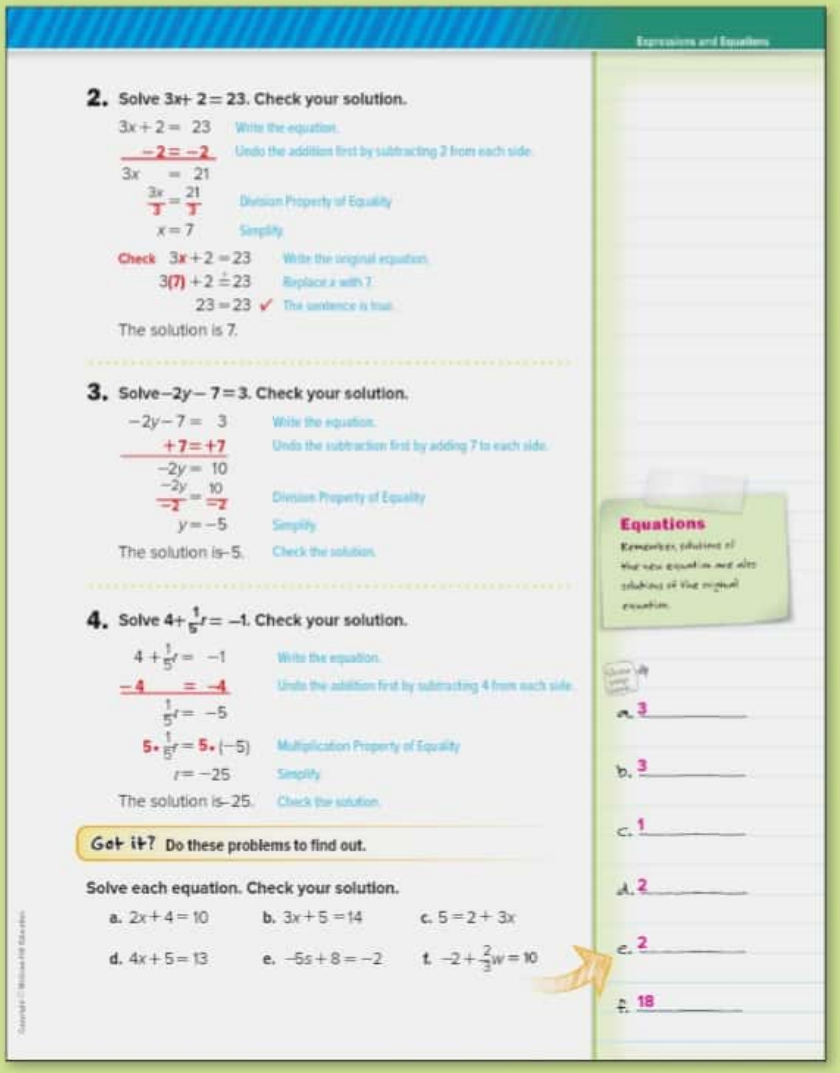
Examples

1. Solve $2x + 3 = 9$. Check your solution.

$2x + 3 = 9$ Write the equation.
$-3 = -3$ Undo the addition first by subtracting 3 from each side.
$2x = 6$
$\frac{2x}{2} = \frac{6}{2}$ Next, undo the multiplication by dividing each side by 2.
$x = 3$ Simplify.

Check $2x + 3 = 9$ Write the original equation.
$2(3) + 3 \stackrel{?}{=} 9$ Replace x with 3.
$9 = 9$ ✓ The sentence is true.

The solution is 3.



Expressions and Equations

2. Solve $3x + 2 = 23$. Check your solution.

$3x + 2 = 23$ Write the equation.
$-2 = -2$ Undo the addition first by subtracting 2 from each side.
$3x = 21$
$\frac{3x}{3} = \frac{21}{3}$ Division Property of Equality
$x = 7$ Simplify.
Check $3x + 2 = 23$ Write the original equation.
$3(7) + 2 \stackrel{?}{=} 23$ Replace x with 7.
$23 = 23$ ✓ The sentence is true.
The solution is 7.

3. Solve $-2y - 7 = 3$. Check your solution.

$-2y - 7 = 3$ Write the equation.
$+7 = +7$ Undo the subtraction first by adding 7 to each side.
$-2y = 10$
$\frac{-2y}{-2} = \frac{10}{-2}$ Division Property of Equality
$y = -5$ Simplify.
The solution is -5. Check the solution.

4. Solve $4 + \frac{1}{5}r = -1$. Check your solution.

$4 + \frac{1}{5}r = -1$ Write the equation.
$-4 = -4$ Undo the addition first by subtracting 4 from each side.
$\frac{1}{5}r = -5$
$5 \cdot \frac{1}{5}r = 5 \cdot (-5)$ Multiplication Property of Equality
$r = -25$ Simplify.
The solution is -25. Check the solution.

Got it? Do these problems to find out.

Solve each equation. Check your solution.

a. $2x + 4 = 10$ b. $3x + 5 = 14$ c. $5 = 2 + 3x$

d. $4x + 5 = 13$ e. $-5s + 8 = -2$ f. $-2 + \frac{2}{3}w = 10$

Equations
Remember, solutions of the new equation are also solutions of the original equation.

a. 3
b. 3
c. 1
d. 2
e. 2
f. 18

Examples

2. Solve a two-step equation.

AL • *What two operations are being performed on the variable?* multiplication and addition

OL • *What operation will you undo first? Why?* Undo the addition by subtracting 2 from both sides; because you reverse the order of operations.

BL • *Write a real-world problem that could be represented by the equation.* See students' work.

Need Another Example?
Solve $6 + 5y = 26$. Check your solution. 4

3. Solve a two-step equation.

AL • *What two operations are being performed on the variable?* multiplication and subtraction

OL • *What operation will you undo first? Why?* Undo the subtraction by adding 7 to both sides; because you reverse the order of operations.

• *What does the equation become after adding 7 to both sides?* $-2y = 10$

BL • *Why is the solution -5 and not 5?* A positive divided by a negative is a negative.

Need Another Example?
Solve $-3c + 9 = 3$. Check your solution. 2

4. Solve a two-step equation.

AL • *What two operations are being performed on the variable?* addition and multiplication

OL • *After subtracting 4, what does the equation become?* $\frac{1}{5}r = -5$

BL • *Why do you multiply by 5 to undo the multiplication of $\frac{1}{5}$?* Multiplying by $\frac{1}{5}$ is the same as dividing by 5. The inverse of dividing by 5 is multiplying by 5.

Need Another Example?
Solve $0 = 6 + \frac{1}{3}t$. Check your solution. -18

Example

5. Write and solve a two-step equation to represent a real-world problem.

AL • *What are you trying to find?* the number of friends at the party

• *What expression represents the cost of the tickets?* $8.5n$

OL • *What equation represents this situation?* $27 + 8.5n = 78$

• *What are the steps to solve the equation?* Subtract 27 from each side, and then divide each side by 8.5.

BL • *How much money was spent on movie tickets?* AED 51

Need Another Example?

Hamad's cell phone plan costs AED 39 per month. Text messages cost an additional AED 0.15 each. If Hamad's cell phone bill last month totaled AED 55.05, write and solve an equation to find the number of text messages he sent.

$0.15m + 39 = 55.05$; 107 text messages

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

AL LA **Roundrobin** Have students go around the room or around a circle responding to specific questions regarding each exercise. Some possible questions are given below.

MP 1, 2, 7

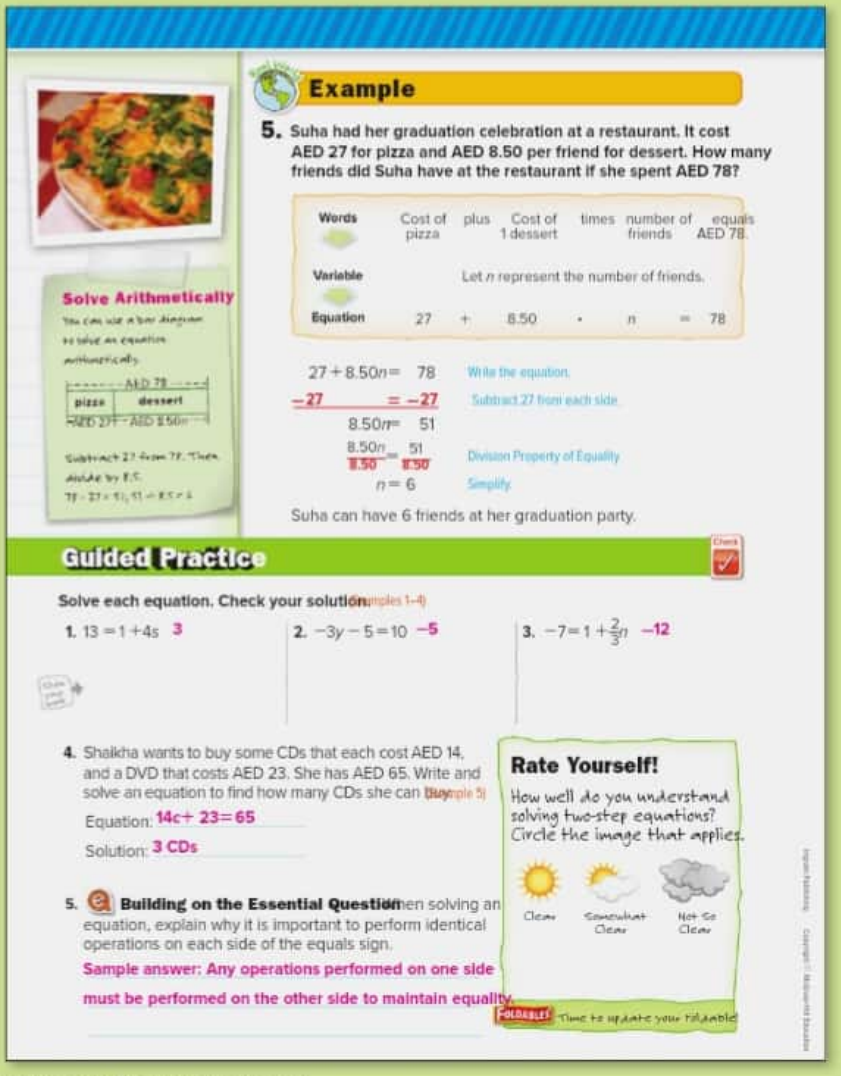
• *What is the coefficient of the variable?* See students' work.

• *What operations are being performed on the variable?* See students' work.

• *Which operation will you undo first?* See students' work.

• *Which operation will you undo second?* See students' work.

• *What is the solution to the equation?* See students' work.



Example

5. Suha had her graduation celebration at a restaurant. It cost AED 27 for pizza and AED 8.50 per friend for dessert. How many friends did Suha have at the restaurant if she spent AED 78?

Words	Cost of pizza	plus	Cost of 1 dessert	times	number of friends	equals	AED 78.
Variable	Let n represent the number of friends.						
Equation	27	+	8.50	·	n	=	78

Solve Arithmetically

You can use a bar diagram to solve an equation arithmetically.

AED 78

pizza	dessert
AED 27	AED 8.50n

Subtract 27 from 78. Then divide by 8.5.

78 − 27 = 51, 51 ÷ 8.5 = 6

$27 + 8.50n = 78$ Write the equation.

$-27 \quad = -27$ Subtract 27 from each side.

$8.50n = 51$

$\frac{8.50n}{8.50} = \frac{51}{8.50}$ Division Property of Equality

$n = 6$ Simplify.

Suha can have 6 friends at her graduation party.

Guided Practice

Solve each equation. Check your solution. (Examples 1–4)

1. $13 = 1 + 4s$ 3

2. $-3y - 5 = 10$ −5

3. $-7 = 1 + \frac{2}{3}n$ −12

4. Shaikha wants to buy some CDs that each cost AED 14, and a DVD that costs AED 23. She has AED 65. Write and solve an equation to find how many CDs she can buy. (Example 5)

Equation: $14c + 23 = 65$

Solution: 3 CDs

5. **Building on the Essential Question** When solving an equation, explain why it is important to perform identical operations on each side of the equals sign.

Sample answer: Any operations performed on one side must be performed on the other side to maintain equality.

Rate Yourself!

How well do you understand solving two-step equations? Circle the image that applies.

Clear | Somewhat Clear | Not So Clear

FOLDABLES Time to update your foldable.

Name _____ My Homework

Independent Practice

Solve each equation. Check your solution. (Examples 1–4)

1. $3x + 1 = 10$ **3**
2. $-3 + 8n = -5$ $-\frac{1}{4}$
3. $4h - 6 = 22$ **7**
4. $-8s + 1 = 33$ **−4**
5. $-4w - 4 = 8$ **−3**
6. $5 + \frac{1}{7}b = -2$ **−49**

7. **Reason Abstractly** Khalid is saving money to buy a bike that costs AED 189. He has saved AED 99 so far. He plans on saving AED 10 each week. In how many weeks will he have enough money to buy the bike? Use a bar diagram to solve arithmetically. Then use an equation to solve algebraically. (Example 5)

189 = 10x + 99; 9 weeks

Solve each equation. Check your solution.

8. $2r - 3.1 = 1.7$ **2.4**
9. $4t + 3.5 = 12.5$ **2.25**
10. $8m - 5.5 = 10.1$ **1.95**

Temperature is usually measured on the Fahrenheit scale (°F) or the Celsius scale (°C). Use the formula $F = \frac{9}{5}C + 32$ to convert from one scale to the other.

a. Convert the temperature for Alaska's record low in July to Celsius. Round to the nearest degree.
−9°C

b. Hawaii's record low temperature is 11°C. Find the difference in degrees Fahrenheit between Hawaii's record low temperature and the record low temperature for Alaska in January.
92.2°F

Alaska Record Low Temperatures (°F) by Month	
January	−80
April	−50
July	16
October	−48

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

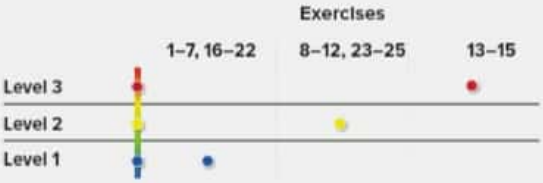
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–11, 13, 14, 16, 29, 30
OL	On Level	1–11 odd, 12–14, 16, 29, 30
BL	Beyond Level	12–17, 29, 30

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	14
2 Reason abstractly and quantitatively.	7
3 Construct viable arguments and critique the reasoning of others.	13, 23
4 Model with mathematics.	12, 15

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a clos ing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students solve the equation $-18\frac{6}{11}b = 96$. 143

12. **Model with Mathematics** Refer to the graphic novel frame below. Jamal figured that they will spend AED 39 for popcorn. Each movie cost AED 19. Write and solve an equation to find how many movies they can purchase.

19x + 39 = 100; x = 3.21; Since it is not possible to purchase part of a movie, they can purchase 3 movies.

H.O.T. Problems Higher Order Thinking

13. **Reason Inductively** Refer to Exercise 11. Is there a temperature in the table at which the number of degrees Celsius is the same as the number of degrees Fahrenheit? If so, find it. If not, explain why not.

No, none of the Fahrenheit temperatures convert to the same temperature in Celsius. Only $-40°F = -40°C$.

14. **Persevere with Problems** Suppose your school is selling magazine subscriptions. Each subscription costs AED 20. The company pays the school half of the total sales in dirhams. The school must also pay a one-time fee of AED 18. Write and solve an equation to determine the fewest number of subscriptions that can be sold to earn a profit of AED 200.

$\frac{1}{2}(20x) - 18 = 200$; 22 subscriptions

15. **Model with Mathematics** Write a real-world problem that can be represented by the equation $\frac{(12 + 14) \times h}{2} = 52$. Then solve the problem.

Sample answer: Tarek found the area of a trapezoid to be 52 square centimeters. One base was 12 centimeters long and the other was 14 centimeters long. What is the height h of the trapezoid?; 4 cm.

Name _____ My Homework

Extra Practice

Solve each equation. Check your solution.

16. $5x + 4 = 19$

$5x + 4 = 19$
$-4 = -4$
$\frac{5x}{5} = \frac{15}{5}$
$x = 3$

17. $6m + 1 = -23$ **−4**

18. $5 + 4d = 37$ **8**

19. $-7y + 3 = -25$ **4**

20. $25 + \frac{11}{12}b = 47$ **24**

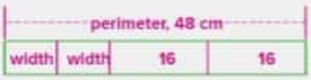
21. $15 - \frac{1}{2}b = -3$ **36**

22. It costs AED 17.50 to enter a petting zoo. Each cup of food to feed the animals is AED 2.50. If you have AED 22.50, how many cups can you buy? Use a bar diagram to solve arithmetically. Then use an equation to solve algebraically.

17.50 + 2.50p = 22.50; 2 cups

23. **Multiple Representations** The perimeter of a rectangle is 48 centimeters. Its length is 16 centimeters. What is the width *w*?

a. Draw a bar diagram that represents this situation.



b. Write and solve an equation that represents this situation.

48 = 32 + 2w; 8 cm

c. How does solving the equation arithmetically compare to solving an equation algebraically?

Sample answer: Using either method, you would subtract first and then divide.

Power Up! Test Practice

Exercises 24 and 25 prepare students for more rigorous thinking needed for the assessment.

24.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly model and solve the equation.
1 point	Students correctly model OR solve the equation.

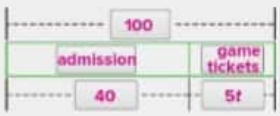
25.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

24. Admission to an amusement park costs AED 40 and game tickets cost AED 5 each. Yousif has AED 100 to pay for admission and game tickets. Select the correct labels to complete the bar diagram that can be used to find the number of game tickets t that Yousif can purchase.

admission
game tickets
5
40
100
$5t$
t



How many game tickets can Yousif purchase? 12 game tickets

25. A rental car company charges a fixed fee of AED 90 plus AED 3 per kilometers. Let c represent the total cost of renting a car and driving it k kilometers.

Write an equation that could be used to find the total cost of renting a car and driving it any number of kilometers. $c = 90 + 3k$

Khalid's family paid AED 798 for their car rental. How many kilometers did they drive? 236 km

Spiral Review

Use the Distributive Property to rewrite each expression.

26. $2(x + 7) =$ $2 \cdot x + 2 \cdot 7$ or $2x + 14$

27. $6(10 + n) =$ $6 \cdot 10 + 6 \cdot n$ or $60 + 6n$

28. $5(k - 4) =$ $5 \cdot k - 5 \cdot 4$ or $5k - 20$

Factor each expression.

29. $5x + 5 \cdot 7 =$ $5(x + 7)$

30. $4n + 4 \cdot 2 =$ $4(n + 2)$

31. $10t + 10 \cdot 3 =$ $10(t + 3)$

32. $7v + 7 \cdot 8 =$ $7(v + 8)$

Expressions and Equations

Inquiry Lab
More Two-Step Equations

inquiry **HOW are equations in $p(x + q) = r$ form different from $px + q = r$ equations?**

MP Mathematical Practices 1, 3, 4

Ahmed has two summer jobs. He delivers newspapers and helps with the gardening. He works at each job three days a week and earns a total of AED 240. The table shows his earnings each day. How much does he earn each day newspaper delivery?

Job	Daily Earnings (AED)
Newspaper delivery	x
Gardening	30

What do you know? gardening daily earnings: AED 30, total daily earnings: AED 240, he works 3 days per week.

What do you need to find? the amount earned each day from delivering newspapers.

Hands-On Activity 1

Step 1 Draw a bar diagram that represents the situation.

Step 2 Write an equation that is modeled by the bar diagram.

$3(\text{AED } x + \text{AED } 30) =$ AED 240

From the diagram, you can see that one third of Ahmed's total earnings is equal to AED x + AED 30. So, AED x + AED 30 $= \frac{\text{AED } 240}{3}$ or AED 80.

Ahmed earns AED 80 − AED 30, or AED 50 each day delivering newspapers.

Focus narrowing the scope

Objective Use models to write and solve two-step equations in $p(x + q) = r$ form.

Coherence connecting within and across grades

Now Students model and solve two-step equations in the form $p(x + q) = r$.

Next Students will solve two-step equations in the form $p(x + q) = r$.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 479.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activity 2.

Materials: algebra tiles, equation mats

Hands-On Activity 1

AL LA **One Stay, One Stray** In pairs, have students complete the activity. Students should discuss how the bar diagram helps them write the equation. Then, have one student stray to another pair to share responses and discuss differences MP 1, 2, 4, 5

BL LA **Pairs Consult** Give students expressions in the $px + q$ form to translate into the form $p(x + q)$ form. Have them identify the property they are using MP 1, 7

Hands-On Activity 2

AL LA **Roundrobin** Have students work in groups of 3 or 4. Have each student be responsible either for reading a step aloud or performing the actions in the step. Have students alternate roles for each step. MP 1, 2, 4, 5

BL LA **Pairs Consult** Have students model the equation $2x + 12 = 16$ using algebra tiles and explain why the models are the same. Have them identify the property that states that the expressions $2(x + 6)$ and $2x + 12$ are equivalent. MP 1, 5, 7

Ismail and his brother bought two sandwiches and two lemonades. The sandwiches cost AED 6 each. They spent a total of AED 16. How much did each lemonade cost?

Hands-On Activity 2

Use algebra tiles to model the situation described above.

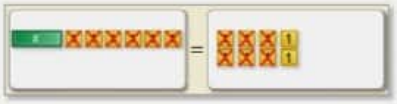
Step 1 Model $2(x + 6) = 16$ using algebra tiles. Use 2 groups of $(x + 6)$ tiles.

$2(x + 6) = 16$

Step 2 Divide the tiles into 2 equal groups on each side of the mat. Remove 1 group from each side.

$x + 6 = 8$

Step 3 Remove the same number of 1-tiles from each side.



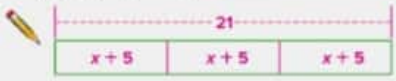
$x = 2$

So, $x =$ 2. Each lemonade costs AED 2.

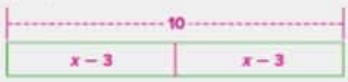
Investigate

Work with a partner to model and solve each equation. Use a bar diagram for Exercises 1 and 2. Use algebra tiles for Exercises 3–6.

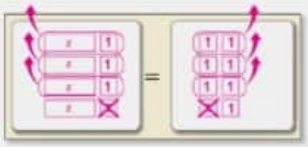
1. $3(x+5)=21$ $x=$ 2



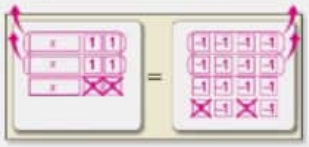
2. $2(x-3)=10$ $x=$ 8



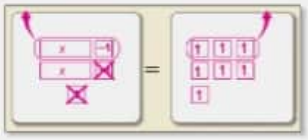
3. $4(x+1)=8$ $x=$ 1



4. $3(x+2)=-12$ $x=$ −6



5. $2(x-1)=6$ $x=$ 4



6. $3(x-4)=-3$ $x=$ 3

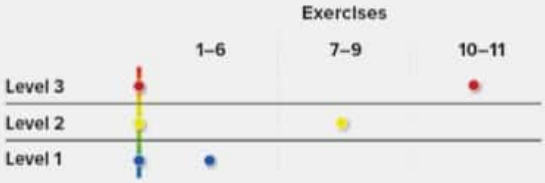
ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Circle the Sage** Designate several "sages" for Exercises 1–6. Sages should be proficient in bar diagrams, using algebra tiles, solving two-step equations, and communication skills. Have the sages stand in the front of the room. Have one student from each group report to a sage, with no two students from the same group reporting to the same sage, if possible. Sages lead the discussion to complete each exercise. Then have students report back to their original groups and compare what they learned from each sage. Students may use manipulatives, such as a paper bag and counters, if algebra tiles are not readily available. MP 1, 2, 5

BL LA **Trade-a-Problem** Have students create their own equation in the form $p(x+q)=r$. Then have them trade equations with a partner. Partners should either draw a bar diagram or use algebra tiles to solve the equation. Each partner checks and verifies the other student's work. MP 1, 2, 5

Analyze and Reflect

AL LA **Pairs to Groups** Arrange students in pairs to complete Ex ercises 7–9. Students may use manipulatives, such as algebra t iles, to complete Exercise 9. Then, have each pair join with an other pair to form a group of 4. Have each pair share their responses and processes they used to complete each exerc ise. Have the group discuss and resolve any discrepan cies. Have groups share their final responses with the class MP 1, 2, 4, 5, 7

Create

BL LA **Share-a-Problem** Have each student share their real-world problem with another student. Each student should read aloud their word problem, while the other student listens carefully. Then have the pair discuss how to solve each problem. MP 1, 3

BL LA **Mix and Match** Have students write equations in the form $p(x + q) = r$ on index cards. Then have them use the Distributive Property to rewrite the same equation without using paren theses on other index cards. Have them shuffle the index cards and lay them face down. Students will play a game in which th ey take turns turning two cards face up. If the equations are equivalent, they remove those cards from the set and e arn a point. If the equations are not equivalent, they return th e cards face down to the pile. The student with the most po ints wins the gam MP 1, 2, 7

Inquiry Students should be able to answer "HOW are equations in $p(x + q) = r$ form different from $px + q = r$ equations?" Check for student understanding and provide guidance, if needed.

Analyze and Reflect

Work with a partner to write and solve an equation that represents each problem.

7. Refer to Activity 1. If Ahmed worked four days a week and made AED 360, how much did he earn delivering newspapers each day?
   $4(x + 30) = 360$; AED 60

8. Refer to Activity 2. If Ismail and his brother spent a total of AED 15, how much did each lemonade cost?
   $2(x + 6) = 15$; AED 1.50

9. MP **Reason Inductively** After modeling an equation using algebra tiles, Shaima used the steps shown below to solve the equation. Write two different equations in $p(x + q) = r$ form that Shaima could have solved.

   Step 1 Divide the tiles into three equal groups on both sides of the mat.

   Step 2 Remove two groups from each side.

   Step 3 Add four 1-tiles to each side.

   Sample equations are given.

   Equation 1: $3(x - 4) = 3$ Equation 2: $3(x - 4) = 6$

Create

10. MP **Model with Mathematics** Write a real-world problem that can be represented by the equation $4(x + 15) = 140$. Then solve the problem.
    Sample answer: Amer and three friends went to a festival. The students spent an equal amount of money and AED 140 altogether. Each student spent AED 15 on rides. How much did each have left to spend?; Each student had AED 20 to spend.

11. Inquiry HOW are equations in $p(x + q) = r$ form different from $px + q = r$ equations?
    Sample answer: The order of operations states to multiply before adding or subtracting. To solve an equation in $p(x + q) = r$ form, first divide to undo the multiplication.

Expressions and Equations

Lesson 5

More Two-Step Equations

Real-World Link

Museums A new exhibit about dinosaurs is being constructed. The exhibit is a rectangle that is 11 meters long. It has a perimeter of 34 meters. Follow the steps to write an equation that can be used to find the width of the museum exhibit.

Essential Question

WHAT does it mean to say two quantities are equal?

Mathematical Practices

1, 2, 3, 4

Step 1 Draw a diagram to help visualize the exhibit.

Label the length and width. Let w represent the width.

Step 2 Write an expression that represents the sum of the length and width of the exhibit. $11 + w$

Step 3 Write an expression that represents twice the sum of the length and width. $2(11 + w)$

Step 4 Write an equation that represents the perimeter of the exhibit. $2(11 + w) = 34$

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Solve two-step equations of the form $p(x) = r$.

Coherence connecting within and across grades

Previous	Now	Next
Students modeled and solved equations in the form $p(x + q) = r$.	Students solve two-step equations in the form $p(x + q) = r$.	Students will model and solve inequalities.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 485.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Teams-Pairs-Solo** Have students work in small teams to complete Steps 1 and 2, ensuring that each team member understands how to generate the expression in Step 2. Then have the team divide into pairs to complete Steps 3 and 4. Then have them rejoin their original team to compare answers. MP 1, 2, 7

Alternate Strategies

AL Review the perimeter formula for rectangles and what perimeter means.

BL Have students explain how the equation in Step 4 would change if the rectangular exhibit was 40 feet long and had a perimeter of 120 feet. MP 1, 3

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Solve two-step equations in the form $p(x) = r$.

AL • *What operation is indicated by the number outside the parentheses?* multiplication

• *How could you undo this multiplication by dividing?* Divide by 3.

OL • *What property allows you to divide both sides by 3?* Division Property of Equality

• *After dividing by 3, what is the final step in solving the equation?* Subtract 5 from both sides.

BL • *Which method do you prefer to use to solve the equation?* See students' work.

Need Another Example?
Solve $2(x + 4) = 18$. 5

2. Solve two-step equations in the form $p(x) = r$.

AL • *What operation is indicated by the number outside the parentheses?* multiplication

• *How could you undo this multiplication by dividing?* Divide by 5.

OL • *What property allows you to divide both sides by 5?* Division Property of Equality

• *After dividing by 5, what is the final step in solving the equation?* Add 2 to both sides.

BL • *Describe another way to solve the equation.* Sample answer: Use the Distributive Property to rewrite the equation without parentheses. The new equation becomes $5n - 10 = -30$. Then add 10 to both sides and divide both sides by 5.

Need Another Example?
Solve $4(5 + b) = -12$. -8

Work Zone

Check Your Work
Remember to plug your solution back into the original equation to see if it makes a true statement.

a. 6

b. 10

c. −13

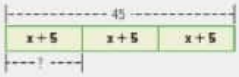
Solve Two-Step Equations

An equation like $2(w - 36) = 114$ is in the form $p(x - q) = r$. It contains two factors, p and $(x - q)$, and is considered a two-step equation. Solve these equations using the properties of equality.

Examples

1. Solve $3(x + 5) = 45$.

Method 1 Solve arithmetically.



Draw a bar diagram. From the diagram, you can see that $x + 5 = 45 \div 3$ or 15. So, $x = 15 - 5$ or 10.

Method 2 Solve algebraically.

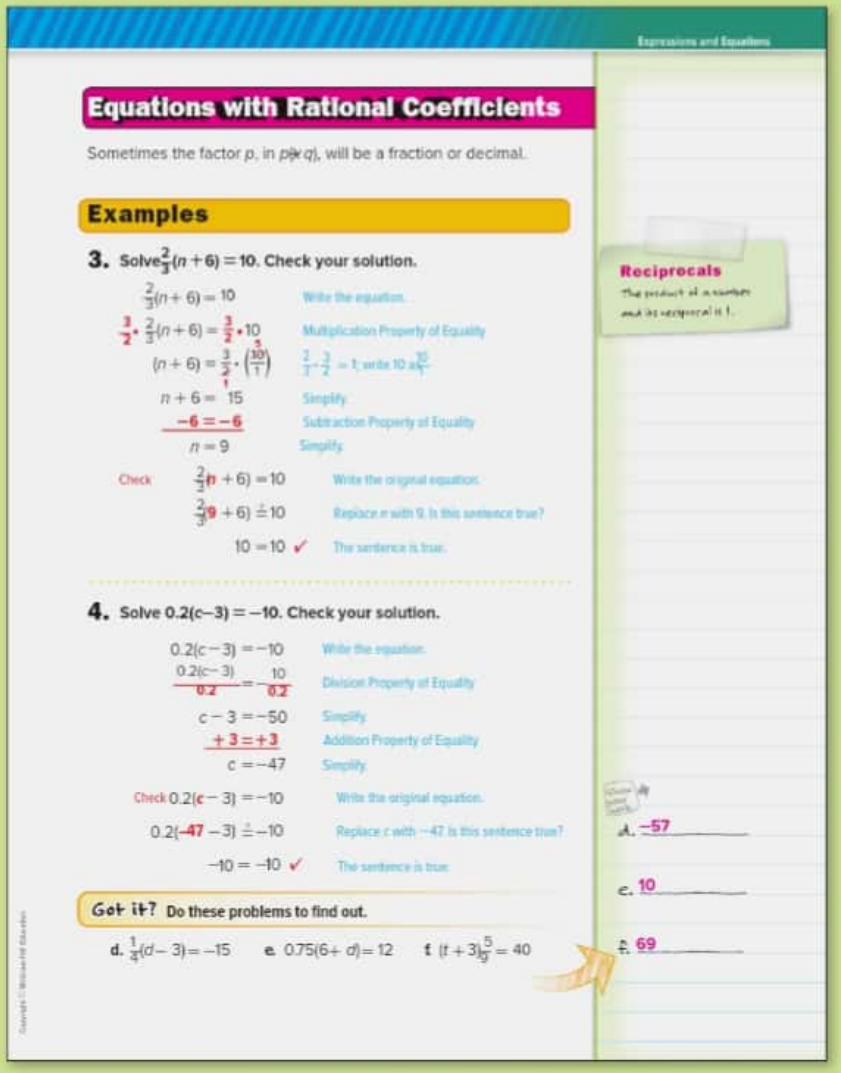
$3(x + 5) = 45$	Write the equation.
$\frac{3(x + 5)}{3} = \frac{45}{3}$	Division Property of Equality
$x + 5 = 15$	Simplify.
$-5 = -5$	Subtraction Property of Equality
$x = 10$	Simplify.

2. Solve $5(n - 2) = -30$.

$5(n - 2) = -30$	Write the equation.
$\frac{5(n - 2)}{5} = \frac{-30}{5}$	Division Property of Equality
$n - 2 = -6$	Simplify.
$+2 = +2$	Addition Property of Equality
$n = -4$	Simplify. Check the solution.

Got it? Do these problems to find out.

a. $2(x + 4) = 20$ b. $3(b - 6) = 12$ c. $-7(6 + d) = 49$



Expressions and Equations

Equations with Rational Coefficients

Sometimes the factor p, in $p(x + q)$, will be a fraction or decimal.

Examples

3. Solve $\frac{2}{3}(n + 6) = 10$. Check your solution.

$\frac{2}{3}(n + 6) = 10$ Write the equation.

$\frac{3}{2} \cdot \frac{2}{3}(n + 6) = \frac{3}{2} \cdot 10$ Multiplication Property of Equality

$(n + 6) = \frac{3}{2} \cdot \left(\frac{10}{1}\right)$ $\frac{2}{3} \cdot \frac{3}{2} = 1$; write 10 as $\frac{10}{1}$.

$n + 6 = 15$ Simplify.

$-6 = -6$ Subtraction Property of Equality

$n = 9$ Simplify.

Check $\frac{2}{3}(n + 6) = 10$ Write the original equation.

$\frac{2}{3}(9 + 6) \stackrel{?}{=} 10$ Replace n with 9. Is this sentence true?

$10 = 10$ ✓ The sentence is true.

4. Solve $0.2(c - 3) = -10$. Check your solution.

$0.2(c - 3) = -10$ Write the equation.

$\frac{0.2(c - 3)}{0.2} = \frac{-10}{0.2}$ Division Property of Equality

$c - 3 = -50$ Simplify.

$+3 = +3$ Addition Property of Equality

$c = -47$ Simplify.

Check $0.2(c - 3) = -10$ Write the original equation.

$0.2(-47 - 3) \stackrel{?}{=} -10$ Replace c with -47. Is this sentence true?

$-10 = -10$ ✓ The sentence is true.

Got it? Do these problems to find out.

d. $\frac{1}{4}(d - 3) = -15$ e. $0.75(6 + d) = 12$ f. $(t + 3)\frac{5}{9} = 40$

Reciprocals

The product of a number and its reciprocal is 1.

a. −57

e. 10

f. 69

Examples

3. Solve two-step equations in the form $p(x) = r$.

AL • *What do you notice about the number being multiplied outside of the parentheses?* It is a fraction.

• *How could you undo this multiplication by $\frac{2}{3}$?* Multiply by its reciprocal.

OL • *What property allows you to multiply by the reciprocal?* Multiplication Property of Equality

• *What is $\frac{3}{2} \cdot 10$?* 15

BL • *Describe another way to solve the equation.* Sample answer: Multiply both sides of the equation by 3 to eliminate the denominator of 3. The new equation becomes $2(n + 6) = 30$. Divide both sides by 2. Then subtract 6 from each side.

Need Another Example?

Solve $\frac{1}{2}(w - 4) = 5$. Check your solution. 14

4. Solve two-step equations in the form $p(x) = r$.

AL • *What do you notice about the number being multiplied outside of the parentheses?* It is a decimal.

• *How could you undo this multiplication by 0.2?* Divide by 0.2.

OL • *What property allows you to divide both sides by 0.2?* Division Property of Equality

• *What is $-10 \div 0.2$?* −50

BL • *Describe another way to solve the equation.* Sample answer: Use the Distributive Property to rewrite the equation without parentheses. The new equation becomes $0.2c - 0.6 = -10$. Then add 0.6 to both sides and divide both sides by 0.2.

Need Another Example?

