UNITED ARAB EMIRATES
MINISTRY OF EDUCATION

YEAR OF
ZAYED

TEACHER EDITION

MATH

6

McGraw-Hill Education

Integrated Math

United Arab Emirates Edition

Mc Graw Hill Education

United Arab Emirates
Ministry of Education

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2021
الإمارات العربية المتحدة
UNITED ARAB EMIRATES
GOLDEN JUBILEE YEAR 50

Teacher Edition

McGraw-Hill Education

Integrated Math

United Arab Emirates Edition

GRADE **6** • VOLUME 2

Mc Graw Hill Education

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"Extensive knowledge and modern science must be acquired. The educational process we see today is in an ongoing and escalating challenge which requires hard work.
We succeeded in entering the third millennium, while we are more confident in ourselves."

H.H. Sheikh Khalifa Bin Zayed Al Nahyan
President of the United Arab Emirates

CONTENTS IN BRIEF

Units organized by domain

This book is organized into units based on groups called domains. The Standards for MP Mathematical Practices are embedded throughout the course.

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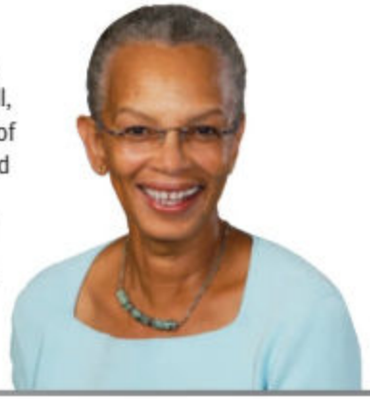

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Mathematical Practices

Mathematical Practices	Student Edition
MP1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	A strong problem-solving strand is present throughout the textbook with an emphasis on strategies in the Problem-Solving Investigation lessons. Look for the **Persevere with Problems** head in the exercises.
MP2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	Students are routinely asked to write an equation or an expression in order to solve a real-world problem. Exercises that emphasize this practice are labeled as **Reason Abstractly**.

Mathematical Practices	Student Edition
MP3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	Students are required to justify their reasoning in problems and to find the errors in samples of other's work. Look for these heads in the exercises: **Justify Conclusions** **Reason Inductively** **Make a Conjecture** **Use a Counterexample** **Find the Error** **Which One Doesn't Belong** **Make a Prediction** **Multiple Representations** **Construct an Argument**
MP4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	Real-world applications in problem solving are woven throughout every lesson. In addition to the real-world examples in each lesson, look for **Model with Mathematics** heads in the exercises.
MP5 Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	In addition to the traditional mathematical tools like estimating, using mental math, or measuring, students are encouraged to use software and the Internet in problem solving. Exercises utilizing this strategy are labeled with **Use Math Tools**.

Mathematical Practices	Student Edition
MP6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	Solutions are not just numbers, but include measurements to give the solution meaning. Look for **Be Precise** heads in the exercises.
MP7 Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	Emphasizing the structure of mathematics is present through use of classifying, explaining, giving examples as well as nonexamples. Exercises that emphasize this practice are labeled with **Identify Structure**.
MP8 Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	Patterns in reasoning are demonstrated throughout leading students to sound mathematical conclusions. Exercises with **Identify Repeated Reasoning** heads exemplify this practice.

UNIT 1 Ratios and Proportional Relationships

UNIT PROJECT PREVIEW
page 2

Chapter 1 Ratios and Rates

Essential Question

HOW do you use equivalent rates in the real world?

Chapter 2
Fractions, Decimals, and Percents

Essential Question

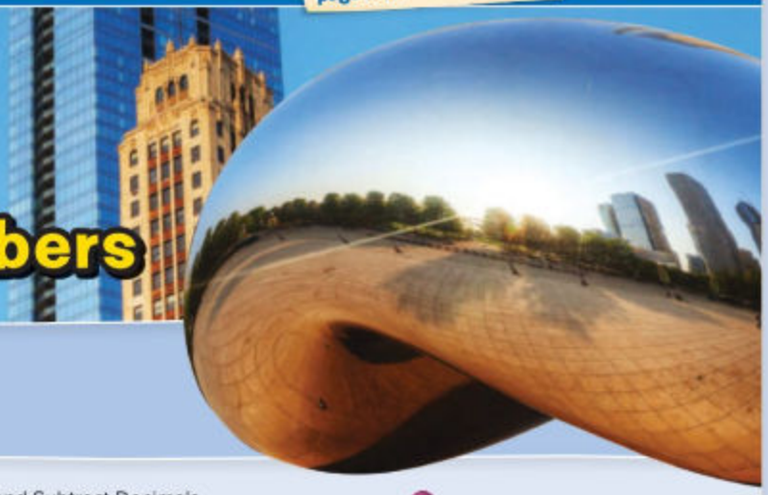
WHEN is it better to use a fraction, a decimal or a percent?

Ratios and Proportional Relationships

UNIT 2 The Number System

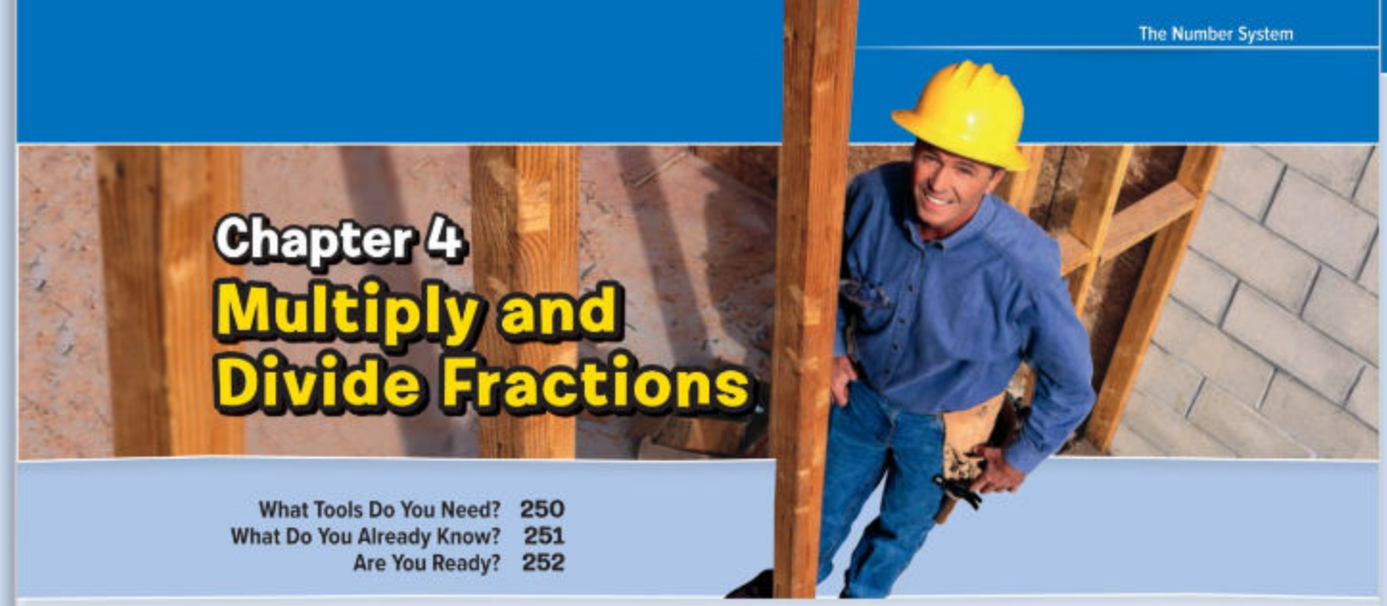
UNIT PROJECT PREVIEW
page 168

Chapter 3
Compute with Multi-Digit Numbers



Essential Question

HOW can estimating be helpful?



Chapter 4 Multiply and Divide Fractions

Essential Question

WHAT does it mean to multiply and divide fractions?

The Number System

Chapter 5
Integers and the Coordinate Plane

Essential Question

HOW are integers and absolute value used in real-world situations?

UNIT 3 Expressions and Equations

UNIT PROJECT PREVIEW
page 418

Chapter 6
Expressions

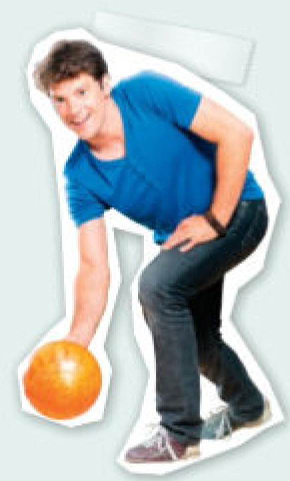
Essential Question

HOW is it helpful to write numbers in different ways?

Chapter 7
Equations

Essential Question

HOW do you determine if two numbers or expressions are equal?



Chapter 8 Functions and Inequalities

Essential Question

HOW are symbols, such as <, >, and =, useful?

UNIT 4 Geometry

UNIT PROJECT PREVIEW

Chapter 9 Area

What Tools Do You Need?
What Do You Already Know?
Are You Ready?

Inquiry Lab: Area of Parallelograms
Lesson 1 Area of Parallelograms

Inquiry Lab: Area of Triangles
Lesson 2 Area of Triangles

Inquiry Lab: Area of Trapezoids
Lesson 3 Area of Trapezoids

Problem-Solving Investigation:
Draw a Diagram

Essential Question

HOW does measurement help you solve problems in everyday life?

Mid-Chapter Check

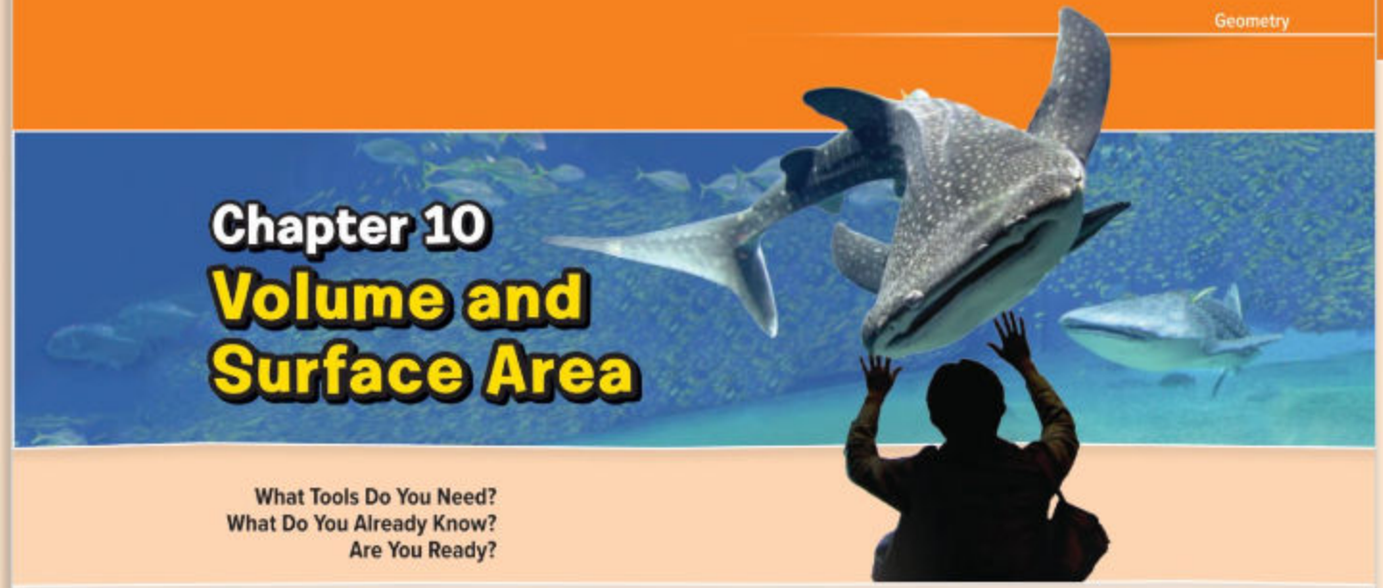
Lesson 4 Changes in Dimensions

Lesson 5 Polygons on the Coordinate Plane

Inquiry Lab: Area of Irregular Figures
Lesson 6 Area of Composite Figures

21st Century Career
in Community Planning

Chapter Review
Performance Task
Reflect



Chapter 10 Volume and Surface Area

What Tools Do You Need?
What Do You Already Know?
Are You Ready?

Inquiry Lab: Volume of Rectangular Prisms
Lesson 1 Volume of Rectangular Prisms

Lesson 2 Volume of Triangular Prisms

Problem-Solving Investigation: Make a Model

Essential Question
HOW is shape important when measuring a figure?

Mid-Chapter Check

Inquiry Lab: Surface Area of Rectangular Prisms
Lesson 3 Surface Area of Rectangular Prisms

Inquiry Lab: Nets of Triangular Prisms
Lesson 4 Surface Area of Triangular Prisms

Inquiry Lab: Nets of Pyramids
Lesson 5 Surface Area of Pyramids

21st Century Career in Interior Design

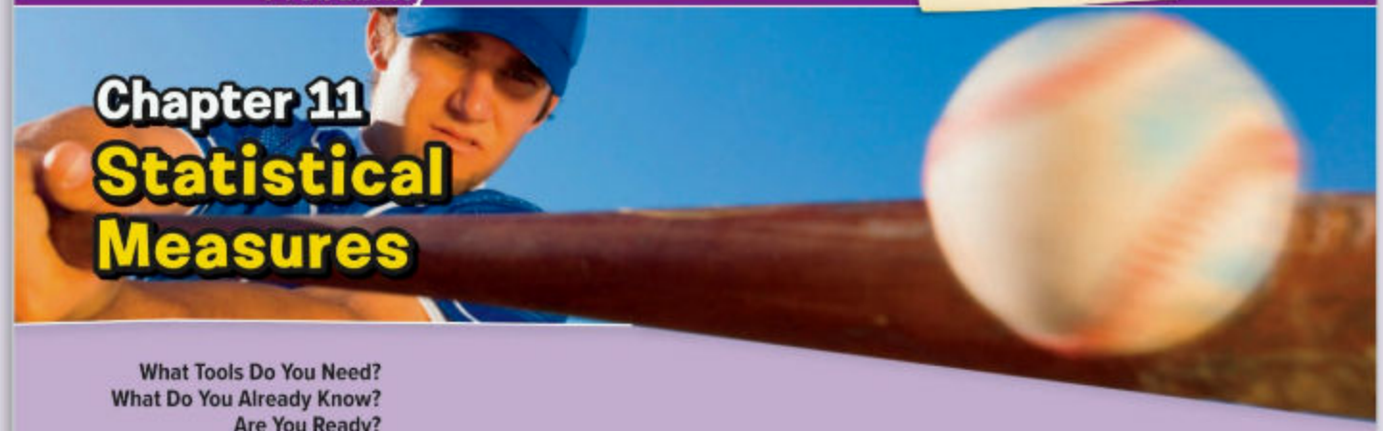
Chapter Review
Performance Task
Reflect

UNIT PROJECT

A New Zoo

UNIT 5 Statistics and Probability

UNIT PROJECT PREVIEW



Chapter 11 Statistical Measures

What Tools Do You Need?
What Do You Already Know?
Are You Ready?

Essential Question

HOW are the mean, median, and mode helpful in describing data?

Chapter 12 Statistical Displays

Essential Question

WHY is it important to carefully evaluate graphs?

UNIT PROJECT

Let's Exercise

This book focuses on four critical areas: (1) using concepts of ratio and rate to solve problems; (2) understanding division of fractions; (3) using expressions and equations; and (4) understanding of statistical reasoning.

Content

Ratios and Proportional Relationships

- Understand ratio concepts and use ratio reasoning to solve problems.

The Number System

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

Geometry

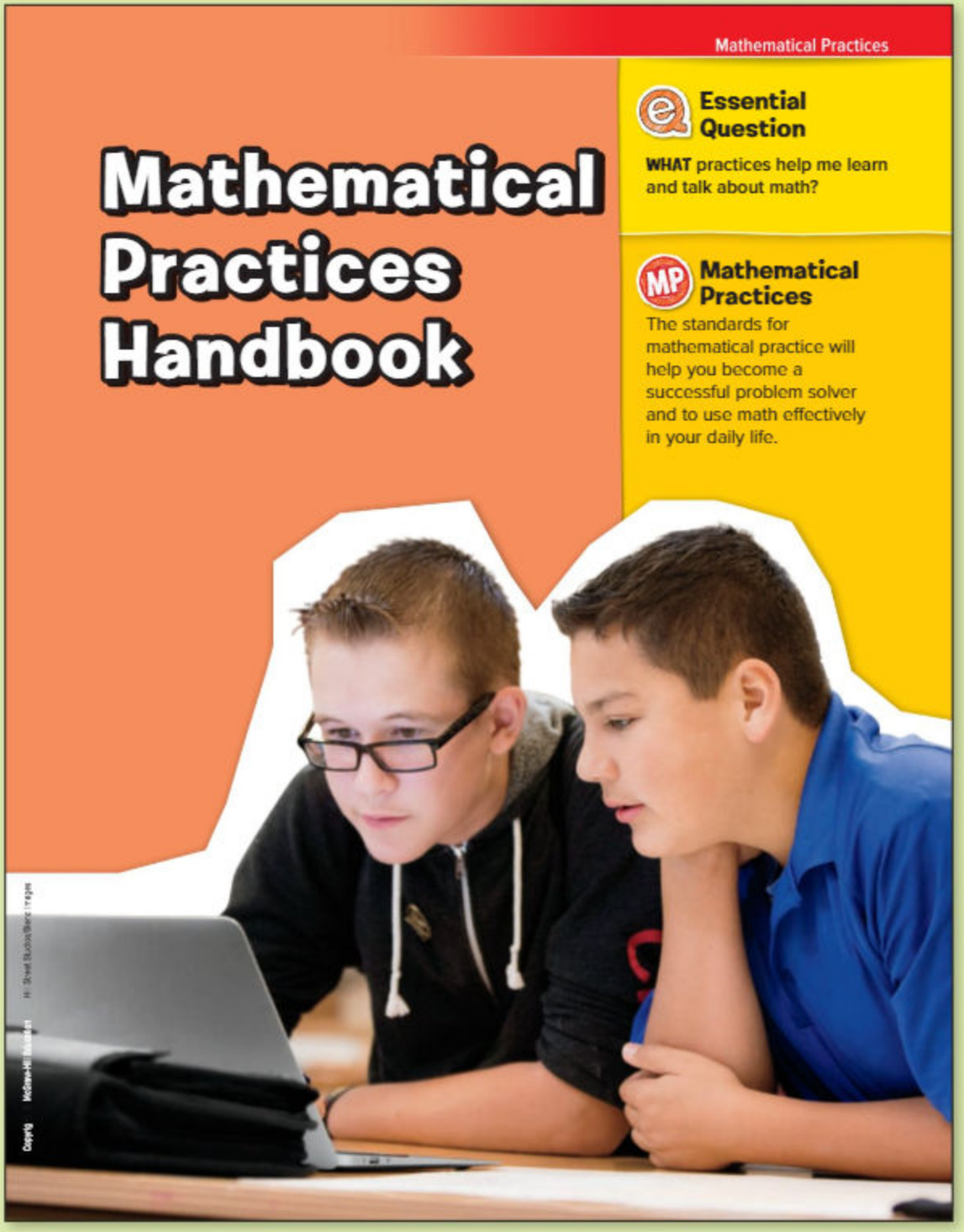
- Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability

- Develop understanding of statistical variability.
- Summarize and describe distributions.

MP Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



What is the MP Mathematical Practices Handbook?

Use the Mathematical Practices Handbook to introduce students to the **Mathematical Practices**.

The Standards for Mathematical Practice describe how students should approach mathematics. The goal of the practice standards is to instill in all students the abilities to be mathematically literate and to create a positive disposition for the importance of using math effectively.

Included in this handbook are activities and exercises that allow students to become familiar with the Mathematical Practices below. Throughout this text, students will see MP to remind them that they are using these Mathematical Practices.

- MP Persevere with Problems
- MP Reason Abstractly and Quantitatively
- MP Construct an Argument
- MP Model with Mathematics
- MP Use Math Tools
- MP Attend to Precision
- MP Make Use of Structure
- MP Use Repeated Reasoning

Building on the Essential Question

At the end of the Mathematical Practices Handbook, students should be able to answer "WHAT practices help me learn and talk about math?"

Throughout this text, refer to the following icons to find differentiated strategies to meet the needs of all learners.

- AL Approaching-Level Learners
- OL On-Level Learners
- BL Beyond-Level Learners
- LA Language Acquisition

What You'll Learn

Throughout this handbook, you will learn about each of these mathematical practices and how they are integrated in the chapters and lessons of this book.

1. **Focus on Mathematical Practice** Persevere with Problems
2. **Focus on Mathematical Practice** Reason Abstractly and Quantitatively
3. **Focus on Mathematical Practice** Construct an Argument
4. **Focus on Mathematical Practice** Model with Mathematics
5. **Focus on Mathematical Practice** Use Math Tools
6. **Focus on Mathematical Practice** Attend to Precision
7. **Focus on Mathematical Practice** Make Use of Structure
8. **Focus on Mathematical Practice** Use Repeated Reasoning

Place a checkmark below the face that expresses how much you know about each Mathematical Practice. Then explain in your own words what it means to you.

 I have no clue.  I've heard of it.  I know it!

Mathematical Practices

Mathematical Practice				What it means to me
1				
2				
3				
4				
5				
6				
7				
8				

MP Focus on Mathematical Practice 1

Persevere with Problems

How do I make sense of problems?

Yolanda will put 72 photos in a scrapbook. She will put the same number of photos on each of 6 pages. Four photos will be in each row. How many rows will be on each page?

As you work on a problem, you may find you need to change how you are solving it. This is called persevering.

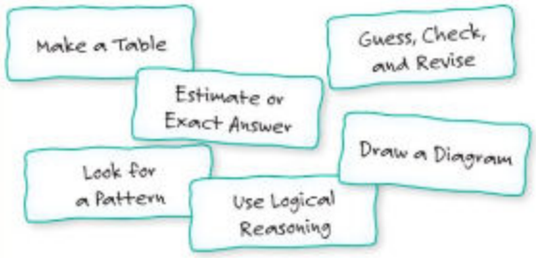
You can use a four-step plan to solve problems. Let's study each step.

MP Mathematical Practice 1

Make sense of problems and persevere in solving them.

1. **Understand** Read the problem. Circle the information that you are given and underline what you are trying to find.

2. **Plan** Decide on an appropriate strategy to use. Some strategies are listed below.



What strategy will you use to solve the problem above?

Sample answer: Draw a Diagram

3. **Solve** Apply your strategy to solve the problem.

There will be three rows on each page.

4. **Check** Determine if your solution is accurate and makes sense. What should you do if your solution doesn't make sense?

Yes it is accurate and makes sense; Try to solve the problem again.

Focus narrowing the scope

Objective Make sense of problems and persevere in solving them.

Coherence connecting within and across grades

Previous
Students multiplied multi-digit whole numbers.

Now
Students make sense of problems and persevere in solving them.

Next
Students will reason abstractly and quantitatively.

Rigor pursuing concepts, fluency, and applications

Mathematically proficient students use a logical process to make sense of problems, understand that there may be more than one way to solve a problem, and alter the process if needed.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Ideas for Use

LA **Group-Pair-Share** Have students work in groups to complete Exercises 1 and 2, ensuring that each member understands. Then have the groups divide into pairs to complete Exercises 3 and 4. Have them reconvene into the larger group to share responses and resolve any discrepancies. MP 1, 3

Alternate Strategy

AL For Exercise 2, provide several examples of problems that can be solved using the strategies listed. Have students discuss why the strategy is the best one. MP 1, 3

2 Practice and Apply

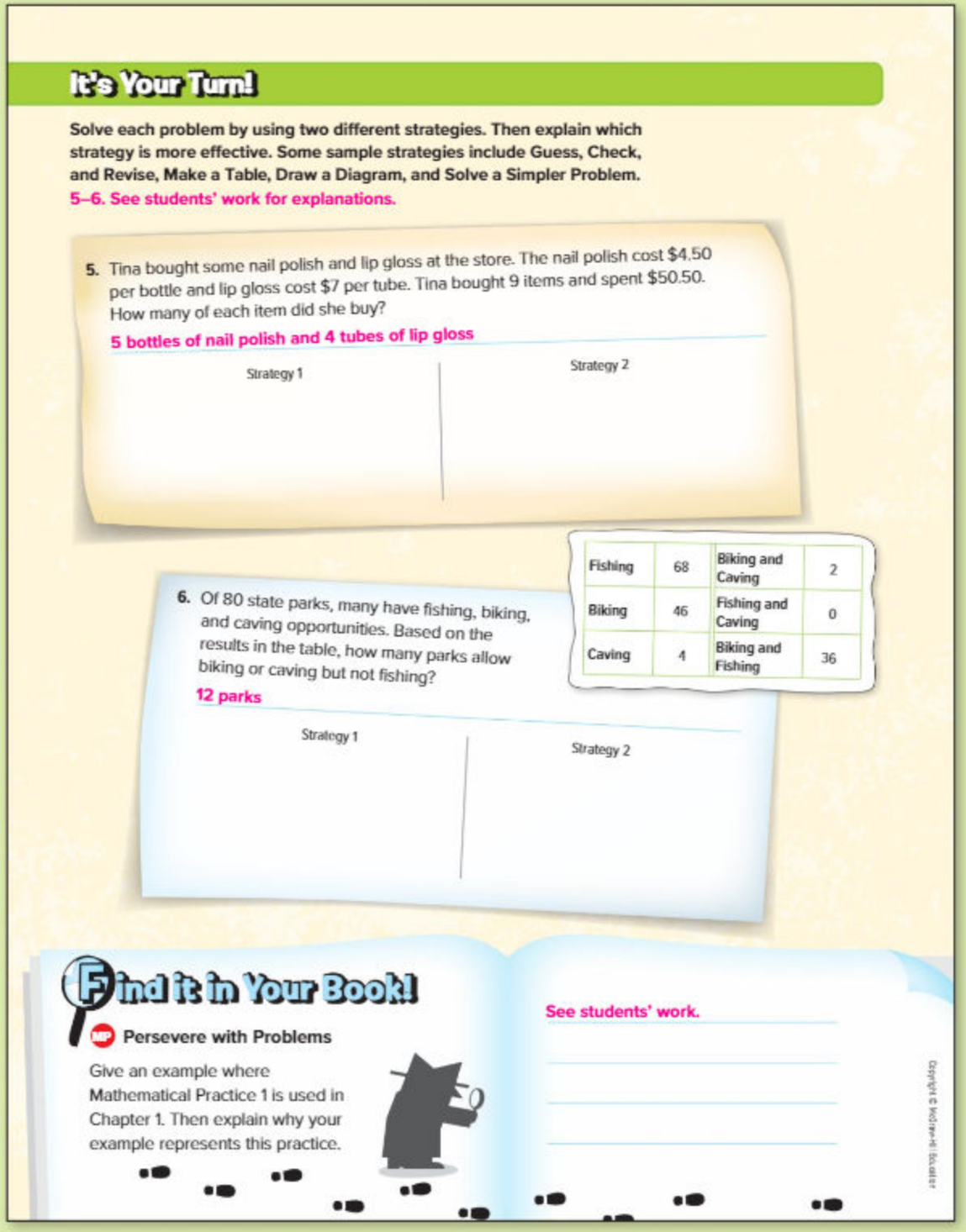
Ideas for Use

LA **Pairs Discussion** Have students work in pairs to discuss how they completed Exercises 5 and 6. Then call on one student to share their responses within a small group or large group discussion. MP 1, 3

Alternate Strategies

AL If students are having difficulty deciding on an appropriate problem-solving strategy, have them confer with a partner to decide on a strategy. Often, more than one strategy will lead to the correct solution, and sometimes, a discussion about an "incorrect" strategy may lead to a "correct" strategy. MP 1, 3, 5

BL LA **Pairs Create** Have students work in pairs to create a Venn diagram for Exercise 6. Then have the pairs write additional questions that can be answered using the Venn diagram. MP 1, 3, 5



It's Your Turn!

Solve each problem by using two different strategies. Then explain which strategy is more effective. Some sample strategies include Guess, Check, and Revise, Make a Table, Draw a Diagram, and Solve a Simpler Problem.

5–6. See students' work for explanations.

5. Tina bought some nail polish and lip gloss at the store. The nail polish cost $4.50 per bottle and lip gloss cost $7 per tube. Tina bought 9 items and spent $50.50. How many of each item did she buy?

5 bottles of nail polish and 4 tubes of lip gloss

Strategy 1

Strategy 2

6. Of 80 state parks, many have fishing, biking, and caving opportunities. Based on the results in the table, how many parks allow biking or caving but not fishing?

Fishing	68	Biking and Caving	2
Biking	46	Fishing and Caving	0
Caving	4	Biking and Fishing	36

12 parks

Strategy 1

Strategy 2

Find it in Your Book!

MP **Persevere with Problems**

Give an example where Mathematical Practice 1 is used in Chapter 1. Then explain why your example represents this practice.

See students' work.

Focus on Mathematical Practice 2

Reason Abstractly and Quantitatively

How can I use numbers to describe situations?

Mathematical Practice 2

Reason abstractly and quantitatively.

Suppose you receive gift cards to an online store for your graduation. Aunt Hilda sends you a $10 gift card, Uncle Norman sends you a $25 gift card, and your grandparents also send you a gift card. You've had your eye on five MP3 downloads, three games, and a wireless game controller. How much does the gift card from your grandparents need to be worth to cover the total cost?

Item	Cost ($)
MP3 download	0.99
Game 1	47.91
Game 2	17.61
Game 3	27.95
Game controller	60.00

You can represent this situation quantitatively, or by using numbers.

1. What numerical expression represents the total cost of all of the items?

   5(0.99) + 47.91 + 17.61 + 27.95 + 60

2. If your grandparents gave you a $100 gift card, will you have enough money to buy everything? Explain.

   no; Sample answer: The total amount of the gift cards would be $135, which is less than the total cost.

3. List three real-world situations that you could represent quantitatively. **See students' work.**

Situation 1	Situation 2	Situation 3

Focus narrowing the scope

Objective Reason abstractly and quantitatively.

Coherence connecting within and across grades

Previous
Students made sense of problems and persevered in solving them.

Now
Students reason abstractly and quantitatively.

Next
Students will construct viable arguments and critique the reasoning of others.

Rigor pursuing concepts, fluency, and applications

Mathematically proficient students can start with a concrete or real-world context and then represent it with abstract numbers or symbols (decontextualize), find a solution, then refer back to the context to check that the solution makes sense (contextualize).

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Ideas for Use

LA **Roundrobin** Have groups of students go around in a circle verbally explaining the skills they learned in math class last year. 1, 3

Alternate Strategy

BL LA **Trade-a-Problem** Have students create their own problem using one of the real-world situations they listed in Exercise 3. Students trade their problems, solve each other's problems, and compare solutions. If the solutions do not agree, students work together to find the errors. 1, 3, 4

2 Practice and Apply

Ideas for Use

LA **Pairs Discussion** Have students work in pairs to complete Exercises 4–6. Have them trade their solutions with another pair of students and discuss any differences. MP 1, 3

Alternate Strategies

AL LA For Exercise 4, have students use grid paper to align the columns and keep track of the location of the decimal point when they perform the calculations. MP 1, 3, 5

BL LA **Pairs Consult** Have pairs use the Internet, or another source, to research a real-world situation in which a relationship was represented using a table or a graph. Then have them represent the relationship using another form (table, graph, or equation). MP 1, 2, 3, 5

It's Your Turn!

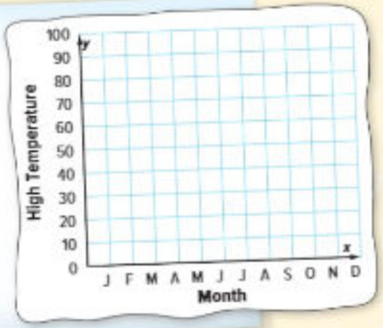
Represent each problem quantitatively.

4. A certain rollercoaster can accommodate 1,600 riders per hour. A digital sign states there are 345 people in line. If you get into line, about how many minutes will it take you to get to the front of the line?

13 min

5. Whew! Was this year the hottest year on record? Research and graph the record monthly high temperatures for your town. Then describe how this year's temperatures compare to the record temperatures.

See students' work for graphs and comparisons.



6. You have just arrived at summer camp. In your cabin, the counselor has each camper perform the super-secret handshake with every other person in the cabin. You and another person are in the cabin, so there is one handshake. Another person arrives, and another, until there are 14 people in the cabin. How many handshakes will there be in all?

91

Find it in Your Book!

MP **Reason Abstractly**

Give an example where Mathematical Practice 2 is used in Chapter 7. Then explain why your example represents this practice.

See students' examples.

Focus on Mathematical Practice 3

Construct an Argument

Have you ever had to explain to someone why you were late?

Mathematical Practice 3

Construct viable arguments and critique the reasoning of others.

Chances are that saying "I don't know" is not going to go over very well. You need to give a good reason why you were late. In math, you often need to construct a viable, or practical, argument or examine the reasoning of others. When you use rules, definitions, or properties, you are using *deductive reasoning*. When you use examples or patterns, you are using *inductive reasoning*.

1. The table shows some of the properties of math. Complete the table by completing the example for each property.

Commutative Property of Addition	Commutative Property of Multiplication
$5 + 8 = \boxed{8} + 5$	$5 \times 8 = 8 \times \boxed{5}$
Associative Property of Addition	**Associative Property of Multiplication**
$6 + (4 + 3) = (\boxed{6} + 4) + 3$	$6 \times (4 \times 3) = (6 \times 4) \times \boxed{3}$
Identity Property of Addition	**Identity Property of Multiplication**
$9 + 0 = \boxed{9}$	$9 \times 1 = \boxed{9}$

2. Complete each step to construct an argument for how you can find 3.5 + 1.7 + 1.5 mentally.

Step	Property
$3.5 + 1.7 + 1.5 = 1.7 + 3.5 + 1.5$	Commutative Property of Addition
$= 1.7 + (3.5 + 1.5)$	Associative Property of Addition
$= 1.7 + 5.0$	Add mentally.
$= 6.7$	Add mentally.

Focus narrowing the scope

Objective Construct viable arguments and critique the reasoning of others.

Coherence connecting within and across grades

Previous
Students reasoned abstractly and quantitatively.

Now
Students construct viable arguments and critique the reasoning of others.

Next
Students will model with mathematics.

Rigor pursuing concepts, fluency, and applications

Mathematically proficient students can clearly communicate their thoughts and defend them using sound mathematical arguments.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Ideas for Use

LA **Roundrobin** Have students in each group go around in a circle verbally generating additional examples for each property listed in the table on page MP7. MP 1, 3, 7

Alternate Strategy

BL LA **Numbered Heads Together** Provide several pairs of numerical expressions. Have students work in groups to determine if the expressions are equivalent based on the properties of math. Call on one group member to share the group's responses. MP 1, 3, 7

2 Practice and Apply

Ideas for Use

LA **Talking Chips** Have students work in small groups to complete Exercises 3–5. Give each student 5 chips. As a group, students discuss and solve each problem. Students must place a chip in the center of the table each time they contribute to the discussion, offering their opinions and/or ideas. After they have used all of their chips, they may no longer contribute to the discussion. All students must use all of their chips. MP 1, 3, 5

Alternate Strategies

AL LA If students are struggling to solve the problems, have them use the four-step problem-solving model for each exercise. MP 1, 3, 5

Step 1 Understand

Step 2 Plan

Step 3 Solve

Step 4 Check

BL LA **Pairs Present** Hold an informal class debate on a topic or topics of your choice. Give students enough time to prepare their arguments. Arguments should be well thought-out and convincing. Have pairs or groups of students present their arguments to the class. MP 1, 3

It's Your Turn!

Solve each problem. Construct a viable argument for your answer.

3. About 80% of the egg rolls sold in U.S. grocery stores are made in Houston, Texas. A skilled wrapper can make 4,000 egg rolls in an 8-hour day. Bettie said this is about one egg roll a minute. Is her claim reasonable? Explain.

no; Sample answer: 1 egg roll a minute is 60 egg rolls an hour × 8 hours is 480 egg rolls.

4. Not all products labeled cheese are all cheese! A cheese product is less than half actual cheese. Cheese food is more than half, but less than all cheese. Arturo said a 1 pound block that contained 9 ounces of cheese should be called a cheese food. Is his claim reasonable? Explain.

yes; Sample answer: one-half of a pound is 8 ounces. Nine ounces is more than half so the food is a cheese food.

5. A department store advertised that for every $25 you spend at the store, you get $2 off your total bill. Nathanial's bill came to $154 before the discount. He figured he would get a $12 discount. Is his claim reasonable? Explain.

yes; Sample answer: $154 is about $150; 150 ÷ 25 = 6 and 6 × 2 = 12.

Find it in Your Book!

MP **Construct an Argument**

Give an example where Mathematical Practice 3 is used in Chapter 1. Then explain why your example represents this practice.

See students' examples.

Focus on Mathematical Practice 4

Model with Mathematics

How many different names do you have?

Mathematical Practice 4
Model with mathematics

Your family, friends, and teachers may call you by different names. You might be Susanna, Suzie, Sue, or Miss Wilson. All of these names still represent *you*. In math, we also use different ways to model the same idea. We can model situations using words, graphs, tables, numbers, symbols, or diagrams.

1. How can you model the number 0.25 using words?

   Sample answer: twenty-five hundredths

2. How can you model the number 0.25 using a fraction?

   $\frac{1}{4}$

3. Suppose you and your friends ordered four pizzas. Together, a total of 3 pizzas were eaten. Complete the graphic organizer to model the fraction of the total pizza that was eaten in different ways.

Diagram	Numbers	Words
$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$	$\frac{3}{4}$, 0.75, 75 %	three-fourths three out of four seventy-five percent

4. Draw a fraction circle to show the relationship between the amount of pizza you and your friends ate and the total amount of pizza ordered. Explain why the fraction circle is a good model.

   Sample drawing:

   Sample answer: It shows the relationship between the parts and the whole.

Focus narrowing the scope

Objective Model with mathematics.

Coherence connecting within and across grades

Previous
Students constructed viable arguments and critiqued the reasoning of others.

Now
Students model with mathematics.

Next
Students will use appropriate tools strategically.

Rigor pursuing concepts, fluency, and applications

Mathematically proficient students explain their thinking or search for patterns using models such as diagrams, drawings, classroom objects, and manipulatives, or geometric, graphical, algebraic, tabular, and statistical models.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Ideas for Use

LA **Team-Pair-Solo** Have students work in small groups to complete Exercises 1–2. Then have them divide into pairs to complete Exercise 3. Have students work individually to complete Exercise 4. Have them regroup to discuss their responses to Exercises 3–4 and resolve any differences. MP 1, 3

Alternate Strategy

AL LA Have students use fraction circles to represent other fractions, such as $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{1}{4}$. MP 1, 3, 5

2 Practice and Apply

Ideas for Use

LA **Think-Pair-Share** Give students 1–2 minutes to think through their responses to Exercise 5. Then have them discuss how they determined their answers with a partner. Ask a volunteer to share the pair's solution with the class. MP 1, 3

Alternate Strategies

AL LA Create a coordinate plane on the floor using masking tape. Call out an ordered pair. Then have a student explain how to find the location of the ordered pair by first walking along the *x*-axis and then along the *y*-axis. Repeat for several other ordered pairs and with different students. MP 1, 3, 5

BL Introduce the distance equation, distance = rate × time. Then have students determine the distance traveled for other times. MP 1, 3, 5

It's Your Turn!

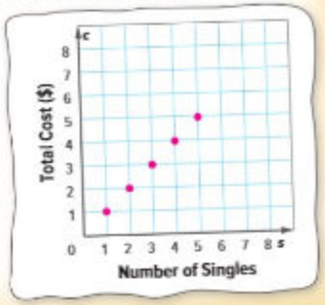
Use the models shown to solve each problem.

5. You are downloading MP3 singles onto your computer. The singles cost $0.99 each.

a. **Tables** Complete the table to show the cost of 1, 2, 3, 4, and 5 singles.

Number of Singles, *s*	Total Cost ($), *c*
1	0.99
2	1.98
3	2.97
4	3.96
5	4.95

b. **Graphs** Graph the ordered pairs (number of singles, total cost) on the coordinate plane.



6. Luna's family is driving along the highway. The table shows the time, in hours, traveled and the total distance, in miles, traveled.

Time, *t* (h)	Distance, *d* (mi)
1	68
2	136
3	204
4	272
5	340

a. **Tables** Use the table to find the total distance traveled after 7 hours. 476 mi

b. **Words** Describe the relationship between time and distance traveled.

Sample answer: Every hour, Luna's family traveled 68 miles.

Find it in Your Book!

MP **Model with Mathematics**

Give an example of where Mathematical Practice 4 is used in Chapter 8. Then explain why your example represents this practice.

See students' examples.

Focus on Mathematical Practice 5

Use Math Tools

How likely is it that a baseball player will hit a homerun using a hockey stick?

Mathematical Practice 5
Use appropriate tools strategically.

Since he is using a hockey stick as a bat, he's not very likely to make any hit, much less a homerun! Similar to using the correct equipment in sports, you also need to use the correct tools in mathematics. So, let's investigate how to choose and use the appropriate tools to solve math problems.

1. To build a hockey goal, you might use a hammer or a drill. While you wouldn't use a hammer or a drill to solve a math problem, you might use some of the tools and strategies listed below. Complete the graphic organizer by adding some of your own tools and strategies. **Sample answers given.**

Math Tools

Tools	
counters	algebra tiles
ruler	computer
compass	calculator

Strategies	
estimation	choose the correct unit
draw a diagram	mental math
look for a pattern	make a prediction

2. Suppose you want to build a hockey goal and need to determine how much the materials will cost. List three tools or techniques that you could use to determine the total cost of the materials.
Sample answer: a diagram, measuring tape, calculator

3. Describe a situation in which you might use estimation.
Sample answer: I can estimate when the problem asks for an approximation and not an exact answer. I can also estimate to check my answer for reasonableness.

Focus narrowing the scope

Objective Use appropriate tools strategically.

Coherence connecting within and across grades

Previous
Students modeled with mathematics.

Now
Students use appropriate tools strategically.

Next
Students will attend to precision.

Rigor pursuing concepts, fluency, and applications

Mathematically proficient students understand the benefits and limitations of using mathematics tools, including estimation and virtual tools, and use them appropriately.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Ideas for Use

LA **Think-Pair-Share** Have students work in pairs. Give them time to think through their solution to each exercise. Have them share their responses with their partner first, then with the entire class. MP 1, 3

Alternate Strategy

BL After Exercise 3, have students describe situations that can be solved with a combination of some of the other tools and strategies. MP 1, 3, 5

2 Practice and Apply

Ideas for Use

LA **Stand Up, Hand Up, Pair Up** For each exercise, pose the question and give think time. Call out, "Stand up, hand up, pair up". Students stand, put their hand up, and pair up with a student. In pairs, students discuss their responses. Repeat for each exercise. Have students pair up with a different student each time. MP 1, 3

Alternate Strategies

AL Before students select the tools and strategies for each exercise, have them ask themselves if they have solved a similar problem. If so, have them describe the tools and strategies they used to solve the problem. MP 1, 3, 5

BL LA **Gallery Walk** Have students work with a partner to create a real-world problem. Post the problems around the room. Students walk around the room and select a problem, not their own. Working with their partner, they determine which tools and strategies can be used to solve the problem. Students are not limited to the tools and strategies listed in the table on the previous page. MP 1, 3, 4, 5

It's Your Turn!

For Exercises 4–7, select the tools and/or strategies you could use to solve each problem. Then write the corresponding letter(s) of each tool or strategy.
Sample answers are given.

Tools	Strategies
A. paper and pencil	F. draw a diagram
B. calculator	G. estimation
C. measuring tape/ruler	H. mental math
D. Internet	I. number sense
E. virtual manipulatives	J. make a prediction

4. Yikes, the supply list for school is huge! You need to buy everything on the list. How much will it all cost?

B, I

5 spiral notebooks	10 pens
3-ring binder	3 pencils
notebook paper	dictionary
4 highlighters	10 folders
scientific calculator	flash drive

5. You want to bake a cake for your best friend's graduation. How much will it cost to buy all of the ingredients?

A, G, H

6. The table shows the different admission prices to an amusement park. You and your family want to spend the day at the amusement park, have lunch, and buy a souvenir. How much money should you take?

A, G

Wonderama! Ticket Prices	
Adult	$22.95
Children (4–11)	$18.95
Seniors	$21.95

Find it in Your Book!

MP **Use Math Tools**

Give an example of where Mathematical Practice 5 is used in Chapter 1. Then explain why your example represents this practice.

See students' examples.

Focus on Mathematical Practice 6

Attend to Precision

How can the everyday meaning of a word help you remember the math meaning?

Mathematical Practice 6
Attend to precision.

Mathematics is not just about numbers. You also use words, symbols, and drawings to communicate mathematics. Being precise in your communication means using mathematical words accurately. Some words are used in everyday life as well as mathematics.

1. Look up the word "annex" in the dictionary. What does it mean to "annex"?
   Sample answers: to join together, to incorporate, to obtain, to add on

2. Give two examples of how the word "annex" is used in everyday life. **Sample answers given.**

Example 1	Example 2
annex a piece of land	annex of a building

3. How is "annex" used in mathematics?
   Sample answer: when you place zeroes to the right of decimals when adding or subtracting

4. Explain how you would solve 24 – 9.34. Use correct mathematical vocabulary in your explanation.
   Sample answer: Since 9.34 has a decimal to the hundredths, I need to annex two zeroes at the end of 24 in order to subtract. My subtraction problem is now 24.00 – 9.34.

Focus narrowing the scope

Objective Attend to precision.

Coherence connecting within and across grades

Previous
Students used appropriate tools strategically.

Now
Students attend to precision.

Next
Students will look for and make use of structure.

Rigor pursuing concepts, fluency, and applications

Mathematically proficient students communicate the language of mathematics precisely, as well as calculate efficiently and accurately.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Ideas for Use

LA **Group-Pair-Share** Have students work in groups to complete Exercise 1, ensuring that each member understands. Then have the groups divide into pairs to complete Exercises 2–4. Have them reconvene into the larger group to share responses and resolve any discrepancies. MP 1, 3

Alternate Strategy

AL Provide students with grid paper that they can use with Exercise 4. MP 1, 5

2 Practice and Apply

Ideas for Use

Rally Coach Have students work in pairs to complete Exercises 5–7. Have one partner complete Exercise 5 while the second partner watches, listens, coaches, and praises. Have them trade roles for Exercises 6 and 7. MP 1, 3

Alternate Strategies

AL Place students that have difficulty with the Exercises into a group that will receive help from the teacher. As individual students become more successful, have them begin to work on their own. MP 1, 3

BL **Find the Fib** Have students work with a partner to create two facts and one fib for each exercise. For example, one fact for Exercise 5 could be that Manny exercised less than an hour each day. One fib for Exercise 5 could be that he exercised more than two hours total. Have pairs share their facts and fibs with another pair of students. Each pair should identify the facts and fibs of the other pair. MP 1, 3, 5

It's Your Turn!

Solve. Then use correct mathematical vocabulary to explain how you solved each problem.

5. The table shows the amount of time Manny exercised this weekend. How long did Manny exercise altogether?

Day	Part of Hour Exercised
Saturday	$\frac{2}{3}$
Sunday	$\frac{5}{6}$

$1\frac{1}{2}$ h; Sample answer: In order to find the sum of two fractions with different denominators, I need to get a common denominator. After finding the sum, I need to rename the improper fraction as a mixed number.

6. Catie is making bracelets for her friends using small rubber bands. She has 364 rubber bands and she wants to make 6 bracelets. How many rubber bands will she use for each bracelet? How many rubber bands will be left over?

60 rubber bands; 4 left over; Sample answer: I need to find 364 ÷ 6 to find the number of rubber bands per bracelet. Since there is a remainder of 4, there are 4 rubber bands left over.

7. When baking, it is important to use exact measures of ingredients. Kevin wants to double the recipe for carrot cake. A single batch requires $1\frac{1}{4}$ cup oil. He estimates he will need about 2 cups of oil. How much oil will he need? Why is estimation not a good strategy when baking?

$2\frac{1}{2}$ c; Sample answer: If Kevin estimates and only uses 2 cups of oil, he will not put enough oil in the cake.

Find it in Your Book!

MP **Be Precise**

Give an example of where Mathematical Practice 6 is used in Chapter 1. Then explain why your example represents this practice.

See students' examples.

Focus on Mathematical Practice 7

Make Use of Structure

Can you think of an easier way to find a solution?

Mathematics is based on patterns and structure. You can look for those patterns to help solve problems. If you know how numbers work together, you can find easier ways to solve problems.

Mathematical Practice 7
Look for and make use of structure.

1. The restaurant receipt shown does not have a total. Use paper and pencil to find the total spent, not including tip.
   $24

Mimi's Diner	
two drinks	$2.50
appetizer	$5.50
entrée	$11.75
dessert	$4.25
Total	

2. What are some other strategies for finding a solution?
   mental math, estimation, calculator

3. Use the terms below to describe how you would solve a problem using mental math or estimation. **Sample answers given.**

compatible numbers rounding properties of numbers

Mental Math	Estimation
Use the properties of numbers to rewrite the problem so you can use compatible numbers to solve.	You can round to compatible numbers or use rounding methods to estimate.

4. Using one of the methods above, explain how you found the total of the restaurant receipt from Exercise 1.
   Sample answer: I know that 2.50 + 5.50 + 11.75 + 4.25 = 2 + 0.50 + 5 + 0.50 + 11 + 0.75 + 4 + 0.25. I then used mental math to rewrite the expression as 2 + 5 + 11 + 4 + 0.50 + 0.50 + 0.75 + .25. This made it easier to add the whole numbers and the decimals.

Focus narrowing the scope

Objective Look for and make use of structure.

Coherence connecting within and across grades

Previous
Students attended to precision.

Now
Students look for and make use of structure.

Next
Students will look for and express regularity in repeated reasoning.

Rigor pursuing concepts, fluency, and applications

Mathematically proficient students look for structure to find easier ways to solve problems.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Ideas for Use

LA **Group-Pair-Share** Have students work in groups to complete Exercises 1 and 2, ensuring that each member understands. Then have the groups divide into pairs to complete Exercises 3 and 4. Have them reconvene into the larger group to share responses and resolve any discrepancies. MP 1, 3

Alternate Strategy

AL Provide students with grid paper that they can use with Exercise 1. MP 1, 5

2 Practice and Apply

Ideas for Use

Pairs Discussion Have students work in pairs to discuss how they completed Exercises 5–7. Then call on one student to share their responses within a small group or large group discussion. MP 1, 3

Alternate Strategies

AL Place students that have difficulty with the Exercises into a group that will receive help from the teacher. As individual students become more successful, have them begin to work on their own. MP 1, 3

BL **Find the Fib** Have students work with a partner to create two facts and one fib for each exercise. For example, one fact for Exercise 5 could be that less than half of the 200 students will have an electronic tablet. One fib for Exercise 5 could be that 20 students in the homeroom do not have an electronic tablet. Have pairs share their facts and fibs with another pair of students. Each pair should identify the facts and fibs of the other pair. MP 1, 3, 5

It's Your Turn!

Explain how you would solve each problem. Then solve.

5. Six of the 24 students in a homeroom have an electronic tablet. Predict how many students out of 200 have an electronic tablet.

Sample answer: Six is one-fourth of 24 and one-fourth of 200 is 50.

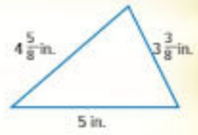
So about 50 students out of 200 students have an electronic tablet.

6. Tucker ran 2.25 kilometers each day for 5 days. How many kilometers did he run altogether?

Sample answer: 2.25 can be written as $2 + 0.25$. By using the Distributive Property, $5 \times (2 + 0.25)$ can be rewritten as $5 \times 2 + 5 \times 0.25$. So, $5 \times 2.25 = 10 + 1.25$ or 11.25; 11.25 km

7. The perimeter of the triangle can be found by adding the lengths of the sides. Find the perimeter of the triangle.

Sample answer: The fractions $\frac{5}{8}$ and $\frac{3}{8}$ have a sum of 1. By using mental math, the perimeter is $4 + 3 + 5 + 1$, or 13 inches.



Find it in Your Book!

MP **Make Use of Structure**

Give an example of where Mathematical Practice 7 is used in Chapter 1. Then explain why your example represents this practice.

See students' examples.

MP Focus on Mathematical Practice 8

Use Repeated Reasoning

How can repeated reasoning help you find shortcuts?

MP Mathematical Practice 8

Look for and express regularity in repeated reasoning.

In mathematics, if you recognize a pattern, it can help you to find shortcuts to solve the rest of the problem or similar problems. When you look for and use patterns, you are using repeated reasoning.

The Taco Hut likes to give away free food on a regular basis to encourage customers to return. The table shows which items they give away for free and how often.

Food	Frequency
Taco	every 4 days
Nachos	every 5 days
Burrito	every 6 days

1. In the first 60 days of the promotion, on which days will a taco be given away for free?
   days 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, and 60
2. In the first 60 days, on which days will nachos be given away for free?
   days 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, and 60
3. In the first 60 days, on which days will a burrito be given away for free?
   days 6, 12, 18, 24, 30, 36, 42, 48, 54, and 60
4. Are there any days in which more than one food item is given away? If so, which days? **yes; On days 12, 24, 36, and 48, both tacos and burritos are free. On days 20, and 40, both tacos and nachos are free. On day 30, both nachos and burritos are free. On day 60, all three items are free.**
5. If all three items were given away today, how many days until all three are given away on the same day? Explain your reasoning.
   60 days; Sample answer: After listing the days the items are given away for free, the next day in common is day 60.

Focus narrowing the scope

Objective Look for and express regularity in repeated reasoning.

Coherence connecting within and across grades

Previous
Students looked for and made use of structure.

Now
Students look for and express regularity in repeated reasoning.

Next
Students will find the GCF and LCM of a set of numbers.

Rigor pursuing concepts, fluency, and applications

Mathematically proficient students recognize and use patterns that can lead to results more quickly and efficiently.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

Ideas for Use

LA **Think-Pair-Share** Have students work in pairs. Give them time to think through their solution to each exercise. Have them share their responses with their partner first, then with the entire class. MP 1, 3

Alternate Strategy

AL Provide students with number lines that they can use with Exercises 1–5. MP 1, 5

2 Practice and Apply

Ideas for Use

Rally Robin In small groups, have students take turns speaking aloud their answers to Exercises 6–8. MP 1, 3

Alternate Strategies

AL **Pairs Check** If students have difficulty using the coordinate plane in Exercise 7, have students work in pairs and have them check their answers. MP 1, 5

BL Have students determine an algebraic algorithm for each set of points for Exercise 7. MP 1, 2, 5

It's Your Turn!

Solve each problem by using repeated reasoning.

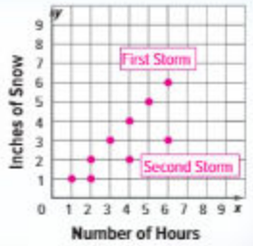
6. Elyse is saving up to buy a new digital camera. She begins with $25 in her savings account and deposits $10 each week. How many more weeks must she save to purchase model X250 than model X150? Explain.

3 weeks; Sample answer: Model X250 costs $30 more. It will take her $30 ÷ $10 or 3 weeks to save an additional $30.

Camera Model	Cost ($)
X150	99
X250	129

7. Anya tracked the amount of snowfall during two different snowstorms. During the first snowstorm, it snowed 1 inch per hour. The second storm produced an inch of snow every two hours. How many more inches of snow did the first storm produce in 6 hours than the second storm? Graph the situation on the coordinate plane.

3 inches



8. The drama club is building a wall for their next production. The wall is 12 feet long. They are placing the studs every 18 inches. If there are studs on each end of the wall, how many studs will they place?

9 studs

Find it in Your Book!

MP **Use Repeated Reasoning**

Give an example of where Mathematical Practice 8 is used in Chapter 1. Then explain why your example represents this practice.

See students' examples.

MP Mathematical Practices Handbook

Review

Use the Mathematical Practices

Solve.

During the summer, you earn money to buy school clothes. The table shows how much money you earn for each job. This summer, you babysat for 32 hours, mowed 10 lawns, and worked at the pool for 20 hours. You spent $256.78 on school clothes.

Job	Money Earned
babysitting	$5.00/hr
lawn mowing	$15.00/lawn
pool	$6.20/hr

a. Do you have any money left? If so, how much? yes; $177.22

b. You want to buy a tablet that costs $399. Do you have enough money left? If not, how many more hours would you need to babysit to have enough to buy the tablet? no; 45 hours

Determine which mathematical practices you used to determine the solution. Shade the circles that apply.

Which MP **Mathematical Practices** did you use?
Shade the circle(s) that applies.

- ① Persevere with Problems
- ② Reason Abstractly
- ③ Construct an Argument
- ④ Model with Mathematics
- ⑤ Use Math Tools
- ⑥ Attend to Precision
- ⑦ Make Use of Structure
- ⑧ Use Repeated Reasoning

Use the Mathematical Practices

Use the next two pages to review each of the **Mathematical Practices** and how students will use them to engage in the content they will encounter throughout this text.

Ideas for Use

LA **Numbered Heads Together** Assign students to a 3- or 4-person learning team. Each member is assigned a number from 1 to 4. Each team completes the problem, making sure that everyone understands. Call on a specific number from one team to present the team's solution to the class. MP 1, 3

Answering the Essential Question

At the end of the Mathematical Practices Handbook, students should be able to answer "WHAT practices help me learn and talk about math?"

Ideas for Use

LA **Think-Pair-Share** Have students work in pairs. Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer. Then have them share their responses with their classmate before they complete the graphic organizer.

MP 1, 3, 5

Reflect

Answering the Essential Question

Use what you learned about the mathematical practices to complete the graphic organizer. Write 3 examples that you could use for each category.
Sample answers are given.

Select Tools and Strategies
1. calculator
2. mental math
3. paper and pencil

Organize Ideas
1. concept map
2. Foldable
3. web

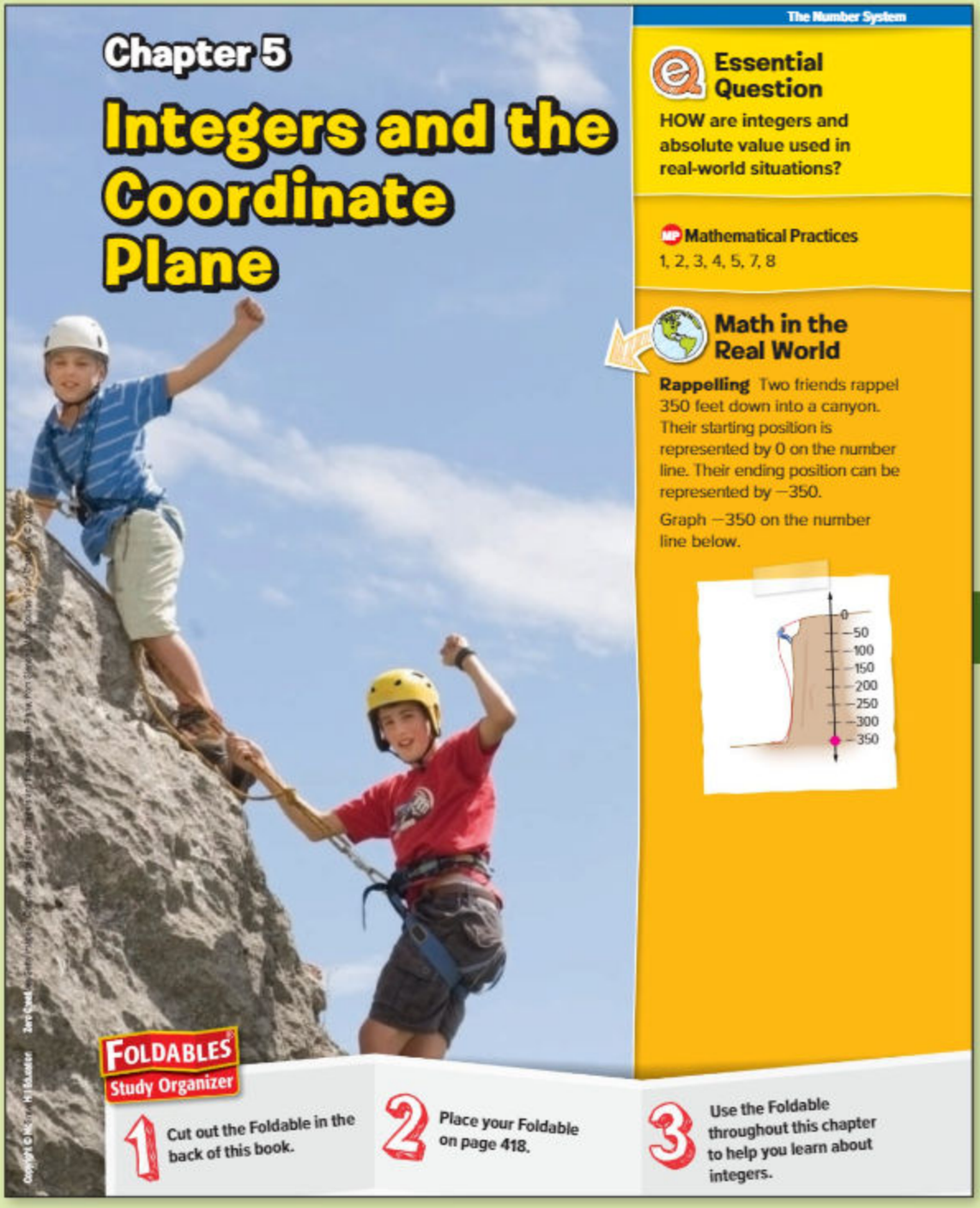
Essential Question
WHAT practices help me learn and talk about math?

Use Multiple Representations
1. graphs
2. tables
3. words

Use a Problem-Solving Model (List the four steps)
1. Understand
2. Plan
3. Solve
4. Check

Answer the Essential Question. WHAT practices help me learn and talk about math?

Sample answer: Organize Ideas helps me organize concepts and terms and Use Multiple Representations helps me communicate ideas by using graphs, tables, and words.



Focus narrowing the scope

This chapter focuses on content from the **Number System**(NS) domain.

Coherence connecting within and across grades

Previous
Students added, subtracted, multiplied, and divided fractions and decimals.

Now
Students use integers, absolute value, and graphing on the coordinate plane to solve real-world problems.

Next
Students will use expressions, equations, and functions to describe relationships.

Rigor pursuing concepts, fluency, and applications

The Levels of Complexity charts located throughout this chapter indicate how the exercises progress from conceptual understanding and procedural skills and fluency, to application and critical thinking.

Launch the Chapter

Math in the Real World

Rappelling Point out to students that key words like *below* and *descend* indicate use of a negative integer.

What Tools Do You Need?

Vocabulary Activity

LA As you proceed through the chapter, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define: Absolute value is the distance between the number and zero on a number line.

Example: The absolute value of −3 is 3.

Ask:

- *What is* | −8 | *?* 8

Review Vocabulary

LA Have students complete the section. Have students use this type of a graphic organizer for all of the vocabulary terms in the chapter. As they progress through each lesson, they can put the organizers together and have their own glossary of all of the vocabulary in the chapter.

What Tools Do You Need?

Vocabulary

absolute value	positive integer
bar notation	quadrants
integer	rational number
negative integer	repeating decimal
opposites	terminating decimal

Review Vocabulary

Using a graphic organizer can help you to remember important vocabulary terms. Fill in the graphic organizer below for the word *decimal*.

Decimal

Definition
A number that has a digit in the tenths place, hundredths place, and beyond.

Math Example
Sample answer: 0.36

Real World Example
Sample answer: decimals are used to represent dirhams and fils in money; AED 3.95

What Do You Already Know?

Place a checkmark below the face that expresses how much you know about each concept. Then scan the chapter to find a definition or example of it. See students' work.

I have no clue. I've heard of it. I know it!

Integers and the Coordinate Plane				
Concept				Definition or Example
absolute value				
comparing integers				
graphing integers on a number line				
graphing ordered pairs				
ordering rational numbers				
rational numbers				

When Will You Use This?

Here is an example of how integers are used in the real world.

Activity Does anyone in your family own a digital camera? If so, how much data does the memory card hold? Use the Internet to determine how many pictures your memory card might hold.

See students' work.

What Do You Already Know?

In this activity students assess their prior knowledge choosing a face to represent their knowledge about concepts in the chapter.

After completing the chapter, have students return to this page and have them reevaluate their knowledge level about the content.

When Will You Use This?

Activity

Students discover the use of integers in real-world situations.

Are You Ready?

Use this page to determine if students have skills that are needed for the chapter.

Quick Review

Students with strong math backgrounds may opt to go directly to the Quick Check.

Quick Check

If students have difficulty with the exercises, present an additional example to clarify any misconceptions.

Exercises 1–3

Is 0.8 less than, greater than, or equal to 0.7? greater than

Exercises 4–7

Is $\frac{4}{5}$ less than, greater than, or equal to $\frac{7}{8}$? less than

Are You Ready?

Try the Quick Check below.

Quick Review

Example 1

Replace the ◯ with <, >, or = to make a true statement.

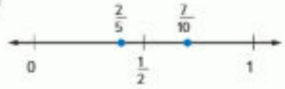
1.6 ◯ 1.3

Since 1.6 is to the right of 1.3, 1.6 > 1.3.

Example 2

Replace the ◯ with <, >, or = to make a true statement.

$\frac{2}{5}$ ◯ $\frac{7}{10}$



Since $\frac{2}{5}$ is less than $\frac{1}{2}$ and $\frac{7}{10}$ is greater than $\frac{1}{2}$, $\frac{2}{5} < \frac{7}{10}$.

Quick Check

Compare Decimals Replace each ◯ with <, >, or = to make a true statement.

1. 4.8 (=) 4.80
2. 7.7 (>) 7.5
3. 1.2 (<) 2.1

Show your work.

Compare Fractions Replace each ◯ with <, >, or = to make a true statement.

4. $\frac{2}{11}$ (<) $\frac{9}{10}$ =
5. $\frac{3}{5}$ (>) $\frac{1}{4}$ =
6. $\frac{2}{3}$ (=) $\frac{4}{6}$ =

7. Omar bought $\frac{2}{3}$ pound of peanuts and $\frac{1}{4}$ pound of walnuts. Did Omar buy more peanuts or more walnuts? peanuts

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4 5 6 7

Inquiry Lab
Integers

Inquiry **HOW can positive and negative values be represented?**

MP Mathematical Practices 1, 3, 4

In coastal regions, some animals live above sea level and other animals live in the ocean. A sea star can be found at an ocean depth of two feet. How can you represent an ocean depth of two feet?

What do you know? A sea star is found at an ocean depth of two feet.

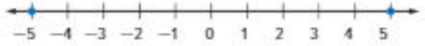
What do you need to find? I need to find how to represent an ocean depth of two feet.

Hands-On Activity

Sea level can be represented with the number 0.

To represent a location above sea level, use a positive number. A positive number can be written with or without a positive sign, such as 5 or +5.

To represent a location below sea level, use a negative number. A negative number is written with a negative sign, such as −5.



Write a number to represent an ocean depth of two feet.

Step 1 Determine if a positive sign or a negative sign should be used.

Since the location is below, or less than, sea level, use a negative sign.

Step 2 Determine which number to use.

Use the number 2 to represent two feet.

So, the number −2 represents an ocean depth of two feet.

Focus narrowing the scope

Objective Use counters to represent integers.

Coherence connecting within and across grades

Now
Students represent situations with integers and graph integers on a number line.

Next
Students will write an integer and its opposite. They will graph integers on a number line.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 344.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Hands-On Activity

AL Using real-world manipulatives, such as candy or money, have one student borrow from another and then "spend" what he borrowed. MP 1, 4

Ask:

- *You have no money left, and you have to pay back what you borrowed. How can we represent how much money you will have?* with a negative number

BL LA Have students work in pairs to answer the following questions. MP 1, 2

Ask:

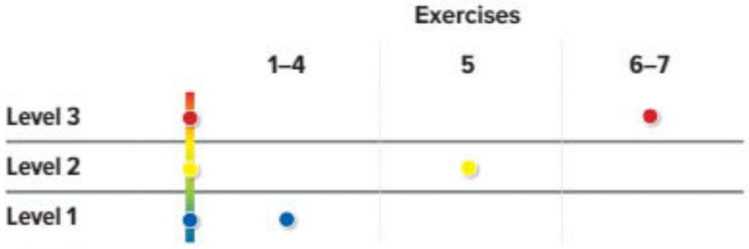
- *Where else might we see negative numbers?* Sample answers: bank account, football statistics, golf, temperatures
- *What other key words could be used to indicate a negative number?* Sample answers: less than, below, withdrawal, loss

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA Before beginning the activity, ask the following questions. Then have students work in pairs to complete Exercises 1–5. MP 1, 6

Ask:

- *In the table, what does the word "above" represent?* a positive number
- *What does the word "below" represent?* a negative number

Create

BL LA **Trade-a-Problem** Have students write their problem on an index card, then trade with a partner and solve the other problem. Have them listen carefully to each other's solutions and discuss any differences. MP 1, 3, 4

Inquiry Students should be able to answer "HOW can positive and negative values be represtented?" Check for student understanding and provide guidance, if needed.

Investigate

Work with a partner. Write the correct number to represent each location in relationship to sea level. The first one is done for you. Then draw a number line to represent each number. 1–4. See students' work for number lines.

	Animal	Elevation (ft)	Above or Below Sea Level	Number
Show your work.	Fiddler Crab	3	above sea level	+3
1.	Eagle's Nest	75	above sea level	+75
2.	Dolphin	10	below sea level	−10
3.	Spider Crab	375	below sea level	−375
4.	Blue Heron	4	above sea level	+4

Analyze and Reflect

5. MP **Reason Inductively** What negative number is the same distance from 0 as the number +4? Explain. Graph both numbers on the number line below.

−4; The number +4 is 4 units to the right of 0 on the number line.

The number −4 is 4 units to the left of zero on the number line.

Create

6. MP **Model with Mathematics** Write about a real-world situation that can be described using the number −6. Describe what the number 0 would represent in the situation. What would the number 6 represent?

Sample answer: Jasem dove 6 feet under the surface of the water. His location in comparison to the water's surface can be described as −6 feet. The surface of the water can be represented by the number 0. The number 6 represents 6 feet above the water.

7. Inquiry HOW can positive and negative values be represented?

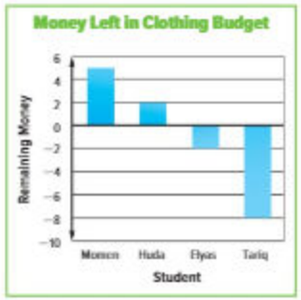
Sample answer: You can represent positive and negative values using positive and negative signs with numbers or on a number line.

Lesson 1

Integers and Graphing

Real-World Link

Money The bar graph shows the amount of money remaining in the clothing budgets of four students at the end of one month. A value of −AED 2 means that someone overspent the budget and owes his or her parents AED 2.



1. What number represents owing AED 8? −8
2. What number represents having AED 5 left? 5
3. Who has the most money left? Who owes the most? Explain.

Momen has AED 5 left, which is the most.

Tariq overspent AED 8 so he owes the most.

Essential Question

HOW are integers and absolute value used in real-world situations?

Vocabulary

integer
negative integer
positive integer

Mathematical Practices
1, 3, 4, 5, 7

Which Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Use integers to represent real-world situations.

Coherence connecting within and across grades

Previous
Students represented situations with an integer. They located integers on a number line.

Now
Students identify a number and its opposite. They will graph integers on a number line.

Next
Students will find the absolute value of an integer.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 349.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Think-Pair-Share** Give students 1–2 minutes to consider the questions. Then ask them to discuss how they determined their answers with a partner. Ask a student to share the pair's solution with the class. MP 1, 3

Alternate Strategies

AL LA If students have difficulty, discuss why spending more money than is budgeted is a negative and spending less is considered positive.

BL Ask students to prepare a budget for a graduation party. Have them compare the categories they used in their budgets and the amount of money allotted. MP 1, 2, 4

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1–3. **Write integers.**

AL
- *In Example 1, does the word "loss" indicate moving forward or moving backward on a football field?* moving backward
- *In Example 2, does the phrase "above normal" indicate more than or less than the normal amount?* more than
- *In Example 3, does the word "deposit" indicate an addition to or a subtraction from the account balance?* an addition to

OL
- *What is the meaning of zero in Example 1? Example 2? Example 3?* In Example 1, zero represents no yards lost or no yards gained. In Example 2, zero represents the normal amount of rain. In Example 3, zero represents no change in the account balance.
- *In Example 1, what would the integer 3 represent?* a gain of 3 yards

BL
- *How would you write an integer for a rainfall of 2 inches below normal?* −2
- *How would you write an integer for a withdrawal of $40 from a savings account?* −40

Need Other Examples?

Write an integer for each situation. Then explain the meaning of zero in each situation.

a. 12 feet below sea level −12; The integer 0 represents something that is at sea level.

b. winning 3 tokens in a game 3; The integer 0 represents winning no tokens.

c. the price of a stock dropping $2 −2; The integer 0 represents the stock neither gaining nor losing value.

Work Zone

Zero

The number zero can have different meanings based on real-world context. Sometimes zero represents an amount that does not change. Zero can also be used to represent real-world ideas, such as sea level.

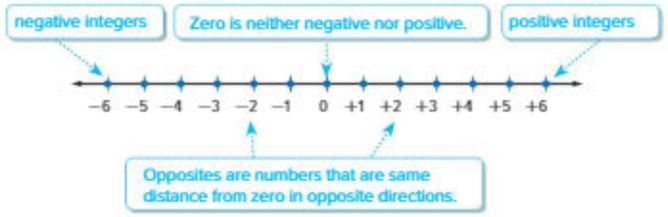
Show your work.

a. 2; The integer 0 represents neither a gain nor a loss.

b. −10; The integer 0 represents 0 degrees.

Use Integers to Represent Data

Positive whole numbers, their opposites, and zero are called **integers**. To represent data that are less than a 0, you can use **negative integers**. A negative integer is written with a − sign. Data that are greater than zero are represented by **positive integers**.



Examples

Write an integer for each situation. Explain the meaning of zero in each situation.

1. **a 10 meters (m) loss**

Because it represents a loss, the integer is −10. In football, the integer 0 represents no meters lost or no meters gained.

2. **4 inches of rain above normal**

Because it represents above, the integer is 4. In this situation, the integer 0 represents the normal amount of rain.

3. **a AED 48 deposit into a savings account**

Because it represents an increase, the integer is 48.

In this situation, the integer 0 represents neither a deposit nor a withdrawal.

Got it? **Do these problems to find out.**

Write an integer for each situation. Explain the meaning of zero in each situation.

a. a gain of AED 2 a share

b. 10 degrees below zero

The Number System

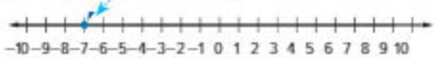
Graph Integers

Integers and sets of integers can be graphed on a horizontal or vertical number line. To graph a point on the number line, draw a point on the number line at its location. A set of integers is written using braces, such as {2, −9, 0}.

Examples

4. **Graph −7 on a number line.**

Draw a number line. Then draw a dot at the location that represents −7.



5. **Graph the set of integers {−4, 2, −1} on a number line.**

Draw a number line. Then draw a dot at the location of each integer.

6. **Graph the set of integers {0, 2, −3} on a number line.**

Draw a number line. Then draw a dot at the location of each integer.

Got it? **Do these problems to find out.**

Graph each set of integers on a number line.

c. {−3, 0, −2, 4}

d. {8, −6, −9, 5}

Examples

4. **Graph an integer on a number line.**

AL • *Are negative numbers located to the right or left of zero?* left

OL • *What is a possible range for the number line we should draw?* Sample answer: −10 to 10

• *How far to the left of 0 should we locate −7?* 7 units

BL • *Name the integer that is the same distance from 0 as −7 but on the opposite side of 0.* 7

• *Give a real-world situation that this graph could represent.* Sample answer: 7 feet below sea level

Need Another Example?
Graph −2 on a number line. See Answer Appendix.

5. **Locate a set of integers on a number line.**

AL • *What would be a good range for your number line?* Sample answer: −5 to 5

• *Which numbers should we place to the left of zero?* −4 and −1

OL • *Which number of the set is the greatest? How far is each number from 0?* 2; 4 units, 2 units, 1 unit

BL • *Name the integer that is the same distance from 0 as −4 but on the opposite side of 0.* 4

Need Another Example?
Graph the set of integers {−3, 1, 0} on a number line.
See Answer Appendix.

6. **Locate a set of integers on a number line.**

AL • *What is a vertical number line?* one that goes up and down

• *What would be a good range for a vertical number line?* Sample answer: 5 to −5

OL • *Which number of the set is the least? How far is each number from 0?* −3; 0 units, 2 units, 3 units

BL • *Other than 0, which number in this set is closest to 0?* 2

Need Another Example?
Graph the set of integers {−4, 1, 3} on a vertical number line. See Answer Appendix.

Example

7. **Graph integers and explain the meaning of them.**

AL
- *What kind of integer represents above par?* positive integer
- *What kind of integer represents below par?* negative integer

OL
- *What does the integer 0 represent in this situation?* par
- *Which scores are below par?* −1, −3

BL
- *Which score is best? Explain.* −3; In golf, the lowest score is best, as you want the fewest strokes possible to get the ball in the hole.
- *Which two scores are the same distance from zero but on opposite sides?* −1 and 1

Need Another Example?

The change in the high temperatures from yesterday to today for different cities are −1°F, −5°F, 4°F, and 2°F. Graph the temperature changes on a number line. Explain the meaning of zero in this situation.

The integer 0 represents no change in high temperature.

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

AL LA **Roundrobin** Have students go around in a circle to generate real-world situations that can be represented by negative integers (losses, below, etc.). One student generates the real-world situation, the next student identifies the integer, and the next student locates the integer on a number line. Repeat as time allows. MP 1, 2, 4, 6

Example

7. **Alaa and her dad played golf on four different days. The data set {−1, +1, −3, +2} shows Alaa's scores in relation to par. Graph the scores. Explain the meaning of zero in this situation.**

Draw a number line. Then draw a dot at the location of each golf score.

The integer 0 represents par.

Guided Practice

Write an integer for each situation. Explain the meaning of zero in each situation. (Examples 1–3)

1. 15-meter (m) gain 15; The integer 0 represents neither a gain nor a loss.
2. loss of 2 hours −2; The integer 0 represents neither a gain nor a loss.

Graph each integer or set of integers on a number line. (Examples 4–6)

3. −2

4. {−1, 1, 0}

5. The data set {+5, 0, −15, +20} shows the number of points Mariam scored on each hand of a card game. Graph the scores. Explain the meaning of zero in this situation. (Example 7)

The integer 0 represents a score of 0.

6. **Building on the Essential Question** How can you use integers to represent data?

Sample answer: Integers can be used to represent a gain or loss, temperatures above or below 0°, or elevations above and below sea level.

Rate Yourself!

How confident are you about integers and graphing? Check the box that applies.

Name _____ My Homework _____

Independent Practice

Write an integer for each situation. Explain the meaning of zero in each situation. (Examples 1–3)

1. 3 kilometers below sea level **−3; The integer 0 represents at sea level.**
2. earning AED 45 **45; The integer 0 represents neither earning nor spending.**
3. moving back 5 spaces on a game board **−5; The integer 0 represents neither moving backward nor moving forward.**

Graph each integer or set of integers on a number line. (Examples 4–6)

4. −5

5. {2, −3, 0, 1}

6. The data set {+4, −1, −2, 0} shows a change in number of state representatives for four states after the last census. Graph the change in number of representatives. Explain the meaning of zero in this situation. (Example 7)

The integer 0 represents neither a gain nor loss in number of representatives.

7. MP **Use Math Tools** The table shows the record low temperatures for several states. Graph the temperatures on a number line.

Record Low Temperature by State (°F)				
AL	AK	CT	NJ	VA
−27	−29	−32	−34	−30

8. MP **Use Math Tools** The table shows the number of points earned for each action in a video game. While playing the video game, Sultan fell in water, jumped over a rock, touched a cactus and climbed a mountain. Graph the number of points he earned for each action on the number line.

Action	Points
fall in water	−10
walk over a bridge	+5
climb mountain	+10
jump over rock	+5
walk through quicksand	−15
touch cactus	−15

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 11–13, 24, 25
OL	On Level	1–5 odd, 7–9, 11–13, 24, 25
BL	Beyond Level	7–13, 24, 25

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	10
3 Construct viable arguments and critique the reasoning of others.	11, 13
4 Model with mathematics.	9, 23
5 Use appropriate tools strategically.	7, 8
7 Look for and make use of structure.	12

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write a positive integer and a negative integer and graph each on a number line, using a separate piece of paper. See students' work.

Watch Out!

Common Error In Exercise 13, students may think that -2 and 3 are only 1 unit apart, since $3 - 2 = 1$. Encourage them to locate the integers on the number line to verify that they are actually 5 units apart. The number -2 is two units to the left of zero and the number 3 is three units to the right of zero, so the distance between them is 5 units.

9. **Model with Mathematics** Complete the graphic organizer by writing words or symbols used to represent positive and negative integers. Sample answers are given.

Positive Integer	Negative Integer
• gain • above • earn • +	• lose • below • spend • −

H.O.T. Problems Higher Order Thinking

10. **Persevere with Problems** A football team receives the ball on their own 10 yard line.

a. They make a gain of 15 yards in the first play. What yard line is the ball on?

the 25 yard line

b. What represents zero in this situation? Explain.

The 10 yard line represents neither a gain nor loss in yards.

11. **Justify Conclusions** The temperature outside is 15°F. If the temperature drops 20°, will the outside temperature be represented by a positive or negative integer? Explain your reasoning.

Negative; Sample answer: A drop of 15° would result in a temperature of 0°F. Since the drop of 20° is greater than 15°, the temperature is below zero and will be represented by a negative integer.

12. **Identify Structure** Describe the characteristics of each set of numbers that make up the set of integers.

Sample answer: Negative integers are to the left of zero on the number line, positive integers are to the right of zero on the number line, and zero is neither positive nor negative.

13. **Reason Inductively** Explain how to find the distance between -2 and 3 on a number line.

Sample answer: Locate -2 and 3 on a number line. Count the number of units between each integer and 0. -2 is 2 units to the left of zero and 3 is 3 units to the right. So, the number of units between -2 and 3 is $2 + 3$ or 5 units.

Name _____ My Homework _____

Extra Practice

Write an integer for each situation. Explain the meaning of zero in each situation.

Homework Help

14. 13° below zero −13; The integer 0 represents zero degrees.

15. spending AED 25 −25; The integer 0 represents neither spending nor earning.

16. 13-meter (m) gain 13; The integer 0 represents neither a gain nor a loss.

Graph each integer or set of integers on a number line.

17. −8

18. {0, −3, 1, −1}

19. {−1, 1, −2}

20. {3, −5, 4, −1}

21. {4, −2, 2}

22. The data set {+3, −1, −2, +1} shows the moves a player made forward or backward in a board game. Graph the moves the player made. Explain the meaning of zero in this situation.

The integer 0 represents a move neither forward nor backward.

23. **Model with Mathematics** The table shows the overnight low temperatures for 5 days in a city. Graph the temperatures on a number line.

Overnight Low Temperatures (°F)				
1	−1	3	−6	0

Power Up! Test Practice

Exercises 24 and 25 prepare students for more rigorous thinking needed for the assessment.

24.

This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Knowledge	DOK2
Mathematical Practices	MP2, MP3
Scoring Rubric	
1 point	Students correctly answer the question.

25.

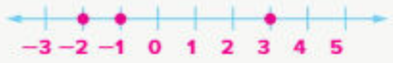
This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP4, MP6
Scoring Rubric	
2 points	Students correctly plot all three points.
1 point	Students correctly plot two of three points.

Power Up! Test Practice

24. Jasir owes his sister Mona AED 15. Explain the meaning of zero in this situation.

The integer 0 represents no longer owing his sister money.

25. On Monday Karima spent AED 2 on lunch. On Tuesday she spent AED 1 on a snack. On Wednesday, her sister gave her AED 3. Graph the integers on the number line.



Spiral Review

Fill in each ◯ with < or > to make the inequality true.

26. 26 (>) 22

27. 11 (<) 13

28. 2.5 (<) 3

29. 44 (>) 4.4

30. 15 (>) 6.8

31. 1.8 (<) 1.9

32. Ali bought $\frac{1}{12}$ kilogram (kg) of cashews and $\frac{5}{6}$ kilogram (kg) of granola. Plot the fractions on the number line. Which quantity is greater? Explain.

$\frac{5}{6}$; It is closer to 1 and greater than $\frac{1}{2}$.

33. The number of raffle tickets the student council sold over three days is shown in the table. How many total raffle tickets did they sell?

104 raffle tickets

Day	Tickets Sold
Wednesday	35
Thursday	23
Friday	46

Inquiry Lab
Absolute Value

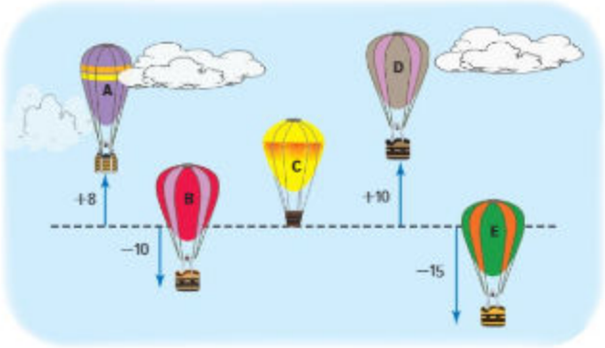
Inquiry **HOW can a number line help you find two integers that are the same distance from zero?**

Mathematical Practices 1, 2, 3, 5

Several hot air balloons were flying at the same height. The dashed line below represents their starting point. Which two balloons moved the same distance but in opposite directions?

Hands-On Activity

In the diagram below, +8 means Balloon A climbed 8 meters (m) and −10 means Balloon B moved down 10 meters (m).



Use the diagram to compare the distance each balloon moved.

Step 1 Complete the chart to compare the distance each balloon moved from the dashed line.

Balloon	Integer	Direction	Distance Moved (ft)
C	0	none	0
D	+10	up	10
E	−15	down	15

Step 2 Determine which two balloons moved the same distance away from the dashed line.

So, Balloon B and Balloon D moved 10 feet from the dashed line.

Focus narrowing the scope

Objective use a number line to explore the absolute value of an integer.

Coherence connecting within and across grades

Now
Students model the absolute value of a number.

Next
Students will write an integer, its opposite, and its absolute value.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 354.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Hands-On Activity

AL LA Use the following activity to help students visualize the movement before asking the given questions. 1, 4

Step 1 Place a number line on the floor.

Step 2 Have two students stand at 0.

Step 3 Tell both students to move 5 units, one student to the left, one student to the right.

Step 4 Ask the following questions.

Ask:

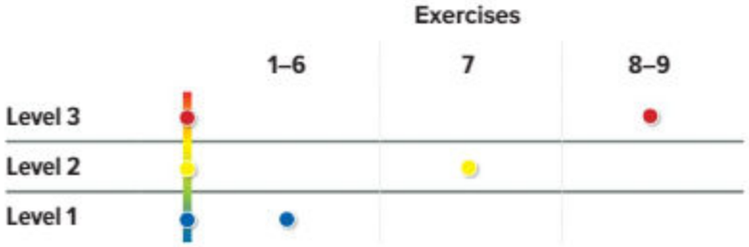
- *On what numbers did the students land?* 5 and −5
- *Because the students walked in* ***opposite*** *directions, what could we call these numbers?* opposites

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Analyze and Reflect

AL If students are struggling with Exercise 7, refer them back to the table they completed in Exercises 3–6.

- *What do you notice about all of the answers in the Distance Between Integer and Zero column and the Distance Between Opposite Integer and Zero column?* Sample answer: The answers are all positive and the same in both columns.

BL Have students work in pairs to answer Exercises 3–6 and answer the following extension question. MP 1, 2, 3

- *Refer to Exercise 7. Will your conclusion change if the number is a fraction or a decimal? Explain.* no; A fraction and its opposite are the same distance from zero, as well as a decimal and its opposite.

Create

inquiry Students should be able to answer "HOW can a number line help you find two integers that are the same distance from zero?" Check for student understanding and provide guidance, if needed.

Investigate

Sample answers: 7–9

MP **Use Math Tools** Use the number line to determine the distance between each integer and zero.

Show your work.

1. −2 2
2. +3 3

Work with a partner to complete the table. The first one is done for you.

	Integer	Distance Between Integer and Zero	Opposite Integer	Distance Between Opposite Integer and Zero
	3	3	−3	3
3.	7	7	−7	7
4.	−11	11	11	11
5.	−13	13	13	13
6.	−21	21	21	21

Analyze and Reflect

7. MP **Reason Inductively** What can you conclude about the distance from zero for both an integer and its opposite?

Both an integer and its opposite are the same distance from zero.

Create

8. MP **Model with Mathematics** The movement of Balloon B in the Activity was represented by the number −10. Write a number to represent the starting point of the balloons. How is this number shown on the diagram?

0; the horizontal dashed line

9. inquiry HOW can a number line help you find two integers that are the same distance from zero?

Plotting integers on a number line can help you find the distance from zero to both positive and negative integers.

Lesson 2

Absolute Value

Vocabulary Start-Up

The distance between a number and 0 on the number line is called its **absolute value**.

1. Each mark on the number line indicates one yard. Draw a tree three yards west of the house. Draw a mailbox three yards east of the house.

2. The distance between the house and the tree is equal to the distance between the house and the mailbox.

3. The tree and the mailbox are in opposite directions from the house.

4. How does the number line above help you to understand absolute value? Sample answer: The house is at 0 on the number line. The tree and the mailbox are the same distance from the house, but in different directions.

Real-World Link

5. **Errands** Mostafa leaves home and walks 4 blocks west to the grocery store to buy milk then returns home. He then walks another 4 blocks east to the Post Office. Compare the distance and the direction of Mostafa's house and the Post Office from the grocery store.
They are both 4 blocks from the store in opposite directions.

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Essential Question

HOW are integers and absolute value used in real-world situations?

Vocabulary

absolute value
opposites

MP Mathematical Practices
1, 2, 3, 4

Focus narrowing the scope

Objective Find the absolute value of an integer.

Coherence connecting within and across grades

Previous
Students wrote an integer to represent a real-world situation and graphed its opposite on a number line.

Now
Students find the absolute value of an integer and simplify expressions involving absolute value.

Next
Students will compare and order integers on a number line.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 359.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA Rally Coach Line-Up In groups of 3, Student 1 explains the answer to Exercise 1, while Student 2 listens and coaches. Student 2 explains to Student 3 why Student 1 completed Exercise 1 correctly or incorrectly, while Student 3 listens and coaches. MP 1, 3

Alternate Strategy

AL If students have difficulty, have pairs of students stand back-to-back and take 3 steps. Have them discuss why they are the same distance from their starting point. Choose one student to be positive 3 and describe the position of the other, using an integer. Have them discuss why the absolute value of 3 and −3 are both 3. MP 1, 3, 4

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Find the opposite of an integer.

AL • *To graph −5 on the number line, in which direction from 0 should we move?* left

• *How far is −5 from zero on a number line?* 5 units

OL • *Which integer is the same distance from 0 as −5, but on opposite sides of 0?* 5

• *To write positive 5, do we need to include the + sign? Explain.* no; It is commonly understood that a number without a sign is positive.

BL • *Give an example of a real-world situation that could be represented by the integer −5.* Sample answer: Adrianne took $5 out of her cashbox.

Need Another Example?
Find the opposite of −12. 12

2. Find the opposite of the opposite of an integer.

AL • *What is the opposite of 4?* −4

• *What is the opposite of −4?* 4

OL • *How far are 4 and the opposite of 4 from zero on a number line?* 4 units

• *What does "opposite" mean in this situation?* Sample answer: opposite sides of zero, opposite signs, opposite direction

BL • *Make a generalization about finding the opposite of the opposite of a number.* The opposite of the opposite of a number is the original number.

• *Identify the opposite of the opposite of the opposite of 4.* −4

Need Another Example?
Identify the opposite of the opposite of 8. 8

Work Zone

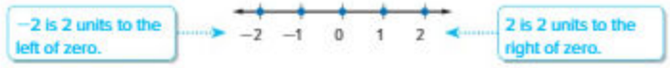
a. −3

b. −2

Find Opposites

Positive numbers, such as 2, are graphed to the right (or above) zero on a number line. Negative numbers, such as −2, are graphed to the left (or below) zero on a number line.

Opposites are numbers that are the same distance from zero in opposite directions. Since 0 is not negative nor positive, 0 is its own opposite. The opposite of the opposite of a number, is the number itself. For example, the opposite of the opposite of 3, −(−3), is 3.

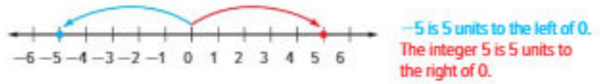


Examples

1. Find the opposite of −5.

Method 1 **Use a number line.**

Draw a number line and graph −5.



So, 5 is the opposite of −5.

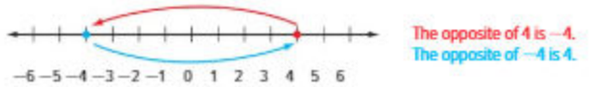
Method 2 **Use symbols.**

The integer −5 uses the negative symbol.

The opposite of a negative symbol is a positive symbol.

So, the opposite of −5 is +5, or 5.

2. Find the opposite of the opposite of 4.



So, 4 is the opposite of the opposite of 4.

Got it? **Do these problems to find out.**

a. What is the opposite of 3?

b. What is the opposite of the opposite of −2?

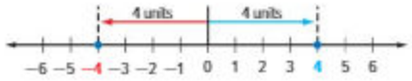
The Number System

Absolute Value

Key Concept

Words The absolute value of a number is the distance between the number and zero on a number line.

Model



Symbols $|4| = 4$ The absolute value of 4 is 4.

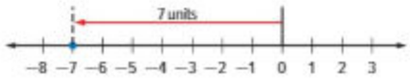
$|-4| = 4$ The absolute value of −4 is 4.

The integers −4 and 4 are each 4 units from 0, even though they are on opposite sides of 0. $|-4|$ is read *absolute value of negative four*.

Absolute Value
Since distance cannot be negative, the absolute value of a number is always positive or zero.

Examples

3. Evaluate $|-7|$.



The graph of −7 is 7 units from 0 on the number line.

So, $|-7| = 7$.

4. Evaluate $|5| + |-6|$.

$|5| + |-6| = 5 + |-6|$ The absolute value of 5 is 5.

$= 5 + 6$ The absolute value of −6 is 6.

$= 11$ Simplify.

5. Evaluate $|-7| - |3|$.

$|-7| - |3| =$ 7 − 3 Find the absolute value of −7 and 3.

$=$ 4 Simplify.

Got it? Do these problems to find out.

c. $|14|$ d. $|-9| + |3|$ e. $|-8| - |-2|$

Show your work.

c. 14

d. 12

e. 6

Examples

3. Evaluate an expression involving absolute value.

AL • *How far is −7 from zero on a number line?* 7 units

• *What do the vertical bars around −7 mean?* absolute value

OL • *When finding the absolute value of a number, to which other number do we compare the original number?* 0

BL • *Give an example of a real-world situation when it would be useful to find the absolute value of a number.* Sample answer: When giving a golf score such as −3, it would be useful to know the score is $|-3|$, or 3 strokes below par.

Need Another Example?
Evaluate $|18|$. 18

4. Evaluate an expression involving absolute value.

AL • *What is $|5|$?* 5 *What is $|-6|$?* 6 *What is $5 + 6$?* 11

OL • *Which do we find first, the absolute value of each number or the sum?* absolute value; the absolute value bars act like grouping symbols

BL • *Would the value of the expression change if it were $|-5| + |6|$? Explain.* no; The value would still be 11 since $|-5| = 5$, $|6| = 6$, and $5 + 6 = 11$.

Need Another Example?
Evaluate $|-12| + |7|$. 19

5. Evaluate an expression involving absolute value.

AL • *What is $|-7|$?* 7 *What is $|3|$?* 3 *What is $7 - 3$?* 4

OL • *Which do we find first, the absolute value of each number, or the difference?* absolute value; the absolute value bars act like grouping symbols

BL • *Would the value of the expression change if it were $|-7| + |3|$?* yes; The operation would change to addition, and $|-7| + |3| = 7 + 3 = 10$.

Need Another Example?
Evaluate $|11| - |-9|$. 2

Example

6. **Write and evaluate the expressions involving absolute value.**

OL • *What expression can be used to find the distance between Nevaeh and the seagull?* |25| + |−15|

• *What expression can be used after absolute values are found?* 25 + 15

BL • *Explain another method that can be used to solve the problem.* Sample answer: Locate the values on a number line and use the number line to find each distance.

Need Another Example?
Anna has −10 points in a video game. Her friend has 12 points in the same video game. What is the difference between the two scores? 22 points

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

AL LA **Pairs Discussion** Have students work in pairs to complete Exercises 3–5. Have them highlight the integers inside each set of absolute value bars. Student 2 determines the absolute value of each. Student 1 completes the problem, if necessary by adding or subtracting. MP 1, 6

Example

6. **A seagull is flying 25 feet above sea level. Nael is diving 15 feet below sea level. What is the distance between Nael and the seagull?**

The expression |25| describes the seagull's distance above sea level. The expression |−15| describes Nael's distance below sea level.

To find the distance, add the absolute values.

|25| + |−15| = 25 + |−15| The absolute value of 25 is 25.
= 25 + 15 The absolute value of −15 is 15.
= 40 Add.

So, the total distance is 40 feet.

Guided Practice

1. What is the opposite of 0? (Example 1)
0

Show your work.

2. What is the opposite of the opposite of 6? (Example 2)
6

Evaluate each expression. (Examples 3–5)

3. |−5| = 5

4. |20| − |−3| = 17

5. |−16| + |−12| = 28

6. A game show contestant lost 15 points. He answered another question incorrectly and lost another 15 points. How many total points has he lost? (Example 6)
30 points

7. **Building on the Essential Question** How can absolute value help you to understand the size of a quantity? Give an example. Sample answer: Absolute value describes the distance of an integer from zero. If an account balance is −30 dirhams, the absolute value 30 describes the size of the debt.

Rate Yourself!

How well do you understand opposites and absolute value? Circle the image that applies.

Clear | Somewhat Clear | Not So Clear

Name _____ My Homework _____

Independent Practice

Find the opposite of each integer. (Example 1)

1. 6 −6
2. −3 3
3. 0 0

Show your work.

Find the opposite of the opposite of each integer. (Example 2)

4. 12 12
5. −9 −9
6. −17 −17

Evaluate each expression. (Examples 3–5)

7. $|-14| =$ 14
8. $|31| - |-1| =$ 30
9. $|-15| + |-6| =$ 21

10. Jasir spent AED 18 on a shirt. Then he spent AED 24 on a pair of pants. What is the total amount he spent? (Example 6) AED 42

11. Laila saw a jelly fish at 6 feet below sea level. She saw a bright blue fish at 10 feet below sea level. What is the distance between the blue fish and the jelly fish? (Example 6) 4 feet

12. STEM The table shows the melting points of various elements. Is the absolute value of the melting point of neon greater than or less than the absolute value of the melting point of hydrogen? less than

Element	Melting Point (°C)
Hydrogen	−259
Neon	−248
Oxygen	−218

13. STEM The surface of Jupiter is made of colorful clouds created by various chemicals in the atmosphere. The temperature at the top of the clouds is −230°F. The temperature below the clouds is 70°F. Which temperature has the lower absolute value? 70°F

MP **Reason Abstractly** **Evaluate each expression.**

14. $-|3| =$ −3
15. $|5 + 9| =$ 14
16. $|17 - 8| =$ 9

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–11, 13, 15, 17, 18, 21–23, 41, 42
OL	On Level	1–11 odd, 12–18, 21–23, 41, 42
BL	Beyond Level	12–23, 41, 42

MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	19, 20
2 Reason abstractly and quantitatively.	14–16, 21, 22, 39, 40
3 Construct viable arguments and critique the reasoning of others.	17, 18, 23
4 Model with mathematics.	10–13

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET
Out the Door

Have students evaluate the expression $|-3| + |5|$. 8

Watch Out!

Common Error In Exercise 17, students may incorrectly remove the absolute value bars without thinking of what absolute value means. Remind them that absolute value cannot be negative, because it represents the distance a number is from 0, and distance can never be negative.

H.O.T. Problems Higher Order Thinking

17. **Find the Error** May is evaluating an expression using absolute value. Find her mistake and correct it.

Absolute value cannot be a negative number.

So, the absolute value of −14 is 14, not −14.

18. **Which One Doesn't Belong?** Identify the phrase that *cannot* be described by the same absolute value as the other three. Explain your reasoning.

a loss of 8 pounds | 8 miles above sea level | giving away AED 8 | 18° below normal

18° below normal; the others have an absolute value of 8.

Persevere with Problems **Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.**

19. The absolute value of a positive integer is a negative integer.

Never; distance cannot be negative.

20. If a and b are integers and $a > b$, then $|a| > |b|$.

Sometimes; if a is 4 and b is −6, then $|4| < |-6|$.

21. **Reason Abstractly** Explain why the absolute value of a number is never negative. Absolute value is distance and distance cannot be negative.

22. **Reason Abstractly** Explain why an account balance less than −40 dirhams represents a debt greater than 40 dirhams.

Sample answer: Absolute value can be used to represent the amount of debt, but the value of the number is based on its position on the number line. So, an account balance less than −40 dirhams, such as −50 dirhams, has a greater absolute value, but a smaller value.

23. **Reason Inductively** Is $-n$ *always*, *sometimes*, or *never* a positive number? Explain your reasoning. sometimes; Sample answer: If n is positive, then $-n$ is negative. If n is negative, then $-n$ is positive.

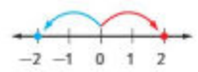
Name _____ My Homework _____

Extra Practice

Find the opposite of each integer.

24. −2 2

Homework Help



25. 15 −15

26. 42 −42

Find the opposite of the opposite of each integer.

27. 9 9

28. 0 0

29. −8 −8

Evaluate each expression.

30. |18| = 18

31. |0| = 0

32. |25| = 25

33. |2| + |−13| = 15

34. |−20| − |17| = 3

35. |−16| − |5| = 11

36. The balance of Baher's account is AED 16. Jamila's account is AED 5 overdrawn. What is the difference between their account balances?

AED 21

37. A football team lost 3 yards on their first play and 6 yards on their second play. How many total yards did they lose?

9 yards

38. The table shows the lowest elevations for several states. Is the absolute value of the lowest elevation of California greater than or less than the absolute value of the lowest elevation of Illinois?

greater than

State	Lowest Elevation (ft)
Oklahoma	289
Illinois	279
Kentucky	257
California	−282

MP **Reason Abstractly** **Evaluate each expression.**

39. −|−10| = −10

40. |13 − 6| = 7

Power Up! Test Practice

Exercises 41 and 42 prepare students for more rigorous thinking needed for the assessment.

41.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

42.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly complete all three number sentences
1 point	Students correctly complete two of the three number sentences

Power Up! Test Practice

41. The table shows the freezing point of different liquids. What liquid's freezing point has the greatest absolute value?

- ☐ water
- ☐ acetic acid
- ☐ linseed oil
- ■ acetone

Liquid	Freezing Point (°F)
Water	32
Acetic Acid	62
Linseed Oil	−4
Acetone	−94

42. Let $x = -1$, $y = -2$, and $z = 1$. Use the variables x, y, and z to complete the following number sentences. **Sample answers:**

a. $|x| < |y|$

b. $|y| > |z|$

c. $|x| = |z|$

Spiral Review

Fill in each ◯ with >, <, or = to make a true statement.

43. 69.23 (<) 69.25

44. 171.10 (>) 171.09

45. 47.74 (=) 47.740

46. Part of a sauce recipe is shown. If all the ingredients are mixed together, how much sauce will be made?

$8\frac{3}{4}$ tsp

6 teaspoons Worcestershire Sauce
$1\frac{1}{2}$ teaspoons seasoned salt
$\frac{1}{4}$ teaspoon onion powder
$\frac{1}{4}$ teaspoon garlic powder

Show your work.

47. Amani's soccer practice starts at a quarter after 4 P.M. and ends at 5 P.M. How many minutes does her soccer practice last?

45 minutes

The Number System

Lesson 3

Compare and Order Integers

Real-World Link

Winter Tripoli is located in Libya. The average temperature for several months is shown on the thermometer.

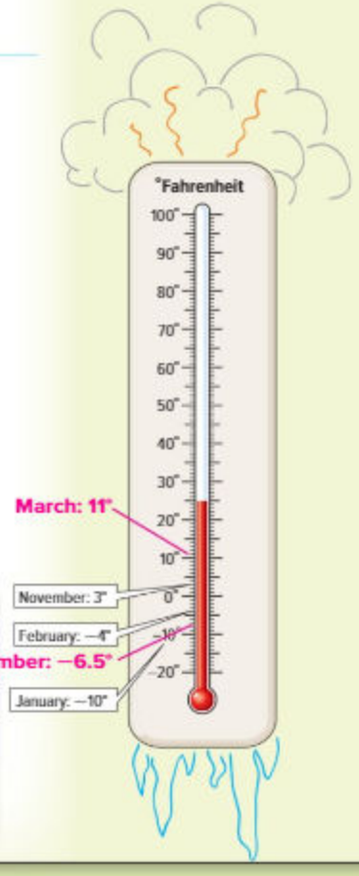
1. The average temperature for December is −6.5°F and the average temperature for March is 11°F. Label December and March on the thermometer.
2. Which months have a greater average temperature than February? November and March
3. Which months have a lower average temperature than November? February, December, and January
4. Complete the inequality to compare the temperatures of November and February.

