لجنة الميكانيك
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# [المكتبة التخصصية]

لجنة الميكانيك
Polytechnic


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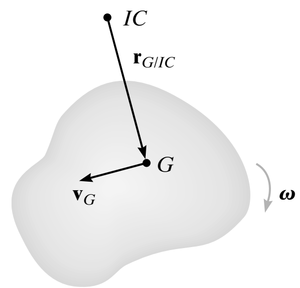
0789434018 MechFet

**•18–1.** At a given instant the body of mass $m$ has an angular velocity $\boldsymbol{\omega}$ and its mass center has a velocity $\mathbf{v}_G$. Show that its kinetic energy can be represented as $T = \frac{1}{2}I_{IC}\omega^2$, where $I_{IC}$ is the moment of inertia of the body computed about the instantaneous axis of zero velocity, located a distance $r_{G/IC}$ from the mass center as shown.




$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\,\omega^2 \qquad \text{where } v_G = \omega r_{G/IC}$$

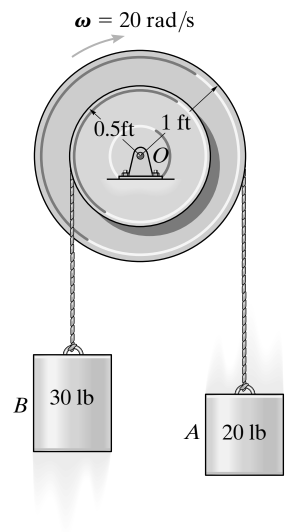
$$= \frac{1}{2}m(\omega r_{G/IC})^2 + \frac{1}{2}I_G\,\omega^2$$

$$= \frac{1}{2}\left(mr_{G/IC}^2 + I_G\right)\omega^2 \qquad \text{However } mr_{G/IC}^2 + I_G = I_{IC}$$

$$= \frac{1}{2}I_{IC}\,\omega^2$$

**Q.E.D.**

**18–2.** The double pulley consists of two parts that are attached to one another. It has a weight of 50 lb and a radius of gyration about its center of $k_O = 0.6$ ft. If it rotates with an angular velocity of 20 rad/s clockwise, determine the kinetic energy of the system. Assume that neither cable slips on the pulley.



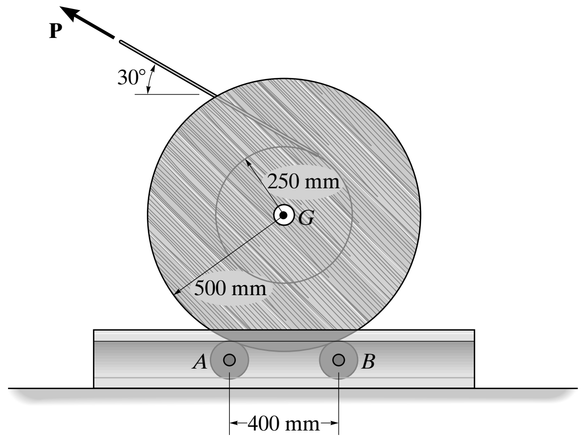

$$T = \frac{1}{2}I_O\,\omega_O^2 + \frac{1}{2}m_A\,v_A^2 + \frac{1}{2}m_B\,v_B^2$$

$$T = \frac{1}{2}\left(\frac{50}{32.2}(0.6)^2\right)(20)^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)\left[(20)(1)\right]^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)\left[(20)(0.5)\right]^2$$

$$= 283 \text{ ft} \cdot \text{lb}$$

**Ans.**

**18–3.** A force of $P = 20$ N is applied to the cable, which causes the 175-kg reel to turn without slipping on the two rollers $A$ and $B$ of the dispenser. Determine the angular velocity of the reel after it has rotated two revolutions starting from rest. Neglect the mass of the cable. Each roller can be considered as an 18-kg cylinder, having a radius of 0.1 m. The radius of gyration of the reel about its center axis is $k_G = 0.42$ m.




System:

$$T_1 + \Sigma U_{1-2} = T_2$$

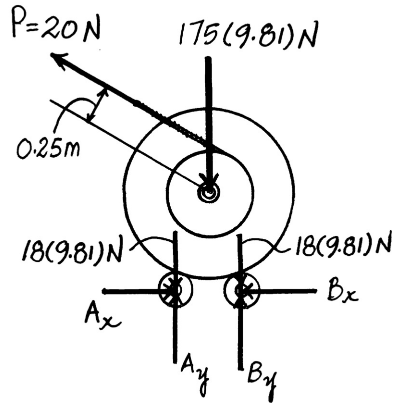
$$[0 + 0 + 0] + 20(2)(2\pi)(0.250) = \frac{1}{2}\left[175(0.42^2)\right]\omega^2 + \frac{2}{2}\left[\frac{1}{2}(18)(0.1)^2\right]\omega_r^2$$

$$v = \omega_r\,(0.1) = \omega(0.5)$$

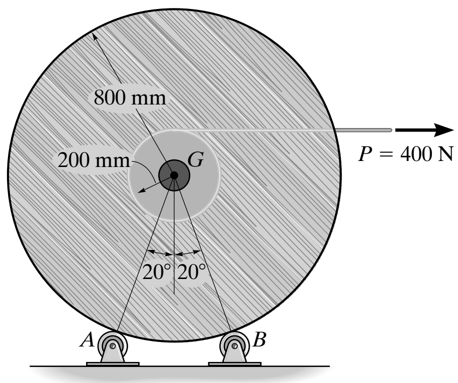
$$\omega_r = 5\omega$$

Solving:

$$\omega = 1.88 \text{ rad/s}$$ **Ans.**



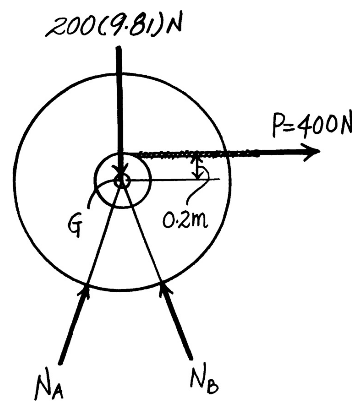

***18–4.** The spool of cable, originally at rest, has a mass of 200 kg and a radius of gyration of $k_G = 325$ mm. If the spool rests on two small rollers $A$ and $B$ and a constant horizontal force of $P = 400$ N is applied to the end of the cable, determine the angular velocity of the spool when 8 m of cable has been unwound. Neglect friction and the mass of the rollers and unwound cable.



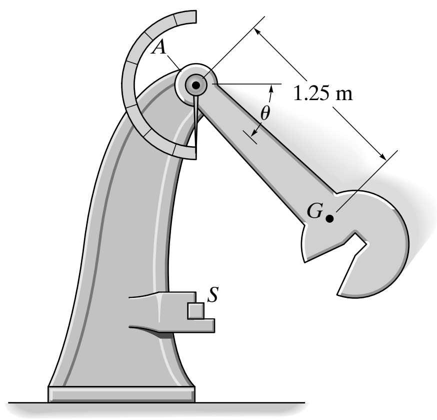

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + (400)(8) = \frac{1}{2}\left[200(0.325)^2\right]\omega_2^2$$

$$\omega_2 = 17.4 \text{ rad/s}$$ **Ans.**



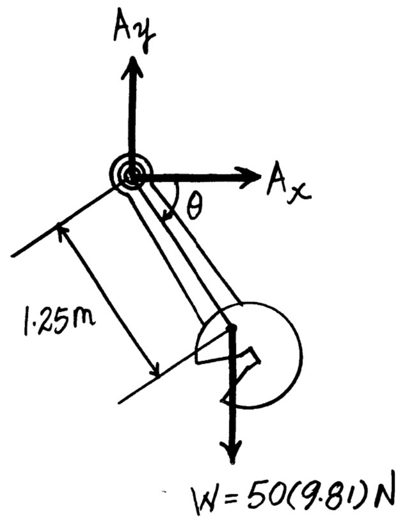
**•18–5.** The pendulum of the Charpy impact machine has a mass of 50 kg and a radius of gyration of $k_A = 1.75$ m. If it is released from rest when $\theta = 0°$, determine its angular velocity just before it strikes the specimen $S$, $\theta = 90°$.



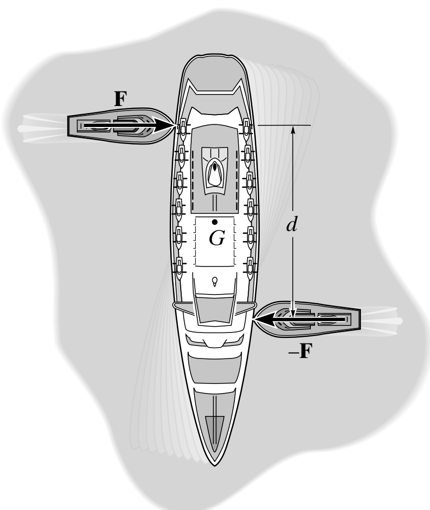

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + (50)(9.81)(1.25) = \frac{1}{2}\left[(50)(1.75)^2\right]\omega_2^2$$

$$\omega_2 = 2.83 \text{ rad/s}$$ **Ans.**




**18–6.** The two tugboats each exert a constant force **F** on the ship. These forces are always directed perpendicular to the ship's centerline. If the ship has a mass $m$ and a radius of gyration about its center of mass $G$ of $k_G$, determine the angular velocity of the ship after it turns 90°. The ship is originally at rest.




**Principle of Work and Energy:** The two tugboats create a couple moment of $M = Fd$ to rotate the ship through an angular displacement of $\theta = \frac{\pi}{2}$ rad. The mass moment of inertia about its mass center is $I_G = mk_G^2$. Applying Eq. 18–14, we have

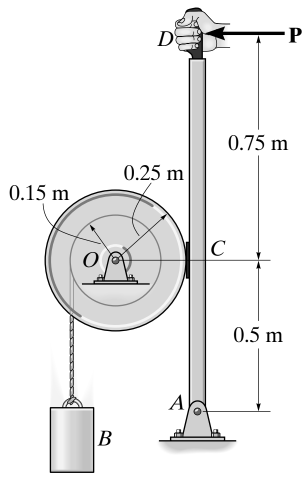
$$T_1 + \sum U_{1-2} = T_2$$

$$0 + M\theta = \frac{1}{2} I_G\, \omega^2$$

$$0 + Fd\left(\frac{\pi}{2}\right) = \frac{1}{2}\left(mk_G^2\right)\omega^2$$

$$\omega = \frac{1}{k_G}\sqrt{\frac{\pi F d}{m}}$$ **Ans.**

**18–7.** The drum has a mass of 50 kg and a radius of gyration about the pin at $O$ of $k_O = 0.23$ m. Starting from rest, the suspended 15-kg block $B$ is allowed to fall 3 m without applying the brake $ACD$. Determine the speed of the block at this instant. If the coefficient of kinetic friction at the brake pad $C$ is $\mu_k = 0.5$, determine the force $\mathbf{P}$ that must be applied at the brake handle which will then stop the block after it descends *another* 3 m. Neglect the thickness of the handle.




Before braking:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 15(9.81)(3) = \frac{1}{2}(15)v_B^2 + \frac{1}{2}\left[50(0.23)^2\right]\left(\frac{v_B}{0.15}\right)^2$$

$$v_B = 2.58 \text{ m/s} \qquad \textbf{Ans.}$$

$$\frac{s_B}{0.15} = \frac{s_C}{0.25}$$

Set $s_B = 3$ m, then $s_C = 5$ m.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 - F(5) + 15(9.81)(6) = 0$$

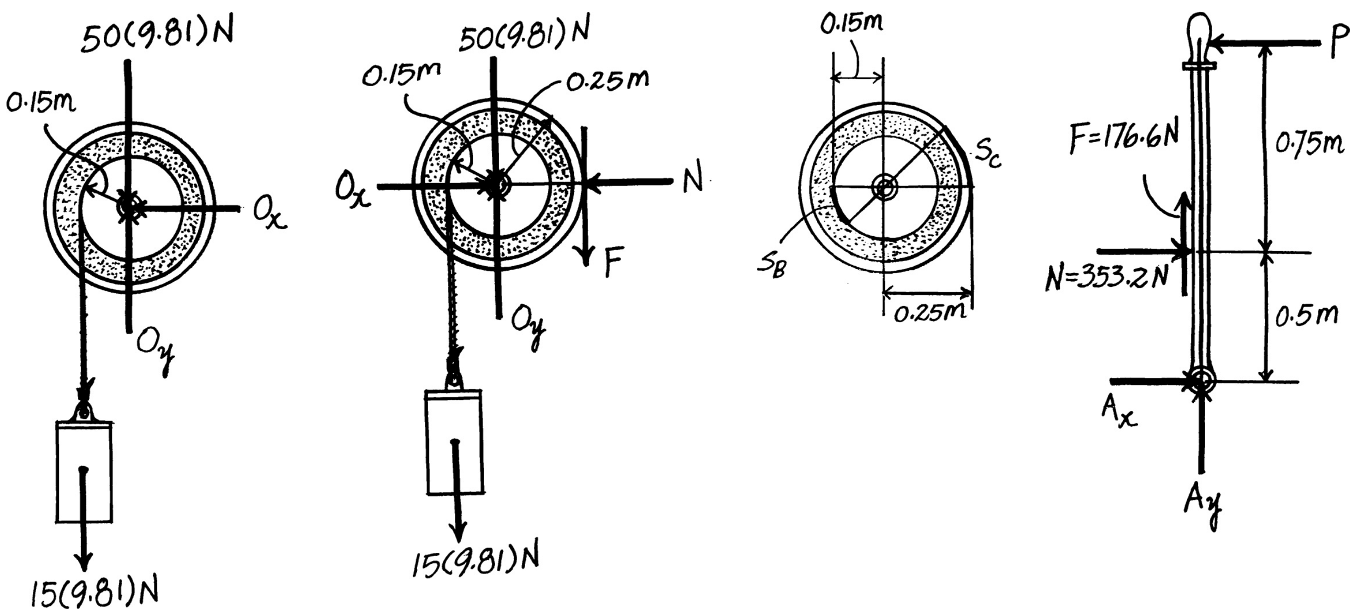
$$F = 176.6 \text{ N}$$

$$N = \frac{176.6}{0.5} = 353.2 \text{ N}$$

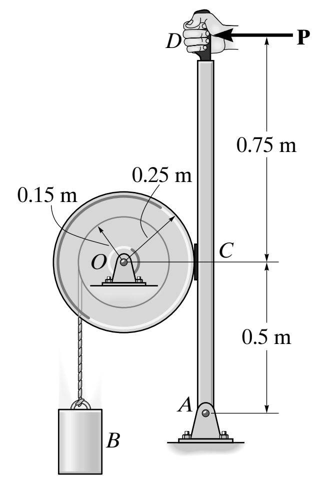
Brake arm:

$\circlearrowleft + \Sigma M_A = 0; \qquad -353.2(0.5) + P(1.25) = 0$

$$P = 141 \text{ N} \qquad \textbf{Ans.}$$



***18–8.** The drum has a mass of 50 kg and a radius of gyration about the pin at $O$ of $k_O = 0.23$ m. If the 15-kg block is moving downward at 3 m/s, and a force of $P = 100$ N is applied to the brake arm, determine how far the block descends from the instant the brake is applied until it stops. Neglect the thickness of the handle. The coefficient of kinetic friction at the brake pad is $\mu_k = 0.5$.




Brake arm:

$$\zeta + \Sigma M_A = 0; \qquad -N(0.5) + 100(1.25) = 0$$

$$N = 250 \text{ N}$$

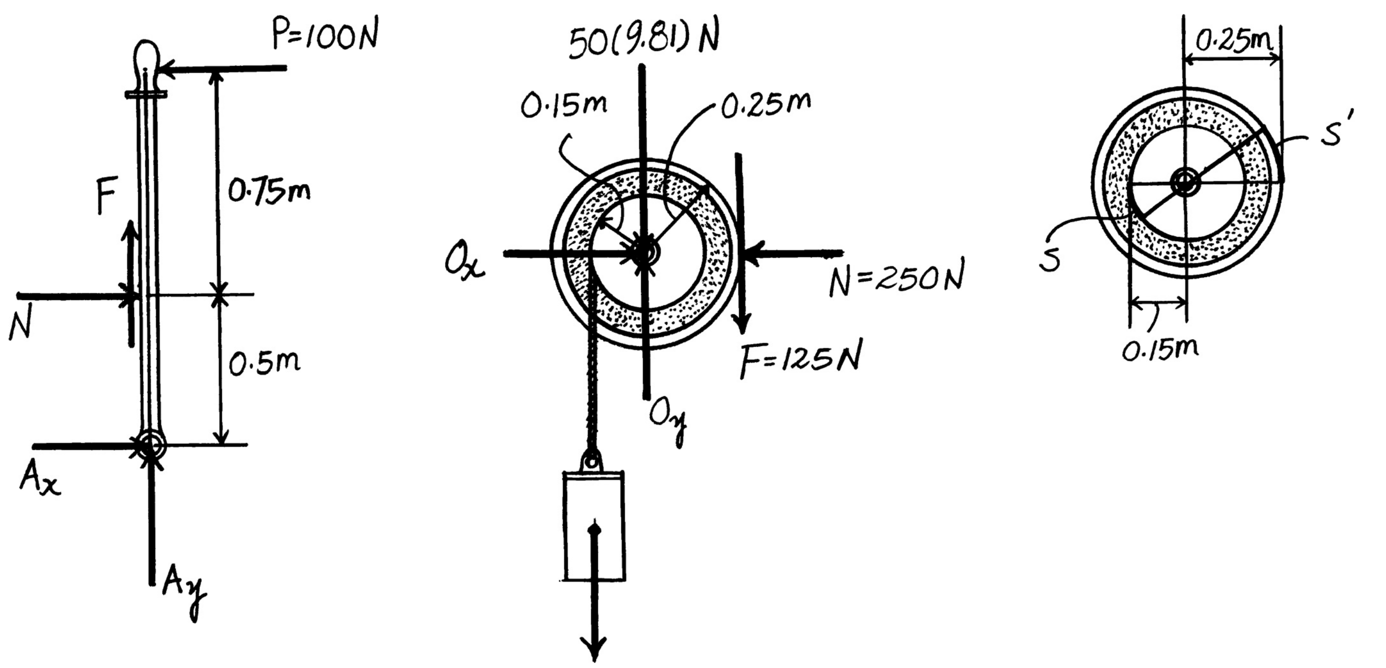
$$F = 0.5(250) = 125 \text{ N}$$

If block descends $s$, then $F$ acts through a distance $s' = s\left(\frac{0.25}{0.15}\right)$.

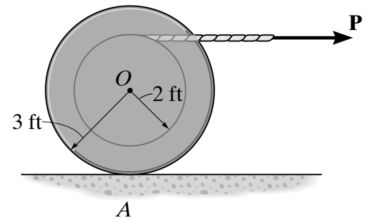
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left[(50)(0.23)^2\right]\left(\frac{3}{0.15}\right)^2 + \frac{1}{2}(15)(3)^2 + 15(9.81)(s) - 125(s)\left(\frac{0.25}{0.15}\right) = 0$$

$$s = 9.75 \text{ m}$$ **Ans.**



**•18–9.** The spool has a weight of 150 lb and a radius of gyration $k_O = 2.25$ ft. If a cord is wrapped around its inner core and the end is pulled with a horizontal force of $P = 40$ lb, determine the angular velocity of the spool after the center $O$ has moved 10 ft to the right. The spool starts from rest and does not slip at $A$ as it rolls. Neglect the mass of the cord.




**Kinematics:** Since the spool rolls without slipping, the instantaneous center of zero velocity is located at point $A$. Thus,

$$v_O = \omega r_{O/IC} = \omega(3)$$

Also, using similar triangles

$$\frac{s_P}{5} = \frac{10}{3} \qquad s_P = 16.67 \text{ ft}$$

**Free-Body Diagram:** The 40 lb force does *positive* work since it acts in the same direction of its displacement $s_P$. The normal reaction $N$ and the weight of the spool do no work since they do not displace. Also, since the spool does not slip, friction does no work.

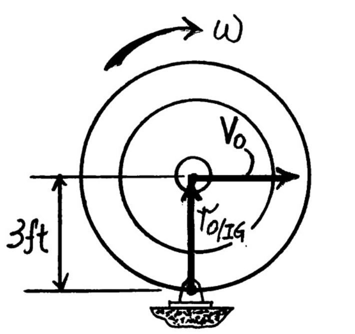
**Principle of Work and Energy:** The mass moment of inertia of the spool about point $O$ is $I_O = mk_O^2 = \left(\frac{150}{32.2}\right)\left(2.25^2\right) = 23.58 \text{ slug} \cdot \text{ft}^2$. Applying Eq. 18–14, we have

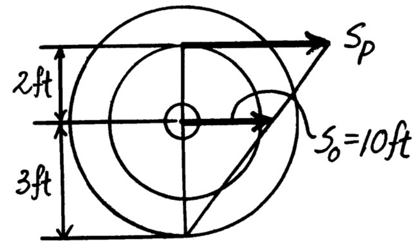
$$T_1 + \sum U_{1-2} = T_2$$

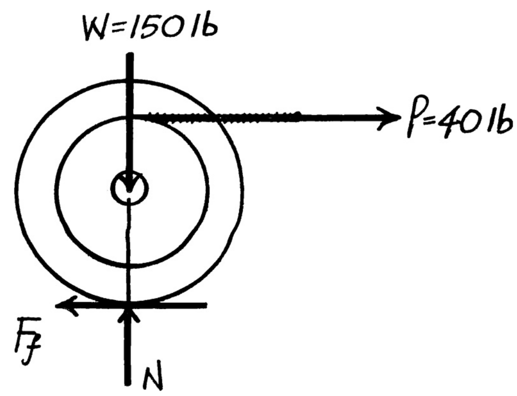
$$0 + P(s_P) = \frac{1}{2} m v_O^2 + \frac{1}{2} I_O \, \omega^2$$

$$0 + 40(16.67) = \frac{1}{2}\left(\frac{150}{32.2}\right)[\omega(3)]^2 + \frac{1}{2}(23.58)\,\omega^2$$

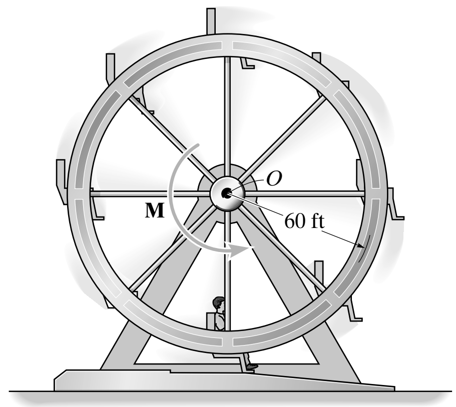
$$\omega = 4.51 \text{ rad/s}$$ **Ans.**



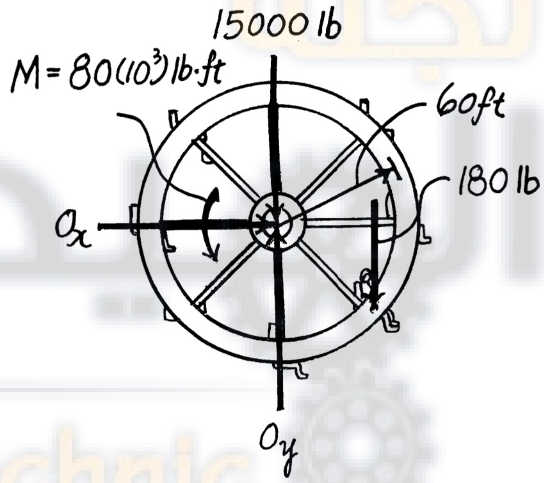
**18–10.** A man having a weight of 180 lb sits in a chair of the Ferris wheel, which, excluding the man, has a weight of 15 000 lb and a radius of gyration $k_O = 37$ ft. If a torque $M = 80(10^3)$ lb·ft is applied about $O$, determine the angular velocity of the wheel after it has rotated 180°. Neglect the weight of the chairs and note that the man remains in an upright position as the wheel rotates. The wheel starts from rest in the position shown.



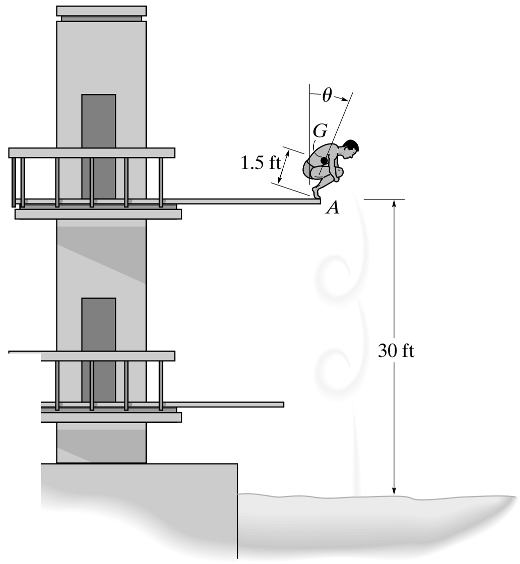

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 80\left(10^3\right)(\pi) - (180)(120) = \frac{1}{2}\left[\left(\frac{15\,000}{32.2}\right)(37)^2\right]\omega^2 + \frac{1}{2}\left(\frac{180}{32.2}\right)(60\omega)^2$$

$$\omega = 0.836 \text{ rad/s}$$ **Ans.**



**18–11.** A man having a weight of 150 lb crouches down on the end of a diving board as shown. In this position the radius of gyration about his center of gravity is $k_G = 1.2$ ft. While holding this position at $\theta = 0°$, he rotates about his toes at $A$ until he loses contact with the board when $\theta = 90°$. If he remains rigid, determine approximately how many revolutions he makes before striking the water after falling 30 ft.