Solve $0.4(w - 7) = 18$. Check your solution. 52

Example

5. Write and solve a two-step equation for a real-world problem.

AL • *What are you trying to find?* the amount of money each boy received

• *What variable is used to represent this unknown?* m

OL • *What does $(m - 15)$ represent?* the amount of money each boy had after spending AED 15

• *What does $3(m - 15)$ represent?* the amount of money altogether after each boy spent AED 15; there are three boys

BL • *What percentage of his money did each boy spend?* AED 15 of AED 25 is 60%

Need Another Example?
Badr bought 3 bags of balloons for a party. He used 8 balloons from each bag. Write and solve an equation to determine how many balloons were originally in each bag if there were 21 balloons left over. $3(b - 8) = 21$; 15 balloons

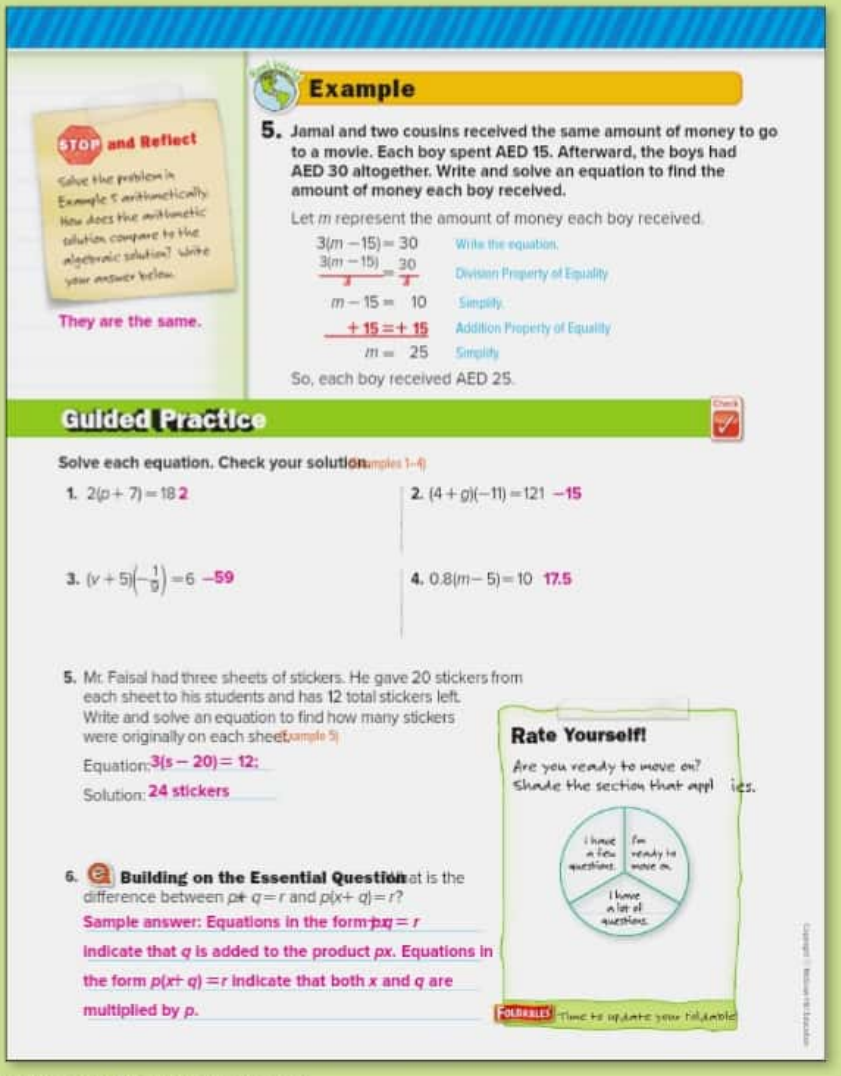
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Rally Coach** Have students work with a partner to complete Exercises 1–6. Have Student 1 talk through their procedure for solving the equation, while Student 2 watches, listens, coaches, and encourages. Have students trade roles for each successive exercise. MP 1, 2, 7

BL LA **Trade-a-Problem** Have students work in pairs to write a real-world problem that can be represented by a two-step equation in the form $p(x + q) = r$. Have them trade their problems with a partner and each partner writes and solve the equation. Have each partner check each other's work. MP 1, 2, 4, 7



STOP and Reflect

Solve the problem in Example 5 arithmetically. How does the arithmetic solution compare to the algebraic solution? Write your answer below.

They are the same.

Example

5. Jamal and two cousins received the same amount of money to go to a movie. Each boy spent AED 15. Afterward, the boys had AED 30 altogether. Write and solve an equation to find the amount of money each boy received.

Let m represent the amount of money each boy received.

$3(m - 15) = 30$ Write the equation.

$\frac{3(m - 15)}{3} = \frac{30}{3}$ Division Property of Equality

$m - 15 = 10$ Simplify.

$+15 = +15$ Addition Property of Equality

$m = 25$ Simplify

So, each boy received AED 25.

Guided Practice

Solve each equation. Check your solution. (Examples 1–4)

1. $2(p + 7) = 18$ 2
2. $(4 + g)(-11) = 121$ −15
3. $(v + 5)\left(-\frac{1}{9}\right) = 6$ −59
4. $0.8(m - 5) = 10$ 17.5

5. Mr. Faisal had three sheets of stickers. He gave 20 stickers from each sheet to his students and has 12 total stickers left. Write and solve an equation to find how many stickers were originally on each sheet. (Example 5)

Equation: $3(s - 20) = 12$;

Solution: 24 stickers

6. **Building on the Essential Question** What is the difference between $px + q = r$ and $p(x + q) = r$?

Sample answer: Equations in the form $px + q = r$ indicate that q is added to the product px. Equations in the form $p(x + q) = r$ indicate that both x and q are multiplied by p.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

FOLDABLES Time to update your foldable

Name _____ My Homework _____

Independent Practice

Solve each equation. Check your solution. (Examples 1–4)

1. $8(s+3)=72$ **6**
2. $-7(z-6)=-70$ **16**
3. $(r+8)(-2)=12$ **−14**
4. $\frac{8}{11}(n-10)=64$ **98**
5. $-0.6(r+0.2)=1.8$ **−3.2**
6. $\left(w-\frac{4}{9}\right)\left(-\frac{2}{3}\right)=-\frac{4}{5}$ **$\frac{74}{45}$ or $\frac{29}{45}$**

7. The length of each side of an equilateral triangle is increased by 5 centimeters, so the perimeter is now 60 centimeters. Write and solve an equation to find the original length of each side of the equilateral triangle. (Example 5)

Equation: **$3(\ell+5)=60$** Solution: **15 cm.**

8. **Multiple Representations** Kwama and three of his friends went to the movies. They originally had a total of AED 40. Each boy had the same amount of money and spent AED 7.50 on a ticket. How much money did each boy have left after buying his ticket?

a. **Model** Draw a bar diagram that represents the situation.

b. **Algebra** Write and solve an equation that represents the situation.
4(x+7.50)= 40; AED 2.50

c. **Words** Explain how you solved your equation.
I divided each side of the equation by 4. Then I subtracted 7.50 from each side.

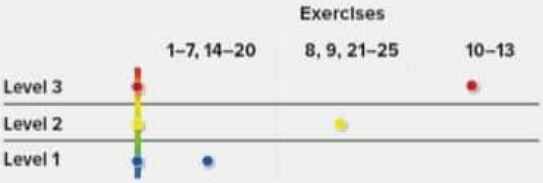
d. **Words** Compare the arithmetic solution and the algebraic solution.
Sample answer: Both solutions use the same inverse operations. Arithmetic solutions use numbers. Algebraic solutions use equations.

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice
The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments
You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9–11, 13, 24, 25
OL	On Level	1–7 odd, 8–11, 13, 24, 25
BL	Beyond Level	8–13, 24, 25

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	12
2 Reason abstractly and quantitatively.	20
3 Construct viable arguments and critique the reasoning of others.	8, 11
4 Model with mathematics.	10
5 Use appropriate tools strategically.	13

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a clos ing formative assessment before dismissing students from your class.

TICKET Out the Door

Solve the equation $4(s + 3) = 52$. 10

Watch Out!

Common Error Students may forget the negative sign when dividing each side of an equation by a negative number. Suggest that they write the negative number inside parentheses.

9. Mrs. Khadija bought one ruler, one compass, and one mechanical pencil at the prices shown in the table for each of her 12 students.

Item	Price (AED)
compass	1.49
mechanical pencil	0.59
ruler	0.49

a. Suppose Mrs. Khadija had 36 fils left after buying the school supplies. Write an equation to find the amount of money Mrs. Khadija initially had to spend on each student.

$12(m - 2.57) = 0.36$

b. Describe a two-step process you could use to solve your equation. Then solve the equation.

Sample answer: I first divided each side by 12 and then added 2.57 to each side; AED 2.60.

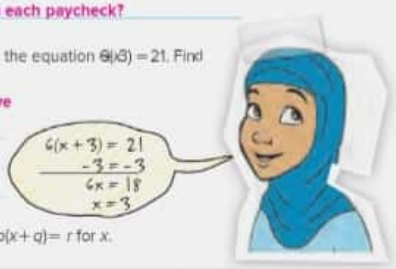
H.O.T. Problems Higher Order Thinking

10. **Model with Mathematics** Write a real-world situation that can be represented by the equation $2(n20) = 110$.

Sample answer: For the past two weeks, Jamal has saved an additional AED 20 from his paycheck. He saved AED 110 during this time period. How much does Jamal typically save from each paycheck?

11. **Find the Error** Maysoun is solving the equation $6(x3) = 21$. Find her mistake and correct it.

Sample answer: Maysoun should have divided by six before subtracting three; $6(x + 3) = 21$, $x + 3 = 3.5$, $x = 3.5 - 3$, $x = 0.5$



12. **Persevere with Problems** Solve $p(x + q) = r$ for x.

$x = \frac{r}{p} - q$

13. **Use Math Tools** Write an equation to represent the bar diagram at the right. Then write a real-world problem that can be represented by the equation and the diagram.

$3(x - 8) = 12$; Sample answer: Fatema bought a new collar for each of her three cats. She paid AED 8 for each collar. Suppose she had AED 12 left. How much money did Fatema have initially to spend on each cat collar?; AED 12 per collar

Name _____ My Homework _____

Extra Practice

Solve each equation. Check your solution.

14. $0.25(3+a) = 0.5$

$0.25(3+a) = 0.5$

$\frac{0.25(3+a)}{0.25} = \frac{0.5}{0.25}$

$3+a = 2$

$a = -1$

15. $12(x-20) = -48$ **16**

16. $-28 = 7(n+3)$ **−7**

17. $(t+9)20 = 140$ **−2**

18. $\frac{5}{9}(8+c) = -20$ **−44**

19. $(d-3)\frac{2}{5} = 30$ **78**

20. Reason Abstractly Abeer bought a necklace for each of her three sisters. She paid AED 7 for each necklace. Suppose she had AED 9 left. Write and solve an equation to find how much money Abeer had initially to spend on each sister.

Equation: $3(m-7) = 9$

Solution: AED 10

Solve each equation. Check your solution.

21. $1\frac{3}{5}(t-6) = -0.4$ $5\frac{3}{4}$ or 5.75

22. $\left(x+5\frac{1}{2}\right)0.75 = \frac{5}{8}$ $-4\frac{2}{3}$ or $-4.\overline{6}$

23. Mr. Khalid bought fruit to make fruit salad. He bought $2\frac{1}{2}$ kilograms of apples and spent AED 4.50 on apples and oranges. Write and solve an equation to determine the number of kilograms of oranges Mr. Khalid bought.

$1.20\left(n+2\frac{1}{2}\right) = 4.50$; 1.25 or $\frac{1}{4}$ kilograms

Fruit	Price per kilogram (AED)
apples	1.20
bananas	0.50
grapes	1.50
oranges	1.20

Power Up! Test Practice

Exercises 24 and 25 prepare students for more rigorous thinking needed for the assessment.

24. This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.

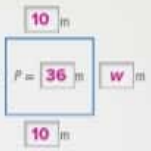
Depth of Knowledge	DOK2
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students complete the model of the classroom, write correct expressions and find the width.
1 point	Students correctly model the classroom and complete two or three of the other four blanks OR students fail to correctly model the classroom, but correctly complete the other blanks.

25. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

24. A rectangular classroom is 10 meters long and has a perimeter of 36 meters. Label the drawing with the correct values to represent the situation. Let w represent the width of the classroom.



Write an expression that represents the sum of the length and width. $w + 10$

Write an expression that represents twice the sum of the length and width. $2(w + 10)$

Write an equation you could use to find the perimeter of the classroom. $2(w + 10) = 36$

What is the width of the classroom? 8 m

25. Which of the following are operations that you could use to solve the equation $p(x - q) = r$ for x? Select all that apply.

- ☐ Subtract q from both sides.
- ☐ Multiply both sides by p.
- ■ Divide both sides by p.
- ■ Add q to both sides.

Spiral Review

Solve each equation.

26. $x + 3 = 5$ 2

27. $x - 2 = -6$ −4

28. $4x = 12$ 3

29. $-6x = -24$ 4

30. $\frac{x}{2} = -1$ −2

31. $\frac{x}{-3} = 1$ −3

Write the number or numbers from the set {−2, −1, 0, 1, 2, 3} that make each statement true.

32. $4m = 12$ 3

33. $y - 1 = 1$ 2

34. $v > 0$ 1, 2, 3

35. $r \leq 0$ −3, −2, −1, 0

Problem-Solving Investigation
Work Backward

Mathematical Practices 1, 3, 4

Case #1 Yard Work

Ayman earned extra money by doing yard work for his neighbor. Then he spent AED 5.50 at the convenience store and four times that amount at the bookstore. Now he has AED 7.75 left.

How much money did Ayman have before he went to the convenience store and the bookstore?

1 Understand *What are the facts?*

You know Ayman has AED 7.75 left. You need to find the amount before his purchases.

2 Plan *What is your strategy to solve this problem?*

Start with the end result and work backward.

3 Solve *How can you apply the strategy?*

He has AED 7.75 left.

Undo the four times AED 5.50 spent at the bookstore.

Since AED 5.50 × 4 is AED 22, add AED 7.75 and AED 22.

AED 7.75
+ AED 22.00
AED 29.75

Undo the AED 5.50 spent at the convenience store.

Add AED 5.50 and AED 29.75.

AED 29.75
+ AED 5.50
AED 35.25

So, Ayman's starting amount was AED 35.25.

4 Check *Does the answer make sense?*

Assume Ayman started with AED 35.25. He spent AED 5.50 and AED 22. He had AED 35 − AED 5.50 − AED 22 or AED 7.75 left. So, AED 35.25 is correct. ✓

Analyze the Strategy

Construct an Argument Describe how to solve a problem by working backward. Sample answer: Begin by taking the last value in the problem and perform opposite operations with each subsequent value until you arrive at the initial value.

Focus narrowing the scope

Objective Solve problems by working backward. This lesson emphasizes **Mathematical Practice 3** Construct an Argument.

Work Backward Working backward is not only useful in problem solving but has a strong link to solving equations. Students have used the reverse process when solving addition, subtraction, and multiplication equations.

Coherence connecting within and across grades

Now Students solve non-routine problems.

Next Students will apply the work backward strategy to solve inequalities.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 491.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

The problems on pages 489 and 490 are intended to be used as a whole-group discussion on how to solve non-routine problems and are designed to provide scaffolded guidance. The problem on page 489 walks students through the solution, while the problem on page 490 asks students to come up with their own solutions.

Case #1 Yard Work

BL Have students extend the problem by having them answer the question below.

Ask:

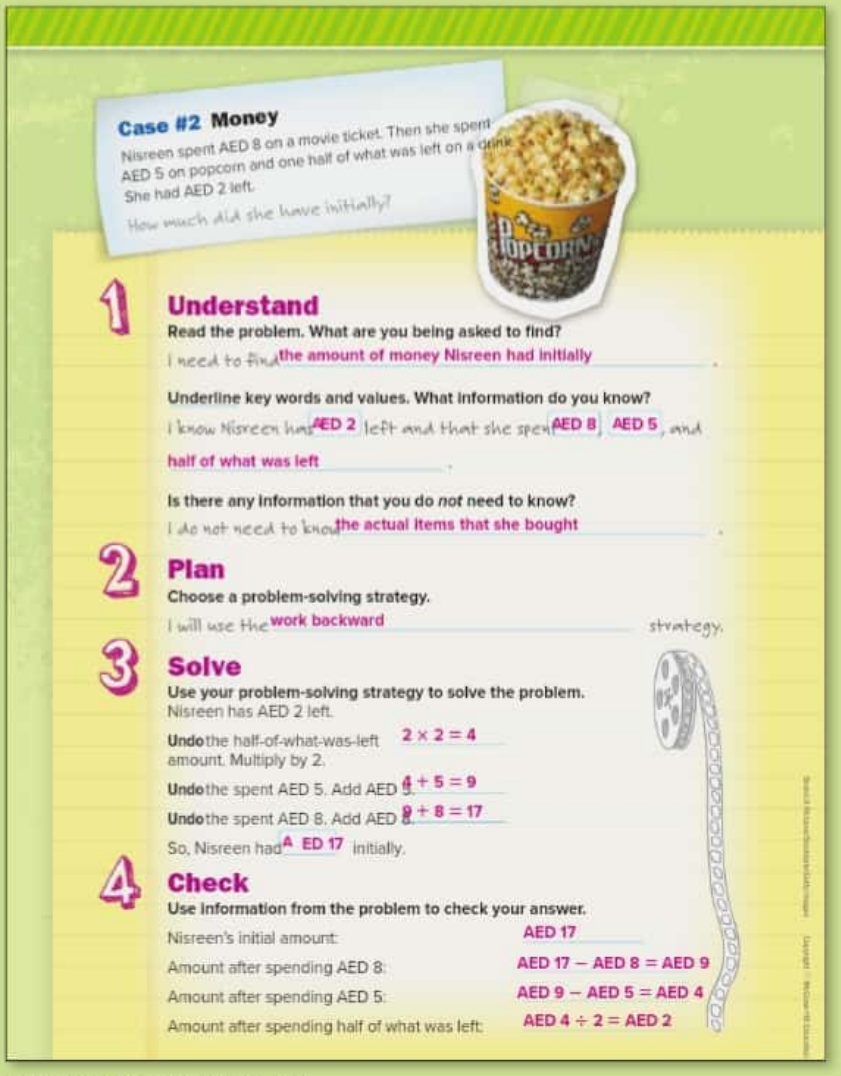
- *Write an equation with a variable that could be used to solve the problem in a different way.* $m - 5.50 - 22 = 7.75$ or $m - 27.50 = 7.75$

Case #2 Money

AL LA **Rally Coach** Have students work in pairs to complete Step 3 using the *work backward* strategy. Have Student 1 complete the first step, speaking out loud, while Student 2 listens carefully, coaches, and praises. Next have Student complete the second step while Student 1 listens carefully, coaches, and praises. Partners take turns until they have completed all of the steps. MP 1, 6

BL LA **Trade-a-Problem** Have students create their own problem that uses the *work backward* strategy. Students trade their problems with a partner, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to determine any errors. MP 1, 4

Need Another Example?
Alia has a brother who is 8 years younger than she is. Her younger sister is 10 years old and is 3 years older than her younger brother. How old is Alia? 18 years old

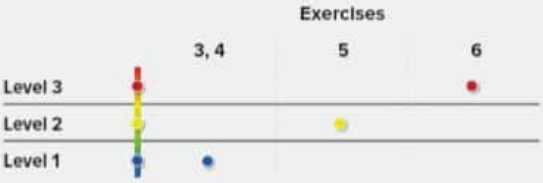


ENGAGE EXPLORE **EXPLAIN** **ELABORATE** EVALUATE

2 Collaborate

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



AL LA **Pairs Discussion** Have students work in pairs to answer the following questions related to Case 4. MP 4, 2, 4

Ask:

- *How can you solve the question?* Sample answer: I can use a proportion and cross multiply. $\frac{3}{28} = \frac{x}{476}$; $x = 51$; $\frac{4}{28} = \frac{x}{476}$; $x = 68$; Of the kites Travis examined, 68 did not have tails and 51 had the wrong color.
- *What is another method to solve the problem? Which method do you prefer? Explain.* Sample answer: I could solve the problem by making a table. The proportion is more efficient and quicker.

BL LA **Think-Pair-Share** Have students work in pairs to answer the following extension questions related to Case 3. MP 1, 2

Ask:

- *Write an equation you could use to solve Case 3.* Sample answer: $2.5h + 26.5 = 979$
- *Pikes Peak is 81.5 meters shorter than 11.5 times the architectural height of the Empire State Building. Find the height of Pikes Peak in feet.* 4,300 m

Expressions and Equations

Inquiry Lab
Solve Inequalities

inquiry HOW is an inequality like an equation? How is it different?

MP Mathematical Practices 1, 2, 3, 4

Mr. Ahmed volunteered to drive Rashid and his friends to the school fundraiser. The car can carry up to 5 people, including the driver. How many friends can ride in the car with Rashid?

What do you know? The car can carry up to 5 people. Mr. Ahmed and Rashid will be in the car.

What do you need to find? the number of friends that can ride in the car with Rashid

Hands-On Activity 1

The real-world situation described above can be represented by the inequality $x + 2 \leq 5$. Let x represent the friends that can ride with Rashid.

$x + 2 \leq 5$

Mr. Ahmed, Rashid, and friends — Maximum number of people

You can use a balance to model and solve the inequality $x + 2 \leq 5$.

Step 1 On one side of a balance, place a paper bag and 2 cubes to model $x + 2$.

Step 2 On the other side of a balance, place 5 cubes.

Add one cube to the bag at a time. Then complete the table.

Number of Friends, x	$x + 2$	Less than or equal to 5?
1	3	yes
2	4	yes
3	5	yes
4	6	No

So, up to 3 friends can ride with Rashid to the school fundraiser.

Focus narrowing the scope

Objective Use models to solve problems involving inequalities.

Coherence connecting within and across grades

Now Students use models to solve one-step inequalities.

Next Students will use Properties of Inequality to solve one-step inequalities.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 495.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activity 1 is intended to be used as a whole-group activity. Activity 1 is designed to provide more guidance than Activity 2.

Hands-On Activity 1

AL LA Explain to students that because the car can hold no more than 5 people, the number of people in the car must be less than or equal to 5.

Ask:

- *What does the symbol ≤ mean?* less than or equal to
- *Is 0 + 2 less than or equal to 5?* yes
- *Is 1 + 2 less than or equal to 5?* yes
- *Is 2 + 2 less than or equal to 5?* yes
- *Is 3 + 2 less than or equal to 5?* yes
- *Is 4 + 2 less than or equal to 5?* no

Hands-On Activity 2

AL LA Tell students to work together to draw the bar diagram and discuss what each part of the diagram represents.
MP 1, 3, 4, 5

Ask:

- *How much does Alia's suitcase weigh without the shoes?* 16 kilograms
- *What is the unknown?* the weight of her shoes
- *What symbol is used to represent less than or equal to?* ≤

An *inequality* is a mathematical sentence that compares quantities. The table shows two examples of inequalities.

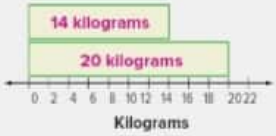
Words	Symbols
x is less than two	$x < 2$
x is greater than or equal to four	$x \geq 4$

To solve an inequality means to find values for the variable that make the sentence true. You can use bar diagrams to solve inequalities.

Hands-On Activity 2

An airline charges for checked luggage that weighs more than 20 kilograms. Alia's suitcase currently weighs 14 kilograms and she still needs to pack her shoes. Find the maximum amount her shoes can weigh so Alia will not be charged a fee.

Step 1 In the bar diagram, write the maximum weight Alia's luggage can be without a fee. Label the weight of Alia's luggage without her shoes.



Step 2 In the bar diagram, write an x beside the bar that represents the weight of Alia's luggage.

The weight of Alia's suitcase plus the weight of her shoes must be less than or equal to the maximum luggage weight.

This can be written as $14 + x \leq 20$.

Using the bar diagram, Alia's shoes cannot weigh more than $20 - 14$ or 6 kilograms.

Investigate

Work with a partner to solve the following problems.

Reason Inductively For Exercises 1–3, assume the paper bag is weightless. Write the inequality represented by each balance. Then write the different possible numbers of cubes in the paper bag if the sides of each balance remain unlevel.

1. 

Inequality: $x + 2 < 7$

Number of Cubes: 0, 1, 2, 3, or 4

2. 

Inequality: $x + 3 < 10$

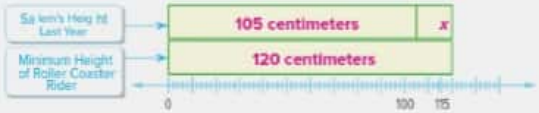
Number of Cubes: 0, 1, 2, 3, 4, 5, or 6

3. 

Inequality: $5 > x + 1$ or $x + 1 < 5$

Number of Cubes: 0, 1, 2, or 3

4. **Reason Abstractly** At an amusement park, roller coaster riders are required to be at least 120 centimeters tall. Last year, Salem was 105 centimeters tall. Complete the bar diagram to determine the number of centimeters x Salem needed to grow this year to be able to ride the roller coaster. Then write an inequality to represent the situation.



So, Salem needed to grow at least 15 centimeters.

Inequality: $105 + x \geq 120$

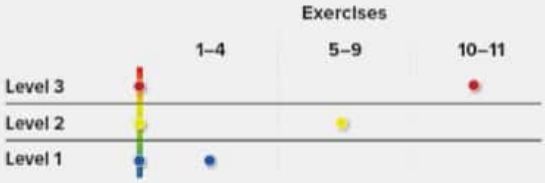
ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Pairs Discussion** Have students work in pairs to complete Exercises 1–4. Have them trade their solutions with another pair of students and discuss any differences. MP 1, 2, 5

BL LA **Trade-a-Problem** Have students create their own problem, similar to Exercises 1–4. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. MP 1, 2, 4

Analyze and Reflect

AL LA **Think-Pair-Share** Have students work in pairs. Give students one minute to think through their responses to Exercises 5–9. Have them share their responses with their partner. Then call on one student to share their response within a small group or large group discussion. MP 1, 2, 4

BL LA **Pairs Discussion** Have students work in pairs to complete Exercises 5–9. Have them trade their solutions with another pair of students and discuss any differences. MP 3, 2

Create

BL LA **Trade-a-Problem** After students create their own real-world situation for Exercise 10, have them trade their problems with their partner. They should solve each other's problems and compare solutions. If the solutions do not agree, students work together to find the error. MP 1, 4

Inquiry Students should be able to answer "HOW is an inequality like an equation? How is it different?" Check for student understanding and provide guidance, if needed.

Analyze and Reflect

Work with a partner to circle the correct inequality for each situation. The first one is done for you.

	Real-World Situation	Inequalities	
	Fawzia wants to score at least 84% on the next history test.	$x \leq 84$	$x \geq 84$
5.	To see a certain movie, you must be at least 13 years old.	$n \leq 13$	$n \geq 13$
6.	Rana has AED 4.99 left on a music download gift card. She has a download costing AED 1.99 in her online shopping cart. How much money does Rana have left to spend?	$x + 1.99 \leq 4.99$	$x + 1.99 > 4.99$
7.	In some countries, teens must be at least 18 years old to obtain a driver's license.	$x < 18$ $x \leq 18$	$x > 18$ $x \geq 18$
8.	Khamis' family budgets a maximum amount of AED 125 per week for groceries. Mr. Khamis already spent AED 40. How much more can Khamis' family spend on groceries?	$x + 40 < 125$ $x + 40 \leq 125$	$x + 40 > 125$ $x + 40 \geq 125$
9.	Sultan pays AED 30 for a ticket to an amusement park. He cannot spend more than AED 150. How much more money can Sultan spend at the amusement park?	$x + 30 < 150$ $x + 30 \leq 150$	$x + 30 > 150$ $x + 30 \geq 150$

Create

10. MP **Model with Mathematics** Write a real-world situation that could be represented by $x + 20 \geq 50$.

Sample answer: Muna wants to save at least AED 50 for a school jacket. If she has saved AED 20, how much more does Muna have to save?

11. Inquiry HOW is an inequality like an equation? How is it different?

Sample answer: An inequality is like an equation because it is a comparison of two quantities. In an equation, the two quantities are equal. In an inequality, one quantity may be less than or greater than the other quantity.

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Solve an inequality.

AL
- *What is the variable?* x
- *What number is on the same side of the inequality sign as the variable?* 3
- *What operation is used between x and 3?* addition
- *How will you undo adding 3?* subtract 3 from each side of the inequality

OL
- *Does subtracting 3 from each side of the inequality change the solution of the inequality?* no
- *What is the first step to solving the inequality?* Subtract 3 from each side of the inequality.
- *Can 7.5 be a solution? Explain.* yes; It is greater than 7.

BL
- *Is 7 part of the solution? Explain.* no; The inequality is greater than, not greater than or equal to.
- *How can you check that the solution is correct?* Replace x with a number greater than 7 in the original inequality, simplify, and determine if the resulting statement is true.

Need Another Example?
Solve $x + 5 > 12$. $x > 7$

Work Zone

Key Concept

Solve Inequalities

Words You can solve inequalities by using the **Addition Property of Inequalities** and the **Subtraction Property of Inequalities**. When you add or subtract the same number from each side of an inequality, the inequality remains true.

Symbols For all numbers a, b, and c.

1. if $a > b$, then $a + c > b + c$ and $a - c > b - c$.
2. if $a < b$, then $a + c < b + c$ and $a - c < b - c$.

Examples

$$\begin{array}{r} 2 < 4 \\ +3 \quad +3 \\ \hline 5 < 7 \end{array} \qquad \begin{array}{r} 6 > 3 \\ -4 \quad -4 \\ \hline 2 > -1 \end{array}$$

An **inequality** is a mathematical sentence that compares quantities. Solving an inequality means finding values for the variable that make the inequality true.

The table below gives some examples of the words you might use when describing different inequalities.

Inequalities				
Words	• is less than • is fewer than	• is greater than • is more than • exceeds	• is less than or equal to • is no more than • is at most	• is greater than or equal to • is no less than • is at least
Symbols	$<$	$>$	$\leq$	$\geq$

Examples

1. Solve $x + 3 > 10$.

$x + 3 > 10$ Write the inequality.
$-3 \quad -3$ Subtract 3 from each side.
$x > 7$ Simplify.

Therefore, the solution is $x > 7$.

You can check this solution by substituting a number greater than 7 into the original inequality. Try using 8.

Check $x + 3 > 10$ Write the inequality.
$8 + 3 \stackrel{?}{>} 10$ Replace x with 8. Is this sentence true?
$11 > 10$ This is a true statement. ✓

Expressions and Equations

2. Solve $-6 \geq n - 5$.

$-6 \geq n - 5$ Write the inequality.

$+5 \quad +5$ Add 5 to each side.

$-1 \geq n$ Simplify.

The solution is $-1 \geq n$ or $n \leq -1$.

You can check this solution by substituting a number less than -1 into the original inequality.

Got it? Do these problems to find out.

Solve each inequality.

a. $a - 3 < 8$ b. $0.4 + y \geq 7$

a. $a < 11$

b. $y \geq 6.6$

Example

3. Solve $a + \frac{1}{2} < 2$. Graph the solution set on a number line.

$a + \frac{1}{2} < 2$ Write the inequality.

$-\frac{1}{2} \quad -\frac{1}{2}$ Subtract $\frac{1}{2}$ from each side.

$a < 1\frac{1}{2}$ Simplify.

The solution is $a < 1\frac{1}{2}$. Check your solution.

Graph the solution.

Place an open dot at $1\frac{1}{2}$. Draw a line and an arrow to the left.

Got it? Do these problems to find out.

Solve each inequality. Graph the solution set on the number line provided.

c. $h + 4 > 4$ d. $x - 6 \leq 4$

c. $h > 0$

d. $x \leq 10$

Open and Closed Dots

When graphing inequalities on a number line, an open dot is used when the value should not be included in the solution, as with > and < inequalities. A closed dot indicates that value is included in the solution, as with ≤ and ≥ inequalities.

Examples

2. Solve an inequality.

AL
- What number is on the same side of the inequality sign as the variable? 5
- What operation is used between n and 5? subtraction
- How will you undo subtracting 5? Add 5 to each side.
- What is $5 + (-6)$? -1

OL
- What is the first step to solve the inequality? Add 5 to each side.
- Is -1 one of the solutions to the inequality? Explain. yes; when you substitute -1 in for n, you get $-6 \geq -6$, which is a true statement.
- Name one other possible solution. Sample answer: -5

BL
- Which is easier to understand, $-1 \geq n$ or $n \leq -1$? Explain. See students' work.

Need Another Example?

Solve $-8 \geq y + 3$. $-11 \geq y$ or $y \leq -11$

3. Solve and graph an inequality.

AL
- What is the first step to solve the inequality? Subtract $\frac{1}{2}$ from each side.
- Does the inequality symbol contain an "or equal to"? no
- Will the circle on the graph be open or closed? open

OL
- To graph the solution on a number line, do you draw an open dot or a closed dot for the inequality? Explain. Since the sign is < and not ≤, you use an open dot because $1\frac{1}{2}$ is not included in the solution.
- Does the arrow point to the right or to the left? left

BL
- Since the values are less than $1\frac{1}{2}$, why isn't the arrow started at 1? Sample answer: The solution includes any values less than $1\frac{1}{2}$; this could include numbers like 1^1.

Need Another Example?

Solve $n + \frac{3}{4} > 2$. Graph the solution set on a number line.

$n > 1\frac{1}{4}$

Example

4. Write and interpret inequalities.

AL • *What do you need to find?* The most Saleh can spend on games.

• *What word or phrase indicates which inequality symbol to use?* the most he can spend

• *What inequality symbol will you use?* ≤

OL • *What inequality can be used to find the amount of money he can spend on games without spending more than AED 18?* $5.5 + x \leq 18$

BL • *If each game costs AED 0.75, what is maximum number of games Saleh can play?* 16

Need Another Example?

Omar took AED 20 to the store to buy a book and some CDs. If he buys a book that costs AED 4.50, write and solve an inequality to find the most he could spend on CDs. Interpret the solution. $4.50 + x \leq 20$; $x \leq 15.50$; The most Omar can spend on CDs is AED 15.50.

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Paired Heads Together** Have students work in pairs to solve Exercises 1–3. Students complete the first exercise, then turn to their partner and discuss their answers, paying particular attention to the graphs. Students should check to make sure the correct dot and arrow direction were used. Students repeat this process for Exercises 2 and 3. MP 1, 2, 5

BL LA **Trade-a-Problem** Have students create their own problem, similar to Exercise 3, using either addition or subtraction, and requiring a graph as part of the answer. Challenge students to use fractions with unlike denominators in their problems. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the error. MP 1, 4, 5

Write Inequalities

Inequalities can be used to represent real-world situations. You will want to first identify a variable to represent the unknown value.

Example

4. Saleh has AED 60 to ride go-karts and play games at the city fair. Suppose the go-karts cost AED 15.50. Write and solve an inequality to find the most he can spend on games.

Words	Cost of go-kart	plus	cost of games	must be less than or equal to	total amount.
Symbols	Let x = the cost of the games.				
Inequality	15.50	+	x	≤	60

$15.5 + x \leq 60$ Write the inequality. (15.50 = 15.5)

$-15.5 \quad -15.5$ Subtract 15.5 from each side.

$x \leq 44.5$ Simplify.

So, the most Saleh can spend on games is AED 44.50.

Guided Practice

Solve each inequality. Graph the solution set on a number line. (Examples 1–3)

1. $6 + h \geq 12$ $h \geq 6$

2. $14 + t > 5$ $t > -9$

3. An elevator can hold 1,300 kilograms or less. Write and solve an inequality that describes how much more weight the elevator can hold if it is currently holding 1,100 kilograms. Interpret the solution. (Example 4)

$x + 1{,}100 \leq 1{,}300$; $x \leq 200$; The elevator can hold 200 kilograms or less.

4. **Building on the Essential Question** Explain when you would use addition and when you would use subtraction to solve an inequality. Sample answer: Use the opposite operation as what appears in the inequality.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

Name _____ My Homework _____

Independent Practice

Solve each inequality. (Examples 1 and 2)

1. $h - 16 \leq -24$ $h \leq -8$
2. $y + 6 \geq -13$ $y \geq -19$
3. $-3 < n - 8$ $5 < n$
4. $3 \leq m + 1.4$ $1.6 \leq m$
5. $x + 0.7 > -0.3$ $x > -1$
6. $w - 8 \geq 5.6$ $w \geq 13.6$

Solve each inequality. Graph the solution set on a number line. (Example 3)

7. $m + 5 \geq -1$ $m \geq -6$

8. $-11 > t + 7$ $-18 > t$ or $t < -18$

Reason Abstractly Write an inequality and solve each problem. For Exercises 11 and 12, interpret the solution. (Example 4)

9. Four more than a number is more than 13.

Inequality: $n + 4 > 13$
Solution: $n > 9$

10. The sum of a number and 19 is at least 8.2.

Inequality: $n + 19 \geq 8.2$
Solution: $n \geq -10.8$

11. The high school football team can have no more than 26 players. Write and solve an inequality to determine how many more players can make the team if the coach has already chosen 17 players.

Inequality: $p + 17 \leq 26$
Solution: $p \leq 9$
Interpretation: Nine additional players or fewer can make the team.

12. Bilal has 1,500 minutes per month on his cell phone plan. How many more minutes can he use if he has already talked for 785 minutes?

Inequality: $785 + m \leq 1{,}500$
Solution: $m \leq 715$
Interpretation: Bilal can talk for an additional 715 minutes or less.

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

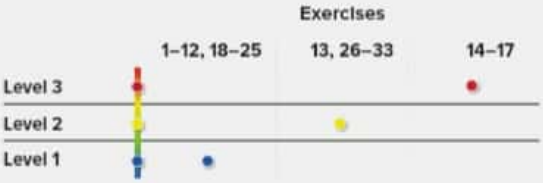
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–15, 17, 32, 33
OL	On Level	1–11 odd, 13–15, 17, 32, 33
BL	Beyond Level	13–17, 32, 33

Watch Out!

Common Error When working with fractions or decimals, stude may neglect to consider the signs. Remind students that signed rational numbers follow the same rules as signed integers. Emphasize the importance of checking their answers.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	16
2 Reason abstractly and quantitatively.	9–12
3 Construct viable arguments and critique the reasoning of others.	14, 17
4 Model with mathematics.	15, 21, 22

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

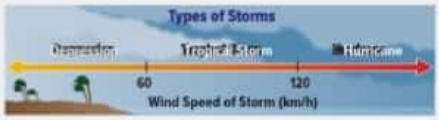
Formative Assessment

Use this activity as a clos ing formative assessment before dismissing students from your class.

TICKET Out the Door

Write an inequality on th e board, such as $6 < 12$. Draw four different graphs lab eled A–D, with only one being the correct graph of the solution set for the inequality. Ask students to write the letter of the graph that shows the correct answer on a small piece of paper and hand it to you as they leave the room. See students' work.

13. Refer to the diagram below.



a. A hurricane has winds that are at least 120 kilometers per hour. Suppose a tropical storm has winds that are 70 kilometers per hour. Write and solve an inequality to find how much the winds must increase before the storm becomes a hurricane.

Inequality: $70 + x \geq 120$ Solution: $x \geq 50$

b. A *major storm* has wind speeds that are at least 180 kilometers per hour. Write and solve an inequality that describes how much greater these wind speeds are than the slowest hurricane.

Inequality: $120 + y \geq 180$ Solution: $y \geq 60$

H.O.T. Problems Higher Order Thinking

14. **Reason Inductively** Compare and contrast the solutions of $a - 3 = 15$ and $a - 3 \geq 15$. Sample answer: $a - 3 = 15$ only has one solution, $a = 18$; $a - 3 \geq 15$ has an infinite number of solutions.

