Appendix G

Worked Example for the Calculation of Volt-drop in a Circuit Containing an Induction Motor

G.1 INTRODUCTION

The following example explains how volt-drop calculations can be carried out. Initially the subject is approached from a rigorous standpoint. Subsequently various simplifications are introduced, their results compared and their appropriateness discussed. The calculation sequence is:-

- Rigorous solution, see a) to p).
- Simplified solution, see q) to t).
- Formulae method based on kVA ratings, see u).
- Graphical estimation, see v).

Figure G.1 is a simplified one-line diagram of a power generation and distribution system that would be suitable for most oil industry power systems that have their own power generating plants, e.g. an off-shore production platform. Figure G.2 is the equivalent diagram showing the basic symbols and configuration needed for the volt-drop calculation process. The example data are given below:-

i) Generator data.
3 generators, each rated at 3.125 MVA at 0.8 PF lagging.
Rated voltage = 13.8 kV, $X'_d = 25\%$, $R_a = 2\%$.

ii) Switchboard data.
Rated voltage = 13.8 kV
Standing load = 900 kW at 0.9 PF lagging ($\cos\phi_{og}$).

iii) Transformer data.
Rated at 3.15 MVA.
Rated voltage ratio = 13.2 : 4 kV. $X_{tpu} = 6\%$, $R_{tpu} = 0.7\%$.

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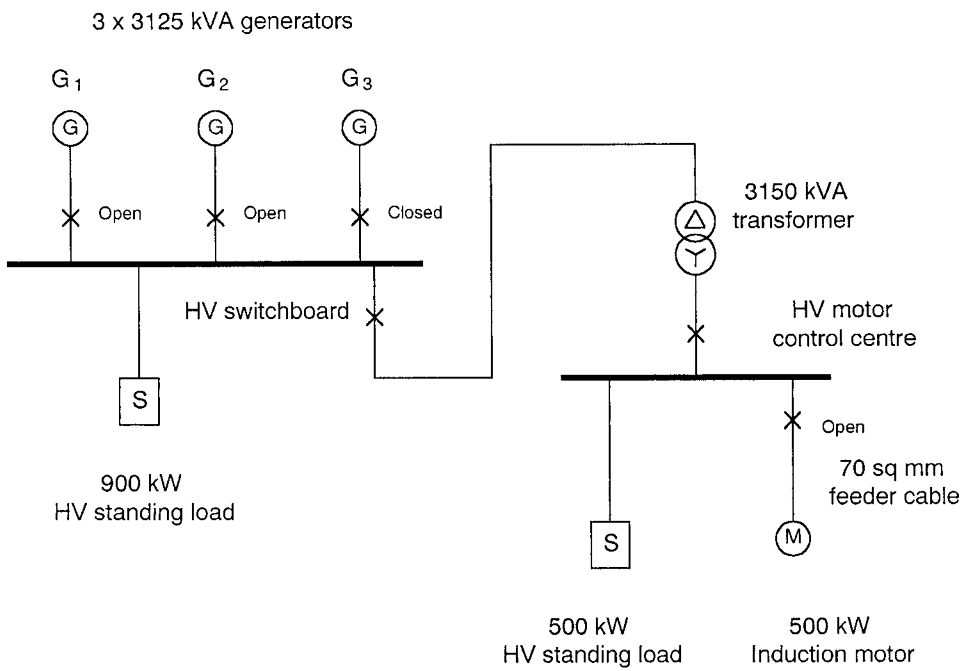

Figure G.1 Simplified one-line diagram for calculating the volt-drop of a 500 kW HV motor.

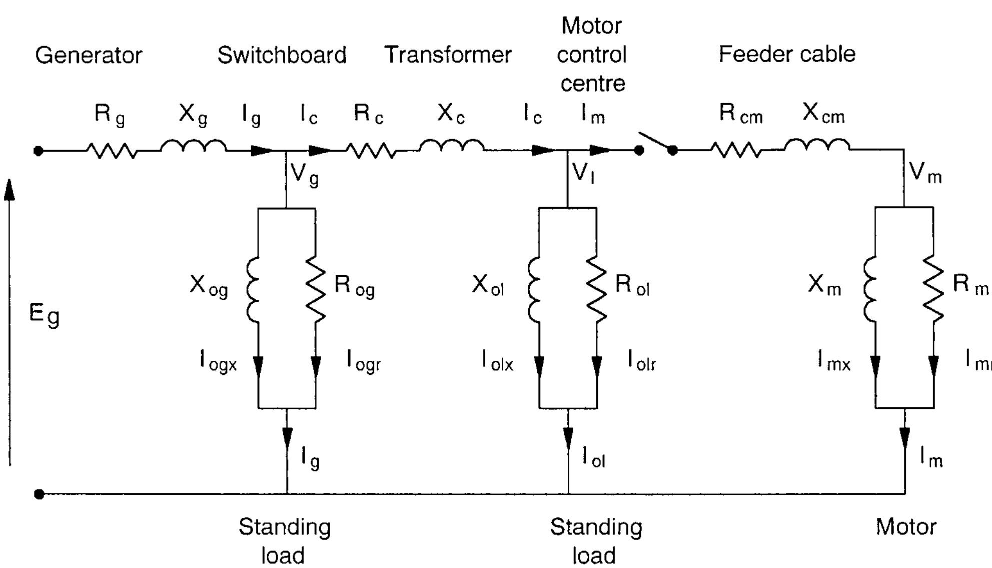

Figure G.2 Basic equivalent circuit for calculating the volt-drop of a 500 kW HV motor.

iv) Motor control centre data.
Rated voltage = 4.16 kV
Standing load = 500 kW at 0.85 PF lagging ($\cos\phi_{ol}$).

v) Motor feeder cable

Conductor size	=	70 mm^2
Conductor temperature	=	75°C
Specific resistance R_{km}	=	0.343 ohms per km
Specific reactance X_{km}	=	0.129 ohms per km
Rated voltage	=	5 kV
Rated frequency	=	60 Hz
Route length	=	1500 m

vi) Motor

Rated voltage	=	4 kV
Rated efficiency	=	95%
Rated power factor	=	0.88 lagging
Starting current	=	5 times the full-load current
Starting power factor	=	0.25 lagging

Convert the data to the system base values.

a) For convenience choose the system base kVA and voltages to be:-

$$\text{System base kVA} = \text{Generator kVA} = 3125$$

$$\text{System base voltage at the switchboard} = 13,800 \text{ volts}$$

$$\begin{aligned}\text{System base voltage at the MCC} &= 13,800 \times \text{transformer ratio}\\ &= 13,800 \times \frac{4000}{13,200} = 4181.8 \text{ volts,}\end{aligned}$$

b) The system base value of the generator impedance $R_g + jX_g$ is the same as that for the generator kVA base.

$$R_g + jX_g = R_a + jX'_d = 0.02 + j0.25 \text{ pu}$$

c) Convert the transformer impedance to the system base values.

$$\begin{aligned}R_c + jX_c &= (R_{pu} + jX_{pu})\frac{(\text{base kVA})\ (\text{trans pri voltage})^2}{(\text{trans kVA})\ (\text{base pri voltage})^2}\\ &= (0.7 + j6.0)\frac{(3125)(13,200)^2}{(3150)(13,800)^2} = 0.00635 + j0.05446 \text{ pu}\end{aligned}$$

d) Switchboard (SWBD) parallel circuit components.

