Section 2

Units

2.1 The Greek alphabet

The Greek alphabet is used extensively to denote engineering quantities. Each letter can have various meanings, depending on the context in which it is used.

Table 2.1 The Greek alphabet

Name	*Symbol*		*Used for*
	Capital	*Lower case*	
alpha	A	α	Angles, angular acceleration
beta	B	β	Angles, coefficients
gamma	Γ	γ	Shear strain, kinematic viscosity
delta	Δ	δ	Differences, damping coefficient
epsilon	E	ε	Linear strain
zeta	Z	ζ	
eta	H	η	Dynamic viscosity, efficiency
theta	Θ	θ	Angles, temperature
iota	I	ι	
kappa	K	κ	Compressibility (fluids)
lambda	Λ	λ	Wavelength, thermal conductivity
mu	M	μ	Coefficient of friction, dynamic viscosity, Poisson's ratio
nu	N	ν	Kinematic viscosity
xi	Ξ	ξ	
omicron	O	o	
pi	Π	π	Mathematical constant
rho	R	ρ	Density
sigma	Σ	σ	Normal stress, standard deviation, sum of
tau	T	τ	Shear stress
upsilon	Υ	υ	
phi	Φ	φ	Angles, heat flowrate, potential energy
chi	x	χ	
psi	Ψ	ψ	Helix angle (gears)
omega	Ω	ω	Angular velocity, solid angle

Engineers' Data Book, Fourth Edition. Clifford Matthews.

2.2 Units systems

Unfortunately, the world of mechanical engineering has not yet achieved uniformity in the system of units it uses. The oldest system is that of British Imperial units – still used in many parts of the world, including the USA. The CGS (or MKS) system is a metric system, still used in some European countries, but is gradually being superseded by the Systeme International (SI) system. Whilst the SI system is understood (more or less) universally, you will still encounter units from the others.

2.2.1 The SI system

The strength of the SI system is its *coherence*. There are four mechanical and two electrical base units, from which all other quantities are derived. The mechanical ones are:

Length:	metre (m)
Mass:	kilogram (kg)
Time:	second (s)
Temperature:	Kelvin (K)

Remember, other units are derived from these; e.g. the Newton (N) is defined as $N = kg\ m/s^2$.

2.2.2 SI prefixes

As a rule, prefixes are applied to the basic SI unit, except for weight, where the prefix is used with the unit gram (g), not the basic SI unit kilogram (kg). Prefixes are not used for units of angular measurement (degrees, radians), time (seconds), or temperature (°C or K).

Prefixes should be chosen in such a way that the numerical value of a unit lies between 0.1 and 1000.

For example	28 kN	rather than	2.8×10^4 N
	1.25 mm	rather than	0.00125 m
	9.3 kPa	rather than	9300 Pa

Table 2.2 SI prefixes

Multiplication factor	*Prefix*	*Symbol*
1 000 000 000 000 000 000 000 000 = 10^{24}	yotta	Y
1 000 000 000 000 000 000 000 = 10^{21}	zetta	Z
1 000 000 000 000 000 000 = 10^{18}	exa	E
1 000 000 000 000 000 = 10^{15}	peta	P
1 000 000 000 000 = 10^{12}	tera	T
1 000 000 000 = 10^{9}	giga	G
1 000 000 = 10^{6}	mega	M
1 000 = 10^{3}	kilo	k
100 = 10^{2}	hecto	h
10 = 10^{1}	deka	da
0.1 = 10^{-1}	deci	d
0.01 = 10^{-2}	centi	c
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p
0.000 000 000 000 001 = 10^{-15}	femto	f
0.000 000 000 000 000 001 = 10^{-18}	atto	a
0.000 000 000 000 000 000 001 = 10^{-21}	zepto	z
0.000 000 000 000 000 000 000 001 = 10^{-24}	yocto	y

2.2.3 Conversions

Units often need to be converted. The least confusing way to do this is by expressing equality:

For example: to convert 600 mm H_2O to Pascals (Pa)

Using 1 mm H_2O = 9.80665 Pa
Add denominators as

$$\frac{1\,\text{mm}\,H_2O}{600\,\text{mm}\,H_2O} = \frac{9.80665\,\text{Pa}}{x\text{Pa}}$$

Solve for x

$$x\,\text{Pa} = \frac{600 \times 9.80665}{1} = 5883.99\,\text{Pa}$$

Hence 600 mm H_2O = 5883.99 Pa

Setting out calculations in this way can help avoid confusion, particularly when they involve large numbers and/or several sequential stages of conversion.

2.3 Units and conversions

2.3.1 Force

The SI unit is the Newton (N) – it is a derived unit.

Table 2.3 Force (F)

Unit	*N*	*lb*	*gf*	*kgf*
1 Newton (N)	1	0.2248	102.0	0.1020
1 pound (lb)	4.448	1	453.6	0.4536
1gram-force (gf)	9.807×10^{-3}	2.205×10^{-3}	1	0.001
1 kilogram-force (kgf)	9.807	2.205	1000	1

Note: Strictly, all the units in the table except the Newton (N) represent weight equivalents of mass, and so depend on g. The true SI unit of force is the Newton (N) which is equivalent to 1 kg m/s^2.

2.3.2 Weight

The true weight of a body is a measure of the gravitational attraction of the earth on it. Since this attraction is a force, the weight of a body is correctly expressed in Newtons (N).

Mass is measured in kilogram (kg)
Force (N) = mass (kg) × g (m/s^2)

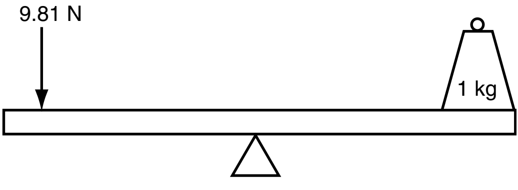


Figure 2.1

1 kg = 2.20462 lbf
1000 kg = 1 tonne (metric) = 0.9842 tons (imperial)
1 ton (US) = 2000 lb = 907.185 kg

Table 2.4 Density (ρ)

Unit	*kg/m³*	*g/cm³*	*lb/ft³*	*lb/in³*
1 kg per m^3	1	0.001	6.243×10^{-2}	3.613×10^{-5}
1g per cm^3	1000	1	62.43	3613×10^{-2}
1 lb per ft^3	16.02	1.602×10^{-2}	1	5.787×10^{-4}
1 lb per in^3	2.768×10^4	27.68	1728	1

2.3.3 Pressure

The SI unit is the Pascal (Pa).

$$1\ \text{Pa} = 1\ \text{N/m}^2$$
$$1\ \text{Pa} = 1.45038 \times 10^{-4}\ \text{lbf/in}^2\ \text{(i.e. psi)}$$

In practice, pressures are measured in MPa, bar, atmospheres, torr or the height of a liquid column, depending on the application.

