15

Harmonic Voltages and Currents

15.1 INTRODUCTION

It is generally understood that the voltages and currents in industrial power systems are sinusoidal quantities with a frequency of usually 50 Hz or 60 Hz. The design of these systems is based on an assumption that the voltages and currents are not distorted by harmonic components. In the majority of power systems this assumption is true and the effects of harmonics can be ignored.

However, occasions do arise when the design must take account of harmonics. Such consideration may be necessary at the beginning of a new project, or for a plant that already exists. In the first case the minimisation of the bad effects of harmonics is reasonably easy to accomplish. The second case for existing plants it is usually more difficult due to constraints that may not be removable or reducible.

The main sources of harmonics in power systems are:-

- Magnetic saturation in the stators and rotors of generators.
- Geometry of the windings in the stators and rotors of generators.
- Magnetic saturation in transformer cores.
- Non-linear consumers such as battery chargers, uninterruptible power supplies, fluorescent light fittings.
- Rectifiers and inverters for major consumers such as DC and AC motors.

The presence of harmonics caused by magnetic saturation and winding geometry of generators and transformers can be minimised from the outset by carefully specifying the design requirements of these equipments before they are purchased. Such specification may incur some small extra cost at the purchasing stage. For example if the operating flux density in these equipments is kept near to or below the knee-point of their saturation characteristics, then this will usually require a greater volume of iron in their magnetic circuits. This in turn will tend to make the equipment larger in its principal dimensions, and therefore more expensive.

The creation of harmonics by minor consumers can usually by minimised or eliminated by the use of shunt-connected capacitors, simple internal filters or smoothing circuits. This is again a matter of specification before purchasing the equipment.

Handbook of Electrical Engineering: For Practitioners in the Oil, Gas and Petrochemical Industry. Alan L. Sheldrake
 ISBN: 0-471-49631-6

The use of rectifiers and inverters for variable speed motor drives is becoming common in the oil industry, especially for large gas compressors and oil pumps. Adding these to an existing power system can create problems that are difficult to solve, even if they are furnished with harmonic filters. Power systems that have long high-voltage feeder cables, such as submarine cables between platforms, are particularly sensitive to harmonic currents created by rectifiers-inverter loads. The amount of shunt capacitance in these cables can be enough to cause a resonant condition at a low multiple of the fundamental e.g. 5, 7, 11, 13. These low frequency harmonics usually exist at a magnitude that cannot be ignored in such situations. This can present the power system engineer with a difficult task in designing a suitable anti-resonant filter. The remainder of this chapter is concerned only with harmonics caused by variable speed motor drives.

The theoretical operations of rectifiers and inverters under steady state and transient conditions are described in many publications, for example References 1 to 6.

Reference 2 also describes the 'on-off' characteristics of the power semiconductors used in the bridges e.g. diodes, thyristors, triads, gate turn-off thyristors and bipolar power transistors. Only the steady state operations of bridges are described herein. For such operations it is assumed that the load is well matched to the rating of the bridge. The remainder of this section is an introduction to the subject of harmonic voltages and currents that are caused by variable speed systems for DC and AC motors. It emphasises the main aspects that affect the supply power systems.

15.2 RECTIFIERS

15.2.1 Diode Bridges

Power rectifiers rated above a few kVA are usually three-phase units and occasionally six-phase units. The bridge elements may be diodes, thyristors (silicon controlled rectifiers) or power transistors operated as switches.

Diode bridges are the simplest and are suitable where the output DC voltage is constant and related to the input AC voltage by a fixed factor. They are well suited to battery chargers, uninterruptible power supplies and cathodic protection units. Figure 15.1 shows the basic element of a three-phase diode bridge, in this case the rectifier elements R_1 to R_6 and diodes, not thyristors as shown.

15.2.1.1 Commutation

The transfer of the load current from one diode to the next is called 'commutation'. This takes place when the potential at the anode of the first diode has fallen to a value equal to the rising potential at the anode of the second diode. Shortly after the transfer is initiated both diodes conduct the current and a temporary short circuit exists across the two phases supplying the diodes. Since the short circuit contains the leakage reactance of the supply transformer, plus the impedance upstream of the transformer, there is sufficient inductance to delay the rise in current in the second diode. Hence the current rises exponentially from zero to a value equal to the DC load current. At this point the commutation is complete and the first diode ceases to conduct. The finite time taken by the commutation process is related to the periodic time of the supply voltage by defining an angle 'u' called the commutation angle. As the load current is increased the commutation time is increased and so the angle u increases. At no-load the angle u is zero. At full-load the angle u is between zero and 60° for properly designed bridges, and in practice u will be in the order of 10° if a good power factor is to be obtained, as shown in Table 15.1.

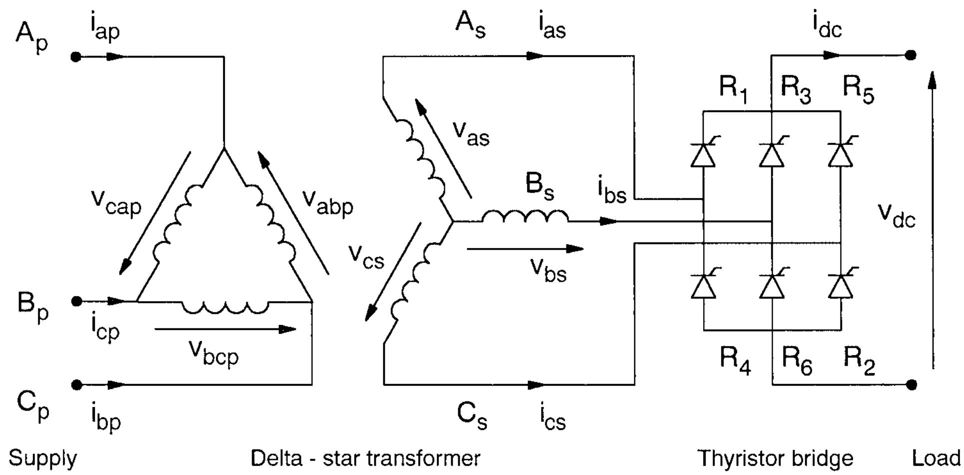

Figure 15.1 Circuit diagram of a six-pulse thyristor bridge.

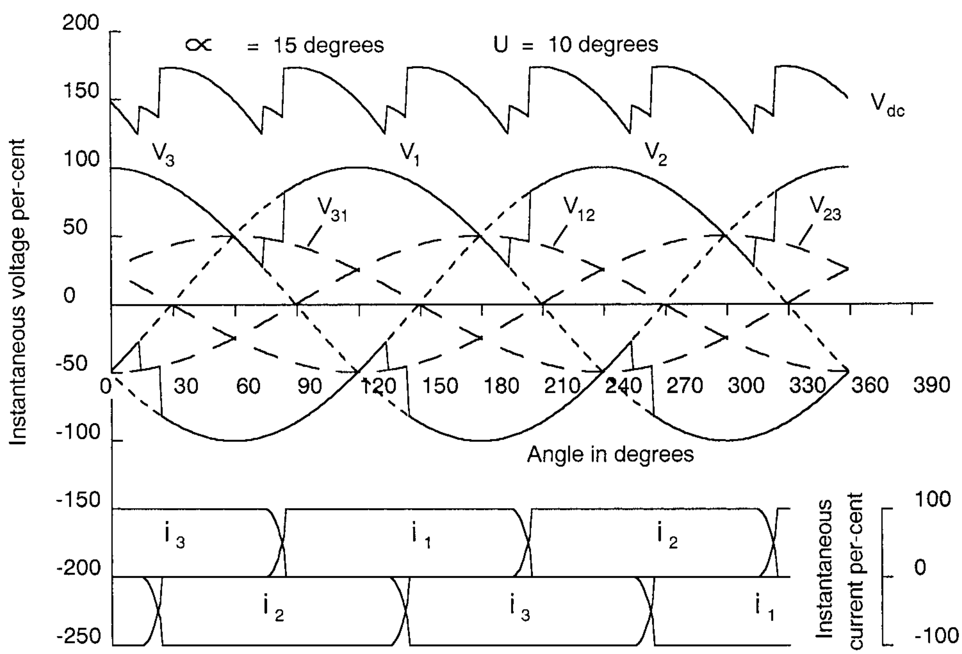

Figure 15.2 Voltage and current in six-pulse thyristor bridge.

When the angle u is within the range of zero to 60° the current in each phase of the supply is discontinuous as it crosses over at its zero value, and is almost trapezoidal, as shown in Figure 15.2.

This type of operation is often called 'Mode 1' and includes load currents that are within the rating of the bridge and its supply transformer. If the bridge needs to carry a higher current then the commutation is modified. The maximum commutation angle u is 60° and thereafter the commutation occurs in the negative half of the bridge because the decaying current has not yet reached zero.

A time delay is incurred until the zero is reached, and the corresponding angle is 'α' which is called the 'delay angle'. This angle occurs from zero to 30°, and the type of operation is called 'Mode 2'.

If the load current is further increased a new condition arises, called 'Mode 3' operation. The decaying current requires more time to reach zero. During this extra time there is a short-term three-phase short circuit and the output voltage is discontinuous at zero for this period. The output voltage appears as a train of saw-toothed pulses. The average value of this voltage is capable of driving more current into the load. As this occurs the delay or 'retardation angle γ' increases until it eventually reaches 60°, at which angle there is a complete three-phase short circuit and the output voltage is zero. During the increase in retardation angle the AC phase currents change their shape from a quazi-trapezium to a pure fundamental sine wave. The AC current is then limited only by the impedance of the transformer and any impedance upstream.

References 7 and 8 describes these commutation process in relation to the use of diode bridges in the main rotor circuits of synchronous generators.

15.2.1.2 Harmonic components

The waveform of the current in the secondary winding, phase AS of Figure 15.1 is shown in Figure 15.2 in relation to its phase voltage. The operating condition is for angle $u = 10°$ in Mode 1, when the delay angles α has a nominal value 15°.

As the three angles u, α and γ increase the current waveform moves to the right of the phase voltage waveform. The centre of the current waveform is approximately the position of the peak value of the fundamental current component. Consequently as the current increases the power factor of the fundamental current decreases. Table 15.1 shows values of the harmonic components of current and the power factor as the retardation angle u is increased from zero to 60°. The fundamental component is taken as unity reference at each value of u.