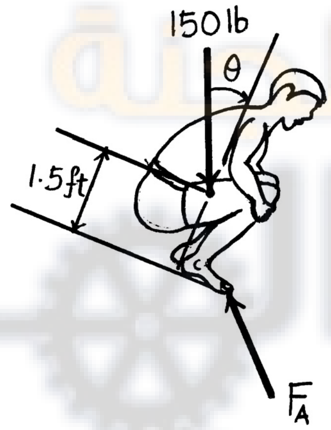

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 150(1.5) = \frac{1}{2}\left(\frac{150}{32.2}\right)(1.5\omega)^2 + \frac{1}{2}\left[\left(\frac{150}{32.2}\right)(1.2)^2\right]\omega^2$$

$$\omega = 5.117 \text{ rad/s}$$

$$v_G = (1.5)(5.117) = 7.675 \text{ ft/s}$$

During the fall no forces act on the man to cause an angular acceleration, so $\alpha = 0$.




$$(+\downarrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$30 = 0 + 7.675t + \frac{1}{2}(32.2)t^2$$

Choosing the positive root,

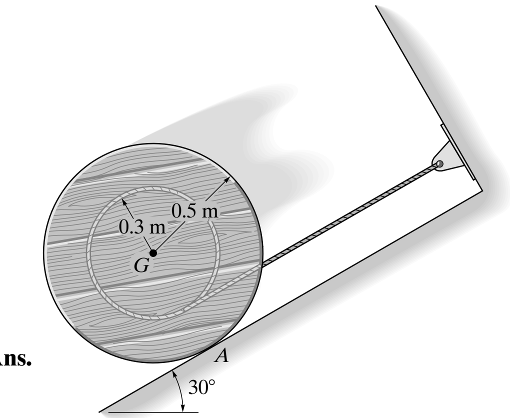
$$t = 1.147 \text{ s}$$

$$(\circlearrowleft +) \qquad \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$

$$\theta = 0 + 5.117(1.147) + 0$$

$$\theta = 5.870 \text{ rad} = 0.934 \text{ rev.} \qquad \textbf{Ans.}$$

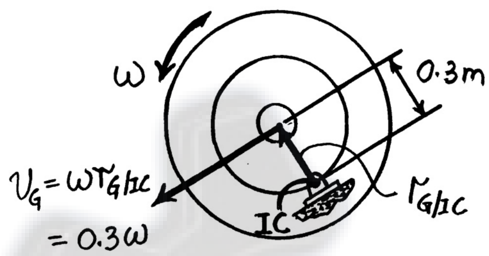
***18–12.** The spool has a mass of 60 kg and a radius of gyration $k_G = 0.3$ m. If it is released from rest, determine how far its center descends down the smooth plane before it attains an angular velocity of $\omega = 6$ rad/s. Neglect friction and the mass of the cord which is wound around the central core.

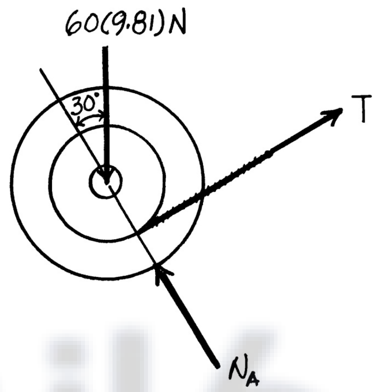



$$T_1 + \Sigma U_{1-2} = T_2$$

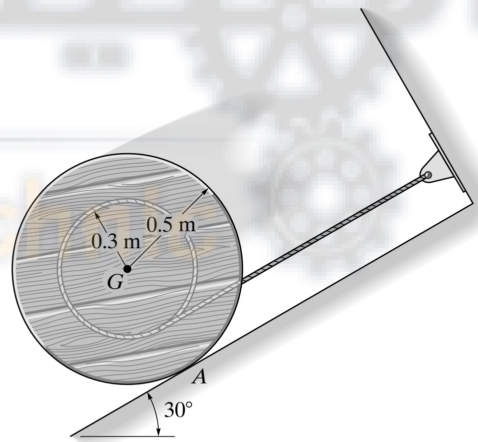
$$0 + 60(9.81)\sin 30°(s) = \frac{1}{2}\left[60(0.3)^2\right](6)^2 + \frac{1}{2}(60)\left[0.3(6)\right]^2$$

$$s = 0.661 \text{ m}$$ **Ans.**




**•18–13.** Solve Prob. 18–12 if the coefficient of kinetic friction between the spool and plane at $A$ is $\mu_k = 0.2$.




$$\frac{s_G}{0.3} = \frac{s_A}{(0.5 - 0.3)}$$

$$s_A = 0.6667 s_G$$

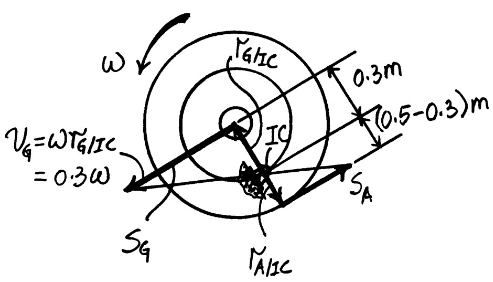
$+\nwarrow \Sigma F_y = 0; \qquad N_A - 60(9.81)\cos 30° = 0$

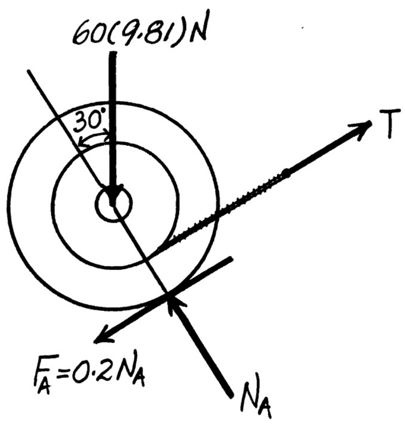
$$N_A = 509.7 \text{ N}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

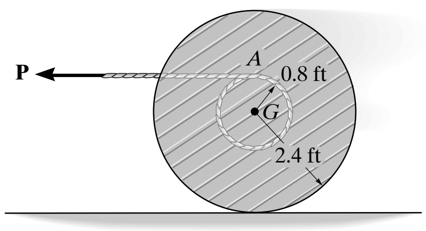
$$0 + 60(9.81)\sin 30°(s_G) - 0.2(509.7)(0.6667 s_G) = \frac{1}{2}\left[60(0.3)^2\right](6)^2 + \frac{1}{2}(60)\left[(0.3)(6)\right]^2$$

$$s_G = 0.859 \text{ m}$$ **Ans.**



**18–14.** The spool has a weight of 500 lb and a radius of gyration of $k_G = 1.75$ ft. A horizontal force of $P = 15$ lb is applied to the cable wrapped around its inner core. If the spool is originally at rest, determine its angular velocity after the mass center $G$ has moved 6 ft to the left. The spool rolls without slipping. Neglect the mass of the cable.



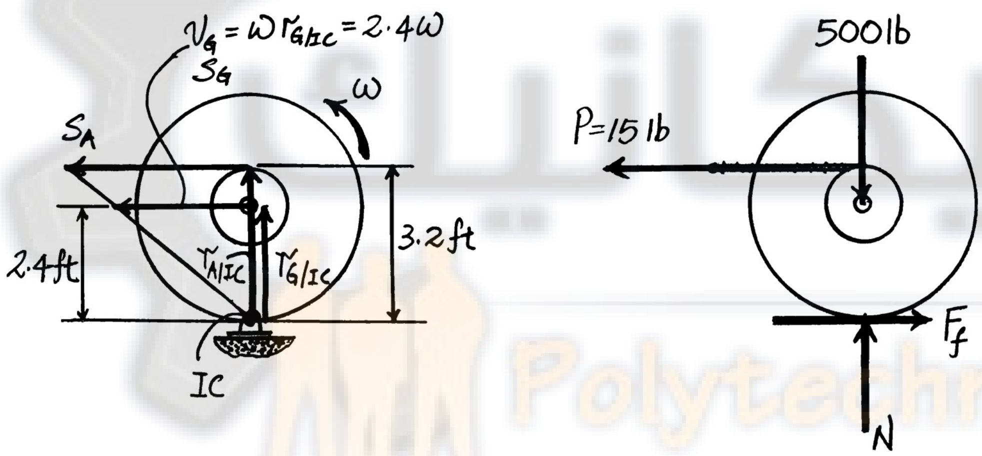

$$\frac{s_G}{2.4} = \frac{s_A}{3.2}$$

For $s_G = 6$ ft, then $s_A = 8$ ft.

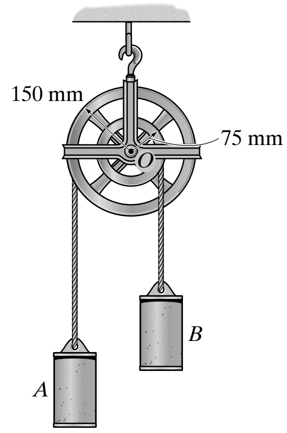
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 15(8) = \frac{1}{2}\left[\left(\frac{500}{32.2}\right)(1.75)^2\right]\omega^2 + \frac{1}{2}\left(\frac{500}{32.2}\right)(2.4\omega)^2$$

$$\omega = 1.32 \text{ rad/s} \qquad \textbf{Ans.}$$



**18–15.** If the system is released from rest, determine the speed of the 20-kg cylinders $A$ and $B$ after $A$ has moved downward a distance of 2 m. The differential pulley has a mass of 15 kg with a radius of gyration about its center of mass of $k_O = 100$ mm.




**Kinetic Energy and Work:** The kinetic energy of the pulley and cylinders $A$ and $B$ is

$$T_P = \frac{1}{2} I_O \omega^2 = \frac{1}{2}\left[15\left(0.1^2\right)\right]\omega^2 = 0.075\omega^2$$

$$T_A = \frac{1}{2} m_A v_A{}^2 = \frac{1}{2}(20)v_A{}^2 = 10v_A{}^2$$

$$T_B = \frac{1}{2} m_B v_B{}^2 = \frac{1}{2}(20)v_B{}^2 = 10v_B{}^2$$

Thus, the kinetic energy of the system is

$$T = T_P + T_A + T_B$$

$$T = 0.075\omega^2 + 10v_A{}^2 + 10v_B{}^2 \qquad \textbf{(1)}$$

However, since the pulley rotates about a fixed axis,

$$\omega = \frac{v_A}{r_A} = \frac{v_A}{0.15} = 6.667v_A$$

then

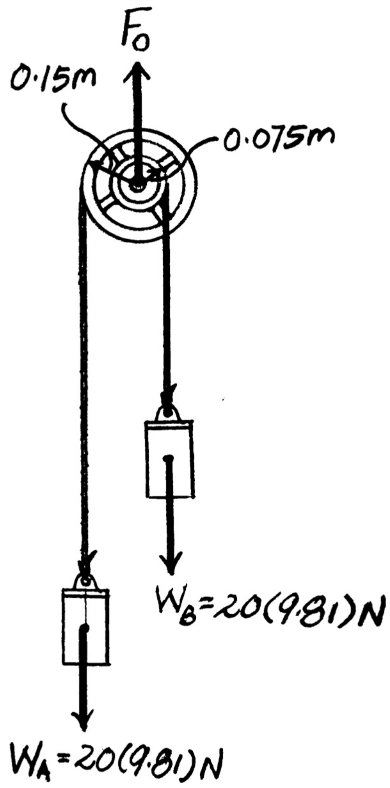
$$v_B = \omega r_B = 6.667v_A\,(0.075) = 0.5v_A$$

Substituting these results into Eq. (1), we obtain

$$T = 15.833v_A{}^2$$

Since the system is initially at rest,

$$T_1 = 0$$




Referring to Fig. *a*, $\mathbf{F}_O$ does no work, while $W_A$ does positive work, and $W_B$ does negative work. Thus,

$$U_A = W_A s_A \qquad\qquad U_B = -W_B s_B$$

Here, $s_A = 2$ m. Thus, the pulley rotates through an angle of $\theta = \frac{s_A}{r_A} = \frac{2}{0.15} = 13.33$ rad. Then, $s_B = r_B\theta = 0.075(13.33) = 1$ m. Thus,

$$U_A = 20(9.81)(2) = 392.4 \text{ J}$$

$$U_B = -20(9.81)(1) = -196.2 \text{ J}$$

**Principle of Work and Energy:**

$$T_1 + \Sigma U_{1-2} = T_2$$

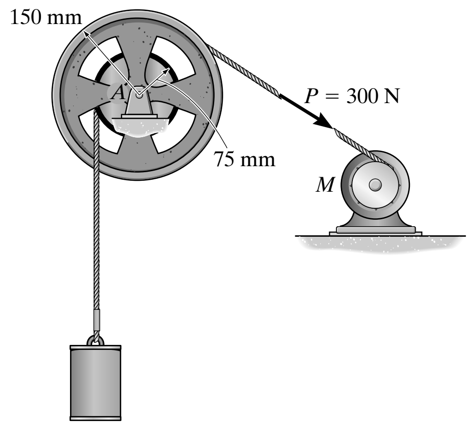
$$0 + \left[392.4 + (-196.2)\right] = 15.833v_A{}^2$$

$$v_A = 3.520 \text{ m/s} = 3.52 \text{ m/s}\downarrow \qquad \textbf{Ans.}$$

Then

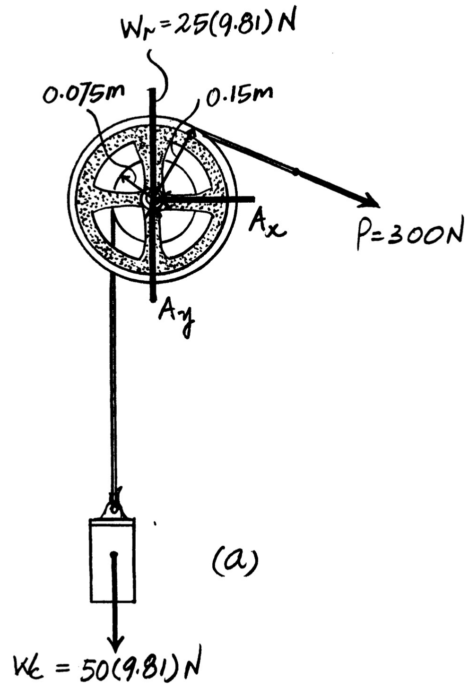
$$v_B = 0.5(3.520) = 1.76 \text{ m/s}\uparrow \qquad \textbf{Ans.}$$

***18–16.** If the motor $M$ exerts a constant force of $P = 300$ N on the cable wrapped around the reel's outer rim, determine the velocity of the 50-kg cylinder after it has traveled a distance of 2 m. Initially, the system is at rest. The reel has a mass of 25 kg, and the radius of gyration about its center of mass $A$ is $k_A = 125$ mm.




**Kinetic Energy and Work:** Since the reel rotates about a fixed axis, $v_C = \omega_r r_C$ or $\omega_r = \dfrac{v_C}{r_C} = \dfrac{v_C}{0.075} = 13.33 v_C$. The mass moment of inertia of the reel about its mass centers is $I_A = m_r k_A{}^2 = 25\left(0.125^2\right) = 0.390625 \text{ kg} \cdot \text{m}^2$. Thus, the kinetic energy of the system is

$$\begin{aligned} T &= T_r + T_C \\ &= \frac{1}{2} I_A \,\omega_r{}^2 + \frac{1}{2} m_C \, v_C{}^2 \\ &= \frac{1}{2}(0.390625)(13.33 v_C)^2 + \frac{1}{2}(50) v_C{}^2 \\ &= 59.72 v_C{}^2 \end{aligned}$$




Since the system is initially at rest, $T_1 = 0$. Referring to Fig. $a$, $\mathbf{A}_y$, $\mathbf{A}_x$, and $\mathbf{W}_r$ do no work, while $\mathbf{P}$ does positive work, and $\mathbf{W}_C$ does negative work. When the cylinder displaces upwards through a distance of $s_C = 2$ m, the wheel rotates $\theta = \dfrac{s_C}{r_C} = \dfrac{2}{0.075} = 26.67$ rad. Thus, $\mathbf{P}$ displaces a distance of $s_P = r_P \theta = 0.15(26.67) = 4$ m. The work done by $\mathbf{P}$ and $\mathbf{W}_C$ is therefore

$$U_P = P s_P = 300(4) = 1200 \text{ J}$$

$$U_{W_C} = -W_C \, s_C = -50(9.81)(2) = -981 \text{ J}$$

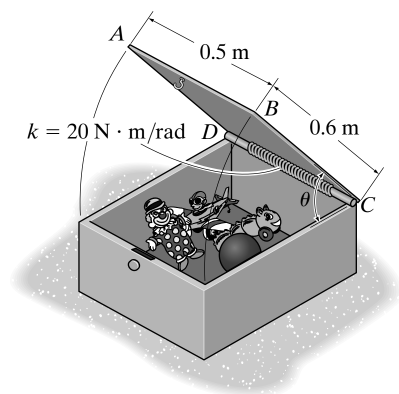
**Principle of Work and Energy:**

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left[1200 + (-981)\right] = 59.72 v_C{}^2$$

$$v_C = 1.91 \text{ m/s}\uparrow$$ **Ans.**

**•18–17.** The 6-kg lid on the box is held in equilibrium by the torsional spring at $\theta = 60°$. If the lid is forced closed, $\theta = 0°$, and then released, determine its angular velocity at the instant it opens to $\theta = 45°$.




**Equilibrium:** Here, $M = k\theta_0 = 20\theta_0$, where $\theta_0$ is the initial angle of twist for the torsional spring. Referring to Fig. *a*, we have

$$+\Sigma M_C = 0; \qquad 6(9.81)\cos 60°(0.3) - 20\theta_0 = 0 \qquad \theta_0 = 0.44145 \text{ rad}$$

**Kinetic Energy and Work:** Since the cover rotates about a fixed axis passing through point $C$, the kinetic energy of the cover can be obtained by applying $T = \frac{1}{2} I_C \omega^2$, where $I_C = \frac{1}{3} mb^2 = \frac{1}{3}(6)\left(0.6^2\right) = 0.72 \text{ kg} \cdot \text{m}^2$. Thus,

$$T = \frac{1}{2} I_C\omega^2 = \frac{1}{2}(0.72)\omega^2 = 0.36\omega^2$$

Since the cover is initially at rest ($\theta = 0°$), $T_1 = 0$. Referring to Fig. *b*, $C_x$ and $C_y$ do no work. **M** does positive work, and $W$ does negative work. When $\theta = 0°$ and $45°$, the angles of twist for the torsional spring are $\theta_1 = 1.489$ rad and $\theta_2 = 1.489 - \frac{\pi}{4} = 0.703$ rad, respectively. Also, when $\theta = 45°$, **W** displaces vertically upward through a distance of $h = 0.3 \sin 45° = 0.2121$ m. Thus, the work done by **M** and **W** are

$$U_M = \int M\, d\theta = \int_{\theta_2}^{\theta_1} 20\theta\, d\theta = 10\theta^2 \Big|_{0.7032 \text{ rad}}^{1.4886 \text{ rad}} = 17.22 \text{ J}$$

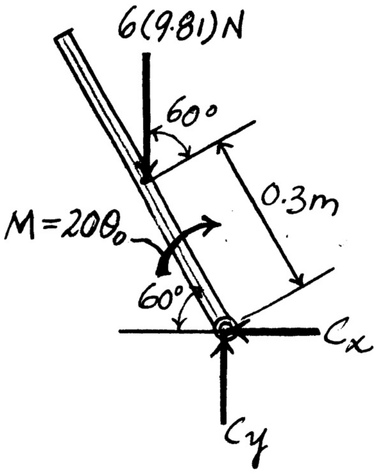
$$U_W = -Wh = -6(9.81)(0.2121) = -12.49 \text{ J}$$

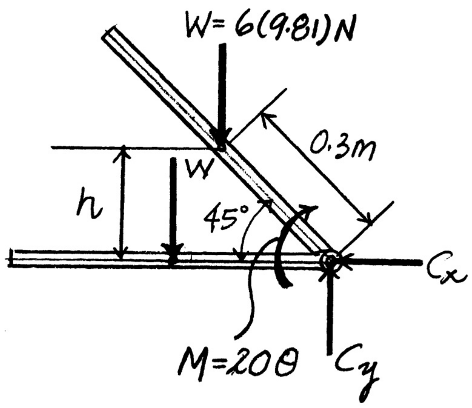
**Principle of Work and Energy:**

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left[17.22 + (-12.49)\right] = 0.36\omega^2$$

$$\omega = 3.62 \text{ rad/s} \qquad \textbf{Ans.}$$

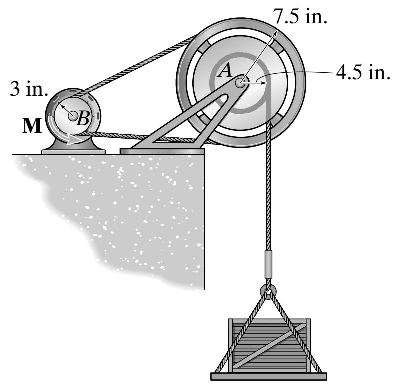





لجنة الميكانيك - الإتجاه الإسلامي

**18–18.** The wheel and the attached reel have a combined weight of 50 lb and a radius of gyration about their center of $k_A = 6$ in. If pulley $B$ attached to the motor is subjected to a torque of $M = 40(2 - e^{-0.1\theta})$ lb · ft, where $\theta$ is in radians, determine the velocity of the 200-lb crate after it has moved upwards a distance of 5 ft, starting from rest. Neglect the mass of pulley $B$.




**Kinetic Energy and Work:** Since the wheel rotates about a fixed axis, $v_C = \omega r_C = \omega(0.375)$. The mass moment of inertia of $A$ about its mass center is $I_A = mk_A{}^2 = \left(\frac{50}{32.2}\right)(0.5^2) = 0.3882$ slug · ft$^2$. Thus, the kinetic energy of the system is

$$T = T_A + T_C$$

$$= \frac{1}{2} I_A \omega^2 + \frac{1}{2} m_C v_C{}^2$$

$$= \frac{1}{2}(0.3882)\omega^2 + \frac{1}{2}\left(\frac{200}{32.2}\right)\left[\omega(0.375)\right]^2$$

$$= 0.6308\omega^2$$

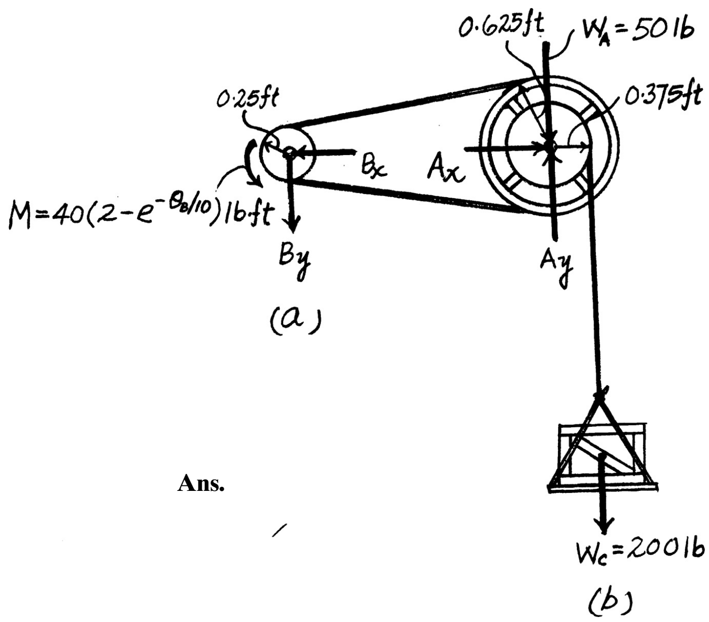
Since the system is initially at rest, $T_1 = 0$. Referring to Fig. $b$, $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{W}_A$ do no work, $\mathbf{M}$ does positive work, and $\mathbf{W}_C$ does negative work. When crate $C$ moves 5 ft upward, wheel $A$ rotates through an angle of $\theta_A = \frac{s_C}{r} = \frac{5}{0.375} = 13.333$ rad. Then, pulley $B$ rotates through an angle of $\theta_B = \frac{r_A}{r_B}\theta_A = \left(\frac{0.625}{0.25}\right)(13.333) = 33.33$ rad. Thus, the work done by $\mathbf{M}$ and $\mathbf{W}_C$ is

$$U_M = \int M d\theta_B = \int_0^{33.33 \text{ rad}} 40\left(2 - e^{-0.1\theta}\right)d\theta$$

$$= \left[40\left(2\theta + 10e^{-0.1\theta}\right)\right]\Big|_0^{33.33 \text{ rad}}$$

$$= 2280.93 \text{ ft} \cdot \text{lb}$$

$$U_{W_C} = -W_C s_C = -200(5) = -1000 \text{ ft} \cdot \text{lb}$$




**Principle of Work and Energy:**

$$T_1 + \Sigma U_{1-2} = T_2$$

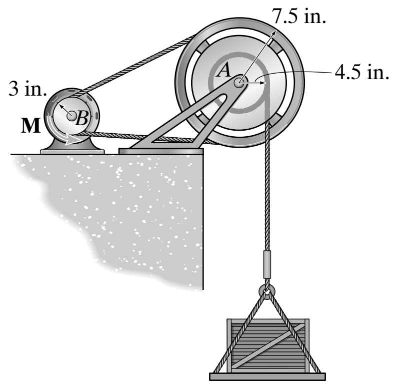
$$0 + [2280.93 - 1000] = 0.6308\omega^2$$

$$\omega = 45.06 \text{ rad/s}$$

Thus,

$$v_C = 45.06(0.375) = 16.9 \text{ ft/s} \uparrow$$ **Ans.**

**18–19.** The wheel and the attached reel have a combined weight of 50 lb and a radius of gyration about their center of $k_A = 6$ in. If pulley $B$ that is attached to the motor is subjected to a torque of $M = 50\text{ lb}\cdot\text{ft}$, determine the velocity of the 200-lb crate after the pulley has turned 5 revolutions. Neglect the mass of the pulley.