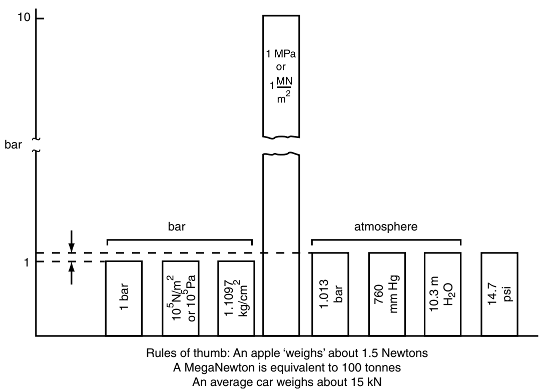


Figure 2.2

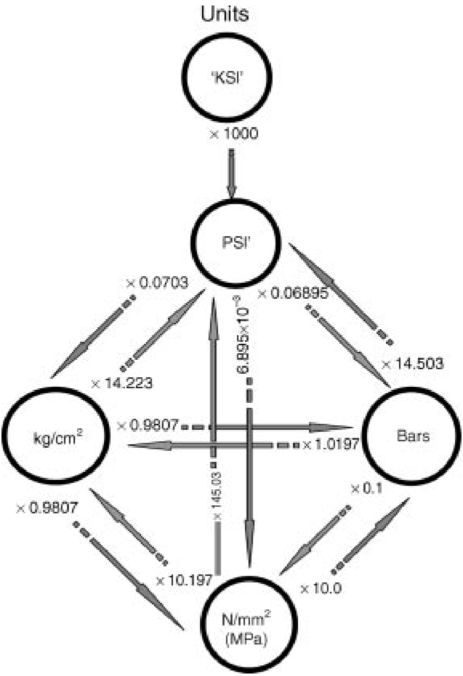


Figure 2.3

Table 2.5 Pressure (*P*)

Unit	Atm	in H_2O	cm Hg	N/m^2(Pa)	lb/in^2(psi)[a]	lb/ft^2
1 atmosphere (atm)	1	406.8	76	1.013×10^5	14.70	2116
1 in of water at 4°C	2.458×10^{-3}	1	0.1868	249.1	3.613×10^{-2}	5.02
1 cm of mercury at 0°C	1.316×10^{-2}	5.353	1	1333	0.1934	27.85
1 N per m^2	9.869×10^{-6}	4.015×10^{-3}	7.501×10^{-4}	1	1.450×10^{-4}	2.089×10^{-2}
1 lb per in^2 (psi)	6.805×10^{-2}	27.68	5.171	6.895×10^3	1	144
1 lb per ft^2	4.725×10^{-4}	0.1922	3.591×10^{-2}	47.88	6.944×10^{-3}	1

[a] Where $g = 9.80665\ m/s^2$.

[b] Note that the United States unit ksi ('kip' per square inch) may be used. 1 ksi = 1000 psi, not 1 kg/square inch.

And for liquid columns:

1 mm Hg = 13.59 mm H_2O = 133.3224 Pa = 1.333224 mbar
1 mm H_2O = 9.80665 Pa
1 torr = 133.3224 Pa

For conversion of liquid column pressures; 1 in = 25.4 mm.

2.3.4 Temperature

The SI unit is degrees Kelvin (K). The most commonly used unit is degrees Celsius (°C).

Absolute zero is defined as 0 K or −273.15 °C, the point at which a perfect gas has zero volume.

The imperial unit of temperature is degrees Fahrenheit (°F).

$$°C = {}^5/_9\ (°F - 32)$$
$$°F = {}^9/_5\ (°C) + 32$$

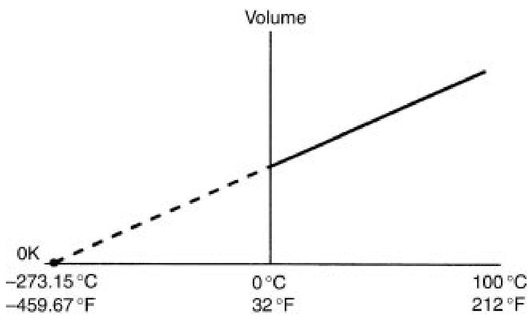


Figure 2.4

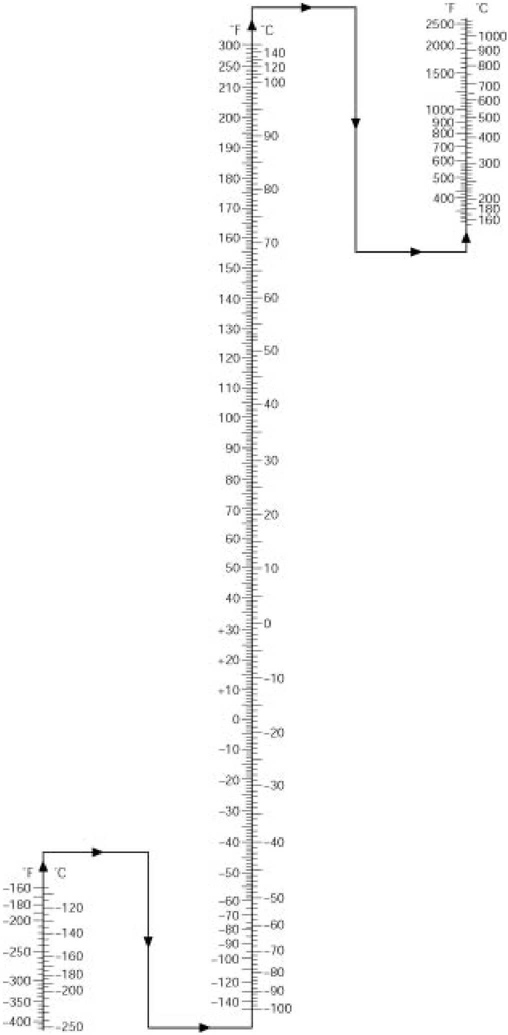


Figure 2.5

2.3.5 Heat energy

The SI unit for heat energy (in fact all forms of energy) is the Joule (J).