15. **Model with Mathematics** Write an addition inequality for the solution set graphed below.

Sample answer: $x + 3 < 25$

16. **Persevere with Problems** Solve $x + b > c$ for x.

$x > c - b$

17. **Reason Inductively** Does the graph shown at the right show the solution set of the inequality $x \geq 2$? If not, explain how you would change the graph to show the actual solution set. Sample answer: The solution is $x \geq -1$, so the graph should have a closed dot above -1 and the arrow should point to the right, not the left.

Name _____ My Homework _____

Extra Practice

Solve each inequality.

18. $10 < b - 8$ $b > 18$

$10 + 8 < b - 8 + 8$

$18 < b$

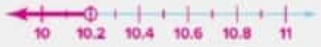
19. $1.2 + m \leq 5.5$ $m \leq 4.3$

20. $c - \frac{1}{4} > -2\frac{1}{2}$ $c > -1\frac{1}{4}$

Model with Mathematics Solve each inequality. Graph the solution set on a number line.

21. $-21 < a - 16$ $-5 < a$

22. $t - 6.2 < 4$ $t < 10.2$



Write an inequality and solve each problem.

23. Eight less than a number is less than 10.

Inequality: $n - 8 < 10$

Solution: $n < 18$

24. The difference between a number and $21\frac{1}{2}$ is no more than $14\frac{1}{4}$.

Inequality: $n - 21\frac{1}{2} \leq 14\frac{1}{4}$

Solution: $n \leq 35\frac{3}{4}$

25. There were a total of 125 cars at a car dealership. A salesperson sold 68 of the cars in one month. Write and solve an inequality that describes how many more cars, at most, the salesman has left to sell. Interpret the solution.

Inequality: $68 + c \leq 125$

Solution: $c \leq 57$

Interpretation: The salesman has 57 cars or less left to sell.

Copy and Solve Solve each inequality. Graph the solution set on a number line. Show your work on a separate sheet of paper. 26–31. See students' work for number lines.

26. $n - \frac{1}{5} \leq \frac{3}{10}$ $n \leq \frac{1}{2}$

27. $6 > x + 3\frac{1}{3}$ $2\frac{2}{3} > x$ or $x < 2\frac{2}{3}$

28. $c + \frac{1}{4} < 5$ $c < 3\frac{3}{4}$

29. $9 \leq m - 2\frac{1}{3}$ $m \geq 11\frac{1}{3}$

30. $\frac{3}{4} + d > 4\frac{1}{2}$ $d > 3\frac{3}{4}$

31. $-\frac{7}{8} \leq n + 3\frac{5}{16}$ $n \geq -4\frac{3}{16}$

Power Up! Test Practice

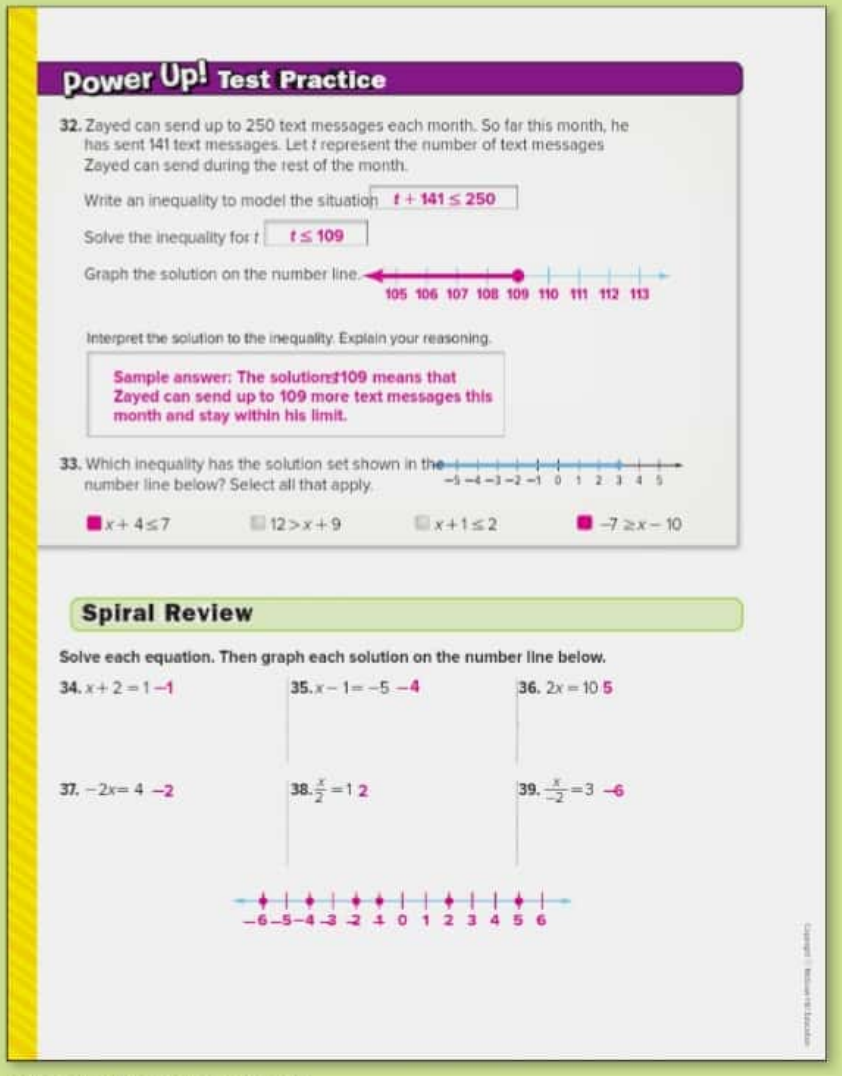
Exercises 32 and 33 prepare students for more rigorous thinking needed for the assessment.

32. This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.

Depth of Knowledge	DOK3
Mathematical Practice	MP1, MP3, MP4
Scoring Rubric	
3 points	Students correctly write, solve, graph, and interpret the solution to an inequality.
2 points	Students correctly complete three of the four parts.
1 point	Students correctly complete two of the four parts.

33. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.



Power Up! Test Practice

32. Zayed can send up to 250 text messages each month. So far this month, he has sent 141 text messages. Let t represent the number of text messages Zayed can send during the rest of the month.

Write an inequality to model the situation $t + 141 \leq 250$

Solve the inequality for t $t \leq 109$

Graph the solution on the number line.

105 106 107 108 109 110 111 112 113

Interpret the solution to the inequality. Explain your reasoning.

Sample answer: The solution $t \leq 109$ means that Zayed can send up to 109 more text messages this month and stay within his limit.

33. Which inequality has the solution set shown in the number line below? Select all that apply.

−5 −4 −3 −2 −1 0 1 2 3 4 5

- [x] $x + 4 \leq 7$
- [ ] $12 > x + 9$
- [ ] $x + 1 \leq 2$
- [x] $-7 \geq x - 10$

Spiral Review

Solve each equation. Then graph each solution on the number line below.

34. $x + 2 = 1$ **−1**

35. $x - 1 = -5$ **−4**

36. $2x = 10$ **5**

37. $-2x = 4$ **−2**

38. $\frac{x}{2} = 1$ **2**

39. $\frac{x}{-2} = 3$ **−6**

−6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6

Expressions and Equations

Lesson 7

Solve Inequalities by Multiplication or Divisio

Real-World Link

Science An astronaut in a space suit weighs about 132 kilograms on Earth, but only 22 kilograms on the Moon.

weight on Earth		weight on Moon
132 kg	>	22 kg

1. If the astronaut and space suit each weighed half as much, would the inequality still be true?

$\frac{132}{2} > \frac{22}{2}$ Divide each side by 2.

66 > 11

Is the inequality still true? Circle yes or no.

Yes No

2. Is the weight of one astronaut greater on Pluto or Earth? Would the weight of 5 astronauts be greater on Pluto or on Earth? Explain by using an inequality.

Earth; Earth; 132 > 30, so 5(132) > 5(30)

Location	Weight of Astronaut (kg)
Earth	132
Moon	22
Pluto	30
Jupiter	360

3. Is the weight of one astronaut greater on Jupiter or on Earth? Would the weight of 5 astronauts be greater on Jupiter or on Earth? Explain by using an inequality.

Jupiter; Jupiter; 360 > 132, so 5(360) > 5(132)

Essential Question

WHAT does it mean to say two quantities are equal?

Vocabulary

Multiplication Property of Inequality

Division Property of Inequality

Mathematical Practices

1, 2, 3, 4, 7

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Solve inequalities by using the Multiplication or Division Properties of Inequality.

Coherence connecting within and across grades

Previous Students solved and graphed one-step addition and subtraction inequalities.

Now Students solve and graph one-step multiplication and division inequalities.

Next Students will solve two-step inequalities.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 509.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL LA **Pairs Discussions** Have students work in pairs. Ask students to extend the Real-World Link by comparing the weights of an astronaut on the moon and on Earth by using an inequality. Then have them write another inequality after multiplying the weights by 2. Pairs should discuss what happened, make other comparisons, and then make a conjecture about multiplying by a negative number. Have them trade their conjectures with another pair of students and discuss any differences. MP 1, 2, 7

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Solve an inequality.

AL • *What is the variable?* x
• *What number is on the same side of the inequality sign as the variable?* 8
• *What operation is used between x and 8?* multiplication
• *What operation will undo multiplication?* division

OL • *If you were solving the equation 8x = 40, what would be your first step?* Divide each side by 8.
• *Based on that, what is the first step to solve this inequality?* Divide each side by 8.

BL • *How can you check that the solution is correct?* Replace *x* with 5 or a number less than 5 in the original inequality, simplify, and determine if the resulting statement is true.

Need Another Example?
Solve $5x > 30$. $x > 6$

2. Solve an inequality.

AL • *What number is on the same side of the inequality sign as the variable?* 2
• *What operation is used between d and 2?* division
• *What operation will undo division?* multiplication

OL • *What is the first step to solve the inequality?* Multiply each side of the inequality by 2.

BL • *Are the statements $d > 14$ and $14 < d$ equivalent? Explain.* yes; Sample answer: They both state that the solution is more than 14.

Need Another Example?
Solve $3 \geq \frac{h}{4}$. $12 \geq h$ or $h \leq 12$

Key Concept

Multiplication and Division Properties of Inequality, Positive Number

Work Zone

Words The **Multiplication Property of Inequality** and the **Division Property of Inequality** state that an inequality remains true when you multiply or divide each side of an inequality by a positive number.

Symbols For all numbers a, b, and c, where $c > 0$,
1. if $a > b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.
2. if $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

These properties are also true for $a \geq b$ and $a \leq b$.

STOP and Reflect
What does the inequality $c > 0$ mean? Explain below.

It means that c is a positive number.

You can solve inequalities by using the Multiplication Property of Inequality and the Division Property of Inequality.

Examples

1. Solve $8x \leq 40$.

$8x \leq 40$ Write the inequality.
$\frac{8x}{8} \leq \frac{40}{8}$ Divide each side by 8.
$x \leq 5$ Simplify.

The solution is $x \leq 5$. You can check this solution by substituting 5 or a number less than 5 into the inequality.

2. Solve $\frac{d}{2} > 7$.

$\frac{d}{2} > 7$ Write the inequality.
$2\left(\frac{d}{2}\right) > 2(7)$ Multiply each side by 2.
$d > 14$ Simplify.

The solution is $d > 14$. You can check this solution by substituting a number greater than 14 into the inequality.

Got It? Do these problems to find out.

a. $x < 10$

b. $42 \geq x$ or $x \leq 42$

a. $4x < 40$

b. $6 \geq \frac{x}{7}$

Multiplication and Division Properties of Inequality, Negative Number

Key Concept

Words When you multiply or divide each side of an inequality by a negative number, the inequality symbol must be reversed for the inequality to remain true.

Symbols For all numbers a, b, and c, where $c \neq 0$,

1. if $a > b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
2. if $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

Examples

$7 > 1$

$-2(7) < -2(1)$ Reverse the symbols.

$-14 < -2$

$\frac{-4}{-4} > \frac{16}{-4}$

$1 > -4$

These properties are also true for $a \geq b$ and $a \leq b$.

STOP and Reflect

What does the inequality $c < 0$ mean? Explain below.

It means that c is a negative number.

Examples

3. Solve $-2g < 10$. Graph the solution set on a number line.

$-2g < 10$ Write the inequality.

$\frac{-2g}{-2} > \frac{10}{-2}$ Divide each side by -2 and reverse the symbol.

$g > -5$ Simplify.

4. Solve $\frac{x}{-3} \leq 4$. Graph the solution set on a number line.

$\frac{x}{-3} \leq 4$ Write the inequality.

$-3\left(\frac{x}{-3}\right) \geq -3(4)$ Multiply each side by -3 and reverse the symbol.

$x \geq -12$ Simplify.

Got it? Do these problems to find out.

c. $\frac{k}{-2} < 9$

c. $k > -18$

Examples

3. Solve a multiplication inequality.

AL
- *What number is on the same side of the inequality sign as the variable?* −2
- *What operation is used between g and −2?* multiplication
- *How will you undo multiplying by −2?* divide each side of the inequality by −2

OL
- *What is the first step to solve the inequality?* Divide each side of the inequality by −2.
- *Since you are dividing by a negative number, what do you need to do?* Reverse the direction of the inequality symbol.

BL
- *Do you draw an open or closed dot on the graph of the solution set? Explain.* open; When the symbol is just > or <, the dot is open.

Need Another Example?
Solve $-4x \leq 4$. Graph the solution set on a number line.
See Answer Appendix.

4. Solve a division inequality.

AL
- *What is the first step to solve the inequality?* Multiply each side by −3.
- *Since you are multiplying by a negative number, what do you need to do?* Reverse the direction of the inequality symbol.

OL
- *Do you need to reverse the inequality symbol? Explain.* Yes; since you are multiplying both sides by a negative number, you need to reverse the symbol.
- *What is the new symbol?* ≥

BL
- *Consider the inequality $\frac{x}{6} \geq -4$. Since you will be multiplying 6 and −4, do you need to reverse the inequality symbol when solving the inequality? Explain.* No; you only need to reverse the symbol when both sides are multiplied by a negative number.

Need Another Example?
Solve $\frac{x}{-4} > -5$. Graph the solution set on a number line.
See Answer Appendix.

Example

5. Write and interpret inequalities.

AL • *What do you need to find?* The number of hours Ling needs to work to make at least AED 120

• *What word or phrase indicates which inequality symbol to use?* at least

• *What inequality symbol will you use?* ≥

OL • *What inequality can be used to find the number of hours Ling must work to earn at least AED 120?* $8x \geq 120$

BL • *Do you need to reverse the inequality symbol? Explain.* No, because you are not dividing both sides by a negative number.

Need Another Example?

A plate weighs $\frac{1}{4}$ pound. A shelf can hold at most 20 pounds. Write and solve an inequality to find how many plates the shelf can hold. Interpret the solution. $\frac{1}{4}x \leq 20$; $x \leq 80$; The shelf can hold at most 80 plates.

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Check** Have students work in pairs to solve Exercise 1. One partner solves the problem while the other coaches, paying particular attention to multiplying or dividing by a negative number. Students switch roles for the next two problems. After the three problems are done, the pairs check their answers with another pair. MP 1, 2, 7

BL LA **Pairs Consult** Have students work in pairs to write a one-step inequality about the graph shown. Then pairs write a real-world problem for the inequality. Pairs trade their problems to check that the solutions do apply to the real-world problems written. MP 1, 2, 4

Example

5. Omar earns AED 8 per hour working at the zoo. Write and solve an inequality that can be used to find how many hours he must work in a week to earn at least AED 120. Interpret the solution.

Words	Amount earned per hour	•	number of hours	is at least	amount earned each week.	
Variable	Let x represent the number of hours.					
Inequality	8	•	x	≥	120	

$8x \geq 120$ Write the inequality

$\frac{8x}{8} \geq \frac{120}{8}$ Divide each side by 8.

$x \geq 15$ Simplify

So, Omar must work at least 15 hours.

Guided Practice

Solve each inequality. Graph the solution set on a number line. (Examples 1–4)

1. $-3n \leq -22$ $n \geq 7\frac{1}{3}$

2. $\frac{t}{4} < -11$ $t > 44$

3. At a sporting game you can get a sandwich for AED 2. You have AED 10 to spend. Write and solve an inequality to find the number of sandwiches you can buy. Interpret the solution. (Example 5) $2x \leq 10$; $x \leq 5$; You can buy five sandwiches or fewer.

4. **Building on the Essential Question** Explain when you should reverse the inequality symbol when solving an inequality. Sample answer: Reverse the symbol when multiplying or dividing by a negative number.

Rate Yourself!

How confident are you about solving multiplication and division inequalities? Check the box that applies.

Name _____ My Homework

Independent Practice

Solve each inequality. (Examples 1 and 2)

1. $6y < 18$ $y < 3$
2. $-3s \geq 33$ $s \leq -11$
3. $60 \leq \frac{m}{3}$ $180 \leq m$
4. $\frac{t}{-2} < 6$ $t > -12$
5. $\frac{m}{-14} \leq -4$ $m \geq 56$
6. $-56 \leq -8x$ $x \leq 7$
7. $12n \leq 54$ $n \leq 4.5$
8. $\frac{h}{9} > \frac{1}{4}$ $h > 2\frac{1}{4}$
9. $\frac{w}{-5} \geq 9$ $w \leq -45$

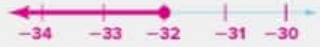
Solve each inequality. Graph the solution set on a number line. (Examples 3 and 4)

10. $4x \geq 36$ $x \geq 9$

11. $20 < 5t$ $4 < t$

12. $\frac{s}{-6} > -16$ $s < 96$

13. $\frac{x}{-4} \geq 8$ $x \leq -32$



14. A pool charges AED 20 each visit, or you can buy a membership. Write and solve an inequality to find how many times a person should use the pool so that a membership is less expensive than paying each time. Interpret the solution. (Example 5)

Inequality: $20x > 500$ Solution: $x > 25$

Interpretation: You would need to use the pool more than 25 times in three months.

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

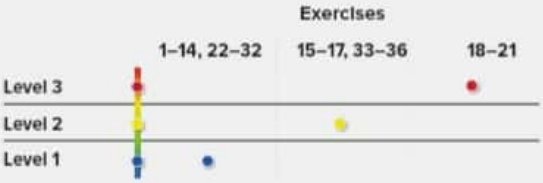
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–15, 17, 18, 20, 35, 36
OL	On Level	1–13 odd, 15–18, 20, 35, 36
BL	Beyond Level	15–21, 35, 36

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	19, 21
2 Reason abstractly and quantitatively.	32
3 Construct viable arguments and critique the reasoning of others.	15, 20
7 Look for and make use of structure.	18

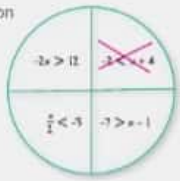
Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write how yesterday's lesson, solving inequalities by adding o r subtracting, helped them with the content of today's lesson. See student's work.

15. **Reason Inductively** Cross out the inequality that does not belong in the organizer shown at the right. Then explain your reasoning.
Sample answer: The inequalities $2x > 12$, $\frac{x}{2} < -3$, and $-7 > x - 1$ are equal to $x < -6$. The inequality $-2 < x + 4$ is equal to $x > -6$.



Write an inequality for each sentence. Then solve the inequality.

16. Sixteen is less than eight times a number.
Inequality: $16 < 8n$
Solution: $2 < n$ or $n > 2$

17. The product of a number and five is at the most 30.
Inequality: $5n \leq 30$
Solution: $n \leq 6$

H.O.T. Problems Higher Order Thinking

18. **Identify Structure** Write two different inequalities that have the solution $y > 6$. One inequality should be solved using multiplication properties, and the other should be solved using division properties.
Sample answer: $7y > 42$ and $\frac{1}{2}y > 3$

19. **Persevere with Problems** You score 15, 16, 17, 14, and 19 points out of 20 possible points on five tests. What must you score on the sixth test to have an average of at least 16 points?
at least a 15

20. **Reason Inductively** The inequalities $3x > 2$ and $9x > 6$ are equivalent inequalities. Write another inequality that is equivalent to $3x > 2$ and $9x > 6$.
Sample answer: $12x > 8$

21. **Persevere with Problems** Consider the inequalities $b \geq 4$ and $b \leq 13$.
a. Graph each inequality on the number line.


b. Do the solution sets of the two inequalities overlap? If so, what does this overlapping area represent?
yes; It represents the solutions that satisfy both inequalities.
c. A compound inequality is an inequality that combines two inequalities. Write a compound inequality for the situation.
$4 \leq b \leq 13$
d. Look back at the graph of the solutions for both inequalities. Make another graph that shows only the solution of the inequality.


Name _____ My Homework _____

Extra Practice

Solve each inequality.

22. $-10n > -20$ $n < 2$

$\frac{-10n}{-10} > \frac{-20}{-10}$

$n < 2$

23. $-7y < 35$ $y > -5$

24. $15 < 3r$ $5 < r$ or $r > 5$

25. $12p \geq -72$ $p \geq -6$

26. $\frac{t}{-7} > 10$ $t < -70$

27. $-8 < \frac{y}{5}$ $-40 < y$ or $y > -40$

Solve each inequality. Graph the solution set on a number line.

28. $\frac{h}{5} \leq -12$ $h \leq -60$

29. $-3w < -39$ $w > 13$

30. $15 < 4x$ $3.75 < x$ or $x > 3.75$

31. $10 \leq \frac{t}{-2}$ $-20 \geq t$ or $t \leq -20$

32. **Reason Abstractly** Each game at a carnival costs AED 5, or you can pay AED 150 and play an unlimited amount of games. Write and solve an inequality to find how many times you should play a game so that the unlimited game play is less expensive than paying each time. Interpret the solution.

Inequality: $5x > 150$ Solution: $x > 30$

Interpretation: A person should play more than 30 games.

Write an inequality for each sentence. Then solve the inequality.

33. The product of a number and 4 is at least −12.

Inequality: $4n \geq -12$

Solution: $n \geq -3$

34. Five times a number is less than −45.

Inequality: $5n < -45$

Solution: $n < -9$

Power Up! Test Practice

Exercises 35 and 36 prepare students for more rigorous thinking needed for the assessment.

35.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

36.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly write, solve, and graph an inequality and list the solution.
1 point	Students correctly complete two or three of the four parts.

Power Up! Test Practice

35. Manal earns AED 7 per hour babysitting. She wants to earn at least AED 105 for a camping trip. Determine if each statement is true or false.

a. The inequality $\frac{b}{7} \geq 105$ models how many hours Manal must babysit to earn at least AED 105. ☐ True ■ False

b. The inequality $7h \geq 105$ models how many hours Manal must babysit to earn at least AED 105. ■ True ☐ False

c. Manal must babysit up to 15 hours in order to earn at least AED 105. ☐ True ■ False

36. Footballs cost AED 24 each at Sports Emporium. Coach Amer can spend at most AED 120 on equipment for the football team. Let b represent the number of footballs Coach Amer can buy.

Write an inequality to model the situation. $24b \leq 120$

Solve the inequality for b. $b \leq 5$

Graph the solution on the number line.

How many footballs can Coach Amer buy? List all of the possible answers.

0, 1, 2, 3, 4, or 5 footballs

Spiral Review

Solve each equation. Check your solution.

37. $5k + 6 = 16$ **2**

38. $-14 = 2x - 8$ **−3**

39. $-4n + 3 = 13$ **−2.5**

40. $25 = 7m + 4$ **3**

41. $10.5 + h = 22.5$ **12**

42. $14n - 32 = 22$ $3\frac{6}{7}$

Expressions and Equations

Lesson 8

Solve Two-Step Inequalities

Real-World Link

Newspapers Amani is placing an ad in the local newspaper for a pottery class. The cost of placing an ad is shown in the table.

Service	Cost (AED)
10-day ad with 3 lines	38.00
each additional line	9.00

1. Complete the equation to find the total cost c of an ad with 4 or more lines. Use x as the variable.

cost of a 10-day add with only 3 lines: 38 + cost of each additional line: 9x = total cost: c

2. How much will it cost to place the ad if it is 5 lines long?
AED 56

3. Suppose Amani can spend only AED 50 on the ad. Does she have enough money to place the ad? Circle yes or no.
yes (no)
If the answer is no, how much more money will Amani need? Explain. She needs AED 6 more. The ad costs AED 56 and she only has AED 50. AED 56 − AED 50 = AED 6.

Essential Question
WHAT does it mean to say two quantities are equal?

Vocabulary
two-step inequality

Mathematical Practices
1, 2, 3, 4, 5

Which MP **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Model and solve two-step inequalities and represent the solution on the number line

Coherence connecting within and across grades

Previous Students solved one-step inequalities.

Now Students model and solve two-step inequalities.

Next Students will write and solve multi-step equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 517.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Think-Pair-Share** Give students one minute to individually think through their responses to Exercises 1–3. Then have them complete the exercises with a partner, ensuring that each student understands how to write the two-step inequality. Call on one pair of students to share their responses with the class MP 1, 2

Alternate Strategies

AL Have students create a table that shows the rates for having a 3-, 4-, and 5-line advertisement print MP 1, 4, 5

BL Have students write an inequality to represent how many lines Amani can place in the ad if she only has AED MP 1, 4

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Solve a two-step inequality and represent the solution on the number line.**

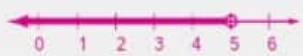
AL • *What are the two operations in the inequality?* multiplication and addition

OL • *What is the coefficient of the variable?* 3

• *Will the dot be open or closed? Explain.* closed; the inequality includes 4

BL • *How many solutions are there to this inequality? Explain.* There are an infinite number of solutions; any value greater than or equal to 4 is a solution.

Need Another Example?
Solve $2x + 1 < 11$. Graph the solution set on a number line.
$x < 5$



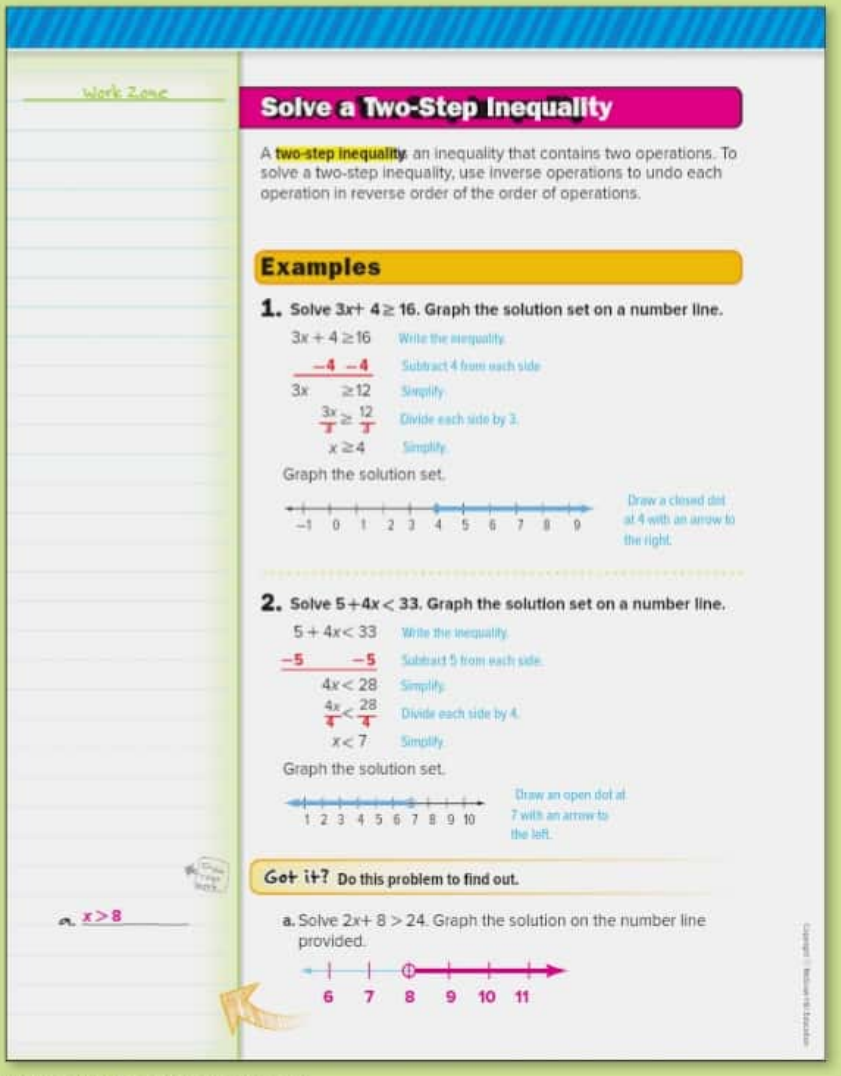
2. **Solve a two-step inequality.**

AL • *What are the inverse operations for these operations?* division and subtraction

OL • *Will the dot be open or closed? Explain.* open; The inequality does not include 7.

BL • *How many solutions are there to this inequality? Explain.* There are an infinite number of solutions; any value less than 7 is a solution.

Need Another Example?
Solve $8 + 3x > 14$. Graph the solution set on a number line.
$x > 2$



Examples

3. Solve $7 - 2x > 11$. Graph the solution set on a number line.

$7 - 2x > 11$ Write the inequality.

$-7 \quad -7$ Subtract 7 from each side.

$-2x > 4$ Simplify.

$\frac{-2x}{-2} < \frac{4}{-2}$ Divide each side by -2. Reverse inequality symbol.

$x < -2$ Simplify. Check your solution.

Graph the solution set.

Draw an open dot at -2 with an arrow to the left.

You can check the solution by substituting a number less than -2 into the original inequality. Try using -3.

Check $7 - 2x > 11$ Write the inequality.

$7 - 2(-3) \stackrel{?}{>} 11$ Replace x with -3. Is the sentence true?

$13 > 11$ This is a true statement. ✓

4. Solve $\frac{x}{2} - 5 < -8$. Graph the solution set on a number line.

$\frac{x}{2} - 5 < -8$ Write the inequality.

$+5 \quad +5$ Add 5 to each side.

$\frac{x}{2} < -3$ Simplify.

$\frac{x}{2}(2) < -3(2)$ Multiply each side by 2.

$x < -6$ Simplify. Check your solution.

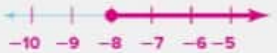
Graph the solution set.

Draw an open dot at -6 with an arrow to the left.

Got it? Do these problems to find out.

Solve each inequality. Graph the solution set on the number line provided.

b. $\frac{x}{2} + 9 \geq 5$



c. $8 - \frac{x}{3} \leq 7$

Solving Inequalities

Remember that if multiplying or dividing by a negative number when solving inequalities, reverse the direction of the inequality symbol.

b. $x \geq -8$

c. $x \geq 3$

Examples

3. Solve a two-step inequality.

AL • *What are the inverse operations for the operations in the inequality?* division and subtraction

OL • *What are the steps to solve the inequality?* Subtract 7 from each side. Then divide each side by -2.

• *Will the dot be open or closed? Explain.* open; The inequality does not include -2.

BL • *How many solutions are there to this inequality? Explain.* There are an infinite number of solutions; any value less than -2 is a solution.

Need Another Example?

Solve $6 - 3x \leq 9$. Graph the solution set on a number line.

$x \geq -1$

4. Solve a two-step inequality.

AL • *What are the two operations in the inequality?* division and subtraction

• *What is the inverse operation for these operations?* multiplication and addition

OL • *What are the steps to solve the inequality?* Add 5 to each side. Then multiply each side by 2.

• *Will the dot be open or closed? Explain.* open; The inequality does not include -6.

BL • *How many solutions are there to this inequality? Explain.* There are an infinite number of solutions; any value less than -6 is a solution.

Need Another Example?

Solve $\frac{x}{4} + 3 \geq 7$. Graph the solution set on a number line.

$x \geq 16$

Example

5. Write and solve an inequality for a real-world problem.

AL
- *What are you trying to find?* How many more games it will take for Jamal to have at least 61 strikes
- *What phrase indicates an inequality?* at least

OL
- *What expression represents 2 strikes per game?* 2g
- *What expression represents the total number of strikes?* 34 + 2g

BL
- *Why does Jamal need 14 more games?* We have to round up from 13.5 because we can't have 0.5 of a game.

Need Another Example?
Khalaf has already earned AED 40 mowing lawns. He earns AED 10 per lawn. Write and solve an inequality to determine how many more lawns he will have to mow to have at least AED 95 for a new lawn mower. Interpret the solution. $40 + 10x \geq 95$, $x \geq 5.5$. Khalaf will have at least AED 95 after mowing 6 more lawns.

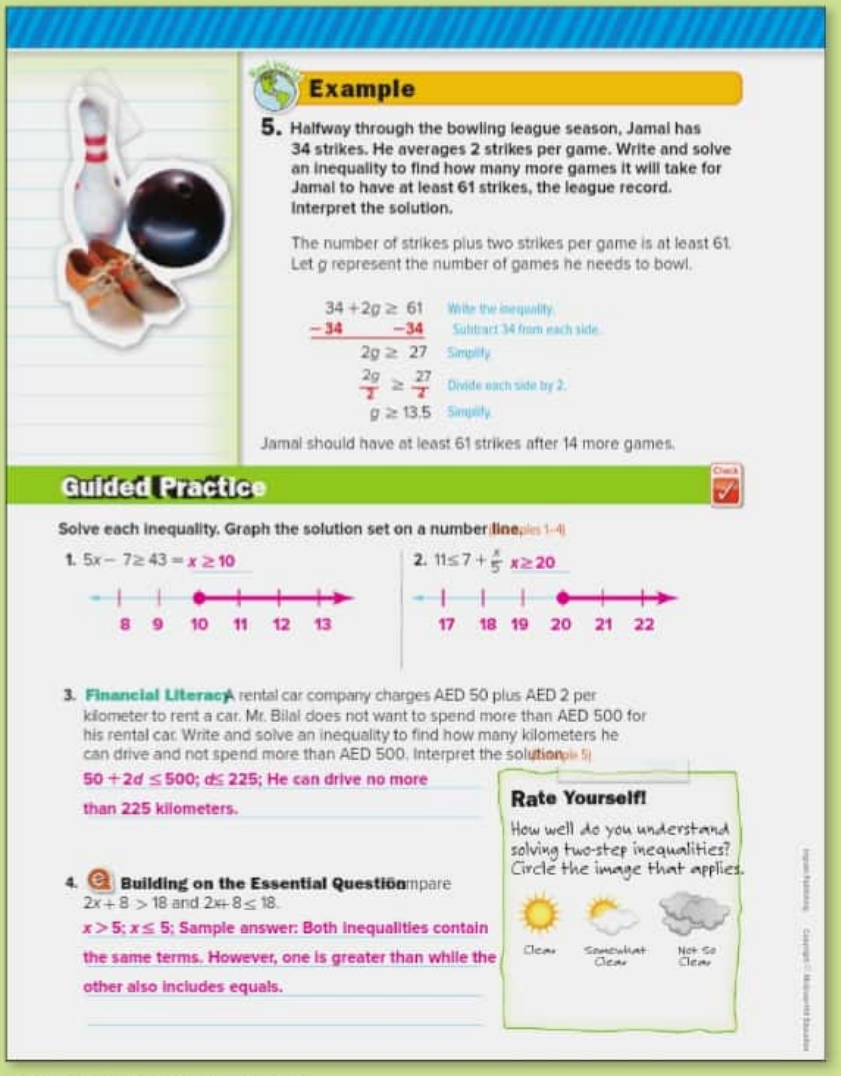
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Talking Chips** Have students work in small groups to discuss and complete Exercises 1–4. Provide each student with 5 chips. Students must place a chip in the center of the table as they contribute to the discussion. After they have used all of their chips, they may not contribute. All students must use all of their chips. MP 1, 2, 5

BL LA **Trade-a-Problem** Have students work in pairs to write a real-world problem that can be represented by a two-step inequality. Have them trade their problems with a partner and each partner writes and solves the inequality. Have each partner check each other's work. MP 1, 2, 4



Example

5. Halfway through the bowling league season, Jamal has 34 strikes. He averages 2 strikes per game. Write and solve an inequality to find how many more games it will take for Jamal to have at least 61 strikes, the league record. Interpret the solution.

The number of strikes plus two strikes per game is at least 61. Let g represent the number of games he needs to bowl.

$34 + 2g \geq 61$ Write the inequality.
$-34 \quad -34$ Subtract 34 from each side.
$2g \geq 27$ Simplify.
$\frac{2g}{2} \geq \frac{27}{2}$ Divide each side by 2.
$g \geq 13.5$ Simplify.

Jamal should have at least 61 strikes after 14 more games.

Guided Practice

Solve each inequality. Graph the solution set on a number line. (Examples 1–4)

1. $5x - 7 \geq 43$ $x \geq 10$

2. $11 \leq 7 + \frac{x}{5}$ $x \geq 20$

3. **Financial Literacy** A rental car company charges AED 50 plus AED 2 per kilometer to rent a car. Mr. Bilal does not want to spend more than AED 500 for his rental car. Write and solve an inequality to find how many kilometers he can drive and not spend more than AED 500. Interpret the solution. (Example 5)

$50 + 2d \leq 500$; $d \leq 225$; He can drive no more than 225 kilometers.

4. **Building on the Essential Question** Compare $2x + 8 > 18$ and $2x + 8 \leq 18$.

$x > 5$; $x \leq 5$; Sample answer: Both inequalities contain the same terms. However, one is greater than while the other also includes equals.

Rate Yourself!

How well do you understand solving two-step inequalities? Circle the image that applies.

Name _____ My Homework

Independent Practice

Solve each inequality. Graph the solution set on a number line. Examples 1–4

1. $6x + 14 \geq 20$ $x \geq 1$

2. $4x - 13 < 11$ $x < 6$

3. $-20 > -2x + 4$ $x > 12$

4. $\frac{x}{13} + 3 \geq 4$ $x \geq 13$

5. Jassim needs at least AED 830 for a new video game system. He has already saved AED 200. He earns AED 30 an hour at his job. Write and solve an inequality to find how many hours he will need to work to buy the system. Interpret the solution. Example 5

Inequality: $200 + 30x \geq 830$ Solution: $x \geq 21$ hours

Interpretation: He will have to work at least 21 hours.

Reason Abstractly Write and solve an inequality for each sentence.