3 > −4

Essential Question

HOW are integers and absolute value used in real-world situations?

MP Mathematical Practices
1, 2, 3, 4, 5



Which MP **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Compare and order integers.

Coherence connecting within and across grades

Previous
Students found the absolute value of integers and graphed them on a number line.

Now
Students compare and order integers using a number line.

Next
Students will find equivalent forms of decimals and fractions.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 367.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Think-Pair-Share** Give students 1–2 minutes to think through their responses to Exercises 1–4. Then have them discuss how they determined their answers with a partner. Ask a volunteer to share the pair's solution with the class. MP 1, 3, 5

Alternate Strategy

AL If students have difficulty locating the temperatures, ask them to draw a vertical number line, from zero to twenty, putting numbers at only 0, 10, and 20. Ask them where to find 17. Then ask them where to find 17.5, then 17.2, and 17.8. Ask them to locate 5 and 7 and how the position of the numbers indicates the larger number. MP 1, 5

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Compare integers.**

AL • *What scale would be appropriate for our number line?* Sample answer: −4 to 14

OL • *If two numbers are graphed on a number line, how can you tell which one is greater?* The number that is farthest to the right on a number line is the greater number.

• *Do the expressions $12 > -4$ and $-4 < 12$ represent the same relationship?* yes

BL • *Without using a number line, how can you compare a positive and a negative number?* The positive number will always be the greater number.

• *Which number has a lesser absolute value?* −4

Need Another Example?

Is $-2 <, >,$ or $= 6$? <

2. **Compare integers in a real-world context.**

AL • *Which number is farther to the left of 0 on a number line?* −5

OL • *Since both numbers are negative, which number is closer to 0 on a number line?* −4

• *Both scores are negative. How can you tell which student has a greater score?* The score that is farther to the right is the greater score.

BL • *Even though $5 > 4$, why is $-4 > -5$?* As numbers get more and more negative, their value decreases.

Need Another Example?

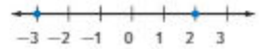
The low temperature for Smithville yesterday was −1°F. The low temperature for Bronson yesterday was 3°F. Write an inequality to compare the temperatures. Explain the meaning of the inequality. $-1 < 3$; The low temperature for Smithville was lower than the low temperature for Bronson.

Work Zone

Compare Integers

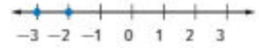
To compare integers, you can compare the signs as well as the magnitude, or size, of the numbers. Greater numbers are graphed farther to the right.

Compare the signs.



Positive numbers are greater than negative numbers. So, $2 > -3$.

Compare the position on the number line.



Since −2 is farther to the right, $-2 > -3$.

Example

Fill in the ◯ with <, >, or = to make a true sentence.

1. 12 ◯ −4

Graph 12 and −4 on a number line. Then compare.

Since 12 is to the right of −4, $12 > -4$.

Got it? **Do these problems to find out.**

a. -3 (>) -5 b. -5 (<) 0 c. 6 (>) -1

Absolute Value

Although −5 is the lesser value in Example 2, it represents the greater point deficit.

$|-5| > |-4|$

Show your work.

2 > −2; The temperature was warmer on Tuesday.

d.

Real World

Example

2. **Mohammad has a score of −4 on a game. Dalia's score is −5. Write an inequality to compare the scores. Explain the meaning of the inequality.**

$-4 > -5$ −4 is farther to the right on a number line than −5.

Since $-4 > -5$, Mohammad has a higher score than Dalia.

Got it? **Do this problem to find out.**

d. The temperature on Tuesday was 2°F. The temperature on Wednesday was −2°F. Write an inequality to compare the temperatures. Explain the meaning of the inequality.

The Number System

Order Integers

You can use a number line to order a set of integers. Integers can be ordered from least to greatest or from greatest to least.

Example

3. **Order the set {−9, 6, −3, 0} from least to greatest.**

Method 1 **Use a number line.**

Graph the numbers on a number line.

The order from left to right is −9, −3, 0, and 6.

Method 2 **Compare signs and values.**

Compare negative numbers. Then compare positive numbers.

The negative integers are −9 and −3. $-9 < -3$

The integer 0 is neither positive nor negative.

The positive integer is 6.

So, the order from least to greatest is −9, −3, 0, and 6.

Got it? **Do these problems to find out.**

e. Order the set {−4, 3, 11, −25} from greatest to least.

f. Order the set {−18, 30, 12, −6, 3} from least to greatest.

Absolute Value

Since absolute value is always positive, it is not used to compare and order integers.

Show your work.

e. 11, 3, −4, −25

f. −18, −6, 3, 12, 30

Example

3. **Order a set of integers.**

AL
- *Which number(s) are negative?* −9 and −3
- *Which number(s) are positive?* 6
- *Are positive numbers greater than negative numbers? Use this knowledge to find the greatest number in this set.* yes; 6 is the greatest number.
- *How could you put the set of integers in order from least to greatest?* Graph the numbers on the same number line. The points from left to right show you the order from least to greatest.

OL
- *Which integer is farthest left on a number line?* −9
- *Which integer is farthest right on a number line?* 6
- *Explain the advantages and disadvantages of each method.* Sample answer: Method 1 is more visual, but Method 2 can often be quicker.

BL
- *Would the order of the integers from least to greatest change if 6 were changed to −6? Explain.* Yes; 6 would no longer be the greatest number. The order from least to greatest would be −9, −6, −3, and 0.
- *Create your own set of integers, including at least 2 negative integers and at least 1 positive integer. Trade sets with a partner. Each student should order the set of numbers from least to greatest, and then from greatest to least.* See students' work.

Need Another Example?

Order {18, 0, −10, 12} from least to greatest. −10, 0, 12, 18

Example

4. Order integers.

AL
- *Are the elevations listed as positive or negative?* negative
- *Do the negative elevations represent above or below sea level?* below sea level

OL
- *Graph the integers on a number line. Which number is farthest left?* −418
- *Which number is farthest right?* −12

BL
- *Explain another way to order the integers without using a number line.* Sample answer: Since all the numbers are negative, you could list them in order from greatest to least absolute value.
- *Which continent has its lowest elevation closest to sea level?* Australia

Need Another Example?

The average daily low temperatures in four northern towns are 6, −14, 10, and −8 degrees Fahrenheit. Order the temperatures from least to greatest. −14, −8, 6, 10

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

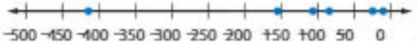
AL LA **Pairs Discussion** Have students work in pairs to complete Exercises 1–5. If students are struggling with Exercise 2, have them write the inequality statement using the lesser number on the left side of the inequality symbol, $-10 < -6$. Then have them rewrite the inequality statement as given in the exercise and have them explain why the inequality symbol is reversed. MP 1, 6, 7

BL LA **Trade-a-Problem** Have students write inequality problems similar to Exercises 1–3, using a combination of positive and negative numbers, using at least one absolute value. Students trade problems, solve them and discuss their solutions. MP 1, 2, 4

Example

4. STEM The table shows the lowest elevation for several continents. Order the elevations from least to greatest.

First, graph each integer. Then, write the integers as they appear on the number line from left to right.



Continent	Lowest Elevation (m)
Africa	−156
Asia	−418
Australia	−12
Europe	−28
North America	−86
South America	−105

The elevations from least to greatest are −418, −156, −105, −86, −28, and −12.

Guided Practice

Check

Fill in each ◯ with <, >, or = to make a true statement. (Example 1)

1. 17 < 31
2. −6 > −10
3. −83 < −38

4. Khaled and his father are scuba diving at −38 feet and Jumeriah has an elevation of −83 feet. Write an inequality to compare the depths. Explain the meaning of the inequality. (Example 2)

   −38 > −83; Jumeriah is located at a greater depth than Khaled's location.

5. STEM The daily low temperatures in Safiyya's hometown last week were 2°C, −9°C, −18°C, −6°C, 3°C, 0°C, and −7°C. Order the temperatures from greatest to least. (Examples 3 and 4)

   3°C, 2°C, 0°C, −6°C, −7°C, −9°C, and −18°C

6. **Building on the Essential Question** How can symbols and absolute value help you to order sets of integers?

   Sample answer: Positive numbers have a greater value than negative numbers. You can use absolute value to determine the distance of a number from 0.

Rate Yourself!

How confident are you about comparing and ordering integers? Shade the ring on the target.

FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

Fill in each ◯ with <, >, or = to make a true statement. (Example 1)

1. -2 (>) -4
2. 1 (>) -3
3. 5 (>) 0

4. Iman is building a house. The basement floor is at −15 feet. The roof of the house is above the ground 25 feet. Write an inequality to compare the heights. Explain the meaning of the inequality. (Example 2)

   −15 < 25; The roof of the house is at a greater height than the basement floor.

5. The low temperature in Albiqaa, Lebanon, one day was −9°F. On the same day, the low temperature in Byblos, Lebanon, was 26°F. Write an inequality to compare the temperatures. Explain the meaning of the inequality. (Example 2)

   −9 < 26; The temperature in Byblos, Lebanon, was warmer.

Order each set of integers from least to greatest. (Example 3)

6. {15, 17, 21, 6, 3}

   3, 6, 15, 17, 21

7. {−55, 143, 18, −79, 44, 101}

   −79, −55, 18, 44, 101, 143

8. The table indicates Zaid's cell phone use over the last four months. Positive values indicate the number of minutes he went over his allotted time, and negative values indicate the number of minutes he was under. Arrange the months from least to most minutes used. (Example 4)

   February, May, April, March

Month	Time (min)
February	−156
March	12
April	0
May	−45

9. MP **Use Math Tools** Refer to the table and the following information. The apparent magnitude of an object measures how bright the object appears to the human eye. A negative magnitude identifies a brighter object than a positive magnitude.

   a. Which object appears the brightest to the human eye?

   Sun

   b. Order the objects from the brightest to the faintest.

   Sun, 100-Watt Bulb, Full Moon, Venus, Andromeda Galaxy, Alpha Centauri

   c. Find the least apparent magnitude of this data set.

   −27

Object	Approximate Apparent Magnitude
100-Watt Bulb	−19
Alpha Centauri	4
Andromeda Galaxy	0
Full Moon	−13
Sun	−27
Venus	−5

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

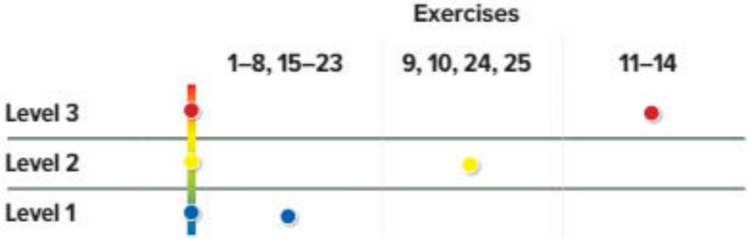
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 11, 12, 24, 25
OL	On Level	1–7 odd, 9–12, 24, 25
BL	Beyond Level	9–14, 24, 25

MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	13, 14
2 Reason abstractly and quantitatively.	12
3 Construct viable arguments and critique the reasoning of others.	10
4 Model with mathematics.	11
5 Use appropriate tools strategically.	9, 23

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

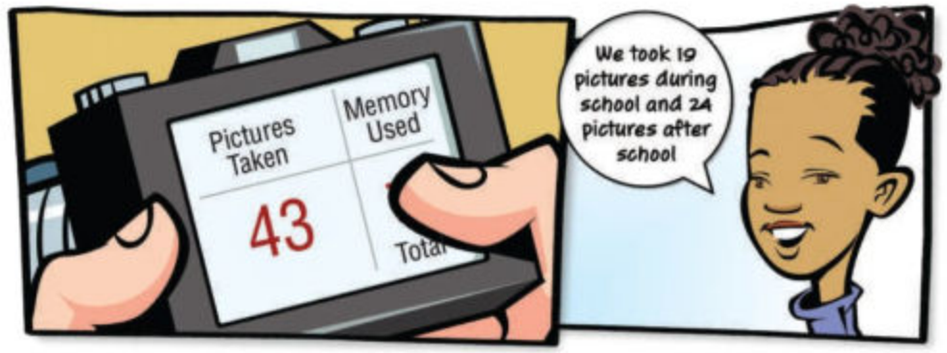
TICKET
Out the Door

Present the following problem to students: Lia has taken 50 cents out of her wallet. Claire has put 35 cents in her wallet. Dean has taken 25 cents out of his wallet. Have students explain how they would compare and order the amounts from least to greatest. See students' work.

Watch Out!

Common Error In Exercise 13, students may incorrectly order the fractions by ignoring their signs. Remind them that all negative numbers are less than zero. Have them sort the numbers by their signs first, prior to ordering the fractions. The negative fractions should all be less than the positive fractions.

10. **Justify Conclusions** Refer to the graphic novel frame below for exercises a–c.



a. The memory card holds 65,536 kilobytes. If each picture is about 760 kilobytes, about how many more pictures can they take? 43 pictures

b. Write an inequality to compare the number of pictures taken during school to the number of pictures taken after school. $19 < 24$

c. Explain the meaning of the inequality. More pictures were taken after school.

H.O.T. Problems Higher Order Thinking

11. **Model with Mathematics** Write a real-world situation to explain the inequality $-\text{AED } 15 < \text{AED } 7$. Sample answer: Moza owes her brother AED 15. Khaled has AED 7. Moza has less money than Khaled.

12. **Reason Abstractly** Explain why -11 is less than -7, but $|-11|$ is greater than $|-7|$. Sample answer: Since -11 is to the left of -7 on a number line, $-11 < -7$. But since -11 is a greater distance from zero than -7, $|-11| > |-7|$.

13. **Persevere with Problems** Order the fractions $-\frac{1}{2}, \frac{5}{2}, -\frac{12}{4}, \frac{1}{6}$, and $\frac{7}{8}$ from least to greatest. $-\frac{12}{4}, -\frac{1}{2}, \frac{1}{6}, \frac{7}{8}, \frac{5}{2}$

14. **Persevere with Problems** Find all integers that make $|n| < 3$ a true statement. Then graph the integers on the number line. $-2, -1, 0, 1, 2$

Name _____ My Homework _____

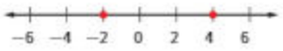
Extra Practice

Fill in each ◯ with <, >, or = to make a true statement.

15. $-2 \bigcirc 4$

$-2 < 4$

Homework Help → Since -2 is to the left of 4, -2 < 4.



16. $-6 \bigcirc 3$

$-6 < 3$

17. $-3 \bigcirc 2$

$-3 < 2$

18. The elevation of a mountain, is 163 meters above sea level. A valley has an elevation of −86 meters. Write an inequality to compare the elevations. Explain the meaning of the inequality.

163 > −86; the mountain has a higher elevation than the valley.

19. Amna owes her sister AED 25. Abdalla's checking account balance is −AED 20. Write an inequality to compare the amounts. Explain the meaning of the inequality.

−20 > −25; Abdalla owes less money than Amna.

Order each set of integers from least to greatest.

20. {14, 1, 6, 23, 7, 5}

1, 5, 6, 7, 14, 23

21. {−221, 63, 54, −89, −71, −10}

−221, −89, −71, −10, 54, 63

22. Jasim, Suhail, and Bahija are all waiting for their trains to arrive. Jasim's train leaves at 5 minutes before noon, Suhail's leaves at 25 minutes after noon, and Bahija's leaves 5 minutes before Suhail's train. Order the three by who will leave first.

Jasim, Bahija, Suhail

23. MP **Use Math Tools** Use the bar graph and the information below. The bar graph gives the scores of four golfers (A, B, C, and D). The numbers indicate scores above and below par.

a. Order the scores on a number line.

b. Which player had the worst score? Explain your answer.

D; Since −4 < −3 < 2 < 6, player D had the most strokes over par.

Power Up! Test Practice

Exercises 24 and 25 prepare students for more rigorous thinking needed for the assessment.

24.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP6
Scoring Rubric	
1 point	Students correctly answer the question.

25.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP2
Scoring Rubric	
2 points	Students correctly order all four players.
1 point	Students correctly consecutively order three of the four players.

Power Up! Test Practice

24. Fadrous (V) was 12 minutes early to class, Dawood (D) was right on time, and Karima (K) was 3 minutes late. Which time line represents the students' arrival to class?

25. The table shows the scores at the end of a game of miniature golf. The integer 0 represents par. Sort the players from fewest shots taken to most shots taken.

Player	Score
Hamza	−6
Batool	2
Manal	−3
Mariam	5

	Player
Fewest Shots	Hamza
	Manal
	Batool
Most Shots	Mariam

Spiral Review

Write each fraction as a decimal.

26. $\frac{3}{4} =$ 0.75

27. $\frac{1}{5} =$ 0.2

28. $\frac{3}{20} =$ 0.15

29. The table shows the heights of Salma's siblings. Who is taller, Farida or Khalil? Compare their heights using the symbol >.

Khalil; $5\frac{5}{6} > 5\frac{1}{4}$

Member	Height (ft)
Farida	$5\frac{1}{4}$
Khalil	$5\frac{5}{6}$

30. Karima and Mazen were given the same math assignment. Karima completes 0.8 of her work in class. Mazen completes 0.75 of his work during class. Who has more homework remaining after class?

Mazen

MP Problem-Solving Investigation

Work Backward

MP Mathematical Practices 1, 3

Case #1 Hit the Slopes!

Mushira and her family are on a ski trip at Mount of Cedar, Lebanon. They returned from the slopes at 6 P.M. By 9 P.M., the temperature had fallen 18° to the day's low temperature of −8°F.

What was the temperature at 6 P.M.?

1 Understand *What are the facts?*

- By 9 P.M., the temperature had fallen 18°.
- The day's low temperature was −8°F.

2 Plan *What is your strategy to solve this problem?*

Work backward from the low temperature at 9 P.M. Use a thermometer diagram to find the temperature at 6 P.M.

3 Solve *How can you apply the strategy?*

Start at −8 °F. Shade the thermometer 18 degrees to find the temperature at 6 P.M.

So, the temperature at 6 P.M. was 10°F.

4 Check *Does the answer make sense?*

−8°F is 8 degrees away from 0°F. 18 − 8 = 10

So, add to 10 to 0°F. Since 0 + 10 = 10, the answer is reasonable.

Analyze the Strategy

MP **Justify Conclusions** The high temperature was 36°F. How far away from −8°F is 36°F? Explain. 44°; Sample answer: The difference between 36° and 0° is 36 and −8° is 8° away from 0°. The total difference is 36 + 8, or 44°.

Focus narrowing the scope

Objective Solve problems by using the work backward strategy. This lesson emphasizes MP **Mathematical Practice 3** Justify Conclusions.

Work Backward Working backward is a useful strategy for solving logic problems and has a strong link to solving equations. In some problems, information is not given in the order in which it must be used to find the solution. Encourage students to read each problem carefully to identify what information is missing and to plan a solution.

Coherence connecting within and across grades

Now
Students apply the content standard to solve non-routine problems.

Next
Students will apply the work backward strategy to solve real-world problems.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 373.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

The problems on pages 371 and 372 are intended to be used as a whole-group discussion on how to solve non-routine problems and are designed to provide scaffolded guidance.

Case #1 Hit the Slopes!

BL Have students extend the problem by having them answer the question below. MP 1, 6

Ask:

- *The next day they went back out to the slopes and by noon, the temperature was 28°. What is the difference between the temperature at noon and the temperature at 9 pm?* 36° difference

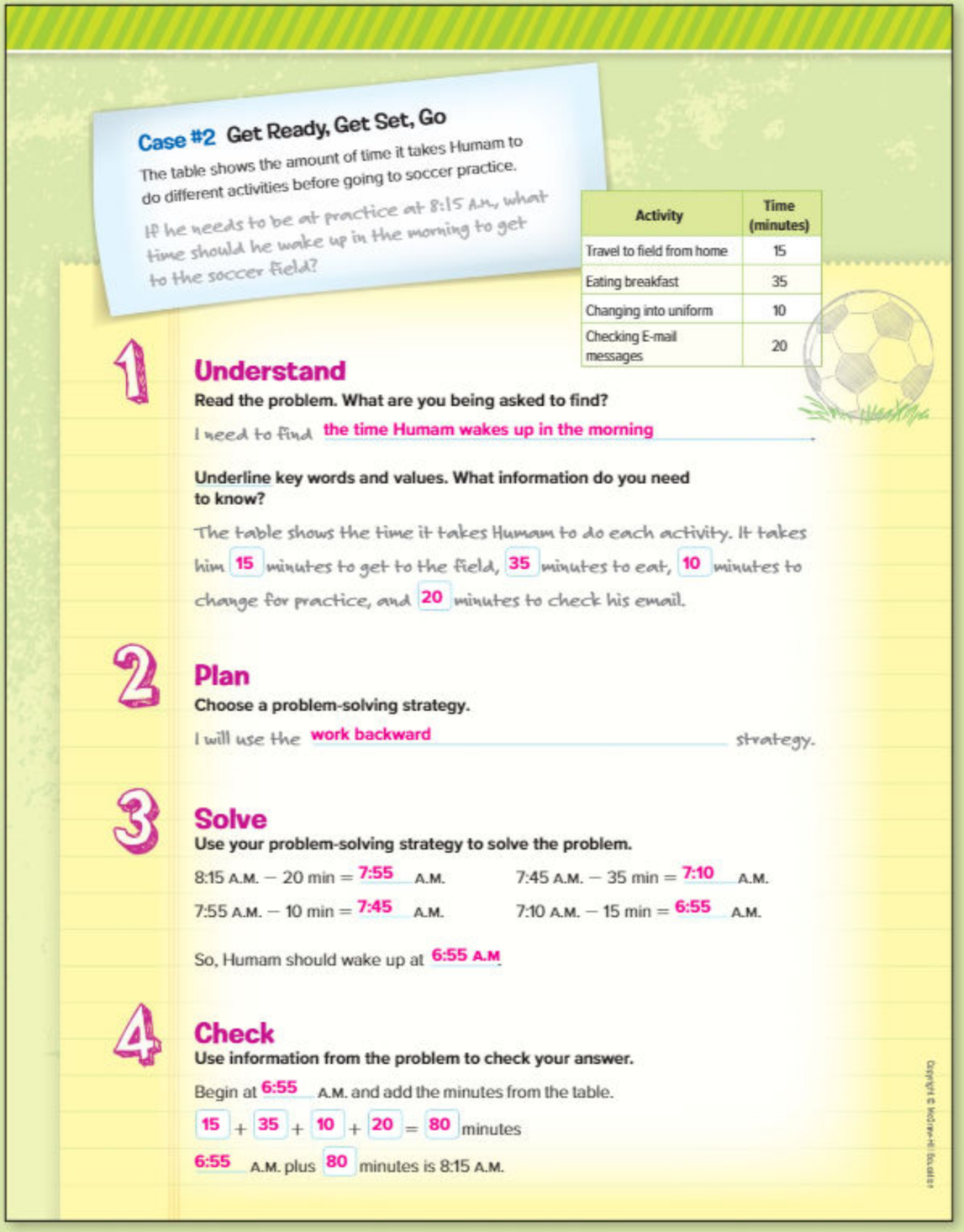
Case #2 Get Ready, Get Set, Go

AL LA **Pairs Discussion** Have students work in pairs to complete *Get Ready, Get Set, Go*. One student should speak aloud their response. The other student should listen carefully to the response and ask for any assistance, if needed. MP 1, 3, 4

BL LA **Numbered Heads Together** Assign students to 4-person learning teams. Each team member is assigned a number from 1 to 4 that represents a step in the four-step plan. Have them solve the problem as a team as each student explains how to complete their assigned step. The other team members should listen carefully, ask clarifying questions, and provide assistance if needed. Call on one number to explain the solution to the class. MP 1, 3, 5

Need Another Example?

During one day, an animal shelter takes in 7 new animals and gets 13 animals adopted as pets. If by the end of the day there are 47 animals in the shelter, how many animals were there at the start of the day? 53



Case #2 Get Ready, Get Set, Go

The table shows the amount of time it takes Humam to do different activities before going to soccer practice.

If he needs to be at practice at 8:15 A.M., what time should he wake up in the morning to get to the soccer field?

Activity	Time (minutes)
Travel to field from home	15
Eating breakfast	35
Changing into uniform	10
Checking E-mail messages	20

1 Understand

Read the problem. What are you being asked to find?

I need to find the time Humam wakes up in the morning

Underline key words and values. What information do you need to know?

The table shows the time it takes Humam to do each activity. It takes him 15 minutes to get to the field, 35 minutes to eat, 10 minutes to change for practice, and 20 minutes to check his email.

2 Plan

Choose a problem-solving strategy.

I will use the work backward strategy.

3 Solve

Use your problem-solving strategy to solve the problem.

8:15 A.M. − 20 min = 7:55 A.M.

7:55 A.M. − 10 min = 7:45 A.M.

7:45 A.M. − 35 min = 7:10 A.M.

7:10 A.M. − 15 min = 6:55 A.M.

So, Humam should wake up at 6:55 A.M.

4 Check

Use information from the problem to check your answer.

Begin at 6:55 A.M. and add the minutes from the table.

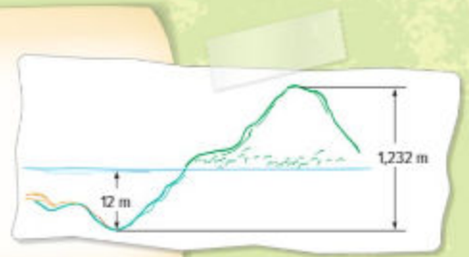
15 + 35 + 10 + 20 = 80 minutes

6:55 A.M. plus 80 minutes is 8:15 A.M.

Work with a small group to solve the following cases.
Show your work on a separate piece of paper.

Case #3 Sea Level

Mr. Eyad went diving along the coral reef in Mount Al Laith. He descended 12 meters below sea level. The difference between this point on the coral reef and the highest point on the island, Mount Al Laith, is 1,232 meters. Two-fifths of the way up the mountain is a ranger station.



How far above sea level is the ranger station?

488 meters

Case #4 Cameras

Adham saved 13 pictures on his digital camera for a total of 12,021.1 KB. He deleted 32 pictures for a total of 29,590.4 KB.

If there are now 108 pictures, how many kilobytes of storage did he use at the beginning?

117,436.9 KB

Case #5 Mystery Number

A number is multiplied by 4, and then 6 is added to the product. The result is 18.

What is the number?

3

Case #6 Ladders

You are standing on the middle rung of a ladder.

If you first climb up 3 rungs, then down 5 rungs, and then up 10 rungs to get onto the top rung, how many rungs are on the ladder?

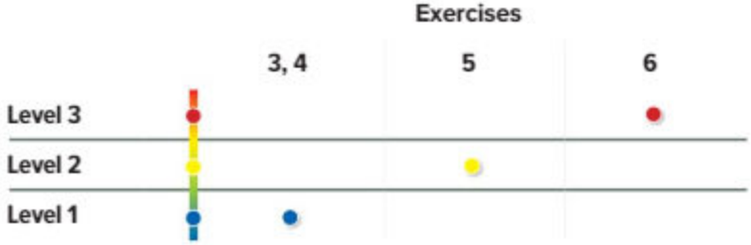
17 rungs

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



AL LA **Think-Pair-Share** Have students work in pairs. Give students one minute to think through their responses to Cases 3–6. Have them share their responses with their partner. Then call on one student to share their response within a small group or large group discussion. MP 1, 3

BL LA **Pairs Discussion** Have students work in pairs to prepare a short oral presentation that explains the *work backward* strategy. As one student presents from each team, have other students listen carefully. After each student finishes, have one student from another pair write down the main points using chart paper. Ask other students from the class to add to the list, if needed. MP 1, 5, 6

Mid-Chapter Check

If students have trouble with Exercises 1–12, they may need help with the following concepts.

Concept	Exercise(s)
opposites (Lesson 2)	1
evaluating expressions with absolute value (Lesson 2)	2, 5–10
graphing integers (Lesson 1)	3, 4
ordering integers (Lesson 3)	11, 12

Vocabulary Activity

LA **Rally Coach** Have students work in pairs to complete Exercise 1. Have Student 1 speak aloud about what a negative integer is, while Student 2 listens, coaches, and encourages. If students are having trouble remembering the definition of a negative integer, have them discuss how the word integer might help them. MP 1, 3, 6

Alternate Strategies

AL LA Have students explain the difference between a positive integer and a negative integer. MP 1, 6

BL LA Have students give a real-world situation for their negative integer. MP 1, 4

Mid-Chapter Check

Vocabulary Check

1. MP **Be Precise** Define *negative integer*. Give an example of a negative integer and then give its opposite. (Lesson 1)

   A negative integer is the opposite of a whole number. It is less than 0.

   −6 is a negative integer and 6 is its opposite.

2. Fill in the blank in the sentence below with the correct term. (Lesson 2)

   The absolute value of the numbers −4 and 4 is 4.

Skills Check and Problem Solving

MP **Model with Mathematics** **Graph each set of integers on a number line.** (Lesson 1)

3. {−4, −6, 0, 3}

4. {2, −3, 1, −1}

Evaluate each expression. (Lesson 2)

5. |−12| = 12
6. |−4| + |−10| = 14
7. |9| + |−2| = 11
8. |13| − |−5| = 8
9. |−16| − |−2| = 14
10. |−15| + |−7| = 22

11. Huda, Buthaina, and Shaima are auditioning for the same role. Huda auditions at 10 minutes before four, Buthaina auditions 30 minutes before Huda, and Shaima auditions at 5 minutes before four. Order the three by who will audition first. (Lesson 3) Buthaina, Huda, Shaima

12. MP **Persevere with Problems** The tables shows the overnight low temperatures for a four-day period. The low temperature on Wednesday was between the second and third highest temperatures of the ones shown, when placed in order. Give a possible temperature for Wednesday. (Lesson 3)

    Sample answer: 0°F

Temperature (°F)	
Thursday	−8
Friday	7
Saturday	18
Sunday	−11

Inquiry Lab
Number Lines

Inquiry **HOW can you use a number line to model and compare positive and negative rational numbers?**

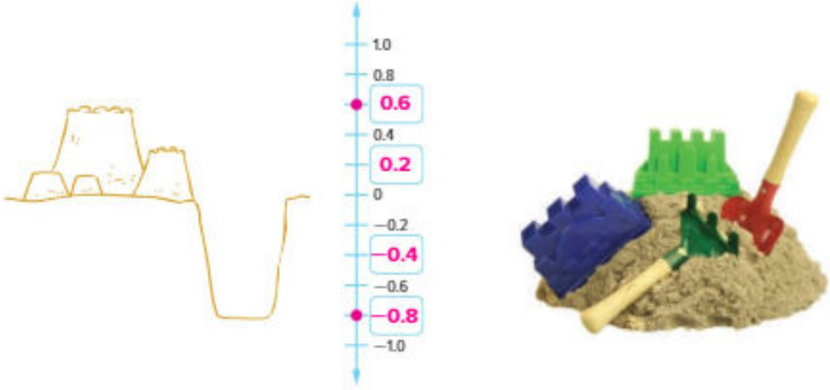
MP Mathematical Practices 1, 3, 4

Mahmoud and Suhail are at the beach. Mahmoud builds a sandcastle 0.6 meter high. Suhail digs a hole in the sand 0.8 meter deep.

Hands-On Activity 1

Just as you can graph integers on a number line, you can graph positive and negative fractions and decimals. Recall that positive numbers are to the right of zero on the number line and negative numbers are to the left of zero.

Step 1 Complete the number line from −1 to 1, with increments of 0.2.



Step 2 The sandcastle is above sea level. Its height is *greater than zero* on the number line, so draw a dot at 0.6 to represent the sandcastle.

Step 3 The hole is below sea level. Its depth is *less than zero* on the number line. So draw a dot at −0.8 to represent the hole.

Focus narrowing the scope

Objective Model rational numbers.

Coherence connecting within and across grades

Now
Students model, graph, and compare positive and negative rational numbers using a number line.

Next
Students will write rational numbers as equivalent fractions and decimals.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 376.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activity 2.

Materials: fraction tiles

Hands-On Activity 1

AL LA As students complete the number line, reinforce visually the idea of absolute value by pointing out that integers such as −1 and 1 are the same distance from zero on the number line and therefore have the same absolute value.

Ask:

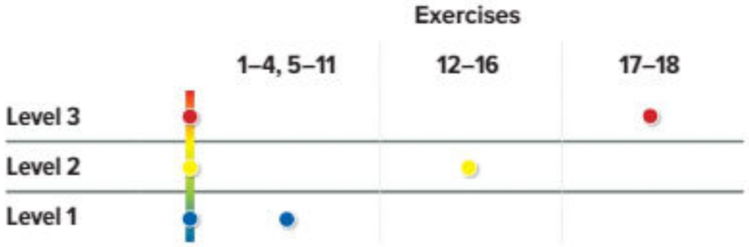
- *How many units from 0 is 0.6?* 0.6 units
- *How many units from 0 is −0.8?* 0.8 units

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Hands-On Activity 2

AL LA **Rally Coach** If students are struggling, have them work with a partner to answer the following questions. Have one student speak the first question aloud and then answer the question, while the other student listens carefully, coaches, and praises the first student. Then students trade roles to answer the second and third questions. MP 1, 3, 5

Ask:

- *Why should the number line be divided into 4 equal parts?* Since you are graphing $-\frac{3}{4}$, the number line should be divided into fourths.
- *From left to right, what do the marks on the number line represent?* $-1, -\frac{3}{4}, -\frac{2}{4}, -\frac{1}{4}$, and 0
- *Which labeled numbers on the number line are greater than $-\frac{3}{4}$? Explain.* $-\frac{2}{4}, -\frac{1}{4}$, and 0; All numbers to the right of $-\frac{3}{4}$ are greater than $-\frac{3}{4}$.

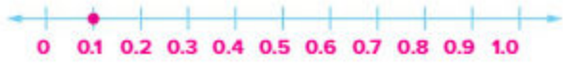
Investigate

Collaborate

MP **Model with Mathematics** Work with a partner. Graph each number on a number line.

1. -2.4

2. 0.1



3. -4.5

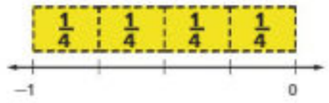
4. -6.8

Hands-On Activity 2

Graph $-\frac{3}{4}$ on a number line.

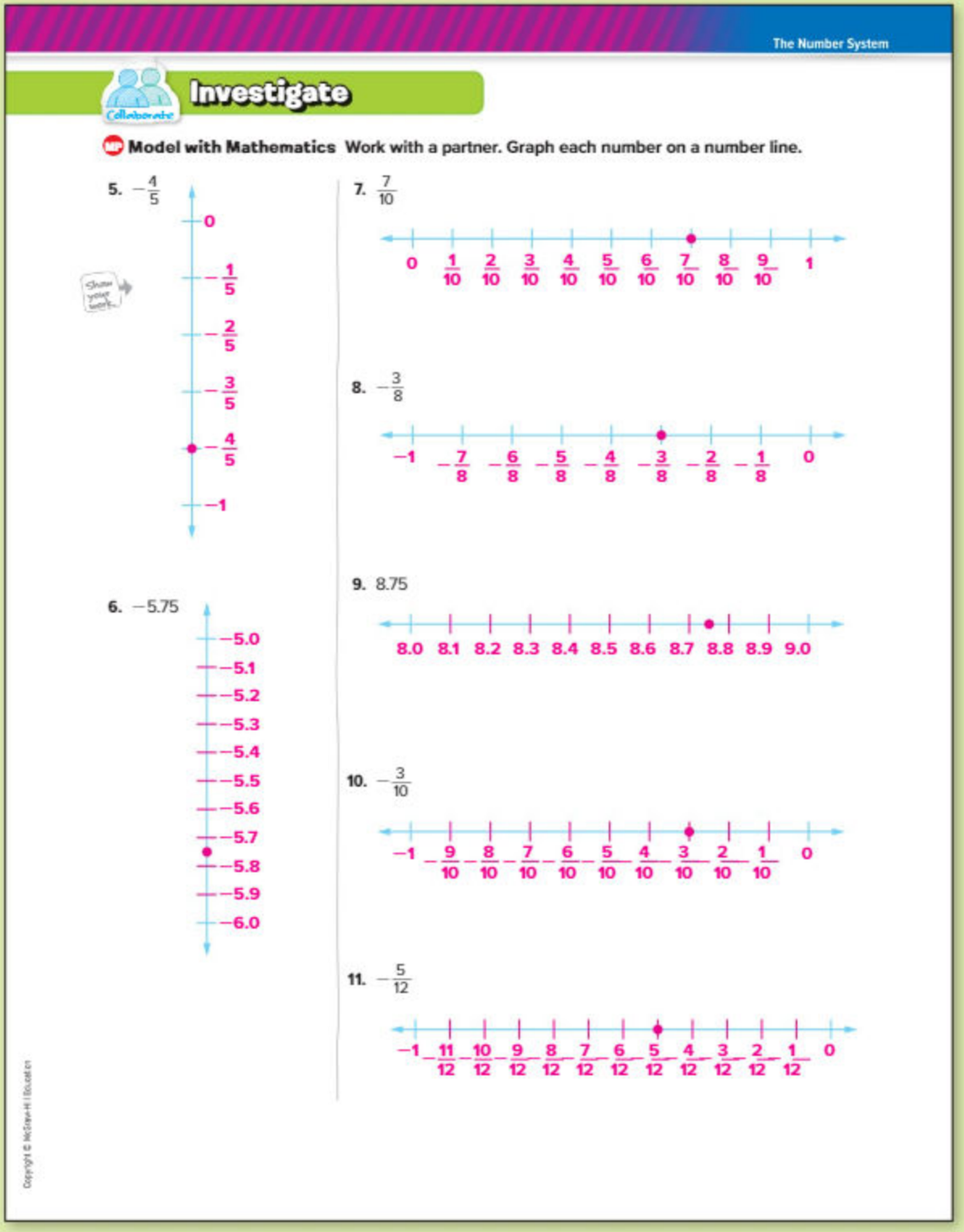
Step 1 Model $-\frac{3}{4}$ using fraction tiles. Draw a number line from -1 to 0.

Since the denominator of the fraction is 4, divide your number line into 4 equal parts.



Step 2 Each mark on the number line represents $\frac{1}{4}$. Label the number line with $-\frac{3}{4}, -\frac{2}{4}$, and $-\frac{1}{4}$.

Step 3 Draw a dot to graph $-\frac{3}{4}$ on the number line above.



Investigate

AL BL LA Pairs Discussion Have students work in pairs to complete Exercises 5–11. Have them trade solutions with another pair of students and discuss any differences. Have them ask each other clarifying questions such as, "Why is the number line divided into eighths in Exercise 8?" Have students carefully listen to each other's responses. MP 1, 5, 7

OL LA Have students create their own rational number that can be located on the number line. Have them present their number and number line to another student, verbally explaining how they located the number on the number line. MP 1, 4, 5

BL Have students work in pairs to answer Exercises 5–11 and answer the following extension questions. MP 1, 4, 6, 7

Ask:

- *Refer to Exercise 8. If the number line is labeled with decimal values, what would be the value at the point you graph?* −0.375
- *Refer to Exercises 10 and 11. Use your number lines to determine which value is greater,* $-\frac{3}{10}$ *or* $-\frac{5}{12}$. $-\frac{3}{10}$

Analyze and Reflect

AL BL LA Pair an Approaching Level student with a Beyond Level student to work together to complete Exercises 12–16. Have students work in pairs to answer Exercises 12–14 and answer the following questions.

Ask:

- *What is the greatest number in the table?* $\frac{4}{5}$
- *What is the least number in the table?* -3.5

Create

AL LA Instead of assigning Exercise 17, have students write a real-world problem that involves a positive and negative value. Have them graph the values used in the problem on a number line. Ask them to identify which value is greater. If needed, give students a prompt such as, "A rabbit lives under ground in a hole and a bird lives above ground in a tree. Create a problem using these animals." MP 1, 2, 4

BL LA **Trade-a-Problem** Using the problem written in Exercise 17, have students trade and solve each other's problem. Have them discuss any similarities and differences. MP 1, 3

inquiry Students should be able to answer "HOW can you use a number line to model and compare positive and negative rational numbers?" Check for student understanding and provide guidance, if needed.

Analyze and Reflect

Work with a partner to complete the table. The first one is done for you.

	Number	Positive or Negative	Greater Than or Less Than Zero	Left or Right of 0 on the Number Line
	-3.5	negative	$<$	left
12.	$+\frac{4}{5}$	positive	$>$	right
13.	$\frac{1}{3}$	negative	$<$	left
14.	$+0.3$	positive	$>$	right

15. MP **Reason Inductively** Which number is greater, 0.3 or -0.7? Explain.

0.3; Sample answer: The number 0.3 is greater than 0 and the number -0.7 is less than zero. So, 0.3 is greater than -0.7.

16. MP **Reason Inductively** Juwairyah thinks that $-\frac{1}{2}$ is greater than $\frac{1}{4}$ because it is farther from zero on the number line. Is her thinking correct? Explain.

No; Sample answer: $-\frac{1}{2}$ is less than $\frac{1}{4}$ because it is to the left on the number line. As the numbers on the number line are read from left to right, the values increase.

Create

Sample answers: 17–18

17. MP **Model with Mathematics** Write a real-world problem that involves a positive and a negative value. Then graph the values used in the problem on a number line. Compare the values.

A rabbit dug a nest 1.2 feet under the ground. A cardinal built his nest in the top of a 10.5-foot tree. Since positive numbers are greater in value than negative numbers, 10.5 is greater than -1.2.

18. inquiry HOW can you use a number line to model and compare positive and negative rational numbers?

Negative values are lesser the farther they are from zero. Positive values are greater the farther they are from zero.

Lesson 4

Terminating and Repeating Decimals

Vocabulary Start-Up

Any number that can be written as a fraction is called a **rational number**. Every rational number can be written as either a **terminating decimal** or a **repeating decimal**.

Draw lines from each word to its matching statement.

Word	Statement
terminating decimal	the decimal form of a rational number which has a repeating digit of zero; 0.625
repeating decimal	the decimal form of a rational number; 0.33333. . .

Essential Question

HOW are integers and absolute value used in real-world situations?

Vocabulary

rational number
terminating decimal
repeating decimal
bar notation

Mathematical Practices
1, 2, 3, 4, 7, 8

Real-World Link

Party Favors Jaber is buying fruit snacks for party favors. He asks the cashier for a half pound of fruit snacks.

1. Express one half as a fraction.
   $\frac{1}{2}$
2. Write the decimal that represents half a pound.
   0.5
3. Suppose Jaber wanted to buy one third of a pound. What decimal would the scale show?
   0.33333. . .

Which Mathematical Practices did you use?
Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Write positive and negative fractions as decimals.

Coherence connecting within and across grades

Previous Students modeled, graphed, and compared fractions and decimals.

Now Students write rational numbers as equivalent fractions or decimals.

Next Students will compare and order rational numbers.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 383.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Stand Up, Hand Up, Pair Up** For each exercise, pose the question and give think time. Call out, "Stand up, hand up, pair up". Students stand, put their hand up, and pair up with a student. In pairs, students discuss their responses. Repeat for each exercise. Have students pair up with a different student each time. MP 1, 3

Alternate Strategy

BL Have students write a table of common fraction-decimal equivalents, such as denominators of 2, 3, 4, 5, 6, 8, 9, and 10. Have them make a generalization of when the decimal form of each fraction will be terminating or repeating. MP 1, 8

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. **Write a fraction as a decimal.**

AL
- *How do you write a fraction as a decimal?* Divide the numerator by the denominator.
- *Why did we annex zeros after 5?* 12 is greater than 5; 12 does not divide 5, but it does divide 50

OL
- *When you find 5 ÷ 12, which digit(s) repeat?* 6
- *How do you show that the 6 repeats?* Write 6 with a bar over it.

BL
- *In how many ways can you classify the number $\frac{5}{12}$?* fraction, repeating decimal, rational number
- *Why will the remainder never be zero?* The digit 6 is repeating, which means that the division will never end.

Need Another Example?

Write $\frac{1}{9}$ as a decimal. $0.\overline{1}$

Watch Out!

Common Error If students incorrectly place the bar when using bar notation, have them write the repeating decimal on grid paper with one digit per square. Have them highlight the first set of number(s) that repeats. Students should rewrite the decimal so the bar is placed over those digits in the decimal.

Work Zone

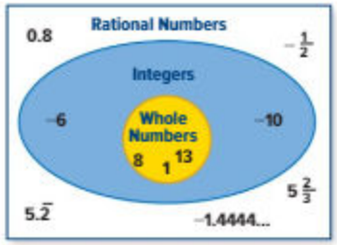
Key Concept

Rational Numbers

Words Rational numbers can be written as fractions.

Algebra $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Model



Fractions, terminating and repeating decimals, percents, and integers are all rational numbers. Every rational number can be expressed as a decimal by dividing the numerator by the denominator.

Rational Number	Repeating Decimal	Terminating Decimal
$\frac{3}{10}$	0.300...	0.3
$\frac{4}{5}$	0.800...	0.8
$\frac{5}{6}$	0.833...	does not terminate

To indicate the number pattern that repeats indefinitely, use bar notation. **Bar notation** is a bar placed over the digits that repeat.

$0.545454... = 0.\overline{54}$ $0.583333... = 0.58\overline{3}$

Example

1. **Write $\frac{5}{12}$ as a decimal.**

$$
\begin{array}{r}
0.4166 \\
12\overline{)5.000} \quad \text{Divide 5 by 12.} \\
-\,48 \\
\hline
20 \\
-\,12 \\
\hline
80 \\
-\,72 \\
\hline
80 \\
-\,72 \\
\hline
8 \quad \text{The remainder will never be zero.}
\end{array}
$$

So, $\frac{5}{12} = 0.4166...$ or $0.41\overline{6}$.

Got it? Do these problems to find out.

Write each fraction as a decimal. Use bar notation if necessary.

a. $\frac{1}{6}$ b. $\frac{8}{9}$ c. $\frac{2}{11}$

Write a Negative Fraction as a Decimal

When writing negative fractions as decimals, the process is the same. Divide as with positive fractions. Write the negative sign in front of the decimal.

Examples

2. **Write $-\frac{2}{9}$ as a decimal.**

$$\begin{array}{r} 0.222 \\ 9\overline{)2.000} \\ -18 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 2 \end{array}$$

Divide 2 by 9.

The remainder will never be zero.

So, $-\frac{2}{9} = -0.222\ldots$ or $-0.\overline{2}$.

3. **Write $-2\frac{2}{3}$ as a decimal.**

$-2\frac{2}{3}$ can be rewritten as $-\frac{8}{3}$.

The mixed number $-2\frac{2}{3}$ can be written as $-2.\overline{6}$.

$$\begin{array}{r} 2.6\ldots \\ 3\overline{)8.0} \\ -6 \\ \hline 20 \\ -18 \\ \hline 2 \end{array}$$

Got it? Do these problems to find out.

Write each fraction as a decimal. Use bar notation if necessary.

d. $-\frac{1}{4}$ e. $-\frac{5}{6}$ f. $-2\frac{1}{6}$

Show your work.

a. $0.1\overline{6}$

b. $0.\overline{8}$

c. $0.\overline{18}$

Repeating Decimals

When dividing, it is sometimes helpful to divide until the repeated pattern is shown at least three times.

d. -0.25

e. $-0.8\overline{3}$

f. $-2.1\overline{6}$

Examples

2. **Write a negative fraction as a decimal.**

AL
- *Is the number a whole number? Explain.* No, whole numbers are not negative and have denominators of 1 when in simplest form.
- *What does the bar over the 2 mean?* That digit repeats when dividing.

OL
- *Is $-\frac{2}{9}$ an integer? How would you classify $-\frac{2}{9}$?* no; rational number
- *How can you divide 2 by 9?* write 2 as 2.0, place the decimal in the quotient, then annex zeroes to continue dividing

BL
- *Explain why you annex zeroes.* Sample answer: 9 does not divide into 2, but annexing a zero will allow 9 to divide into 20.
- *Explain why the number is rational, but not an integer.* It is not of the set {...−3, −2, −1, 0, 1, 2, 3, ...}

Need Another Example?

Write $-\frac{11}{12}$ as a decimal. $-0.91\overline{6}$

3. **Write a negative fraction as a decimal.**

AL
- *Is the number a whole number? What does the bar over the 6 mean?* no; That digit repeats when dividing.

OL
- *Explain why the number is a rational number.* Sample answer: Rational numbers include repeating decimals.

BL
- *Explain why the number is rational, but not an integer or a whole number.* It is not of the integer set {...−3, −2, −1, 0, 1, 2, 3 ...} and not included in the whole number set {0, 1, 2, 3 ...}
- *Could you classify an improper fraction as a rational number? an integer? a whole number? Explain with examples.* Sample answer: an improper fraction could be classified as a rational number, such as $-\frac{11}{2}$, as an integer, such as $-\frac{6}{3}$ or $\frac{10}{5}$, and some could be whole numbers, such as $\frac{4}{1}$ or $\frac{15}{3}$.

Need Another Example?

Write $-4\frac{5}{6}$ as a decimal. $-4.8\overline{3}$

Example

4. Find the average.

AL • *In the fraction, what does 34 mean?* 34 made free-throws

• *In the fraction, what does 44 mean?* 44 attempted free-throws

OL • *How would you round 0.77272727 to the nearest thousandth?* Look at the value in the ten-thousandths place. Since this value is 5 or more, round the digits in the thousandths place up one to 3.

BL • *If Frankie wants his free-throw average to be 80%, or 0.8, how many more free-throws will he have to make in a row? Explain.* He will need to make 6 free-throws out of his next 6 attempts. This will change his average to $\frac{40}{50}$ or 0.8.

Need Another Example?

Anna made 15 strikes out of 55 balls rolled during a bowling contest. To the nearest thousandth, what is her strike average in the contest? 0.273

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Numbered Heads Together** Have students work in groups of 3–4 to work as a team to solve each problem, making sure that everyone understands. Then call on a random student to explain the solution. MP 1, 3

BL LA **Pairs Discussion** Have students work in pairs to generate the repeating decimals that are equivalent to fractions with denominators of 9, such as $\frac{1}{9}$, $\frac{2}{9}$, $\frac{3}{9}$, and so on. Then have them describe the pattern and predict what the equivalent repeating decimal would be for $\frac{8}{9}$. MP 1, 7, 8

9. $0.\overline{4}$

Show your work.

Example

4. **Farid made 34 out of 44 free throws this season. To the nearest thousandth, what is his free-throw average?**

Using a calculator, divide 34 by 44.

34 ÷ 44 ENTER 0.77272727

To the nearest thousandth, his free-throw average is 0.773.

Got it? **Do this problem to find out.**

g. Of nine students surveyed, four said they prefer exercising in the morning rather than in the evening. Express this fraction as a decimal. Use bar notation if necessary.

Guided Practice

Write each fraction as a decimal. Use bar notation if necessary. (Examples 1–3)

1. $\frac{7}{9} = 0.\overline{7}$
2. $-\frac{1}{33} = -0.\overline{03}$
3. $-2\frac{5}{6} = -2.8\overline{3}$
4. $\frac{10}{15} = 0.\overline{6}$
5. $-\frac{4}{5} = -0.8$
6. $1\frac{5}{9} = 1.\overline{5}$

7. Maryam bought $\frac{2}{3}$ yard of fabric to make a new purse. Write the amount of fabric she used as a decimal. (Example 4)

$0.\overline{6}$

8. **Building on the Essential Question** How are repeating decimals used in real-world situations?

Sample answer: Repeating decimals can be used to describe real-world situations, such as batting averages.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

Name _____ My Homework _____

Independent Practice

Write each fraction as a decimal. Use bar notation if necessary.
(Examples 1–3)

1. $\frac{7}{15} = 0.4\overline{6}$
2. $\frac{8}{18} = 0.\overline{4}$
3. $-\frac{8}{12} = -0.\overline{6}$

Show your work.

4. $-\frac{6}{7} = -0.\overline{857142}$
5. $3\frac{15}{44} = 3.34\overline{09}$
6. $-2\frac{5}{22} = -2.2\overline{27}$

7. Safaa had 34 out of 99 hits when she was at bat during the softball season. What was her batting average? (Example 4)
$0.\overline{34}$

8. Shadi and his friends ate $3\frac{1}{6}$ pizzas. Write this amount as a decimal. (Example 4)
$3.1\overline{6}$

Write each decimal as a fraction or mixed number in simplest form.

9. $-0.9 = -\frac{9}{10}$
10. $-0.85 = -\frac{17}{20}$
11. $-3.8 = -3\frac{4}{5}$

Evaluate each expression.

12. $|-2.3| = 2.3$
13. $\left|\frac{4}{13}\right| = \frac{4}{13}$
14. $\left|-8\frac{7}{11}\right| = 8\frac{7}{11}$

15. STEM There are over 2,700 species of snakes in the world. Over 600 species are venomous. Write the fraction of species that are *not* venomous as a decimal. $0.\overline{7}$

16. **Justify Conclusions** The ratio of the distance around a circle to the distance across a circle through its center is represented by the number π. The number π is a decimal that does not repeat. The fraction $\frac{22}{7}$ is sometimes used as an estimate for π. Is $\frac{22}{7}$ a repeating decimal? Explain.
yes; The fraction $\frac{22}{7}$ can be represented as $3.\overline{142857}$ by dividing with a calculator.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice
The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments
You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–8, 9–17 odd, 18, 20, 21, 33, 34
OL	On Level	1–7 odd, 9–18, 20, 21, 33, 34
BL	Beyond Level	9–21, 33, 34

MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	19
2 Reason abstractly and quantitatively.	17
3 Construct viable arguments and critique the reasoning of others.	16, 18, 20
8 Look for and express regularity in repeated reasoning.	21, 32

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

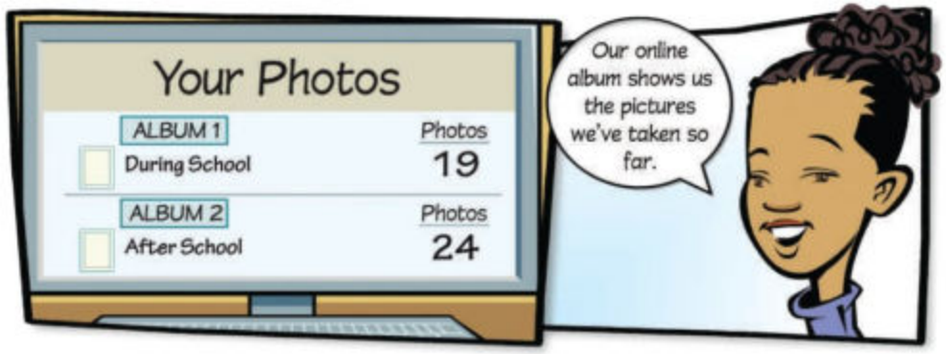
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET
Out the Door

Have students explain how the previous lesson helped them understand this lesson on writing different forms of rational numbers. Give them the following writing prompts. See students' work.

- In the previous lesson, I learned...
- In this lesson, I learned...
- What I learned in the previous lesson helped me with this lesson because...

17. **Reason Abstractly** Refer to the graphic novel frame below for Exercises a–b.



a. How many total photos were taken? 43

b. What fraction of the photos were taken after school? Write this fraction as a decimal. Round to the nearest thousandth. $\frac{24}{43}$; 0.558

H.O.T. Problems Higher Order Thinking

18. **Justify Conclusions** Name a number that is a rational number, but not an integer. Justify your response. Sample answer: $-\frac{2}{3}$; The number is negative, but because the denominator is not 1, it is not an integer.

19. **Persevere with Problems** Predict whether or not the decimal equivalent to $\frac{17}{36}$ is terminating. Explain your reasoning. Check your prediction with a calculator. $\frac{17}{36}$ is not a terminating decimal since decimals are based on powers of 10 and 36 is not a factor of any power of 10.

20. **Which One Doesn't Belong?** Identify the decimal equivalent that does *not* have the same characteristic as the other three. Explain.

$\frac{1}{12}$ $\frac{2}{12}$ $\frac{3}{12}$ $\frac{4}{12}$

$\frac{3}{12}$; Its decimal equivalent is a terminating decimal.

21. **Identify Repeated Reasoning** Find the decimal equivalents for $\frac{1}{11}$, $\frac{2}{11}$, and $\frac{3}{11}$. Use the pattern to mentally find the decimal equivalents for $\frac{7}{11}$ and $\frac{8}{11}$. $\frac{1}{11} = 0.\overline{09}$, $\frac{2}{11} = 0.\overline{18}$, $\frac{3}{11} = 0.\overline{27}$; The digits that are repeated are equal to the numerator times 9. So, $\frac{7}{11} = 0.\overline{63}$ and $\frac{8}{11} = 0.\overline{72}$.

Name _____ My Homework _____

Extra Practice

Write each fraction as a decimal. Use bar notation if necessary.

22. $\frac{32}{75} =$ $0.42\overline{6}$

Homework Help

$$\begin{array}{r} 0.42\overline{6} \\ 75\overline{)32.000} \\ -300 \\ \hline 200 \\ -150 \\ \hline 500 \\ -450 \\ \hline 50 \end{array}$$

23. $\frac{3}{11} =$ $0.\overline{27}$

24. $-\frac{5}{8} =$ -0.625

25. $-\frac{7}{10} =$ -0.7

26. $2\frac{5}{7} =$ $2.\overline{714285}$

27. $-1\frac{80}{99} =$ $-1.\overline{80}$

28. Hassan answered 61 out of 66 questions correctly on a test. What is his test average to the nearest thousandth? 0.924

Write each decimal as a fraction or mixed number in simplest form.

29. $-0.15 =$ $-\frac{3}{20}$

30. $-7.75 =$ $-7\frac{3}{4}$

31. $-12.54 =$ $-12\frac{27}{50}$

32. MP **Identify Repeated Reasoning** The table shows the decimal equivalent to fractions with a denominator of 7.

Fraction	Decimal	Fraction	Decimal
$\frac{1}{7}$	$0.\overline{142857}$	$\frac{4}{7}$	$0.\overline{571428}$
$\frac{2}{7}$	$0.\overline{285714}$	$\frac{5}{7}$	$0.\overline{714285}$
$\frac{3}{7}$	$0.\overline{428571}$	$\frac{6}{7}$	$0.\overline{857142}$

a. What do you notice about the pattern of the six repeated numbers?

The same six numbers are repeated in the same order but at different starting points.

b. Using the decimals, add the first half of each pattern to the numbers in the last half. For example, $\frac{1}{7} = 0.\overline{142857}$, so add 142 + 857.

What pattern do you notice? All have a sum of 999.

c. Using a calculator, try the same experiment with $\frac{5}{13}$. Is the result the same? Justify your reasoning. yes; $\frac{5}{13} = 0.\overline{384615}$ and 384 + 615 = 999

Power Up! Test Practice

Exercises 33 and 34 prepare students for more rigorous thinking needed for the assessment.

33.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

34.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

33. Write a decimal to represent the shaded portion of the figure.

$0.1\overline{6}$

34. Which of the following fractions are equivalent to $0.\overline{3}$?
Select all that apply.

- ■ $\frac{1}{3}$
- ■ $\frac{3}{9}$
- ☐ $\frac{3}{10}$
- ■ $\frac{11}{33}$

Spiral Review

Fill in each ◯ with < or > to make a true statement.

35. 4,556 < 4,565

36. 8,698 > 8,689

37. 47,872 > 47,871

38. 26,525 > 26,522

39. 1,123,004 < 1,123,040

40. 5,776,050 > 5,775,005

41. The table shows the number of miles Mariam walked for two weeks. Compare the distances using the < symbol.

$5.691 < 5.78$

Week	Number of Miles
1	5.78
2	5.691

42. The table shows the amount of different colored paints in a bin in art class. Compare the amount of blue and orange paint using the > symbol.

$47.394 > 47.362$

Color	Number of Ounces
Blue	47.362
Green	47.637
Orange	47.394
Yellow	47.583

Compare and Order Rational Numbers

Real-World Link

Insects The lengths of several common types of insects are shown in the table.

Insect	Length (in.)
Green June Beetle	$\frac{3}{4}$
Cricket	$\frac{1}{1}$
Fire ant	$\frac{1}{3}$
Firefly	$\frac{3}{4}$
Housefly	$\frac{1}{4}$
Japanese beetle	$\frac{1}{2}$
Mosquito	$\frac{5}{8}$

1. Which of the insects is the longest?
   the cricket

2. Shade each fraction strip to represent the lengths of a fire ant and a housefly. Which is longer, the fire ant or housefly?
   fire ant

3. How many of the insects are longer than 0.5 inch?
   4 insects

4. Order the lengths of a housefly, a Green June beetle, and a fire ant from the shortest to longest.
   $\frac{1}{4}$ in., $\frac{1}{3}$ in., $\frac{3}{4}$ in.

Essential Question

HOW are integers and absolute value used in real-world situations?

MP Mathematical Practices
1, 2, 3, 4, 5, 7

Which MP **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Compare and order rational numbers.

Coherence connecting within and across grades

Previous
Students wrote rational numbers as equivalent fractions and decimals.

Now
Students compare and order rational numbers.

Next
Students will graph ordered pairs that include rational numbers.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 391.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Pairs Discussion** Have students work the problems in pairs to determine the answer to the question and describe how they found the solution. MP 1, 3

Alternate Strategy

AL LA Provide students with the fractional model that shows thirds and fourths for the housefly and the fire ant. Have students determine the lengths of the insects as decimals and compare the numerators and denominators for fractions greater or less than one-half. Ask them to look at decimal values for $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$, and describe the relationship between the denominator of a unit fraction and the value of the number. MP 1, 4, 7

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Compare decimals.

AL • *What do the symbols <, > mean?* less than; greater than

OL • *Explain how to compare the numbers without using a number line.* −1.2 is negative and 0.8 is positive. Negative numbers are less than positive numbers.

BL • *Which number is closer to 0? Explain.* 0.8; 0.8 is 0.8 units from 0 and −1.2 is 1.2 units from 0.

Need Another Example?
Is −0.4 <, >, or = 1.1? <

2. Compare decimals.

OL • *If you were comparing −1.40 and −1.4, which symbol would you use to compare? Explain.* =; Sample answer: They are equal.

BL • *Which number is closer to 0?* −1.25

Need Another Example?
Is −3.15 <, >, or = −3.25? >

3. Compare fractions.

AL • *How do the denominators compare?* 8 is half of 16

OL • *Which number is farthest left on the number line?* $-\frac{6}{16}$ or $-\frac{3}{8}$

• *Why rewrite using the common denominator?* So you can more easily compare by comparing numerators

BL • *Explain another method you could use to compare the fractions.* Rewrite each fraction as a decimal, and then compare the decimals.

Need Another Example?
Is $-\frac{2}{5}$ <, >, or = $-\frac{4}{15}$? <

Work Zone

Least Common Multiple (LCM)
Find the LCM of 8 and 16.
8: 8, 16, 24
16: 16, 32, 48
The LCM is 16.

Compare Decimals and Fractions

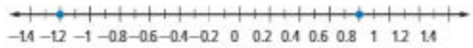
Positive and negative rational numbers can be represented on a number line. You can use a number line to help you compare and order rational numbers.

Examples

Fill in each ◯ with <, >, or = to make a true statement.

1. −1.2 ◯ 0.8

Graph the decimals on a number line.

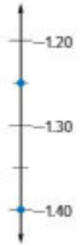


Since −1.2 is to the left of 0.8, −1.2 < 0.8.

2. −1.40 ◯ −1.25

Graph the decimals on a number line.

Since −1.40 is below −1.25, −1.40 < −1.25.



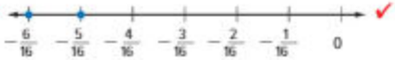
3. $-\frac{3}{8}$ ◯ $-\frac{5}{16}$

Rename the fractions using the least common denominator.

$-\frac{3}{8} = -\frac{3 \times 2}{8 \times 2} = -\frac{6}{16}$ $\quad -\frac{5}{16} = -\frac{5 \times 1}{16 \times 1} = -\frac{5}{16}$

Since $-6 < -5$, $-\frac{6}{16} < -\frac{5}{16}$ and $-\frac{3}{8} < -\frac{5}{16}$.

Check ✓



Got it? Do these problems to find out.

a. 3.1 (>) −3.7

b. −4.5 (<) −4.49

c. $\frac{9}{16}$ (<) $\frac{12}{16}$

d. $-\frac{7}{10}$ (>) $-\frac{4}{5}$

The Number System

Compare and Order Rational Numbers

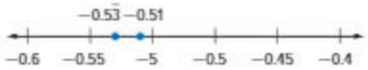
To compare and order rational numbers, first write them in the same form.

Examples

Fill in each ◯ with <, >, or = to make a true statement.

4. $-0.51 \bigcirc -\frac{8}{15}$

Rename $-\frac{8}{15}$ as a decimal. Then graph both decimals on a number line.



$-\frac{8}{15} = -0.5\overline{3}$

Since -0.51 is to the right of $-0.5\overline{3}$ on the number line, $-0.51 > -\frac{8}{15}$.

5. **Order the set $\left\{-2.46, -2\frac{22}{25}, -2\frac{1}{10}\right\}$ from least to greatest.**

Write $-2\frac{22}{25}$ and $-2\frac{1}{10}$ as decimals to the hundredths place.

$-2\frac{22}{25} = -2.88$ $\quad -2\frac{1}{10} = -2.1$

Graph the decimals on the number line.

From least to greatest, the order is $-2\frac{22}{25}$, -2.46, and $-2\frac{1}{10}$.

Got it? **Do these problems to find out.**

Fill in each ◯ with <, >, or = to make a true statement.

e. $-3\frac{5}{8} \bigcirc -3.625$ = =

f. $\frac{3}{7} \bigcirc 0.413$ >

g. Order the set $\left\{-7\frac{13}{20}, -7.78, -7\frac{17}{100}\right\}$ from greatest to least.

STOP and Reflect

How could you represent that -8.3 feet is deeper than -5.7 feet? Explain.

−8.3 < −5.7; On a number line, −8.3 is to the left of −5.7.

Show your work.

g. $-7\frac{17}{100}, -7\frac{13}{20}, -7.78$

Examples

4. **Compare rational numbers.**

AL
- *How can you write $-\frac{8}{15}$ as a decimal?* Divide 8 by 15 and write the decimal as a negative number.
- *How can you represent the rational number $-\frac{8}{15}$ as a repeating decimal?* $-0.5\overline{3}$

OL
- *How do you know that both numbers are close to each other?* Sample answer: They are both approximately equal to −0.5.

BL
- *Without graphing the rational numbers on a number line, explain another way that you can compare them.* Sample answer: Since both numbers have the same tenths value, compare the hundredths values: $-0.01 > -0.0\overline{3}$. Therefore, $-0.51 > -0.5\overline{3}$.

Need Another Example?

Is -0.79 <, >, or = $-\frac{9}{11}$? >

5. **Order a set of rational numbers.**

AL
- *Is -2.46 <, >, or = -2.1?* <
- *Is -2.88 <, >, or = -2.46?* <

OL
- *What is $-2\frac{22}{25}$ written as a decimal? $-2\frac{1}{10}$?* −2.88; −2.1
- *When graphed on a number line, which of the three numbers is farthest left? farthest right?* −2.88; −2.1

BL
- *If $-2\frac{3}{5}$ was added to the set of rational numbers, how would you order the set from least to greatest?* $-2\frac{22}{25}, -2\frac{3}{5}, -2.46, -2\frac{1}{10}$
- *Give a general rule for ordering rational numbers?* Sample answer: Write each value in the same form, either fractions or decimals. All fractions should have a common denominator; all decimals should extend to the same place value.

Need Another Example?

Order the set $\left\{-3\frac{1}{5}, -3\frac{12}{25}, -3.3\right\}$ from least to greatest.

$-3\frac{12}{25}, -3.3, -3\frac{1}{5}$

Example

6. Order a set of rational numbers.

AL
- *Which numbers are not written as decimals?* $3\frac{1}{4}$ and $-1\frac{7}{10}$
- *How do you know which number is the greatest just by looking at the table?* Sample answer: $3\frac{1}{4}$ is the greatest number because it is positive and greater than the other positive number, 1.7.

OL
- *Who's plant shows the greatest difference from the average?* Ricky
- *What value would represent the "average" on a number line?* 0

BL
- *Explain why you would rewrite the fraction values as decimals.* Sample answer: You would have to write all the values with a common denominator, decimal values are easier to compare.

Need Another Example?

The lengths of four different snakes in the reptile exhibit at the zoo are $2\frac{5}{6}$ feet, $3.\overline{6}$ feet, $2\frac{2}{3}$ feet, and 3.625 feet. Order the lengths from least to greatest. $2\frac{2}{3}$ ft, $2\frac{5}{6}$ ft, 3.625 ft, $3.\overline{6}$ ft

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

AL LA **Three-Step Interview** Place students in pairs. Student 1 interviews Student 2 using the interview questions below. Student 2 then interviews Student 1 using the same questions, but for the next exercise. Call on either student to explain the solution to a random exercise. MP 1, 3

- What are the steps, in order, needed to solve this problem?
- What is the solution to the problem? How did you determine this?

Real World

Example

6. **Mr. Badr's science class is growing plants under different conditions. The table shows the difference from the average for some students' plants. Order the differences from least to greatest.**

Student	Difference (in.)
Basem	$3\frac{1}{4}$
Dana	-2.2
Khaled	1.7
Lubna	$-1\frac{7}{10}$

Express each number as a decimal.

Basem's plant: $3\frac{1}{4} = 3.25$ Dana's plant: -2.2

Khaled's plant: 1.7 Lubna's plant: $-1\frac{7}{10} = -1.7$

From least to greatest, the differences are -2.2, $-1\frac{7}{10}$, 1.7, and $3\frac{1}{4}$.

Guided Practice

Check

Fill in each ◯ with <, >, or = to make a true statement. (Examples 1–4)

1. $9.7 \bigcirc -10.3$ **>**
2. $\frac{5}{8} \bigcirc -\frac{3}{8}$ **>**
3. $-6.7 \bigcirc -6\frac{7}{10}$ **=**
4. $-\frac{5}{6} \bigcirc -0.94$ **>**

Show your work.

Order the following sets of numbers from least to greatest. (Example 5)

5. $\left\{-3\frac{1}{3}, 3.3, -3\frac{3}{4}, 3.5\right\}$ $-3\frac{3}{4}, -3\frac{1}{3}, 3.3, 3.5$
6. $\left\{2.\overline{1}, -2.1, 2\frac{1}{11}, -2\right\}$ $-2.1, -2, 2\frac{1}{11}, 2.\overline{1}$

7. **Financial Literacy** Saeed recorded these amounts in his checkbook: −AED 6.50, AED 7.00, −AED 6.75, and AED 7.25. Order these amounts from least to greatest. (Example 6)

   −AED 6.75, −AED 6.50, AED 7.00, AED 7.25

8. **Building on the Essential Question** How can a number line help in ordering rational numbers?

   Sample answer: On a horizontal number line, numbers are shown from least to greatest from left to right.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

Fill in each ◯ with <, >, or = to make a true statement. (Examples 1–4)

1. $\frac{5}{4}$ (>) $-\frac{1}{4}$

2. $-6\frac{1}{3}$ (>) -6.375

3. $-\frac{3}{5}$ (=) -0.6

4. $-9\frac{2}{7}$ (>) -9.3

Show your work.

Order each set of numbers from least to greatest. (Example 5)

5. $\left\{2.8, -2\frac{3}{4}, 3\frac{1}{8}, -2.\overline{2}\right\}$ $-2\frac{3}{4}, -2.\overline{2}, 2.8, 3\frac{1}{8}$

6. $\left\{\frac{2}{3}, -0.6, 0.65, \frac{4}{5}\right\}$ $-0.6\ 0.65, \frac{2}{3}, \frac{4}{5}$

7. **Financial Literacy** The change in four stocks during a day are: $-4\frac{1}{2}$, 5.6, $-2\frac{3}{8}$, and 1.35.

   Order the changes from least to greatest. (Example 6)

   $-4\frac{1}{2}, -2\frac{3}{8}, 1.35, 5.6$

8. MP **Multiple Representations** Consider the inequality $-3.5 < -1.5$.

   a. **Words** Write a real-world problem that could be represented by the inequality.

   Sample answer: The low temperature in City A was −3.5 °F and the low temperature in City B was −1.5 °F. Which city was warmer?

   b. **Number Line** Graph −3.5 and −1.5 on the number line.

   c. **Symbols** Use the symbol > to compare −3.5 and −1.5.

   $-1.5 > -3.5$

9. For a STEM competition, Mariam constructed a model rocket. The rocket can reach an average height of 545 feet. Find the differences between the average height and the actual heights reached. Then write them as positive and negative rational numbers. Order the differences from least to greatest.

   $-10.8, -9.7, 9.0, 11.4$

Trials	Actual Height (ft)
1	534.2
2	556.4
3	554.0
4	535.3

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 11–13, 15, 29, 30
OL	On Level	1–7 odd, 8–13, 15, 29, 30
BL	Beyond Level	8–15, 29, 30

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	14
2 Reason abstractly and quantitatively.	13
3 Construct viable arguments and critique the reasoning of others.	8, 11, 12, 24
4 Model with mathematics.	15
7 Look for and make use of structure.	10

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

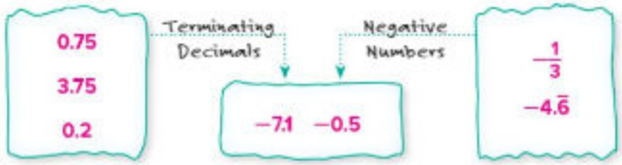
TICKET
Out the Door

Have students write a set of rational numbers that are in order from least to greatest. Their list should contain at least one terminating decimal, at least one fraction, and at least two negative numbers. See students' work.

Watch Out!

Common Error Some students compare rational numbers incorrectly because they assume that decimals with fewer places are less than decimals with more places. Have students annex zeros when comparing decimals so they can compare the values extended to the same place value.

10. **Identify Structure** Fill in the diagram with appropriate numbers. Sample answers are given.



H.O.T. Problems Higher Order Thinking

11. **Reason Inductively** Determine whether the following statement is *always*, *sometimes*, or *never* true. Give examples to justify your answer.
If x and y are both greater than zero and $x > y$, then $-x < -y$.
always; The greater a number is, the farther away from zero. Therefore, its opposite will also be farther from zero.

12. **Justify Conclusions** Determine whether the fractions $-\frac{4}{5}$, $-\frac{4}{6}$, $-\frac{4}{7}$, and $-\frac{4}{8}$ are arranged in order from least to greatest. Explain.
yes; When positive, if the numerators are equal, the larger the denominator, the smaller the fraction. Therefore, when negative, if the numerators are equal, the greater the denominator, the greater the fraction.

13. **Reason Abstractly** Explain why -0.33 is greater than $-0.\overline{33}$.
The first decimal is a terminating decimal, so its thousandths place is zero. The second decimal has a repeating digit of 3, so its thousandths place is 3. $-0.330 > -0.\overline{333}$

14. **Persevere with Problems** Compare the set $\left\{-0.\overline{7}, -0.\overline{6}, -\frac{7}{9}, -\frac{2}{3}\right\}$. Explain your answer.
Sample answer: $-0.\overline{7} = -\frac{7}{9}$ and $-0.\overline{6} = -\frac{2}{3}$; So, both $-\frac{2}{3}$ and $-0.\overline{6}$ are greater than $-\frac{7}{9}$ and $-0.\overline{7}$.

15. **Model with Mathematics** Write a real-world problem in which you would order a set of four rational numbers.
Sample answer: The temperature of a freezer changed throughout a day as the door was opened and shut. The temperatures were −11°F, 13°F, −12°F, and 15°F. Order the set of temperatures from least to greatest.; −12°F, −11°F, 13°F, 15°F

Name _____ My Homework _____

Extra Practice

Fill in each ◯ with <, >, or = to make a true statement.

16. $-18.6 \bigcirc -18.06$ (<)

Homework Help

17. $-4.08 \bigcirc -4.7$ (>)

18. $-\frac{3}{7} \bigcirc -\frac{2}{5}$ (<)

19. $-3.375 \bigcirc -3\frac{4}{10}$ (>)

20. $-5\frac{1}{5} \bigcirc -5.2$ (=)

21. $-8\frac{2}{5} \bigcirc -8.3$ (<)

Order the following sets of numbers from least to greatest.

22. $\left\{\frac{1}{8}, -0.02\overline{5}, 0.2, -\frac{1}{7}\right\}$ $-\frac{1}{7}, -0.02\overline{5}, \frac{1}{8}, 0.2$

23. $\left\{1.25, 1\frac{3}{4}, 1.2\overline{5}, 1\frac{1}{5}\right\}$ $1\frac{1}{5}, 1.25, 1.2\overline{5}, 1\frac{3}{4}$

24. MP **Reason Inductively** The average amount of time Bassam spent in-line skating for one week was 34 minutes. During the next week, the difference between the average time and actual time spent skating was 4.2 minutes, $-5\frac{1}{3}$ minutes, $-2\frac{1}{2}$ minutes, and 3.75 minutes.

Order these differences from least to greatest. $-5\frac{1}{3}, -2\frac{1}{2}, 3.75, 4.2$

Fill in each ◯ with <, >, or = to make a true statement.

25. $-4\frac{4}{5} \bigcirc -4.\overline{7}$ (<)

26. $-3.2\overline{5} \bigcirc -3.\overline{2}$ (<)

27. $-5.\overline{31} \bigcirc -5.\overline{313}$ (>)

28. The table shows the profit or loss of the after-school snack stand.

Day	Profit or Loss	(AED)
1	Profit	7.50
2	Loss	3.50
3	Loss	6.00
4	Profit	4.50

a. Write each profit as a positive number and each loss as a negative number.

Day 1: AED 7.50, Day 2: −AED 3.50, Day 3: −AED 6.00, Day 4: AED 4.50

b. Order the numbers from least to greatest.

−AED 6.00, −AED 3.50, AED 4.50, AED 7.50

Power Up! Test Practice

Exercises 29 and 30 prepare students for more rigorous thinking needed for the assessment.

29.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP6
Scoring Rubric	
1 point	Students correctly answer each part of the question.

30.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP2
Scoring Rubric	
2 points	Students correctly order all four differences.
1 point	Students correctly consecutively order three of the four differences.

Power Up! Test Practice

29. Refer to the number line. Determine if each inequality about the coordinates of the points is true or false.

a. $B > M$	☐ True	■ False
b. $F > B$	■ True	☐ False
c. $M < F$	☐ True	■ False
d. $F < M$	■ True	☐ False

30. Student Council's goal was to raise AED 50 each week for 4 weeks to have enough money for the school ceremony. The table shows the difference between the goal and the actual amount raised. Sort the amounts from least to greatest.

Week	1	2	3	4
Difference (AED)	5.50	−6.25	7.80	−2.45

	Amount
Least	−AED 6.25
	−AED 2.45
	AED 5.50
Greatest	AED 7.80

Spiral Review

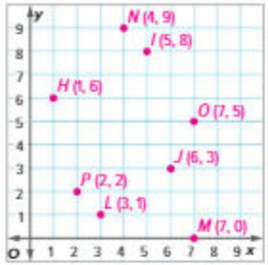
Graph the points on the coordinate plane.

31. $H(1, 6)$ **32.** $M(7, 0)$

33. $I(5, 8)$ **34.** $N(4, 9)$

35. $J(6, 3)$ **36.** $O(7, 5)$

37. $L(3, 1)$ **38.** $P(2, 2)$



39. Graph the point on the number line that represents $\frac{3}{10}$ and label it A.

Lesson 6

The Coordinate Plane

Real-World Link

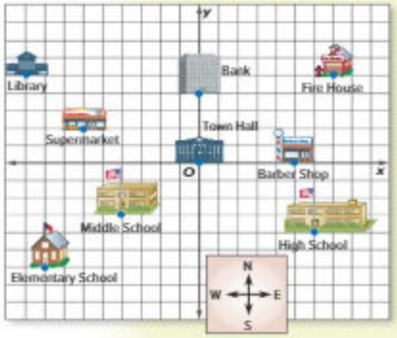
Maps The map shows the layout of a small town. The locations of buildings are described in respect to the town hall. Each unit on the grid represents one block.

1. Describe the location of the barber shop in relation to the town hall.
   The barber shop is 5 blocks east of the town hall.

2. What building is located 7 blocks east and 5 blocks north of the town hall? the fire house

3. Fairouz is at the library. Describe how many blocks and in what direction she should travel to get to the supermarket. 3 blocks south and 3 blocks east

4. Town Hall and the bank are both located on the same vertical number line. The number 0 represents the location of Town Hall on the number line. What number represents the location of the bank? 4



Essential Question

HOW are integers and absolute value used in real-world situations?

Vocabulary

quadrants

Mathematical Practices

1, 3, 4, 5, 7

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Graph ordered pairs on the coordinate plane.

Coherence connecting within and across grades

Previous
Students graphed and identified points in the first quadrant.

Now
Students write ordered pairs to represent points in all four quadrants.

Next
Students will graph points and their reflections in all four quadrants.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 399.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL LA **Pairs Discussion** Have students investigate the use of latitude and longitude in navigation to locate a position on Earth. They should relate what they learn to graphing points. Have each pair report their findings to the rest of the class. MP 1, 2, 4

Alternate Strategy

AL Have students create a poster of a coordinate plane and label the following parts: MP 1, 6

- origin
- quadrant I
- *x*-axis
- *y*-axis
- *A*(1, 4)
- *B*(0, 4)

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Write an ordered pair.**

AL • *In what quadrant is point C located?* Quadrant I

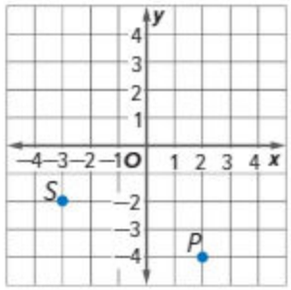
• *Is point C located above or below the x-axis?* above

OL • *How many units right along the x-axis do you need to move to be directly below point C?* $1\frac{1}{2}$ units

• *How many units up do you need to move to be at point C?* 1 unit

BL • *What is true about the coordinates of ordered pairs that are located in Quadrant I?* Both coordinates are always positive.

Need Another Example?
Identify the ordered pair that names point *P*. Then identify the quadrant in which it is located.
(2, −4), IV



2. **Identify a point.**

AL • *Is the x-coordinate positive or negative?* negative

• *Is the y-coordinate positive or negative?* negative

OL • *Since the x-coordinate is negative, which direction do you move from the origin? How many units?* left; $1\frac{1}{2}$ units

• *Since the y-coordinate is negative, which direction do you move from the origin? How many units?* down; 1 unit

BL • *How are points B and C related to each other?* They are the same distance from the origin.

continued on page 397

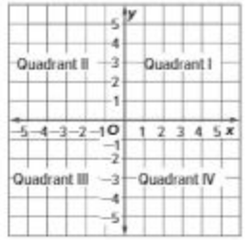
Work Zone

Ordered Pairs
A point located on the x-axis will have a y-coordinate of 0. A point located on the y-axis will have an x-coordinate of 0. Points located on an axis are not in any quadrant.

Identify Points and Ordered Pairs

A coordinate plane is formed when the *x*-axis and *y*-axis intersect at their zero points. The axes separate the coordinate plane into four regions called **quadrants**.

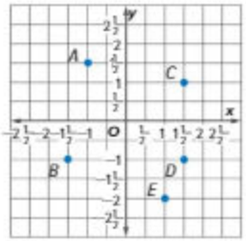
You can use the location on the plane or use the *x*-coordinates and *y*-coordinates to identify the quadrant in which a point is located.



Quadrant	*x*-coordinate	*y*-coordinate	Example
I	positive	positive	(2, 5)
II	negative	positive	(−2, 5)
III	negative	negative	(−2, −5)
IV	positive	negative	(2, −5)

Examples

1. **Identify the ordered pair that names point C. Then identify the quadrant in which it is located.**



Step 1 Start at the origin. Move right on the *x*-axis. The *x*-coordinate of point *C* is $1\frac{1}{2}$.

Step 2 Move up the *y*-axis. The *y*-coordinate is 1.

Point *C* is located at $\left(1\frac{1}{2}, 1\right)$. Both coordinates are positive.

So, point *C* is in Quadrant I.

2. **Identify the point located at $\left(-1\frac{1}{2}, -1\right)$. Then identify the quadrant in which it is located.**

Step 1 Start at the origin. Move left on the *x*-axis. The *x*-coordinate is $-1\frac{1}{2}$.

Step 2 Move down the *y*-axis. The *y*-coordinate is −1.

Point *B* is located at $\left(-1\frac{1}{2}, -1\right)$. Both coordinates are negative.

So, point *B* is in Quadrant III.

Got it? Do these problems to find out.

a. Refer to Example 1. Identify the ordered pair that names point *A*. Then identify the quadrant in which it is located.

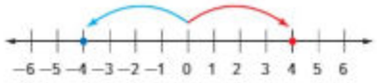
b. Refer to Example 1. Identify the point located at (1, −2). Then identify the quadrant in which it is located.

a. $\left(-1, 1\frac{1}{2}\right)$; II

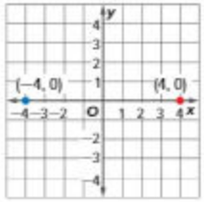
b. point E; IV

Reflections on the Coordinate Plane

You can use what you know about number lines and opposites to compare locations on the coordinate plane. Consider the number line and coordinate plane below. The number line shows that −4 and 4 are opposites.



The coordinate plane shows that the points (−4, 0) and (4, 0) are the same distance from the *y*-axis in opposite directions. So, they are *reflected* across the *y*-axis. Notice that the *y*-coordinates did not change and that the *x*-coordinates are opposites.

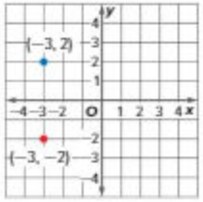


Example

3. **Name the ordered pair that is a reflection of (−3, 2) across the *x*-axis.**

To reflect across the *x*-axis, keep the same *x*-coordinate, −3, and take the opposite of the *y*-coordinate. The opposite of +2 is −2.

So, (−3, 2) reflected across the *x*-axis is located at (−3, −2).



Got it? Do these problems to find out.

Name the ordered pair that is a reflection of each point across the *x*-axis.

c. (1, −4) d. (−2, 5) e. (−3, −1)

c. (1, 4)

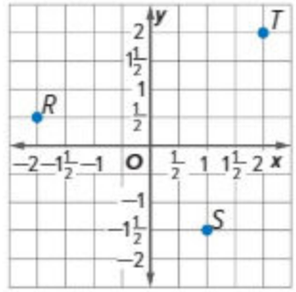
d. (−2, −5)

e. (−3, 1)

Show your work.

Need Another Example?

Identify the point located at $\left(-2, \frac{1}{2}\right)$. Then identify the quadrant in which it is located. point *R*, II



Example

3. **Reflect a point over the *x*-axis.**

AL • *Is the x-axis the vertical axis or the horizontal axis?* horizontal

• *When reflecting a point across the x-axis, which coordinate stays the same?* the *x*-coordinate

OL • *What do you need to do to the y-coordinate of the original point to find the y-coordinate of the reflected point?* take the opposite of the *y*-coordinate

• *What is the opposite of the y-coordinate?* −2

BL • *Where have you heard the word reflection before? How does that relate to reflection on the coordinate plane?* Sample answer: using mirrors; The reflection of a point on the coordinate plane is the mirror image of the original point.

Need Another Example?

Name the ordered pair that is a reflection of (2, −4) over the *x*-axis. (2, 4)

Example

4. Reflect points over the *y*-axis.

AL • *Is the y-axis the vertical axis or the horizontal axis?* vertical

• *When reflecting a point across the y-axis, which coordinate stays the same?* the *y*-coordinate

OL • *What do you need to do to the x-coordinate of the original point to find the x-coordinate of the reflected point?* take the opposite of the *x*-coordinate

• *What is the opposite of the x-coordinate?* 4

BL • *For a different fence, suppose Kendall puts a fencepost at (−2, 2). What are the coordinates of the other locations of the posts if she finds them by reflecting over the x- and y-axes?* (−2, −2), (2, 2), and (2, −2)

Need Another Example?

Mr. Hinds is planning a new square vegetable garden in his backyard. One corner of the garden is located at (−3, 2). What is the location of the corner that reflects (−3, 2) over the *y*-axis? (3, 2)

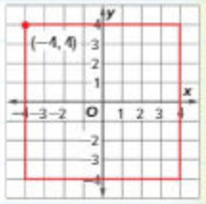
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Numbered Heads Together** Have students work in teams of 3–4 to solve each problem, making sure that everyone understands the signs, the order of the numbers in the pairs, and that counting the first square occurs at the end of the square. Students should be certain that everyone understands the position of the quadrants and that some points are on an axis and not in a quadrant. Exs. 1, 6, 7

BL LA **Trade-a-Problem** Have students use an existing game, like chess, or design a game that uses the concept of a coordinate plane. They should assign an origin to the grid and write directions based on using the coordinate plane. They can exchange with other students and play their games. Exs. 1, 2, 4



f. (4, −4)

Real World

Example

4. **Amira is building a square fence. He places fence posts at the locations indicated on the grid. What is the location of the post that reflects (−4, 4) across the *y*-axis?**

To reflect across the *y*-axis, keep the same *y*-coordinate, 4.

The opposite of the *x*-coordinate, −4, is 4.

So, (−4, 4) reflected across the *y*-axis is (4, 4).

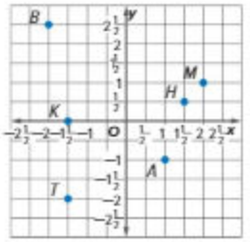
Got it? **Do this problem to find out.**

f. Amira also placed a fence post at (−4, −4). What is the location of the post that reflects (−4, −4) across the *y*-axis?

Guided Practice

Check

Identify the ordered pair that names each point or the name of each point. Then identify the quadrant in which it is located. (Examples 1 and 2)



1. T $\left(-1\frac{1}{2}, -2\right)$; III

2. $\left(-1\frac{1}{2}, 0\right)$ *K*; none

3. $\left(-2, 2\frac{1}{2}\right)$ *B*; II

4. Refer to the diagram of a school. (Examples 3 and 4)

a. What is located at the reflection of (−3, −4) across the *y*-axis. What are the coordinates of this location? the gym; (3, −4)

b. What is located at the reflection of the science labs across the *x*-axis? What are the coordinates of this location? the art studio; (−3, 2)

5. **Building on the Essential Question** How are number lines and the coordinate plane related?

The coordinate plane is the intersection of a vertical and horizontal number line.

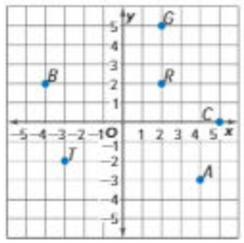
Rate Yourself!

Are you ready to move on? Shade the section that applies.

Name _____ My Homework _____

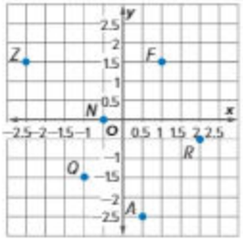
Independent Practice

Identify the ordered pair that names each point. Then identify the quadrant in which it is located. (Example 1)



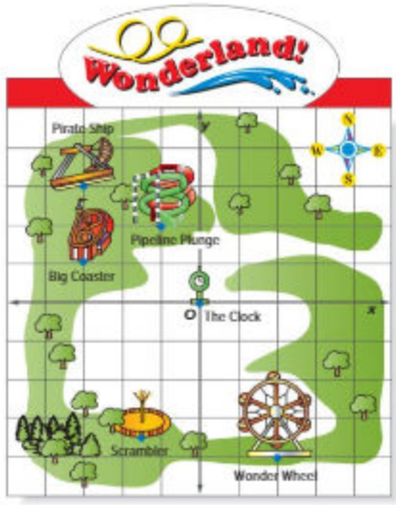
1. *R* — (2, 2); I
2. *G* — (2, 5); I
3. *B* — (−4, 2); II
4. *T* — (−3, −2); III
5. *C* — (5, 0); none
6. *A* — (4, −3); IV

Identify the name of each point. Then identify the quadrant in which it is located. (Example 2)



7. (−2.5, 1.5) — *Z*; II
8. (1, 1.5) — *F*; I
9. (0.5, −2.5) — *A*; IV
10. (2, −0.5) — *R*; IV
11. (−0.5, 0) — *N*; none
12. (−1, −1.5) — *Q*; III

13. **Use Math Tools** Refer to the map of Wonderland Park. (Examples 3 and 4)



a. What is located closest to the origin?
The Clock

b. Liza is standing at (2, 4). What is located at the reflection of (2, 4) across the *x*-axis? What are the coordinates of this location?
the Wonder Wheel; (2, −4)

c. What is located at the reflection of (3, 1) across the *y*-axis? What are the coordinates of this location? the Big Coaster; (−3, 1)

d. The Pipeline Plunge is reflected across the *x*-axis. What are the coordinates of its new location? (−1, −2)

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

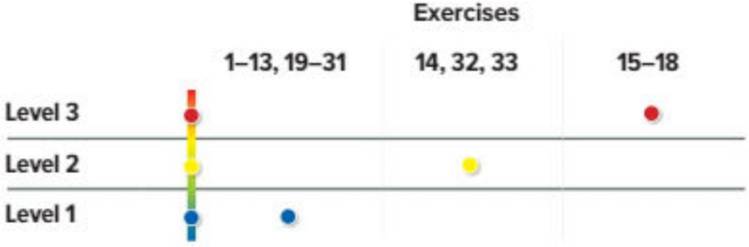
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–13, 17, 18, 32, 33
OL	On Level	1–13 odd, 14, 17, 18, 32, 33
BL	Beyond Level	14–18, 32, 33

MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	15, 16
3 Construct viable arguments and critique the reasoning of others.	17
4 Model with mathematics.	18, 31
5 Use appropriate tools strategically.	13
7 Look for and make use of structure.	14

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students name the ordered pair that is a reflection of (5, −4) over the *y*-axis. (−5, −4)

14. **Identify Structure** Fill in the graphic organizer below. Consider the point (−3, 2).

Action	Result
opposite of −3	3
point (−3, 2) reflected across the *y*-axis	(3, 2)
point (−3, 2) reflected across the *x*-axis	(−3, −2)

H.O.T. Problems Higher Order Thinking

Persevere with Problems **Without graphing, identify the quadrant(s) for which each of the following statements is true for any point (*x*, *y*). Justify your response.**

15. The *x*- and *y*-coordinates have the same sign.
Quadrants I and III; Sample answer: In Quadrant I, both coordinates are positive and in Quadrant III, both coordinates are negative.

16. The *x*- and *y*-coordinates have opposite signs. Quadrants II and IV; Sample answer: In both Quadrants II and IV, the coordinates have different signs.

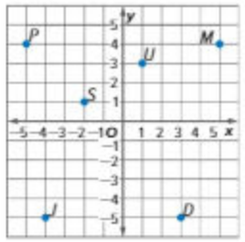
17. **Reason Inductively** Does the order of the numbers in an ordered pair matter when naming a point? Can that point be represented by more than one ordered pair? Sample answer: The first coordinate corresponds to a number on the *x*-axis. The second coordinate corresponds to a number on the *y*-axis. A point is defined by only one ordered pair.

18. **Model with Mathematics** A parallelogram is graphed on a coordinate plane so that two points are in the first quadrant and two points are in the third quadrant. What are possible coordinates of the vertices of the parallelogram?
Sample answer: (2, 4), (4, 4), (−2, −6), (−4, −6)

Name _____ My Homework _____

Extra Practice

Identify the ordered pair that names each point. Then identify the quadrant in which it is located.



19. *U*

Homework Help

(1, 3); I

Both numbers are positive so it is in the first quadrant.

20. *D*

(3, −5); IV

21. *S*

(−2, 1); II

22. *P*

(−5, 4); II

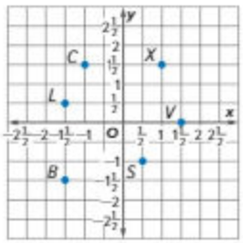
23. *J*

(−4, −5); III

24. *M*

(5, 4); I

Identify the name of each point. Then identify the quadrant in which it is located.



25. $\left(-1\frac{1}{2}, \frac{1}{2}\right)$

L; II

26. $\left(1, 1\frac{1}{2}\right)$

X; I

27. $\left(\frac{1}{2}, -1\right)$

S; IV

28. $\left(1\frac{1}{2}, 0\right)$

V; none

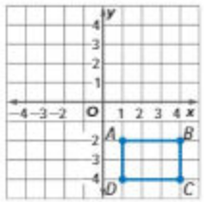
29. $\left(-1\frac{1}{2}, -1\frac{1}{2}\right)$

B; III

30. $\left(-1, 1\frac{1}{2}\right)$

C; II

31. MP **Model with Mathematics** Fadi is making a model of a park. He has the basketball court drawn on his model.



a. The swing set is located at the reflection of point *B* across the *x*-axis. What ordered pair describes the location of the swing set?

(4, 2)

b. The slide is located at the reflection of point *C* across the *x*-axis. What ordered pair describes the location of the slide?

(4, 4)

c. A water fountain is located at the reflection of point *D* across the *y*-axis. What ordered pair describes the location of the water fountain?

(−1, −4)

Power Up! Test Practice

Exercises 32 and 33 prepare students for more rigorous thinking needed for the assessment.

32.

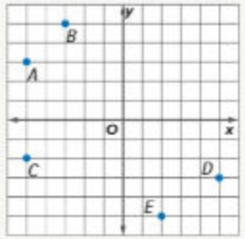
This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP5
Scoring Rubric	
2 points	Students correctly plot all five points.
1 point	Students correctly plot three or four of the five points.

33.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP5
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

32. Identify the ordered pair that names each point. Then identify the quadrant in which it is located.



a. *A* (−5, 3); II

b. *B* (−3, 5); II

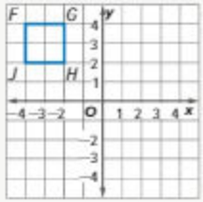
c. *C* (−5, −2); III

d. *D* (5, −3); IV

e. *E* (2, −5); IV

33. Write the ordered pair that repesents the reflection of point *J* across the *y*-axis.

(4, 2)



Spiral Review

Represent the set of numbers as decimals on the number line.

34. $\left\{5\frac{3}{10}, 5\frac{1}{10}, 5\right\}$

35. $\left\{3\frac{1}{10}, 2\frac{7}{10}, 2\frac{9}{10}\right\}$

36. Draw a line of symmetry on the figure shown.

Sample answer is given.

37. The table shows how many magazines three co-workers sold in one month. How many magazines did they sell in total?

19 magazines

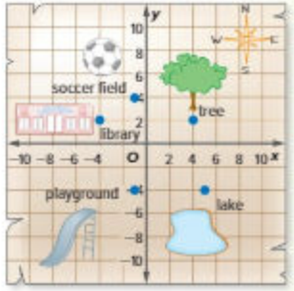
Name	Number of Magazines
Jamila	12
Dawood	0
Kamal	7

Lesson 7

Graph on the Coordinate Plane

Real-World Link

Scavenger Hunt Maryam hid the clues to a scavenger hunt for her hiking club. Use the map to show where she hid the clues. Identify the location of each clue.



1. The first clue is hidden near a tree. What ordered pair describes its location?
   (4, 2)
2. Maryam hid the next clue at a location that is reflected across the *y*-axis from the tree. Where is it hidden?
   library
3. She then walks 3 blocks east and 2 blocks north to place the next clue. Where is it hidden?
   soccer field
4. The next clue is at a location that is reflected across the *x*-axis from Clue 3. Where is it hidden?
   playground
5. Maryam then hid the next clue under a rock by the lake. How many blocks east did she walk to the lake?
   6 blocks
6. The final clue tells the hikers to walk 5 blocks north and three blocks east to find the prize. What ordered pair describes the location of the prize? **(8, 1)**

Essential Question

HOW are integers and absolute value used in real-world situations?

Mathematical Practices
1, 2, 3, 4, 7

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

- ① Persevere with Problems
- ② Reason Abstractly
- ③ Construct an Argument
- ④ Model with Mathematics
- ⑤ Use Math Tools
- ⑥ Attend to Precision
- ⑦ Make Use of Structure
- ⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Graph ordered pairs on the coordinate plane.

Coherence connecting within and across grades

Previous
Students identified and named points in all four quadrants.

Now
Students graph points in all four quadrants.

Next
Students will graph and find the perimeter and area of polygons on the coordinate plane.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 407.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL **Think-Pair-Share** Give student pairs one minute to think about and discuss a response to the following questions. MP 1, 7

- *Why is it important that when naming a point, the pairs are ordered?*
- *Can you think of a rule to remember the order of an ordered pair?*

Alternate Strategy

AL Use the classroom as a grid. Have a student stand at each location on the map. Have one student represent Maria. Other students give her directions to travel from the clue to clue. MP 1, 4

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Graph an ordered pair.**

AL • *What is the x-coordinate of point M? What is the y-coordinate of point M?* −3; 5

OL • *Since the x-coordinate is negative, do you move left or right along the x-axis from the origin? How many units?* left; 3 units

• *Since the y-coordinate is positive, do you move up or down along the y-axis from the origin? How many units?* up; 5 units

BL • *What are the coordinates of a point located in Quadrant I that is the same distance from the origin as point M?* Sample answer: (3, 5)

Need Another Example?

Graph point *A* at (−4, 3). See Answer Appendix.

2. **Graph an ordered pair.**

AL • *The x-coordinate is* $-2\frac{1}{2}$. *Where is* $-2\frac{1}{2}$ *located on the x-axis?* halfway between −2 and −3

• *The y-coordinate is* $-3\frac{1}{2}$. *Where is* $-3\frac{1}{2}$ *located on the y-axis?* halfway between −3 and −4

OL • *Since the x-coordinate is negative, do you move left or right along the x-axis from the origin? How many units?* left; $2\frac{1}{2}$ units

• *Since the y-coordinate is negative, do you move up or down along the y-axis from the origin? How many units?* down; $3\frac{1}{2}$ units

BL • *What are the coordinates of a point located in Quadrant I that is the same distance from the origin as point N?* Sample answer: $\left(2\frac{1}{2}, 3\frac{1}{2}\right)$

Need Another Example?

Graph point *B* at $\left(1\frac{1}{2}, -2\right)$. See Answer Appendix.

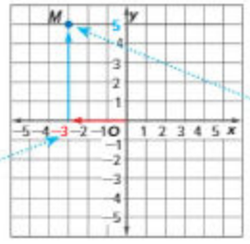
Work Zone

Graph Ordered Pairs

To graph an ordered pair, draw a dot at the point that corresponds to the coordinates.

Examples

1. **Graph point *M* at (−3, 5).**



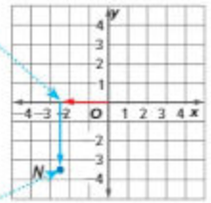
Start at the origin. The x-coordinate is −3. So, move 3 units to the left.

Next, since the y-coordinate is 5, move 5 units up. Draw a dot.

2. **Graph point *N* at** $\left(-2\frac{1}{2}, -3\frac{1}{2}\right)$.

The x-coordinate $-2\frac{1}{2}$ is between −2 and −3.

Start at the origin and move $2\frac{1}{2}$ units left.



The y-coordinate $-3\frac{1}{2}$ is between −3 and −4.

Next, move $3\frac{1}{2}$ units down. Draw a dot.

Got it? **Do these problems to find out.**

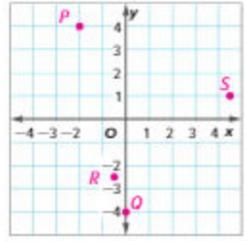
Graph and label each point on the coordinate plane below.

a. *P*(−2, 4)

b. *Q*(0, −4)

c. $R\left(-\frac{1}{2}, -2\frac{1}{2}\right)$

d. *S*(4.5, 1)



The Number System

Graph Reflections on the Coordinate Plane

You can graph points that are reflected across the *x*- and *y*-axes. Remember that points reflected across the *x*-axis will have the same *x*-coordinates and their *y*-coordinates will be opposites. Points reflected across the *y*-axis will have the same *y*-coordinates and their *x*-coordinates will be opposites.

Symbols
Use the notation *A′* to label the reflection of a point *A*.

Examples

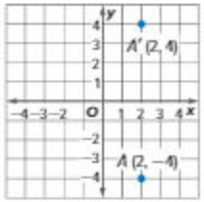
3. **Graph *A*(2, −4). Then graph its reflection across the *x*-axis.**

Graph point *A*.

To reflect across the *x*-axis, keep the same *x*-coordinate, 2, and take the opposite of the *y*-coordinate.

The opposite of −4 is 4.

So, point *A* reflected across the *x*-axis is located at point *A′*(2, 4). Graph point *A′*.



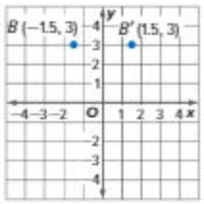
4. **Graph *B*(−1.5, 3). Then graph its reflection across the *y*-axis.**

Graph point *B*.

To reflect across the *y*-axis, keep the same *y*-coordinate and take the opposite of the *x*-coordinate.

The opposite of −1.5 is 1.5.

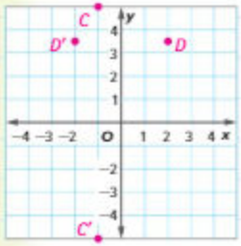
So, point *B* reflected across the *y*-axis is point *B′*(1.5, 3).