Convert to system base values.

$$\text{SWBD load kVA} = S_{og} = \frac{\text{SWBD load power}}{\text{SWBD load power factor}}$$

$$= \frac{P_{og}}{\cos\phi_{og}} = \frac{900 \times 1000}{0.9} = 1000 \times 10^3 \text{ VA}$$

$$\text{SWBD load kVA/phase} = S_{ogp} = \frac{1000 \times 1000}{3} = 333 \times 10^3 \text{ VA}$$

$$\text{SWBD load power/phase} = P_{ogp} = \frac{900 \times 1000}{3} = 300 \times 10^3 \text{ VA}$$

$$\text{SWBD load reactive power/phase} = Q_{ogp} = \sqrt{(S_{ogp}{}^2 - S_{ogp}{}^2)}$$

$$= 145.29 \times 10^3 \text{ VAr}$$

$$\text{Ohmic resistance per phase} = R_{ogp} = \left(\frac{\text{Phase voltage}}{\text{Phase active power}}\right)^2$$

$$= \left(\frac{V_{og}}{P_{ogp}}\right)^2 = \frac{13,800 \times 13,800}{3 \times 300 \times 10^3} \times 10^6$$

$$= 211.6 \text{ ohms per phase}$$

$$\text{Ohmic reactance per phase} = X_{ogp} = \left(\frac{\text{Phase voltage}}{\text{Phase reactive power}}\right)^2$$

$$= \left(\frac{V_{og}}{Q_{ogp}}\right)^2 = \frac{13,800 \times 13,800}{3 \times 145.29 \times 10^3} \times 10^6$$

$$= 436.92 \text{ ohms per phase}$$

Convert to the system base impedance values.

$$\text{System impedance in per-unit} = \text{Ohmic impedance} = \frac{(\text{load base kVA})\ (\text{system base kVA})}{(\text{system base voltage})^2(\text{load base kVA})}$$

Hence

$$R_{og} = \frac{(211.6 \times 1000 \times 1000)(3125 \times 1000)}{(13,800)^2(1000 \times 1000)} = 3.4722 \text{ pu}$$

And

$$X_{og} = \frac{(436.92 \times 1000 \times 1000)(3125 \times 1000)}{(13,800)^2(1000 \times 1000)} = 7.1696 \text{ pu}$$

These are the parallel elements of the load in per-unit at the system base.

e) Motor control centre (MCC) parallel circuit components.

Convert to system base values.

$$\text{MCC load kVA} = S_{ol} = \frac{\text{MCC load power}}{\text{MCC load power factor}}$$

$$= \frac{P_{ol}}{\cos\phi_{ol}} = \frac{500 \times 1000}{0.85} = 588.235 \times 10^3 \text{ VA}$$

$$\text{MCC load kVA/phase} = S_{olp} = \frac{588.235 \times 1000}{3} = 196.078 \times 10^3 \text{ VA}$$

$$\text{MCC load power/phase} = P_{olp} = \frac{500 \times 1000}{3} = 166.667 \times 10^3 \text{ VA}$$

$$\text{MCC load reactive power/phase} = Q_{olp} = \sqrt{(S_{olp}{}^2 - S_{olp}{}^2)}$$

$$= 103.29 \times 10^3 \text{ VAr}$$

$$\text{Ohmic resistance per phase} = R_{olp} = \left(\frac{\text{Phase voltage}}{\text{Phase active power}}\right)^2 =$$

$$= \left(\frac{V_{ol}}{P_{olp}}\right)^2 = \frac{4160 \times 4160}{3 \times 166.67 \times 10^3} \times 10^6$$

$$= 34.61 \text{ ohms per phase}$$

$$\text{Ohmic reactance per phase} = X_{olp} = \left(\frac{\text{Phase voltage}}{\text{Phase reactive power}}\right)^2$$

$$= \left(\frac{V_{ol}}{Q_{olp}}\right)^2 = \frac{41,600 \times 4160}{3 \times 103.29 \times 10^3} \times 10^6$$

$$= 55.848 \text{ ohms per phase}$$

Convert to the system base impedance values.

$$\text{System impedance in per-unit} = \text{Ohmic impedance} = \frac{(\text{load base kVA})\,(\text{system base kVA})}{(\text{system base voltage})^2(\text{load base kVA})}$$

Hence

$$R_{ol} = \frac{(34.61 \times 588.235 \times 1000)(3125 \times 1000)}{(4160)^2(588.235 \times 1000)} = 6.2498 \text{ pu}$$

and

$$X_{ol} = \frac{(55.848 \times 588.235 \times 1000)(3125 \times 1000)}{(4160)^2(588.235 \times 1000)} = 10.085 \text{ pu}$$

These are the parallel elements of the load in per-unit at the system base.

f) Motor feeder cable. Convert to system base.

A 70 mm^2 three-core 5 kV cable has an ohmic impedance of $0.343 + j0.129$ ohms per kilometre per phase and a current rating of 250 A. Hence the total ohmic impedance is $0.5145 + j0.1935$ ohms per phase.

$$\text{The total VA rating for the cable} = \sqrt{3} \times \text{Rated line voltage} \times \text{Rated phase current}$$
$$= \sqrt{3} \times 5000 \times 250 = 2.165 \times 10^6 \text{ VA}$$
$$\text{The VA rating for the cable per phase} = 0.3333 \times 2.165 \times 10^6 = 721.67 \times 10^3$$
$$\text{The 1.0 pu impedance of the cable per phase} = V_{olp} = \frac{5000}{\sqrt{3} \times 250}$$
$$= 11.547 \text{ ohms per phase.}$$

Hence the per-unit impedance of this particular cable at its own base is:-

$$R_{pu} + jX_{pu} = \frac{0.5145 + j0.1935}{11.547} = 0.04456 + j0.01676 \text{ pu}$$

Convert this impedance to the system base:-

$$R_{cm} + jX_{cm} = (R_{pu} + jX_{pu})\frac{(\text{base kVA})\ (\text{cable rated voltage})^2}{(\text{cable kVA})\ (\text{system base voltage})^2}$$
$$= \frac{(0.04456 + j0.01676)(3125)(5000)^2}{(2.165 \times 10^6)(13,800)^2}$$
$$= 0.00844 + j0.003175 \text{ pu}$$

g) Motor running conditions (suffix 'r')

$$\text{Motor rated voltage} = 4000.0 \text{ volts}$$
$$\text{Motor system base voltage} = 4181.8 \text{ volts}$$
$$\text{Motor terminal voltage} = 4160.0 \text{ volts}$$
$$\text{Input power to each phase} = \frac{\text{Rated power output}}{3 \times \text{efficiency}}$$
$$P_{mrp} = \frac{500}{3 \times 0.95} \times 10^3 = 175.44 \text{ kW}$$
$$\text{Input VA to each phase} = \frac{\text{Rated power input}}{\text{Power factor}}$$
$$S_{mrp} = \frac{175.44}{0.88} \times 10^3 = 199.36 \text{ kVA}$$
$$\text{Input VAr to each phase} = \sqrt{(S_{mrp}{}^2 - S_{mrp}{}^2)}$$
$$Q_{mrp} = 1000.0 \times \sqrt{(199.36^2 - 175.44^2)} = 94.68 \text{ kVAr}$$

At the motor rating base the phase ohmic resistance R_{mrp} is:-

$$R_{olp} = \left(\frac{\text{Phase voltage}}{\text{Phase active power}}\right)^2$$
$$= \frac{4000 \times 4000}{3 \times 175.44 \times 10^3} = 30.4 \text{ ohms per phase}$$

Similarly the phase ohmic reactance X_{mrp} is:-

$$X_{olp} = \left(\frac{\text{Phase voltage}}{\text{Phase reactive power}}\right)^2$$
$$= \frac{4000 \times 4000}{3 \times 94.68 \times 10^3} = 56.32 \text{ ohms per phase}$$

Convert this impedance to the motor per-unit base.

$$\text{The 1.0 pu motor kVA per phase} = S_{mrp} = 199.36$$
$$\text{The 1.0 pu motor impedance per phase} = Z_{mrp}$$

$$Z_{mrp} = \left(\frac{\text{Phase voltage}}{\text{Phase VA}}\right)^2$$
$$= \frac{4000 \times 4000}{3 \times 199.36 \times 10^3} = 26.75 \text{ ohms per phase}$$

Hence the per-unit motor running resistance is R_{olppu}:-

$$R_{olppu} = \frac{R_{mrp}}{Z_{mrp}} = \frac{30.4}{26.75} = 1.136 \text{ pu per phase}$$

And the per-unit motor running reactance is X_{olppu}:-

$$X_{olppu} = \frac{X_{mrp}}{Z_{mrp}} = \frac{56.32}{26.75} = 2.105 \text{ pu per phase}$$