6. Three times a number increased by four is less than −62.
$3x + 4 < -62; x < -22$

7. The quotient of a number and 5 increased by one is at most 7.
$\frac{x}{5} + 1 \leq 7; x \geq -30$

8. The quotient of a number and 3 minus two is at least −12.
$\frac{x}{3} - 2 \geq -12; x \geq -30$

9. The product of −2 and a number minus six is greater than −18.
$-2x - 6 > -18; x < 6$

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

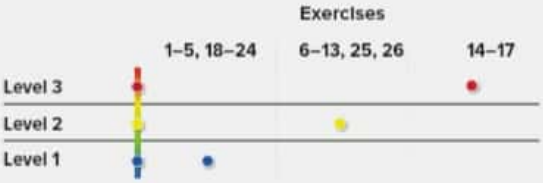
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7–13 odd, 14, 16, 17, 25, 26
OL	On Level	1–5 odd, 6–14, 16, 17, 25, 26
BL	Beyond Level	6–17, 25, 26

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	15
2 Reason abstractly and quantitatively.	6–9, 23
4 Model with mathematics.	14, 17
5 Use appropriate tools strategically.	16

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a clos ing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write abo ut the similarities and differences between solving two-st ep equations and two-step inequalities. See students' work.

Watch Out!

Common Error Some students may forget that they are solving an inequality and use an equals sign. Remind them that each step of their solution should contain an inequality symbol as the one in the original inequality, or the reverse symbol if they multiplied or divided by a negative number.

Write a two-step inequality that could be represented by each number line.

10. 

Sample answer: $4x + 1 > 53$

11. 

Sample answer: $-2x + 5 > -7$

12. 

Sample answer: $2x - 6 \leq 186$

13. 

Sample answer: $\frac{x}{2} + 5 \geq 30$

H.O.T. Problems Higher Order Thinking

14. **Model with Mathematics** Write a real-world example that could be solved by using the inequality $4x + 8 \geq 32$. Then solve the inequality.
Sample answer: Hala plans on spending at least AED 32 on a new purse. She has already saved AED 8. If she earns AED 4 an hour tutoring, how many hours will she need to tutor to save at least AED 32?; at least 6 hours

15. **Persevere with Problems** In five games, you score 16, 12, 15, 13, and 17 points. Write and solve an inequality to determine how many points you must score in the sixth game to have an average of at least 15 points.
$\frac{73 + x}{6} \geq 15$; $x \geq 17$; at least 17 points

16. **Use Math Tools** Solve $-x + 6 > -(2x + 4)$. Then graph the solution set on the number line.
Solution: $x > -10$

17. **Model with Mathematics** Write and solve a real-world problem that can be represented by the inequality $4(2.8) \leq 45$.
Sample answer: At the electronics store, CDs were marked down AED 2.80 from their regular price. Fahd has AED 45 to spend on 4 CDs. How much can he spend on each CD?; AED 14.05

Name _____ My Homework _____

Extra Practice

Solve each inequality. Graph the solution set on a number line.

18. $4x - 15 \leq 5$ $x \leq 5$

$4x - 15 \leq 5$
$+15 \quad +15$
$4x \leq 20$
$x \leq 5$

19. $-73 \geq 15 + 11x$ $x \leq -8$

20. $\frac{x}{5} - 2 > 1$ $x > 15$

21. $9 \leq \frac{x}{11} + 6$ $x \geq 42$

22. Ayesha is starting a babysitting business. She spent AED 26 to make signs to advertise. She charges an initial fee of AED 5 and then AED 3 for each hour of service. Write and solve an inequality to find the number of hours she will have to babysit to make a profit. Interpret the solution.

Inequality: $5 + 3h > 26$ Solution: $h > 7$

Interpretation: **Ayesha must babysit more than 7 hours to make a profit.**

23. **Reason Abstractly** As a salesperson, Ahmed earns AED 550 per week plus AED 30 per sale. This week, he wants his pay to be at least AED 850. Write and solve an inequality for the number of sales Ahmed needs to make. Interpret the solution.

Inequality: $550 + 30s \geq 850$ Solution: $s \geq 10$

Interpretation: **Ahmed needs to make at least 10 sales for his pay to be AED 850.**

24. Jamal and his sister went to the movies. They had AED 34 altogether and spent AED 9.50 per ticket. Jamal and his sister bought the same snacks. Write and solve an inequality for the amount that each person spent on snacks. Interpret the solution.

Inequality: $19 + 2a \leq 34$ Solution: $a \leq 7.5$

Interpretation: **Each person spent a maximum of AED 7.50 on snacks.**

Power Up! Test Practice

Exercises 25 and 26 prepare students for more rigorous thinking needed for the assessment.

25.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

26.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly model the inequality and solve it.
1 point	Students correctly model the inequality OR solve it.

Power Up! Test Practice

25. Which of the following are operations that you should use to solve $-2x - 5 < 7$ for x? Select all that apply.

- ☐ Subtract 7 from both sides.
- ☐ Add 5 to both sides.
- ☐ Divide both sides by -2.
- ☐ Reverse the inequality symbol.

26. The table shows the cost of renting a jet ski.

Rental Period	Cost (AED)
First hour	AED 55
Each additional 15-minutes	AED 10

Abdalla can spend no more than AED 105 on a jet ski rental. Let x represent the number of additional 15-minute increments. Fill in each box to write an inequality to represent the situation.

55	+	10x	≤	105

10x	10
55x	55
105x	105
<	≤
>	≥

What is the greatest length of time Abdalla can rent the jet ski?

135 minutes, or 2 hours 15 minutes

Spiral Review

Solve and graph each inequality.

27. $n + 1 > -2$

Solution: $n > -3$

28. $-2y > 12$

Solution: $y < -6$

29. $\frac{t}{-2} > -2$

Solution: $t < 2$

Solve each equation. Check your solution.

30. $5y + 6 = 46$ 8

31. $-4k - 1 = 47$ −12

32. $5 = 8m + 1$ $\frac{1}{2}$

33. Ali's dad is 30 years of age. He is 2 years more than four times Ali's age m. Write and solve a two-step equation to determine Ali's age.

Equation: $4m + 2 = 30$ Solution: 7 years

21ST CENTURY CAREER in Veterinary Medicine

Veterinary Technician

If you love being around animals, enjoy working with your hands, and are good at analyzing problems, a challenging career in veterinary medicine might be a perfect fit for you. Veterinary technicians help veterinarians by helping to diagnose and treat medical conditions. They may work in private clinics, animal hospitals, zoos, aquariums, or wildlife rehabilitation centers.

Is This the Career for You?

Are you interested in a career as a veterinary technician? Take some of the following courses in high school.

- Algebra
- Animal Science
- Biology
- Chemistry
- Veterinary Assisting

Find out how math rela tes to Medicine.

Focus narrowing the scope

Objective Apply mathematics to problems arising in the workplace.
This lesson emphasizes **Mathematical Practice 4** Model with Mathematics.

Coherence connecting within and across grades

Previous
Students wrote and solved equations to represent real-world problems.

Now
Students apply the content standard to solve problems in the workplace.

Rigor pursuing concepts, fluency, and applications

See the Career Project on page 522.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ask students to read the information on the student page about veterinary medicine and answer the following questions.

Ask:

- *What does a veterinary technician do?* helps the veterinarian by recording case histories, conducting medical tests, and helping to diagnose and treat animals
- *What kinds of classes should you take to be a veterinary technician?* Algebra, Animal Sciences and Services, Biology, Veterinary Assisting, Chemistry

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** EVALUATE

2 Collaborate

AL LA **Trade-a-Problem** Have students create their own problem using the information in the tables. Have students trade their problems with a partner, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find any errors. MP 1, 2, 4

BL LA **Pairs Discussion** Have students work in pairs to answer the following extension question. MP 1, 2, 4

Ask:

- *Cotton-top tamarins can cover up to 1.1 kilometers per day while searching for food. If they move about 0.15 kilometer per hour, write and solve an equation to determine about how many hours it takes them to travel 1.1 kilometers.*
  $1.1 = 0.15t$; about 7.3 hours

Career Portfolio

When students complete this page, have them add it to their Career Portfolio.

Career Facts

A similar career to a veterinary technician is a veterinary technologist. Both careers involve similar job responsibilities, but different educational requirements. Veterinary technicians attend accredited, two-year veterinary technology programs and usually earn Associate degrees. Veterinary technologists attend four year programs and usually earn Bachelor's degrees.

Vet Techs Don't Monkey Around

For each problem, use the information in the tables to write an equation. Then solve the equation.

1. The minimum tail length of an emperor tamarin is 4 centimeters greater than that of a golden lion tamarin. What is the minimum tail length of a golden lion tamarin?
   $35 = t + 4$; 31 cm
2. The minimum body length of a golden lion tamarin is 8.5 centimeters less than the maximum body length. What is the maximum body length? $20 = \ell - 8.5$; 28.5 cm
3. Tamarins live an average of 15 years. This is 13 years less than the years that one tamarin in captivity lived. How long did the tamarin in captivity live? $15 = y - 13$; 28 yr
4. The maximum weight of a golden lion tamarin is about 1.97 times the maximum weight of an emperor tamarin. What is the maximum weight of an emperor tamarin? Round to the nearest tenth.
   $790 = 1.97w$; 401 g
5. For an emperor tamarin, the max imum total length, including the body and t ail, is 67.5 centimeters. What is the m aximum body length of an emperor tamarin?
   $b + 42 = 67.5$; 25.5 cm.

Golden Lion Tamarin Monkeys

Measure	Minimum	Maximum
Body length	20 cm	ℓ
Tail length	t	39 cm
Weight	360 g	790 g

Emperor Tamarin Monkeys

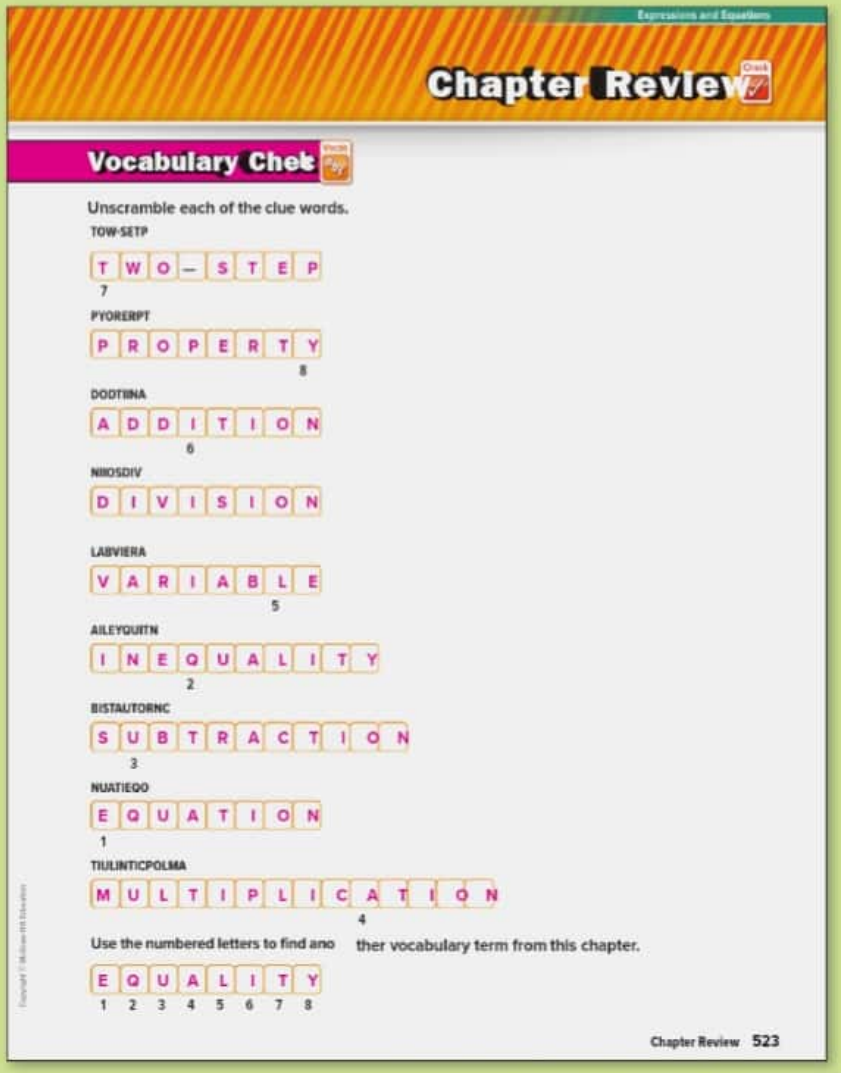
Measure	Minimum	Maximum
Body length	23 cm	b
Tail length	35 cm	42 cm
Weight	303 g	w

Career Project

It's time to update your career portfolio! Go online and research a career as a veterinary technician. Include brief descriptions of the work environment, education and training requirements, and the job outlook.

Do you think you would enjoy a career as a veterinary technician? Why or why not?

- _____
- _____
- _____
- _____
- _____



Expressions and Equations

Chapter Review

Vocabulary Check

Unscramble each of the clue words.

TOW-SETP

T W O – S T E P
7

PYORERPT

P R O P E R T Y
8

DODTIINA

A D D I T I O N
6

NIROSDIV

D I V I S I O N

LABVIERA

V A R I A B L E
5

AILEYQUITN

I N E Q U A L I T Y
2

BISTAUTORNC

S U B T R A C T I O N
3

NUATIEQO

E Q U A T I O N
1

TIULINTICPOLMA

M U L T I P L I C A T I O N
4

Use the numbered letters to find another vocabulary term from this chapter.

E Q U A L I T Y
1 2 3 4 5 6 7 8

Vocabulary Check

LA **Find the Fib** Have students work in small groups to complete the Vocabulary Check. Have students write two facts and one fib, relating to the vocabulary terms. For example, one fact could be that a variable represents an unknown quantity. One fib could be that $t = 15$ is an example of a two-step equation. Then have each student read aloud what they wrote to the group. The group will discuss each fact and fib, identify the fibs, and correct them so that they become facts.
MP 1, 3, 6

Alternate Strategy

AL LA To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include the following terms.

- Addition Property of Equality (Lesson 1)
- Addition Property of Inequality (Lesson 6)
- Division Property of Equality (Lesson 2)
- Division Property of Inequality (Lesson 7)
- Multiplication Property of Equality (Lesson 2)
- Multiplication Property of Inequality (Lesson 7)
- Subtraction Property of Equality (Lesson 1)
- Subtraction Property of Inequality (Lesson 6)
- two-step equation (Lesson 4)
- two-step inequality (Lesson 8)
- variable (Lesson 5–1)

Key Conc ept Check

FOLDABLES LA A completed Foldable for this chapter should include a review of solving two-step equations.

If you choose not to use this Foldable, have students write a brief review of the Key Concepts found throughout the chapter and give an example of each.

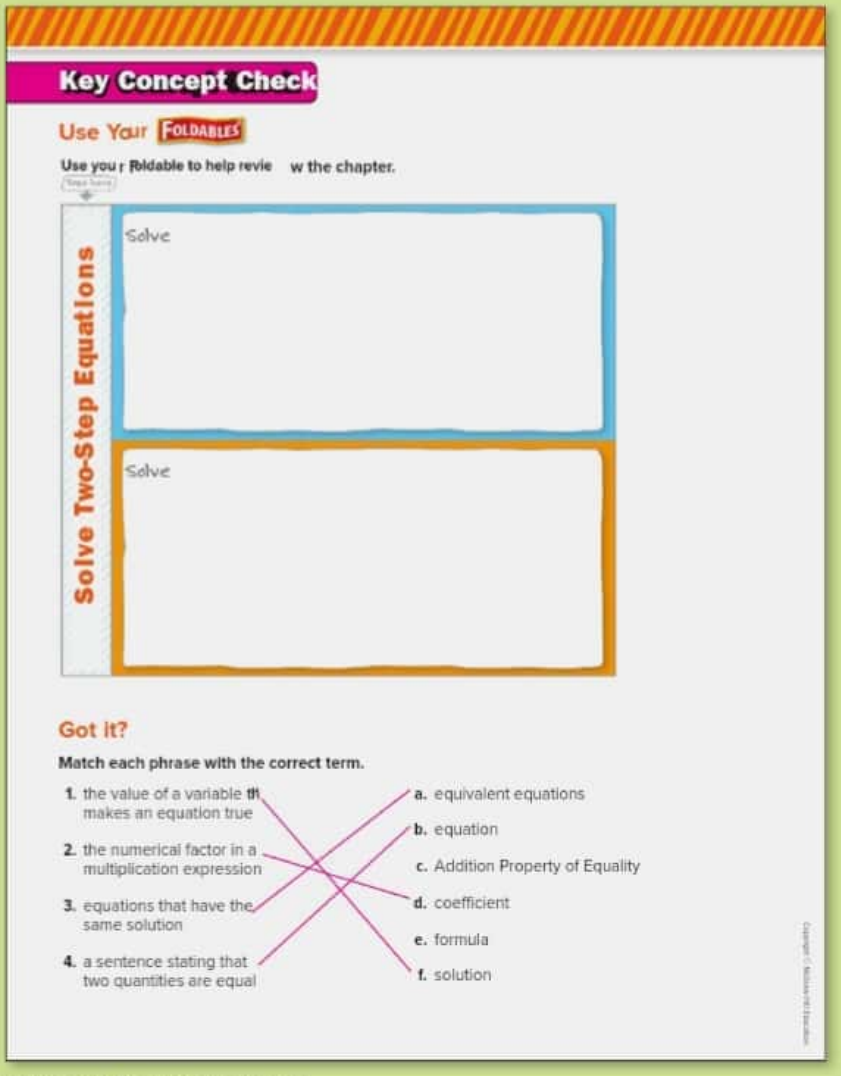
Ideas for Use

LA Have students work in pairs to discuss their Foldables. Have them practice speaking in a collaborative setting by sharing how they have completed their Foldable thus far and how they could finish it. Have each student complete their Foldable and trade with their partner to discuss any similarities and differences MP 1, 3, 5

Got It?

If students have trouble with Exercises 1–4, they may need help with the following concept(s).

Concept	Exercise(s)
equations (Lesson 1)	1, 3, 4
coefficients (Lesson 2)	2



Key Concept Check

Use Your FOLDABLES

Use your Foldable to help review the chapter.

Solve Two-Step Equations

Solve

Solve

Got It?

Match each phrase with the correct term.

1. the value of a variable that makes an equation true
2. the numerical factor in a multiplication expression
3. equations that have the same solution
4. a sentence stating that two quantities are equal

a. equivalent equations
b. equation
c. Addition Property of Equality
d. coefficient
e. formula
f. solution

Geometry

Essential Question

HOW can you use different measurements to solve real-life problems?

Chapter 7

Geometric Figures

Geometric shapes can be drawn freehand, with a ruler and protractor, or using technology. In this chapter, you will draw two- and three-dimensional figures. You will also solve problems involving scale drawings of geometric figures.

Chapter 8

Measure Figures

Real-life problems involving area, surface area, and volume can be solved by using formulas. In this chapter, you will use formulas to find the area and circumference of a circle and to find the surface area and volume of prisms and pyramids.

Essential Question

At the end of this unit, students should be able to answer "How can you use different measurements to solve real-life problems?"

Each chapter explores a different essential question that assists students in answering the unit question. The lessons in each chapter include exercises that lead students to various aspects of the essential question.

Draw, construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
3. Describe the two-dimensional figures that result from slicing threedimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

continued on page 530

Focus narrowing the scope

This chapter focuses on **Geometry**.

Coherence connecting within and across grades

Previous	Now	Next
Students simplified expressions and solved equations.	Students create and draw two- and three-dimensional figures.	Students will use various methods to measure figures.

Rigor pursuing concepts, fluency, and applications

The Levels of Complexity charts located throughout this chapter indicate how the exercises progress from conceptual understanding and procedural skills and fluency, to application and critical thinking.

Launch the Chapter

Math in the Real World

Robots Tell students that to find the length of the robot's arm on the drawing, they can write and solve a proportion.

What Tools Do You Need

Vocabulary Activity

LA As you proceed through the chapter, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define: Supplementary angles are two angles with measures that have a sum of 180°.

Example: $m\angle A = 120°$ and $m\angle B = 60°$, so they are supplementary angles.

Ask:

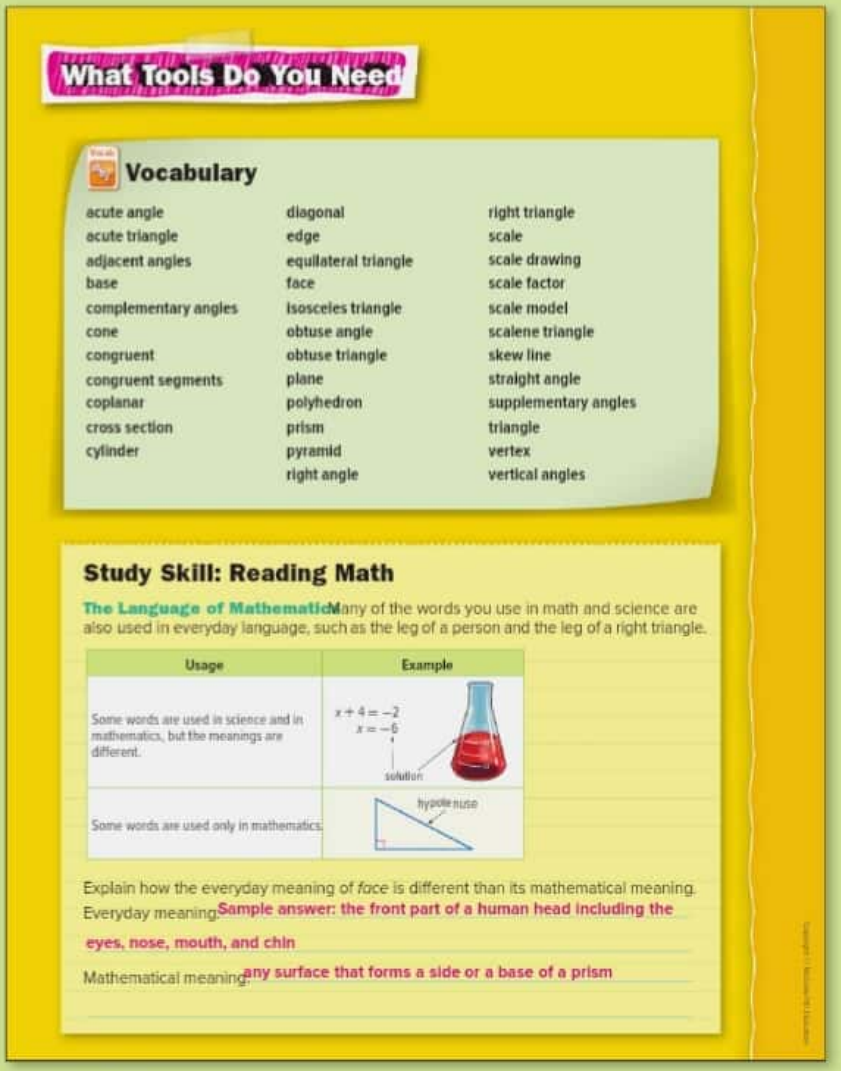
- *What is the measure of an angle supplementary to an angle with a measure of 56°?* 124°

Reading Math

LA Have students read the Reading Math section. Students will learn about mathematical vocabulary that also have meanings in science or everyday usage. Have students read about the language of mathematics. Have them suggest words that they believe have meaning in both mathematics and in everyday or scientific use.

Ask:

- *What are the everyday and mathematical meanings for "side"?* Sample answer: a position held in a dispute; a segment that forms part of a geometric figure



What Tools Do You Need

Vocabulary

acute angle	diagonal	right triangle
acute triangle	edge	scale
adjacent angles	equilateral triangle	scale drawing
base	face	scale factor
complementary angles	isosceles triangle	scale model
cone	obtuse angle	scalene triangle
congruent	obtuse triangle	skew line
congruent segments	plane	straight angle
coplanar	polyhedron	supplementary angles
cross section	prism	triangle
cylinder	pyramid	vertex
	right angle	vertical angles

Study Skill: Reading Math

The Language of Mathematics Many of the words you use in math and science are also used in everyday language, such as the leg of a person and the leg of a right triangle.

Usage	Example
Some words are used in science and in mathematics, but the meanings are different.	$x + 4 = -2$, $x = -6$, solution
Some words are used only in mathematics.	hypotenuse

Explain how the everyday meaning of *face* is different than its mathematical meaning.

Everyday meaning: Sample answer: the front part of a human head including the eyes, nose, mouth, and chin

Mathematical meaning: any surface that forms a side or a base of a prism

Are You Ready

Use this page to determine if students have skills that are needed for the chapter.

Quick Review

Students with strong math backgrounds may opt to go directly to Quick Check.

Example	Skill
1	Find Measures of Angles
2	Area of a Triangle

Quick Check

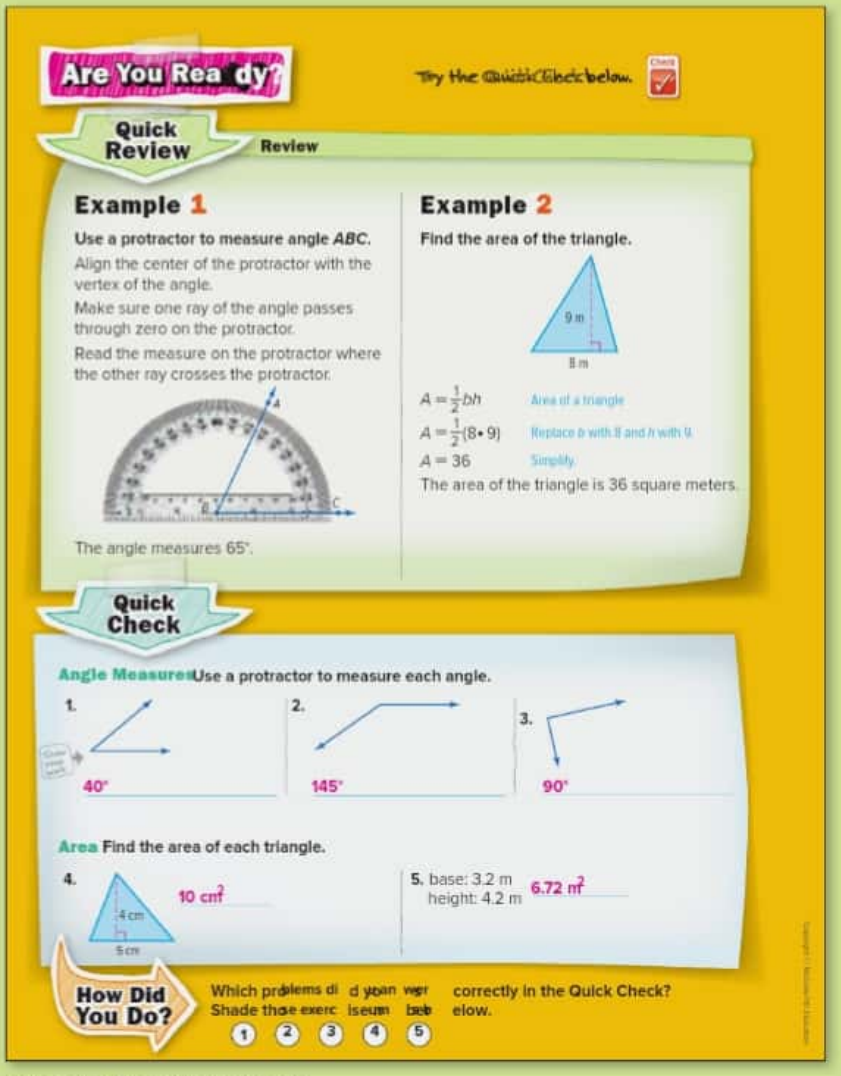
If students have difficulty with the exercises, present an additional example to clarify any misconceptions.

Exercises 1–3

Use a protractor to find the measure of the angle shown. 30°

Exercises 4–5

Find the area of a triangle with a base of 8 meters and a height of 5.2 meters. 20.8 m²



Geometry

Lesson 1

Classify Angles

Vocabulary Start-Up

An angle is formed by two rays that share a common endpoint. The **vertex** is the point where the two rays meet.

Complete the table by drawing the hands of a clock to represent each angle. Sample answers are given.

Type of Angle			
Right	Acute	Obtuse	Straight
exactly 90°	less than 90°	greater than 90°	exactly 180°

Essential Question

HOW does geometry help us describe real-world objects?

Vocabulary

vertex
right angle
acute angle
obtuse angle
straight angle
vertical angles
congruent
adjacent angles

Math Symbols

∠
≅

Mathematical Practices

1, 3, 4, 7

Real-World Link

The angle formed by a bike ramp is shown.

1. What type of angle is formed?
   acute
2. Estimate the measure of the angle.
   Sample answer: 20°

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Classify angles and identify vertical and adjacent angles.

Coherence connecting within and across grades

Previous	Now	Next
Students identified types of two-dimensional figures.	Students identify and classify different types of angles.	Students will compare angle relationships.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 539.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Pairs Discussion** Have students work with a partner to complete the table and complete Exercises 1 and 2 in the Real-World Link. Have students draw several different possibilities for each type of angle. Call on one pair of students to share their responses with the class. MP 1, 6

Alternate Strategy

AL LA Have students work in pairs to create vocabulary cards using the terms *vertex*, *acute*, *right*, *obtuse*, and *straight*. Have students draw an example for each word. Have them use these cards as a reference throughout the chapter. MP 1, 4, 6

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

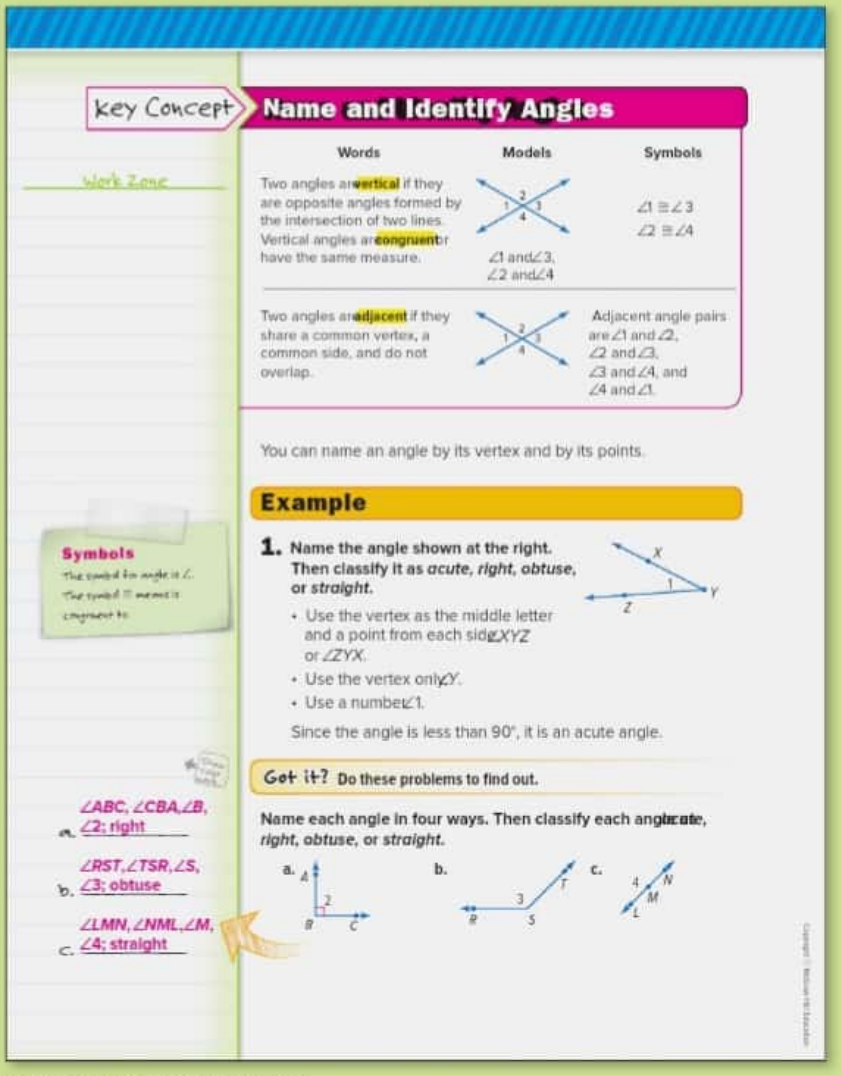
1. Name and classify angles.

AL
- *How would you name the angle using a number?* ∠1
- *What is the vertex of the angle?* Y
- *What is a vertex?* the point common to two sides of an angle

OL
- *How can you name the angle using only its vertex?* ∠Y
- *How can you name the angle using its vertex and a point from each ray?* ∠XYZ or ∠ZYX
- *How would you classify the angle?* an acute angle

BL
- *Estimate the measure of ∠1.* Sample answer: about 30°
- *If there were another point on ray YX, and this point was labeled P, give two other possibilities for naming this angle.* ∠PYZ or ∠ZYP

Need Another Example?
Name the angle shown in four ways. Then classify it as *acute*, *right*, *obtuse*, or *straight*. ∠FGH, ∠HGF, ∠G, ∠2; acute



Example

2. **Identify a pair of vertical angles and adjacent angles in the diagram at the right. Justify your response.**

Since ∠2 and ∠4 are opposite angles formed by the intersection of two lines, they are vertical angles.

Since ∠1 and ∠2 share a common side and vertex, and they do not overlap, they are adjacent angles.

Got it? **Do this problem to find out.**

d. Refer to the diagram in Example 2. Identify different pairs vertical and adjacent angles. Justify your response.

d. Sample answer: ∠1 and ∠3 are vertical angles because ∠1 and ∠3 are opposite angles formed by the intersection of two lines. ∠2 and ∠3 are adjacent angles because ∠2 and ∠3 share a common vertex, a common side, and do not overlap.

Find a Missing Measure

You can use what you learned about vertical and adjacent angles to find the value of a missing measure.

Example

3. **What is the value of *x* in the figure?**

The angle labeled $(2x + 2)°$ and the angle labeled 130° are vertical angles.

Since vertical angles are congruent, $(2x + 2)°$ equals 130°.

$2x + 2 = 130$ Write the equation.

$-2 = -2$ Subtract 2 from each side.

$\frac{2x}{2} = \frac{128}{2}$ Divide each side by 2.

$x = 64$

So, the value of *x* is 64.

Got it? **Do this problem to find out.**

e. What is the value of *y* in the figure in Example 2?

e. 20

Examples

2. **Identify vertical angles and adjacent angles.**

AL • *Which pairs of angles are vertical?* ∠1 and ∠3, ∠2 and ∠4

• *What kind of angle is formed by ∠1 and ∠2?* straight angle

OL • *What do you know about the measures of ∠1 and ∠3, and ∠2 and ∠4?* They have the same measure.

BL • *If m∠1 = 75°, what is m∠2? Explain.* 105°, because ∠1 and ∠2 form a straight line and the sum is equal to 180°.

Need Another Example?

Identify a pair of vertical angles and adjacent angles in the diagram. Justify your response. Sample answer: ∠3 and ∠5 are vertical because they are opposite angles formed by intersecting lines; ∠4 and ∠5 are adjacent because they share a common side and do not overlap.

3. **Find missing measures.**

AL • *What type of angles are the 130° angle and the angle labeled (2x + 2)°?* They are vertical angles.

OL • *What is true about the measures of vertical angles?* They are equal.

• *What angles are adjacent to the 50° angle?* the ones with measures of 130° and $(2x + 2)°$

BL • *What equation could you use to solve for y?* $3y - 10 = 50$

• *What is the value of y?* $y = 20$

Need Another Example?

What is the value of *x* in the figure? 60

Example

4. Write and solve an equation to find a missing measure.

AL • *What do you know about the two labeled angles?* They are adjacent angles that form a straight angle.

• *What is the degree value of a straight angle?* 180°

OL • *What is the equation that we can use to find the value of x?* $115 + 5x = 180$

• *What is the measure of the angle labeled (5x)? Explain.* 65°; Since $x = 13$ and $5 \times 13 = 65$, the measure of the angle labeled (5x) is 65°.

BL • *Explain why you set the addition equation equal to 180°.* The line and ray drawn (not the streets themselves), form adjacent angles that together make a straight angle of 180°.

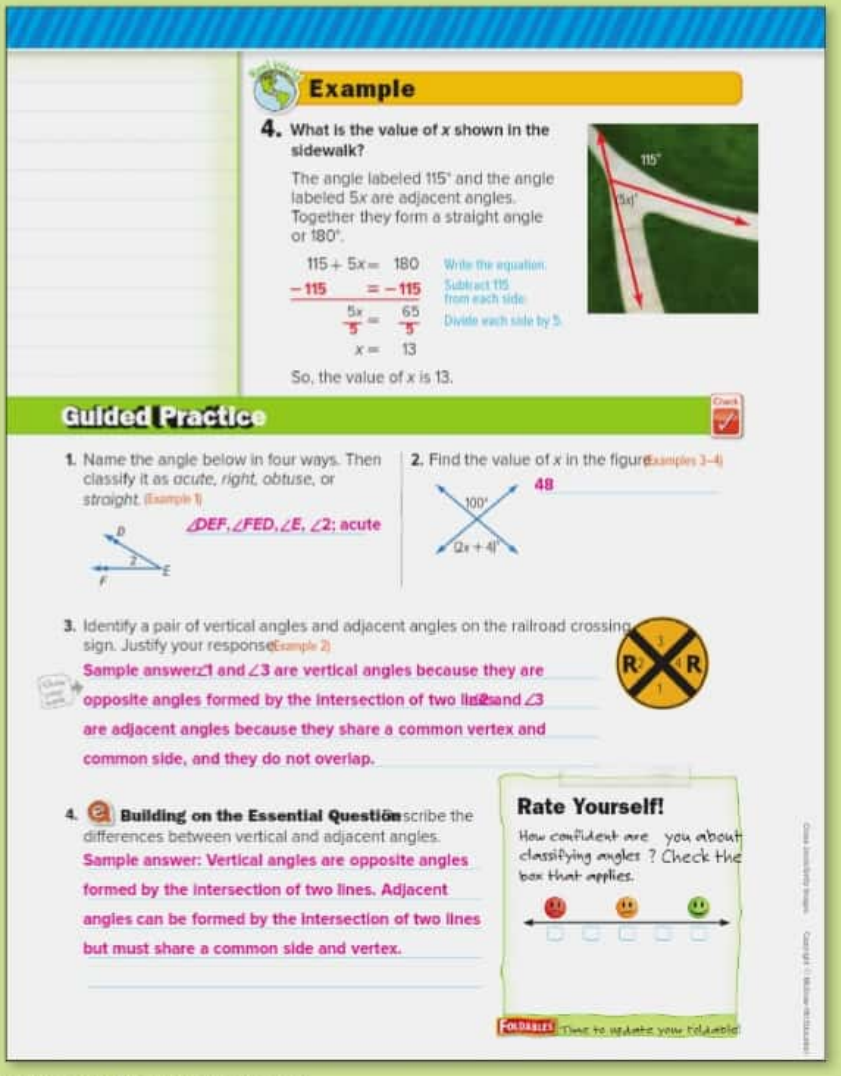
Need Another Example?
What is the value of x in the figure shown? 10

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

AL LA **Pairs Consult** Have students work in pairs to complete Exercises 1–3. Prior to completing the exercises, you may wish to have them create a graphic organizer or other visual display to help them record the types of angles and angle relationships they learned in this lesson. They can use this as a reference to aid them in completing the exercises.
MP 1, 5, 6



Example

4. What is the value of x shown in the sidewalk?

The angle labeled 115° and the angle labeled 5x are adjacent angles. Together they form a straight angle or 180°.