Got it? **Do these problems to find out.**

e. Graph *C*(−1, 5). Then graph its reflection across the *x*-axis.

f. Graph $D\left(2, 3\frac{1}{2}\right)$. Then graph its reflection across the *y*-axis.



Examples

3. **Graph and reflect a point.**

AL • *How would you graph point A on a coordinate plane?* Move 2 units right from the origin then move 4 units down

• *When reflecting a point across the x-axis, which coordinate stays the same?* the *x*-coordinate

OL • *What do you need to do to the y-coordinate of the original point to find the y-coordinate of the reflected point?* take the opposite of the *y*-coordinate

• *What is the opposite of the y-coordinate?* 4

• *What are the coordinates of the reflected point?* (2, 4)

BL • *Why do you think they use the notation A′ to label the reflected point of A?* Sample answer: The prime notation shows that the point is related to point *A*, but different.

Need Another Example?
Graph point *L*(−3, 4). Then graph the point that is a reflection of point *L* over the *x*-axis. See Answer Appendix.

4. **Graph and reflect a point.**

AL • *How would you graph point B on a coordinate plane?* Move 1.5 units left from the origin then move 3 units up

• *When reflecting a point across the y-axis, which coordinate stays the same?* the *y*-coordinate

OL • *What do you need to do to the x-coordinate of the original point to find the x-coordinate of the reflected point?* take the opposite of the *x*-coordinate

• *What are the coordinates of the reflected point?* (1.5, 3)

BL • *Suppose you reflected point B over the x-axis and the y-axis, then connected the points. What figure would you have? Be as specific as possible.* a right scalene triangle

Need Another Example?
Graph point *M*(2, −2.5). Then graph the point that is a reflection of point *M* over the *y*-axis. See Answer Appendix.

Example

5. Reflect points over the *y*-axis.

AL • *How would you graph (2, 4)?* Move 2 units right from the origin and then move 4 units up.

• *How would you graph (2, −2)?* Move 2 units right from the origin and then move 2 units down.

OL • *What are the coordinates of the ordered pair that is a reflection of (2, −2) over the y-axis?* (−2, −2)

• *What are the coordinates of the ordered pair that is a reflection of (−2, −2) over the x-axis?* (−2, 2)

BL • *Suppose the (2, −2) is labeled as point X. What do you think the name of the point (−2, 2) is labeled? Explain your reasoning.* Sample answer: *X″*; When you reflect a point, you add a prime symbol. Since this point was reflected twice, two prime symbols would be added.

Need Another Example?
Martin is designing a formation for the marching band. One corner of the figure is graphed at (4, 2) and another corner is graphed at (−4, 0). He reflects (4, 2) over the *x*-axis. Then he reflects the new point over the *y*-axis to graph the last corner. What figure will the marching band make? trapezoid; See Answer Appendix for graph.

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

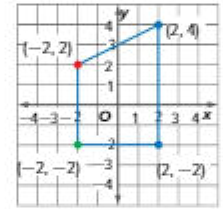
If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Discussion** Have students work in pairs to complete Exercise 1. Have them trade their graph with another pair of students and discuss any differences. MP 1, 4

BL LA **Trade-a-Problem** Choose a location in the school from which to measure, and have students indicate the position of a feature in the building, or on the grounds, using a number of feet east or west and north or south. Have them trade descriptions and find the locations. MP 1, 4

Real-World Example

5. **Mr. Ayman is using a coordinate plane to design a logo. He graphs points at (2, 4) and (2, −2). He reflects (2, −2) across the *y*-axis. Then he reflects the new point across the *x*-axis. What figure is Mr. Ayman using for his logo?**



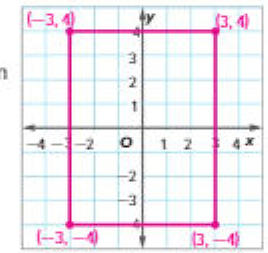
Graph (2, 4) and (2, −2). (2, −2) reflected across the *y*-axis is (−2, −2).

Graph (−2, −2). (−2, −2) reflected across the *x*-axis is (−2, 2). Graph (−2, 2).

So, the figure is a trapezoid.

Got it? Do this problem to find out.

g. Shahira is drawing a map of the school. Her room is at (−3, 4) and the gym is at (3, 4). The library is a reflection of (3, 4) across the *x*-axis. This point is reflected across the *y*-axis to graph the office. What figure is graphed on the map?

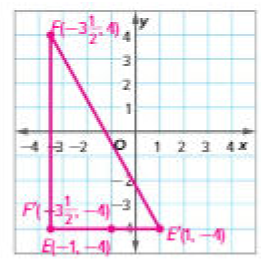


Show your work.

g. rectangle

Guided Practice

1. Use a coordinate plane to represent Yasmin's stone garden. Graph points $E(-1, -4)$ and $F\left(-3\frac{1}{2}, 4\right)$. Then reflect point *E* across the *y*-axis and point *F* across the *x*-axis. What is the shape of her stone garden? (Examples 1–5)



Show your work.

triangle

2. **Building on the Essential Question** How can the coordinate plane be used to represent geometric figures?

Sample answer: You can graph and connect the points to represent geometric figures on the coordinate plane.

Rate Yourself!

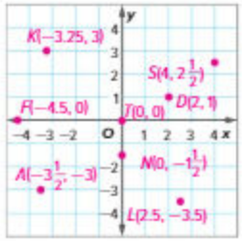
How confident are you about graphing on the coordinate plane? Check the box that applies.

Name _____ My Homework _____

Independent Practice

Graph and label each point on the coordinate plane to the right.
(Examples 1 and 2)

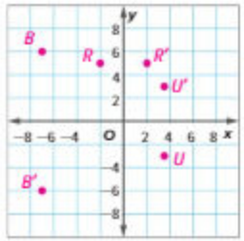
1. $T(0, 0)$
2. $D(2, 1)$
3. $K(-3.25, 3)$
4. $N\left(0, -1\frac{1}{2}\right)$
5. $F(-4.5, 0)$
6. $A\left(-3\frac{1}{2}, -3\right)$
7. $L(2.5, -3.5)$
8. $S\left(4, 2\frac{1}{2}\right)$



9. Graph $U(3.5, -3)$ on the coordinate plane to the right. Then graph its reflection across the x-axis. (Example 3)

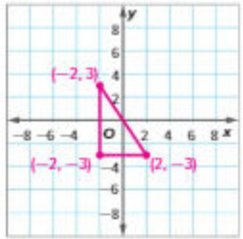
10. Graph $B(-7, 6)$ on the coordinate plane on the right. Then graph its reflection across the x-axis. (Example 3)

11. Graph $R(-2, 5)$ on the coordinate plane to the right. Then graph its reflection across the y-axis. (Example 4)



12. Amna is drawing a map of the park. She graphs the entrance at $(2, -3)$. She reflects $(2, -3)$ across the y-axis. Then Amna reflects the new point across the x-axis. What figure is graphed on the map? (Example 5)

triangle



13. A point is reflected across the y-axis. The new point is located at $(-4.25, -1.75)$. Write the ordered pair that represents the original point. **(4.25, −1.75)**

14. **Model with Mathematics** A point is reflected across the x-axis. The new point is $(-7.5, 6)$. What is the distance between the two points?

12 units

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

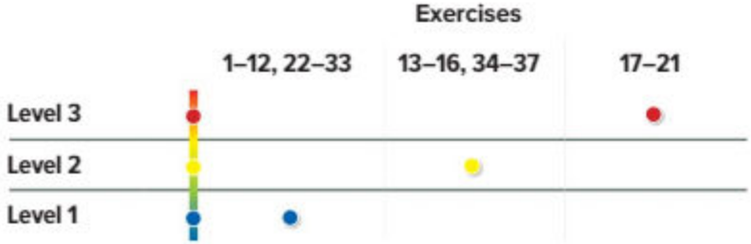
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–13, 15, 17, 36, 37
OL	On Level	1–11 odd, 13–17, 36, 37
BL	Beyond Level	13–21, 36, 37

MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	18–21
4 Model with mathematics.	14, 35
7 Look for and make use of structure.	17

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

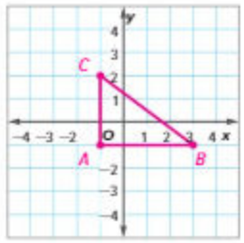
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students describe how they would graph (−3, −2) on a coordinate plane. Sample answer: From the origin, go left 3 units and then down 2 units to plot the point.

15. On a coordinate plane, draw triangle ABC with vertices $A(-1, -1)$, $B(3, -1)$, and $C(-1, 2)$. Find the area of the triangle in square units.

6 square units



16. The points (4, 3) and (−4, 0) are graphed on a coordinate plane. The point (4, 3) is reflected across the x- and y-axes. If all four points are connected, what figure is graphed?

trapezoid

H.O.T. Problems Higher Order Thinking

17. **Identify Structure** Three vertices of a quadrilateral are (−1 −1), (1, 2), and (5, −1). What are the coordinates of two vertices that will form two different parallelograms? Sample answer: (7, 2), (−5, 2)

Persevere with Problems **Determine whether each statement is *sometimes*, *always*, or *never* true. Give an example or a counterexample.**

18. When a point is reflected across the y-axis, the new point has a negative x-coordinate. sometimes; Sample answer: The x-coordinate of the new point will be negative if the x-coordinate of the original point is positive.

19. The point (x, y) is reflected across the x-axis. Then the new point is reflected across the y-axis. The location of the point after both reflections is $(-x, -y)$. always; The y-coordinate will be the opposite of the original following the reflection across the x-axis. The x-coordinate will be the opposite of the original following the reflection across the y-axis.

20. The x-coordinate of a point that lies on the x-axis is negative.
sometimes; Sample answer: If the point is located to the left of the origin, the x-coordinate is negative (−2, 0), if the point is located to the right of the origin, the x-coordinate is positive (2, 0).

21. The x-coordinate of a point that lies on the y-axis is positive.
never; The x-coordinate of any point on the y-axis is always zero.

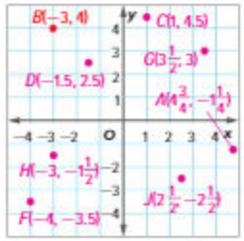
Name _____ My Homework _____

Extra Practice

Graph and label each point on the coordinate plane to the right.

22. $B(-3, 4)$ The x-coordinate is -3. The y-coordinate is 4.

23. $D(-1.5, 2.5)$



24. $A\left(4\frac{3}{4}, -1\frac{1}{4}\right)$

25. $J\left(2\frac{1}{2}, -2\frac{1}{2}\right)$

26. $C(1, 4.5)$

27. $F(-4, -3.5)$

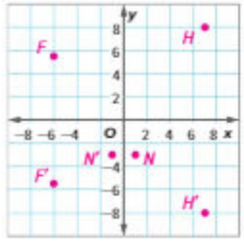
28. $G\left(3\frac{1}{2}, 3\right)$

29. $H\left(-3, -1\frac{1}{2}\right)$

30. Graph $N(1, -3)$ on the coordinate plane to the right. Then graph its reflection across the y-axis.

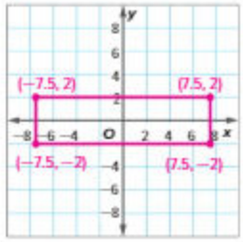
31. Graph $H(7, 8)$ on the coordinate plane on the right. Then graph its reflection across the x-axis.

32. Graph $F(-6, 5.5)$ on the coordinate plane to the right. Then graph its reflection across the x-axis.



33. Mahmoud is drawing a plan for his vegetable garden. He graphs one corner at $(-7.5, 2)$ and one corner at $(7.5, 2)$. He reflects $(-7.5, 2)$ across the x-axis. Then Mahmoud reflects the new point across the y-axis. What shape is the vegetable garden?

rectangle



34. A point is reflected across the x-axis. The new point is located at $(4.75, -2.25)$. Write the ordered pair that represents the original point.

(4.75, 2.25)

35. MP **Model with Mathematics** A point is reflected across the x-axis. The new point is $(5, -3.5)$. What is the distance between the two points?

7 units

Power Up! Test Practice

Exercises 36 and 37 prepare students for more rigorous thinking needed for the assessment.

36.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK3
Mathematical Practices	MP1, MP4, MP6
Scoring Rubric	
2 points	Students correctly draw the triangle and find the area.
1 point	Students correctly draw the triangle, but fail to find the area OR students have errors in their construction but a correct area based on those errors.

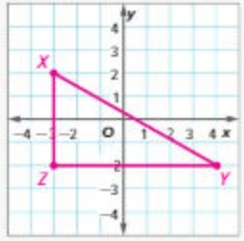
37.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP5
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

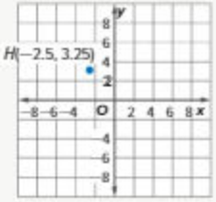
36. Draw triangle XYZ with vertices $X(-3, 2)$, $Y(4, -2)$, and $Z(-3, -2)$ on the coordinate plane. Then find the area of the triangle in square units.

14 square units



37. What are the coordinates of point H after it is reflected across the x-axis, and then reflected across the y-axis?

(2.5, −3.25)



Spiral Review

Multiply.

38. $1 \times 1 \times 1 =$ 1

39. $3 \times 3 \times 3 =$ 27

40. $6 \times 6 \times 6 =$ 216

41. Use the geometric pattern below to find the number of unit squares in the next figure.

25

42. Aya saved a total of AED 210. Each week she saved the same amount of money. She has been saving for 7 weeks. How much money did Aya save each week?

AED 30

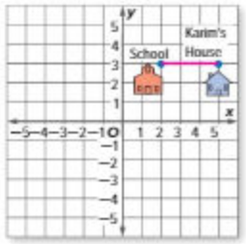
Inquiry Lab

Find Distance on the Coordinate Plane

Mathematical Practices 1, 3, 4

Inquiry **WHAT is the relationship between coordinates and distance?**

Karim's house and school are each shown on the map. What is the distance between the two points?



What do you know? I can use the map to find the location of Karim's house and the school.

What do you need to find? the distance between Karim's house and the school

Hands-On Activity 1

Find the distance between Karim's house and the school.

Step 1 Find the coordinates of Karim's house and the school.

Karim's House: (5, 3) School: (2, 3)

Step 2 Write the *x*-coordinates for each location in the table.

Location	*x*-coordinate
house	5
school	2

Step 3 Find the difference between the *x*-coordinates.

3 units

Step 4 Draw a line connecting the points in the coordinate plane above. How many units are between the points? 3 units

How does this compare to your answer from Step 3?

They are the same.

So, there are 3 units between Karim's house and the school.

Focus narrowing the scope

Objective Find the distance between two points on the coordinate plane.

Coherence connecting within and across grades

Now
Students find the distance between two points with the same *x*- or *y*- coordinates.

Next
Students use the Distance Formula to find the distance between any two points on the coordinate plane.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 412.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance than Activity 2.

Hands-On Activity 1

AL LA As students complete the Activity, remind students that the *x*-coordinate is the first number in the ordered pair and the *y*-coordinate is the second number.

Ask:

- *How many units right along the x-axis do you need to move to be directly below Taylor's house? below the school?* 5 units; 2 units
- *How many units up do you need to move to be at Taylor's house? at the school?* 3 units; 3 units
- *Are the points on the same horizontal or vertical line?* horizontal

Hands-On Activity 2

AL LA Remind students that distance is always positive. Since they are finding the distance between the two points they will need to find the absolute value of the distance each point is from the *x*-axis.

Ask:

- *How many units left along the x-axis do you need to move to be directly below point A?* 2 units
- *How many units up do you need to move to be at point A?* 3 units
- *How many units left along the x-axis do you need to move to be directly above point B?* 2 units
- *How many units down do you need to move to be at point B?* 4 units
- *Are the points on the same horizontal or vertical line?* vertical

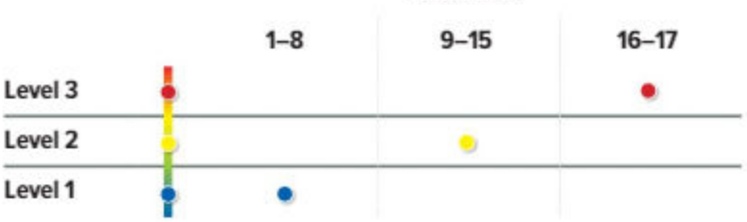
ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

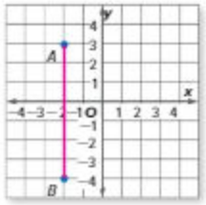


Hands-On Activity 2

Find the distance between point *A* and point *B* on the coordinate plane.

Step 1 Find the coordinates of each point.

Point *A*: (−2, 3) Point *B*: (−2, −4)



Step 2 Count the number of units between each *y*-coordinate and the *x*-axis.

Point	*y*-coordinate	Distance from *x*-axis
A	3	3
B	−4	4

Step 3 To find the distance between the two points, add the distance from the *x*-axis to each point.

3 + 4 = 7

Step 4 Draw a line connecting the points. How many units are between the points? 7 units

How does this compare to your answer from Step 3?

They are the same.

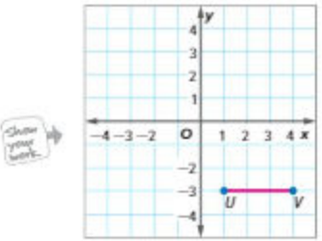
So, the distance between point *A* and point *B* is 7 units.

Investigate

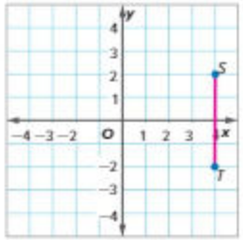
Collaborate

MP **Model with Mathematics** Work with a partner. Draw a line between each pair of points. Then find the distance between the points.

1. 3 units



2. 4 units



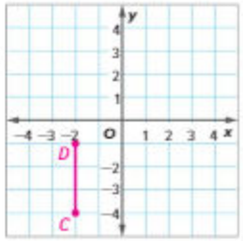
The Number System

Investigate

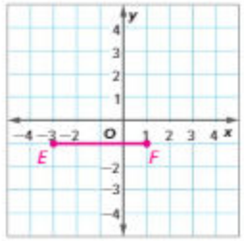
MP Model with Mathematics Work with a partner. Plot each pair of points on the coordinate plane. Then find the distance between the points.

3. $C(-2, -4)$, $D(-2, -1)$ **3 units**

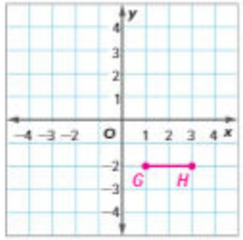
Show your work.



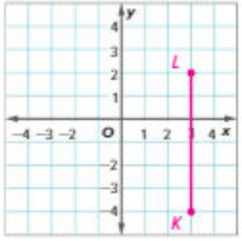
4. $E(-3, -1)$, $F(1, -1)$ **4 units**



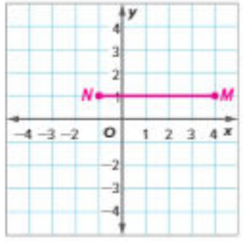
5. $G(1, -2)$, $H(3, -2)$ **2 units**



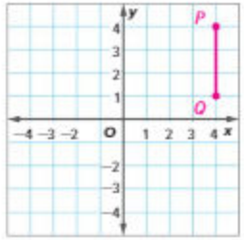
6. $K(3, -4)$, $L(3, 2)$ **6 units**



7. $M(4, 1)$, $N(-1, 1)$ **5 units**



8. $P(4, 4)$, $Q(4, 1)$ **3 units**



Investigate

AL LA Pairs Discussion Have students work in pairs to complete Exercises 1–8. Remind students to count units at the end of the grid square. If students are having trouble accurately counting units, suggest that they use the axis scales to check that their lengths are correct. For example, in Exercise 1, the segment extends from 1 to 4, so UV is 3 units long. In Exercise 2, the segment extends 2 units to the x-axis, and then 2 units beyond, so ST is 4 units long. **1, 4, 6, 7**

AL LA Trade-a-Problem Have students write a problem similar to Exercises 3–8. Have them trade their problem with another student and solve. Ask the pairs to discuss when they might find distance between points outside of math class. **MP 1, 4**

Analyze and Reflect

AL BL LA Pair an Approaching Level student with a Beyond Level student to work together to complete Exercises 9–15.

OL Have students work in pairs to answer Exercises 13–15 and answer the following question.

Ask:

- *Is it easier to use your rule to find the distance between the points or to graph the points and count the units?*

Create

AL LA Instead of assigning Exercise 17, have students write two ordered pairs that either have the same *x*-coordinates or the same *y*-coordinates. Have them explain how they would find the distance between the two points. MP 1, 4, 7

BL LA **Trade-a-Problem** Using the problem written in Exercise 16, have students trade and solve each other's problem. Have them discuss any similarities and differences. MP 1, 3

Inquiry Students should be able to answer "WHAT is the relationship between coordinates and distance?" Check for student understanding and provide guidance, if needed.

Analyze and Reflect

Sample answers: 13–16

Work with a partner to complete the table below. Use your answers from Exercises 3–6. The first one is done for you.

	Exercise	Coordinates Used	Horizontal or Vertical Line?	Same or Different Quadrant?	Line Length
	2	2 and −2	vertical	different	4 units
9.	3	−4 and −1	vertical	same	3 units
10.	4	−3 and 1	horizontal	different	4 units
11.	5	1 and 3	horizontal	same	2 units
12.	6	−4 and 2	vertical	different	6 units

13. Compare your answers from Exercises 10 and 11. What is the relationship between the coordinates used and the length of each line?
In Exercise 10, you could add the absolute values of −3 and 1 to find the length of the line. In Exercise 11, you could subtract the absolute values of 3 and 1 to find the length of the line.

14. Name the coordinates of two points that have the same *x*-coordinates and are 8 units apart. **(2, 5) and (2, −3)**

15. MP **Reason Inductively** Use absolute value to write a rule for determining the distance between two points on a coordinate plane that have the same *x*-coordinate. **The distance can be found by finding the absolute value of the difference between their y-coordinates if both are positive or if both are negative. If one y-coordinate is positive and the other is negative, add the absolute values.**

Create

16. MP **Model with Mathematics** Write and solve a real-world problem that involves determining distance on a coordinate plane. **A map shows Aisha's house at (5, −7) and the library at (5, 3). What is the map distance between Aisha's house and the library? 10 units**

17. Inquiry WHAT is the relationship between coordinates and distance?
To find the distance between two points on a horizontal line, use their *x*-coordinates. To find the distance between two points on a vertical line, use their *y*-coordinates.

21ST CENTURY CAREER in Art

Scientific Illustrator

If you are artistic and have a strong interest in science, you should think about a career as a scientific illustrator. Scientific illustrators combine their artistic abilities with their scientific backgrounds to draw scientifically accurate images. Karen Carr, a wildlife and natural history artist, has artwork in scientific publications, museums, and zoos. To draw animals that are extinct, she examines fossils, talks to scientists, and uses measurements and proportions from scientific literature.

Is This the Career for You?

Are you interested in a career as a scientific illustrator? Take some of the following courses in high school.

- Algebra
- Biology
- Geometry
- Life/Figure Drawing
- Physics

Find out how math relates to a career in Art.

Focus narrowing the scope

Objective Apply mathematics to problems arising in the workplace.

This lesson emphasizes **Mathematical Practice 4** Model with Mathematics.

Coherence connecting within and across grades

Previous
Students compared and ordered rational numbers.

Now
Students apply the content standard to solve problems in the workplace.

Rigor pursuing concepts, fluency, and applications

See the Career Project on page 416.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ask students to read the information on the student page about scientific illustrators, and answer the following questions.

Ask:

- *What do you think scientific illustrators draw?* Sample answers: bugs, plants, body parts
- *Where do scientific illustrators get the information they need to draw accurate images?* Sample answer: They might examine fossils, or use the measurements they find in literature.
- *What classes help students prepare for a career in scientific illustration?* Sample answers: Algebra, Biology, Geometry, and Life/Figure Drawing

2 Collaborate

AL LA Trade-a-Problem In groups of 2 or 4, have students write a real-world problem using the dinosaur measurements in the table. Have students trade their problems, solve each other's problem, and discuss the solutions. MP 1, 2, 4

BL LA Roundrobin Assign students to a 2- or 3-person learning team. Each team member will solve an exercise until all of Exercises 1–6 are complete. Have each team discuss their results to find errors and check their solutions. MP 1, 6

Career Portfolio

When students complete this page, have them add it to their Career Portfolio.

Career Facts

Today, scientific illustrators must be proficient not only in using pens, pencils, and brushes, but also in computer-generated illustrations, computer animation, and three-dimensional rendering.

MP You be the Scientific Illustrator!

Use the information in the table to solve each problem. Write in simplest form.

1. Write the length and height of an Argentinosaurus as decimals. Use bar notation if necessary.
   114.83̄ ft, 24.1 ft
2. How much taller was a Velociraptor than a Microraptor? Write your answer as a decimal. 2.32 ft
3. Which is greater, the height of the Argentinosaurus or the length of the Camptosaurus? height of Argentinosaurus
4. How much longer was a Camptosaurus than a Velociraptor? Plot your answer on the number line.
   $10\frac{1}{2}$

5. Compare the heights of all four dinosaurs. Order them from least to greatest.
   $\frac{24}{25}$, $3\frac{7}{25}$, $11\frac{4}{5}$, $24\frac{1}{10}$
6. An artist is creating a mural in which a Microraptor is $1\frac{1}{2}$ times the actual size. What is the length of the dinosaur in the mural? $3\frac{15}{16}$ ft

Dinosaur Measurements

Dinosaur	Length (ft)	Height (ft)
Argentinosaurus	$114\frac{5}{6}$	$24\frac{1}{10}$
Camptosaurus	$16\frac{2}{5}$	$11\frac{4}{5}$
Microraptor	$2\frac{5}{8}$	$\frac{24}{25}$
Velociraptor	$5\frac{9}{10}$	$3\frac{7}{25}$

MP Career Project

It's time to update your career portfolio! Investigate the education and training requirements for a career as a scientific illustrator.

What are some short-term goals you need to achieve to become a scientific illustrator?

-
-
-
-

Chapter Review

Vocabulary Check

Complete the puzzle by unscrambling the letters below to reveal words from the vocabulary list at the beginning of the chapter.

Q	U	A	D	R	A	N	T	S	:		T	H	E		F	O	U	R
			R	E	G	I	O	N	S		I	N		A				
	C	O	O	R	D	I	N	A	T	E		P	L	A	N	E		
	S	E	P	A	R	A	T	E	D		B	Y		T	H	E		
X	-	A	X	I	S		A	N	D		Y	-	A	X	I	S	.	

			D	I	A		N	N										
		O	R	A	D	N	T	A	S		I	N		A	H	S		
	C	A	O	R	G	A	A	N	T		Y	H	E	T	F	E		
X	U	A	X	E	S	I	T	S	D		B	Y	L	A	N	E		
Q	S	E	P	R	R	I	O	E	D	E	T	P	A	X	I	O	U	R

Complete each sentence using the vocabulary list at the beginning of the chapter.

1. A rational number is a number that can be written as a fraction.
2. A number that is less than zero is a negative integer.
3. A number that is greater than zero is a positive integer.
4. The absolute value of a number is the distance between the number and zero on a number line.
5. The division of a terminating decimal ends.
6. A decimal whose digits repeat in groups of one or more is a repeating decimal.

Vocabulary Check

LA **Numbered Heads Together** Assign students to a 3- or 4-person learning team. Each member is assigned a number from 1 to 4. Each team completes the Vocabulary Check, making sure every team member understands the terms and definitions. Call on a specific number from one team to present the team's solutions to the class. MP 1, 3, 6

Alternate Strategy

AL LA To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include the following terms.

- absolute value (Lesson 2)
- negative integer (Lesson 1)
- positive integer (Lesson 1)
- rational number (Lesson 4)
- repeating decimal (Lesson 5)
- terminating decimal (Lesson 5)

Key Concept Check

FOLDABLES LA A completed Foldable for this chapter should include a review of comparing and ordering rational numbers.

If you choose not to use this Foldable, have students write a brief review of the Key Concepts found throughout the chapter and give an example of each.

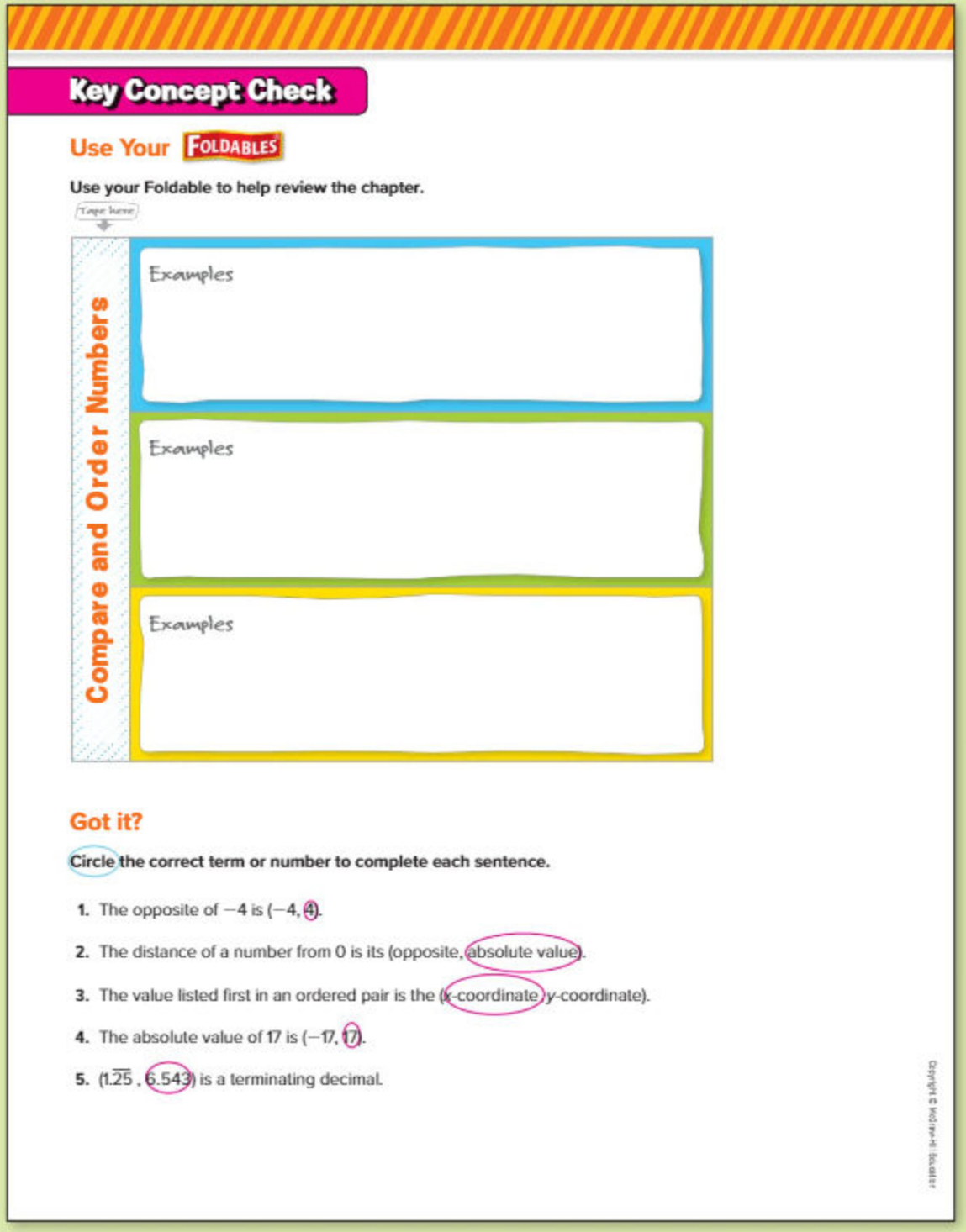
Ideas for Use

LA Have students work in pairs to discuss their Foldables. Have them practice speaking in a collaborative setting by sharing how they have completed their Foldable thus far and how they could finish it. Have each student complete their Foldable and trade with their partner to discuss any similarities and differences. MP 1, 2, 5

Got It?

If students have trouble with Exercises 1–5, they may need help with the following concept(s).

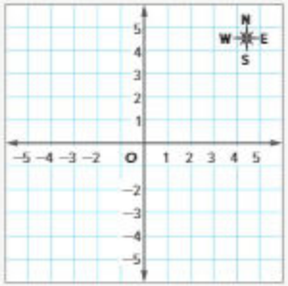
Concept	Exercise(s)
opposites (Lesson 1)	1
absolute value (Lesson 2)	2, 4
coordinate plane (Lessons 6 and 7)	3
terminating and repeating decimals (Lesson 4)	5



Power Up! Performance Task

City Grid

Maps are often represented on coordinate planes like the one shown below. Each unit on the grid represents one mile. Example: A building located at (0, 2) is 2 miles away from a building located at (0, 4).



Write your answers on another piece of paper. Show all of your work to receive full credit.

Part A
There are six buildings that need to be plotted on a city grid: City Hall (0, 1), Hamad Stadium (3, 2), Al Ansar High School (−4, 1), Al Ansar Mall (4, −3), Abdulla Bin Alzubair School (−4, 3), and Date's Mill (−3, −4). Place a point on the graph to represent each building and label it with the correct building name.

Part B
Which building is farther to the west: Abdulla Bin Alzubair School or the Date's Mill? Explain your answer in words and by writing an inequality using the proper coordinates and either the greater than (<) or less than (>) sign.

Part C
All roads in the city travel either north and south or east and west. How many miles must the city's residents travel to go from the Hamad Stadium to Date's Mill? Explain. Your explanation should include absolute value.

Power Up! Performance Task

This Performance-Based Assessment requires students to solve multi-step problems through abstract reasoning, precision, and perseverance. This practice scenario can be used to help students prepare for the thinking skills that will be used on the Assessment.

A complete scoring rubric with answers to the Exercises can be found in the back of the book.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the **Building on the Essential Question** exercises found in each lesson of the chapter.

- How can you use integers to represent data?
- How can absolute value help you to understand the size of a quantity? Give an example.
- How can symbols and absolute value help you to order sets of integers?
- How are repeating decimals used in real-world situations?
- How can a number line help in ordering rational numbers?
- How are number lines and the coordinate plane related?
- How can the coordinate plane be used to represent geometric figures?

Ideas for Use

LA **Think-Pair-Share** Have students work in pairs. Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer. Then have them share their responses with their partner before they complete the graphic organizer.

MP 1, 5

Reflect

Answering the Essential Question

Use what you learned about integers and the coordinate plane to complete the graphic organizer.

Essential Question

HOW are integers and absolute value used in real-world situations?

Vocabulary	Definition
integer	Any number of the set {... −4, −3, −2, −1, 0, 1, 2, 3, 4 ...} where ... means continues without end.
absolute value	The distance between a number and zero on a number line.

Describe a real-world situation that can be represented by the absolute value of 27.
Sample answer: Mona deposited AED 27 in her savings. The amount of change in her account is represented by 27.

Describe a real-world situation that can be represented by the absolute value of −16?
Sample answer: The temperature in Fargo, North Dakota, was −16°F. The absolute value, 16, represents the number of degrees below 0°F.

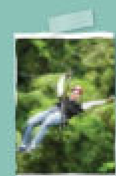
Answer the Essential Question. HOW are integers and absolute value used in real-world situations?

See students' work.

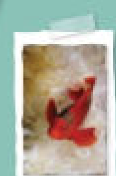
UNIT 3
Expressions and Equations

Essential Question
HOW can you communicate mathematical ideas effectively?

Chapter 6
Expressions
Numerical and algebraic expressions can be used to represent and solve real-world problems. In this chapter, you will write and evaluate expressions and apply the properties of operations to generate equivalent expressions.



Chapter 7
Equations
Variables are used to represent an unknown number in an expression or equation. In this chapter, you will write and solve one-variable addition, subtraction, multiplication, and division equations.



Chapter 8
Functions and Inequalities
Functions can be represented using words, equations, tables, and graphs. In this chapter, you will represent and analyze the relationship between two variables using functions. You will also write, graph, and solve one-variable inequalities.

Essential Question
At the end of this unit, students should be able to answer "How can you communicate mathematical ideas effectively?"

Each chapter explores a different essential question that assists students in answering the unit question. The lessons in each chapter include exercises that lead students to various aspects of the essential question.

Apply and extend previous understandings of arithmetic to algebraic expressions.

1. Write and evaluate numerical expressions involving whole-number exponents.
2. Write, read, and evaluate expressions in which letters stand for numbers.
   a. Write expressions that record operations with numbers and with letters standing for numbers.
   b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.

continued on page 424

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

3. Apply the properties of operations to generate equivalent expressions.
4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Unit Project Preview

Have students record their resuls in a table.Then they can graph the ordered pairs from the table.

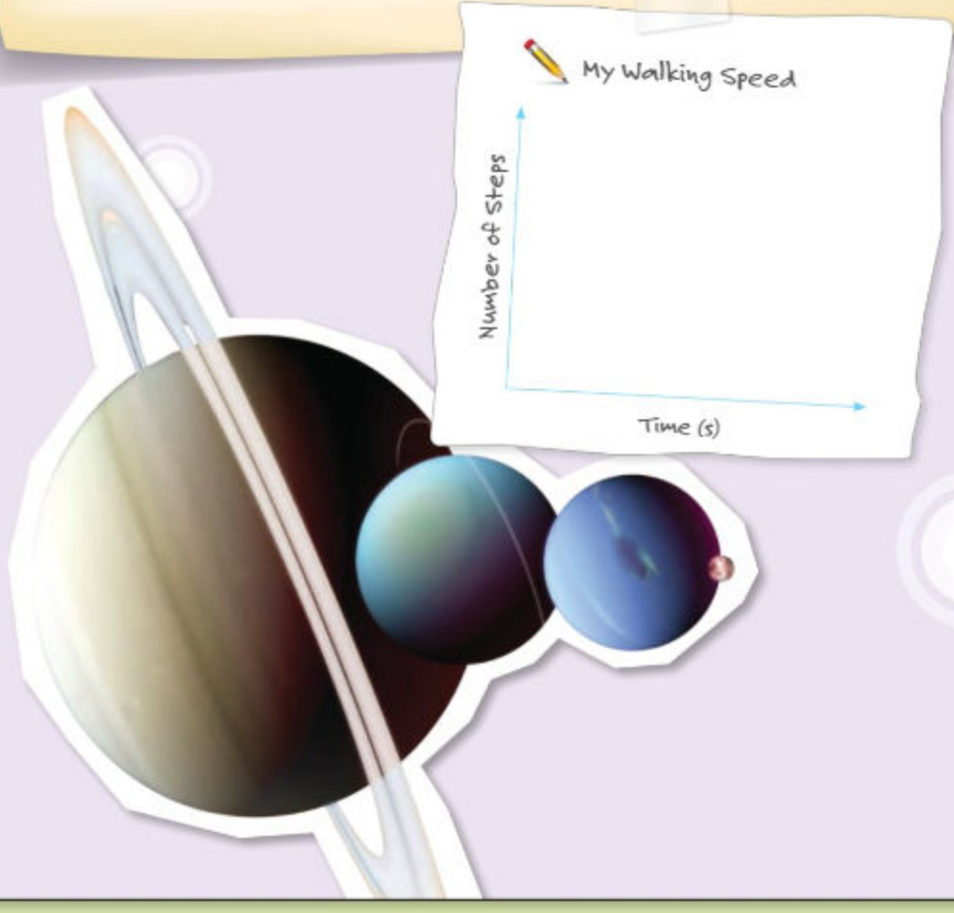
The Unit Project can be found on pages 649–650.

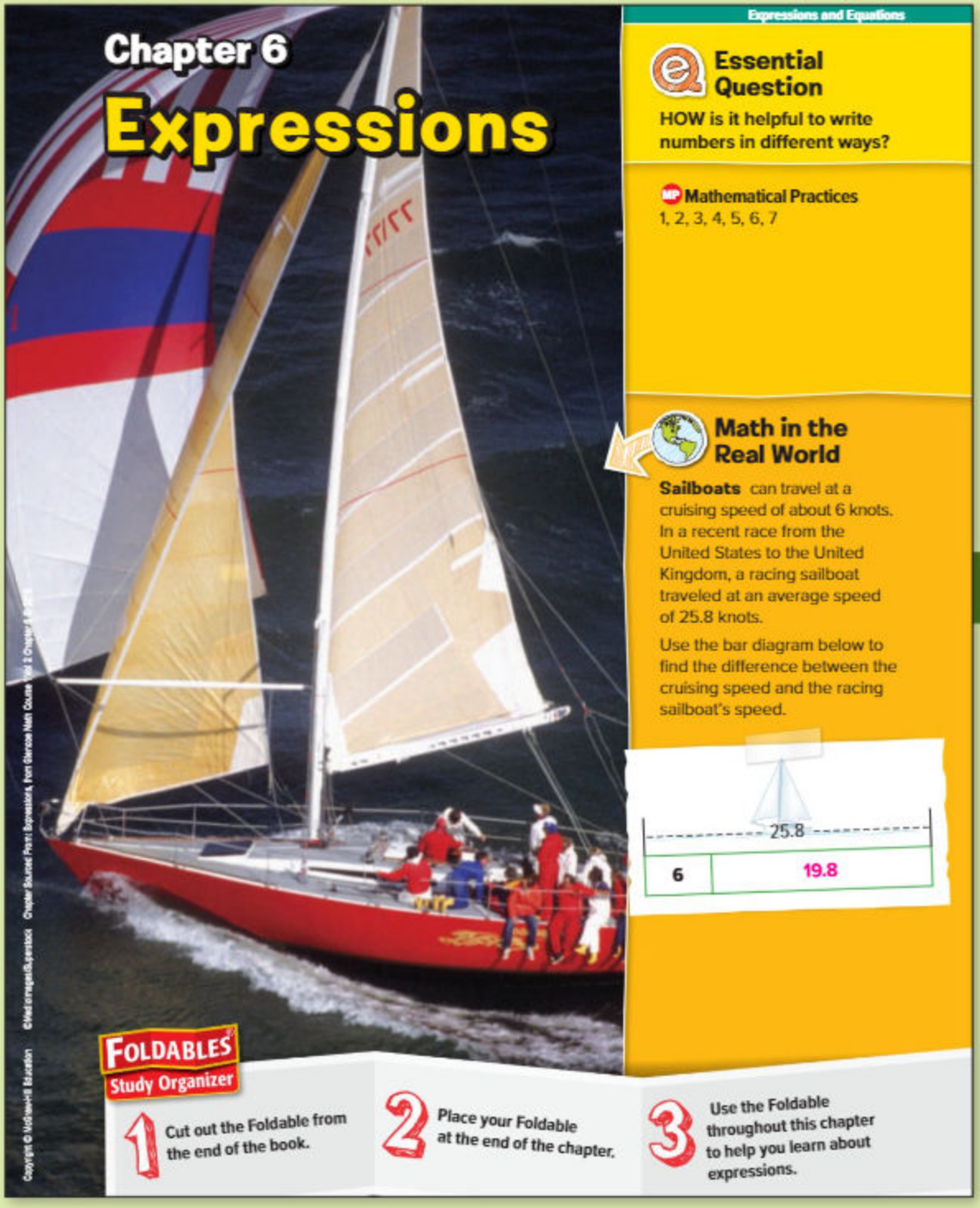
Unit Project Preview

It's Out of This World The speed at which a planet orbits the sun or a moon orbits a planet is called *orbital velocity* or *orbital speed*. Each planet and moon in our solar system has a different average orbital speed.

Work with a partner. Take turns counting the number of steps you each take while walking in a circle for 10 seconds. Then use the information to find the approximate number of steps you take in 20, 30, and 40 seconds. Write and graph the ordered pairs to represent your walking speed.

At the end of Chapter 8, you'll complete a project to compare two planets' orbits around the sun. Their speed is out of this world!





Focus narrowing the scope

This chapter focuses on **Expressions and Equations**.

Coherence connecting within and across grades

Previous
Students compared and ordered integers and graphed integers on the coordinate plane.

Now
Students write and simplify expressions using properties.

Next
Students will solve one-step equations and inequalities.

Rigor pursuing concepts, fluency, and applications

The Levels of Complexity charts located throughout this chapter indicate how the exercises progress from conceptual understanding and procedural skills and fluency, to application and critical thinking.

Launch the Chapter

Math in the Real World

Sailboats To help students with the bar diagram, remind them that the sum of the cruising speed and some number is the average speed of the racing sailboat. This number is the difference between the two speeds.

What Tools Do You Need?

Vocabulary Activity

LA As you proceed through the chapter, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define: An algebraic expression is a combination of variables, numbers, and at least one operation.

Example: $5h - 7$

Ask:

- *What are some examples of algebraic expressions?* Sample answers: $x + 13$; $10 - d$

Reading Math

LA Have students read the *Meaning of Division* section. Students explore the meaning of division as seen in different contexts in word problems. They examine meanings such as *share*, *to take away equal amounts*, and *to find how many times greater*. Have students read about the other meanings of division.

Ask:

- *How is one meaning of division, to put objects into equal groups, similar to another meaning to share equally?* Sample answer: If you are sharing equally, you are giving an equal amount to each group or person.
- *What division sentence could you write for the word problem under "to take away equal amounts"?* $26 \div 6.5 = n$
- *What is another term for "to take away equal amounts"?* repeated subtraction
- *How do you know that you should divide and not subtract to find the answer for the third word problem?* Sample answer: The question is, "About how many times as long..." The word times indicates that you should divide the greater length, 4,160 miles, by the lesser length, 1,900 miles.

What Tools Do You Need?

Vocabulary

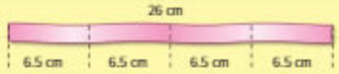
algebra	defining the variable	like terms
algebraic expression	Distributive Property	numerical expression
Associative Properties	equivalent expressions	perfect square
base	evaluate	powers
coefficient	exponent	properties
Commutative Properties	factor the expression	term
constant	Identity Properties	variable

Study Skill: Reading Math

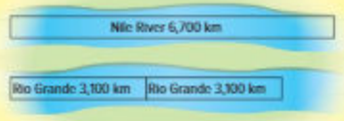
Meaning of Division Look for these other meanings when you are solving a word problem.

- **To share:**
  Ibrahim and his friend are going to share 3 apples equally. How many apples will each boy have?

- **To take away equal amounts:**
  Eiman is making bookmarks from a piece of ribbon. Each bookmark is 6.5 centimeters long. How many bookmarks can she make from a piece of ribbon that is 26 centimeters long?



- **To find how many times greater:**
  The Nile River, the longest river on Earth, is 6,700 kilometers long. The Rio Grande River is 3,100 kilometers long. About how many times as long is the Nile as the Rio Grande?



Practice

Identify the meaning of division shown in each problem. Then solve the problem.

1. Ahmed's family wants to buy a flat-screen television that costs AED 1,200. They plan to pay in six equal payments. What will be the amount of each payment?

   sharing; AED 200

2. A full-grown blue whale can weigh 150 tons. An adult African elephant weighs about 5 tons. How many times as great does a blue whale weigh as an African elephant?

   how many times greater; 30 times

What Do You Already Know?

Read each statement. Decide whether you agree (A) or disagree (D). Place a checkmark in the appropriate column and then justify your reasoning. See students' work.

Expressions			
Statement	A	D	Why?
You must follow the order of operations to find the value of a numerical expression.			
A variable is a symbol used to represent an operation.			
The phrase 4 less than x is written as $4 - x$.			
The multiplicative identity is 0.			
Subtraction is a commutative operation.			
The Distributive Property combines addition and multiplication.			

When Will You Use This?

Here is an example of how expressions are used in the real world.

Activity Use the Internet to find the cost of admission to a science museum. How much will your admission ticket cost? Are there any other events at the museum that you would pay to see?

See students' work.

What Do You Already Know?

In this activity students assess their prior knowledge by determining whether they agree or disagree with each statement about concepts in this chapter.

- You may want to add a third option of "I don't know" for those students who do not have any prior knowledge of the content of the statement.
- After completing the chapter, have students return to this page and see if any of their responses would change now that they have finished the chapter.

When Will You Use This?

Activity 1

Students learn how to apply the use of expressions to model and solve a real-world situation.

Activity 2

Use the Graphic Novel to help students learn about using expressions to solve problems.

Ask:

- *What mathematical operations can be used to solve this problem?* Sample answer: addition

Are You Ready?

Use this page to determine if students have skills that are needed for the chapter.

Quick Review

Students with strong math backgrounds may opt to go directly to the Quick Check.

REVIEW	
Example	**Skill**
1	Multiply whole numbers.
2	Add and subtract fractions.

Quick Check

If students have difficulty with the exercises, present an additional example to clarify misconceptions they may have.

Exercises 1–3

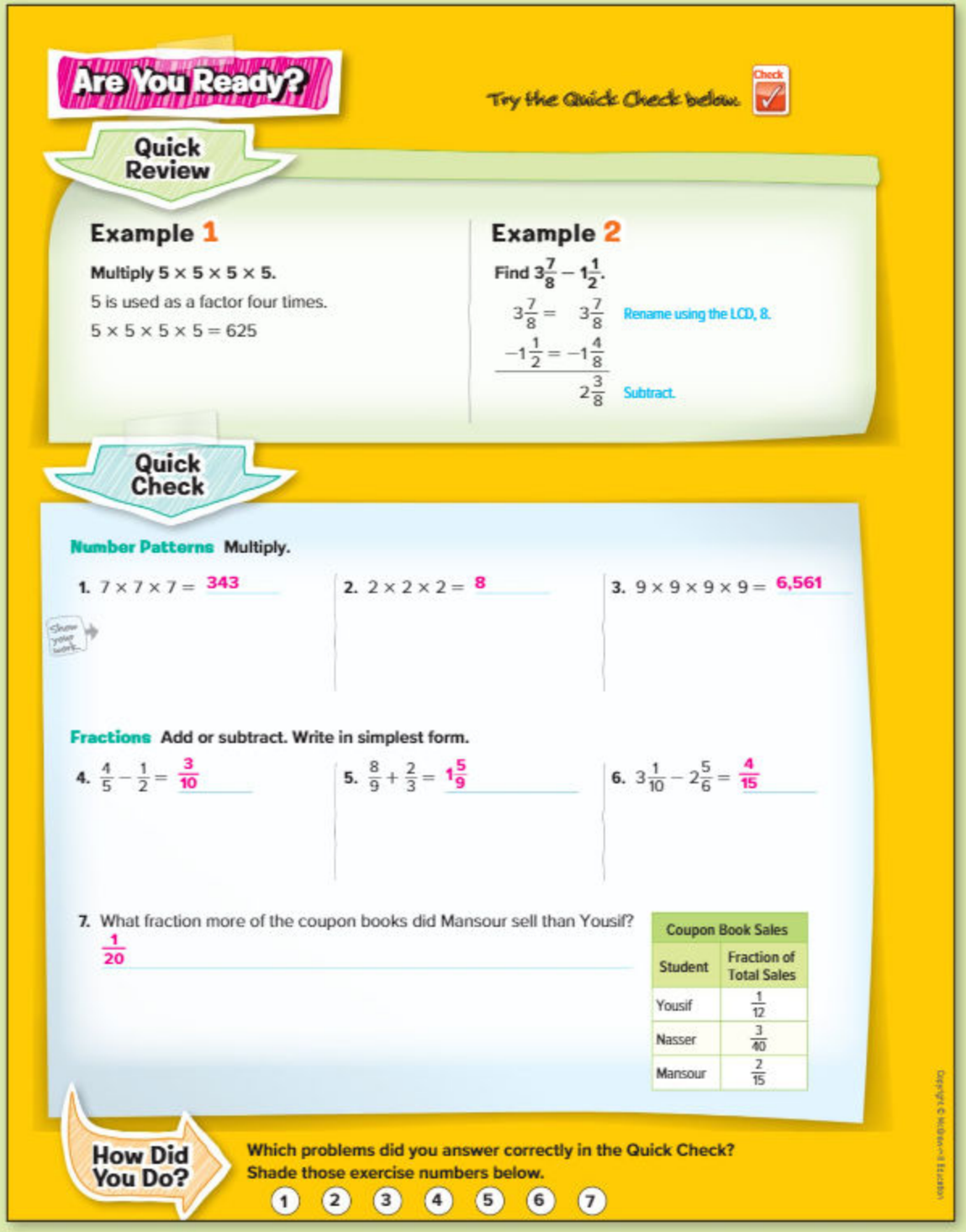
Evaluate $6 \times 6 \times 6 \times 6 \times 6$. 7,776

Exercises 4–7

Find $4\frac{3}{8} - 2\frac{1}{4}$. $2\frac{1}{8}$

Track Your Progress

Prior to beginning this chapter, have your students go to pages xix–xxii to rate their current knowledge. At the end of the chapter, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge of the key ideas has increased.



Are You Ready?

Try the Quick Check below.

Quick Review

Example 1

Multiply $5 \times 5 \times 5 \times 5$.

5 is used as a factor four times.

$5 \times 5 \times 5 \times 5 = 625$

Example 2

Find $3\frac{7}{8} - 1\frac{1}{2}$.

$3\frac{7}{8} = 3\frac{7}{8}$ Rename using the LCD, 8.

$-1\frac{1}{2} = -1\frac{4}{8}$

$2\frac{3}{8}$ Subtract.

Quick Check

Number Patterns **Multiply.**

1. $7 \times 7 \times 7 =$ 343

Show your work.

2. $2 \times 2 \times 2 =$ 8

3. $9 \times 9 \times 9 \times 9 =$ 6,561

Fractions **Add or subtract. Write in simplest form.**

4. $\frac{4}{5} - \frac{1}{2} = \frac{3}{10}$

5. $\frac{8}{9} + \frac{2}{3} = 1\frac{5}{9}$

6. $3\frac{1}{10} - 2\frac{5}{6} = \frac{4}{15}$

7. What fraction more of the coupon books did Mansour sell than Yousif?

$\frac{1}{20}$

Coupon Book Sales	
Student	**Fraction of Total Sales**
Yousif	$\frac{1}{12}$
Nasser	$\frac{3}{40}$
Mansour	$\frac{2}{15}$

How Did You Do?

Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4 5 6 7

Inquiry Lab

Structure of Expressions

Inquiry **HOW can you identify the parts of an expression using mathematical terms?**

Mathematical Practices 1, 3, 4

Fitness Fortress recycles plastic water bottles. On Tuesday, 8 bottles were placed in the bins. On Wednesday, 8 more bottles were recycled.

Hands-On Activity 1

You can use an expression to represent the number of bottles that were recycled. An *expression* consists of a combination of numbers and operations. Each *term* of an expression is separated by a plus or minus sign.

Step 1 Use a bar diagram to represent the number of bottles recycled on Tuesday. Use a second bar diagram to represent the number of bottles recycled on Wednesday.

Tuesday	8 bottles
Wednesday	8 bottles

Step 2 The addition expression $8 + 8$ represents the total. How many terms are in the expression? **2**

Does the expression represent a *sum*, *product*, or *quotient*?

sum

Step 3 The multiplication expression 2×8 also represents the total. How many terms are in the expression? **1**

Does the expression represent a *sum*, *product*, or *quotient*?

product

Investigate

Collaborate

Work with a partner. Rewrite each sum as a product. Then identify the number of terms in each expression.

1. $14 + 14 =$ **2×14**
   Sum: **2 terms**
   Product: **1 term**

2. $92 + 92 + 92 =$ **3×92**
   Sum: **3 terms**
   Product: **1 term**

Focus narrowing the scope

Objective Explore parts of an expression.

Coherence connecting within and across grades

Now
Students use models to identify the different parts of a numerical expression.

Next
Students will write and evaluate expressions involving whole number exponents.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 427.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activity 2.

Hands-On Activity 1

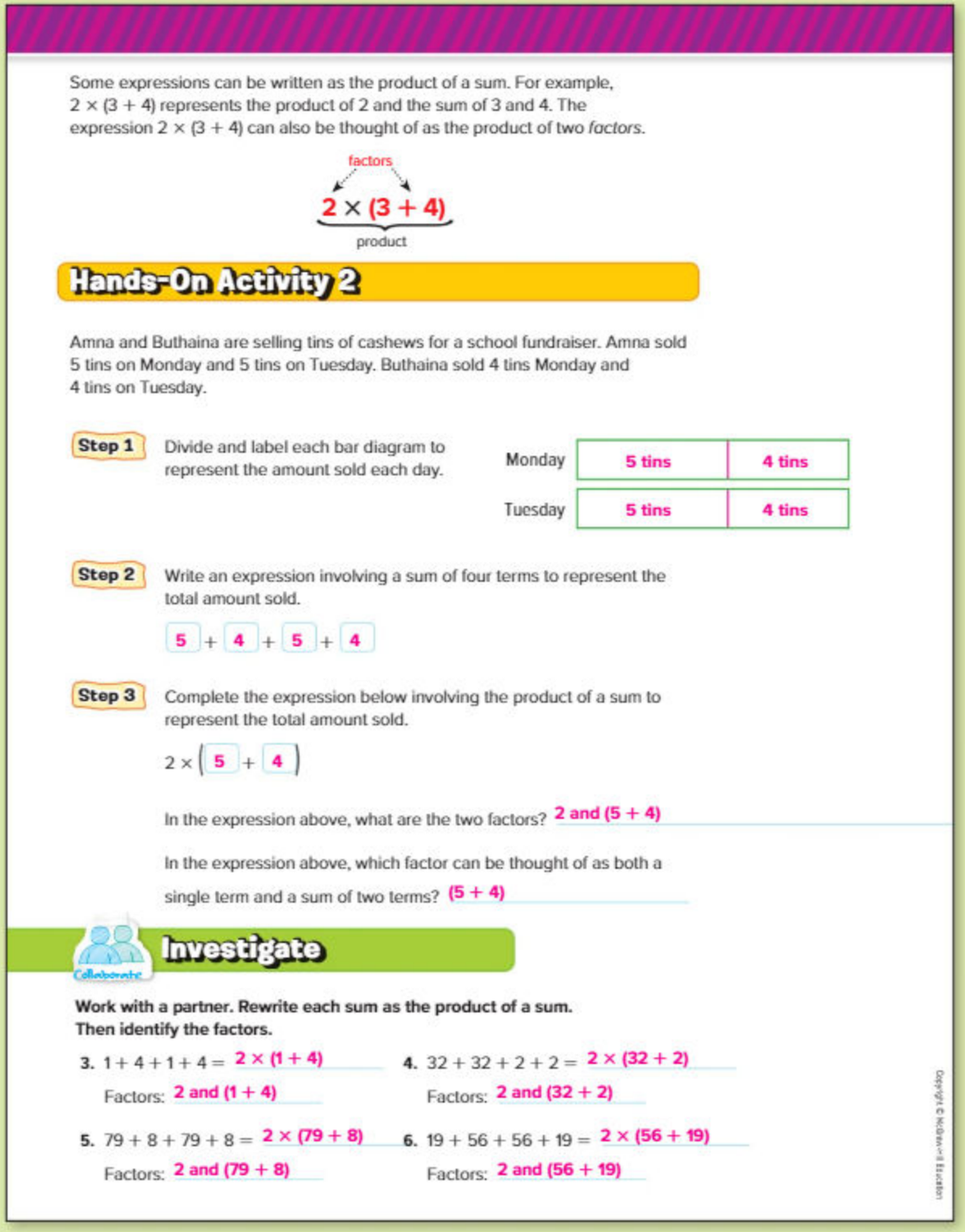
AL LA Pairs Discussion Before beginning the activity, have students discuss with their partner how multiplication is repeated addition. Have them use manipulatives, such as counters, to demonstrate several multiplication problems and their repeated addition equivalents (e.g., $8 + 8 = 2 \times 8$). MP 1, 4, 5

BL LA Trade-a-Problem Have students work in pairs to draw a bar diagram that shows how a repeated addition expression is equivalent to a multiplication expression. Then have them trade their bar diagrams with another pair of students. Each pair should correctly write the repeated addition and multiplication expressions. MP 1, 4, 5

Hands-On Activity 2

AL LA **Think-Pair-Share** Have students work in pairs. Give them about one minute to think through how they would create their bar diagrams for Activity 2. Then have them create the bar diagrams, cut them out, and tape them to their student page. Have them trade pages with their partner. Each partner should verify that the bar diagram created is accurate. Have students discuss and resolve any differences. Then have them agree upon a bar diagram to draw in their textbooks. Call on one pair of students to share their responses with the class. MP 1, 3, 4, 5

BL LA **Pairs Discussion** Have students continue working with the same partner they did in the activity above. Have them extend Exercise 6 to identify the properties of mathematics that allow them to write $19 + 56 + 19 + 56$ as $2 \times (19 + 56)$. MP 1, 3, 7



Some expressions can be written as the product of a sum. For example, $2 \times (3 + 4)$ represents the product of 2 and the sum of 3 and 4. The expression $2 \times (3 + 4)$ can also be thought of as the product of two *factors*.

factors

$2 \times (3 + 4)$

product

Hands-On Activity 2

Amna and Buthaina are selling tins of cashews for a school fundraiser. Amna sold 5 tins on Monday and 5 tins on Tuesday. Buthaina sold 4 tins Monday and 4 tins on Tuesday.

Step 1 Divide and label each bar diagram to represent the amount sold each day.

Monday	5 tins	4 tins
Tuesday	5 tins	4 tins

Step 2 Write an expression involving a sum of four terms to represent the total amount sold.

5 + 4 + 5 + 4

Step 3 Complete the expression below involving the product of a sum to represent the total amount sold.

$2 \times (5 + 4)$

In the expression above, what are the two factors? 2 and (5 + 4)

In the expression above, which factor can be thought of as both a single term and a sum of two terms? (5 + 4)

Investigate

Collaborate

Work with a partner. Rewrite each sum as the product of a sum. Then identify the factors.

3. $1 + 4 + 1 + 4 =$ 2 × (1 + 4)
   Factors: 2 and (1 + 4)

4. $32 + 32 + 2 + 2 =$ 2 × (32 + 2)
   Factors: 2 and (32 + 2)

5. $79 + 8 + 79 + 8 =$ 2 × (79 + 8)
   Factors: 2 and (79 + 8)

6. $19 + 56 + 56 + 19 =$ 2 × (56 + 19)
   Factors: 2 and (56 + 19)

Investigate

Work with a partner. Represent each expression using bar diagrams.

7. $5 + 5$

5	5

8. $9 + 9$

9	9

Work with a partner. Represent each expression using bar diagrams. Then identify the factors.

9. $2 \times (3 + 1)$

Factors: 2 and (3 + 1)

Which factor is also a sum?

(3 + 1)

3	1

3	1

10. $2 \times (5 + 2)$

Factors: 2 and (5 + 2)

Which factor is also a sum?

(5 + 2)

5	2

5	2

Work with a partner. Represent each diagram as a sum.

11. 17 + 17

17	17

12. 74 + 74

74	74

Work with a partner. Represent each diagram as the product of a sum. Then identify the factors.

13. Product: 2 × (5 + 8)

Factors: 2 and (5 + 8)

Which factor is also a sum?

(5 + 8)

5	8

5	8

14. Product: 2 × (54 + 58)

Factors: 2 and (54 + 58)

Which factor is also a sum?

(54 + 58)

54	58

54	58

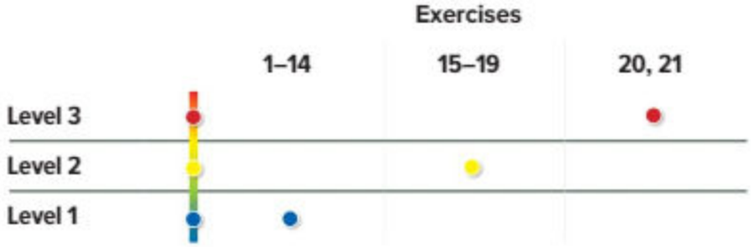
ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Team-Pair-Solo** Have students work together in a four-person team to complete Exercises 7 and 8, ensuring that each student understands how to draw the bar diagrams. Then have teams divide into pairs to complete Exercises 9 and 10. Finally, have students complete Exercises 11–14 individually. Then have them rejoin their original team to discuss responses and share solutions. MP 1, 3, 4, 5

BL LA **Pairs Present** For Exercises 7–10, have students work in pairs to decide on the accurate bar diagrams to model the expressions. Have them prepare a brief oral presentation to explain to the class why they chose that model and how it shows whether or not the expressions are equivalent. MP 1, 3, 4, 5

Analyze and Reflect

AL LA **Pairs Discussion** Work together as a class to complete Exercises 15–18. Then have students work with a partner to complete Exercise 19. MP 1, 3

Ask:

- *What does the word "sum" mean?* answer for an addition problem
- *What does the word "product" mean?* answer for a multiplication problem
- *What choices do we have that contain a product?* *a* and *d*

BL LA **Pairs Discussion** For Exercises 15–18, have students work with a partner to generate two more examples of each statement using different numbers. MP 1, 4

Create

BL LA **Trade-a-Problem** For Exercise 20, have students trade the expressions they wrote to verify that they meet the criteria and are equivalent. MP 1, 3

Inquiry Students should be able to answer "HOW can you identify the parts of an expression using mathematical terms?" Check for student understanding and provide guidance, if needed.

Analyze and Reflect

Work with a partner to match each description to the correct expression. The first one is already done for you.

	Description	Expression
15.	This expression is a sum of two terms.	a. $(1 + 2) \times 2$
16.	This expression can be thought of as a product of two factors. One of the factors is the sum of 6 and 4.	b. $6 + 6$
17.	This expression can be thought of as a product of two factors. One of the factors is the sum of 1 and 2.	c. $14 \div 7$
18.	This expression is the quotient of 14 and 7.	d. $(6 + 4) \times 2$

19. MP **Reason Inductively** Ayesha wrote the expression $2 \times (31 + 47)$. She states that the expression is a product and that the expression $(31 + 47)$ is a factor. Ahmed states that the expression $(31 + 47)$ is a sum of two terms. Who is correct? Explain. **both; Sample answer: In the expression $2 \times (31 + 47)$, $(31 + 47)$ can be thought of as a factor or as the sum of two terms, 31 and 47.**

Create

Sample answers: 20–21

20. MP **Model with Mathematics** Write an expression and a real-world problem for the situation modeled to the right.

4 kilograms	6 kilograms
4 kilograms	6 kilograms

$2 \times (4 + 6)$; Last week, Hiyam bought 4 kilograms of apples and 4 kilograms of oranges. This week, Hiyam bought 6 kilograms of apples and 6 kilograms of oranges.

21. Inquiry HOW can you identify the parts of an expression using mathematical terms?

Each term of an expression is separated by a minus sign or a plus sign. Symbols, such as +, ÷, and × help you to identify the expression as a sum, quotient, or product.

Lesson 1

Powers and Exponents

Vocabulary Start-Up

A product of like factors can be written in exponential form using an exponent and a base. The **base** is the number used as a factor. The **exponent** tells how many times a base is used as a factor.

1. Fill in the boxes with the words *factors*, *exponent*, and *base*.

$10 \times 10 = 10^2$

factors — base — exponent

2. Give an example of an exponent.

Sample answer: The 2 in 4^2 is an exponent.

3. Write the definition of exponent in your own words.

Sample answer: In a power, the number that tells how many times the base is used as a factor.

Essential Question

HOW is it helpful to write numbers in different ways?

Vocabulary

base
exponent
powers
perfect square

Mathematical Practices
1, 3, 4, 6, 8

Real-World Link

MP3 players come in different storage sizes, such as 2GB, 4GB, or 16GB, where GB means gigabyte. One gigabyte is equal to $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ bytes.

What is this number written with exponents? 10^9

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Represent numbers using exponents.

Coherence connecting within and across grades

Previous Students used models to identify the parts of numerical expressions.

Now Students write and simplify expressions using whole number exponents.

Next Students will use order of operations to simplify numerical expressions.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 433.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Trade-a-Problem** Have each student complete Exercises 1–3 individually. Then have them trade their responses with a partner to discuss solutions and resolve any disagreements. MP 1, 3

Alternate Strategies

AL LA Have students make a list of the different meanings of the term *base* they have used or seen in the real world and compare and contrast it to the mathematical meaning. MP 1, 3, 6

BL Have students discuss why 10^3 does not have the same value as 3^{10}. MP 1, 3

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Write a product as a power.**

AL
- *Which number is repeatedly multiplied?* 6
- *How many times is that number multiplied?* 4

OL
- *What is the base in this expression?* 6
- *How many times is it used as a factor?* 4 times
- *What will be the exponent?* 4

BL
- *What is the value of the expression?* 1,296

Need Another Example?
Write $8 \times 8 \times 8 \times 8 \times 8$ using an exponent. 8^5

2. **Write a product as a power.**

AL
- *What is a base?* a number used as a factor
- *What is an exponent?* a number that tells how many times its base is used as a factor

OL
- *What is the base in this expression?* 4
- *How many times is it used as a factor?* 3 times
- *What will be the exponent?* 3

BL
- *What is the value of the expression?* 64
- *If two more 4s are multiplied by the current expression, write the new equivalent expression using an exponent.* 4^5

Need Another Example?
Write $7 \times 7 \times 7$ using an exponent. 7^3

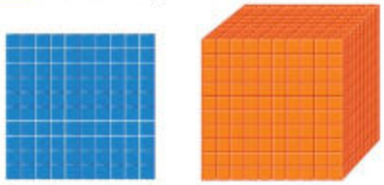
Work Zone

a. 7^4

b. 9^7

Write Products as Powers

Numbers expressed using exponents are called **powers**. For example, 100 is a power of 10 because it can be written as 10^2. Numbers like 100 are **perfect squares** because they are the squares of whole numbers.



$10 \times 10 = 100$
$10^2 = 100$

$10 \times 10 \times 10 = 1{,}000$
$10^3 = 1{,}000$

Perfect cubes are numbers with three identical whole number factors such as $4 \times 4 \times 4 = 64$. So, the number 64 is a perfect cube.

Examples

1. **Write $6 \times 6 \times 6 \times 6$ using an exponent.**

$6 \times 6 \times 6 \times 6 = 6^4$ 6 is used as a factor four times.

2. **Write $4 \times 4 \times 4$ using an exponent.**

The factor 4 is the base.

The factor is multiplied 3 times.

The exponent is 3.

So, $4 \times 4 \times 4$ can be written as 4^3.

Got It? **Do these problems to find out.**

Write each product using an exponent.

a. $7 \times 7 \times 7 \times 7$

b. $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$

Expressions and Equations

Write Powers as Products

To write powers as products, determine the base and the exponent. The base of 10^2 is 10 and the exponent is 2. To read powers, consider the exponent. The power 10^2 is read as *ten squared* and 10^3 is read as *ten cubed*.

Examples

3. **Write 5^2 as a product of the same factor. Then find the value.**

The base is 5. The exponent is 2. So, 5 is used as a factor two times.

$5^2 = 5 \times 5$ Write 5^2 as a product.

$= 25$ Multiply 5 by itself.

4. **Write 1.5^3 as a product of the same factor. Then find the value.**

The base is 1.5. The exponent is 3. So, 1.5 is used as a factor three times.

$1.5^3 = 1.5 \times 1.5 \times 1.5$ Write 1.5^3 as a product.

$= 3.375$ Multiply.

5. **Write $\left(\frac{1}{2}\right)^3$ as a product of the same factor. Then find the value.**

The base is $\frac{1}{2}$. The exponent is 3. So $\frac{1}{2}$ is used as a factor three times.

$\left(\frac{1}{2}\right)^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ Write $\left(\frac{1}{2}\right)^3$ as a product.

$= \frac{1}{8}$ Multiply.

Got It? **Do these problems to find out.**

Write each power as a product of the same factor. Then find the value.

c. 10^5 d. 2.1^2 e. $\left(\frac{1}{4}\right)^2$

Notation

In Example 5, the fraction $\frac{1}{2}$ is set in parentheses to note that the entire fraction is the base

$\left(\frac{1}{2}\right)^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Without the parentheses, it is understood that the base is only the numerator of the fraction.

$\frac{1^3}{2} = \frac{1 \times 1 \times 1}{2} = \frac{1}{2}$

Show your work.

c. $10 \times 10 \times 10 \times 10 \times 10$; 100,000

d. 2.1×2.1; 4.41

e. $\frac{1}{4} \times \frac{1}{4}$; $\frac{1}{16}$

Examples

3. **Write a power as a product.**

AL • *What is the base?* 5 *What is the exponent?* 2

OL • *How many times will 5 be used as a factor?* 2 times

• *What is the value of this expression?* $5^2 = 25$

BL • *Is 25 a perfect square? Explain.* Yes; 25 can be expressed as 5×5, which makes it a perfect square.

Need Another Example?

Write 3^5 as a product of the same factor. Then find the value.

$3 \times 3 \times 3 \times 3 \times 3$; 243

4. **Write a power as a product.**

AL • *What is the base?* 1.5 *What is the exponent?* 3

OL • *How many times will 1.5 be used as a factor?* 3 times

• *What is the value of this expression?* $1.5^3 = 3.375$

BL • *How many decimal places will be in the product? Explain.* 3 decimal places; The total number of decimal places of the factors determines how many will be in the product: $1 + 1 + 1 = 3$.

Need Another Example?

Write 2.4^3 as a product of the same factor. Then find the value. $2.4 \times 2.4 \times 2.4$; 13.824

5. **Write a power as a product.**

AL • *What is the base?* $\frac{1}{2}$ *What is the exponent?* 3

OL • *How many times will $\frac{1}{2}$ be used as a factor?* 3 times

• *What is the value of this expression?* $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

BL • *Will the product be greater than or less than $\frac{1}{2}$? Explain.* less than; Multiplying a number by a fraction that is between 0 and 1 will yield a product that is less than the number.

Need Another Example?

Write $\left(\frac{1}{4}\right)^2$ as a product of the same factor. Then find the value.

$\frac{1}{4} \times \frac{1}{4}$; $\frac{1}{16}$

Example

6. Find the value of a power.

AL • *What is the problem asking you to find?* how many liters of water the aquarium holds

• *What information does the problem give?* The aquarium holds around 7^4 liters of water.

• *How can you solve the problem?* Find the value of 7^4.

OL • *How can you write 7^4 as a product of the same factor?* $7 \times 7 \times 7 \times 7$

• *What is this product* 2,401

BL • *How else could you express the product?* Sample answer: $49 \times 49 = 2{,}401$

• *If it costs the zoo AED0.02 per liter of water, how much will it cost to fill the aquarium?* AED48.02

Need Another Example?

Giant squids live more than 8^3 meters below sea level. How far below sea level does a giant squid live? 512 m

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Discussion** Have students work in pairs to complete Exercises 1–7. Ask them to discuss why 2^5 is not the same as 2×5 in Exercise 4. Have a pair of students volunteer to share their responses with the class. MP 1, 3

BL LA **Trade-a-Problem** Have each student write two expressions involving exponents for their partner to determine each value. The first expression should use a whole number as the base, and the second should use a fraction between 0 and 1 as the base. After they have traded problems and found the values, have the group discuss when the value was greater than the base, and when it was less than the base. MP 1, 3, 8

Real World

Example

6. STEM The zoo has an aquarium that holds around 7^4 liters of water. About how many liters of water does the aquarium hold?

$7^4 = 7 \times 7 \times 7 \times 7$ Write 7^4 as a product.

$= 2{,}401$ Multiply.

So, the aquarium holds about 2,401 liters of water.

Got It? Do this problem to find out.

f. STEM Michigan has more than 10^4 inland lakes. Find the value of 10^4.

f. 10,000 lakes

Guided Practice

Check

Write each product using an exponent. (Examples 1 and 2)

1. $8 \times 8 \times 8 =$ 8^3

2. $1 \times 1 \times 1 \times 1 \times 1 =$ 1^5

Show your work.

Write each power as a product of the same factor. Then find the value. (Examples 3–5)

3. $\left(\frac{1}{7}\right)^3 =$ $\frac{1}{7} \times \frac{1}{7} \times \frac{1}{7} = \frac{1}{343}$

4. $2^5 =$ $2 \times 2 \times 2 \times 2 \times 2$; 32

5. $1.4^2 =$ 1.4×1.4; 1.96

6. Coal mines have shafts that can be as much as 5^3 meters deep. About how many meters deep into Earth's crust are these shafts? (Example 6)

about 125 meters

7. **Building on the Essential Question** How is using exponents helpful? Sample answer: A product of like factors can be written in a simpler, shorter format using exponents. For example $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$ can be written as 9^8.

Rate Yourself!

How confident are you about powers and exponents? Shade the ring on the target.

Name _____ My Homework _____

Independent Practice

Write each product using an exponent. (Examples 1 and 2)

1. $6 \times 6 =$ 6^2

Show your work.

2. $1 \times 1 \times 1 =$ 1^3

3. $5 \times 5 \times 5 \times 5 \times 5 \times 5 =$ 5^6

4. $12 \times 12 =$ 12^2

27. $27 \times 27 \times 27 \times 27 =$ 27^4

6. $15 \times 15 \times 15 =$ 15^3

Write each power as a product of the same factor. Then find the value. (Examples 3–5)

$6^4 =$ $6 \times 6 \times 6 \times 6$; 1,296

8. $0.5^3 =$ $0.5 \times 0.5 \times 0.5$; 0.125

$\left(\frac{1}{8}\right)^2 =$ $\frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$

10. **Identify Repeated Reasoning** A byte is a basic unit of measurement for information storage involving computers. (Example 6)

a. A kilobyte is equal to 10^3 bytes. Write 10^3 as a product of the same factor. Then find the value.
$10 \times 10 \times 10$; 1,000

b. A megabyte is equal to 10^6 bytes. Write 10^6 as a product of the same factor. Then find the value.
$10 \times 10 \times 10 \times 10 \times 10 \times 10$; 1,000,000

c. How many more bytes of information are in a gigabyte than a megabyte? 999,000,000 bytes

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–11, 13, 14, 16, 17, 30, 31
OL	On Level	1–9 odd, 11–14, 16, 17, 30, 31
BL	Beyond Level	11–17, 30, 31

MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	15
3 Construct viable arguments and critique the reasoning of others.	17
4 Model with mathematics.	14
6 Attend to precision.	16, 27
8 Look for and express regularity in repeated reasoning.	10

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write the value of 11^4 on a piece of paper.
14,641

Watch Out!

Common Error Watch for students who find the value of powers by multiplying the base and the exponent. Suggest to students that they first write the power as a product of the same factor, and then find the value.

Find the value of each expression.

11. $0.5^4 + 1 =$ 1.0625

12. $3.2^3 \times 10 =$ 327.68

13. $10.3^3 + 8 =$ 1,100.727

H.O.T. Problems Higher Order Thinking

14. **Model with Mathematics** Write a power whose value is greater than 1,000. Sample answer: 50^2

15. **Persevere with Problems** Use the table to solve.

Powers of 2	Powers of 4	Powers of 10
$2^4 = 16$	$4^4 = 256$	$10^4 = 10{,}000$
$2^3 = 8$	$4^3 = 64$	$10^3 = 1{,}000$
$2^2 = 4$	$4^2 = 16$	$10^2 = 100$
$2^1 = 2$	$4^1 = 4$	$10^1 = 10$
$2^0 = 1$	$4^0 = 1$	$10^0 = 1$

a. Describe the pattern for the powers of 2.
Write the values of 2^1 and 2^0 in the table.
The next values are found by dividing the previous power by 2.

b. Describe the pattern for the powers of 4.
Write the values of 4^1 and 4^0 in the table. The next values are found by dividing the previous power by 4.

c. Describe the pattern for the powers of 10. Write the values of 10^1 and 10^0 in the table. The next values are found by dividing the previous power by 10.

d. Write a rule for finding the value of any base with an exponent of 0.
Any nonzero number with an exponent of 0 has a value of 1.

16. **Be Precise** Multiplication is defined as repeated addition. Use the word repeated to define exponential form. Justify your reasoning.
Sample answer: Exponential form is a short way to write repeated multiplication of a common factor such as $5 \times 5 \times 5 \times 5 = 5^4$.

17. **Reason Inductively** Suppose the population of the United States is about 230 million. Is this number closer to 10^7 or 10^8? Explain your reasoning.
10^8; Sample answer: $10^7 = 10{,}000{,}000$ and $10^8 = 100{,}000{,}000$. 100,000,000 is much closer to 230,000,000 than 10,000,000.

Name _____ My Homework _____

Extra Practice

Write each product using an exponent.

18. $6 \times 6 \times 6 =$ 6^3

Homework Help

The factor 6 is used 3 times.
The base is 6.
The exponent is 3.

19. $10 \times 10 \times 10 =$ 10^3

20. $32 \times 32 \times 32 \times 32 =$ 32^4

21. $9 \times 9 =$ 9^2

22. $7 \times 7 \times 7 \times 7 \times 7 \times 7 =$ 7^6

23. $13 \times 13 \times 13 \times 13 \times 13 =$ 13^5

Write each power as a product of the same factor. Then find the value.

24. $3^7 =$ $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$; 2,187

25. $0.06^2 =$ 0.06×0.06; 0.0036

26. $\left(\frac{1}{4}\right)^3 =$ $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$

27. MP **Be Precise** The baseball infield at the right has an area of 30^2 square meters. What is the area of the infield?

900 square meters

28. Last week Bakery Marvels baked 5^5 muffins. How many muffins did Bakery Marvels bake?

3,125 muffins

29. Usama ran 3.5^3 kilometers in the month of January. **How many kilometers did** Usama run in January? 42.875 kilometers

Power Up! Test Practice

Exercises 30 and 31 prepare students for more rigorous thinking needed when taking the assessment.

30.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly assign all four values and find the number of kilometers traveled.
1 point	Students correctly assign all four values but fail to find the number of kilometers traveled OR students fail to correctly assign all four values but find the number of kilometers traveled.

31.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP8
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

30. Mrs. Eiman traveled about 8 × 8 × 8 × 8 kilometers from Ohio to Hawaii. Select values to complete the model below to show the repeated multiplication as a power.

2	4
6	8

The base is 8.

The exponent is 4.

The repeated multiplication can be expressed as the power: 8^{4}.

About how many kilometers did Mrs. Eiman travel? 4,096 km

31. Asma used counters to make the pattern shown below.

She continues the pattern for several more figures. Which of the following accurately describe how many counters she uses in different figures? Select all that apply.

- ■ There are 25 counters in the 5th figure.
- ■ There are 81 counters in the 9th figure.
- ☐ There are 42 counters in the 6th figure.
- ■ There are 121 counters in the 11th figure.

Spiral Review

Multiply or divide.

32. 6 × 8 = 48

33. 64 ÷ 8 = 8

34. 42 ÷ 7 = 6

35. All video games are on sale at The Game House for AED 29 each. How much will Nasser pay for 3 video games?

AED 87

36. Usama and two of his friends carpooled on a visit to the zoo. The cost of admission was AED 12 per person. Parking cost AED 7 per car. How much did the group pay on their visit to the zoo? AED 43

Lesson 2

Numerical Expressions

Real-World Link

Snacks The table shows the cost of different snacks at a concession stand at the school hockey game.

Item	Price (AED)
Popcorn	2
Juice or Soda	1
chicken sandwiches	4

1.  = AED 6

2.  = AED 16

3. Find the total cost of buying 3 boxes of popcorn and 4 chicken sandwiches.

   AED 22

4. What operations could you use in Exercises 1–2? Explain how to find the answer to Exercise 3 using these operations.

   multiplication and addition; Sample answer: multiply the price of each item by the amount purchased. Then add the products.

Essential Question

HOW is it helpful to write numbers in different ways?

Vocabulary

numerical expression

order of operations

Mathematical Practices

1, 2, 3, 4, 5

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Find the value of expressions using order of operations.

Coherence connecting within and across grades

Previous
Students wrote and simplified numerical expressions with whole number exponents.

Now
Students simplify numerical expressions using the order of operations.

Next
Students will write and simplify algebraic expressions.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 441.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Pairs Discussion** Have students work in pairs. Have Student 1 determine the cost of the popcorn and Student 2 determine the cost of the chicken sandwiches. Have them explain why the multiplication must come before the addition when determining the total cost. MP 1, 3, 8

Alternate Strategy

AL LA Help students write the expression 3×2 for the popcorn and 4×4 for the chicken sandwiches. Have them write the sum in the form $3 \times 2 + 4 \times 4$ and ask them to identify the cost of the popcorn and the cost of chicken sandwiches. MP 1, 4

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Use the order of operations to find the value of a numerical expression.**

AL
- *Does the expression contain grouping symbols?* no
- *Does the expression contain exponents?* no
- *Does the expression contain multiplication or division?* no
- *Does the expression contain addition or subtraction?* yes

OL
- *What operation should you perform first?* $10 - 2$
- *What operation should you perform next?* $8 + 8$

BL
- *What would the value of the expression be if we performed the addition first? How does this demonstrate the need for an accepted order of operations?* $10 - 10 = 0$; Sample answer: The result was different. Everyone needs to agree upon the same order in which operations are performed so that everyone gets the same results.

Need Another Example?
Find the value of $30 - 10 + 9$. 29

2. **Use the order of operations to find the value of a numerical expression.**

AL
- *Does the expression contain multiplication or division? If so, which one?* yes; multiplication

OL
- *What should you evaluate first?* 3×5
- *Then what should you do?* Add 4 to the product.

BL
- *Would the result be the same if you added 4 and 3 first? Explain.* no; The result would be 35, which is not an equivalent expression.

Need Another Example?
Find the value of $4 + 10 \times 3$. 34

Key Concept

Order of Operations

1. Simplify the expressions inside grouping symbols, like parentheses.
2. Find the value of all powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

Work Zone

A **numerical expression** like $3 \times 2 + 4 \times 4$ is a combination of numbers and operations. The **order of operations** tells you which operation to perform first so that everyone finds the same value for an expression.

Examples

Find the value of each expression.

1. $10 - 2 + 8$

There are no grouping symbols or powers.
There are no multiplication or division symbols.
Add and subtract in order from left to right.

$10 - 2 + 8 = 8 + 8$ Subtract 2 from 10 first.
$= 16$ Add 8 and 8.

2. $4 + 3 \times 5$

There are no grouping symbols or powers.
Multiply before adding.

$4 + 3 \times 5 = 4 + 15$ Multiply 3 and 5.
$= 19$ Add 4 and 15.

Got It? **Do these problems to find out.**

a. $10 + 2 \times 15$ b. $16 \div 2 \times 4$

a. 40

b. 32

Parentheses and Exponents

Expressions inside grouping symbols, such as parentheses are simplified first. Follow the order of operations inside parentheses. For example in the expression $3 + (4^2 + 5)$, you will need to find the value of the power, 4^2, before you can add the expression inside the parentheses.

Examples

Find the value of each expression.

3. $20 \div 4 + 17 \times (9 - 6)$

$20 \div 4 + 17 \times (9 - 6) = 20 \div 4 + 17 \times 3$ Subtract 6 from 9.

$= 5 + 17 \times 3$ Divide 20 by 4.

$= 5 + 51$ Multiply 17 by 3.

$= 56$ Add 5 and 51.

4. $3 \times 6^2 + 4$

$3 \times 6^2 + 4 = 3 \times 36 + 4$ Find 6^2.

$= 108 + 4$ Multiply 3 and 36.

$= 112$ Add 108 and 4.

5. $5 + (8^2 - 2) \times 2$

$5 + (8^2 - 2) \times 2 = 5 + (64 - 2) \times 2$ Simplify the exponent.

$= 5 + 62 \times 2$ Simplify inside parentheses.

$= 5 + 124$ Multiply.

$= 129$ Add.

Got It? Do these problems to find out.

c. $25 \times (5 - 2) \div 5 - 12$

d. $24 \div (2^3 + 4)$

STOP and Reflect

Why is it important to have the order of operations?

Sample answer: The order of operations is used to ensure that there is only one correct answer.

Show your work.

c. 3

d. 2

Examples

3. **Use the order of operations to find the value of a numerical expression.**

AL • *Does the expression contain grouping symbols?* yes

• *What is inside the grouping symbols?* $9 - 6$

OL • *What should you evaluate first?* $9 - 6$

BL • *After evaluating within grouping symbols, could you evaluate the expression from left to right and obtain the same result? Explain.* no; The multiplication at the end of the expression needs to be completed before the addition.

Need Another Example?

Find the value of $90 \div 3 + (3 - 2) - 20$. 11

4. **Use the order of operations to find the value of a numerical expression.**

AL • *Does the expression contain grouping symbols? exponents?* no; yes

OL • *What should you evaluate first?* 6^2 *What do you need to evaluate next?* 3×36 *What is the last step?* $108 + 4$

BL • *Why do we multiply before adding?* Multiplication comes before addition in the order of operations.

Need Another Example?

Find the value of $4^3 + 5 \times 2 - 1$. 73

5. **Use the order of operations to find the value of a numerical expression.**

AL • *Does the expression contain grouping symbols?* yes

• *What should you evaluate first?* 8^2

OL • *What do you need to evaluate next?* Subtract 2 from 64.

• *Should you add 5 and 62 or multiply 62 and 2 next?* Multiply 62 and 2.

BL • *Why do we multiply before adding?* Multiplication comes before addition in the order of operations.

Need Another Example?

Find the value of $3 \times (2^3 + 5) - 7$. 32

Example

6. **Use order of operations to solve a real-world problem.**

AL • *What do you need to find?* the total cost of the items

OL • *How will you represent the cost of the lotions?* 5×5 or 5^2 *candles?* 2×7 *lip balms?* 4×2

• *What expression can you write to solve the problem?* $5 \times 5 + 2 \times 7 + 4 \times 2$

BL • *If you have AED40, do you have enough money to buy the items? If not, how much more money do you need?* No, I would need AED7 more.

Need Another Example?

Salem and two of his friends need to buy soccer equipment. Each person buys a ball, a pair of shin guards, and a pair of cleats. Write an expression for the total cost of the equipment they buy altogether. Then find the total cost.

$3 \times$ AED25 $+ 3 \times$ AED15 $+ 3 \times$ AED18; AED174

Cost of Soccer Equipment			
Item	Cleats	Balls	Shin Guards
Cost (AED)	25	15	18

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

AL LA **Roundrobin** Have students complete Exercises 1–3 in a small group. Have each student provide one of the steps in the order of operations. Then have the rest of the group show agreement (thumbs up) or disagreement (thumbs down). Then have the next student provide the next step. Repeat until all of the operations have been performed. 1, 3

Example

6. **Write an expression for the total cost of 5 lotions, 2 candles, and 4 lip balms. Find the total cost.**

Cost of Items			
Item	Lotion	Candle	Lip balm
Cost (AED)	5	7	2

$5 \times$ AED $5 + 2 \times$ AED $7 + 4 \times$ AED 2

$= 5^2 + 2 \times 7 + 4 \times 2$

$= 25 + 2 \times 7 + 4 \times 2$ Simplify 5^2 to find the cost of the lotions.

$= 25 + 14 + 4 \times 2$ Multiply 2 and 7 to find the cost of the candles.

$= 25 + 14 + 8$ Multiply 4 and 2 to find the cost of the lip balms.

$= 47$

The total cost of the items is AED 47.

Got It? **Do these problems to find out.**

e. Ayman and 3 friends are at the mall. Each person buys a pretzel for AED 4, sauce for AED 1, and a drink for AED 2. Write an expression for the total and find the total cost.

Show your work.

e. $4 \times$ AED $4 + 4 \times$ AED $1 + 4 \times$ AED 2; AED 28

Guided Practice

Check

Find the value of each expression. (Examples 1–5)

1. $9 + 3 - 5 =$ 7

Show your work.

2. $(26 + 5) \times 2 - 15 =$ 47

3. $5^2 + 8 \div 2 =$ 29

4. **Financial Literacy** Tickets to a play cost AED 10 for members and AED 24 for nonmembers. Write an expression to find the total cost of 4 nonmember tickets and 2 member tickets. Then find the total cost. (Example 6)

$4 \times 24 + 2 \times 10$; AED 116

5. **Building on the Essential Question** How are grouping symbols helpful in simplifying expressions correctly?

Sample answer: Grouping symbols like parentheses help identify the expression(s) that must first be simplified.

Rate Yourself!

How well do you understand order of operations? Circle the image that applies.

Name _____ My Homework _____

Independent Practice

Find the value of each expression. (Examples 1–5)

1. $8 + 4 - 3 =$ 9

Show your work.

2. $38 - 19 + 12 =$ 31

3. $7 + 9 \times (3 + 8) =$ 106

4. $15 - 2^3 \div 4 =$ 13

$55 \div 11 + 7 \times (2 + 14) =$ 117

6. $5^3 - 12 \div 3 =$ 121

7. $8 \times (2^4 - 3) + 8 =$ 112

8. $9 + 4^3 \times (20 - 8) \div 2 + 6 =$ 399

9. **Financial Literacy** Ayman and four friends go to the movies. Each person buys a movie ticket for AED 7, a snack for AED 5, and a drink for AED 2. Write an expression for the total cost of the trip to the movies. Then find the total cost. (Example 6)

5 × AED 7 + 5 × AED 5 + 5 × AED 2; AED 70

10. **Financial Literacy** Badr's family went to a concert together. They purchased 4 concert tickets for AED 25 each, 3 T-shirts for AED 15 each, and a poster for AED 10. Write an expression for the total cost. Then find the total cost. (Example 6)

4 × AED 25 + 3 × AED 15 + AED 10; AED 155

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

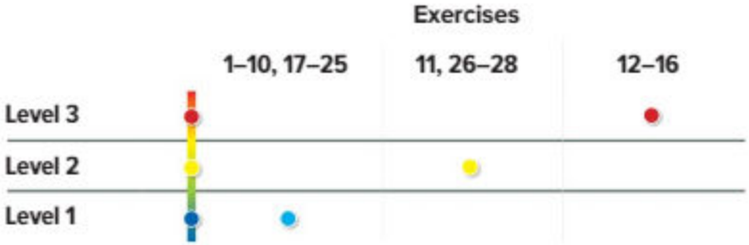
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–13, 15, 16, 27, 28
OL	On Level	1–9 odd, 11–13, 15, 16, 27, 28
BL	Beyond Level	11–16, 27, 28

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	14
3 Construct viable arguments and critique the reasoning of others.	12, 13, 16, 26
5 Use appropriate tools strategically.	11, 15

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Give students the following expression: $3^2 + (21 - 9) \times 6 \div 2$. Ask them to explain the steps they would use to find the value of the expression. Sample answer: First, find the value of 21 − 9 which is 12. Then evaluate 3^2. Multiply 12 by 6, which is 72. Divide 72 by 2, which is 36. Then add 9 and 36, which is 45.

Watch Out!

Common Error Watch for students who think that you must perform operations in an expression in the order in which they appear. You may want to suggest to students that when they first read a problem, they underline or circle the multiplication and division to remind them when to perform those operations.

Use Math Tools A wholesaler sells rolls of fruit snacks in two sizes of bags. The table shows the number of rolls that come in each bag. Write an expression that could be used to determine the number of rolls in 3 large bags and 2 small bags. Then find the number of rolls.

Bag	Number of Rolls
Large	10
Small	5

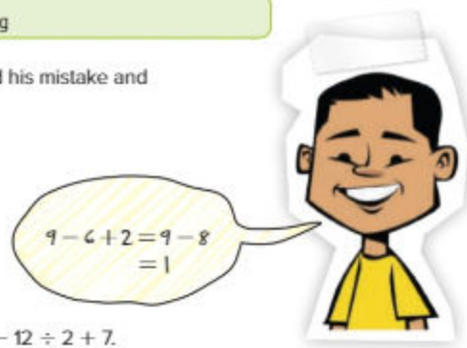
$3 \times 10 + 2 \times 5$; 40 rolls

H.O.T. Problems Higher Order Thinking

12. **Find the Error** Bilal is finding $9 - 6 + 2$. Find his mistake and correct it.

Bilal did not add and/or subtract in order from left to right.

$9 - 6 + 2 = 3 + 2 = 5$



13. **Reason Inductively** Use the expression $34 - 12 \div 2 + 7$.

a. Place parentheses in the expression so that the value of the expression is 18. $(34 - 12) \div 2 + 7$

b. Place parentheses in the expression to find a value other than 18. Then find the value of the new expression. Sample answer:

$34 - (12 \div 2) + 7 = 34 - 6 + 7 = 28 + 7 = 35$

14. **Persevere with Problems** Write an expression with a value of 12. It should contain four numbers and two different operations. Sample answer:

$35 \div 5 + 10 \div 2$

15. **Use Math Tools** Place parentheses in each equation, if needed, to make each equation true.

a. $7 + 3 \times 2 + 4 = 25$ $7 + 3 \times (2 + 4) = 25$

b. $8^2 \div 4 \times 8 = 2$ $8^2 \div (4 \times 8) = 2$

c. $16 + 8 - 5 \times 2 = 14$ parentheses not needed

16. **Which One Doesn't Belong?** Which expression does not belong with the other three? Justify your response.

$6^2 - 9$ 3^3 $(5 + 4)^2 \div 3$ $4 \times 5 + 9$

$4 \times 5 + 9$; The other expressions have a value of 27.

Name _____ My Homework _____

Extra Practice

Find the value of each expression.

17. $9 + 12 - 15 =$ 6

Homework Help → $9 + 12 - 15 = 21 - 15$
$= 6$

18. $22 - 17 + 8 =$ 13

19. $(9 + 2) \times 6 - 5 =$ 61

20. $27 \div (3 + 6) \times 5 - 12 =$ 3

21. $26 + 6^2 \div 4 =$ 35

22. $22 \div 2 \times 3^2 =$ 99

23. $12 \div 4 + (5^2 - 6) =$ 22

24. $96 \div 4^2 + (25 \times 2) - 15 - 3 =$ 38

25. Financial Literacy Admission to a circus is AED 16 for adults and AED 8 for children. Write an expression to find the total cost of 3 adult tickets and 8 children's tickets. Then find the total cost.

$3 \times 16 + 8^2$; AED 112

26. MP Reason Inductively Amani is making caramel apples. She has $2\frac{1}{2}$ bags of apples. One full bag has 8 apples, and each apple weighs 150 grams. Write an expression that could be used to find the total number of grams of apples Amani has. Then find the total number of grams.

$2\frac{1}{2} \times 8 \times 5$; 3000 g

Power Up! Test Practice

Exercises 27 and 28 prepare students for more rigorous thinking needed when taking the assessment.

27.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

28.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly model the number sentence and find the number of party favors.
1 point	Students correctly model the number sentence OR find the number of party favors.

Power Up! Test Practice

27. Badria wants to buy 4 pencils and 3 notebooks. Which of the following represent the total cost? Select all that apply.

Pencils	AED 0.50
Notebooks	AED 2.25

- [x] 4(AED 0.50) + 3(AED 2.25)
- [ ] 7(AED 0.50 + AED 2.25)
- [x] AED 8.75
- [ ] AED 19.25

28. Jassim has $3\frac{2}{5}$ boxes of party favors. One full box contains 15 bags of favors, and each bag has 3 favors in it. In addition to this he has 7 extra party favors that are not in bags or boxes.

Select the correct operation to model a numerical expression for the total number of party favors.

$3\frac{2}{5}$ [×] 15 [×] 3 [+] 7

× ÷ + −

How many party favors does Jassim have in all?

160 party favors

Spiral Review

Find the missing number.

29. 131 + 9 = 140

30. 360 − 6 = 354

31. 14 + 210 = 224

32. Use skip counting and the number line to find the missing number.

3 × 4 = 12

33. Halima earns AED 7 an hour babysitting and AED 8 an hour for cleaning the house. Last week she babysat for 3 hours and cleaned for 2 hours.

How much did Halima earn last week? AED 37

Lesson 3

Algebra: Variables and Expressions

Vocabulary Start-Up

Algebra is a language of symbols including variables. A **variable** is a symbol, usually a letter, used to represent a number.

Scan the lesson to complete the graphic organizer.
Sample answers are given.

Math Meaning	Everyday Meaning
A symbol, usually a letter, used to represent a number.	able to change or vary, as in variable winds
variable	
Example	**Non-example**
x, *y*, *a*	3, 6, −10

Essential Question

HOW is it helpful to write numbers in different ways?

Vocabulary

algebra
variable
algebraic expression
evaluate

Mathematical Practices
1, 2, 3, 4, 6

Real-World Link

A box contains an unknown number of markers. There are 2 markers outside the box. The total number of markers is represented by the bar diagram below.

unknown number of markers	2 markers

1. Suppose there are 14 markers in the box. Find the total number of markers. Explain your answer. 16 markers; Sample answer: There are 14 markers in the box plus two outside the box. 14 + 2 = 16

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Evaluate algebraic expressions.

Coherence connecting within and across grades

Previous
Students simplified numerical expressions using the order of operations.

Now
Students will simplify algebraic expressions.

Next
Students will write algebraic expressions.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 449.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Think-Pair-Share** Have each student think about situations in daily life where they have seen the use of variables, a letter or symbol standing for a number. Then have them discuss their answers with a partner. MP 1, 2, 3

Alternate Strategy

AL Ask students to imagine that they each have a handful of pennies and then they receive five more. The number of pennies each student has could be represented as HANDFUL + 5. Ask them why the HANDFUL can be considered a variable. Ask them to choose just one letter to represent the HANDFUL and rewrite the expression. MP 1, 2, 3, 5

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Evaluate a one-step algebraic expression.**

AL • *What operation does the expression involve?* addition

• *What is the value of b?* 25

OL • *To evaluate the expression, what should you do?* Replace *b* with 25 and then add.

BL • *Will the value of the expression change if it is b + 16? Explain.* no; The value will be 41 because of the Commutative Property of Addition.

Need Another Example?
Evaluate 20 + *c* if *c* = 5. 25

2. **Evaluate a one-step algebraic expression.**

AL • *What is a variable?* a letter used to represent a number

• *How many variables does this expression contain?* 2

OL • *To evaluate the expression, what should you do?* Replace *x* with 64 and *y* with 27, and then subtract.

BL • *Will the value of the expression change if it is y − x? Explain.* yes; The answer will be −37 because subtraction is not commutative.

Need Another Example?
Evaluate *p* − *q* if *p* = 14 and *q* = 13. 1

3. **Evaluate a one-step algebraic expression.**

AL • *What is the value of x?* $\frac{1}{2}$

OL • *To evaluate the expression, what should you do?* Replace *x* with $\frac{1}{2}$ and then multiply.

BL • *What other way(s) can the expression be written?* 6 • *x* or 6(*x*)

Need Another Example?
Evaluate 9*x* if *x* = 4. 36

Work Zone

a. 14

b. 2

c. 24

d. 3

Evaluate One-Step Expressions

Algebraic expressions contain at least one variable and at least one operation. For example, the expression *n* + 2 represents *the sum of an unknown number and two.*

Any letter can be used as a variable. → $n + 2$

The letter *x* is often used as a variable. To avoid confusion with the symbol ×, there are other ways to show multiplication.

$5 \cdot x$	$5(x)$	$5x$
5 times *x*	5 times *x*	5 times *x*

The variables in an expression can be replaced with any number. Once the variables have been replaced, you can **evaluate**, or find the value of, the algebraic expression.

Examples

1. **Evaluate 16 + *b* if *b* = 25.**

$16 + b = 16 + 25$ Replace *b* with 25.
$= 41$ Add 16 and 25.

2. **Evaluate *x* − *y* if *x* = 64 and *y* = 27.**

$x - y = 64 - 27$ Replace *x* with 64 and *y* with 27.
$= 37$ Subtract 27 from 64.

3. **Evaluate 6*x* if $x = \frac{1}{2}$.**

$6x = 6 \cdot \frac{1}{2}$ Replace *x* with $\frac{1}{2}$.
$= 3$ Multiply 6 and $\frac{1}{2}$.

Got It? **Do these problems to find out.**

Evaluate each expression if *a* = 6, *b* = 4, and $c = \frac{1}{3}$.

a. $a + 8$ **b.** $a - b$ **c.** $a \cdot b$ **d.** $9c$

Expressions and Equations

Evaluate Multi-Step Expressions

To evaluate multi-step expressions, replace each variable with the correct value and follow the order of operations.

Examples

4. Evaluate $5t + 4$ if $t = 3$.

$5t + 4 = 5 \cdot 3 + 4$ Replace t with 3.

$= 15 + 4$ Multiply 5 and 3.

$= 19$ Add 15 and 4.

5. Evaluate $4x^2$ if $x = \frac{1}{8}$.

$4x^2 = 4 \cdot \left(\frac{1}{8}\right)^2$ Replace x with $\frac{1}{8}$.

$= 4 \cdot \frac{1}{64}$ Simplify $\left(\frac{1}{8}\right)^2$.

$= \frac{1}{16}$ Multiply.

6. Evaluate $10a + 7$ if $a = \frac{1}{5}$.

$10a + 7 = 10\left(\frac{1}{5}\right) + 7$ Replace a with $\frac{1}{5}$.

$= 2 + 7$ Multiply 10 and $\frac{1}{5}$.

$= 9$ Add.

Got It? Do these problems to find out.

Evaluate each expression if $d = 12$ and $e = \frac{1}{3}$.

e. $2d - 5$

f. $50 - 3d$

g. $9e^2$

Show your work.

e. 19

f. 14

g. 1

Examples

4. Evaluate a multi-step algebraic expression.

AL • *What is the value of t?* 3

OL • *To evaluate the expression, what are the steps in order?* Replace t with 3. Multiply 5 and 3. Add 15 and 4.

• *Why do we multiply before we add?* Multiplication comes before addition in the order of operations.

BL • *Why is the value of 5t not equal to 53 when t = 3?* $5t$ represents the product of 5 and t, not a two-digit number with 5 in the tens place and t in the ones place.

Need Another Example?

Evaluate $4p - 6$ if $p = 3$. 6

5. Evaluate a multi-step algebraic expression.

AL • *What is the value of x?* $\frac{1}{8}$

• *How is this example different from Example 4?* It contains an exponent.

OL • *To evaluate the expression, what should you do first?* Replace x with $\frac{1}{8}$.