Table 2.6 Heat energy

Unit	*J*	*Btu*	*ft lb*	*hph*	*Cal*	*kWh*
1 Joule (J)	1	9.481×10^{-4}	0.7376	3.725×10^{-7}	0.2389	2.778×10^{-7}
1 British thermal unit (Btu)	1055	1	777.9	3.929×10^{-4}	252	2.93×10^{-4}
1 foot-pound (ft lb)	1.356	1.285×10^{-3}	1	5.051×10^{-7}	0.3239	3.766×10^{-7}
1 horsepower-hour (hph)	2.685×10^{6}	2545	1.98×10^{6}	1	6.414×10^{5}	0 7457
1 calorie (cal)	4.187	3.968×10^{-3}	3.087	1.559×10^{-6}	1	1.163×10^{-6}
1 kilowatt hour (kWh)	3.6×10^{6}	3413	2.655×10^{6}	1.341	8.601×10^{5}	1

Specific energy is measured in Joules per kilogram (J/kg).

$$1\ \text{J/kg} = 0.429923 \times 10^{-3}\ \text{Btu/lb}$$

Specific heat capacity is measured in Joules per kilogram Kelvin (J/kg K).

$$1\ \text{J/kg K} = 0.238846 \times 10^{-3}\ \text{Btu/lb °F}$$

$$1\ \text{kcal/kg K} = 4186.8\ \text{J/kg K}$$

Heat flowrate is also defined as power, with the SI unit of Watts (W).

$$1\ \text{W} = 3.41214\ \text{Btu/h} = 0.238846\ \text{cal/s}$$

2.3.6 Power

The Watt is a small quantity of power, so kW is normally used.

Table 2.7 Power (*P*)

Unit	*Btu/h*	*Btu/s*	*ft-lb/s*	*hp*	*cal/s*	*kW*	*W*
1 Btu/h	1	2.778×10^{-4}	0.2161	3.929×10^{-4}	7.000×10^{-2}	2.930×10^{-4}	0.2930
1 Btu/s	3600	1	777.9	1.414	252.0	1.055	1.055×10^{-3}
1 ft-lb/s	4.628	1.286×10^{-3}	1	1.818×10^{-3}	0.3239	1.356×10^{-3}	1.356
1 hp	2545	0.7069	550	1	178.2	0.7457	745.7
1 cal/s	14.29	0.3950	3.087	5.613×10^{-3}	1	4.186×10^{-3}	4.186
1 kW	3413	0.9481	737.6	1.341	238.9	1	1000
1 W	3.413	9.481×10^{-4}	0.7376	1.341×10^{-3}	0.2389	0.001	1

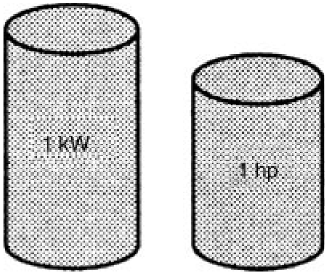


Figure 2.6

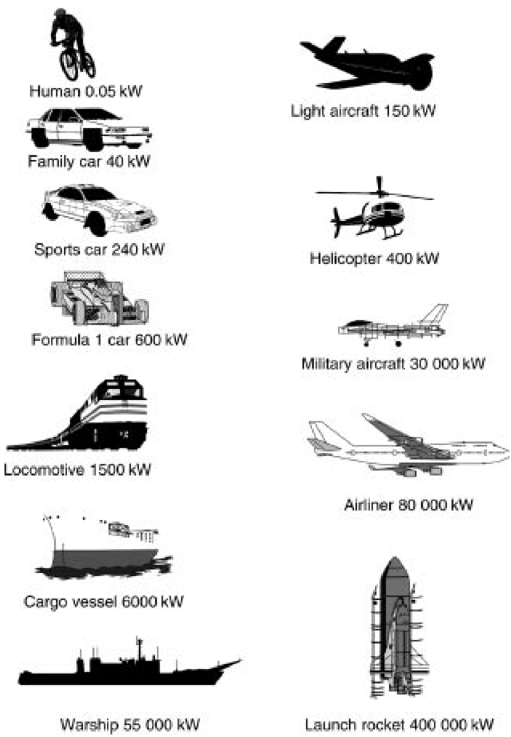


Figure 2.7 Comparative power outputs

2.3.7 Flow

The SI unit of volume flowrate is m^3/s.

$1\ m^3/s = 219.969$ UK gall/s $= 1000$ litres/s
$1\ m^3/h = 2.77778 \times 10^{-4}\ m^3/s$
1 UK gall/min $= 7.57682 \times 10^{-5}\ m^3/s$
1 UK gall = 4.546 litres

Figure 2.8

The SI unit of mass flowrate is kg/s.

1 kg/s = 2.20462 lb/s = 3.54314 ton (imp)/h
1 US gall = 3.785 litres

2.3.8 Torque

The SI unit of torque is the Newton metre (N.m). You may also see this referred to as 'moment of force'.

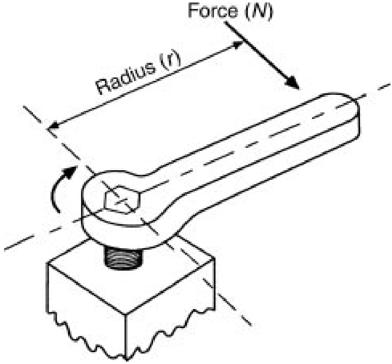


Figure 2.9

1 N.m = 0.737 lbf ft (i.e. 'foot pounds')
1 kgfm = 9.81 N.m

2.3.9 Stress

Stress is measured in Pascals – the same SI unit used for pressure, although it is a different physical quantity. 1 Pa is an impractical small unit so MPa is normally used.

$1\ \text{MPa} = 1\ \text{MN/m}^2 = 1\ \text{N/mm}^2$
$1\ \text{kgf/mm}^2 = 9.80665\ \text{MPa}$

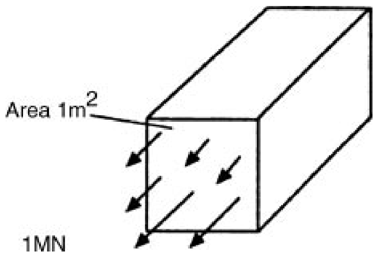


Figure 2.10

2.3.10 Linear velocity (speed)

The SI unit is metres per second (m/s).

Table 2.8 Velocity (*v*)

Unit	*ft/s*	*km/h*	*m/s*	*mile/h*	*cm/s*
1 ft per s	1	1.097	0.3048	0.6818	30.48
1 km per h	0.9113	1	0.2778	0.6214	27.78
1 m per s	3.281	3.600	1	2.237	100
1 mile per h	1.467	1.609	0.4470	1	44.70
1 cm per s	3.281×10^{-2}	3.600×10^{-2}	0.0100	2.237×10^{-2}	1

2.3.11 Acceleration

The SI unit of acceleration is metres per second squared (m/s^2).

$1\ \text{m/s}^2 = 3.28084\ \text{ft/s}^2$

Standard gravity (g) is normally taken as $9.81\ \text{m/s}^2$.

2.3.12 Angular velocity

The SI unit is radians per second (rad/s).

$1\ \text{rad/s} = 0.159155\ \text{rev/s} = 57.2958\ \text{degree/s}$

The radian is the SI unit used for plane angles.