15.2.2 Thyristor Bridges

Thyristors used in rectifier and inverter bridges are usually of two types. The first type is a three-terminal semiconductor that can only be turned 'on' by a control or 'firing' signal applied to its

Table 15.1. Operating modes of a three-phase diode bridge

Mode	Rectifier angles			Approximate	
	u	α	γ	Power factor angle ϕ	Power factor $\cos\phi$
1	0	0	0	0	1.0
1	20	0	0	13.1	0.974
1	45	0	0	29.6	0.870
1	60	0	0	39.1	0.776
2	60	15	0	50.3	0.639
2	60	30	0	63.0	0.454
3	60	30	15	74.6	0.266
3	60	30	30	82.9	0.124
3	60	30	60	90.0	0.0

'gate'. It cannot be turned 'off' by the control signal. It can only be turned 'off' by forcing the anode current to zero, which is achieved by a special circuit that is connected across the anode and cathode, see References 6 and 9. This was the first type to be developed. In recent years a second type has been developed that can be turned 'off' by applying a reversed polarity control signal to the gate. This device is usually called a 'gate turn off' thyristor or GTO. Both devices are either in their fully 'on' state or their fully 'off' state when operating in normal bridge circuits. There is not an intermediate state such as found with transistors.

Thyristor bridges are used where the DC output voltage needs to be varied. For example for control purposes such as varying the speed of motors or for protective purposes such as limiting the maximum DC output current that can flow when an external short circuit occurs.

The basic circuit of a thyristor bridge is almost the same as that for a diode bridge. The essential differences are the replacement of the diode elements by thyristor elements, the inclusion of a controlled firing system for the thyristor gates, and in some cases the application of forced commutation circuits, see Figure 15.1.

15.2.2.1 Commutation

The commutation processes for Mode 1 operation of delay and current transfer are essentially the same as the diode bridge, except that the delay angle α is now controlled instead of occurring naturally and can be extended to 90° from 60°. The current transfer occurs in the same manner and gives rise to the same angle u.

Control of the triggering pulses to the thyristors needs to be carefully managed when the commutation is in Modes 2 and 3, otherwise the operation of the bridge may become unstable, see Chapter 7 of Reference 1.

The normal control range of the delay angle α is from zero to 90°, over which the average DC output voltage decreases from its maximum value to zero. In a good design of the bridge, with an appropriate reactance in the supply transformer and enough inductance in the DC load circuit, the practical operating region is ensured to be within the Mode 1 operating range. If the load is a motor then it will produce an emf that has a magnitude roughly in proportion to the shaft speed. During transient disturbances there may be a wide mismatch between the output voltage of the bridge and the emf within the motor. The mismatch will cause a large current to flow, e.g. if the motor suddenly stalls, which may drive the bridge into a Mode 2 or 3 operation unless the protective control circuits rapidly take corrective action to prevent such operation.

15.2.2.2 Harmonic components

The shape of the waveform for the AC current in the supply lines to the bridge will be the same as that for the diode bridge. Hence the harmonic analysis will yield the same results for practical operating conditions. Table 15.2 shows the harmonic components for the range of u between zero and 60°. The fundamental component is taken as reference.

15.2.2.3 Distortion upstream of the bridge

The installation of a rectifier bridge that has a relatively high power rating with respect to its supply will cause significant distortion to the supply line currents and line voltages.

Table 15.2. Variation of harmonic coefficients with the commutation angle u

Harmonic number	Magnitude of the coefficient b_n at different values of u in degrees							
u	0.01	0.25	1.0	5.0	10.0	20.0	40.0	60.0
1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
5	0.2001	0.2001	0.2001	0.1986	0.1941	0.1766	0.1152	0.0400
7	0.1429	0.1429	0.1429	0.1409	0.1345	0.1106	0.0384	0.0204
11	0.0911	0.0910	0.0910	0.0878	0.0779	0.0449	0.0156	0.0083
13	0.0771	0.0771	0.0771	0.0732	0.0618	0.0262	0.0171	0.059
17	0.0591	0.0590	0.0590	0.0540	0.0398	0.0035	0.0035	0.035
19	0.0529	0.0529	0.0529	0.0473	0.0320	0.0028	0.0028	0.0028
23	0.0438	0.0438	0.0438	0.0370	0.0199	0.0085	0.0055	0.0019
25	0.0403	0.0403	0.0403	0.0331	0.0153	0.0088	0.0031	0.0016
29	0.0349	0.0349	0.0344	0.0266	0.080	0.0066	0.0023	0.0012
31	0.0326	0.0327	0.0327	0.0239	0.0052	0.0047	0.0031	0.0011

The near-rectangular line currents will produce volt-drops in the series resistance-reactance cables, overhead lines and transformers. These volt-drops will be non-sinusoidal and will distort the waveform at their intermediate points of connection. At such points there may be a switchboard or distribution board and the loads connected to them will experience the distorted voltage waveform.

The line voltage waveform at the primary terminals of the transformer that feeds the bridge will be distorted by the short commutation pulses. These are often called ‘notches’. At the thyristors or diodes the notches have a near-zero base due to the temporary short circuit during the commutation. Immediately upstream of these elements is the impedance of the transformer, and beyond that the impedance to the main source of supply. A potential divider circuit exists between the bridge elements and the source of supply. Consequently the higher the transformer impedance the lower will be the impact of the commutation notches. Suppose the bridge is fed from a motor control centre that has its own feeder transformer. Since the feeder transformer and its upstream circuit has a finite impedance, there will be a certain amount of distortion to the voltages at the busbars of the motor control centre. The notching distortion injects high frequency currents into all the loads and instrumentation connected to the busbars. In many situations the loads are not sensitive to this form of distortion, but a few in a particular situation may be adversely affected, especially power factor correction capacitors and capacitors in fluorescent light fittings (if fitted). Retrofitting filters to an existing set of loads on a switchboard or motor control centre may be a difficult task to complete satisfactorily. Some instrumentation within or supplied from the switchgear may be requiring timings pulses or triggering signals that are derived from the busbar voltages. These signals may be disrupted by the presence of notching distortion.

The presence of high frequency harmonics in the power supply lines leaving the switchgear can cause mutual coupling to electronic and telecommunication cables if they are routed in close proximity to the power cables. This can occur especially if the cable racks run parallel to each other over an appreciable distance. As a ‘rule-of-thumb’ guide, derived from Table 13.1, the spacing (d) between power and electronic cables should be at least,

$$d \geq 300 + 1.75 I_n \text{ millimetres}$$

Where I_n is the current rating of the power cable.

The spacing need not be greater than about 1000 mm unless the parallel route length is very long.

15.2.3 Power Transistor Bridges

In recent times there has been a rapid development in the design of high-power transistors, to such an extent that they are feasible alternatives to thyristors for many applications. The main advantage of transistors is that they can be switched ‘on’ and ‘off’ at any point in the conducting half-cycle that can appear across their emitter and collector terminals. They must be protected against the reversal of voltage when the second half-cycle appears across the terminals. It is therefore possible to synthesise the waveforms in such a manner as to reduce the harmonic distortion at the supply terminals to a low level.

Although a power transistor can be controlled over its whole operating range from being fully ‘off’ to being fully ‘on’, it is not usually operated in the intermediate state. This is because the inherent resistance of the device in the intermediate state causes a very large amount of heat to be developed in the transistor itself, which if not properly conducted away from the transistor will cause thermal instability and permanent damage. In the ‘off’ state the current in the transistor is negligibly small and its collector-to-emitter voltage will be high. Hence the product of voltage and current will be very small. When the transistor is fully ‘on’ the current will be high and the collector-to-emitter voltage will be small, but not negligible. Hence the power dissipated by the product of a high current and a small voltage will again be small, but a definite amount of heat will be dissipated. This amount can normally be conducted away by using standard designs of air fins or ‘heat sinks’. See also Reference 9.

15.2.4 DC Motors

15.2.4.1 Voltages and currents

Variable speed DC motors are mainly used in the oil industry for driving drilling equipment such as the drill string, draw-works, mud pumps, cement pumps, winches and the propulsion systems in semi-submersible rigs and barges. They are typically rated at approximately 800 kW, 750 volts, and several motors may be operated mechanically in parallel e.g. the draw-works motors. Each bridge that supplies a motor has a typical current rating of 2250 amps. Within its control system is a manually adjustable current limiting potentiometer to safeguard the bridge and to limit the torque produced by the motor. The bridges are fed from a three-phase 600 volt power source which is usually earthed by a high resistance fault detection device, that gives an alarm but does not trip the source.

Assume that the secondary phase-to-neutral emf of the supply transformer is E and the fundamental reactance of each phase winding is X_l, and the DC load current is I_d, then for Mode 1 operation the DC output voltage V_d is,

$$V_d = \frac{3\sqrt{6}E}{\pi}\cos\alpha - \frac{3X_c I_d}{\pi} = I_d R + E_m \tag{15.1}$$

Where R is the DC circuit resistance.
E_m is the emf in the motor armature.
$X_c = 2X_l$ is the commutating reactance.

An alternative expression for V_d in terms of the commutation angle u, is,

$$V_d = \frac{3\sqrt{6}E}{2\pi}(\cos\alpha - \cos(\alpha + u)) \tag{15.2}$$

Hence it can be seen that u is a function of I_d, as will be shown below.

The factor $3\sqrt{6}/\pi$ applies to a three-phase bridge and is derived from,

$$V_{do} = 2E\left(1 - \cos\left(\frac{4\pi}{n}\right)\right)^{1/2}\left(\frac{n}{\pi}\right)\sin\left(\frac{\pi}{n}\right)$$

Where n is the ripple number, in the above case $n = 3$

and V_{do} is the average ripple voltage at no-load.

The current I_d can also be given as a function of α and u,

$$I_d = \frac{\sqrt{6}E}{2X_c}(\cos\alpha - \cos(\alpha + u)) \tag{15.3}$$

It can be seen from (15.1) that for a given delay angle α the output voltage has declining or 'drooping' value as the DC current rises, for example as the load on a motor is increased causing it to slow down and to reduce its emf. Figure 15.3 shows a family of curves of output voltage against current, as a function of angle α.

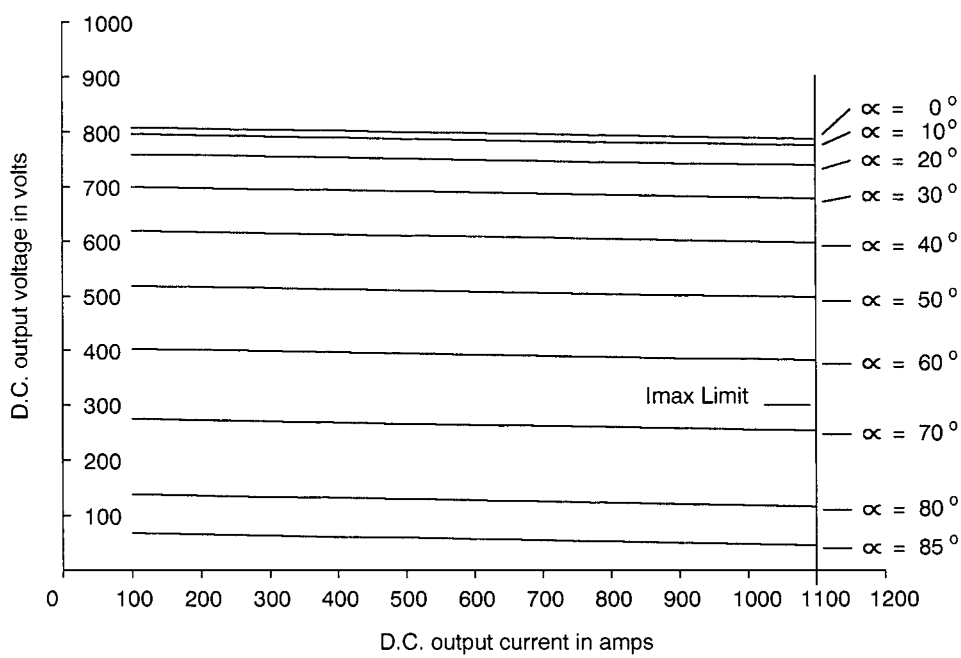

Figure 15.3 Voltage versus current regulation of DC thyristor bridge used for a drilling system DC motor.