Where R_{olppu} and X_{olppu} are parallel components representing the motor during the full-load running condition. Convert this impedance to the system base at the motor system voltage of 4181.8 volts.

$$R_{mr} + jX_{mr} = (R_{mrppu} + jX_{mrppu})\frac{(\text{base kVA})\ (\text{motor rated voltage})^2}{(\text{motor kVA})\ (\text{system base voltage})^2}$$
$$= \frac{(1.136 + j2.105)(3125)(4000)^2}{(3 \times 199.36 \times 10^3)(4181.8)^2}$$
$$= 5.4324 + j10.065 \text{ pu}$$

h) Motor running conditions (suffix 's')

$$\text{Rated current to each phase} = \frac{\text{Rated input VA}}{\sqrt{3} \times \text{Rated motor voltage}}$$
$$P_{mrp} = \frac{598.08}{\sqrt{3} \times 4000} \times 10^3 = 86.33 \text{ amps}$$
$$\text{Starting current} = 5 \times \text{Rated current} = 431.63 \text{ amps}$$

The starting impedance Z_{msp} is:-

$$Z_{msp} = \frac{\text{Phase voltage}}{\text{Starting current}}$$
$$= \frac{4000}{\sqrt{3} \times 431.64} = 5.35 \text{ ohms per phase}$$

The starting resistance R_{msp} (parallel branch) is:-

$$Z_{msp} = \frac{Z_{msp}}{\cos\phi_s} = \frac{5.35}{0.25} = 21.4 \text{ ohms per phase}$$

The starting reactance X_{msp} (parallel branch) is:-

$$X_{msp} = \frac{Z_{msp}}{\sin\phi_s} = \frac{5.35}{0.9682} = 5.526 \text{ ohms per phase}$$

Hence the per-unit motor starting resistance is R_{msppu}:-

$$R_{msppu} = \frac{R_{msp}}{Z_{msp}} = \frac{21.4}{26.75} = 0.80 \text{ pu per phase}$$

And the per-unit motor starting reactance is X_{msppu}:-

$$X_{msppu} = \frac{X_{msp}}{Z_{msp}} = \frac{5.526}{26.75} = 0.2066 \text{ pu per phase}$$

Where, R_{msppu} and X_{msppu} are parallel components representing the motor during the starting condition. Convert this impedance to the system base at the motor system voltage of 4181.8 volts.

$$R_{ms} + jX_{ms} = (R_{msppu} + jX_{msppu})\frac{(\text{base kVA})\ (\text{motor rated voltage})^2}{(\text{motor kVA})\ (\text{system base voltage})^2}$$
$$= \frac{(0.8 + j0.2066)(3125)(4000)^2}{(598.08 \times 10^3)(4181.8)^2}$$
$$= 0.38244 + j0.9875 \text{ pu}$$

i) Summary of the results thus far.

The data to be used in the per-unit circuit diagram in Figure G.2 are:-

Generator	R_g	=	0.2 pu
	X_g	=	0.25 pu
SWBD parallel load	R_{og}	=	3.4722 pu
	X_{og}	=	7.1676 pu
Transformer	R_c	=	0.00635 pu
	X_c	=	0.05446 pu
MCC parallel load	R_{ol}	=	6.2498 pu
	X_{ol}	=	10.085 pu

Motor feeder cable	R_{cm} =	0.00844 pu
	X_{cm} =	0.003175 pu
Motor running	R_{mr} =	5.43024 pu
	X_{mr} =	10.065 pu
Motor starting	R_{ms} =	3.8244 pu
	X_{ms} =	0.9875 pu

j) Rigorous solution

The sequence of calculations is as follows:-

- Initial conditions, using suffix '*o*'.
- Running conditions, using suffix '*n*'.
- Starting conditions, using suffix '*s*'.
- Compare the calculated voltages and find the volt-drops.
- Design comments.

k) Initial conditions

The motor starter is open and the generator terminal voltage is 1.0 per-unit.

Hence,

$$V_{go} = 1.0 + j0.0 \text{ pu.}$$

Find the initial values of I_c and V_l, i.e. I_{co} and V_{lo}, noting that $I_m = 0.0$

At the MCC the parallel load is R_{ol} in parallel with X_{ol}.

Convert the parallel load into a series load of $R_{ol1} + jX_{ol1}$.

The formulae for this conversion are:-

$$R_{oll} = \frac{R_{ol}X_{ol}^2}{R_{ol}^2 + X_{ol}^2} \quad \text{pu per phase}$$

$$X_{oll} = \frac{X_{ol}R_{ol}^2}{R_{ol}^2 + X_{ol}^2} \quad \text{pu per phase}$$

Where

$$R_{ol} = 6.2498 \text{ and } X_{ol} = j10.085$$

Hence,

$$Z_{oll} = R_{oll} + jX_{oll} = 4.5156 + j2.7985 \text{ pu}$$

The impedance seen at the SWBD is

$$Z_{ol1} + Z_c = 0.00635 + 4.5156 + j(0.05446 + 2.7985)$$

$$= 4.5220 + j2.8530 \text{ pu}$$

$$Z_{oll} = \frac{V_{go}}{Z_{oll} + Z_c} = \frac{1.0 + j0.0}{4.5220 + j2.8530}$$

$$= 0.1582 - j0.0998 \text{ pu}$$

$$V_{lo} = \frac{V_{go}Z_{oll}}{Z_{oll} + Z_c} = 0.9936 - j0.0080 \text{ pu,}$$

which has a magnitude of 0.9936 pu.

Find the initial emf, E_o, of the generator.

At the SWBD the parallel load is R_{og} in parallel with X_{og}.

Convert the parallel load into a series load of $R_{ogl} + jX_{ogl}$.

$$R_{og} = 3.4722 \text{ pu and } X_{og} = j7.1696 \text{ pu},$$

hence Z_{ogl} is:-

$$Z_{ogl} = R_{ogl} + jX_{ogl} = 2.8125 + j1.3622 \text{ pu}.$$

The initial load current I_{ogo} is:-

$$I_{ogo} = \frac{V_{go}}{Z_{ogl}} = \frac{(1.0 + j0.0)(2.8125 - j1.3621)}{2.8125^2 + 1.3621^2}$$
$$= 0.288 - j0.1395 \text{ pu}.$$

The total initial generator current I_{go} is:-

$$I_{go} = I_{ogo} + I_{co} = 0.1582 - j0.0998 + 0.288 - j0.1395$$
$$= 0.4462 - j0.2393 \text{ pu}.$$

Hence,

$$E_o = V_{go} + I_{go}Z_g = 1.0 + j0 + (0.4461 - j0.2393)(0.02 + j0.25) \text{ pu}$$
$$= 1.0687 + j0.1068 \text{ pu, which has a magnitude of } 1.0741 \text{ pu}.$$

l) Running conditions

The motor starter is closed and the generator emf is 1.0741 per-unit.

Assume the rated impedance for the motor since this will give a worst-case running impedance for it. (The 500 kW motor will be over-sized in any case with respect to the driven machine by about 10% and so the actual impedance will be about 10% higher than the rated impedance.)