• *What do you need to do next?* Find $\left(\frac{1}{8}\right)^2$. *What is the last step?* Multiply 4 and $\frac{1}{64}$.

BL • *If you multiplied 4 and $\frac{1}{8}$ before squaring the number, would the result be the same? Explain.* no; The result would be $\frac{1}{4}$.

Need Another Example?

Evaluate $2x^2$ if $x = \frac{3}{4}$. $1\frac{1}{8}$

6. Evaluate a multi-step algebraic expression.

AL • *What is the value of a?* $\frac{1}{5}$

OL • *To evaluate the expression, what should you do first?* Replace a with $\frac{1}{5}$.

• *What are the rest of the steps in order?* Multiply 10 and $\frac{1}{5}$. Then add 2 and 7.

BL • *How can you find one-fifth of 10 mentally?* Divide 10 by 5. One-fifth of 10 is 2.

Need Another Example?

Evaluate $4m + 12$ if $m = \frac{1}{2}$. 14

Example

7. Evaluate multi-step algebraic expressions.

AL
- *What do you need to find?* the surface area of the box
- *What is the measure of each side length of the box?* $\frac{1}{2}$ m

OL
- *What will you replace s with in the expression? Why?* $\frac{1}{2}$; *s* represents the side length and the side length is $\frac{1}{2}$ meter.
- *What product does the expression* $\left(\frac{1}{2}\right)^2$ *result in?* $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

BL
- *What expression is used to give the area of a square where s is the side length?* s^2 *How is the expression* $6s^2$ *related to the expression for the area of a square?* Sample answer: A cube has 6 sides, so to find the total area of the cube, multiply the area of one side, s^2, by 6.

Need Another Example?

The amount of money Suhaila will need to pay for 5 binders using a AED2 coupon can be represented by the expression $5x - 2$, where x is the cost of each binder. Find the amount of her purchase if each binder is AED4. AED18

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Roundrobin** Have students work with a partner, taking turns evaluating the expressions in Exercises 1–6. Have one student rewrite the expression replacing the variable with its value. Then the second student performs the operations. Then have them complete Exercises 7 and 8. MP 1, 2, 3

BL LA **Trade-a-Problem** Have students write an algebraic expression involving more than one operation. Have them trade their expressions with a partner. Each partner evaluates the expression for a chosen value of the variable. Have partners check each other's work. MP 1, 2, 3

Real World

Example

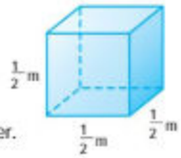
7. Khalil is wrapping a gift for his brother's graduation. The box has side lengths that are $\frac{1}{2}$ meter. Use the expression $6s^2$, where s represents the length of a side, to find the surface area of the box he is wrapping. Write your answer in square meter.

$6s^2 = 6 \cdot \left(\frac{1}{2}\right)^2$ Replace s with $\frac{1}{2}$.

$= 6 \cdot \frac{1}{4}$ Simplify $\left(\frac{1}{2}\right)^2$.

$= \frac{6}{4}$ or $1\frac{1}{2}$ Multiply.

So, the surface area of the box is $1\frac{1}{2}$ square meter.



Guided Practice

Check

Evaluate each expression if $m = 4$, $z = 9$, and $r = \frac{1}{6}$. (Examples 1–6)

1. $3 + m$ 7

Show your work.

2. $z - m$ 5

3. $12r$ 2

4. $4m - 2$ 14

5. $60r - 4$ 6

6. $3r^2$ $\frac{1}{12}$

7. The amount of money that remains from a 20-dirham bill after Hamdah buys 4 party favors for p dirhams each is $20 - 4p$. Find the amount remaining if each favor cost AED 3. (Example 7)

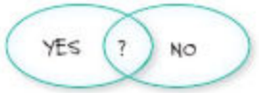
AED 8

8. **Building on the Essential Question** How are numerical expressions and algebraic expressions different?

Sample answer: Numerical expressions include only numerical values and operations. Algebraic expressions can include numerical values, operations, and variables.

Rate Yourself!

Are you ready to move on? Shade the section that applies.



Name _____ My Homework _____

Independent Practice

Evaluate each expression if $m = 2$, $n = 16$, and $p = \frac{1}{3}$. (Examples 1–6)

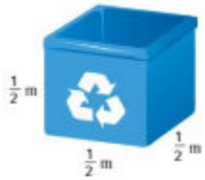
1. $m + 10$ 12
2. $n \div 4$ 4
3. $m + n$ 18
4. $6m - 1$ 11

Show your work.

5. $3p$ 1
6. $12p$ 4
7. $12m - 4$ 20
8. $9p^2$ 1

9. A paper recycling bin has the dimensions shown. Use the expression s^3, where s represents the length of a side, to find the volume of the bin. Write your answer in cubic meters. (Example 7)

$\frac{1}{8}$ m^3



10. **Model with Mathematics** Refer to the graphic novel frame below for Exercises **a–b**.

a. What is the total cost for one individual admission and one individual movie pass on Family Night? AED 14.50

b. The expression $14.50x$ can be used to find the total cost for x tickets on Family Night for admission and the movie. What is the cost for 3 tickets? AED 43.50

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–9, 11–15 odd, 17, 18, 33, 34
OL	On Level	1–9 odd, 10–15, 17, 18, 33, 34
BL	Beyond Level	10–18, 33, 34

Watch Out!

Common Error In Exercises 1–8, students may forget to apply the order of operations. Remind them to multiply before adding or subtracting.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	16
2 Reason abstractly and quantitatively.	17
3 Construct viable arguments and critique the reasoning of others.	18
4 Model with mathematics.	10
6 Attend to precision.	15, 28

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students evaluate the expression $5m + 11$ if $m = 3$.
26

Financial Literacy Badr earns AED 13.50 per hour. His company deducts 23% of his pay each week for taxes. Badr uses the expression $0.77(13.50h)$ to compute his earnings after taxes for the hours h he works. What will be his earnings after taxes, if he works 40 hours? AED 415.80

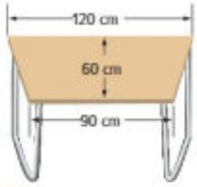
Evaluate each expression if $x = 3$, $y = 12$, and $z = 8$.

12. $4z + 8 - 6$ 34

13. $7z \div 4 + 5x$ 29

14. $y^2 \div (3z)$ 6

15. Be Precise To find the area of a trapezoid, use the expression $\frac{1}{2}h(b_1 + b_2)$, where h represents the height, b_1 represents the length of the top base, and b_2 represents the length of the bottom base. What is the area of the trapezoidal table? 6,300 cm²



H.O.T. Problems Higher Order Thinking

16. Persevere with Problems Faleh and Huda each have a calculator. Huda starts at 100 and subtracts 7 each time. Faleh starts at zero and adds 3 each time. If they press the keys at the same time, will their displays ever show the same number? If so, what is the number? yes; 30

17. Reason Abstractly Describe the difference between algebraic expressions and numerical expressions.

Sample answer: Both numerical expressions and algebraic expressions use operations. An algebraic expression, such as $6 + a$, includes numbers and variables, where a numerical expression, such as $6 + 3$, only includes numbers.

18. Justify Conclusions Complete the table of values to evaluate $5n$ and 5^n for the given values of n. Which will be greater when $n > 5$? Justify your response.

n	1	2	3	4
$5n$	$5 \times 1 = 5$	$5 \times 2 = 10$	$5 \times 3 = 15$	$5 \times 4 = 20$
5^n	$5^1 = 5$	$5^2 = 25$	$5^3 = 125$	$5^4 = 625$

5^n; Sample answer: The value of the expression 5^n grows at a faster rate than it does for $5n$ because each value is being multiplied by 5. In $5n$, 5 is added to each value. 5^n is repeated multiplication while $5n$ is repeated addition.

Name _____ My Homework _____

Extra Practice

Evaluate each expression if $m = 2$, $n = 16$, and $g = \frac{1}{5}$.

19. $n + 8$ 24

$n + 8 = 16 + 8$
$= 24$

20. $12 \div m$ 6

21. $n - m$ 14

22. $2n - 6$ 26

23. $15g$ 3

24. $45g$ 9

25. $7m + 8$ 22

26. $50g^2$ 2

27. Financial Literacy Bilal earns AED 7 per hour plus AED 1.50 for each pizza delivery. The expression $7h + 1.50d$ can be used to find the total earnings after h hours and d deliveries have been made. How much money will Bilal earn after working 15 hours and making 8 deliveries?

AED 117

28. MP Be Precise As a member of a music club, you can order CDs for AED 14.99 each. The music club also charges AED 4.99 for each shipment. The expression $14.99n + 4.99$ represents the cost of n CDs. Find the total cost for ordering 3 CDs.

AED 49.96

Evaluate each expression if $a = \frac{1}{2}$, $b = 15$, and $c = 9$.

29. $c^2 + a$ $81\frac{1}{2}$

30. $2ac$ 9

31. $b^2 - 5c$ 180

32. What is the value of $st \div (6r)$ if $r = 5$, $s = 32$, and $t = 45$?

48

Power Up! Test Practice

Exercises 33 and 34 prepare students for more rigorous thinking needed when taking the assessment.

33.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

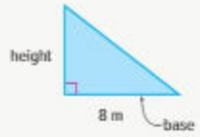
34.

This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Knowledge	DOK3
Mathematical Practices	MP1, MP2
Scoring Rubric	
2 points	Students find a correct expression and explain their response.
1 point	Students find a correct expression OR explain their response.

Power Up! Test Practice

33. The height of the triangle shown can be found using the expression $48 \div b$, where b is the base of the triangle.

What is the height of the triangle? **6 m**



34. The table shows the total medal counts for the top 6 medal winning countries at the 2012 Summer Olympic Games. The top 6 countries earned a total of 421 medals at the Games. Based on the information in the table, write an expression that represents the total amount of medals earned by these countries.

356 + *x*

How could you use your expression and the information given to determine the number of medals Great Britain earned? Explain your reasoning.

Sample answer: I know that 356 + *x* equals 421. I can use the Guess, Check, and Revise strategy to find the value of *x*.

Total Medal Count

Country	Number of Medals
United States	104
China	88
Russia	82
Great Britain	*x*
Germany	44
Japan	38

Spiral Review

Write the symbol <, >, or = for each description.

35. equal to **=**

36. greater than **>**

37. less than **<**

38. Write a number sentence to show that *two plus four equals six*.

2 + 4 = 6

39. Write a number sentence to show *the sum of fourteen and eight is twenty-two*.

14 + 8 = 22

40. Houriyya skied three times farther than her brother Hamdan. Hamdan skied four kilometers. How far did Houriyya ski?

12 kilometers

Inquiry Lab
Write Expressions

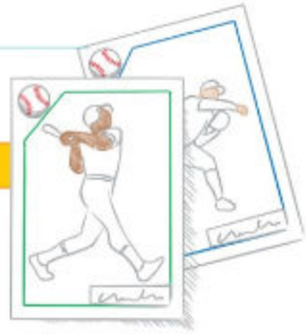
Inquiry **HOW can bar diagrams help you to write expressions in which letters stand for numbers?**

MP Mathematical Practices 1, 3, 4

Khalid has 6 more baseball cards than Ahmed. Write an algebraic expression to represent the number of baseball cards Khalid has.

What do you know? Khalid has 6 more baseball cards than Ahmed.

What do you need to know? the number of baseball cards Khalid has



Hands-On Activity 1

Algebraic expressions are similar to numerical expressions.

Step 1 Ahmed has an unknown number of baseball cards *c*. Use a bar diagram to show Ahmed's cards.

Khalaf	*c* cards

Step 2 Khalid has 6 more baseball cards than Ahmed. Complete the bar diagram below to show how **many baseball cards Khalid has.**

Khalid	*c* cards	6 cards

So, Khalid has *c* + 6 baseball cards.

Recall that the terms of an expression are separated by addition or subtraction signs.

How many terms are in the expression? 2

Does the expression represent a *sum*, *difference*, *product*, or *quotient*?

sum

Focus narrowing the scope

Objective Use models to write expresssions.

Coherence moving from concrete to abstract

Now Students use models to write algebraic expressions to represent real-world situations.

Next Students will write algebraic expressions to represent real-world situations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 455.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1, 2, and 3 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activities 2 and 3.

Hands-On Activity 1

AL LA **Pairs Discussion** Have students work in pairs to demonstrate the scenario using real cards. Use an undetermined number of cards in a stack for Ahmed and show 6 cards for Khalid. Have students assign a variable to the stack, and then show how they would represent adding 6 more cards. MP 1, 4, 6, 7

BL LA **Trade-a-Problem** Have students write a real-world problem similar to the one given and model how they would express the scenario as an expression. Then have them trade problems and expressions with a partner to check their work. MP 1, 4, 6

Hands-On Activity 2

AL LA **Numbered Heads Together** Have students work in groups of 3–4 to complete Activity 2, ensuring that each student understands how the bar diagram shows "10 fewer messages". Each student is assigned a number. Upon completion of the activity, call on one numbered student to share their group's algebraic expression and response to the questions. Call on a different numbered student to share their group's response to the questions. Then call on a third numbered student to respond to the question below. MP 1, 3, 4, 5, 6

Ask:

- *What operation will we need to use to show that Khalifa sent fewer messages?* subtraction

BL LA **Pairs Discussion** Have students extend the activity by having them draw a bar diagram to represent the expression $m + 4$ and another bar diagram to represent the expression $m - 4$. Have them explain why the bar diagrams look different. MP 1, 3, 4, 6

Hands-On Activity 3

AL BL LA **I Have/ Who Has?** After completing the activity, prepare a deck of cards with each card having a question on one side and an answer to a different question on the other side. For example, one side has "$8x$" written on it, the other side has "7 less than a number" written on it. The next card would have "$n - 7$" (the answer to the previous question) on one side and "the quotient of 12 and 3" on the other. Repeat for all cards. Distribute one card to each student, giving them time to read and understand the two expressions on their card. Select one student to start. He or she will stand up and say, "Who has...?" and ask for what is on the card, for example, "Who has 7 less than a number?" The student with the answer to the question will stand up and say, "I have..." and give the solution, then flip their card over and say, "Who has...?" Continue the game until all questions have been answered. The game should end with the first student answering the last question. MP 1, 2, 6, 7, 8

Hands-On Activity 2

Khalifa sent 10 fewer messages in July than in August. Write an algebraic expression to represent the number of text messages Khalifa sent in July.

Step 1 Khalifa sent an unknown number of messages m in August. Label the bar diagram to represent the messages Khalifa sent in August.

Step 2 Khalifa sent 10 fewer messages in July. Label the bar diagram to show the messages Khalifa sent in July.

So, Khalifa sent m – 10 messages in July.

How many terms are in the expression? 2

Does the expression represent a *sum, difference, product,* or *quotient?*

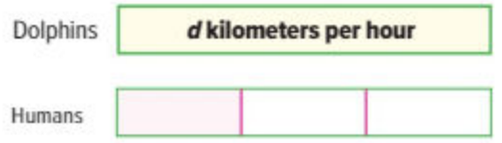
difference

Hands-On Activity 3

A bottlenose dolphin can swim d kilometers per hour. Humans swim one-third as fast as dolphins. Write an algebraic expression that could be used to find out how fast humans can swim.

Step 1 Dolphins can swim an unknown number of kilometers per hour d. Use a bar diagram to represent the speed a dolphin swims.

Step 2 Humans swim one-third as fast as dolphins. Divide and shade a second bar diagram to represent the speed humans can swim.



So, humans can swim $d \div 3$ kilometers per hour.

How many terms are in the expression? 1

Does the expression represent a *sum, difference, product,* or *quotient?*

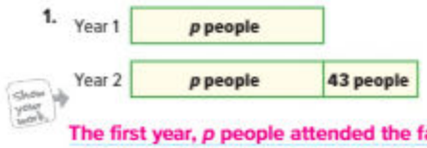
quotient

Investigate

Sample answers: 1–6

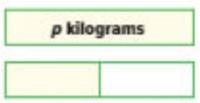
Work with a partner. Write a real-world problem and algebraic expression for each situation modeled.

1.



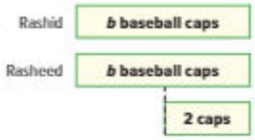
The first year, p people attended the family reunion. The second year, 43 more people attended. Write an algebraic expression to represent the number of people that attended the second year; $p + 43$.

2.



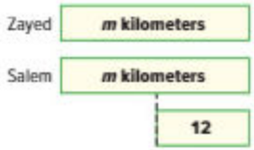
Wafa bought a bag of apples weighing p kilograms. She bought a bag of oranges which weighed one half that of the apples. Write an algebraic expression to represent the weight of the bag of oranges; $\frac{p}{2}$.

3.



Rashid has b baseball caps. Rasheed has two less than Rashid. Write an algebraic expression to represent the number of Rasheed's baseball caps; $b - 2$.

4.



Zayed has an area of m square square kilometers. Salem has an area that is 12 square square kilometers less than Zayed. Write an algebraic expression to represent the area of Salem in square square kilometers; $m - 12$.

5.

Saeed talked m minutes on his cell phone. Sultan talked one-fourth as many minutes as Saeed. Write an algebraic expression to represent the number of minutes Sultan talked on her cell phone; $\frac{m}{4}$.

6.

In sixth grade, Hassan was h centimeters tall. In seventh grade, he was 2 centimeters taller. Write an algebraic expression to represent Hassan's height in seventh grade; $h + 5$.

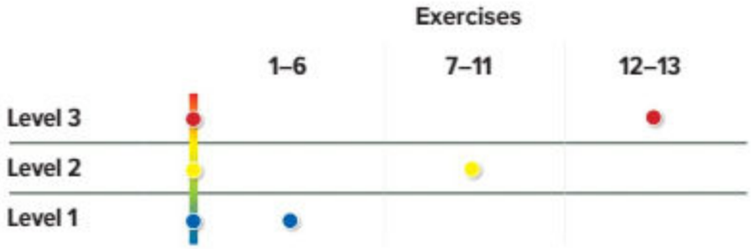
ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL BL LA **Team-Pair-Solo** Have students work in four-person teams to complete Exercises 1 and 2, making sure each team has both Approaching Level students and Beyond Level students. Each team member is responsible to ask questions and ensure that all team members understand. Then have them work in partners to complete Exercises 3 and 4. Finally, have them work individually to complete Exercises 5 and 6. Have them check their work with their previous partner and discuss any differences in solutions. 1, 2, 3, 5

Analyze and Reflect

AL LA **Pairs Check** Complete Exercise 7 together as a class. Then have students work in pairs to complete Exercises 8–11. Have them trade their papers with another pair of students to check their work. Have them respond to the following questions. MP 1, 4, 6

Ask:

- *What verbal phrases represent subtraction?* Sample answers: less than, minus, difference, decreased
- *What verbal phrases represent multiplication?* Sample answers: product, times, double, triple
- *What verbal phrases represent addition?* Sample answers: more than, plus, sum, increased
- *What verbal phrases represent division?* Sample answers: quotient, half, one-third

BL LA **Roundrobin** Have students work in pairs or teams to complete Exercises 7–10, with the first person completing Exercise 7, the second person checking Exercise 7 for accuracy, then completing Exercise 8, and so on. Have students complete Exercise 11 on their own. Have them discuss with a partner how many different ways they could represent this phrase. MP 1, 3, 4, 6

Create

inquiry Students should be able to answer "HOW can bar diagrams help you to write expressions in which letters stand for numbers?" Check for student understanding and provide guidance, if needed.

Analyze and Reflect

Work with a partner to complete the table. The first one is done for you.

	Algebraic Expression	Word Phrase	Model
	$a + 8$	the sum of a number and 8	a | 8
7.	$r - 4$	4 less than a number	r | 4
8.	$5w$	the product of 5 and a number	w | w | w | w | w
9.	$\frac{c}{3}$	the quotient of a number and 3	c | | |
10.	$7 + m$	7 increased by a number	7 | m

11. MP **Reason Inductively** Write an algebraic expression that represents *a number y divided by 10.* $y \div 10$

Create

12. MP **Model with Mathematics** Write a real-world situation and an algebraic expression that is represented by the bar diagram.

w	w	w	3

Sample answer: The number of text messages Mahmoud received was three times as many as Mansour, plus three more; $3w + 3$

13. inquiry HOW can bar diagrams help you to write expressions in which letters stand for numbers?

Sample answer: Bar diagrams show the relationship between the letters that stand for numbers and the values of given numbers.

Lesson 4

Algebra: Write Expressions

Real-World Link

Airports Missouri has 8 major commercial airports. California has 24 major commercial airports.

1. **Alabama has 4 fewer airports than Missouri.**
   a. Underline the key math word in the problem.
   b. Circle the operation you would use to determine how many airports are located in Alabama. Explain.

   \+ (−) × ÷

   The key word is fewer, which means subtract.

2. **California has three times as many airports as Georgia.**
   a. Underline the key math words in the problem.
   b. Circle the operation you would use to find how many airports Georgia has. Explain.

   \+ − × (÷)

   You know the number of airports California has. You also know that it has 3 times the number in Georgia. To undo multiplication, you use division.

3. Missouri has two times as many airports as Ohio. How many airports does Ohio have?

   8 ÷ 2 = 4

Essential Question

HOW is it helpful to write numbers in different ways?

Vocabulary

defining the variable

Mathematical Practices

1, 2, 3, 4, 6

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Write verbal phrases as simple algebraic expressions.

Coherence connecting within and across grades

Previous
Students used the order of operations to evaluate algebraic expressions.

Now
Students write algebraic expressions to represent real-world situations.

Next
Students will write and simplify algebraic expressions using mathematical properties.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 461.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Think-Pair-Share** Give students about one minute to think through their responses to Exercises 1–3. Then have them discuss their responses with a partner. MP 1, 3

Alternate Strategies

AL LA Have students explain why the inverse operation is used in Exercises 2 and 3, but not in Exercise 1. MP 1, 3

BL Have students rewrite the statement in Exercise 2 so that the operation in the statement is division. MP 1, 2, 3

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Write the phrase as an algebraic expression.**

AL • *What is a variable?* a letter used to represent a number

• *What information is unknown?* the number of dirhams Saleh has earned

• *How can you represent this unknown information?* with a variable

OL • *Since dirhams is the unknown amount, what letter could you use to represent this?* d

• *How will you represent 8 dirhams more than d?* $d + 8$

BL • *Can you write the expression as $d + 8$ or $8 + d$? Explain.* yes; Addition is commutative.

Need Another Example?

Write the phrase *two dirhams less than the price of a movie ticket* as an algebraic expression. $m - 2$

2. **Write the phrase as an algebraic expression.**

AL • *What information is unknown?* the original price

• *How can you represent the unknown information?* with a variable

OL • *Since price is the unknown amount, what letter could you use to represent this?* p

• *How will you represent 10 dirhams less than p?* $p - 10$

BL • *Can you write the expression as $10 - p$? Explain.* no; Subtraction is not commutative.

• *What verbal phrase would $10 - p$ represent?* Sample answer: p dirhams less than 10 dirhams

Need Another Example?

Write the phrase *five years less than her aunt's age* as an algebraic expression. $a - 5$

Work Zone

Less Than

You can write ten more than a number as either $10 + p$ or $p + 10$. But ten less than a number can only be written as $p - 10$.

Write Phrases as Algebraic Expressions

To write verbal phrases as algebraic expressions, follow the steps below. In the second step, **defining the variable**, choose a variable and decide what it represents.

Words	Describe the situation. Use only the most important words.
Variable	Choose a variable to represent the unknown quantity.
Expression	Translate your verbal phrase into an algebraic expression.

Examples

Write each phrase as an algebraic expression.

1. ***eight dirhams more than Saleh earned***

Words	eight dirhams more than Saleh earned
Variable	Let d represent the number of dirhams Saleh earned.
Model	Saleh's earnings: d; d | 8
Expression	The expression is $d + 8$.

2. ***ten dirhams less than the original price***

Words	ten dirhams less than the original price
Variable	Let p represent the original price.
Model	original price: p; p | AED10
Expression	The expression is $p - 10$.

3. *four times the number of liters*

Words four times the number of liters

Variable Let l represent the number of liters.

Model number of liters: l | l | l | l

Expression The expression is $4l$.

Got It? Do these problems to find out.

a. four points fewer than the Eagles scored

b. 12 times the number of meters

c. the total cost of a shirt and an AED 8 pair of socks

a. $b - 4$

b. $12m$

c. $s + 8$

Write Two-Step Expressions

Two-step expressions contain two different operations.

Example

4. **Write the phrase *5 less than 3 times the number of points* as an algebraic expression.**

Words 5 less than 3 times the number of points

Variable Let p represent the number of points.

Model number of points: p | p | p | 5

Expression The expression is $3p - 5$.

Got It? Do this problem to find out.

d. Write the phrase *AED 3 more than four times the cost of a pretzel* as an algebraic expression.

d. c = cost of a pretzel; $4c + 3$

Examples

3. **Write the phrase as an algebraic expression.**

AL
- *What information is unknown?* the number of liters
- *How can you represent the unknown information?* with the variable l

OL
- *What operation does the phrase represent? How do you know?* multiplication; The phrase uses *times*, which indicates multiplication.
- *How will you represent 4 times the number of gallons?* $4l$

BL
- *How can you write the verbal phrase in a different way that represents the same algebraic expression?* Sample answer: the product of the number of liters and 4

Need Another Example?

Write the phrase *three times the number of yards* as an algebraic expression. $3y$

4. **Write the phrase as an algebraic expression.**

AL
- *What information is unknown?* the number of points
- *How can you represent the unknown information?* with the variable p

OL
- *How many operations are indicated by the phrase?* 2 operations
- *What part of the phrase indicates multiplication?* *times*
- *How will you represent 3 times the number of points?* $3p$
- *What part of the phrase indicates subtraction?* *less than*
- *How will you represent 5 less than 3p?* $3p - 5$

BL
- *Could you also write the expression as $5 - 3p$? Explain.* no; Subtraction is not commutative.

Need Another Example?

Write the phrase *2 less than 6 times the cat's weight* as an algebraic expression. $6w - 2$

Example

5. **Use an algebraic expression to solve a real-world problem.**

AL
- *What do you need to find?* the total amount Amal spent
- *How much did the magazine cost?* AED5
- *Besides the magazine, what else did Amal buy?* 2 bottles of nail polish

OL
- *How can you represent the cost of the two bottles of nail polish?* $2d$
- *How can you represent adding the cost of the magazine?* $+ 5$
- *What expression represents the total cost?* $2d + 5$
- *If you know that $d = 3$, how will you solve the problem?*
  Replace d with 3 and evaluate the expression using the order of operations.

BL
- *Can the expression also be written as $5 + 2d$? Explain.* yes; Addition is commutative.

Need Another Example?
Huda made 5 fewer than three times the number of bracelets that Hana made. Write an expression to represent the number of bracelets Huda made. Then determine the number of bracelets Huda made if Hana made 12 bracelets. $3s - 5$; 31 bracelets

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

AL LA **Roundrobin** Have students work in groups of 3–4. Have Student 1 highlight the variable in the problem and choose a letter to represent it. Have Student 2 determine what operation(s) is/are being performed on the variable and have Student 3 write the algebraic expression. Student 4, if available, determines if there is a different way to write the expression. MP 1, 2, 3, 4

Example

5. **Amal bought a magazine for AED 5, and 2 bottles of nail polish. Write an expression to represent the total amount she spent. Then find the total amount if each bottle of nail polish cost AED 3.**

Step 1 The nail polish costs an unknown amount. Use d to represent the cost of the nail polish.

Step 2 She bought 2 bottles of polish plus a magazine.

total amount	d dirhams	d dirhams	AED 5

The expression is $2 \times d + 5$ or $2d + 5$.

$2d + 5 = 2(3) + 5$ Replace d with 3.
$= 6 + 5$ Multiply.
$= 11$ Add.

So, the total amount is AED 11.

Guided Practice

Define a variable and write each phrase as an algebraic expression.
(Examples 1–4)

1. four times more money than Tarek saved m = money Tarek saved; $4m$
2. half as many pages as Amer read b = the number of pages Amer read; $\frac{b}{2}$
3. the width of a box that is 4 inches less than the length ℓ = the length of the box; $\ell - 4$
4. the cost of 5 CDs and a AED 12 DVD c = cost of a CD; $5c + 12$
5. Abdulrahman bought a box of popcorn for AED 3.50 and three medium drinks. Define a variable and write an expression to represent the total amount they spent. Then find the total amount if one drink costs AED 1.50. (Example 5)
   d = cost of one drink; $3d + 3.50$; AED 8.00
6. **Building on the Essential Question** How can writing phrases as algebraic expressions help you solve problems?
   Sample answer: Key words and phrases, such as *four times as many*, can help you to determine which operation to use in an expression in order to solve a problem.

Rate Yourself!

- ☐ I understand how to write algebraic expressions.
  Great! You're ready to move on!
- ☐ I still have some questions about writing algebraic expressions.

Name _____ My Homework _____

Independent Practice

Define a variable and write each phrase as an algebraic expression.

(Examples 1–4)

1. six meters less than the width

   w = the width; $w - 6$

2. 6 hours more per week than Mahmoud studies

   h = the number of hours Mahmoud studies; $h + 6$

3. six years less than Eiman's age

   t = Eiman's age; $t - 6$

4. 2 less than one-third of the points that the Panthers scored

   p = the number of points the Panthers scored; $\frac{1}{3}p - 2$ or $(p \div 3) - 2$

5. The United States House of Representatives has 35 more members than four times the number of members in the United States Senate. Define a variable and write an expression to represent the number of members in the House of Representatives. Then find the number of members in the House of Representatives, if there are 100 members in the Senate.

   (Example 5)

   s = the number in the Senate; $4s + 35$; 435 members

6. MP **Multiple Representations** Amna uses the table to help her convert measurements when she is sewing.

Number of Inches	12	24	36	48
Number of Meters	1	2	3	4

   a. **Words** Describe the relationship between the number of inches and the number of meters.

   Sample answer: The number of meters is one twelfth the number of inches.

   b. **Symbols** Write an expression for the number of meters in x inches.

   $\frac{1}{12}x$ or $x \div 12$

   c. **Numbers** Find the number of meters in 252 inches.

   21 ft

7. MP **Be Precise** An inch is equal to about 2.54 centimeters. Write an expression which estimates the number of centimeters in x inches. Then estimate the number of centimeters in 12 inches.

   $2.54x$; 30.48 cm

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7, 9, 10, 12–14, 23, 24
OL	On Level	1–5 odd, 6–10, 12–14, 23, 24
BL	Beyond Level	6–14, 23, 24

MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	11
2 Reason abstractly and quantitatively.	22
3 Construct viable arguments and critique the reasoning of others.	6, 10, 12–14
6 Attend to precision.	7

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students write the phrase *eleven less than four times the number of text messages* as an algebraic expression.
Sample answer: $4t - 11$

Watch Out!

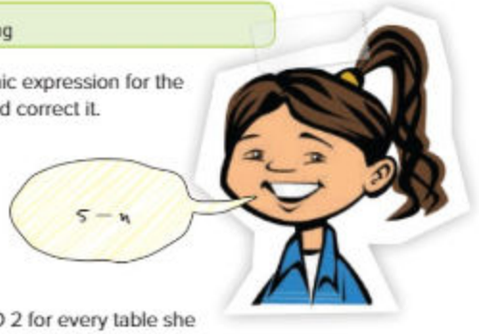
Common Error In Exercise 10, Buthaina correctly determined the operation but confused the order of the terms. Remind students to analyze the wording of problems carefully.

8. **Financial Literacy** On a recent day, a Euro was equal to about 1.2 UAE dirhams. Write an expression which estimates the number of dirhams in x Euros. Then estimate the number of UAE dirhams equal to 25 Euros.
1.2x; AED 30

9. Abdulraheem is 2 years older than one third his sister Abeer's age. Their sister Ayesha is four years younger than 2 times Abdulraheem's age. Define a variable and write an expression to represent Abdulraheem's age. Then find Abdulraheem's age and Ayesha's age if Abeer is 63 years old.
m = Abeer's age; $\frac{1}{3}m + 2$; Abdulraheem is 23 years old and Aiysha is 42 years old.

H.O.T. Problems Higher Order Thinking

10. **Find the Error** Buthaina is writing an algebraic expression for the phrase *5 less than a number*. Find her mistake and correct it.
Sample answer: She is indicating *a number less than 5*; $n - 5$ is the correct expression.



11. **Persevere with Problems** Hessa earns AED 2 for every table she serves plus 20% of the total customer order. Define a variable and write an expression to represent the amount of money she earns for one table.
c = total customer order; $2 + 0.2c$

12. **Justify Conclusions** If n represents the amount of songs stored on an MP3 player, analyze the meaning of the expressions $n + 7$, $n - 2$, $4n$, and $n \div 2$.
Sample answer: 7 more songs, 2 fewer songs, 4 times the songs, and half the number of songs.

13. **Justify Conclusions** Determine whether the statement below is *always*, *sometimes*, or *never* true. Justify your reasoning.

The expressions $x - 3$ and $y - 3$ represent the same value.

sometimes; Sample answer: $x - 3$ and $y - 3$ represent the sample values only when $x = y$.

14. **Reason Inductively** Suppose x is an odd number. Write an expression to represent each of the following:
   a. The odd number immediately following x. **$x + 2$**
   b. The odd number immediately preceding x. **$x - 2$**

Name _____ My Homework _____

Extra Practice

Define a variable and write each phrase as an algebraic expression.

15. four times as many apples a = the number of apples; $4 \times a$ or $4a$

Homework Help

16. ten more shoes than Fatheya s = number of Fatheya shoes; $s + 10$

17. AED5 dirhams less on dinner than Ibrahim spent j = the cost of Ibrahim's dinner; $j - 5$

18. 3 more than twice as many ringtones as Fawzia m = the number of ringtones Fawzia has; $2m + 3$

19. Hessa goes bowling on Saturday afternoons. She bowls three games and pays for shoe rental. Define a variable and write an expression to represent the total cost Hessa pays. Then find the total cost if one game costs AED 4. b = the cost of one game; $3b + 2$; AED 14

Bowl-A-Rama	
One Game	■
Shoe Rental	AED 2

20. Jassim bought a pizza for AED 12.75 and four medium drinks at Pauli's Pizza. Define a variable and write an expression to represent the total amount of money he spent. Then find the total cost if one drink costs AED 3.

c = cost of one drink; $4c + 12.75$; AED 24.75

21. Lamya's music library has 17 more than two times the number of songs than her brother Zayed's music library. Define a variable and write an expression to represent the number of songs in Lamya's music library. Then find the number of songs in Lamya's library if Zayed has 5 songs in his library.

s = number of songs in Zayed library; $2s + 17$; 27 songs

22. MP **Reason Abstractly** Halima has 3 more than one half as many purses as Ayesha. Define a variable and write an expression to represent the number of purses in Halima's collection. Then find the number of purses in Halima's collection if Ayesha has 12 purses.

p = number of purses in Ayesha's collection; $\frac{1}{2}p + 3$; Halima has 9 purses.

Power Up! Test Practice

Exercises 23 and 24 prepare students for more rigorous thinking needed when taking the assessment.

23.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP3
Scoring Rubric	
1 point	Students correctly answer each part of the question.

24.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK3
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly answer each part of the question.
1 point	Students correctly answer 3 of the four parts.

Power Up! Test Practice

23. Abdulaziz and his friends bought game tokens for AED 15 and three admission tickets to Fun Palace. Let t represent the cost of an admission ticket. Determine if each statement below is true or false.

a. The expression $3(t + 15)$ represents the total amount that Abdulaziz and his friends spent. ☐ True ■ False

b. If each admission ticket costs AED 2.50, then Abdulaziz and his friends spent AED 22.50 in all. ■ True ☐ False

24. The table below shows the relationship between meters and yards.

Number of Meters	3	6	9	12	15
Number of Yards	1	2	3	4	5

Fill in each box to write an algebraic expression to represent the number of meters in any number of yards.

f	$\frac{1}{3}$
y	meters
3	yards

Words 3 times the number of yards.

Variable Let y represent the number of yards.

Model y y y

Expression The number of meters in y yards is given by the expression 3 y.

Spiral Review

Evaluate each expression.

25. $7 + 0.8 =$ 7.8

26. $8.3 \times 1 =$ 8.3

27. $3.5 + (4 + 7) =$ 14.5

26. Hamdah ran five kilometers each day for seven days. Houriyya ran seven kilometers each day for five days. Did the girls run the same distance? Explain.

yes; Sample answer: Since $5 \times 7 = 35$ and $7 \times 5 = 35$, both girls ran a total of 35 kilometers.

MP Problem-Solving Investigation

Act It Out

Case #1 Table Trouble

Lamis is arranging tables for her volleyball banquet. The rectangular tables can seat up to 6 people. She can line up tables to seat more people.

How many people can be seated using four tables?

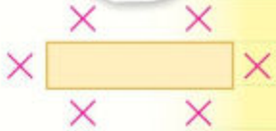
MP Mathematical Practices 1, 3, 4

1 Understand *What are the facts?*

Each rectangular table can seat up to 6 people.

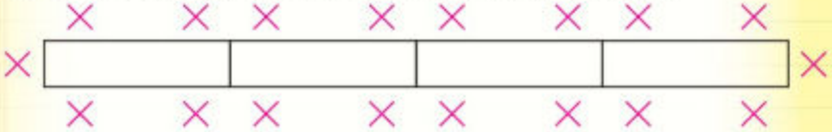
2 Plan *What is your strategy to solve this problem?*

Use the rectangle to represent one table. Use counters to represent each seat. Draw an X to show where each counter was placed.



3 Solve *How can you apply the strategy?*

Act out the situation to find the number seats at four tables. Use counters to represent each seat. Draw an X to show where each counter was placed.



Four tables can seat 18 people.

4 Check *Does the answer make sense?*

Use the expression $4x + 2$, where x represents the number of tables.

So, 4 × 4 + 2 = 18. ✓

Analyze the Strategy

MP **Reason Inductively** Explain how the Act It Out strategy could help you check the reasonableness of answers. Sample answer: It is easier to determine if my answers are reasonable.

Focus narrowing the scope

Objective Solve problems by acting them out. This lesson emphasizes MP **Mathematical Practice 3** Construct viable arguments and critique the reasoning of others.

Act It Out Students can use a variety of materials to act problems out. Have a variety of materials on hand such as masking tape, construction paper, toothpicks, counters, and math tiles for students to use to solve the exercises.

Coherence connecting within and across grades

Now
Students solve non-routine problems.

Next
Students will apply the act it out strategy to write equivalent expressions..

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 467.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

The problems on pages 465 and 466 are intended to be used as a whole-group discussion on how to solve non-routine problems and are designed to provide scaffolded guidance. The problem on page 465 walks students through the solution, while the problem on page 466 asks students to come up with their own solutions.

Case #1 Table Trouble

BL Have students extend the problem by having them answer the question below.

Ask:

- *How many people will 6 tables, lined up end to end, seat?* 26

Case #2 Step It Up

AL LA **Cooperative Play** Break students into 4-person learning teams. Provide each team with a variety of materials such as counters, constructions paper, math tiles, and masking tape. Explain to students that each team should use one or more of the given materials in order to act out the problem. After students have solved the problem, discuss as a class the results and how they were obtained. MP 1, 3, 4, 5

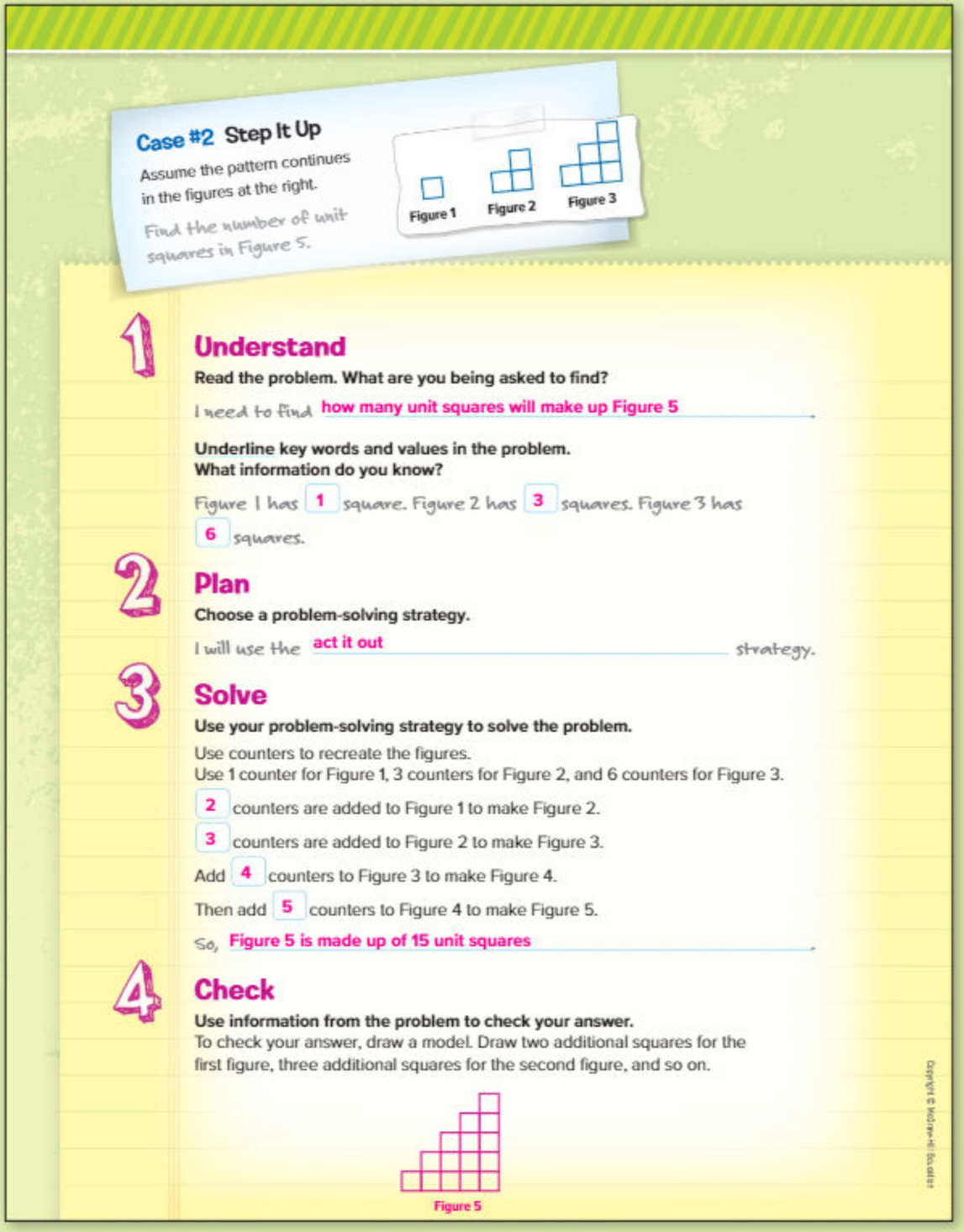
BL LA **Class Discussion** After students complete the *Step It Up* problem, use the following question to guide a whole group or small group discussion. MP 1, 3

Ask:

- *Why does the act it out strategy help you solve this problem?* Sample answer: There are a few things that need to be understood in order to solve the problem. Acting it out helps to visualize everything all at once to come up with a more accurate answer.

Need Another Example?

Lamya is making a scrapbook. She wants to make a border in ribbon around each of the pages. Each page uses 98 centimeters of ribbon. She has a spool of ribbon that is 22.75 meters long. How many bordered pages can Lamya make? 23 pages



Case #2 Step It Up

Assume the pattern continues in the figures at the right.

Find the number of unit squares in Figure 5.

Figure 1 Figure 2 Figure 3

1 Understand

Read the problem. What are you being asked to find?

I need to find how many unit squares will make up Figure 5

Underline key words and values in the problem.
What information do you know?

Figure 1 has 1 square. Figure 2 has 3 squares. Figure 3 has 6 squares.

2 Plan

Choose a problem-solving strategy.

I will use the act it out strategy.

3 Solve

Use your problem-solving strategy to solve the problem.

Use counters to recreate the figures.
Use 1 counter for Figure 1, 3 counters for Figure 2, and 6 counters for Figure 3.

2 counters are added to Figure 1 to make Figure 2.

3 counters are added to Figure 2 to make Figure 3.

Add 4 counters to Figure 3 to make Figure 4.

Then add 5 counters to Figure 4 to make Figure 5.

So, Figure 5 is made up of 15 unit squares

4 Check

Use information from the problem to check your answer.

To check your answer, draw a model. Draw two additional squares for the first figure, three additional squares for the second figure, and so on.

Figure 5

Work with a small group to solve the following cases.
Show your work on a separate piece of paper.

Case #3 Teams

Twenty-four students will be divided into four equal-size teams. Each student will count off, beginning with the number 1 as the first team.

If Tarek is the eleventh student to count off, to which team number will he be assigned?

Team 3

Case #4 Savings

Khadija has AED 5.38 in her savings account. Each week she adds AED 2.93.

How much money does Khadija have after 5 weeks? after n weeks?

AED 20.03; $2.93n + 5.38$

Case #5 Vacations

The UAE tourism agency surveyed people on their favorite vacation cities. Half of the people said Dubai, $\frac{1}{4}$ said Abu Dhabi, $\frac{1}{8}$ said Ajman, $\frac{1}{16}$ responded Al Ain, $\frac{1}{32}$ said Sharjah, and the rest said Umm Al Quwain.

If 22 people said Umm Al Quwain, how many people responded Dubai? 352 people

Use any strategy!

Case #6 School

The birth months of the students in Mr. Salem's geography class are shown.

Birth Months		
June	July	April
March	July	June
October	May	August
June	April	October
May	October	April
September	December	January

What is the difference in the percentage of students born in June than in August? Round to the nearest whole percent.

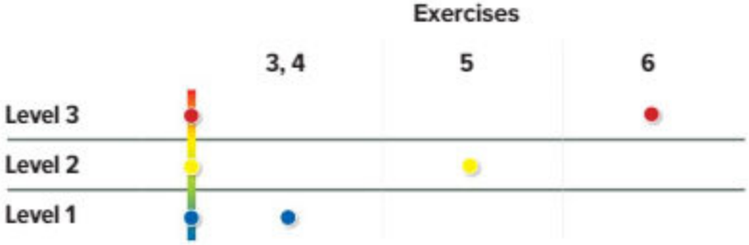
11%

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** EVALUATE

2 Collaborate

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



AL LA **All Write Roundrobin** Have students work in teams of four to solve Cases 3–6. For Case 3, have one student talk through the solution process and speak their answer aloud. If teammates agree, all students record the answer on their paper. Repeat for Cases 4–6, alternating students for each exercise. MP 1, 3

BL LA **Trade-a-Problem** Have students write a real-world problem that requires them to use the *act it out* strategy. Be sure that students write and give a solution to their problem. Have students trade their problems with a partner. Select a problem or two to share with the class and allow volunteers from the class to act out the problem in order to solve it. MP 1, 3, 4

Mid-Chapter Check

If students have trouble with Exercises 1–9, they may need help with the following concepts.

Concept	Exercise(s)
powers (Lesson 1)	1, 3, 4
evaluating algebraic expressions (Lesson 3)	5–7
writing algebraic expressions (Lesson 4)	8
order of operations (Lesson 2)	2, 9

Vocabulary Activity

LA **Numbered Heads Together** Have students work in small groups to complete Exercise 1. Each student is assigned a number. Students are responsible to ensure that each group member understands the meaning of a powers. Students should ask each other for clarification and assistance, as needed. Call on one numbered student to share their definition with the class. MP 1, 3, 8

Alternate Strategy

AL Have students explain the difference between 3^4 and 3×4. MP 1, 3, 7

Mid-Chapter Check

Vocabulary Check

1. MP **Be Precise** Define *powers*. Provide an example of power with an exponent of 2. (Lesson 1)

   Numbers expressed using exponents are called powers

   Sample answer: 13^2.

2. Fill in the blank in the sentence below with the correct term. (Lesson 2)

   The order of operations tells you which operation to perform first so that everyone finds the same value for an expression.

Skills Check and Problem Solving

Write each power as a product of the same factor. Then find the value. (Lesson 1)

3. $7^2 =$ 7 × 7; 49

4. $5^5 =$ 5 × 5 × 5 × 5 × 5; 3,125

Evaluate each expression if $x = 6$. (Lesson 3)

5. $x + 11$ 17

6. $4(x - 5)$ 4

7. $2x \div 6$ 2

8. MP **Reason Abstractly** Khawla is 8 years younger than her sister Rana. Rana is y years old. Write an algebraic expression that describes Khawla's age. (Lesson 4)

   $y - 8$

9. MP **Reason Abstractly** The prices per kilogram of different types of nuts are shown. Write an expression that can be used to find the total cost of 2 kilograms of peanuts, 3 kilograms of cashews, and 1 kilogram of almonds, all for 20% off. (Lesson 2)

   0.80(2 × 3.95 + 3 × 4.25 + 5.99)

Lesson 5

Algebra: Properties

Real-World Link

Baking Manal and Muna are baking cookies for a bake sale fundraiser. Manal baked 6 sheets with 10 cookies each and Muna baked 10 sheets with 6 cookies each.

1. How many total cookies can Manal bake?
   6 × 10 = 60
2. How many total cookies did Muna bake?
   10 × 6 = 60
3. What do you notice about your answers for Exercises 1 and 2?
   The answers are the same.
4. What do these exercises suggest about the order in which factors are multiplied?
   The order does not matter.

Essential Question

HOW is it helpful to write numbers in different ways?

Vocabulary

properties
Commutative Properties
Associative Properties
Identity Properties
equivalent expressions

Mathematical Practices
1, 2, 3, 4, 5

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Use properties to simplify expressions.

Coherence connecting within and across grades

Previous
Students wrote and simplified numerical and algebraic expressions.

Now
Students write equivalent expressions using the properties of operations.

Next
Students will write equivalent expressions using the Distributive Property.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 473.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Pairs Discussion** Have students work in pairs to complete Exercises 1–4, ensuring that each partner understands. Call on one set of pairs to share their responses with the class. MP 1, 3

Alternate Strategies

AL LA Have students use a grid to draw a rectangle 6 units by 10 units and give the total number of squares. Repeat for a rectangle 10 units by 6 units. Discuss why the order does not matter. MP 1, 4, 5

BL Have students identify the property that allows them to multiply two numbers in any order. MP 1, 3, 7

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Determine whether two expressions are equivalent.

AL • *How are the numbers and operation for both expressions alike?* They are the same. *How are they different?* They are grouped differently.

OL • *What property states that the way in which numbers are grouped does not affect their sum?* Associative Property of Addition

BL • *If you were not sure if the expressions were equivalent, how could you check?* Sample answer: I could find the value of each expression to see if they are equal. Both expressions equal 28, so they are equivalent.

Need Another Example?

Determine if the expressions $5 \times (6 \times 10)$ and $(5 \times 6) \times 10$ are equivalent. If so, tell what property is applied. If not, explain why. yes; Associative Property

2. Determine whether two expressions are equivalent.

AL • *How are the numbers and operation for both expressions alike?* They are the same. *How are they different?* They are grouped differently.

OL • *When subtracting numbers, does grouping them differently change the final value?* yes

BL • *If you were not sure if the expressions were equivalent, how could you check?* Sample answer: I could find the value of each expression to see if they are equal.

Need Another Example?

Determine if the expressions $100 \div (25 \div 5)$ and $(100 \div 25) \div 5$ are equivalent. If so, tell what property is applied. If not, explain why. no; Sample answer: The first expression is equal to 20 and the second expression is equal to 0.8.

Key Concept

Use Properties to Compare Expressions

Work Zone

Commutative Properties The order in which two numbers are added or multiplied does not change their sum or product.

$7 + 9 = 9 + 7$ $4 \cdot 6 = 6 \cdot 4$

$a + b = b + a$ $a \cdot b = b \cdot a$

Associative Properties The way in which three numbers are grouped when they are added or multiplied does not change their sum or product.

$3 + (9 + 4) = (3 + 9) + 4$ $8 \cdot (5 \cdot 7) = (8 \cdot 5) \cdot 7$

$a + (b + c) = (a + b) + c$ $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

Identity Properties The sum of an addend and 0 is the addend. The product of a factor and 1 is the factor.

$13 + 0 = 13$ $7 \cdot 1 = 7$

$a + 0 = a$ $a \cdot 1 = a$

Properties are statements that are true for any number. The expressions 6×10 and 10×6 are called **equivalent expressions** because they have the same value. This illustrates the Commutative Property.

Examples

Determine whether the two expressions are equivalent. If so, tell what property is applied. If not, explain why.

1. $15 + (5 + 8)$ and $(15 + 5) + 8$

The numbers are grouped differently. They are equivalent by the Associative Property.

Use an $=$ sign to compare the expressions.

So, $15 + (5 + 8) = (15 + 5) + 8$.

2. $(20 - 12) - 3$ and $20 - (12 - 3)$

The expressions are not equivalent because the Associative Property is not true for subtraction.

Use the $\neq$ sign to show the expressions are not equivalent.

So, $(20 - 12) - 3 \neq 20 - (12 - 3)$.

Determine whether the two expressions are equivalent. If so, tell what property is applied. If not, explain why.

3. **34 + 0 and 34**

The expressions are equivalent by the Identity Property.

So, 34 + 0 = 34.

4. **20 ÷ 5 and 5 ÷ 20**

The expressions are not equivalent because the Commutative Property does not hold for division.

So, 20 ÷ 5 ≠ 5 ÷ 20.

Got It? **Do these problems to find out.**

a. 5 × (6 × 3) and (5 × 6) × 3 **b.** 27 ÷ 3 and 3 ÷ 27

Use Properties to Solve Problems

Properties can also be used to write equivalent expressions and to solve problems.

Example

5. **In a recent season, the football team had 15 guards, 4 forwards, and 3 centers on their roster. Write two equivalent expressions using the Associative Property that can be used to find the total number of players on their roster.**

The Associative Property states that the grouping of numbers when they are added does not change the sum, so 15 + (4 + 3) is the same as (15 + 4) + 3.

Got It? **Do this problem to find out.**

c. **Financial Literacy** Maha earned AED 7 babysitting and AED 12 from another job. Write two equivalent expressions using the Commutative Property that can be used to find the total amount she earned.

Division

The Commutative Property does not hold for division. To prove this, simplify the expressions in Example 4.

$20 \div 5 = 4$

$5 \div 20 = \frac{1}{4}$

Since 4 is not equal to $\frac{1}{4}$, expressions are not equivalent.

Show your work.

a. yes; Associative Property

b. No; the first quotient is 9, the second is $\frac{1}{9}$.

c. AED 7 + AED 12 and AED 12 + AED 7

Examples

3. **Determine whether two expressions are equivalent.**

AL • *What is the sum of 34 and 0?* 34

OL • *What property does this demonstrate?* Identity Property

• *Are the expressions equivalent?* yes

BL • *Could you write the expression as 0 + 34? Explain.* yes; Addition is commutative.

Need Another Example?

Determine if the expressions 15 and 1 × 15 are equivalent. If so, tell what property is applied. If not, explain why. yes; Identity Property

4. **Determine whether two expressions are equivalent.**

AL • *How are the expressions alike?* They have the same numbers and operation. *How are they different?* the order in which they are divided

OL • *Does the order in which you divide matter?* yes

BL • *Give another example that demonstrates that division is not commutative.* Sample answer: 16 ÷ 2 = 8, but 2 ÷ 16 ≠ 8

Need Another Example?

Determine if the expression 12 × 4 and 4 × 12 are equivalent. If so, tell what property is applied. If not, explain why. yes; Commutative Property

5. **Write equivalent expressions.**

AL • *How can you determine the total?* Add 15, 4, and 3.

OL • *Can you group the numbers differently and get the same sum?* yes

• *What property allows you to do this?* Associative Property

BL • *Write another equivalent expression that can be used to find the total number of players on the Jayhawks' roster.* Sample answer: (3 + 15) + 4

Need Another Example?

Mr. Abdulaziz has budgeted AED150 per day for his hotel and meals during his six-day vacation. Write two equivalent expressions using the Commutative Property that can be used to find how much he will spend. 150 × 6 and 6 × 150

Example

6. Evaluate the expression using properties.

AL
- *What do you need to find?* the area of the triangle
- *What expression are you given for the area of a triangle?* $\frac{1}{2}bh$

OL
- *How will you find the area?* Evaluate the given formula.
- *With what will you replace b? Why?* 15; It is the length of the base of the triangle.
- *With what will you replace h? Why?* 12; It is the height of the triangle.

BL
- *Why is the Commutative Property performed?* Sample answer: It is used so that you can multiply $\frac{1}{2}$ by 12, which is easier mental math.
- *Why is the area given in square feet?* Sample answer: When finding the area of something, you are multiplying units, and unit × unit = unit^2, or square units.

Need Another Example?

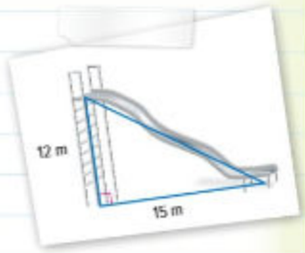
The formula for the volume of a rectangular prism is $V = \ell wh$, where ℓ is the length, w is the width, and h is the height of the prism. Find the volume of a prism with a length of 10 meters, a width of 9 meters, and a height of $\frac{1}{2}$ meter. 45 ft^3

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

AL LA **Pairs Discussion** For Exercises 1–4, have one student find the value of the first expression. The other student finds the value of the second expression. Have them discuss why they should expect the values to be the same.
In Exercise 5, ask them to determine all possible groupings and decide which allows for the easiest mental calculations.
MP 1, 3, 8



Real World

Example

6. The area of a triangle can be found using the expression $\frac{1}{2}bh$, where b is the base and h is the height. Find the area of the triangle shown at the left.

$\frac{1}{2}bh = \frac{1}{2}(15)(12)$ Replace b with 15 and h with 12.

$= \frac{1}{2}(12)(15)$ Commutative Property

$= 6(15)$ Multiply. $\frac{1}{2} \times 12 = 6$

$= 90$ Multiply.

The area of the triangle is 90 square meters.

Got It? Do this problem to find out.

d. **Financial Literacy** Moza earned AED 6 an hour while working 11 hours over the weekend. She put $\frac{1}{3}$ of what she earned in a savings account. Find how much she put into the account.

d. AED 22

Guided Practice

Check

Determine whether the two expressions are equivalent. If so, tell what property is applied. If not, explain why. (Examples 1–4)

1. $(35 + 17) + 43$ and $35 + (17 + 43)$ yes; Associative Property
2. $(25 - 9) - 5$ and $25 - (9 - 5)$ No; the first expression is equal to 11 and the second is equal to 21.
3. 59×1 and 59 yes; Identity Property
4. At a gymnastics meet, a gymnast scored an 8.95 on the vault and a 9.2 on the uneven bars. Write two equivalent expressions that could be used to find her total score. (Example 5)
   8.95 + 9.2 and 9.2 + 8.95
5. Halima bought suntan lotion for AED 12, sunglasses for AED 15, and a towel for AED 18. Use the Associative Property to mentally find the total of her purchases. (Example 6) AED 45; The expression 12 + 15 + 18 can be rewritten as (18 + 12) + 15.
6. **Building on the Essential Question** How can using properties help you to simplify expressions?
   Sample answer: The properties can help you to mentally solve problems.

Rate Yourself!

How confident are you about using properties? Check the box that applies.

Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

Determine whether the two expressions are equivalent. If so, tell what property is applied. If not, explain why. (Examples 1–4)

1. $(8 + 27) + 52$ and $8 + (27 + 52)$ yes; Associative Property

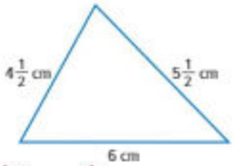
2. $(3 \cdot 6) \cdot 9$ and $3 \cdot (6 \cdot 9)$ yes; Associative Property

3. $72 - (63 - 8)$ and $(72 - 63) - 8$ no; The first expression is equal to 17 and the second is equal to 1.

4. $36 \div (12 \div 3)$ and $(36 \div 12) \div 3$ No; the first expression is equal to 9 and the second is equal to 1.

5. $0 + 32$ and 0 No; the first expression is equal to 32, not 0.

6. STEM Find the perimeter of the triangle shown. (Example 6)



$\left(4\frac{1}{2} + 5\frac{1}{2}\right) + 6 = 16$ cm

7. Each day, about 75,000 people visit Paris, France. Use the Commutative Property to write two equivalent expressions that could be used to find the number of people that visit over a 5-day period. (Example 5)

75,000 • 5 and 5 • 75,000

Use one or more properties to rewrite each expression as an expression that does not use parentheses.

8. $(y + 1) + 4 =$ $y + 5$

9. $(6 \cdot r) \cdot 7 =$ $42r$

Find the value of x that makes a true statement.

10. $24 + x = 24$ 0

11. $17 + x = 3 + 17$ 3

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

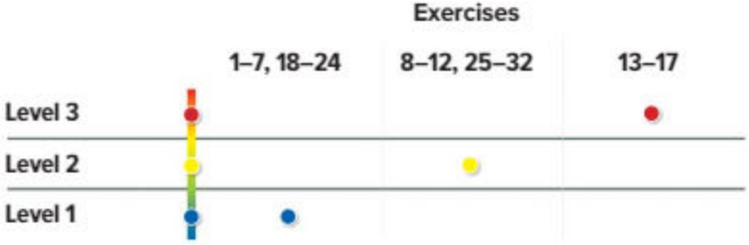
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 11, 13, 14, 16, 17, 31, 32
OL	On Level	1–7 odd, 8–14, 16, 17, 31, 32
BL	Beyond Level	8–17, 31, 32

MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	15
2 Reason abstractly and quantitatively.	12, 13
3 Construct viable arguments and critique the reasoning of others.	14, 16, 17
5 Use appropriate tools strategically.	23

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET
Out the Door

Have students use the Associative Property to write an expression equivalent to 23 + (7 + 18) that makes it easier to add the numbers mentally. (23 + 7) + 18

Watch Out!

Common Error Students may confuse the Identity Property of Addition and the Identity Property of Multiplication. Remind students that the sum of an addend and 0 is the addend and the product of a factor and 1 is the factor.

12. **Reason Abstractly** The graphic shows the driving distance between certain cities in Florida.

a. Write a number sentence that compares the mileage from Miami to Jacksonville to Tampa, and the mileage from Tampa to Jacksonville to Miami.

338 + 188 = 188 + 338

b. Refer to part a. Name the property that is illustrated by this sentence.

Commutative Property

H.O.T. Problems Higher Order Thinking

13. **Reason Abstractly** Write two equivalent expressions that illustrate the Associative Property of Addition. Sample answer: 12 + (8 + 5) and (12 + 8) + 5

14. **Construct an Argument** Determine whether (18 + 35) × 4 = 18 + 35 × 4 is *true* or *false*. Explain. false; Using the order of operations, (18 + 35) • 4 = 212 and 18 + 35 • 4 = 158.

15. **Persevere with Problems** A *counterexample* is an example showing that a statement is not true. Provide a counterexample to the following statement.

Division of whole numbers is commutative.

Sample answer: 24 ÷ 12 = 2 and 12 ÷ 24 = 0.5

16. **Justify Conclusions** Do (4 + 9) + 5 = (9 + 4) + 5 and (4 + 9) + 5 = 4 + (9 + 5) illustrate the same property? Justify your response.

no; Sample answer: The first sentence illustrates the Commutative Property because the order of the numbers in the grouping symbols changes. The second sentence illustrates the Associative Property because the numbers that are grouped together change.

17. **Reason Inductively** How can the Associative Property be used to mentally find 48 + 82?

Sample answer: Rewrite 48 + 82 as 48 + (52 + 30). By using the Associative Property, 48 + (52 + 30) = (48 + 52) + 30. So, 48 + 82 = 130.

Name _____ My Homework _____

Extra Practice

Determine whether the two expressions are equivalent. If so, tell what property is applied. If not, explain why.

Homework Help

18. 64 + 0 and 64 yes; Identity Property

19. 23 • 1 and 23 yes; Identity Property

20. 8 ÷ 2 and 2 ÷ 8 No; the first expression is equal to 4 and the second is equal to $\frac{1}{4}$ or 0.25.

21. 46 + 15 and 15 + 46 yes; Commutative Property

22. 13 • 1 and 1 No; the first expression is equal to 13, not 1.

23. **Use Math Tools** Reham's mother hosted a dinner party. The table shows the costs. Use the Associative Property to write two equivalent expressions that could be used to find the total amount spent.

Sample answer: (12 + 24) + 6 and 12 + (24 + 6)

Party Costs	
Item	**Cost (AED)**
Cake	12
Chicken sandwiches and beefburgers	24
Drinks	6

24. Maysa sold 37 necklaces for AED 20 each at the craft fair. She is going to donate half of the money she earned to charity. Use the Commutative Property to mentally find how much money she will donate. Explain the steps you used.

The expressions $\frac{1}{2}(37)(20)$ and $\frac{1}{2}(20)(37)$ are equivalent. Either one can be used to find how much money she will donate. Half of 20 is 10 and 10 • 37 is 370. She will donate AED 370.

Use one or more properties to rewrite each expression as an expression that does not use parentheses.

25. 2 + (x + 4) = x + 6

26. 4 + (b + 0) = b + 4

27. 1 • (n • 8) = 8n

28. 20 • (6 • y) = 120y

29. (6 + m) + 9 m + 15

30. (w • 12) • 3 36w

Power Up! Test Practice

Exercises 31 and 32 prepare students for more rigorous thinking needed when taking the assessment.

31.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

32.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly assign all seven values.
1 point	Students correctly assign five to six of the seven values.

Power Up! Test Practice

31. The table shows the number of desks in each classroom at three different schools. Which expressions represent the total number of desks? Select all that apply.

- [x] $2 \times (12 \times 25) + 15 \times 20$
- [ ] $2 \times (12 + 12 + 15)$
- [x] $(2 \times 12) \times 25 + 15 \times 20$
- [x] $15 \times 20 + (2 \times 12) \times 25$

School	Number of Classrooms	Number of Desks per Classroom
Al Saada	12	25
Al Ofoq	12	25
Al Wahda	15	20

32. Determine if the two expressions in each pair are equivalent. If they are equivalent, select the property that is illustrated.

Associative Property
Commutative Property
Identity Property

	Equivalent?	Property
$5 + 0$ and 5	Yes	Identity Property
12×2 and 2×12	Yes	Commutative Property
$16 - 3$ and $3 - 16$	No	
$3 + (1 + 9)$ and $(3 + 1) + 9$	Yes	Associative Property

Spiral Review

Write each number in expanded form.

33. $15 =$ 10 + 5

34. $37 =$ 30 + 7

35. $209 =$ 200 + 9

36. Halima had AED 10 bills and AED 1 coins in her wallet. She used seven bills and coins to buy a pair of shoes for AED 43. How many of each type of bill or coin did she spend?

four AED 10 bills and three AED 1 coins

37. Jamal has 3 10-fils coins. Hareb has 5 10-fils coins. They put their money into a donation box for a local pet shelter. What is the value of the money they added to the donation box? Explain.

80 fils; Since 3 coins + 5 coins = 8 coins and 8 coins × 10 fils = 80 fils, the value of the money is 80 fils.

Inquiry Lab
The Distributive Property

Inquiry **HOW can you use models to evaluate and compare expressions?**

MP Mathematical Practices 1, 3, 5

Three friends are going to a concert at the fair. They will each pay for admission to the fair, which is AED 6, and admission to the concert, which is AED 22. What is the total that the three friends will spend?

Hands-On Activity 1

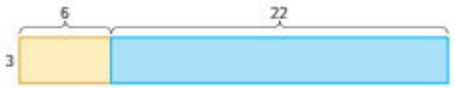
Step 1 Write an expression to represent the amount spent in dirhams.

$$3(6 + 22)$$

3 friends; 6 fair admission; 22 concert

Step 2 Use area models to evaluate the expression.

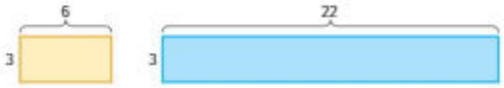
Method 1 Add the lengths. Then multiply.



$3(6 + 22) = 3(28)$

$= 84$

Method 2 Find each area. Then add.



$3 \cdot 6 + 3 \cdot 22 = 18 + 66$

$= 84$

Since both expressions are equal to 84, they are equivalent.

So, $3(6 + 22) = 3 \cdot 6 + 3 \cdot 22$.

Focus narrowing the scope

Objective Model the Distributive Property.

Coherence moving from concrete to abstract

Now
Students use area models and algebra tiles to model and use the Distributive Property.

Next
Students will use the Distributive Property to write equivalent expressions.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 479.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance to students than Activity 2.

Materials: algebra tiles

Hands-On Activity 1

AL LA Remind students that the area diagrams represent the total cost of 3 tickets to both the fair and the concert. If students have trouble with the area models, have them use algebra tiles to model the situation. MP 1, 3, 4

Ask:

- *Refer to Step 2. Are the expressions in Method 1 and Method 2 equivalent? Explain.* yes; Sample answer: Each expression is equal to 84.
- *How are Method 1 and Method 2 different?* Sample answer: The order in which the numbers are multiplied and added is different.

Hands-On Activity 2

AL LA If students have difficulty with modeling these expressions, have them first model expressions like $x + 5$, $x + 1$, and $2x + 6$. Then they can move on to the activity that involves using the Distributive Property. MP 1, 4, 6

Ask:

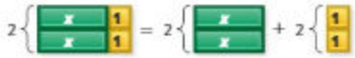
- *Refer to Step 1. What does the 2 outside the parentheses represent?* 2 sets of $2x + 1$
- *Does the model represent this?* yes
- *Refer to Step 2. When you group the like tiles, what are you doing to the expression?* simplifying the expression

Hands-On Activity 2

Tools

You can also use algebra tiles to model expressions with variables. Refer to the set of algebra tiles below.

Just like 2(3) means 2 groups of 3, $2(x + 1)$ means 2 groups of $x + 1$.



Use algebra tiles to tell whether the expressions $2(2x + 1)$ and $4x + 2$ are equivalent.

Step 1 Model the expression $2(2x + 1)$.

There are 2 groups with $2x + 1$ in each group.

Step 2 Group like tiles together.

The model shows 4 x-tiles and 2 integer tiles.

Both models have the same number of x-tiles and the same number of integer tiles.

So, the expression $2(2x + 1)$ is equivalent to the expression $4x + 2$.

Investigate

Work with a partner. Draw area models to show that the pairs of expressions are equivalent.

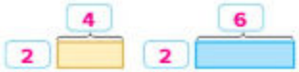
1. $2(4 + 6)$ and $(2 \cdot 4) + (2 \cdot 6)$

$2(4 + 6) = 2($ 10 $)$

$= $ 20

$(2 \cdot 4) + (2 \cdot 6) =$ 8 $+$ 12

$=$ 20



2. $4(3 + 2)$ and $(4 \cdot 3) + (4 \cdot 2)$

$4(3 + 2) = 4($ 5 $)$

$= $ 20

$(4 \cdot 3) + (4 \cdot 2) =$ 12 $+$ 8

$=$ 20

3. $6(20 + 3)$ and $(6 \cdot 20) + (6 \cdot 3)$

$6(20 + 3) = 6($ 23 $)$

$= $ 138

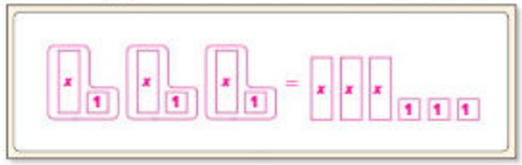
$(6 \cdot 20) + (6 \cdot 3) =$ 120 $+$ 18

$=$ 138

Use algebra tiles to tell whether the pairs of expressions are equivalent.

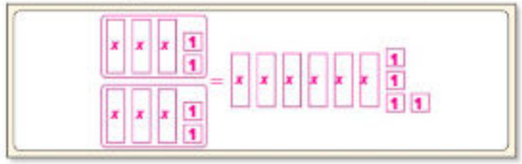
4. $3(x + 1)$ and $3x + 3$ yes

$3(x + 1)$: 3 x-tiles, 3 integer tiles $3x + 3$: 3 x-tiles, 3 integer tiles



5. $2(3x + 2)$ and $6x + 4$ yes

$2(3x + 2)$: 6 x-tiles, 4 integer tiles $6x + 4$: 6 x-tiles, 4 integer tiles



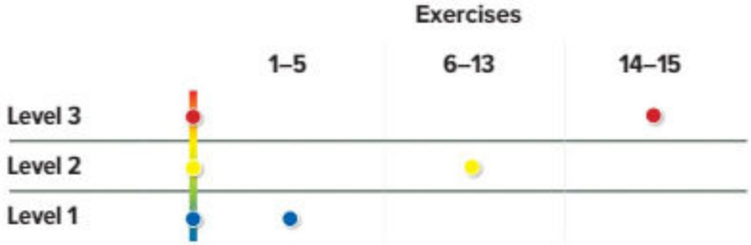
ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Think-Pair-Share** Have students work in pairs. Give students one minute to think through their answers to Exercises 1–5. Have them share their responses with their partner. Then call on one student to share their response within a small group or large group discussion.
MP 1, 3, 6

BL LA For Exercises 1–5, have students choose area models or algebra tiles to model the expressions, then explain to the class why they chose that model and how it shows whether or not the expressions are equivalent.
MP 1, 3, 4

Analyze and Reflect

AL LA Work together as a class to complete Exercises 6–12. Then have students work with a partner to complete Exercise 13. MP 1, 7, 8

Ask:

- *What do you notice about the original expression and the rewritten expression?* The rewritten expression is the sum of each addend inside the parentheses multiplied by the number outside of the parentheses.

Create

BL LA **Trade-a-Problem** For Exercise 14, have students write an expression that includes parentheses. Then have them write a real-world situation that could represent that expression. Have students trade their word problems to verify that they meet the criteria. MP 1, 2, 6, 7

Inquiry Students should be able to answer "HOW can you use models to evaluate and compare expressions?" Check for student understanding and provide guidance, if needed.

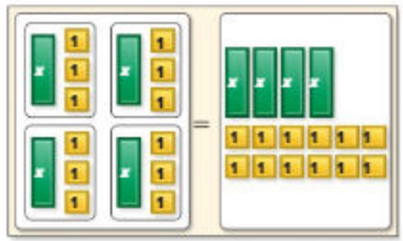
Analyze and Reflect

Sample answers: 13–15

Work with a partner to complete the table. Use a model if needed. The first one is done for you.

	Expression	Rewrite the expression.	Evaluate.
	2(4 + 1)	2(4) + 2(1)	10
6.	7(8 + 4)	7(8) + 7(4)	84
7.	9(3 + 9)	9(3) + 9(9)	108
8.	5(3 + 5)	5(3) + 5(5)	40
9.	2(24 + 6)	2(24) + 2(6)	60
10.	3(16 + 5)	3(16) + 3(5)	63
11.	4(8 + 7)	4(8) + 4(7)	60
12.	6(22 + 9)	6(22) + 6(9)	186

13. MP **Use Math Tools** A friend decides that $4(x + 3) = 4x + 3$. Use the algebra tiles at the right to explain to your friend that $4(x + 3) = 4x + 12$.



Count the total number of each type of tile. This corresponds to the expression $4x + 12$, not $4x + 3$. So, $4(x + 3) = 4x + 12$.

Create

14. MP **Model with Mathematics** Write a real-world word problem that could be represented by the expression 3(23). Then explain how you could solve the problem mentally. Sample answer: Find the total cost of three tickets to the water park if each ticket costs AED 23; Think of 3(23) as 3(20 + 3). Since 3(20 + 3) = 3(20) + 3(3), multiply 3 by 20 and 3 by 3. Then add 60 and 9, which is 69.

15. Inquiry HOW can you use models to evaluate and compare expressions?
An area model can show the relationships between the values in the expressions. Algebra tiles can be used to group like expressions and compare the two expressions.

Lesson 6

The Distributive Property

Real-World Link

Baseball Three friends went to a baseball game. Each ticket cost AED 20 and all three friends bought a baseball hat for AED 15 each.

1. What does the expression 3(20 + 15) represent?

   3 represents: three friends

   20 represents: the price of a ticket

   15 represents: the price of a baseball hat

2. Evaluate the expression in Exercise 1.

   (20 + 15) = 35

   35 × 3 = 105

3. What does the expression 3 × 20 + 3 × 15 represent?

   3 × 20 represents: the price of three tickets

   3 × 15 represents: the price of three hats

4. Evaluate the expression 3 × 20 + 3 × 15.

   3 × 20 = 60

   3 × 15 = 45

   60 + 45 = 105

5. What do you notice about the answers to Exercises 2 and 4?

   The answer to 3(20 + 15) is the same as the answer to 3 × 20 + 3 × 15.

Which MP **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Essential Question

HOW is it helpful to write numbers in different ways?

Vocabulary

Distributive Property

factor the expression

Mathematical Practices

1, 3, 4, 5, 6, 7, 8

Focus narrowing the scope

Objective Use the Distributive Property to compute multiplication problems mentally and to rewrite algebraic expressions.

Coherence connecting within and across grades

Previous
Students wrote equivalent expressions using the properties of operations.

Now
Students write equivalent expressions using the Distributive Property.

Next
Students will use models to determine if two expressions are equivalent.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 485.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Roundrobin** In small groups, have students go around the group, each student providing an answer to one part of each exercise. Other students are to listen carefully and ask for support if they do not understand. Groups are responsible to ensure that all members understand. MP 1, 3

Alternate Strategy

AL LA Have groups of students use counters to demonstrate the expressions 3(2 + 5) and 3 × 2 + 3 × 5 and explain why the totals are the same. MP 1, 3, 4, 5

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Compute a product mentally.**

AL • *What two operations are combined with the Distributive Property?* addition and multiplication

• *What operation is represented by parentheses?* multiplication

OL • *How can $4\frac{1}{3}$ be rewritten as a sum?* $4 + \frac{1}{3}$

• *Which numbers are each multiplied by 9 when you distribute?* 4 and $\frac{1}{3}$

BL • *Why is using the Distributive Property helpful in finding the value of this expression?* Calculating 9×4 and $9 \times \frac{1}{3}$, and then adding, is easier than calculating $9 \times 4\frac{1}{3}$.

Need Another Example?

Find $4 \times 10\frac{1}{2}$ mentally using the Distributive Property.

$4(10) + 4\left(\frac{1}{2}\right) = 42$

2. **Write an equivalent expression.**

AL • *What number do you need to distribute?* 2

• *Do you distribute the 2 to both terms or only one term?* both terms

OL • *What is 2 times x?* $2x$

• *What is 2 times 3?* 6

• *Explain how the model represents the problem.* Sample answer: Each mat represents $x + 3$. There are two mats, since this sum is being multiplied by 2. By adding $x + 3$ and $x + 3$, the model can be simplified to represent $2x + 6$.

BL • *Why is the expression $2(x + 3)$ not equivalent to $2x + 3$?* The 2 needs to be distributed, or multiplied by, each of the terms inside the parentheses, not just the x.

Need Another Example?

Use the Distributive Property to rewrite $5(x + 6)$. $5x + 30$

Key Concept **Distributive Property**

Words To multiply a sum by a number, multiply each addend by the number outside the parentheses.

Example

Numbers	Algebra
$2(7 + 4) = 2 \times 7 + 2 \times 4$	$a(b + c) = ab + ac$

Work Zone

The expressions $3(20 + 15)$ and $3 \times 20 + 3 \times 15$ show how the **Distributive Property** combines addition and multiplication.

Example

1. **Find $9 \times 4\frac{1}{3}$ mentally using the Distributive Property.**

$9 \times 4\frac{1}{3} = 9\left(4 + \frac{1}{3}\right)$ Write $4\frac{1}{3}$ as $4 + \frac{1}{3}$.

$= 9(4) + 9\left(\frac{1}{3}\right)$ Distributive Property

$= 36 + 3$ Multiply.

$= 39$ Add.

Show your work.

Got It? **Do these problems to find out.**

Find each product mentally. Show the steps you used.

a. $5 \times 2\frac{3}{5}$ b. $12 \times 2\frac{1}{4}$ c. 2×3.6

a. $5(2) + 5\left(\frac{3}{5}\right) = 13$

b. $12(2) + 12\left(\frac{1}{4}\right) = 27$

c. $2(3) + 2(0.6) = 7.2$

Example

Tools

2. **Use the Distributive Property to rewrite $2(x + 3)$.**

$2(x + 3) = 2(x) + 2(3)$ Distributive Property

$= 2x + 6$ Multiply.

Got It? **Do these problems to find out.**

Use the Distributive Property to rewrite each expression.

d. $8(x + 3)$ e. $5(9 + x)$ f. $2(x + 3)$

d. $8x + 24$

e. $45 + 5x$

f. $2x + 6$

Expressions and Equations

Example

3. **Maysoun is making a pair of earrings and a bracelet for four friends. Each pair of earrings uses 4.5 centimeters of wire and each bracelet uses 13 centimeters. Write two equivalent expressions and then find how much total wire is needed.**

Using the Distributive Property, $4(4.5) + 4(13)$ and $4(4.5 + 13)$ are equivalent expressions.

$4(4.5) + 4(13) = 18 + 52 \quad 4(4.5 + 13) = 4(17.5)$
$= 70 \qquad\qquad = 70$

So, Maysoun needs 70 centimeters of wire.

Got It? **Do this problem to find out.**

g. Each day, Eissa lifts weights for 10 minutes and runs on the treadmill for 25 minutes. Write two equivalent expressions and then find the total minutes that Eissa exercises in 7 days.

Show your work.

g. 7(10) + 7(25) =
7(10 + 25) = 245 min

Factor an Expression

When numeric or algebraic expressions are written as a product of their factors, the process is called **factoring the expression**.

Example

4. **Factor 12 + 8.**

$12 = 2 \cdot 2 \cdot 3$ Write the prime factorization of 12 and 8.
$8 = 2 \cdot 2 \cdot 2$ Circle the common factors.

The GCF of 12 and 8 is $2 \cdot 2$ or 4.

Write each term as a product of the GCF and its remaining factor. Then use the Distributive Property to *factor out* the GCF.

$12 + 8 = 4(3) + 4(2)$ Rewrite each term using the GCF.
$= 4(3 + 2)$ Distributive Property

So, $12 + 8 = 4(3 + 2)$.

Got It? **Do these problems to find out.**

Factor each expression.

h. $9 + 21$ **i.** $14 + 28$ **j.** $80 + 56$

Prime Factorization
The prime factorization of an algebraic expression contains both the prime factors and any variable factors. For example, the prime factorization of $6x$ is $2 \cdot 3 \cdot x$.

h. 3(3 + 7)

i. 14(1 + 2)

j. 8(10 + 7)

Examples

3. **Write equivalent expressions.**

AL
- *How many sets of earrings and bracelets is she making?* 4
- *What is the length of wire used for the earrings?* 4.5 cm
- *What is the length of wire used for the bracelets?* 13 cm

OL
- *How would you find the total amount of wire needed for one pair of earrings and one bracelet?* Add the amounts together.
- *By what number should you multiply this sum? Explain.* 4; Maysoun is making a set of jewelry for 4 friends.

BL
- *Why do both expressions give the same result?* The expressions are equivalent due to the Distributive Property.

Need Another Example?
Khawla is baking 5 batches each of cookies and cupcakes. The cookies use 2 cups of flour per batch and the cupcakes use 3 cups of flour per batch. Write two equivalent expressions and then find the total cups of flour that are needed.
$5(2 + 3) = 5(2) + 5(3) = 25$ cups

4. **Factor to write an equivalent expression.**

AL
- *When you factor a number, are you writing it as a sum or product of its factors?* product
- *How can you write the prime factorization of a number?* Sample answer: make a factor tree

OL
- *What is the prime factorization of 12?* $2 \times 2 \times 3$ *8?* $2 \times 2 \times 2$
- *What is the GCF of 12 and 8?* 4
- *Why do we factor out the GCF?* Sample answer: Each number is a product of 4 and some other number. We factor out the GCF to write an equivalent expression in the form of $4(x + y)$.

BL
- *Is there another way of factoring 12 + 8 to generate a different equivalent expression?* yes; Sample answer: $2(6 + 4)$.

Need Another Example?
Factor $18 + 9$. $9(2 + 1)$

Example

5. Factor to write an equivalent expression.

AL
- *How can you write 3x as a product of its factors?* 3 • x
- *What is the prime factorization of 3x?* 3 • x
- *What is the prime factorization of 15?* 3 • 5

OL
- *What is the GCF of 3x and 15?* 3
- *Once we factor out the GCF of 3x and 15, what are we left with inside the parentheses?* x + 5

BL
- *How can you check the answer?* Use the Distributive Property to write an expression equivalent to 3(x + 5) without the parentheses.
- *How could you factor the expression if the second term was 15x instead of 15? Explain.* 3x(1 + 5); The GCF would be 3x, not 3.

Need Another Example?
Factor 4x + 20. 4(x + 5)

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Think-Pair-Share** Give students 1–2 minutes to think about how they would complete each exercise. Then have them share their responses with a partner. Be sure they identify the GCF in Exercises 5 and 6. Have a different set of partners explain how to complete one of the exercises to the class. MP 1, 3, 7

BL LA **Trade-a-Problem** Have students write two different expressions involving the sum of more than two addends where the Distributive Property could be applied. Have them trade expressions with a partner. Each partner writes an equivalent expression. MP 1, 3, 8

k. 4(4 + x)

l. 7(x + 6)

m. 6(6x + 5)

Example

5. Factor 3x + 15.

3x = 3 • x Write the prime factorization of 15 and 3x.
15 = 3 • 5 Circle the common factors.

The GCF of 3x and 15 is 3.

3x + 15 = 3(x) + 3(5) Rewrite each term using the GCF.
= 3(x + 5) Distributive Property

So, 3(x + 5) = 3x + 15.

Got It? Do these problems to find out.

Factor each expression.

k. 16 + 4x **l.** 7x + 42 **m.** 36x + 30

Guided Practice

1. Find $9 \times 8\frac{2}{3}$ mentally. Show the steps you used. (Example 1) 9(8) + 9(2/3) = 78

Use the Distributive Property to rewrite each algebraic expression. (Example 2)

2. 3(x + 1) = 3x + 3

3. 5(x + 8) = 5x + 40

4. 4(x + 6) = 4x + 24

Factor each expression. (Examples 4 and 5)

5. 25 + 60 = 5(5 + 12)

6. 4x + 40 = 4(x + 10)

7. Financial Literacy Six friends are going to the funfair. The cost of one admission is AED 25, and the cost for one ride on the Ferris wheel is AED 10. Write two equivalent expressions and then find the total cost. (Example 3)
6(25 + 10) = 6(25) + 6(10) = AED 210

8. Building on the Essential Question How can the Distributive Property help you to rewrite expressions?
Sample answer: You can rewrite a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor.

Rate Yourself!

How well do you understand the Distributive Property? Circle the image that applies.

Clear Somewhat Clear Not So Clear

Name _____ My Homework _____

Independent Practice

Find each product mentally. Show the steps you used. (Example 1)

1. $9 \times 44 =$ 9(40) + 9(4) = 396

2. $4 \times 5\frac{1}{8} =$ $4(5) + 4\left(\frac{1}{8}\right) = 20\frac{1}{2}$

3. $7 \times 3.8 =$ 7(3) + 7(0.8) = 26.6

Show your work.

Use the Distributive Property to rewrite each algebraic expression. (Example 2)

4. $8(x + 7) =$ 8x + 56

5. $6(11 + x) =$ 66 + 6x

6. $8(x + 1) =$ 8x + 8

7. **Identify Repeated Reasoning** A coyote can run up to 69 kilometers per hour while a rabbit can run up to 56 kilometers per hour. Write two equivalent expressions and then find how many more kilometers a coyote can run in six hours than a rabbit at these rates. (Example 3)

6(69) − 6(56) = 6(69 − 56); 78 km

Factor each expression. (Examples 4 and 5)

8. $8 + 16 =$ 8(1 + 2)

9. $54 + 24 =$ 6(9 + 4)

10. $63 + 81 =$ 9(7 + 9)

11. $11x + 55 =$ 11(x + 5)

12. $32 + 16x =$ 16(2 + x)

13. $77x + 21 =$ 7(11x + 3)

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–13, 16–18, 33, 34
OL	On Level	1–13 odd, 14, 16–18, 33, 34
BL	Beyond Level	14–18, 33, 34

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	15
3 Construct viable arguments and critique the reasoning of others.	17, 18
4 Model with mathematics.	14
6 Attend to precision.	25, 26
7 Look for and make use of structure.	16
8 Look for and express regularity in repeated reasoning.	7

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

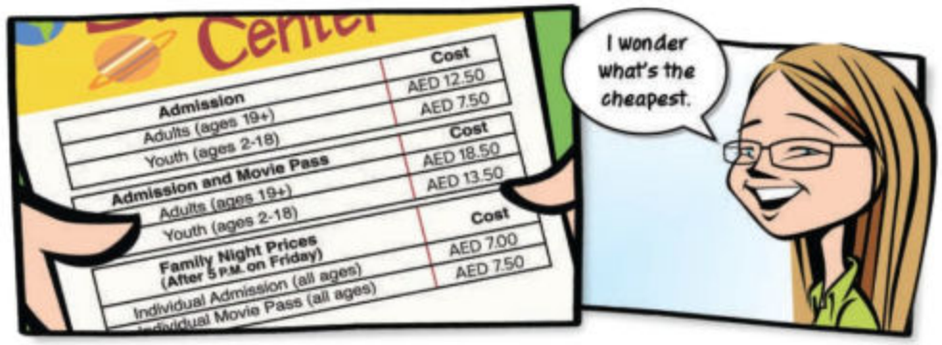
TICKET
Out the Door

Have students write an expression equivalent to $4(x + 7)$ using the Distributive Property. $4x + 28$

Watch Out!

Common Error Watch for students who do not multiply each addend by the number outside the parentheses when using the Distributive Property. Suggest to students that they draw arrows from the number outside the parentheses to each addend.

14. **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–b.



a. Write two equivalent expressions that demonstrate the Distributive Property for the cost of x tickets for admission and movie passes on Family Night. $x(7.00 + 7.50)$ and $x(7) + x(7.50)$

b. Is it less expensive for a youth to pay regular admission with a movie pass or go on Family Night? Explain. It is cheaper to pay regular admission. The total cost for one person is AED 13.50 versus AED 14.50 on Family Night.

H.O.T. Problems Higher Order Thinking

15. **Persevere with Problems** Evaluate the expression 0.1(3.7) mentally. Justify your response using the Distributive Property. 0.37; Sample answer: $0.1(3.7) = 0.1(3) + 0.1(0.7) = 0.3 + 0.07 = 0.37$

16. **Identify Structure** Write two equivalent expressions involving decimals that illustrate the Distributive Property. Sample answer: $3(4.8)$ and $3(4) + 3(0.8)$

17. **Construct an Argument** A friend rewrote the expression $5(x + 2)$ as $5x + 2$. Write a few sentences to your friend explaining the error. Then, rewrite the expression $5(x + 2)$ correctly. Sample answer: The friend did not multiply 5 and 2. The expression $5(x + 2) = 5x + 10$.

18. **Reason Inductively** Explain why $3(5x)$ is not equivalent to $(3 \cdot 5)(3 \cdot x)$. Sample answer: The Distributive Property combines addition and multiplication. The expression $3(5x)$ is one term with three factors, and does not involve addition. So, $3(5x) = 15x$.

Name ______________________ My Homework ______________________

Extra Practice

Find each product mentally. Show the steps you used.

19. $4 \times 38 =$ 152

Homework Help

$4(30) + 4(8)$
$= 120 + 32$
$= 152$

20. $11 \times 27 =$ 297

$11(20) + 11(7)$
$= 220 + 77$
$= 297$

21. $3 \times 3.9 =$ 11.7

$3(3) + 3(0.9)$
$= 9 + 2.7$
$= 11.7$

Use the Distributive Property to rewrite each algebraic expression.

22. $4(x + 2) =$ $4x + 8$

23. $3(x + 7) =$ $3x + 21$

24. $5(2x + 7) =$ $10x + 35$

25. **Be Precise** Mrs. Sumayya bought 9 folders and 9 notebooks. The cost of each folder was AED 2.50. Each notebook cost AED 4. Write two equivalent expressions and then find the total cost.

$9(2.50 + 4) = 9(2.50) + 9(4)$; AED 58.50

26. **Be Precise** Five friends bought admission tickets to the museum and a box lunch. The cost of each admission ticket was AED 11.75. Each box lunch cost AED 20. Write two equivalent expressions and then find the total cost.

$5(11.75 + 20) = 5(11.75) + 5(20)$; AED 158.75

Factor each expression.

27. $27 + 12 =$ $3(9 + 4)$

28. $12 + 36 =$ $12(1 + 3)$

29. $16 + 20 =$ $4(4 + 5)$

30. $2x + 8 =$ $2(x + 4)$

31. $30 + 12x =$ $6(5 + 2x)$

32. $42x + 49 =$ $7(6x + 7)$

Power Up! Test Practice

Exercises 33 and 34 prepare students for more rigorous thinking needed when taking the assessment.

33.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

34.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly model the expression and find the total amount spent.
1 point	Students correctly model the expression OR find the total amount spent.

Power Up! Test Practice

33. Determine if each statement illustrates the Distributive Property. Select yes or no.

a. $7x + 1 = 7(x + 1)$	☐ yes	■ no
b. $3x + 6 = 3(x + 2)$	■ yes	☐ no
c. $5(x + 4) = 5x + 20$	■ yes	☐ no
d. $9(x + 4) = 9x + 4$	☐ yes	■ no

34. Salem and three of his friends ate lunch together at a deli. Each person ordered a sandwich and a drink.

Item	Cost (AED)
Sandwich	2.75
Drink	1.25

Fill in each box to write an expression to represent the amount they spent altogether.

4 × (2.75 + 1.25)

1.25	1.50	2
2.75	3	4

How much did Salem and his friends spend altogether? AED 16

Spiral Review

Evaluate each expression.

35. $4 + 5.23 + 3 =$ 12.23

36. $4 \times 0 \times 9.17 =$ 0

37. $1.8 \times 1 \times 2 =$ 3.6

38. Suha and her sister Shaikha recorded the amount they saved each week for a month. How much did each person save? Use the information in the table to compare the total amount that Suha saved to the total amount Shaikha saved. AED 65; Since 20 + 15 + 10 + 20 = 15 + 20 + 10 + 20, they each saved the same amount.

Week	Suha's Savings (AED)	Shaikha's Savings (AED)
1	20	15
2	15	20
3	10	10
4	20	20

39. Each bottle holds 500 milliliters of water. Bottles are packaged in 4 rows of 6 bottles. How many milliliters of water are in each package? 12,000 milliliters

Inquiry Lab

Equivalent Expressions

Inquiry **HOW do you know that two expressions are equivalent?**

MP Mathematical Practices 1, 3, 4

Faris and his friends bought tickets for the dirt bike rally. The cost of each ticket was x dirhams. Faris bought 2 tickets on Saturday and 3 tickets on Sunday. They paid AED 4 for parking. The expression $2x + 4 + 3x$ represents the total cost in dirhams of the dirt bike rally.

Hands-On Activity

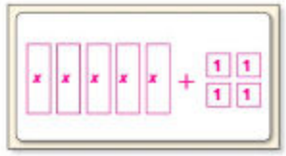
Tools

Simplify the expression $2x + 4 + 3x$ using algebra tiles.

Step 1 Choose tiles to represent each addend. Use [2] x-tiles to model $2x$, [4] 1 tiles to model 4, and [3] x-tiles to model $3x$.

Step 2 Find the like terms. The like terms are [2x] and [3x] because they are both x-tiles. There are a total of [5] x-tiles and four 1-tiles.

Step 3 Draw the algebra tiles in the space below, placing all like terms together.



Step 4 Rewrite the expression using addition to combine the like terms. Add $2x$ and $3x$.

So, $2x + 4 + 3x =$ [5x] + [4].

Rearrange the algebra tiles to determine if $2x + 4 + 3x$ is equivalent to $4x + x + 4$. Are they equivalent expressions? [yes]

Focus narrowing the scope

Objective Use models to simplify algebraic expressions.

Coherence moving from concrete to abstract

Now Students will use models to determine if two expressions are equivalent.

Next Students will write equivalent expressions by simplifying.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 490.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Materials: algebra tiles

Hands-On Activity

AL LA **Cooperative Play** Provide students with algebra tiles. Give students time to play with the tiles and discover how they will be used in the activity. Have pairs of students work together to complete each step in the activity. MP 1, 4, 6

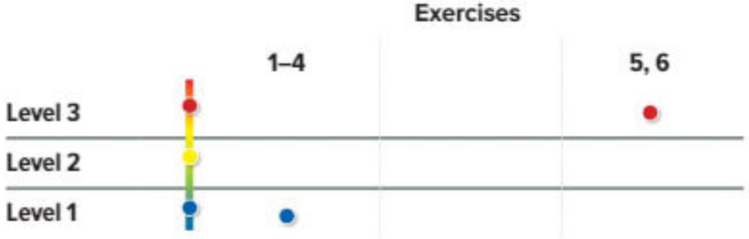
BL LA **Cooperative Play** Provide students with algebra tiles. Challenge them to model the expression $2x + 4 + 3x$ without looking at the diagram in their texts. Have them determine on their own how to rearrange the tiles to generate an equivalent expression. MP 1, 2, 4, 5, 6

2 Collaborate

The **Investigate** section is intended to be used as a small-group investigation. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL **LA** **Pairs Discussion** Have students work with a partner to complete Exercises 1–4. Have them discuss and respond to the following question. MP 1, 2, 3, 7

Ask:

- *Without using algebra tiles, how can you tell how many total x-tiles you will use to model the expression in Exercise 1?* Sample answer: Because all 3 terms use *x*-tiles, we can add together the terms, so I know that I will use 6 total *x*-tiles.

Create

BL **LA** **Pairs Check** In pairs, have students extend Exercise 5 by responding to the following question. MP 1, 3, 7

Ask:

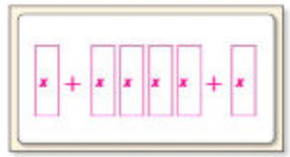
- *If Nahla is 8 years old, how old are Hassan and Noura?? Explain.* Hassan is 8 + 4, or 12 years old. Noura? is 3(8 + 4), or 36 years old.

inquiry Students should be able to answer "HOW do you know that two expressions are equivalent?" Check for student understanding and provide guidance, if needed.

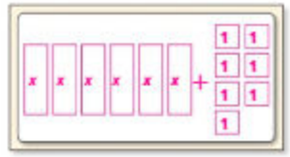
Investigate

Work with a partner. Simplify each expression using algebra tiles. Draw algebra tile models to represent each expression.

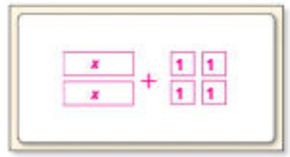
1. $x + 4x + x =$ 6x



2. $4x + 7 + 2x =$ 6x + 7

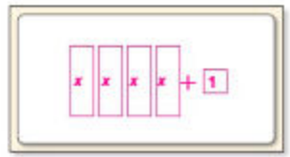


3. $2(x + 2) =$ 2x + 4



4. Determine if the expressions $x + 1 + 3x$ and $4x + 1$ are equivalent using algebra tiles. Draw your tiles at the right.

They are equivalent.



Create

5. MP **Model with Mathematics** Nahla is *x* years old. Her brother Hassan is 4 years older than her. Noura is 3 times as old as Hassan. Write and simplify an expression that represents Noura's age. Explain. 3(x + 4) = 3x + 12; Sample answer: If Nahla is *x* years old, Hassan's age can be represented with the expression *x* + 4. To find Noura's age, multiply the expression that represents Hassan's age by 3.

6. inquiry How do you know that two expressions are equivalent?

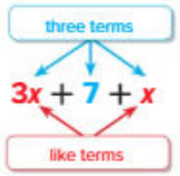
Sample answer: The expressions 2(*x* + 1) and 2*x* + 2 are equivalent because both can be modeled using 2 *x*-tiles and 2 integer tiles. They have the same value.

Lesson 7

Equivalent Expressions

Vocabulary Start-Up

When addition or subtraction signs separate an algebraic expression into parts, each part is called a **term**. The numerical factor of a term that contains a variable is called the **coefficient**. A term without a variable is called a **constant**. **Like terms** are terms that contain the same variables, such as x, $2x$, and $3x$.

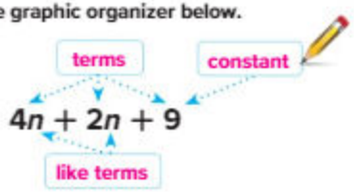


The three terms are $3x$, 7, and x.

The terms $3x$ and x are like terms because they have the same variable, x.

The constant is 7.

Label the graphic organizer below.



Essential Question

HOW is it helpful to write numbers in different ways?

Vocabulary

term
coefficient
constant
like terms

Mathematical Practices

1, 3, 4, 5, 7

Real-World Link

Games Ayoub's mother gave him a computer game and AED 10 for his graduation. His aunt gave him two computer games and AED 5. The expression $x + 10 + 2x + 5$, where x represents the cost of each game, can be used to represent Ayoub's graduation gifts.

1. What is the coefficient of the term $2x$? 2
2. How many terms are in the expression $x + 10 + 2x + 5$? 4

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Use properties to simplify expressions.

Coherence connecting within and across grades

Previous
Students used models to determine if two expressions were equivalent.

Now
Students write equivalent expressions using the properties of operations.

Next
Students will solve one-step equations and inequalities.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 495.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Pairs Discussion** Have students complete the Vocabulary activity and Real-World Link, ensuring that each student understands the meaning of each vocabulary term. MP 1, 3, 6

Alternate Strategies

AL LA Have students discuss how the word *constantly* helps them to remember the meaning of *constant* in mathematics. MP 1, 3, 6

BL Have students write an expression containing three terms, two of which are like terms. There should be at least one variable with a coefficient of 1. MP 1, 2, 3, 4

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Simplify an expression with one variable.**

AL • *What operation do the parentheses indicate?* multiplication

OL • *What property allows you to regroup multiplication?* Associative Property

• *What is 4 • 6?* 24 *What is 24 • x?* 24*x*

BL • *Would the simplified expression be the same if the original expression was 4(6x)? Explain.* yes; 4(6*x*) = (4 • 6)*x* or 24*x*

Need Another Example?

Simplify the expression 7(3*y*). 21*y*

2. **Simplify an expression with one variable.**

AL • *What is the cost of admission?* AED*x*

• *What is the cost to see the mummy exhibit?* AED1

• *How many friends will view the mummy exhibit?* 3

• *How many friends will not view the mummy exhibit?* 1

OL • *What expression represents the cost to get into the museum and see the exhibit?* *x* + 1 *What expression represents this cost for the three friends altogether?* 3(*x* + 1)

• *How will you find the total cost for all 4 friends?* Add *x* to 3(*x* + 1).

BL • *If admission is AED7, how much will it cost all 4 friends?* AED31

Need Another Example?

Four friends are going to a baseball game. It will cost AED*x* each to get into the game and three of the friends will buy a hot dog for AED2 each. Write and simplify an expression that represents the total cost for the four friends.

3(*x* + 2) + *x*; 4*x* + 6

Work Zone

Equivalent Expressions

Two expressions are equivalent when the expressions have the same value, no matter what value is substituted for *x*. So, 24*x* is equivalent to 4(6*x*).

Show your work.

a. 33*x*

b. 3*x*

c. 8*x* + 8

d. 4(*x* + 10) + 2*x*; AED 6*x* + AED 40

Simplify Expressions with One Variable

To simplify an algebraic expression, use properties to write an equivalent expression that has no like terms and no parentheses.

Numbers	Variables
3 + 3 = 2(3) or 6	*x* + *x* = 2*x*

Example

1. **Simplify the expression 4(6*x*).**

4(6*x*) = 4 • (6 • *x*) Parentheses indicate multiplication.

= (4 • 6) • *x* Associative Property

= 24*x* Multiply 4 and 6.

Got It? **Do these problems to find out.**

Simplify each expression.

a. (3 • *x*) • 11 b. *x* + *x* + *x* c. 7*x* + 8 + *x*

Real World Example

2. **Three friends will pay AED *x* each for admission to the museum plus AED 10 each to view the mummy exhibit. A fourth friend will pay admission but will not view the mummy exhibit. Write and simplify an expression that represents the total cost.**

The expression 3(*x* + 10) + *x* represents the total cost.

3(*x* + 10): cost of admission and exhibit for three friends

x: cost of admission for the fourth friend

3(*x* + 10) + *x* = 3*x* + 30 + *x* Distributive Property

= 3*x* + *x* + 30 Commutative Property

= 4*x* + 30 Combine like terms.

So, the total cost is AED 4*x* + AED 30.

Got It? **Do this problem to find out.**

d. Write and simplify an expression for the total cost of six friends to go to the museum if only four friends view the mummy exhibit.

Expressions and Equations

Simplify Expressions with Two Variables

Properties can be used to simplify or to factor expressions with two variables.

Compare the effects of operations on numbers to the effects of operations on variables.

Numbers	Variables
$3 + 3 + 4 = 2(3) + 4$	$x + x + y = 2x + y$

Examples

3. **Simplify the expression $(14y + x) + 22y$.**

$(14y + x) + 22y = (x + 14y) + 22y$ Commutative Property

$= x + (14y + 22y)$ Associative Property

$= x + 36y$ Combine like terms.

4. **Simplify $4(2x + y)$ using the Distributive Property.**

$4(2x + y) = 4(2x) + 4(y)$ Distributive Property

$= 8x + 4y$ Multiply.

5. **Factor $27x + 18y$.**

Step 1 Find the GCF of $27x$ and $18y$.