**Kinetic Energy and Work:** Since the wheel at $A$ rotates about a fixed axis, $v_C = \omega r_C = \omega(0.375)$. The mass moment of inertia of wheel $A$ about its mass center is $I_A = mk_A{}^2 = \left(\frac{50}{32.2}\right)(0.5^2) = 0.3882\text{ slug}\cdot\text{ft}^2$. Thus, the kinetic energy of the system is

$$T = T_A + T_C$$

$$= \frac{1}{2} I_A\,\omega^2 + \frac{1}{2} m_C\, v_C{}^2$$

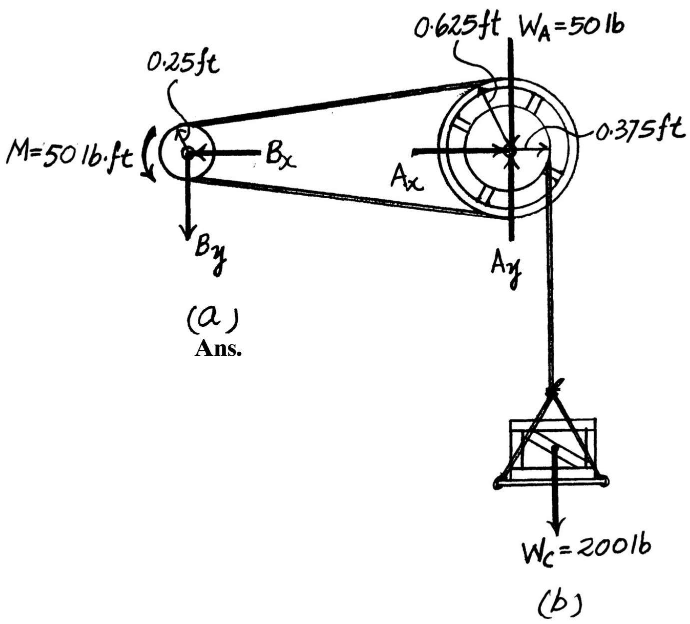
$$= \frac{1}{2}(0.3882)\omega^2 + \frac{1}{2}\left(\frac{200}{32.2}\right)\left[\omega(0.375)\right]^2$$

$$= 0.6308\omega^2$$

Since the system is initially at rest, $T_1 = 0$. Referring to Fig. $b$, $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{W}_A$ do no work, $\mathbf{M}$ does positive work, and $\mathbf{W}_C$ does negative work. When pulley $B$ rotates $\theta_B = (5\text{ rev})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 10\pi\text{ rad}$, the wheel rotates through an angle of $\theta_A = \frac{r_B}{r_A}\theta_B = \left(\frac{0.25}{0.625}\right)(10\pi) = 4\pi$. Thus, the crate displaces upwards through a distance of $s_C = r_C\,\theta_A = 0.375(4\pi) = 1.5\pi$ ft. Thus, the work done by $\mathbf{M}$ and $\mathbf{W}_C$ is

$$U_M = M\theta_B = 50(10\pi) = 500\pi\text{ ft}\cdot\text{lb}$$

$$U_{W_C} = -W_C\, s_C = -200(1.5\pi) = -300\pi\text{ ft}\cdot\text{lb}$$




**Principle of Work and Energy:**

$$T_1 + \Sigma U_{1-2} = T_2$$

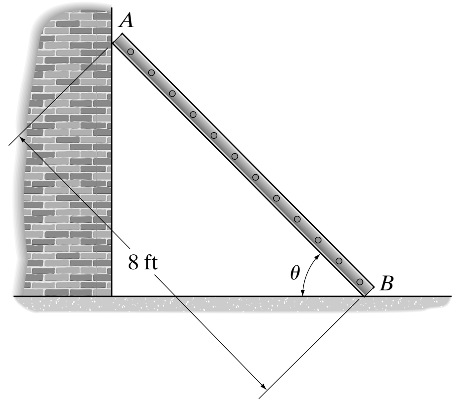
$$0 + [500\pi - 300\pi] = 0.6308\omega^2$$

$$\omega = 31.56\text{ rad/s}$$

Thus,

$$v_C = 31.56(0.375) = 11.8\text{ ft/s}\uparrow$$ **Ans.**

***18–20.** The 30-lb ladder is placed against the wall at an angle of $\theta = 45°$ as shown. If it is released from rest, determine its angular velocity at the instant just before $\theta = 0°$. Neglect friction and assume the ladder is a uniform slender rod.




**Kinetic Energy and Work:** Referring to Fig. *a*,

$$(v_G)_2 = \omega_2 r_{G/IC} = \omega_2(4)$$

The mass moment of inertia of the ladder about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{30}{32.2}\right)(8^2) = 4.969\ \text{slug}\cdot\text{ft}^2$. Thus, the final kinetic energy is

$$T_2 = \frac{1}{2}m(v_G)_2{}^2 + \frac{1}{2}I_G\omega_2{}^2$$

$$= \frac{1}{2}\left(\frac{30}{32.2}\right)\left[\omega_2(4)\right]^2 + \frac{1}{2}(4.969)\omega_2{}^2$$

$$= 9.938\omega_2{}^2$$

Since the ladder is initially at rest, $T_1 = 0$. Referring to Fig. *b*, $\mathbf{N}_A$ and $\mathbf{N}_B$ do no work, while $\mathbf{W}$ does positive work. When $\theta = 0°$, $\mathbf{W}$ displaces vertically through a distance of $h = 4 \sin 45°\ \text{ft} = 2.828\ \text{ft}$. Thus, the work done by $\mathbf{W}$ is

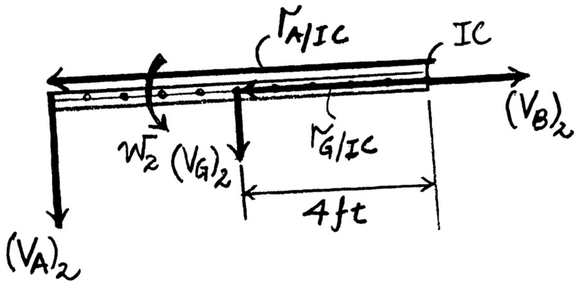
$$U_W = Wh = 30(2.828) = 84.85\ \text{ft}\cdot\text{lb}$$

**Principle of Work and Energy:**

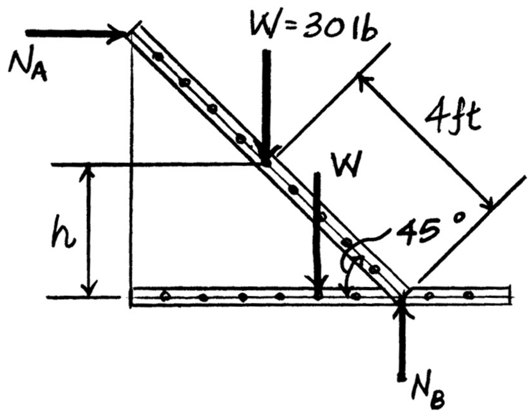
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 84.85 = 9.938\omega_2{}^2$$

$$\omega_2 = 2.92\ \text{rad/s}$$ **Ans.**

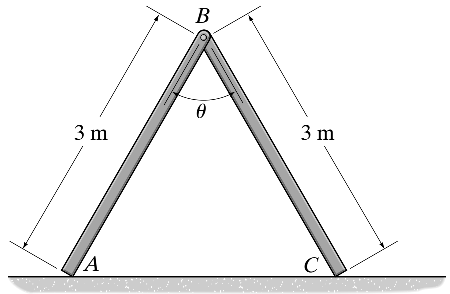



(a)




(b)

**•18–21.** Determine the angular velocity of the two 10-kg rods when $\theta = 180°$ if they are released from rest in the position $\theta = 60°$. Neglect friction.




**Kinetic Energy and Work:** Due to symmetry, the velocity of point $B$ is directed along the vertical, as shown in Fig. *a*. Also, $(\omega_{AB})_2 = (\omega_{BC})_2 = \omega_2$ and $\left(v_{G_{AB}}\right)_2 = \left(v_{G_{BC}}\right)_2 = (v_G)_2$. Here, $(v_G)_2 = \omega_2 r_{G/IC} = \omega_2(1.5)$. The mass moment of inertia of the rods about their respective mass centers is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(10)\left(3^2\right) = 7.5\ \text{kg}\cdot\text{m}^2$. Thus, the final kinetic energy is

$$T_2 = (T_{AB})_2 + (T_{BC})_2$$

$$= 2\left[\frac{1}{2}m(v_G)_2{}^2 + \frac{1}{2}I_G\omega_2{}^2\right]$$

$$= 2\left[\frac{1}{2}(10)\left[\omega_2(1.5)\right]^2 + \frac{1}{2}(7.5)\omega_2{}^2\right]$$

$$= 30\omega_2{}^2$$

Since the system is initially at rest, $T_1 = 0$. Referring to Fig. *b*, **N** does no work, while **W** does positive work. When $\theta = 180°$, **W** displaces vertically downward through a distance of $h = 1.5\cos 30° = 1.2990$ m. Thus, the work done by **W** is

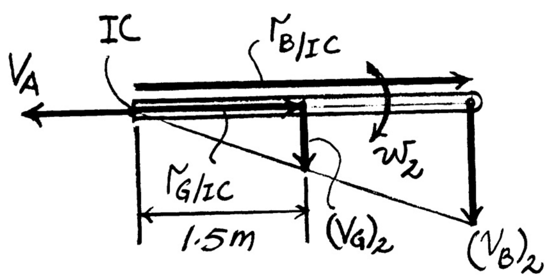
$$U_W = Wh = 10(9.81)(1.2990) = 127.44\ \text{J}$$

**Principle of Work and Energy:**

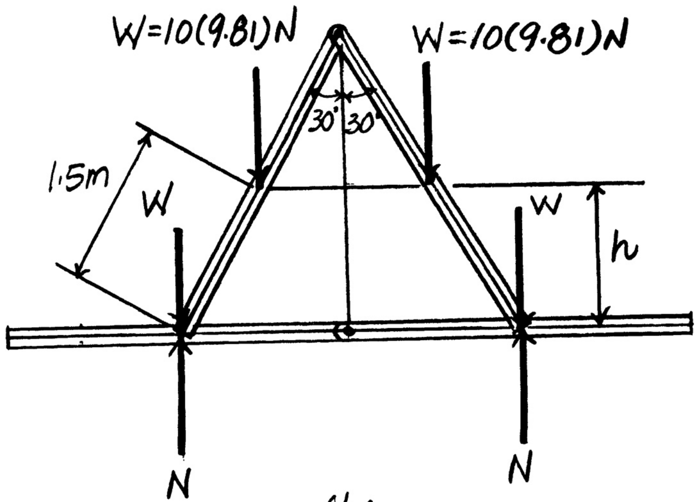
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 2(127.44) = 30\omega_2{}^2$$

$$\omega_2 = 2.91\ \text{rad/s} \qquad \textbf{Ans.}$$

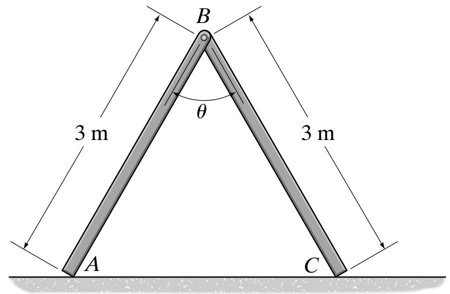



(a)




(b)

**18–22.** Determine the angular velocity of the two 10-kg rods when $\theta = 90°$ if they are released from rest in the position $\theta = 60°$. Neglect friction.




**Kinetic Energy and Work:** Due to symmetry, the velocity of point $B$ is directed along the vertical, as shown in Fig. *a*. Also, $(\omega_{AB})_2 = (\omega_{BC})_2 = \omega_2$ and $\left(v_{G_{AB}}\right)_2 = \left(v_{G_{BC}}\right)_2 = (v_G)_2$. From the geometry of this diagram, $r_{G/IC} = 1.5$ m. Thus, $(v_G)_2 = \omega_2 r_{G/IC} = \omega_2(1.5)$. The mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}(10)\left(3^2\right) = 7.5\ \text{kg}\cdot\text{m}^2$. Thus, the final kinetic energy is

$$T_2 = (T_{AB})_2 + (T_{BC})_2$$

$$= 2\left[\frac{1}{2}m(v_G)_2{}^2 + \frac{1}{2}I_G\omega_2{}^2\right]$$

$$= 2\left[\frac{1}{2}(10)[\omega_2(1.5)]^2 + \frac{1}{2}(7.5)\omega_2{}^2\right]$$

$$= 30\omega_2{}^2$$

Since the system is initially at rest, $T_1 = 0$. Referring to Fig. *b*, **N** does no work, while **W** does positive work. When $\theta = 90°$, **W** displaces vertically downward through a distance of $h = 1.5\cos 30° - 1.5\cos 45° = 0.2384$ m. Thus, the work done by **W** is

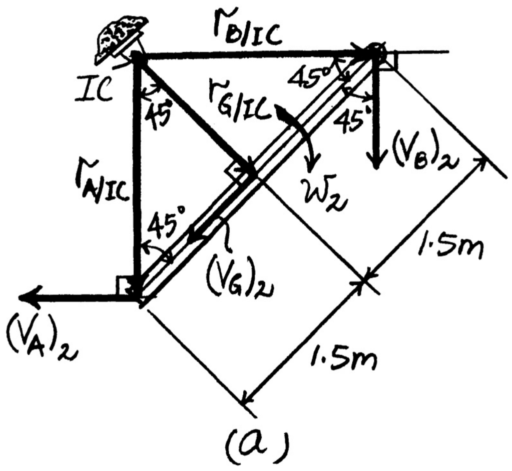
$$U_W = Wh = 10(9.81)(0.2384) = 23.38\ \text{J}$$

**Principle of Work and Energy:**

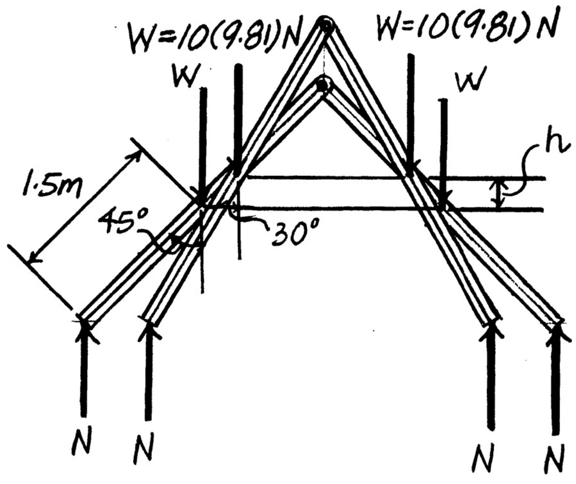
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 2(23.38) = 30\omega_2{}^2$$

$$\omega_2 = 1.25\ \text{rad/s}$$ **Ans.**

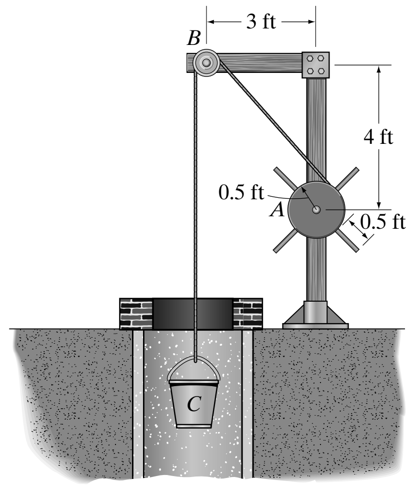



(a)




(b)

**18–23.** If the 50-lb bucket is released from rest, determine its velocity after it has fallen a distance of 10 ft. The windlass $A$ can be considered as a 30-lb cylinder, while the spokes are slender rods, each having a weight of 2 lb. Neglect the pulley's weight.




**Kinetic Energy and Work:** Since the windlass rotates about a fixed axis, $v_C = \omega_A r_A$ or $\omega_A = \dfrac{v_C}{r_A} = \dfrac{v_C}{0.5} = 2v_C$. The mass moment of inertia of the windlass about its mass center is

$$I_A = \frac{1}{2}\left(\frac{30}{32.2}\right)(0.5^2) + 4\left[\frac{1}{12}\left(\frac{2}{32.2}\right)(0.5^2) + \frac{2}{32.2}(0.75^2)\right] = 0.2614\ \text{slug}\cdot\text{ft}^2$$

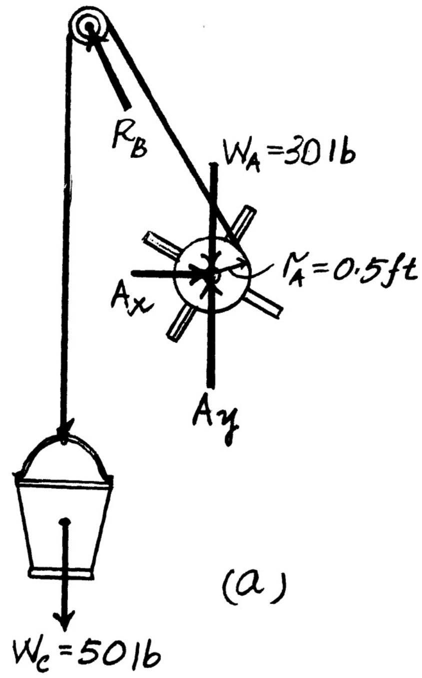
Thus, the kinetic energy of the system is

$$T = T_A + T_C$$

$$= \frac{1}{2}I_A\omega^2 + \frac{1}{2}m_C v_C{}^2$$

$$= \frac{1}{2}(0.2614)(2v_C)^2 + \frac{1}{2}\left(\frac{50}{32.2}\right)v_C{}^2$$

$$= 1.2992v_C{}^2$$




Since the system is initially at rest, $T_1 = 0$. Referring to Fig. $a$, $\mathbf{W}_A$, $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{R}_B$ do no work, while $\mathbf{W}_C$ does positive work. Thus, the work done by $\mathbf{W}_C$, when it displaces vertically downward through a distance of $s_C = 10$ ft, is

$$U_{W_C} = W_C s_C = 50(10) = 500\ \text{ft}\cdot\text{lb}$$

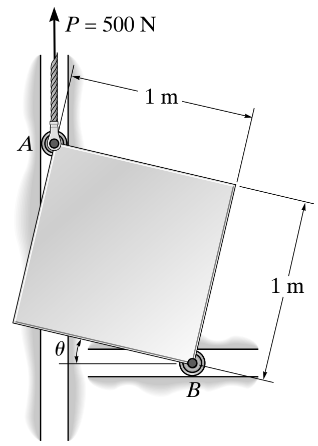
**Principle of Work and Energy:**

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 500 = 1.2992v_C{}^2$$

$$v_C = 19.6\ \text{ft/s}$$ **Ans.**

***18–24.** If corner $A$ of the 60-kg plate is subjected to a vertical force of $P = 500$ N, and the plate is released from rest when $\theta = 0°$, determine the angular velocity of the plate when $\theta = 45°$.




**Kinetic Energy and Work:** Since the plate is initially at rest, $T_1 = 0$. Referring to Fig. $a$,

$$(v_G)_2 = \omega_2 r_{G/IC} = \omega_2(1\cos 45°) = 0.7071\omega_2$$

The mass moment of inertia of the plate about its mass center is $I_G = \frac{1}{12}m\left(a^2 + b^2\right) = \frac{1}{12}(60)\left(1^2 + 1^2\right) = 10\text{ kg}\cdot\text{m}^2$. Thus, the final kinetic energy is

$$T_2 = \frac{1}{2}m(v_G)_2{}^2 + \frac{1}{2}I_G\omega_2{}^2$$

$$= \frac{1}{2}m(60)(0.7071\omega_2)^2 + \frac{1}{2}(10)\omega_2{}^2$$

$$= 20\omega_2{}^2$$

Referring to Fig. $b$, $\mathbf{N}_A$ and $\mathbf{N}_B$ do no work, while $\mathbf{P}$ does positive work, and $\mathbf{W}$ does negative work. When $\theta = 45°$, $\mathbf{W}$ and $\mathbf{P}$ displace upwards through a distance of $h = 1\cos 45° - 0.5 = 0.2071$ m and $s_P = 2(1\cos 45°) - 1 = 0.4142$ m. Thus, the work done by $\mathbf{P}$ and $\mathbf{W}$ is

$$U_P = Ps_P = 500(0.4142) = 207.11\text{ J}$$

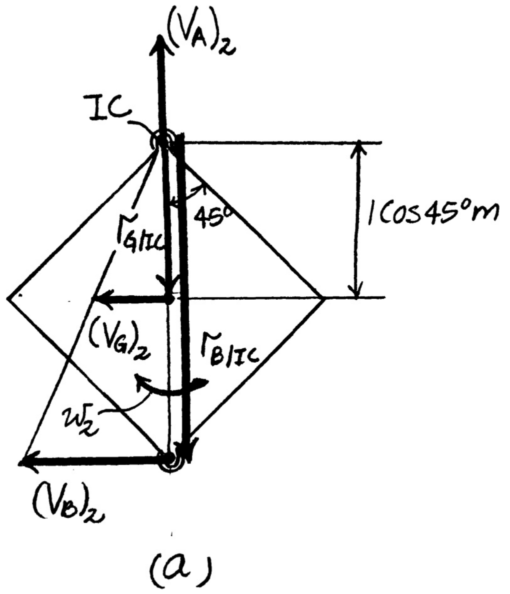
$$U_W = -Wh = -60(9.81)(0.2071) = -121.90\text{ J}$$

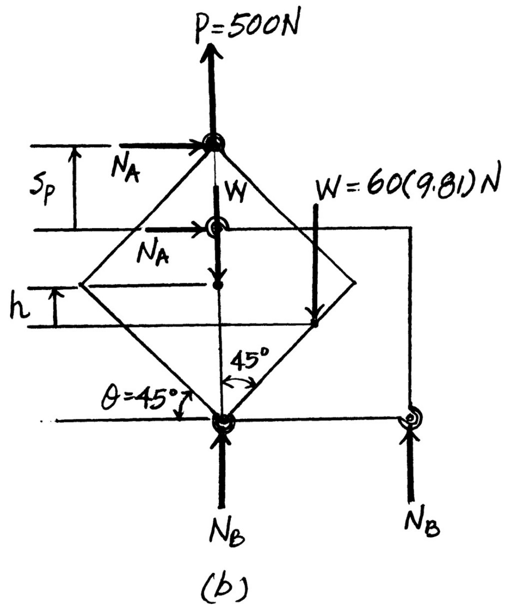
**Principle of Work and Energy:**

$$T_1 + \Sigma U_{1\text{-}2} = T_2$$

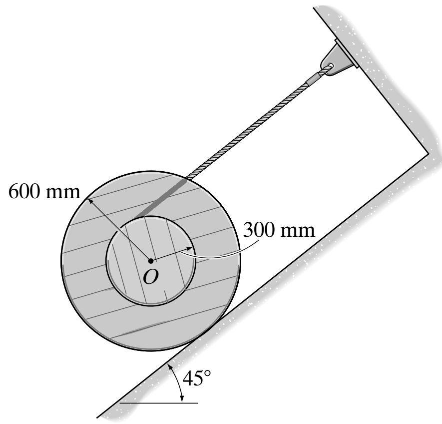
$$0 + [207.11 - 121.90] = 20\omega_2{}^2$$

$$\omega_2 = 2.06\text{ rad/s} \qquad \textbf{Ans.}$$



**•18–25.** The spool has a mass of 100 kg and a radius of gyration of 400 mm about its center of mass $O$. If it is released from rest, determine its angular velocity after its center $O$ has moved down the plane a distance of 2 m. The contact surface between the spool and the inclined plane is smooth.




**Kinetic Energy and Work:** Referring to Fig. *a*,

$$v_O = \omega r_{O/IC} = \omega(0.3)$$

The mass moment of inertia of the spool about its mass center is $I_O = mk_O{}^2 = 100\left(0.4^2\right) = 16 \text{ kg} \cdot \text{m}^2$. Thus, the final kinetic energy of the spool is

$$T = \frac{1}{2}mv_O{}^2 + \frac{1}{2}I_O\omega^2$$

$$= \frac{1}{2}(100)[\omega(0.3)]^2 + \frac{1}{2}(16)\omega^2$$

$$= 12.5\omega^2$$

Since the spool is initially at rest, $T_1 = 0$. Referring to Fig. *b*, **T** and **N** do no work, while **W** does positive work. When the center of the spool moves down the plane through a distance of $s_O = 2$ m, **W** displaces vertically downward $h = s_O \cos 45° = 2 \cos 45° = 1.4142$ m. Thus, the work done by **W** is

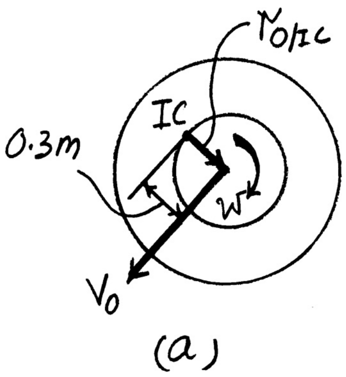
$$U_W = Wh = 100(9.81)(1.4142) = 1387.34 \text{ N}$$

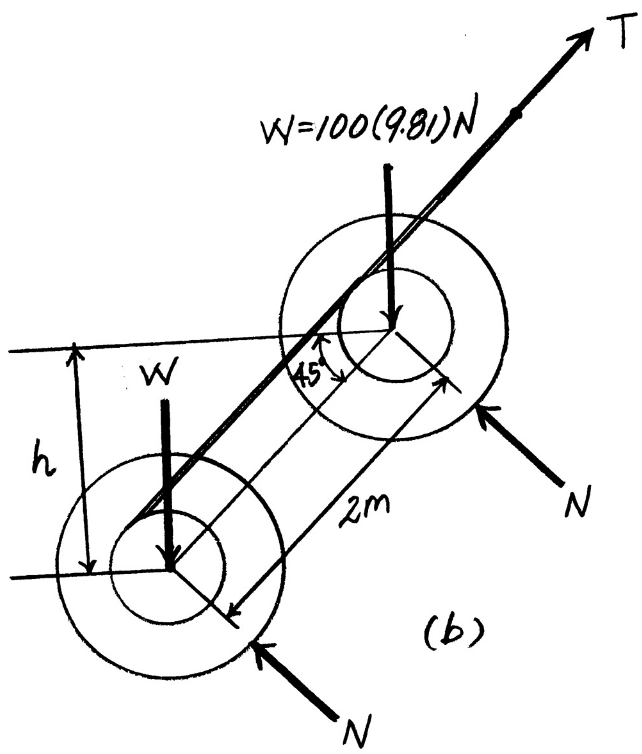
**Principle of Work and Energy:**

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 1387.34 = 12.5\omega^2$$

$$\omega = 10.5 \text{ rad/s} \qquad \textbf{Ans.}$$

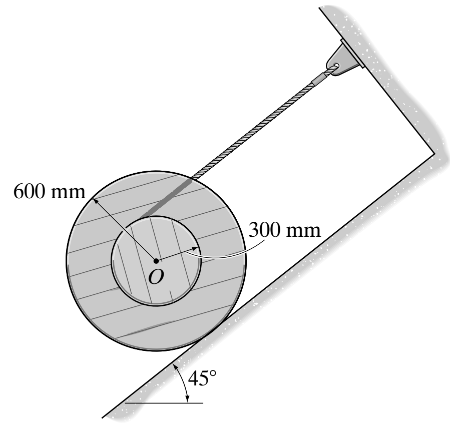





لجنة الميكانيك - الإتجاه الإسلامي

**18–26.** The spool has a mass of 100 kg and a radius of gyration of 400 mm about its center of mass *O*. If it is released from rest, determine its angular velocity after its center *O* has moved down the plane a distance of 2 m. The coefficient of kinetic friction between the spool and the inclined plane is $\mu_k = 0.15$.