The parallel impedance of the running motor is Z_{mn}:-

$$R_{mn} = 5.4324 \text{ and } X_{mn} = j10.065 \text{ pu}$$

The series impedance of the running motor is Z_{mnl}:-

$$Z_{mnl} = R_{mnl} + jX_{mnl} = 4.2069 + j2.2706 \text{ pu}$$

Now add the feeder cable impedance in series to obtain the total series impedance between the MCC and the motor. Call this total impedance Z_{mnlc}.

$$\begin{aligned} Z_{\text{mnlc}} &= R_{\text{mnlc}} + jX_{\text{mnlc}} = R_{\text{mnlc}} + R_{\text{cm}} + j(X_{\text{mnlc}} + X_{\text{cm}}) \\ &= 0.00844 + 4.2069 + j(0.003175 + 2.2706) \\ &= 4.2153 + j2.2738 \text{ pu} \end{aligned}$$

The total load on the MCC consists of the static load Z_{ol1} (series components) in parallel with the cable and motor Z_{mnlc} (series components). The total impedance Z_{oln} is therefore:-

$$Z_{\text{oln}} = R_{\text{oln}} + jX_{\text{oln}} = \frac{Z_{\text{oll}} \times Z_{\text{mnlc}}}{Z_{\text{oll}} + Z_{\text{mnlc}}} = 2.1828 + j1.2590 \text{ pu}$$

The impedance seen at the SWBD for the cable, motor and MCC load is Z_{cn}:-

$$\begin{aligned} Z_{\text{cn}} = Z_{\text{oln}} + Z_c &= 2.1828 + j1.2590 + 0.00635 + j0.05446 \\ &= 2.1891 + j1.3135 \text{ pu} \end{aligned}$$

This impedance is in parallel with that of the local load Z_{og} on the SWBD. The total equivalent load on SWBD is Z_{ogn} where:-

$$Z_{\text{ogn}} = R_{\text{ogn}} + jX_{\text{ogn}} = \frac{Z_{\text{ogl}} \times Z_{\text{cn}}}{Z_{\text{ogl}} + Z_{\text{cn}}} = 1.2341 + j0.6746 \text{ pu}$$

Hence the total impedance seen by the generator emf E_{o} is Z_{gn}:-

$$\begin{aligned} Z_{\text{ogn}} &= R_{\text{g}} + R_{\text{ogn}} + j(X_{\text{g}} + X_{\text{ogn}}) \\ &= 0.02 + 1.2341 + j(0.25 + 0.6746) \\ &= 1.2541 + j0.9246 \text{ pu} \end{aligned}$$

The current in the generator I_{gn} is:-

$$I_{\text{gn}} = \frac{E_{\text{o}}}{Z_{\text{gn}}} = \frac{1.0687 - j0.1068}{1.2541 + j0.9246} = 0.5928 - j0.3519 \text{ pu}$$

Hence the terminal voltage of the generator V_{gn} is:-

$$\begin{aligned} V_{\text{gn}} &= \frac{E_{\text{o}} Z_{\text{ogn}}}{Z_{\text{gn}}} = \frac{(1.0687 + j0.1068)(1.2341 + j0.6746)}{1.2541 + j0.9246} \\ &= 0.9689 - j0.0344 \text{ pu, which has a magnitude of } 0.9695 \text{ pu.} \end{aligned}$$

Similarly the voltage of the MCC V_{ln} is:-

$$\begin{aligned} V_{\text{ln}} &= \frac{V_{\text{gn}} Z_{\text{oln}}}{Z_{\text{cn}}} = \frac{(0.9689 - j0.0344)(2.1828 + j1.259)}{2.1891 + j1.3135} \\ &= 0.9556 - j0.0504 \text{ pu, which has a magnitude of } 0.9570 \text{ pu.} \end{aligned}$$

m) Starting conditions

The motor starter is closed. Repeat the procedure as for l) the running conditions, but with the starting impedance using the suffix 's' for starting.

The parallel impedance of the running motor is Z_{mn}:-

$$R_{ms} = 3.8244 \text{ and } X_{ms} = j0.9875 \text{ pu}$$

The series impedance of the running motor is Z_{msl}:-

$$Z_{msl} = R_{msl} + jX_{msl} = 0.2390 + j0.9257 \text{ pu}$$

Now add the feeder cable impedance in series to obtain the total series impedance between the MCC and the motor. Call this total impedance Z_{mlc}.

$$\begin{aligned} Z_{mslc} &= R_{mslc} + jX_{mslc} = R_{mslc} + R_{cm} + j(X_{mslc} + X_{cm}) \\ &= 0.00844 + 0.2390 + j(0.003175 + 0.9257) \\ &= 0.2475 + j0.9289 \text{ pu} \end{aligned}$$

The total load on the MCC consists of the static load Z_{ol1} (series components) in parallel with the cable and motor Z_{mnsc} (series components). The total impedance Z_{ols} is therefore:-

$$Z_{ols} = R_{ols} + jX_{ols} = \frac{Z_{oll} \times Z_{mslc}}{Z_{oll} + Z_{mnsc}} = 0.3050 + j0.7874 \text{ pu}$$

The impedance seen at the SWBD for the cable, motor and MCC load is Z_{cs}:-

$$\begin{aligned} Z_{cs} = Z_{ols} + Z_c &= 0.3050 + j0.7874 + 0.00635 + j0.05446 \\ &= 0.3114 + j0.8418 \text{ pu} \end{aligned}$$

This impedance is in parallel with that of the local load Z_{og} on the SWBD. The total equivalent load on SWBD is Z_{ogs} where:-

$$Z_{ogs} = R_{ogs} + jX_{ogs} = \frac{Z_{ogl} \times Z_{cs}}{Z_{ogl} + Z_{cs}} = 0.3631 + j0.6375 \text{ pu}$$

Hence the total impedance seen by the generator emf E_o is Z_{gs}:-

$$\begin{aligned} Z_{ogs} &= R_g + R_{ogs} + j(X_g + X_{ogs}) \\ &= 0.02 + 0.3631 + j(0.25 + 0.6375) \\ &= 0.3831 + j0.8875 \text{ pu} \end{aligned}$$

The current in the generator I_{gs} is:-

$$I_{gns} = \frac{E_o}{Z_{gs}} = \frac{1.0687 - j0.1068}{0.5395 + j0.9713} = 0.5395 - j0.9713 \text{ pu}$$

Hence the terminal voltage of the generator V_{gs} is:-

$$V_{gs} = \frac{E_o Z_{ogs}}{Z_{gn} s} = \frac{(1.0687 + j0.1068)(0.3631 + j0.6375)}{0.3838 + j0.8892}$$

$$= 0.8151 - j0.0087 \text{ pu, which has a magnitude of } 0.8152 \text{ pu.}$$

Similarly the voltage of the MCC V_{ls} is:-

$$V_{ls} = \frac{V_{gs} Z_{ols}}{Z_{cn} S} = \frac{(0.8151 - j0.0087)(0.3050 + j0.7874)}{0.3114 + j0.8418}$$

$$= 0.7660 - j0.0199 \text{ pu, which has a magnitude of } 0.7669 \text{ pu.}$$

Similarly the motor voltage V_{ms} is:-

$$V_{ms} = \frac{V_{ls} Z_{msl}}{Z_{msl} + Z_{cm}} = \frac{(0.7660 - j0.0199)(0.2390 + j0.9257)}{0.2390 + j0.9257 + 0.00844 + j0.003175}$$

$$= 0.7626 - j0.0140 \text{ pu, which has a magnitude of } 0.7627 \text{ pu.}$$

n) Calculate the percentage volt-drops

The customary method of defining volt-drop is in percentage terms as follows:-

$$\text{Volt-drop in percent} = \frac{\text{No-load voltage} - \text{Loaded voltage}}{\text{No-load voltage}} \times 100\%$$

Where the no-load voltage is the service voltage that exists before the change in load is applied and the loaded voltage is the service voltage during the application of the change in load. For example, when a motor is being started there are two aspects to consider. Firstly, the situation at the motor terminals since this determines the ability of the motor to create enough torque during the starting period and, secondly, at the MCC since this influences the performance of existing loads and their contactor coils. Other parts in the power system could be examined in a similar manner, e.g. at the generator terminals and its switchboard. The motor example above may be used to illustrate these comments:-

- Motor terminal volt-drop in percent.

  No-load voltage = pre-disturbance voltage at the MCC.

  Loaded voltage = voltage at the motor terminals at starting or running of the motor.

$$\text{Volt-drop at starting\%} = \frac{V_{lo} - V_{ms}}{V_{lo}} \times 100\%$$

$$= \frac{0.9936 - 0.7627}{0.9936} \times 100\% = 23.24\%$$

$$\text{Volt-drop at running\%} = \frac{V_{slo} - V_{mn}}{V_{lo}} \times 100\%$$

$$= \frac{0.9936 - 0.9552}{0.9936} \times 100\% = 3.86\%$$

- MCC terminal voltage in percent.