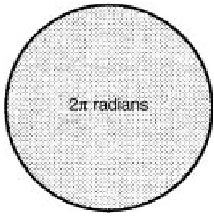


Figure 2.11

A complete circle is 2π radians
A quarter-circle (90°)is $\pi/2$ or 1.57 radians
1 degree $= \pi/180$ radians

2.3.13 Volume and capacity

The SI unit is cubic metres (m^3), but many imperial units are still in use.

$1\ m^3 = 35.3147\ ft^3 = 61\ 023.7\ in^3$

2.3.14 Area

The SI unit is square metres (m^2) but many imperial units are still in use.

Table 2.9 Area (*A*)

Unit	*sq in*	*Sq ft*	*Sq yd*	*sq mile*	*cm^2*	*dm^2*	*m^2*	*A*	*ha*	*km^2*
1 square inch	1	–	–	–	6.452	0.06452	–	–	–	–
1 square foot	144	1	0.1111	–	929	9.29	0.0929	–	–	–
1 square yard	1296	9	1	–	8361	83.61	0.8361	–	–	–
1 square mile	–	–	–	1	–	–	–	–	259	2.59
1cm^2	0.155	–	–	–	1	0.01	–	–	–	–
1dm^2	15.5	0.1076	0.01196	–	100	1	0.01	–	–	–
1m^2	1550	10.76	1.196	–	10000	100	1	0.01	–	–
1a	–	1076	119.6	–	–	10000	100	1	0.01	–
1ha	–	–	–	–	–	–	10000	100	1	0.01
1km^2	–	–	–	0.3861	–	–	–	10000	100	1

Other metric units of area:

Japan:	1 tsubo	$= 3.306\,m^2$
	1 se	$= 0.9917\,a$
	1 ho-ri	$= 15.42\,km^2$
Russia:	1 kwadr. archin	$= 0.5058\,m^2$
	1 kwadr. saschen	$= 4.5522\,m^2$
	1 dessjatine	$= 1.0925\,ha$
	1 kwadr. werst	$= 1.138\,km^2$

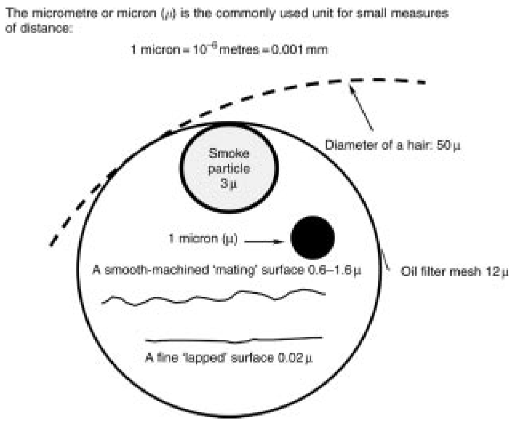


Figure 2.12 Making sense of microns (μ)

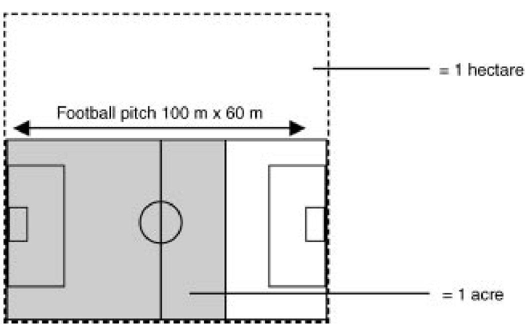


Figure 2.13

2.3.15 Viscosity

Dynamic viscosity (μ) is measured in the SI system in Pascal seconds (Pa s).

$$1 \text{ Pa s} = 1 \text{ N s/m}^2 = 1 \text{ kg/m s}$$

A common unit from another units system is the centipoise (cP), or standard imperial units may be used:

Table 2.10 Dynamic viscosity (μ)

Unit	*Centipoise*	*poise*	*kgf-s/m²*	*lb-s/ft²*	*kg/m-s*	*lbm/ft-s*
1 centipoise	1	10^{-2}	1.020×10^{-4}	2.089×10^{-5}	10^{-3}	6.720×10^{-4}
1 poise	100	1	1.020×10^{-2}	2.089×10^{-3}	0.100	6.720×10^{-2}
1 N-s per m^2	9.807×10^3	98.07	1	0.2048	9.807	6.590
1 lb (force)-s per ft^2	4.788×10^4	4.788×10^2	4.882	1	47.88	32 174
1 kg per m-s	10^3	10	0.1020	2.089×10^{-2}	1	0.6720
1 lb (mass) per ft-s	1.488×10^3	14.88	0.1518	3.108×10^{-2}	1.488	1

Kinematic viscosity *(ν)* is a function of dynamic viscosity.

Kinematic viscosity = dynamic viscosity/density, i.e. $\nu = \mu/\rho$ The SI unit is m^2/s. Other imperial and CGS units are also used.

$$1 \text{ m}^2/\text{s} = 10.7639 \text{ ft}^2/\text{s} = 5.58001 \times 10^6 \text{ in}^2/\text{h}$$
$$1 \text{ Stoke (St)} = 100 \text{ centistokes (cSt)} = 10^{-4} \text{ m}^2/\text{s}$$

2.4 Consistency of units

Within any system of units, the consistency of units forms a 'quick check' of the validity of equations. The units must match on both sides.

Example:

To check kinematic viscosity $(\nu) = \dfrac{\text{dynamic viscosity}(\mu)}{\text{density }(\rho)}$

$$\frac{\text{m}^2}{\text{s}} = \frac{\text{Ns}}{\text{m}^2} \times \frac{\text{m}^3}{\text{kg}}$$

Replacing N with kgm/s^2

$$\frac{\text{m}^2}{\text{s}} = \frac{\text{kgm s}}{\text{s}^2\text{m}^2} \times \frac{\text{m}^3}{\text{kg}}$$

Cancelling gives

$$\frac{\mathrm{m}^2}{\mathrm{s}} = \frac{\mathrm{m}^4\mathrm{s}}{\mathrm{s}^2\mathrm{m}^2} = \frac{\mathrm{m}^2}{\mathrm{s}}$$

OK, units match.

2.4.1 Foolproof conversions: using unity brackets

When converting between units it is easy to make mistakes by dividing by a conversion factor instead of multiplying, or vice versa. The best way to avoid this is by using the technique of unity brackets.