**Kinetic Energy and Work:** Referring to Fig. *a*,

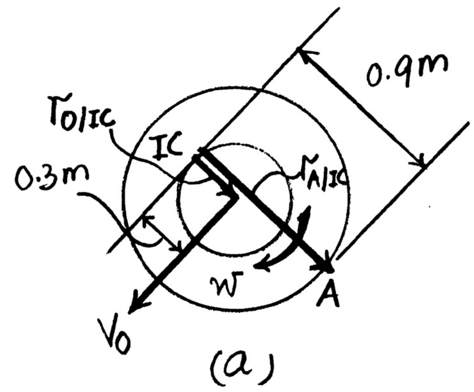
$$v_O = \omega r_{O/IC} = \omega(0.3)$$

The mass moment of inertia of the spool about its mass center is $I_O = mk_O{}^2 = 100(0.4^2) = 16\ \text{kg}\cdot\text{m}^2$. Thus, the kinetic energy of the spool is

$$T = \frac{1}{2}mv_O{}^2 + \frac{1}{2}I_O\omega^2$$

$$= \frac{1}{2}(100)\left[\omega(0.3)\right]^2 + \frac{1}{2}(16)\omega^2$$

$$= 12.5\omega^2$$



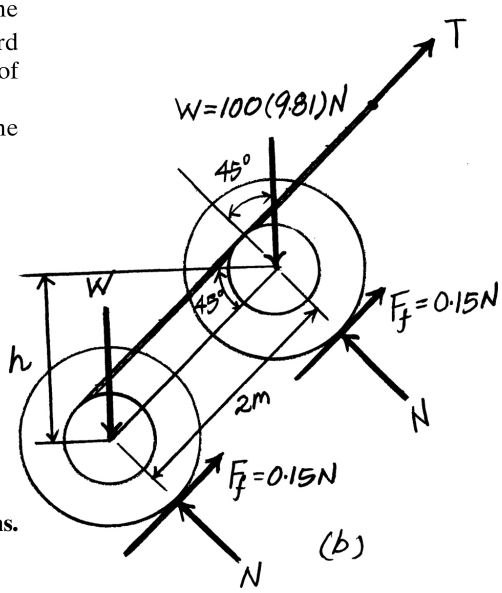

Since the spool is initially at rest, $T_1 = 0$. Referring to Fig. *b*, **T** and **N** do no work, while **W** does positive work, and $\mathbf{F}_f$ does negative work. Since the spool slips at the contact point on the inclined plane, $F_f = \mu_k N = 0.15N$, where **N** can be obtained using the equation of motion,

$$\Sigma F_{y'} = m(a_a)_{y'}; \qquad N - 100(9.81)\cos 45° = 0 \qquad N = 693.67\ \text{N}$$

Thus, $F_f = 0.15(693.67) = 104.05$ N. When the center of the spool moves down the inclined plane through a distance of $s_O = 2$ m, **W** displaces vertically downward $h = s_O \sin 45° = 2\sin 45° = 1.4142$ m. Also, the contact point *A* on the outer rim of the spool travels a distance of $s_A = \left(\frac{r_{A/IC}}{r_{O/IC}}\right)s_O = \frac{0.9}{0.3}(2) = 6$ m, Fig. *a*. Thus, the work done by **W** and $\mathbf{F}_f$ is

$$U_W = Wh = 100(9.81)(1.4142) = 1387.34\ \text{J}$$

$$U_{F_f} = -F_f s_A = -104.05(6) = -624.30\ \text{J}$$



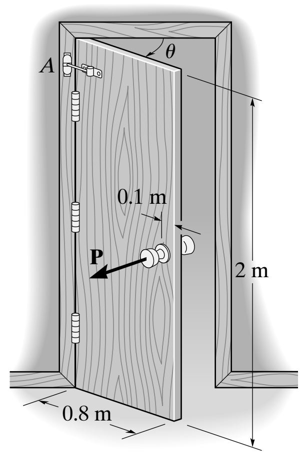

**Principle of Work and Energy:**

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [1387.34 - 624.30] = 12.5\omega^2$$

$$\omega = 7.81\ \text{rad/s} \qquad \textbf{Ans.}$$

**18–27.** The uniform door has a mass of 20 kg and can be treated as a thin plate having the dimensions shown. If it is connected to a torsional spring at $A$, which has a stiffness of $k = 80\ \text{N}\cdot\text{m/rad}$, determine the required initial twist of the spring in radians so that the door has an angular velocity of 12 rad/s when it closes at $\theta = 0°$ after being opened at $\theta = 90°$ and released from rest. *Hint:* For a torsional spring $M = k\theta$, when $k$ is the stiffness and $\theta$ is the angle of twist.

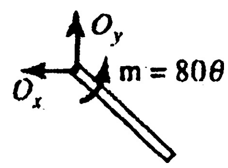



$$T_1 + \Sigma U_{1-2} = T_2$$

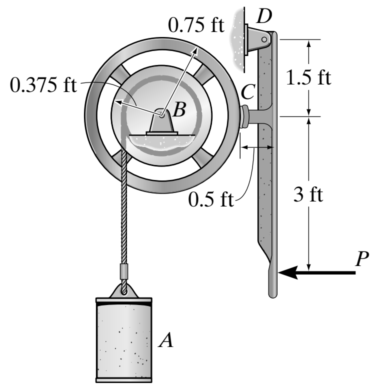
$$0 + \int_{\theta_O}^{\theta_O+\frac{\pi}{2}} 80\theta\, d\theta = \frac{1}{2}\left[\frac{1}{3}(20)(0.8)^2\right](12)^2$$

$$40\left[\left(\theta_O + \tfrac{\pi}{2}\right)^2 - \theta_0^2\right] = 307.2$$

$$\theta_O = 1.66 \text{ rad}$$ **Ans.**



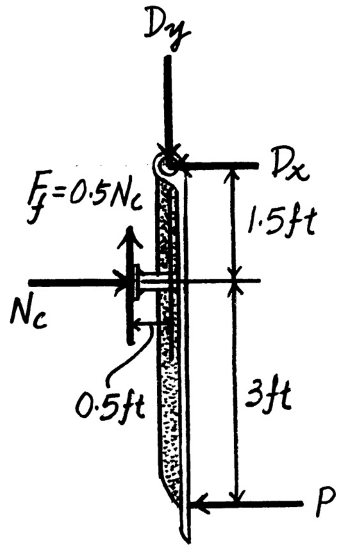
***18–28.** The 50-lb cylinder $A$ is descending with a speed of 20 ft/s when the brake is applied. If wheel $B$ must be brought to a stop after it has rotated 5 revolutions, determine the constant force **P** that must be applied to the brake arm. The coefficient of kinetic friction between the brake pad $C$ and the wheel is $\mu_k = 0.5$. The wheel's weight is 25 lb, and the radius of gyration about its center of mass is $k = 0.6$ ft.




**Equilibrium:** Referring to Fig. $a$, we have

$$\circlearrowleft + \Sigma M_D = 0; \qquad N_C(1.5) - 0.5N_C(0.5) - P(4.5) = 0$$

$$N_C = 3.6\,P$$




(a)

**Kinetic Energy and Work:** Since the wheel rotates about a fixed axis, $(\omega_B)_1 = \dfrac{(v_A)_1}{r_A} = \dfrac{20}{0.375} = 53.33$ rad/s. The mass moment of inertia of the wheel about its mass center is $I_B = m_B k^2 = \dfrac{25}{32.2}\left(0.6^2\right) = 0.2795\ \text{slug}\cdot\text{ft}^2$. Thus, the initial kinetic energy of the system is

$$T_1 = (T_A)_1 + (T_B)_1$$

$$= \frac{1}{2}m_A(v_A)_1^2 + \frac{1}{2}I_B(\omega_B)_1^2$$

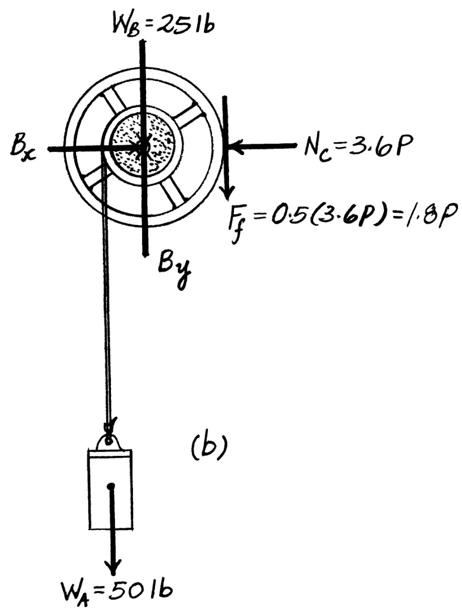
$$= \frac{1}{2}\left(\frac{50}{32.2}\right)\left(20^2\right) + \frac{1}{2}(0.2795)\left(53.33^2\right)$$

$$= 708.07\ \text{ft}\cdot\text{lb}$$

Since the system is brought to rest, $T_2 = 0$. Referring to Fig. $b$, $\mathbf{B}_x$, $\mathbf{B}_y$, $\mathbf{W}_B$, and $\mathbf{N}_C$ do no work, while $\mathbf{W}_A$ does positive work, and $\mathbf{F}_f$ does negative work. When wheel $B$ rotates through the angle $\theta = (5\ \text{rev})\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 10\pi$ rad, $\mathbf{W}_A$ displaces $s_A = r_A\theta = 0.375(10\pi) = 3.75\pi$ ft vertically downward, and the contact point $C$ on the outer rim of the wheel travels a distance of $s_C = r_B\theta = 0.75(10\pi) = 7.5\pi$. Thus, the work done by $\mathbf{W}_A$ and $\mathbf{F}_f$ is

$$U_{W_A} = W_A s_A = 50(3.75\pi) = 187.5\pi\ \text{ft}\cdot\text{lb}$$

$$U_{F_f} = -F_f s_C = -1.8P(7.5\pi) = -13.5\pi P$$




(b)

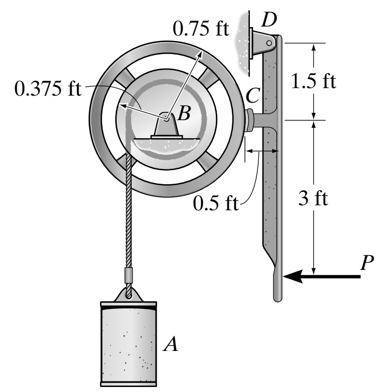
**Principle of Work and Energy:**

$$T_1 + \Sigma U_{1-2} = T_2$$

$$708.07 + [187.5\pi - 13.5\pi P] = 0$$

$$P = 30.6\ \text{lb}$$ **Ans.**

**•18–29.** When a force of $P = 30$ lb is applied to the brake arm, the 50-lb cylinder $A$ is descending with a speed of 20 ft/s. Determine the number of revolutions wheel $B$ will rotate before it is brought to a stop. The coefficient of kinetic friction between the brake pad $C$ and the wheel is $\mu_k = 0.5$. The wheel's weight is 25 lb, and the radius of gyration about its center of mass is $k = 0.6$ ft.




**Equilibrium:** Referring to Fig. $a$,

$$\zeta + \Sigma M_D = 0; \qquad N_C(1.5) - 0.5N_C(0.5) - 30(4.5) = 0$$

$$N_C = 108 \text{ lb}$$

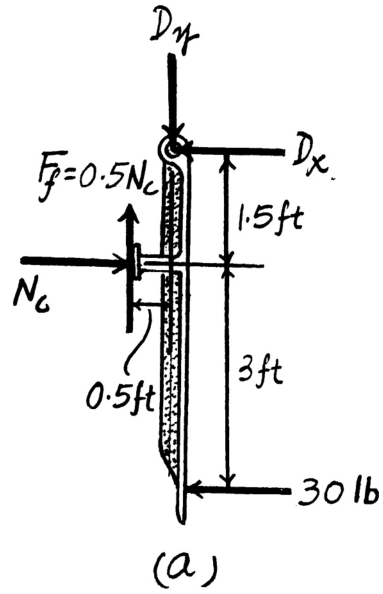
**Kinetic Energy and Work:** Since the wheel rotates about a fixed axis, $(\omega_B)_1 = \dfrac{(v_A)_1}{r_A} = \dfrac{20}{0.375} = 53.33$ rad/s. The mass moment of inertia of the wheel about its mass center is $I_B = m_B k^2 = \dfrac{25}{32.2}\left(0.6^2\right) = 0.2795 \text{ slug}\cdot\text{ft}^2$. Thus, the initial kinetic energy of the system is

$$T_1 = (T_A)_1 + (T_B)_1$$

$$= \frac{1}{2}m_A(v_A)_1^2 + \frac{1}{2}I_B(\omega_B)_1^2$$

$$= \frac{1}{2}\left(\frac{50}{32.2}\right)\left(20^2\right) + \frac{1}{2}(0.2795)\left(53.33^2\right)$$

$$= 708.07 \text{ ft}\cdot\text{lb}$$




Since the system is brought to rest, $T_2 = 0$. Referring to Fig. $b$, $\mathbf{B}_x$, $\mathbf{B}_y$, $\mathbf{W}_B$, and $N_C$ do no work, while $\mathbf{W}_A$ does positive work, and $\mathbf{F}_f$ does negative work. When wheel $B$ rotates through the angle $\theta$, $\mathbf{W}_A$ displaces $s_A = r_A\theta = 0.375\theta$ and the contact point on the outer rim of the wheel travels a distance of $s_C = r_B\theta = 0.75\theta$. Thus, the work done by $\mathbf{W}_A$ and $\mathbf{F}_f$ are

$$U_{W_A} = W_A s_A = 50(0.375\theta) = 18.75\theta$$

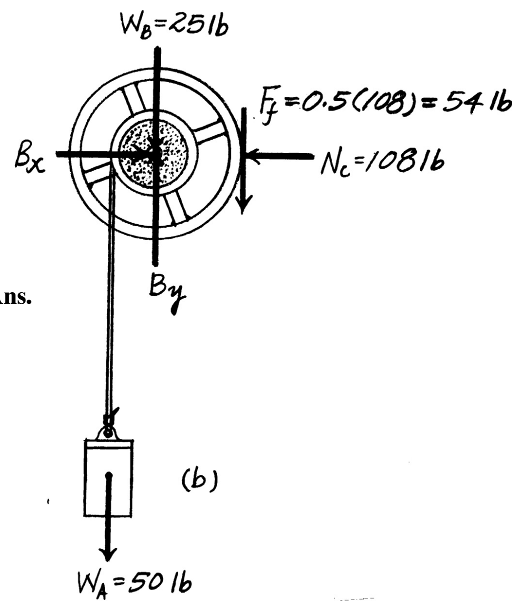
$$U_{F_f} = -F_f s_C = -0.5(108)(0.75\theta) = -40.5\theta$$

**Principle of Work and Energy:**

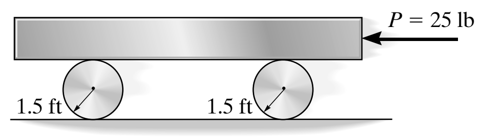
$$T_1 + \Sigma U_{1-2} = T_2$$

$$708.07 + [18.75\theta - 40.5\theta] = 0$$

$$\theta = 32.55 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi}\right) = 5.18 \text{ rev}$$ **Ans.**



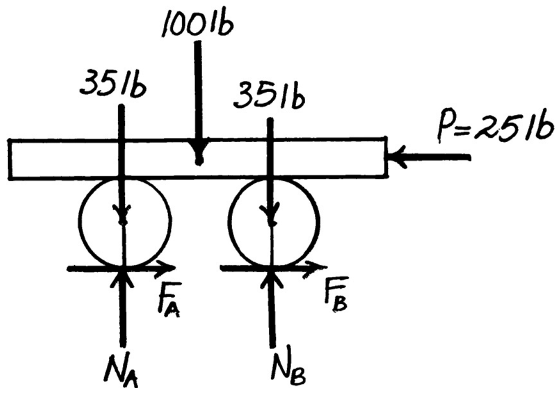
**18–30.** The 100-lb block is transported a short distance by using two cylindrical rollers, each having a weight of 35 lb. If a horizontal force $P = 25$ lb is applied to the block, determine the block's speed after it has been displaced 2 ft to the left. Originally the block is at rest. No slipping occurs.

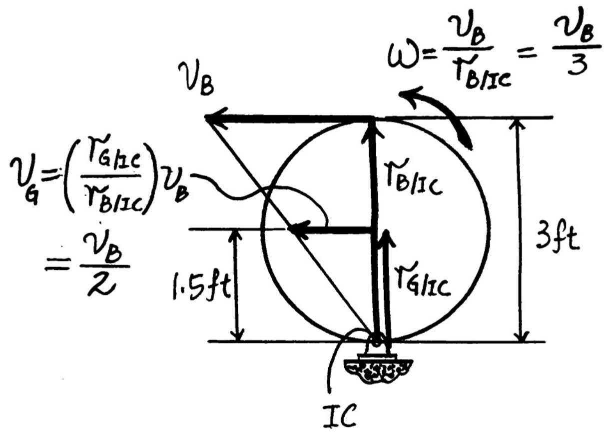



$$T_1 + \Sigma U_{1-2} = T_2$$

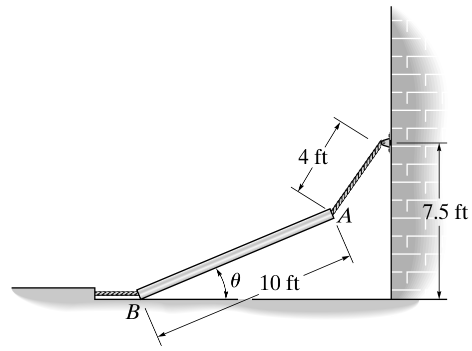
$$0 + 25(2) = \frac{1}{2}\left(\frac{100}{32.2}\right)(v_B)^2 + 2\left[\frac{1}{2}\left(\frac{35}{32.2}\right)\left(\frac{v_B}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{2}\left(\frac{35}{32.2}\right)(1.5)^2\right)\left(\frac{v_B}{3}\right)^2\right]$$

$$v_B = 5.05 \text{ ft/s} \qquad \textbf{Ans.}$$




**18–31.** The slender beam having a weight of 150 lb is supported by two cables. If the cable at end $B$ is cut so that the beam is released from rest when $\theta = 30°$, determine the speed at which end $A$ strikes the wall. Neglect friction at $B$.

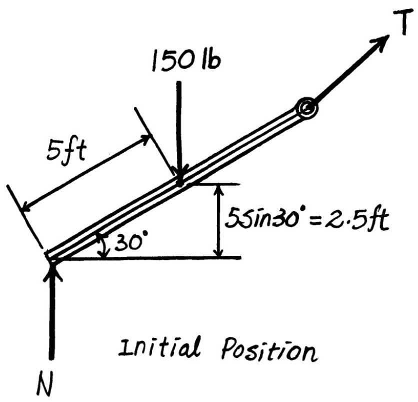


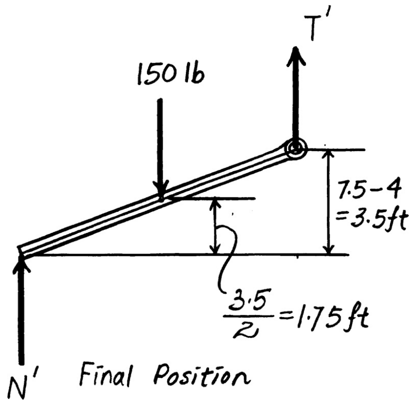

In the final position, the rod is in translation since the $IC$ is at infinity.

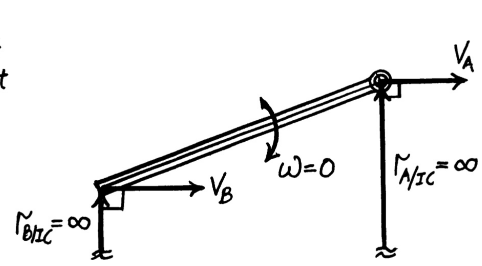
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 150(2.5 - 1.75) = \frac{1}{2}\left(\frac{150}{32.2}\right)v_G^2$$

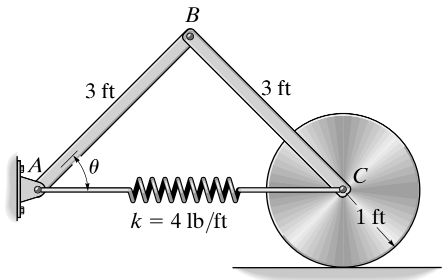
$$v_G = v_A = 6.95 \text{ ft/s} \qquad \textbf{Ans.}$$



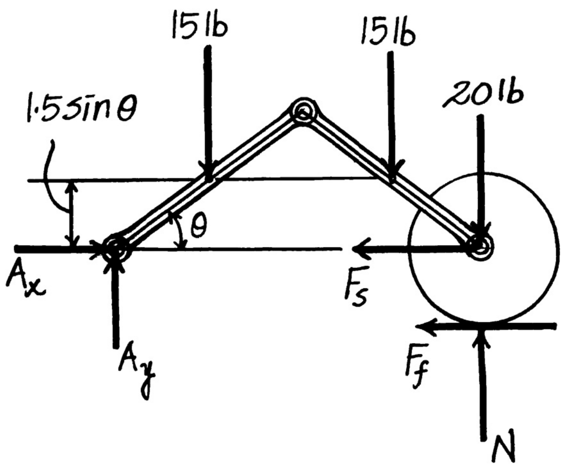
***18–32.** The assembly consists of two 15-lb slender rods and a 20-lb disk. If the spring is unstretched when $\theta = 45°$ and the assembly is released from rest at this position, determine the angular velocity of rod $AB$ at the instant $\theta = 0°$. The disk rolls without slipping.

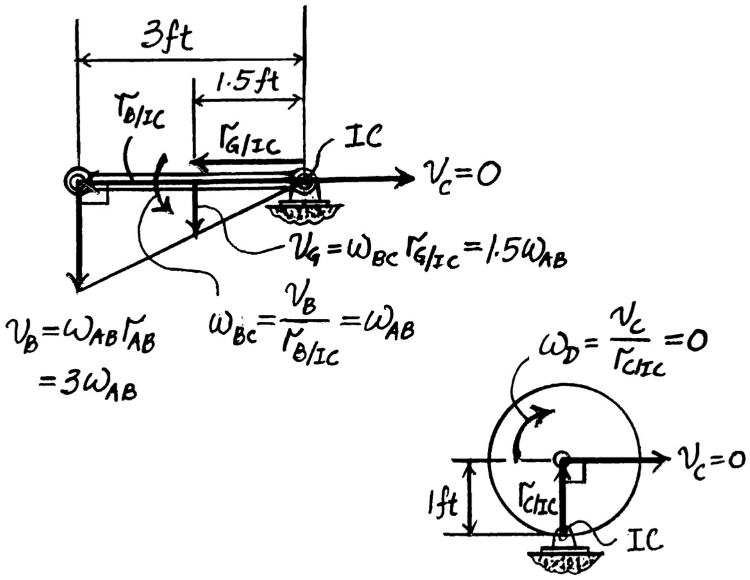



$$T_1 + \Sigma U_{1-2} = T_2$$

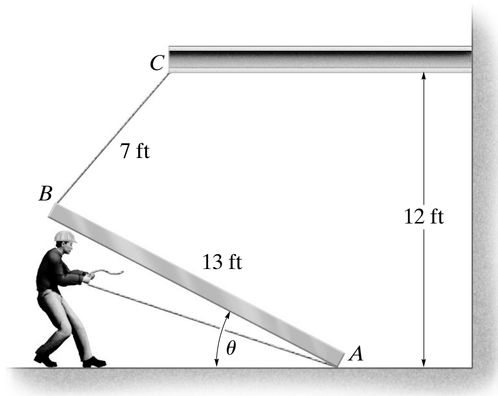
$$[0 + 0] + 2(15)(1.5) \sin 45° - \frac{1}{2}(4)\left[6 - 2(3) \cos 45°\right]^2 = 2\left[\frac{1}{2}\left(\frac{1}{3}\left(\frac{15}{32.2}\right)(3)^2\right)\omega_{AB}^2\right]$$

$$\omega_{AB} = 4.28 \text{ rad/s}$$ **Ans.**



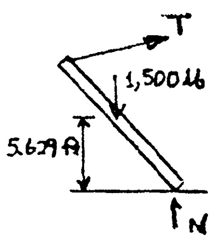

**18–33.** The beam has a weight of 1500 lb and is being raised to a vertical position by pulling very slowly on its bottom end $A$. If the cord fails when $\theta = 60°$ and the beam is essentially at rest, determine the speed of $A$ at the instant cord $BC$ becomes vertical. Neglect friction and the mass of the cords, and treat the beam as a slender rod.