  No-load voltage = pre-disturbance voltage at the MCC.

  Loaded voltage = voltage at the MCC at starting or running of the motor.

$$\text{Volt-drop at starting\%} = \frac{V_{lo} - V_{ls}}{V_{lo}} \times 100\%$$

$$= \frac{0.9936 - 0.7669}{0.9936} \times 100\% = 22.82\%$$

$$\text{Volt-drop at running\%} = \frac{V_{lo} - V_{ln}}{V_{lo}} \times 100\%$$

$$= \frac{0.9936 - 0.9570}{0.9936} \times 100\% = 3.68\%$$

- Generator and SWBD terminal voltage in percent.

  No-load voltage = pre-disturbance voltage at the SWBD.

  Loaded voltage = voltage at the SWBD at starting or running of the motor.

$$\text{Volt-drop at starting\%} = \frac{V_{go} - V_{gs}}{V_{go}} \times 100\%$$

$$= \frac{1.0 - 0.8152}{1.0} \times 100\% = 18.48\%$$

$$\text{Volt-drop at running\%} = \frac{V_{go} - V_{gn}}{V_{go}} \times 100\%$$

$$= \frac{1.0 - 0.9695}{1.0} \times 100\% = 3.05\%$$

o) Examine the actual volt-drops

Although the percentage volt-drops are now known, and they give an indication of the seriousness of the volt-drop by simple inspection, what is important as far as each piece of equipment is concerned is the actual voltage on its terminals in volts. This is especially important when the rated voltage of the equipment is different from the nominal operating value as in the above example. Consider each component.

- The motor.

Rated voltage	=	4000.0 volts
Nominal operating system voltage	=	4181.8 volts
Starting voltage received	=	4181.8 × 0.7627 volts
	=	3189.5 volts = 79.74% of the rated value
Running voltage received	=	4181.8 × 0.9552 volts
	=	3994.4 volts = 99.86% of the rated value

- The motor control centre.

Rated voltage	=	4160.0 volts
Nominal operating system voltage	=	4181.8 volts

Starting voltage at the busbars	=	4181.8 × 0.7669 volts
	=	3207.0 volts = 77.09% of the rated value
Running voltage at the busbars	=	4181.8 × 0.9570 volts
	=	4002.0 volts = 96.20% of the rated value

- The generator switchboard.

Rated voltage	=	13800.0 volts
Nominal operating system voltage	=	13800.0 volts
Starting voltage at the busbars	=	13800.0 × 0.8152 volts
	=	11249 volts = 81.52% of the rated value
Running voltage at the busbars	=	13800.0 × 0.9695 volts
	=	13379 volts = 96.95% of the rated value

The motor may have been specified for a starting voltage drop of 15% and a running voltage drop of 2.5%. In the example the voltage received by the motor during starting would be 79.74% and so the voltage drop of 20.26% would have been excessive. However, the running voltage drop would be 0.14% which is well within the specified value. The MCC could experience problems with its contactor coils during motor starting due to the voltage drop being too large. The contactors on existing energised circuits could fail to hold in once the busbar voltage drops below 75%. The actual voltage during starting of 77.09% would be just sufficient for reliable operation. The running voltage would be well within specification for a motor control centre, i.e. only 3.8% volt-drop. If the feeder transformer was fitted with a tap-changing device then the actual running voltage could be maintained at a value nearer to its nominal value. The generator switchboard volt-drop of 18.48% at starting is just about acceptable, but well within limits during the running situation.

p) Design comments

From the results it can be seen that direct-on-line starting of the motor is only just possible when only one generator is available. High volt-drops occur during the starting period. However, several corrective measures can be taken:-

- Recalculate the volt-drops for the cases where two and three generators are running before the motor is started direct-on-line. If the results are satisfactory then an operating restriction can be imposed that at least two generators should be running initially.
- Recalculate using a ‘reduced voltage’ starting method, e.g. a Korndorfer starter, and one running generator. In this case also add the impedance of the starting device to the impedance of the motor feeder cable, and account for any transformer voltage ratio that may be present.
- Reduce the transient reactance of the generators to say 0.15 per-unit and recalculate the results.
- Reduce the starting current to running current ratio of the motor to say four times and recalculate the results.
- Consider a combination of the above measures.

The calculation process is lengthy if attempted by manual methods and is best programmed in a small desktop computer that can handle complex numbers. Such a programming exercise is simple to achieve. In order to screen various alternative cases it is possible to make some valid simplifications in the proposed system and to use a simpler calculation method.

q) Simplified solution

In the proposed system, used as the example above, it is acceptable to ignore the cable impedance $R_{cm} + jX_{cm}$ and the transformer impedance $R_c + jX_c$ for approximate calculation purposes. This is only allowable for the following reasons:-

- The power system distribution cables or overhead lines are well rated for their current duty and are short in length.
- The series impedances for cables are usually small in comparison with the transient reactances and load impedances, but this is not always the case with low voltage situations where for example the route lengths of cables can be relatively long.

Figure G.3 shows a simplified form of Figure G.2, where Z_o is the equivalent impedance of Z_{ol} in parallel with Z_{og} and is calculated as follows:-

$$R_o = \frac{1}{\dfrac{1}{R_{ol}} + \dfrac{1}{R_{og}}} = 2.2321 \text{ pu}$$

$$X_o = \frac{1}{\dfrac{1}{X_{ol}} + \dfrac{1}{X_{og}}} = 4.1903 \text{ pu}$$

r) Initial conditions

The motor starter is open. The motor terminal V_{mo} equals the generator terminal V_{go} voltage, which is 1.0 per-unit.

Hence,

$$V_{mo} = V_{go} = 1.0 + j0.0 \text{ pu.}$$

Find the initial values of I_c and V_l, i.e. I_{co} and V_{lo}, noting that $I_m = 0.0$

At the MCC the parallel load is R_o in parallel with X_o.

Convert the parallel load into a series load of R_{ol} + j X_{ol}.

The conversions are:-

$$R_{ol} = \frac{R_o X_o^2}{R_o^2 + X_o^2} \text{ pu per phase}$$

$$X_{ol} = \frac{X_o R_o^2}{R_o^2 + X_o^2} \text{ pu per phase}$$

Where R_o and X_o were found above.

Hence

$$Z_{ol} = R_{ol} + jX_{ol} = 1.7388 + j0.9262 \text{ pu}$$

$$I_{oo} = I_{go} = \frac{V_{mo}}{Z_{ol}} = \frac{1.0 + j0.0}{1.7388 + j0.9262} = 0.448 - j0.2386 \text{ pu}$$

This compares well with I_{go} found in the rigorous case.

$$E_o = V_{go} + I_{oo} Z_g = 1.0 + j0 + (0.448 - j0.2386)(0.02 + j0.25) \text{ pu}$$
$$= 1.0686 + j0.1072 \text{ pu, which has a magnitude of } 1.0740 \text{ pu.}$$

Which is within 0.01% of the rigorous case.

s) Running conditions

The motor starter is closed and the generator emf is 1.0740 per-unit.