$27x = 3 \cdot 3 \cdot 3 \cdot x$ Write the prime factorization of $27x$ and $18y$.

$18y = 2 \cdot 3 \cdot 3 \cdot y$ Circle the common factors.

The GCF of $27x$ and $18y$ is $3 \cdot 3$ or 9.

Step 2 Write each term as a product of the GCF and its remaining factor. Then use the Distributive Property to *factor out* the GCF.

$27x + 18y = 9(3x) + 9(2y)$ Rewrite each term using the GCF.

$= 9(3x + 2y)$ Distributive Property

Got It? **Do these problems to find out.**

e. Simplify $3x + 9y + 2x$.

f. Simplify $7(3x + y)$.

g. Factor $12x + 8y$.

Show your work.

e. $5x + 9y$

f. $21x + 7y$

g. $4(3x + 2y)$

Examples

3. **Simplify an expression with two variables.**

AL
- *What are like terms?* terms that have the same variable
- *What are the like terms in this expression?* $14y$ and $22y$

OL
- *What property allows you to change the order in which numbers are added?* Commutative Property
- *What property allows you to regroup numbers being added?* Associative Property
- *What is $14y + 22y$?* $36y$

BL
- *Could the simplified expression also be written as $36y + x$? Explain.* yes; Addition is commutative.

Need Another Example?

Simplify the expression $(7x + y) + 15x$. $y + 22x$

4. **Simplify an expression with two variables.**

AL
- *What operation is indicated by the parentheses?* multiplication
- *Will the number 4 be multiplied only by $2x$? Explain.* No; 4 will also be multiplied by y.

OL
- *What is $4 \cdot 2x$?* $8x$ *What is $4 \cdot y$?* $4y$

BL
- *Explain why the expression cannot be simplified to $12xy$.* $8x$ and $4y$ are not like terms, so they cannot be added.

Need Another Example?

Simplify the expression $9(x + 3y)$. $9x + 27y$

5. **Factor an expression.**

AL
- *What is the prime factorization of $27x$? $18y$?* $3 \cdot 3 \cdot 3 \cdot x$; $2 \cdot 3 \cdot 3 \cdot y$

OL
- *What is the GCF of $27x$ and $18y$? How did you find this?* 9; Each number has $3 \cdot 3$, or 9, as a common factor.
- *When we factor out the GCF, what are we left with inside the parentheses?* $3x + 2y$

BL
- *How can you check to see if you factored correctly?* Use the Distributive Property to simplify $9(3x + 2y)$. When you distribute the 9, the product is $27x + 18y$, the original expression.

Need Another Example?

Factor $16x + 8y$. $8(2x + y)$

Example

6. **Write and simplify an expression to solve a real-world problem.**

AL
- *How many apples are in each basket?* 3 *What expression represents the cost of 3 apples?* $3(a)$ or $3a$
- *How many pears are in each basket?* 1 *What expression represents the cost of 1 pear?* p

OL
- *What expression represents the apples and pears in one basket?* $3a + p$
- *What do you need to do to find the total cost of 5 baskets?* Multiply $3a + p$ by 5.
- *What is* $5 \times 3a$*?* $15a$ *What is* $5 \times p$*?* $5p$

BL
- *How would the simplified expression change for the cost of 5 baskets if each basket also included 2 mangoes?* $15a + 5p + 10m$

Need Another Example?

A can of tennis balls comes with y yellow balls and x orange balls. Write and simplify an expression that represents the total of each color in 6 cans of tennis balls. $6(y + x)$; $6y + 6x$

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Discussion** Have students work in pairs to complete the Exercises. In Exercises 1–3, have them use highlighters to highlight any like terms. Ask them to discuss why Exercises 1 and 3 do not contain any like terms. MP 1, 3, 5

BL LA **Pairs Discussion** Have students work with a partner to extend Exercise 6 to determine the total cost if each bottle of nail polish costs AED1.99 and each tube of lip gloss costs AED2.25. MP 1, 3, 7

Example

6. **The farmer's market sells fruit baskets. Each basket has 3 apples and 1 pear. Use a to represent the cost of each apple and p to represent the cost of each pear. Write and simplify an expression that represents the total cost of 5 baskets.**

Use the expression $3a + p$ to represent the cost of each basket.

Use $5(3a + p)$ to represent the cost of 5 baskets.

Use the Distributive Property to rewrite $5(3a + p)$.

$5(3a + p) = 5(3a) + 5(p)$ Distributive Property

$= 15a + 5p$ Multiply.

So, the total cost of five baskets is $15a + 5p$.

Guided Practice

Simplify each expression. (Examples 1, 3, and 4)

1. $5(6x) =$ $30x$

2. $2x + 5y + 7x =$ $9x + 5y$

3. $4(2x + 5y) =$ $8x + 20y$

Show your work.

4. Factor $35x + 28y$. (Example 5) $7(5x + 4y)$

5. Ayesha bought five skirts at AED x each. Three of the five skirts came with a matching top for an additional AED 9 each. Write and simplify an expression that represents the total cost of her purchase. (Example 2)

   $3(x + 9) + 2x$; AED $5x +$ AED 27

6. The gift bag from Claire Cosmetics includes 5 bottles of nail polish and 2 tubes of lip gloss. Use p to represent the cost of each bottle of nail polish and g to represent the cost of each tube of lip gloss. Write and simplify an expression that represents the total cost of 8 gift bags. (Example 6)

   $8(5p + 2g) = 40p + 16g$

7. **Building on the Essential Question** How can properties help to write equivalent algebraic expressions?

   Sample answer: To find equivalent algebraic expressions, apply the properties and combine like terms, if needed.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

Name _____ My Homework _____

Independent Practice

Simplify each expression. (Examples 1, 3, and 4)

1. $x + 4x + 6x =$ **11x**

2. $3x + 4x + 5x =$ **12x**

3. $9(5x) =$ **45x**

Show your work.

4. $3x + 8y + 13x =$ **16x + 8y**

5. $7(3x + 5y) =$ **21x + 35y**

6. $3x + 6x + 2x =$ **11x**

Factor each expression. (Example 5)

7. $24x + 18y =$ **6(4x + 3y)**

8. $16x + 40y =$ **8(2x + 5y)**

9. Eight friends went to a hockey game. The price of admission per person was AED x. Four of the friends paid an extra AED 6 each for a player guide book. Write and simplify an expression that represents the total cost. (Example 2)

4(x + 6) + 4x; AED 8x + AED 24

10. Hala is x years old. Her sister, Hidaya, is six years older than she is. Their mother is twice as old as Hidaya. Their aunt, Huda, is x years older than their mother. Write and simplify an expression that represents Huda's age in years. (Example 2)

2(x + 6) + x; 3x + 12

11. A DVD box set includes 3 thriller movies and 2 comedies. Use t to represent the cost of each thriller and c to represent the cost of each comedy. Write and simplify an expression that represents the total cost of 6 box sets. (Example 6)

6(3t + 2c) = 18t + 12c

12. A fall candle gift set has 4 vanilla candles and 6 pumpkin spice candles. Use v to represent the cost of each vanilla candle and p to represent the cost of each pumpkin candle. Write and simplify an expression that represents the total cost of 4 sets. (Example 6)

4(4v + 6p) = 16v + 24p

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercise references of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–13, 15–17, 20, 34, 35
OL	On Level	1–11 odd, 13–17, 20, 34, 35
BL	Beyond Level	13–20, 34, 35

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	18, 19
2 Reason abstractly and quantitatively.	20
3 Construct viable arguments and critique the reasoning of others.	17
5 Use appropriate tools strategically.	15, 29
7 Look for and make use of structure.	16

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students simplify the expression $2y + y + 9y$. **12y**

Watch Out!

Common Error Students may incorrectly combine like terms when the coefficient is 1. Remind students that $x = 1x$ according to the Identity Property. You may wish to have students rewrite any terms that have a coefficient of 1.

Find the value of *y* that makes each equation true for all values of *x*.

13. $3x + 6x = yx$ **9**

14. $x + 5 + 11x = 12x + y$ **5**

15. **Use Math Tools** Pizza Palace charges AED *x* for a large cheese pizza and an additional fee based on the number of toppings ordered.

Pizza Palace Prices	
Pizza	**Price (AED)**
large cheese	*x*
add 1 topping	add AED0.75
add 2 toppings	add AED1.50
add 3 toppings	add AED2.25
add 4 toppings	add AED3.00

a. Two large cheese pizzas and three large beef pepperoni pizzas are ordered. Write and simplify an expression that represents the total cost. **3(*x* + 0.75) + 2*x*; AED 5*x* + AED 2.25**

b. Write and simplify an expression that represents the total cost of eight large pizzas, if two are cheese and six have four toppings each.
6(*x* + 3) + 2*x*; AED 8*x* + AED 18

c. Hana orders three large cheese pizzas, a large beef pepperoni and mushroom pizza, and a large green pepper and onion pizza. Write and simplify an expression that represents the total cost.
2(*x* + 1.50) + 3*x*; AED 5*x* + AED 3

H.O.T. Problems Higher Order Thinking

16. **Identify Structure** Write an expression that, when simplified, is equivalent to $15x + 7$. **Sample answer: 8*x* + 7 + 7*x***

17. **Reason Inductively** Explain why the expressions $y + y + y$ and $3y$ are equivalent.
Sample answer: The expressions are equivalent because they name the same number regardless of which number stands for *y*.

Persevere with Problems **For Exercises 18 and 19, simplify each expression.**

18. $7x + 5(x + 3) + 4x + x + 2$ **17*x* + 17**

19. $6 + 2(x + 8) + 3x + 11 + x$ **6*x* + 33**

20. **Reason Abstractly** The algebraic expression shown below is missing two whole-number constants. Determine the constants so that the expression simplifies to $14x + 11$.

$4x + 8(x +$ **1** $) +$ **3** $+ 2x$

Name _____ My Homework _____

Extra Practice

Simplify each expression.

21. $4x + 2x + 3x =$ 9x

$4x + 2x + 3x = (4x + 2x) + 3x$
$= 6x + 3x$
$= 9x$

22. $2x + 8x + 4x =$ 14x

23. $7(3x) =$ 21x

24. $8y + 4x + 6y =$ 14y + 4x

25. $4(7x + 5y) =$ 28x + 20y

26. $6x + 2x =$ 8x

Factor each expression.

27. $10x + 15y =$ 5(2x + 3y)

28. $35x + 63y =$ 7(5x + 9y)

29. **Use Math Tools** Four friends went to see a movie. Each ticket cost AED x. The table shows the prices of several items at the theater. They bought four large pretzels and four bottles of water. Write and simplify an expression that represents the total cost of tickets and snacks or beverages.

4(x + 3 + 2); AED 4x + AED 20

Snack or Beverage	Price
large popcorn	AED 4
large pretzel	AED 3
small soda	AED 2
bottle of water	AED 2

30. Seven friends have similar cell phone plans. The price of each plan is AED x. Three of the seven friends pay an extra AED 4 per month for unlimited text messaging. Write and simplify an expression that represents the total cost of the seven plans.

3(x + 4) + 4x; AED 7x + AED 12

31. A set of glassware includes 5 tall glasses and 3 juice glasses. Use t to represent the cost of each tall glass and j to represent the cost of each juice glass. Write and simplify an expression that represents the total cost of 4 sets.

4(5t + 3j) = 20t + 12j

Identify the terms, like terms, coefficients, and constants in each expression.

32. $4y + 5 + 3y$

terms: 4y, 3y, 5; like terms: 4y, 3y;

coefficients 4, 3; constant: 5

33. $2x + 3y + x + 7$

terms: 2x, 3y, x, 7; like terms: 2x, x;

coefficients: 1, 2, 3; constant: 7

Power Up! Test Practice

Exercises 34 and 35 prepare students for more rigorous thinking needed when taking the assessment.

34.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP2
Scoring Rubric	
2 points	Students answer each part of the question.
1 point	Students correctly answer four of the five parts of the question.

35.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

34. Use the expression $3n + 5p + 2 + n$.

List all of the terms of the expression. **3n, 5p, 2, n**

List the like terms of the expression. **3n, n**

List the coefficients of the expression. **3, 5, 1**

List the constants of the expression. **2**

What is the simplified form of the expression? **4n + 5p + 2**

35. A shipping company charges x dirhams to ship a package that weighs up to 1 kilogram and an additional fee based on the additional weight of the package.

Write and simplify an expression that represents the total cost of shipping 2 packages that each weigh 0.75 kilograms, 3 packages that each weigh 2.3 kilograms, and 1 package that weighs 4.2 kilograms.

2x + 3(x + 3) + (x + 6); 6x + 15

Shipping Prices	
Weight	**Price (AED)**
Up to 1 kg	x
Up to 2 kg	add AED 1.50
Up to 3 kg	add AED 3.00
Up to 4 kg	add AED 4.50
Up to 5 kg	add AED 6.00

Spiral Review

Find the missing number that makes the sentence true.

36. $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{\boxed{1}}{8}$

37. $\frac{4}{7} = \frac{2}{7} + \frac{\boxed{2}}{7}$

38. $2\frac{5}{9} = 2 + \frac{\boxed{5}}{9}$

39. Find the missing number in the pattern below.

14, 21, **28**, 35, 42, ...

40. Soccer balls cost AED 18 each. Complete the table and use a pattern to find the cost of 2, 3, and 4 soccer balls.

Number of Soccer Balls	Addition Pattern	Total Cost (AED)
1	18	AED 18
2	18 + 18	AED 36
3	18 + **18** + **18**	AED 54
4	18 + **18** + **18** + **18**	AED 72

21ST CENTURY CAREER in Engineering

Water Slide Engineer

Do you love riding the twisting, turning, plunging slides at water parks? Do you have ideas that would make them more fun and exciting? If so, you should think about a career designing water slides! Water slide engineers apply engineering principles, the newest technology, and their creativity to design state-of-the-art water slides that are both innovative and safe. These engineers are responsible for designing not only the winding flumes that riders slide down, but also the pumping systems that allow the slides to have the appropriate flow of water.

College & Career READINESS

Is This the Career for You?

Are you interested in a career as a water slide engineer? Take some of the following courses in high school.

- Algebra
- Computer-Aided Drafting
- Engineering Calculus
- Engineering Technology
- Physics

Find out how math relates to a career in Engineering.

Focus narrowing the scope

Objective Apply mathematics to problems arising in the workplace.

This lesson emphasizes **Mathematical Practice 3** Construct viable arguments and critique the reasoning of others.

Coherence connecting within and across grades

Previous
Students wrote and simplified numerical and algebraic expressions.

Now
Students apply the content standard to solve problems in the workplace.

Rigor pursuing concepts, fluency, and applications

See the Career Project on page 500.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ask students to read the information on the student page about space weather forecasters and answer the following questions.

Ask:

- *What is a water slide engineer's job?* Sample answers: to design innovative and safe water slides, to design the water flumes and the pumping systems for the water
- *What kinds of classes should you take to be a water slide engineer?* Algebra, Computer-Aided Drafting, Engineering Calculus, Engineering Technology, and Physics

2 Collaborate

AL LA **Pairs Discussion** Have students work in pairs to complete Exercises 1–4. As they work together to answer each exercise, have them respond to the following questions. MP 1, 3

Ask:

- *In Exercise 1, which operation is used to find the gallons of water pumped out in any number of minutes?* multiplication
- *In Exercise 4, which factor should be split into a sum to solve the problem mentally?* $2\frac{1}{2}$

BL LA **Think-Pair-Write** Have students individually complete Exercises 1–4. In pairs, have students choose two exercises to discuss with their partner. Students check each other's solutions and discuss and resolve any differences. Then have them write and solve their own real-world problem that involves the facts from the table. MP 1, 3, 4

Career Portfolio

When students complete this page, have them add it to their Career Portfolio.

Career Facts

Riders on The Black Hole experience strobe lights, lasers, a shimmering water wall, and sudden changes in temperature. The technology available today makes it so that these special effects can occur in random sequences. As a result, people have a different experience each time.

MP It's a Slippery Ride!

Use the information in the table to solve each problem.

1. The table shows the relationship between the number of minutes and the liters of water pumped out on The Black Hole. Write an expression to determine the number of liters pumped out for any number of minutes.

Number of Minutes (m)	Water Pumped Out (l)
3	3,000
6	6,000
9	9,000

$1{,}000m$

2. Refer to the fact about Big Thunder. Define a variable. Then write an expression that could be used to find the number of meters that riders travel in any number of seconds.
Let s represent the number of seconds; $30s$

3. Write two equivalent expressions that could be used to find the number of liters of water pumped out of the Crush 'n' Gusher ater 90 seconds. Then determine the number of liters pumped in 90 seconds.
$90(20) + 90(3)$; 90×23; 2,070 gal

4. Explain how you could use the Distributive Property to find how many liters of water are pumped out of The Black Hole in $2\frac{1}{2}$ minutes.
$2\frac{1}{2}(1{,}000)$
$= 2(1{,}000) + \frac{1}{2}(1{,}000)$
$= 2{,}000 + 500 = 2{,}500$ gal

Water Slides	
Water Slide, Park	**Fact**
Big Thunder, Rapids Water Park	At the steepest drop, riders travel about 9 meters per second
The Black Hole, Wet 'n wild	Riders plummet 150 meters as water is pumped out at 4,000 liters per minute
Crush 'n' Gusher, Typhoon Lagoon	The water jet nozzle on each slide pumps out about 92 liters of water per second.
Gulf Scream, Adventure Island	Riders hurl down a 63-meter slide at 40 kilometers per hour.

MP Career Project

It's time to update your career portfolio!
Find three water slides in the country. Use a spreadsheet to compare several features of the slides, such as the longest drop, total length, and gallons of water pumped. Describe how you, as a water slide engineer, would have designed the slides differently.

List several challenges associated with this career.

-
-
-
-
-

Chapter Review

Vocabulary Check

Complete the crossword puzzle using the vocabulary list at the beginning of the chapter.

Across

1. an expression which combines variables, numbers, and at least one operation
4. a mathematical language of symbols, including variables
7. numbers expressed using exponents
8. an expression which combines numbers and operations
9. in a power, the number used as a factor
10. expressions that have the same value
12. each part of an algebraic expression separated by a plus or minus sign
13. the numerical factor of a term that contains a variable

Down

2. to find the value of an algebraic expression
3. numbers with square roots that are whole numbers
5. terms that contain the same variables to the same power
6. in a power, the number that tells how many times the base is used as a factor
11. a symbol used to represent a number

Vocabulary Check

LA **Rally Coach** Have students work in pairs to complete the Vocabulary Check. Student 1 completes the first exercise while Student 2 watches, listens, coaches, and praises. Have students alternate roles for each exercise. MP 1, 3, 6

Alternate Strategy

AL LA To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include the following terms.

- algebra (Lesson 3)
- algebraic expression (Lesson 3)
- base (Lesson 1)
- coefficient (Lesson 7)
- equivalent expressions (Lesson 5)
- evaluate (Lesson 3)
- exponent (Lesson 1)
- like terms (Lesson 7)
- numerical expression (Lesson 2)
- perfect square (Lesson 1)
- powers (Lesson 1)
- term (Lesson 7)
- variable (Lesson 3)

Key Concept Check

FOLDABLES LA

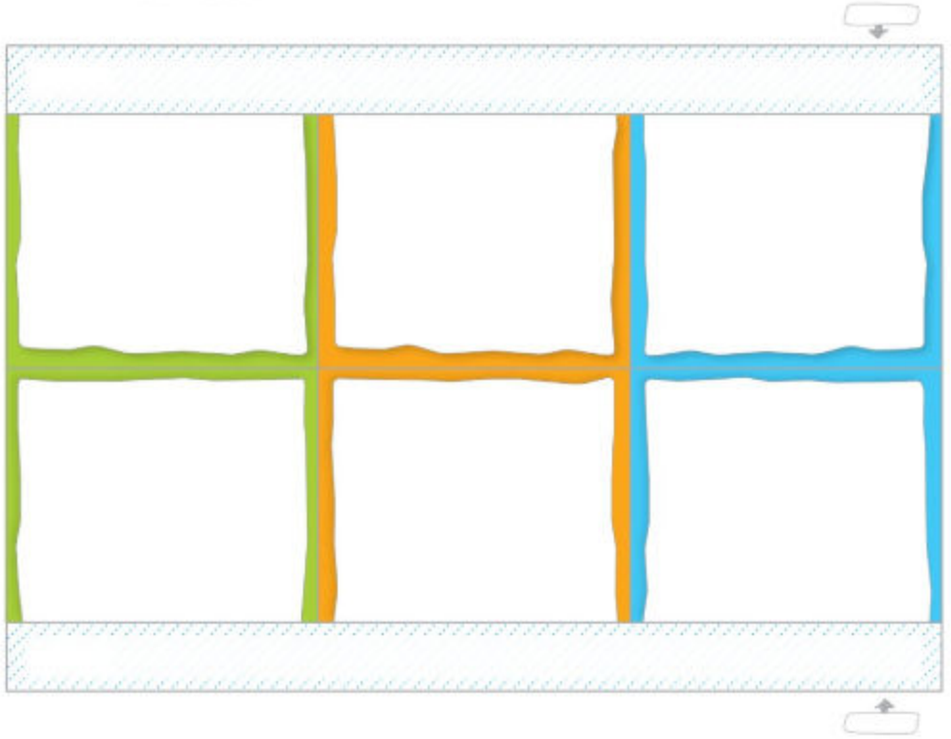
LA

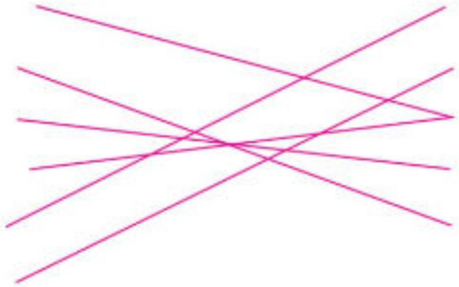
MP 1, 3, 4, 5

Concept	Exercise(s)
Distributive Property (Lesson 6)	1, 3, 4, 6
factoring expressions (Lesson 7)	2, 5

Key Concept Check

FOLDABLES



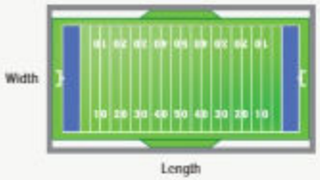


Expressions and Equations

Power Up! Performance Task

Cross Country Tryouts

The local high school is having tryouts for the cross country team. The school does not have a track, so the runners run around the school's football field. The cross country coach determines that the width of the field is seventy meters shorter than the length.



Write your answers on another piece of paper. Show all of your work to receive full credit.

Part A

Write an expression that represents the perimeter of the football field. Let x represent the length of the football field. Include parentheses in your expression. Next, write an equivalent expression that does not include parentheses. What property or properties did you use to simplify? Explain.

Part B

The cross-country coach later determines that the length of the football field is 120 meters. All students must run five laps. Using your answer from Part A, determine the actual number of yards that each athlete must run in the tryouts. In order to make the team, students must complete the laps in 6 minutes. How quickly must they run each lap?

Part C

Ismail is the manager of the football team, and he has been assigned the task of painting a mascot in the middle of the field. The painting fits neatly in the shape of a square with one side of the painting equal to five meters. The area of square is given by the formula $A = s^2$, where s is the length of a side. What is the area of the painting in square meters?

Power Up! Performance Task

This Performance-Based Assessment requires students to solve multi-step problems through abstract reasoning, precision, and perseverance. This practice scenario can be used to help students prepare for the thinking skills that will be used on the Assessment.

A complete scoring rubric with answers to the Exercises can be found on page PT1.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the **Building on the Essential Question** exercises found in each lesson of the chapter.

- How is using exponents helpful? (p. 436)
- How are grouping symbols helpful in simplifying expressions correctly. (p. 444)
- How are numerical expressions and algebraic expressions different? (p. 452)
- How can writing phrases as algebraic expressions help you solve problems? (p. 464)
- How can using properties help you to simplify expressions? (p. 476)
- How can the Distributive Property help you to rewrite expressions? (p. 488)
- How can properties help to write equivalent algebraic expressions? (p. 498)

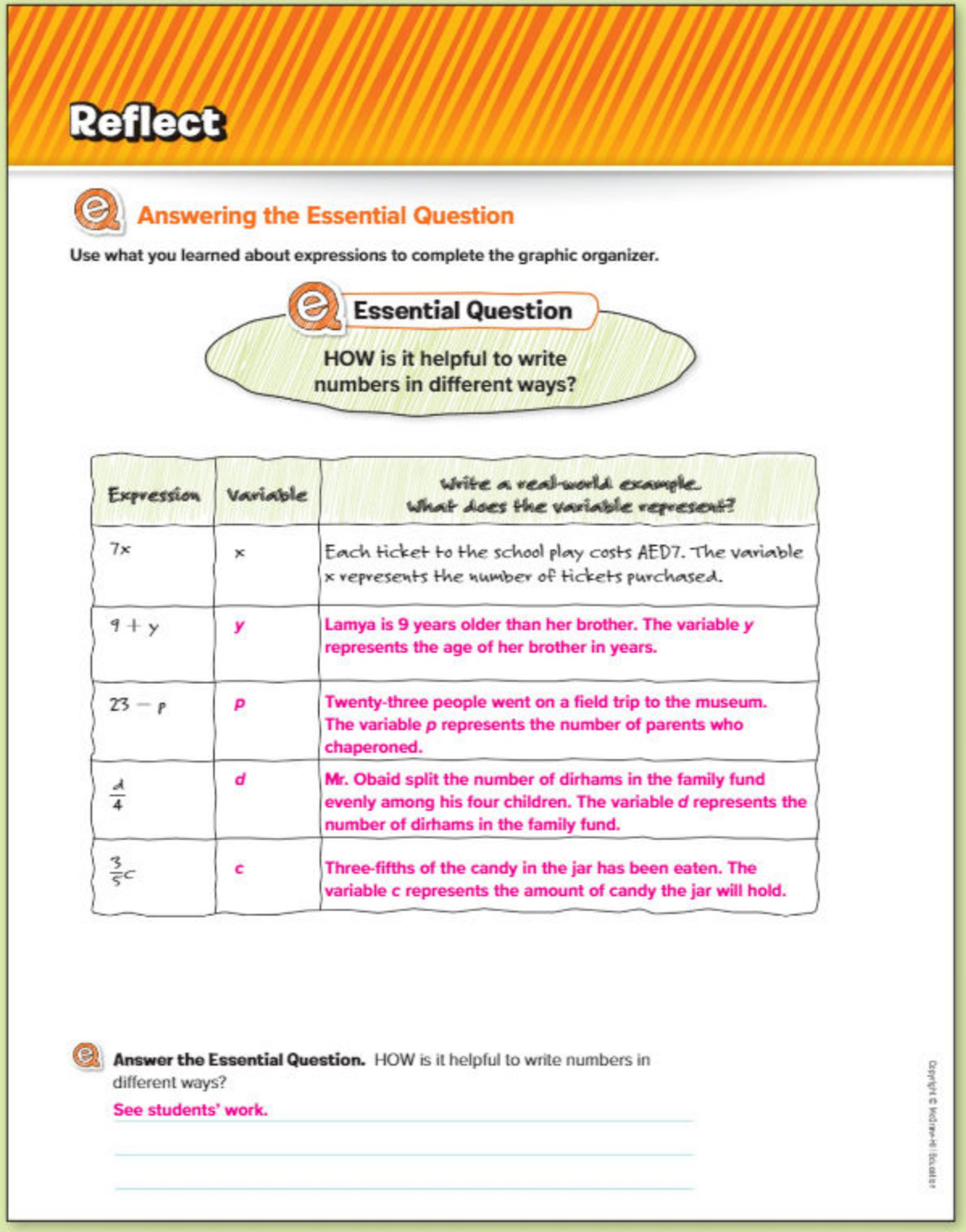
Ideas for Use

LA **Think-Pair-Share** Have students work in pairs. Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer. Then have them share their responses with their partner before they complete the graphic organizer.

MP 1, 3, 4, 5

Track Your Progress

Have your students return to pages xix–xxii to rate their knowledge. They should see that their knowledge of the key ideas has increased now that they have completed this chapter.



Reflect

Answering the Essential Question

Use what you learned about expressions to complete the graphic organizer.

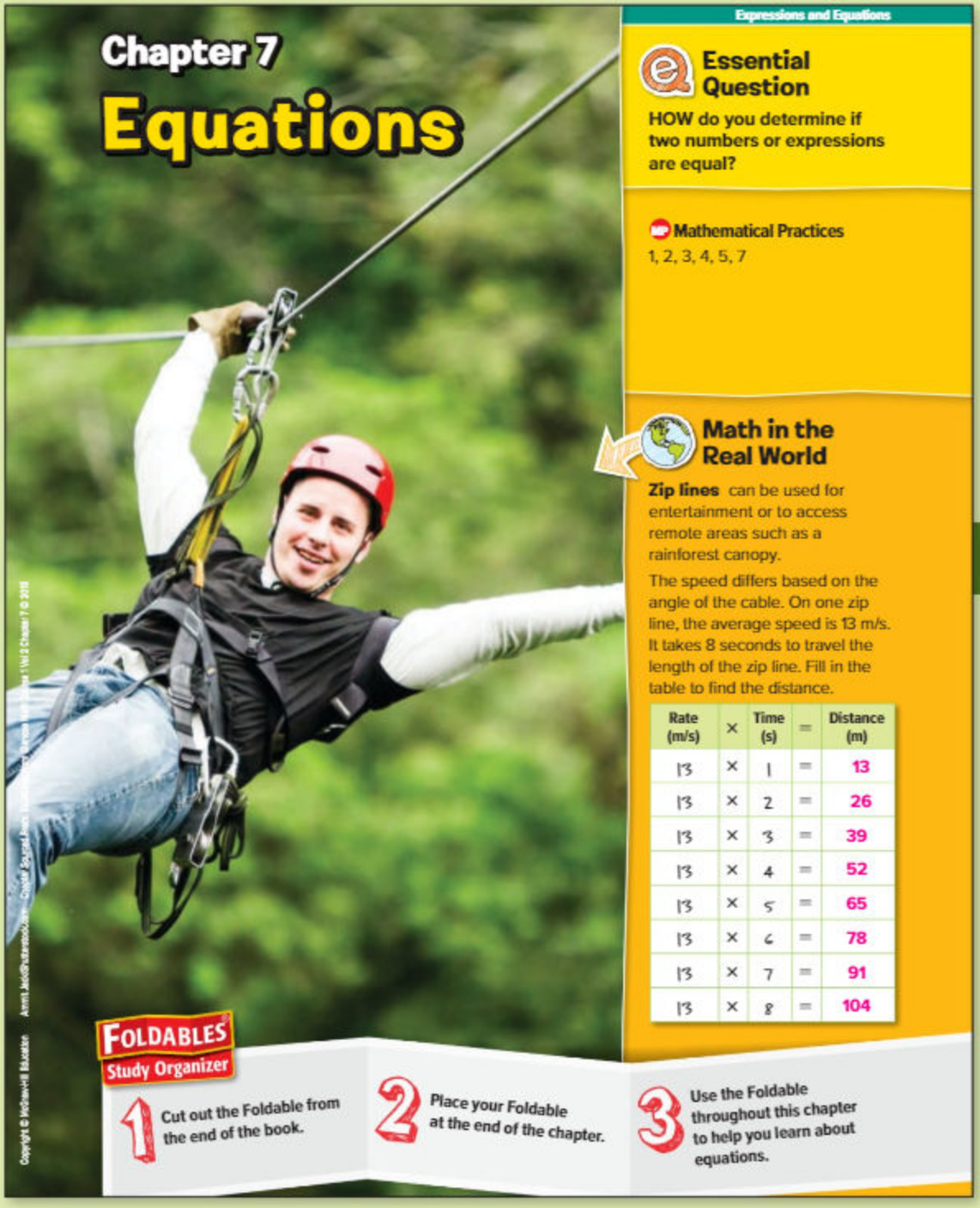
Essential Question

HOW is it helpful to write numbers in different ways?

Expression	Variable	Write a real-world example. What does the variable represent?
$7x$	x	Each ticket to the school play costs AED7. The variable x represents the number of tickets purchased.
$9 + y$	y	Lamya is 9 years older than her brother. The variable y represents the age of her brother in years.
$23 - p$	p	Twenty-three people went on a field trip to the museum. The variable p represents the number of parents who chaperoned.
$\frac{d}{4}$	d	Mr. Obaid split the number of dirhams in the family fund evenly among his four children. The variable d represents the number of dirhams in the family fund.
$\frac{3}{5}c$	c	Three-fifths of the candy in the jar has been eaten. The variable c represents the amount of candy the jar will hold.

Answer the Essential Question. HOW is it helpful to write numbers in different ways?

See students' work.



Focus narrowing the scope

This chapter focuses on **Expressions and Equations**.

Coherence connecting within and across grades

Previous
Students used properties of operations to write and evaluate expressions.

Now
Students write and solve one-step equations.

Next
Students will graph and solve functions, and write and solve inequalities.

Rigor pursuing concepts, fluency, and applications

The Levels of Complexity charts located throughout this chapter indicate how the exercises progress from conceptual understanding and procedural skills and fluency, to application and critical thinking.

Launch the Chapter

Math in the Real World

Zip Lines Explain to students that you can find the total distance traveled after any amount of time by multiplying the rate by the time.

What Tools Do You Need?

Vocabulary Activity

LA As you proceed through the chapter, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define: A coefficient is the numerical factor of a term that contains a variable.

Example: The coefficient in the mathematical sentence $20x + 4$ is 20.

Ask:

- *What is the coefficient in $5x - 3$?* 5

Studying Math

LA Have students read the Studying Math section. Students will learn to solve a long word problem by rewriting and simplifying the problem. Have students read the problem and the steps taken to solve it.

Ask:

- *Why do you rewrite the problem two times?* Sample answer: Rewriting the problem two times helps me define the question I need to answer and the information I need to use to answer it.

Have students read Exercise 1.

- *How would you first rewrite this problem?* Sample answer: Fahd has saved AED80 and plans to save AED5 each week. What is the total amount he will have saved after any number of weeks?

Have students read Exercise 2.

- *What will the variable represent in this problem?* Sample answer: The variable will represent any number of miles.

What Tools Do You Need?

Vocabulary

Addition Property of Equality	inverse operations
Division Property of Equality	Multiplication Property of Equality
equals sign	solution
equation	solve
expressions	Subtraction Property of Equality

Study Skill: Studying Math

Simplify the Problem Read the problem carefully to determine what information is needed to solve the problem.

Step 1 **Read the problem.**

Nasir wants to order several pairs of running shorts from an online store. They cost AED 14 each, and there is a one-time shipping fee of AED 7. What is the total cost of buying any number of pairs of shorts?

Step 2 **Rewrite the problem to make it simpler. Keep all of the important information but use fewer words.**

Nasir wants to buy some _____ that cost _____ each plus a shipping fee of _____. What is the total cost for any number of pairs of shorts?

Step 3 **Rewrite the problem using even fewer words. Write a variable for the unknown.**

The total cost of x shorts is _____ + _____.

Step 4 **Translate the words into an expression.**

_____

Use the method above to write an expression for each problem.

1. Fahd is saving money to buy a bicycle. He has already saved AED 80 and plans to save an additional AED 5 each week. Find the total amount he has saved after any number of weeks.

_____

2. A taxi company charges AED 1.50 per kilometer plus a AED 10 fee. What is the total cost of a taxi ride for any number of kilometers?

_____

What Do You Already Know?

Place a checkmark below the face that expresses how much you know about each concept. Then scan the chapter to find a definition or example of it.

I have no clue. I've heard of it. I know it!

Equations				
Concept				Definition or Example
inverse operations				
solving addition equations				
solving division equations				
solving multiplication equations				
solving subtraction equations				
writing equations				

When Will You Use This?

Here is an example of how equations are used in the real world.

Activity Describe a rewards system that you would use to earn a party for your class. What type of party would you want to have? How would you earn points? How long would you have to earn points? _____

_____

_____

_____

_____

What Do You Already Know?

In this activity students assess their prior knowledge choosing a face to represent their knowledge about concepts in the chapter.

After completing the chapter, have students return to this page and have them reevaluate their knowledge level about the content.

When Will You Use This?

Activity

Students learn how to create equations to develop a reward system.

Are You Ready?

Use this page to determine if students have skills that are needed for the chapter.

Quick Review

Students with strong math backgrounds may opt to go directly to the Quick Check.

REVIEW	
Example	**Skill**
1	Subtract decimals.
2	Subtract fractions.

Quick Check

If students have difficulty with the exercises, present an additional example to clarify any misconceptions.

Exercises 1–3

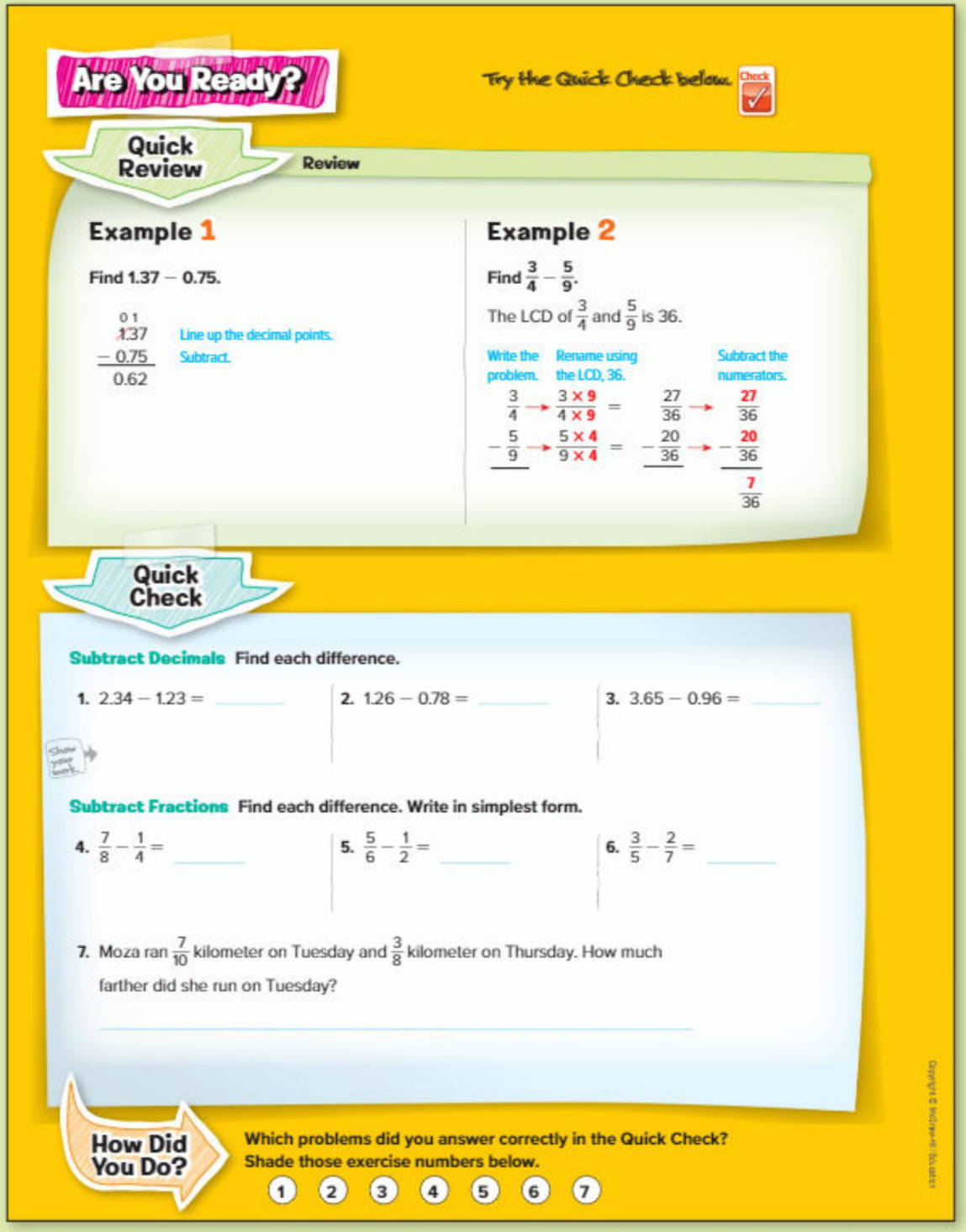
Find $1.87 - 0.63$. 1.24

Exercises 4–7

Find $\frac{2}{3} - \frac{4}{9}$. $\frac{2}{9}$

Track Your Progress

Prior to beginning this chapter, have your students go to pages xix–xxii to rate their current knowledge. At the end of the chapter, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge of the key ideas has increased.



Lesson 1

Equations

Vocabulary Start-Up

An **equation** is a mathematical sentence showing two expressions are equal. An equation contains an **equals sign**, =.

Equation	Expression
Definition A mathematical sentence showing two expressions are equal.	Definition A combination of numbers and operations which might contain variables.
Example $5 + x = 13$	Example $5 + x - 16$

How are an equation and an expression similar?
They both contain numbers and operations and may contain variables.

How are an equation and an expression different?
An equation has an equals sign, but an expression does not.

Essential Question

HOW do you determine if two numbers or expressions are equal?

Vocabulary

equation
equals sign
solve
solution

Mathematical Practices
1, 2, 3, 4, 7

Real-World Link

Shopping Maysoun bought a package of 6 pair of socks. She writes the equation below to find how much she paid per pair. Circle the *solution* of the equation.

$6x$ = AED 9

AED 0.50 (AED 1.50) AED 2.00

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Solve equations by using mental math and the *guess*, *check*, and *revise* strategy.

Coherence connecting within and across grades

Previous Students used properties of operations to create expressions.

Now Students solve equations mentally.

Next Students will write and solve one-step equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 513.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Paired Heads Together** Have students work in pairs to complete the graphic organizer. Have each student choose to become the spokesperson for the term *equation* or *expression*. Choose one pair of students to present their responses, having one student explain the definition and examples for *equation* and the other student explain the definition and examples for *expression*. MP 1, 2, 5

Alternate Strategy

AL LA Have students discuss with a partner how the letters *equa* in *equals* and *equation* help them to remember the definition of an equation. MP 1, 3

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Find values that make the equation true.**

AL • *What operation is performed on the variable?* addition

OL • *Replace a with 3. Is the sum 11?* no *Replace a with 4. Is the sum 11?* yes *Replace a with 5. Is the sum 11?* no

BL • *Explain how you could use subtraction to solve the equation.* Sample answer: You could subtract 7 from both sides of the equation. $11 - 7 = 4$

Need Another Example?

Is 5, 6, or 7 the solution of the equation $4 + b = 10$? 6

2. **Solve the equation mentally.**

AL • *What operation is performed on the variable?* subtraction

OL • *How can you use mental math to solve the equation?* Sample answer: $10 - 7 = 3$, so the solution is 10.

BL • *Explain how you can solve the equation using addition.* Sample answer: I can add 7 to both sides of the equation. Since $3 + 7 = 10$, the solution is 10.

Need Another Example?

Solve $m - 5 = 2$ mentally. 7

3. **Solve the equation mentally.**

AL • *What do you need to find?* the cost of the kneepads

OL • *What equation is used to represent the situation?* $45 + k = 63$

BL • *How can you use subtraction to solve the equation?* Sample answer: Subtract 45 from both sides. Since $63 - 45 = 18$, the solution is 18.

Need Another Example?

Mazen and Ayoub have 72 marbles together. Ayoub has 12 marbles. Use the *guess, check, and revise* strategy to solve the equation $72 - d = 12$ to find d, the number of marbles that Mazen has. 60 marbles

Work Zone

STOP and Reflect

How can you check if your solution to an equation is correct?

Substitute the solution for the variable and see if it makes the sentence true.

Solve Addition and Subtraction Equations Mentally

When you replace a variable with a value that results in a true sentence, you **solve** the equation. That value for the variable is the **solution** of the equation.

$2 + x = 9$ — The value for the variable that results in a true sentence is 7. So, 7 is the solution.

$2 + 7 = 9$

$9 = 9$ — This sentence is true.

Examples

1. **Is 3, 4, or 5 the solution of the equation $a + 7 = 11$?**

Value of a	$a + 7 \stackrel{?}{=} 11$	Are Both Sides Equal?
3	$3 + 7 \stackrel{?}{=} 11$ $10 \neq 11$	no
4	$4 + 7 \stackrel{?}{=} 11$ $11 = 11$	yes ✓
5	$5 + 7 \stackrel{?}{=} 11$ $12 \neq 11$	no

The solution is 4.

2. **Solve $g - 7 = 3$ mentally.**

$g - 7 = 3$ — Think What number minus 7 equals 3?

$10 - 7 = 3$ — You know that $10 - 7 = 3$.

$3 = 3$

The solution is 10.

3. **The total cost of a pair of skates and kneepads is AED 63. The skates cost AED 45. Use the guess, check, and revise strategy to solve the equation $45 + k = 63$ to find k, the cost of the kneepads.**

Use the *guess, check, and revise* strategy.

Try 14.

$45 + k = 63$

$45 + 14 \stackrel{?}{=} 63$

$59 \neq 63$

Try 16.

$45 + k = 63$

$45 + 16 \stackrel{?}{=} 63$

$61 \neq 63$

Try 18.

$45 + k = 63$

$45 + 18 \stackrel{?}{=} 63$

$63 = 63$ ✓

So, the kneepads cost AED 18.

Got it? Do these problems to find out.

a. Is 4, 5, or 6 the solution of the equation $c + 8 = 13$?

b. Solve $9 - x = 2$ mentally.

c. The difference between an ostrich's speed and a chicken's speed is 50 kilometers per hour. An ostrich can run at a speed of 64 kilometers per hour. Use mental math or the *guess, check, and revise* strategy to solve the equation $64 - c = 50$ to find c, the speed a chicken can run.

a. 5

b. 7

c. 14 kilometers per hour

Solve Multiplication and Division Equations Mentally

Multiplication and division equations are solved in a similar way to addition and subtraction equations.

Examples

4. **Is 3, 4, or 5 the solution of the equation $18 = 6z$?**

Value of z	$18 \stackrel{?}{=} 6z$	Are Both Sides Equal?
3	$18 \stackrel{?}{=} 6 \cdot 3$ $18 = 18$	yes ✓
4	$18 \stackrel{?}{=} 6 \cdot 4$ $18 \neq 24$	no
5	$18 \stackrel{?}{=} 6 \cdot 5$ $18 \neq 30$	no

The solution is 3.

5. **Solve $16 \div s = 8$ mentally.**

$16 \div s = 8$ Think 16 divided by what number equals 8?

$16 \div 2 = 8$ You know that $16 \div 2 = 8$.

$8 = 8$

The solution is 2.

Got it? Do these problems to find out.

d. Is 2, 3, or 4 the solution of the equation $4n = 16$?

e. Solve $24 \div w = 8$ mentally.

d. 4

e. 3

Examples

4. **Find values that make the equation true.**

AL
- *What is the variable in the equation?* z
- *What operation is performed on the variable?* multiplication

OL
- *Replace z with 3. Is the product 18?* yes
- *Replace z with 4. Is the product 18?* no
- *Replace z with 5. Is the product 18?* no
- *Which value makes the solution true?* 3
- *What is the solution to the equation?* 3

BL
- *What is the inverse operation of multiplication?* division
- *Explain how you could use division to solve the equation.* Sample answer: You could divide both sides of the equation by 6 to find the value of z.

Need Another Example?
Is 3, 4, or 5 the solution of the equation $16 = 4s$? 4

5. **Solve the equation mentally.**

AL
- *What is the variable?* s
- *What operation is performed in the equation?* Division

OL
- *How can you use mental math to determine the value of s?* Sample answer: I know that 16 divided by 2 equals 8, so the value of s must be 2.
- *If someone told you the solution was 4, how would you explain to them that this was incorrect?* Sample answer: $16 \div 4 \neq 8$

BL
- *Dividing by 2 is the same as multiplying by what number?* $\frac{1}{2}$
- *If the equation was $s \div 16 = 8$, what would you do to solve the equation? What is the solution?* Sample answer: Multiplication and division are inverse operations, so multiply 8 by 16. The solution is 128.

Need Another Example?
Solve $20 \div d = 5$ mentally. 4

Examples

6. Solve the equation mentally.

AL • *What do you need to find?* the number of packages of gum that Fahd bought

• *What equation is used to represent the situation?* $8 \cdot p = 72$

OL • *How do you know that 7 is not the solution to the equation?* The product of 8 and 7 is not equal to 72.

• *How do you know that 8 is not the solution to the equation?* The product of 8 and 8 is not equal to 72.

BL • *Explain how you could use division to solve the equation.*
Sample answer: Since multiplication and division are inverse operations, you could divide both sides by 8 to find p.

Need Another Example?
Ayoub has 60 baseball cards. There are 10 baseball cards in each package. Use the *guess, check, and revise* strategy to solve the equation $10 \cdot b = 60$ to find b, the number of packages of cards Ayoub has. 6 packages

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Think-Pair-Share** Give students about one minute to think about how they would respond to Exercises 1–7. Then have them pair with a partner to complete the exercises. Call on one pair of students to explain their responses to the class. MP 1, 3

BL LA **Numbered Heads Together** In groups of 3 or 4, have students think of alternative methods they could use to find the solution to the equations in Exercises 1–6. For example, they may discuss the *work backward* strategy or using inverse operations. Have them compare and contrast the method(s) with the ones presented in this lesson. Ask them which method they prefer. MP 1, 3

Example

6. **Fahd bought 72 sticks of gum. There are 8 sticks of gum in each package. Use the guess, check, and revise strategy to solve the equation $8 \cdot p = 72$ to find p, the number of packages Fahd bought.**

Use the *guess, check, and revise* strategy.

Try 7.	Try 8.	Try 9.
$8 \cdot p = 72$	$8 \cdot p = 72$	$8 \cdot p = 72$
$8 \cdot 7 \stackrel{?}{=} 72$	$8 \cdot 8 \stackrel{?}{=} 72$	$8 \cdot 9 \stackrel{?}{=} 72$
$56 \neq 72$	$64 \neq 72$	$72 = 72$ ✓

So, Fahd bought 9 packages of gum.

Guided Practice

Identify the solution of each equation from the list given. (Examples 1 and 4)

1. $9 + w = 17$; 7, 8, 9 8

Show your work.

2. $8 \div c = 8$; 0, 1, 2 1

Solve each equation mentally. (Examples 2 and 5)

3. $x - 11 = 23$ 34

4. $4x = 32$ 8

5. Mississippi and Georgia have a total of 21 electoral votes. Mississippi has 6 electoral votes. Use mental math or the *guess, check, and revise* strategy to solve the equation $6 + g = 21$ to find g, the number of electoral votes Georgia has. (Example 3)
15 votes

6. Huda and her sister collect stickers. Huda has 220 stickers in her sticker collection. Her sister has 55 stickers in her collection. Huda has how many times as many stickers as her sister? Use mental math or the *guess, check, and revise* strategy to solve the equation $55x = 220$. (Example 6)
$x = 4$

7. **Building on the Essential Question** How do you solve an equation? By finding a value for the variable that makes the equation true.

Rate Yourself!

I understand how to solve equations.

Great! You're ready to move on!

I still have some questions about solve equations.

FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

Identify the solution of each equation from the list given. (Examples 1 and 4)

1. $29 + d = 54$; 24, 25, 26 **25**

Show your work.

2. $35 = 45 - n$; 10, 11, 12 **10**

3. $6w = 30$; 5, 6, 7 **5**

4. $x \div 7 = 3$; 20, 21, 22 **21**

Solve each equation mentally. (Examples 2 and 5)

5. $m + 4 = 17$ **13**

6. $12 = 24 - y$ **12**

7. $15 - b = 12$ **3**

8. $10t = 90$ **9**

9. $22 \div y = 2$ **11**

10. $54 = 6b$ **9**

MP **Identify Structure** **For Exercises 11–13, solve using mental math or the guess, check, and revise strategy.** (Examples 3 and 6)

11. One season, the Heroes won 20 games. They played a total of 25 games. Use the equation $20 + g = 25$ to find g, the number of games the team lost.

5 games

12. Five friends earn a total of AED 50 doing yard work in their neighborhood. Each friend earns the same amount. Use the equation $5f = 50$ to find f, the amount that each friend earns. **AED 10**

13. Last year, 700 students attended a cycle 2 school. This year, there are 665 students. Use the equation $700 - d = 665$ to find d, the decrease in the number of students from last year to this year.

35 students

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–16, 19, 20, 34, 35
OL	On Level	1–13 odd, 14–16, 19, 20, 34, 35
BL	Beyond Level	14–20, 34, 35

Watch Out!

Common Error Students may add instead of subtracting, or vice versa, when solving equations. Remind students to pay close attention to the sign in each equation. Have them check their solution by reading the completed addition or subtraction equation to make sure it is correct.

MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	17, 18
2 Reason abstractly and quantitatively.	15, 19
3 Construct viable arguments and critique the reasoning of others.	14, 16
4 Model with mathematics.	20
7 Look for and make use of structure.	11–13, 31–33

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students solve the equation $x + 5 = 5$. Ask students to explain how they solved it. 0; Sample answer: I used mental math to find the value of x true.

H.O.T. Problems Higher Order Thinking

14. **Reason Inductively** What 3 consecutive even numbers added together equal 42? Use the equation $n + (n + 2) + (n + 4) = 42$ to help you solve. 12, 14, 16

15. **Reason Abstractly** Give an example of an equation that has a solution of 5. Sample answer: $m + 8 = 13$

16. **Reason Inductively** Tell whether the statement below is *always*, *sometimes*, or *never* true.

Equations like $a + 4 = 8$ and $4 - m = 2$ have exactly one solution.

always

Persevere with Problems Tell whether each statement is true **or** false. **Then explain your reasoning.**

17. In $m + 8$, the variable m can have any value.

true; Since $m + 8$ is not equal to any specific value, there are no restrictions placed upon the value of m.

18. In $m + 8 = 12$, the variable m can have any value and be a solution.

false; This is an equation, so both sides of the equation must equal the same value. Therefore, $m + 8$ must equal 12 and m can only have one solution, 4.

19. **Reason Abstractly** Distinguish between expressions and equations algebraically, by providing an example of an algebraic expression and an example of an algebraic equation.

Sample answer: $14 + x$ is an algebraic expression. $14 + x = 20$ is an algebraic equation.

20. **Model with Mathematics** Write a real-world problem in which you would solve the equation $a + 12 = 30$.

Sample answer: Yousif has 12 baseball cards. Yousif and Nasser have a total of 30 baseball cards. Solve the equation $a + 12 = 30$ to find the number of baseball cards that Nasser has.

Name _____ My Homework _____

Extra Practice

Identify the solution of each equation from the list given.

21. $a + 15 = 23$; 6, 7, 8 8

Try 6. $6 + 15 \neq 23$ Try 7. $7 + 15 \neq 23$ Try 8. $8 + 15 = 23$ ✓

Homework Help

22. $19 = p - 12$; 29, 30, 31 31

23. $63 = 9k$; 6, 7, 8 7

24. $36 \div s = 4$; 9, 10, 11 9

Solve each equation mentally.

25. $j + 7 = 13$ 6

26. $22 = 30 - m$ 8

27. $25 - k = 20$ 5

28. $5m = 25$ 5

29. $d \div 3 = 6$ 18

30. $24 = 12k$ 2

MP **Identify Structure** **For Exercises 31–33, solve using mental math or the** guess, check, and revise **strategy.**

31. Ayesha made 36 cookies. She gave away 28 cookies. Use the equation $28 + c = 36$ to find c, the number of cookies she kept.

8 cookies

32. Sultan's family ate a total of 12 sandwiches at a cookout. Each family member ate 2 sandwiches. Use the equation $2m = 12$ to find m, the number of members in Sultan's family. 6 members

33. A bottlenose dolphin is 96 inches long. There are 12 inches in 1 foot. Use the equation $12d = 96$ to find d, the length of the bottlenose dolphin in feet.

8 ft

Power Up! Test Practice

Exercises 34 and 35 prepare students for more rigorous thinking needed for the assessment.

34.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

35.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

Power Up! Test Practice

34. Select the correct solution for each equation.

a. Yousif bought a box of 12 golf balls for AED 18. Solve the equation $12x$ = AED 18 to find the price of each golf ball.

☐ AED 1.25 ■ AED 1.50 ☐ AED 1.75

b. Najla is 5 years older than Amer. Najla is 16 years old. Solve the equation $r + 5 = 16$ to find Amer's age.

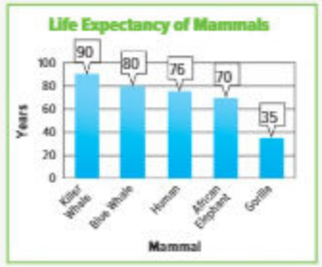
■ 11 years ☐ 16 years ☐ 21 years

c. Mr. Saeed divides 72 students into 12 equal groups. Solve the equation $\frac{72}{s} = 12$ to find the number of students in each group.

■ 6 students ☐ 8 students ☐ 84 students

35. The graph shows the life expectancy of certain mammals. Write and solve an equation to find the difference d in the number of years a blue whale lives and the number of years a gorilla lives.

35 + d = 80; 45 years



Spiral Review

Add.

36. 56 + 89 = 145

37. 37 + 26 = 63

38. 95 + 48 = 143

39. 29 + 86 = 115

40. 64 + 48 = 112

41. 31 + 62 = 93

42. The table shows the number of exhibition tickets the art club sold during the beginning of the week. On Thursday, the art club sold what they sold on Sunday and Tuesday together. How many tickets did they sell on Thursday? 96 exhibition tickets

Day	Tickets Sold
Sunday	42
Monday	67
Tuesday	54

Inquiry Lab

Solve and Write Addition Equations

Inquiry **HOW do you solve addition equations using models?**

MP Mathematical Practices 1, 3, 4,

Omar played two baseball games last weekend. He got 7 hits in all. He had 3 hits in the first game. How many hits did he get in the second game?

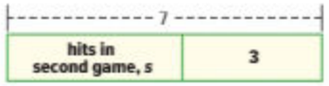
What do you know? Omar played two baseball games last weekend. He got 7 hits in all. He had 3 hits in the first game.

What do you need to find? the number of hits Omar had in the second game

Hands-On Activity 1

Step 1 Define a variable. Use the variable s to represent the number of hits Omar had in the second game.

Step 2 Use a bar diagram to help write the equation.



The total length of the diagram represents the total number of hits.

The 3 represents the number of hits in the first game.

$s + 3 = 7$

Step 3 Work backward. Rewrite the equation as a subtraction sentence and solve.

$7 - 3 = s$

So, Omar had 4 hits in the second game.

Focus narrowing the scope

Objective Solve addition equations using models.

Coherence connecting within and across grades

Now
Students use bar diagrams and cups and counters to solve one-step addition equations.

Next
Students will use Properties of Equality to solve addition equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 518.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance than Activity 2.

Materials: cups, counters, equation mats

Hands-On Activity 1

AL LA Make sure students understand that the variable s represents the unknown number of hits in the second game. Since Bryan got three hits in the first game, the model for the total number of hits is three longer than s. Remind students that the number on top of the bar diagram is the total number of hits, 7. Since the sum of $s + 3$ is 7, s is 4. MP 1, 4, 5

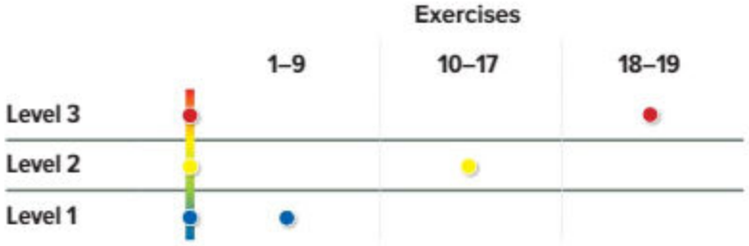
BL Omit Activities 1 and 2 and proceed directly to the Investigate section.

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Pairs Discussion** Have students work in pairs to complete Exercises 1 and 2. Have them trade their solutions with another pair of students and discuss any differences. MP 1, 3, 5

BL LA **Trade-a-Problem** Have students create their own problem, similar to Exercises 1 and 2. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. MP 1, 3, 4

Hands-On Activity 2

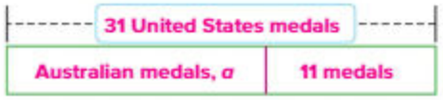
AL LA Explain that the cup represents an unknown number x because you do not know what is inside the cup. In Step 2, one counter is crossed out on each side of the mat. This was done in order to get the cup by itself.

Investigate

Work with a partner. Write and solve an addition equation using a bar diagram.

1. In the 2008 Summer Olympics, the United States won 11 more medals in swimming than Australia. The United States won a total of 31 medals. Find the number of medals won by Australia.

   $a + 11 = 31$; $a = 20$ medals

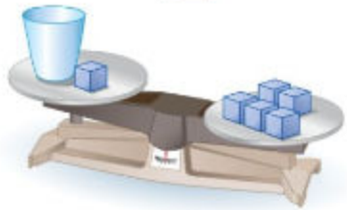
2. A lion can run 80 kilometers per hour. This is 30 kilometers per hour faster than a house cat. Find the speed of a house cat.

   $c + 30 = 80$; $c = 50$ kmph

Hands-On Activity 2

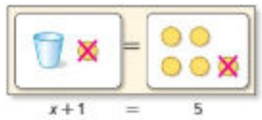
An equation is like a balance. The quantity on the left side of the equals sign is balanced with the quantity on the right.



To solve an addition equation using cups and counters, subtract the same number of counters from each side of the mat so that the equation remains balanced.

Solve $x + 1 = 5$ using cups and counters.

Step 1 Model the equation. Use a cup to represent x.



Step 2 Use the model above. Cross out 1 counter from each side so that the cup is by itself.

Step 3 There are 4 counters remaining on the right side, so $x =$ 4.

So, the solution is 4.

Check $x + 1 = 5$ Write the original equation.

$4 + 1 \stackrel{?}{=} 5$ Replace x with your solution.

$5 = 5$ Is the sentence true? yes

Expressions and Equations

Investigate

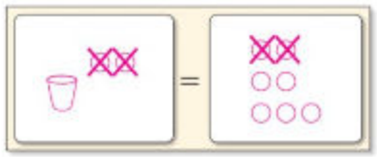
Work with a partner. Solve each equation using cups and counters. Draw cups and counters to show your work.

3. $1 + x = 8$

$x =$ 7

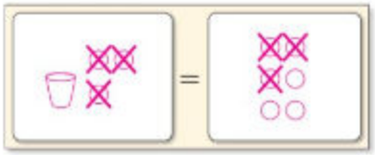
4. $x + 2 = 7$

$x =$ 5



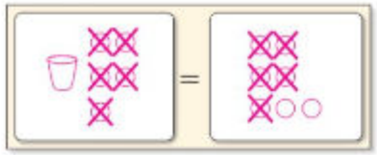
5. $3 + x = 6$

$x =$ 3



6. $x + 5 = 7$

$x =$ 2



Work with a partner. Write a real-world problem that can be represented by the equation. Then solve each addition equation using the model of your choice.

7. $9 = x + 3$

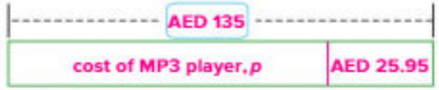
Sample answer: Suhaila is three years older than her sister Yasmin. Suhaila is 9 years old. How old is Yasmin? 6 years old

8. $4 + x = 6$

Sample answer: Mohammad had 4 video games and bought more with his allowance. He now has 6 video games. How many games did he buy? 2

9. Ayoub bought an MP3 player. He spent the rest of his money on an Internet music subscription for AED 25.95. If he started with AED 135, how much was the MP3 player? Write and solve an equation using a bar diagram.

$p + 25.95 = 135$; $p =$ AED 109.05



Investigate

AL LA **Think-Pair-Share** Have students work in pairs. Give students one minute to think through their answers to Exercises 3–9. Have them share their responses with their partner. Then call on one student to share their response within a small group or large group discussion. MP 1, 3

BL LA For Exercises 3–9, ask students to explain how they could model and solve each equation using algebra tiles instead of cups and counters. MP 1, 3, 5

Analyze and Reflect

AL LA **Rally Coach** For Exercises 10–15, have students work in pairs. Partner A should answer the first Exercise while Partner B watches, listens, coaches, and praises. Partner B solves the next Exercise while Partner A watches, listens, coaches, and praises. Partners take turns until the Exercises are complete. MP 1, 3

BL LA After completing Exercises 10–17, ask students how they think you might solve an equation that involves subtraction instead of addition. MP 1, 2

Create

BL LA **Trade-a-Problem** For Exercise 18, have students trade their word problems, solve each other's problem using the diagram, and compare solutions. Ask them if there are any other ways to solve the problem. MP 1, 4, 6

Inquiry Students should be able to answer "HOW do you solve addition equations using models?" Check for student understanding and provide guidance, if needed.

Analyze and Reflect

Work with a partner to complete the table. The first one is done for you.

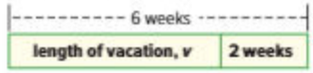
	Addition Equation	Subtraction Sentence	Solution
	$x + 1 = 3$	$3 - 1 = x$	$x = 2$
10.	$y + 9 = 12$	$12 - 9 = y$	$y = 3$
11.	$14 = 7 + m$	$14 - 7 = m$	$m = 7$
12.	$8 + f = 20$	$20 - 8 = f$	$f = 12$
13.	$47 = 17 + v$	$47 - 17 = v$	$v = 30$
14.	$100 + c = 129$	$129 - 100 = c$	$c = 29$
15.	$h + 89.4 = 97.4$	$97.4 - 89.4 = h$	$h = 8$

16. MP **Reason Inductively** Write a rule that you can use to solve an addition equation without using models. Sample answer: The answer to an addition equation is found by subtracting the number being added to the variable from the number on the other side of the equals sign.

17. How can the number family 3, 4, 7 help you to solve the equation $3 + x = 7$? Sample answer: If you know that $3 + 4 = 7$, you can mentally add to determine that $x = 4$. You can also use the subtraction sentence $7 - 3 = 4$ to determine that $x = 4$.

Create

18. MP **Model with Mathematics** Write a real-world problem for the equation modeled below. Then write the equation and solve.



Sample answer: Wafa earns 6 weeks of vacation time each year. She has already used 2 weeks. How many weeks of vacation time does she have left this year? $v + 2 = 6$; $v = 4$ weeks

19. Inquiry HOW do you solve addition equations using models?
Sample answer: You can solve an addition equation using a bar diagram or cups and counters. Both of these provide a visual to help determine what operation can be used to solve the equation.

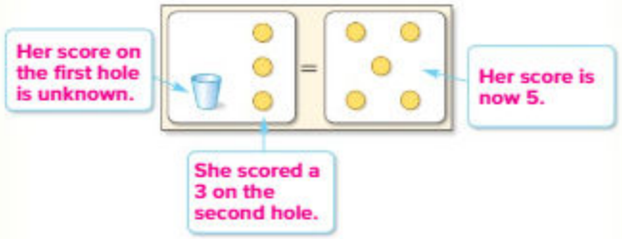
Expressions and Equations

Lesson 2

Solve and Write Addition Equations

Real-World Link

Miniature Golf On the second hole of miniature golf, it took Maysoun 3 putts to sink the golf ball. Her score is now 5. She represents this situation with cups and counters.



Essential Question

HOW do you determine if two numbers or expressions are equal?

Vocabulary

inverse operations
Subtraction Property of Equality

Mathematical Practices
1, 2, 3, 4, 5

1. Fill in the boxes above using the phrases below:
   - Her score on the first hole is unknown.
   - Her score is now 5.
   - She scored a 3 on the second hole.

2. Write the addition equation shown in the figure.
   $3 + x = 5$

3. Explain how to solve the equation.
   Subtract three counters from each side.

4. What was Maysoun's score on the first hole? 2

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Solve and write addition equations.
Materials: cups, counters, equation mats

Coherence connecting within and across grades

Previous
Students determined if a particular number was a solution to an addition equation.

Now
Students use Properties of Equality to solve addition equations.

Next
Students will use Properties of Equality to solve other types of equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 525.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL LA **Find the Fib** Individually, students write down two correct uses of the word Inverse and one fib. Then they present all three to a larger group. The job of the group is to identify the fib. MP 1, 3

Alternate Strategy

AL If students are having difficulty modeling the situation with cups and counters, give them a blank bar diagram and have them fill in the parts. They can answer the questions using the bar diagram. MP 1, 5

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Solve an addition equation by subtracting.**

AL
- *Is 8 a known value or an unknown?* known
- *How would you represent 8 in the equation?* use 8 counters
- *Is x a known value or an unknown?* unknown
- *How would you represent x in the equation?* use a cup
- *Is 3 a known value or an unknown?* known
- *How would you represent 3 in the equation?* use 3 counters

OL
- *How would you model the equation?* Place 8 counters on one side of the mat and a cup and 3 counters on the other side of the mat.
- *How many counters need to be removed from each side so that the cup is by itself?* 3

BL
- *Why do you use subtraction to undo addition?* Addition and subtraction are inverse operations so one will undo the other.

Need Another Example?
Solve $x + 4 = 5$. Check your solution. 1

Work Zone

a. 3

b. 1

c. 9.25

Solve an Equation By Subtracting

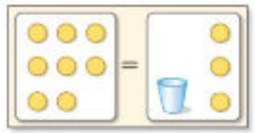
In Lesson 1, you mentally solved equations. Another way is to use **inverse operations**, which *undo* each other. For example, to solve an addition equation, use subtraction.

Example

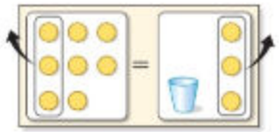
1. **Solve $8 = x + 3$. Check your solution.**

Method 1 **Use models.**

Model the equation using counters for the numbers and a cup for the variable.



Remove 3 counters from each side.



There are 5 counters remaining.

Method 2 **Use symbols.**

$8 = x + 3$ Write the equation.

$-3 = \quad -3$ Subtract 3 from each side to "undo" the addition of 3 on the right.

$5 = x$

Check

$8 = x + 3$ Write the equation.

$8 \stackrel{?}{=} 5 + 3$ Replace x with 5.

$8 = 8$ ✓ This sentence is true.

Using either method, the solution is 5.

Got it? **Do these problems to find out.**

Solve each equation. Check your solution.

a. $c + 2 = 5$ **b.** $6 = x + 5$ **c.** $3.5 + y = 12.75$

Subtraction Property of Equality

Key Concept

Words If you subtract the same number from each side of an equation, the two sides remain equal.

Examples

Numbers	Algebra
$5 = 5$	$x + 2 = 3$
$-3 = -3$	$-2 = -2$
$2 = 2$	$x = 1$

When you solve an equation by subtracting the same number from each side of the equation, you are using the **Subtraction Property of Equality**.

Example

2. **Obaid and Tariq have 245.5 downloaded minutes of music. If Obaid has 132 minutes, how many belong to Tariq? Write and solve an addition equation to find how many minutes belong to Tariq.**

Words Obaid and Tariq have 245.5 minutes of music.

Variable Let t represent the number of minutes that belong to Tariq.

Bar Diagram

Equation $132 + t = 245.5$

$132 + t = 245.5$ Write the equation.

$-132 \quad = -132$ Subtract 132 from each side.

$t = 113.5$ Simplify.

So, 113.5 minutes belong to Tariq.

Check $132 + 113.5 = 245.5$ ✓

Got it? **Do this problem to find out.**

d. Suppose Obaid had 147.5 minutes of the 245.5 that were downloaded. Write and solve an addition equation to find how many minutes belong to Tariq.

Checking Solutions

You should always check your solution. You hardly know immediately whether your solution is correct or not.

Show your work.

d. $147.5 + t = 245.5$; 98 minutes

Examples

2. **Write and solve an addition equation.**

AL
- *What do you need to find?* the number of minutes that belong to Tariq
- *What do you know?* Obaid has 132 minutes and they have a total of 245.5 minutes to use.
- *What operation will you use to write the equation? Explain.* addition; Sample answer: The word total tells me to add numbers together.

OL
- *What equation could be used to represent the situation?* $132 + t = 245.5$
- *Why do you subtract 132 from each side?* Sample answer: You want to get the variable t alone on one side, so you want to eliminate the number with it, 132.

BL
- *When you justify a step in a solution, you provide an acceptable reason for performing the step. What justifies subtracting 132 from each side of the equation?* the Subtraction Property of Equality

Need Another Example?

Hiyam and Hana went swimming. Hiyam swam 575 meters. This is 150 more meters than Hana swam. Write and solve an addition equation to find the number of meters that Hana swam.

$150 + x = 575$; 425 m

Examples

3. Write and solve an addition equation.

AL • *What do you need to find?* the weight of the average female gorilla

• *What do you know?* the average weight of a male gorilla is 172 kilograms, this weight is 82 kilograms more than the average weight of a female gorilla

OL • *What equation could be used to represent this situation?* $82 + w = 172$

BL • *How do you check your answer?* Sample answer: Substitute the solution for w in the original equation and see if the equation is true. If it is, then the solution is correct.

Need Another Example?

Pennsylvania is 280 miles from east to west. This is 120 miles more than the length of the state from north to south. Write and solve an equation to find the length of Pennsylvania from north to south. $x + 190 = 450$; 260 km

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Team-Pair-Solo** Have students work in groups of 4 to complete Exercises 1 and 2. Then have them pair up to work on Exercise 3. Finally, students work alone to complete Exercise 4. Students return to the larger group to compare answers. If the solutions do not agree, students work together to find the errors. MP 1, 6

BL LA **Trade-a-Problem** Have students create their own problem, similar to Exercises 3 and 4. Challenge students to write problems that use fractions and mixed numbers. Students trade problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. See students' work. MP 1, 4

Example

3. A male gorilla weighs 172 kilograms on average. This is 82 kilograms more than the weight of the average female gorilla. Write and solve an addition equation to find the weight of an average female gorilla.

Words	82 kilograms plus the weight of an average female gorilla is 172 kilograms.
Variable	Let w represent the weight of an average female gorilla.
Bar Diagram	172 kilograms: 82 kilograms | weight, w
Equation	$82 + w = 172$

$82 + w = 172$ Write the equation.

$-82 \quad = -82$ Subtract 82 from each side.

$w = 90$ $172 - 82 = 90$

So, an average female gorilla weighs 90 kilograms.

Check $82 + 90 = 172$ ✓

Guided Practice

Solve each equation. Check your solution. (Example 1)

1. $y + 7 = 10$ **3**

2. $10 = 6 + e$ **4**

3. A board that measures 19.5 meters in length is cut into two pieces. One piece measures 7.2 meters. Write and solve an equation to find the length of the other piece. (Example 2)

   $x + 7.2 = 19.5$; 12.3 m

4. It takes 43 facial muscles to frown. This is 26 more muscles than it takes to smile. Write and solve an equation to find the number of muscles it takes to smile. (Example 3)

   $x + 26 = 43$; 17 muscles

5. **Building on the Essential Question** How can the Subtraction Property of Equality be used to solve addition equations?

   Sample answer: It allows you to subtract the same number from each side of the equation.

Rate Yourself!

How confident are you about writing and solving addition equations? Shade the ring on the target.

FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

Solve each equation. Check your solution. (Example 1)

1. $c + 3 = 6$ **3**

2. $9 = 2 + x$ **7**

3. $7 + a = 9$ **2**

Show your work.

4. Ahmed and Eissa together have AED 756.80. If Ahmed has AED 489.50, how much does Eissa have? Write and solve an addition equation to find how much money belongs to Eissa. (Example 2) **489.50 + p = 756.80; AED 267.30**

5. The average length of a King Cobra is 300 centimeters, which is 55 centimeters longer than a Black Mamba. Write and solve an addition equation to find the average length of a Black Mamba. (Example 3) **m + 22 = 118; 245 centimeters.**

6. MP **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–b.

a. If Reham has already earned 30 points, write and solve an addition equation to find the number of points she still needs.
30 + p = 50; 20 points

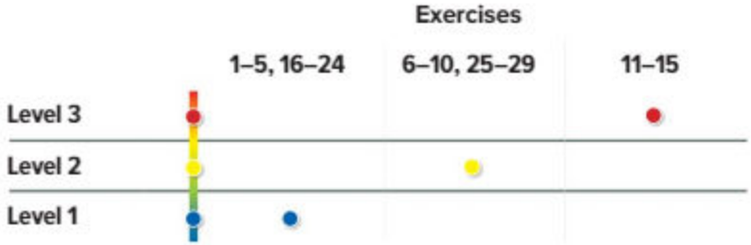
b. Suppose Muna has already earned 36 points. Write and solve an addition equation to find the number of points she still needs to earn for the pizza party. **36 + p = 50; 14 points**

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice
The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments
You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7, 9, 11, 13–15, 28, 29
OL	On Level	1–5 odd, 6–11, 13–15, 28, 29
BL	Beyond Level	6–15, 28, 29

Watch Out!

Common Error If students have trouble solving equations in which the variable is on the right side of the equals sign, demonstrate how they can rewrite the equation to show the variable on the left side of the equals sign.

MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	12
2 Reason abstractly and quantitatively.	15
3 Construct viable arguments and critique the reasoning of others.	11, 13, 14
4 Model with mathematics.	6
5 Use appropriate tools strategically.	24

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students solve the equation $x + 4 = 17$. 13

Solve each equation. Check your solution.

7. $a + \frac{1}{10} = \frac{5}{10}$ **2/5**

8. $m + \frac{1}{3} = \frac{2}{3}$ **1/3**

9. $\frac{3}{4} = x + \frac{1}{2}$ **1/4**

10. $\frac{7}{8} = y + \frac{1}{4}$ **5/8**

H.O.T. Problems Higher Order Thinking

11. **Reason Inductively** Write two different addition equations that have 12 as the solution. **Sample answers: 56 = 44 + x; 36 = 24 + m**

12. **Persevere with Problems** In the equation $x + y = 5$, the value for x is a whole number greater than 2 but less than 6. Determine the possible solutions for y. **0, 1, 2**

13. **Which One Doesn't Belong?** Identify the equation that does not belong with the other three. Explain your reasoning.

$6 + x = 9$ $\quad$ $15 = x + 12$ $\quad$ $x + 9 = 11$ $\quad$ $7 + x = 10$

x + 9 = 11; The solution for the other equations is 3.

14. **Find the Error** Maysa is solving the equation $x + 12 = 31$. Find her mistake and correct it.

$x + 12 = 31$
$\quad + 12 = +12$
$x \quad = 43$

Sample answer: She should have subtracted 12 from each side; 19

15. **Reason Abstractly** Suppose $x + y = 13$ and the value of x increases by 4. If their sum remains the same, what must happen to the value of y?

The value of y decreases by 4.

Name _____ My Homework _____

Extra Practice

Solve each equation. Check your solution.

16. $x + 5 = 11$

Homework Help

$x + 5 = 11$
$-5 = -5$
$x = 6$

17. $7 = 4 + y$ **3**

18. $5 + g = 6$ **1**

19. $d + 3 = 8$ **5**

20. $x + 4 = 6$ **2**

21. $3 + f = 8$ **5**

22. Khalid and Khalaf together have 386 trading cards. If Khalid has 221 trading cards, how many does Khalaf have? Write and solve an addition equation to find how many trading cards are Khalaf's.

221 + c = 386; 165 trading cards

23. Adnan is 160 centimeters tall, which is 23 centimeters taller than his cousin, Salem. Write and solve an addition equation to find Salem's height.

9 + x = 63; 137 centimeters

24. MP **Use Math Tools** The table shows the heights of three monster trucks. Bigfoot 5 is 1.5 meters taller than Bigfoot 2. Write and solve an addition equation to find the height of Bigfoot 2. **15.4 = x + 4.9; 3.2 m**

Truck	Height (m)
Bigfoot 5	4.7
Swamp Thing	3.7
Bigfoot 2	■

Solve each equation. Check your solution.

25. $t + \frac{8}{10} = \frac{9}{10}$ **$\frac{1}{10}$**

26. $\frac{5}{8} + n = \frac{7}{8}$ **$\frac{1}{4}$**

27. $t + \frac{1}{4} = \frac{3}{4}$ **$\frac{1}{2}$**

Power Up! Test Practice

Exercises 28 and 29 prepare students for more rigorous thinking needed for the assessment.

28.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK3
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly model the equation, identify the equation, and solve the equation.
1 point	Students correctly identify and solve the equation but fail to correctly model the equation OR students correctly model and solve the equation but fail to identify the equation OR students correctly model and identify the equation but fail to correctly solve the equation.

29.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

Power Up! Test Practice

28. Nasser has scored 174 points this basketball season. This is 29 points more than Hassan has scored. Select the correct items to complete the bar diagram representing the number of points Hassan has scored this season.

Hassan's points, x
29 points
174 points

What equation is modeled by the bar diagram? $29 + x = 174$

How many points has Hassan scored? 145 points

29. Faris wants to buy a skateboard that costs AED 85. He has already saved AED 15. Fill in the box to complete each statement.

a. The equation $x + 15 = 85$ can be used to find the amount of money Faris still needs to save to buy the skateboard.

b. Faris still needs to save AED 70 to buy the skateboard.

Spiral Review

Subtract.

30. $22 - 8 =$ 14

31. $72 - 34 =$ 38

32. $34 - 19 =$ 15

33. $51 - 32 =$ 19

34. $66 - 14 =$ 52

35. $49 - 32 =$ 17

36. The table shows the distances three friends hiked. How much farther did Fatheya hike than Ali? 0.4 km

Name	Distance Hiked (km)
Ali	1.85
Faleh	2.55
Fatheya	2.25

Inquiry Lab

Solve and Write Subtraction Equations

Inquiry **HOW do you solve subtraction equations using models?**

Mathematical Practices 1, 3, 4

Husam gave 5 trading cards to his sister. Now he has 41 cards. How many cards did he have originally?

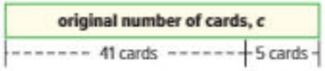
What do you know? Husam gave away 5 cards and he has 41 cards left.

What do you need to find? the number of cards Husam had originally

Hands-On Activity

Step 1 Define a variable. Use the variable c to represent the number of cards Husam had originally.

Step 2 Use a bar diagram to help write the equation.



The total length of the diagram shows the total number of cards.

The number 41 represents the number of cards he has left.

The number 5 represents the number of cards he gave his sister.

$c - 5 = 41$

Step 3 Work backward. Rewrite the equation as an addition sentence and solve.

$41 + 5 = c$

So, Husam originally had 46 trading cards.

Focus narrowing the scope

Objective Solve subtraction equations using models.

Coherence connecting within and across grades

Now
Students use bar diagrams to solve one-step subtraction equations.

Next
Students will use Properties of Equality to solve subtraction equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 530.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Hands-On Activity

AL LA Make sure students understand that the variable c represents the total number of cards in Husam's collection before he gave away any cards. Explain that the difference between the total number of cards in Husam's collection and the number of cards Husam gave away is the number of cards he has left. The equation $c - 5 = 41$ represents this problem.

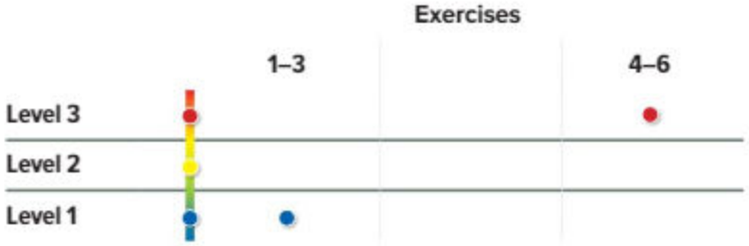
BL Omit the Activity and proceed directly to the Investigate section.

2 Collaborate

The **Investigate** section is intended to be used as a small-group investigation. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Pairs Discussion** Have students work in pairs to complete Exercises 1–3. Have them trade their solutions with another pair of students and discuss any differences. MP 1, 3

Create

BL LA **Find the Fib** Designate one person within a group of four to write a problem that does not represent the bar diagram in Exercise 5. Have groups trade problems with another group to see if the groups can identify the incorrect word problem in each group. MP 1, 3, 4

inquiry Students should be able to answer "HOW do you solve subtraction equations using models?" Check for student understanding and provide guidance, if needed.

Investigate

Work with a partner. Write and solve a subtraction equation using a bar diagram.

1. Maysa gave her friend Lamis 8 beads and was left with 37 beads. How many did she have originally?

   $b - 8 = 37$; 45 beads

2. Khamis has AED 12 after buying a snack at the mall. The snack cost AED 5. How much money did Khamis have originally?

   $s - 5 = 12$; AED 17

3. The Martin County Cat Shelter placed 8 cats with new owners on Monday. On Tuesday, 31 cats remained at the shelter. How many cats were at the shelter originally?

   $c - 8 = 31$; 39 cats

Create

4. MP **Reason Inductively** Write a rule for solving equations like $x - 4 = 7$.

   Sample answer: Add the number that is subtracted from x to each side of the equation.

5. MP **Model with Mathematics** Write a real-world subtraction problem for the equation modeled below. Then write the equation and solve.

   Sample answer: A salesman drove 67 fewer kilometers on Tuesday than he did on Monday. If he drove 128 kilometers on Tuesday, how many kilometers did he drive on Monday? $m - 67 = 128$; $m = 195$ kilometers

6. inquiry HOW do you solve subtraction equations using models?

   Sample answer: You can solve a subtraction equation using a bar diagram. The bar diagram shows the relationship between the parts and the total amount.

Lesson 3

Solve and Write Subtraction Equations

Real-World Link

Bowling Ayesha's bowling score was 39 points less than Huda's. Ayesha's score was 109.

1. Let s represent Huda's score. Write an equation for *39 points less than Huda's score is equal to 109.*

   $s - 39 = 109$

2. Use the number line to find Huda's score by counting forward.

   $s =$ 148

3. What operation does counting forward suggest?

   addition

4. Would it be reasonable to use cups and counters to solve this equation? Explain.

   no; Sample answer: Too many counters would need to be used to model the equation.

Essential Question

HOW do you determine if two numbers or expressions are equal?

Vocabulary

Addition Property of Equality

Mathematical Practices

1, 3, 4, 5

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Solve and write subtraction equations.
Materials: balance

Coherence connecting within and across grades

Previous Students modeled and solved one-step addition equations.

Now Students use Properties of Equality to solve one-step subtraction equations.

Next Students will use Properties of Equality to solve other equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 535.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

AL LA **Round Robin** Have students work in pairs to extend the pattern on the number line by counting forward. MP 1, 8

Alternate Strategy

AL Give students a balance and unit cubes. Have students place ten unit cubes on each side. Ask the following questions. MP 1, 4

- *What does the balance tell you about the cubes?*
- *What happens if you remove 2 cubes from the left side?*
- *What do you have to do to the right side to balance it?*
- *What would you have to do if you added 5 cubes to one side?*

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Example

1. **Solve a subtraction equation.**

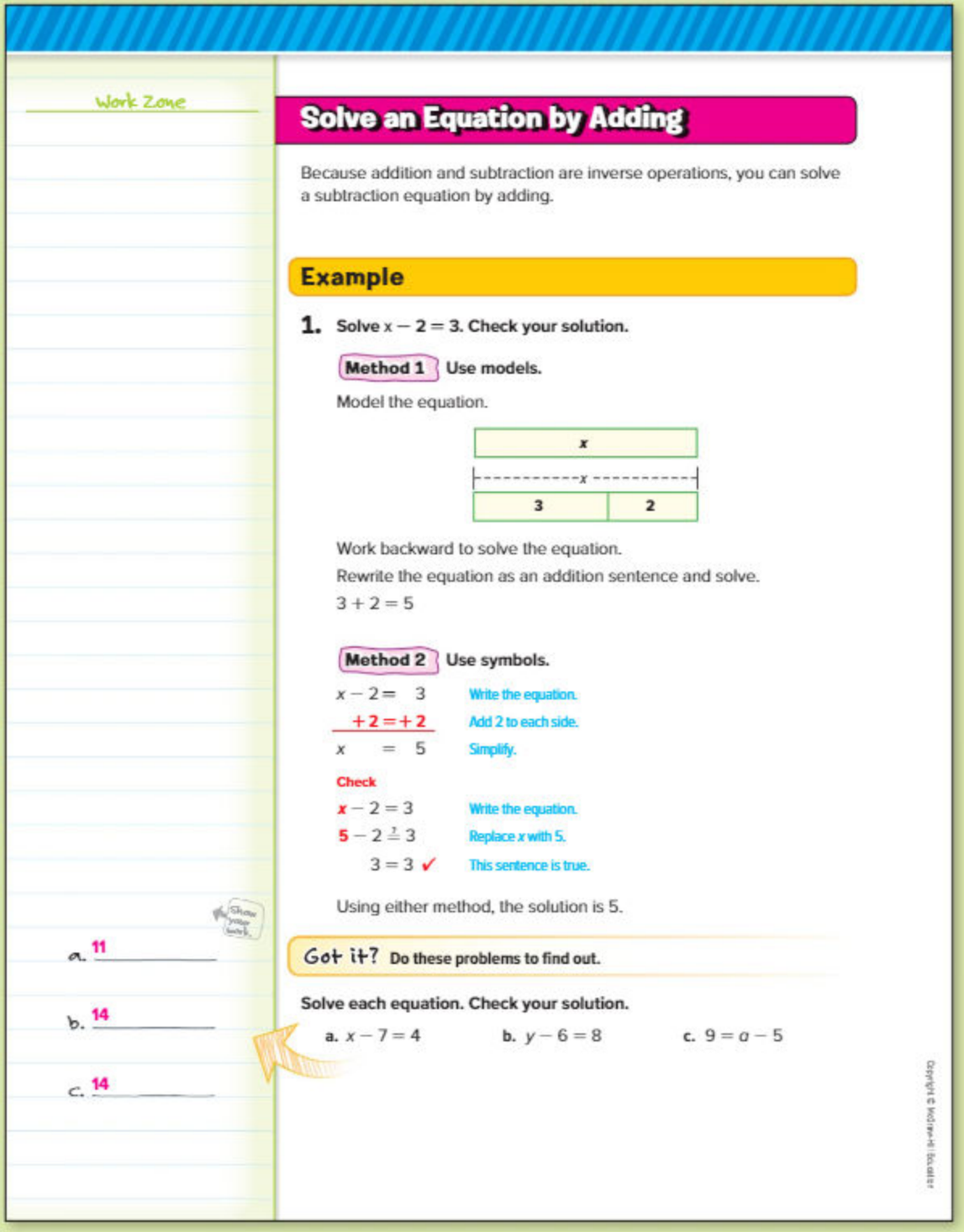
AL
- *How would you model the equation using a bar diagram?* use the bar diagram to show that the sum of 3 and 2 is x

- *How does a bar diagram help model the equation?* Sample answer: It helps to show that the subtraction equation can be rewritten as an addition sentence.

OL
- *What operation is used to "undo" subtraction?* addition
- *What number would you add to both sides?* 2

BL
- *How can you check your solution?* Replace x with 3 in the original equation and check to see if the resulting sentence is true.

Need Another Example?
Solve $x - 5 = 10$. Check your solution. 15



Work Zone

Solve an Equation by Adding

Because addition and subtraction are inverse operations, you can solve a subtraction equation by adding.

Example

1. **Solve $x - 2 = 3$. Check your solution.**

Method 1 **Use models.**

Model the equation.

Work backward to solve the equation.

Rewrite the equation as an addition sentence and solve.

$3 + 2 = 5$

Method 2 **Use symbols.**

$x - 2 = 3$	Write the equation.
$+2 = +2$	Add 2 to each side.
$x = 5$	Simplify.

Check

$x - 2 = 3$	Write the equation.
$5 - 2 \stackrel{?}{=} 3$	Replace x with 5.
$3 = 3$ ✓	This sentence is true.

Using either method, the solution is 5.

Got it? **Do these problems to find out.**

Solve each equation. Check your solution.

a. $x - 7 = 4$ **b.** $y - 6 = 8$ **c.** $9 = a - 5$

a. 11

b. 14

c. 14

Addition Property of Equality

Key Concept

Words If you add the same number to each side of an equation, the two sides remain equal.

Examples

Numbers	Algebra
$5 = 5$	$x - 2 = 3$
$+3 = +3$	$+2 = +2$
$8 = 8$	$x = 5$

When you solve an equation by adding the same number to each side of the equation, you are using the **Addition Property of Equality**.

Example

2. STEM **At age 25, Gherman Titov of Russia was the youngest person to travel into space. This is 52 years less than the oldest person to travel in space, John Glenn. How old was John Glenn? Write and solve a subtraction equation.**

Words Oldest age minus youngest age is 52 years.

Variable Let a represent the oldest age in space.

Bar Diagram

Equation $a - 25 = 52$

$a - 25 = 52$ Write the equation.

$+25 = +25$ Add 25 to each side.

$a = 77$ Simplify.

John Glenn was 77 years old.

Check $77 - 25 = 52$ ✓

Got it? **Do this problem to find out.**

d. Alia's height is 10 centimeters less than Laila's height. Alia is 148 centimeters. Write and solve a subtraction equation to find Laila's height.

Show your work.

d. $h - 4 = 58$; 158 centimete

Example

2. Write and solve a subtraction equation.

AL
- *What do you need to find?* John Glenn's age when he traveled into space
- *How many years older than Gherman Titov was John Glenn?* 52 years
- *What operation will you use to write the equation?* subtraction

OL
- *What equation could be used to represent the situation?* $a - 25 = 52$
- *What does a represent?* the age of the oldest person to travel in space
- *Why do you add 25 to each side?* Sample answer: You want to get the variable t alone on one side, so you want to eliminate the number with it, 25.

BL
- *When you justify a step in a solution, you provide an acceptable reason for performing the step. What justifies adding 25 to each side of the equation?* the Addition Property of Equality

Need Another Example?

The difference between the number of people that visit an amusement park on a Saturday and the number of people that visit the park on a Wednesday is 1,465. There are 4,320 visitors on Wednesday. How many visitors are there on Saturday? Write and solve a subtraction equation. $1{,}465 = x - 4{,}320$; 5,785 visitors

Example

3. Write and solve a subtraction equation.

AL • *What do you need to find?* the cost of the bicycle

• *What do you know?* the cost of the rollerblades and that the rollerblades cost AED70.25 less than the bicycle

OL • *What equation could be used to represent this situation?* $b - 70.25 = 43.50$

• *What does b represent?* the cost of the bicycle

BL • *Does the Addition Property apply if the numbers are fractions or decimals? Explain.* yes; Sample answer: It doesn't matter what form the numbers are in, if you add the same amount to both sides of the equation, the two sides remain equal.

Need Another Example?

After you withdraw AED50 from your savings account, the balance is AED124. Write and solve a subtraction equation to find your starting balance. $b - 50 = 124$; AED174

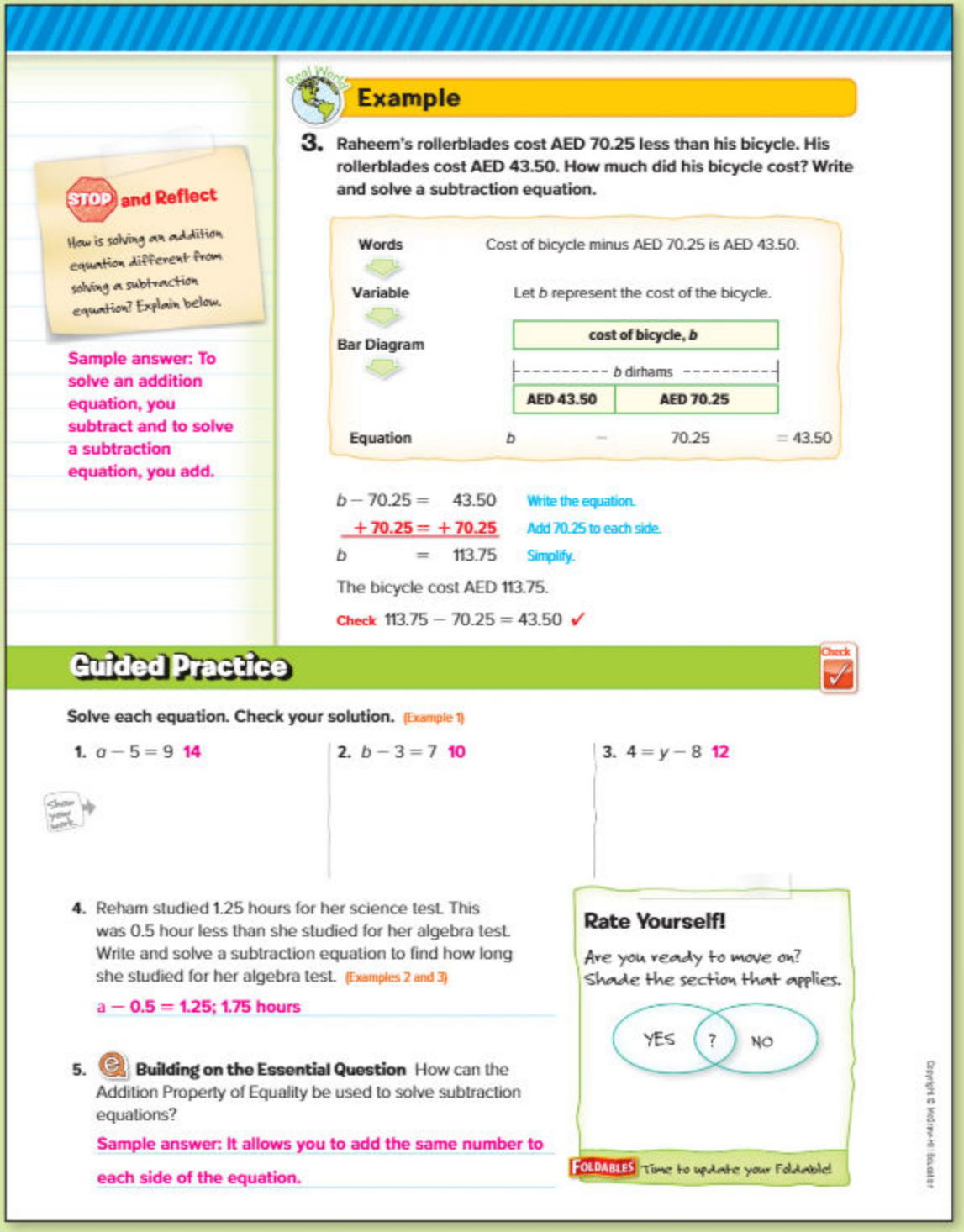
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Think-Pair-Share** Have students work in pairs. Give students a few minutes to work through their solutions to Exercises 1–4. Have them share their solutions with their partner. Their partner should then check their solutions using the check format in the text. Then call on students to share their solution for each Exercise. MP 1, 3

BL LA **Three Stay, One Stray** Arrange students in groups of 3 or 4 to complete Exercises 1–4. After each exercise, have one student from each group stray to another group to discuss ideas, procedures and results. Have students return to their original group to discuss their findings. Repeat for each exercise. Call on students to share results with the rest of the class. MP 1, 3



Name _____ My Homework _____

Independent Practice

Solve each equation. Check your solution. (Examples 1 and 3)

1. $c - 1 = 8$ 9

2. $t - 7 = 2$ 9

3. $1 = g - 3$ 4

Show your work.

4. $a - 2.1 = 5.8$ 7.9

5. $a - 1.1 = 2.3$ 3.4

6. $4.6 = e - 3.2$ 7.8

7. Ismail is 15 years old. This is 6 years younger than his sister Fawzia. Write and solve a subtraction equation to find Fawzia's age. (Example 2)

a − 6 = 15; 21 years old

8. A CD costs AED 14.95. This is AED 7.55 less than the cost of a DVD. Write and solve a subtraction equation to find the cost of the DVD. (Example 3)

d − 7.55 = 14.95; AED 22.50

9. If $b - 10 = 5$, what is the value of $b + 6$? 21

Solve each equation. Check your solution.

10. $m - \frac{1}{3} = \frac{2}{3}$ 1

11. $n - \frac{1}{4} = \frac{3}{4}$ 1

12. $s - \frac{1}{3} = \frac{7}{9}$ $\frac{10}{9}$ or $1\frac{1}{9}$

13. Lamya spent her graduation money on a video game that cost AED 24, a controller for AED 13, and a memory card for AED 16. The total tax was AED 3. Write and solve a subtraction equation to find how much money Lamya gave the cashier if she received AED 4 in change.

x − 56 = 4; AED 60

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–8, 9–13 odd, 15, 16, 18, 32, 33
OL	On Level	1–7 odd, 9–16, 18, 32, 33
BL	Beyond Level	9–18, 32, 33

Watch Out!

Common Error In Exercise 13, make sure students understand that they must first find the sum of all of Alejandra's purchases, including the tax, before writing the subtraction equation.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	17
3 Construct viable arguments and critique the reasoning of others.	14, 15, 18
4 Model with mathematics.	16
5 Use appropriate tools strategically.	25

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

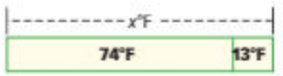
TICKET
Out the Door

Tell students that the next lesson involves solving multiplication equations. Ask them to write how they think the lessons on solving addition and subtraction equations will help them in understanding this lesson. See students' work.

Watch Out!

Common Error In Exercise 15, Khawla did not use the inverse operation (addition) to solve the subtraction equation. She should add 6 to each side.

14. **Multiple Representations** The bar diagram represents a subtraction equation. 14a–c. Sample answers given.



a. **Words** Write a real-world problem that can be represented by the bar diagram. The high temperature yesterday was 23°C, which was 7°C less than the high temperature today. What was the high temperature today?

b. **Algebra** Write a subtraction equation that can be represented by the bar diagram. $x - 7 = 23$

c. **Numbers** Solve the equation you wrote in part b. 30°C

H.O.T. Problems Higher Order Thinking

15. **Find the Error** Khawla is explaining how to solve the equation $d - 6 = 4$. Find her mistake and correct it. Khawla did not perform the inverse operation. Add 6 to each side to undo subtracting 6.

16. **Model with Mathematics** Write a real-world problem that could be represented by $d - 32 = 64$. Sample answer: I have d dirhams. After paying my sister AED 32, I have AED 64 left. How much money did I have to start with?

17. **Persevere with Problems** Another type of subtraction equation is $16 - b = 7$. Explain how you would solve this equation then solve it. Sample answer: I would use what I know about fact families to rewrite the equation $b + 7 = 16$. The solution is 9.

18. **Reason Inductively** Which of the following is true concerning $x - 5 = 13$? I

I To find the value of x, add 5 to each side.
II To find the value of x, subtract 5 from each side.
III To find the value of x, add 13 to each side.
IV To find the value of x, subtract 13 from each side.

Name _____ My Homework _____

Extra Practice

Solve each equation. Check your solution.

19. $f - 1 = 5$

Homework Help

$f - 1 = 5$
$+1 = +1$
$f = 6$

20. $2 = e - 1$ **3**

21. $r - 3 = 1$ **4**

22. $z - 6.3 = 2.1$ **8.4**

23. $t - 9.25 = 5.45$ **14.7**

24. $k - 32.9 = 16.5$ **49.4**

25. **Use Math Tools** North Carolina has 12 less electoral votes than Florida. Write and solve a subtraction equation to find the number of electoral votes for Florida. **15 = v − 12; 27 votes**

Electoral Votes	
State	Number of Votes
Florida	■
North Carolina	15

26. Badr's cat weighs 4.7 kilograms. This is 11 kilograms less than the weight of his goat. Write and solve a subtraction equation to find the weight of Badr's goat. **d − 11 = 4.7; 15.7 kg**

27. Find the value of t if $t - 7 = 12$. **19**

Solve each equation. Check your solution.

28. $s - \frac{1}{2} = \frac{1}{2}$ **1**

29. $h - \frac{1}{4} = \frac{1}{4}$ $\frac{1}{2}$

30. $c - 1 = \frac{3}{4}$ $1\frac{3}{4}$

31. At a movie, Rashid bought a medium popcorn for AED 4, a small drink for AED 3, and a box of fruit snacks for AED 5. Write and solve a subtraction equation to find how much money Rashid gave the cashier if he received AED 3 in change.

x − 12 = 3; AED 15

Power Up! Test Practice

Exercises 32 and 33 prepare students for more rigorous thinking needed for the assessment.

32.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK3
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly model, identify, and solve the equation.
1 point	Students correctly identify and solve the equation but fail to correctly model the equation OR students correctly model and solve the equation but fail to identify the equation OR students correctly model and identify the equation but fail to correctly solve the equation.

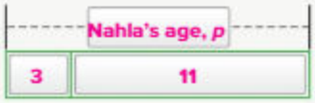
33.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP3
Scoring Rubric	
1 point	Students correctly answer each part of the question.

Power Up! Test Practice

32. Zayed's age is 3 years less than Nahla's age. Zayed is 11 years old. Select the correct items to complete the bar diagram below representing Nahla's age.

Nahla's age, p
3
11



What equation is modeled by the bar diagram?

p − 3 = 11 or p − 11 = 3

How old is Nahla? 14 years old

33. Mahmoud bought a pair of shoes and the shirt shown. The cost of the shirt was AED 42 less than the price of the shoes. Let s represent the price of the shoes. Determine if each statement is true or false.

a. The equation $s - 22 = 42$ models the situation. ■ True ☐ False

b. The equation $42 - s = 22$ models the situation. ☐ True ■ False

c. The cost of the shoes was AED 64. ■ True ☐ False

Spiral Review

Multiply.

34. 63 × 8 = 504

35. 19 × 6 = 114

36. 27 × 5 = 135

37. 13 × 8 = 104

38. 36 × 4 = 144

39. 21 × 3 = 63

40. The Zayed's Cat shelter has 3 calico cats for every gray cat. If they have 9 calico cats available, how many gray cats do they have?