$115 + 5x = 180$ Write the equation.
$-115 \quad = -115$ Subtract 115 from each side.
$\frac{5x}{5} = \frac{65}{5}$ Divide each side by 5.
$x = 13$

So, the value of x is 13.

Guided Practice

1. Name the angle below in four ways. Then classify it as *acute, right, obtuse,* or *straight.* (Example 1)
∠DEF, ∠FED, ∠E, ∠2; acute

2. Find the value of x in the figure. (Examples 3–4)
48

3. Identify a pair of vertical angles and adjacent angles on the railroad crossing sign. Justify your response. (Example 2)
Sample answer: ∠1 and ∠3 are vertical angles because they are opposite angles formed by the intersection of two lines. ∠2 and ∠3 are adjacent angles because they share a common vertex and common side, and they do not overlap.

4. **Building on the Essential Question** Describe the differences between vertical and adjacent angles.
Sample answer: Vertical angles are opposite angles formed by the intersection of two lines. Adjacent angles can be formed by the intersection of two lines but must share a common side and vertex.

Rate Yourself!
How confident are you about classifying angles? Check the box that applies.

FOLDABLES Time to update your foldable!

Name _____ My Homework _____

Independent Practice

Name each angle in four ways. Then classify the angle as *acute*, *right*, *obtuse*, or *straight*. Example 1

1. 

∠ABC, ∠CBA, ∠B, ∠4; acute

2. 

∠DEF, ∠FED, ∠E, ∠5; right

3. 

∠MNP, ∠PNM, ∠N, ∠1; obtuse

Identify Structure Refer to the diagram at the right. Identify each angle pair as *adjacent*, *vertical*, or *neither*. Example 2

4. ∠2 and ∠5 vertical

5. ∠4 and ∠6 neither

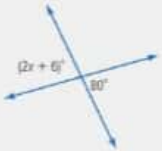
6. ∠3 and ∠4 adjacent

7. ∠5 and ∠6 adjacent

8. ∠1 and ∠3 neither

9. ∠1 and ∠4 vertical

10. What is the value of x in the figure at the right? 37 Examples 3 and 4



11. What is the value of x in the figure at the right? 11 Examples 3 and 4

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

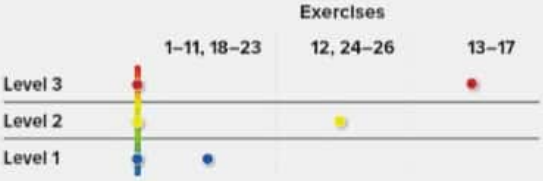
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–11, 13, 14, 17, 25, 26
OL	On Level	1–11 odd, 12–14, 17, 25, 26
BL	Beyond Level	12–17, 25, 26

Watch Out!

Common Error When more than two lines intersect, students may mistakenly identify angle pairs as being vertical when they are neither vertical nor adjacent. When classifying angle pairs with multiple intersecting lines, have students first identify the lines that intersect to create each angle. If the same straigh lines are used to create both angles, then the angles are vertica

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	15, 16
3 Construct viable arguments and critique the reasoning of others.	14, 17
4 Model with mathematics.	13
7 Look for and make use of structure.	4–9, 24

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Ask students to use their own words to write a definition for vertical and adjacent angles. See students' work.

12. Angles ABC and DBE are vertical angles. If the measure of $\angle ABC$ is 40°, what is the measure of $\angle ABD$?
140°

H.O.T. Problems Higher Order Thinking

13. **Model with Mathematics** Draw examples of angles that represent real-world objects. Be sure to include at least three of the following angles: acute, right, obtuse, straight, vertical, and adjacent. Verify by measuring the angles.
See students' work.

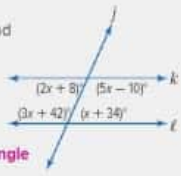
14. **Reason Inductively** Explain how you can use a protractor to measure the angle shown. Find the measure of the angle.
Sample answer: Since a full circle measures 360°, measure the smaller angle, then subtract that measure from 360; 320°.

Persevere with Problems Determine whether each statement is *true* or *false*. If the statement is true, provide a diagram to support it. If the statement is false, explain why.

15. A pair of obtuse angles can also be vertical angles.
True; Sample answer:

16. A pair of straight angles can also be adjacent angles.
True; Sample answer:

17. **Reason Inductively** Lines ℓ and k shown at the right are parallel and are intersected by line j. Explain how you can write and solve equations to find the measure of each angle. Then find the measure of each angle.
Sample answer: Each pair of angles is adjacent and forms a straight angle. So $(2x + 8) + (5x - 10) = 180$ and $(3x + 42) + (x + 34) = 180$. When you solve both equations, $x = 26$. The angle measures are 60°, 120°, 120°, and 60°.



Name _____ My Homework _____

Extra Practice

Name each angle in four ways. Then classify the angle as *acute*, *right*, *obtuse*, or *straight*.

18.

∠MNP, ∠PNM, ∠N, ∠7;
straight

19.

∠HKI, ∠IKH, ∠K, ∠8;
obtuse

20.

∠RTS, ∠STR, ∠T, ∠9;
acute

21. The corner where the states of Utah, Arizona, New Mexico, and Colorado meet is called the Four Corners.

a. Identify a pair of vertical angles. Justify your response.

Sample answer: ∠1 and ∠3; Since ∠1 and ∠3 are opposite angles formed by the intersection of two lines, they are vertical angles.

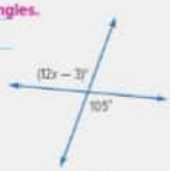
b. Identify a pair of adjacent angles. Justify your response.

Sample answer: ∠1 and ∠2; Since ∠1 and ∠2 share a common vertex, a common side, and do not overlap, they are adjacent angles.

22. What is the value of x in the figure at the right?

26

23. What is the value of x in the figure at the right?



9

24. **Identify Structure** The John Hancock Center in Chicago is shown at the right. Classify each pair of angles.

a. ∠1 and ∠2 adjacent

b. ∠2 and ∠4 vertical

c. ∠3 and ∠4 adjacent

d. ∠1 and ∠3 vertical

e. If the measure of ∠2 is 66°, what are the measures of the other angles? ∠1 measures 114°, ∠4 measures 66°, ∠3 measures 114°

Power Up! Test Practice

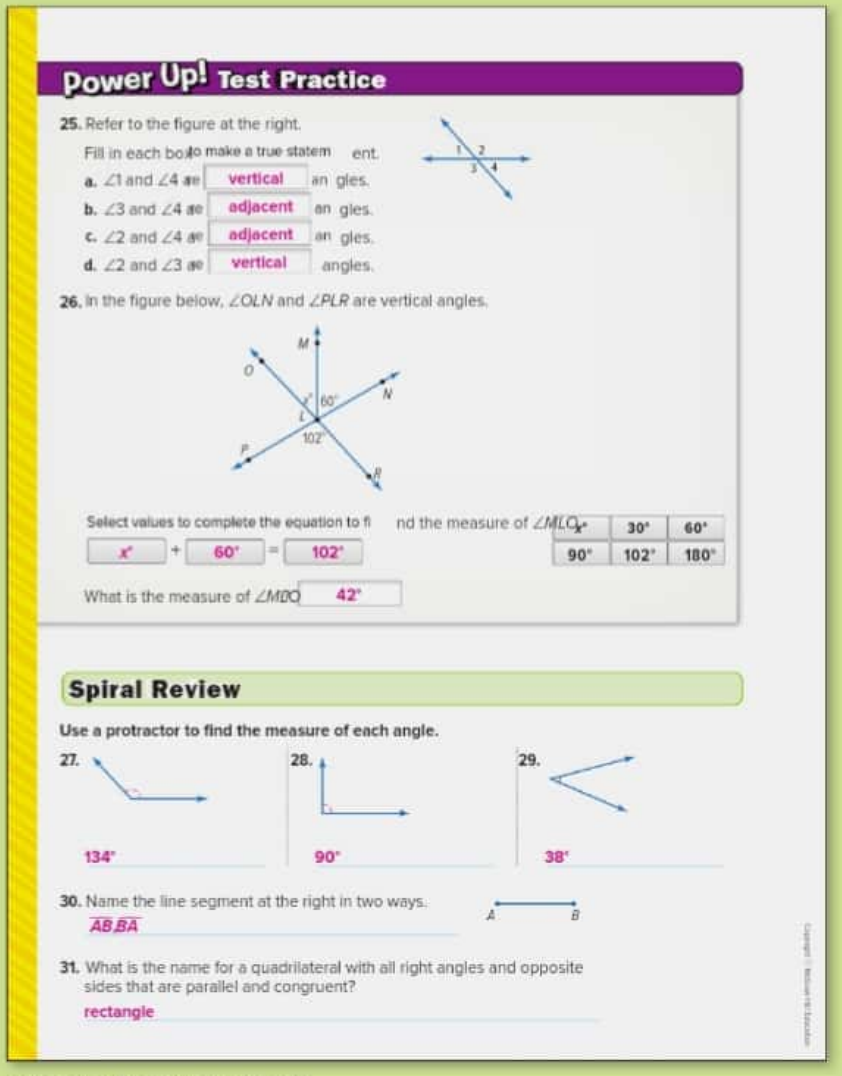
Exercises 25 and 26 prepare students for more rigorous thinking needed for the assessment.

25. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

26. This test item requires students to reason abstractly and quantitatively when problem solving.

Depth of Knowledge	DOK1
Mathematical Practice	MP1, MP4
Scoring Rubric	
2 points	Students correctly model the equation and find the measure of the angle.
1 point	Students correctly model the equation OR find the measure of the angle.



Power Up! Test Practice

25. Refer to the figure at the right.

Fill in each box to make a true statement.

a. ∠1 and ∠4 are vertical angles.

b. ∠3 and ∠4 are adjacent angles.

c. ∠2 and ∠4 are adjacent angles.

d. ∠2 and ∠3 are vertical angles.

26. In the figure below, ∠*OLN* and ∠*PLR* are vertical angles.

Select values to complete the equation to find the measure of ∠*MLQ*.

$x°$	30°	60°
90°	102°	180°

$x°$ + 60° = 102°

What is the measure of ∠*MLQ*? 42°

Spiral Review

Use a protractor to find the measure of each angle.

27. 134°

28. 90°

29. 38°

30. Name the line segment at the right in two ways.

$\overline{AB}$, $\overline{BA}$

31. What is the name for a quadrilateral with all right angles and opposite sides that are parallel and congruent?

rectangle

Geometry

Lesson 2

Complementary and Supplementary Angles

Real-World Link

Bridges Engineers use angles to construct bridges. The Golden Gate Bridge is created by combining angles as shown.

Essential Question

HOW does geometry help us describe real-world objects?

Vocabulary

complementary angles
supplementary angles

Math Symbols

$m\angle 1$

Mathematical Practices

1, 3, 4, 7

1. What types of angles make up the two angles marked in the drawing of the bridge? right angles

2. What is the sum of the two angles marked in the drawing of the bridge? 180°

3. In the space below, draw a figure that contains two angles that have a sum of 90°. Sample answer:

Which Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Identify pairs of complementary and supplementary angles.

Coherence connecting within and across grades

Previous
Students identified and classified vertical and adjacent angles.

Now
Students identify complementary and supplementary angles and find missing angle measures.

Next
Students will find missing angles measures of triangles.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 547.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Pairs Discussion** Have students work with a partner to complete Exercises 1–3. For Exercise 3, have them brainstorm different ways to draw the figure. Have them list several pairs of angle measures with a sum of 90°. MP 1, 5, 6

Alternate Strategy

AL LA Have students explain the meanings of *complementary and supplementary* outside of mathematics. Have them relate these meanings to the meanings of the terms within mathematics. Have them come up with a way that will help them remember the meanings of the terms as they relate to angle relationships. MP 1, 6

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Identify complementary and supplementary angles.**

AL • *What kind of line do ∠1 and ∠2 form?* a straight line

OL • *Are the angles complementary, supplementary, or neither? Explain.* supplementary; They form a straight angle, so the sum of their angle measures is 180°.

BL • *Estimate the measures of each angle.* ∠1 ≈ 135°, ∠2 ≈ 45°

• *Suppose the m∠1 = 135°, what is m∠2? Explain.* $m\angle 1 + m\angle 2 = 180°$; therefore, $m\angle 2 = 45°$

Need Another Example?
Identify the pair of angles as *complementary, supplementary,* or *neither.* supplementary

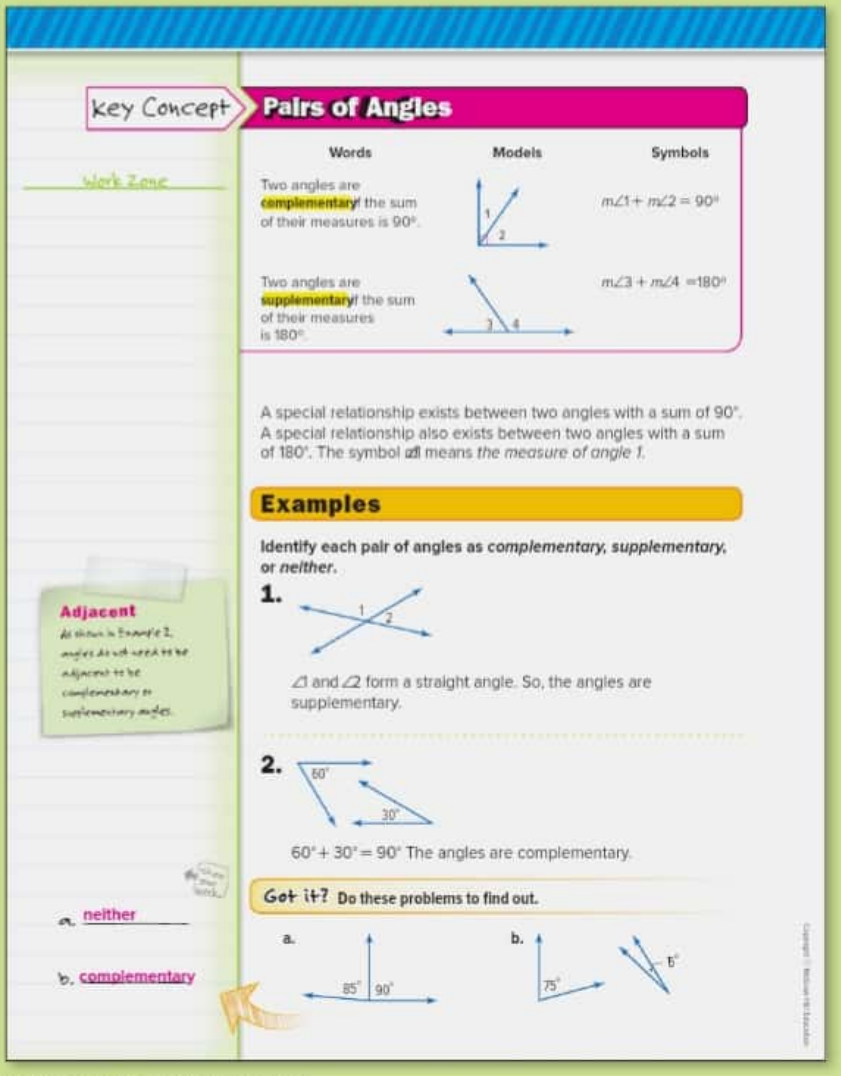
2. **Identify complementary and supplementary angles.**

AL • *Are the angles adjacent? Explain.* no; They do not share a common vertex.

OL • *Are the angles complementary, supplementary, or neither? Explain.* complementary; The sum of their angle measures is 90°.

BL • *Can two angles be complementary, if they are not adjacent? Explain.* Yes, as long as the sum of their measures equals 90°.

Need Another Example?
Identify ∠1 and ∠2 as *complementary, supplementary,* or *neither.* complementary



Find a Missing Measure

You can use angle relationships to find missing measures.

Examples

3. **Find the value of *x*.**

Since the two angles form a right angle, they are complementary.

Words	The sum of the measures of $\angle ABC$ and $\angle CBD$	is	90°.
Variable	Let $2x$ represent the measure of $\angle CBD$.		
Equation	$28 + 2x$	$=$	90

$28 + 2x = 90$ Write the equation.

$-28 \quad = -28$ Subtract 28 from each side.

$\frac{2x}{2} = \frac{62}{2}$ Divide each side by 2.

$x = 31$

So, the value of x is 31.

4. **The angles shown are supplementary. Find the value of *x*.**

$123 + 3x = 180$ Write the equation.

$-123 \quad = -123$ Subtract 123 from each side.

$\frac{3x}{3} = \frac{57}{3}$ Divide each side by 3.

$x = 19$

So, the value of x is 19.

Got it? **Do this problem to find out.**

c. Find the value of *x*.

STOP and Reflect

Circle true or false.
The sum of two angles that are supplementary is 180°.
True False

c. 45

Examples

3. **Find missing measures.**

AL • *Are the angles complementary or supplementary?* complementary

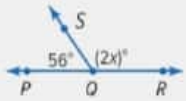
• *What is the sum of the measures of the two angles?* 90°

OL • *What equation can we use to find the value of x?* $28 + 2x = 90$

• *What steps do we use to solve the equation?* First, subtract 28 from both sides. Then divide both sides by 2.

BL • *If the value of x is 31, what is m∠CBD? Explain.* You have to substitute the value back into the expression $2x$; $m\angle CBD = 62°$.

Need Another Example?
Find the value of *x*. 62



4. **Find missing measures.**

AL • *If the angles are supplementary, what is the sum of their measures?* 180°

• *What is the measure of the angle we are given?* 123°

• *What is the algebraic expression for the other angle measure we are given?* $(3x)°$

OL • *What equation can we use to find the value of x?* $123 + 3x = 180$

• *What steps do we use to solve the equation?* First, subtract 123 from both sides. Then divide both sides by 3.

BL • *How can you tell, by looking at the diagrams of the angles, that the angles are not complementary?* Sample answer: One of the angles is obtuse, which means that angle measure is already greater than 90°.

Need Another Example?
The angles shown are complementary. Find the value of *x*. 9

Example

5. Find missing measures.

AL • *If the angles are supplementary, what is their sum?* 180°

• *What is the algebraic expression for the other angle measure we are given?* (10*x*)°

OL • *What equation can we use to find the value of x?* 80 + 10*x* = 180

• *What steps do we use to solve the equation?* First, subtract 80 from both sides. Then divide both sides by 10.

BL • *What is the measure of the angle labeled (10x)?* 100°

Need Another Example?
The hour, minute, and second hands make up two adjacent angles that form a right angle in the clock shown.
Find the value of *x*. 3

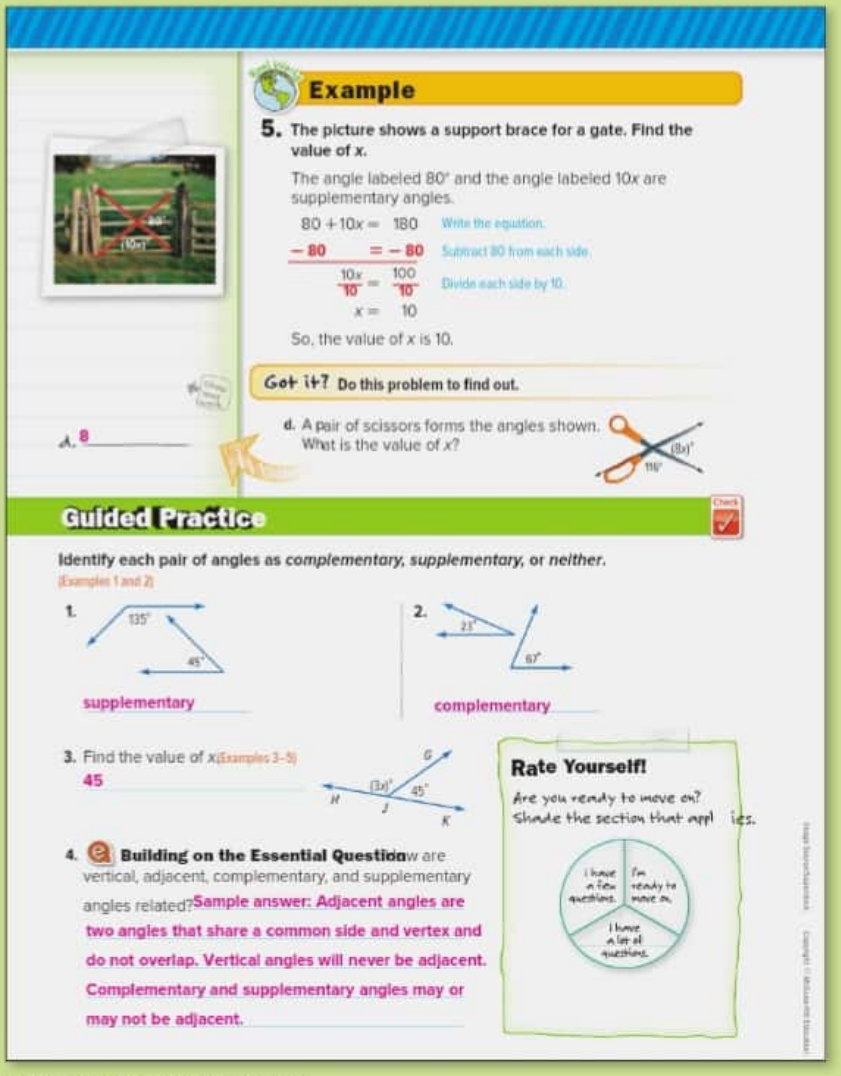
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Check** Have students complete Exercises 1–4 in pairs. Then have them join another pair to check their work and discuss and resolve any discrepancies. MP 1, 6

BL LA **Trade-a-Problem** Have students draw their own diagram using either complementary or supplementary angles. Have them provide the numerical measure of one of the angles. The other angle should be an algebraic expression involving a coefficient that is not 1. Have them trade diagrams with a partner. Each student writes and solves an equation to determine the value of *x* and then finds the measure of the unknown angle. MP 1, 2, 7



Example

5. The picture shows a support brace for a gate. Find the value of *x*.

The angle labeled 80° and the angle labeled 10*x* are supplementary angles.

$80 + 10x = 180$ Write the equation.

$-80 \quad = -80$ Subtract 80 from each side.

$\frac{10x}{10} = \frac{100}{10}$ Divide each side by 10.

$x = 10$

So, the value of *x* is 10.

Got it? Do this problem to find out.

d. A pair of scissors forms the angles shown. What is the value of *x*? (8*x*)°, 116°

A. 8

Guided Practice

Identify each pair of angles as *complementary*, *supplementary*, or *neither*. (Examples 1 and 2)

1. 135°, 45° — supplementary

2. 23°, 67° — complementary

3. Find the value of *x*. (Examples 3–5) 45

4. **Building on the Essential Question** How are vertical, adjacent, complementary, and supplementary angles related? Sample answer: Adjacent angles are two angles that share a common side and vertex and do not overlap. Vertical angles will never be adjacent. Complementary and supplementary angles may or may not be adjacent.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

Name _____ My Homework

Independent Practice

Identify each pair of angles as *complementary, supplementary,* or *neither*.
(Examples 1 and 2)

1. 

neither

2. 

supplementary

3. 

supplementary

Find the measure of *x* in each figure. (Examples 3 and 4)

4. 

25

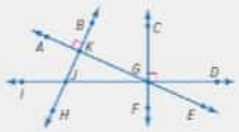
5. 

20

6. $\angle A$ and $\angle B$ are complementary angles. The measure of $\angle B$ is $(4x)°$, and the measure of $\angle A$ is 50°. What is the value of *x*? (Example 5)
10

A skateboard ramp forms a 42° angle as shown. Find the value of *x*. (Example 5)
23

Use the figure at the right to name the following.



8. a pair of supplementary angles
Sample answer: $\angle IJK$, $\angle KJG$

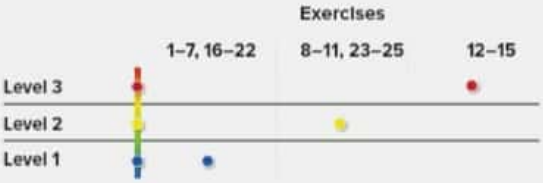
9. a pair of complementary angles
Sample answer: $\angle CGK$, $\angle KGJ$

10. a pair of vertical angles
Sample answer: $\angle AKB$, $\angle JKG$

3 Practice and Apply

Independent Practice and Extra Practice
The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments
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Differentiated Homework Options		
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OL	On Level	1–7 odd, 8–12, 15, 24, 25
BL	Beyond Level	8–15, 24, 25

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	13, 14
3 Construct viable arguments and critique the reasoning of others.	12, 15, 23
7 Look for and make use of structure.	21, 22

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a clos ing formative assessment before dismissing students from your class.

TICKET Out the Door

Tell students that ∠1 and ∠2 are complementary, m∠1 measures 48°, and ∠2 measures $(3x)°$. Have students write and solve an equation to find the value of x.

$48 + 3x = 90; x = 14$

Watch Out!

Common Error Students may confuse the terms *complementary* and *supplementary*. Provide students with a mnemonic device to help them remember the definition of supplementary, such as supplementary: straight.

11. Use the figure at the right.

a. Are ∠1 and ∠2 vertical angles, adjacent angles, or neither? And ∠3? ∠1 and ∠3?

adjacent; adjacent; vertical

b. Write an equation representing the sum of $m\angle 1$ and $m\angle 2$. Then write an equation representing the sum of $m\angle 2$ and $m\angle 3$.

$m\angle 1 + m\angle 2 = 180°$; $m\angle 2 + m\angle 3 = 180°$

c. Solve the equations you wrote in part **b** for $m\angle 1$ and $m\angle 3$, respectively. What do you notice?

$m\angle 1 = 180° - m\angle 2$; $m\angle 3 = 180° - m\angle 2$; Sample answer: $m\angle 1$ and $m\angle 3$ are equal.

d. **Make a Conjecture** Use your answer from part **c** to make a conjecture as to the relationship between vertical angles.

Sample answer: Vertical angles are congruent.

H.O.T. Problems Higher Order Thinking

12. **Reason Inductively** When a basketball hits a hard, level surface, it bounces off at the same angle at which it hits. Use the figure to find the angle at which the ball hit the floor.

55°

13. **Persevere with Problems** Find the measure of each angle in the given situation.

a. complementary angles E and F, where $m\angle E = (x - 10)°$ and $m\angle F = (x + 2)°$

$m\angle E = 39°$; $m\angle F = 51°$

b. supplementary angles B and C, where $m\angle B = (2x - 40)°$ and $m\angle C = (2x + 20)°$

$m\angle B = 60°$; $m\angle C = 120°$

14. **Persevere with Problems** In the figure shown, the sum of the measures of ∠YXZ and ∠WXV is 75°. What is the measure of ∠ZXW?

105°

15. **Reason Inductively** Is the statement below *always*, *sometimes*, or *never* true? Explain.

If two angles are right angles, they must be supplementary.

Sample answer: Right angles have a measure of 90°, so two right angles will always have a sum of 180°. This is the definition of supplementary angles.

Name _____ My Homework _____

Extra Practice

Identify each pair of angles as *complementary, supplementary,* or *neither*.

16. 

∠1 and ∠2 form a straight angle. So, the angles are supplementary.

17. 

complementary

18. 

neither

19. ∠*J* and ∠*K* are supplementary. The measure of ∠*J* is (9*x*)° and the measure of ∠*K* is 45°. What is the value of *x*?

15

20. ∠*C* and ∠*D* are complementary. The measure of ∠*C* is (4*x*)° and the measure of ∠*D* is 26°. What is the value of *x*?

16

Identify Structure **Determine whether each statement is *always, sometimes,* or *never* true. Explain your reasoning.**

21. Two obtuse angles are supplementary.

never; Sample answer: Since an obtuse angle is greater than 90°, the sum of two obtuse angles must be greater than, not equal to, 180°.

22. Two vertical angles are complementary.

sometimes; Sample answer: If the measure of each angle is 45°, then the two angles are complementary.

23. **Multiple Representations** Line *a* passes through (1, 4) and (-4, −1). Line *b* passes through (-3, 4) and (2, −1).

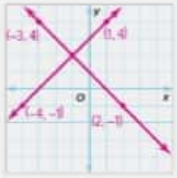
a. **Graphs** Graph each line on the same coordinate plane.

b. **Words** Describe the lines.

The lines appear to be perpendicular.

c. **Numbers** What is the slope of each line?

line *a*: 1; line *b*: −1



Power Up! Test Practice

Exercises 24 and 25 prepare students for more rigorous thinking needed for the assessment.

24. This test item requires students to reason abstractly and quantitatively when problem solving.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

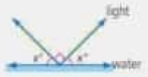
25. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly model the equation and find each measure.
1 point	Students correctly model the equation but fail to find both measures OR students find both measures but fail to correctly model the equation.

Power Up! Test Practice

24. Which angle pairs below are s upplementary? Select a ll that apply.

25. The angle at which the light ray hits the water is equal to the angle at which the light ray is reflected from the water.



Select values to complete the equation below to find the value of x.

$2 \times$ [$x°$] $+$ [$90°$] $=$ [$180°$]

$x°$
45°
60°
90°
180°

What is the m easure of the angle at which the light ray hits the water ?

45°

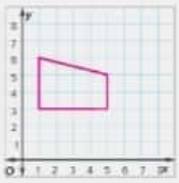
What is the me asure of the angle at which the light ray is reflected from the water?

45°

Spiral Review

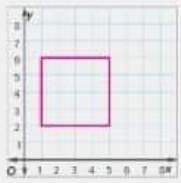
Graph each figure with the given vertices on the coordinate plane. Then classify each figure.

26. (1, 3), (1, 6), (5, 5), and (5, 3)



trapezoid

27. (1, 2), (5, 2), (5, 6), and (1, 6)



square

Inquiry Lab
Create Triangles

inquiry **WHAT do you notice about the measures of the sides or the measures of the angles that form triangles?**

MP Mathematical Practices 1, 3, 5, 8

Salem has a sailboat. The sail on his boat is in the shape of a triangle with side lengths of 6 meters, 8 meters, and 10 meters. These dimensions work to form a triangle, but not just any three lengths form a triangle. Complete the Activity below to determine which side lengths form triangles.

Hands-On Activity 1

Step 1 Measure and cut several plastic straws into lengths that equal 3, 4, 4, 5, 8, 8, 8, 13, 15, 15, 15, and 15 centimeters.

Step 2 Arrange three of the pieces that each measure 15 centimeters to see if you can form a triangle.

So, you can form a triangle with side lengths of 15 centimeters, 15 centimeters, and 15 centimeters.

Step 3 Continue using pieces of straw to try to form triangles using the different combinations of side lengths given. Determine whether or not the lengths form a triangle. Complete the table.

Side 1	Side 2	Side 3	Do the sides form a triangle?
15 cm	15 cm	15 cm	yes
3 cm	4 cm	5 cm	yes
8 cm	8 cm	13 cm	yes
3 cm	4 cm	8 cm	no
4 cm	4 cm	5 cm	yes
8 cm	3 cm	15 cm	no
4 cm	8 cm	15 cm	no

Focus narrowing the scope

Objective Draw triangles using given angles or side lengths.

Materials: plastic drinking straws, patty paper, protractor

Coherence connecting within and across grades

Now
Students use manipulatives to try to create triangles from given angle measurements or given side measurements.

Next
Students will find the measure of an unknown angle in a triangle.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 552.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activity 1 is intended to be used as a whole-group activity. Activity 1 is designed to provide more guidance than Activity 2

Hands-On Activity 1

AL LA Explain to students that when measuring the straws, it is very important that they measure their straws carefully. Precise measurements are important in completing this Activity. MP 1, 6

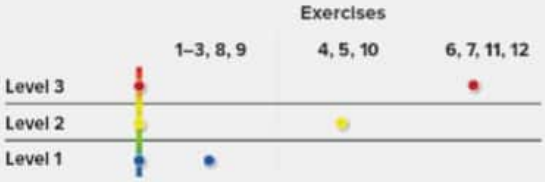
BL Omit Activities 1 and 2 and proceed directly to the Investigate section.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Pairs Discussion** Have students work in pairs to complete E xercises 1–3. Have them trade their solutions with another pa ir of students and discuss any differen MP 1, 5

Analyze and Reflect

AL LA **Think-Pair-Share** Have students work in pairs. Give students o ne minute to think through their responses to Exercises 4 and 5. Have them share their responses with their partner. T hen call on one student to share their response within a small g roup or large group discussi MP 1, 7

Create

BL LA **Brainstorm** Have students brainstorm different side lengths that might make a triangle. Have students explain why or why not their measurements could be used to make a triangle. MP 1, 7

Investigate

Work with a partner. Try to create triangles using the given side lengths. Circle *yes* if you can make a triangle or *no* if you cannot.

1. 5 cm, 8 cm, 15 cm — Yes or (No)
2. 13 cm, 8 cm, 15 cm — (Yes) or No
3. 13 cm, 4 cm, 4 cm — Yes or (No)

Analyze and Reflect

Work with a partner.

4. The table below contains the dimensions you used in Step 3 of the Activity. Transfer your results from the investigation into the fourth column and then complete the fifth column.

Side 1	Side 2	Side 3	Do the sides form a triangle?	Is Side 1 + Side 2 greater than or less than Side 3?
15 cm	15 cm	15 cm	yes	greater than
3 cm	4 cm	5 cm	yes	greater than
8 cm	8 cm	13 cm	yes	greater than
3 cm	4 cm	8 cm	no	less than
4 cm	4 cm	5 cm	yes	greater than
8 cm	3 cm	15 cm	no	less than
4 cm	8 cm	15 cm	no	less than

5. What do you notice about the figures with a Side 1 and Side 2 sum that is less than the length of Side 3? Sample answer: They do not form triangles.

Create

6. Can you create a triangle that has the same shape as the triangle in the Activity, but different side lengths? Explain.
Yes; 8 cm, 8 cm, 8 cm

7. MP **Reason Inductively** Could you form a triangle using the side lengths of 7, 8, and 25 centimeters? Explain. No; Sample answer: To form a triangle, the sum of any two sides must be greater than the length of the third side. 7 + 8 is not greater than 25.

Investigate

AL LA **Talking Chips** Have students work in groups of 3–4 to complete Exercises 8 and 9. Provide each student with 4 talking chips. As students discuss each exercise, they should place a chip in the center of the table every time they verbally contribute a response or ask a clarifying question. After they have placed all of their chips in the center of the table, they may not contribute. Students must use all of their chips. MP 5

BL LA **Circle the Sage** Separate students into groups of 4. Teacher should select a "sage" to lead the discussion for each group through Exercises 8 and 9. The sage should be a student that is proficient in triangle sums and is able to communicate ideas to the group and lead a discussion. Sages are responsible for ensuring that each team member contributes to the discussion and that each team member understands how to complete the exercises. MP 1, 7

Analyze and Reflect

AL LA **Pairs Check** Have students work in pairs to complete Exercise 10. Have students discuss differences in responses and make corrections, if necessary. Have pairs share their responses and reasoning with the class. MP 3, 8

Create

BL LA **Trade-a-Problem** Have students draw and measure three angles on their own. Students trade their angles with a partner and determine if the angles could be used to form a triangle. If they do not form a triangle, students work together to find an angle that would work. MP 1, 5, 7

Inquiry Students should be able to answer "WHAT do you notice about the measures of the sides or the measures of the angles that form triangles?" Check for student understanding and provide guidance, if needed.

Investigate

Work with a partner.

8. Draw another 60° angle on a piece of patty paper. Describe the angles and side lengths of the figure you form using three 60° angles.
   The figure is a triangle with three equal angles and three equal side lengths.

9. Draw angles measuring 20°, 70°, and 90° on pieces of patty paper.
   a. Do the angles form a triangle? **yes**
   b. Can you create more than one triangle that is the same shape with different side lengths? What are the side lengths of your triangle?
   yes; Student answers will vary.

Analyze and Reflect

10. MP **Identify Repeated Reasoning** Refer back to the table in Step 3 of Activity 2. Compare the sum of the angle measures. Describe any patterns that are found.
    To form a triangle, the sum of the angles must equal 180°.

Create

11. MP **Use Math Tools** Use a protractor to measure the three angles below. Would you be able to form a triangle from these angles? Explain.
    Yes. The angles measure 65°, 25°, 90°. The sum of these angles equals 180°.

12. Inquiry WHAT do you notice about the measures of the sides or the measures of the angles that form triangles?
    The sum of the lengths of any two sides of a triangle must be greater than the length of the third side. The sum of the three angles of a triangle must equal 180°.

Geometry

Lesson 3

Triangles

Real-World Link

Ramps Mazen practices jumping on a ski ramp. The front of the ramp is a triangle like the one shown below.

1. Draw an X through the type of angle that is not shown in the triangle.

right acute obtuse

2. Measure the unknown angle. Describe the relationship between the 80° angle and the unknown angle.

10°; Sample answer: The angles are complementary.

3. Draw a triangle with one obtuse angle.

Sample answer:

4. Is it possible to draw a triangle with two obtuse angles? Explain.

No; When you have two obtuse angles the rays of the angles can not connect.

Essential Question

HOW does geometry help us describe real-world objects?