A unity bracket is a term consisting of a numerator and denominator in different units which has a value of unity.

$$e.g. \left[\frac{2.205\ \mathrm{lbs}}{\mathrm{kg}}\right] \text{ or } \left[\frac{\mathrm{kg}}{2.205\ \mathrm{lbs}}\right] \text{ are unity brackets}$$

$$\text{as are } \left[\frac{25.4\ \mathrm{mm}}{\mathrm{in}}\right] \text{ or } \left[\frac{\mathrm{in}}{25.4\ \mathrm{mm}}\right] \text{ or } \left[\frac{\mathrm{Atmosphere}}{10\,1\,325\ \mathrm{Pa}}\right]$$

Remember that as the value of the bracket is unity it has no effect on any term that multiplies.

Example: Convert the density of steel $\rho = 0.29\ \mathrm{lb/in}^3$ to $\mathrm{kg/m}^3$

Step 1: State the initial value: $\rho = \dfrac{0.29\ \mathrm{lb}}{\mathrm{in}^3}$

Step 2: Apply the 'weight' unity bracket:

$$\rho = \frac{0.29\ \mathrm{lb}}{\mathrm{in}^3}\left[\frac{\mathrm{kg}}{2.205\ \mathrm{lb}}\right]$$

Step 3: Then apply the 'dimension' unity brackets (cubed):

$$\rho = \frac{0.29\ \mathrm{lb}}{\mathrm{in}^3}\left[\frac{\mathrm{kg}}{2.205\ \mathrm{lb}}\right]\left[\frac{\mathrm{in}}{25.4\ \mathrm{mm}}\right]^3\left[\frac{1000\ \mathrm{mm}}{\mathrm{m}}\right]^3$$

Step 4: Expand* and cancel:

$$\rho = \frac{0.29\,\text{lb}}{\text{in}^3}\left[\frac{\text{kg}}{2.205\,\text{lb}}\right]\left[\frac{\text{in}^3}{(25.4)^3\text{mm}^3}\right]\left[\frac{(100)^3\text{mm}^3}{\text{m}^3}\right]$$

$$\rho = \frac{0.29\,\text{kg}(1000)^3}{2.205(25.4)^3\text{m}^3}$$

$$\rho = 8025.8\,\text{kg/m}^3 : \text{ Answer}$$

* Take care to use the correct algebraic rules for the expansion. For example:

$$(a.b)^N = a^N.b^N \textit{ not } a.b^N$$

So, for example, $\left(\frac{1000\,\text{mm}}{\text{m}}\right)^3$ expands to $\frac{(1000)^3.(\text{mm})^3}{(\text{m})^3}$

Unity brackets can be used for all unit conversions provided you follow the rules for algebra correctly.

2.4.2 Imperial-metric conversions

Table 2.11 Imperial–metric conversions

Fraction (in)	*Decimal (in)*	*Millimetre (mm)*
1/64	0.01562	0.39687
1/32	0.03125	0.79375
3/64	0.04687	1.19062
1/16	0.06250	1.58750
5/64	0.07812	1.98437
3/32	0.09375	2.38125
7/64	0.10937	2.77812
1/8	0.12500	3.17500
9/64	0.14062	3.57187
5/32	0.15625	3.96875
11/64	0.17187	4.36562
3/16	0.18750	4.76250
13/64	0.20312	5.15937
7/32	0.21875	5.55625
15/64	0.23437	5.95312
1/4	0.25000	6.35000
17/64	0.26562	6.74687

Table 2.11 (Cont.)

Fraction (in)	*Decimal (in)*	*Millimetre (mm)*
9/32	0.28125	7.14375
19/64	0.29687	5.54062
15/16	0.31250	7.93750
21/64	0.32812	8.33437
11/32	0.34375	8.73125
23/64	0.35937	9.12812
3/8	0.37500	9.52500
25/64	0.39062	9.92187
13/32	0.40625	10.31875
27/64	0.42187	10.71562
7/16	0.43750	11.11250
29/64	0.45312	11.50937
15/32	0.46875	11.90625
31/64	0.48437	12.30312
1/2	0.50000	12.70000
33/64	0.51562	13.09687
17/32	0.53125	13.49375
35/64	0.54687	13.89062
9/16	0.56250	14.28750
37/64	0.57812	14.68437
19/32	0.59375	15.08125
39/64	0.60937	15.47812
5/8	0.62500	15.87500
41/64	0.64062	16.27187
21/32	0.65625	16.66875
43/64	0.67187	17.06562
11/16	0.68750	17.46250
45/64	0.70312	17.85937
23/32	0.71875	18.25625
47/64	0.73437	18.65312
3/4	0.75000	19.05000
49/64	0.76562	19.44687
25/32	0.78125	19.84375
51/64	0.79687	20.24062
13/16	0.81250	20.63750
53/64	0.82812	21.03437
27/32	0.84375	21.43125
55/64	0.85937	21.82812
7/8	0.87500	22.22500
57/64	0.89062	22.62187
29/32	0.90625	23.01875
59/64	0.92187	23.41562
15/16	0.93750	23.81250
61/64	0.95312	24.20937
31/12	0.96875	24.60625
63/64	0.98437	25.00312
1	1.00000	25.40000

2.5 Dimensional analysis

2.5.1 Dimensional analysis (DA) – what is it?

DA is a technique based on the idea that one physical quantity is related to others in a precise mathematical way.

2.5.2 What is it used for?

It is used for:

- Checking the validity of equations;
- Finding the arrangement of variables in a formula;
- Helping to tackle problems that do not possess a complete theoretical solution – particularly those involving fluid mechanics.

2.5.3 Primary and secondary quantities

These are quantities which are absolutely independent of each other. They are:

M Mass
L Length
T Time

For example: Velocity (v) is represented by length divided by time, and this is shown by:

$$[v] = \frac{L}{T}$$

Note the square brackets denoting the 'dimension of'.

Table 2.12 Dimensional analysis – quantities

Quantity	*Dimensions*
Mass (m)	M
Length (l)	L
Time (t)	T
Area (A)	L^2
Volume (V)	L^3
First moment of area (I)	L^3
Second moment of area (I)	L^4
Velocity (v)	LT^{-1}
Acceleration (a)	LT^{-2}

Table 2.12 (Cont.)

Quantity	*Dimensions*
Angular velocity (v)	T^{-1}
Angular acceleration (α)	T^{-2}
Frequency (f)	T^{-1}
Force (F)	MLT^{-2}
Stress (Pressure) (σ, p)	$ML^{-1}T^{-2}$
Torque (T)	ML^2T^{-2}
Modulus of elasticity (E)	$ML^{-1}T^{-2}$
Work (W)	ML^2T^{-2}
Power (P)	ML^2T^{-3}
Density (ρ)	ML^{-3}
Dynamic viscosity (μ)	$ML^{-1}T^{-1}$
Kinematic viscosity (ν)	L^2T^{-1}

Hence velocity is called a secondary quantity because it can be expressed in terms of primary quantities.