The power factor of the fundamental phase current in the reference phase of the secondary winding can be found from the in-phase and quadrature Fourier coefficients of the current. Let these be a_1 and b_1 respectively. Hence the fundamental instantaneous current is,

$$\begin{aligned} i_1 &= \hat{I}_1(a_1 \sin \omega t + b_1 \cos \omega t) \\ &= \hat{I}_1 c_1 \sin(\omega t + \varnothing_1) \end{aligned}$$

Where the power factor is $\cos \varnothing_1$, and the suffix 1 refers to the fundament component.

Reference 4 gives an expression for a_1 and b_1 in terms of the angles α and u that is suitable for Mode 1 operation,

$$a_1 = \cos \alpha + \cos(u + \alpha) \tag{15.4}$$

and

$$b_1 = \frac{\sin(2\alpha + 2u) - \sin 2\alpha - 2u}{2[\cos \alpha - \cos(u + \alpha)]} \tag{15.5}$$

where α and u are in radians.

From which,

$$c_1 = \sqrt{{a_1}^2 + {b_1}^2} \tag{15.6}$$

and

$$\cos \varnothing_1 = \frac{a_1}{c_1}$$

and

$$u = \cos^{-1}\left(\frac{\pi R - 3X_c}{\pi R + 3X_c}\right) \text{ radians} \tag{15.7}$$

The fundamental components of the rms current I in the phases of the secondary winding are,

Real part,

$$I_r = \frac{I_d}{\pi}\sqrt{\frac{3}{2}}a_1 \tag{15.8}$$

and

Imaginary part,

$$I_i = \frac{I_d}{\pi}\sqrt{\frac{3}{2}}b_1 \tag{15.9}$$

and the rms magnitude is,

$$I = \frac{I_d}{\pi}\sqrt{\frac{3}{2}}c_1 \tag{15.10}$$

The coefficient c_1 has a maximum value of 2 when α is zero and the commutation angle u is assumed to be negligibly small.

In this case the maximum rms value of I is,

$$I_{\max} = \frac{I_d}{\pi}\sqrt{\frac{3}{2}}2 = \frac{\sqrt{6}}{\pi}I_d \tag{15.11}$$

15.2.4.2 Active and reactive power

The rectifying elements of the bridge are assumed to be free of ohmic power losses. Therefore the power input to the DC motor must be equal to the AC power input to the bridge. Hence the sum of the active power in each phase of the supply transformer must equal the motor input power.

The input power P_d to the motor is,

$$P_d = V_d I_d \tag{15.12}$$

The output volt-amperes of the transformer is,

$$S_{\text{sec}} = 3EI = \frac{3EI_d}{\pi}\sqrt{\frac{3}{2}}c_1 \tag{15.13}$$

The active and reactive powers at the output of the transformer are,

$$P_{\text{sec}} = \frac{3EI_d}{\pi}\sqrt{\frac{3}{2}}a_1 \tag{15.14}$$

and

$$Q_{\text{sec}} = \frac{3EI_d}{\pi}\sqrt{\frac{3}{2}}b_1 \tag{15.15}$$

The power factor of the fundamental current I is,

$$\cos \emptyset_1 = \frac{P_{\text{sec}}}{S_{\text{sec}}} = \frac{a_1}{c_1} \tag{15.16}$$

15.2.4.2.1 Worked example

Certain operations that take place when drilling oil wells require the DC motors to operate at reduced speed and to produce a moderate or high torque, e.g. reaming holes, running casing, stuck pipe removal, working over a well. Consider an example where a draw-works is running casing and several series-wound motors operate in parallel to drive the line drum.

The motor design details are

Rated output power	750 kW
Rated efficiency	93%
Rated voltage	750 volts
Rated current	1075 amps
Rated speed	975 rev/min
Rated torque	7350 nm
Armature and field circuit resistance (hot)	0.0488 ohms
Armature and field circuit inductance	0.006 henry

The motor running details are,

Running output power	217.8 kW
Running voltage	323.3 volts
Running current	761.1 amps
Running speed	400 rev/min
Running torque	5200 nm
Running input power	246.1 kW

The transformer that feeds the bridge has the following ratings,

Rated kVA	6000
Voltage ratio, volts/volts	11,000/600
Leakage reactance in per-unit	0.04
Leakage reactance at 346 volts/phase	0.0024 ohms

Commutating reactance = 2 × 0.0024 = 0.0048 ohms.

For (15.1) the variables and parameters are,

$$\begin{aligned} V_d &= 323.1 \text{ volts} \\ E &= 346.0 \text{ volts} \\ I_d &= 761.1 \text{ amps} \\ X &= 0.0048 \text{ ohms} \end{aligned}$$

Therefore,

$$\begin{aligned} 323.1 &= \frac{3\sqrt{6} \times 346}{\pi} \cos\alpha - \frac{3 \times 0.0048 \times 761.1}{\pi} \\ &= 810.285 \cos\alpha - 3.4886 \\ \cos\alpha &= \frac{326.59}{810.29} = 0.4033 \\ \alpha &= 66.215 \text{ degrees} \end{aligned}$$

From (15.2)

$$\begin{aligned} 323.1 &= \frac{3\sqrt{6} \times 346}{2\pi}(0.4033 + \cos(66.215 + u)) \\ 0.7975 &= 0.4033 + \cos(66.215 + u) \\ u &= 66.215 - 65.677 = 0.538 \text{ degrees} \\ &= 0.00939 \text{ radians} \end{aligned}$$

From (15.4)

$$\begin{aligned} a_1 &= \cos 66.215 + \cos 66.753 \\ &= 0.4033 + 0.3947 = 0.7980 \end{aligned}$$

$$b_1 = \frac{\sin(2 \times 66.753) - \sin(2 \times 66.215) - 2 \times 0.00939}{2(0.4033 - 0.3947)}$$
$$= \frac{0.7253 - 0.7381 - 0.01878}{0.0172}$$
$$= -1.834 \text{ indicating a lagging power factor}$$

From (15.8), (15.9) and (15.10)

$$I_r = +\frac{761.1}{\pi}\sqrt{\frac{3}{2}} \quad 0.798 = +236.8 \text{ amps}$$
$$I_i = -\frac{761.1}{\pi}\sqrt{\frac{3}{2}} \quad 1.834 = -544.2 \text{ amps}$$

and

$$I = \frac{761.1}{\pi}\sqrt{\frac{3}{2}} \quad (0.7680^2 + 1.834^2)^{1/2}$$
$$= 593.46 \text{ amps per phase}$$

From (15.13), (15.14) and (15.15) the volt-amperes at the bridge AC terminals are,

$$S_{sec} = P_{sec} + jQ_{sec}$$

Where

$$P_{sec} = \frac{3 \times 346.0 \times 761.1 \times 0.7980 \times 1.2247}{3.1415926}$$
$$= 246.09 \text{ kW}$$

and

$$Q_{sec} = \frac{3 \times 346.0 \times 761.1 \times 1.834 \times 1.2247}{3.1415926}$$
$$= 565.52 \text{ kVA}_r$$

and

$$S_{sec} = 616.75 \text{ kVA}$$

The power factor of the fundamental current is,

$$\cos \varnothing_1 = \frac{246.09}{616.75} = 0.3990 \text{ lagging}$$

or

$$\frac{a_1}{c_1} = \frac{0.7980}{\sqrt{0.7980^2 + 1.834^2}} = 0.3990 \text{ lagging}$$

Note, a 'rule-of-thumb' expression for the power factor is,

$$\cos \varnothing_1 \simeq 0.7\frac{\omega_o}{\omega_n} + 0.2$$

Where ω_o is the running speed of the motor and ω_n is the rated speed of the motor.

Hence,

$$\cos \varnothing_1 \simeq \left(0.7 \times \frac{400}{975}\right) + 0.2$$
$$= 0.4872 \quad \text{which is a little optimistic but a satisfactory estimate.}$$

15.3 HARMONIC CONTENT OF THE SUPPLY SIDE CURRENTS

15.3.1 Simplified Waveform of a Six-pulse Bridge

In a well-designed rectifier-load system the inductance in the DC circuit may be assumed to be sufficiently large to completely smooth the DC current. In practice the smoothing is not perfect but adequate for the performance of the bridge. In the ideal situation the shape of the current in the three lines that supply the bridge are rectangular in shape, when the commutation angle u is assumed to be zero. A positive rectangle of duration 120° is followed by a pause of zero value and a duration of 60°. A second rectangle of negative magnitude follows in the same form as the positive rectangle. In this simplified situation only the magnitude of the rectangle changes with loading of the bridge, the sides of the rectangles do not change shape or position relative to each other. Hence the harmonic components of the AC currents remain constant with loading.

For the simplified situation the harmonic coefficients of the AC currents are only odd coefficients, and all triple coefficients are absent. The coefficients may be summarised as,

$$\frac{I_n}{I_1} = \frac{1}{n}, \quad \text{for } n = 5, 7, 11, 13, 17, 19 \text{ etc.}$$
$$n = 6k \pm 1$$

Where $k = 1, 2, 3, \ldots, \infty$. The lowest harmonic present is the fifth.

For the purpose of Fourier analysis assume that the positive 120° rectangle is placed with the centre at $\pi/2$ on the x-axis, and the centre of the negative rectangle at $3\pi/2$. The analysis will yield only coefficients for the sine terms. Assume the amplitude $i_{\max}$ of the rectangle is 1.0. The Fourier integration yields the harmonic coefficients as,

$$b_n = \frac{1}{n\pi}\left(\cos\frac{\pi n}{6} - \cos\frac{5\pi n}{6} - \cos\frac{7\pi n}{6} + \cos\frac{11\pi n}{6}\right) \text{ and } a_n = 0$$

$$i(\omega t) = i_{\max}\sum_{n=1}^{n=\infty} b_n \sin \omega t \tag{15.17}$$

Let b_n be denoted as b_{n120} for use in sub-section 15.3.4.