Vocabulary

acute triangle
right triangle
obtuse triangle
scalene triangle
isosceles triangle
equilateral triangle
triangle
congruent segments

Math Symbols

△

Mathematical Practices

1, 2, 3, 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Identify and classify triangles and find missing angle measures.

Coherence connecting within and across grades

Previous
Students used properties of angles to find missing angle measures.

Now
Students classify different types of triangles and find missing angle measures in triangles.

Next
Students will classify quadrilaterals and find missing angle measures of quadrilaterals.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 559.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Think-Pair-Draw** Give students one minute to think through their responses to Exercises 1–4. Then have them discuss their ideas and responses with a partner. Call on one set of partners to explain their responses to the class. MP 1, 5, 7

Alternate Strategy

BL Have students explain whether it is possible to draw a triangle with two right triangles. Have them verify their response by using logical reasoning. MP 1, 3, 7

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Draw and classify triangles.

AL
- *Describe what you know about an obtuse angle.* Sample answer: Its angle measure is greater than 90° and less than 180°.
- *How can you draw an obtuse angle?* Sample answer: Use a ruler or straightedge to draw one side. Then use a protractor to draw a second side so that the angle formed is more than a right angle but less than a straight angle.

OL
- *If a triangle has one obtuse angle, what types of angles are the other two angles?* acute angles
- *Since the triangle has one obtuse angle, how would you classify the triangle by its angles?* obtuse
- *Since the triangle has no congruent sides, how can you classify the triangle by its sides?* scalene

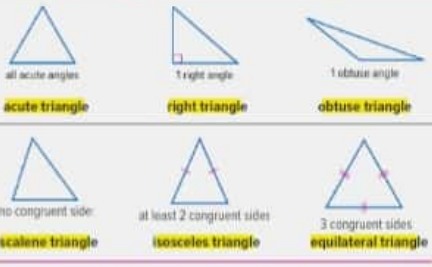
BL
- *When drawing the triangle, how can you be certain that it will be scalene?* Sample answer: make sure that the three sides drawn are all different lengths

Need Another Example?
Draw a triangle with three acute angles and two congruent sides. Then classify the triangle. Sample answer: acute isosceles

Key Concept **Classify Triangles**

Work Zone

Congruent Segments
The tick marks on the sides of the triangle indicate that those sides are congruent.



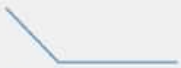
A **triangle** is a figure with three sides and three angles. The symbol for triangle is △.

Every triangle has at least two acute angles. One way you can classify a triangle is by using the third angle. Another way to classify triangles is by their sides. Sides with the same length are **congruent segments**.

Example

1. **Draw a triangle with one obtuse angle and no congruent sides. Then classify the triangle.**

Draw an obtuse angle.
The two segments of the angle should have different lengths.



Connect the two segments to form a triangle.

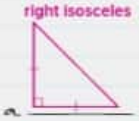
The triangle is an obtuse scalene triangle.

Got it? Do this problem to find out.

Draw a triangle that satisfies the set of conditions below. Then classify the triangle.

a. a triangle with one right angle and two congruent sides

Sample answer: right isosceles



Example

2. **Classify the triangle on the house by its angles and by its sides.**

The triangle has one obtuse angle and two congruent sides. So, it is an obtuse isosceles triangle.

Got it? Do this problem to find out.

b. Classify the triangle shown by its angles and by its sides.

STOP and Reflect

How would you classify a triangle with a right angle and two congruent sides?

right isosceles

b. right scalene

Angles of a Triangle

Key Concept

Words The sum of the measures of the angles of a triangle is 180°.

Algebra $x + y + z = 180$

You can write and solve an equation to find the missing angle measure of a triangle.

Example

3. **Find $m\angle Z$.**

The sum of the angle measures in a triangle is 180°.

$m\angle Z + 43° + 119° = 180°$ Write the equation.

$m\angle Z + 162° = 180°$ Simplify.

$-162° = -162°$ Subtract 162° from each side.

$m\angle Z = 18°$

So, $m\angle Z$ is 18°.

Got it? Do this problem to find out.

c. In $\triangle ABC$, if $m\angle A = 25°$ and $m\angle B = 108°$, what is $m\angle C$?

c. 47°

Examples

2. **Classify triangles.**

AL • *What do the tic marks indicate?* The tic marks indicate congruent sides.

• *What type of triangle has two congruent sides?* isosceles

OL • *What type of angle does the top angle appear to be?* obtuse

• *If one angle in a triangle is obtuse, what do you know about the other two angles?* They have to be acute.

BL • *What do the congruent sides indicate about the angles that are opposite each side?* Angles that are opposite congruent sides are also congruent.

Need Another Example?
Classify the marked triangle by its angles and by its sides. right scalene

3. **Find missing measures.**

AL • *What is the sum of the measures of the angles in a triangle?* 180°

• *What is the sum of the measures of the two known angles?* $119° + 43° = 162°$

OL • *What equation can you write to use to find the value of $\angle Z$?* $m\angle Z + 43 + 119 = 180$

• *What steps can you use to solve the equation?* First, add 43 and 119, which is 162. Then subtract 162 from each side.

BL • *Classify this triangle by its angles and by its sides.* obtuse, scalene

Need Another Example?
In triangle ABC, $m\angle B = 50°$ and $m\angle C = 80°$. What is $m\angle A$? 50°

Example

4. Find missing measures.

AL • *What is the sum of the angles of a triang* 180°

• *What angle measures are you give* 110° and 35°

OL • *What equation can you write to find the value of x?*
$x + 110 + 35 = 180$

• *What are the steps you can do to solve the equation?*
First, add 110 and 35, which is 145. Then subtract 145 from each side.

BL • *Using what you know about the angle measures of this triangle, classify the triangle by both its angles and its sides.* obtuse, isosceles

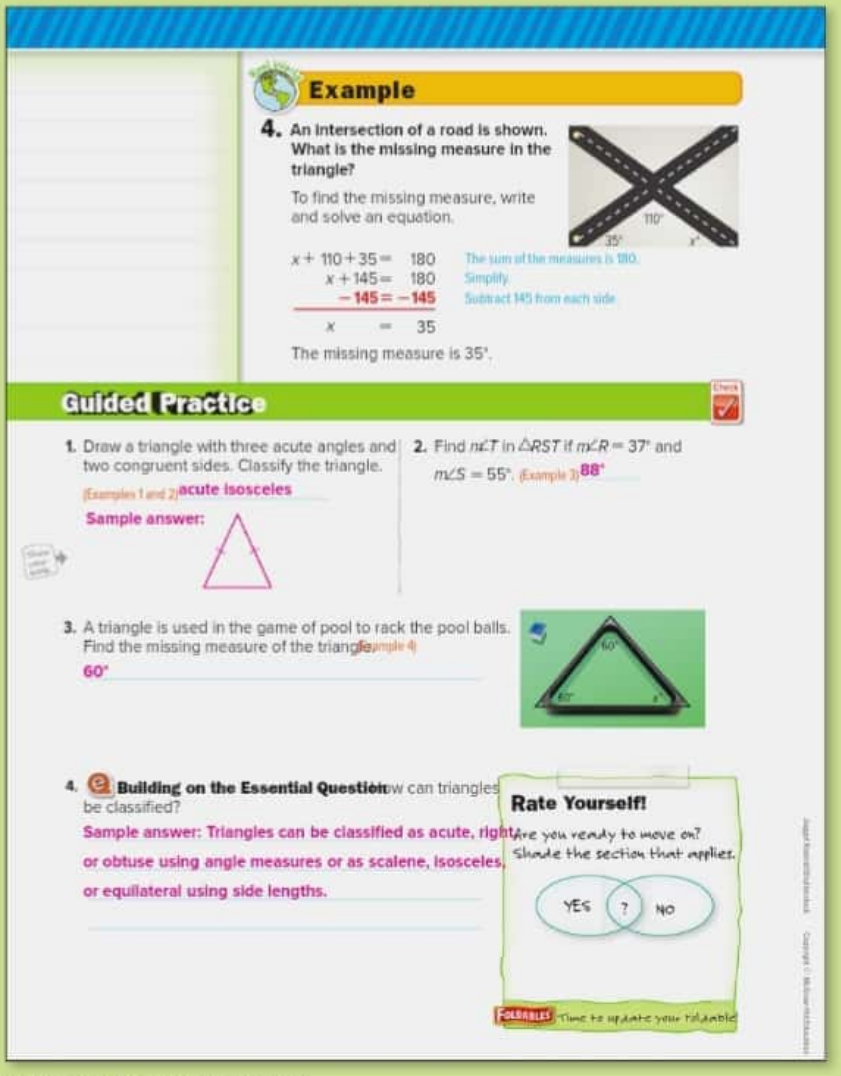
Need Another Example?
Find the value of x in the triangle shown. 32

Guided Practice

Formative A ssessment Use these exercises to assess students' un derstanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

AL LA **Pair-to-Pair to Compare** Arrange students in pairs to complete Exercises 1–4. Students may benefit from drawing the triangle described in Exercise 2. Have students switch partners to form new pairs to compare their solutions to each exercise. This step may be repeated. Then call on one student to share their response to a given exerc 1, 5, 7



Example

4. An intersection of a road is shown. What is the missing measure in the triangle?

To find the missing measure, write and solve an equation.

$x + 110 + 35 = 180$ The sum of the measures is 180.
$x + 145 = 180$ Simplify.
$-145 = -145$ Subtract 145 from each side.
$x = 35$

The missing measure is 35°.

Guided Practice

1. Draw a triangle with three acute angles and two congruent sides. Classify the triangle. (Examples 1 and 2) acute isosceles

Sample answer:

2. Find $m\angle T$ in $\triangle RST$ if $m\angle R = 37°$ and $m\angle S = 55°$. (Example 3) 88°

3. A triangle is used in the game of pool to rack the pool balls. Find the missing measure of the triangle. (Example 4)

60°

4. **Building on the Essential Question** How can triangles be classified?

Sample answer: Triangles can be classified as acute, right or obtuse using angle measures or as scalene, isosceles, or equilateral using side lengths.

Rate Yourself!
Are you ready to move on? Shade the section that applies.

FOLDABLES Time to update your foldable

Name _____ My Homework

Independent Practice

Draw a triangle that satisfies each set of conditions. Then classify the triangle. (Example 1)

1. a triangle with three acute angles and three congruent sides
Sample answer: acute equilateral

2. a triangle with one right angle and no congruent sides
Sample answer: right scalene

Classify the marked triangle by its angles and by its sides. (Example 2)

3. 
acute equilateral

4. 
acute equilateral

5. 
obtuse isosceles

Find the value of *x*. (Examples 3 and 4)

6. 
60

7. 
118

8. 
27

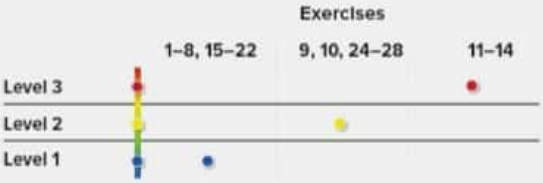
9. **Model with Mathematics** Refer to the graphic novel below. Classify the triangle formed by the cabin, ropes course, and mess hall by its angles and sides.
acute isosceles

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice
The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments
You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 12–14, 27, 28
OL	On Level	1–9 odd, 10, 12–14, 27, 28
BL	Beyond Level	9–14, 27, 28

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	11
2 Reason abstractly and quantitatively.	24–26
3 Construct viable arguments and critique the reasoning of others.	13, 14
4 Model with mathematics.	9, 12

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a clos ing formative assessment before dismissing students from your class.

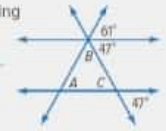
TICKET Out the Door

Have students explain h ow classifying angles helps to classify a triangle as *ac ute*, *right*, or *obtu* See students' work.

Watch Out!

Common Error Students need to read the instruction line and problems carefully. Students may not find the measure of the angle in the triangle and only find the value of x. Some problems ask for the value of x, while others ask them to find the missing measure.

10. Triangle ABC is formed by two parallel lines and two other intersecting lines. Find the measure of each angle A, B, and C of the triangle.



$m\angle A = 61°$, $m\angle B = 72°$, $m\angle C = 47°$

H.O.T. Problems Higher Order Thinking

11. **Persevere with Problems** Apply what you know about triangles to write and solve equations to find the missing angle measures in the figure.

$125 + a = 180$, so $a = 55$; $a + b + 60 = 180$, so $b = 65$; $60 + d = 90$; so $d = 30$; $c + d + 90 = 180$, so $c = 60$

12. **Model with Mathematics** Draw an acute scalene triangle. Describe the angles and sides of the triangle.

Sample answer: All of the angles are acute and there are no congruent sides.

13. **Justify Conclusions** Determine whether each statement is *sometimes*, *always*, or *never* true. Justify your answer.

a. It is possible for a triangle to have two right angles.

never; Sample answer: The sum of the interior angles of a triangle is 180°. Two right angles have a sum of 180°. This means the third angle would equal 0°, which is not possible.

b. It is possible for a triangle to have two obtuse angles.

never; Sample answer: The sum of the interior angles of a triangle is 180°. The measure of an obtuse angle is greater than 90°. So, a triangle cannot have more than one obtuse angle.

14. **Reason Inductively** Hareb says that an equilateral triangle is sometimes an obtuse triangle. Noura says that an equilateral triangle is always an acute triangle. Is either of them correct? Explain your reasoning.

Noura sample answer: An equilateral triangle has all sides congruent. So, each angle measures 60. Therefore, an equilateral triangle is always acute.

Name _____ My Homework _____

Extra Practice

Classify the marked triangle in each object by its angles and by its sides.

15.

The triangle has all acute angles and two congruent sides. It is an acute isosceles triangle.

16.

obtuse isosceles

17.

right scalene

Draw a triangle that satisfies each set of conditions. Then classify the triangle.

18. a triangle with three acute angles and no congruent sides
acute scalene
Sample answer:

19. a triangle with one obtuse angle and two congruent sides
obtuse isosceles
Sample answer:

Find the value of *x*.

20.

65

21.

90

22.

37

23. Find $m\angle Q$ in $\triangle QRS$ if $m\angle R = 25°$ and $m\angle S = 102°$. 53°

Reason Abstractly Find the value of *x* in each triangle.

24.

60

25.

30

26.

77.5

Power Up! Test Practice

Exercises 27 and 28 prepare students for more rigorous thinking needed for the assessment.

27. This test item requires students to reason abstractly and quantitatively when problem solving.

Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP7
Scoring Rubric	
1 point	Students correctly answer each part of the question.

28. This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.

Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly model the right triangle and find the missing angle.
1 point	Students correctly model the right triangle OR find the missing angle.

Power Up! Test Practice

27. Refer to the figure shown. Determine if each statement is true or false.

a. To find $m\angle R$, subtract 30° from 90°. ☐ True ■ Fals
b. The measure of $\angle R$ is 120°. ■ True ☐ Fals
c. Triangle RST is an acute triangle. ☐ True ■ False

28. In a right triangle, the measure of one of the angles is 43°. Sketch a diagram to represent this situation. **Sample answer:**

What is the measure of the other angle? 47°

Spiral Review

Find the area of each figure.

29. 25 cm²

30. 32 m²

31. 35 cm²

32. 25 m²

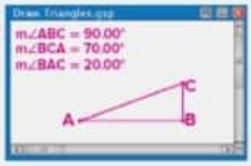
33. 9 m²

34. 36 cm²

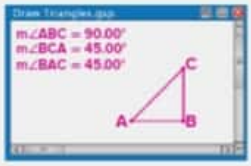
Investigate

MP Use Math Tools Work with a partner to construct each triangle. Once you have constructed a triangle, draw the text and image that appears on your display.

1. $\angle ABC = 90°$
   $\angle BCA = 70°$
   $\angle BAC = 20°$



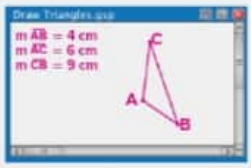
2. $\angle ABC = 90°$
   $\angle BCA = 45°$
   $\angle BAC = 45°$



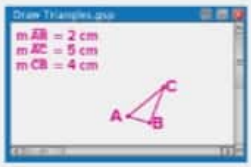
3. Explain the steps you would take to create a triangle if you were given the measures of all three angles.

Sample answer: Use a geometry program to draw and measure three angles of a triangle. Then drag one or more vertices to get the specified angle measurements.

4. $\overline{AB} = 4$ centimeters
   $\overline{AC} = 6$ centimeters
   $\overline{CB} = 9$ centimeters



5. $\overline{AB} = 2$ centimeters
   $\overline{AC} = 5$ centimeters
   $\overline{CB} = 4$ centimeters



6. **MP Justify Conclusions** Explain the steps you would take to create a triangle if you were given the lengths of all three sides.

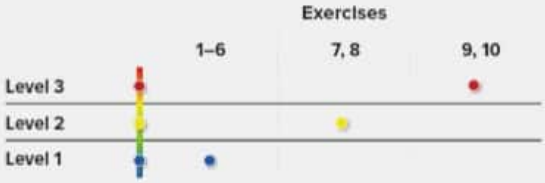
Sample answer: Use a geometry program to draw and measure the three sides of a triangle. Then drag one or more vertices to get the specified side lengths.

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Team-Pair-Solo** Have students work in small teams to complete Exercises 1–3. If the technology is unavailable, have students use protractors to construct triangles with the specified angle measurements for each exercise. Have students label each vertex. Students should name each angle with its corresponding measurement. Have the teams divide into pairs to complete Exercises 4 and 5. Then have students work alone to complete Exercise 6. Upon completion, have them rejoin their original team to verify answers and discuss and compare solutions. MP 1, 5, 6

BL LA **Pairs Present** Have students work with a partner to come up with a rule for how to find the third angle measure of a triangle when the first two angle measures are known. Have them use illustrations and drawings, including The Geometer's Sketchpad®, if available. Have them present their results to the class. MP 1, 7

Analyze and Reflect

AL LA **Pairs Discussion** Have students discuss and complete Exercises 7 and 8. If students are having trouble answering the questions, have them respond to the following question. MP 1, 5, 7

Ask:

- *What is the sum of the interior angles of any triang* 180°

Create

BL LA **Pairs Present** Have students work with a partner to complete Exercise 9. Have them prepare a brief oral presentation showing how they determined the sum of the angle measures of any quadrilateral. Have them include illustrations and examples in their presentation. Then have them present their results to the class. MP 1, 5, 7, 8

Inquiry Students should be able to answer "HOW can you use technology to draw geometric shapes?" Check for student understanding and provide guidance, if needed.

Analyze and Reflec

Work with a partner to answer each of the following questions.

7. Is it possible to use dynamic geometry software to draw a triangle with angles of 50°, 65°, and 70°? Explain.

   No; the sum of the three interior angles of a triangle is always 180°.

8. Is it possible to use dynamic geometry software to draw a triangle with side measures of 3, 6, and 10 centimeters? Explain.

   No; the sum of the lengths of the two shorter sides must be greater than the longer side in order to form a triangle.

Create

9. MP **Reason Inductively** You know the rule to find the sum of the interior angles of a triangle. Does a similar rule exist for the sum of the interior angles of a quadrilateral? Use dynamic geometry software to draw four different quadrilaterals and complete the table below to find out. (*Hint:* Do not draw more than one square or rectangle.)

   Sample answer: The sum of the interior angles of a quadrilateral is 360°.

   See students' work for angle measurements.

	$m\angle 1$	$m\angle 2$	$m\angle 3$	$m\angle 4$	Sum of Angles
Quadrilateral 1					360°
Quadrilateral 2					360°
Quadrilateral 3					360°
Quadrilateral 4					360°

10. Inquiry HOW can you use technology to draw geometric shapes?

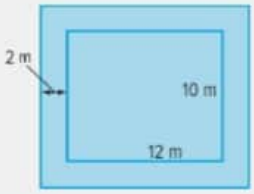
    Sample answer: You can use the Straightedge Tool to draw segments that form a polygon, such as a triangle. You can use the measure tool to check the measures of sides and angles. Then you can adjust the shape.

Case #2 Portraits

AL LA **Trade-a-Problem** Have students create a problem similar to the Portraits problem. Allow them to work in pairs. Have students take turns where one student speaks aloud the problem and their solution while the other listens and asks clarifying questions to help answer the problem. MP 1, 4, 5

BL LA **Draw-Pair-Share** Have students complete the exercise individually and draw their model on a piece of paper. Have students work in pairs to share their models and any differences. MP 1, 5

Need Another Example?
A rectangular garden is 12 meters long and 10 meters wide. A stone path around the garden is 2 meters wide. How much area is covered by the garden and the path? Use the *make a model* strategy. 224 m²



Case #2 Portraits

Muna created a portrait that is 25 centimeters wide by 32.5 centimeters long. She wants to put it in a frame that is 5.5 centimeters wide on each side.

What is the area of the framed portrait?

1 Understand

Read the problem. What are you being asked to find?

I need to find the area of the framed portrait.

2 Plan

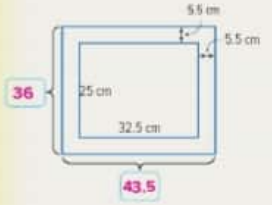
What is your strategy to solve this problem?

I will use the make a model strategy.

3 Solve

How can you apply the strategy?

I will draw a model of the portrait and the frame and label the dimensions.



The inner rectangle is the portrait and the outer rectangle is the frame.

Label the combined length and width of the portrait and the frame.

The area of the framed portrait is 43.5 centimeters × 36 centimeters or 1,566 square centimeters.

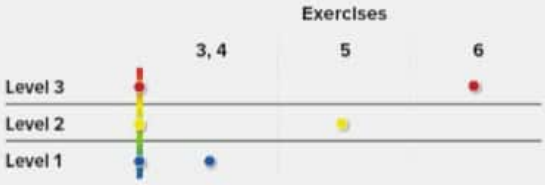
4 Check

Estimate the product of the length and width of the framed portrait to determine if your answer is reasonable.

Sample answer: 40 × 35 = 1,400. Since 1,400 is close to my answer, my answer is reasonable.

2 Collaborate

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



AL LA **Pairs Check** Have students work in pairs to complete Cases 3–6. One partner solves the problem while the other listens and coaches. Students switch roles until all exercises are complete. After every two problems, the pairs check their answers with another pair and resolve any differences. MP 1, 5

BL LA **Stand Up, Hand Up, Pair Up** Give students time to complete Cases 3–6 individually. Call out, "Stand up, hand up, pair up." As students do so, allow them to choose a partner in a different area of the classroom to discuss their answers. Use the following questions to help guide the discussion. MP 1, 5

Ask:

- *How does the diagram in Case 4 help you solve the problem?* Sample answer: The diagram helps you find the area of the pool by using tiles in the diagram.
- *What type of model could you use to solve Case 5?* Sample answer: You could use a table and list to find what language each person takes.

Work with a small group to solve the following cases. Show your work on a separate piece of paper.

Case #3 Tables

Members of the Student Council are setting up tables end-to-end for an awards banquet. Each table will seat one person on each side.

How many square tables will they need to put together for 32 people? Explain your method.

15 tables; 2 people can sit on the ends. Then divide the remaining people by 2. $(32 - 2) \div 2 = 15$.

Case #4 Tile

The diagram shows the design of a tile border around a rectangular swimming pool that measures 10.5 meters by 6 meters. Each tile is a square measuring 1.5 meters on each side.

Explain a method you could use to find the area of just the tile border. Then solve.

Subtract the area of the pool from the total area. $198 - 63 = 135$ m²

Case #5 Classes

Fatima, Sara and Ghaya take French, Spanish, and German. No person's language class begins with the same letter as their first name. Sara's best friend takes French.

Which language does each person take?

Fatima: Spanish; Sara: German; Ghaya: French

Case #6 Money

Badria received money for her graduation. She let Laila borrow AED 4.50 and spent half of the remaining money. The next day she received AED 10 from her uncle. After spending AED 12.50 at the movies, she still had AED 7.75 left.

How much money did she receive for her graduation?

AED 25

Mid-Chapter Check

If students have trouble with Exercises 1–7, they may need help with the following concepts.

Concept	Exercise(s)
complementary angles (Lesson 2)	1
classifying triangles (Lesson 3)	2, 7
vertical angles (Lesson 1)	3, 5
supplementary angles (Lesson 2)	4
find missing angle measures in triangles (Lesson 3)	6

Vocabulary Activity

LA **Numbered Heads Together** Have students work in a small group to complete Exercise 1. Each student is assigned a number. Students are responsible to ensure that each group member understands the meaning of attributes of similar figures. Students should ask each other for clarification and assistance, as needed. Call on one numbered student to share their definition with the class. MP 1, 6

Alternate Strategies

AL Have students look up the definition of attribute. To help students understand the term, have them list attributes of themselves. Then transition them back to attributes of angles and triangles. MP 1, 6

BL Have students verbally explain the method they would use to analyze angles they encounter in the real-world to determine if they are vertical angles, complementary angles or supplementary angles. MP 1, 4

Mid-Chapter Check

Vocabulary Check

1. **Be Precise** Define *complementary angles*. Give an example of two angles that would be complementary. (Lesson 2)
   Two angles are complementary if the sum of their measures is 90°. Sample answer: Two 45° angles would be complementary.

2. Fill in the blank in the sentence below with the correct term. (Lesson 3)
   A right scalene triangle is made up of one right angle and no congruent sides.

Skills Check and Problem Solving

Refer to the figure below for Exercises 3–5. (Lessons 1 and 2)

3. Identify a pair of vertical angles.
   Sample answer: ∠1 and ∠3

4. Identify a pair of supplementary angles.
   Sample answer: ∠1 and ∠2

5. Suppose $m\angle 1 = 127°$. Find the measures of the other angles.
   $m\angle 2 = 53°$ $m\angle 3 = 127°$ $m\angle 4 = 53°$

6. What is $m\angle A$ in $\triangle ABC$ if $m\angle B = 35°$ and $m\angle C = 52°$?
   93°

7. **Reason Inductively** Classify the triangle that satisfies each set of conditions.
   a. one right angle and two congruent sides: right isosceles
   b. one obtuse angle and no congruent sides: obtuse scalene
   c. three acute angles and three congruent sides: acute equilateral

Inquiry Lab

Investigate Online Maps and Scale Drawings

HOW is the zoom feature of an online map like the scale of a drawing?

Mathematical Practices 1, 3, 4, 5

Maps and blueprints are scale drawings of the locations and buildings they represent. Unlike maps printed on paper, online map services allow user the opportunity to view a location from different distances.

Hands-On Activity 1

Step 1 Use the online map service provided to you by your teacher. Locate your school on a map.

Step 2 Measure the length of the scale bar in centimeters on the online map. Find the scale distance of the map. Write these values in the Original View table in Step 4.

Step 3 Click on the satellite or aerial view. Use the zoom feature to zoom in until your school shows up on the map.

Step 4 Measure the length of the scale bar in centimeters. Find the new scale distance for the map. Write these values in the Zoom View table. **Answers will vary depending on the online map program used. Sample answers are given.**

Original View	
Scale Bar	2.8 cm
Scale Distance	800 m

Zoom View	
Scale Bar	2.8 cm
Scale Distance	30 m

What happens when you use the zoom feature?

Sample answer: The map zooms in and zooms out.

Describe the appearance of the map as you zoomed in.

Sample answer: The map images became larger.

Focus narrowing the scope

Objective Use online maps to reproduce a scale drawing at a different scale.

Coherence connecting within and across grades

Now Students will solve real-world problems involving scale drawings.

Next Students will solve mathematical and real-world problems involving scale drawings.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 572.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activity 2.

Materials: computer with Internet access

Hands-On Activity 1

AL LA **Teammates Consult** Have students work in small teams to complete Activity 1. Have each student alternate leading the discussion by reading each step aloud and guiding the rest of the team in completing that step. Then have the team discuss and respond to the questions. MP 1, 5, 6

BL LA **Pairs Discussion** Have students determine how many times they need to zoom in to reach a certain scale distance. MP 1, 6

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

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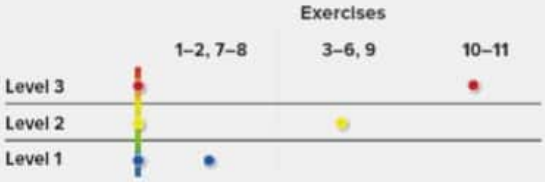


Investigate

AL LA **Pairs Check** Have students work in pairs to complete Exercises 1 and 2. Then have them trade online maps wi th another pair of students to check each other's work. MP 1, 5, 6

Analyze and Reflect

BL LA **Teams Present** Have students work in a small team of 3–4 students to complete Exercises 3–6 and prepare a brief oral presentation about how the zoom feature on an online map affects the scale of the map. Have them present to the class while the rest of the class listens carefully and asks for clarifying questions at the en MP 1, 6

Investigate

MP **Use Math Tools** Work with a partner to answer the following questions about using an online map service.

1. Locate the local public library on the map. Write the scale bar and scale distance values in the Original View table below Exercise 2.

2. Click on the satellite or aerial view. Use the zoom feature to zoom in until the building shows up on the map. Write the scale bar and scale distance values in the Zoom View table.

Original View	
Scale Bar	2.8 cm
Scale Distance	400 m

Zoom View	
Scale Bar	2.8 cm
Scale Distance	200 m

Answers will vary depending on the online map program used and location selected. Sample answers are given.

Analyze and Reflect

Work with a partner to answer the following questions about using an online map.

3. Refer to Activity 1. Write a rate $\frac{\text{scale bar}}{\text{scale distance}}$ for the original view and the zoom view. Sample answers given.

Original View: $\frac{2.8 \text{ centimeters}}{800 \text{ meters}}$ Zoom View: $\frac{2.8 \text{ centimeters}}{30 \text{ meters}}$

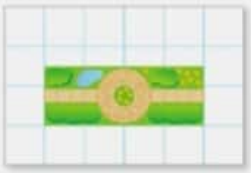
4. How many times bigger is the zoom view?
Sample answer: The scale distance of the original view is about 27 times larger than the scale distance of the zoom view. The zoom view is about 27 times bigger.

5. How does zooming in affect the scale on the map?
Sample answer: As you zoom in, the scale distance decreases.

6. When using the zoom feature on an online map, what changes and what stays the same? Sample answer: The scale distance changes as you zoom in or zoom out. Zooming in will result in the images on the map becoming larger. The window in which you view the map always stays the same.

Hands-On Activity 2

The diagram shown represents a garden. The scale is 1 centimeter = 30 meters. That means that each square on the grid measures 1 centimeter by 1 centimeter or 30 meters by 30 meters.



Step 1 Write the length and width of the drawing of the garden.

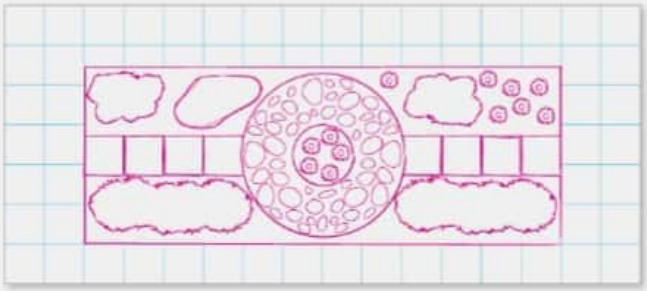
Length: 4 centimeters Width: 1.5 centimeters

Step 2 Use the scale to find the dimensions of the garden.

Length: 120 meters Width: 45 meters

Step 3 On the grid below, draw the garden so that the scale is 1 centimeter = 10 meters. Write the dimensions of your drawing.

Length: 12 centimeters Width: 4.5 centimeters



Step 4 Use the scale on your drawing to compute the dimensions of the garden. How do the dimensions compare to the dimensions in Step 2?

Length: 120 meters Width: 45 meters

The dimensions are the same.

Hands-On Activity 2

AL LA **Three-Step Interview** Have students work in a team of 3–4 students. Student 1 reads aloud the boldface directions for Activity 2, then interviews Student 2, asking Student 2 to find the length and width of the drawing of the garden to complete Step 1. Then Student 2 interviews Student 3 by asking them to use the scale to find the dimensions of the garden in Step 2. Student 3 interviews Student 4 (or Student 1 if there are only 3 students in the group) to complete Step 3. Then the team works together to complete Step 4 MP 1, 5, 6

BL LA **Pairs Discussion** Have students work in pairs to extend Activity 2 by reproducing the scale drawing using a new scale. Have them decide on a scale and use grid paper to draw the new scale drawing. Have them write the dimensions of their drawing and use the scale to compute the actual dimensions of the garden and compare it to the actual dimensions of the garden in Activity MP 1, 5, 6

Ask:

- *How do you know that your scale and drawing are accurate?*
  Sample answer: By finding the actual dimensions of the garden using our scale and drawing, they should be the same dimensions of the garden in Activity 2.

Investigate

AL LA **Think-Pair-Draw** Have students work in pairs to complete Exercises 7–8. Give them one minute to think through how they would draw the baseball diamond in Exercise 7 using the new scale. Then have them discuss their thinking with their partner, without actually completing the drawing. Finally, have students work individually to complete the drawing and share their drawings with their partner to resolve any differences. MP 1, 6

Ask:

- *How do you know that the drawing, using the new scale, will be smaller than the original drawing?* Sample answer: The new scale has 1 unit = 30 meters, so each unit on the grid will represent a greater distance; therefore, the drawing will be smaller.

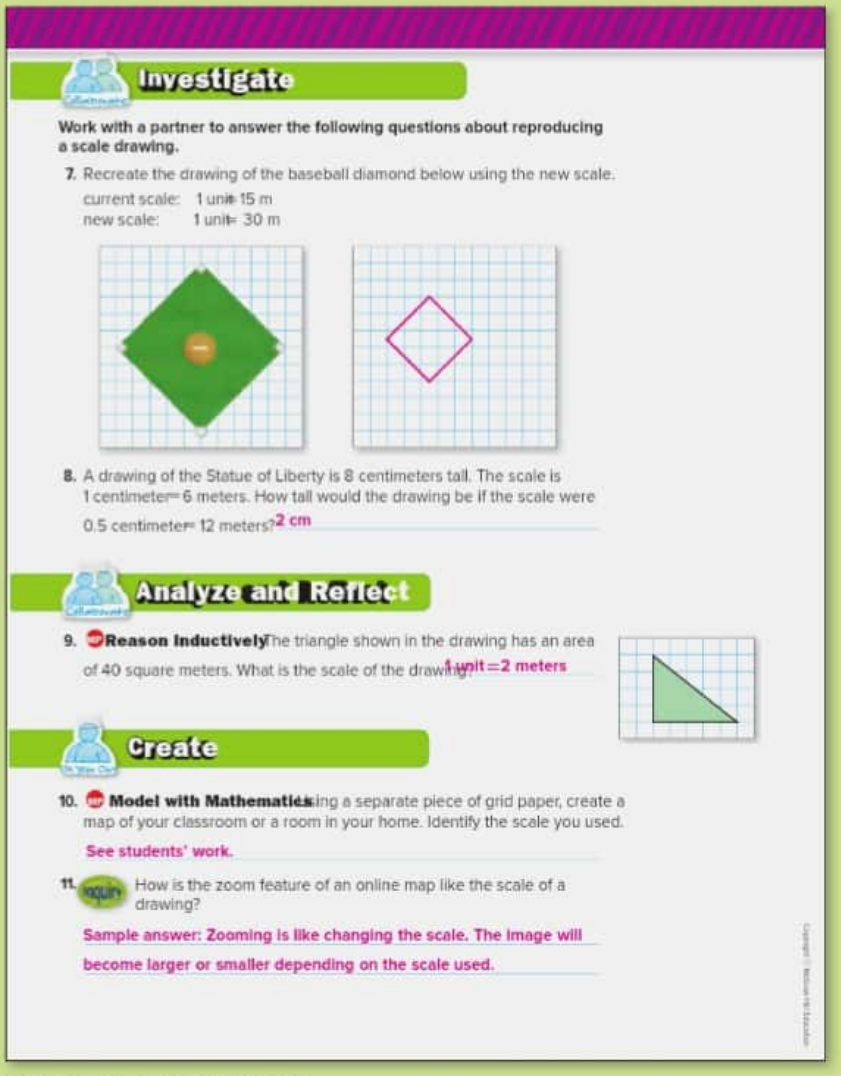
Analyze and Reflect

BL LA **Find the Fib** Have students with a partner to write two facts and one fib for Exercise 9. For example, one fact could be the scale of the drawing is 1 unit = 2 meters. One fib could be that the triangle is an equilateral triangle. Have students trade papers with another set of paired students. Each pair identifies the facts and fib of the other pair. MP 1, 3, 6

Create

AL LA **Gallery Walk** Have students work in pairs to complete Exercise 10. Then post the scale drawings around the room. Have students walk around the room and select a scale drawing, other than their own. Have them determine the actual measurements of the room indicated in the drawing. MP 3, 4

Inquiry Students should be able to answer "HOW is the zoom feature of an online map like the scale of a drawing?" Check for student understanding and provide guidance, if needed.



Investigate

Work with a partner to answer the following questions about reproducing a scale drawing.

7. Recreate the drawing of the baseball diamond below using the new scale.

current scale: 1 unit = 15 m
new scale: 1 unit = 30 m

8. A drawing of the Statue of Liberty is 8 centimeters tall. The scale is 1 centimeter = 6 meters. How tall would the drawing be if the scale were 0.5 centimeter = 12 meters? 2 cm

Analyze and Reflect

9. MP **Reason Inductively** The triangle shown in the drawing has an area of 40 square meters. What is the scale of the drawing? 1 unit = 2 meters

Create

10. MP **Model with Mathematics** Using a separate piece of grid paper, create a map of your classroom or a room in your home. Identify the scale you used.

See students' work.

11. Inquiry How is the zoom feature of an online map like the scale of a drawing?

Sample answer: Zooming is like changing the scale. The image will become larger or smaller depending on the scale used.

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. Find the actual distance.