2.5.4 An example of deriving formulae using DA

To find the formulae for periodic time (t) of a simple pendulum we can assume that t is related in some way to m, l, and g, i.e.

$$t = \Phi\{m, l, g\}$$

Introducing a numerical constant C and some possible exponentials gives:

$$t = Cm^a l^b g^d$$

C is a dimensionless constant so, in dimensional analysis terms this equation becomes

$$[t] = [m^a l^b g^d]$$

Substitute primary dimensions gives:

$$\begin{aligned} T &= M^a L^b (LT^{-2})^d \\ &= M^a L^{b+d} T^{-2d} \end{aligned}$$

In order for the equation to balance

For M, a must = 0
For L, b + d = 0
For T, −2d = 1
Giving $b = {}^1/_2$ and d $= -{}^1/_2$

So we know the formula is now written:

$$t = \mathrm{C}l^{1/2}g^{-1/2}$$
$$\text{or} \quad t = \mathrm{C}\sqrt{\frac{l}{g}} : \text{the answer}$$

Note how dimensional analysis can give you the 'form' of the formula but not the numerical value of the constant C.

Note also how the technique has shown us that the mass (m) of the pendulum bob does not affect the periodic time (t) (i.e. because a = 0).

2.6 Essential engineering mathematics

2.6.1 *Powers and roots*

$$a^n.a^m = a^{n+m} \quad \frac{a^n}{a^m} = a^{n-m} \quad ab^n = a^n b^n \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$(a^n)^m = (d^m)^n = a^{nm} \quad (\sqrt[n]{a})^n = a \quad a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{n/m} = \sqrt[m]{a^n} \quad n\sqrt{ab} = n\sqrt{a}.n\sqrt{b}$$

2.6.2 *Logarithms*

$$\log_a a = 1 \quad \log_a 1 = 0 \; (\log_a M)N = \log_a M + \log_a N$$

$$(\log_b N) = \frac{\log_a N}{\log_a b} \quad \log_b b^N = N \quad b^{\log_b N} = N$$

2.6.3 *The quadratic equation*

A quadratic equation is one in the form $\mathrm{a}x^2 + \mathrm{b}x + \mathrm{c} = 0$ Where a, b, and c are constants.

$$\text{The solution is : } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2.6.4 Trigonometric functions

$\sin\alpha = \frac{y}{r}$

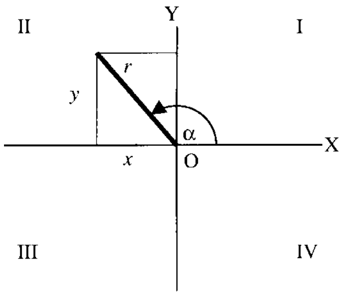
$\cos\alpha = \frac{x}{r}$

$\tan\alpha = \frac{y}{x}$

$\cot\alpha = \frac{x}{y}$

$\sec\alpha = \frac{r}{x}$

$\operatorname{cosec}\alpha = \frac{r}{y}$



The signs of these functions depend on which quadrant they are in:

Quadrant	*Sin*	*Cos*	*Tan*	*Cot*	*Sec*	*Cosec*
I	+	+	+	+	+	+
II	+	−	−	−	−	+
III	−	−	+	+	−	−
IV	−	+	−	−	+	−

2.6.5 Trig functions of common angles

	0°	*30°*	*45°*	*60°*	*90°*
Sin	0	$\frac{1}{2}$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
Cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$\frac{1}{2}$	0
Tan	0	$\sqrt{3}/3$	1	$\sqrt{3}$	∞
Cot	∞	$\sqrt{3}$	1	$\sqrt{3}/3$	0
Sec	1	$2\sqrt{3}/3$	$\sqrt{2}$	2	∞
Cosec	∞	2	$\sqrt{2}$	$2\sqrt{3}/3$	1

$$\sin\alpha = \frac{1}{\mathrm{cosec}\alpha} \quad \cos\alpha = \frac{1}{\sec\alpha} \quad \tan\alpha = \frac{1}{\cot\alpha} = \frac{\sin\alpha}{\cos\alpha}$$

$$\sin^2\alpha + \cos^2\alpha = 1 \quad \sec^2\alpha - \tan^2\alpha = 1$$
$$\mathrm{cosec}^2\alpha - \cot^2\alpha = 1$$

2.6.6 Differential calculus

Derivatives	*Integrals*
$\frac{\mathrm{d}}{\mathrm{d}x}(u \pm v \pm \ldots) = \frac{\mathrm{d}u}{\mathrm{d}x} \pm \frac{\mathrm{d}v}{\mathrm{d}x} \pm \ldots$	$\int \mathrm{d}f(x) = f(x) + C$
$\frac{\mathrm{d}}{\mathrm{d}x}(uv) = \frac{u\mathrm{d}v}{\mathrm{d}x} + \frac{v\mathrm{d}u}{\mathrm{d}x}$	$\mathrm{d}\int f(x)\mathrm{d}x = f(x)\mathrm{d}x$
$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{u}{v}\right) = \frac{1}{v}\frac{\mathrm{d}u}{\mathrm{d}x} - \frac{u}{v^2}\frac{\mathrm{d}v}{\mathrm{d}x}$	$\int af(x)\mathrm{d}x = a\int f(x)\mathrm{d}x$ (a = constant)
$\frac{\mathrm{d}}{\mathrm{d}x}(u^n) = nu^{n-1}\frac{\mathrm{d}u}{\mathrm{d}x}$	$\int u\mathrm{d}v = uv - \int v\mathrm{d}u$
$\frac{\mathrm{d}}{\mathrm{d}x}(\ln u) = \frac{1}{u}\frac{\mathrm{d}u}{\mathrm{d}x}$	$\int u^n\mathrm{d}u = \frac{u^{n+1}}{n+1} + C \quad (n \neq -1)$
$\frac{\mathrm{d}}{\mathrm{d}x}(\tan u) = \sec^2 u\frac{\mathrm{d}u}{\mathrm{d}x}$	$\int \frac{\mathrm{d}u}{u} = \ln u + C$
$\frac{\mathrm{d}}{\mathrm{d}x}(\sin u) = \cos u\frac{\mathrm{d}u}{\mathrm{d}x}$	$\int e^u\mathrm{d}u = e^u + C$
$\frac{\mathrm{d}}{\mathrm{d}x}(\cos u) = -\sin u\frac{\mathrm{d}u}{\mathrm{d}x}$	$\int \sin\,\mathrm{d}u = -\cos u + C$
	$\int \cos u\,\mathrm{d}u = \sin u + C$
	$\int \tan u\,\mathrm{d}u = -\ln\cos u + C$

2.7 Maths and the real world?

2.7.1 What's it all about?

'Please sir, what use is this?' Fair question. Most people who are forced to use maths have little idea what it is really about. This also applies to people who are quite good at it and to many who teach it, or do little else. To them all, it is seen as an obscure and rather tiresome series of symbols and enforced equations surrounded by a bewildering number of different ways to put various numbers in to obtain (sometimes) the answer that you are supposed to get.