The lowest harmonic present is the fifth.

The value of the fundamental coefficient b_1 is,

$$b_1 = \frac{1}{\pi}\left(4\frac{\sqrt{3}}{2}\right) = \frac{2\sqrt{3}}{\pi}$$

15.3.2 Simplified Commutation Delay

In practice the commutation delay angle is in the order of a few degrees. When the waveform of AC current is drawn it is difficult to distinguish a difference between a sloping straight line and an exponential line for the 'vertical' faces of the waveform. For this reason it is acceptable to assume a straight line and treat the waveform as a trapezium, as for example, in Reference 1, Chapter 9. Figures 15.4 and 15.5 show a trapezoidal waveform for two values of commutation angle $u = 20^\circ$ and $u = 50^\circ$. It can be seen that as u increases from zero the right-hand side face moves to the right and reduces the zero valued gap from 60° to zero. As a result the coefficient of each harmonic component diminishes from $1/n$ to $1/n^2$, which may be expected because a trapezium is a closer approximation to a sine wave than the rectangular pulse. Table 15.2 shows the reduction in coefficient magnitudes as the commutation angle u increases over its theoretical range. The method of calculation was by numerical integration, as described for example in References 10 and 11, which is sufficiently accurate for practical purposes. It can be seen that for practical values of u the approximation of commutation by a sloping straight line can even be ignored, and the simple rectangle pulse is adequate for all practical steady state loading of the bridge.

15.3.3 Fourier Coefficients of the Line Current Waveform

The Fourier coefficients of the line current waveform for the sine and cosine components can be found by integrating the waveform over any period of π, or 360°. The waveform is shown in Figure 15.4 or

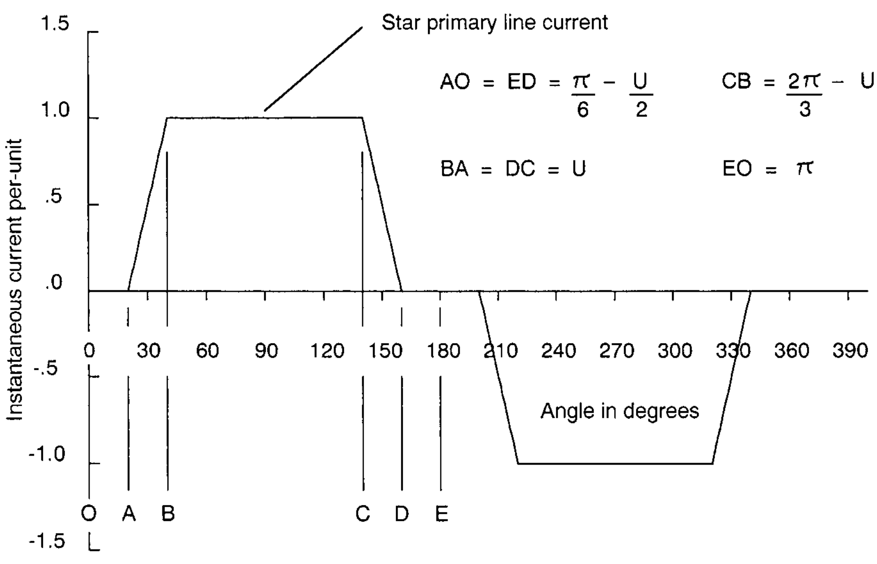

Figure 15.4 Trapezoidal current in the supply side of a six-pulse thyristor bridge, with the commutation angle $u = 20^\circ$.

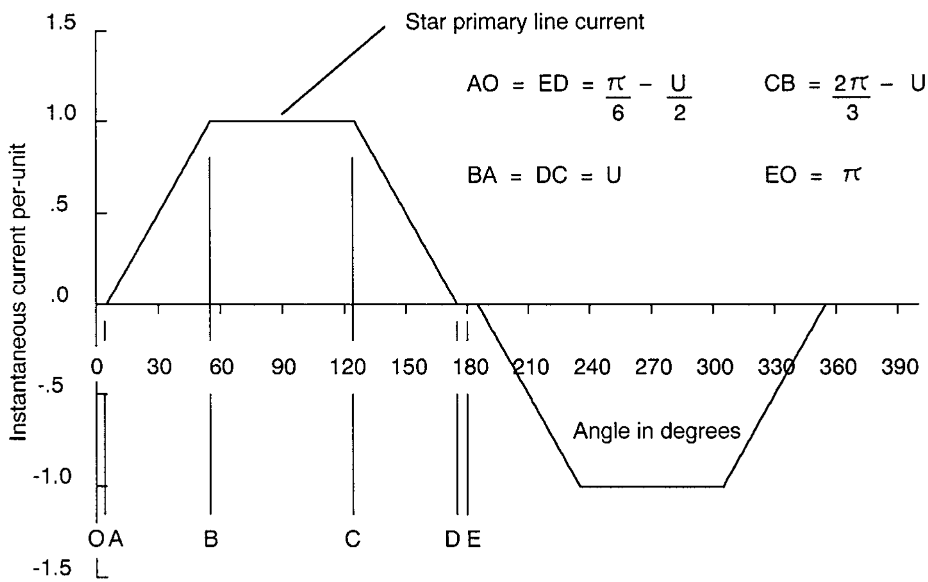

Figure 15.5 Trapezoidal current in the supply side of a six-pulse thyristor bridge, with the commutation angle $u = 50^\circ$.

15.5 for a value of the commutation angle u between zero and 60°. This is the current in the phase A terminal of the bridge. If the first point on the waveform is placed at the origin of the X-Y axes then the waveform will contain both sine and cosine terms. Alternatively the waveform can be advanced, so that the centre of the positive half-wave coincides with $\pi/2$ or 90°, and the centre of the negative half-wave coincides with $3\pi/2$ or 270°. This simplifies the analysis and yields only coefficients for the sine terms.

The Fourier integration is carried out as six sequential parts along the X-axis, i.e. A to B, B to C, C to D for the positive half-wave and similarly for the negative half-wave. If the maximum instantaneous value of the current is $i_{\max}$ then the result of the integration yields the following expression for the sine coefficients.

$$b_{nu} = \frac{2}{un^2\pi}\left(\sin\frac{un}{2}\right)\left(\cos\frac{n\pi}{6} - \cos\frac{5n\pi}{6} - \cos\frac{7n\pi}{6} + \cos\frac{11n\pi}{6}\right) \tag{15.18}$$

$$a_{nu} = 0$$

For which,

$$i_a(\omega t) = i_{\max}\sum_{j=1}^{n} b_{ju}\sin j\omega t$$

It is found from the integration that all even harmonics and those multiples of three are not present in the waveform. Hence n has the following value for a six-phase bridge:

$$n = 1, 5, 7, 11, 13, 17, 19 \text{ etc.}$$

$$\text{or } 1, 6k \pm 1 \text{ for } k = 1 \text{ to infinity.} \tag{15.19}$$

The average value of the waveform is zero because it is symmetrical about the Y-axis, and so the coefficient a_o for the average value is zero. The sinusoidal function in the coefficient a_n varies with the commutation angle u and approaches a limiting value when u is small,

$$\text{As } u \to 0, \quad \frac{2\sin\dfrac{un}{2}}{un^2} \to \frac{1}{n}$$

Which applies to a rectangular waveform. When u is 60° the sinusoidal function has an absolute value of,

$$u = 60^\circ, \quad \left|\frac{2\sin\dfrac{un}{2}}{un^2}\right| = \frac{3}{\pi n^2}$$

$$= \frac{0.9549}{n^2}$$

Therefore the magnitude of all the harmonics decrease as u increases, which is a reasonable expectation since the waveform more closely resembles a sine wave.

The magnitude of the sum of the four cosine terms in (15.18) is $2\sqrt{3}$ for all values of k in (15.19), otherwise the magnitude is zero.

Table 15.2 shows the magnitudes of b_n after scaling them by $1/b_1$, i.e. creating $b_1 = 1.0$ as reference.

15.3.3.1 Worked example

Consider a 250 kW DC motor fed by a rectifier system. The line voltage is 415 volts at 50 Hz. The rectifier is fed by a 400 kVA transformer which has an unusually high impedance of $0.0 + 24.5\%$. Assume the motor rated efficiency is 0.9 per unit. Assume the motor terminal voltage is 262.3 volts and its total current is 425 amps.

Phase voltage of the supply $E = \dfrac{415}{\sqrt{3}} = 239.6$ volts.

Open-circuit DC voltage of the rectifier $V_{do} = \dfrac{3\sqrt{6}}{\pi}(239.6) = 560.45$ volts.

The supply current

$$I_{ac} = \frac{2I_d}{\pi}\sqrt{\frac{3}{2}} = 0.7797\ I_d$$

$$= 0.7797 \times 425 = 331.37 \text{ amps}$$

The transformer rated current $= \dfrac{400{,}000}{\sqrt{3} \times 415} = 556.48$ amps

1 pu impedance $= \dfrac{239.6}{556.48} = 0.4306$ ohms/phase

Therefore the commutating reactance $= 2.0 \times 0.245 \times 0.4306 = 0.211$ ohms/phase $= X_c$

$$V_d = V_{do}\cos\alpha - \frac{3X_c I_d}{\pi}$$

$$262.3 = 560.45\cos\alpha - \frac{3 \times 0.211 \times 425.0}{\pi}$$

$$= 560.45\cos\alpha - 43.685$$

Therefore

$$\cos\alpha = \frac{262.3 + 85.623}{560.45} = 0.6208$$

$$\alpha = 51.63^\circ$$

Also

$$V_d = \frac{V_{do}}{2}(\cos\alpha + \cos(\alpha + u))$$

$$262.3 = \frac{560.45}{2}(0.6208 + \cos(51.63 + u))$$

$$\cos(51.63 + u) = 0.3152$$

$$51.63 + u = 71.626$$

$$u = 20^\circ$$

The resulting waveform is shown in Figure 15.4.

15.3.4 Simplified Waveform of a 12-pulse Bridge

The six-pulse rectifier bridges can be connected in such a manner as to produce a 12-pulse DC output voltage. The average value of DC ripple voltage is thereby reduced. From the AC power system point of view the magnitude of the harmonic components is reduced and some harmonics are eliminated. Figure 15.6 shows a typical circuit of a 12-pulse bridge.

The upper bridge is fed by a Dyll delta-star transformer T_u which has a 30° phase shift between the primary and secondary line currents. The lower transformer T_l has zero phase shift. See sub-section 6.4 for an explanation of phase shifts in transformer windings.