3 gray cats

MP Problem-Solving Investigation

Guess, Check, and Revise

MP Mathematical Practices 1, 3, 4

Case #1 Smart Money

Ibrahim used AED 20 bills and AED 10 bills to pay for his AED 100 guitar lesson.

If he paid with 8 bills, how many of each bill did he use?

1 Understand What are the facts?

- Ibrahim paid with 8 bills that add to AED 100.
- The money was in AED 20 bills and AED 10 bills.

2 Plan What is your strategy to solve this problem?

Make a guess until you find an answer that makes sense for the problem.

3 Solve How can you apply the strategy?

Use addends that have a sum of 8 to find the number of AED 20 and AED 10 bills.

Number of AED 20 bills	Number of AED 10 bills	Total Amount	Compare to AED 100
1	7	1(AED 20) + 7(AED 10) = AED 90	less
2	6	2(AED 20) + 6(AED 10) = AED 100	equal
3	5	3(AED 20) + 5(AED 10) = AED 110	more
4	4	4(AED 20) + 4(AED 10) = AED 120	more

Ibrahim paid with 2 AED 20 bills and 6 AED 10 bills.

4 Check Does the answer make sense?

The other combinations are either less than or greater than AED 100.

Analyze the Strategy

MP **Reason Inductively** Ghaya received AED 100 in AED 10 and AED 5 bills, including eight AED 10 bills. Use the equation $x + 80 = 100$ to find how much money x was given to her in AED 5 bills. How many AED 5 bills did she receive?

$x = 20$; four AED 5 bills

Focus narrowing the scope

Objective Solve problems by using the guess, check, and revise strategy.
This lesson emphasizes MP **Mathematical Practice 3** Reason Inductively.

Guess, Check, and Revise This is an important strategy for problem solving, and is especially useful in taking multiple-choice tests. Sometimes the easiest way to determine a solution to a problem is to make an educated guess, check the answer for reasonableness, and then revise the guess until the correct answer is determined.

Coherence connecting within and across grades

Now
Students solve non-routine problems.

Next
Students will apply the Guess, Check, and Revise strategy to solve problems.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 541.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

The problems on pages 539 and 540 are intended to be used as a whole-group discussion on how to solve non-routine problems and are designed to provide scaffolded guidance.

Case #1 Smart Money

BL Have students extend the problem by having them answer the question below.

Ask:

- *Explain how using the Guess, Check, and Revise strategy helped you to solve the problem.* See students' responses.

Case #2 Anime Adventure

AL LA **Pairs Discussion** Have students work in pairs to respond to the following questions. MP 1, 5

Ask:

- *Is there any information you do not need to know from the problem?* what kind of books are being sold
- *What other problem-solving strategy could you use to solve the problem?* Sample answer: make a table

BL LA **Find the Fib** Have students work in pairs to write two facts and one fib about the graphic novel problem. For example, one fact could be that bought 2 used packages of graphic novels. One fib could be that the total number of graphic novels in 2 used packages is 15. MP 1, 3

Need Another Example?

Mahmoud is younger than Mohammed. Each of their ages is a different prime number. The sum of their ages is a prime number that is at most 13. What are the possible combinations of their ages?

The possible ages are 2 and 3, 2 and 5, or 2 and 11.

Case #2 Anime Adventure

A book store sells used graphic novels in packages of 5 and new graphic novels in packages of 3.

If Houriyya buys a total of 16 graphic novels, how many packages of new and used graphic novels does she buy?

1 Understand

Read the problem. What are you being asked to find?

I need to find a combination of new and used packages that totals 16 graphic novels.

Underline key words and values in the problem.
What information do you know?

The used novels come in packages of 5 and the new novels come in packages of 3. Houriyya buys 16 graphic novels.

Is there any information that you do not need to know?

I do not need to know what kind of books are being sold.

2 Plan

Choose a problem-solving strategy.

I will use the guess, check, and revise strategy.

3 Solve

Use your problem-solving strategy to solve the problem. Make a guess.

2 used packages and 1 new package: 2 (5) + 1 (3); 13 < 16

3 used packages and 2 new packages: 3 (5) + 2 (3); 21 > 16

2 used packages and 2 new packages: 2 (5) + 2 (3); 16 = 16

So, Hourriya used and 2 new packages of graphic novels.

4 Check

Use information from the problem to check your answer.

Make a list of multiples of 3 and a list of multiples of 5. Look for a combination of these multiples that add up to 16.

multiples of 3: 3, (6), 9, 12, 15 multiples of 5: 5, (10), 15, 20, 25

6 + 10 = 16

Expressions and Equations

Collaborate

Work with a small group to solve the following cases.
Show your work on a separate piece of paper.

Case #3 Quizzes

On a science quiz, Abdulraheem earned 18 points. There are six problems worth 2 points each and two problems worth 4 points each.

Find the number of problems of each type Abdulraheem answered correctly.

five problems worth 2 points each and two problems worth 4 points each

Case #4 Numbers

Suha is thinking of four numbers from 1 through 9 with a sum of 18. Each number is used only once.

Find the numbers.

Sample answer: 2, 4, 5, and 7

Case #5 Equations

Use the symbols +, −, ×, or ÷ to make the following equation true. Use each symbol only once.

3 ■ 4 ■ 6 ■ 1 = 18

3 × 4 + 6 ÷ 1 = 18

Use any strategy!

Case #6 Money

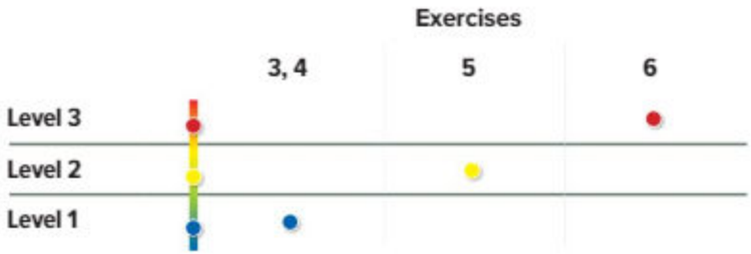
Rasheed is saving money to buy a new graphics card for his computer that costs AED 260.

If he is saving AED 18 a month and already has AED 134, in how many more months will he have enough money for the graphics card?

7 months

2 Collaborate

Levels of Complexity
The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



AL LA **Rally Coach** Have students work in pairs to complete Cases 3–6. For each exercise, have one student read aloud the problem and talk through their solution process, while the other student watches, listens, and encourages. Have students alternate roles for each successive case. 1, 3

BL LA **Trade-a-Problem** Have students write a real-world problem that requires students to use the *guess, check, and revise* strategy. Have students trade problems with a partner. Student 1 reads Student 2's problem aloud. Student 1 explains how to solve the problem. Student 2 listens and gives approval or disapproval to the solution and the solution process. If there are discrepancies, the pair works together to resolve them. Have students switch roles. 1, 3, 4

Mid-Chapter Check

If students have trouble with Exercises 1–12, they may need help with the following concepts.

Concept	Exercise(s)
solve equations mentally (Lesson 1)	1, 3, 4
write and solve addition equations (Lesson 2)	2, 5–7, 12
write and solve subtraction equations (Lesson 3)	8–11

Vocabulary Activity

LA **Think-Pair-Share** Have students work in pairs to complete Exercise 1. Give them about one minute to individually think through their response. Then have them share their responses with a partner. Call on one set of pairs to share their responses with the class. MP 1, 3, 6

Alternate Strategy

AL Have students explain how expressions and equations are different when written as real-world situations.

Mid-Chapter Check

Vocabulary Check

1. Define *equation*. Give an example of an equation and an example of an expression. Use a variable in each example. (Lesson 1)

   Sample answer: An equation is a mathematical sentence showing two expressions are equal. $2 + x = 6$ is an equation. $2 + x$ is an expression.

2. Fill in the blank in the sentence below with the correct term. (Lesson 2)

   You can solve equations using inverse operations, which undo each other.

Skills Check and Problem Solving

Circle the solution of the equation from the list given. (Lesson 1)

3. $x + 22 = 27$; 5, 6, 7

4. $17 + n = 24$; 6, 7, 8

Solve each equation. Check your solution. (Lessons 2 and 3)

5. $63 + d = 105$ 42

6. $h + 7.9 = 13$ 5.1

7. $a + 1.6 = 2.1$ 0.5

8. $p - 13 = 29$ 42

9. $y - 9 = 26$ 35

10. $r - 5\frac{1}{6} = 10$ $15\frac{1}{6}$

11. MP **Use Math Tools** The difference between the water levels for high and low tide was 110 centimeters. Write and solve an equation to find the water level at high tide. (Lesson 3)

    $x - 6 = 110$; 116 cm

12. MP **Persevere with Problems** If $x + 9.8 = 14.7$, what is the value of $8(x - 3.7)$? (Lesson 2)

    9.6

Inquiry Lab

Solve and Write Multiplication Equations

Inquiry **HOW do you solve multiplication equations using models?**

Mathematical Practices 1, 3, 4

In 5 days, Amna ran a total of 10 kilometers. She ran the same amount each day. How much did she run each day?

What do you know? Amna ran a total of 10 kilometers in 5 days.

What do you need to find? the number of kilometers she ran each day

Hands-On Activity 1

Step 1 Define a variable. Use the variable d to represent the distance run in one day.

Step 2 Use a bar diagram to help write the equation.

The total length of the diagram shows the total kilometers ran.

The variable d appears in the diagram 5 times.

5 d = 10

Step 3 Work backward. Rewrite the equation as a division sentence and solve.

10 ÷ 5 = d

So, Amna ran 2 kilometers each day.

Focus narrowing the scope

Objective Solve multiplication equations using models.

Coherence connecting within and across grades

Now
Students use bar diagrams and cups and counters to solve one-step multiplication equations.

Next
Students will use Properties of Equality to solve multiplication equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 544.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

Activities 1 and 2 are intended to be used as whole-group activities. Activity 1 is designed to provide more guidance than Activity 2.

Materials: cups, counters

Hands-On Activity 1

AL LA Make sure students understand that d represents the number of kilometers Amna ran in one day. Since she ran the same distance each day, the total distance she ran can be represented as $d + d + d + d + d$, or $5d$.

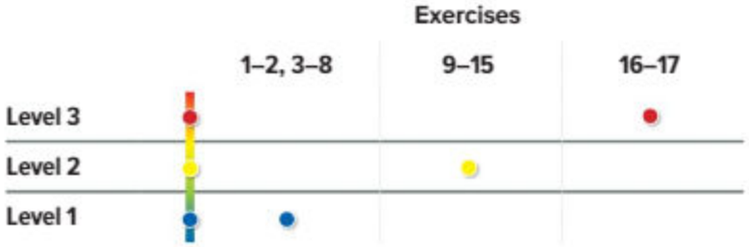
BL Omit Activities 1 and 2 and proceed directly to the Investigate section.

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Pairs Discussion** Have students work in pairs to complete Exercises 1 and 2. Have them trade their solutions with another pair of students and discuss any differences. MP 1, 3, 4

BL LA **Trade-a-Problem** Have students create their own problem, similar to Exercises 1 and 2. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. MP 1, 3, 4

Hands-On Activity 2

AL LA Remind students that one cup is represented by x in the equation. Since the equation shows $3x$, the model shows 3 cups. Emphasize that to solve the equation, each cup must contain the same number of counters. In this equation, $x = 4$ because each cup contains 4 counters.

Investigate

Work with a partner. Define the variable. Then write and solve a multiplication equation using a bar diagram.

1. Suppose Amna ran 12 kilometers in four days. If she ran the same distance d each day, how many kilometers did she run in one day?

d represents the distance in one day; 4d = 12; 3 km.

2. Buthaina has owned her cell phone for 8 months, which is twice as long as her sister Houriyya has owned her cell phone. How many months m has Houriyya had her cell phone?

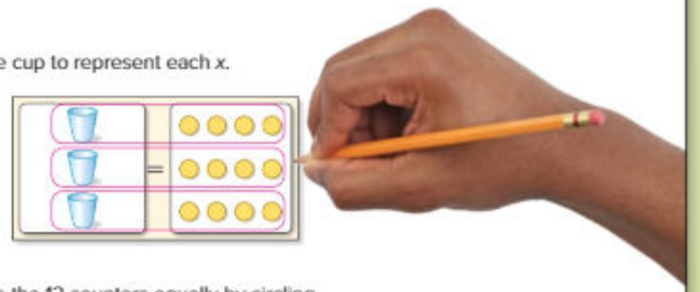
m represents the number of months Houriyya has had her cell phone; 2m = 8; 4 months

Hands-On Activity 2

Tools

Solve 3x = 12. Check your solution.

Step 1 Model the equation. Use one cup to represent each x.



Step 2 Use the model above. Divide the 12 counters equally by circling 3 groups. There are 4 counters in each group.

So, the solution is 4.

Check $3(x) = 12$ Write the original equation.

$3(4) \stackrel{?}{=} 12$ Replace x with your solution.

$12 = 12$ Is the sentence true? yes

Expressions and Equations

Investigate

Work with a partner. Solve each equation using cups and counters.

3. $4n = 8$

$n =$ 2

4. $3x = 9$

$x =$ 3

5. $10 = 5x$

$x =$ 2

6. $6x = 12$

$x =$ 2

Define a variable. Then write and solve a multiplication equation using a bar diagram.

7. The average lifespan of a horse is 40 years, which is five times longer than the average lifespan of a hamster. Use the bar diagram below to find the average lifespan of a hamster. Label each section of the diagram. g represents the lifespan of a hamster; $5g = 40$; 8 years

8. Humaid is saving an equal amount each week for 4 weeks to buy a AED 40 video game. Use the bar diagram below to find how much he is saving each week. Label each section of the diagram.

s represents how much he is saving each week; $4s = 40$; AED 10

Investigate

AL LA **Think-Pair-Share** Have students work in pairs. Give students one minute to think through their answers to Exercises 3–8. Have them share their responses with their partner. Then call on one student to share their response within a small group or large group discussion. MP 1, 3

BL LA For Exercises 3–8, ask students to explain how they could model and solve each equation using algebra tiles instead of cups and counters or bar diagrams. MP 1, 3, 5

Analyze and Reflect

AL LA Numbered Heads Together Assign students to 3– or 4–person learning teams. Each member is assigned a number from 1 to 4. Each team completes Exercises 9–15, making sure that every member understands. Call on a specific number from one team to present the team's solutions to the class. MP 1, 3

BL LA Mix and Match Give a small group of students one set of index cards with different bar diagrams on them and another set of index cards with real-world situations that match the bar diagrams. Have students mix the cards together and place them face down on the table. Students can play a match game with the cards. Whoever collects the most matches wins the game. MP 1, 5

Create

BL LA Trade-a-Problem For Exercise 16, have students trade their word problems, solve each other's problem using the diagram, and compare solutions. Ask them if there are any other ways to solve the problem. MP 1, 3

Inquiry Students should be able to answer "HOW do you solve multiplication equations using models?" Check for student understanding and provide guidance, if needed.

Analyze and Reflect

Work with a partner to complete the table. The first one is done for you.

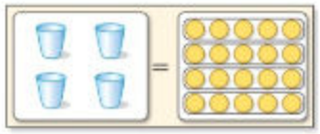
	Multiplication Equation	Coefficient	Variable	Product	Division Sentence	Solution
	=				÷ =	=
9.	=	3	y	21	21 ÷ 3 = y	= 7
10.	=	5	m	45	45 ÷ 5 = m	= 9
11.	=	8	d	48	48 ÷ 8 = d	= 6
12.	=	16	f	32	32 ÷ 16 = f	= 2
13.	=	13	b	39	39 ÷ 13 = b	= 3

14. MP **Reason Inductively** Write a rule for solving equations like $2x = 24$ without using models. Use a related division sentence to explain your answer.

Sample answer: Find the factor that, when multiplied by 2, is equal to 24 by using a related division sentence. Since $24 \div 2 = 12$, $x = 12$.

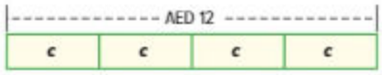
15. Write and solve an equation to represent the situation modeled below.

$4x = 20$; $x = 5$



Create

16. MP **Model with Mathematics** Write a real-world problem for the equation modeled below. Then write the equation and solve.



Sample answer: Huda is buying croissants from the local bakery. She purchased 4 croissants for AED 12. What was the cost of each croissant? $4c = 12$; AED 3

17. Inquiry HOW do you solve multiplication equations using models?

Sample answer: You can solve a multiplication equation using a bar diagram or cups and counters. In the bar diagram, the total is represented by the whole bar. The factors are represented by the number of parts, and the variable.

Lesson 4

Solve and Write Multiplication Equations

Vocabulary Start-Up

The equation $3x = 9$ is a multiplication equation. In $3x$, 3 is the coefficient of x because it is the number by which x is multiplied.

Fill in the table. The first one is done for you.

Prefix	Root Word	New Word	Meaning
co-	pilot	copilot	the second pilot that flies with the primary pilot of the plane
co-	author	coauthor	write something with someone else
co-	operate	cooperate	work together
co-	efficient	coefficient	when a numerical factor contains a variable, the number with the variable is the coefficient

Essential Question

HOW do you determine if two numbers or expressions are equal?

Vocabulary

Division Property of Equality

Mathematical Practices

1, 2, 3, 4, 5

Real-World Link

Ringtones Khalifa is downloading ringtones. The cost to download each ringtone is AED 2. When Khalifa is finished he has spent a total of AED 10. Let x represent the number of ringtones. What does the expression $2x$ represent?

the cost of downloading x ringtones

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Solve and write multiplication equations.

Coherence connecting within and across grades

Previous
Students solved one step addition and subtraction equations.

Now
Students use Properties of Equality to solve multiplication equations.

Next
Students will use Properties of Equality to solve other equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 551.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL LA **Circle the Sage** Have a group of students research what the prefix *co-* means and how it changes the root words pilot, author, and operate. Then have them research the etymology of *coefficient*. They become the sages and spread out in the room. The rest of the class surrounds a different sage to learn what they know. MP 1, 6, 7, 8

Alternate Strategy

AL Supply students with a dictionary or access to online resources to find the definitions of the words in the chart. Then, based on the activity, ask students how they think the prefix *co-* changes the meaning of a root word. MP 1, 7

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Solve a multiplication equation.**

AL • *How would you model the equation using a bar diagram?* Label a bar diagram with a total of 10, then divide it into 2 equal sections.

• *What is the value of each group?* 5

OL • *What operation is used to "undo" multiplication?* division

• *By what number would you divide both sides?* 2

BL • *What is the first step to solve the equation?* Divide each side by 2.

Need Another Example?
Solve $6x = 18$. Check your solution. 3

2. **Solve a multiplication equation.**

AL • *What is the coefficient of x?* 3

• *What do you need to "undo" to get x alone on one side?* multiplying *x* by 3

• *How will you undo that?* Divide both sides by 3.

OL • *What is the first step to solve the equation?* Divide each side by 3.

• *What is the solution to the equation?* $x = 2$

BL • *Dividing by a number is the same thing as multiplying by what number?* the reciprocal

• *How can you solve the equation* $5x = \frac{5}{6}$. Sample answer: Multiply by the reciprocal, $\frac{1}{5}$.

Need Another Example?
Solve $4x = 24$. Check your solution. 6

Work Zone

a. 5

b. 2

c. 7

Solve a Multiplication Equation

A multiplication equation is an equation like $2x = 10$ because the variable x is multiplied by 2. Multiplication and division are inverse operations. So, to solve a multiplication equation, use division.

Examples

1. **Solve $2x = 10$. Check your solution.**

$2x = 10$ Write the equation.

$\frac{2x}{2} = \frac{10}{2}$ Divide each side by the coefficient 2.

$x = 5$

Check $2x = 10$ Write the original equation.

$2(5) \stackrel{?}{=} 10$ Replace x with 5.

$10 = 10$ This sentence is true. ✓

2. **Solve $3x = 6$. Check your solution.**

Fill in the boxes below.

$3x = 6$ Write the equation.

$\frac{3x}{\boxed{3}} = \frac{6}{\boxed{3}}$ Divide each side by the coefficient 3.

$x = \boxed{2}$

Check $3\boxed{x} = 6$ Write the original equation.

$3\left(\boxed{2}\right) \stackrel{?}{=} 6$ Replace x with 2.

$\boxed{6} = 6$ This sentence is true. ✓

Got it? **Do these problems to find out.**

Solve each equation. Check your solution.

a. $3x = 15$ **b.** $8 = 4x$ **c.** $2x = 14$

Expressions and Equations

Division Property of Equality

Key Concept

Words If you divide each side of an equation by the same nonzero number, the two sides remain equal.

Examples

Numbers	Algebra
$18 = 18$	$3x = 12$
$\frac{18}{6} = \frac{18}{6}$	$\frac{3x}{3} = \frac{12}{3}$
$3 = 3$	$x = 4$

When you solve an equation by dividing both sides of the equation by the same number, you are using the **Division Property of Equality**.

Example

3. **Mohammad and some friends shared the cost of a package of blank CDs. The package cost AED 24 and each person contributed AED 6. How many people shared the cost of the CDs?**

Words Amount each contributed times number of people equals cost of CDs.

Variable Let x represent the number of people that contributed money.

Bar Diagram

The number of sections is unknown, but each section represents AED 6.

Equation $6 \cdot x = 24$

$6x = 24$ Write the equation.

$\frac{6x}{6} = \frac{24}{6}$ Divide each side by 6.

$x = 4$ Simplify.

Check $6 \times 4 = 24$ ✓

There were 4 people who split the cost of the CDs.

Got it? **Do this problem to find out.**

d. In 2004, Pen Hadow and Simon Murray walked 1,100 kilometers to the South Pole. The trip took 58 days. Suppose they traveled the same distance each day. Write and solve a multiplication equation to find about how many kilometers they traveled each day.

STOP and Reflect

What is the coefficient in the equation in Example 3?

6

Show your work.

$58m = 1{,}100$; about 19 km

d. 19 km

Example

3. **Write and solve a multiplication equation.**

AL
- *What do you need to find?* the number of people that shared the cost of the CDs
- *What was the total cost?* AED24
- *How much did each person contribute?* AED6

OL
- *What equation could be used to represent the situation?* $6x = 24$
- *What does x represent?* the number of people that contributed money
- *What is the first step in solving the equation?* Divide each side by 6.

BL
- *What property justifies dividing each side by 6?* the Division Property of Equality

Need Another Example?

Majed uses 4 cups of flour to make 2 loaves of banana bread. Write and solve a multiplication equation to find the number of cups of flour that are needed to make one loaf of bread. $2x = 4$; 2 cups

Example

4. Write and solve a multiplication equation.

AL
- *What is the coefficient?* 3.28
- *How would you divide 19.68 by 3.28?* Move the decimal point two places to the right in each number. Then divide 1968 by 328.

OL
- *What is different about this equation from the first few examples?* The coefficient is a decimal.
- *Would you use the same method to solve the equation? Explain.* yes; Sample answer: The Division Property of Equality applies to all nonzero numbers.

BL
- *Based on what you have learned, make a conjecture about what the Multiplication Property of Equality is and when you would use it.* Sample answer: You can multiply both sides of an equation by the same number and the two sides remain equal; when solving a division equation.

Need Another Example?

Solve $8.34x = 41.7$. Check your solution. 5

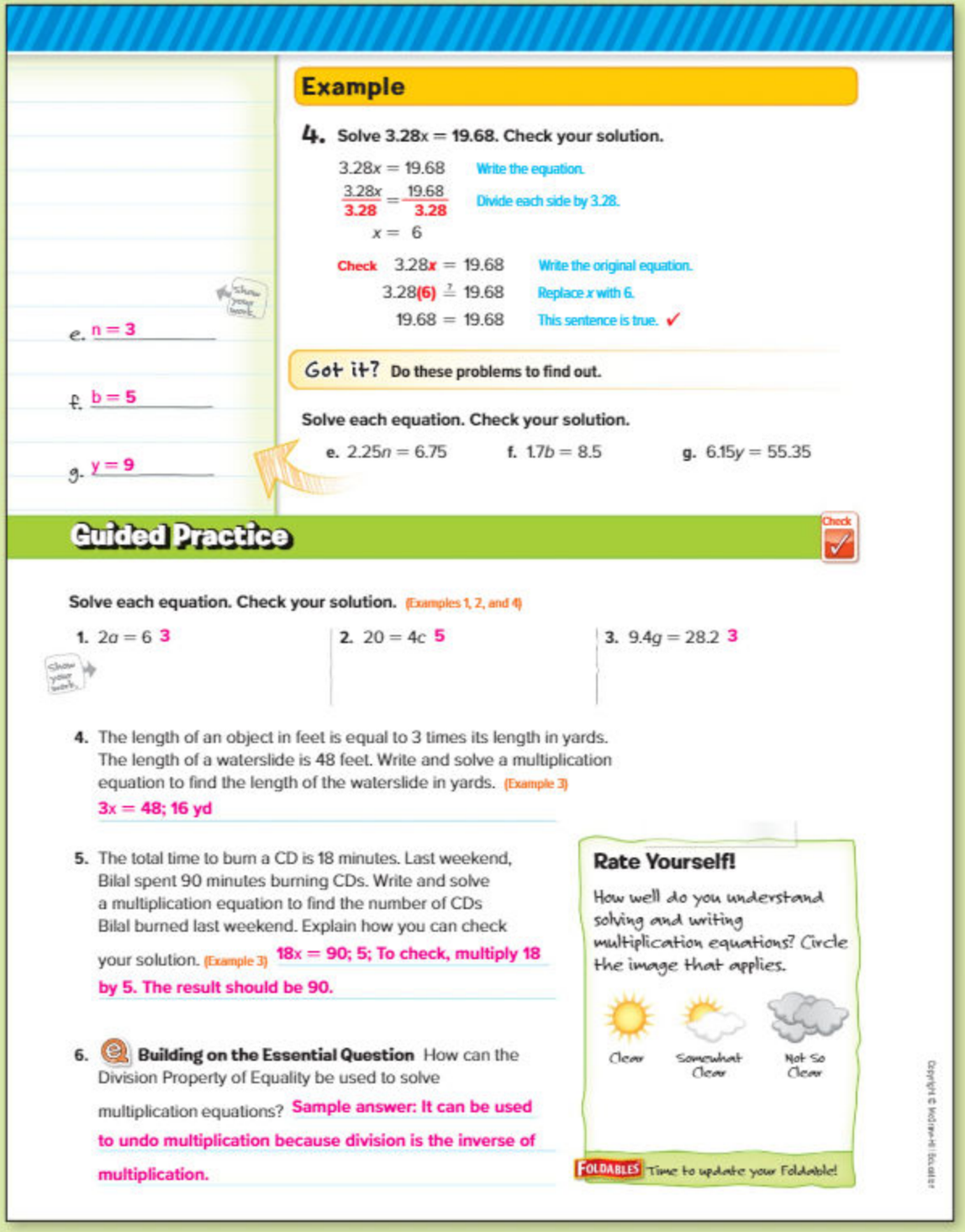
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Check** Have students work in pairs. One student solves Exercise 1 while the other coaches. Students switch roles for the next problem. After every two problems, the pairs check their answers with another pair. MP 1, 3

BL LA **Trade-a-Problem** Have students create their own problem, similar to Exercise 5. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. Challenge students to include fractions or mixed numbers in their problems. MP 1, 3, 4



e. $n = 3$

f. $b = 5$

g. $y = 9$

Example

4. Solve $3.28x = 19.68$. Check your solution.

$3.28x = 19.68$ Write the equation.

$\frac{3.28x}{3.28} = \frac{19.68}{3.28}$ Divide each side by 3.28.

$x = 6$

Check $3.28x = 19.68$ Write the original equation.

$3.28(6) \stackrel{?}{=} 19.68$ Replace x with 6.

$19.68 = 19.68$ This sentence is true. ✓

Got it? Do these problems to find out.

Solve each equation. Check your solution.

e. $2.25n = 6.75$ f. $1.7b = 8.5$ g. $6.15y = 55.35$

Guided Practice

Solve each equation. Check your solution. (Examples 1, 2, and 4)

1. $2a = 6$ 3
2. $20 = 4c$ 5
3. $9.4g = 28.2$ 3

4. The length of an object in feet is equal to 3 times its length in yards. The length of a waterslide is 48 feet. Write and solve a multiplication equation to find the length of the waterslide in yards. (Example 3) $3x = 48$; 16 yd

5. The total time to burn a CD is 18 minutes. Last weekend, Bilal spent 90 minutes burning CDs. Write and solve a multiplication equation to find the number of CDs Bilal burned last weekend. Explain how you can check your solution. (Example 3) $18x = 90$; 5; To check, multiply 18 by 5. The result should be 90.

6. **Building on the Essential Question** How can the Division Property of Equality be used to solve multiplication equations? Sample answer: It can be used to undo multiplication because division is the inverse of multiplication.

Rate Yourself!

How well do you understand solving and writing multiplication equations? Circle the image that applies.

Clear Somewhat Clear Not So Clear

FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

Solve each equation. Check your solution. (Examples 1, 2, and 4)

1. $4g = 24$ **6**

Show your work.

2. $5d = 30$ **6**

3. $36 = 6e$ **6**

4. $1.5x = 3$ **2**

5. $2.5y = 5$ **2**

6. $8.1 = 0.9a$ **9**

7. A jewelry store is selling a set of 4 pairs of gemstone earrings for AED 58, including tax. Shaikha and three of her friends want to buy the set so each could have one pair of earrings. Write and solve a multiplication equation to find how much each person should pay. (Example 3)

4e = 58; AED 14.50

Solve each equation. Check your solution.

8. $39 = 1\frac{3}{10}b$ **30**

9. $\frac{1}{2}e = \frac{1}{4}$ **$\frac{1}{2}$**

10. $\frac{2}{5}g = \frac{3}{5}$ **$1\frac{1}{2}$**

MP **Use Math Tools** Use the table that shows football data.

a. Morten Andersen played in the NFL for 25 years. Write and solve an equation to find how many points he averaged each year.

25p = 2,544; 107.76 points

b. Jason Hanson played in the NFL for 20 years. Write and solve an equation to find how many points he averaged each year.

20p = 2,150; 107.5 points

Top NFL Kickers	
Player	**Career Points**
Morten Andersen	2,544
Gary Anderson	2,434
Jason Hanson	2,150
John Carney	2,062
Adam Vinatieri	2,006

12. STEM An average person's heart beats about 103,680 times a day. Write and solve an equation to find about how many times the average person's heart beats in one minute.

1,440x = 103,680; 72 beats

ENGAGE EXPLORE **EXPLAIN** **ELABORATE** **EVALUATE**

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9–13 odd, 14, 15, 17, 35, 36
OL	On Level	1–7 odd, 8–15, 17, 35, 36
BL	Beyond Level	8–17, 35, 36

Watch Out!

Common Error Make sure students understand that they use the same strategy to solve equations when the products or coefficients are fractions or decimals as when they are counting numbers.

MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	16
2 Reason abstractly and quantitatively.	28
3 Construct viable arguments and critique the reasoning of others.	14, 15
4 Model with mathematics.	13, 17
5 Use appropriate tools strategically.	11

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET
Out the Door

Have students solve the following equation and explain how they arrived at a solution: $6s = 42$. 7; I divided both sides of the equation by 6 in order to get the variable by itself.

Watch Out!

Common Error In Exercise 14, Hamed did not initially divide both sides by the coefficient, 5. At the first step, he should have divided $5x$ by 5 as well.

13. **Model with Mathematics** Problems involving constant speed can be solved by the formula distance = rate × time. Usama's family traveled 436 kilometers on a road trip last weekend. They drove for 4 hours. What was the rate at which Usama's family traveled? Write and solve a multiplication equation.

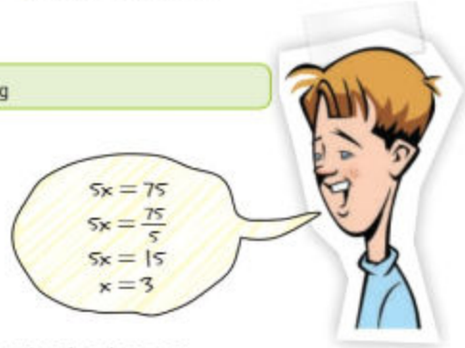
distance	=	rate	×	time
436 kilometers		r		4 hours

Usama's family traveled an average rate of 109 kilometers per hour.

H.O.T. Problems Higher Order Thinking

14. **Find the Error** Hamed is solving $5x = 75$. Find his mistake and correct it.

He did not divide each side by 5; $x = 15$.



15. **Which One Doesn't Belong?** Identify the equation that does not belong with the other three. Explain your reasoning.

$5x = 20$ $4b = 7$ $8w = 32$ $12y = 48$

$4b = 7$; The solution for the other equations is 4.

16. **Persevere with Problems** Explain how you know that the equations $\frac{1}{4} = 2x$ and $\frac{1}{4} \div x = 2$ have the same solution. Then, find the solution.

Sample answer: If you divide each side of the equation $\frac{1}{4} = 2x$ by x, you will have the equation $\frac{1}{4} \div x = 2$. Thus, the equations are equivalent, as long as $x \neq 0$. So, they have the same solution $\frac{1}{8}$. Replacing x with $\frac{1}{8}$ into each equation makes each equation true.

17. **Model with Mathematics** Write a real-world problem that could be represented by the equation $4r = 240$. Then solve the equation and interpret the solution.

Sample answer: Ahmad's family traveled 240 kilometers in 4 hours. What was their average speed?; 60 kilometers per hour; Ahmad's family traveled an average of 60 kilometers per hour.

Name _____ My Homework _____

Extra Practice

Solve each equation. Check your solution.

18. $4c = 16$

Homework Help

$4c = 16$

$\frac{4c}{4} = \frac{16}{4}$

$c = 4$

19. $5t = 25$ **5**

20. $5a = 15$ **3**

21. $3f = 12$ **4**

22. $21 = 3g$ **7**

23. $6x = 12$ **2**

24. $5.9q = 23.6$ **4**

25. $2.55d = 17.85$ **7**

26. $6.5a = 32.5$ **5**

27. Majed's family drove 2,800 kilometers across the United States on their vacation. If it took a total of 28 hours, write and solve a multiplication equation to find their average speed in miles per hour.

2,800 = 28r; 100 kmph

28. MP **Reason Abstractly** Four friends went bowling one afternoon. Use the table that shows the bowling data.

Player	Score
Omar	320
Khalid	366
Abdalla	522
Abdulkarim	488

a. Khalid bowled 3 games. Write and solve an equation to find how many points he averaged each game. **3p = 366; 122 points**

b. Abdalla bowled 5 games. Write and solve an equation to find how many points he averaged each game. **5p = 522; 104.4 points**

Copy and Solve **Solve each equation. Show your work on a separate piece of paper.**

29. $1\frac{2}{5}x = 7$ **5**

30. $3\frac{1}{2}r = 28$ **8**

31. $2\frac{1}{4}w = 6\frac{3}{4}$ **3**

32. $2\frac{3}{4}a = 19\frac{1}{4}$ **7**

33. $1\frac{1}{2}c = 6$ **4**

34. $3\frac{3}{4}m = 33\frac{3}{4}$ **9**

Power Up! Test Practice

Exercises 35 and 36 prepare students for more rigorous thinking needed for the assessment.

35.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP3
Scoring Rubric	
1 point	Students correctly answer each part of the question.

36.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly model the equation and solve the equation.
1 point	Students correctly model the equation OR solve the equation.

Power Up! Test Practice

35. Mr. Hasan bikes at a constant speed of 19 kilometers per hour. He wants to find the number of hours it will take him to bike 86 kilometers. Determine if each statement is true or false.

- **a.** To find the number of hours, subtract 19 from 86. ☐ True ■ False
- **b.** To find the number of hours, divide 86 by 19. ■ True ☐ False
- **c.** It will take Mr. Hasan 5 hours to bike 86 kilometers. ☐ True ■ False

36. The table shows some of the nutritional information for a bottle of iced tea. Moza wants to determine how many grams of sugar are in each serving. Let s represent the grams of sugar in each serving. Select the correct values to model the situation with a multiplication equation.

Nutritional Facts (2 Servings)
Calories: 80
Total Fat: 0 grams
Sodium: 50 milligrams
Sugars: 64 grams

2 × s = 64

0	50	80
2	64	s

How many grams of sugar are in each serving? 32 g

Spiral Review

Divide.

37. 138 ÷ 6 = 23

38. 80 ÷ 5 = 16

39. 208 ÷ 4 = 52

40. 217 ÷ 7 = 31

41. 216 ÷ 24 = 9

42. 378 ÷ 6 = 63

43. The table shows the cost of concessions at a concert. Tarek spent AED 31.50 buying popcorn for his class. How many bags of popcorn did Tarek buy?

21 bags

Item	Cost (AED)
Nachos	AED 3.00
Popcorn	AED 1.50
Water	AED 2.00

44. After dinner, $\frac{3}{4}$ of a pie remains. If Shaima eats $\frac{1}{6}$ of the remaining pie, how much of the total pie does Shaima eat? $\frac{1}{8}$ pie

Inquiry Lab

Solve and Write Division Equations

Inquiry **HOW do you solve division equations using models?**

Mathematical Practices 1, 3, 4

Four friends decided to split the cost of season concert tickets equally. Each person paid AED 35. Find the total cost of the season concert tickets.

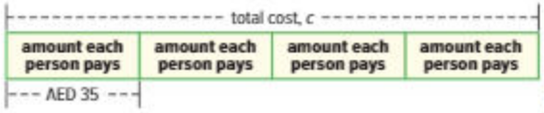
What do you know? Four people split the cost equally and paid AED 35 each.

What do you need to find? the total cost of the tickets

Hands-On Activity

Step 1 Define a variable. Use the variable c to represent the total cost of the tickets.

Step 2 Use a bar diagram to help write the equation.



The total length of the diagram shows the total cost.

The number 35 represents the amount each person paid.

There are four equal sections because four friends split the cost equally.

$c \div 4 = 35$

Step 3 Work backward. Rewrite the equation as a multiplication sentence and solve.

$35 \times 4 = c$

So, the total cost of the season tickets was AED 140.

Focus narrowing the scope

Objective Solve division equations using models.

Coherence connecting within and across grades

Now
Students use bar diagrams to solve one-step division equations.

Next
Students will use Properties of Equality to solve division equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 560.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Hands-On Activity

AL LA Students should understand that the unknown value in this expression is the total cost of the season concert tickets. Four people shared this total cost, so the bar diagram is divided into four equal parts. Remind students that this division may be written as $c \div 4$. However, in algebraic equations it is usually written as $\frac{c}{4}$.

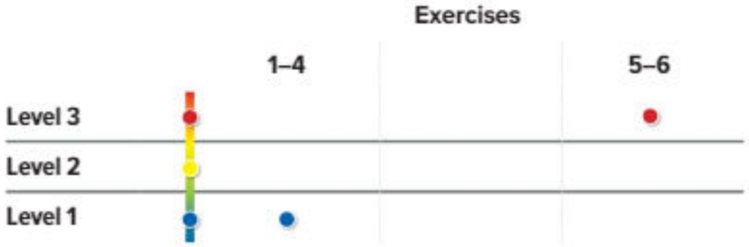
BL Omit the Activity and proceed directly to the Investigate section.

2 Collaborate

The **Investigate** section is intended to be used as a small-group investigation. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Pairs Discussion** Have students work in pairs to complete Exercises 1–4. Have them trade their solutions with another pair of students and discuss any differences. MP 1, 4

Create

BL LA **Trade-a-Problem** Have students create their own problem, similar to Exercise 5. Students trade their problems, solve each other's problem, and compare solutions. If the solutions do not agree, students work together to find the errors. MP 1, 3, 4

inquiry Students should be able to answer "HOW do you solve division equations using models?" Check for student understanding and provide guidance, if needed.

Investigate

MP **Model with Mathematics** **Work with a partner. Write and solve a division equation using a bar diagram.**

1. Three teachers went to a conference. They shared the cost of gasoline g equally. Each teacher paid AED 38.50. Draw a bar diagram to find the total cost of gasoline.

   $g \div 3 = \text{AED } 38.50$; $g = \text{AED } 115.50$

2. Manal has completed 8 math exercises e. This is one fourth of the assignment. How many exercises were assigned?

   $e \div 4 = 8$; $e = 32$ exercises

3. Hareb bought a shirt for $\frac{1}{2}$ off. He paid AED 21.75 for the shirt s. Draw a bar diagram to find the original cost of the shirt.

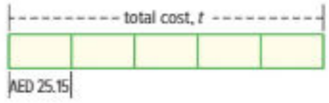
   $s \div 2 = \text{AED } 21.75$; $s = \text{AED } 43.50$

4. Six friends are sharing the cost for a pizza party p equally. Each person paid AED 15.25. Find the total cost of the pizza party.

   $p \div 6 = \text{AED } 15.25$; $p = \text{AED } 91.50$

Create

5. MP **Model with Mathematics** Write a real-world division problem for the equation modeled below. Then write the equation and solve.



   Sample answer: Noura and four of her friends split the cost of catering for a dinner party. If each person paid AED 25.15, what was the total cost of the catering? $t \div 5 = \text{AED } 25.15$; $t = \text{AED } 125.75$

6. inquiry HOW do you solve division equations using models?

   Sample answer: You can solve a division equation using a bar diagram. The bar diagram shows the relationship between the value of each part and the total amount.

Lesson 5

Solve and Write Division Equations

Real-World Link

Allowances Sindiyya spends AED 50 a month on snacks at school, which is one fourth of her monthly allowance. Complete the questions below to find Sindiyya's monthly allowance.

1. Draw a bar diagram to represent AED 50 as one fourth of Sindiyya's monthly allowance.

2. What is Sindiyya's monthly allowance? AED 200

3. What operation did you use to find Sindiyya's allowance? multiplication

4. How can you check your answer to determine if it is accurate? I can use inverse operations and divide AED 200 by 4 to check to see if the answer is accurate.

Essential Question

HOW do you determine if two numbers or expressions are equal?

Vocabulary

Multiplication Property of Equality

Mathematical Practices
1, 2, 3, 4, 7

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Solve and write division equations.

Coherence connecting within and across grades

Previous Students used Properties of Equality to solve one-step equations.

Now Students use Properties of Equality to solve division equations.

Next Students will use Properties of Equality to solve two-step equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 561.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

BL LA **Trade-a-Problem** Have students create their own problem similar to the Real-World Link. Students trade their problems, solve each other's problem and compare solutions. If the solutions do not agree, students work together to find the errors. If students are ready, challenge them to create a two-step situation and solve it using a bar diagram. MP 1, 3, 4

Alternate Strategy

AL Give students at least 20 counters. Students can then put five counters in four different piles to find Sindiyya total monthly allowance. MP 1, 4

ENGAGE EXPLORE **EXPLAIN** ELABORATE EVALUATE

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Solve a division equation.**

AL
- *What is the variable?* a
- *What is the number paired with the variable?* 3
- *What operation is involved?* division
- *How do you "undo" division?* multiplication

OL
- *What operation did you use to solve a multiplication equation in the previous lesson?* division
- *What operation would you use to "undo" division?* multiplication
- *By what number would you multiply both sides?* 3

BL
- *How can* $\frac{a}{3}$ *be rewritten as the product of a fraction and a variable?* $\frac{1}{3}a$
- *If you multiply* $\frac{1}{3}a$ *by 3, what do you need to do to the other side of the equation?* Multiply by 3.

Need Another Example?

Solve $\frac{p}{7} = 5$. Check your solution. 35

Work Zone

Show your work.

a. 72

b. 32

c. 45

d. 60

Solve Division Equations

In the situation on the previous page, equation $\frac{a}{4} = 50$, where a represents the monthly allowance, means the monthly allowance *divided by* 4 equals AED 50. Since multiplication and division are inverse operations, use multiplication to solve division equations.

Example

1. **Solve $\frac{a}{3} = 7$. Check your solution.**

Method 1 **Use models.**

Model the equation.

Solve the equation. Work backward.

Since $\frac{a}{3} = 7$, $7 \times 3 = a$. So, $a = 21$.

Method 2 **Use symbols.**

$\frac{a}{3} = 7$	Write the equation.
$\frac{a}{3}(3) = 7(3)$	Multiply each side by 3.
$a = 21$	Simplify.
Check $\frac{a}{3} = 7$	Write the original equation.
$\frac{21}{3} \stackrel{?}{=} 7$	Replace a with 21.
$7 = 7$	This is a true sentence. ✓

Using either method, the solution is 21.

Got it? **Do these problems to find out.**

Solve each equation. Check your solution.

a. $\frac{x}{8} = 9$ b. $\frac{y}{4} = 8$

c. $\frac{m}{5} = 9$ d. $30 = \frac{b}{2}$

Expressions and Equations

Multiplication Property of Equality

Key Concept

Words If you multiply each side of an equation by the same nonzero number, the two sides remain equal.

Examples

Numbers	Algebra
$3 = 3$	$\frac{x}{4} = 7$
$3(6) = 3(6)$	$\frac{x}{4}(4) = 7(4)$
$18 = 18$	$x = 28$

When you solve an equation by multiplying each side of the equation by the same number, you are using the **Multiplication Property of Equality**.

STOP and Reflect

How is solving a multiplication equation similar to solving a division equation? How is it different? Explain below.

Sample answer: Both equations are solved by using the inverse relationship between multiplication and division. Division is used to solve multiplication equations and multiplication is used to solve division equations.

Example

2. **The weight of an object on the Moon is one sixth that of its weight on Earth. If an object weighs 35 kilograms on the Moon, write and solve a division equation to find its weight on Earth.**

Words Weight of object on Earth divided by 6 equals weight on Moon.

Variable Let w represent the weight of the object on Earth.

Bar Diagram

Equation $\frac{w}{6} = 35$

$\frac{w}{6} = 35$ Write the equation.

$\frac{w}{6}(6) = 35(6)$ Multiply each side by 6.

$w = 210$ $6 \times 35 = 210$

The object weighs 210 kilograms on Earth.

Got it? **Do this problem to find out.**

e. Hassan picked a total of 60 apples in $\frac{1}{3}$ hour. Write and solve a division equation to find how many apples Hassan could pick in 1 hour.

Show your work.

e. $\frac{a}{3} = 60$; 180 apples

Examples

2. **Write and solve a division equation.**

AL
- *What do you need to find?* the weight of an object on Earth
- *What was the weight of the object on the moon?* 35 kg
- *What number will you multiply the weight on moon by to get the weight on Earth?* 6

OL
- *What equation could be used to represent the situation?* $\frac{w}{6} = 35$ or $\frac{1}{6}w = 35$
- *What does w represent?* the weight of the object on Earth
- *What is the first step in solving the equation?* Multiply each side by 6

BL
- *What would you need to do to the weight of the object on the moon to find the weight of the object on Earth?* multiply by 6

Need Another Example?

One fourth of the number of students in the sixth grade participate in an after-school sport. If 42 students play sports, write and solve a division equation to find the number of students in the sixth grade. $\frac{s}{4} = 42$; 168 students

Examples

3. Write and solve a division equation.

AL
- *What do you need to find?* the length of ribbon she needs to buy
- *How long is each piece of ribbon?* 8.5 cm.
- *How many pieces are needed?* 16

OL
- *What equation could be used to represent this situation?* $\frac{r}{8.5} = 16$
- *What does the r represent?* the length of the ribbon she needs to buy
- *What is the first step in solving the equation?* Multiply each side by 8.5.

BL
- *Equivalent equations are equations that have the same solution. Are the equations* $\frac{r}{8.5} = 16$ *and* $\frac{r}{16} = 8.5$ *equivalent? Explain.* yes; Sample answer: Both solutions are 8.5×16 or 136.

Need Another Example?

Fahd cut a board into 45 centimeter long pieces to make a book shelf. He could get 4 pieces out of the board. Write and solve a division equation to find the length of the original board. $\frac{b}{45} = 4$; 180 cm.

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Pairs Discussion** Have students work in pairs to complete Exercises 1–5. Students discuss how to solve the problem before actually solving it. Have them trade their solutions with another pair of students and discuss any differences. MP 1, 3

BL LA **Pairs Discussion** Have students work in pairs to first discuss how they could use the Multiplication Property of Equality to solve the equation $5x = 42$, and then to solve the equation. Have them trade their ideas with another pair of students and discuss any differences. MP 1, 3

f. $\frac{s}{4.5} = 8$; 36

Real World

Example

3. **Amal is buying ribbon for costumes. She wants to divide the ribbon into 8.5 centimeter pieces for 16 costumes. Write and solve a division equation to find the length of ribbon Amal should buy.**

Let r represent the length of ribbon Amal should buy.

$\frac{r}{8.5} = 16$ Write the equation.

$\frac{r}{8.5}(8.5) = 16(8.5)$ Multiply each side by 8.5.

$r = 136$ $8.5 \times 16 = 136$

Amal should buy 136 centimeters of ribbon.

Got it? **Do this problem to find out.**

f. Fatema is baking a pie. She wants 4.5 strawberries in each serving for 8 people. Write and solve a division equation to find how many strawberries Fatema will need.

Guided Practice

Check

Solve each equation. Check your solution. (Example 1)

1. $\frac{m}{6} = 10$ 60
2. $\frac{k}{5} = 11$ 55
3. $\frac{v}{13} = 14$ 182

4. Sumayya and Abeer are sharing a pack of stickers. Each girl gets 11 stickers. Write and solve a division equation to find how many total stickers there are. (Example 2) $\frac{s}{11} = 2$; 22 stickers

5. Sultan is buying a beef roast. He wants to divide it into 200-gram servings for 12 people. Write and solve a division equation to find what size beef roast Sultan should buy. (Example 3) $\frac{h}{200} = 12$; 2,400 g

6. **Building on the Essential Question** When solving an equation, why is it necessary to perform the same operation on each side of the equals sign? Sample answer: To maintain equality, an operation performed on one side of an equation must also be performed on the other side.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

Solve each equation. Check your solution. (Examples 1 and 3)

1. $5 = \frac{p}{4}$ **20**

2. $17 = \frac{w}{6}$ **102**

3. $4.7 = \frac{g}{3.2}$ **15.04**

Show your work.

Write and solve a division equation to solve each problem. (Examples 2 and 3)

4. Khadija is buying party favors. She has a budget of AED 2.75 a person for 6 people. How much can Khadija spend on party favors?
$\frac{f}{2.75} = 6$; **AED 16.50**

1-1 Sally baked 3 dozen oatmeal raisin cookies for the bake sale at school. This is one fourth the number of dozens of cookies she baked in all. How many dozens of cookies did she bake in all?
$\frac{x}{4} = 3$; **12 dozen**

6. MP **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–b.

a. If Reham has earned 30 points, write and solve a multiplication equation to find how many books she needs to read. **$20 = 5x$; 4 books**

b. Suppose Reham has read 7 books. Write and solve a division equation to find the number of points she has earned. **$\frac{x}{5} = 7$; 35 points**

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7–9, 11, 25, 26
OL	On Level	1–5 odd, 6–9, 11, 25, 26
BL	Beyond Level	6–11, 25, 26

Watch Out!

Common Error Some students may multiply both sides of the equation by the quotient, rather than the divisor. Suggest that students read the problem aloud to remind them which number is the divisor.

MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	10
2 Reason abstractly and quantitatively.	8
3 Construct viable arguments and critique the reasoning of others.	9, 11, 24
4 Model with mathematics.	6
7 Look for and make use of structure.	7, 21–23

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Ask students to solve $\frac{t}{9} = 4$ and check the solution.

36; $\frac{36}{9} = 4$

7. **Identify Structure** Write the property used to solve each type of equation.

+ Subtraction Property of Equality	− Addition Property of Equality
× Division Property of Equality	÷ Multiplication Property of Equality

H.O.T. Problems Higher Order Thinking

8. **Reason Abstractly** Write a division equation that has a solution of 42.

Sample answer: $\frac{x}{7} = 6$

9. **Reason Inductively** *True* or *false*: $\frac{x}{3}$ is equivalent to $\frac{1}{3}x$. Explain your reasoning.

True; Sample answer: Dividing by 3 is the same as multiplying by $\frac{1}{3}$.

10. **Persevere with Problems** Explain how you would solve $\frac{16}{c} = 8$. Then solve the equation.

Sample answer: Multiply both sides of the equation by c, then divide both sides of the equation by 8; 2.

11. **Multiple Representations** Every autumn, the North American Monarch butterfly migrates up to 4,800 kilometers to California and Mexico where it hibernates until early spring. The butterfly travels on average 80 kilometers per day.

a. **Algebra** Write an equation that represents the distance d a butterfly will travel in t days. $d = 80t$

b. **Tables** Use the equation to complete the table.

c. **Words** Use the pattern in the table to determine how many days it will take the butterfly to travel 4,000 kilometers. 50 days

Time (days)	1	2	3	4	5
Distance (kilometers)	50	100	150	200	250

Name _____ My Homework _____

Extra Practice

Solve each equation. Check your solution.

12. $4 = \frac{r}{8}$

Homework Help

$4 = \frac{r}{8}$

$4(8) = \frac{r}{8}(8)$

$32 = r$

13. $12 = \frac{q}{7}$ **84**

14. $18 = \frac{r}{2}$ **36**

15. $\frac{h}{13} = 13$ **169**

16. $\frac{j}{12} = 11$ **132**

17. $\frac{z}{7} = 8$ **56**

18. $\frac{c}{0.2} = 7$ **1.4**

19. $\frac{d}{12} = 0.25$ **3**

20. $\frac{m}{16} = 0.5$ **8**

MP **Identify Structure** **Write and solve a division equation to solve each problem.**

21. One third of a bird's eggs hatched. If 2 eggs hatched, how many eggs did the bird lay? $\frac{x}{3} = 2$; **6 eggs**

22. Ibrahim is purchasing a board to build a bookcase. He wants to divide the board into 53-centimeter sections. He needs 6 sections. What size board does Ibrahim need? $\frac{p}{53} = 6$; **320 cm**

23. Jamal is cutting a piece of rope into fourths. If each piece is 16 centimeters long, what is the length of the entire rope? $\frac{r}{4} = 16$; **64 cm.**

24. MP **Justify Conclusions** A model plane is $\frac{1}{50}$ the size of the actual plane. If the model plane is 70 centimeters long, how long is the actual plane? Explain your reasoning to a classmate. $\frac{p}{50} = 70$; **3,500 cm.; The length of the actual plane divided by 48 equals the length of the model plane.**

Power Up! Test Practice

Exercises 25 and 26 prepare students for more rigorous thinking needed for the assessment.

25.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

26.

This test item requires students to support their reasoning or evaluate the reasoning of others by justifying their response and constructing arguments.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP3, MP4
Scoring Rubric	
2 points	Students correctly model the equation, solve the equation and explain their response.
1 point	Students correctly model and solve the equation but fail to explain their response OR students solve the equation and explain their response but fail to correctly model the equation.

Power Up! Test Practice

25. Bilal does chores to earn money in the summer. The table shows the amount he earns per chore. Bilal weeded the garden 6 times over the summer. Write and solve a division equation to find how much he earned weeding the garden.

Chore	Amount Earned (AED)
mow lawn	AED 10
wash car	AED 5
weed garden	AED 8

$\frac{x}{6} = 8$; AED 48

26. Hamdah ran 6 kilometers in 1 week. This was one third of what she ran in the month. Let m represent the number of kilometers Hamdah ran in the month. Select the correct values to model the situation with a division equation.

$\frac{m}{3} = 6$

1, 3, 6, 7, m

How many kilometers did Hamdah run in the month? Explain how you can check your answer.

18 kilometers; Sample answer: Find one third of 18 kilometers and check to see if this is equal to 6 kilometers: $18 \div 3 = 6$ kilometers.

Spiral Review

Fill in each ◯ with <, >, or = to make a true sentence.

27. 6.5 (>) 5.2

28. 1.9 (>) 1.7

29. 2.2 (=) 2.2

30. 5.6 (<) 6.5

31. 4.2 (>) 3.9

32. 5.5 (<) 5.7

33. The table shows the number of inches in different number of feet.

How many inches are in 5 feet? 60 in.

Feet	Inches
1	12
2	24
3	36
4	48

34. Describe the pattern shown below. Then find the next number in the pattern.

4, 8, 12, 16, 20, 24, . . .

Each number is found by adding 4 to the previous number; 28

21ST CENTURY CAREER in Music

Sound Engineer

Do you enjoy using electronics to make music sound better? If so, you might want to explore a career in sound engineering. Sound engineers, or audio technicians, prepare the sound equipment for recording sessions and live concert performances. They are responsible for operating consoles and other equipment to control, replay, and mix sound from various sources. Sound engineers adjust the microphones, amplifiers, and levels of various instrument and voice tones so that everything sounds great together.

Is This the Career for You?

Are you interested in a career as a sound engineer? Take some of the following courses in high school.

- Algebra
- Electronic Technology
- Music and Computers
- Physics
- Sound Engineering

Find out how math relates to a career in Music.

Focus narrowing the scope

Objective Apply mathematics to problems arising in the workplace.

This lesson emphasizes **Mathematical Practice 4** Model with Mathematics.

Coherence connecting within and across grades

Previous
Students wrote and solved equations to solve problems.

Now
Students apply the content standard to solve problems in the workplace.

Rigor pursuing concepts, fluency, and applications

See the Career Project on page 566.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ask students to read the information on the student page about sound engineers and answer the following questions.

Ask:

- *How do sound engineers help musicians?* Sample answers: they prepare the equipment for recording; they mix sounds; they adjust microphones
- *What courses should you take in high school to prepare for a career as a sound engineer?* Sample answers: Physics, Music, Algebra, Electronic Technology

2 Collaborate

AL LA **Paired Heads Together** Have students work in pairs to complete Exercises 1–5. Each student is numbered 1 or 2. Pairs must work together to ensure that each student understands how to complete each exercise. Each student is responsible to ask for help, if needed. Upon completion, call on one numbered student to share their responses with the class. MP 1, 7

BL LA **Think-Pair-Write** Give students time to individually think about how they could create a real-world problem using the information given in the table and diagram. Then have students pair up and discuss their ideas to decide upon and generate their real-world problem. Each pair trades problems with another pair and solves the other pair's problems. MP 1, 3, 4

Career Portfolio

When students complete this page, have them add it to their Career Portfolio.

Career Facts

Sound engineers also work in sporting arenas, theaters, and in movie productions. According to the U.S. Bureau of Labor Statistics, the best way to prepare for a career in sound engineering is to receive training in broadcast technology, electronics, or computer networking.

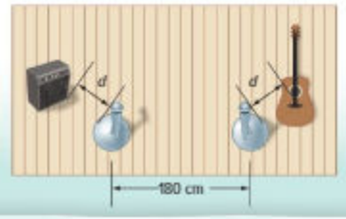
MP Amping the Band!

Use the information in the table and the diagram to solve each problem.

1. In the diagram, the distance between the microphones is 180 centimeters. This is 3 times the distance d from each microphone to the sound source. Write an equation that represents this situation. $6 = 3d$
2. Solve the equation that you wrote in Exercise 1. Explain the solution. 2; The distance from each microphone to the sound source is 60 cm.
3. The distance from the microphone to the acoustic guitar sound hole is about 11 centimeters less than what it should be. Write an equation that models this situation. $7.5 = x - 27.5$
4. Solve the equation that you wrote in Exercise 3. Explain the solution. 35; The distance from the microphone to the acoustic guitar sound hole should be 35 cm.
5. The microphone is about 9 times farther from the electric guitar amplifier than it should be to produce a natural, well-balanced sound. Write and solve an equation to find how far from the amplifier the microphone should be placed. $90 = 9x$; 10 cm.

Microphone Mistakes

Sound Source	Location of Microphone	Resulting Sound
Acoustic guitar	7.5 centimeters from sound hole	very bassy
Electric guitar amplifier	90 centimeters from amp	thin, reduced bass



MP Career Project

It's time to update your career portfolio! Go online and research careers in sound engineering. Make a list of the advantages and disadvantages of working in that field.

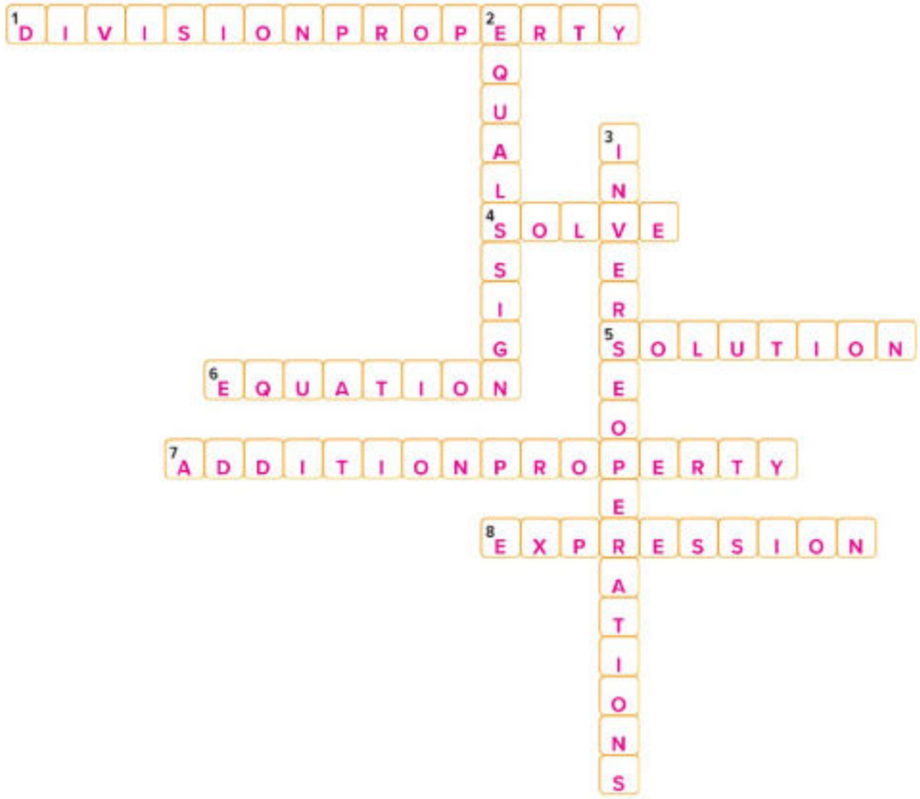
List several challenges associated with this career.

-
-
-
-
-

Chapter Review

Vocabulary Check

Write the correct term for each clue in the crossword puzzle.



Across

1. property of equality used to solve multiplication equations
4. replace a variable with a value that results in a true sentence
5. the value of a variable that makes an equation true
6. mathematical sentence showing two expressions are equal
7. property of equality used to solve subtraction equations
8. a combination of numbers, variables, and at least one operation

Down

2. a symbol of equality
3. operations which undo each other

Vocabulary Check

LA **Popcorn Share** Give students time to individually complete the exercises in the Vocabulary Check. After all students have completed the exercises, call out, "Popcorn, Exercise 1," and allow students to quickly and voluntarily pop up from their chairs and, one at a time, share their answer. Discuss with the class any incorrect answers or discrepancies. Repeat for the remaining exercises. 1, 6

Alternate Strategy

AL LA To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include the following terms.

- Addition Property of Equality (Lesson 3)
- Division Property of Equality (Lesson 4)
- equals sign (Lesson 1)
- equation (Lesson 1)
- expression (Lesson 1)
- inverse operations (Lesson 2)
- solution (Lesson 1)
- solve (Lesson 1)

Key Concept Check

FOLDABLES LA A completed Foldable for this chapter should include a review of equations and inequalities.

If you choose not to use this Foldable, have students write a brief review of the Key Concepts found throughout the chapter and give an example of each.

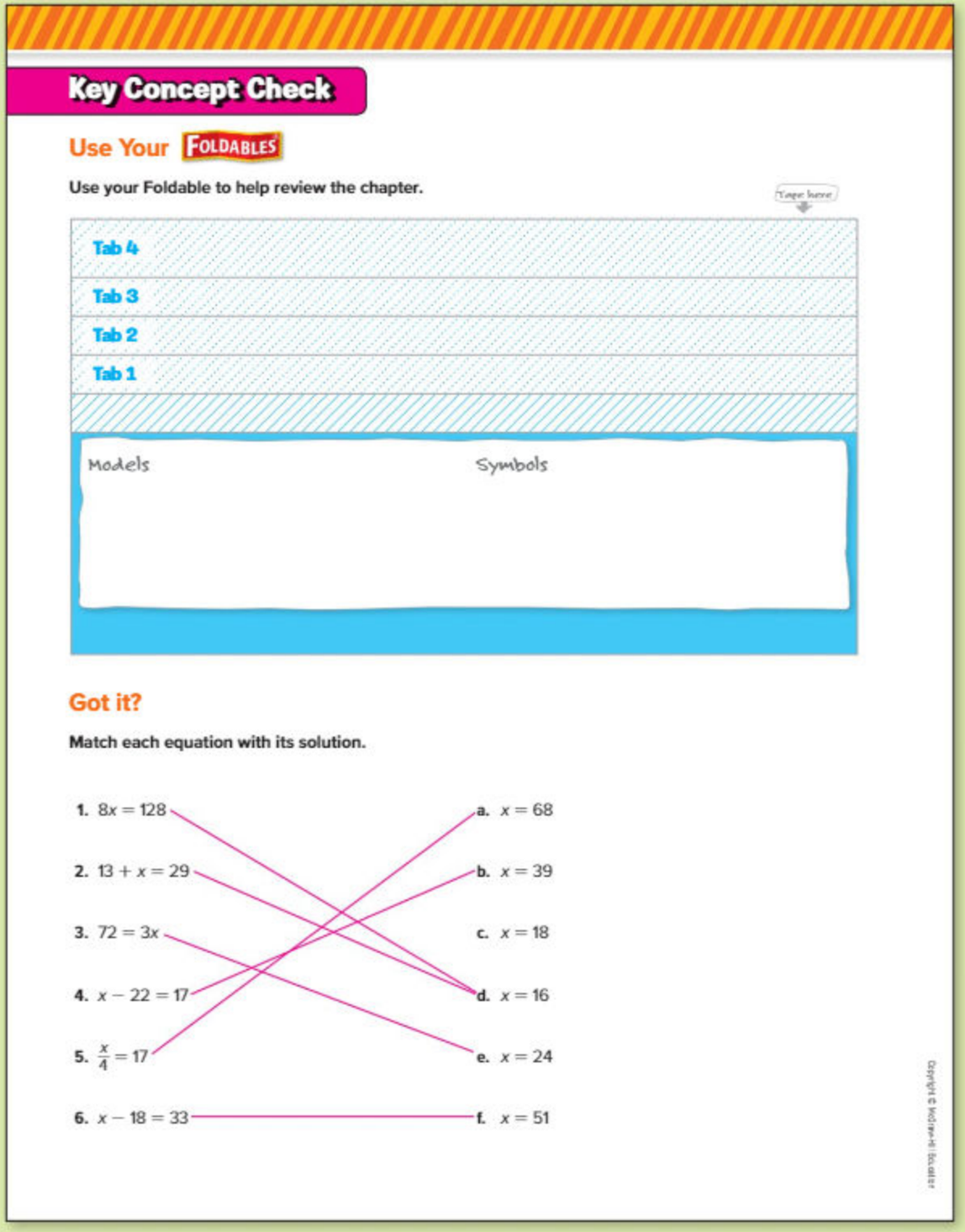
Ideas for Use

LA Have students work in pairs to discuss their Foldables. Have them practice speaking in a collaborative setting by sharing how they have completed their Foldable thus far and how they could finish it. Have each student complete their Foldable and trade with their partner to discuss any similarities and differences. MP 1, 3, 5

Got It?

If students have trouble with Exercises 1–6, they may need help with the following concept(s).

Concept	Exercise(s)
addition equations (Lesson 2)	2
subtraction equations (Lesson 3)	4, 6
multiplication equations (Lesson 4)	1, 3
division equations (Lesson 5)	5



Expressions and Equations

Power Up! Performance Task

Study Buddies

Eiman and Asma spend the weekend studying for upcoming tests. They start with math, since that is their favorite subject. The table lists their scores for the first three math tests of the semester.

Student	Test #1	Test #2	Test #3
Eiman	75	100	100
Asma	92	x	88

Write your answers on another piece of paper. Show all of your work to receive full credit.

Part A
Asma cannot remember what she scored on the second test, but she knows that the sum of the three tests is 270. Write and solve an addition equation to determine what she scored on her second test.

Part B
An A grade will be given to students having at least 450 total test points. There are two more tests to take before the semester is over. Eiman wants to know what she needs to score on the next two tests to finish with an A. Write and solve an equation to determine what score she needs to average on the next two tests if each question is worth 1 point. Explain your reasoning.

Part C
Consider the equation $5x = 8$. Write a scenario pertaining to the girls' studying that is represented by this equation. Solve the equation and explain what the answer represents.

Power Up! Performance Task

This Performance-Based Assessment requires students to solve multi-step problems through abstract reasoning, precision, and perseverance. This practice scenario can be used to help students prepare for the thinking skills that will be used on the Assessment.

A complete scoring rubric with answers to the Exercises can be found on page PT2.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the **Building on the Essential Question** exercises found in each lesson of the chapter.

- How do you solve an equation? (p. 516)
- How can the Subtraction Property of Equality be used to solve addition equations? (p. 528)
- How can the Addition Property of Equality be used to solve subtraction problems? (p. 538)
- How can the Division Property of Equality be used to solve multiplication problems? (p. 554)
- When solving an equation, why is it necessary to perform the same operation on each side of the equals sign? (p. 564)

Ideas for Use

LA **Think-Pair-Share** Have students work in pairs. Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer. Then have them share their responses with their classmate before they complete the graphic organizer.
MP 1, 3, 5

Track Your Progress

Have your students return to pages xix–xxii to rate their knowledge. They should see that their knowledge of the key ideas has increased now that they have completed this chapter.

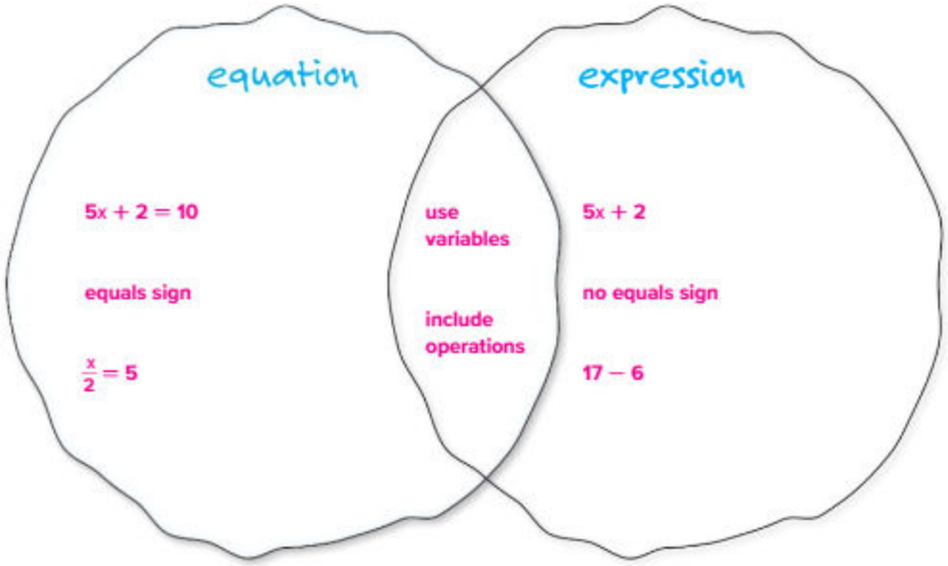
Reflect

Answering the Essential Question

Use what you learned about expressions and equations to complete the graphic organizer. Sample answers are given.

Essential Question

HOW do you determine if two numbers or expressions are equal?



Answer the Essential Question. HOW do you determine if two numbers or expressions are equal?

See students' work.

Focus narrowing the scope

This chapter focuses on **Expressions and Equations**.

Coherence connecting within and across grades

Previous
Students simplified expressions and solved equations.

Now
Students write and solve functions and inequalities.

Next
Students will find the area of polygons and composite figures.

Rigor pursuing concepts, fluency, and applications

The Levels of Complexity charts located throughout this chapter indicate how the exercises progress from conceptual understanding and procedural skills and fluency, to application and critical thinking.

Launch the Chapter

Math in the Real World

Ocean Life Explain to students that they need to use < or > to compare the two values. The symbol < represents *less than* and the symbol > represents *greater than*.

What Tools Do You Need?

Vocabulary Activity

LA As you proceed through the chapter, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define: A multiplicative relationship uses multiplication to compare the independent and dependent quantities of a relationship.

Example: $y = 7x$

Ask:

- *How would you check to see if the equation $y = 12x$ is a multiplicative relationship?* Sample answer: make a table

Writing Math

LA Have students read the Describe Data section.

Ask:

- *How would you describe the data for watermelon gum in the survey?* Sample answer: Two more people favored watermelon gum than cinnamon gum.
- *In Exercise 1, how would you describe the data for the cockroach?* Sample answer: Twelve fewer people voted for the cockroach as their least favorite bug than the spider.

What Tools Do You Need?

Vocabulary

arithmetic sequence	independent variable
dependent variable	inequality
function	linear function
function rule	sequence
function table	term
geometric sequence	

Study Skill: Writing Math

Describe Data

When you *describe* something, you represent it in words.

Fahd surveyed his class to find their favorite flavor of sugarless gum. Describe the data.

- Eight more people favor peppermint gum over cinnamon gum.
- The total number of people surveyed is 40.

These statements describe the data. What other ways can you describe the data? Sample answer: The data could also be described using measures of center or comparing other flavors to one another.

Favorite Flavor of Sugarless Gum	
Flavor	**Number**
Cinnamon	10
Peppermint	18
Watermelon	12

Describe the data below.

1.

Least Favorite "Bug"	
Kind	**Number**
Centipede	2
Cockroach	18
Spider	30

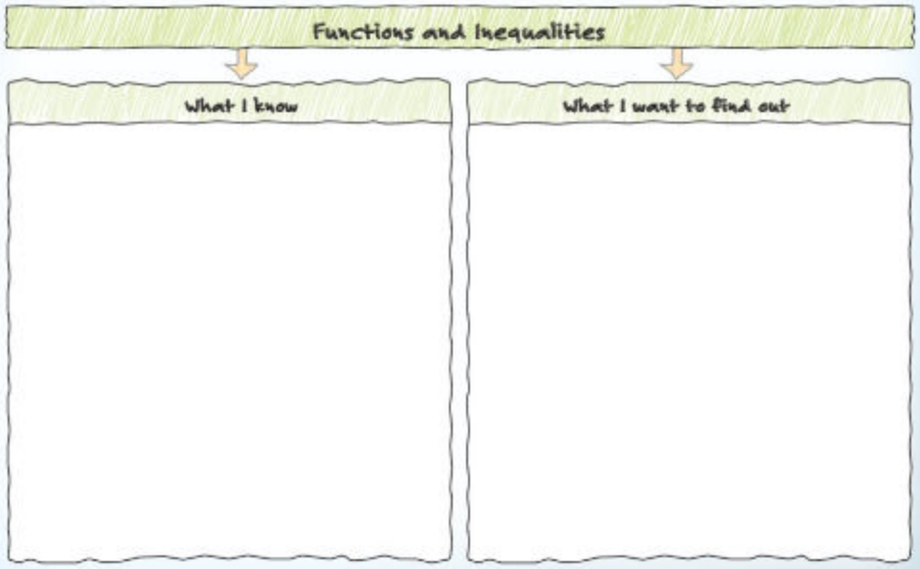
Sample answer: Thirty people named spiders as their least favorite bug. The total number of people surveyed was 50.

2. **Swimming**

Sample answer: The swimmer completed 8 laps in the first week. The swimmer swam 7 more laps in Week 3 than in Week 2.

What Do You Already Know?

List three things you already know about functions and inequalities in the first section. Then list three things you would like to learn about functions and inequalities in the second section. See students' work.



When Will You Use This?

Here is an example of how inequalities are used in the real world.

Activity Ask your parents to help you research the cost of an upcoming concert in your region. Give the concert and the cost of one ticket. Are there any additional fees? If so, how much are they?

See students' work.

What Do You Already Know?

In this activity, students assess their prior knowledge by listing three things they already know and three things they would like to learn about concepts in the chapter.

- You may want to add a third option of "I don't know" for those students who do not have any prior knowledge of the topic.
- After completing the chapter, have students return to this page and have them add three new facts that they learned about the topic.

When Will You Use This?

Activity

Students discover the connection of functions to real-world situations through research.

Are You Ready?

Use this page to determine if students have skills that are needed for the chapter.

Quick Review

Students with strong math backgrounds may opt to go directly to the Quick Check.

REVIEW	
Example	**Skill**
1	Compare Whole Numbers.
2	Solve One-Step Equations.

Quick Review

If students have difficulty with the exercises, present an additional example to clarify any misconceptions.

Exercises 1–4

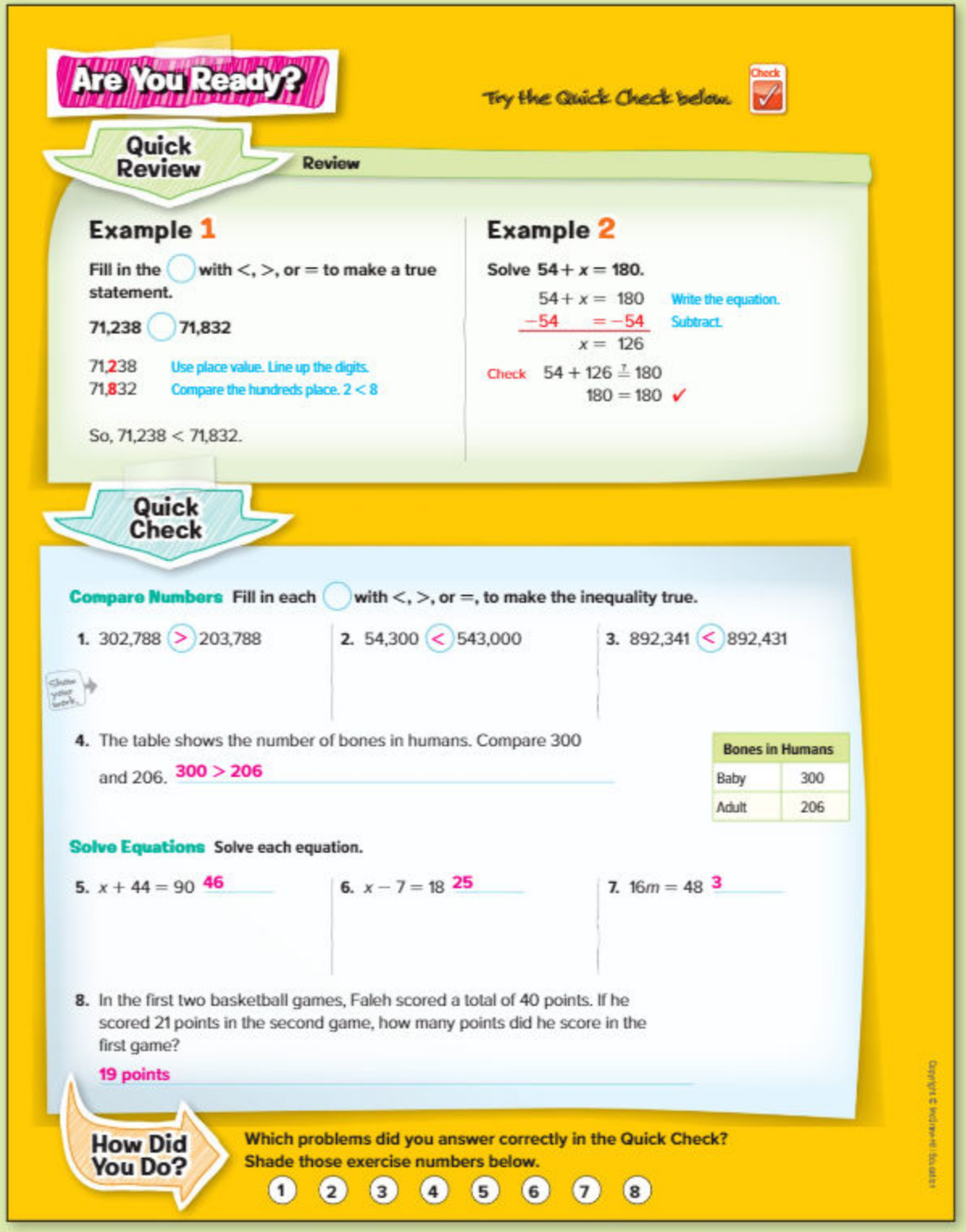
Is 26,351 <, >, or = 26,531? <

Exercises 5–7

Solve $6m = 24$. 4

Track Your Progress

Prior to beginning this chapter, have your students go to pages xix–xxii to rate their current knowledge. At the end of the chapter, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge of the key ideas has increased.



Lesson 1

Function Tables

Real-World Link

Science A ruby-throated hummingbird beats its wings about 52 beats per second.

1. Make a table showing show many times this bird beats its wings in 2 seconds.

Number of Seconds (s)	s · 52	Wing Beats
2	2 · 52	104

2. Make a table to show how many times it beats its wings in 6 seconds.

Number of Seconds (s)	s · 52	Wing Beats
6	6 · 52	312

3. Make a table to show how many times it beats its wings in 20 seconds.

Number of Seconds (s)	s · 52	Wing Beats
20	20 · 52	1,040

4. A giant hummingbird beats its wings about 10 times per second. Make a table to show how many times the Giant Hummingbird beats its wings in 3 seconds.

Number of Seconds (s)	s · 10	Wing Beats
3	3 · 10	30

Essential Question

HOW are symbols, such as <, >, and =, useful?

Vocabulary

function
function rule
function table
independent variable
dependent variable

Mathematical Practices
1, 3, 4, 5

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Complete function tables for given function rules.

Coherence connecting within and across grades

Previous
Students solved and wrote one-step equations.

Now
Students complete function tables, finding the input and the output.

Next
Students will find a rule to represent arithmetic sequences.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 579.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Chapter

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Numbered Heads Together** Have students work in small groups to complete Exercises 1–4. Assign each student to a number. Groups are responsible to ensure that each student understands. Call on one numbered student to share their group's responses. MP 1, 5

Alternate Strategies

AL LA Show a table starting with 1 second, 2 seconds, 3 seconds, and so on. Ask the students why they can just multiply the number of seconds by 52. MP 1, 5

BL LA Have students find the number of times a ruby-throated hummingbird beats its wings in 1 minute, 2 minutes, 3 minutes, and *m* minutes. MP 1, 5

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Complete a function table.**

AL • *What are the input values?* 10, 12, 14

• *Why do you think these values are called input values?* Sample answer: These are the values that you "put in" for the variable.

• *If the output is 7 more than the input, what is the function rule?* $x + 7$

OL • *How can you find each output value?* Add 7 to each input value.

• *Which quantity is the dependent quantity?* the output values, 17, 19, and 21

BL • *If the output is 25, what is the input value for this rule? Explain how you found the input value.* 18; Subtract 7 from 25.

Need Another Example?
The output is 6 more than the input. Complete a function table for this relation. See Answer Appendix.

2. **Complete a function table.**

AL • *What are the input values?* 8, 10, 12

• *If the output is 5 times the input, what is the function rule?* $5x$

OL • *How can you find each output value?* Multiply each input value by 5.

• *Which quantity is the dependent quantity?* the output values; 40, 50, and 60

BL • *If the output is 20, what is the input value for this rule? Explain how you found the input value.* 4; Divide 20 by 5.

Need Another Example?
The output is 3 times the input. Complete a function table for this relation. See Answer Appendix.

Work Zone

STOP and Reflect
What values were used for the independent variable in Example 1? Answer below.

10, 12, and 14

Find the Output for a Function Table

A **function** is a relation that assigns exactly one output value to one input value. The number of wing beats (output) depends on the number of seconds (input). The **function rule** describes the relationship between each input and output. You can organize the input-output values and the function rule in a **function table**.

In a function, the input value is also known as the **independent variable**, since it can be any number you choose. The value of the output depends upon the input value, so the output value is known as the **dependent variable**.

Examples

1. **The output is 7 more than the input. Complete a function table for this relation.**

The function rule is $x + 7$. Add 7 to each input.

Input (x)	$x + 7$	Output
10		
12		
14		

→

Input (x)	$x + 7$	Output
10	$10 + 7$	17
12	$12 + 7$	19
14	$14 + 7$	21

2. **The output is 5 times the input. Complete a function table for this relation.**

The function rule is $5x$. Multiply each input by 5.

Input (x)	$5x$	Output
8		
10		
12		

→

Input (x)	$5x$	Output
8	$5 \cdot 8$	40
10	$5 \cdot 10$	50
12	$5 \cdot 12$	60

Got it? **Do these problems to find out.**

a.

Input (x)	$x - 4$	Output
4	$4 - 4$	0
7	$7 - 4$	3
10	$10 - 4$	6

b.

Input (x)	$3x$	Output
0	$3(0)$	0
2	$3(2)$	6
5	$3(5)$	15

Find the Input for a Function Table

The input and output of a function table can be represented as a set of ordered pairs, or a *relation*. In this lesson, the *x*-values represent the input and the *y*-values represent the output.

Example

3. **Find the input for the function table.**

Input (x)	$3x$	Output
		6
		15
		21

Use the *work backward* strategy to determine the input. If the output is found by multiplying by 3, then the input is found by dividing by 3.

The input values are $6 \div 3$ or 2, $15 \div 3$ or 5, and $21 \div 3$ or 7.

Got it? **Do these problems to find out.**

c.

Input (x)	$2x - 1$	Output
1	$2(1) - 1$	1
2	$2(2) - 1$	3
3	$2(3) - 1$	5

d.

Input (x)	$3x + 2$	Output
5	$3(5) + 2$	17
6	$3(6) + 2$	20
9	$3(9) + 2$	29

Example

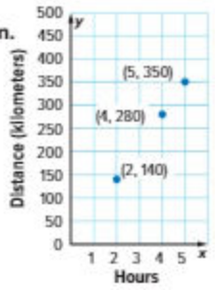
4. **Rashid's family is traveling at a rate of 70 kilometers per hour. The function rule that represents this situation is $70x$, where x is the number of hours. Make a table to find how many hours they have driven at 140 kilometers, 280 kilometers, and 350 kilometers. Then graph the function.**

Input (x)	$70x$	Output (y)
2	70(2)	140
4	70(4)	280
5	70(5)	350

Use the *work backward* strategy. Divide each output by 70.

The missing input values are $140 \div 70$ or 2, $280 \div 70$ or 4, and $350 \div 70$ or 5.

The input and output values are the ordered pairs (x, y). Plot each ordered pair on the graph.



Examples

3. **Find the input value given the function rule and output value.**

AL • *What are the output values?* 6, 15, 21

• *What is the function rule?* $3x$

OL • *How can you find each input value? Explain.* Divide each output by 3; Sample answer: Since the output is found by multiplying the input by 3, you can find the input by dividing the output by 3.

• *What are the input values?* 2, 5, 7

• *Which values are the independent quantities?* 2, 5, and 7 (the input values)

BL • *Give a real-world situation that this table could represent.* Sample answer: Cashews cost AED3 per kilogram. How many kilograms are purchased if the total cost is AED6, AED15, or AED21? 2 kg; 5 kg; 7 kg

Need Another Example?
Find the input for the function table.

Input (x)	$x - 3$	Output
10	$10 - 3$	7
8	$8 - 3$	5
5	$5 - 3$	2

4. **Use a function table.**

AL • *What is the function rule?* $70x$

• *What are the output values?* 140, 280, 350

OL • *How can you find each input value?* Divide each output by 70.

• *What are the input values?* 2, 4, 5

BL • *If the distance to their destination is 525 kilometers, how long will it take the Rashid family to reach their destination? Explain.* 7.5 h; 525 km ÷ 70 km/h = 7.5 h

Need Another Example?
A large pizza at The Pizza Palace costs AED9. The total cost of the pizzas is represented by the rule $9x$ where x is the number of pizzas ordered. Make a table to determine how many pizzas were ordered for total costs of AED108, AED126, and AED153. Then graph the ordered pairs. See Answer Appendix.

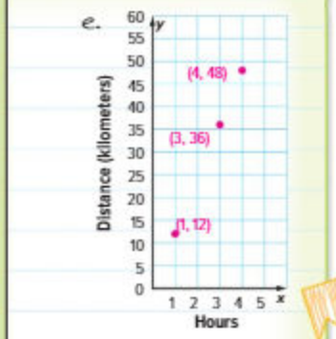
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Team-Pair-Solo** Have students work as a small team to complete Exercise 1, ensuring that each team member understands how to complete the function table to show the cost of 2, 3, and 4 kilograms of jelly beans. Then have teams divide into pairs to complete Exercise 2. Finally, have students work individually to complete Exercise 3. MP 1, 5

BL LA **Pairs Discussion** Have students extend Exercise 2 to compare the rule $4x$ to the rule that represents another person, Abdulrahman, who hikes at a rate of 5 kilometers per hour. Have students create a table for Abdulrahman's rule, $5x$. Then have students graph the relationship on the same coordinate plane as the rule for Majed. Have them compare and contrast the tables and graphs. MP 1, 2, 3, 5



Got it? Do this problem to find out.

e. Noura bikes 12 kilometers per hour. The function rule that represents this situation is $12x$, where x is the number of hours. Make a table to find how many hours she has biked when she has gone 12, 36, and 48 kilometers. Then graph the function.

Input (x)	12x	Output (y)
1	12(1)	12
3	12(3)	36
4	12(4)	48

Guided Practice

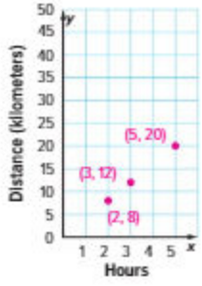
Check

1. Usama is buying jelly beans. In bulk, they cost AED 3 per kilogram, and a candy dish costs AED 2. The function rule, $3x + 2$ where x is the number of kilograms, can be used to find the total cost of x kilograms of jelly beans and 1 dish. Make a table that shows the total cost of buying 2, 3, or 4 kilograms of jelly beans and 1 dish. (Examples 1 and 2)

Kilograms (x)	3x + 2	Cost (AED) (y)
2	3(2) + 2	8
3	3(3) + 2	11
4	3(4) + 2	14

2. Majed hikes 4 kilometers per hour. The function rule that represents this situation is $4x$, where x is the number of hours. Make a table to find how many hours he has hiked when he has gone 8, 12, and 20 kilometers. Then graph the function. (Examples 3 and 4)

Hours (x)	4x	Kilometers (y)
2	4(2)	8
3	4(3)	12
5	4(5)	20



3. **Building on the Essential Question** How can a function table help you find input or output?

Sample answer: When data is organized, I can use the function rule and the input to find the output or work backward using the output and the function rule to find the input.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

Use Math Tools Complete each function table. (Examples 1–3)

1.

Input (x)	3x + 5	Output
0	3(0) + 5	5
3	3(3) + 5	14
9	3(9) + 5	32

2.

Input (x)	x − 4	Output
4	4 − 4	0
8	8 − 4	4
11	11 − 4	7

3.

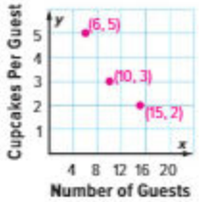
Input (x)	x + 2	Output
0	0 + 2	2
1	1 + 2	3
6	6 + 2	8

4.

Input (x)	2x + 4	Output
7	2(7) + 4	18
9	2(9) + 4	22
15	2(15) + 4	34

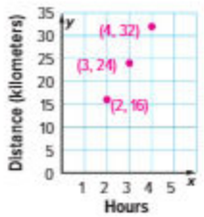
5. Hana has a total of 30 cupcakes for her guests. The function rule, 30 ÷ x where x is the number of guests, can be used to find the number of cupcakes per guest. Make a table of values that shows the number of cupcakes each guest will get if there are 6, 10, or 15 guests. Then graph the function. (Examples 1 and 2)

Number of Guests (x)	30 ÷ x	Cupcakes per Guest (y)
6	30 ÷ 6	5
10	30 ÷ 10	3
15	30 ÷ 15	2



6. Huda rollerblades 8 kilometers in one hour. The function rule that represents this situation is 8x, where x is the number of hours. Make a table to find how many hours she has skated when she has traveled 16, 24, and 32 kilometers. Then graph the function. (Examples 3 and 4)

Hours (x)	8x	Kilometers (y)
2	8(2)	16
3	8(3)	24
4	8(4)	32



7. Refer to Exercise 6. How many kilometers would Huda travel if she skated for 7 hours? 56 kilometers

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–8, 10–13, 20, 21
OL	On Level	1–5 odd, 7, 8, 10–13, 20, 21
BL	Beyond Level	7–13, 20, 21

Watch Out!

Common Error When using the work backward strategy on a two-step rule, some students may undo operations in the incorrect order. Remind students that to undo the order of operations, they need to go in reverse order. Instead of multiplying and dividing first, you do these last.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	9
3 Construct viable arguments and critique the reasoning of others.	8, 10, 11, 12
4 Model with mathematics.	13
5 Use appropriate tools strategically.	1–4, 14–17

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

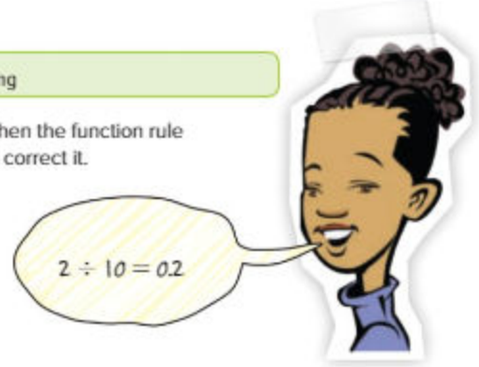
TICKET Out the Door

Have students create a table to represent this situation. Fawzia is 5 years older than her brother Amer. Fawzia's age, in years, is $x + 5$, where x is Amer's age, in years. See students' work.

Watch Out!

Find the Error In Exercise 8, Nahla switched the dividend and divisor in the function rule. She should have divided by the input value of 2 instead of by 10. Have students complete a 3-column function table in which the rule is the middle column. This should help them see that the divisor of the rule is the input value.

H.O.T. Problems Higher Order Thinking



8. **Find the Error** Nahla is finding the output when the function rule is $10 \div x$ and the input is 2. Find her mistake and correct it.

She divided the input by 10 instead of dividing 10 by the input. $10 \div 2 = 5$

9. **Persevere with Problems** Around 223 million Americans keep containers filled with coins in their home. Suppose each of the 223 million people started putting their coins back into circulation at a rate of AED 10 per year. Create a function table that shows the amount of money that would be recirculated in 1, 2, and 3 years.

Years (x)	223 million × AED 10 × x
1	AED 2,230,000,000
2	AED 4,460,000,000
3	AED 6,690,000,000

10. **Reason Inductively** Explain how to find the input given a function rule and output.

To find the input, work backward by performing the rule using the reverse of the order of operations.

11. **Justify Conclusions** Given the rule $x \div n$, describe the values of n for which the output value will be greater than the input value. Justify your response.

any number between 0 and 1; Sample answer: When you divide by a fraction, you multiply by the reciprocal. If the fraction is between 0 and 1, the reciprocal is greater than 1.

12. **Reason Inductively** Compare and contrast the tables used in this lesson to ratio tables.

Sample answer: When the rule is multiplication or division, the inputs and outputs form equivalent ratios. When the rule is addition or subtraction, the inputs and outputs do not form equivalent ratios.

13. **Model with Mathematics** Write a real-world problem that can be represented by a rule and a table using division.

Sample answer: Laila is making quilts for a charity. She has 48 meters of fabric to make quilts. Make a table that shows the number of quilts she can make that use 2, 3, and 4 meters of fabric.

Name _____ My Homework _____

Extra Practice

MP Use Math Tools Complete each function table.

Homework Help

14.

Input (x)	$x + 3$	Output
0	0 + 3	3
2	2 + 3	5
4	4 + 3	7

15.

Input (x)	$4x + 2$	Output
1	4(1) + 2	6
3	4(3) + 2	14
6	4(6) + 2	26

16.

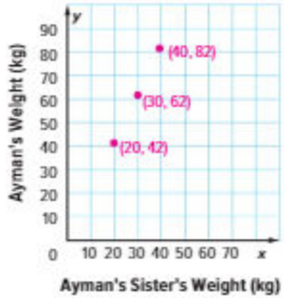
Input (x)	$x - 1$	Output
1	1 − 1	0
3	3 − 1	2
5	5 − 1	4

17.

Input (x)	$2x - 6$	Output
3	2(3) − 6	0
6	2(6) − 6	6
9	2(9) − 6	12

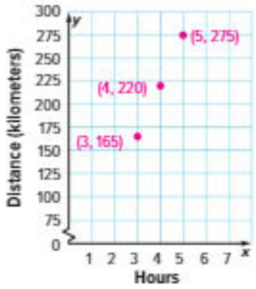
18. Ayman weighs 2 kilograms more than twice his sister's weight. The function rule, $2x + 2$ where x is his sister's weight, can be used to find Ayman's weight. Make a table of values that show Ayman's weight when his sister is 20, 30, and 40 kilograms. Then graph the function.

Ayman's Sister's Weight (x)	$2x + 2$	Ayman's Weight (y)
20	2(20) + 2	42
30	2(30) + 2	62
40	2(40) + 2	82



19. Jamal's family drove at a rate of 55 kilometers per hour. The function rule that represents this situation is $55x$, where x is the number of hours. Make a table to find how many hours they have traveled when they have driven 165, 220, and 275 kilometers. Then graph the function.

Hours (x)	$55x$	Kilometers (y)
3	55(3)	165
4	55(4)	220
5	55(5)	275



Power Up! Test Practice

Exercises 20 and 21 prepare students for more rigorous thinking needed for the assessment.

20.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP3, MP4, MP5
Scoring Rubric	
2 points	Students correctly complete the table and find how many points would be earned.
1 point	Students correctly complete the table but fail to find how many points would be earned OR students have a mathematical error that is carried through the table and the number of points is found based on the error OR students correctly complete 7–9 of the blanks in the table and may or may not correctly find the number of points.

21.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

Power Up! Test Practice

20. In U.S. football, a touchdown is worth 6 points. Complete the table that shows the points earned for scoring 1, 2, 3, 4, and 5 touchdowns.

How many points would a team earn for scoring 8 touchdowns?

48 points

Number of Touchdowns (x)	$6x$	Points (y)
1	6(1)	6
2	6(2)	12
3	6(3)	18
4	6(4)	24
5	6(5)	30

21. Refer to the function table at the right. Determine if each statement is true or false.

a. The output value when $x = 5$ is 3. ☐ True ■ False

b. The output value when $x = 6$ is 13. ■ True ☐ False

c. The output value when $x = 7$ is 16. ■ True ☐ False

Input (x)	$3x - 5$	Output (y)
5	3(5) − 5	
6	3(6) − 5	
7	3(7) − 5	

Spiral Review

Find the next number in the pattern using the given rule.

22. Add 3: 2, 5, 8, 11, . . . 14

23. Subtract 2: 10, 8, 6, 4, . . . 2

24. Multiply by 2: 2, 4, 8, 16 . . . 32

25. Subtract 7: 84, 77, 70, 63, . . . 56

26. Multiply by 2: 3, 6, 12, 24, . . . 48

27. Add 15: 12, 27, 42, 57, . . . 72

28. Ms. Halima is buying pencils for her class. What is the cost if she buys 24 pencils? AED 4.80

29. Salem and Zayed both start a savings account in May. Salem saves AED 2 each month and Zayed saves AED 4 each month. What do you notice about the amount in each account each month?

Zayed has twice as much money each month.

Month	Salem's Account (AED)	Zayed's Account (AED)
May	2	4
June	4	8
July	6	12

Lesson 2

Function Rules

Vocabulary Start-Up

A **sequence** is a list of numbers in a specific order. Each number in the list is called a **term** of the sequence.

Arithmetic sequences can be found by adding the same number to the previous term. In a **geometric sequence**, each term is found by multiplying the previous term by the same number.

Essential Question

HOW are symbols, such as <, >, and =, useful?

Vocabulary

sequence
term
arithmetic sequence
geometric sequence

Mathematical Practices
1, 3, 4, 7

Compare arithmetic sequences and geometric sequences.

Sample answers are given.

arithmetic sequence	geometric sequence
Definition: A sequence in which the difference between any two consecutive terms is the same.	Definition: A sequence in which each term is found by multiplying the previous term by the same number.
Example: 3, 6, 9, 12	Example: 3, 6, 12, 24

Real-World Link

Delivery The China Palace sells lunch specials for AED 6 with a delivery charge of AED 5 per order. Fill in the table with the next three numbers in the sequence.

Specials	1	2	3	4	5	6	7
Cost (AED)	11	17	23	29	35	41	47

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Extend and describe sequences using algebraic expressions.

Coherence connecting within and across grades

Previous
Students completed function tables given a rule.

Now
Students find a rule to describe the relationships between the terms of sequences.

Next
Students will represent functions using tables, graphs, and equations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 587.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Talking Chips** Have students work in small groups. Provide each student with 3 chips. Have students discuss the Vocabulary Start-Up and Real-World Link. Students must place one chip on the table as they verbally contribute to the discussion. Students may not talk after they have used all of their chips, and they must use all of their chips. MP 1, 3, 5

Alternate Strategy

AL LA Have students practice saying *arithmetic* aloud. Point out that the pronunciation of the word arithmetic is different with arithmetic sequences than the study of the four operations. MP 1, 6

2 Teach the Concept

Ask the scaffolded questions to differentiate instruction.

Examples

1. **Describe the relationship between the terms in an arithmetic sequence.**

AL • *How can you find each next term in an arithmetic sequence?* Add the same number to the previous term.

OL • *What is the relationship between the terms?* Each term is 7 more than the previous term.

• *How can you find the next three terms?* Add 7; $28 + 7 = 35$, $35 + 7 = 42$, $42 + 7 = 49$

BL • *Will the number 91 eventually be in this sequence? Explain.* yes; $49 + 7 = 56$; $56 + 7 = 63$; $63 + 7 = 70$; $70 + 7 = 77$; $77 + 7 = 84$; $84 + 7 = 91$

Need Another Example?

Describe the relationship between the terms in the arithmetic sequence 4, 7, 10, 13, ... Then write the next three terms. Each term is found by adding 3 to the previous term; 16, 19, 22.

2. **Describe the relationship between the terms in a geometric sequence.**

AL • *How can you find each next term in a geometric sequence?* Multiply the same number by the previous term.

OL • *What is the relationship between the terms?* Each term is 2 times the previous term.

• *How can you find the next three terms?* Multiply by 2; $16 \times 2 = 32$, $32 \times 2 = 64$, $64 \times 2 = 128$.

BL • *Does the number 514 belong in this sequence? Explain.* no; $128 \times 2 = 256$; $256 \times 2 = 512$; $512 \times 2 = 1{,}024$, so 514 is not in this sequence.

Need Another Example?

Describe the relationship between the terms in the geometric sequence 3, 12, 48, 192, ... Then write the next three terms. Each term is found by multiplying the previous term by 4; 768; 3,072; 12,288.

Work Zone

a. Each term is found by adding 15 to the previous term; 60, 75, 90

b. Each term is found by subtracting 0.5 from the previous term; 2.5, 2, 1.5

c. Each term is found by multiplying the previous term by 3; 81, 243, 729

d. Each term is found by multiplying the previous term by 2; 48, 96, 192

Arithmetic and Geometric Sequences

Determining if a sequence is arithmetic or geometric can help you find the pattern. When you know the pattern, you can continue the sequence to find missing terms.

Examples

1. **Describe the relationship between the terms in the arithmetic sequence 7, 14, 21, 28, ... Then write the next three terms.**

7, 14, 21, 28, ...
+7 +7 +7

Each term is found by adding 7 to the previous term. Continue the pattern to find the next three terms.

$28 + 7 = 35$ $35 + 7 = 42$ $42 + 7 = 49$

The next three terms are 35, 42, and 49.

2. **Describe the relationship between the terms in the geometric sequence 2, 4, 8, 16, ... Then write the next three terms.**

2, 4, 8, 16, ...
×2 ×2 ×2

Each term is found by multiplying the previous term by two. Continue the pattern to find the next three terms.

$16 \times 2 = 32$ $32 \times 2 = 64$ $64 \times 2 = 128$

The next three terms are 32, 64, and 128.

Got it? **Do these problems to find out.**

a. 0, 15, 30, 45, ...
b. 4.5, 4, 3.5, 3, ...
c. 1, 3, 9, 27, ...
d. 3, 6, 12, 24, ...

Expressions and Equations

Find a Rule

A sequence can also be shown in a table. The table gives both the position of each term in the list and the value of the term.

List

8, 16, 24, 32, ...

Table

Position	1	2	3	4
Value of Term	8	16	24	32

You can write an algebraic expression to describe a sequence. The value of each term can be described as a function of its position in the sequence.

In the table above, the position can be considered the input, and the value of the term as the output.

Example

3. **Use words and symbols to describe the value of each term as a function of its position. Then find the value of the tenth term.**

Position	1	2	3	4	n
Value of Term	3	6	9	12	■

Notice that the value of each term is 3 times its position number. So, the value of the term in position n is $3n$.

Position	Multiply by 3	Value of Term
1	1×3	3
2	2×3	6
3	3×3	9
4	4×3	12
n	$n \times 3$	$3n$

Now find the value of the tenth term.

$3n = 3 \cdot 10$ Replace n with 10.

$= 30$ Multiply.

The value of the tenth term in the sequence is 30.

Work Backward
You can check your rule by working backward. Divide each term by 3 to check the position.

Got it? **Do these problems to find out.**

Use words and symbols to describe the value of each term as a function of its position. Then find the value of the eighth term.

e.

Position	2	3	4	5	n
Value of Term	12	18	24	30	■

f.