Good news. The reason you find maths awkward is simply because it is *abstract*. There's no reason to be surprised at this – lots of things are – language, for example, is abstract, and you use it all the time. Think about this explanation.

- Maths is an abstract depiction of *nature*.

Thinking of it as a depiction of nature is the first essential step. Think of the other way of doing it – Art, which is also a depiction of nature, and you might find it easier. These two systems are all there are, it's just that most people have little problem with accepting that a painting of a tree represents a tree, but find it more difficult to conceive that a jumble of numbers, symbols and equations can equally represent what a tree is, and does. This is the difficulty with maths – your mind is better tuned to looking at pictures and images and things rather than equations, because it is easier.

Why does maths depict nature?
Because nature is a 100% rule-based game. Everything that exists, and happens, does so because it has passed the test of compliance with an unbreakable set of rules, as they stand. Anything that doesn't comply can't exist, or happen, so simply looking around you provides first-hand evidence that enough things comply with these rules to result in all the things you can see, hear and feel.

How many of these rules are there?
Millions of millions without a doubt. Some are simple and others are almost infinitely complex. Think of them as the rules of a complex game, like rugby or cricket. The simpler rules are quite adequate at deciding that when a cricket ball is caught before it hits the ground the batsman is out. More complex rules conclude what should happen if the umpire's hat falls off and knocks over the stumps assisted, or not, by a disoriented pigeon. The game functions under these rules, hence proving their existence and effectiveness.

The rules of nature are a little more complex. They have to be good for billions of items and trillions of actions. They have to cover genetics, mechanics, acoustics, optics, aesthetics (that's an interesting one) and all the others, acting as an immutable, always-correct lowest common denominator of the world as it is.

How do we know that these rules involve maths?

Because the simplest ones, that we can observe directly, seem to work. Two boxes, each containing three apples spookily always results in a total of $2 \times 3 = 6$ apples. This is the simple test that proves the relationship – count the apples and you get 6, then multiple 2×3 and you get 6. See? It works. The rules are not all that simple of course, but the matching continues:

- Atoms and molecules arrange themselves, without help or persuasion, into patterns that can be described by fairly simple formulae. This is encouraging – success at this lowest scale probably means that bigger and more complex things will be the same.

Throw a ball in the air and, before very long, it will stop and come down. By applying our previous logic chain, it happens, so the rules must allow it. Maths is an abstract depiction of the rules, so there *must* be a mathematical way to describe what is happening. All you have to do is find it. Once you have found it think of the advantages: it will tell you what will happen if the ball is twice as heavy, thrown twice as high, or at an angle, so there will be no need to waste time trying it out in practice.

Now it's time for the big step. Once you accept that all the rules of everything can be depicted by maths you are ready to use it to find out things that are impossible to find out in any other way. You can predict what will happen in things too small to see, or in places to which you have no access (the planets, the sun and other suns). You have the tools to manage invisible things such as electromagnetic waves (radio, ultraviolet, infrared and the rest). Once you know the rules they follow, they are under your control (oh, don't forget, the rules are governing you, as well).

Where is this book containing all the rules?

There isn't one. The discovery of the rules is ongoing – it's doubtful if we've discovered 0.00001% of them, but its a standing target, because they are all there, static and unchanging, waiting to be discovered.

Ok, how do I use the rules?

Maths is used for everything that involves any of the 'big four' parameters shown in Fig. 2.14.

- Quantity
- Structures
- Space
- Change

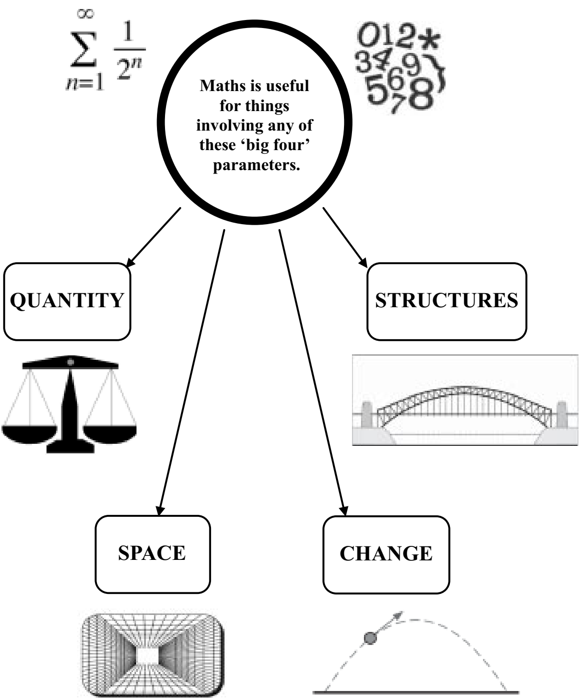


Figure 2.14 The main uses of maths

It won't give you a guaranteed answer to everything, because it is an abstract depiction of reality rather than a perfect one. It is perfect at its rule-based core but it is our use of it that brings the imperfection, because we didn't write the rules. Pure maths is more important than applied maths because there couldn't be the second without the first. Applied maths is used in engineering throughout the world as an essential tool in the design of things and processes, and therefore is practically more useful. This is why maths is in your degree syllabus.

2.7.2 Why bother with calculus?

Let's be honest, 99.999% people in the world don't understand the first thing about calculus – nor do they need to. Moving on to the mechanical engineering world where the percentage is a bit higher (but not much) you soon find that using it (mainly to pass exams) is an altogether different thing from actually understanding what it is all about.

In an engineering career, perhaps the only reason for getting involved with calculus is that you have to. Fortunately, most engineers won't need to. It is only in the higher echelons of engineering technology (fundamental design, for example) that you will need it. In this top 1% of the industry it becomes an issue – leaving the remaining 99% (which of course is most of it) to continue without it, once the necessary exams have been passed and forgotten.

Does all maths involve calculus?

Certainly not – most of it doesn't. It is difficult to put an accurate value on it but perhaps 70–80% of maths does not need calculus in any major way. Trigonometry for example, being mainly concerned with lengths, distances and angles between objects, doesn't require much calculus in the mainstream 80% of the subject.

Contrast this with the fact that virtually all engineering-based exams feature calculus heavily. Students are left with little choice but to study it in some considerable depth. Derivatives and integrals are memorized and regurgitated, elaborate solutions rehearsed for line-by-line presentation and the (hopefully correct) answers presented, with a flourish, at the end.

So what *is* calculus all about?