AL
- *What is the scale?* 1 cm = 40 kilometers
- *What do we need to find?* the actual distance between the cities

OL
- *What is the distance on the map?* about 4 cm
- *What proportion can we use?*

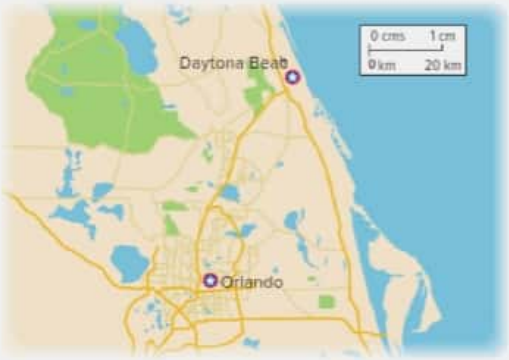
$$\frac{1 \text{ cm}}{40 \text{ kilometers}} = \frac{4 \text{ cm}}{d \text{ kilometers}}$$

- *What is the actual distance between the cities?* about 160 kilometers

BL
- *What is another way we can set up the proportion?*

$$\frac{1 \text{ cm}}{4 \text{ cm}} = \frac{40 \text{ kilometers}}{d \text{ kilometers}}$$

Need Another Example?
Refer to the map of Florida below. What is the actual distance between Daytona Beach and Orlando? Use a ruler to measure.
about 83 kilometers



Work Zone

Scale
A map scale can be written in different ways including the following:
1 cm = 40 km
1 cm : 40 km
1 cm / 40 km

about 128 km

Use a Scale Drawing or a Scale Model

Scale drawings and **scale models** are used to represent objects that are too large or too small to be drawn or built at actual size. **Scale** gives the ratio that compares the measurements of the drawing or model to the measurements of the real object. The measurements on a drawing or model are proportional to the measurements on the actual object.

Example

1. What is the actual distance between Hagerstown and Annapolis?

Step 1 Use a centimeter ruler to find the map distance between the two cities. The map distance is about 4 centimeters.

Step 2 Write and solve a proportion using the scale. Let d represent the actual distance between the cities.

	Scale	Length	
map →	1 centimeter	4 centimeter	← map
actual →	40 kilometers	d kilometers	← actual

$$1 \times d = 40 \times 4 \quad \text{Cross products}$$

$$d = 160 \quad \text{Simplify.}$$

The distance between the cities is about 160 kilometers.

Got it? Do this problem to find out.

a. On the map of Arkansas shown, find the actual distance between Clarksville and Little Rock. Use a ruler to measure.

Example

2. A graphic artist is creating an advertisement for this cell phone. If she uses a scale of 5 centimeters = 1 centimeter, what is the length of the cell phone on the advertisement?

Write a proportion using the scale. Let a represent the length of the advertisement cell phone.

	Scale	Length	
advertisement →	$\frac{5 \text{ centimeters}}{1 \text{ centimeter}}$	$\frac{a \text{ centimeters}}{10 \text{ centimeters}}$	← advertisement
actual →			← actual

$5 \cdot 10 = 1 \cdot a$ Cross products

$50 = a$ Simplify

The length of the cell phone on the advertisement is 50 centimeters long.

Got it? Do this problem to find out.

b. A scooter is 1 meter long. Find the length of a scale model of the scooter if the scale is 1 centimeter = 10 centimeters.

Scale

The scale is the ratio of the drawing/model measure to the actual measure. It is not always the ratio of a smaller measure to a larger measure.

b. 10 cm

Find a Scale Factor

A scale written as a ratio without units in simplest form is called the **scale factor**.

Example

3. Find the scale factor of a model sailboat if the scale is 1 centimeter = 0.75 meter.

$\frac{1 \text{ centimeter}}{0.75 \text{ meter}} = \frac{1 \text{ centimeter}}{75 \text{ centimeters}}$ Convert 0.75 meter to centimeters.

$= \frac{1}{75}$ Divide out the common units.

The scale factor is $\frac{1}{75}$.

Got it? Do this problem to find out.

c. What is the scale factor of a model car if the scale is 1 centimeter = 0.25 meter?

c. $\frac{1}{25}$

Examples

2. Find the distance on the scale drawing.

AL
- *What is the scale?* 5 centimeters = 1 centimeter
- *What do we need to find?* the length of the cell phone on the advertisement
- *Will the length on the ad be less than or greater than the actual length?* greater than; the scale is 5 centimeters = 1 centimeter

OL
- *What proportion can we use?* $\frac{5 \text{ centimeters}}{1 \text{ centimeter}} = \frac{a \text{ centimeters}}{10 \text{ centimeters}}$
- *What is the length of the cell phone on the advertisement?* 50 cm

BL
- *If the width of the actual cell phone was 4 cm what would be the width of the cell phone on the advertisement?* 20 centimeters

Need Another Example?

An artist is painting a large mural of flowers on the side of a school. If she uses the scale 4 centimeters = 1 centimeter, how large will the mural painting of a rose bloom be if the actual height of the rose bloom is 16 centimeters? 64 centimeters

3. Find the scale factor.

AL
- *How is the scale factor different from the scale?* The scale factor is written as a ratio, without units, in simplest form.
- *What is the scale?* 1 centimeter = 0.75 meter

OL
- *Write the scale as a ratio.* $\frac{1 \text{ centimeter}}{0.75 \text{ meter}}$
- *How do we eliminate the units?* Convert 0.75 meter to centimeters.
- *What is the ratio written without units?* $\frac{1}{75}$
- *Where does the 75 come from?* 0.75 meter = 75 centimeters

BL
- *If the model sailboat was 20 centimeters in length, what is the actual length of the sailboat in meters? in centimeters?* 15 m; 1,500 cm
- *How does the scale factor help you determine the actual length?* 1 centimeter on the model is 75 centimeters on the actual sailboat, so 20 centimeters × 75 = 1,500 centimeters

Need Another Example?

Find the scale factor of a blueprint if the scale is 1 centimeter = 0.30 meter. $\frac{1}{30}$

Example

4. Solve a problem involving a scale drawing.

AL • *What is the scale?* 1 centimeter = 2 meters

• *What do we need to find?* the area of Bedroom 1

OL • *How do you know that this problem requires multiple steps to solve?* First we need to find the dimensions of Bedroom 1. Then we need to find the area of Bedroom 1.

• *To find the length of Bedroom 1, why do we put the variable in the denominator of the second fraction?* It corresponds to 2 meters in the scale.

BL • *What is the actual perimeter of Bedroom 2?* 20 meters

Need Another Example?
Refer to the diagram on the student page. What is the actual area of the hallway? 12 m^2

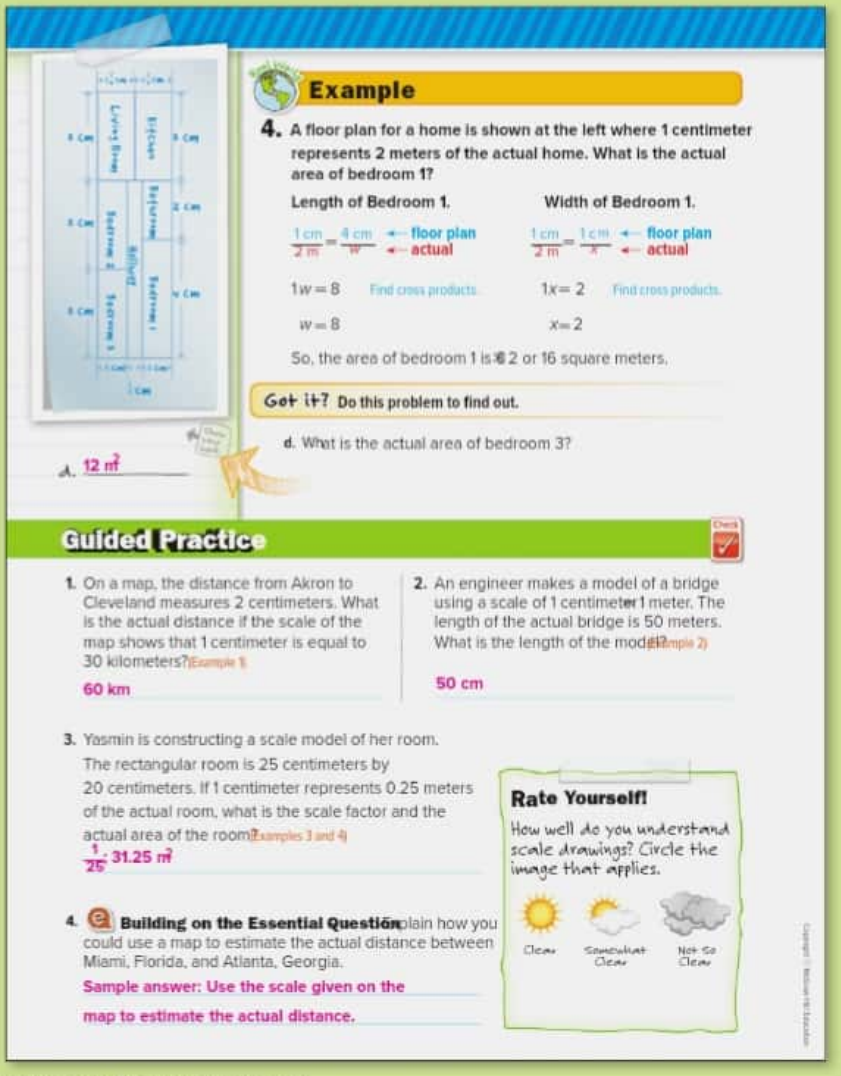
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Consult** Have students work in pairs to complete Exercises 1–4. For Exercises 1–3, have them create a drawing to aid them in completing each problem. 1, 4

BL LA **Trade-a-Problem** Have students work in pairs to create their own multi-step real-world problem involving a scale drawing. Have them trade their problems with another pair of students. Each pair solves the other pair's problem. Then have them meet back together to discuss and resolve any differences. 1, 4



Example

4. A floor plan for a home is shown at the left where 1 centimeter represents 2 meters of the actual home. What is the actual area of bedroom 1?

Length of Bedroom 1.

$\frac{1 \text{ cm}}{2 \text{ m}} = \frac{4 \text{ cm}}{w}$ ← floor plan ← actual

$1w = 8$ Find cross products.

$w = 8$

Width of Bedroom 1.

$\frac{1 \text{ cm}}{2 \text{ m}} = \frac{1 \text{ cm}}{x}$ ← floor plan ← actual

$1x = 2$ Find cross products.

$x = 2$

So, the area of bedroom 1 is 8 · 2 or 16 square meters.

Got it? Do this problem to find out.

d. What is the actual area of bedroom 3?

d. 12 m^2

Guided Practice

1. On a map, the distance from Akron to Cleveland measures 2 centimeters. What is the actual distance if the scale of the map shows that 1 centimeter is equal to 30 kilometers? (Example 1)

60 km

2. An engineer makes a model of a bridge using a scale of 1 centimeter to 1 meter. The length of the actual bridge is 50 meters. What is the length of the model? (Example 2)

50 cm

3. Yasmin is constructing a scale model of her room. The rectangular room is 25 centimeters by 20 centimeters. If 1 centimeter represents 0.25 meters of the actual room, what is the scale factor and the actual area of the room? (Examples 3 and 4)

$\frac{1}{25}$; 31.25 m^2

4. **Building on the Essential Question** Explain how you could use a map to estimate the actual distance between Miami, Florida, and Atlanta, Georgia.

Sample answer: Use the scale given on the map to estimate the actual distance.

Rate Yourself!

How well do you understand scale drawings? Circle the image that applies.

Clear · Somewhat Clear · Not So Clear

Example

4. Solve a problem involving a scale drawing.

AL • *What is the scale?* 1 centimeter = 2 meters

• *What do we need to find?* the area of Bedroom 1

OL • *How do you know that this problem requires multiple steps to solve?* First we need to find the dimensions of Bedroom 1. Then we need to find the area of Bedroom 1.

• *To find the length of Bedroom 1, why do we put the variable in the denominator of the second fraction?* It corresponds to 2 meters in the scale.

BL • *What is the actual perimeter of Bedroom 2?* 20 meters

Need Another Example?
Refer to the diagram on the student page. What is the actual area of the hallway? 12 m^2

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

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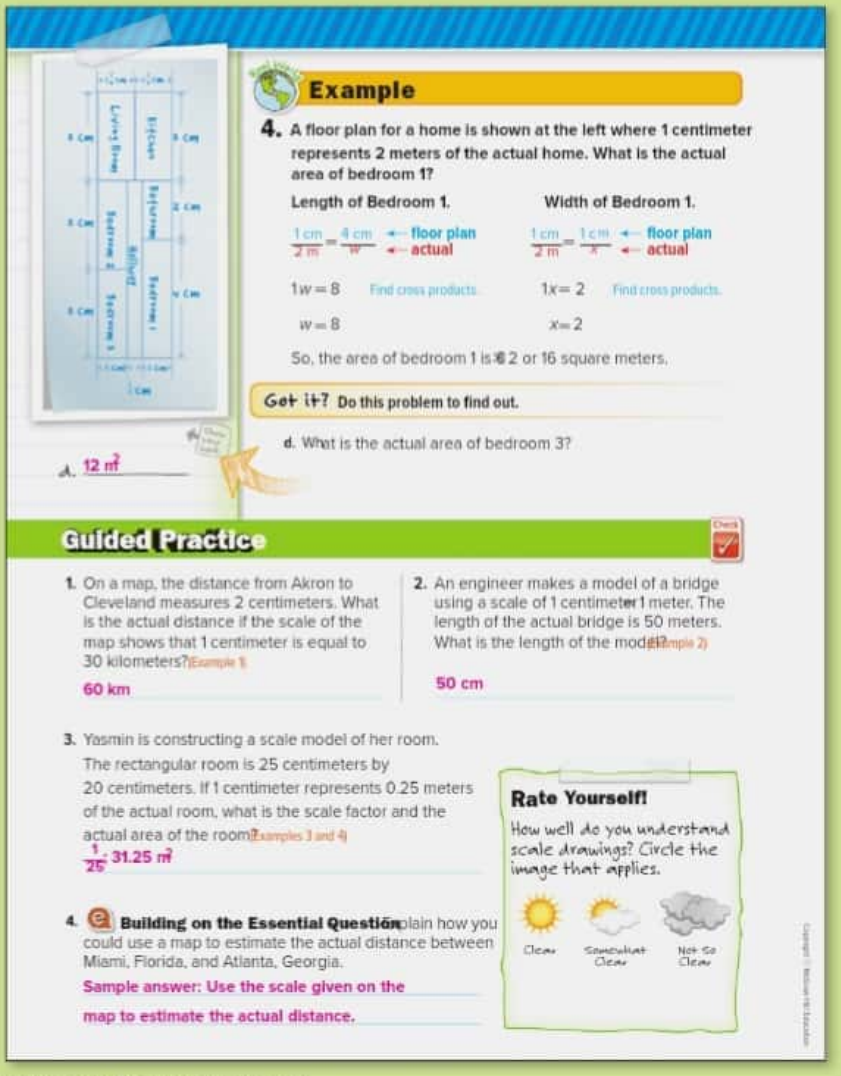


Name _____ My Homework

Independent Practice

Use Math Tools Find the actual distance between each pair of locations in South Carolina. Use a ruler to measure. Example 1 **1–2. Approximate distances are given.**

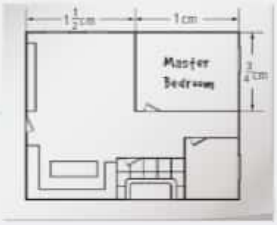
1. Columbia and Charleston 164 km
2. Hollywood and Sumter 130 km

Find the length of each model. Then find the scale factor. Examples 2 and 3

3. 0.5 cm = 1.5 m
12 cm; $\frac{1}{300}$

4. 1 cm = 1 m
26 cm; $\frac{1}{100}$

5. A model of an apartment is shown where 1 centimeter represents 4 meters in the actual apartment. Find the actual area of the master bedroom. Example 4
12 m²



ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

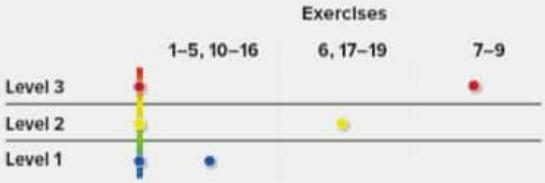
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7–9, 18, 19
OL	On Level	1–5 odd, 6–9, 18, 19
BL	Beyond Level	6–9, 18, 19

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
2 Reason abstractly and quantitatively.	8
3 Construct viable arguments and critique the reasoning of others.	9
4 Model with mathematics.	6, 7
5 Use appropriate tools strategically.	1, 2, 10–13

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a clos ing formative assessment before dismissing students from your class.

TICKET Out the Door

Tell students that a map has a scale of 1 centimeter to 80 kilometers. Have studen ts write how they could find the actual distance between two points shown on the m See students' work.

Watch Out!

Common Error Students may incorrectly set up their proportions to find the measurements for the actual object. Have students check their answers to make sure they are reasonable.

6. **Model with Mathematics** Refer to the graphic novel frames below. The scale on the map shows that 1 centimeter is equal to 75 meters. If the red line represents the path they took, how far have Ahmed, Mohammad, and Mahmoud traveled since they left the lake? Each square on the map is 1 centimeter long.

1,275 meters

H.O.T. Problems Higher Order Thinking

7. **Model with Mathematics** On the grid paper, create a scale drawing of a room in your home. Include the scale that you used. See students' work.

8. **Reason Abstractly** A statue of a camel was made using a scale of 3 centimeters = 1 centimeters. Write an expression to represent the height of the statue if the camel is x centimeters in height. Then find its actual height if the height of the statue is 579 centimeters.

$3x$; about 1.93 meters or 1 meter 93 centimeters

9. **Justify Conclusions** Determine whether the following statement is *always*, *sometimes*, or *never* true. Justify your reasoning.

If the scale factor of a scale drawing is greater than one, the scale drawing is larger than the actual object.

always; Sample answer: A scale factor of $\frac{3}{1}$ means that 3 units of the drawing is equal to 1 unit of the object, so the scale drawing or model will be larger than the actual object.

Name _____ My Homework

Extra Practice

Use Math Tools Find the actual distance between each pair of cities in New Mexico. Use a ruler to measure. **10–13.** Approximate distances are given.

10. Carlsbad and Artesia 50 km

$\frac{1\text{ cm}}{25\text{ km}} = \frac{2\text{ cm}}{d\text{ km}}$

$1 \times d = 25 \times 2$

$d = 50$

11. Hobbs and Eunice 30 km

12. Artesia and Eunice 130 km

13. Lovington and Carlsbad 102.5 km

14. Find the length of the model. Then find the scale factor. The length of an actual bird is shown at the right.
29.4 cm; $\frac{2}{1}$

Copy and Solve Show your work on a separate piece of paper.

15. A model of a tree is made using a scale of 1 centimeter = 3 meters. What is the height of the actual tree if the height of the model is 11 centimeters. 33 meters

16. A map of Bakersfield has a scale of 1 centimeter = 3.2 kilometers. If the city is 13 centimeters across on the map, what is the actual distance across the city? 41.6 kilometers

17. Tarek is creating a scale drawing of the area of his school. The rectangular drawing shows the length as 50 centimeters and the width as 47.5 centimeters. The drawing uses a scale of 1 centimeter = 36 centimeters. What is the actual area of the school in square meters? 3127.8 m^2

Power Up! Test Practice

Exercises 18 and 19 prepare students for more rigorous thinking needed for the assessment.

18. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

19. This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.

Depth of Knowledge	DOK3
Mathematical Practices	MP1, MP3
Scoring Rubric	
2 points	Students find the dimensions and explain their answer.
1 point	Students find the dimensions but fail to explain their answer OR students have a mathematical error in finding the dimensions but explain their answer based on the mathematical error.

Power Up! Test Practice

18. A landscape designer created a scale drawing of a garden as shown. The actual width of the bench is 2 meters, and the height is 1 meter. Fill in each box to complete the following statements.

a. The scale of the drawing is 1 centimeter(s) : 1 meter.

b. The height of the scale drawing is 1 centimeter(s).

19. A scale drawing of a doctor's office is shown. What are the actual dimensions of the doctor's office? Explain how you found your answer.

12 m by 18 m; Sample answer: Set up and solve proportions to find the actual length and width: $\frac{1\text{ cm}}{6\text{ m}} = \frac{2\text{ cm}}{w\text{ m}}$ and $\frac{1\text{ cm}}{6\text{ m}} = \frac{3\text{ cm}}{\ell\text{ m}}$

Spiral Review

20. A carpenter sawed a piece of wood into 3 pieces. The ratio of wood pieces is 1:3:6. The longest piece is 0.75 m longer than the shortest piece. Use the *draw a diagram* strategy to find the length of the original piece.

1.5 m

Solve each proportion.

21. $\frac{2}{5} = \frac{b}{25}$ 10

22. $\frac{3}{7} = \frac{a}{49}$ 21

23. $\frac{2}{9} = \frac{x}{99}$ 22

24. Hala has 60 sports cards. This is at least six more than three times as many cards as Nisreen. Write and solve an inequality to represent the situation.

$3a + 6 \leq 60;\ a \leq 18$

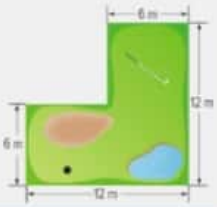
Geometry

Inquiry Lab
Scale Drawings

Mathematical Practices 1, 3, 5

Inquiry **WHAT happens to the size of a scale drawing when it is reproduced using a different scale?**

The owner of the miniature golf course wants to create a sign with an image of the 18th hole on it. Use the dimensions shown to create a scale drawing using the Geometer's Sketchpad®. Use the scale 1 centimeter = 3 meters.



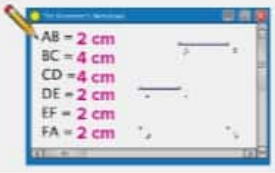
Hands-On Activity

Step 1 Determine the length the 6 meter side and the 12 meter side will be in the drawing.

Scale	Length	Scale	Length
$\frac{1 \text{ cm}}{3 \text{ m}} = \frac{x \text{ cm}}{6 \text{ m}}$		$\frac{1 \text{ cm}}{3 \text{ m}} = \frac{x \text{ cm}}{12 \text{ m}}$	
$1 \cdot 6 = 3 \cdot x$		$1 \cdot 12 = 3 \cdot x$	
$x =$ 2		$x =$ 4	

So, the 6 meter side will be 2 centimeters and the 12 meter side will be 4 centimeters in the drawing.

Step 2 Create the drawing using a dynamic geometry software. Then fill in the correct length for each line segment.



Focus narrowing the scope

Objective Use Geometer's Sketchpad® to calculate measurements for scale drawings.

Materials: computer with Geometer's Sketchpad® software or other dynamic geometry software

Coherence connecting within and across grades

Now Students use Geometer's Sketchpad® to calculate measurements for scale drawings.

Next Students will use scale factor to reduce and enlarge figures on the coordinate plane.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 584.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The Activity is intended to be used as a whole-group activity.

Hands-On Activity

AL LA Explain to students that the scale is 1 centimeter = 3 meters because they need to use a unit that makes sense for a scale drawing.

Ask:

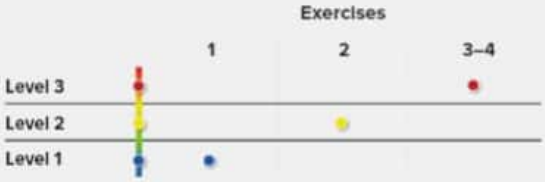
- *What ratio represents the length of the 6-meter side in the Sketchpad drawing to the actual length of the miniature golf course?* $\frac{x \text{ cm}}{6 \text{ m}}$
- *How will you find the length?* Write a proportion, cross multiply, and then simplify.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Pairs Discussion** Have students work in pairs to complete E xercise 1. Have them show their solutions to another pa ir of students and discuss any differen MP 1, 5

Analyze and Reflect

BL LA **Gallery Walk** Have students work in pairs to draw a scale dra wing of something in the classroom. Then post the scale dra wings around the room. Have students walk around the room and select a scale drawing. Have them determine the actual m easurements of the item shown in the drawing.
MP 1, 4, 5

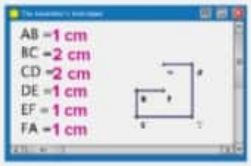
Create

Inquiry Students should be able to answer "WHAT happens to the size of a scale drawing when it is reproduced using a different scale?" Check for student understanding and provide guidance, if needed.

Investigate

Work with a partner. Use a dynamic geometry software.

1. MP **Use Math Tools** The owner wants a different size image of the 18th hole to place on the scorecards. Use the scale 1 centimeter = 6 meters. Fill in the new lengths of the line segments and draw the new scale drawing on the screen below. (*Hint:* You don't have to redraw the figure. Try clicking and dragging on the sides of your first drawing to adjust the side lengths.)



Analyze and Reflect

2. What happened to the size of the scale drawing when the scale changed from 1 centimeter = 3 meters to 1 centimeter = 6 meters?
The drawing became smaller.

Create

3. MP **Reason Inductively** Suppose you drew the miniature golf hole again at the scale 1 centimeter = 2 meters. Would the size of your drawing be larger or smaller than the drawing in the Activity? Explain.
Sample answer: The drawing would be larger because one centimeter would represent a lesser length of the actual golf hole.

4. Inquiry WHAT happens to the size of a scale drawing when it is reproduced using a different scale?
Sample answer: The drawing will be larger if a single unit of represents a lesser length. The drawing will be smaller if a single unit represents a greater length.

Geometry

Lesson 5

Draw Three-Dimensional Figures

Real-World Link

New York City In art class, Rasheed studied buildings known for their unusual architecture. He studied the Flatiron Building shown.

Three-dimensional figures, such as the Flatiron Building, have length, width, and height. They can be viewed from different perspectives, including the *side* view and the *top* view.

Essential Question

HOW does geometry help us describe real-world objects?

Mathematical Practices

1, 3, 4

1. What is the two-dimensional figure that makes up the side view?
   rectangle
2. What is the two-dimensional figure that makes up the top view?
   triangle
3. Sketch the side view of the Flatiron Building.
4. Sketch the top view of the Flatiron Building.

5. The top view, side view, and front view of a three-dimensional figure are shown below. Sketch the figure.

top 
side 
front 

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Draw three-dimensional figures given the top, side, and front views.

Coherence connecting within and across grades

Previous	Now	Next
Students found the volume and surface area of three-dimensional figures.	Students draw different views of three-dimensional figures.	Students will examine the shapes of different cross sections of three-dimensional figures.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 589.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL Before beginning the Real-World Link, give students concrete objects such as a water glass, a can, or a geometric solid. Ask them to hold the object at eye level, draw the two-dimensional figure they see, and label what side they were viewing. They should repeat the process for different views MP 1

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Draw top, front, and side views of a three-dimensional figure.**

AL • *How many line segments does the top have?* 3

• *What two-dimensional figure has three line segments?* triangle

• *What shape is the top and bottom of the figure?* triangle

OL • *What figure would you see from above?* triangle *the side?* rectangle *the front?* rectangle

BL • *Name the figure in Example 1.* triangular prism

2. **Draw top, front, and side views of a three-dimensional figure.**

AL • *Form a cone with a piece of paper. Fold the paper so the bottom is flat, then hold the cone so you can view it from above. What shape do you see?* a circle

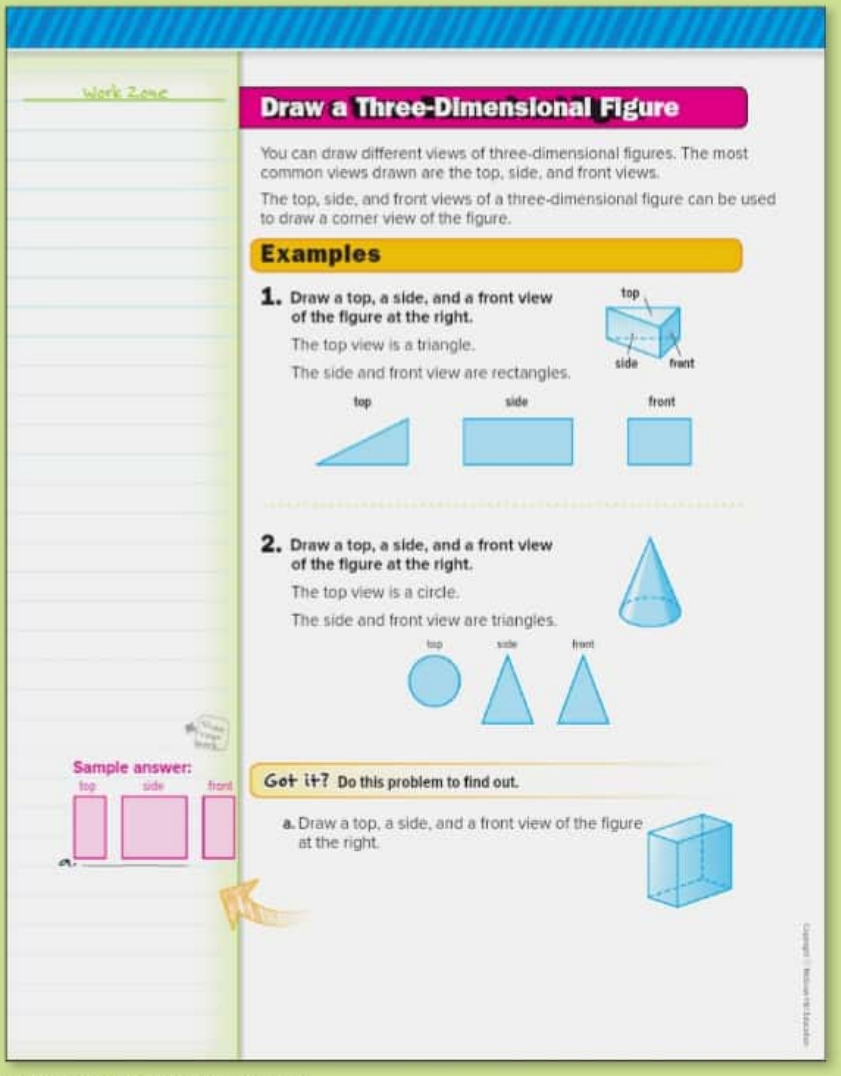
• *Look at the cone from the front. What shape do you see?* a triangle

• *What shape is the side?* a triangle

OL • *What figure would you see from above?* circle *the side?* triangle *the front?* triangle

BL • *Name the figure in Example 2.* cone

Need Another Example?
Draw a top, a side, and a front view of the figure. See Answer Appendix.



Work Zone

Draw a Three-Dimensional Figure

You can draw different views of three-dimensional figures. The most common views drawn are the top, side, and front views.

The top, side, and front views of a three-dimensional figure can be used to draw a corner view of the figure.

Examples

1. **Draw a top, a side, and a front view of the figure at the right.**

The top view is a triangle.

The side and front view are rectangles.

top side front

top side front

2. **Draw a top, a side, and a front view of the figure at the right.**

The top view is a circle.

The side and front view are triangles.

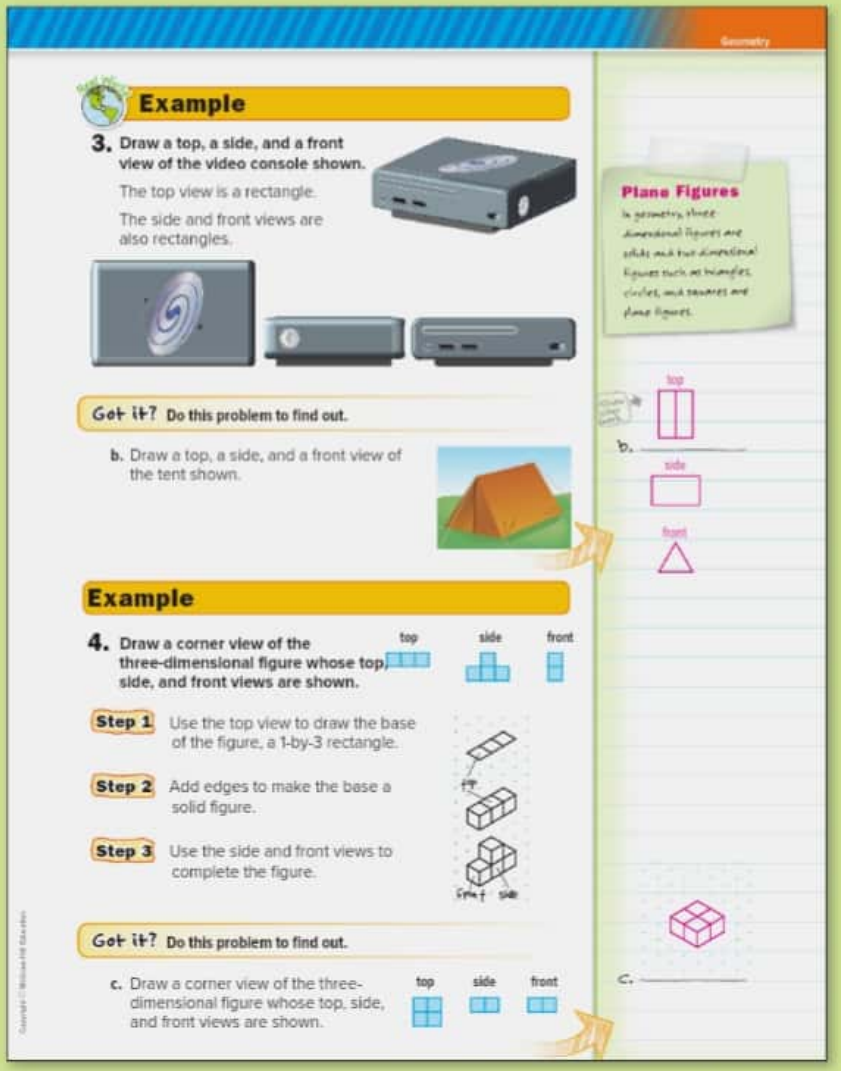
top side front

Sample answer:

top side front

Got it? **Do this problem to find out.**

a. Draw a top, a side, and a front view of the figure at the right.



Example

3. Draw a top, a side, and a front view of the video console shown.

The top view is a rectangle.

The side and front views are also rectangles.

Got it? Do this problem to find out.

b. Draw a top, a side, and a front view of the tent shown.

Example

4. Draw a corner view of the three-dimensional figure whose top, side, and front views are shown.

top side front

Step 1 Use the top view to draw the base of the figure, a 1-by-3 rectangle.

Step 2 Add edges to make the base a solid figure.

Step 3 Use the side and front views to complete the figure.

Got it? Do this problem to find out.

c. Draw a corner view of the three-dimensional figure whose top, side, and front views are shown.

top side front

Plane Figures

In geometry, three-dimensional figures are solids and two-dimensional figures such as triangles, circles, and squares are plane figures.

top

b. _____

side

front

c. _____

Examples

3. Draw top, front, and side views of a three-dimensional figure.

AL • *What figure do you see from above?* rectangle *the side?* rectangle *the front?* rectangle

OL • *Name the figure.* rectangular prism

BL • *Which faces are congruent?* Opposite faces are congruent; top and bottom, left side and right side, front and back

• *What other characteristics do the opposite faces share?* Sample answer: They are parallel.

Need Another Example?
Draw a top, a side, and a front view of the figure. See Answer Appendix.

4. Draw a three-dimensional figure.

AL • *Use centimeter cubes to build the figure. How do you know the figure is two cubes tall in the middle?* The middle of the side view is two tall.

• *Why is the front view two cubes tall?* It shows that the middle

OL • *What part of the figure does the top view represent?* the base

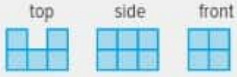
• *What are the dimensions of the rectangle?* 1 by 3

• *How does the side view help you complete the drawing?* The side view shows the only part that is 2 cubes tall is the middle.

BL • *How does using isometric dot paper make it easier to draw a corner view of a three-dimensional figure?* See students' work.

Need Another Example?
Draw a corner view of the three-dimensional figure whose top, side, and front views are shown. See Answer Appendix.

top side front



Example

5. Draw a three-dimensional figure.

AL • *Use centimeter cubes to build the figure. How does the top view help you build it?* The top view shows the base of the figure.

OL • *What part of the figure does the top view represent?* the base

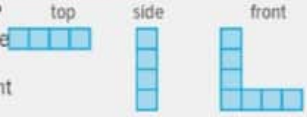
• *What are the dimensions of the rectangle?* 2-by-4

• *How does the side view help you complete the drawing?* The side view shows the only parts that are 2 cubes tall are at the back.

BL • *Could you draw this figure using only the top and front views? Explain.* No, the side view shows that the figure is only one cube tall in some sections.

Need Another Example? Draw a corner view of the three-dimensional figure whose top, side, and front views are shown.

top side front



See Answer Appendix.

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

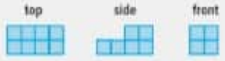
AL LA **Paired Heads Together** Have students work in pairs to solve Exercises 1 and 2. Allow students to use concrete materials as needed. Students complete the first exercise, then turn to their partner and discuss their answers. Students repeat this process for Exercise 2, building the figure with cubes first before drawing it on dot paper. VP 1, 4

BL LA **Trade-a-Problem** Have students create their own top, side, and front view of a figure, similar to Exercise 2. Students trade their drawings, draw each other's figure, and compare solutions. If the solutions do not agree, students work together to find the error. VP 1, 4

Example

5. Draw a corner view of the three-dimensional figure whose top view, side view, and front view are shown.

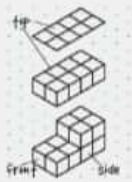
top side front



Step 1 Use the top view to draw the base of the figure, a 2-by-4 rectangle.

Step 2 Add edges to make the base a solid figure.

Step 3 Use the side and front views to complete the figure.



Guided Practice

1. Draw a top, a side, and a front view of the figure. (Examples 1–3)

top side front

2. Draw a corner view of the three-dimensional figure whose top view, side view, and front view are shown. (Examples 4–5)

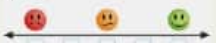
top side front

3. **Building on the Essential Question** How does drawing the different views of a three-dimensional figure help you have a better understanding of the figure?

Sample answer: The different views help you to see what two-dimensional shapes make up the three-dimensional figure.

Rate Yourself!

How confident are you about drawing three-dimensional figures? Check the box that applies.