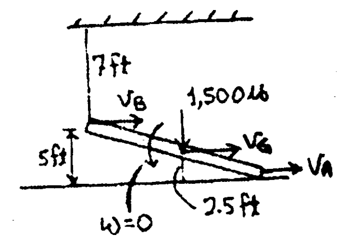



$$T_1 + \Sigma U_{1-2} = T_2$$

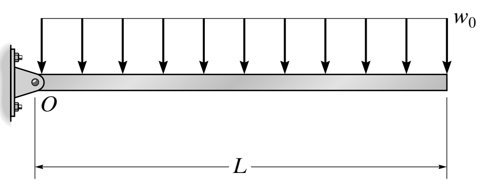
$$0 + 1500(5.629) - 1500(2.5) = \frac{1}{2}\left(\frac{1500}{32.2}\right)(v_G)^2$$

$$v_G = v_A = 14.2 \text{ ft/s}$$ **Ans.**



**18–34.** The uniform slender bar that has a mass $m$ and a length $L$ is subjected to a uniform distributed load $w_0$, which is always directed perpendicular to the axis of the bar. If the bar is released from rest from the position shown, determine its angular velocity at the instant it has rotated 90°. Solve the problem for rotation in (a) the horizontal plane, and (b) the vertical plane.




a)

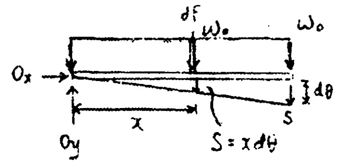
$$T_1 + \Sigma U_{1-2} = T_2$$

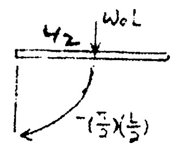
$$[0] + \int_0^{\frac{\pi}{2}} \int_0^{L} (w_0\,dx)(x\,d\theta) = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2$$

$$\int_0^{\frac{\pi}{2}} \frac{w_0 L^2}{2} d\theta = \frac{1}{6}mL^2\omega^2$$

$$\frac{w_0 L^2}{2}\left(\frac{\pi}{2}\right) = \frac{1}{6}mL^2\omega^2$$

$$\omega = \sqrt{\frac{3\pi}{2}\left(\frac{w_0}{m}\right)}$$ **Ans.**




**Note:** The work of the distributed load can also be determined from its resultant.

$$U_{1-2} = w_0 L\left(\frac{\pi}{2}\right)\left(\frac{L}{2}\right) = \frac{w_0}{4}\pi L^2$$

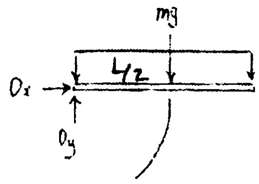
b)

$$T_1 + \Sigma U_{1-2} = T_2$$

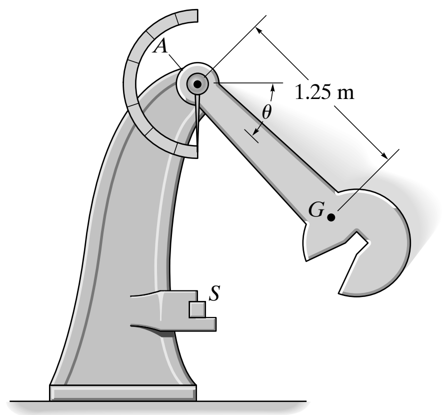
$$[0] + \frac{w_0}{4}\pi L^2 + mg\left(\frac{L}{2}\right) = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2$$

$$\omega^2 = \frac{3}{2}\frac{w_0\pi L}{mL} + \frac{mg(6)}{2mL}$$

$$\omega = \sqrt{\frac{3\pi}{2}\frac{w_0}{m} + \frac{3g}{L}}$$ **Ans.**




**18–35.** Solve Prob. 18–5 using the conservation of energy equation.



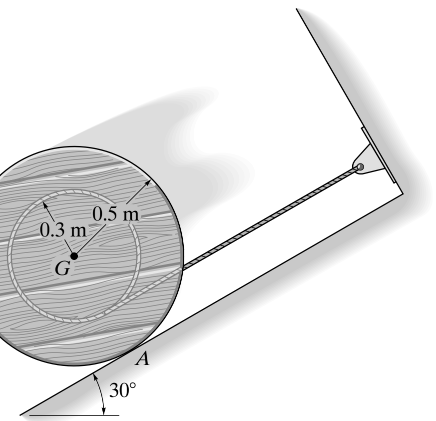

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 50(9.81)(1.25) = \frac{1}{2}\left[50(1.75)^2\right]\omega^2 + 0$$

$$\omega = 2.83 \text{ rad/s}$$ **Ans.**

***18–36.** Solve Prob. 18–12 using the conservation of energy equation.

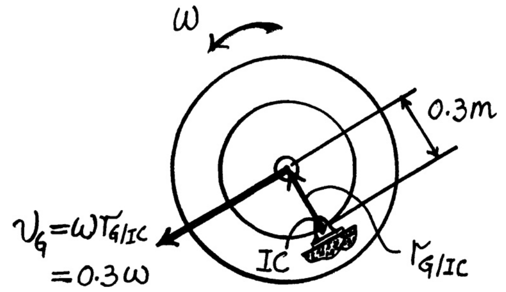



Datum at lowest point through $G$.

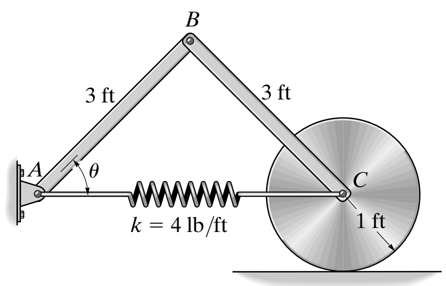
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 60(9.81)(s \sin 30°) = \frac{1}{2}\left[60(0.3)^2\right](6)^2 + \frac{1}{2}(60)\left[(0.3)(6)\right]^2 + 0$$

$$s = 0.661 \text{ m}$$ **Ans.**




**•18–37.** Solve Prob. 18–32 using the conservation of energy equation.

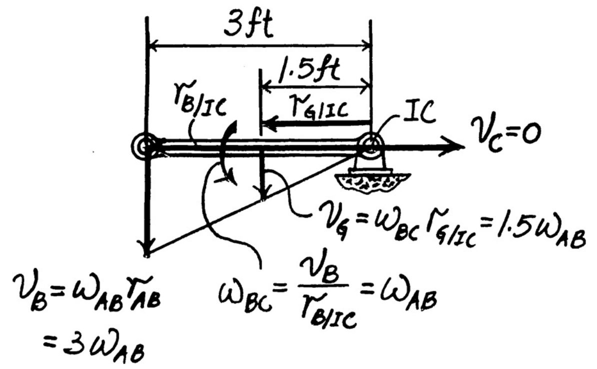


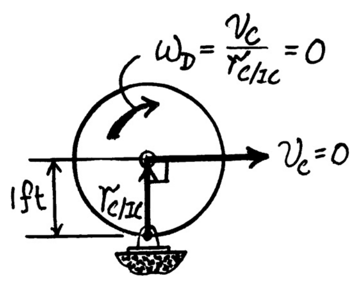

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

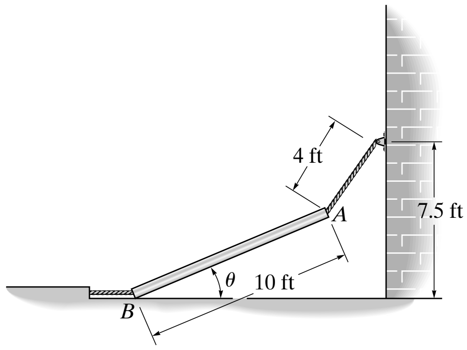
$$0 + 2\left[15(1.5 \sin 45°)\right] = 2\left[\frac{1}{2}\left(\frac{1}{3}\left(\frac{15}{32.2}\right)(3)^2\right)\omega_{AB}^2\right] + \frac{1}{2}(4)\left[6 - 2(3 \cos 45°)\right]^2 + 0$$

$$\omega_{AB} = 4.28 \text{ rad/s}$$ **Ans.**



**18–38.** Solve Prob. 18–31 using the conservation of energy equation.



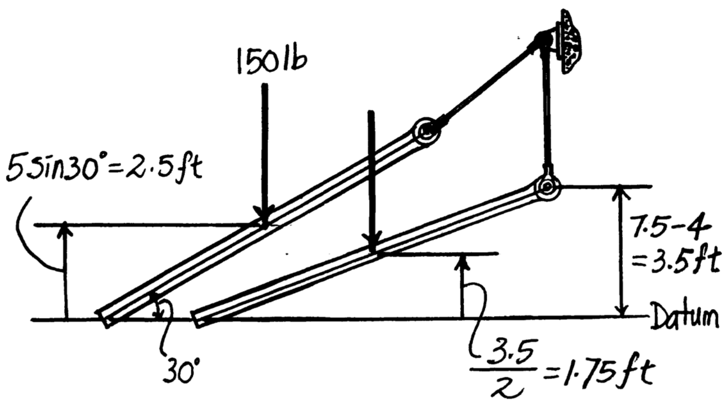

$$T_1 + V_1 = T_2 + V_2$$

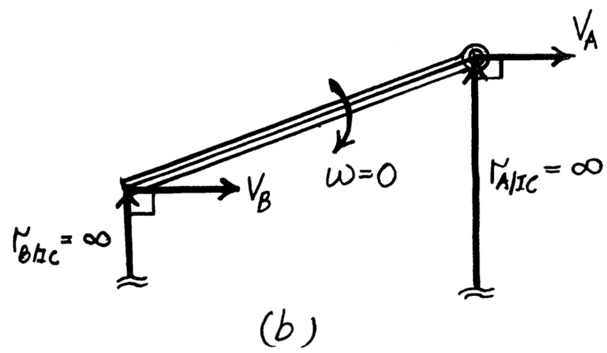
$$0 + 150(2.5) = \frac{1}{2}\left(\frac{150}{32.2}\right)v_G^2 + 150(1.75)$$

$$v_G = 6.95 \text{ ft/s}$$

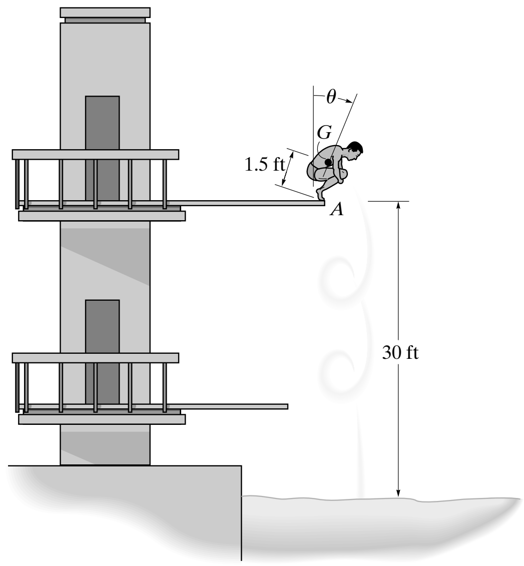
Since the rod is in translation at the final instant, then

$$v_A = 6.95 \text{ ft/s} \qquad \textbf{Ans.}$$




**18–39.** Solve Prob. 18–11 using the conservation of energy equation.




Datum at $A$.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 150(1.5) = \frac{1}{2}\left[\left(\frac{150}{32.2}\right)(1.2)^2\right]\omega^2 + \frac{1}{2}\left(\frac{150}{32.2}\right)(1.5\omega)^2 + 0$$

$$\omega = 5.117 \text{ rad/s}$$

Time to fall:

$$s = s_0 + v_0 t + \tfrac{1}{2}a_c t^2$$

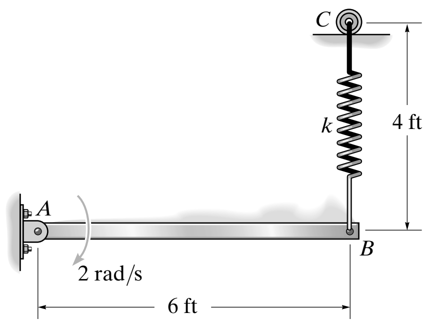
$$30 = 0 + 1.5(5.117)t + \frac{1}{2}(32.2)t^2$$

Choosing the positive root: $t = 1.147$ s

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$

$$\theta = 0 + 5.117(1.147) + 0 = 5.870 \text{ rad} = 0.934 \text{ rev.} \qquad \textbf{Ans.}$$

***18–40.** At the instant shown, the 50-lb bar rotates clockwise at 2 rad/s. The spring attached to its end always remains vertical due to the roller guide at *C*. If the spring has an unstretched length of 2 ft and a stiffness of $k = 6$ lb/ft, determine the angular velocity of the bar the instant it has rotated 30° clockwise.



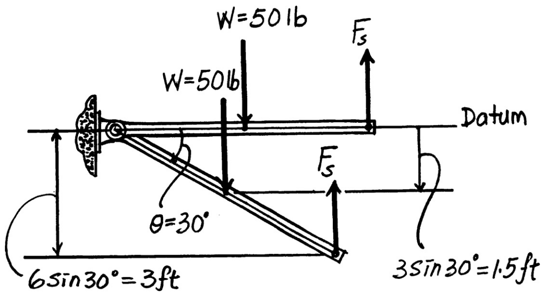

Datum through *A*.

$$T_1 + V_1 = T_2 + V_2$$

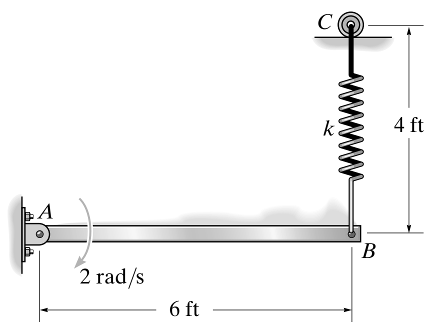
$$\frac{1}{2}\left[\frac{1}{3}\left(\frac{50}{32.2}\right)(6)^2\right](2)^2 + \frac{1}{2}(6)(4-2)^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{50}{32.2}\right)(6)^2\right]\omega^2$$

$$+\frac{1}{2}(6)(7-2)^2 - 50(1.5)$$

$$\omega = 2.30 \text{ rad/s} \qquad \textbf{Ans.}$$




**•18–41.** At the instant shown, the 50-lb bar rotates clockwise at 2 rad/s. The spring attached to its end always remains vertical due to the roller guide at *C*. If the spring has an unstretched length of 2 ft and a stiffness of $k = 12$ lb/ft, determine the angle $\theta$, measured from the horizontal, to which the bar rotates before it momentarily stops.




$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{3}\left(\frac{50}{32.2}\right)(6)^2\right](2)^2 + \frac{1}{2}(12)(4-2)^2 = 0 + \frac{1}{2}(12)(4 + 6\sin\theta - 2)^2 - 50(3\sin\theta)$$

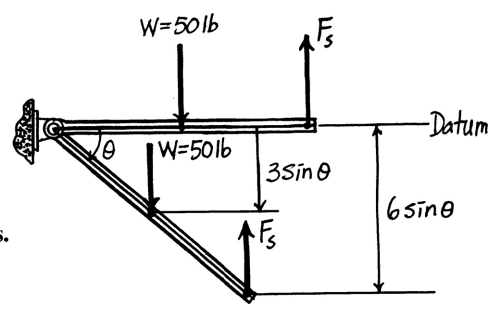
$$61.2671 = 24(1 + 3\sin\theta)^2 - 150\sin\theta$$

$$37.2671 = -6\sin\theta + 216\sin^2\theta$$

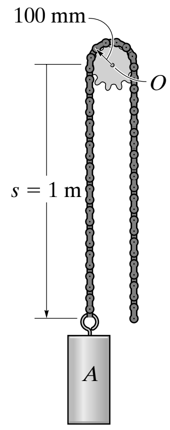
Set $x = \sin\theta$, and solve the quadratic equation for the positive root:

$$\sin\theta = 0.4295$$

$$\theta = 25.4° \qquad \textbf{Ans.}$$



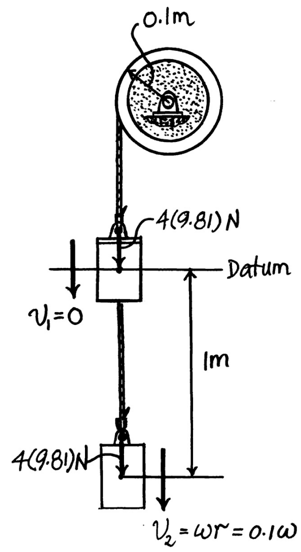
**18–42.** A chain that has a negligible mass is draped over the sprocket which has a mass of 2 kg and a radius of gyration of $k_O = 50$ mm. If the 4-kg block $A$ is released from rest from the position $s = 1$ m, determine the angular velocity of the sprocket at the instant $s = 2$ m.



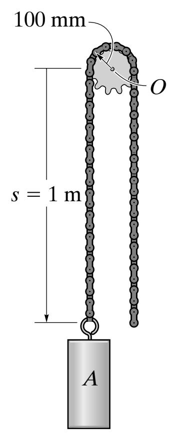

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 + 0 = \frac{1}{2}(4)(0.1\,\omega)^2 + \frac{1}{2}\left[2(0.05)^2\right]\omega^2 - 4(9.81)(1)$$

$$\omega = 41.8 \text{ rad/s} \qquad \textbf{Ans.}$$




**18–43.** Solve Prob. 18–42 if the chain has a mass per unit length of 0.8 kg/m. For the calculation neglect the portion of the chain that wraps over the sprocket.

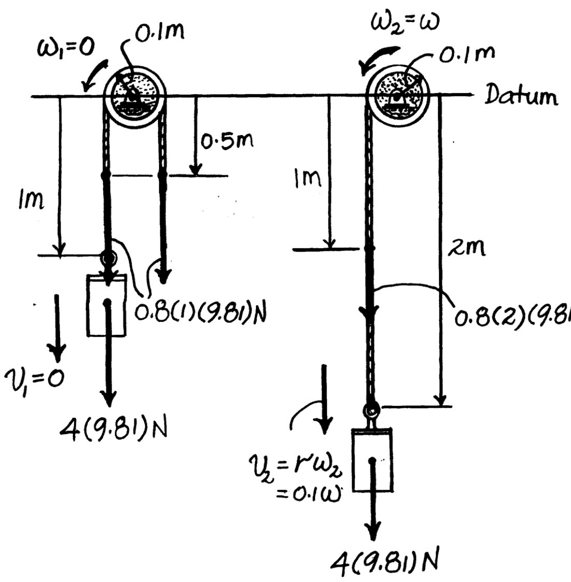



$$T_1 + V_1 = T_2 + V_2$$

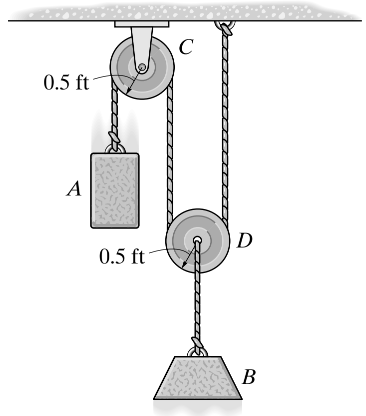
$$0 - 4(9.81)(1) - 2\left[0.8(1)(9.81)(0.5)\right] = \frac{1}{2}(4)(0.1\,\omega)^2 + \frac{1}{2}\left[2(0.05)^2\right]\omega^2$$

$$+ \frac{1}{2}(0.8)(2)(0.1\,\omega)^2 - 4(9.81)(2) - 0.8(2)(9.81)(1)$$

$$\omega = 39.3 \text{ rad/s} \qquad \textbf{Ans.}$$



***18–44.** The system consists of 60-lb and 20-lb blocks *A* and *B*, respectively, and 5-lb pulleys *C* and *D* that can be treated as thin disks. Determine the speed of block *A* after block *B* has risen 5 ft, starting from rest. Assume that the cord does not slip on the pulleys, and neglect the mass of the cord.




**Kinematics:** The speed of block *A* and *B* can be related using the position coordinate equation.

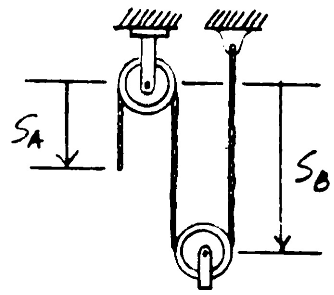
$$s_A + 2s_B = l \quad \textbf{(1)}$$

$$\Delta s_A + 2\Delta s_B = 0 \qquad \Delta s_A + 2(5) = 0 \qquad \Delta s_A = -10 \text{ ft} = 10 \text{ ft}\downarrow$$

Taking time derivative of Eq. (1), we have

$$v_A + 2v_B = 0 \qquad v_B = -0.5v_A$$

**Potential Energy:** Datum is set at fixed pulley *C*. When blocks *A* and *B* (pulley *D*) are at their initial position, their centers of gravity are located at $s_A$ and $s_B$. Their initial gravitational potential energies are $-60s_A$, $-20s_B$, and $-5s_B$. When block *B* (pulley *D*) rises 5 ft, block *A* decends 10 ft. Thus, the final position of blocks *A* and *B* (pulley *D*) are $(s_A + 10)$ ft and $(s_B - 5)$ ft *below* datum. Hence, their respective final gravitational potential energy are $-60(s_A + 10)$, $-20(s_B - 5)$, and $-5(s_B - 5)$. Thus, the initial and final potential energy are




$$V_1 = -60s_A - 20s_B - 5s_B = -60s_A - 25s_B$$

$$V_2 = -60(s_A + 10) - 20(s_B - 5) - 5(s_B - 5) = -60s_A - 25s_B - 475$$

**Kinetic Energy:** The mass moment inertia of the pulley about its mass center is $I_G = \frac{1}{2}\left(\frac{5}{32.2}\right)(0.5^2) = 0.01941 \text{ slug}\cdot\text{ft}^2$. Since pulley *D* rolls without slipping, $\omega_D = \frac{v_B}{r_D} = \frac{v_B}{0.5} = 2v_B = 2(-0.5v_A) = -v_A$. Pulley *C* rotates about the fixed point hence $\omega_C = \frac{v_A}{r_C} = \frac{v_A}{0.5} = 2v_A$. Since the system is at initially rest, the initial kinectic energy is $T_1 = 0$. The final kinetic energy is given by

$$T_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 + \frac{1}{2}m_D v_B^2 + \frac{1}{2}I_G\omega_D^2 + \frac{1}{2}I_G\,\omega_C^2$$

$$= \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)(-0.5v_A)^2 + \frac{1}{2}\left(\frac{5}{32.2}\right)(-0.5v_A)^2$$

$$+ \frac{1}{2}(0.01941)(-v_A)^2 + \frac{1}{2}(0.01941)(2v_A)^2$$

$$= 1.0773v_A^2$$

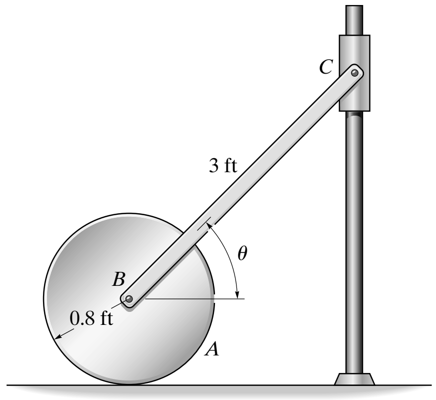
**Conservation of Energy:** Applying Eq. 18–19, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-60s_A - 25s_B) = 1.0773v_A^2 + (-60s_A - 25s_B - 475)$$

$$v_4 = 21.0 \text{ ft/s} \quad \textbf{Ans.}$$

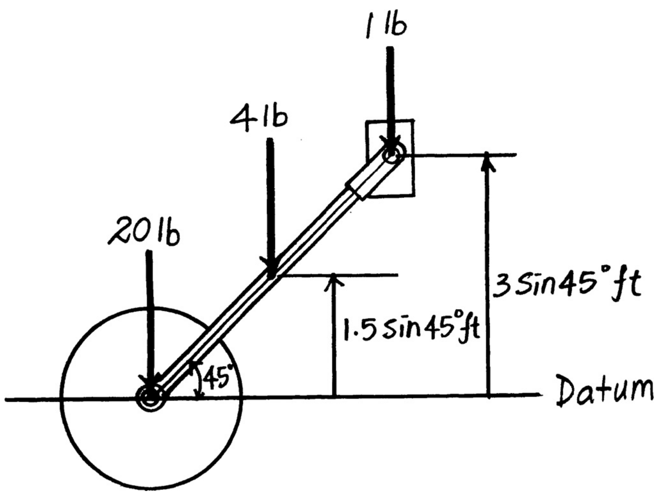
**•18–45.** The system consists of a 20-lb disk $A$, 4-lb slender rod $BC$, and a 1-lb smooth collar $C$. If the disk rolls without slipping, determine the velocity of the collar at the instant the rod becomes horizontal, i.e., $\theta = 0°$. The system is released from rest when $\theta = 45°$.