The primary currents for transformer T_u are added as follows,

$$I_{12} = I_1 - I_2$$

$$I_{23} = I_2 - I_3$$

$$I_{31} = I_3 - I_1$$

Where, I_{12} etc. can be either the rms values or the instantaneous values, but displaced by their appropriate phase angles, i.e. 0°, −120° and −240°.

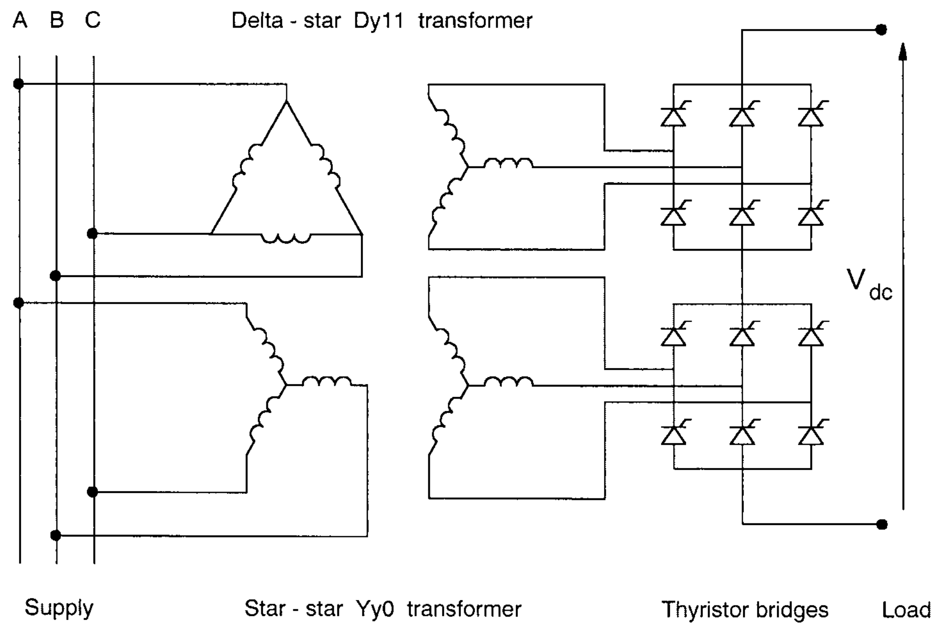

Figure 15.6 Circuit diagram of a 12-pulse thyristor bridge.

For example, let $i_1 = I\ \sin\omega t$ and $i_2 = I\ \sin(\omega t - 120^\circ)$ be the fundamental instantaneous currents, then i_{12} becomes,

$$\begin{aligned}
i_{12} &= I(\sin\omega t - \sin(\omega t - 120^\circ)) \\
&= I(\sin\omega t - \sin\omega t\cos(-120^\circ) - \cos\omega t\sin(-120^\circ)) \\
&= I(\sin\omega t + 0.866\cos\omega t + 0.5\sin\omega t) \\
&= I(+1.5\sin\omega t + 0.866\cos\omega t) \\
&= \sqrt{3}I\sin(\omega t + 30^\circ)
\end{aligned}$$

In order to obtain the full benefit of harmonic cancellation the two bridges must be controlled in a common manner. The control system will enable the fundamental current in both supply lines of the same phase to be in-phase, i.e. the star primary line current must be in-phase with the delta primary line current. See Reference 12, Chapter 3 which emphasises this aspect. The controlled firing of the delta-star bridge T_u cancels the 30° degree phase shift of the transformer. From the Fourier analysis point of view this can be achieved by adding $a + 30^\circ$ phase shift to the delta primary line current.

In sub-section 15.3.1 the line current of the star-star bridge T_l was the same as the phase current, both having the shape of the 120° rectangle wave form. When the phase currents are combined to produce the delta line current the waveform consists of two parts. The first part is a full, rectangular wave, which can be called the '180° rectangle waveform'. The second part is a narrow rectangular wave. The width of this rectangle is 60°, hence call this the '60° rectangle waveform'. The two parts have the same magnitude, which is 1.0 per unit for the analysis. In both waveforms the rectangles are centred at $\pi/2$ and $3\pi/2$, as described in sub-section 15.3.1. The harmonic coefficients for the two parts are,

Part 1. For the 180° rectangle waveform,

$$b_{n180} = \frac{4}{\pi n}, \text{ the fundamental } b_{1180} = \frac{4}{\pi}$$

Part 2. For the 60° rectangle waveform,

$$b_{n60} = \frac{2}{\pi n}\left(\cos\frac{2\pi n}{6} - \cos\frac{4\pi n}{6} - \cos\frac{8\pi n}{6} + \cos\frac{10\pi n}{6}\right)$$

The value of the fundamental coefficient b_{160} is,

$$b_{160} = \frac{1}{\pi}(4)\frac{1}{2} = \frac{2}{\pi}$$

The magnitude of the two parts is divided by $\sqrt{3}$ to obtain the primary line current of the delta-star transformer. The result is then added to the line current of the star-star transformer. The total magnitude of the supply line harmonic coefficient b_{nsum} is given by,

$$\begin{aligned} b_{\text{nsum}} = \frac{1}{\pi n}\Bigg[& \frac{4}{\sqrt{3}} + \cos\frac{\pi n}{6} + \frac{1}{\sqrt{3}}\cos\frac{2\pi n}{6} \\ & - \frac{1}{\sqrt{3}}\cos\frac{4\pi n}{6} - \cos\frac{5\pi n}{6} - \cos\frac{7\pi n}{6} \\ & - \frac{1}{\sqrt{3}}\cos\frac{8\pi n}{6} + \frac{1}{\sqrt{3}}\cos\frac{10\pi n}{6} + \cos\frac{11\pi n}{6}\Bigg] \end{aligned}$$

and

$$i_{\text{sum}}(\omega t) = i_{\max}\sum_{n=1}^{n=\infty} b_{\text{nsum}} \sin n\,\omega t$$

The value of the fundamental coefficient $b_{1\text{sum}}$ is,

$$b_{1\text{sum}} = \frac{1}{\pi}\left(\frac{4}{\sqrt{3}} + \frac{4\sqrt{3}}{2} + \frac{2}{\sqrt{3}}\right) = \frac{4\sqrt{3}}{\pi}$$

The fundamental coefficients from the 180°, 120° and 60° waveforms are found to be in the ratio 2:$\sqrt{3}$:1 respectively. The fundamental coefficient of the supply current is double the magnitude of the 120° waveform coefficient, which is the desired result.

The 180° waveform contains triplen harmonics for n taking odd values. The 60° waveform also contains the same triplen harmonics but with opposite signs, which therefore cancel those in the 180° waveform. None of the waveforms contain even harmonics.

The following harmonics are contained in the waveform,

$$n = 12\,k \pm 1$$

Where $k = 1, 2, 3, \ldots, \infty$. The lowest harmonic present is the eleventh.

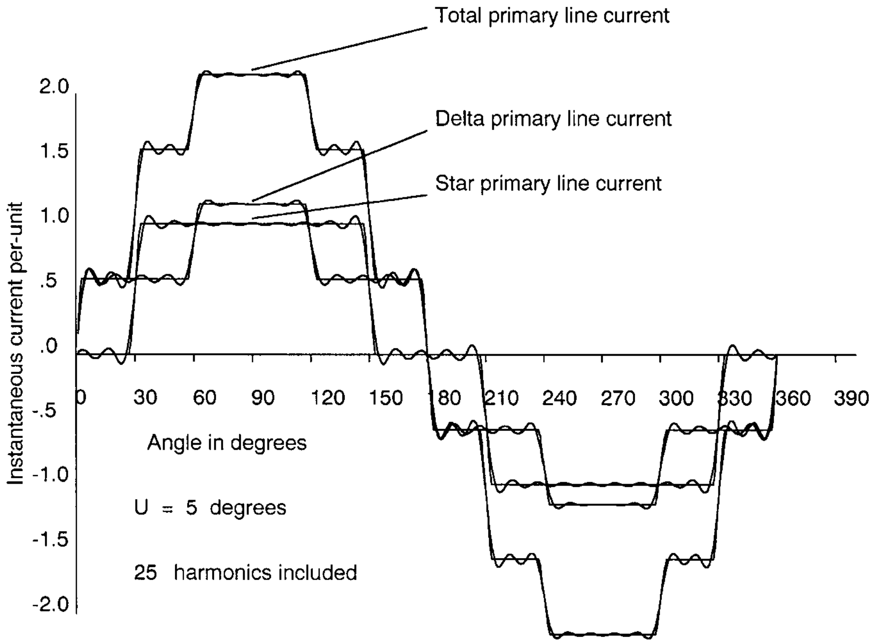

Figure 15.7 Primary line currents in the transformers feeding a 12-pulse thyristor bridge, with the commutation angle $u = 5°$. The waveforms are composed of 25 harmonics, some of which are zero in magnitude.

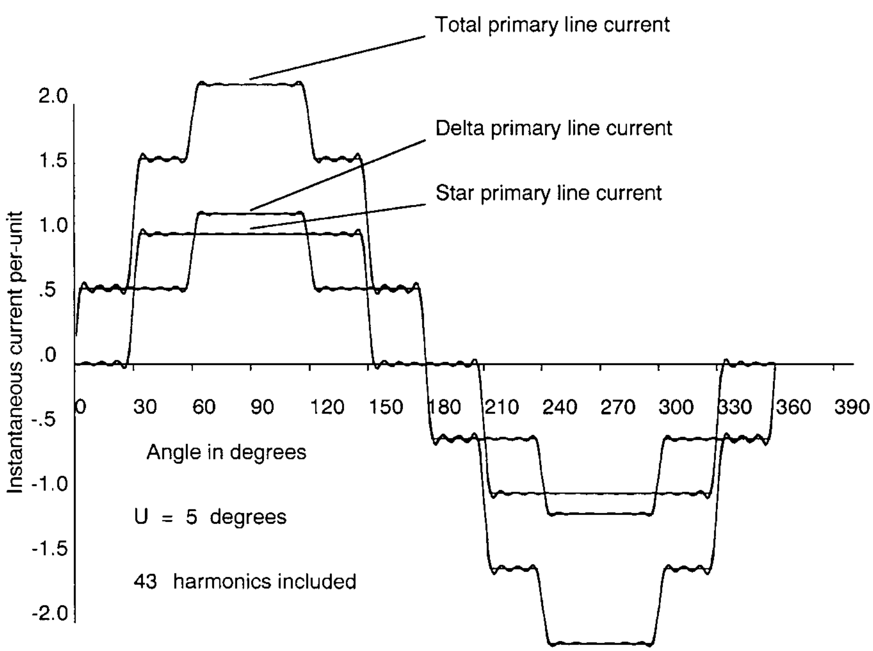

Figure 15.8 Primary line currents in the transformers feeding a 12-pulse thyristor bridge, with the commutation angle $u = 5°$. The waveforms are composed of 43 harmonics, some of which are zero in magnitude.