Position	3	4	5	6	n
Value of Term	7	8	9	10	■

Show your work.

e. 6 times its position number; $6n$; 48

f. add 4 to the position number; $n + 4$; 12

Examples

3. **Find a function rule.**

AL
- *What are the positions given in the table?* 1, 2, 3, 4, and n
- *What are the values of the terms given in the table?* 3, 6, 9, 12, and an unknown value
- *What are you trying to find?* the value of the term for position n and the value of the tenth term

OL
- *How can you describe the relationship between the value of the term and its position?* Each value is three times its position.
- *How can you write "three times its position" as an algebraic rule?* $3(n)$ or $3n$
- *How can you find the value of the tenth term?* Replace n with 10 in the expression $3n$ and multiply.

BL
- *Examine the "Value of Term" column of the table. Is this an arithmetic or geometric sequence? Explain.* arithmetic; Each term is found by adding 3 to the previous term.
- *What is the position number for a term value of 141 in this relationship? Explain.* 47; By working backward, $141 \div 3 = 47$.

Need Another Example?

Use words and symbols to describe the value of each term in relation to its position. Then find the value of the eighth term. 2 more than its position number; $n + 2$; 10

Position	4	5	6	7	n
Value of Term	6	7	8	9	■

Examples

4. Find a function rule.

AL • *What is the problem asking you to find?* a rule for the number of necklaces Badria can make in x hours

• *How do the number of necklaces change for each hour?* The number of necklaces increases by 2 each hour.

OL • *How do we know that the rule includes the expression $2x$?* The number of necklaces increases by 2 each hour.

• *How do you know that the rule $2x + 1$ does not represent this situation?* Sample answer: If the rule was $2x + 1$, then the number of necklaces after 1 hour would be $2(1) + 1$, or 3, which is not true.

• *How can we test the rule $2x + 3$?* Use the *guess, check, and revise* strategy.

BL • *Suppose the number of necklaces were 8, 10, and 12 for 1, 2, and 3 hours, respectively. What would be the rule that represents this new situation?* $2x + 6$

Need Another Example?
The table shows the number of loaves of bread that a baker can bake, based on the number of hours he works. Write a rule to determine the number of loaves of bread he can make in x hours. $5x + 2$

Hours (x)	Number of Loaves
1	7
2	12
3	17
x	■

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activity below.

AL LA **Pairs Discussion** Have students work in pairs to first determine whether each sequence in Exercises 1–3 is arithmetic, geometric, or neither. Then have them describe the relationship, find the rule, and extend the pattern. MP 1, 2, 3

STOP and Reflect
Find the rule for the following sequence:
5, 9, 13, 17, . . .

$4x + 1$

Real World

Example

4. **The table shows the number of necklaces Badria can make, based on the number of hours she works. Write a function rule to find the number of necklaces she can make in x hours.**

To find the rule, determine the function.

Notice that the values 5, 7, 9, ... increase by 2, so the rule includes $2x$. If the rule were simply $2x$, then the number of necklaces in 1 hour would be 2. But this value is 5, which is three more than $2x$.

To test the rule $2x + 3$, use the *guess, check, and revise* strategy.

Row 1: $2x + 3 = 2(1) + 3 = 2 + 3$ or 5

Row 3: $2x + 3 = 2(3) + 3 = 6 + 3$ or 9

The rule $2x + 3$ represents the function table.

Hours (x)	Number of Necklaces
1	5
2	7
3	9
x	■

Guided Practice

Check

1. Describe the relationship between the terms in the sequence 13, 26, 52, 104, ... Then write the next three terms in the sequence. (Examples 1 and 2)
Each term is found by multiplying the previous term by 2; 208, 416, 832

2. Use words and symbols to describe the value of each term as a function of its position. Then find the value of the fifteenth term in the sequence. (Example 3)

Position	1	2	3	4	n
Value of Term	2	4	6	8	■

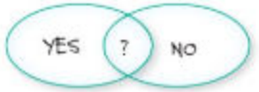
multiply the position number by 2; $2n$; 30

3. The table at the right shows the fee for overdue books at a library, based on the number of weeks the book is overdue. Write a function rule to find the fee for a book that is x weeks overdue. (Example 4) $2x + 1$

Weeks Overdue (x)	Fee (AED)
1	3
2	5
3	7
4	9
x	■

4. **Building on the Essential Question** What is the difference between an arithmetic sequence and a geometric sequence? Sample answer: Both are numerical patterns, but arithmetic sequences are additive and geometric sequences are multiplicative.

Rate Yourself!
Are you ready to move on? Shade the section that applies.



Name _____ My Homework _____

Independent Practice

Use words and symbols to describe the value of each term as a function of its position. Then find the value of the twelfth term in the sequence. (Examples 1–3)

1.

Position	3	4	5	6	n
Value of Term	12	13	14	15	■

add 9 to the position number; $n + 9$; 21

2.

Position	2	3	4	5	n
Value of Term	24	36	48	60	■

multiply the position number by 12; $12n$; 144

3. Describe the relationship between the terms in the sequence 6, 18, 54, 162, ... Then write the next three terms in the sequence. (Example 2)

Sample answer: This is a geometric sequence. Each term is found by multiplying the previous term by 3; 486, 1,458, 4,374

4. The table shows the amount it costs to rock climb at an indoor rock climbing facility, based on the number of hours. What is the rule to find the amount charged to rock climb for x hours? (Example 4)

$8x + 5$

Time (x)	Amount (AED)
1	13
2	21
3	29
4	37
x	■

MP **Identify Structure** **Determine how the next term in each sequence can be found. Then find the next two terms in the sequence.**

5. 4, 16, 28, 40, ...

Show your work.

add 12; 52, 64

6. 1.5, 3.9, 6.3, 8.7, ...

add 2.4; 11.1, 13.5

7. $2\frac{1}{4}, 2\frac{3}{4}, 3\frac{1}{4}, 3\frac{3}{4}, \ldots$

add $\frac{1}{2}$; $4\frac{1}{4}$, $4\frac{3}{4}$

Find the missing number in each sequence.

8. 30, $24\frac{1}{2}$, 19, $13\frac{1}{2}$, ...

9. 43.8, 36.7, 29.6, 22.5, ...

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–4, 5–13 odd, 14, 16, 26, 27
OL	On Level	1, 3, 5–14, 16, 26, 27
BL	Beyond Level	5–16, 26, 27

Watch Out!

Common Error If students wrote the wrong rule in Exercise 4, have them check it by testing it on the first set of input-output values. Suggest that they modify the rule so it works for the first set of input-output values and then test it on the other set.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	15
3 Construct viable arguments and critique the reasoning of others.	14, 16
7 Look for and make use of structure.	5–7, 21–23

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET
Out the Door

Display the following table. Ask students to find the value of the tenth position. 15

Position	10	11	12	13	14
Value of Term	■	16	17	18	19

State whether each sequence is arithmetic or geometric. Then find the next two terms in the sequence.

10. 1, 6, 36, 216

geometric sequence; 1,296, 7,776

11. 0.75, 1.75, 2.75, 3.75

arithmetic sequence; 4.75, 5.75

12. 0, 13, 26, 39

arithmetic sequence; 52, 65

Faris is stacking cereal boxes to create a store display. The number of boxes in each row is shown in the table. Is the pattern an example of an arithmetic sequence or a geometric sequence? Explain. How many boxes will be in row 5?

arithmetic sequence; Each term is found by adding 2 to the previous term.; 10 + 2 = 12; 12 boxes

Row	Number of Boxes
1	4
2	6
3	8
4	10
5	■

H.O.T. Problems Higher Order Thinking

14. **Reason Inductively** Create a sequence in which $1\frac{1}{4}$ is added to each number.

Sample answer: 1, $2\frac{1}{4}$, $3\frac{1}{2}$, $4\frac{3}{4}$, ...

15. **Persevere with Problems** Refer to the table below. Use words and symbols to generalize the relationship of each term as a function of its position. Then determine the value of the term when $n = 100$.

Position	1	2	3	4	5	n
Value of Term	1	4	9	16	25	■

The value of each term is the square of its position; n^2; 10,000

16. **Justify Conclusions** What is the rule to find the value of the missing term in the sequence in the table at the right? Justify your response.

4(x) − 3; Sample answer: The values 1, 5, 9, 13, and 17 increase by 4, so the rule includes 4(x). When the input is 1, the output is 1, which is 3 less than 4. So, the rule is 4(x) − 3.

Position, x	Value of Term
1	1
2	5
3	9
4	13
5	17
x	■

Name _____ My Homework _____

Extra Practice

Use words and symbols to describe the value of each term as a function of its position. Then find the value of the twelfth term in the sequence.

17.

Position	6	7	8	9	n
Value of Term	2	3	4	5	■

Homework Help

subtract 4 from the position number; $n - 4$; 8

Look at position 6 and the value of the term 2 is 4 less than 6, so try subtracting 4 from the other position numbers listed. The function rule is $n - 4$. $12 - 4 = 8$

18.

Position	1	2	3	4	n
Value of Term	5	10	15	20	■

multiply the position number by 5; $5n$; 60

19. Describe the relationship between the terms in the sequence 4, 12, 36, 108, ... Then write the next three terms in the sequence.

Each term is found by multiplying the previous term by 3;

324, 972, 2,916

20. The table shows the cost of a pizza based on the number of toppings. Write a function rule to find the cost for a pizza with x toppings.

$2x + 10$

Number of Toppings (x)	Cost (AED)
1	12
2	14
3	16
4	18

MP **Identify Structure** **Determine how the next term in each sequence can be found. Then find the next two terms in the sequence.**

21. 1, 4, 7, 10, ...

add 3; 13, 16

22. 2.3, 3.2, 4.1, 5.0, ...

add 0.9; 5.9, 6.8

23. $1\frac{1}{2}$, 3, $4\frac{1}{2}$, 6, ...

add $1\frac{1}{2}$; $7\frac{1}{2}$, 9

Find the missing number in each sequence.

24. 7, $11\frac{1}{2}$, 16, $20\frac{1}{2}$, ...

25. 14.6, 19.3, 24, 28.7, ...

Power Up! Test Practice

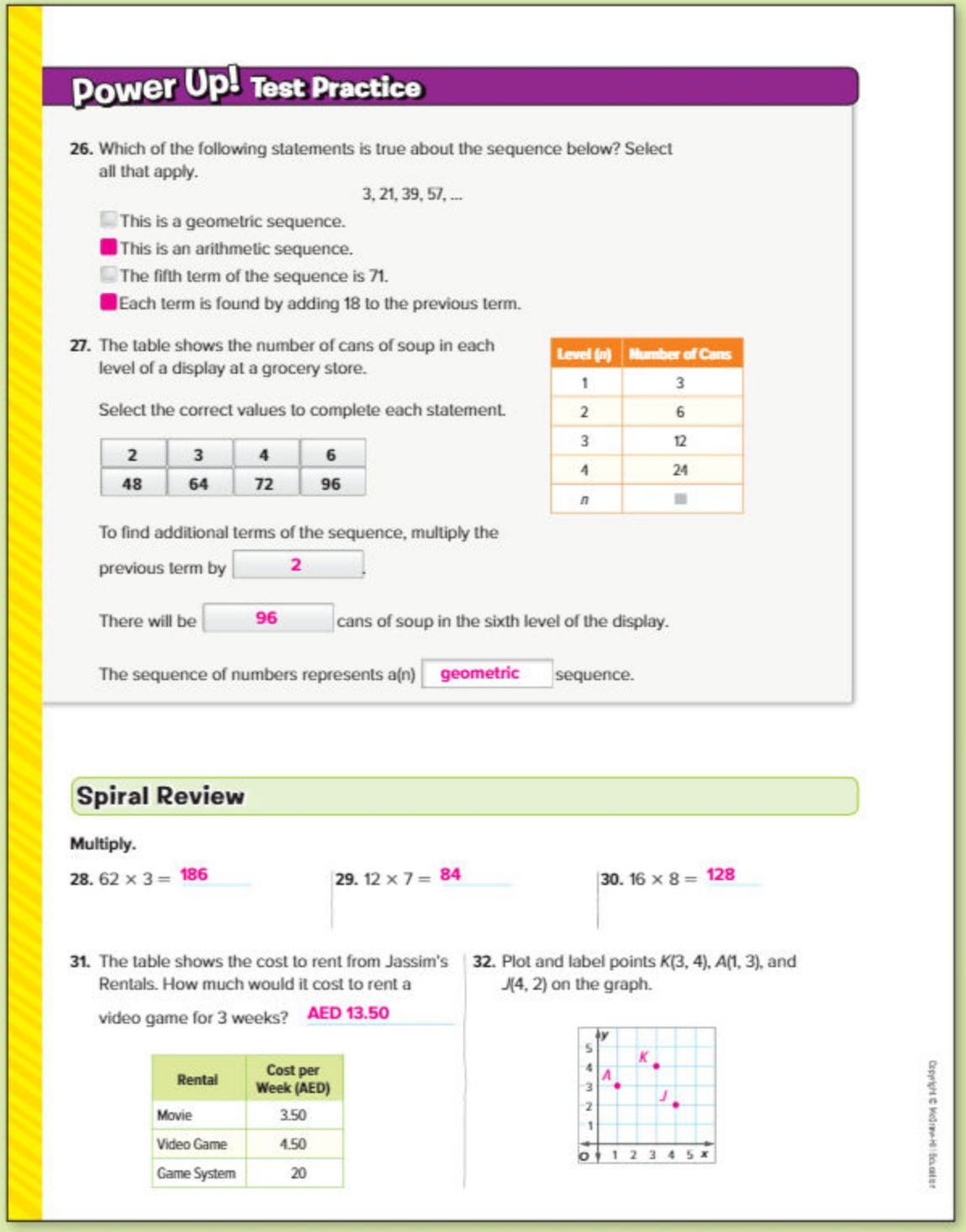
Exercises 26 and 27 prepare students for more rigorous thinking needed for the assessment.

26.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

27.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP7
Scoring Rubric	
2 points	Students correctly complete each sentence and identify the sequence as geometric.
1 point	Students correctly complete each sentence OR identify the sequence as geometric.



Power Up! Test Practice

26. Which of the following statements is true about the sequence below? Select all that apply.

3, 21, 39, 57, ...

- ☐ This is a geometric sequence.
- ■ This is an arithmetic sequence.
- ☐ The fifth term of the sequence is 71.
- ■ Each term is found by adding 18 to the previous term.

27. The table shows the number of cans of soup in each level of a display at a grocery store.

Level (n)	Number of Cans
1	3
2	6
3	12
4	24
n	■

Select the correct values to complete each statement.

2	3	4	6
48	64	72	96

To find additional terms of the sequence, multiply the previous term by [2].

There will be [96] cans of soup in the sixth level of the display.

The sequence of numbers represents a(n) [geometric] sequence.

Spiral Review

Multiply.

28. $62 \times 3 =$ 186

29. $12 \times 7 =$ 84

30. $16 \times 8 =$ 128

31. The table shows the cost to rent from Jassim's Rentals. How much would it cost to rent a video game for 3 weeks? AED 13.50

Rental	Cost per Week (AED)
Movie	3.50
Video Game	4.50
Game System	20

32. Plot and label points $K(3, 4)$, $A(1, 3)$, and $J(4, 2)$ on the graph.

Lesson 3

Functions and Equations

Vocabulary Start-Up

A **linear function** is a function whose graph is a line.

Linear	Everyday Definition of Function
having the form of or resembling a line	The purpose for which something is designed or exists.
linear function	
What do you notice about the graph? It forms a line. 	Math Definition of Function A relationship that assigns exactly one output value to one input value.

Essential Question

HOW are symbols, such as <, >, and =, useful?

Vocabulary

linear function

Mathematical Practices

1, 3, 4, 8

Real-World Link

Babysitting The table shows the amount of money Asma earns based on the number of hours she babysits.

1. Write a sentence that describes the relationship between the number of hours she babysits and her earnings.
   She earns AED 6 for every hour she babysits.
2. Does she earn the same amount each hour?
   Explain. Yes; her earnings increase by the same amount for every hour she babysits.

Hours Babysitting	Earnings (AED)
1	6
2	12
3	18
4	24

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

- (1) Persevere with Problems
- (2) Reason Abstractly
- (3) Construct an Argument
- (4) Model with Mathematics
- (5) Use Math Tools
- (6) Attend to Precision
- (7) Make Use of Structure
- (8) Use Repeated Reasoning

Focus narrowing the scope

Objective Construct and analyze different verbal, tabular, graphical, and algebraic representations of functions.

Coherence connecting within and across grades

Previous Students found a rule to describe the relationship between the terms of sequences.

Now Students represent functions using tables, graphs, and equations.

Next Students will construct and analyze multiple representations of functions.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 595.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Teammates Consult** Have students work in a small group to complete the graphic organizer, making sure that each group member understands and can explain what the parts of the term *linear function* mean in everyday life and in math. Then call on one group to share their responses with the class. MP 1, 3, 5

Alternate Strategy

AL LA Have students highlight the letters *line* in the term *linear* to have them remember what the graph of a linear function looks like. MP 1, 6

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Write an equation to represent a function.**

AL
- *Which variable represents the input values?* x
- *Which variable represents the output values?* y

OL
- *What is the relationship between the input values and the output values?* Each output value is 9 times the corresponding input value.
- *What equation represents this relationship?* $y = 9x$

BL
- *What is the difference between the equations $y = 9x$ and $x = 9y$? How do you know which equation correctly represents this relationship?* The equation $y = 9x$ is the correct equation because it shows that the output value, y, is 9 times the input value, x. The equation $x = 9y$ is incorrect because the input value, x, is not 9 times the output value, y.

Need Another Example?

Write an equation to represent the function shown in the table.

$y = 12x$

Input, x	1	2	3	4	5
Output, y	12	24	36	48	60

Work Zone

STOP and Reflect

In the equation $d = 36t$, where d is the distance traveled and t is the time, which variable is independent and which is dependent? Explain below.

Time, t, is the independent variable because the time multiplied by the constant will give the distance, the dependent variable.

a. $y = 16x$

Write an Equation to Represent a Function

You can use an equation to represent a function. The input, or independent variable, represents the x-value, and the output, or dependent variable, represents the y-value. An equation expresses the dependent variable in terms of the independent variable.

Example

1. **Write an equation to represent the function shown in the table.**

Input, x	1	2	3	4	5
Output, y	9	18	27	36	45

Input, x	Multiply by 9	Output, y
1	1×9	9
2	2×9	18
3	3×9	27
4	4×9	36
5	5×9	45

+9 +9 +9 +9

The value of y is equal to 9 times the value of x. So, the equation that represents the function is $y = 9x$.

Got it? **Do this problem to find out.**

a. Write an equation to represent the function shown in the table.

Input, x	1	2	3	4	5
Output, y	16	32	48	64	80

Graph Linear Functions

You can also graph a function. If the graph is a line, the function is then called a *linear equation*. When graphing the function, the input is the x-coordinate and the output is the y-coordinate.

(*input*, *output*) → (x, y)

Expressions and Equations

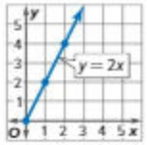
Example

2. Graph $y = 2x$.

Step 1 Make a table of ordered pairs. Select any three values for x. Substitute these values for x to find y.

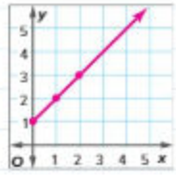
x	$2x$	y	(x, y)
0	2(0)	0	(0, 0)
1	2(1)	2	(1, 2)
2	2(2)	4	(2, 4)

Step 2 Graph each ordered pair. Draw a line through each point.



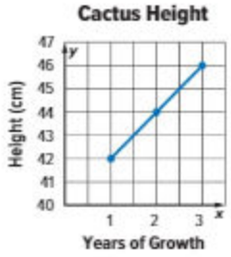
Got it? Do these problems to find out.

b. $y = x + 1$ c. $y = 3x + 2$



Examples

Eissa constructed the graph shown, which shows the height of his cactus after several years of growth.



3. **Make a function table for the input-output values.**

The three input values are 1, 2, and 3. The corresponding output values are 42, 44, and 46.

Input (x)	Output (y)
1	42
2	44
3	46

4. **Write an equation from the graph that could be used to find the height y of the cactus after x years.**

Since the output values increase by 2, the equation includes $2x$. Each output value is 40 more than twice the input. So, the equation is $y = 2x + 40$.

Examples

2. **Graph a linear function.**

AL • *Write the equation $y = 2x$ using words.* Sample answer: The output, y, is 2 times the input, x.

OL • *How can you make a table to represent this function?* Select any three values for x. Then replace those values into the rule $y = 2x$ to generate the y-values.

• *How can you represent this function with a graph?* Graph the ordered pairs from the table and draw a line that connects the points.

BL • *Does the ordered pair (7, 16) fit this relationship? Explain.* no; The output value of 16 is not equal to 2 times the input value of 7.

Need Another Example?

Graph $y = x + 2$. See Answer Appendix.

3–4. **Make a function table and write a function to represent a graph.**

AL • *What are the three input values?* 1, 2, and 3

• *What are the three output values?* 42, 44, and 46

OL • *How do the output values change?* They increase by 2.

• *How do you know the equation includes $2x$?* The output values increase by 2.

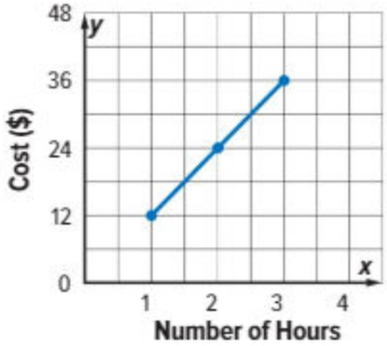
• *How does each output value compare to twice each input value?* Each output is 40 more than twice the input.

BL • *How tall was the cactus after 4.5 years?* 49 centimeters

Need Another Example?

Rana constructed the graph at the right that shows the cost to rent a canoe after several hours. Make a function table for the input-output values. Then write an equation from the graph that could be used to find the cost y to rent a canoe for x hours.

See Answer Appendix.



Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Team-Pair-Solo** Have students work as a small team to complete Exercise 1, ensuring that each student understands how to write a function equation from a table. Then have the team divide into pairs to complete Exercise 2, ensuring that each student understands how to graph a function. Then have students work individually to complete Exercises 3 and 4. Upon completion, have them rejoin their original team to discuss and compare solutions. MP 1, 2, 3, 5

BL LA **Gallery Walk** Have students create multiple representations to display a function; for example, one in which the output value is 6 times the input value. Students should create a table, a graph, and write an equation. Have them display their multiple representations around the room, omitting the equation. Then have students walk around the room and select a function. Have them write the equation that represents this function and compare it to the equation that was written by the creator of that function. MP 1, 2, 3, 5

Magazines (x)	Total (y)
1	20
2	25
3	30
4	35

d. $y = 15 + 5x$

Got it? Do this problem to find out.

d. The graph shows the total amount y that you spend if you buy one book and x magazines. Make a function table for the input-output values. Write an equation from the graph that could be used to find the total amount y if you buy one book and x magazines.

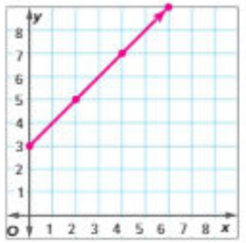
Guided Practice

1. Write an equation to represent the function shown in the table. (Example 1)

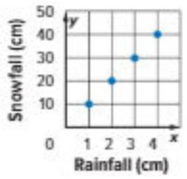
Input (x)	0	1	2	3	4
Output (y)	0	4	8	12	16

$y = 4x$

2. Graph the function $y = x + 3$. (Example 2)



3. The graph below shows the number of centimeters of rainfall x equivalent to centimeters of snow y. Make a function table for the input-output values. Write an equation from the graph that can be used to find the total centimeters of snow y equivalent to centimeters of rain x. (Examples 3 and 4)



Rain (x)	Snow (y)
1	10
2	20
3	30
4	40

$y = 10x$

4. **Building on the Essential Question** How are ordered pairs of a function used to create the graph of the function?

Sample answer: Each set of ordered pairs can be plotted on a coordinate plane. A line is then drawn through each point.

Rate Yourself!

How confident are you about finding the equation of a function? Check the box that applies.

FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

Write an equation to represent each function. (Example 1)

1.

Input (x)	1	2	3	4	5
Output (y)	6	12	18	24	30

$y = 6x$

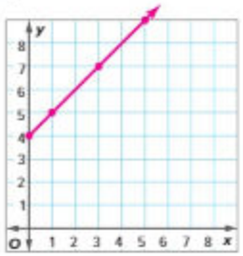
2.

Input (x)	0	1	2	3	4
Output (y)	0	15	30	45	60

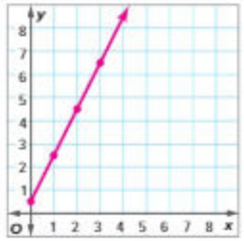
$y = 15x$

Graph each equation. (Example 2)

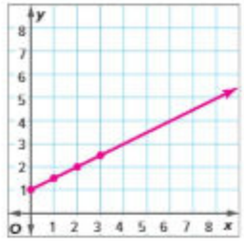
3. $y = x + 4$



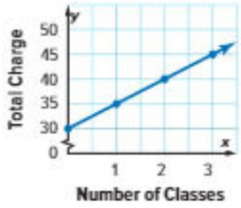
4. $y = 2x + 0.5$



5. $y = 0.5x + 1$



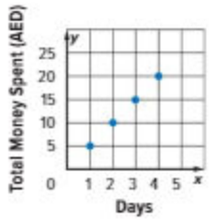
6. The graph shows the charges for a health club in a month. Make a function table for the input-output values. Write an equation that can be used to find the total charge y for the number of x classes. (Examples 3 and 4)



Input (x)	0	1	2	3
Output (y)	30	35	40	45

$y = 30 + 5x$

7. The graph shows the amount of money Sally spent on lunch. Make a function table for the input-output values. Write an equation that can be used to find the money spent y for any number of days x. (Examples 3 and 4)



Input (x)	1	2	3	4
Output (y)	5	10	15	20

$y = 5x$

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 19, 20
OL	On Level	1–7 odd, 8, 9, 19, 20
BL	Beyond Level	8–11, 19, 20

Watch Out!

Common Error When graphing a linear function, students may only graph two ordered pairs since any two points will form a line. Encourage students to graph at least three ordered pairs to check their work and reduce errors.

MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	10, 11
3 Construct viable arguments and critique the reasoning of others.	8
4 Model with mathematics.	9
8 Look for and express regularity in repeated reasoning.	12, 13

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students describe the procedures they would use to graph $y = x + 3$. See students' work.

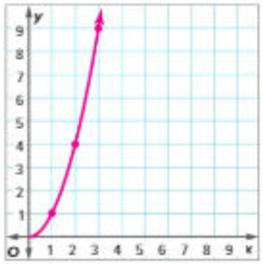
8. **Multiple Representations** The table shows the area of a square with the given side length.

Side Length (x)	Area of Square (y)
1	1
2	4
3	9
4	16

a. **Variables** Write an equation that could represent the function table.

$y = x \cdot x$ or $y = x^2$

b. **Graphs** Graph the function.



c. **Words** Is this a linear function? Explain.

no; The graph is curved; it does not form a line.

H.O.T. Problems Higher Order Thinking

9. **Model with Mathematics** Write about a real-world situation that can be represented by the equation $y = 7x$. Be sure to explain what the variables represent in the situation. Sample answer: Jassim is saving AED 7 per week to buy a new DVD player. The variable y represents the total amount he has saved. The variable x represents the number of weeks.

10. **Persevere with Problems** Write an equation to represent the function in the table shown below. $y = \frac{1}{2}x - 3$

Input (x)	6	8	10	12	14	16
Output (y)	0	1	2	3	4	5

11. **Persevere with Problems** The inverse of a relationship can be found by switching the coordinates in each ordered pair. Complete the table for three input and output values of $y = x + 3$ and its inverse. Then use the table to write an equation of the inverse of $y = x + 3$. $y = x - 3$

Sample answer

$y = x + 3$			
Input (x)	1	2	3
Output (y)	4	5	6

Inverse of $y = x + 3$			
Input (x)	4	5	6
Output (y)	1	2	3

Name _____ My Homework _____

Extra Practice

MP Identify Repeated Reasoning Write an equation to represent each function.

12.

Input (x)	0	1	2	3	4
Output (y)	0	11	22	33	44

$y = 11x$

Each output y is 11 times each input x.

13.

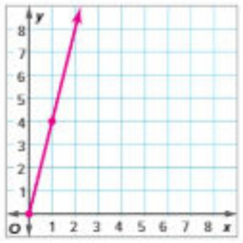
Input (x)	1	2	3	4	5
Output (y)	10	20	30	40	50

$y = 10x$

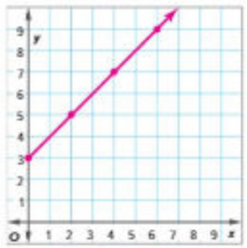
Homework Help

Graph each equation.

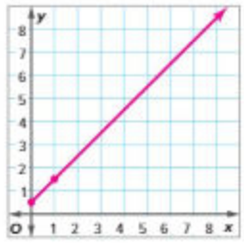
14. $y = 4x$



15. $y = 0.5x$

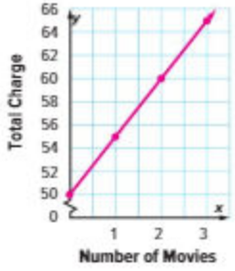


16. $y = x + 0.5$

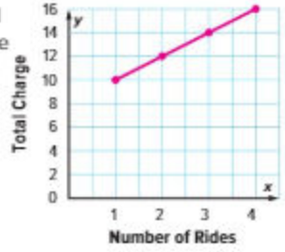


17. A company charges AED 50 per month for satellite television service plus an additional AED 5 for each movie ordered. The equation $y = 50 + 5x$ describes the total amount y a customer will pay if they order x movies. Graph the function.

Show your work



18. A fair charges an admission fee of AED 8. Each ride is an additional AED 2. The equation $y = 8 + 2x$ describes the total charge y for the number of rides x. Graph the function.



Power Up! Common Core Test Practice

Exercises 19 and 20 prepare students for more rigorous thinking needed for the assessments.

19.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

20.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students correctly place all four equations.
1 point	Students correctly place three of the four equations.

Power Up! Test Practice

19. The table shows the total cost of admission to a zoo for different numbers of guests. Determine if each statement is true or false.

a. The total admission for 12 guests is AED 84. ■ True ☐ False

b. The equation $y = 7x$ can be used to find the total admission for x guests. ■ True ☐ False

c. The total admission for 10 guests is AED 63. ☐ True ■ False

Number of Guests, x	Total Cost (AED), y
1	7
2	14
3	21
4	28

20. Match each function table to the correct equation.

Input (x)	1	2	3	4	5
Output (y)	9	10	11	12	13

equation: $y = x + 8$

Input (x)	1	2	3	4	5
Output (y)	7	14	21	28	35

equation: $y = 7x$

Input (x)	1	2	3	4	5
Output (y)	5	10	15	20	25

equation: $y = 5x$

Input (x)	1	2	3	4	5
Output (y)	5	6	7	8	9

equation: $y = x + 4$

$y = 5x$

$y = 7x$

$y = x + 8$

$y = x + 4$

Spiral Review

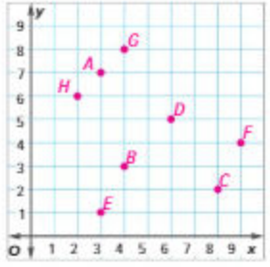
Graph and label each point.

21. $A(3, 7)$ **22.** $B(4, 3)$

23. $C(8, 2)$ **24.** $D(6, 5)$

25. $E(3, 1)$ **26.** $F(9, 4)$

27. $G(4, 8)$ **28.** $H(2, 6)$



29. Sumayya studied 20 minutes on Monday, 45 minutes on Tuesday, 30 minutes on Wednesday, and 45 minutes on Thursday. Organize this information in the table. How long did she study these four days? 2 hours and 20 minutes

Day	Time Studied (min)
Monday	20
Tuesday	45
Wednesday	30
Thursday	45

30. Ibrahim bought 3 notebooks for AED 5.85. How much did each notebook cost? AED 1.95

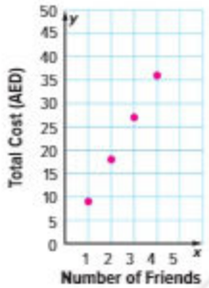
Lesson 4

Multiple Representations of Functions

Real-World Link

Museum A group of friends are going to the museum. Each friend must pay an admission price of AED 9.

Total Cost of Admission	
Number of Friends, x	Total Cost (AED), y
1	9
2	18
3	27
4	36



Essential Question

HOW are symbols, such as $<$, $>$, and $=$, useful?

MP Mathematical Practices
1, 2, 3, 4

1. Complete the table and graph the ordered pairs (number of friends, total cost).

2. Describe the graph.
The ordered pairs lie on a line.

3. Write an equation to find the cost of n tickets.
$y = 9x$, where y represents the total cost and x represents the number of friends

4. List the ordered pair for the cost when 5 friends go to the museum. Describe the location.
(5, 45); It is 5 units to the right and 45 units up.

Which MP **Mathematical Practices** did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Focus narrowing the scope

Objective Construct and analyze different verbal, tabular, graphical, and algebraic representations of functions.

Coherence connecting within and across grades

Previous
Students represented functions using tables, graphs, and equations.

Now
Students construct and analyze multiple representations of functions.

Next
Students will solve inequalities by using mental math.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 603.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Roundrobin** In groups of 4, have one pair of students complete Exercises 1 and 2, and the other pair complete Exercises 3 and 4. Ask them how the table, graph, and equation all represent the same relationship between the number of friends and the total cost in dollars.
MP 1, 2, 3, 5

Alternate Strategy

AL LA Have the students plot each point as they determine the total cost. Have them discuss the relationship of the y-coordinate to the x-coordinate and how this relationship helps them to write the equation in Exercise 3.
MP 1, 2, 3, 5

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Represent functions with words and equations.**

AL • *What do you need to write?* an equation to represent the total cost *t* of *p* pies

• *What is the cost of one pie?* AED5

OL • *How can you find the total cost of 4 pies?* Multiply AED5 by 4.

• *How can you find the total cost of p pies?* Multiply AED5 by *p*.

BL • *Why is writing an equation useful?* Sample answer: We can use the equation to quickly find the total cost for any number of pies.

Need Another Example?

A goat's average heart rate is about 70 beats per minute. Write an equation to find the total amount of heart beats *b* in *m* minutes. $b = 70m$

2. **Represent functions with words and equations.**

AL • *What do you need to write?* an equation to represent the total number of breaths *b* in *m* minutes

• *How many breaths does the average adult take in one minute?* 14 breaths

OL • *How can you find the total breaths in 10 minutes?* Multiply 14 by 10.

• *How can you find the total breaths in m minutes?* Multiply 14 by *m*.

BL • *A newborn baby takes 30–60 breaths per minute. Write two equations, to represent the minimum and maximum number of breaths b a baby takes in m minutes.* $b = 30m$; $b = 60m$

Need Another Example?

There are 20 5-fils coins in one dirham. Write an equation to find the total number of 5-fils coins *n* in *d* dirham. $n = 20d$

Key Concept

Represent Functions Using Words and Equations

Work Zone

Words	A runner's distance in a marathon is equal to 8 kilometers per hour times the number of hours.
Equation	$d = 8t$

Words and equations can be used to describe functions. For example, when a rate is expressed in words, it can be written as an equation with variables. When you write an equation, determine what variables to use to represent different quantities.

Variables

You can use any letter as a variable in an equation. If you graph the equation, make sure to label the axes with the correct variable.

Examples

1. **The drama club is holding a bake sale. They are charging AED5 for each pie they sell. Write an equation to find the total amount earned *t* for selling *p* pies.**

Words	Total earned equals AED 5 times the number of pies sold.
Variable	Let *t* represent the total earned and *p* represent the number of pies sold.
Equation	$t = 5 \cdot p$

So, the equation is $t = 5p$.

2. **In a science report, Nisreen finds that the average adult breathes 14 times each minute when not active. Write an equation to find the total breaths *b* a non-active person takes in *m* minutes.**

Let *b* represent the total breaths and *m* represent the number of minutes.

The number of total breaths equals 14 times the number of minutes.

So, the equation is $b = 14m$.

Got it? **Do these problems to find out.**

a. $d = 8h$

b. $c = 36h$

a. A mouse can travel 8 kilometers per hour. Write an equation to find the total distance *d* a mouse can travel in *h* hours.

b. Najla can make 36 cookies each hour. Write an equation to find the total number of cookies *c* that she can make in *h* hours.

Expressions and Equations

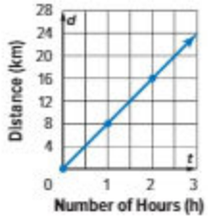
Represent Functions Using Tables and Graphs

Key Concept

Table

Time (h), t	Distance (km), d
0	0
1	8
2	16

Graph



Tables and graphs can also be used to represent functions.

Examples

The Student Council is holding a car wash to raise money. They are charging AED 7 for each car they wash.

3. **Write an equation and make a function table to show the relationship between the number of cars washed c and the total amount earned t.**

Cars Washed, c	$7c$	Total Earned (AED), t
1	1×7	7
2	2×7	14
3	3×7	21
4	4×7	28

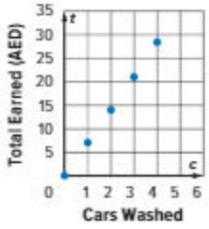
Using the assigned variables, the total earned t equals AED 7 times the number of cars washed c. So, the equation is $t = 7c$.

The total earned (output) is equal to AED 7 times the number of cars washed (input).

Write $7c$ in the middle column of the table.

4. **Graph the ordered pairs. Analyze the graph.**

Find the ordered pairs (c, t). The ordered pairs are (1, 7), (2, 14), (3, 21), and (4, 28). Now graph the ordered pairs.



The graph is linear because the amount earned increases by AED 7 for each car washed.

STOP and Reflect

What are the independent and dependent variables in Example 3? Explain below.

independent variable: c; dependent variable: t; Sample answer: The independent variable is the input and determines the output, or dependent variable.

Examples

3–4. **Use multiple representations of functions.**

AL
- *How much do they charge to wash one car?* AED7
- *What are the input values?* 1, 2, 3, 4
- *What are the output values?* AED7, AED14, AED21, AED28

OL
- *What is the relationship between each input value and its corresponding output value?* The output value is 7 times the input value.
- *What equation can be used to determine the total money earned t for washing c cars?* $t = 7c$
- *Using the information in the table, what is the set of ordered pairs?* (1, 7), (2, 14), (3, 21), (4, 28)
- *Is the graph linear? Explain.* yes; The points fall on a straight line.

BL
- *If the Student Council earned $168 at the end of the day, how many cars did they wash?* 24 cars
- *Suppose the Student Council earned AED30 in donations in addition to the money earned for all the cars they washed. What equation represents the total amount earned t for washing c cars?* $t = 7c + 30$
- *Use the equation you wrote above to find the total earned altogether for washing 15 cars.* AED135

Need Another Example?

Ali sells handmade notebooks. He charges AED25 for each book. Write an equation and make a function table to show the relationship between the total amount earned in dollars t for selling b books. Then graph the ordered pairs and analyze the graph. See Answer Appendix.

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

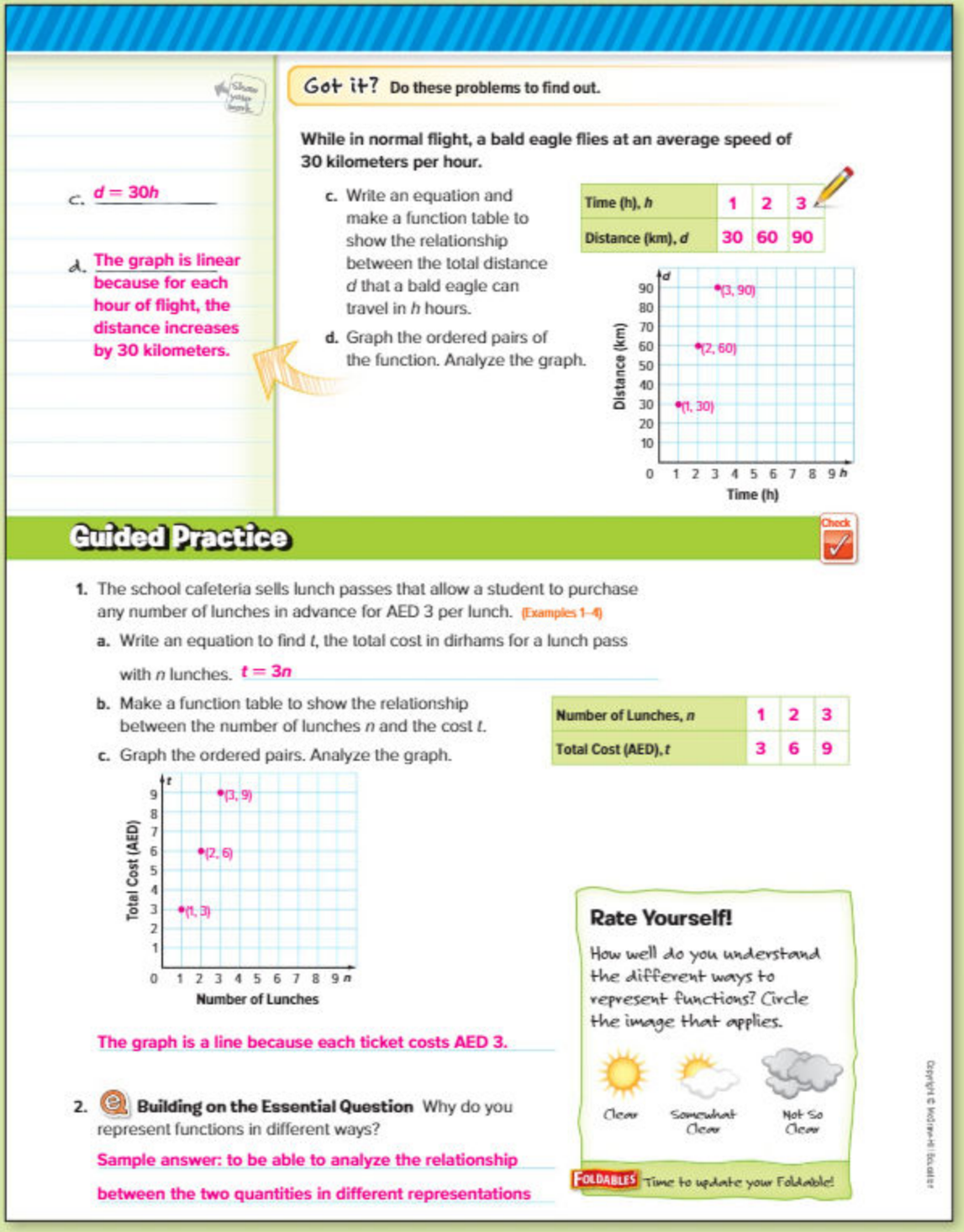
If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Roundrobin** Have students work in small groups to complete Exercises 1 and 2. Have each student contribute to one part of the exercises. For example, Student 1 writes the equation for Exercise 1a. Student 2 creates the table for Exercise 1b. Student 3 graphs the ordered pairs for Exercise 1c. Student 4, if available, leads the discussion for Exercise 2. If Student 4 is not available, have Student 1 lead the discussion for Exercise 2. HMP 1, 3, 5

BL LA **Pairs Discussion** Have students work in pairs to alter the situation in Exercise 1 and determine how the equation, table, and graph would be changed. For example, one alteration could be that the cafeteria increased their price to $5 per lunch. HMP 1, 2, 5

Watch Out!

Common Error Watch for students who switch the placement of the variables when they write the equation. Suggest that they check their equation by substituting ordered pairs into the equation and checking for a true sentence.



Got it? Do these problems to find out.

While in normal flight, a bald eagle flies at an average speed of 30 kilometers per hour.

c. Write an equation and make a function table to show the relationship between the total distance d that a bald eagle can travel in h hours.

d. Graph the ordered pairs of the function. Analyze the graph.

Time (h), h	1	2	3
Distance (km), d	30	60	90

c. $d = 30h$

d. The graph is linear because for each hour of flight, the distance increases by 30 kilometers.

Guided Practice

1. The school cafeteria sells lunch passes that allow a student to purchase any number of lunches in advance for AED 3 per lunch. (Examples 1–4)

   a. Write an equation to find t, the total cost in dirhams for a lunch pass with n lunches. $t = 3n$

   b. Make a function table to show the relationship between the number of lunches n and the cost t.

   c. Graph the ordered pairs. Analyze the graph.

Number of Lunches, n	1	2	3
Total Cost (AED), t	3	6	9

The graph is a line because each ticket costs AED 3.

2. **Building on the Essential Question** Why do you represent functions in different ways?

Sample answer: to be able to analyze the relationship between the two quantities in different representations

Rate Yourself!

How well do you understand the different ways to represent functions? Circle the image that applies.

Clear Somewhat Clear Not So Clear

FOLDABLES Time to update your Foldable!

Name _____ My Homework _____

Independent Practice

An African elephant eats 200 kilograms of vegetation each day. (Examples 1–4)

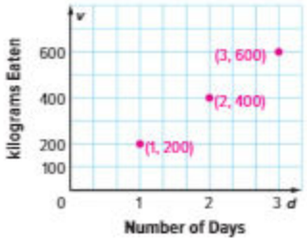
a. Write an equation to find v, the number of kilograms of vegetation an African elephant eats in d days. $v = 400d$

b. Make a table to show the relationship between the number of kilograms v an African elephant eats in days d.

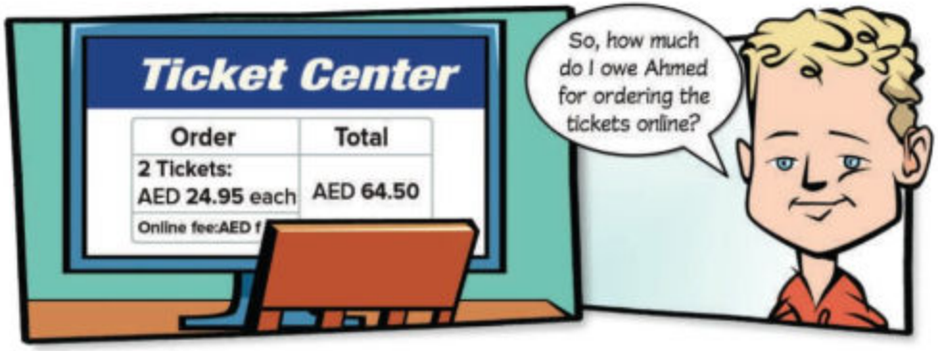
Number of Days, d	1	2	3
Kilograms Eaten, v	200	400	600

c. Graph the ordered pairs. Analyze the graph.

The graph is a line because with each day the amount of vegetation increases by 200.



2. **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–c.



a. Let f represent the cost of ordering each ticket online. Write an equation that could be used to find the cost of ordering each ticket online.

$49.90 + 2f = 64.50$

b. Solve the equation from part a. AED 7.30

c. Another friend wants to go to the concert. What is the total cost of ordering three tickets online?

AED 96.75

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

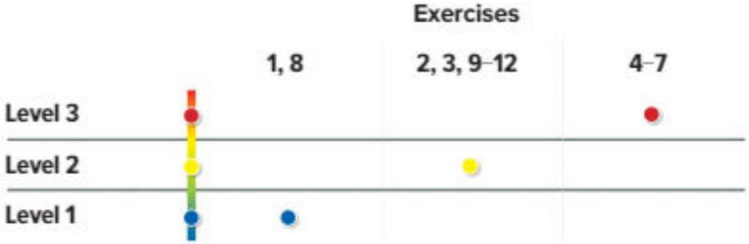
3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–4, 6, 7, 11, 12
OL	On Level	1–4, 6, 7, 11, 12
BL	Beyond Level	2–7, 11, 12

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	5
2 Reason abstractly and quantitatively.	4, 7
3 Construct viable arguments and critique the reasoning of others.	10
4 Model with mathematics.	2, 6

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET
Out the Door

Have students write how the concepts in earlier lessons on tables, rules, graphs, and equations helped with this lesson on multiple representations of functions.

Use the following writing prompts. See students' work.

- In previous lessons, I learned...
- In this lesson, I learned...
- What I learned in previous lessons helped me with the concepts in this lesson because...

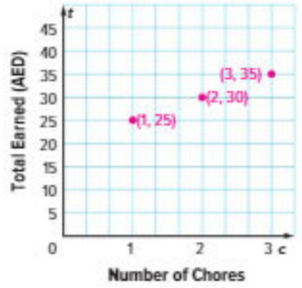
Khalid receives AED 20 per week for allowance and earns an additional AED 5 for each chore he completes.

a. Write an equation to find t, the total amount earned for c chores in one week. $t = 20 + 5c$; where t represents the total earned and c represents the number of chores

b. Make a function table to show the relationship between the number of chores completed c and the total amount earned t in one week if Khalid completes 1, 2, or 3 chores.

Number of Chores, c	1	2	3
Total Earned (AED), t	25	30	35

c. Graph the ordered pairs.



d. How much will Khalid earn if he completes 5 chores in one week? AED 45

e. Identify the independent and dependent variables. The independent variable is the number of chores and the dependent variable is the total earned.

H.O.T. Problems Higher Order Thinking

4. **Reason Abstractly** What would the graph of $y = x$ look like? Name three ordered pairs that lie on the line. straight line; Sample answer: (0, 0), (1, 1), and (2, 2)

5. **Persevere with Problems** Boards 4 U charges AED 10 per hour to rent a snowboard while Slopes charges AED 12 per hour. Will the cost to rent snowboards at each place ever be the same for the same number of hours after zero hours? If so, for what number of hours? no; The graphs of the lines will never meet other than at zero hours.

6. **Model with Mathematics** Write a real-world problem in which you could graph a function. Sample answer: A cable company charges a AED 50 start up fee and AED 60 a month.

7. **Reason Abstractly** A movie rental club charges a one-time fee of AED 25 to join and AED 2 for every movie rented. Write an equation that represents the cost of joining the club and renting any number of movies. $c = 25 + 2m$

Name _____ My Homework _____

Extra Practice

8. In a video game, each player earns 5 points for reaching the next level and 15 points for each coin collected.

Homework Help

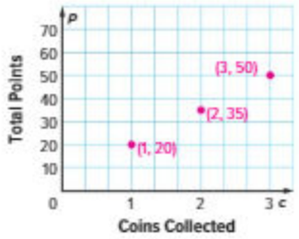
a. Write an equation to find p, the total points for collecting c coins after reaching the next level. $p = 5 + 15c$

Total points p equals 15 times the number of coins c collected plus 5 points for reaching the next level. So, the equation is $p = 5 + 15c$.

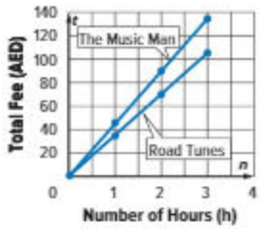
b. Make a table to show the relationship between the number of coins collected c and the total points p.

Number of Coins, c	1	2	3
Total Points, p	20	35	50

c. Graph the ordered pairs. Analyze the graph. The graph is a line because each coin is worth the same amount.



9. Two music players charge different rates. The Music Man charges AED 45 per hour and Road Tunes charges AED 35 per hour. Write equations to represent the total cost t of hiring either music player for any number of hours n. Music Man: $t = 45n$; Road Tunes: $t = 35n$; where t represents the total cost and n represents the number of hours



Copy and Solve For Exercise 10, show your work on a separate piece of paper. See margin.

10. MP **Construct an Argument** A catering service offers lasagna and chicken parmesan. Each pan of lasagna serves 24 people.

a. Write an equation to represent the number of people n served by any number of pans p of lasagna.

b. Make a function table to show the relationship between the number of pans p and the number of people served n.

c. Graph the ordered pairs.

d. The same catering company offers chicken parmesan that serves 16 people per pan. How many more people would 5 pans of lasagna serve than 5 pans of chicken parmesan? Explain your reasoning to a classmate.

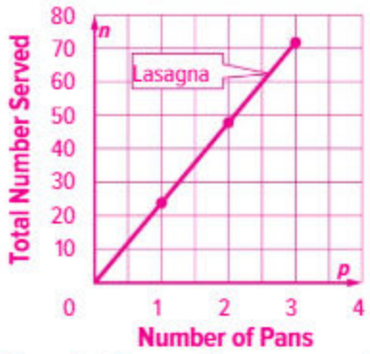
Additional Answers

10a. $24p = n$

10b.

Number of Pans (p)	0	1	2	3
People Served (n)	0	24	48	72

10c.



10d. 40 people; 5 pans of lasagna serve 120 people and 5 pans of chicken parmesan serve 80 people.

Power Up! Test Practice

Exercises 11 and 12 prepare students for more rigorous thinking needed for the assessment.

11.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
2 points	Students write and solve the equation.
1 point	Students write OR solve the equation.

12.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK3
Mathematical Practices	MP1, MP4
Scoring Rubric	
2 points	Students correctly complete the table, graph the points, and write the equation.
1 point	Students complete the table and graph, but fail to write the equation OR students complete the table and write the equation, but fail to graph OR students incorrectly fill in the table, but graph and write an equation based on that error.

Power Up! Test Practice

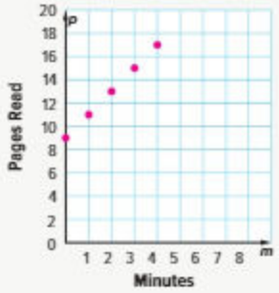
11. For each table Humaid waits on at a restaurant, he is paid AED 4.00 plus 18% of the total bill. Let b represent the amount of the total bill and let m represent the total amount of money Humaid earns.

Write an equation that could be used to find the total amount of money Humaid earns per table. $m = 0.18b + 4$

If a table has a total bill of AED 35, how much will Humaid earn? AED 10.30

12. Hamdan read 9 pages of a book last night. While riding the bus to school this morning, he reads an additional 2 pages each minute. Complete the table below showing how many total pages Hamdan will have read after m minutes of reading on the bus. Then graph the ordered pairs on the coordinate plane.

Minutes (m)	Pages Read (p)
0	9
1	11
2	13
3	15
4	17



Write an equation to represent the situation. $p = 2m + 9$

Spiral Review

Fill in each ◯ with < or > to make a true statement.

13. 116 < 161

14. 63 > 61

15. 105 < 115

16. 50 < 500

17. 12 > 1.2

18. 44 < 49

19. Omar swam 13 laps on Sunday, 12 laps on Monday, 16 laps on Tuesday, 15 laps on Wednesday, and 10 laps on Thursday. Graph each of these numbers on the number line. Which day did he swim the greatest number of laps? Wednesday

Problem-Solving Investigation
Make a Table

Mathematical Practices 1, 3, 4

Case #1 Splitting Up

Blue-green algae is a type of bacteria that can double its population by splitting up to four times in one day.

If it grows at this rate, how many bacteria will be formed at the end of one day?

1 Understand *What are the facts?*

Blue-green algae can double its population up to four times in one day.

2 Plan *What is your strategy to solve this problem?*

Make a table to display and organize the information.

3 Solve *How can you apply the strategy?*

Follow the pattern to find the total number of bacteria after 1 day.

Day Number	Number of Times Split	Total Number of Bacteria	
1	0	1	← ×2
1	1	2	← ×2
1	2	4	← ×2
1	3	8	← ×2
1	4	16	← ×2

4 Check *Does the answer make sense?*

Use the equation $t = 2^n$ where n represents the number of times the bacteria split and t represents the total number of bacteria. $2^4 = 16$

Analyze the Strategy

Justify Conclusions If the bacteria continue to grow at this rate, would the number of bacteria be over 1,000 within a week? Explain. Yes; Sample answer: The number of bacteria would be greater than 1,000 by the end of Day 3.

Focus narrowing the scope

Objective Solve problems by making a table. This lesson emphasizes **Mathematical Practice 4** Model with Mathematics.

Make a Table Students make a table to organize data given in a problem. By recording values that represent relationships between changing quantities, students use tables to solve problems that require extending given information.

Coherence connecting within and across grades

Now
Students solve non-routine problems.

Next
Students will apply the make a table strategy to solve problems.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 609.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

The problems on pages 607 and 608 are intended to be used as a whole-group discussion on how to solve non-routine problems and are designed to provide scaffolded guidance.

Case #1 Splitting Up

BL Extend the problem by asking the question below.

Ask:

- *If the rule $t = 2^n$ represents the total number of bacteria t after n splits, write a rule that represents the total number of bacteria t at the end of d days. Assume the bacteria split four times each day. Explain.* Sample answer: $t = 2^{(4d)}$; At the end of one day, the rule $t = 2^4$ gives the total number of bacteria. At the end of two days, the rule $t = 2^8$ gives the total number of bacteria. The exponent is equivalent to $4d$.

Case #2 Game On!

AL LA **Think-Pair-Share** After students answer the problem individually, have them pair up and share their responses aloud. Students should discuss any differences and make sure the correct answer is obtained. MP 1, 3, 6

BL LA **Pairs Discussion** Have students work in pairs to answer the following extension question. MP 1, 5

Ask:

- *How does making a table help you solve the problem?* Sample answer: Making a table helps organize the information for Hareb and Husam to easily see when they both will have the same score.

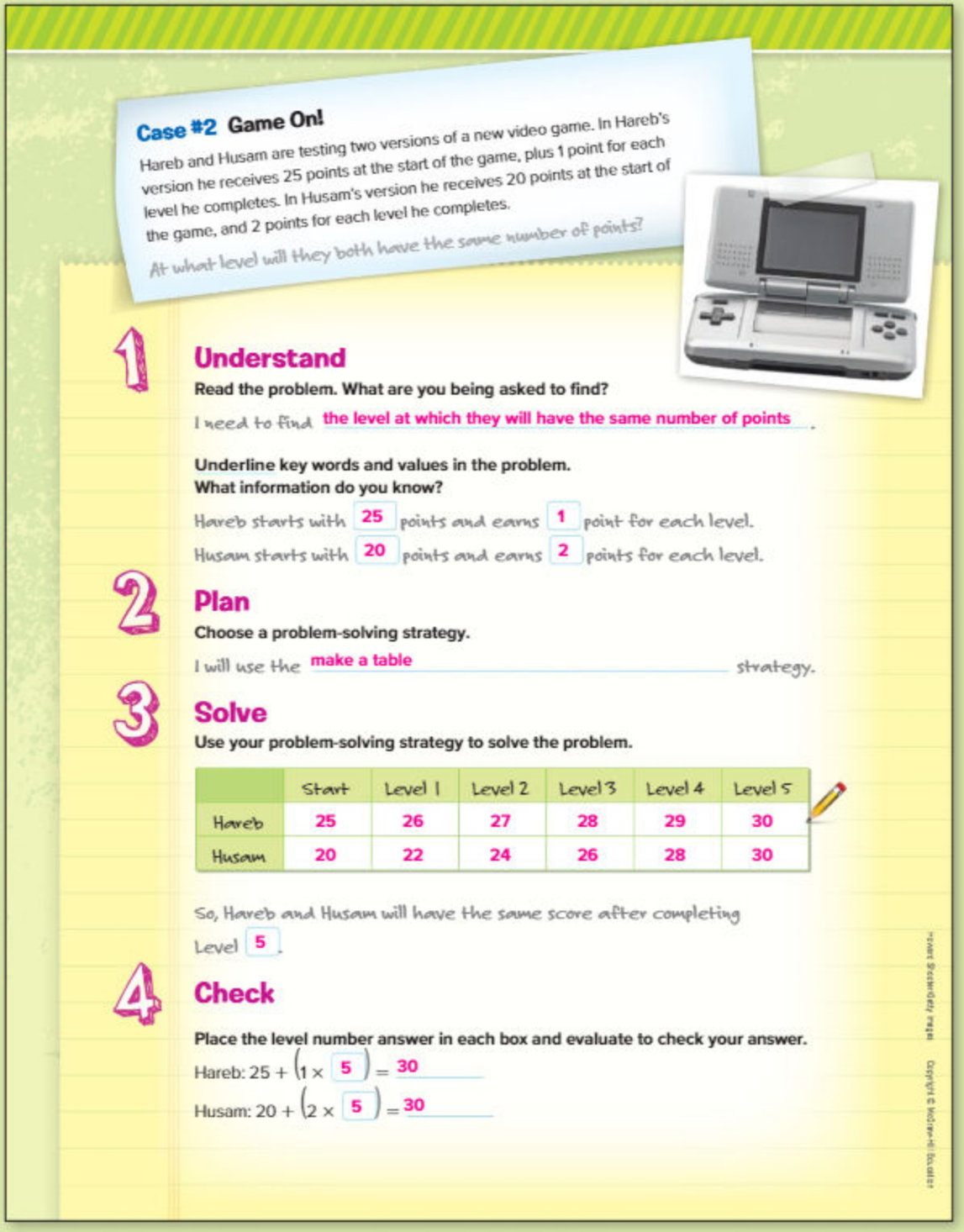
Need Another Example?

Mohammad wants to rent a karaoke machine for a family reunion. The prices to rent the machines from two different companies are shown. For how many days must he rent the machine for the cost from each place to be the same?

Company	Deposit	Cost per Day
Obaid's Music	AED5	AED1.25
Karaoke Korner	AED4	AED1.50

Number of Days	Total Cost (AED)	
	Obaid's Music	Karaoke Korner
0	0	0
1	$5 + 1.25(1) = 6.25$	$4 + 1.50(1) = 5.50$
2	$5 + 1.25(2) = 7.50$	$4 + 1.50(2) = 7.00$
3	$5 + 1.25(3) = 8.75$	$4 + 1.50(3) = 8.50$
4	$5 + 1.25(4) = 10.00$	$4 + 1.50(4) = 10.00$

He must rent the machine for 4 days for the cost to be the same.



Collaborate

Work with a small group to solve the following cases.
Show your work on a separate piece of paper.

Case #3 Geometry

Determine how many cubes are used in each step.

Make a table to find the number of cubes in the seventh step.

35 cubes

Case #4 Car Rental

Hassan needs to rent a car for 9 days to take on vacation. The cost of renting a car is AED 66 per day, AED 15.99 for insurance, and AED 42.50 to fill up the gas tank.

Find the total cost of his rental car.

AED 652.49

Case #5 Numbers

The difference between two whole numbers is 14. Their product is 1,800.

What are the two numbers?

50 and 36

Case #6 Money

The admission for a fair is AED 6 for adults, AED 4 for children, and AED 3 for senior citizens. Twelve people paid a total of AED 50 for admission.

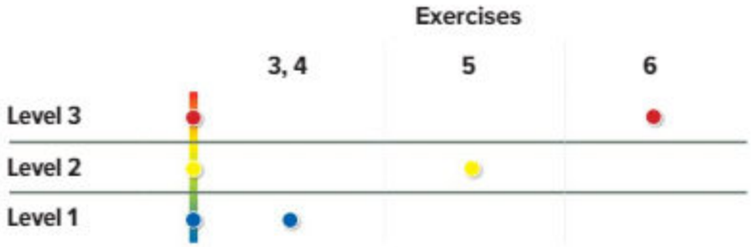
If 8 children attended, how many adults and senior citizens attended?

2 adults, 2 senior citizens

2 Collaborate

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



AL LA **3-Step Interview** After students complete Cases 3–6, have students work in pairs and interview one another by asking the following questions. After each student has been interviewed, in small groups, have each partner introduce their teammate and summarize the information that was shared during the interview. 1, 3, 5

Ask:

- *How did you decide what information from the drawing in Case 3 should be placed in the table?* Sample answer: I counted the number of cubes in the first step and then counted the number of cubes in the second step. From the relationship of the two steps, I created a table that shows the relationship of the number of steps to the number of cubes in each step.
- *What exercises could be solved using another problem-solving strategy? What strategy?* Sample answer: Case 5; make a pattern. Case 6; guess, check, and revise.

BL LA **Trade-a-Problem** Have students choose one of the Cases and write a problem similar to that case. Then have them trade their problems with another student to solve the case. Have students share their answers with each other and work to resolve any differences. 1, 3, 4

Mid-Chapter Check

If students have trouble with Exercises 1–9, they may need help with the following concepts.

Concept	Exercise(s)
sequences (Lesson 2)	1
function tables (Lesson 1)	2–4
finding a function rule (Lesson 2)	5–7
writing equations to represent functions (Lesson 4)	8
using functions (Lesson 3)	9

Vocabulary Activity

LA **Rally Coach** Have students work in pairs to complete Exercises 1 and 2. Have Student 1 speak aloud their response to Exercise 1, while Student 2 listens, coaches, and encourages. Then have students trade roles for Exercise 2. If students are having trouble remembering the definition of a sequence, have them discuss how the word sequence is used in everyday life. MP 1, 3

Alternate Strategy

AL Give several examples and nonexamples of arithmetic and geometric sequences, using index cards. Have students sort the sequences into three piles, arithmetic, geometric, and neither. Have them justify why they chose to sort the sequences into their respective piles. MP 1, 3, 7

Mid-Chapter Check

Vocabulary Check

1. Define *sequence*. Give an example of an arithmetic and a geometric sequence. (Lesson 2)

   A sequence is a list of numbers in a specific order; Sample answer: 2, 4, 6, 8... is an arithmetic sequence and 2, 4, 8, 16... is a geometric sequence.

2. Fill in the blank in the sentence below with the correct term. (Lesson 1)

   A function is a relation that assigns exactly one output value to one input value.

Skills Check and Problem Solving

Complete each function table. (Lesson 1)

3.

Input (x)	$2x + 6$	Output
0	2(0) + 6	6
1	2(1) + 6	8
2	2(2) + 6	10

4.

Input (x)	$3x + 1$	Output
0	3(0) + 1	1
1	3(1) + 1	4
2	3(2) + 1	7

MP **Identify Structure** **Find the rule for each function table.** (Lesson 2)

5. $2x$

Input (x)	Output
3	6
4	8
5	10

6. $4x - 1$

Input (x)	Output
1	3
2	7
3	11

7. $3x + 2$

Input (x)	Output
2	8
3	11
4	14

8. Ali reads an average of 21 pages each day. Write an equation to represent the number of pages read after any number of days. (Lesson 4)

   $p = 21d$

9. MP **Reason Abstractly** The table shows the cost of renting an inner tube to use at the Wave-a-Rama Water Park. Explain how to write an equation to represent the data in the table. Then give the equation for the data. (Lesson 3)

   Sample answer: First find the difference in the output values (cost). Each value increases by 5.50, so the equation includes $5.5x$. Since each output is exactly 5.5 times the input value, the equation is $y = 5.5x$.

Input (x)	Cost (y)
2	AED 11.00
3	AED 16.50
4	AED 22.00

Inquiry Lab
Inequalities

Inquiry **HOW can bar diagrams help you to compare quantities?**

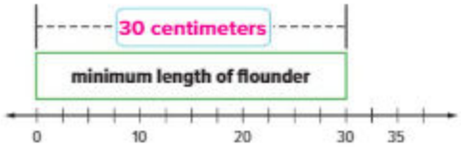
MP Mathematical Practices 1, 3, 4

In saltwater fishing, any flounder that is caught may be kept if it is greater than or equal to 30 centimeters long. Any flounder shorter than that must be released back into the water. Ismail caught a flounder that is 35 centimeters long. He wants to know if he can keep the fish.

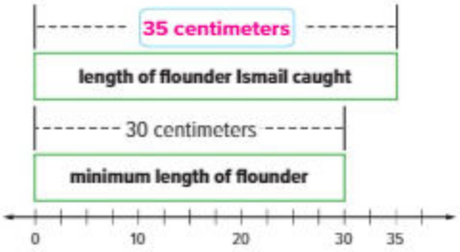
Hands-On Activity

An *inequality* is a mathematical sentence that compares quantities. An inequality like $x < 7$ or $x > 5$ can be written to express how a variable compares to a number.

Step 1 Label the minimum length of flounders that may be kept.



Step 2 Label the length of the flounder Ismail caught on the top bar diagram.



The bar representing Ismail's fish is longer than the bar representing the minimum length that can be kept.

So, Ismail can keep the fish.

Focus narrowing the scope

Objective Model inequalities using bar diagrams.

Coherence connecting within and across grades

Now
Students will use bar diagrams to model one-variable, one-step inequalities that represent problems.

Next
Students will solve inequalities by finding if the given value(s) make(s) one-variable, one-step inequalities true.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 612.

Inquiry At the end of this lab, students should be able to answer "HOW can bar diagrams help you to compare quantities?"

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Hands-On Activity

AL LA **Pairs Discussion** Have students work with a partner to complete the activity. Have them practice saying the term *inequality* aloud and explain how the prefix *in-* helps them to remember the meaning of the term *inequality*. Then have them respond to the following questions upon completion of Steps 1 and 2. MP 1, 3, 6

Ask:

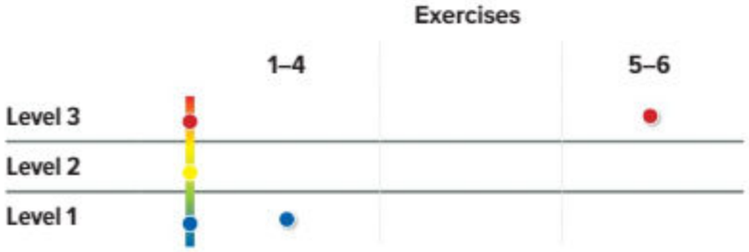
- *Give some sizes of the fish Ismail would have to throw back.* Sample answers: 29 cm, 27 cm, 25 cm, 10 cm
- *What will Ismail need to do if the fish is exactly 30 cm long? Explain.* He will get to keep the fish because the inequality is "greater than or equal to".

2 Collaborate

The **Investigate** section is intended to be used as a small-group investigation. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Think-Pair-Write** Have students read Exercises 1 and 2 individually. Give them one minute to think through how they would draw bar diagrams for each. Then have them work with a partner to share their responses. MP 1, 3, 4, 5

Create

BL LA **Pairs to Groups** Allow students in pairs to complete Exercise 5. Then, have pairs team up to form groups of 4 to respond to the following question. MP 1, 3

Ask:

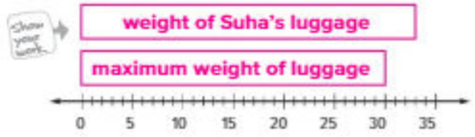
- *Determine if the x-values 5, 7, or 9 are true for the inequality $x < 8$.* The values of 5 and 7 are true, but a value of 9 is not true.

Inquiry Students should be able to answer "HOW can bar diagrams help you to compare quantities?" Check for student understanding and provide guidance, if needed.

Investigate

MP **Model with Mathematics** **Work with a partner. Draw bar diagrams to solve each problem.**

1. For flights within the United States, luggage must be no more than 30 kilograms. Suha's luggage weighs 33 kilograms. Can she take the luggage on her flight? no



2. Adnan needs at least 20 minutes between the end of his soccer practice and the start of his dentist appointment. His practice ends at 4:30 and his appointment is at 5:00.

Does he have enough time? yes

3. MP **Reason Inductively** Which inequality is used when the situation involves a "minimum"? Explain. greater than; Sample answer: By using the term "minimum", the situation is asking for the least value possible, so all other values that make the inequality true would be greater than or equal to the minimum.

4. MP **Reason Inductively** Which inequality is used when the situation involves a "maximum"? Explain. less than; Sample answer: By using the term "maximum", the situation is asking for the greatest number possible, so all other values would be less than or equal to the maximum.

Create

5. MP **Reason Inductively** Write a rule for determining possible values of a variable in an inequality. Sample answer: Using a number line, determine where the possible values lie in relation to the stated value. If the possible value is to the left, it is less than. If it is to the right, it is greater than.

6. Inquiry HOW can bar diagrams help you to compare quantities? Sample answer: The length of two bar diagrams can help you determine if two quantities are equal or if one amount is greater than or less than the other.

Lesson 5

Inequalities

Vocabulary Start-Up

An **inequality** is a mathematical sentence that compares quantities.

Definition	Symbols
A mathematical sentence indicating that two quantities are not equal.	>, <, ≥, and ≤
inequality	
Example	**Nonexample**
Sample answer: $6 + x > 13$	Sample answer: $6x = 18$

Essential Question

HOW are symbols, such as <, >, and =, useful?

Vocabulary

inequality

Mathematical Practices

1, 2, 3, 4, 6, 7

Real-World Link

Compare the following using < or >.

1. the score after 2 goals is (<) the score after 3 goals
2. the cost to download 10 songs is (>) the cost to download 2 songs
3. the outside temperature in summer is (>) the outside temperature in winter
4. the height of a 1st grade student is (<) the height of a 6th grade student
5. the time to eat lunch is (>) the time to brush your teeth

Which **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Solve inequalities by using mental math and the guess, check, and revise strategy.

Coherence connecting within and across grades

Previous
Students modeled one-variable, one-step inequalities using bar diagrams.

Now
Students solve inequalities by finding if given value(s) make(s) inequalities true.

Next
Students will write and graph one-variable, one-step inequalities.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 617.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Roundrobin** Have students work in small groups to complete the graphic organizer. Have each student be responsible for providing responses for one of the sections. Then have the group discuss each section and decide together what their final response should be. Have them record their final response. MP 1, 3, 5

Alternate Strategy

BL Have students interchange the equals sign in the equation $15 + 17 = 32$ with an inequality symbol that would still result in a true statement. Have them justify their response. MP 1, 3

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. **Determine the solution of an inequality.**

AL
- *Using words, what inequality symbol is being used?* less than
- *What operation is being performed on the variable?* addition

OL
- *Suppose the less than sign was an equals sign. What would you do to determine the solution of the equation?* Replace f with each value and simplify to see which value(s) make(s) a true equation.
- *How would you determine which value is a solution of the inequality?* Replace f with each value and simplify to see which value(s) make(s) a true inequality.
- *Is $6 + 2 < 9$?* yes *Is $7 + 2 < 9$?* no *Is $8 + 2 < 9$?* no

BL
- *If $f = 9$, would the inequality be true? Explain.* No; 11 is not less than 9.
- *If $f = 4$, would the inequality be true?* yes
- *If $f = 0$, would the inequality be true?* yes
- *Using words, explain what numbers would make this a true statement.* any number less than 7

Need Another Example?

Of the given values 3, 4, or 5, determine which value(s) make(s) the inequality $m + 9 > 13$ true. 5

Key Concept **Inequalities**

Symbols	$<$	$>$	$\leq$	$\geq$
Words	• is less than • is fewer than	• is greater than • is more than	• is less than or equal to • is at most	• is greater than or equal to • is at least
Examples	$3 < 5$	$8 > 4$	$7 \leq 10$	$12 \geq 9$

Work Zone

Inequalities can be solved by finding values of the variables that make the inequality true.

Example

1. **Of the numbers 6, 7, or 8, which is a solution of the inequality $f + 2 < 9$?**

Replace f with each of the numbers.

$f + 2 < 9$ Write the inequality.
$6 + 2 \stackrel{?}{<} 9$ Replace f with 6.
$8 < 9$ ✓ This is a true statement.

$f + 2 < 9$ Write the inequality.
$7 + 2 \stackrel{?}{<} 9$ Replace f with 7.
$9 < 9$ ✗ This is not a true statement.

$f + 2 < 9$ Write the inequality.
$8 + 2 \stackrel{?}{<} 9$ Replace f with 8.
$10 < 9$ ✗ This is not a true statement.

Since the number 6 is the only value that makes a true statement, 6 is a solution of the inequality.

Got it? **Do this problem to find out.**

a. Of the numbers 8, 9, or 10, which is a solution of the inequality $n - 3 > 6$?

a. 10

Determine Solutions of an Inequality

Since an inequality uses greater than and less than symbols, one-variable inequalities have infinitely many solutions. For example, any rational number greater than 4 will make the inequality $x > 4$ true.

Examples

Is the given value a solution of the inequality?

2. $x + 3 > 9, x = 4$

$x + 3 > 9$ Write the inequality.

$4 + 3 \stackrel{?}{>} 9$ Replace x with 4.

$7 \not> 9$ Simplify.

Since 7 is not greater than 9, 4 is not a solution.

3. $12 \leq 18 - y, y = 6$

$12 \leq 18 - y$ Write the inequality.

$12 \stackrel{?}{\leq} 18 - 6$ Replace y with 6.

$12 \leq 12$ Simplify.

Since $12 = 12$, 12 is a solution.

4. $17 \geq 11 + x, x = 8$

$17 \geq 11 + x$ Write the inequality.

$17 \stackrel{?}{\geq} 11 +$ 8 Replace x with 8.

$17 \not\geq$ 19 Simplify.

Since 17 is not greater than or equal to 19, 8 is not a solution.

Got it? Do these problems to find out.

b. $a + 7 > 15, a = 9$

c. $22 \leq 15 + b, b = 6$

d. $n - 4 < 6, n = 10$

e. $12 \geq 5 + g, g = 7$

STOP and Reflect

Name two solutions of the inequality $12 > 6 + y$.

Sample answer: 4 and 5

Show your work.

b. yes

c. no

d. no

e. yes

Examples

2. **Determine the solution of an inequality.**

AL
- *Using words, what inequality symbol is being used?* greater than
- *What value must the simplified expression $x + 3$ be greater than?* 9

OL
- *How can you determine whether or not 4 is a solution of the inequality?* Replace x with 4 and simplify.
- *Is $4 + 3 > 9$?* no
- *Is 4 a solution of the inequality?* no

BL
- *What value would be a solution of the inequality?* Sample answer: 7
- *Using words, explain what numbers would make this a true statement.* any number greater than 6

Need Another Example?

In the inequality $a + 6 < 12$, is $a = 5$ a solution? yes

3–4. **Determine the solution of an inequality.**

AL
- *Using words, what inequality symbol is being used in Example 3? Example 4?* less than or equal to; greater than or equal to
- *What does the inequality in Example 3 mean, using words? Example 4?* 12 is less than or equal to the difference of 18 and y; 17 is greater than or equal to the sum of 11 and x.

OL
- *Refer to Example 3. How would you determine whether or not 6 is a solution of the inequality?* Replace y with 6 and simplify. *Is $12 \leq 18 - 6$?* yes *Is 6 a solution of the inequality?* yes
- *Refer to Example 4. How would you determine whether or not 8 is a solution of the inequality?* Replace x with 8 and simplify. *Is $17 \geq 11 + 8$?* no *Is 8 a solution of the inequality?* no

BL
- *Give another value of y that is a solution of the inequality in Example 3.* Sample answer: 4

Need Other Examples?

In the inequality $18 < 21 - b$, is $b = 4$ a solution? no

In the inequality $15 \geq 22 - c$, is $c = 7$ a solution? yes

Examples

5. Determine the solution of an inequality.

AL
- *How will Tarek receive a bonus?* If he makes more than 20 balloon bouquets in one month, he will receive a bonus.
- *What does the table show?* the number of balloons Tarek sold in July, August, September, and October

OL
- *How would you determine in which months Tarek sold more than 20 balloons?* Replace b in the inequality $b > 20$ with each of the values given in the table and simplify.
- *Did he earn a bonus in August? Explain.* No; 12 is not greater than 20.

BL
- *If Tarek made exactly 20 balloon bouquets one month, would he receive the bonus? Explain.* no; he must make *more than* 20 balloon bouquets.

Need Another Example?

A state park recorded the number of cars entering the park on certain days last week. On which day(s) did more than 65 cars enter the park? Use the inequality $c > 65$, where c represents the number of cars to solve. Wednesday

Day	Number of Cars
Monday	43
Wednesday	66
Friday	37

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Team-Pair-Solo** Have students work in small teams to complete Exercises 1 and 3. Then have them divide into pairs to complete Exercise 2. Have students complete Exercise 4 on their own, then compare their answers with their partner. MP 1

BL LA **Pairs Discussion** Have students work with a partner to determine three more solutions to each inequality in Exercises 1–5 that are not already listed. Have them verify their solutions using substitution. MP 1

Real World

Example

5. Tarek works at a gift shop. He receives a bonus if he makes more than 20 balloon bouquets in a month. Which months did Tarek receive a bonus? Use the inequality $b > 20$, where b represents the number of balloon bouquets made each month, to solve.

Balloon Sales	
Month	Number Sold
July	25
August	12
September	18
October	32

Use the *guess, check, and revise* strategy.

Try 25. $b > 20$; $25 > 20$ Yes

Try 12. $b > 20$; $12 > 20$ No

Try 18. $b > 20$; $18 > 20$ No

Try 32. $b > 20$; $32 > 20$ Yes

So, Tarek received a bonus in July and October.

Guided Practice

Check

Determine which number is a solution of the inequality. (Example 1)

1. $9 + a < 17$; 7, 8, 9 — 7

Show your work.

2. $b - 10 > 5$; 14, 15, 16 — 16

Is the given value a solution of the inequality? (Examples 2–4)

3. $x - 5 < 5, x = 15$ — no

4. $32 \geq 8n, n = 3$ — yes

5. If the bakery sells more than 45 bagels in a day, they make a profit. Use the inequality $b > 45$ to determine which days the bakery makes a profit. (Example 5)

Day	Number of Bagels Sold
Monday	18
Tuesday	25
Wednesday	21
Thursday	36
Friday	50
Saturday	48
Sunday	40

Friday and Saturday

6. **Building on the Essential Question** How can mental math help you find solutions to inequalities?

Mental math can help determine if a certain number makes the inequality true.

Rate Yourself!

- ☐ I understand how to solve inequalities.
  - Great! You're ready to move on!
- ☐ I still have some questions about solving inequalities.

Name _____ My Homework _____

Independent Practice

Determine which number is a solution of the inequality. (Example 1)

1. $1 + f < 7$; 5, 6, 7 **5**

2. $g - 3 > 4$; 6, 7, 8 **8**

Show your work.

Is the given value a solution of the inequality? (Examples 2–4)

3. $q - 2 > 16, q = 20$ **yes**

4. $t - 7 < 10, t = 28$ **no**

5. The table shows the number of different types of roller coasters in the United States. An amusement park wants to build a new roller coaster. They will only build a roller coaster if there are less than 10 of that type in the United States. Use the inequality $r < 10$, where r is the number of a certain type of roller coaster, to determine which type(s) can be built. (Example 5)

stand up or suspended

Type	Number
Sit down (steel)	530
Sit down (wood)	112
Inverted	43
Flying	10
Stand up	8
Suspended	5

6. The table shows the number of different types of movies in Zayed's collection. He wants to buy a new movie to add to his collection. He only wants to buy a movie if he already has more than 15 movies of that type. Use the inequality $m > 15$, where m is the number of the type of movie, to determine which type(s) he can buy. (Example 5)

action or comedy

Movie Type	Number
Action	18
Comedy	24
Drama	12
Thriller	15

7. The number of text messages Shaima sent each month is shown in the table. She can send no more than 55 messages each month without being charged. Use the inequality $t \leq 55$, where t is the number of text messages in a month, to determine in which months she exceeded her limit. If each additional text costs AED 0.25, how much was Shaima charged from January to April?

Jan. and Feb.; AED 0.75

Month	Text Messages
January	56
February	57
March	55
April	51

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 11, 12, 21, 22
OL	On Level	1–5 odd, 7–9, 11, 12, 21, 22
BL	Beyond Level	7–13, 21, 22

Watch Out!

Common Error Students may not recognize that a number is a solution of an inequality when the symbol $\leq$ or $\geq$ is used and the two numbers on either side of the inequality are equal to each other. Remind students that these symbols mean that the two numbers can be less than/greater than or equal to each other. Either one of these conditions can be solutions of the inequality.

MATHEMATICAL PRACTICES	
Emphasis On	**Exercise(s)**
1 Make sense of problems and persevere in solving them.	10, 13
2 Reason abstractly and quantitatively.	11
3 Construct viable arguments and critique the reasoning of others.	9, 12
6 Attend to precision.	20
7 Look for and make use of structure.	8

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET Out the Door

Have students determine which of the values 11, 12, or 13 make the inequality $x + 5 < 17$ true. **11**

8. **Identify Structure** Use one-variable equations and inequalities to fill in the graphic organizer. **Sample answers are given.**

	Equation	Inequality
Example	$x + 3 = 10$	$x + 8 > 20$
Number of Solutions	one	infinitely many

H.O.T. Problems Higher Order Thinking

9. **Reason Inductively** State three numbers that are solutions to the inequality $x + 1 \leq 5$. **Sample answer: 0, 1, and 2**

10. **Persevere with Problems** If $x = 2$, is the following inequality *true* or *false*? Explain.

$$\frac{112}{8} + x \geq 15 + 4x - 7$$

true; $\frac{112}{8} + 2 \geq 15 + 4(2) - 7$, so $16 \geq 16$

11. **Reason Abstractly** If $a > b$ and $b > c$, what is true about the relationship between a and c? Explain your reasoning.

$a > c$; Sample answer: If $a > b$, then it is to the right of b on the number line. If $b > c$, then it is to the right of c on the number line. Therefore, a is to the right of c on the number line.

12. **Construct an Argument** Explain why inequalities of the form $x > c$ or $x < c$, where c is any rational number, have infinitely many solutions.

Sample answer: In $x > c$, any rational number greater than c would make the inequality true. In $x < c$, any rational number less than c would make the inequality true.

13. **Persevere with Problems** Analyze the relationship between the inequalities in each pair of inequalities below. Then write the integers that are solutions to each pair of inequalities.

a. $y > 4$ and $y \leq 6$ **5 and 6**

b. $x \geq -3$ and $x < 0$ **−3, −2, and −1**

c. $m < 5$ and $m > 3$ **4**

d. $r < -1$ and $r > 0$ **none**

Name _____ My Homework _____

Extra Practice

Determine which number is a solution of the inequality.

14. $5 - h \geq 2$; 3, 4, 5 3

Homework Help

Try 3. $5 - 3 \stackrel{?}{\geq} 2$ $2 \geq 2$ ✓

Try 4. $5 - 4 \stackrel{?}{\geq} 2$ $1 \geq 2$ ✗

Try 5. $5 - 5 \stackrel{?}{\geq} 2$ $0 \geq 2$ ✗

15. $j + 8 \leq 8$; 0, 1, 2 0

Is the given value a solution of the inequality?

16. $25 \geq 5u$, $u = 5$ yes

17. $13 \leq 4v$, $v = 3$ no

18. Mr. Saleh recorded the number of sandwiches sold in his deli on one day. If he sells more than 25 of a type of sandwich, he orders more meat from the butcher. Use the inequality $s > 25$, where s is the number of sandwiches sold, to determine which meats he needs to order. chicken and turkey

Sandwich	Number Sold
Club	25
Chicken	30
Roast beef	22
Turkey	28

19. The height of each member of a family is listed in the table. In order to ride a certain roller coaster at an amusement park, you must be at least 135 centimeters tall. Use the inequality $h \geq 135$, where h is a family member's height, to determine who can ride the roller coaster.

Amna, Ayman, and Hassan

Name	Height (cm)
Amna	165
Ayman	135
Hessa	122
Nasser	130
Hassan	177

20. **Be Precise** Obaid subscribes to a service where he can download up to five free ringtones each month. Each ringtone after that costs AED 3.50 each. During which months did Obaid exceed the plan? How much is Obaid's additional cost in 6 months? February and April; AED 14.00

Month	Ringtones
January	5
February	6
March	4
April	8
May	5
June	4

Power Up! Test Practice

Exercises 21 and 22 prepare students for more rigorous thinking needed for the assessments.

21.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer the question.

22.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP7
Scoring Rubric	
2 points	Students correctly assign all six people.
1 point	Students correctly assign five of the six people.

Power Up! Test Practice

21. The number of moons for some of the planets are shown in the table.

Planets	Moons	Planets	Moons
Earth	1	Uranus	27
Mars	2	Saturn	47
Neptune	13	Jupiter	63

Let m represent the number of moons for a planet. Which of the following planets have moons that represent solutions of the inequality $m > 27$? Select all that apply.

- [x] Jupiter
- [ ] Earth
- [x] Saturn
- [ ] Uranus

22. The inequality $h \geq 48$, where h is a person's height in centimeters, can be used to determine who can ride the Screaming Eagle roller coaster. The table shows the heights of some friends who want to ride the roller coaster.

Name	Height (cm)
Ibrahim	122
Khalid	140
Khalaf	132
Mohammad	120
Sultan	118
Khamis	110

Complete the chart to show who is and who is not able to ride the roller coaster.

Able to Ride	Not Able to Ride
Ibrahim Khalid Khalaf Mohammad	Sultan Khamis

Spiral Review

Write an expression to represent each situation.

23. Najat had 5 stickers and her sister gave her 3 stickers. 5 + 3

24. There were 7 lemons on the lemon tree. Then 2 fell off the tree. 7 − 2

25. Salem had 5 packages of sandwiches that each contained 8 sandwiches. 5 × 8

26. The distance 4 friends walked is shown in the table. Graph the numbers on the number line. Who walked the shortest distance?

Abdulkarim

Name	Kilometers Walked
Abdalla	2.5
Abdulkarim	1.5
Mansour	3
Abdulaziz	2

27. In one week, Abdulraheem read 4 books and Amer read 6 books. Fill in the blanks to compare the number of books they read.

6 > 4

Expressions and Equations

Lesson 6

Write and Graph Inequalities

Real-World Link

Fair Look at the situations below. Circle the numbers that are possible answers in each situation.

1. Abeer spent more than AED 5 at the arcade.

   1 2 3 4 5 (6) (7) (8) (9) (10) (11) (12) (13) (14) (15)

2. [illegible]

3. [illegible]

4. [illegible]

5. [illegible]

6. [illegible]

7. [illegible]

   Sample answer: I circled all the numbers on either side of the given number.

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

Essential Question

HOW are symbols, such as <, >, and =, useful?

MP Mathematical Practices
1, 3, 4, 5, 6

Focus narrowing the scope

Objective Write and graph inequalities.

Coherence connecting within and across grades

Previous Students solved inequalities by finding if given values made them true.

Now Students write inequalities and graph them on the number line.

Next Students will model and solve one-variable, one-step inequalities.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 625.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Roundrobin** Have students work in small groups to complete Exercises 1–7. For Exercises 1–6, have each student take turns providing a possible number for each situation. MP 1

Alternate Strategies

AL LA Have students discuss why the number 5 was not circled in Exercise 1, the number 6 not circled in Exercise 2, and so on. MP 1, 3

BL LA Ask students to discuss the word unique and how it might apply to mathematics. Ask them if the equation $x - 5 = 12$ has a unique solution. Then ask them if the inequality $x - 5 > 12$ has a unique solution. MP 1, 3, 6

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1–3. **Write an inequality.**

AL
- *Refer to Example 1. What word or phrase indicates which symbol to use?* must be over
- *What inequality symbol is used to represent "must be over"?* >
- *Refer to Example 2. What word or phrase indicates which symbol to use?* less than
- *What inequality symbol is used to represent "less than"?* <
- *Refer to Example 3. What word or phrase indicates which symbol to use?* at least
- *What inequality symbol is used to represent "at least"?* ≥

OL
- *Refer to Example 1. Would the value 12 satisfy this inequality? Explain.* no; The age must be more than (over) 12 years, not equal to 12 years.
- *Refer to Example 2. Would the value 14.2 satisfy this inequality? Explain.* no; The height must be less than 14.2 hands, not equal to 14.2 hands.
- *Refer to Example 3. If someone is 16 years old, are they able to get a driver's license? Explain.* Yes, the inequality symbol ≥ indicates that the value can equal 16.

BL
- *Rewrite the wording of the condition in Example 1 so that the inequality is $a \geq 12$.* Sample answer: You must be at least 12 years old to ride the go-karts.

Need Other Examples?

Write an inequality for each sentence.

a. It costs more than AED5 to ride the Ferris wheel. $c > 5$

b. The cat weighs under 220 kilograms. $k < 220$

c. You must be at least 15 years old to take adult swimming lessons. $a \geq 15$

Work Zone

STOP and Reflect

Which inequality symbol represents "is at most"?

≤

Write Inequalities

You can write an inequality to represent a situation.

Examples

Write an inequality for each sentence.

1. **You must be over 12 years old to ride the go-karts.**

Words	Your age	is over	12.
Variable		Let a = your age.	
Inequality	a	>	12

The inequality is $a > 12$.

2. **A pony is less than 14.2 hands tall.**

Words	A pony	is less than	14.2.
Variable		Let p = the height of the pony	
Inequality	p	<	14.2

The inequality is $p < 14.2$.

3. **You must be at least 18 years old to have a driver's license.**

Words	Your age	is at least	18 years.
Variable		Let a = your age.	
Inequality	a	≥	18

The inequality is $a \geq 18$.

Got it? Do these problems to find out.

Write an inequality for each sentence.

a. You must be older than 13 to play in the basketball league.

b. To use one stamp, your domestic letter must weigh under 100 grams.

c. You must be over 120 centimeters tall to ride the roller coaster.

d. You must be at least 18 years old to drive.

a. $a > 13$

b. $m < 3.5$

c. $h > 48$

d. $a \geq 18$

Graph an Inequality

Inequalities can be graphed on a number line. Sometimes, it is impossible to show all the values that make an inequality true. The graph helps you see the values that make the inequality true.

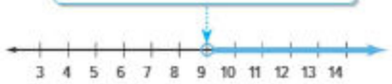
Examples

Graph each inequality on a number line.

4. $n > 9$

Place an open dot at 9. Then draw a line and an arrow to the right.

The open dot means the number 9 is *not* included in the graph.



The values that lie on the line make the sentence true. All numbers greater than 9 make the sentence true.

5. $n \leq 10$

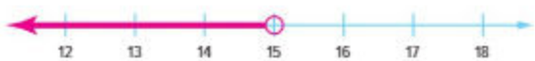
Place a closed dot at 10. Then draw a line and an arrow to the left.

The closed dot means the number 10 *is* included in the graph.

All numbers 10 and less make the sentence true.

Got it? Do these problems to find out.

e. $a < 15$



f. $b \geq 7$

Graphing Inequalities

When inequalities are graphed, an open dot means the number is not included ($<$ or $>$) and a closed dot means it is included ($\leq$ or $\geq$).

Examples

4. Graph an inequality on the number line.

AL
- *How would you read the inequality?* The value n is greater than 9.
- *Could 9 be a solution of the inequality? Explain.* No; 9 is not greater than 9.

OL
- *Do you draw an open dot or a closed dot to represent the inequality? Explain.* Use an open dot because 9 is not included in the inequality.
- *Does the arrow point to the left or the right?* right

BL
- *How many solutions does this inequality have? Explain.* An infinite number, since any number greater than 9 is a solution.

Need Another Example?

Graph the inequality $n < 6$ on a number line.

5. Graph an inequality on the number line.

AL
- *How would you read the inequality?* The value n is less than or equal to 10.
- *Could 10 be a solution of the inequality? Explain.* Yes; 10 is less than or equal to 10.

OL
- *Do you draw an open dot or a closed dot for the inequality? Explain.* Use a closed dot because 10 is included in the inequality.
- *Does the arrow point to the left or the right?* left

BL
- *Give an example of a real-world problem that this inequality could represent.* Sample answer: To order a meal from the children's menu at a restaurant, you must be 10 years old or younger.
- *Explain how a number line allows you to easily see the solutions of an inequality.* Sample answer: You can see all the values included on the shaded line as the solutions.

Need Another Example?

Graph the inequality $n \geq 5$ on a number line.

Examples

6. **Write and graph an inequality.**

AL
- *What can the variable s represent?* the speed on the street
- *What word or phrase indicates which symbol to use?* no more than
- *What inequality symbol is used to represent "no more than"?* $\leq$

OL
- *What inequality represents this situation?* $s \leq 25$
- *When you represent the solution on a number line, will the dot be open or closed? Explain.* Since the sign is $\leq$, you use a closed dot because 25 is included in the inequality.
- *Does the arrow point to the left or the right?* left

BL
- *How would the real-world situation be altered if the inequality was $s < 25$?* Sample answer: The speed on the street must be less than 25 kilometers per hour.

Need Another Example?

You must be at least 48 inches tall to ride a certain roller coaster. Write and graph an inequality to describe the possible heights that can ride the roller coaser. $h \geq 48$

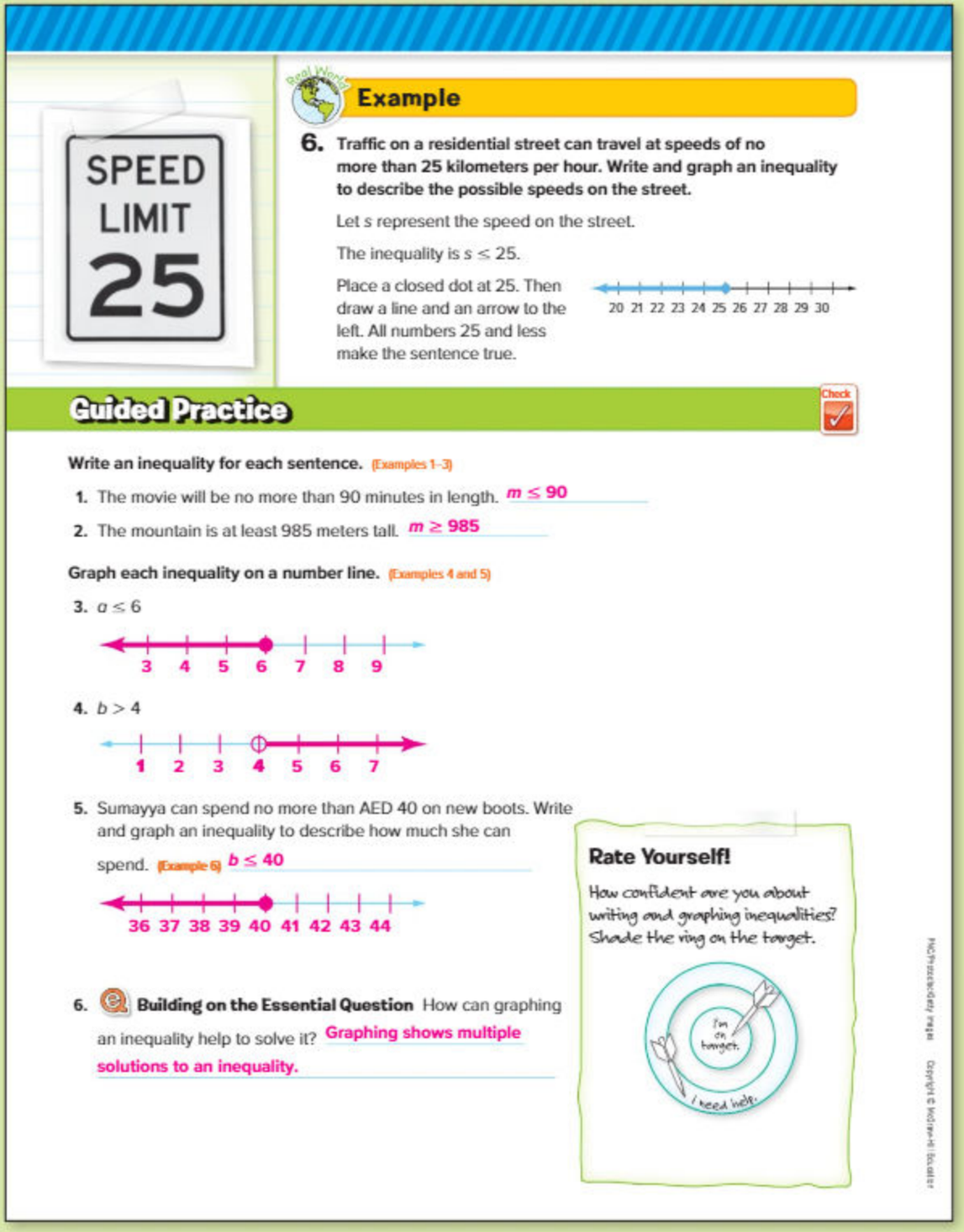
Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Think-Pair-Share** Have pairs of students think about each of Exercises 1 and 2 and generate two numbers, one that is a solution and one that is not. Then have the pairs write each inequality. MP 1, 4

BL LA **Pairs Discussion** Have students work in pairs to generate real-world situations in which inequalities could be used. Ask them to share their situations with the class. MP 1, 4



Name _____ My Homework _____

Independent Practice

Write an inequality for each sentence. (Examples 1–3)

1. Swim practice will be no more than 35 laps. $p \leq 35$
2. Hassan ran for less than 5 kilometers. $r < 5$

H The occupancy of the room must be less than 437 people. $p < 437$

Graph each inequality on a number line. (Examples 4 and 5)

4. $f > 1$

5. $x \leq 5$

6. $y \geq 4$

H 7 A rewritable compact disc must have less than 20 songs on it. Write and graph an inequality to describe how many songs can be on the disc. (Example 6)

$s < 20$

8. MP **Be Precise** Fill in the information in the table. The first is done for you.

Symbol	Words	Open or closed dot on number line?
>	greater than	open dot
≥	greater than or equal to	closed dot
<	less than	open dot
≤	less than or equal to	closed dot

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–7, 9, 11, 12, 21, 22
OL	On Level	1–7 odd, 8, 9, 11, 12, 21, 22
BL	Beyond Level	8–12, 21, 22

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	10
3 Construct viable arguments and critique the reasoning of others.	9, 11
4 Model with mathematics.	12
5 Use appropriate tools strategically.	20
6 Attend to precision.	8

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment
Use this activity as a closing formative assessment before dismissing students from your class.

TICKET
Out the Door

Have students write an inequality to represent the following situation.

Faris is at least 170 centimeters tall. Sample answer: $m \geq 68$

Watch Out!

Common Error In Exercise 9, Nabila used the symbol for the words *less than or equal to* to write the inequality. Have students review the meaning of *at least*. Encourage them to think of situations in which they would use the words and then figure out if it would mean *less than or equal to* or *greater than or equal to*.

H.O.T. Problems Higher Order Thinking

9. **Find the Error** Nabila is writing an inequality for the expression *at least 10 hours of community service*. Find her mistake and correct it.

She used the incorrect symbol. "at least" means the values will be larger than 10, but include 10; $c \geq 10$

10. **Persevere with Problems** Name three solutions of the inequality $w \leq \frac{4}{5}$. Then justify your response using a number line.

Sample answer: $\frac{1}{5}, \frac{1}{4}, \frac{1}{3}$

11. **Justify Conclusions** Explain the difference between graphing an inequality with a closed dot and one with an open dot. Use examples to support your reasoning. Sample answer: When an inequality uses the greater than or less than symbols, it does not include the number given. So, $x > 5$ and $x < 7$ do not include 5 or 7 respectively. When the greater than or equal to and less than or equal to symbols are used, the given numbers are included. So, $x \geq 5$ and $x \leq 7$ include 5 and 7, respectively.

12. **Model with Mathematics** Graph the solution to each set of inequalities on a number line.

a. $x > 5$ and $x < 8$

b. $y \geq -2$ and $y < 7$

c. $t < 3$ or $t \geq 6$

d. $w \leq -5$ or $w \geq 0$

Name _____ My Homework _____

Extra Practice

Write an inequality for each sentence.

13. You cannot spend more than 50 dirhams. $s \leq 50$

Homework Help

Let s represent what you can spend. Cannot spend more means you can spend less than or equal to 50 dirhams.

14. More than 800 fans attended the opening soccer game. $f > 800$

15. The heavyweight division is greater than 200 kilograms. $h > 200$

Graph each inequality on a number line.

16. $g < 6$

17. $z > 18$

18. $h \geq 3$

19. On a certain day, the temperature in Bismarck, North Dakota, was below −15°C. Write and graph an inequality to describe the possible temperatures.

$t < -15$

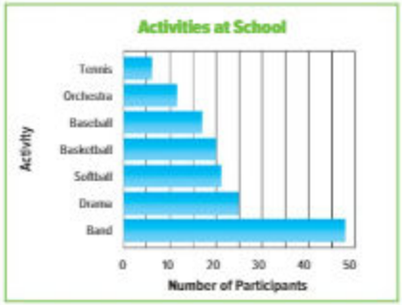
20. MP **Use Math Tools** The graph shows the number of students who participate in some of the activities offered in a cycle 2 school.

a. Which activities have more than 20 participants? at least 20? fewer than 19?

softball, drama, band; basketball, softball, drama, band; tennis, orchestra, baseball

b. Write an inequality comparing the number of orchestra participants and the number of tennis participants.

$12 > 6$



Power Up! Test Practice

Exercises 21 and 22 prepare students for more rigorous thinking needed for the assessments.

21.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK1
Mathematical Practice	MP1
Scoring Rubric	
1 point	Students correctly answer each part of the question.

22.

This test item requires students to analyze and solve complex real-world problems through the use of mathematical tools and models.	
Depth of Knowledge	DOK3
Mathematical Practices	MP3, MP4
Scoring Rubric	
2 points	Students correctly write, graph, and explain the inequality.
1 point	Students correctly write and graph the inequality, but fail to explain it OR students correctly graph and explain, but fail to write the inequality OR students correctly write and explain, but fail to graph the inequality.

Power Up! Test Practice

21. The table shows the number of different kinds of sports equipment sold at a sporting goods store.

Type	Number Sold in Store
Baseball	33
Basketball	n
Football	8
Hockey puck	3
Softball	21

The number of basketballs sold n is greater than the number of softballs sold. Determine if each statement is true or false.

a. The inequality $n > 21$ represents the situation. ■ True ☐ False

b. The store sold more footballs than basketballs. ☐ True ■ False

c. The store could have sold 22 basketballs. ■ True ☐ False

22. Mazen has less than 65 pages of his book left to read. Let p represent the number of pages left to read.

Write an inequality to represent this situation.

$p < 65$

Graph the inequality on the number line.

Did you use a closed dot or an open dot at 65 on the number line?

Explain your reasoning.

open dot; Sample answer: 65 is not included as a solution to the inequality.

Spiral Review

Evaluate each expression.