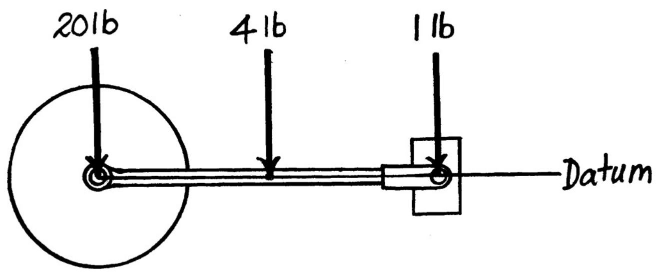


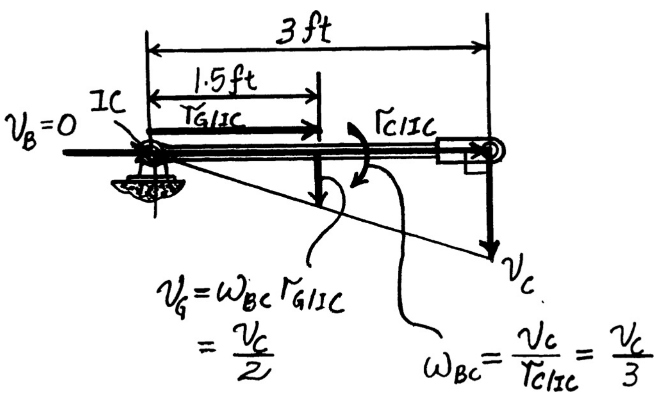

$$T_1 + V_1 = T_2 + V_2$$

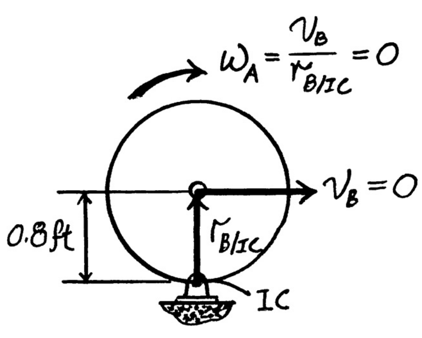
$$0 + 4(1.5 \sin 45°) + 1(3 \sin 45°) = \frac{1}{2}\left[\frac{1}{3}\left(\frac{4}{32.2}\right)(3)^2\right]\left(\frac{v_C}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{32.2}\right)(v_C)^2 + 0$$

$$v_C = 13.3 \text{ ft/s}$$ **Ans.**

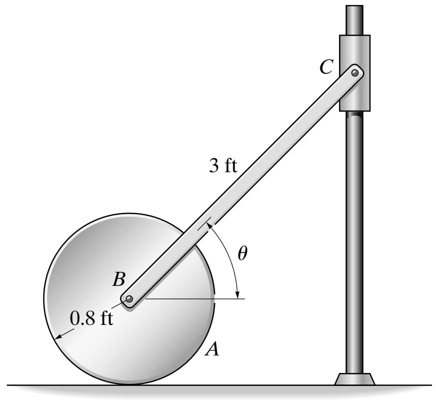


**18–46.** The system consists of a 20-lb disk $A$, 4-lb slender rod $BC$, and a 1-lb smooth collar $C$. If the disk rolls without slipping, determine the velocity of the collar at the instant $\theta = 30°$. The system is released from rest when $\theta = 45°$.




$$v_B = 0.8\omega_A$$

$$\omega_{BC} = \frac{v_B}{1.5} = \frac{v_C}{2.598} = \frac{v_G}{1.5}$$

Thus,

$$v_B = v_G = 1.5\omega_{BC} \qquad v_C = 2.598\omega_{BC}$$

$$\omega_A = 1.875\,\omega_{BC}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 4(1.5 \sin 45°) + 1(3 \sin 45°)$$

$$= \frac{1}{2}\left[\frac{1}{2}\left(\frac{20}{32.2}\right)(0.8)^2\right](1.875\omega_{BC})^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)(1.5\,\omega_{BC})^2$$

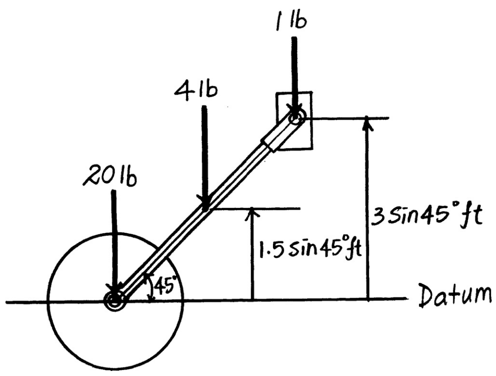
$$+\frac{1}{2}\left[\frac{1}{12}\left(\frac{4}{32.2}\right)(3)^2\right]\omega_{BC}^2 + \frac{1}{2}\left(\frac{4}{32.2}\right)(1.5\omega_{BC})^2$$

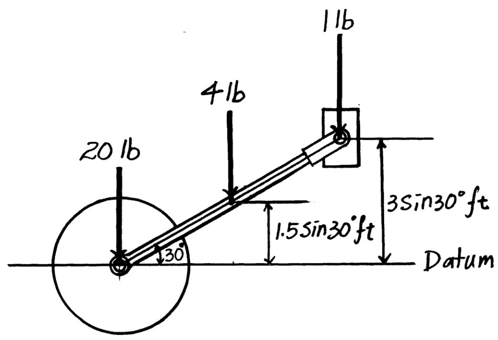
$$+\frac{1}{2}\left(\frac{1}{32.2}\right)(2.598\omega_{BC})^2 + 4(1.5 \sin 30°) + 1(3 \sin 30°)$$

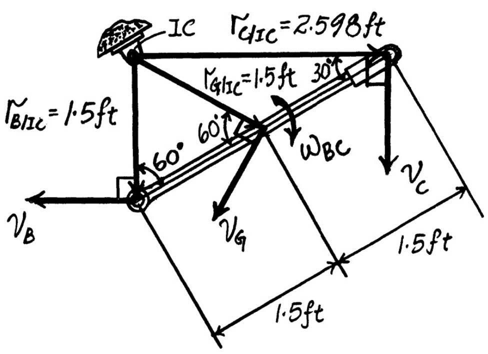
$$\omega_{BC} = 1.180 \text{ rad/s}$$

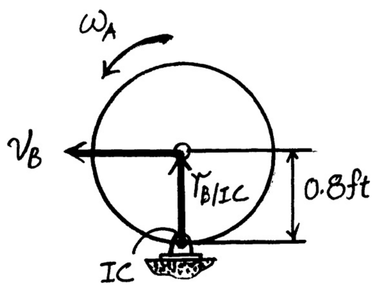
Thus,

$$v_C = 2.598(1.180) = 3.07 \text{ ft/s}$$ **Ans.**

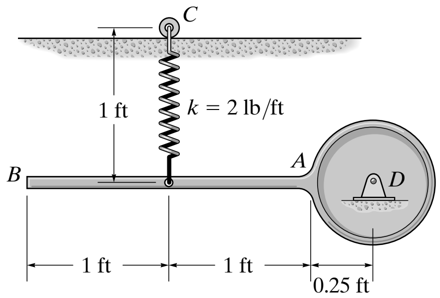


**18–47.** The pendulum consists of a 2-lb rod *BA* and a 6-lb disk. The spring is stretched 0.3 ft when the rod is horizontal as shown. If the pendulum is released from rest and rotates about point *D*, determine its angular velocity at the instant the rod becomes vertical. The roller at *C* allows the spring to remain vertical as the rod falls.




**Potential Energy:** Datum is set at point *O*. When rod *AB* is at vertical position, its center of gravity is located 1.25 ft *below* the datum. Its gravitational potential energy at this position is $-2(1.25)$ ft · lb. The initial and final stretch of the spring are 0.3 ft and $(1.25 + 0.3)$ ft = 1.55 ft, respectively. Hence, the initial and final elastic potential energy are $\frac{1}{2}(2)(0.3^2) = 0.09$ lb · ft and $\frac{1}{2}(2)(1.55^2) = 2.4025$ lb · ft. Thus,

$$V_1 = 0.09 \text{ lb}\cdot\text{ft} \qquad V_2 = 2.4025 + [-2(1.25)] = -0.0975 \text{ lb}\cdot\text{ft}$$

**Kinetic Energy:** The mass moment inertia for rod *AB* and the disk about point *O* are

$$(I_{AB})_O = \frac{1}{12}\left(\frac{2}{32.2}\right)(2^2) + \left(\frac{2}{32.2}\right)(1.25^2) = 0.1178 \text{ slug}\cdot\text{ft}^2$$

and

$$(I_D)_O = \frac{1}{2}\left(\frac{6}{32.2}\right)(0.25^2) = 0.005823 \text{ slug}\cdot\text{ft}^2$$

Since rod *AB* and the disk are initially at rest, the initial kinetic energy is $T_1 = 0$. The final kinetic energy is given by

$$T_2 = \frac{1}{2}(I_{AB})_O\,\omega^2 + \frac{1}{2}(I_D)_O\,\omega^2$$

$$= \frac{1}{2}(0.1178)\,\omega^2 + \frac{1}{2}(0.005823)\,\omega^2$$

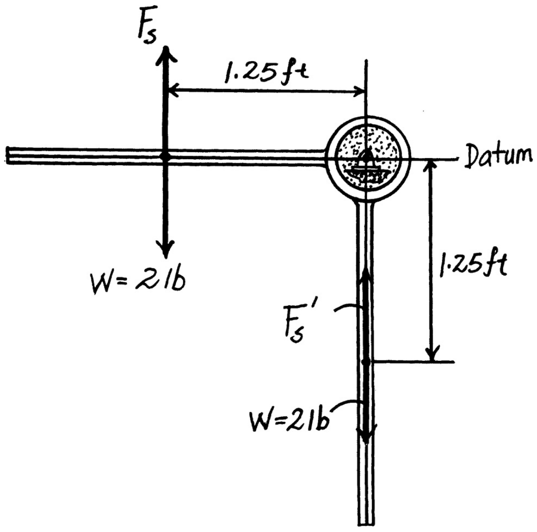
$$= 0.06179\,\omega^2$$

**Conservation of Energy:** Applying Eq. 18–19, we have

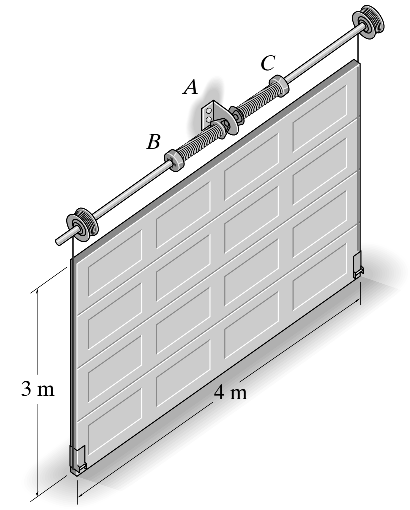
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0.09 = 0.06179\,\omega^2 + (-0.0975)$$

$$\omega = 1.74 \text{ rad/s} \qquad \textbf{Ans.}$$



***18–48.** The uniform garage door has a mass of 150 kg and is guided along smooth tracks at its ends. Lifting is done using the two springs, each of which is attached to the anchor bracket at $A$ and to the counterbalance shaft at $B$ and $C$. As the door is raised, the springs begin to unwind from the shaft, thereby assisting the lift. If each spring provides a torsional moment of $M = (0.7\theta)\ \text{N}\cdot\text{m}$, where $\theta$ is in radians, determine the angle $\theta_0$ at which both the left-wound and right-wound spring should be attached so that the door is completely balanced by the springs, i.e., when the door is in the vertical position and is given a slight force upwards, the springs will lift the door along the side tracks to the horizontal plane with no final angular velocity. *Note:* The elastic potential energy of a torsional spring is $V_e = \frac{1}{2}k\theta^2$, where $M = k\theta$ and in this case $k = 0.7\ \text{N}\cdot\text{m/rad}$.

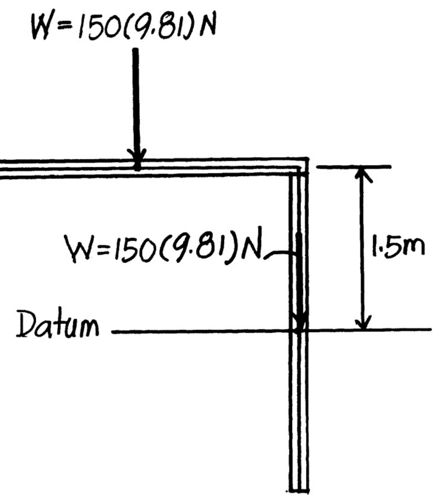



Datum at initial position.

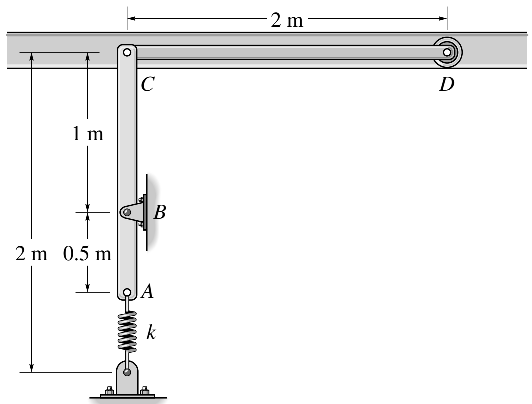
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}(0.7)\theta_0^2\right] + 0 = 0 + 150(9.81)(1.5)$$

$$\theta_0 = 56.15 \text{ rad} = 8.94 \text{ rev.}$$ **Ans.**



**•18–49.** The garage door *CD* has a mass of 50 kg and can be treated as a thin plate. Determine the required unstretched length of each of the two side springs when the door is in the open position, so that when the door falls freely from the open position it comes to rest when it reaches the fully closed position, i.e., when *AC* rotates 180°. Each of the two side springs has a stiffness of $k = 350$ N/m. Neglect the mass of the side bars *AC*.



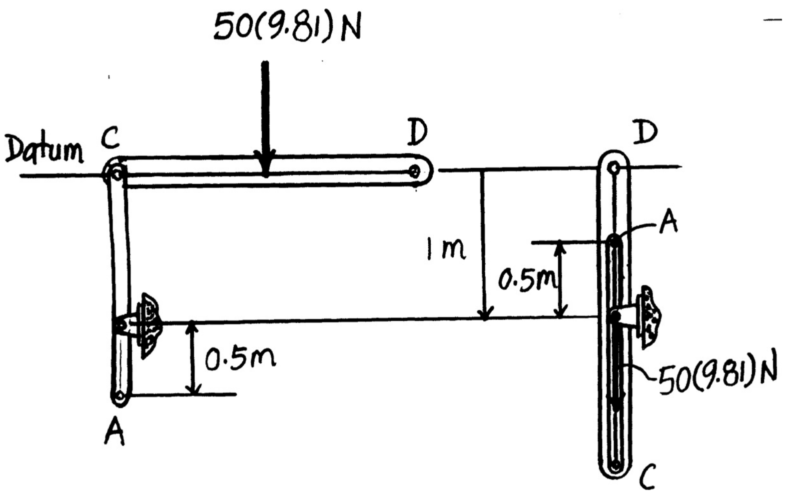

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}(350)(x_1)^2\right] = 0 + 2\left[\frac{1}{2}(350)(x_1 + 1)^2\right] - 50(9.81)(1)$$

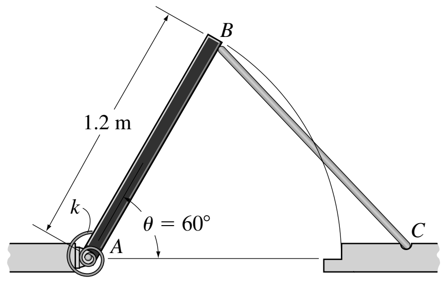
$$x_1 = 0.201 \text{ m}$$

Thus,

$$l_0 = 0.5 \text{ m} - 0.201 \text{ m} = 299 \text{ mm}$$ **Ans.**



**18–50.** The uniform rectangular door panel has a mass of 25 kg and is held in equilibrium above the horizontal at the position $\theta = 60°$ by rod $BC$. Determine the required stiffness of the torsional spring at $A$, so that the door's angular velocity becomes zero when the door reaches the closed position ($\theta = 0°$) once the supporting rod $BC$ is removed. The spring is undeformed when $\theta = 60°$.




**Potential Energy:** With reference to the datum in Fig. *a*, the gravitational potential energy of the panel at positions (1) and (2) is

$$(V_g)_1 = W(y_G)_1 = 25(9.81)(0.6 \sin 60°) = 127.44 \text{ J}$$

$$(V_g)_2 = W(y_G)_2 = 25(9.81)(0) = 0$$

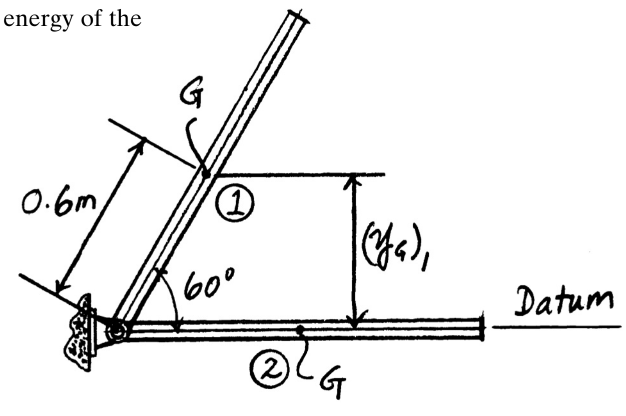
Since the spring is initially untwisted, $(V_e)_1 = 0$. The elastic potential energy of the spring when $\theta = 60° = \frac{\pi}{3}$ rad is

$$(V_e)_2 = \frac{1}{2}k\theta^2 = \frac{1}{2}(k)\left(\frac{\pi}{3}\right)^2 = \frac{\pi^2}{18}k$$

Thus, the potential energy of the panel is

$$V_1 = (V_g)_1 + (V_e)_1 = 127.44 + 0 = 127.44 \text{ J}$$

$$V_2 = (V_g)_2 + (V_e)_2 = 0 + \frac{\pi^2}{18}k = \frac{\pi^2}{18}k$$



(a)

**Kinetic Energy.** Since the rod is at rest at position (1) and is required to stop when it is at position (2), $T_1 = T_2 = 0$.

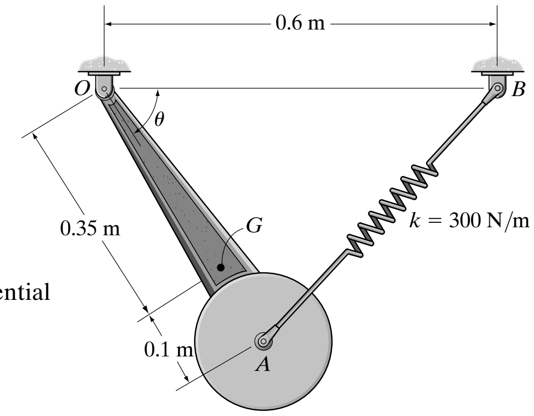
**Conservation of Energy.**

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 127.44 = 0 + \frac{\pi^2}{18}k$$

$$k = 232 \text{ N} \cdot \text{m / rad}$$ **Ans.**

**18–51.** The 30 kg pendulum has its mass center at $G$ and a radius of gyration about point $G$ of $k_G = 300$ mm. If it is released from rest when $\theta = 0°$, determine its angular velocity at the instant $\theta = 90°$. Spring $AB$ has a stiffness of $k = 300$ N/m and is unstretched when $\theta = 0°$.




**Potential Energy:** With reference to the datum in Fig. *a*, the gravitational potential energy of the pendulum at positions (1) and (2) is

$$(V_g)_1 = W(y_G)_1 = 30(9.81)(0) = 0$$

$$(V_g)_2 = -W(y_G)_2 = -30(9.81)(0.35) = -103.005 \text{ J}$$

Since the spring is unstretched initially, $(V_e)_1 = 0$. When $\theta = 90°$, the spring stretches $s = AB - A'B = \sqrt{0.45^2 + 0.6^2} - 0.15 = 0.6$ m. Thus,

$$(V_e)_2 = \frac{1}{2}ks^2 = \frac{1}{2}(300)\left(0.6^2\right) = 54 \text{ J}$$

and

$$V_1 = \left(V_g\right)_1 + (V_e)_1 = 0$$

$$V_2 = \left(V_g\right)_2 + (V_e)_2 = -103.005 + 54 = -49.005 \text{ J}$$

**Kinetic Energy:** Since the pendulum rotates about a fixed axis, $v_G = \omega r_G = \omega(0.35)$. The mass moment of inertia of the pendulum about its mass center is $I_G = mk_G{}^2 = 30\left(0.3^2\right) = 2.7 \text{ kg} \cdot \text{m}^2$. Thus, the kinetic energy of the pendulum is

$$T = \frac{1}{2}mv_G{}^2 + \frac{1}{2}I_G\omega^2$$

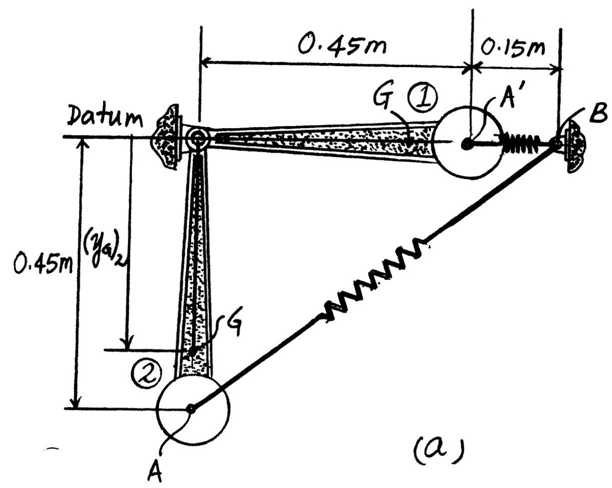
$$= \frac{1}{2}(30)\left[\omega(0.35)\right]^2 + \frac{1}{2}(2.7)\omega^2 = 3.1875\omega^2$$

**Conservation of Energy:**

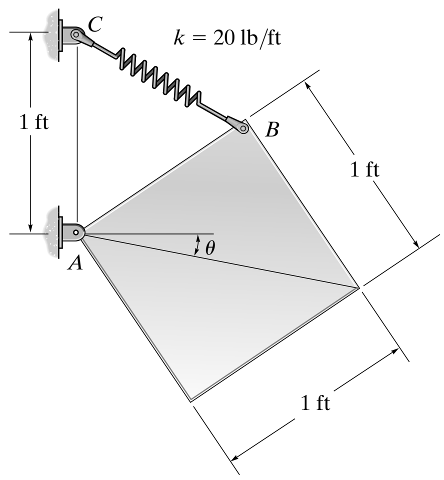
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 3.1875\omega^2 - 49.005$$

$$\omega = 3.92 \text{ rad/s}$$ **Ans.**



***18–52.** The 50-lb square plate is pinned at corner $A$ and attached to a spring having a stiffness of $k = 20$ lb/ft. If the plate is released from rest when $\theta = 0°$, determine its angular velocity when $\theta = 90°$. The spring is unstretched when $\theta = 0°$.




**Potential Energy:** With reference to the datum shown in Fig. *a*, the gravitational potential energy of the plate at position (1) and (2) is

$$\left(V_g\right)_1 = W(y_G)_1 = 50(0) = 0$$

$$\left(V_g\right)_2 = -W(y_G)_2 = -50(1\cos 45°) = -35.36 \text{ lb}\cdot\text{ft}$$

Since the spring is initially unstretched, $(V_e)_1 = 0$. When the plate is at position (2), the spring stetches $s = BC - B'C = 2[1\cos 22.5°] - 2(1\cos 67.5°) = 1.082$ ft. Therefore,

$$(V_e)_2 = \frac{1}{2}ks^2 = \frac{1}{2}(20)\left(1.082^2\right) = 11.72 \text{ lb}\cdot\text{ft}$$

Thus,

$$V_1 = \left(V_g\right)_1 + \left(V_e\right)_1 = 0 + 0 = 0$$

$$V_2 = \left(V_g\right)_2 + (V_e)_2 = -35.36 + 11.72 = -23.64 \text{ ft}\cdot\text{lb}$$

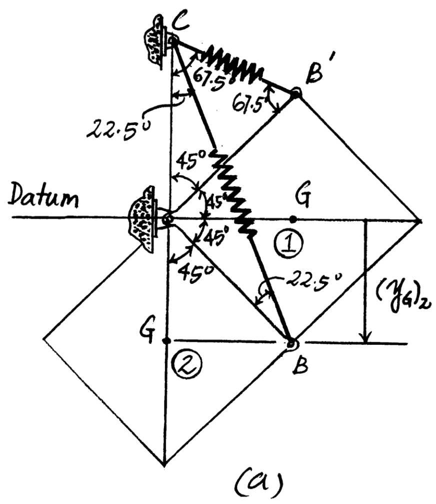
**Kinetic Energy:** Since the plate rotates about a fixed axis passing through point $A$, its kinetic energy can be determined from $T = \frac{1}{2}I_A\,\omega^2$, where

$$I_A = \frac{1}{12}\left(\frac{50}{32.2}\right)\left(1^2 + 1^2\right) + \frac{50}{32.2}(1\cos 45°)^2 = 1.035 \text{ slug}\cdot\text{ft}^2$$

Thus,

$$T = \frac{1}{2}I_A\,\omega^2 = \frac{1}{2}(1.035)\omega^2 = 0.5176\omega^2$$

Since the plate is initially at rest $T_1 = 0$.



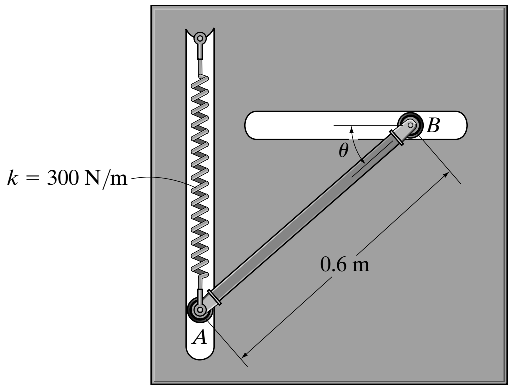

**Conservation of Energy:**

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0.5176\omega^2 - 23.64$$

$$\omega = 6.76 \text{ rad/s} \qquad \textbf{Ans.}$$

**•18–53.** A spring having a stiffness of $k = 300$ N/m is attached to the end of the 15-kg rod, and it is unstretched when $\theta = 0°$. If the rod is released from rest when $\theta = 0°$, determine its angular velocity at the instant $\theta = 30°$. The motion is in the vertical plane.