Name _____ My Homework

Independent Practice

Draw a top, a side, and a front view of each figure. (Examples 1–2)

1.

top side front

2.

top side front

3. Draw a top, a side, and a front view of the eraser shown. (Example 3)

top side front

Draw a corner view of each three-dimensional figure whose top view, side view, and front view are shown. (Examples 4–5)

4. top side front

5. top side front

6. Name a real-world object that has a top view of a triangle, and a side and front view that are each rectangles. **See students' work.**

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

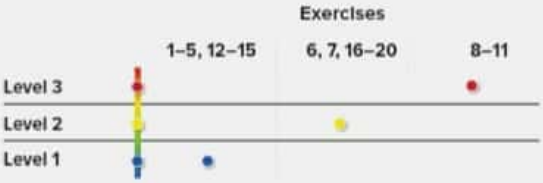
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7–9, 11, 19, 20
OL	On Level	1–5 odd, 6–9, 11, 19, 20
BL	Beyond Level	6–11, 19, 20

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	10
3 Construct viable arguments and critique the reasoning of others.	9, 11, 18
4 Model with mathematics.	7, 8

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a clos ing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write how knowing how to draw three-dimensional figures hel ps in finding the volume of three-dimensional figures. See students' work.

7. **Model with Mathematics** The Quetzalcoatl pyramid in Mexico is shown. Use the photo to sketch views from the top, side, and front of the pyramid.

H.O.T. Problems Higher Order Thinking

8. **Model with Mathematics** Choose an object in your classroom or in your home. Sketch any view of the object. Choose among a top, a side, or a front view. See students' work.

9. **Which One Doesn't Belong?** Identify the figure that does not have the same characteristic as the other three. Explain your reasoning.

triangle; it is the only two-dimensional figure.

10. **Persevere with Problems** Draw a three-dimensional figure in which the front and top views each have a line of symmetry but the side view does not. Sample answer:

11. **Reason Inductively** Determine whether each statement is *always*, *sometimes*, or *never* true.

a. The bases of a cylinder have different radii. never

b. Two planes intersect in a single point. never

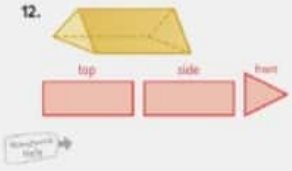
c. Three planes do not intersect in a point. sometimes

Name _____ My Homework _____

Extra Practice

Draw a top, a side, and a front view of each figure.

12.



13.

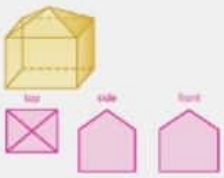
Draw a corner view of each three-dimensional figure whose top view, side view, and and front view are shown.

14.

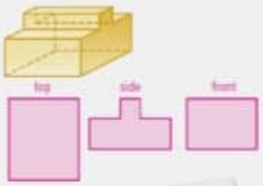
15.

Draw a top, a side, and a front view of each figure.

16.



17.



18. **Find the Error** Ahmed drew the side, top, and front view of the figure shown at the right. Find his mistake and correct it.

The top and side view should be switched.

Watch Out!

Find the Error In Exercise 18, Ahmed mixed up the top view and the side view. Suggest to students that they imagine lookin down on the figure from above to see the correct top view of the drawing.

Power Up! Test Practice

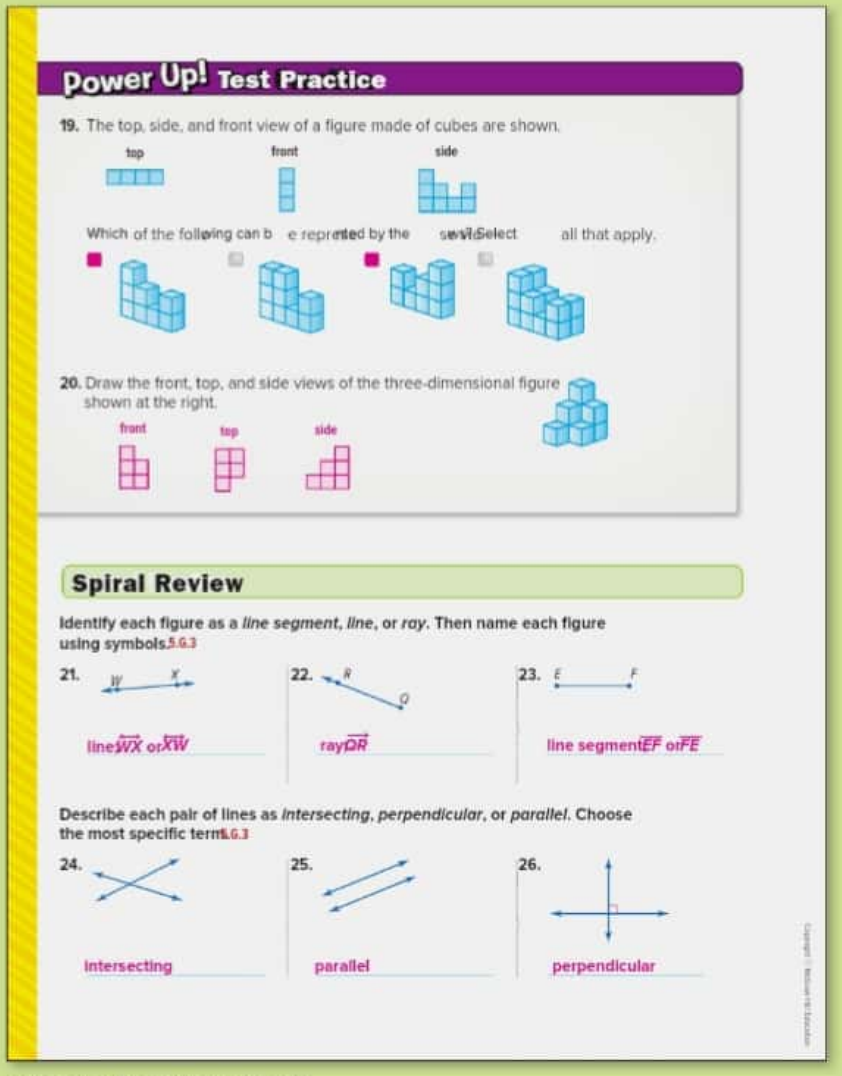
Exercises 19 and 20 prepare students for more rigorous thinking needed for the assessment.

19. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP4
Scoring Rubric	
1 point	Students correctly answer the question.

20. This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.

Depth of Knowledge	DOK3
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly draw each view of the figure.
1 point	Students correctly draw two of the views of the figure.



Power Up! Test Practice

19. The top, side, and front view of a figure made of cubes are shown.

top | front | side

Which of the following can be represented by the views? Select all that apply.

20. Draw the front, top, and side views of the three-dimensional figure shown at the right.

front | top | side

Spiral Review

Identify each figure as a *line segment*, *line*, or *ray*. Then name each figure using symbols. 5.G.3

21. line $\overleftrightarrow{WX}$ or $\overleftrightarrow{XW}$

22. ray $\overrightarrow{QR}$

23. line segment $\overline{EF}$ or $\overline{FE}$

Describe each pair of lines as *intersecting*, *perpendicular*, or *parallel*. Choose the most specific term. 5.G.3

24. intersecting

25. parallel

26. perpendicular

Lesson 6

Cross Sections

Vocabulary Start-Up

A prism is a three-dimensional figure with at least two parallel, congruent faces called bases that are polygons. A pyramid is a three-dimensional figure with one base that is a polygon. Its other faces are triangles.

Write *prism* or *pyramid* on the line below each figure.

prism

pyramid

pyramid

prism

Real-World Link

The Heroes Arts Club is shown below. Is the shape of the building a *prism* or *pyramid*? Explain.

pyramid; Sample answer: The shape has one base with triangular faces.

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Essential Question

HOW does geometry help us describe real-world objects?

Vocabulary

- prism
- bases
- pyramid
- plane
- coplanar
- parallel
- polyhedron
- edge
- face
- vertex
- diagonal
- skew lines
- cylinder
- cone
- cross section

Mathematical Practices 1, 3, 4

Focus narrowing the scope

Objective Identify and draw three-dimensional figures.

Coherence connecting within and across grades

Previous
Students drew different views of three-dimensional figures.

Now
Students name parts of three-dimensional figures and identify the shape of a cross section of the figure.

Next
Students will draw and use nets of three-dimensional figures to find surface area.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 597.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Circle the Sage** Poll the class on which students know the vocabulary words in the list. Have those students, the sages, spread out in the room. The rest of the class each surround a sage. The sage explains what he or she knows, while the other students listen and ask questions. Students return to their seats and report on what they learned MP 1, 6

Alternate Strategy

AL Have students work in pairs. Provide each pair with a net of the objects in the activity. Students cut out the figures and tape the edges together. Then discuss how to name the figures and what the base(s) might be. MP 1, 3, 5

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

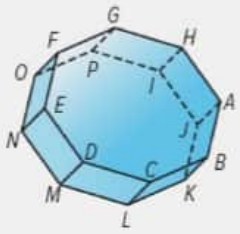
1. Identify figures.

AL
- *How many bases does the figure have?* 1
- *What shape is the base?* pentagon
- *Is the base considered one of the faces?* yes
- *How many faces does the figure have?* 6

OL
- *Does the figure have parallel bases?* no
- *Is the figure a prism or pyramid?* pyramid
- *Identify the pyramid.* pentagonal pyramid

BL
- *What are skew lines?* Lines in the same plane that do not intersect.
- *Since they do not intersect, are $\overline{RS}$ and $\overline{TU}$ skew lines? Explain.* No; they are not skew because the lines that contain those segments do intersect.

Need Another Example?
Identify the figure. Then name the bases, faces, edges, and vertices. See Answer Appendix.



Work Zone

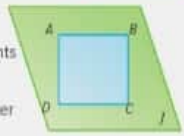
Polygons

The table below lists some common names of polygons.

Sides	Name
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon

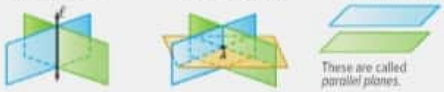
Identify Three-Dimensional Figures

A **plane** is a flat surface that goes on forever in all directions. The figure at the right shows rectangle *ABCD*. Line segments *AB* and *DC* are **coplanar** because they lie in the same plane. They are also **parallel** because they will never intersect, no matter how far they are extended.



Just as two lines in a plane can intersect or be parallel, there are different ways that planes may be related in space.

Intersect in a Line | **Intersect at a Point** | **No Intersection**

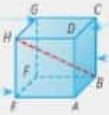


These are called *parallel planes*.

Intersecting planes can form three-dimensional figures. A **polyhedron** is a three-dimensional figure with flat surfaces that are polygons. Prisms and pyramids are both polyhedrons. Some terms associated with three-dimensional figures are *edge*, *face*, *vertex*, and *diagonal*.

Edge where two planes intersect in a line

Face a flat surface

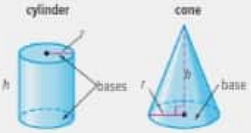


Vertex where three or more planes intersect at a point

Diagonal a line segment whose endpoints are vertices that are neither adjacent nor on the same face

Notice in the figure above $\overline{EC}$ and $\overline{DA}$ do not intersect. These segments are not parallel because they do not lie in the same plane. Lines that do not intersect and are not coplanar are called **skew lines**.

There are also solids that are not polyhedrons. A **cylinder** is a three-dimensional figure with two parallel congruent circular bases connected by a curved surface. A **cone** has one circular base connected by a curved side to a single vertex.



Examples

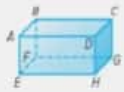
Identify the figure. Name the bases, faces, edges, and vertices. Then, identify a pair of skew lines.

1.

The figure has one base that is a pentagon, so it is a pentagonal pyramid.

base: RSTUV
faces: RSTUV, QVR, QRS, QST, QTU, QUV
edges: $\overline{QR}$, $\overline{QS}$, $\overline{QT}$, $\overline{QU}$, $\overline{QV}$, $\overline{VR}$, $\overline{RS}$, $\overline{ST}$, $\overline{TU}$, $\overline{UV}$
vertices: Q, R, S, T, U, V
skew lines: $\overline{QV}$ and $\overline{TS}$

2.



The figure has rectangular bases that are parallel and congruent, so it is a rectangular prism.

bases: ABCD and EFGH, ABFE and DCGH, ADHE and BCGF
faces: ABCD, EFGH, ABFE, DCGH, ADHE, BCGF
edges: $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AD}$, $\overline{EF}$, $\overline{FG}$, $\overline{GH}$, $\overline{EH}$, $\overline{AE}$, $\overline{BF}$, $\overline{CG}$, $\overline{DH}$
vertices: A, B, C, D, E, F, G, H
skew lines: $\overline{AE}$ and $\overline{FG}$

Common Error

In the drawing of a rectangular prism, the bases do not have to be on the top and bottom. Any two parallel rectangles are bases. In a triangular pyramid, any face is a base.

Got it? Do this problem to find out.

a.

figure name: triangular pyramid
base: JML
faces: JML, JKM, KML, JKL
edges: $\overline{JK}$, $\overline{KL}$, $\overline{JL}$, $\overline{JM}$, $\overline{KM}$, $\overline{LM}$
vertices: J, K, L, M
skew lines: Sample answer: $\overline{KM}$ and $\overline{JL}$

Identify Cross Sections

The intersection of a solid and a plane is called a cross section of the solid.

Example

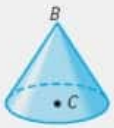
2. Identify figures.

AL
- How many bases does the figure have? 2
- What shape are the bases? rectangles
- Are the bases considered two of the faces? yes
- How many faces does the figure have? 6
- Two faces that can be considered bases are ABCD and EFGH. Are there any other faces that can be called bases? If so, what are they? yes; ABFE and DCGH, ADHE and BCGF

OL
- Does the figure have parallel bases? yes
- How many different sets of bases can this figure have? 3
- Is the figure a prism or pyramid? prism
- Identify the prism. rectangular prism
- $\overline{AE}$ and $\overline{FG}$ are two skew lines. Name another pair of skew lines. Sample answer: $\overline{BF}$ and $\overline{GH}$

BL
- In three-dimensional space, can there be skew planes? Explain your reasoning. no; Sample answer: Lines are two-dimensional figures, so they need three-dimensional space to be skew. Planes would need four-dimensional space in order to be skew.

Need Another Example?
Identify the figure. Then name the bases, faces, edges, and vertices. See Answer Appendix.



Example

3. Identify cross sections.

AL • *In your own words, what is a cross section?* See students' work.

• *What is the shape of the base of the pyramid?* square

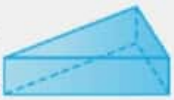
• *If you slice the pyramid parallel to the base, what shape will result?* square

OL • *If you slice a square pyramid at an angle, what shape will you see?* trapezoid

BL • *Describe a real-world example of the use of a cross section.* See students' work; examples may include cutting down a tree and counting the rings, or slicing vegetables.

Need Another Example?

Describe the shape resulting from a vertical, angled, and horizontal cross section of a triangular prism. vertical: rectangle; angled: trapezoid; horizontal: triangle



Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

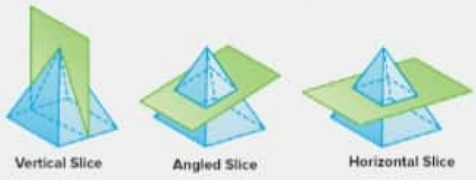
If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Rally Table** Have students work in teams to solve Exercise 1. Allow students to use concrete materials as needed. Students take turns naming the bases, faces, edges, vertices, and skew lines for Exercise 1. Have them trade their solutions with another team and discuss any differences. Remind students that $\triangle QRS$ is the same as $\triangle RTQ$. 4

BL LA **Pairs Discussion** Have students work in pairs to research the Platonic solids and construct the solids out of paper. Then have them discuss and draw vertical and horizontal cross sections of the solids. Have them trade their drawings with another pair of students and discuss any differences. Remind students that the cross sections may look different depending on how the solid is sitting. 4

Example

3. Describe the shape resulting from a vertical, angled, and horizontal cross section of a square pyramid.



Vertical Slice — The cross section is a triangle.

Angled Slice — The cross section is a trapezoid.

Horizontal Slice — The cross section is a square.

Got it? Do this problem to find out.

b. Describe the shape resulting from a vertical, angled, and horizontal cross section of a cylinder.

b. Sample answer: vertical: rectangle; angled: oval; horizontal: circle

Guided Practice

1. Identify the figure. Then name the bases, faces, edges, and vertices. Then, identify a pair of skew lines. (Examples 1–2)

figure name: triangular prism

bases: QRS and TUV

faces: QRS, TUV, QSVT, SRUV, RQTU

edges: QR, RS, QS, TU, UV, TV, QT, RU, SV

vertices: Q, R, S, T, U, V

skew lines: Sample answer: SV and TU

2. Describe the shape resulting from the cross section shown. (Example 3) triangle

3. **Building on the Essential Question** How can knowing the shape of the base of a three-dimensional figure help you name the figure?

Sample answer: Some three-dimensional figures are named by their base.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

Name _____ My Homework _____

Independent Practice

Identify each figure. Name the bases, faces, edges, and vertices. Then, identify a pair of skew lines. Examples 1–2

1.

figure name: triangular pyramid
bases: ACD
faces: ACD, ABD, ABC, DBC
edges: $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AD}$, $\overline{AC}$, $\overline{BD}$
vertices: A, B, C, D
skew lines: Sample answer: $\overline{BD}$ and $\overline{AC}$

2.

figure name: hexagonal pyramid
bases: MNOPQR
faces: MNOPQR, LMN, LNO, LOP, LPQ, LQR, LRM
edges: $\overline{LM}$, $\overline{LN}$, $\overline{LO}$, $\overline{LP}$, $\overline{LQ}$, $\overline{LR}$, $\overline{MN}$, $\overline{NO}$, $\overline{OP}$, $\overline{PQ}$, $\overline{QR}$, $\overline{RM}$
vertices: L, M, N, O, P, Q, R
skew lines: Sample answer: $\overline{LM}$ and $\overline{PO}$

Describe the shape resulting from each cross section. Example 3

3.

rectangle

4.

triangle

5.

triangle

6. A basketball is shaped like a sphere.

a. Draw a basketball with a vertical, angled, and horizontal slice.

vertical: angled: horizontal:

b. Describe the cross section made by each slice.

All cross sections are circles.

c. Is the basketball a polyhedron? Explain.

No. It has no flat surfaces that are polygons.

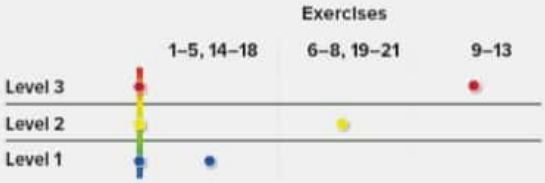
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



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You can use the table below that includes exercises fo all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7–9, 20, 21
OL	On Level	1–5 odd, 6–9, 20, 21
BL	Beyond Level	6–13, 20, 21

Watch Out!

Common Error Students may have trouble with naming all the faces of a solid. Suggest students start with the first letter of the alphabet used to name a vertex. Name each side with the letters moving from one side to the other in a clockwise manner.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	10–13
3 Construct viable arguments and critique the reasoning of others.	7, 19
4 Model with mathematics.	9

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a clos ing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students explain h ow to identify skew lines in a figure.

See students' work.

7. **Use a Counterexample** State whether the following conjecture is *true* or *false*. If *false*, provide a counterexample.

Two planes in three-dimensional space can intersect at one point.

False; two planes intersect at a line, which is an infinite number of points.

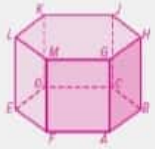
8. Draw and label a hexagonal prism. Then identify each of the following.

Sample answers are given.

a. parallel planes GHJKLM and ABCDEF

b. skew lines JH and LE

c. intersecting planes GHBA and ABCDEF



H.O.T. Problems Higher Order Thinking

9. **Model with Mathematics** Draw the cross sections of a polyhedron, cylinder, or cone. Exchange papers with another student. Identify the three-dimensional figures represented by the cross sections.

See students' work.

Persevere with Problems Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

10. A pyramid has parallel faces.

never; A pyramid has all faces, except the base, intersecting at one vertex.

11. A prism has 2 bases and 4 faces.

sometimes; A rectangular prism has 2 bases and 4 faces, but a triangular prism has 2 bases and 3 faces.

12. A parallelogram cannot be a cross section of a triangular prism.

always; The cross section of a triangular prism will be a triangle, rectangle, or trapezoid.

13. A pyramid has a rectangular base.

sometimes; A triangular pyramid has a triangle for its base.

Name _____ My Homework _____

Extra Practice

Identify each figure. Then name the bases, faces, edges, and vertices. Then identify a pair of skew lines.

14.

figure name: triangular prism

bases: $\triangle RSV$ and $\triangle UTW$

faces: $\triangle RSV$, $\triangle UTW$, $RSTU$, $SVWT$, $VRUW$

edges: $\overline{RS}$, $\overline{SV}$, $\overline{RV}$, $\overline{UT}$, $\overline{TW}$, $\overline{UW}$, $\overline{RU}$, $\overline{VW}$, $\overline{ST}$

vertices: R, S, T, U, V, W

skew lines: Sample answer $\overline{TU}$ and $\overline{RV}$

15.

figure name: rectangular prism

bases: ABCD, EFGH, ABFE, DCGH, ADHE, BCGF

faces: ABCD, EFGH, ABFE, DCGH, ADHE, BCGF

edges: $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AD}$, $\overline{EF}$, $\overline{FG}$, $\overline{GH}$, $\overline{EH}$, $\overline{AE}$, $\overline{BF}$, $\overline{CG}$, $\overline{DH}$

vertices: A, B, C, D, E, F, G, H

skew lines: Sample answers $\overline{DH}$ and $\overline{GF}$

Describe the shape resulting from each cross section.

16.

square

17.

curve

18.

triangle

19. Find the Error Sally is identifying the figure at the right. Find her mistake and correct it.

Because there are two parallel, congruent triangular bases, it is a triangular prism.

Watch Out!

Find the Error In Exercise 19, Sally incorrectly named the figure. Suggest to students that they think of pyramids as having a common point, or vertex, at which all faces meet except the base. Have students think of prisms as having two bases that are opposite and parallel to each other.

Power Up! Test Practice

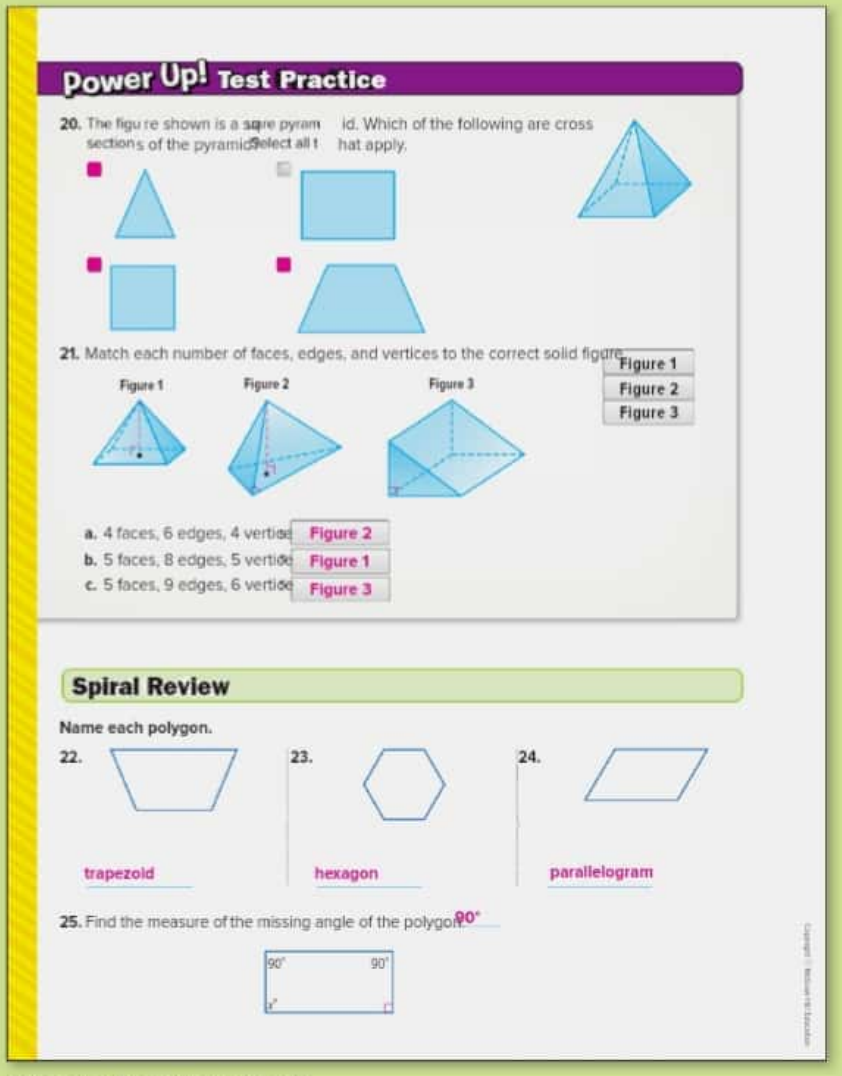
Exercises 20 and 21 prepare students for more rigorous thinking needed for the assessment.

20. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

21. This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.

Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly match all three figures.
1 point	Students correctly match two of the three figures.



Power Up! Test Practice

20. The figure shown is a square pyramid. Which of the following are cross sections of the pyramid? Select all that apply.

21. Match each number of faces, edges, and vertices to the correct solid figure.

Figure 1 Figure 2 Figure 3

a. 4 faces, 6 edges, 4 vertices — Figure 2
b. 5 faces, 8 edges, 5 vertices — Figure 1
c. 5 faces, 9 edges, 6 vertices — Figure 3

Spiral Review

Name each polygon.

22. trapezoid

23. hexagon

24. parallelogram

25. Find the measure of the missing angle of the polygon. 90°

Focus narrowing the scope

Objective Apply mathematics to problems arising in the workplace. This lesson emphasizes **Mathematical Practice 4** Model with Mathematics.

Coherence connecting within and across grades

Previous
Students solved problems involving scale drawings.

Now
Students apply the content standard to solve problems in the workplace.

Rigor pursuing concepts, fluency, and applications

See the Career Project on page 602.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ask students to read the information on the student page about roller coaster designers and answer the following questions.

Ask:

- *What kinds of classes should you take to be a roller coaster designer?* Sample answer: math and science courses like Algebra, Physics, Calculus, Geometry, Trigonometry
- *What are some important things a roller coaster designer needs to consider when designing a new coaster?* Sample answer: how to make the coaster exciting, but still safe

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** EVALUATE

2 Collaborate

AL LA **Think-Write-Share** Have students write a real-world word problem using information from the table. Then have students work in pairs to share their problem and discuss any changes that may need to be made to their word problem and/or solution. MP 1, 4

BL LA **Timed Roundrobin** Divide the class into team of 3 or 4. Give students five minutes to complete Exercises 1–5. Each student contributes at least once in the completion of each of the exercises. When the teams have finished, have students in a whole class setting or a large group setting discuss the solution. MP 1, 6

Career Portfolio

When students complete this page, have them add it to their Career Portfolio.

Career Facts

Design engineers use tricks to make coasters more exciting. For example, when a coaster travels close to the ground, the ground acts as a reference point and makes riders feel like they are going faster than they really are. In another example, in 2007 designers updated SheiKra so that its cars became "floorless." Although the coaster's 200-foot, 90-degree drop did not change, it became even more thrilling because there was no floor against which riders could brace themselves.

A Thrilling Ride

Use the information in the table to solve each problem.

1. In a scale drawing of SheiKra, a designer uses a scale of 1 centimeter = 2 meters. What is the height of the roller coaster in the drawing? 30 cm
2. On a model of Montu, the height of the loop is 32 centimeters. What is the scale? 1 cm = 1 m
3. In a scale drawing of Montu, the height of the roller coaster is 25 centimeters. What is the scale factor? $\frac{1}{200}$
4. SheiKra has a hill that goes through a tunnel. On a model of the roller coaster, the hill is 60 centimeters tall and the scale is 1 centimeter = 0.75 meter. What is the actual height of the tunnel hill? 45 m
5. An engineer is building a model of SheiKra. She wants the model to be about 80 centimeters high. Choose an appropriate scale for the model. Then use it to find the loop height of the model. Sample answer: 1 cm = 0.75 m; 60 cm

Busch Gardens Tampa

Roller Coaster	Coaster Height (m)	Loop Height (m)
SheiKra	60	45
Montu	50	32

Career Project

It's time to update your career portfolio! Describe a roller coaster that you, as a roller coaster designer, would create. Include the height and angle of the tallest drop, the total length, maximum speed, number of loops and tunnels, and color scheme. Be sure to include the name of your roller coaster.

What problem-solving skills might you use as a roller coaster designer?

-
-
-
-
-

Chapter Review

Vocabulary Check

Complete the crossword puzzle using the vocabulary list at the beginning of the chapter.

Across

2. a three-dimensional figure with two parallel, congruent bases that are polygons
4. a triangle with an angle greater than 90 degrees and less than 180 degrees
8. a three-dimensional figure with two parallel congruent circular bases connected by a curved surface
11. a triangle with three congruent sides
12. segments with the same length
14. used to represent an object that is too large to be built at actual size (2 words)
15. two angles with a sum of 90 degrees

Down

1. angles that share a common vertex, a common side, and do not overlap
3. two angles with a sum of 180 degrees
5. a figure with three sides and three angles
6. the ratio that compares the measurements of a model and the real object
7. opposite angles that are formed by the intersection of two lines
8. a three-dimensional figure with one circular base connected by a curved side to a single vertex
9. an angle less than 90 degrees
10. where two rays meet to form an angle
13. a 90 degree angle

Vocabulary Check

LA **Paired Heads Together** Have students work in pairs to complete the Vocabulary Check exercises, ensuring that each partner understands. Each student is assigned a number, 1 or 2. Randomly call on a numbered student to share their responses with the class. MP 1, 6

Alternate Strategy

AL LA To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include the following terms. MP 1, 6

- acute angle (Lesson 1)
- adjacent angles (Lesson 1)
- complementary angles (Lesson 2)
- cone (Lesson 6)
- congruent segments (Lesson 3)
- cylinder (Lesson 6)
- equilateral triangle (Lesson 3)
- obtuse angle (Lesson 1)
- prism (Lesson 6)
- right angle (Lesson 1)
- scale (Lesson 4)
- scale model (Lesson 4)
- supplementary angles (Lesson 2)
- triangle (Lesson 3)
- vertex (Lesson 1)
- vertical angles (Lesson 1)

Key Concept Check

FOLDABLES LA A completed Foldable for this chapter should include a review of similar figures and scale drawings.

If you choose not to use this Foldable, have students write a brief review of the Key Concepts found throughout the chapter and give an example of each.

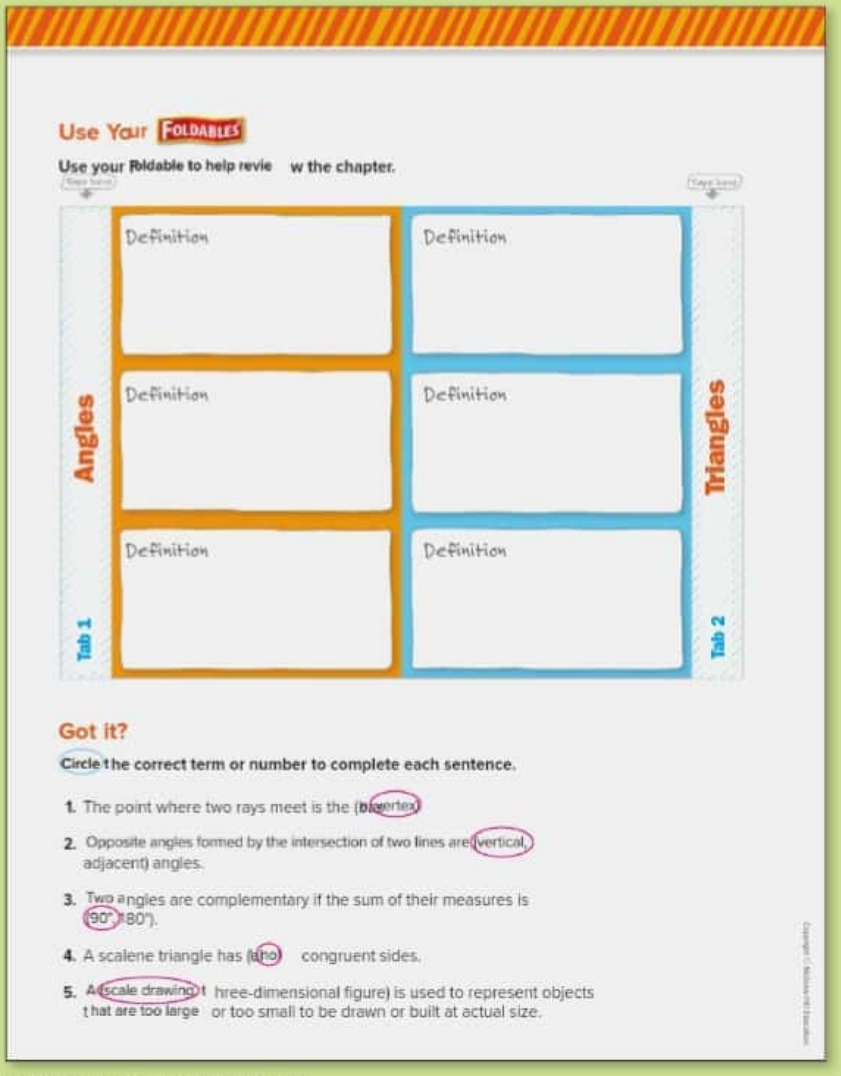
Ideas for Use

LA Have students work in pairs to discuss their Foldables. Have them practice speaking in a collaborative setting by sharing how they have completed their Foldable thus far and how they could finish it. Have each student complete their Foldable and trade with their partner to discuss any similarities and differences MP 1, 3, 5

Got It?

If students have trouble with Exercises 1–5, they may need help with the following concept(s).

Concept	Exercise(s)
vertical angles (Lesson 1)	1, 2
complementary angles (Lesson 2)	3
classifying triangles (Lesson 3)	4
scale drawings (Lesson 4)	5



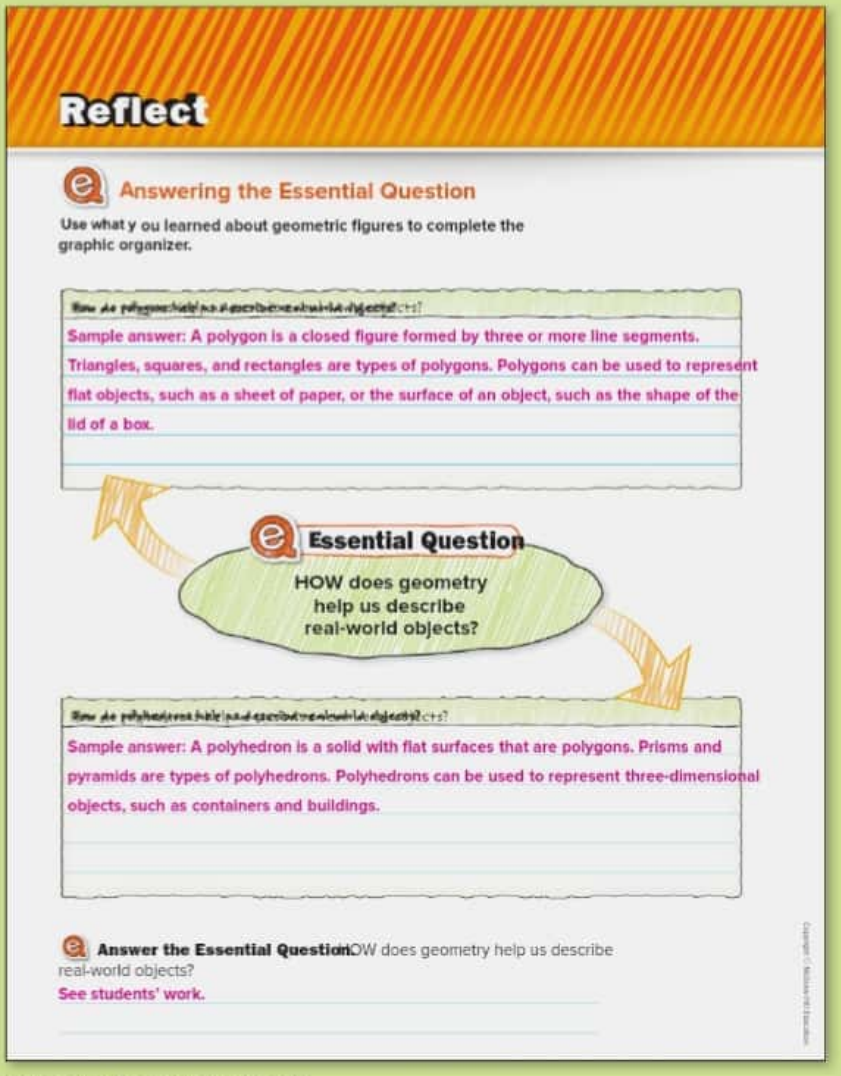
Answering the Essential Question

Before answering the Essential Question, have students review their answers to the **Building on the Essential Question** exercises found in each lesson of the chapter.

- What are the differences between vertical and adjacent angles. (p. 538)
- How are vertical, adjacent, complementary, and supplementary angles related? (p. 546)
- How can triangles be classified? (p. 558)
- How can you use a map to estimate the actual distance between Miami, Florida, and Atlanta, Georgia? (p. 578)
- How does drawing the different views of a three-dimensional figure help you have a better understanding of the figure? (p. 588)
- How can knowing the shape of the base of a three-dimensional figure help you name the figure? (p. 596)

Ideas for Use

LA **Think-Pair-Share** Have students work in pairs. Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer and answer the Essential Question. Then have them share their responses with their partner, ensuring that each partner understands. Have pairs volunteer to share their responses with the class. MP 1, 4



Reflect

Answering the Essential Question

Use what you learned about geometric figures to complete the graphic organizer.

How do polygons help us describe real-world objects?

Sample answer: A polygon is a closed figure formed by three or more line segments. Triangles, squares, and rectangles are types of polygons. Polygons can be used to represent flat objects, such as a sheet of paper, or the surface of an object, such as the shape of the lid of a box.

Essential Question

HOW does geometry help us describe real-world objects?

How do polyhedrons help us describe real-world objects?

Sample answer: A polyhedron is a solid with flat surfaces that are polygons. Prisms and pyramids are types of polyhedrons. Polyhedrons can be used to represent three-dimensional objects, such as containers and buildings.

Answer the Essential Question HOW does geometry help us describe real-world objects?

See students' work.