Easy, you need calculus to understand things that *change*. One of the most common types of change is physical movement – the distance from a moving object to some static point changes with time, so there is a change.

As you would expect, simple movement patterns (in a straight line or circle for example) are not too difficult to understand but it gets more difficult when you try to describe the curved path taken when you throw a ball, or the path of the earth round the sun, or the movement of atoms in a metal, as you heat it up.

Things don't have to *physically* move for the concept of 'movement' to be involved. Look at the shape in Fig. 2.15. As it is a 3-dimensional shape, it is not easy to calculate the volume of such a strange shape. The answer lies in thinking of it as a simple 2-dimensional flat shape rotated about the axis *y–y*. This will produce the 3-D volume. Note that, physically, nothing has actually been rotated (there has been no movement that you could watch), the movement has taken place conceptually.

Here's another type of movement that is non-real. How would you solve this problem?

$$3+2 \times 6 = ?$$

You would do it, knowingly or unknowingly, in steps. From the 'BODMAS' rules of maths (i.e. Brackets – Other operations – Division – Multiplication – Addition – Subtraction) you would do it in the following steps:

Step 1: Rewrite it as $3 + (2 \times 6) = ?$
Step 2: Calculate the terms in brackets $3 + 12 = ?$
Step 3 Do the sum: $3 + 12 = 15$ [*answer*]

See how you moved through the steps? – that's the *movement*. Nothing physical, but movement just the same.

Suspend belief for a moment and imagine that moving forward through steps 1 to 3 is called something special but meaningless (*differentiation*) – that's just the name we have given to the idea of moving forward. If you always want to go only from step 1 to step 3, that's all you need. Now turn things around – suppose you already had the answer of 15 but were

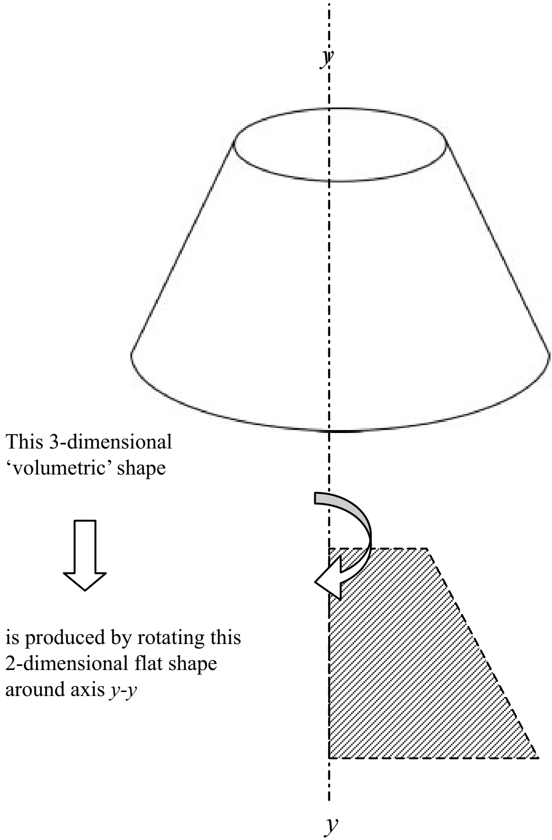


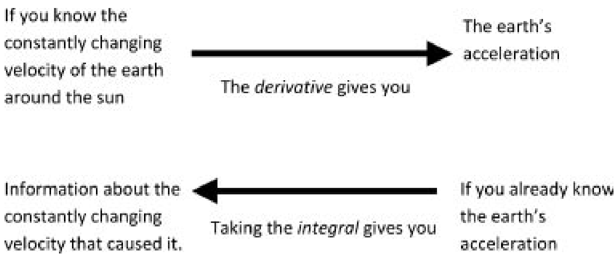
Figure 2.15 Volumes of revolution

curious as to where it came from. Your interest may be driven by the urge to have this answer again (if you liked it) or to stop it appearing again, if it causes you problems. You know that it is linked to other numbers but need to know which ones, and how. To do this you would need to move backwards. Once again, we'll give this a random name – call it *integration* – the opposite of differentiation, the name we gave to moving forward.

In the final act – combining the two concepts of dealing with rates of change with the two main and opposite options of moving forward or backwards – we have it. This is calculus. Catch it before it slips away:

- You have a constantly changing function and want to find out about its rate of change. This is the *derivative*.
  OR
- You already know the rate of change of a system and want to find the given values that describe the system's input. You get this by working backwards – called *integration*.

Or, perhaps examples will explain it better:



Very clever – how does it do that?

Put formally, it relies on the fact that you can get the answer to something by using a set of approximations of increasing accuracy. Sounds convincing, but how does this translate into practice? The whole thing is based on the concept that anything can be broken into infinitesimally small pieces and that these pieces, as they are small, must be simple. Although this is the basis of calculus, it disappears rapidly from sight as soon as you start actually doing it. It is easy to forget about it, but it lives behind the scenes, controlling everything.

How small is infinitesimal?

Almost infinitely small. On a scale of 93 million miles between the earth and the sun, one of the infinitesimal distances could be the thickness of a hair. It is this smallness that makes the whole thing work. On the scale of infinitesimal smallness, irregular shapes (that are difficult to deal with) can be approximated by regular ones which *can* be dealt with. By adding all the small regular shapes together to make the larger shape, the large shape suddenly becomes manageable. Figure 2.16 shows the idea.

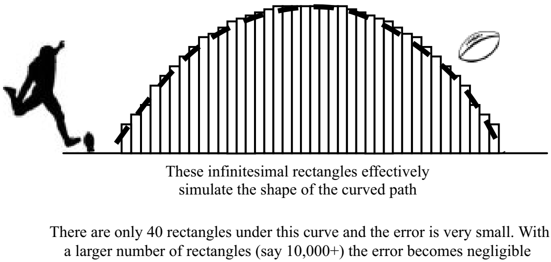


Figure 2.16 A curve described by multiple rectangles

If you think of the large irregular shape as being caused by something in the real world that continually changes direction with time, such as the path of a cannonball fired off a cliff at a passing ship, then you can see the advantages.

Being human, we like to have things expressed as real numbers before we can deal with them. The thing that does this is the concept of *limits*. Limits capture the small-scale behaviour of infinitesimal points of a curve, for example, whilst translating it into real numbers. Think of this when you see the limits on an integration (at the top and bottom of the squiggly line) – that is what the integral sign is doing. You don't see this change from 'real-world things' to 'real-world numbers' when you are finding a derivative, but it is there, just the same, hiding behind the symbols.

Reminder – why do we need calculus?

We need calculus to deal with anything that involves a system that is in a state of constant change. Without calculus it is impossible to predict how that system has previously changed to reach its present state or the state it will adopt in the future.