The parallel impedance of the running motor is Z_{mn}:-

$$R_{mn} = 5.4324 \text{ and } X_{mn} = j10.065 \text{ pu}$$

The series impedance of the running motor is Z_{mnl}:-

$$Z_{mnl} = R_{mnl} + jX_{mnl} = 4.2069 + j2.2706 \text{ pu}$$

The total load resistance on the SWBD is R_{ln} where:-

$$R_{ln} = \frac{R_o \times R_{mn}}{R_o + R_{mn}} = 1.5821 \text{ pu}$$

The total load reactance on the SWBD is X_{ln} where:-

$$X_{ln} = \frac{X_o \times X_{mn}}{X_o + X_{mn}} = 2.9586 \text{ pu}$$

The series equivalent resistance is R_{ogn}:-

$$R_{ogn} = \frac{2.9586 \times 2.9586 \times 1.5821}{2.9586^2 + 1.5821^2} = 1.2303 \text{ pu}$$

The series equivalent reactance is X_{ogn}:-

$$X_{ogn} = \frac{2.9586 \times 1.5821 \times 1.5821}{2.9586^2 + 1.5821^2} = 0.6579 \text{ pu}$$

The total impedance seen by the generator emf E_o is Z_{gn}:-

$$Z_{gn} = R_g + R_{ogn} + j(X_g + X_{ogn})$$
$$= 0.02 + 1.2303 + j(0.25 + 0.6579)$$
$$= 1.2503 + j0.9079 \text{ pu}$$

The current in the generator I_{gn} is:-

$$I_{gn} = E_o = \frac{(1.0686 - j0.1072)(1.2503 - j0.9079)}{2.3875}$$
$$= 0.6004 - j0.3502 \text{ pu}$$

Hence the terminal voltage of the generator V_{gn} is:-

$$V_{gn} = \frac{E_o Z_{ogn}}{Z_{gn}}$$
$$= 0.9691 - j0.0359 \text{ pu, which has a magnitude of } 0.9697 \text{ pu.}$$

t) Starting conditions

The motor starter is closed. Repeat the procedure as for s) but use the motor starting impedance, and using the suffix '*s*' for starting.

The parallel impedance of the running motor is Z_{mn}:-

$$R_{ms} = 3.8244 \text{ and } X_{ms} = j0.9875 \text{ pu}$$

The total load resistance on the SWBD is R_{ls} where:-

$$R_{ls} = \frac{R_o \times R_{ms}}{R_o + R_{ms}} = 1.4095 \text{ pu}$$

The total load reactance on the SWBD is X_{ls} where:-

$$X_{ls} = \frac{X_o \times X_{ms}}{X_o + X_{ms}} = 0.7991 \text{ pu}$$

The series equivalent resistance is R_{ogs}:-

$$R_{ogs} = \frac{0.7991 \times 0.7991 \times 1.4095}{0.7991^2 + 1.4095^2} = 0.3429 \text{ pu}$$

The series equivalent reactance is X_{ogs}:-

$$X_{ogs} = \frac{0.7991 \times 1.4095 \times 1.4095}{0.7991^2 + 1.4095^2} = 0.6047 \text{ pu}$$

The total impedance seen by the generator emf E_o is Z_{gs}:-

$$Z_{gs} = R_g + R_{ogs} + j(X_g + X_{ogs})$$
$$= 0.02 + 0.3429 + j(0.25 + 0.6047)$$
$$= 0.3629 + j0.8547 \text{ pu}$$

The current in the generator I_{gs} is:-

$$I_{gns} = E_o = \frac{(1.0686 - j0.1072)(0.3629 - j0.8547)}{0.8622}$$
$$= 0.5560 - j1.0142 \text{ pu}$$

Hence the terminal voltage of the generator V_{gs} is:-

$$V_{gs} = I_{gs} Z_{ogs}$$
$$= 0.8040 - j0.0115 \text{ pu, which has a magnitude of } 0.8040 \text{ pu.}$$

This is nearly equal to V_{ms}.

The voltage V_{gs} is within 1.5% of the rigorous case but too optimistic for the motor voltage. However, most of the volt-drop is due to the generator impedance in either case and so once some cases have been screened in this way then the more accurate method may be applied to the serious cases. Since the result is optimistic it therefore requires a safety margin of 2% to 5% to be added when this method is used. The percentage volt-drops can be calculated as follows:-

- Generator and motor terminal volt-drop in percent.

$$\text{Volt-drop at starting\%} = \frac{V_{mo} - V_{ms}}{V_{mo}} \times 100\%$$
$$= \frac{1.0 - 0.8040}{1.0} \times 100\% = 19.6\%$$

which is about 4% better than the rigorous case.

u) Formular method based on kVA rating

The simplification of the power system can be generalised by using a formular method. The simplified system can be represented by Figure G.3, where:-

Z_g is the source impedance, e.g. generator transient reactance.
Z_m is the motor impedance (Z_{mr} for running and Z_{ms} for starting).
Z_l is the standing load impedance, Z_o in Figure G.3.

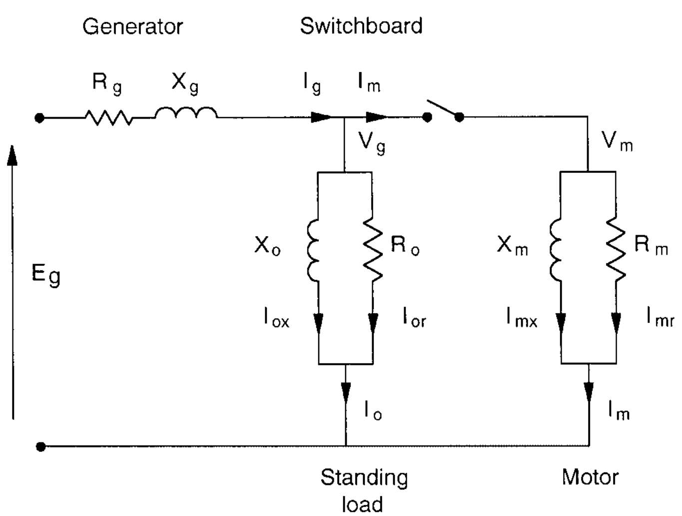

Figure G.3 Reduced equivalent circuit for calculating the volt-drop in a 500 kW HV motor.

All the impedances are in their complex form $R + jX$.

The simplifications made in t) have been applied viz:-

- Cable impedances have been ignored.
- Transformer impedances have been ignored.
- All the standing loads are grouped at the generator terminals.

The initial conditions are easily calculated. The terminal voltage V_o is known and assumed to be $1.0 + j0.0$ per unit. The motor starter is open. The initial circuit consists of Z_l in series with Z_g and is fed by E_o. The initial load current I_l is I_{lo}.

$$I_{lo} = \frac{V_o}{Z_l} \text{ and } E_o = V_o + I_{lo}$$

Therefore it consists of Z_l in series with Z_g and is fed by E_o. The initial load current I_l is I_{lo}.

$$E_o = V_o \left(1 + \frac{V_o}{Z_l}\right)$$

The general case for the running conditions are also easily calculated. The motor starter is closed. The motor and load impedance are then connected in parallel. The total of these impedances is Z_{lm} in series with Z_g and is fed by E_o. The initial load current I_l where:-

$$Z_{lm} = \frac{Z_l Z_m}{Z_l + Z_m}$$

and

$$V = \frac{E_o Z_{lm}}{Z_{lm} + Z_g}$$
$$= \frac{V_o(1 + Z_g)Z_{lm}}{Z_l(Z_{lm} + Z_g)}$$

Let

$$a = (Z_l + Z_g)Z_{lm}$$

and

$$b = (Z_{lm} + Z_g)Z_l$$

Therefore

$$V = \frac{aV_o}{b}$$

$$\text{The Percentage volt-drop}\,\Delta V = \left(\frac{V_o - V}{V}\right) \times 100\%$$

$$\Delta V = \left(V_o - \frac{aV_o}{b}\right) \times 100\%$$

Note that

$$\frac{a}{b}V_o = \frac{Z_g(Z_l - Z_{lm})}{(Z_l + Z_g)Z_{lm}}$$

Substitute for

$$Z_{lm} = \frac{Z_l Z_m}{Z_l + Z_m}$$

Hence,

$$\frac{Z_l - Z_{lm}}{Z_l + Z_m} = \frac{Z_l}{Z_m}$$

The percentage volt-drop

$$\Delta V = \left(\frac{Z_g Z_l}{(Z_l + Z_g)Z_m}\right) \times 100\%$$

The volt-drop ΔV is only of interest in its magnitude.

Therefore

$$|\Delta V| = \frac{|Z_g||Z_l|}{|Z_l + Z_g||Z_m|} \times 100\%$$

Which makes the calculation of volt-drop much easier. However, all these impedances must be correctly reduced to the common system base as follows. It can be shown that the actual parameters may be easily converted to their per-unit system base parameters. The motor, load and generator impedances can be represented in terms of their kVA, or MVA, and voltage bases.

$$Z_g = \frac{Z_{gen} S_{base} V_{gen} V_{gen}}{S_{gen} V_{gbase}{}^2} \quad \text{pu}$$

$$Z_m = \frac{S_{base} V_{motor} V_{motor}}{S_{motor} V_{mbase}{}^2} \quad \text{pu}$$

$$Z_l = \frac{S_{base}}{S_{load}} \text{ pu}$$

Where V_{gbase} is the system base voltage at the generator, e.g. 13 800 volts in the example.