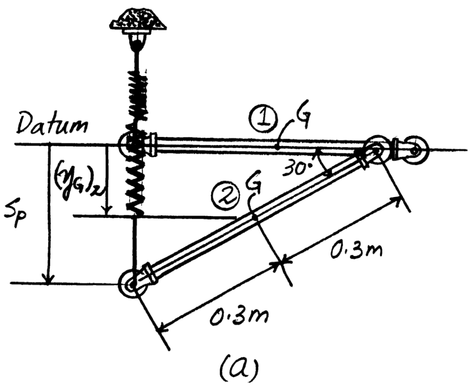

**Potential Energy:** With reference to the datum in Fig. *a*, the gravitational potential energy of the rod at positions (1) and (2) is

$$\left(V_g\right)_1 = W(y_G)_1 = 15(9.81)(0) = 0$$

$$\left(V_g\right)_2 = -W(y_G)_2 = -15(9.81)(0.3 \sin 30°) = -22.0725 \text{ J}$$

Since the spring is initially unstretched, $(V_e)_1 = 0$. When $\theta = 30°$, the stretch of the spring is $s_P = 0.6 \sin 30° = 0.3$ m. Thus, the final elastic potential energy of the spring is

$$\left(V_e\right)_2 = \frac{1}{2} k s_P{}^2 = \frac{1}{2}(300)\left(0.3^2\right) = 13.5 \text{ J}$$

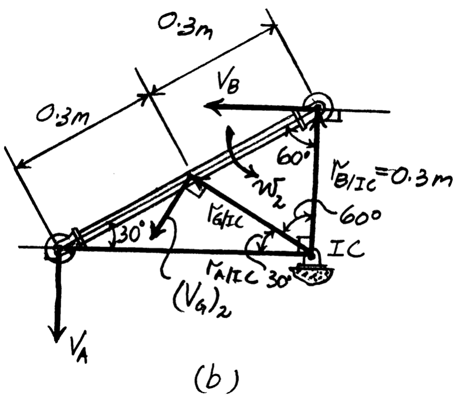



Thus,

$$V_1 = (V_g)_1 + (V_e)_1 = 0 + 0 = 0$$

$$V_2 = (V_g)_2 + (V_e)_2 = -22.0725 + 13.5 = -8.5725 \text{ J}$$

**Kinetic Energy:** Since the rod is initially at rest, $T_1 = 0$. From the geometry shown in Fig. *b*, $r_{G/IC} = 0.3$ m. Thus, $(V_G)_2 = \omega_2 r_{G/IC} = \omega_2\,(0.3)$. The mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12} ml^2 = \frac{1}{12}(15)\left(0.6^2\right) = 0.45 \text{ kg} \cdot \text{m}^2$. Thus, the final kinetic energy of the rod is




$$T_2 = \frac{1}{2} m(v_G)_2{}^2 + \frac{1}{2} I_G \omega_2{}^2$$

$$= \frac{1}{2}(15)\left[\omega_2\left(0.3\right)\right]^2 + \frac{1}{2}\left(0.45\right)\omega_2{}^2$$

$$= 0.9\omega_2{}^2$$

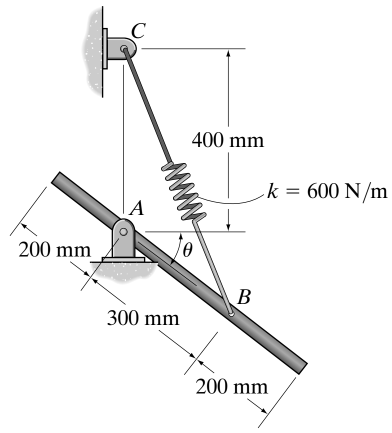
**Conservation of Energy:**

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0.9\omega_2{}^2 - 8.5725$$

$$\omega_2 = 3.09 \text{ rad/s}$$ **Ans.**

**18–54.** If the 6-kg rod is released from rest at $\theta = 30°$, determine the angular velocity of the rod at the instant $\theta = 0°$. The attached spring has a stiffness of $k = 600$ N/m, with an unstretched length of 300 mm.

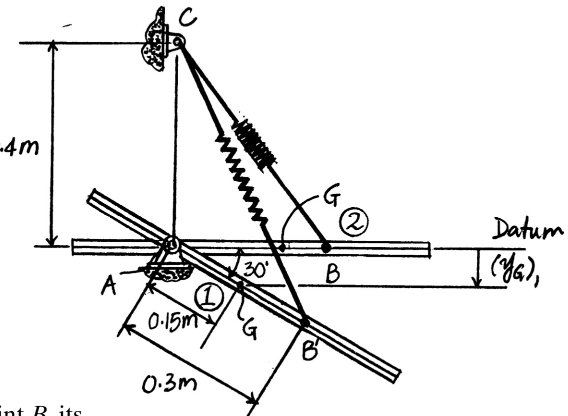



**Potential Energy:** With reference to the datum in Fig. *a*, the gravitational potential energy of the rod at positions (1) and (2) is

$$\left(V_g\right)_1 = -W(y_G)_1 = -6(9.81)(0.15 \sin 30°) = -4.4145 \text{ J}$$

$$\left(V_g\right)_2 = W(y_G)_2 = 6(9.81)(0) = 0$$

The stretch of the spring when the rod is in positions (1) and (2) is $s_1 = B'C - l_0 = \sqrt{0.3^2 + 0.4^2 - 2(0.3)(0.4)\cos 120°} - 0.3 = 0.3083$ m and $s_2 = BC - l_0 = \sqrt{0.3^2 + 0.4^2} - 0.3 = 0.2$ m. Thus, the initial and final elastic potential energy of the spring is




(a)

$$\left(V_e\right)_1 = \frac{1}{2}ks_1{}^2 = \frac{1}{2}(600)\left(0.3083^2\right) = 28.510 \text{ J}$$

$$\left(V_e\right)_2 = \frac{1}{2}ks_2{}^2 = \frac{1}{2}(600)\left(0.2^2\right) = 12 \text{ J}$$

Thus,

$$V_1 = (V_g)_1 + (V_e)_1 = -4.4145 + 28.510 = 24.096 \text{ J}$$

$$V_2 = (V_g)_2 + (V_e)_2 = 0 + 12 = 12 \text{ J}$$

**Kinetic Energy:** Since the rod rotates about a fixed axis passing through point $B$, its kinetic energy can be determined from $T = \frac{1}{2} I_B\omega^2$, where

$$I_B = \frac{1}{12}(6)\left(0.7^2\right) + 6\left(0.15^2\right) = 0.38 \text{ kg} \cdot \text{m}^2$$

Thus,

$$T = \frac{1}{2} I_B\omega^2 = \frac{1}{2}(0.38)\omega^2 = 0.19\omega^2$$

Since the rod is initially at rest, $T_1 = 0$.

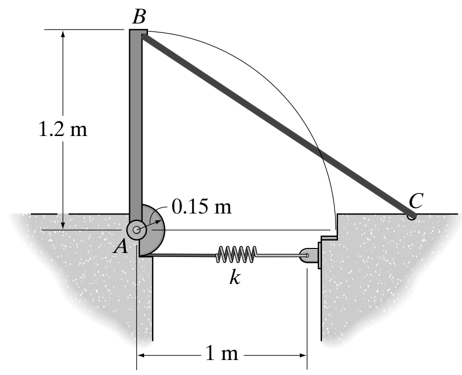
**Conservation of Energy:**

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 24.096 = 0.19\omega^2 + 12$$

$$\omega = 7.98 \text{ rad/s}$$ **Ans.**

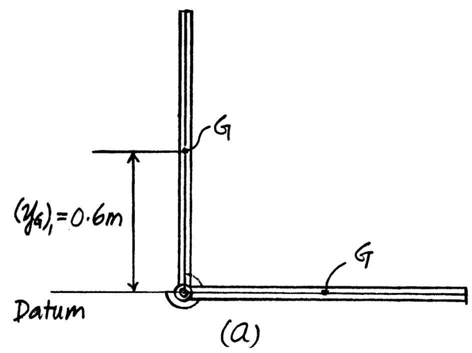
**18–55.** The 50-kg rectangular door panel is held in the vertical position by rod $CB$. When the rod is removed, the panel closes due to its own weight. The motion of the panel is controlled by a spring attached to a cable that wraps around the half pulley. To reduce excessive slamming, the door panel's angular velocity is limited to 0.5 rad/s at the instant of closure. Determine the minimum stiffness $k$ of the spring if the spring is unstretched when the panel is in the vertical position. Neglect the half pulley's mass.




**Potential Energy:** With reference to the datum in Fig. $a$, the gravitational potential energy of the door panel at its open and closed positions is

$$\left(V_g\right)_1 = W(y_G)_1 = 50(9.81)(0.6) = 294.3 \text{ J}$$

$$\left(V_g\right)_2 = W(y_G)_2 = 50(9.81)(0) = 0$$




Since the spring is unstretched when the door panel is at the open position, $(V_e)_1 = 0$. When the door is closed, the half pulley rotates through and angle of $\theta = \dfrac{\pi}{2}$ rad. Thus, the spring stretches $s = r\theta = 0.15\left(\dfrac{\pi}{2}\right) = 0.075\pi$ m. Then,

$$(V_e)_2 = \frac{1}{2}ks^2 = \frac{1}{2}k(0.075\pi)^2 = 0.0028125\pi^2 k$$

Thus,

$$V_1 = \left(V_g\right)_1 + (V_e)_1 = 294.3 + 0 = 294.3 \text{ J}$$

$$V_2 = \left(V_g\right)_2 + (V_e)_2 = 0 + 0.0028125\pi^2 k = 0.0028125\pi^2 k$$

**Kinetic Energy:** Since the door panel rotates about a fixed axis passing through point $A$, its kinetic energy can be determined from $T = \dfrac{1}{2} I_A\,\omega^2$, where

$$I_A = \frac{1}{12}(50)\left(1.2^2\right) + 50\left(0.6^2\right) = 24 \text{ kg}\cdot\text{m}^2$$

Thus,

$$T = \frac{1}{2} I_A\omega^2 = \frac{1}{2}(24)\omega^2 = 12\omega^2$$

Since the door panel is at rest in the open position and required to have an angular velocity of $\omega = 0.5$ rad/s at closure, then

$$T_1 = 0 \qquad\qquad T_2 = 12\left(0.5^2\right) = 3 \text{ J}$$

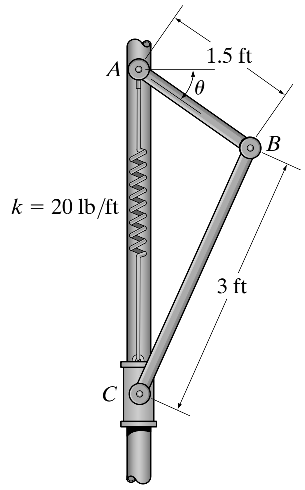
**Conservation of Energy:**

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 294.3 = 3 + 0.0028125\pi^2 k$$

$$k = 10494.17 \text{ N/m} = 10.5 \text{ kN/m}$$ **Ans.**

***18–56.** Rods $AB$ and $BC$ have weights of 15 lb and 30 lb, respectively. Collar $C$, which slides freely along the smooth vertical guide, has a weight of 5 lb. If the system is released from rest when $\theta = 0°$, determine the angular velocity of the rods when $\theta = 90°$. The attached spring is unstretched when $\theta = 0°$.




**Potential Energy:** From the geometry in Fig. $a$, $\theta = \sin^{-1}\left(\frac{1.5}{3}\right) = 30°$. With reference to the datum, the initial and final gravitational potential energy of the system is

$$\left(V_g\right)_1 = W_{AB}\,(y_{G1})_1 - W_{BC}\,(y_{G2})_1 - W_C\,(y_{G3})_1$$

$$= 15(0) - 30(1.5\cos 30°) - 5(3\cos 30°)$$

$$= -51.96\ \text{ft}\cdot\text{lb}$$

$$\left(V_g\right)_2 = -W_{AB}\,(y_{G1})_2 - W_{BC}\,(y_{G2})_2 - W_C\,(y_{G3})_2$$

$$= -15(0.75) - 30(3) - 5(4.5)$$

$$= -123.75\ \text{ft}\cdot\text{lb}$$

Since the spring is initially unstretched, $(V_e)_1 = 0$. When $\theta = 90°$, the spring stretches $s = 4.5 - 3\cos 30° = 1.902$ ft. Thus,

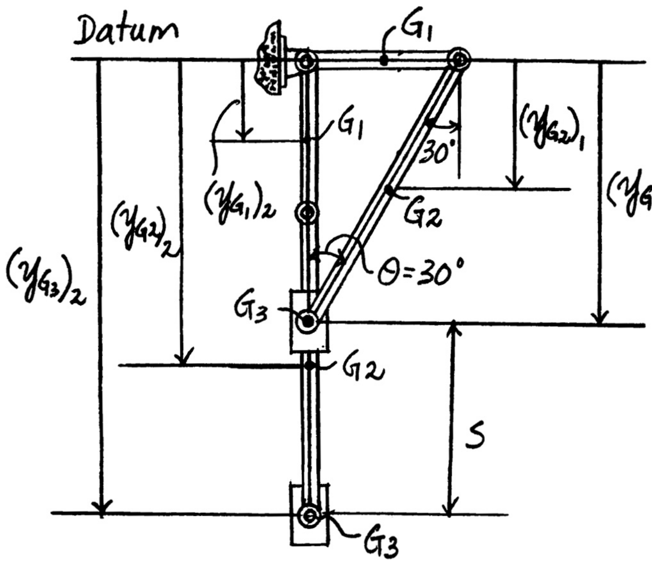
$$(V_e)_2 = \frac{1}{2}ks^2 = \frac{1}{2}(20)\left(1.902^2\right) = 36.17\ \text{ft}\cdot\text{lb}$$

And,

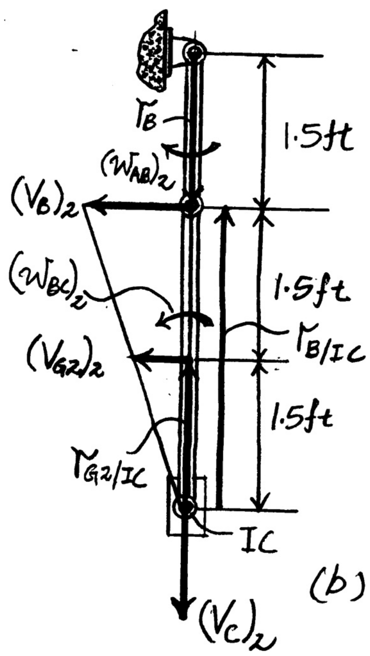
$$V_1 = \left(V_g\right)_1 + (V_e)_1 = -51.96 + 0 = -51.96\ \text{ft}\cdot\text{lb}$$

$$V_2 = \left(V_g\right)_2 + (V_e)_2 = -123.75 + 36.17 = -87.58\ \text{ft}\cdot\text{lb}$$

**Kinetic Energy:** Since the system is initially at rest $T_1 = 0$. Referring to Fig. $b$, $(v_B)_2 = (\omega_{AB})_2\, r_B = (\omega_{AB})_2(1.5)$. Then $(\omega_{BC})_2 = \frac{(v_B)_2}{r_{B/IC}} = \frac{(\omega_{AB})_2(1.5)}{3} = 0.5(\omega_{AB})_2$.



(a)



(b)

***18–56. Continued**

Subsequently, $(v_{G2})_2 = (\omega_{BC})_2\, r_{G2/IC} = 0.5(\omega_{AB})_2(1.5) = 0.75(\omega_{AB})_2$. Since point $C$ is located at the $IC$, $v_C = 0$. The mass moments of inertia of $AB$ about point $A$ and $BC$ about its mass center are $(I_{AB})_A = \frac{1}{3}ml^2 = \frac{1}{3}\left(\frac{15}{32.2}\right)(1.5^2) = 0.3494 \text{ slug/ft}^2$ and $(I_{BC})_{G2} = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{30}{32.2}\right)(3^2) = 0.6988 \text{ slug/ft}^2$. Thus, the final kinetic energy of the system is

$$T_2 = T_{AB} + T_{BC} + T_C$$

$$= \frac{1}{2}(I_{AB})_A(\omega_{AB})_2{}^2 + \left[\frac{1}{2}m_{BC}(v_{G2})^2 + \frac{1}{2}(I_{BC})_{G2}(\omega_{BC})_2{}^2\right] + \frac{1}{2}m_C v_C{}^2$$

$$= \frac{1}{2}(0.3494)(\omega_{AB})_2{}^2 + \left[\frac{1}{2}\left(\frac{30}{32.2}\right)\left[0.75(\omega_{AB})_2\right]^2 + \frac{1}{2}(0.6988)\left[0.5(\omega_{AB})_2\right]^2\right] + 0$$

$$= 0.5241(\omega_{AB})_2{}^2$$

**Conservation of Energy:**

$$T_1 + V_1 = T_2 + V_2$$

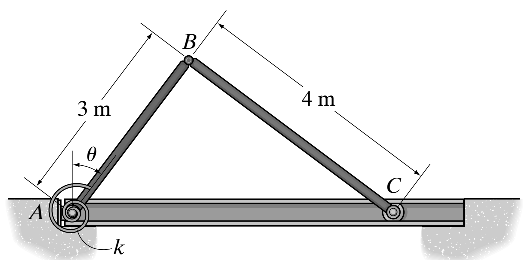
$$0 - 51.96 = 0.5241(\omega_{AB})_2{}^2 - 87.58$$

$$(\omega_{AB})_2 = 8.244 \text{ rad/s} = 8.24 \text{ rad/s} \qquad \textbf{Ans.}$$

Thus,

$$(\omega_{BC})_2 = 0.5(8.244) = 4.12 \text{ rad/s} \qquad \textbf{Ans.}$$

**•18–57.** Determine the stiffness $k$ of the torsional spring at $A$, so that if the bars are released from rest when $\theta = 0°$, bar $AB$ has an angular velocity of 0.5 rad/s at the closed position, $\theta = 90°$. The spring is uncoiled when $\theta = 0°$. The bars have a mass per unit length of 10 kg/m.




**Potential Energy:** With reference to the datum in Fig. $a$, the gravitational potential energy of the system at its open and closed positions is

$$(V_g)_1 = W_{AB}(y_{G1})_1 + W_{BC}(y_{G2})_1$$

$$= 10(3)(9.81)(1.5) + 10(4)(9.81)(1.5) = 1030.5 \text{ J}$$

$$(V_g)_2 = W_{AB}(y_{G1})_2 + W_{BC}(y_{G2})_2$$

$$= 10(3)(9.81)(0) + 10(4)(9.81)(0) = 0$$

**•18–57. Continued**

Since the spring is initially uncoiled, $(V_e)_1 = 0$. When the panels are in the closed position, the coiled angle of the spring is $\theta = \dfrac{\pi}{2}$ rad. Thus,

$$(V_e)_2 = \frac{1}{2}k\theta^2 = \frac{1}{2}k\left(\frac{\pi}{2}\right)^2 = \frac{\pi^2}{8}k$$

And so,

$$V_1 = (V_g)_1 + (V_e)_1 = 1030.5 + 0 = 1030.5\text{ J}$$

$$V_2 = (V_g)_2 + (V_e)_2 = 0 + \frac{\pi^2}{8}k = \frac{\pi^2}{8}k$$

**Kinetic Energy:** Since the system is initially at rest, $T_1 = 0$. Referring to Fig. $b$, $(v_B)_2 = (\omega_{AB})_2 r_B = 0.5(3) = 1.5$ m/s. Then, $(\omega_{BC})_2 = \dfrac{(v_B)_2}{r_{B/IC}} = \dfrac{1.5}{4} = 0.375$ rad/s. Subsequently, $(v_G)_2 = (\omega_{BC})_2 r_{G2/IC} = 0.375(2) = 0.75$ m/s. The mass moments of inertia of $AB$ about point $A$ and $BC$ about its mass center are

$$(I_{AB})_A = \frac{1}{3}ml^2 = \frac{1}{3}[10(3)](3^2) = 90\text{ kg}\cdot\text{m}^2$$

and

$$(I_{BC})_{G2} = \frac{1}{12}ml^2 = \frac{1}{12}[10(4)](4^2) = 53.33\text{ kg}\cdot\text{m}^2$$

Thus,

$$T_2 = \frac{1}{2}(I_{AB})_A(\omega_{AB})_2{}^2 + \left[\frac{1}{2}m_{BC}(v_{G2})^2 + \frac{1}{2}(I_{BC})_{G2}(\omega_{BC})_2{}^2\right]$$

$$= \frac{1}{2}(90)(0.5^2) + \left[\frac{1}{2}[10(4)](0.75^2) + \frac{1}{2}(53.33)(0.375^2)\right]$$

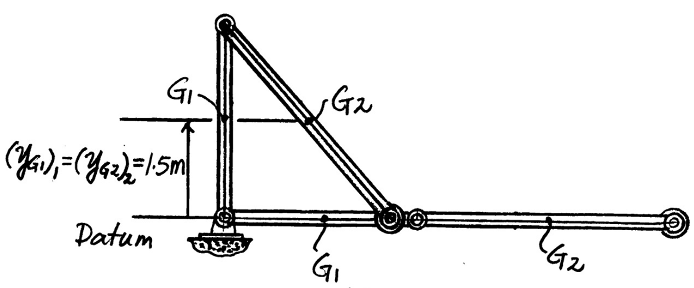
$$= 26.25\text{ J}$$

**Conservation of Energy:**

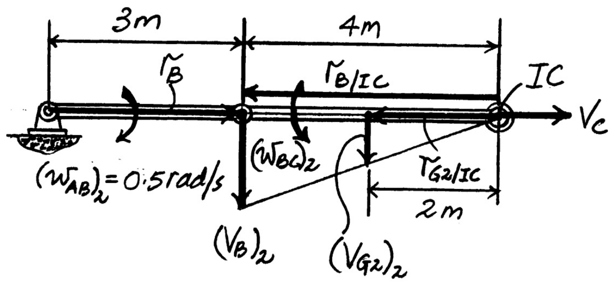
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 1030.5 = 26.25 + \frac{\pi^2}{8}k$$

$$k = 814\text{ N}\cdot\text{m/rad} \qquad \textbf{Ans.}$$

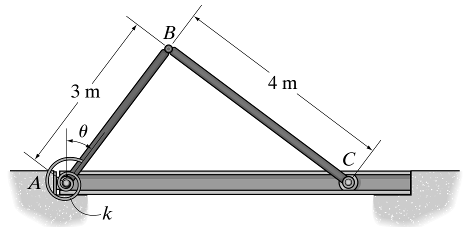



(a)




(b)

**18–58.** The torsional spring at $A$ has a stiffness of $k = 900\ \text{N}\cdot\text{m/rad}$ and is uncoiled when $\theta = 0°$. Determine the angular velocity of the bars, $AB$ and $BC$, when $\theta = 0°$, if they are released from rest at the closed position, $\theta = 90°$. The bars have a mass per unit length of 10 kg/m.