The effect of the commutation angle u on the 180°, 60° and supply current waveforms is the same as found in sub-section 15.3.2 for the 120° waveform. Therefore each coefficient b_n becomes,

$$b_{nu} = \frac{2b_n}{un} \sin\left(\frac{un}{2}\right)$$

Figures 15.7 and 15.8 show the following waveforms for $u = 5°$ for the first 25 and 43 harmonics included, some being naturally zero.

- Star primary line current or 120° waveform.
- Delta primary line current or the sum of the 180° and 60° waveforms.
- Total primary line current.

15.4 INVERTERS

15.4.1 Basic Method of Operation

Inversion is the process by which a DC voltage is changed into an AC voltage by the use of a set of switches. The following illustrates the method of operation of a simple single-phase 'square-wave' inverter. Consider Figure 15.9.

The four switches T_1, T_2, T_3 and T_4, are controlled in their fully 'on' and fully 'off' modes, in a sequence that causes the current I_{ac} and hence voltage V_{ac} to flow in one direction, to fall to zero, to flow in the opposite direction and again to fall to zero. The conduction of current in the load from A to B is achieved by closing T_1 and T_2, and keeping T_3 and T_4 open. The conduction from B to A is the reversed process, T_3 and T_4 are closed and T_1 and T_2 are kept open. The capacitors, diodes

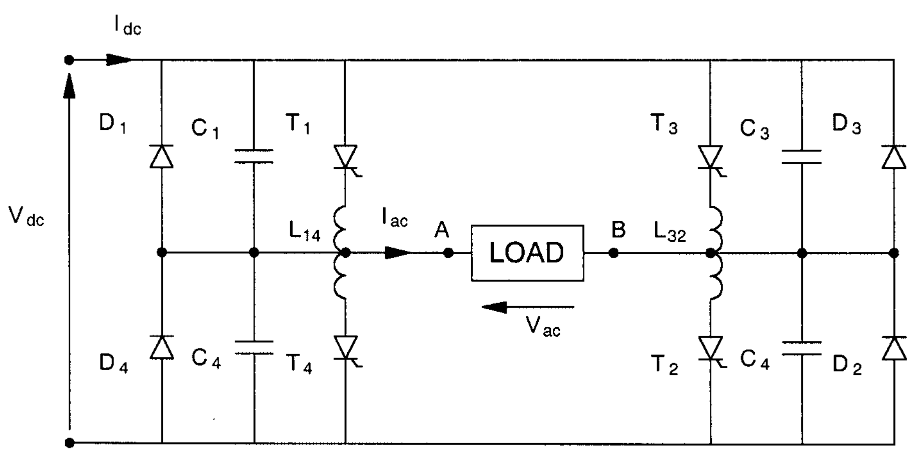

Figure 15.9 Circuit diagram of a single-phase square-wave inverter.

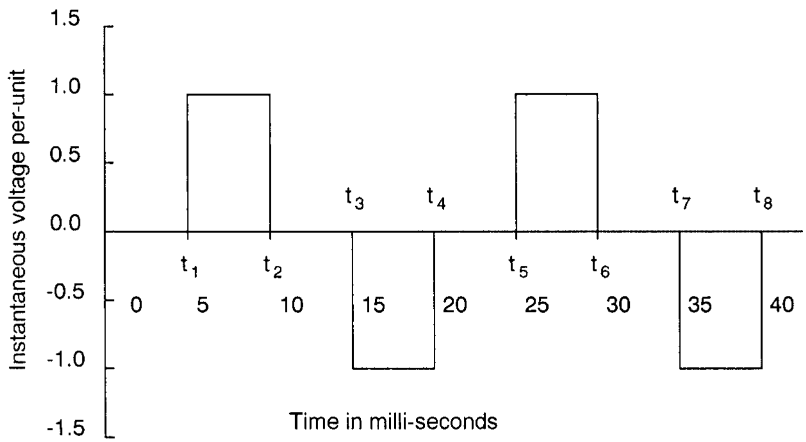

Figure 15.10 Waveform of the load current in a single-phase quazi-square-wave inverter.

and the centre-tapped inductor are used to provide forced commutation where the 'off' state is not controllable. Figure 15.10 shows the voltage applied to the load.

The method described above can be modified to operate as a three-phase inverter. Single and three-phase inverters operating in this manner form the basis for many types of uninterruptible power supplies (UPSs), and variable speed drives for AC motors.

15.4.2 Three-phase Power Inversion

High-power inverters were initially developed for the long-distance transmission of power from a three-phase source to a remote three-phase sink using a DC overhead transmission line or cable. Early DC power transmission used mercury arc thyratrons (gas-filled values or tubes), which functioned in a manner very similar to the early types of thyristors. The 'on' state of the valves was controllable, but the 'off' state was determined by natural commutation made available by the sinusoidal voltages of the sink power system, see Reference 13. A brief description of three-phase inverters follows.

There are two basic types of high-power inverters that are used to supply AC induction or synchronous motors, see References 2 and 9.

- Voltage source inverter.
- Current source inverter.

The voltage source inverter was the first to be developed for the control of induction motors. It consists of a supply rectifier, a DC link inductor, a DC link capacitor and an inverter for the motor. The inductor provides some smoothing of the DC current and short-circuit current limiting for the supply rectifier elements. The capacitor is relatively large and stores sufficient charge to provide current into the inverter. It also provides smoothing of the DC current. Figure 14.3 shows the basic configuration.

The inverter bridge switches the DC voltage across the lines of the motor. The waveform appearing across the lines is a 120° rectangle, similar to the current waveform described in sub-section 15.3.2 for the star-star rectifier transformer, but with vertical sides. Hence the voltage is switched and the load current responds. The Fourier coefficients for this waveform are the same as for (15.17) except that V replaces I and V_{max} replaces I_{max}. The harmonic content is,

- Lowest harmonic is the fifth.
- Zero even harmonics.
- Zero triplen harmonics.

Current source inverters differ from voltage source inverters in two basic ways, the DC link inductor is made large enough to provide an almost constant current, and there is no DC link capacitor. The inverter therefore switches the current and the load voltage responds. The switching of current requires a commutation process to take place, and so the current waveform for each line current to the load has approximately a trapezoidal shape. The shaping of the waveform is described in sub-section 15.2.1. The commutation angle u is therefore inherent in the inverter operation. The line current waveforms are shown in Figure 15.4. It can be seen that these are 120° trapeziums and therefore the harmonic analysis is the same as that applied in sub-section 15.3.2 for a six-phase rectifier bridge for its line currents. Since the currents appearing at the lines of the motor are switched by the commutation process the inductances of the motor create a rapid 'rate of rise' of voltage across themselves. The terminal voltage of the motor will therefore contain a proportion of these 'noisy voltages', and some form of suppression of overvoltages may be necessary.

15.4.3 Induction Motor Fed from a Voltage Source Inverter

If an induction motor is running in a stable steady state with a low slip, then the various fundamental currents and voltages within the motor can be calculated from the conventional equivalent circuit. When the motor is supplied from a source of harmonic voltages the impedance elements in the circuit need to be modified to account for the frequency of each harmonic that is present. The various reactances are directly proportional to the harmonic frequency. The stator and rotor resistances may be assumed constant, although in practice they will increase with the frequency, the rotor more than the stator, see Reference 9, Figure 1.26 therein.

If the harmonic content of the applied voltage is known in terms of magnitudes and phase shifts of the components, then the circuit can be solved for each frequency. The result for each branch current or voltage will be the sum of all their harmonic components plus their fundamentals.

Before the calculation can be made the slip for each frequency needs to be found when the shaft is running at its normally loaded conditions i.e. near to the synchronous speed of the fundamental frequency. The slip s_n for the harmonic frequency nf_1 is given by,

$$s_n = \frac{n - (1 - s_1)}{n} \quad \text{for } n = 1,7,13 \text{ etc.}$$

$$s_n = \frac{n + (1 - s_1)}{n} \quad \text{for } n = 5,11,17 \text{ etc.}$$

As explained in Reference 2, Chapter 6.

For motors that normally operate at slips in the order of 0.5% to 3.0% the values of s_n for n, greater than unity are approximately given by,

$$s_n \simeq \frac{n-1}{n} \text{ for } n = 7, 13, 19 \text{ etc.}$$
$$= 0.8571, 0.9231, 0.9474 \text{ etc.}$$
$$s_n \simeq \frac{n+1}{n} \text{ for } n = 5, 11, 17 \text{ etc.}$$
$$= 1.2, 1.0909, 1.0588 \text{ etc.}$$

The rotor resistance that represents the winding and the load is R_2/s_1 for the fundamental. For the harmonic resistance to be found simply replace s_1 by s_n.

15.4.3.1 Worked example

A 250 kW three-phase, four-pole, 50 Hz, 415 V, induction motor is fed from a voltage source inverter. The motor has the following parameters for its star-wound windings. Find the currents and air-gap voltage in the circuit.

R_1	Stator resistance	0.0053 ohms
X_1	Stator leakage reactance	0.0470 ohms
R_2	Rotor resistance at full-load	0.0045 ohms
X_2	Rotor reactance at full-load	0.1113 ohms
X_m	Magnetising reactance	2.9310 ohms
s_1	Full-load slip	0.00738 pu
	Full-load efficiency	0.983 pu
	Full-load power factor	0.902 pu
	Full-load current	392.26 amps

The equivalent circuit fed from a Thevenin voltage source is shown in Figure 15.11.

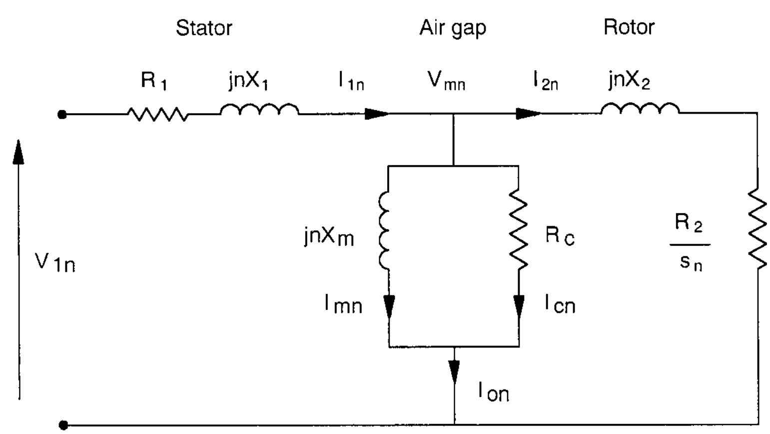

Figure 15.11 Equivalent circuit of an induction motor when fed from a voltage source that contains harmonics.