V_{mbase} is the system base voltage at the motor or MCC, e.g. 4181.8 volts in the example.

$V_{gbase}, V_{mbase}, S_{base}, V_{gen}$ and V_{motor} are real or scalar numbers.

$Z_{gen}, S_{gen}, S_{motor}$ and S_{load} are complex numbers and S_{motor} has to be chosen for the starting or running case.

Example. Consider the data used for the rigorous case for starting the motor.

$$Z_g = 0.02 + j0.25 \text{ and so } |Z_g| = 0.2508 \text{ pu}$$

$$Z_l = Z_{ol} = 1.7388 + j0.9262 \text{ and so } |Z_g| = 1.9701 \text{ pu}$$

$$Z_{lm} = Z_{ml} = 0.2391 + j0.9259 \text{ and so } |Z_g| = 0.9563 \text{ pu}$$

$$Z_l + Z_g = 1.7588 + j1.1762 \text{ and so } |Z_l + Z_g| = 2.1158 \text{ pu}$$

Therefore

$$|\Delta V| = \frac{0.2508 \times 1.9701}{2.1158 \times 0.9563} \times 100 = 24.42\%$$

Which compares pessimistically with the simple case (19.6%) but closely with the rigorous case (23.24 %).

v) Graphical estimation

This sub-section develops a simple graphical method for quickly estimating volt-drops for direct-on-line starting situations. The following data forms the basis of the graphical results:

- The generator data.

$$Z_g = 0.1, 0.15, 0.2, 0.25 \text{ and } 0.3 \text{ pu}$$
$$R_g = 0.0 \text{ pu}$$
$$S_{base} = S_{gen} = 10,000 \text{ kVA}$$
$$\text{Rated power factor} = 0.8 \text{ lagging}$$

- The standing load data.

$$\text{Rated power factor} = 0.9 \text{ lagging}$$
$$\text{Load} = 0.0, 50.0 \text{ and } 80.0\% \text{ of } S_{gen}$$
$$|Z_l| = 1.25, 2.0 \text{ and infinity (no-load) pu}$$

Where

$$Z_l = 1.25(0.9 + j0.436) \text{ for } 80\% \text{ load}$$
$$= 2.00(0.9 + j0.436) \text{ for } 50\% \text{ load}$$
$$= \infty(0.9 + j0.436) \text{ for } 0\% \text{ load}$$

- Motor data.

Table G.1 shows the appropriate data for a range of four-pole high voltage motors.
Example. Consider a 630 kW motor and a 80% standing load.

$$|Z_g| = 0.25 \text{ pu}$$
$$|Z_m| = 0.1713 \times \frac{0.2508}{747.81} = 2.2907 \text{ pu}$$
$$Z_l = 1.25(0.9 + j0.436) = |1.25| \text{ pu}$$
$$|Z_l + Z_g| = |j0.25 + 1.25(0.9 + j0.436)| = |1.457| \text{ pu}$$

Therefore

$$|\Delta V| = \frac{0.25 \times 1.25}{1.457 \times 2.2907} \times 100 = 9.363\%$$

Table G.1. Motor data for graphical estimation of volt-drop

Motor rating (kW)	Efficiency (per-unit)	Power factor at full load	kVA rating at full load	I_s/I_n ratio	Power factor at starting	kVA at starting	Z_{ms} at starting per-unit
315	0.9455	0.8603	387.2	6.787	0.217	2628.3	0.1473
430	0.9537	0.8715	517.4	6.445	0.219	3334.3	0.1552
630	0.9595	0.8780	747.8	5.838	0.208	4365.6	0.1713
720	0.9608	0.8788	852.6	5.619	0.202	4790.9	0.1780
800	0.9617	0.8791	946.3	5.453	0.196	5158.6	0.1834
1,100	0.9638	0.8780	1299.8	5.000	0.179	6498.5	0.2000
1,500	0.9654	0.8756	1774.4	4.661	0.162	8270.0	0.2145
2,500	0.9680	0.8722	2961.0	4.347	0.137	12,872	0.2300
5,000	0.9717	0.8742	5886.6	4.397	0.111	25,883	0.2274
6,300	0.9726	0.8763	7392.3	4.527	0.104	33,461	0.2209
8,000	0.9730	0.8786	9358.1	4.712	0.096	44,093	0.2122
11,000	0.9727	0.8806	12,843	5.017	0.086	64,440	0.1993

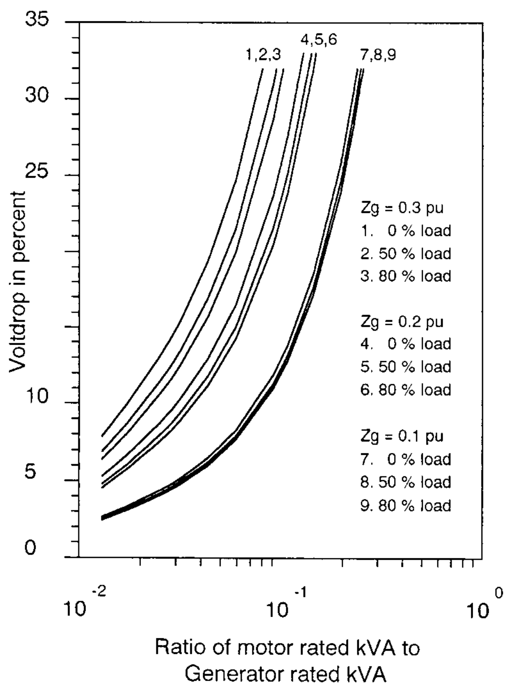

Figure G.4 Volt-drop when starting an induction motor. Volt-drop in per-unit versus the ratio of the motor kVA rating to the generation kVA capacity, for different values of generator per transient impedance Z_g and standing load.

Figure G.4 shows the results of all the cases given in Table G.1. The volt-drop $|\Delta V|$ is plotted against the ratio S_{motor}/S_{gen} so that a generalised presentation may be used. Note that these graphs can be used for most cases where generators up to about 30 MVA are present. Extrapolations can be used with confidence for generators above 30 MVA which have transient

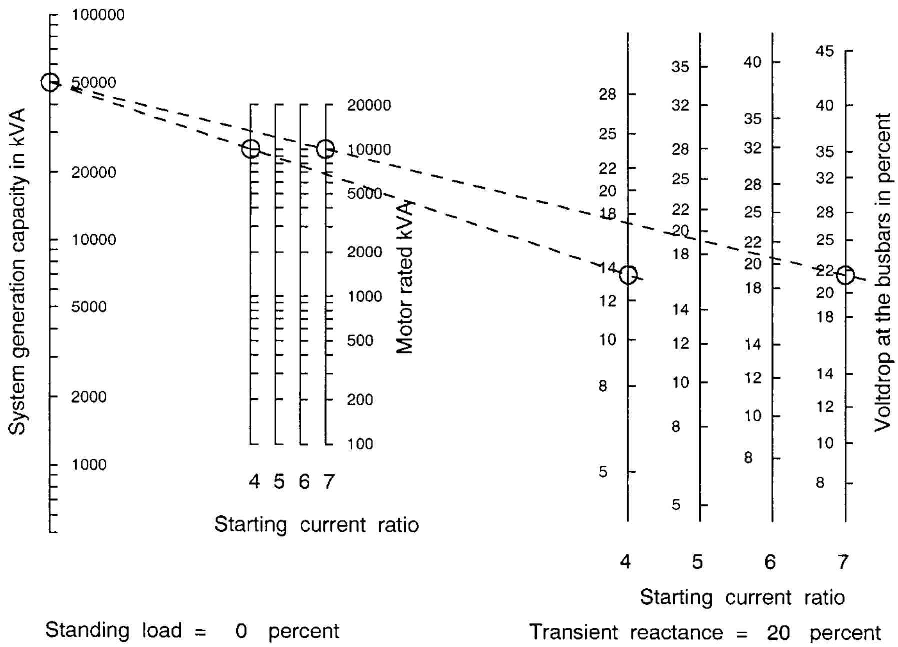

Figure G.5 Volt-drop when starting an induction motor. Nomograph for the volt-drop in per-unit versus the ratio of the motor kVA rating to the generation kVA capacity, for a generator transient impedance Z_g of 0.2 pu and zero standing load. The motor can have different ratios of starting current to running current.