**Potential Energy:** With reference to the datum in Fig. $a$, the gravitational potential energy of the system at its open and closed positions is

$$(V_g)_1 = W_{AB}(y_{G1})_1 + W_{BC}(y_{G2})_1$$

$$= 10(3)(9.81)(0) + 10(4)(9.81)(0) = 0$$

$$(V_g)_2 = W_{AB}(y_{G1})_2 + W_{BC}(y_{G2})_2$$

$$= 10(3)(9.81)(1.5) + 10(4)(9.81)(1.5) = 1030.05\ \text{J}$$

When the panel is in the closed position, the coiled angle of the spring is $\theta = \dfrac{\pi}{2}$ rad. Thus,

$$(V_e)_1 = \frac{1}{2}k\theta^2 = \frac{1}{2}(900)\left(\frac{\pi}{2}\right)^2 = 112.5\pi^2\ \text{J}$$

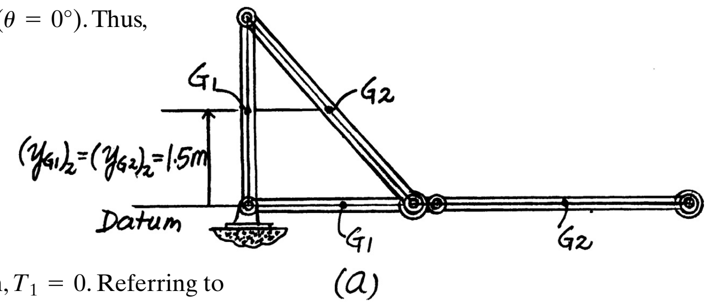
The spring is uncoiled when the panel is in the open position ($\theta = 0°$). Thus,

$$(V_e)_2 = 0$$

And so,

$$V_1 = (V_g)_1 + (V_e)_1 = 0 + 112.5\pi^2 = 112.5\pi^2\ \text{J}$$

$$V_2 = (V_g)_2 + (V_e)_2 = 1030.05 + 0 = 1030.05\ \text{J}$$




**Kinetic Energy:** Since the panel is at rest in the closed position, $T_1 = 0$. Referring to Fig. $b$, the $IC$ for $BC$ is located at infinity. Thus,

$$(\omega_{BC})_2 = 0 \qquad \textbf{Ans.}$$

Then,

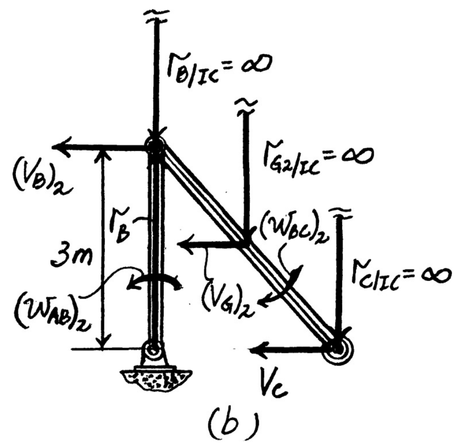
$$(v_G)_2 = (v_B)_2 = (\omega_{AB})_2 r_B = (\omega_{AB})_2(3)$$

The mass moments of inertia of $AB$ about point $A$ and $BC$ about its mass center are

$$(I_{AB})_A = \frac{1}{3}ml^2 = \frac{1}{3}[10(3)](3^2) = 90\ \text{kg}\cdot\text{m}^2$$

and

$$(I_{BC})_{G2} = \frac{1}{12}ml^2 = \frac{1}{12}[10(4)](4^2) = 53.33\ \text{kg}\cdot\text{m}^2$$




Thus,

$$T_2 = \frac{1}{2}(I_{AB})_A(\omega_{AB})_2{}^2 + \frac{1}{2}m_{BC}(v_{G2})^2$$

$$= \frac{1}{2}(90)(\omega_{AB})_2{}^2 + \frac{1}{2}[10(4)]\left[(\omega_{AB})_2(3)\right]^2$$

$$= 225(\omega_{AB})_2{}^2$$

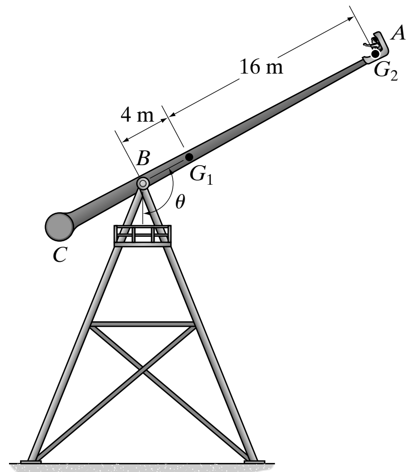
**Conservation of Energy:**

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 112.5\pi^2 = 225(\omega_{AB})_2{}^2 + 1030.05$$

$$(\omega_{AB})_2 = 0.597\ \text{rad/s} \qquad \textbf{Ans.}$$

**18–59.** The arm and seat of the amusement-park ride have a mass of 1.5 Mg, with the center of mass located at point $G_1$. The passenger seated at $A$ has a mass of 125 kg, with the center of mass located at $G_2$ If the arm is raised to a position where $\theta = 150°$ and released from rest, determine the speed of the passenger at the instant $\theta = 0°$. The arm has a radius of gyration of $k_{G1} = 12$ m about its center of mass $G_1$. Neglect the size of the passenger.




**Potential Energy:** With reference to the datum shown in Fig. *a*, the gravitational potential energy of the system at position (1) and (2) is

$$V_1 = \left(V_g\right)_1 = W_1(y_{G1})_1 + W_2(y_{G2})_1$$

$$= 1500(9.81)(4 \sin 60°) + 125(9.81)(20 \sin 60°)$$

$$= 72\,213.53 \text{ J}$$

$$V_2 = \left(V_g\right)_2 = -W_1(y_{G1})_2 - W_2(y_{G2})_2$$

$$= -1500(9.81)(4) - 125(9.81)(20)$$

$$= -83\,385 \text{ J}$$

**Kinetic Energy:** Since the arm rotates about a fixed axis passing through $B$, $v_{G1} = \omega r_{G1} = \omega(4)$ and $v_{G2} = \omega r_{G2} = \omega(20)$. The mass moment of inertia of the arm about its mass center is $I_{G1} = m_1 k_{G1}^2 = 1500\left(12^2\right) = 216\,000 \text{ kg} \cdot \text{m}^2$. Thus, the kinetic energy of the system is

$$T = \left[\frac{1}{2} m_1 (v_G)_1{}^2 + \frac{1}{2} I_{G1}\omega^2\right] + \frac{1}{2} m_2 (v_{G2})^2$$

$$= \left[\frac{1}{2}(1500)[\omega(4)]^2 + \frac{1}{2}(216\,000)\omega^2\right] + \frac{1}{2}(125)[\omega(20)]^2$$

$$= 145\,000\omega^2$$

Since the system is initially at rest, $T_1 = 0$.

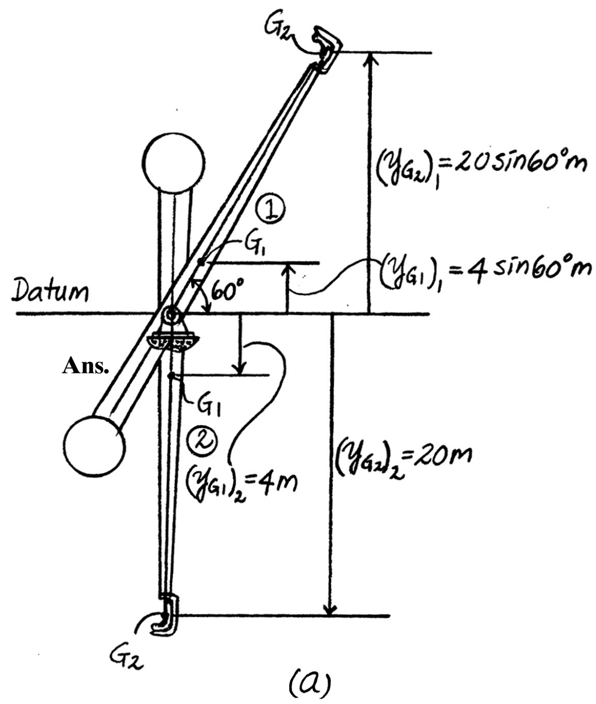
**Conservation of Energy:**

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 72\,213.53 = 145\,000\omega^2 - 83\,385$$

$$\omega = 1.0359 \text{ rad/s}$$

$$v = \omega r = (1.0359 \text{ rad/s})(20 \text{ m}) = 20.7 \text{ m/s}$$ **Ans.**

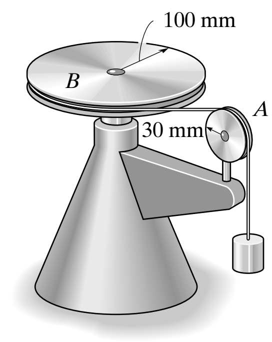


(a)



لجنة الميكانيك - الإتجاه الإسلامي

**18–60.** The assembly consists of a 3-kg pulley $A$ and 10-kg pulley $B$. If a 2-kg block is suspended from the cord, determine the block's speed after it descends 0.5 m starting from rest. Neglect the mass of the cord and treat the pulleys as thin disks. No slipping occurs.




$$T_1 + V_1 = T_2 + V_2$$

$$[0 + 0 + 0] + [0] = \frac{1}{2}\left[\frac{1}{2}(3)(0.03)^2\right]\omega_A^2 + \frac{1}{2}\left[\frac{1}{2}(10)(0.1)^2\right]\omega_B^2 + \frac{1}{2}(2)(v_C)^2 - 2(9.81)(0.5)$$

$$v_C = \omega_B(0.1) = 0.03\,\omega_A$$

Thus,

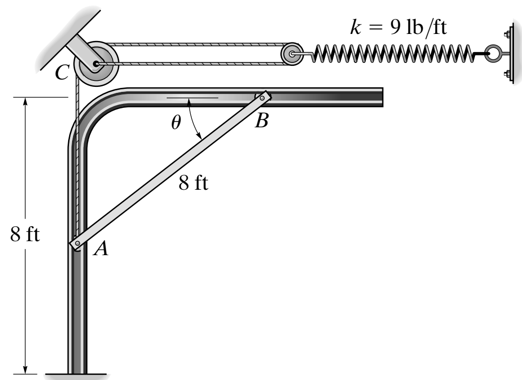
$$\omega_B = 10\,v_C$$

$$\omega = 33.33 v_C$$

Substituting and solving yields,

$$v_C = 1.52 \text{ m/s}$$ **Ans.**

**•18–61.** The motion of the uniform 80-lb garage door is guided at its ends by the track. Determine the required initial stretch in the spring when the door is open, $\theta = 0°$, so that when it falls freely it comes to rest when it just reaches the fully closed position, $\theta = 90°$. Assume the door can be treated as a thin plate, and there is a spring and pulley system on each of the two sides of the door.




$$s_A + 2s_s = l$$

$$\Delta s_A = -2\Delta s_s$$

$$8 \text{ ft} = -2\Delta s_s$$

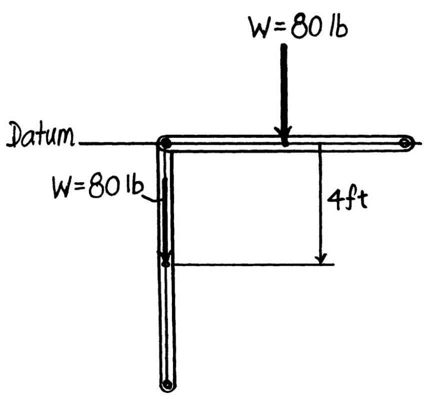
$$\Delta s_s = -4 \text{ ft}$$

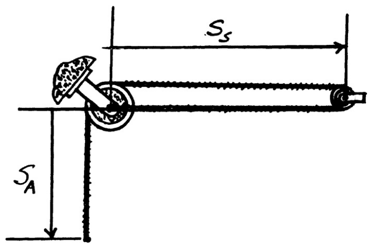
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}(9)s^2\right] = 0 - 80(4) + 2\left[\frac{1}{2}(9)(4 + s)^2\right]$$

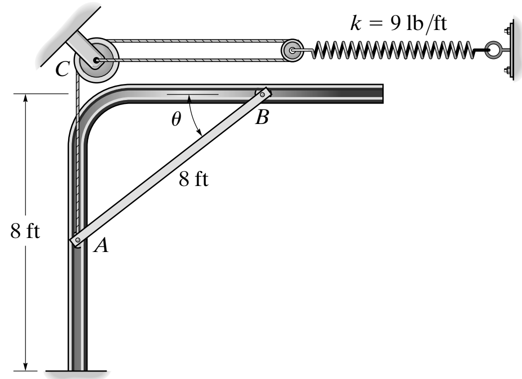
$$9s^2 = -320 + 9(16 + 8s + s^2)$$

$$s = 2.44 \text{ ft}$$ **Ans.**



**18–62.** The motion of the uniform 80-lb garage door is guided at its ends by the track. If it is released from rest at $\theta = 0°$, determine the door's angular velocity at the instant $\theta = 30°$. The spring is originally stretched 1 ft when the door is held open, $\theta = 0°$. Assume the door can be treated as a thin plate, and there is a spring and pulley system on each of the two sides of the door.




$$v_G = 4\omega$$

$$s_A + 2s_s = l$$

$$\Delta s_A = -2\Delta s_s$$

$$4\text{ ft} = -2\Delta s_s$$

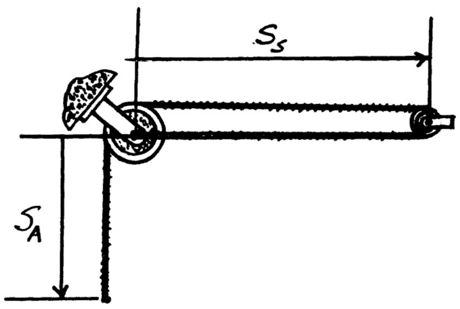
$$\Delta s_s = -2\text{ ft}$$

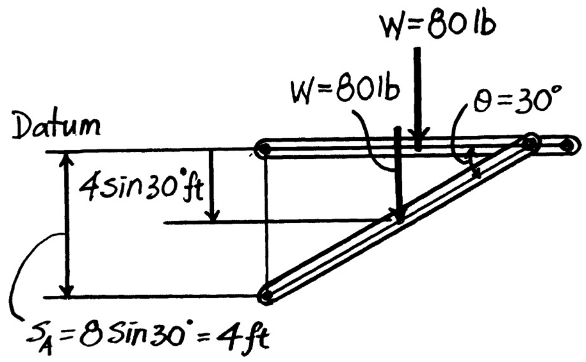
$$T_1 + V_1 = T_2 + V_2$$

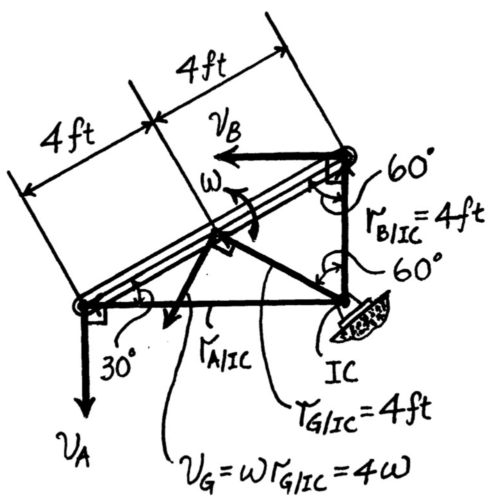
$$0 + 2\left[\frac{1}{2}(9)(1)^2\right] = \frac{1}{2}\left(\frac{80}{32.2}\right)(4\omega)^2 + \frac{1}{2}\left[\frac{1}{12}\left(\frac{80}{32.2}\right)(8)^2\right]\omega^2 - 80(4\sin 30°)$$

$$+ 2\left[\frac{1}{2}(9)(2+1)^2\right]$$

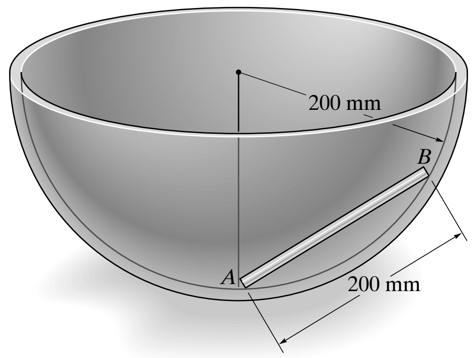
$$\omega = 1.82\text{ rad/s}$$ **Ans.**



**18–63.** The 500-g rod $AB$ rests along the smooth inner surface of a hemispherical bowl. If the rod is released from rest from the position shown, determine its angular velocity at the instant it swings downward and becomes horizontal.




Select datum at the bottom of the bowl.

$$\theta = \sin^{-1}\left(\frac{0.1}{0.2}\right) = 30°$$

$$h = 0.1 \sin 30° = 0.05$$

$$CE = \sqrt{(0.2)^2 - (0.1)^2} = 0.1732 \text{ m}$$

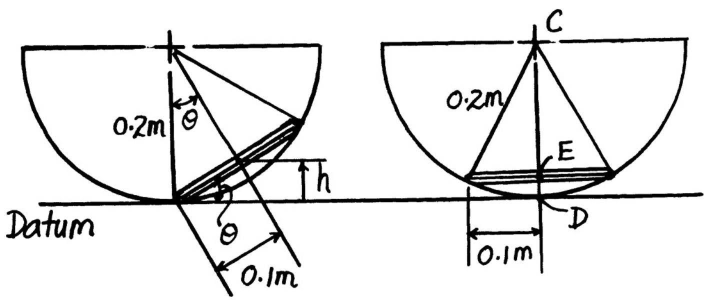
$$ED = 0.2 - 0.1732 = 0.02679$$

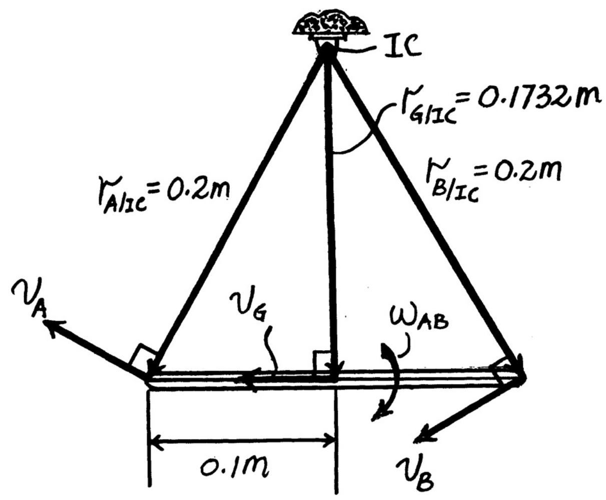
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0.5)(9.81)(0.05) = \frac{1}{2}\left[\frac{1}{12}(0.5)(0.2)^2\right]\omega_{AB}^2 + \frac{1}{2}(0.5)(v_G)^2 + (0.5)(9.81)(0.02679)$$

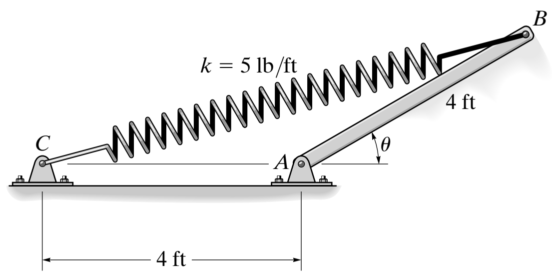
Since $v_G = 0.1732\omega_{AB}$

$$\omega_{AB} = 3.70 \text{ rad/s}$$ **Ans.**




***18–64.** The 25-lb slender rod $AB$ is attached to spring $BC$ which has an unstretched length of 4 ft. If the rod is released from rest when $\theta = 30°$, determine its angular velocity at the instant $\theta = 90°$.

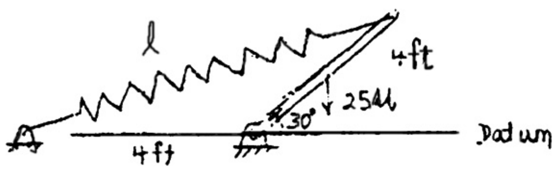



$$l = \sqrt{(4)^2 + (4)^2 - 2(4)(4)\cos 150°} = 7.727 \text{ ft}$$

$$T_1 + V_1 = T_2 + V_2$$

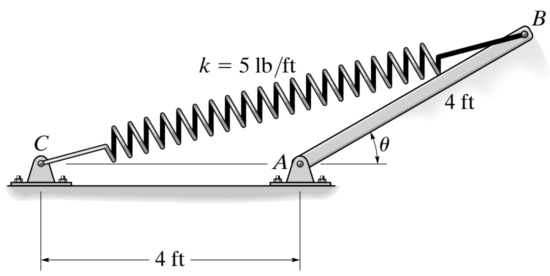
$$0 + 25(2)\sin 30° + \frac{1}{2}(5)(7.727 - 4)^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{25}{32.2}\right)(4)^2\right]\omega^2 + 25(2)$$

$$+ \frac{1}{2}(5)(4\sqrt{2} - 4)^2$$




$$\omega = 1.18 \text{ rad/s}$$ **Ans.**

**•18–65.** The 25-lb slender rod $AB$ is attached to spring $BC$ which has an unstretched length of 4 ft. If the rod is released from rest when $\theta = 30°$, determine the angular velocity of the rod the instant the spring becomes unstretched.

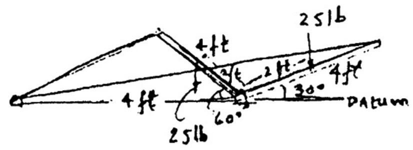



$$l = \sqrt{(4)^2 + (4)^2 - 2(4)(4)\cos 150°} = 7.727 \text{ ft}$$

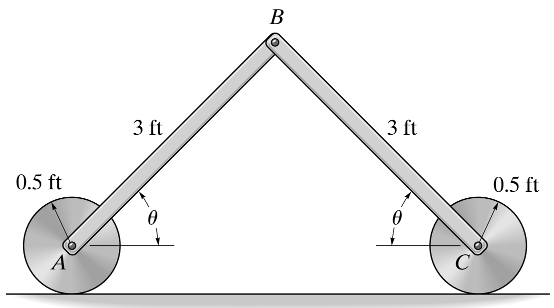
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 25(2)\sin 30° + \frac{1}{2}(5)(7.727 - 4)^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{25}{32.2}\right)(4)^2\right]\omega^2 + 25(2)(\sin 60°) + 0$$

$$\omega = 2.82 \text{ rad/s} \qquad \textbf{Ans.}$$



**18–66.** The assembly consists of two 8-lb bars which are pin connected to the two 10-lb disks. If the bars are released from rest when $\theta = 60°$, determine their angular velocities at the instant $\theta = 0°$. Assume the disks roll without slipping.

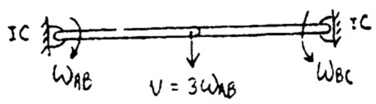


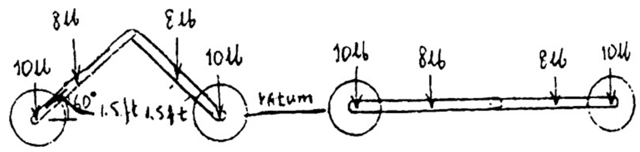

$$\omega_{AB} = \omega_{BC}$$

$$T_1 + V_1 = T_2 + V_2$$

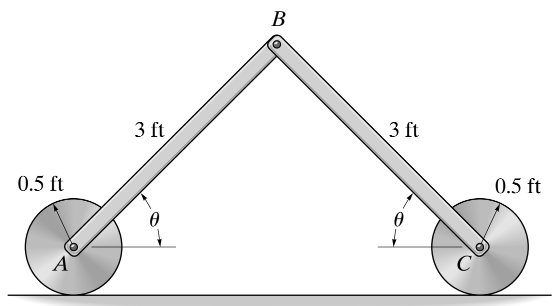
$$[0] + 2(8)(1.5\sin 60°) = 2\left[\frac{1}{2}\left(\frac{1}{3}\right)\left(\frac{8}{32.2}\right)(3)^2\omega^2\right] + [0]$$

$$\omega = 5.28 \text{ rad/s} \qquad \textbf{Ans.}$$





**18–67.** The assembly consists of two 8-lb bars which are pin connected to the two 10-lb disks. If the bars are released from rest when $\theta = 60°$, determine their angular velocities at the instant $\theta = 30°$. Assume the disks roll without slipping.




$$\omega_D = \frac{v_A}{0.5} \qquad v_A = \omega_{AB}(1.5)$$

$$\omega_D = 3\omega_{AB} \qquad v_G = 1.5\omega_{AB}$$

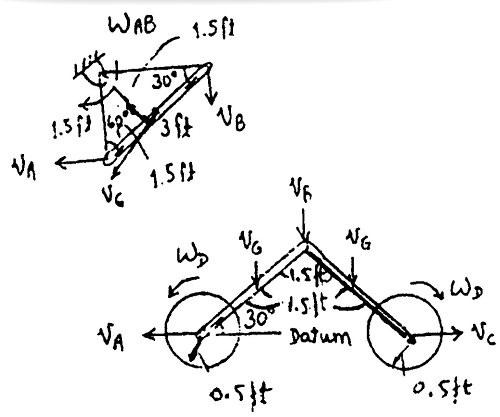
$$T_1 + V_1 = T_2 + V_2$$

$$[0 + 0] + 2[8(1.5\sin 60°)]$$

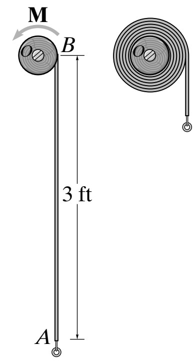
$$= 2\left[\frac{1}{2}\left\{\frac{1}{2}\left(\frac{10}{32.2}\right)(0.5)^2\right\}(3\omega_{AB})^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)\{\omega_{AB}(1.5)\}^2\right.$$

$$\left. + \frac{1}{2}\left(\frac{8}{32.2}\right)(1.5\omega_{AB})^2 + \frac{1}{2}\left\{\frac{1}{12}\left(\frac{8}{32.2}\right)(3)^2\right\}(\omega_{AB})^2\right] + 2[8(1.5\sin 30°)]$$

$$\omega_{AB} = 2.21 \text{ rad/s} \qquad \textbf{Ans.}$$

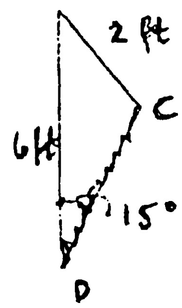


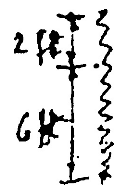
***18–68.** The uniform window shade $AB$ has a total weight of 0.4 lb. When it is released, it winds up around the spring-loaded core $O$. Motion is caused by a spring within the core, which is coiled so that it exerts a torque $M = 0.3(10^{-3})\theta$ lb · ft, where $\theta$ is in radians, on the core. If the shade is released from rest, determine the angular velocity of the core at the instant the shade is completely rolled up, i.e., after 12 revolutions. When this occurs, the spring becomes uncoiled and the radius of gyration of the shade about the axle at $O$ is $k_O = 0.9$ in. *Note:* The elastic potential energy of the torsional spring is $V_e = \frac{1}{2}k\theta^2$, where $M = k\theta$ and $k = 0.3(10^{-3})$ lb · ft/rad.




$$(2)^2 = (6)^2 + (CD)^2 - 2(6)(CD)\cos 15°$$

$$CD^2 - 11.591CD + 32 = 0$$




Selecting the smaller root:

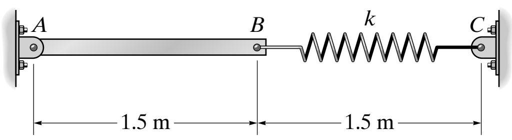
$$CD = 4.5352 \text{ ft}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + 2\left[\frac{1}{2}(k)(8 - 4.5352)^2\right] - 200(6)$$

$$k = 100 \text{ lb/ft}$$ **Ans.**

**18–69.** When the slender 10-kg bar $AB$ is horizontal it is at rest and the spring is unstretched. Determine the stiffness $k$ of the spring so that the motion of the bar is momentarily stopped when it has rotated clockwise 90°.




$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + \frac{1}{2}(k)(3.3541 - 1.5)^2 - 98.1\left(\frac{1.5}{2}\right)$$

$$k = 42.8 \text{ N/m}$$ **Ans.**