Assume the inverter equivalent output phase-to-neutral voltage consists of a 180° rectangle waveform plus a 60° rectangle waveform. The complete waveform has a harmonic content of,

$$\begin{aligned}
b_n &= \frac{4}{\pi n} + \frac{2}{\pi n}\left(\cos\frac{2\pi n}{6} - \cos\frac{4\pi n}{6} - \cos\frac{8\pi n}{6} + \cos\frac{10\pi n}{6}\right) \\
&= \frac{4}{\pi n} + \frac{2}{\pi n}(1.155) \qquad \text{for } n = 1, 5, 7, 11, 13, \text{ etc.} \\
&= \frac{6.31}{\pi n} = \frac{2.0085}{n} \\
\text{And} \qquad &= \frac{4}{\pi n} + \frac{2}{\pi n}(-2.309) \qquad \text{for } n = 3, 9, 15, 21 \text{ etc.} \\
&= \frac{-0.618}{\pi n} = \frac{-0.1967}{n}
\end{aligned}$$

The rms value of the fundamental phase-to-neutral voltage is $415/\sqrt{3} = 239.6$ volts. Therefore its peak value is $239.6\sqrt{2} = 338.85$ volts which corresponds to b_1 having a value of 2.0085. The peak values of the first 17 harmonic components of the phase-to-neutral voltage are given below in Table 15.3.

The triplen harmonics can be ignored because the motor has a star-wound stator winding.

Consider the fifth harmonic situation

The rotor resistance becomes	$0.0045/1.2 = 0.00375$
The rotor reactance becomes	$0.1113 \times 5 = 0.5565$
The stator reactance becomes	$0.0470 \times 5 = 0.2350$
The magnetising reactance becomes	$2.9310 \times 5 = 14.655$

Assume the stator resistance to be constant.

Table 15.3. Peak values of harmonic voltage components

Harmonic number	Peak value and sign of the component voltage
1	338.85
3	−11.06
5	67.77
7	48.41
9	−3.69
11	30.80
13	26.06
15	−2.21
17	19.93
19	17.83

The combined admittance of the rotor and magnetising impedances become,

$$Y_{m2} = \frac{-j}{nX_m} + \frac{\dfrac{R_2}{S_n} - jnX_2}{\left(\dfrac{R_2}{s_n}\right)^2 + n^2 {X_2}^2}$$
$$= 0.01211 - j\ 1.8650 \text{ ohms}$$
$$Z_{m2} = \frac{1}{Y_{m2}} = 0.00348 + j\ 0.5362 \text{ ohms}$$

Add the stator impedance.

$$Z_{1m2} = R_1 + jnX_1 + Z_{m2} = 0.00878 + j\ 0.7711 \text{ ohms}$$

All the harmonics of the supply voltage have zero phase shift (except the triplens which are anti-phase). The fifth harmonic supply voltage V_{5n} is $67.77/\sqrt{2}$ volts (rms). Supply this voltage to the circuit. The supply current is,

$$I_{15} = \frac{V_5}{Z_{1m2}} = \frac{67.77 + j\ 0.0}{\sqrt{2}(0.00878 + j\ 0.7711)}$$
$$= 0.7075 - j\ 62.1377 \quad \text{amps}$$

The volt-drop across the stator impedance is,

$$V_{1m5} = (0.0053 + j\ 0.2350)(0.7075 - j\ 62.1377)$$
$$= 14.606 - j\ 0.163 \quad \text{volts}$$

The air-gap voltage V_{m5} becomes,

$$V_{m5} = V_{15} - V_{1m5}$$
$$= 47.921 - 14.606 + j\ 0.163$$
$$= 33.315 + j\ 0.163 \quad \text{volts}$$

The magnetising current I_{m5} is,

$$I_{m5} = \frac{V_{m5}}{j_n X_m} = \frac{33.315 + j\ 0.163}{0.0 + j\ 14.665}$$
$$= 0.0111 - j\ 2.272 \quad \text{amps}$$

Hence the rotor current I_{25} becomes,

$$I_{25} = I_{15} - I_{m5}$$
$$= 0.7075 - j\ 62.1377 - 0.0111 + j\ 2.272$$
$$= 0.6964 - j\ 59.866 \quad \text{amps}$$

Table 15.4. Harmonic rms currents and voltages in a star-wound induction motor that is fed from a voltage source inverter

Harmonic number	Stator current		Rotor current		Magnetising current		Air-gap voltage	
	Mag. (Amps)	Angle (Degrees)	Mag. (Amps)	Angle (Degrees)	Mag. (Amps)	Angle (Degrees)	Mag. (Amps)	Angle (Degrees)
1	392.28	−25.60	371.51	−14.27	78.56	−89.07	230.27	−3.93
5	62.14	−89.35	59.86	−89.33	2.27	−89.72	33.32	0.28
7	31.70	−89.46	30.54	−89.45	1.16	−89.83	23.80	0.17
11	12.84	−89.69	12.37	−89.69	0.47	−89.88	15.14	0.12
13	9.19	−89.72	8.86	−89.71	0.37	−89.90	12.81	0.09
17	5.38	−89.80	5.18	−89.79	0.20	−89.92	9.80	0.08
19	4.30	−89.81	4.14	−89.80	0.16	−89.93	8.77	0.07
23	2.94	−89.85	2.83	−89.85	0.11	−89.94	7.24	0.06
25	2.49	−89.86	2.40	−89.85	0.09	−89.95	6.66	0.05
29	1.85	−89.88	1.78	−89.88	0.07	−89.96	5.74	0.05
31	1.62	−89.89	1.56	−89.88	0.06	−89.96	5.37	0.04

If the above calculations are made for all the active harmonics then their results can be added and the waveforms synthesised. Table 15.4 summarises the results.

Figure 15.12 shows the synthesised currents and air-gap voltage using the first 61 harmonics.

15.4.3.2 Worked example

The same motor as used in the 'worked example' of sub-section 15.4.3.1 is fed from a current source inverter. Find the currents and air-gap voltage in the circuit.

The equivalent circuit fed from a constant current source is shown in Figure 15.11, wherein I_{1n} is the source current instead of V_{1n}.

Assume the inverter output line current consists of a 120° rectangle wave and that the commutation angle u is small enough to be ignored. The complete waveform has a harmonic content of,

$$\begin{aligned} b_n &= \frac{1}{\pi n}\left(\cos\frac{\pi n}{6} - \cos\frac{5\pi n}{6} - \cos\frac{7\pi n}{6} + \cos\frac{11\pi n}{6}\right) \\ &= \frac{3.464}{n\pi} \quad \text{for } n = 1, 5, 7, 11, 13 \text{ etc.} \\ &= \frac{1.1026}{n} \end{aligned}$$

The rms value of the fundamental line current is 392.26 amps. Therefore its peak value is $392.26\sqrt{2} = 554.74$ amps which corresponds to b_1 having a value of 1.1026. The peak values of the harmonic components of the line current are given below in Table 15.5.

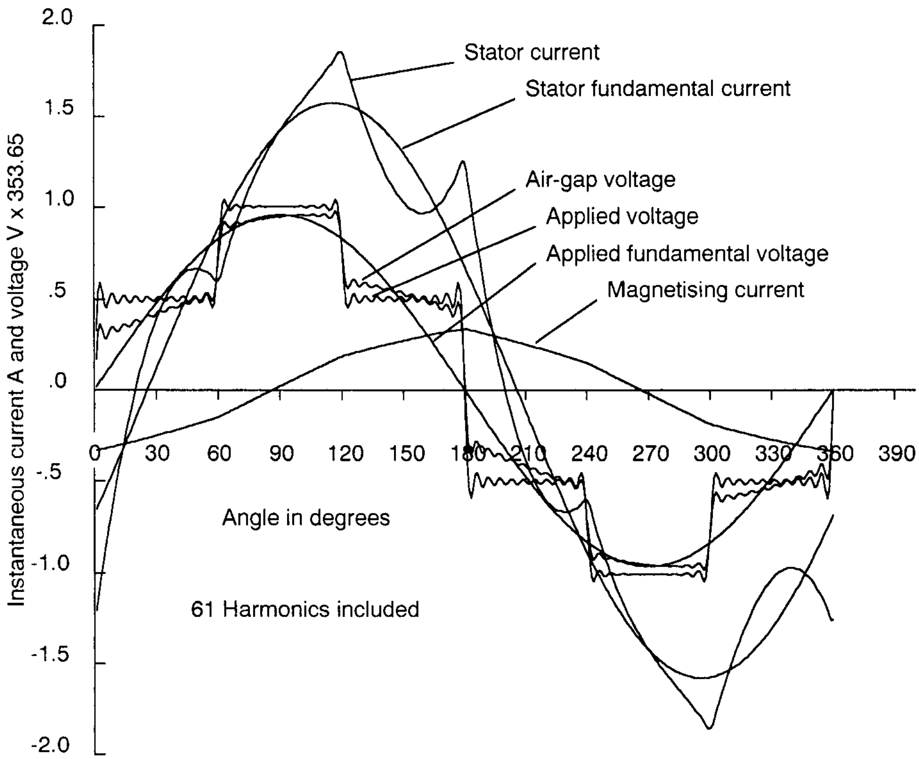

Figure 15.12 Instantaneous voltages and currents in an induction motor supplied from a voltage source inverter.

Table 15.5. Peak values of harmonic current components

Harmonic number	Peak value and sign of the component current
1	+554.74
5	−110.95
7	−79.25
11	+50.43
13	+42.67
17	−32.63
19	−29.20

From the Worked Example 15.4.3.1 the combined impedance of the rotor and magnetising branches is $Z_{m2} = 0.00348 + j\ 0.5362$ ohms. Adding the stator impedance gives,

$$Z_{1m2} = 0.00878 + j\ 0.7711 \quad \text{ohms}$$

All the harmonics of the supply current have either zero or 180° phase shift as seen in Table 15.5. The fifth harmonic supply current I_{15} is $-110.95/\sqrt{2}$ amps (rms). Inject this current into the circuit.

The supply voltage becomes,

$$\begin{aligned}V_{15} &= I_{15}Z_{1m2}\\ &= -\frac{110.95}{\sqrt{2}}(0.00878 + j\ 0.7711)\\ &= -0.6888 - j\ 60.496 \text{ volts}\end{aligned}$$

The voltage across the stator impedance is,

$$\begin{aligned}V_{1m5} &= (0.0053 + j\ 0.2350)(-78.453 - j\ 0)\\ &= -0.4158 - j\ 18.4365\end{aligned}$$

The air-gap voltage V_{m5} becomes,

$$\begin{aligned}V_{m5} &= V_{15} - V_{1m5}\\ &= -0.6888 - j\ 60.496 + 0.4158 + j\ 18.4365\\ &= -0.273 - j\ 42.059 \quad \text{volts}\end{aligned}$$

The magnetising current I_{m5} is,

$$\begin{aligned}I_{m5} &= \frac{V_{m5}}{j_n X_m} = \frac{-0.273 - j\ 42.059}{0.0 + j\ 14.655}\\ &= -2.8699 + j\ 0.0186 \quad \text{amps}\end{aligned}$$

Hence the rotor current I_{25} becomes,

$$\begin{aligned}I_{25} &= I_{15} - I_{m5}\\ &= -78.453 + j\ 0 + 2.8699 - j\ 0.0186\\ &= -75.565 - j\ 0.0186 \quad \text{amps}\end{aligned}$$

If the above calculations are made for all the active harmonics then their results can be added and the waveforms synthesised. Table 15.6 summarises the results.