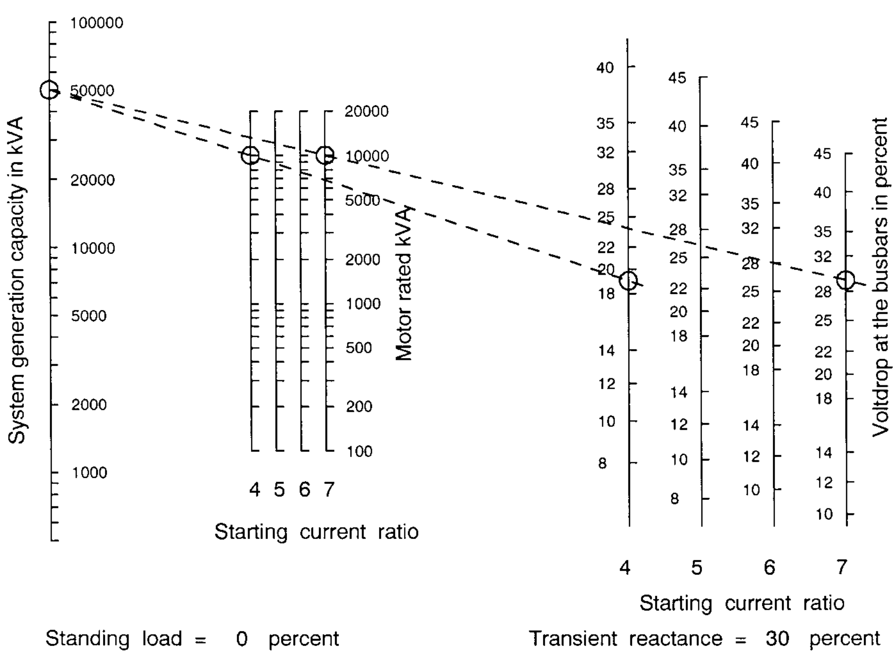

Figure G.6 Volt-drop when starting an induction motor. Nomograph for the volt-drop in per-unit versus the ratio of the motor kVA rating to the generation kVA capacity, for a generator transient impedance Z_g of 0.3 pu and zero standing load. The motor can have different ratios of starting current to running current.

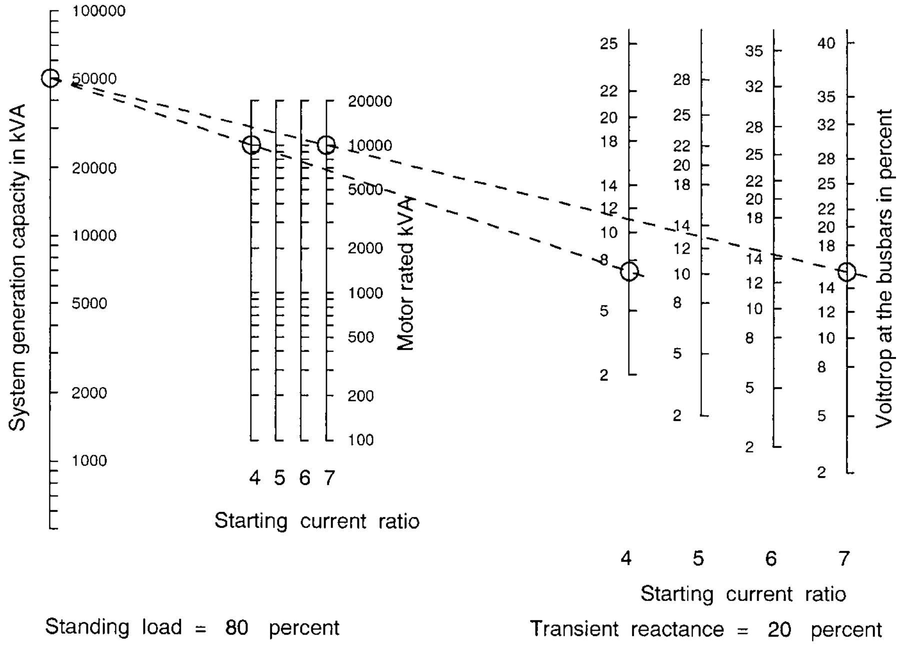

Figure G.7 Volt-drop when starting an induction motor. Nomograph for the volt-drop in per-unit versus the ratio of the motor kVA rating to the generation kVA capacity, for a generator transient impedance Z_g of 0.2 pu and 0.8 pu standing load. The motor can have different ratios of starting current to running current.

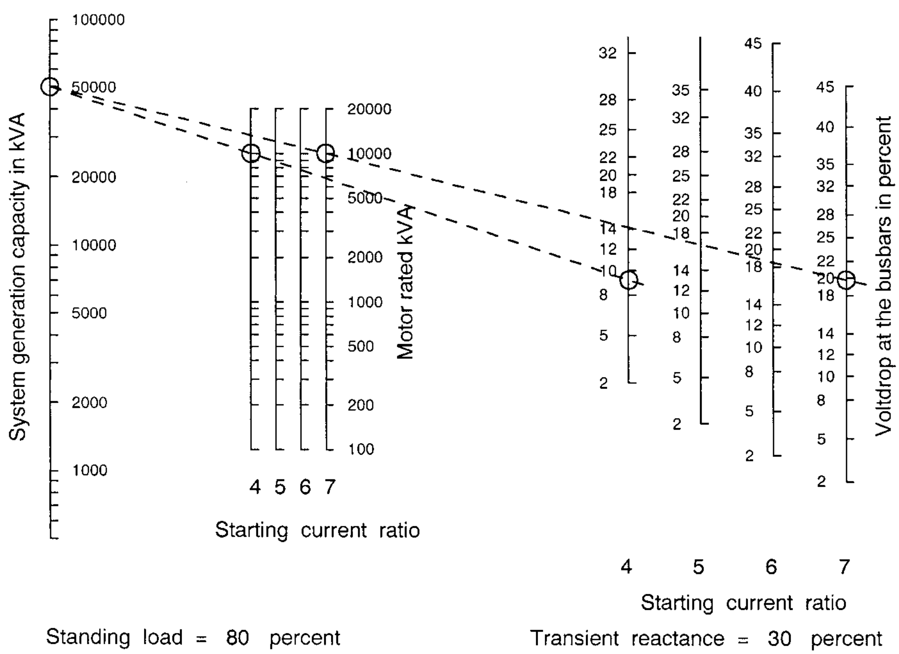

Figure G.8 Volt-drop when starting an induction. Nomograph for the volt-drop in per-unit versus the ratio of the motor kVA rating to the generation kVA capacity, for a generator transient impedance Z_g of 0.3 pu and 0.8 pu standing load. The motor can have different ratios of starting current to running current.

reactances between 15% and 25%, and for motors up to about 15 MW. The main parameter of the motor is the starting-to-running current ratio, which should not fall below 4 for the extrapolation to be valid.

The results can be represented in a more comprehensive manner by using a nomograph, as shown in Figures G.5, 6, 7 and 8. Each nomograph caters for four different starting-to running current ratios, i.e. 4, 5, 6 and 7. References 1 and 2 describe how to draw a nomograph.

REFERENCES

1. Alexander S. Levens, Nomographs. John Wiley & Sons (1948 and 1959). Library of Congress Card No. 59-11819.
2. S. Brodetsky, A first course in nomography. G. Bell and Sons Ltd (reprinted 1938).