Figure 15.13 shows the synthesised currents and air-gap voltage using the first 91 harmonics.

15.5 FILTERING OF POWER LINE HARMONICS

In modern oil industry power systems there is a probability that one or more variable speed systems will be present. When the system engineer designs or modifies a power system he will need to take full account of the effect of the harmonics that will be injected into the system from the rectifier part of the variable speed drive, see also sub-section 15.1.

The most frequently used reference document based on European practice that makes recommendations on the levels of harmonics that can be tolerated in LV and HV systems is Reference 14,

Table 15.6. Harmonic rms currents and voltages in a star-wound induction motor that is fed from a current source inverter

Harmonic number	Stator current		Rotor current		Magnetising current		Air-gap voltage	
	Mag. (Amps)	Angle (Degrees)	Mag. (Amps)	Angle (Degrees)	Mag. (Amps)	Angle (Degrees)	Mag. (Amps)	Angle (Degrees)
1	392.28	0	370.54	11.33	78.36	−68.32	229.67	21.68
5	78.45	180.0	75.58	180.014	2.87	179.63	42.06	269.63
7	56.01	180.0	53.99	180.014	2.05	179.63	42.06	269.63
11	35.66	0	34.35	0.007	1.30	−0.186	42.06	89.81
13	30.67	0	29.07	0.007	1.10	−0.186	42.06	89.81
17	23.07	180.0	22.23	180.005	0.84	179.88	42.06	269.88
19	20.65	180.0	19.89	180.005	0.76	179.88	42.06	269.88
23	17.05	0	16.43	0.004	0.62	−0.093	42.06	89.91
25	15.69	0	15.12	0.004	0.57	−0.093	42.06	89.91
29	13.53	180.0	13.03	180.003	0.49	179.93	42.76	269.93
31	12.65	180.0	12.19	180.003	0.46	179.93	42.76	269.93

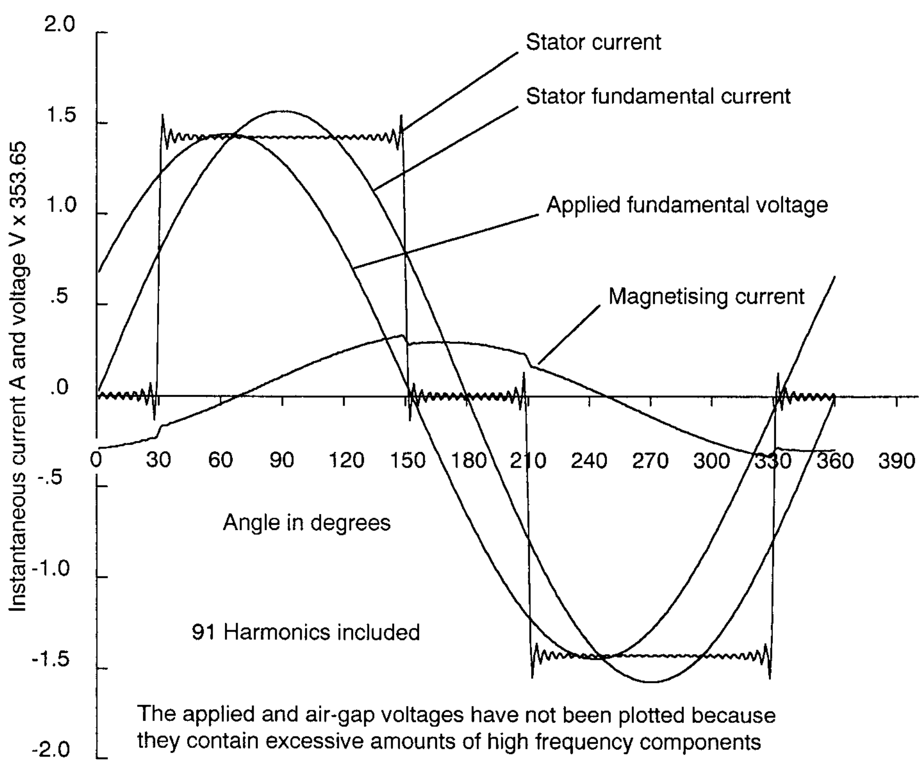

Figure 15.13 Instantaneous voltages and currents in an induction motor supplied from a current source inverter.

which at the time of preparing this book was still in its original form. In addition the Reference 15 is used. When an oil company owns or operates a power system, which is independent of other power systems such as a 'national grid', it may prepare its own specifications to cover the tolerable levels of harmonic voltages and currents. Such specifications may be more or less as strict as the recommendations made in References 14 and 15. In Reference 12 there is a comprehensive description of national standards that apply in various European countries, USA, Scandinavia, Australia and New Zealand. Most of these standards refer to six-pulse and 12-pulse converters and applied in 1985 when the reference was published. Some but not all of these standards have been revised since 1985, e.g. IEEE519 was revised in 1997. The standards described in Reference 12 are not directly comparable with each other because different criteria are used e.g. current in amps, current in percent, kVA converter ratings, individual harmonics, odd and even harmonics, total harmonic distortion and short-circuit rating. These criteria have been applied to public or 'national grid' power systems. For smaller self-contained power systems such as those used in offshore platforms the criteria that use actual current or kVA levels, rather than percentage levels, may prove to be too generous for the HV parts of the system. The G5/3 document offers recommendations for both actual currents in amps and voltages in percent. The recommendations based on percentage voltage are a popular choice in the oil industry. Table 15.7 summarises these recommendations to cover typical voltages in the oil industry.

The design of filters to reduce or eliminate harmonics from the system connected upstream of the source of harmonics is a specialised subject and the results will depend on many factors such as,

a) Proximity of the harmonic source to the source of main power, e.g. an HV converter connected onto the main generation busbars of an oil and gas gathering plant.
b) The type of converter i.e. six or 12 pulse. (Drilling rigs usually have six-pulse converters.)
c) The number of converters that will be operating at the same time.
d) The likely variations in the fundamental frequency during typical operating conditions of the plant.
e) Whether there are long HV feeder cables, e.g. an offshore platform supplied by power from the shore base or another platform some reasonable distance away. The cable capacitance may be sufficient to accentuate the effect of one or more of the lower order harmonics, see Reference 16.
f) Power dissipation from the filter may be a significant factor if it is to be placed indoors in a confined space.

Factors a) and d) are interrelated due to the scheduling for the number of generators that will be needed to operate for a particular plant condition. Generators and motors can be represented by their sub-transient impedance when harmonic studies are being carried out. These impedances and those of

Table 15.7. Recommended harmonic levels in relation to the system voltage level

Switchgear rated voltage (volts)	Total harmonic voltage distortion (%)	Individual harmonic voltage distortion (%)	
		Odd	Even
300 to 1000	5.0	4.0	2.0
1000 to 15,000	4.0	3.0	1.75
5000 to 40,000	3.5	2.5	1.5
40,000 to 80,000	3.0	2.0	1.0
80,000 to 132,000	1.5	1.0	0.5

Table 15.8. Protection alarms and indications for a high-voltage variable speed drive

Function	Protection	Alarm	Indication
1. Open circuit, short circuit and earth faults in the rectifier including the faults in the rectifier including the DC link	X		
2. Open circuit, short circuit and earth faults in the inverter including the cable and motor	X		
3. Overcurrent due to commutation failure in the rectifier	X		
4. Overcurrent due to commutation failure in the inverter	X		
5. Undervoltage at the output of the inverter	X		
6. Overfrequency at the output of the inverter	X		
7. Undervoltage at the input of the rectifier	X		
8. Enclosure over-temperatures		X	
9. Control system faults	X	X	
10. Harmonic filter faults	X	X	
11. Single-phase operation of the motor	X	X	
12. Overcurrent of the motor	X	X	X
13. Winding temperature of the motor	X	X	X
14. Winding temperature of power transformers	X	X	X
15. Supply line voltages			X
16. Supply line currents			X
17. DC link voltage			X
18. DC link current			X
19. Motor line voltages			X
20. Motor line current			X
21. Set-point frequency of inverter			X
22. Actual frequency of inverter			X
23. Motor speed			X

local transformers may be sufficient to cause sensitivity in the performance of the filter with variations in system loading. This is less of a problem where the plant is supplied from an overhead transmission line, and the upstream MVA capacity is large compared with the total demand of the plant.

Drilling rigs and low capacity AC variable speed drives are usually six-phase systems. If these are known to be needed at the conceptual stage of a project then their effects on other equipment can be taken into account reasonably easily. For example the specifications that are prepared for equipment connected to the 'distorted' network can include a full description of the harmonics that will be present. In most cases the manufacturer will be able to include some form of local filtering or add some extra capacity to the equipment offered e.g. larger motor rating so that the extra heat can be accommodated.

Droop governed generators will give a system frequency that varies with the power loading on their network. Some generating plants do not have the generator set-points available for manual or automatic adjustment. Consider a 50 Hz system with 4% droop governing at no-load the frequency may be preset to 51 Hz for each generator. As the loading is increased the frequency will fall to 49 Hz when all the connected generators are fully loaded. If there is another generator available and it is then switched into the system it will take its share of the common load and the frequency will settle at some value above 49 Hz. It can be seen that in this situation a variation of 1 Hz is very likely to be experienced.

If a sharply tuned filter system is used wherein the 'Q-factor' in each series resonant branch is high e.g. 30 or more, then a variation of $n \times 1$ Hz either side of the tuned frequency f_n may be unacceptable.

In practice the filter elements could be tapped with small increments but this would be expensive if some form of automatic control of the tappings were to be used. A more practical solution would be to control the governor set-points at the generator in a simultaneous manner, by using a form of integral control to maintain the system frequency within a narrower band. Reducing the droop settings would not achieve the desired result.

15.6 PROTECTION, ALARMS AND INDICATION

A high-voltage variable speed motor will usually drive an important pump or compressor which must remain in a serviceable condition, and not be subject to lengthy shut downs due to poor performance or serious failure of its major components. Modern systems will usually contain a micro-computer to process alarms, to give visual information, to communicate to external facilities and to safely shut down the system in the event of a serious or progressive fault being detected.

Table 15.8 lists the typical protection, alarms and indications that would be provided in the system.

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