23. $8(2) - 11 =$ 5

24. $7 + 2(2) =$ 11

25. $3(5) - 7 =$ 8

26. $19 - 2(3) =$ 13

27. $3(4) - 7 =$ 5

28. $28 - 4(4) =$ 12

29. Graph 32, 30, 29, and 34 on the number line below.

30. Graph 13, 15, 9, and 11 on the number line below.

Inquiry Lab

Solve One-Step Inequalities

Inquiry **HOW can you use bar diagrams to solve one-step inequalities?**

MP Mathematical Practices 1, 3, 4

In a recent Kentucky Derby, the total weight a horse could carry was less than 57 kilograms. A jockey weighs a certain number of kilograms and his equipment weighs 4 kilograms. How much could the jockey weigh?

What do you know? The total weight is less than 57 kilograms and the equipment is 4 kilograms.

What do you need to find? the weight of the jockey

Hands-On Activity

You already learned that you can add or subtract the same quantity to each side of an equation when solving it. This is also true for inequalities.

Step 1 Model and solve the inequality $x + 9 < 126$ using a bar diagram. Place a dashed line on 126.

Step 2 The symbol is $<$, so a box is drawn to the left of 126.

Step 3 The bar represents $x +$ 9. Label the bar diagram below.

The section of the bar labeled x must be less than 117 for the inequality to be true. So, $x <$ 117.

Focus narrowing the scope

Objective Model and solve one-step addition and subtraction inequalities.

Coherence connecting within and across grades

Now
Students will model and solve one-variable, one-step addition and subtraction inequalities.

Next
Students will solve one-variable, one-step inequalities involving the four operations.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 630.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lab

The activity is intended to be used as a whole-group activity.

Hands-On Activity

AL LA **Think-Pair-Share** Give students 1–2 minutes to read through the steps in the activity, thinking about how they would respond to Step 3. Then have students work in pairs to discuss and complete each step. Call on one pair of students to share their responses with the class. MP 1, 3

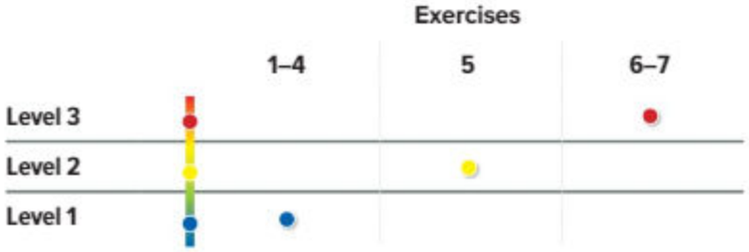
BL LA **Find the Fib** Have students work with a partner to generate two facts and one fib about the inequality represented in the activity. For example, one fact could be that the disc jockey could weigh 50 kilograms. One fib could be that the disc jockey could weigh 53 kilograms. Have students exchange their facts and fibs with another pair to correctly identify each fact and fib. MP 1, 2, 3, 6

2 Collaborate

The **Investigate** and **Analyze and Reflect** sections are intended to be used as small-group investigations. The **Create** section is intended to be used as independent exercises.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.



Investigate

AL LA **Team-Pair-Solo** Have students work in small teams to complete Exercise 1, ensuring that each team member understands how to draw a bar diagram. Then have students divide into pairs to complete Exercise 3, ensuring that each student understands the *guess, check,* and *revise* strategy. Have students work individually to complete Exercises 2 and 4. Have them rejoin their original team to compare their responses. MP 1, 3, 4, 5

Create

AL LA **Roundrobin** Have students work in small groups to complete Exercise 6. Have each student contribute one part of the real-world problem, such as what the variable x could represent, what the number 6 could represent, what the inequality symbol could represent, and what the number 25 could represent. MP 1, 4, 6

Inquiry Students should be able to answer "HOW can you use bar diagrams to solve one-step inequalities?" Check for student understanding and provide guidance, if needed.

Investigate

Work with a partner to solve each problem by using a model.

1. Maysoun sent x text messages before lunch. She sent another 4 text messages after lunch. She sent less than 7 text messages today. How many text messages could she have sent before lunch? Write your answer as an inequality. $x < 3$

2. A player with five personal fouls cannot stay in the game. Amer has already earned two personal fouls. How many more personal fouls x could he earn and still stay in the game? Write your answer as an inequality.

$x < 3$

Work with a partner to solve by using the *guess, check, and revise* strategy. Find the least or greatest number that makes the inequality true.

3. $x - 5 \leq 1$ 6

4. $x + 3 \geq 8$ 5

Analyze and Reflect

Sample answers: 5–7

5. MP **Reason Inductively** Explain how you could solve the inequality $x + 7 \leq 12$ using the *guess, check, and revise* strategy. Then solve. Replace x with different values to check the largest number that will make the inequality true. So, $x \leq 5$.

Create

6. MP **Model with Mathematics** Write and solve a word problem using the inequality $x + 6 \leq 25$. Eissa can spend up to AED 25 at the fair. Admission is AED 6. How much can he spend on rides and snacks? $x \leq 19$; Eissa can spend up to AED 19 on rides and snacks.

7. Inquiry HOW can you use bar diagrams to solve one-step inequalities?

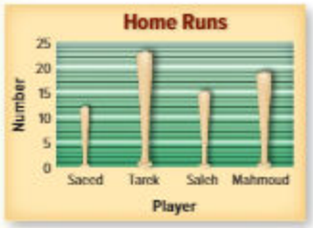
Bar diagrams help you to determine the value of the whole as well as the possible values of each part.

Lesson 7

Solve One-Step Inequalities

Real-World Link

Baseball The graph shows the number of home runs that the top hitters on the baseball team hit last season.



Essential Question

HOW are symbols, such as <, >, and =, useful?

MP Mathematical Practices
1, 3, 4

1. Write an inequality that compares the number of home runs Saleh hit to the number of home runs Saeed hit.

   15 > 12

2. Write an inequality that compares the number of home runs Mahmoud hit to the number of home runs Tarek hit.

   19 < 23

3. Suppose Mahmoud and Tarek each hit 3 more home runs. Write a new inequality that compares the number of home runs Mahmoud and Tarek hit.

   22 < 26

Which MP **Mathematical Practices** did you use? Shade the circle(s) that applies.

① Persevere with Problems	⑤ Use Math Tools
② Reason Abstractly	⑥ Attend to Precision
③ Construct an Argument	⑦ Make Use of Structure
④ Model with Mathematics	⑧ Use Repeated Reasoning

Focus narrowing the scope

Objective Solve one-step inequalities.

Coherence connecting within and across grades

Previous
Students modeled and solved one-step inequalities using addition and subtraction.

Now
Students solve one-step inequalities involving the four operations.

Next
Students will solve equations that include geometric concepts.

Rigor pursuing concepts, fluency, and applications

See the Levels of Complexity chart on page 635.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ideas for Use

You may wish to launch the lesson using a whole group, small group, think-pair-share activity, or independent activity.

LA **Paired Heads Together** Have students work in pairs to complete Exercises 1–3, ensuring that each student understands and asks for help. Assign each student a number. Call on one numbered student to share their responses with the class. MP 1, 5

Alternate Strategy

BL Have students alter Exercise 3 so that Mahmoud and Tarek each tripled the number of home runs that they hit. Have them write a new inequality that compares the number of home runs Mahmoud and Tarek hit. MP 1, 5

2 Teach the Concept

Ask the scaffolded questions for each example to differentiate instruction.

Examples

1. Solve an addition inequality and represent the solution on a number line.

AL • *How do you undo addition?* subtraction

OL • *How do you solve the inequality?* Subtract 7 from each side.

• *Do you draw an open dot or a closed dot?* closed dot

• *Does the arrow point to the left or the right?* right

BL • *How is solving an inequality similar to solving an equation?* It can be solved by performing the inverse operation to both sides of the inequality.

Need Another Example?

Solve $n + 2 > 8$. Graph the solution on a number line. $n > 6$

2. Solve a subtraction inequality and represent the solution on a number line.

AL • *How do you undo subtraction?* addition

OL • *How do you solve the inequality?* Add 3 to each side.

• *Do you draw an open dot or a closed dot?* open dot

• *Does the arrow point to the left or the right?* left

BL • *How is solving an inequality different than solving an equation?* The solution of an equation is one value, whereas the solution of an inequality is a large range of values, often an infinite number of values.

Need Another Example?

Solve $y - 3 \leq 4$. Graph the solution on a number line. $y \leq 7$

Key Concept

Use Addition and Subtraction Properties to Solve Inequalities

Work Zone

Words When you add or subtract the same number from each side of an inequality, the inequality remains true.

Example

$5 < 9$ and $11 > 6$

$+4 \quad +4$ and $-3 \quad -3$

$9 < 13$ and $8 > 3$

These properties are also true for $\leq$ and $\geq$.

Examples

1. Solve $x + 7 \geq 10$. Graph the solution on a number line.

$x + 7 \geq 10$ Write the inequality.

$-7 \quad -7$ Subtract 7 from each side.

$x \geq 3$ Simplify.

The solution is $x \geq 3$. To graph it, draw a closed dot at 3 and draw an arrow to the right on the number line.

2. Solve $x - 3 < 9$. Graph the solution on a number line.

$x - 3 < 9$ Write the inequality.

$+3 \quad +3$ Add 3 to each side.

$x < 12$ Simplify.

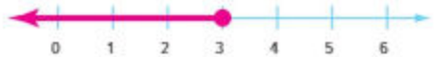
The solution is $x < 12$. To graph it, draw an open dot on 12 and draw an arrow to the left on the number line.

Got it? Do these problems to find out.

a. $n \leq 3$

b. $y > 12$

a. $n + 2 \leq 5$



b. $y - 3 > 9$

Expressions and Equations

Use Multiplication and Division Properties to Solve Inequalities

Key Concept

Words When you multiply or divide each side of an inequality by the same *positive* number, the inequality remains true.

Example

$5 < 10$ $\qquad$ $16 > 12$

$5 \times 2 < 10 \times 2$ $\qquad$ $\frac{16}{2} > \frac{12}{2}$

$10 < 20$ $\qquad$ $8 > 6$

These properties are also true for $\leq$ and $\geq$.

Examples

3. **Solve $5x \leq 45$. Graph the solution on a number line.**

$5x \leq 45$ Write the inequality.

$\frac{5x}{5} \leq \frac{45}{5}$ Divide each side by 5.

$x \leq 9$ Simplify.

The solution is $x \leq 9$.

4. **Solve $\frac{x}{8} > 3$. Graph the solution on a number line.**

$\frac{x}{8} > 3$ Write the inequality.

$\frac{x}{8}(8) > 3(8)$ Multiply each side by 8.

$x > 24$ Simplify.

The solution is $x > 24$.

Got it? **Do these problems to find out.**

c. $10x < 80$

d. $\frac{x}{6} \geq 7$

Checking Solutions

You can check your solutions by substituting numbers into the inequality and testing to verify that it holds true.

Show your work.

c. $x < 8$

d. $x \geq 42$

Examples

3. **Solve a multiplication inequality and represent the solution on a number line.**

AL • *What operation is being performed on the variable?* multiplication

• *How do you undo multiplication?* division

OL • *How do you solve the inequality?* Divide each side by 5.

• *Do you draw an open dot or a closed dot for the inequality?* closed dot

BL • *What value(s) could you use to check the solution of the inequality? Explain.* Sample answer: Let $x = 2$; $5(2) \leq 45$; $10 \leq 45$, which is a true statement.

Need Another Example?

Solve $3x < 21$. Graph the solution on a number line. $x < 7$

4. **Solve a division inequality and represent the solution on a number line.**

AL • *What operation is being performed on the variable?* division

• *How do you undo division?* multiplication

OL • *How do you solve the inequality?* Multiply each side by 8.

• *Do you draw an open dot or a closed dot for the inequality?* open dot

BL • *Give a value of x that satisfies the inequality.* Sample answer: $x = 32$

Need Another Example?

Solve $\frac{x}{6} \leq 7$. Graph the solution on a number line. $x \leq 42$

Examples

5. Write and solve an inequality.

AL • *What word or phrase indicates which symbol to use?* does not want to spend more than

• *What is an equivalent way to say that Lamya does not want to spend more than AED42, without using the word "not"?* Sample answer: Lamya wants to spend less than or equal to AED42.

OL • *What inequality symbol should we use?* $\leq$

• *Write the inequality.* $7c \leq 42$

BL • *Can Lamya spend exactly AED6 on each party favor bag? Explain.* yes; If she spends exactly AED6 on each bag, she will spend a total of AED42, which is included in the inequality.

Need Another Example?

Omar is taking 3 of his friends to a baseball game. He has no more than AED24 to spend on snacks. Write and solve an inequality to find the most he can spend on snacks for each of them. $4s \leq 24$; $s \leq 6$; Omar can spend a maximum of AED6 on each friend.

Guided Practice

Formative Assessment Use these exercises to assess students' understanding of the concepts in this lesson.

If some of your students are not ready for assignments, use the differentiated activities below.

AL LA **Think-Pair-Share** Give students a few minutes to think about how they would respond to each exercise. Then have them share their responses with a partner and work together to complete each exercise. Call on one pair to share their responses with the class. MP 1, 3

BL LA **Pairs Discussion** Have students write an inequality to represent the following situation. Nisreen needs AED100. How much more does she need if she already has AED86 in her purse? Have students discuss their answers. MP 1, 3

Words to Symbols

Remember, at most translates to $\leq$, while at least translates to $\geq$.

Real World

Example

5. **Lamya is making bags of party favors for each of the 7 friends attending her dinner party. She does not want to spend more than AED 42 on the party favors. Write and solve an inequality to find the maximum cost for each party favor bag.**

Let c represent the cost for each bag of party favors.

7 times the cost of each bag must be no more than AED 42.

$7c \leq 42$ Write the inequality.

$\frac{7c}{7} \leq \frac{42}{7}$ Divide each side by 7.

$c \leq 6$ Simplify.

Lamya can spend a maximum of AED 6 on each party favor bag.

Guided Practice

Check

Solve each inequality. Graph the solution on a number line. (Examples 1–4)

1. $h - 6 \geq 13$ $h \geq 19$

Show your work.

2. $5y > 30$ $y > 6$

3. Maha's parents give her AED 10 per week for lunch money. She cannot decide whether she wants to buy or pack her lunch. If a hot lunch at school costs AED 2, write and solve an inequality to find the maximum number of times per week Maha can buy her lunch. (Example 5)

$2x \leq 10$; $x \leq 5$; Maha can buy her lunch a maximum of 5 times.

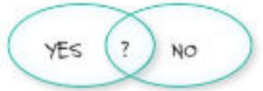
4. Tino's Pizza charges AED 9 for a cheese pizza. Najat has AED 45 to buy pizza for the Spanish Club. Write and solve an inequality to find the maximum number of pizzas that Najat can buy. (Example 5) $9p \leq 45$; $p \leq 5$; Najat can buy a maximum of 5 pizzas.

5. **Building on the Essential Question** How is solving an inequality similar to solving an equation?

Sample answer: You can use addition, subtraction, multiplication, and division properties to solve both.

Rate Yourself!

Are you ready to move on? Shade the section that applies.



Name _____ My Homework _____

Independent Practice

Solve each inequality. Graph the solution on a number line. (Examples 1–4)

1. $2 + y \leq 3$ $y \leq 1$

2. $w - 1 < 4$ $w < 5$

3. $7x > 56$ $x > 8$

4. $\frac{d}{3} \leq 2$ $d \leq 6$

5. A company charges AED 0.10 for each letter engraved. Obaid plans to spend no more than AED 5.00 on the engraving on a jewelry box. Write and solve an inequality to find the maximum number of letters he can have engraved. (Example 5)

$0.1x \leq 5.00$; $x \leq 50$; The maximum is 50 letters.

6. **Model with Mathematics** Refer to the graphic novel frame below for Exercises a–b.

a. Suppose Ayoub has AED 65 to spend on his ticket and some shirts. He already spent AED 32.25 on his ticket and fee. Write an inequality that could be used to find the maximum number of shirts he can buy.

$14.50x \leq 32.75$

b. What is the maximum number of shirts he can buy?

2 shirts

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

3 Practice and Apply

Independent Practice and Extra Practice

The Independent Practice pages are meant to be used as the homework assignment. The Extra Practice page can be used for additional reinforcement or as a second-day assignment.

Levels of Complexity

The levels of the exercises progress from 1 to 3, with Level 1 indicating the lowest level of complexity.

Suggested Assignments

You can use the table below that includes exercises of all complexity levels to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	1–5, 7, 9, 11, 12, 23, 24
OL	On Level	1–5 odd, 6–9, 11, 12, 23, 24
BL	Beyond Level	6–12, 23, 24

Watch Out!

Common Error Students often use an open or closed dot incorrectly when graphing the solutions of an inequality. Remind students that an open dot means that the solution is not part of the graph and a closed dot means that it is part of the graph.

MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	10
3 Construct viable arguments and critique the reasoning of others.	11
4 Model with mathematics.	6, 9, 12, 21, 22

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and apply mathematics to real-world situations.

Formative Assessment

Use this activity as a closing formative assessment before dismissing students from your class.

TICKET
Out the Door

Have students describe the procedure they would use to solve $3x < 39$. Then have them solve the inequality. Sample answer: Divide both sides of the inequality by 3. The solution is $x < 13$.

Solve each inequality. Graph the solution on a number line.

$p - \frac{7}{12} > \frac{3}{10}$ $p > \frac{53}{60}$

8. $f + 0.3 < 1.7$ $f < 1.4$

H.O.T. Problems Higher Order Thinking

9. **Model with Mathematics** Write a word problem that would have the solution $p \leq 21$.

Sample answer: An airplane can hold 53 passengers and there are currently 32 passengers on board. How many more passengers can board the airplane?

10. **Persevere with Problems** In three math tests, you have scored 91, 95, and 88 points. You are about to take your next test. Suppose you want to have an average score of at least 90 points after all four tests. Explain a method you could use to find the score you must receive in order to average at least 90 points. Then find the least score.

Sample answer: The sum of all of the scores divided by 4 must be at least 90 and the words at *least* mean greater than or equal to. If I solve the inequality $\frac{91 + 95 + 88 + x}{4} \geq 90$, I can find the least score; 86 points

11. **Construct an Argument** Does the order of the quantities in an inequality matter? Explain.

yes; Sample answer: $x > 5$ is not the same relationship as $5 > x$. However, $x > 5$ is the same relationship as $5 < x$.

12. **Model with Mathematics** Write a real-world problem and an inequality that can be represented by the number line below.

$x > -2$; Sample answer: A certain brand of sleeping bag is certified to keep the user warm in temperatures greater than −19°C. At what temperatures will the sleeping bag keep the user warm?

Name _____ My Homework _____

Extra Practice

Solve each inequality. Graph the solution on a number line.

13. $a + 4 < 9$ $a < 5$

Homework Help

$a + 4 < 9$
$-4 \quad -4$
$a < 5$

14. $x - 8 \geq 13$ $x \geq 21$

15. $d + 13 \geq 22$ $d \geq 9$

16. $25t \leq 100$ $t \leq 4$

17. $\frac{g}{2} < 6$ $g < 12$

18. $\frac{r}{9} > 8$ $r > 72$

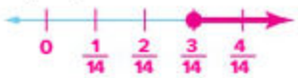
19. A community needs to raise at least AED 5,000 to build a new skateboarding park. They are selling backpacks for AED 25 each to raise the money. Write and solve an inequality to determine the minimum number of backpacks they need to sell in order to reach this goal.

$25b \geq 5{,}000$; $b \geq 200$; They need to sell a minimum of 200 backpacks.

20. A sales associate at a computer store receives a bonus of AED 100 for every computer he sells. He wants to make AED 2,500 in bonuses next month. Write and solve an inequality to find the minimum number of computers he must sell. **$100x \geq 2{,}500$; $x \geq 25$; He must sell at least 25 computers.**

MP Model with Mathematics **Solve each inequality. Graph the solution on a number line.**

21. $n + \frac{2}{7} \geq \frac{1}{2}$ $n \geq \frac{3}{14}$



22. $0.2g > 1.8$ $g > 9$

Power Up! Test Practice

Exercises 23 and 24 prepare students for more rigorous thinking needed for the assessments.

23.

This test item requires students to explain and apply mathematical concepts and solve problems with precision, while making use of structure.	
Depth of Knowledge	DOK1
Mathematical Practices	MP1, MP2
Scoring Rubric	
1 point	Students correctly answer the question.

24.

This test item requires students to reason abstractly and quantitatively when problem solving.	
Depth of Knowledge	DOK2
Mathematical Practices	MP1, MP2
Scoring Rubric	
2 points	Students correctly write and solve the inequality.
1 point	Students correctly write OR solve the inequality.

Power Up! Test Practice

23. Use the graph of the inequality shown below.

Which of the following inequalities have the solution shown on the number line? Select all that apply.

☐ $n + 3 < 8$ ■ $y + 1 > 6$ ■ $z - 4 > 1$ ☐ $c - 7 > 12$

24. The table shows a comparison of male and female ability in the long jump. Moza could jump no farther than 10 centimeters more than the average distance for females. Let j represent the distance that Moza could jump.

Gender	Distance
Male	315 cm
Female	250 cm

Write an inequality to represent the situation.

$j - 10 \text{ cm} \leq 250 \text{ cm}$

How far could Moza jump?

no farther than 260 cm

Spiral Review

Multiply.

25. $12 \times 12 =$ 144

26. $9 \times 13 =$ 117

27. $16 \times 12 =$ 192

28. $8.5 \times 6 =$ 51

29. $13.2 \times 5 =$ 66

30. $7 \times 11.5 =$ 80.5

31. Fatema is painting several boards for scenery for the school play.

What is the area of the board shown? 2 m^2

32. Fatheya is painting her room. She knows that three of her bedroom walls are a total of 28 square meters. The fourth wall in her room measures 2.50 meters wide and 3 meters tall. How much total area will Fatheya need to paint? 35 m^2

21ST CENTURY CAREER in Atmospheric Science

Meteorologist

Have you ever wondered how forecasters can predict severe storms such as hurricanes before they occur? Keeping track of changes in air pressure is one method that they use. Meteorologists study Earth's air pressure, temperature, humidity, and wind velocity. They use complex computer models to process and analyze weather data and to make accurate forecasts. In addition to understanding the processes of Earth's atmosphere, meteorologists must have a solid background in mathematics, computer science, and physics.

Is This the Career for You?

Are you interested in a career as a meteorologist? Take some of the following courses in high school.

- Algebra
- Calculus
- Earth and Its Environment
- Environmental Science
- Physics

Turn the page to find out how math relates to a career in Atmospheric Science.

Focus narrowing the scope

Objective Apply mathematics to problems arising in the workplace.

This lesson emphasizes MP **Mathematical Practice 4** Model with Mathematics

Coherence connecting within and across grades

Previous
Students wrote and solved one-step equations and inequalities.

Now
Students apply the content standard to solve problems in the workplace.

Rigor pursuing concepts, fluency, and applications

See the Career Project on page 640.

ENGAGE EXPLORE EXPLAIN ELABORATE EVALUATE

1 Launch the Lesson

Ask students to read the information on the student page about meteorologists and answer the following questions.

Ask:

- *What courses should you take in high school to become a meteorologist?* Algebra, Calculus, Earth and Its Environment, Environmental Science, Physics
- *How do meteorologists predict storms?* They track changes in air pressure, temperature, humidity, and wind velocity. They use computer models to analyze the data.

College & Career READINESS

2 Collaborate

AL LA Pairs Discussion If students are struggling with the real-world connection in Exercises 1–5, have them work in pairs to discuss the exercises and how to solve them. MP 1, 3

Ask:

- *In Exercise 1, what portion of the diagram do you need to refer to in order to be able to write the inequality?* The ocean water will be above 80°F for at least 200 feet.
- *In Exercise 2, what inequality symbol will represent the phrase "at least"?* ≥
- *In Exercise 3, what inequality symbol will represent the phrase "up to about"?* ≤

BL LA Trade-a-Problem Have students write a real-world problem similar to any of Exercises 1–5. Then allow students to trade their problems with a partner and discuss their responses and correct any mistakes. MP 1, 3, 4

Career Portfolio

When students complete this page, have them add it to their Career Portfolio.

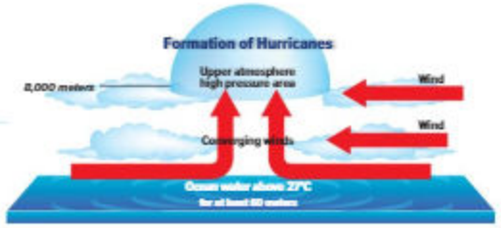
Career Facts

According to the National Oceanic and Atmospheric Association, more deaths occur by lightning than by hurricanes and tornadoes combined. Lightning flashes about 25 million times each year in the United States.

MP The Pressure is On!

Use the information in the diagram and the table to solve each problem.

1. Write an inequality representing the temperature t of the ocean water during the formation of a hurricane. $t > 27$
2. Write an inequality representing the depth d of the water that must be greater than 27°C in order for a hurricane to form. $d \geq 60$
3. The air needs to be humid up to about 5,500 meters for a hurricane to form. Write an inequality to represent this altitude a of the air above the ocean. $a \leq 5{,}500$
4. Air pressure decreases during a storm. The difference between the normal air pressure n and the air pressure during the 1935 Florida Keys hurricane was greater than 121 millibars. Write and solve an inequality to find the normal air pressure in the Florida Keys before the hurricane.
   $n - 892 > 121$; $n > 1{,}013$ mb
5. The air pressure of Hurricane Katrina at landfall was greater than 17 millibars plus the air pressure p before landfall. Write and solve an inequality to find the air pressure of the storm before landfall.
   $920 > p + 17$; $p < 903$ mb



Top 5 Most Intense Hurricanes at Landfall in the U.S.

Rank	Hurricane	Pressure (millibars)
1	Florida Keys, (Labor Day), 1935	892
2	Hurricane Camille, 1969	909
3	Hurricane Katrina, 2005	920
4	Hurricane Andrew, 1992	922
5	Texas (Indianola), 1886	925

MP Career Project

It's time to update your career portfolio! Interview a meteorologist at a local television station. Be sure to ask what he or she likes most about being a meteorologist and what is most challenging. Include all the interview questions and answers in your portfolio.

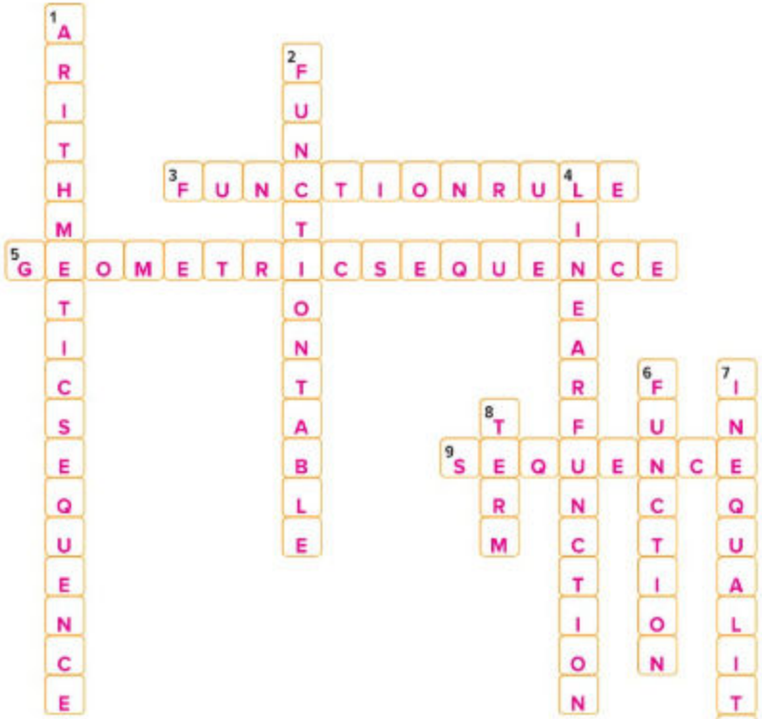
What skills would you need to improve to succeed in this career?

- _____
- _____
- _____
- _____
- _____

Chapter Review

Vocabulary Check

Write the correct term for each clue in the crossword puzzle.



Across

3. an expression that describes the relationship between each input and output
5. found by multiplying the previous term by the same number
9. a list of numbers in a specific order

Down

1. found by adding the same number to the previous term
2. a table organizing the input, rule, and output of a function
4. a function that forms a line when graphed
6. a relationship that assigns exactly one output value to one input value
7. a mathematical sentence indicating that two quantities are not equal
8. each number in a sequence

Vocabulary Check

LA **Popcorn Share** Complete the Vocabulary Check as a class or in small groups. Read each crossword puzzle entry and call out, "Popcorn." A student quickly and voluntarily pops up from their chair and responds to the clue. After the group agrees, allow each student to record the answer. Continue on until the Vocabulary Check is complete. MP 1, 3, 5, 6

Alternate Strategy

AL LA To help students, you may wish to give them a vocabulary list from which they can choose their answers. A vocabulary list for this activity would include the following terms.

- arithmetic sequence (Lesson 2)
- function (Lesson 1)
- function rule (Lesson 1)
- function table (Lesson 1)
- geometric sequence (Lesson 2)
- inequality (Lesson 5)
- linear function (Lesson 3)
- sequence (Lesson 2)
- term (Lesson 2)

Key Concept Check

FOLDABLES LA A completed Foldable for this chapter should include a review of functions.

If you choose not to use this Foldable, have students write a brief review of the Key Concepts found throughout the chapter and give an example of each.

Ideas for Use

LA **Think-Pair-Share** Have students work in pairs to discuss their Foldables. Have them practice speaking in a collaborative setting by sharing how they have completed their Foldable thus far and how they could finish it. Have each student complete their Foldable and trade with their partner to discuss any similarities and differences. MP 1, 3, 5

Got It?

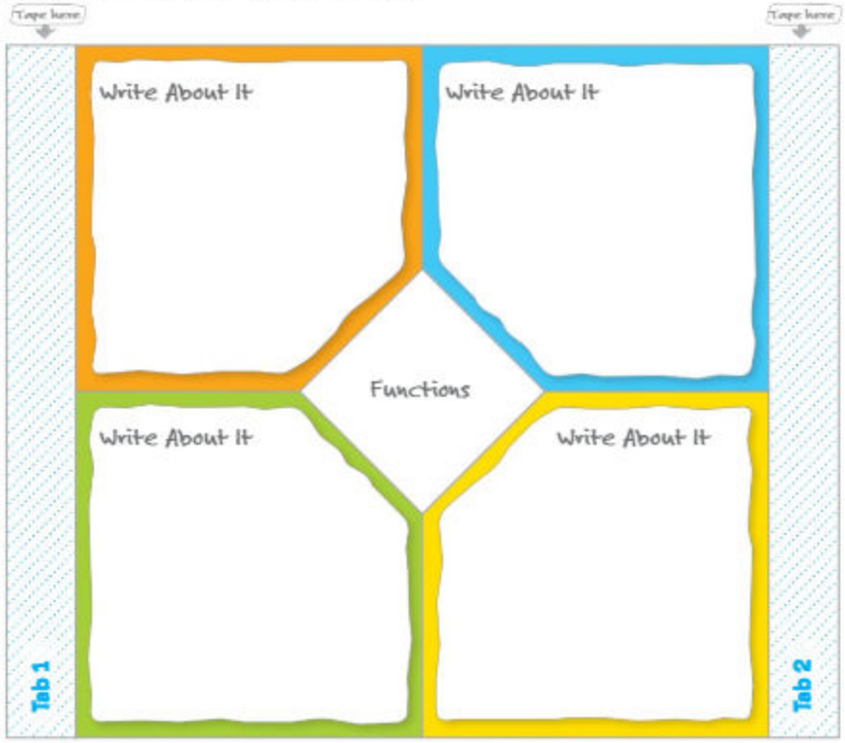
If students have trouble with Exercises 1–5, they may need help with the following concept(s).

Concept	Exercise(s)
function tables (Lesson 1)	2, 4, 5
sequences (Lesson 2)	1, 3
inequalities (Lesson 5)	5

Key Concept Check

Use Your FOLDABLES

Use your Foldable to help review the chapter.



Got it?

Circle the correct term or number to complete each sentence.

1. The next number in the sequence 12, 15, 18, 21, . . . is (24, 27).
2. The output of a function is the (independent, dependent) variable.
3. A(n) (arithmetic, geometric) sequence can be found by multiplying each previous term by the same number.
4. The input of a function is the (independent, dependent) variable.
5. A(n) (inequality, function) is a relation that assigns exactly one output value to one input value.

Expressions and Equations

Power Up! Performance Task

Remodeling Project

Mr. Yousif is installing a new kitchen floor using white and brown tiles. The relationship between the number of brown tiles and the number of white tiles is shown in the table.

White (w)	1	2	3	4	5	6
Brown (b)	4	6	8	10	?	?

Write your answers on another piece of paper. Show all of your work to receive full credit.

Part A

Fill in the missing values based on the pattern in the table. Write an equation that represents the relationship between the white tiles and the brown tiles. Let b represent brown tiles and w represent white tiles.

Part B

Each tile costs AED 12. Determine the cost of 60 tiles, 80 tiles, 100 tiles, and 120 tiles. Write a set of ordered pairs (number of tiles, total cost) to represent the data. Then graph the ordered pairs.

Part C

Mr. Yousif budgeted AED 1,200 for tiles. His sketch of the floor requires him to use 38 white tiles. Each white tile costs AED 12. Write and solve an inequality to find the maximum amount he can spend on brown tiles.

Part D

The tile store has three different brown tiles Mr. Yousif can use. The tile prices are shown in the table. Which tile(s) can he purchase to stay in his budget and meet the design for 38 white tiles? Explain your reasoning.

Tile A	Tile B	Tile C
AED 9.75 per tile	Tiles are AED 11 per tile if fewer than 50 tiles are purchased. For 50 or more tiles, tiles are AED 9.50 per tile.	Tiles are purchased by the box. There are 24 tiles per box. Each box is AED 185.

Power Up! Performance Task

This Performance-Based Assessment requires students to solve multi-step problems through abstract reasoning, precision, and perseverance. This practice scenario can be used to help students prepare for the thinking skills that will be used on the Assessment.

A complete scoring rubric with answers to the Exercises can be found on page PT3.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the **Building on the Essential Question** exercises found in each lesson of the chapter.

- How can a function table help you find input or output? (p. 582)
- What is the difference between an arithmetic sequence and a geometric sequence? (p. 590)
- How are ordered pairs of a function used to create the graph of the function? (p. 598)
- Why do you represent functions in different ways? (p. 606)
- How can mental math help you find solutions to inequalities? (p. 620)
- How can graphing an inequality help to solve it? (p. 628)
- How is solving an inequality similar to solving an equation? (p. 638)

Ideas for Use

LA **Think-Pair-Share** Have students work in pairs. Pose the Essential Question. Give students about one minute to think about how they could complete the graphic organizer. Then have them share their responses with their partner before they complete the graphic organizer.
MP 1, 3, 5, 6

Track Your Progress

Have your students return to pages xix–xxii to rate their knowledge. They should see that their knowledge of the key ideas has increased now that they have completed this chapter.

Reflect

Answering the Essential Question

Use what you learned about inequalities to complete the graphic organizer.

Essential Question

HOW are symbols, such as <, >, and =, useful?

<	>	=
What does it mean? less than	What does it mean? greater than	What does it mean? equal to
Mathematical Example $5 < 8$	Mathematical Example $8 > 5$	Mathematical Example $2 + 3 = 3 + 2$
Real-World Example Suhaila read 5 books last month. Yasmin read 8 books last month. Suhaila read fewer books last month than Yasmin.	Real-World Example Hidaya has AED 8. Hala has AED 5. Hidaya has more money than Hala.	Real-World Example Yousif has 2 red pens and 3 blue pens. Saeed has 3 black pens and 2 purple pens. Yousif and Saeed both have 5 pens.

Answer the Essential Question. HOW are symbols, such as <, >, and =, useful?

See students' work.

UNIT PROJECT

It's Out of This World How fast do objects in our solar system travel through space? Let's explore the orbital speed of different planets and satellites! In this project you will:

- **Collaborate** with your classmates as you investigate the orbital speed of three planets.
- **Share** the results of your research in a creative way.
- **Reflect** on how you communicate mathematical ideas effectively.

Collaborate

Go Online Work with your group to research and complete each activity. You will use your results in the Share section on the following page.

1. Choose three planets in our solar system. Use the Internet to research each planet and find its average orbital speed in miles per second or kilometers per second. Organize the information in a table.

2. Find and record the orbital distance traveled in 1, 2, and 3 seconds for each planet you chose in Exercise 1. Then describe how the orbital distance of each planet changes with time.

3. For your three planets, list the ordered pairs representing (time, distance). Graph each set of ordered pairs on a coordinate plane and connect each set of points with a line. Compare the graphs. Then write equations to represent each relationship.

4. Research artificial satellites, such as the Hubble Space Telescope, that are orbiting Earth. Use the Internet to research three different satellites and determine the purpose of those satellites. Write a summary of your findings.

5. For each satellite you found in Exercise 4, find and record its average orbital speed in miles per second or kilometers per second. Organize the information in a table. Compare the orbital speeds.

Launch the Project

Objective Research the average orbital speeds of three planets and describe the relationship between time and distance using ordered pairs, equations, and words.

It's Out of This World!

This project is designed to be completed by a group of 4 or 5 students over several days or several weeks. It utilizes concepts from the Expressions and Equations domain. You may choose to complete this project after completing the chapters within this domain.

Collaborate

Have students work in teams to research information about three planets. Together, they should be able to gather the necessary information to answer Exercises 1–5. Students should show their work on a separate piece of paper.

Share

After each group gives their presentation, discuss which planets have the fastest and which have the slowest orbital speeds. Discuss how you can determine this by comparing the ordered pairs and the equations for the different planets.

21st Century Skills

You may want your students to connect their projects to a 21st century skill. Check out the suggestion below and on the student page.

Connect with Science

Health Literacy to find a Web site that calculates weights on other planets. Enter your weight, or the weight of a known object, and examine how it differs on each planet. Write a paragraph summarizing your findings.

Reflect

Students should work on their own to reflect on how the chapters from this unit and the objective of the project relate to the Essential Question.

Share

With your group, decide on a way to present what you have learned from each of the activities. Some suggestions are listed below, but you can also think of other creative ways to present your information. Remember to show how you used math to complete each of the activities of this project!

- Create a presentation using the data you collected. Your presentation should include a spreadsheet, graph, and one other visual display.
- Write an article that would be published in a magazine from the perspective of a scientist. Include any important information that you found while researching the orbital speed of each planet.

Check out the note on the right to connect this project with other subjects.

Connect with Social Studies

Global Awareness Research the history of space exploration and write a summary of your findings. Some questions to consider are:

- What have scientists discovered recently about the solar system?
- Which countries have contributed the most to space exploration?

Reflect

6. **Answer the Essential Question** How can you communicate mathematical ideas effectively?

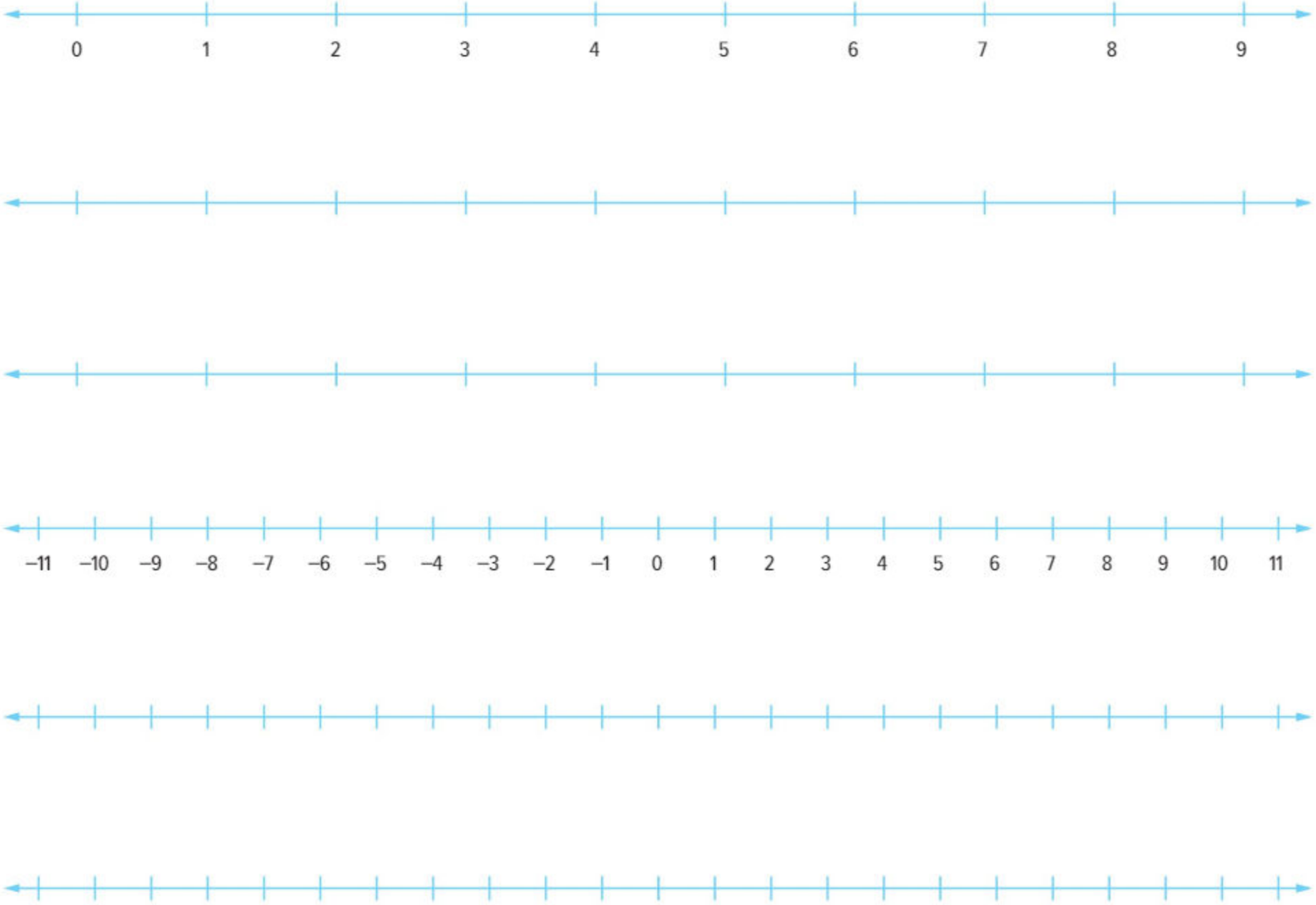
   a. How did you use what you learned about expressions and equations to communicate mathematical ideas effectively in this project?

   See students' work.

   b. How did you use what you learned about functions and inequalities to communicate mathematical ideas effectively in this project?

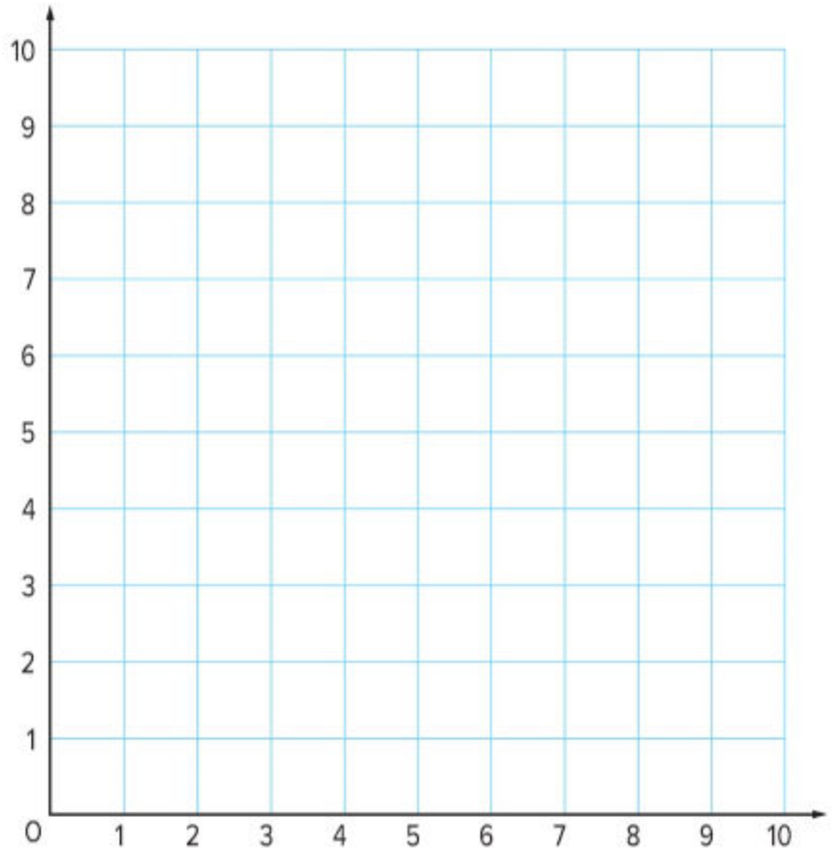
   See students' work.

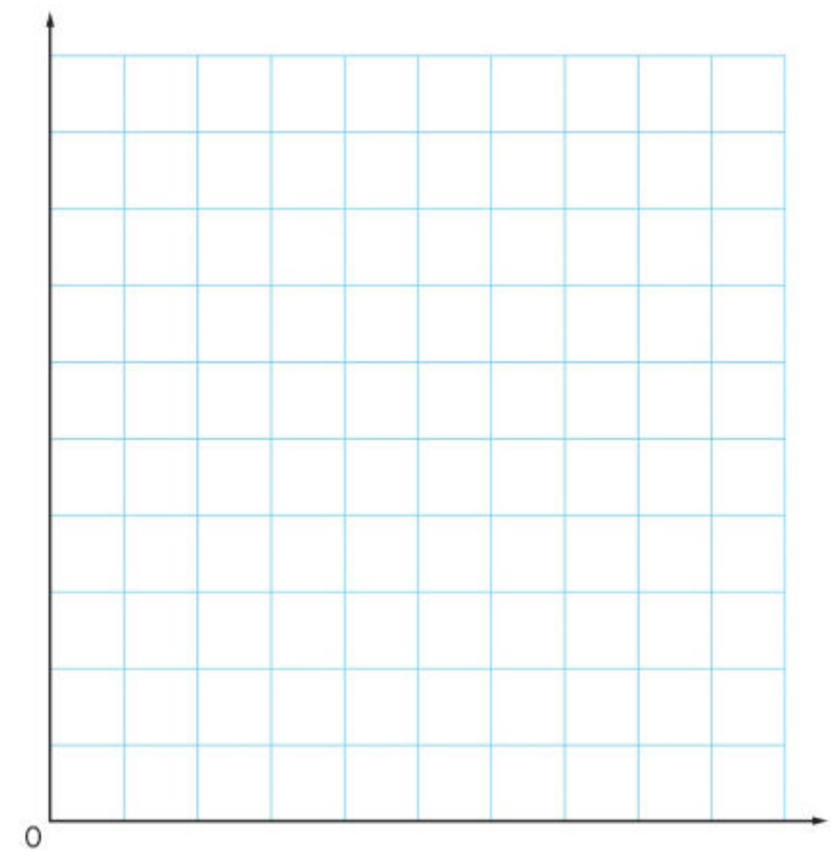
Name _____

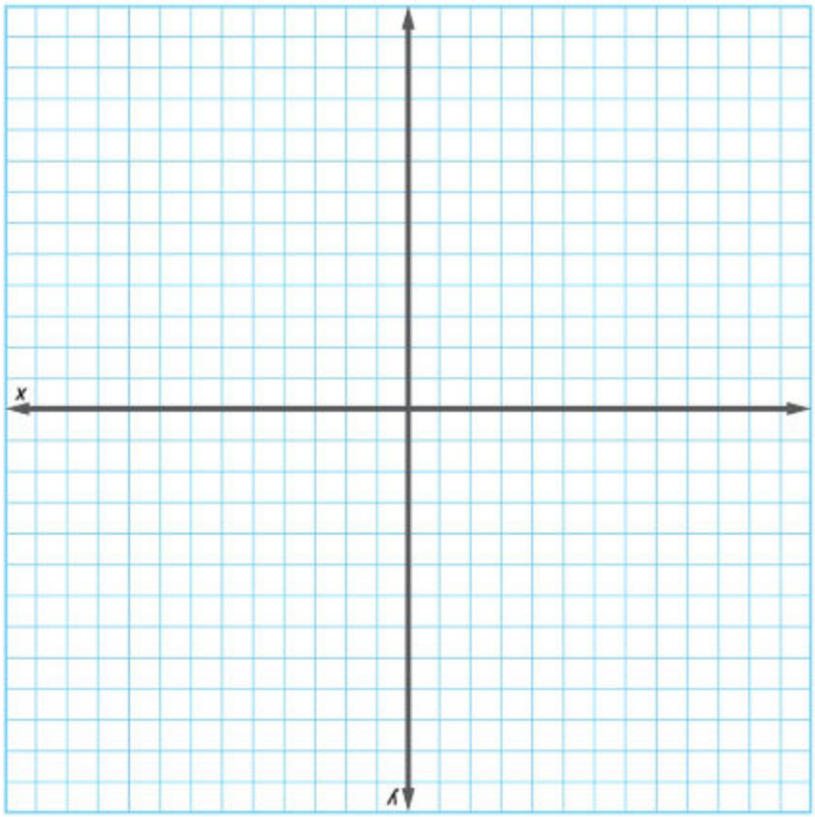


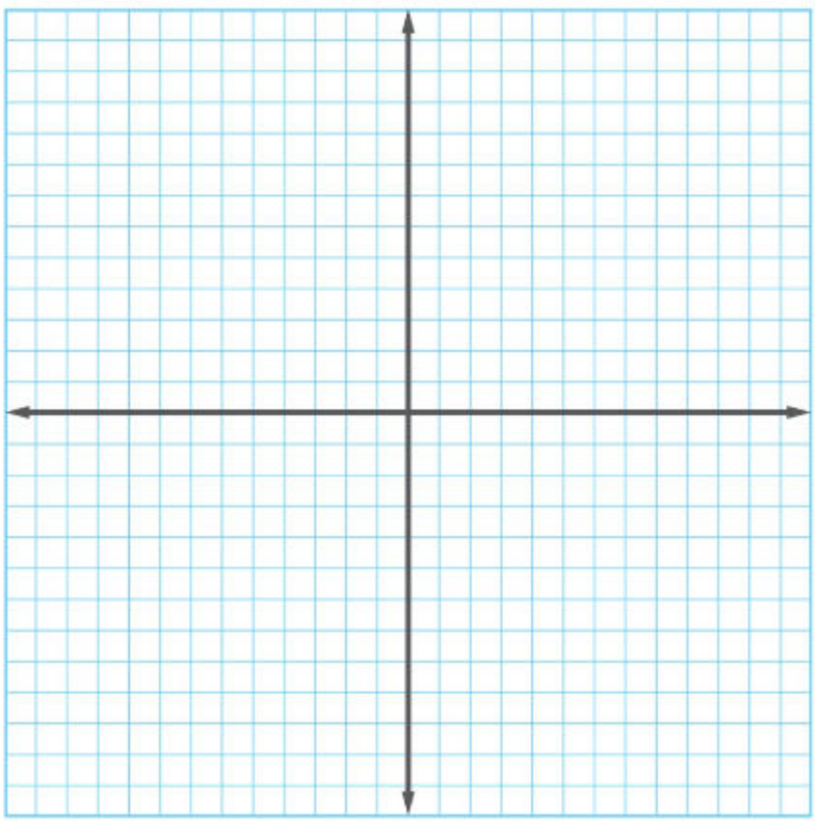
WM2 First Quadrant Grids

Name _____



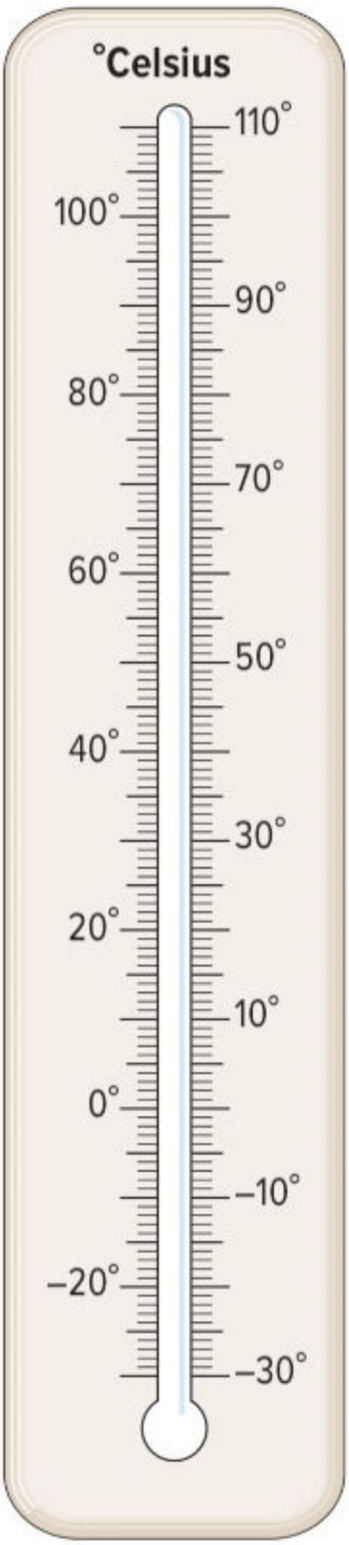






Name _____

Name _____


°Celsius
110°
100°
90°
80°
70°
60°
50°
40°
30°
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10°
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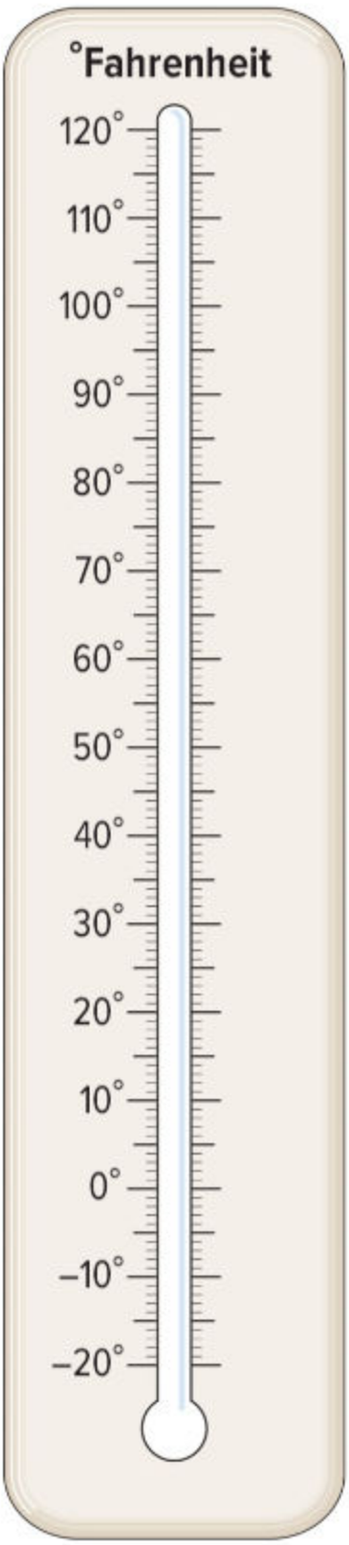
−20°
−30°


°Fahrenheit
120°
110°
100°
90°
80°
70°
60°
50°
40°
30°
20°
10°
0°
−10°
−20°

FOLDABLES Study Organizers

What Are Foldables and How Do I Create Them?

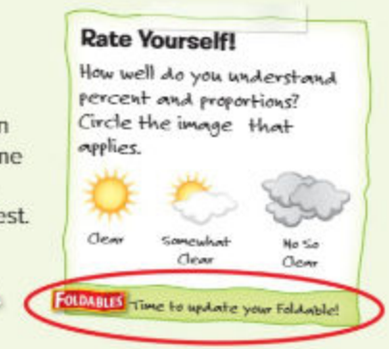
Foldables are three-dimensional graphic organizers that help you create study guides for each chapter in your book.

Step 1 Go to the back of your book to find the Foldable for the chapter you are currently studying. Follow the cutting and assembly instructions at the top of the page.

Step 2 Go to the Key Concept Check at the end of the chapter you are currently studying. Match up the tabs and attach your Foldable to this page. Dotted tabs show where to place your Foldable. Striped tabs indicate where to tape the Foldable.

How Will I Know When to Use My Foldable?

When it's time to work on your Foldable, you will see a Foldables logo at the bottom of the **Rate Yourself!** box on the Guided Practice pages. This lets you know that it is time to update it with concepts from that lesson. Once you've completed your Foldable, use it to study for the chapter test.



How Do I Complete My Foldable?

No two Foldables in your book will look alike. However, some will ask you to fill in similar information. Below are some of the instructions you'll see as you complete your Foldable. **HAVE FUN** learning math using Foldables!

Instructions and what they mean

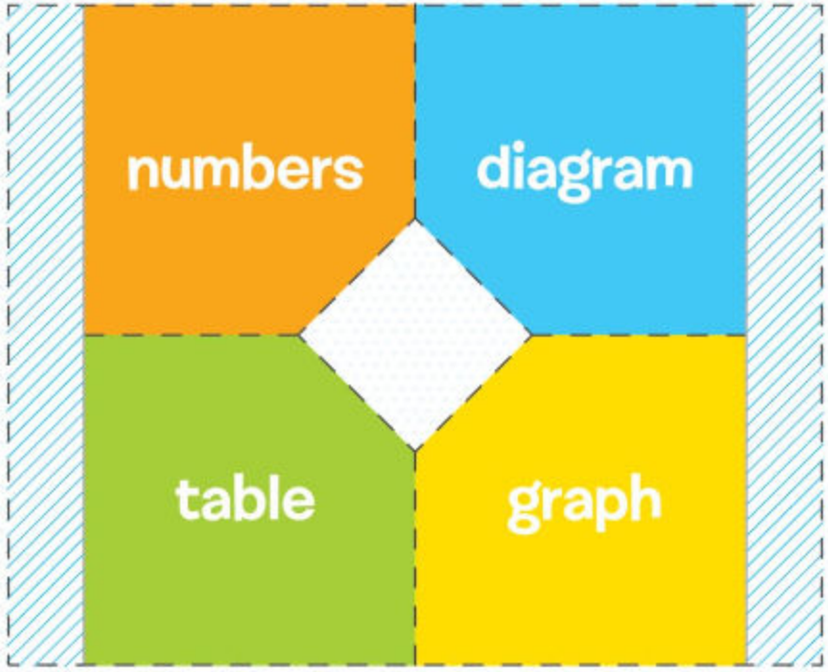
Best Used to...	Complete the sentence explaining when the concept should be used.
Definition	Write a definition in your own words.
Description	Describe the concept using words.
Equation	Write an equation that uses the concept. You may use one already in the text or you can make up your own.
Example	Write an example about the concept. You may use one already in the text or you can make up your own.
Formulas	Write a formula that uses the concept. You may use one already in the text.
How do I ...?	Explain the steps involved in the concept.
Models	Draw a model to illustrate the concept.
Picture	Draw a picture to illustrate the concept.
Solve	
Algebraically	Write and solve an equation that uses the concept.
Symbols	Write or use the symbols that pertain to the concept.
Write About It	Write a definition or description in your own words.
Words	Write the words that pertain to the concept.

Meet Foldables Author Dinah Zike

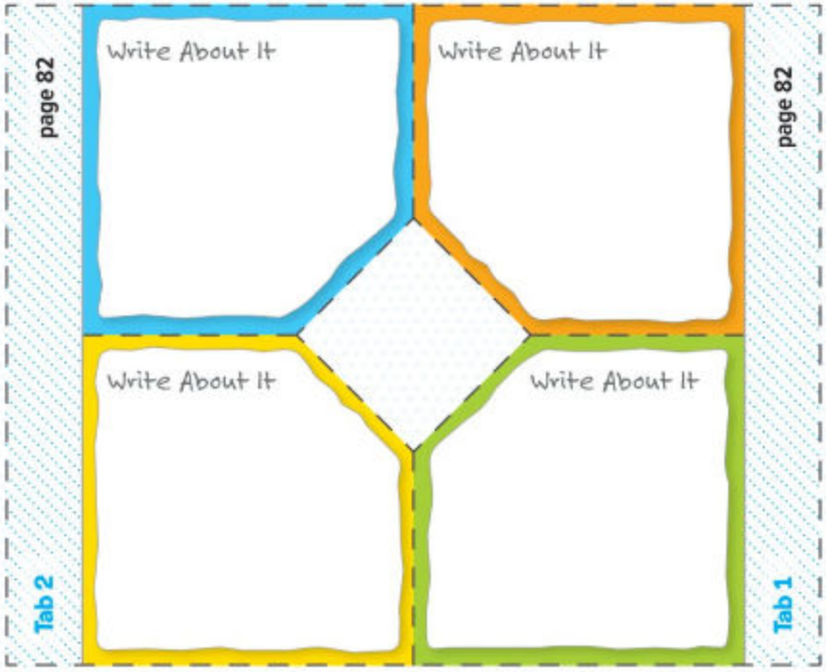
Dinah Zike is known for designing hands-on manipulatives that are used nationally and internationally by teachers and parents. Dinah is an explosion of energy and ideas. Her excitement and joy for learning inspires everyone she touches.

Use this Foldable with Chapter 1.

cut on all dashed lines fold on all solid lines tape to page 82 FOLDABLES



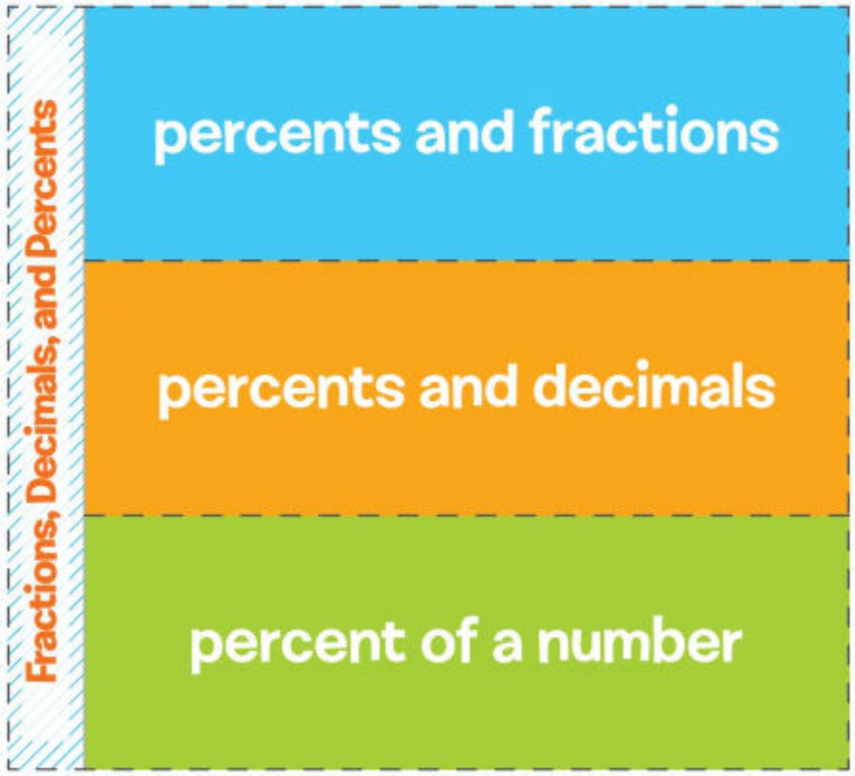
cut on all dashed lines fold on all solid lines tape to page 82 FOLDABLES



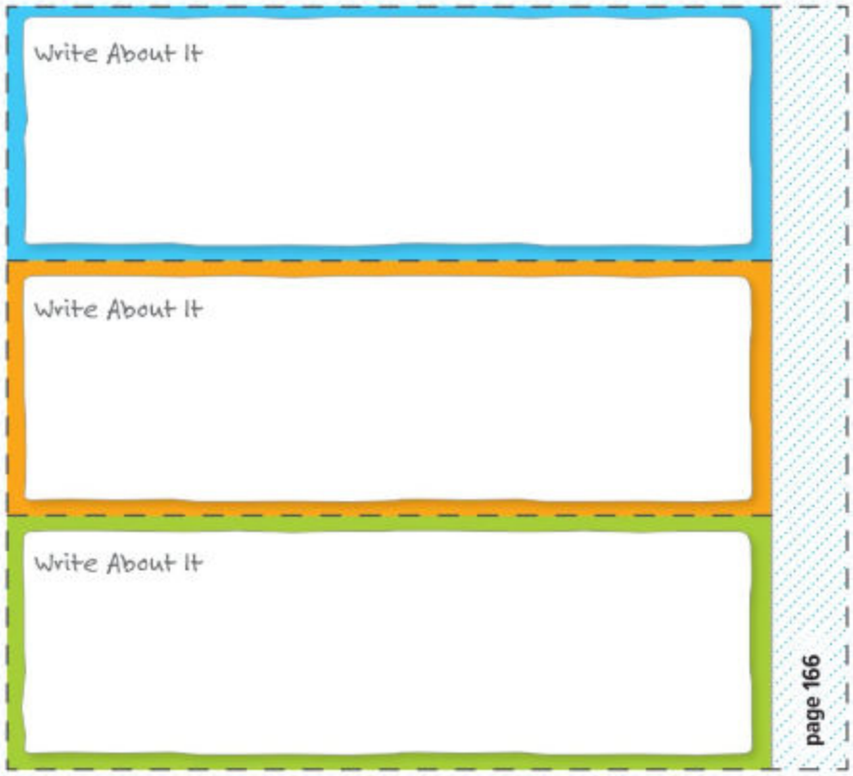
Use this Foldable with Chapter 1.

Use this Foldable with Chapter 2.

cut on all dashed lines fold on all solid lines tape to page 166 FOLDABLES



cut on all dashed lines fold on all solid lines tape to page 166 FOLDABLES



Use this Foldable with Chapter 2.

Use this Foldable with Chapter 3.

cut on all dashed lines fold on all solid lines tape to page 250 FOLDABLES

Divide with Decimals

decimal ÷ whole number	decimal ÷ decimal

Use this Foldable with Chapter 3.

page 250

How do I divide a decimal by a whole number?

How do I divide a decimal by a decimal?

Use this Foldable with Chapter 4.

cut on all dashed lines fold on all solid lines tape to page 336 FOLDABLES

Multiply and Divide Fractions

multiply	divide
Example	Example
fraction × whole number	whole number ÷ fraction
Example	Example
fraction × fraction	fraction ÷ fraction

page 336 **Tab 3**

How do I multiply a fraction by a whole number?

How do I divide a whole number by a fraction?

page 336 **Tab 2**

How do I multiply a fraction by a fraction?

How do I divide a fraction by a fraction?

page 336 **Tab 1**

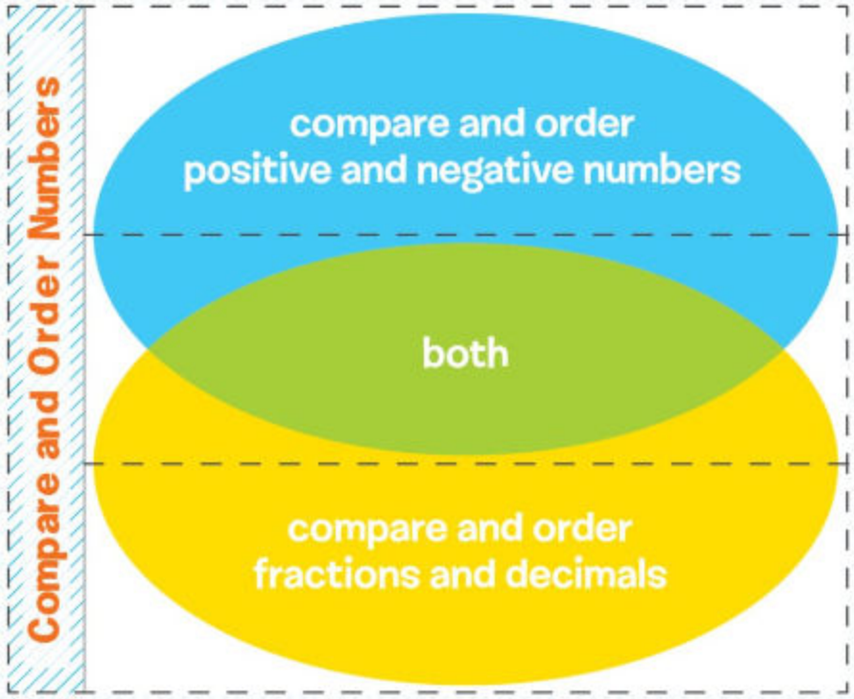
How do I multiply a fraction by a mixed number?

How do I divide a mixed number by a fraction?

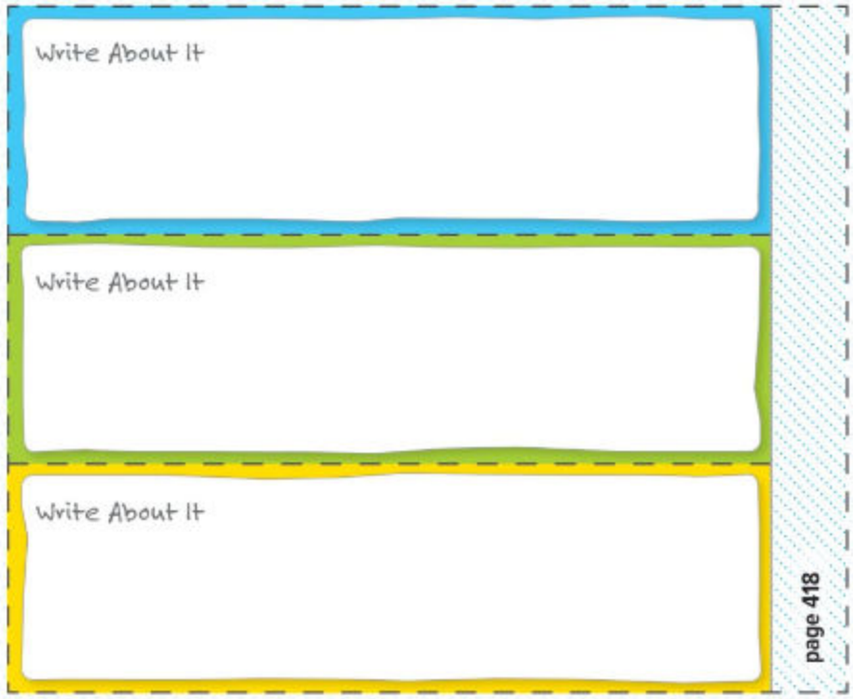
Use this Foldable with Chapter 4.

Use this Foldable with Chapter 5.

cut on all dashed lines fold on all solid lines tape to page 418 FOLDABLES



cut on all dashed lines fold on all solid lines tape to page 418 FOLDABLES

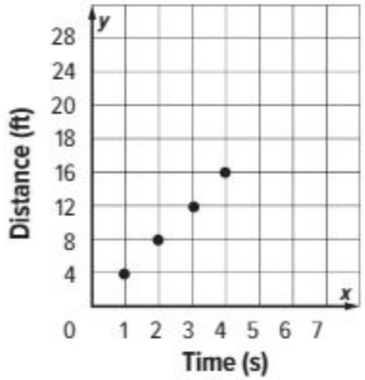


Use this Foldable with Chapter 5.

Chapter 1 Ratios and Rates

Lesson 1-5 Need Another Example?

1–2.

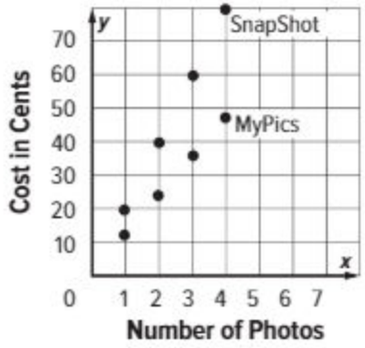


The graph shows that the distance increases by 4 feet for each second that Kaylee travels on the scooter.

3–5.

SnapShot		
Photos, *x*	**Cost in Cents, *y***	**(*x*, *y*)**
1	20	(1, 20)
2	40	(2, 40)
3	60	(3, 60)
4	80	(4, 80)

My Pics		
Photos, *x*	**Cost in Cents, *y***	**(*x*, *y*)**
1	12	(1, 12)
2	24	(2, 24)
3	36	(3, 36)
4	48	(4, 48)



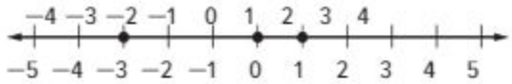
Sample answer: The costs for the services are closer together when the number of photos is small. As the number of photos increases, the cost for SnapShot increases at a faster rate than MyPics.

Chapter 5 Integers and the Coordinate Plane

Lesson 5-1 Need Another Example?

4.

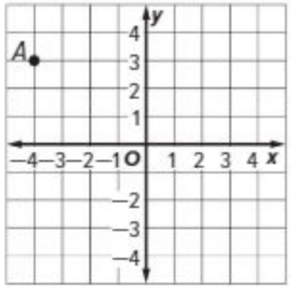
5.



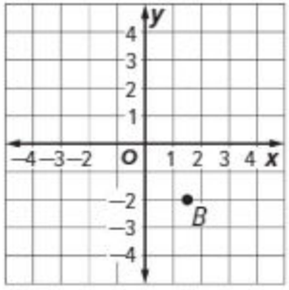
6.

Lesson 5-7 Need Another Example?

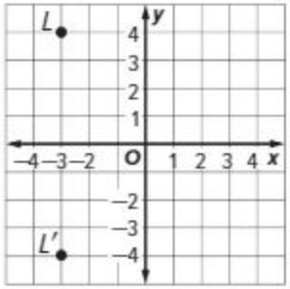
1.



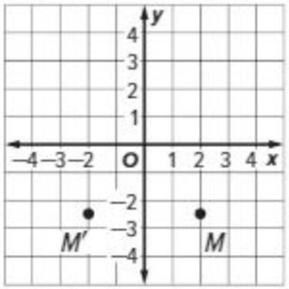
2.



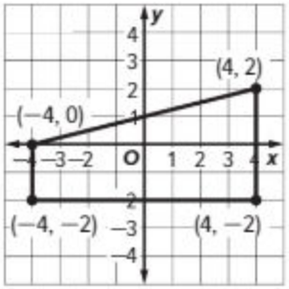
3.


L
L′
y
x
O
4
3
2
1
−4
−3
−2
−2
−3
−4
1
2
3
4

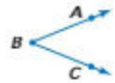
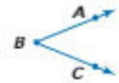
4.


M′
M
y
x
O
4
3
2
1
−4
−3
−2
−2
−3
−4
1
2
3
4

5.


(4, 2)
(−4, 0)
(−4, −2)
(4, −2)
y
x
O
4
3
2
1
−4
−3
−2
−3
−4
1
2
3

Glossary/القاموس

English	العربية
Aa	
absolute value The distance between a number and zero on a number line.	**القيمة المطلقة** هي المسافة بين عدد وصفر على خط الأعداد.
acute angle An angle with a measure greater than 0° and less than 90°. 	**الزاوية الحادة** هي زاوية بقياس أكبر من الدرجة 0 وأصغر من الدرجة 90. 
acute triangle A triangle having three acute angles. 	**المثلث الحاد** هو مثلث يحتوي على ثلاث زوايا حادة. 
Addition Property of Equality If you add the same number to each side of an equation, the two sides remain equal.	**خاصية الجمع في المعادلات** إذا أضفت العدد نفسه لكل طرف في معادلة، يظل الطرفان متساويين.
algebra A mathematical language of symbols, including variables.	**الجبر** هو لغة الرموز الرياضية، بما في ذلك المتغيرات.
algebraic expression A combination of variables, numbers, and at least one operation.	**العبارة الجبرية** هي عبارة عن مجموعة من المتغيرات والأرقام وعملية واحدة على الأقل.
analyze To use observations to describe and compare data.	**التحليل** هو استخدام الملاحظات لوصف ومقارنة البيانات.
angle Two rays with a common endpoint form an angle. The rays and vertex are used to name the angle.  ∠ABC, ∠CBA, or ∠B	**الزاوية** هي شعاعان لهما نقطة مشتركة تشكل زاوية. ويتم استخدام الشعاعين والرأس في تسمية الزاوية.  ∠B أو ∠CBA ،∠ABC

arithmetic sequence A sequence in which the difference between any two consecutive terms is the same.

المتتالية الحسابية هي متتالية يكون فيها الفرق بين أي حدين متتاليين متساويًا.

Associative Property The way in which numbers are grouped does not change the sum or product.

خاصية التجميع هي الطريقة التي يتم فيها تجميع الأعداد بحيث لا يتغير المجموع أو الناتج.

average The sum of two or more quantities divided by the number of quantities; the mean.

المتوسط هو مجموع كميتين أو أكثر مقسومًا على عدد الكميات؛ ويُطلق عليه المتوسط الحسابي.

Bb

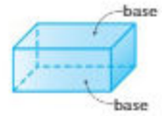
bar notation A bar placed over digits that repeat to indicate a number pattern that repeats indefinitely.

رمز العدد الدوري هو خط يوضع فوق الأرقام التي تتكرر للإ شارة إلى نمط الرقم الذي يتكرر بشكل غير محدد.

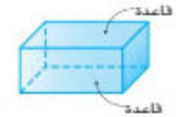
base Any side of a parallelogram.

القاعدة هي أي ضلع في متوازي الأضلاع.

base One of the two parallel congruent faces of a prism.



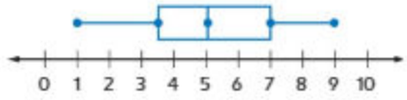
القاعدة هي أحد الوجهين المتطابقين المتوازيين في المنشور.



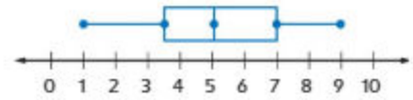
base In a power, the number used as a factor. In 10^3, the base is 10. That is, $10^3 = 10 \times 10 \times 10$.

الأساس في عملية الرفع، هو العدد الذي يتم استخدامه كعامل. في المثال 10^3، يكون الأساس هو العدد 10. بمعنى أن، $10^3 = 10 \times 10 \times 10$.

box plot A diagram that is constructed using five values.



مخطط الصندوق ذي العارضين هو رسم تخطيطي يتم بناؤه باستخدام خمس قيم.



Cc

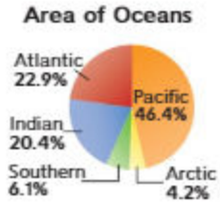
center The given point from which all points on a circle are the same distance.

المركز هو النقطة المعلومة التي تبعد عنها جميع النقاط على الدائرة نفس المسافة.

circle The set of all points in a plane that are the same distance from a given point called the center.

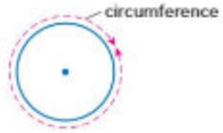
الدائرة هي مجموعة النقاط في المستوى التي لها البعد نفسه عن نقطة معلومة تُسمى المركز.

circle graph A graph that shows data as parts of a whole. In a circle graph, the percents add up to 100.



الرسم البياني الدائري هو رسم بياني يوضح البيانات كأجزاء من الكل. في الرسم البياني الدائري، يكون مجموع النسب المئوية 100.

circumference The distance around a circle.



محيط الدائرة هو المسافة حول الدائرة.

cluster Data that are grouped closely together.

التجمع هو البيانات المجمعة بالقرب من بعضها.

coefficient The numerical factor of a term that contains a variable.

المعامل هو عامل عددي للحد الذي يحتوي على متغير.

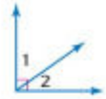
Commutative Property The order in which numbers are added or multiplied does not change the sum or product.

خاصية التبديل هي الترتيب الذي يتم به إضافة أو ضرب الأرقام بحيث لا يتغير المجموع أو الناتج.

compatible numbers Numbers that are easy to use to perform computations mentally.

العددان المتوافقان هما العددان اللذان يسهل استخدامهما في إجراء العمليات الحسابية ذهنيًا.

complementary angles Two angles are complementary if the sum of their measures is 90°.

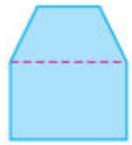


∠1 and ∠2 are complementary angles.

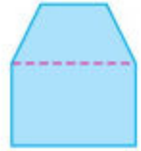
الزاويتان المتتامتان تكون الزاويتان متتامتين إذا كان مجموع قياسهما يساوي 90 درجة.

∠1 و∠2 هما زاويتان متتامتان

composite figure A figure made of triangles, quadrilaterals, semicircles, and other two-dimensional figures.



الشكل المركب هو شكل مركب من مثلثات وأشكال رباعية الأضلاع وأنصاف دائرية وغيرها من الأشكال ثنائية الأبعاد.



congruent Having the same measure.

التطابق هو وجود نفس القياس.

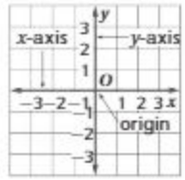
congruent figures Figures that have the same size and same shape; corresponding sides and angles have equal measures.

الأشكال المتطابقة هي الأشكال التي لها نفس القياس ونفس الشكل؛ ويكون لها أضلاع وزوايا متناظرة وقياسات متساوية.

constant A term without a variable.

الثابت هو حد لا يحتوي على متغير.

coordinate plane A plane in which a horizontal number line and a vertical number line intersect at their zero points.



المستوى الإحداثي هو مستوى يكون فيه خط الأعداد الأفقي وخط الأعداد الرأسي متقاطعين في النقاط الصفرية.

corresponding sides The sides of similar figures that "match."

الأضلاع المتناظرة هي أضلاع لأشكال متشابهة و«متطابقة».

cubic units Used to measure volume. Tells the number of cubes of a given size it will take to fill a three-dimensional figure.

3 cubic units

الوحدات المكعبة هي وحدات تستخدم لقياس الحجم. كما أنها تخبرنا بعدد المكعبات التي يستخدمها حجم معين لملء شكل ثلاثي الأبعاد.

3 وحدات مكعبة

Dd

data Information, often numerical, which is gathered for statistical purposes.

البيانات هي معلومات وغالبا ما تكون رقمية، ويتم جمعها لأغراض إحصائية.

decagon A polygon having ten sides.

عشاري الأضلاع هو مضلع له عشرة أضلاع.

defining the variable Choosing a variable and deciding what the variable represents.

تعريف المتغير هو اختيار متغير وتحديد ما يمثله هذا المتغير.

dependent variable The variable in a relation with a value that depends on the value of the independent variable.

المتغير التابع هو المتغيرالذي يرتبط بالقيمة التي تعتمد على قيمة المتغير المستقل.

diameter The distance across a circle through its center.

قطر الدائرة هو المسافة المارة بالمركز داخل الدائرة.

dimensional analysis The process of including units of measurement **when you compute.**

التحليل البعدي هو عملية تضمين وحدات قياس أثناء إجراء العمليات الحسابية.

distribution The arrangement of data values.

التوزيع هو ترتيب قيم البيانات.

Distributive Property To multiply a sum by a number, multiply each addend by the number outside the parentheses.

خاصية التوزيع هي ضرب مجموع في عدد ، وضرب كل حد جمعي في العدد الموجود خارج الأقواس.

Division Property of Equality If you divide each side of an equation by the same nonzero number, the two sides remain equal.

خاصية القسمة في المعادلة في حالة قسمة حدَيْ أية معادلة في نفس العدد غير الصفري، فسوف يظل الحدان متساويين.

dot plot A diagram that shows the frequency of data on a number line. Also known as a line plot.

الرسم البياني بالنقاط هو رسم تخطيطي يظهر تكرار البيانات على خط الأعداد. ويعرف أيضًا باسم التمثيل البياني بالنقاط المجمّعة.

Ee

equals sign A symbol of equality, =.

علامة يساوي هي رمز المساواة، =.

equation A mathematical sentence showing two expressions are equal. An equation contains an equals sign, =.

المعادلة هي عبارة رياضية تحتوي على تعبيرين رياضيين متساويين. وتحتوي المعادلة على علامة يساوي، =.

equilateral triangle A triangle having three congruent sides.

المثلث متساوي الأضلاع هو مثلث يحتوي على ثلاثة أضلاع متطابقة.

equivalent expressions Expressions that have the same value.

التعبيرات المتكافئة هي التعبيرات التي لها نفس القيمة.

equivalent ratios Ratios that express the same relationship between two quantities.

النسب المتكافئة هي النسب التي تعبّر عن علاقة مماثلة بين كميتين.

evaluate To find the value of an algebraic expression by replacing variables with numbers.

إيجاد القيمة يتم إيجاد القيمة لمعرفة قيمة تعبير جبري عن طريق استبدال المتغيرات بالأعداد.

exponent In a power, the number that tells how many times the base is used as a factor. In 5^3, the exponent is 3. That is, $5^3 = 5 \times 5 \times 5$.

الأس في عملية الرفع، هو العدد الذي يوضح عدد المرات التي يتم فيها استخدام الأساس كعامل. وفي 5^3، يكون الأس 3. بمعنى أن $5^3 = 5 \times 5 \times 5$.

Ff

face A flat surface.

الوجه هو أي سطح مستو.

factor the expression The process of writing numeric or algebraic expressions as a product of their factors.

عامل التعبير هو عملية كتابة تعبيرات رقمية أو جبرية في شكل ناتج لعواملها.

first quartile For a data set with median M, the first quartile is the median of the data values less than M.

الرّبيع الأول في مجموعة البيانات ذات الوسيط (M)، يكون الرّبيع الأ ول عبارة عن وسيط لقيم البيانات التي تقل عن قيمة الوسيط.

formula An equation that shows the relationship among certain quantities.

الصيغة هي معادلة تظهر العلاقة بين كميات معينة.

fraction A number that represents part of a whole or part of a set.
$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{3}{4}$

الكسر هو عدد يمثل جزءًا من كل أو جزءًا من مجموعة.
$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{3}{4}$

frequency distribution How many pieces of data are in each interval.

توزيع التكرار هو عدد تكرار البيانات في كل فترة.

frequency table A table that shows the number of pieces of data that fall within the given intervals.

جدول التكرار هو جدول يوضح عدد البيانات التي تقع ضمن فترة معين.

function A relationship that assigns exactly one output value to one input value.

الدالة هي علاقة تحدد بالضبط قيمة مخرج واحد بالنسبة لقيمة مدخل واحد.

function rule An expression that describes the relationship between each input and output.

قاعدة الدالة هو تعبير يصف العلاقة بين كل مدخل ومخرج.

function table A table organizing the input, rule, and output of a function.

جدول الدالة هو جدول ينظم المدخل والقاعدة والمخرج بالدالة.

Gg

gap An empty space or interval in a set of data.

الفجوة هي مساحة فارغة أو فاصل في مجموعة من البيانات.

geometric sequence A sequence in which each term is found by multiplying the previous term by the same number.

المتتالية الهندسية هي متتالية يتم فيها إيجاد كل حد عن طريق ضرب الحد السابق في عدد ثابت.

graph To place a dot at a point named by an ordered pair.

الرسم البياني هو وضع علامة عند نقطة يحددها زوج مرتب.

Greatest Common Factor (GCF) The greatest of the common factors of two or more numbers.

العامل المشترك الأكبر (GCF) هو أكبر عامل مشترك لعددين أو أكثر.

The greatest common factor of 12, 18, and 30 is 6.

العامل المشترك الأكبر للأعداد 12، 18 و30 هو 6.

Hh

height The shortest distance from the base of a parallelogram to its opposite side.

الارتفاع هو أقصر مسافة من قاعدة متوازي أضلاع إلى الضلع المقابل.

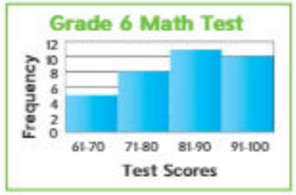
heptagon A polygon having seven sides.

سباعي الأضلاع هو مضلع له سبعة أضلاع.

hexagon A polygon having six sides.

سداسي الأضلاع هو مضلع له ستة أضلاع.

histogram A type of bar graph used to display numerical data that have been organized into equal intervals.



المدرج الإحصائي هو نوع من التمثيل البياني بالأعمدة تُستخدم فيه الأ عمدة لعرض بيانات عددية منظمة على فترات متساوية.

li

Identity Properties Properties that state that the sum of any number and 0 equals the number and that the product of any number and 1 equals the number.

خواص المحايد هي الخواص التي تحدد أن مجموع أي عدد و0 يساوي نفس العدد وأن حاصل ضرب أي عدد و1 يساوي نفس العدد.

independent variable The variable in a function with a value that is subject to choice.

المتغير المستقل هو متغير في الدالة تخضع قيمته للاختيار.

inequality A mathematical sentence indicating that two quantities are not equal.

المتباينة هي عبارة رياضية تشير إلى أن كميتين غير متساويتين.

integer Any number from the set {... −4, −3, −2, −1, 0, 1, 2, 3, 4 ...} where ... means *continues without end.*

العدد الصحيح أي عدد ضمن المجموعة {... −4, −3, −2, −1, 0, 1, 2, 3, 4...} حيث يعني «...» *استمرار المجموعة دون نهاية.*

interquartile range A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set.

المدى الرُبيعي هو مقياس التنوع في مجموعة من البيانات الرقمية، وهو عبارة عن المسافة بين الربيع الأول والثالث لمجموعة البيانات.

intersecting lines *Lines* that meet or cross at a common *point.*

الخطوط المستقيمة المتقاطعة هي *الخطوط المستقيمة* التي تلتقي أو تتقاطع في *نقطة* مشتركة.

interval The difference between successive values on a scale.

الفترة هو الفرق بين القيم المتعاقبة في مقياس ما.

inverse operations Operations which *undo* each other. For example, addition and subtraction are inverse operations.

العمليات العكسية هي العمليات التي تلغي بعضها البعض. على سبيل المثال، يعتبر الجمع والطرح عمليات عكسية.

isosceles triangle A triangle having at least two congruent sides.

المثلث متساوي الساقين هو مثلث يحتوي على ضلعين متطابقين على الأقل.

Ll

lateral face Any face that is not a base.

الوجه الجانبي هو أي وجه غير القاعدة.

least common denominator (LCD) The least common multiple of the denominators of two or more fractions.

المقام المشترك الأصغر (LCD) أصغر مضاعف مشترك في مقامي كسرين أو أكثر.

least common multiple (LCM) The smallest whole number greater than 0 that is a common multiple of each of two or more numbers.

المضاعف المشترك الأصغر (LCM) هو أصغر عدد كلي أكبر من 0 وهو عبارة عن المضاعف المشترك لكل عددين أو أكثر.

The LCM of 2 and 3 is 6.

المضاعف المشترك الأصغر للعدد 2 و3 هو 6.

leaves The digits of the least place value of data in a stem-and-leaf plot.

الأوراق هي الأرقام الأقل منزلة مكانية من البيانات في مخطط الساق والأوراق.

like terms Terms that contain the same variable(s) to the same power.

الحدود المتشابهة هي حدود تتكون من (المتغير) المتغيرات نفسها ومرفوعة لنفس الأس.

line A set of *points* that form a straight path that goes on forever in opposite directions.

الخط المستقيم هو مجموعة من *النقاط* التي تشكل مسارًا مستقيمًا يتجه دائمًا في اتجاهين متعاكسين.

linear function A function that forms a line when graphed.

المعادلة الخطية هي دالة تشكل خطًا عند رسمها بيانيًا.

line graph A graph used to show how a set of data changes over a period of time.

الرسم البياني الخطي هو رسم بياني يستخدم لإظهار مدى تغير مجموعة البيانات بمرور الوقت.

line of symmetry A line that divides a figure into two halves that are reflections of each other.

خط التناظر هو خط مستقيم يقسم الشكل إلى نصفين بحيث يمثل كل نصف انعكاسًا للآخر.

line plot A diagram that shows the frequency of data on a number line. Also known as a dot plot.

تمثيل بياني بالنقاط المجمّعة هو رسم تخطيطي يظهر تكرار البيانات على خط الأعداد. ويعرف أيضًا باسم الرسم البياني بالنقاط.

line segment A part of a *line* that connects two points.

القطعة المستقيمة هي جزء من خط *المستقيم* يصل بين نقطتين.

line symmetry Figures that match exactly when folded in half have line symmetry.

التناظر المحوري يحدث التناظر المحوري في حالة الأشكال التي تتطابق تمامًا عند طيها إلى نصفين.

Mm

mean The sum of the numbers in a set of data divided by the number of pieces of data.

المتوسط الحسابي هو مجموع الأعداد في مجموعة من البيانات مقسومًا على عدد البيانات.

mean absolute deviation A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values.

متوسط الانحراف المطلق هو قياس التباين في مجموعة من البيانات الرقمية التي يتم حسابها بإضافة مسافات بين كل قيمة بيانات والمتوسط الحسابي، ثم قسمة الناتج على عدد قيم البيانات.

measures of center Numbers that are used to describe the center of a set of data. These measures include the mean, median, and mode.

المقياس المركزي هي أعداد تستخدم لوصف مركز مجموعة من البيانات. ويشمل هذا المقياس الوسط الحسابي والوسيط، والمنوال.

measures of variation A measure used to describe the distribution of data.

مقاييس التباين هي مقياس يستخدم لوصف توزيع البيانات.

median A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values.

الوسيط هو مقياس مركزي في مجموعة من البيانات الرقمية. وسيط قائمة القيم هو القيمة التي تظهر في مركز البيانات المصنفة من القائمة أو المتوسط الحسابي للقيمتين المركزيتين، إذا كانت القائمة تحتوي على قيم بعدد زوجي.

mode The number(s) or item(s) that appear most often in a set of data.

المنوال هو العدد (الأعداد) أو العنصر (العناصر) الأكثر تكرارًا في مجموعة من البيانات.

Multiplication Property of Equality If you multiply each side of an equation by the same nonzero number, the two sides remain equal.

خاصية الضرب في المعادلة في حالة ضرب حدَيْ أية معادلة في نفس العدد غير الصفري، فسوف يظل الحدان متساويين.

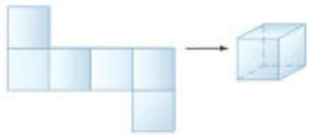
Nn

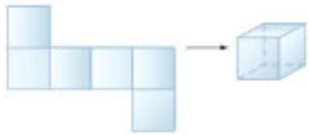
negative integer A number that is less than zero. It is written with a − sign.

العدد الصحيح السالب هو أي عدد أصغر من صفر. وتصاحبه علامة −.

net A two-dimensional figure that can be used to build a three-dimensional figure.

الشبكة هو شكل ثنائي الأبعاد يمكن استخدامه لبناء شكل ثلاثي الأبعاد.





nonagon A polygon having nine sides.

تساعي الأضلاع هو مضلع له تسعة أضلاع.

numerical expression A combination of numbers and operations.

التعبير العددي هو مزيج من الأرقام والعمليات.

Oo

obtuse angle Any angle that measures greater than 90° but less than 180°.

الزاوية المنفرجة هي أي زاوية يكون قياسها أكبر من 90 درجة وأصغر من 180 درجة.

obtuse triangle A triangle having one obtuse angle.

المثلث منفرج الزاوية هو مثلث إحدى زواياه منفرجة.

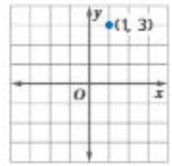
octagon A polygon having eight sides.

ثماني الأضلاع هو مضلع له ثمانية أضلاع.

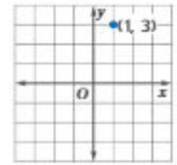
opposites Two integers are opposites if they are represented on the number line by points that are the same distance from zero, but on opposite sides of zero. The sum of two opposites is zero.

العددان المتقابلان هما عددان صحيحان يكونان معكوسين في حالة تمثيليهما على خط الأعداد من النقاط التي تكون على مسافة واحدة من الصفر، ولكنها على ضلعين متقابلين من الصفر. ويكون مجموع العددين المتقابلين صفرًا.

ordered pair A pair of numbers used to locate a point on the coordinate plane. The ordered pair is written in the form (*x*-coordinate, *y*-coordinate).



الزوج المرتب هو زوج يُستخدم لتحديد نقطة على المستوى الإحداثي. ويكتب الزوج المرتب على شكل (الإحداثي Y، والإحداثي X).



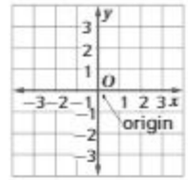
order of operations The rules that tell which operation to perform first when more than one operation is used.

1. Simplify the expressions inside grouping symbols, like parentheses.
2. Find the value of all powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

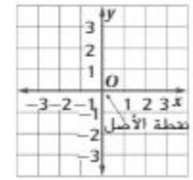
ترتيب العمليات هو القواعد التي تحدد أيًا من العمليات تنفذ أولا عند وجود أكثر من عملية.

1. تبسيط التعبيرات داخل رموز التجميع، مثل الأقواس.
2. البحث عن قيمة جميع الأسس.
3. الضرب والقسمة بالترتيب من اليسار إلى اليمين.
4. الإضافة والطرح بالترتيب من اليسار إلى اليمين.

origin The point of intersection of the x-axis and y-axis on a coordinate plane.



نقطة الأصل هي نقطة تقاطع المحور x مع المحور y في المستوى الإحداثي.



outlier A value that is much greater than or much less than than the other values in a set of data.

القيمة المتطرفة هي قيمة أكبر بكثير أو أقل بكثير من القيم الأخرى في مجموعة من البيانات.

Pp

parallel lines Lines in a plane that never intersect.

الخطوط المتوازية هي خطوط في مستوى واحد ولا تتقاطع أبدًا.

parallelogram A quadrilateral with opposite sides parallel and opposite sides congruent.

متوازي الأضلاع هو شكل رباعي الأضلاع فيه كل ضلعين متقابلين متطابقان.

peak The most frequently occurring value in a line plot.

القمة هي القيمة الأكثر تكرارًا في تمثيل بياني بالنقاط المجمّعة.

pentagon A polygon having five sides.

خماسي الأضلاع هو مضلع له خمسة أضلاع.

percent A ratio that compares a number to 100.

النسبة المئوية هي نسبة تقارن بين العدد و100.

percent proportion One ratio or fraction that compares part of a quantity to the whole quantity. The other ratio is the equivalent percent written as a fraction with a denominator of 100.

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

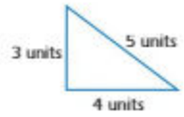
مقدار النسبة المئوية هو نسبة أو كسر يقارن جزءًا من كمية بمجمل الكمية. وتمثل النسبة الأخرى النسبة المئوية المكافئة المكتوبة في شكل كسر مقامه 100.

$$\frac{\text{الجزء}}{\text{الكل}} = \frac{\text{النسبة المئوية}}{100}$$

perfect square Numbers with square roots that are whole numbers. 25 is a perfect square because the square root of 25 is 5.

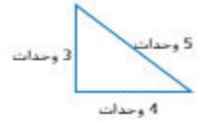
المربع الكامل هو الأعداد التي لها جذور تربيعية عبارة عن أعداد كلية. ويُعد العدد 25 مربعًا كاملاً لأن الجذر التربيعي للعدد 25 هو 5.

perimeter The distance around a figure.



$P = 3 + 4 + 5 = 12$ units

المحيط هو المسافة حول شكل ما.



المحيط = 3 + 4 + 5
= 12 وحدة

pi The ratio of the circumference of a circle to its diameter. The Greek letter π represents this number. The value of pi is always 3.1415926....

π هي نسبة محيط الدائرة إلى قطرها. ويمثل الحرف اليوناني π هذا العدد. ودائمًا ما تكون قيمة π تساوي 3.1415926....

plane A flat surface that goes on forever in all directions.

المستوى هو سطح مستوى يمتد في كل الاتجاهات دائمًا.

point An exact location in space that is represented by a dot.

النقطة هي مكان محدد في الفضاء، ممثلة على شكل نقطة.

polygon A simple closed figure formed by three or more straight line segments.

المضلع هو شكل مغلق بسيط مكون من ثلاث قطع مستقيمة أو أكثر.

population The entire group of items or individuals from which the samples under consideration are taken.

المجتمع الإحصائي هو مجموعة كاملة من العناصر أو الأفراد التي يتم منها أخذ العينات قيد البحث.

positive integer A number that is greater than zero. It can be written with or without a + sign.

العدد الصحيح الموجب هو عدد أكبر من صفر. ويمكن كتابته بعلامة + أو بدونها.

powers Numbers expressed using exponents. The power 3^2 is read *three to the second power*, or *three squared*.

القوة هو التعبير عن الأعداد باستخدام الأسس. فالعدد 3^2 يقرأ ثلاثة *مرفوع إلى الأس اثنين*، أو ثلاثة تربيع.

prism A three-dimensional figure with at least three rectangular lateral faces and top and bottom faces parallel.

المنشور هو شكل ثلاثي الأبعاد يحتوي على ثلاثة أوجه جانبية مستطيلة على الأقل والأوجه العلوية والسفلية متوازية.

properties Statements that are true for any number.

الخواص هي البيانات التي تعتبر حقيقية بالنسبة لأي عدد.

proportion An equation stating that two ratios or rates are equivalent.

التناسب هو معادلة تنص على أن نسبتين أو معدلين متكافئان.

pyramid A three-dimensional figure with at least three triangular sides that meet at a common vertex and only one base that is a polygon.

الهرم هو شكل ثلاثي الأبعاد يحتوي على الأقل على ثلاثة أضلاع على شكل مثلثات تلتقي في قمة مشتركة وقاعدة واحدة فقط مضلعة.

Qq

quadrants The four regions in a coordinate plane separated by the x-axis and y-axis.

الأرباع هي المناطق الأربع في مستوى إحداثي يفصله المحور الأفقي x والمحور الرأسي y.

quadrilateral A closed figure having four sides and four angles.

رباعي الأضلاع هو شكل مغلق يحتوي على أربعة أضلاع وأربع زوايا.

quartiles Values that divide a data set into four equal parts.

الرّبيعات هي قيم تقسم مجموعة من البيانات إلى أربعة أجزاء متساوية.

Rr

radical sign The symbol used to indicate a nonnegative square root, $\sqrt{\ }$.

رمز الجذر هو الرمز المستخدم للدلالة على جذر تربيعي غير سالب، $\sqrt{\ }$.

radius The distance from the center to any point on the circle.

نصف القطر هو المسافة من مركز الدائرة إلى أي نقطة على الدائرة.

range The difference between the greatest number and the least number in a set of data.

المدى هو الفرق بين أكبر عدد وأقل عدد في مجموعة من البيانات.

rate A ratio comparing two quantities with different kinds of units.

المعدل هو النسبة بين كميتين من أنواع مختلفة من الوحدات.

rate of change A rate that describes how one quantity changes in relation to another. A rate of change is usually expressed as a unit rate.

معدل التغير هو معدل يصف كيف تتغير كمية بالنسبة لكمية أخرى. وعادة ما يتم التعبير عن معدل التغير بمعدل الوحدة.

ratio A comparison of two quantities by division. The ratio of 2 to 3 can be stated as 2 out of 3, 2 to 3, 2 : 3, or $\frac{2}{3}$.

المعدل هو مقارنة تتم بين كميتين من خلال قسمتهما. يمكن التعبير عن النسبة 2 إلى 3 باعتبارها 2 من 3، 2 إلى 3، 2 : 3، أو $\frac{2}{3}$.

rational number A number that can be written as a fraction.

العدد النسبي هو العدد الذي يمكن كتابته ككسر.

ratio table A table with columns filled with pairs of numbers that have the same ratio.

جدول النسب هو جدول يحتوي على أعمدة بها زوجان من الأعداد لهما النسبة نفسها.

ray A line that has one endpoint and goes on forever in only one direction.

الشعاع هو خط مستقيم يحتوي على نقطة نهاية ويسير في اتجاه واحد دون نهاية.

reciprocals Any two numbers that have a product of 1.
Since $\frac{5}{6} \times \frac{6}{5} = 1$, $\frac{5}{6}$ and $\frac{6}{5}$ are reciprocals.

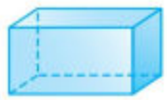
المعكوس الضربي هو مصطلح يُطلق على أي عددين حاصل ضربهما يساوي 1.
بما أن $\frac{5}{6} \times \frac{6}{5} = 1$، إذًا $\frac{5}{6}$ و$\frac{6}{5}$ هما معكوسان ضربيان.

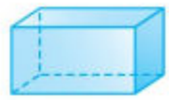
rectangle A parallelogram having four right angles.

المستطيل هو متوازي أضلاع مكون من أربع زوايا قائمة.

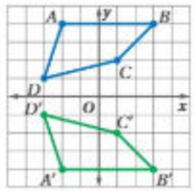
rectangular prism A prism that has rectangular bases.

المنشور المستطيل هو منشور يحتوي على قواعد مستطيلة.

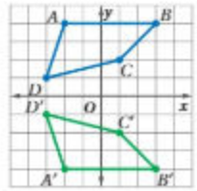




reflection The mirror image produced by flipping a figure over a line.



الانعكاس هو صورة المرآة التي ينتجها قلب شكل عبر خط مستقيم.



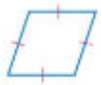
relation A set of ordered pairs such as (1, 3), (2, 4), and (3, 5). A relation can also be shown in a table or a graph.

العلاقة هي مجموعة من أزواج مرتبة مثل (1, 3) و(2, 4) و(3, 5). ويمكن أيضًا إظهار هذه العلاقة في شكل جدول أو رسم بياني.

repeating decimal The decimal form of a rational number.

الكسر العشري الدوري هو الصيغة العشرية من العدد النسبي.

rhombus A parallelogram having four congruent sides.



المعين هو متوازي أضلاع مكون من أربعة أضلاع متطابقة.

right angle An angle that measures exactly 90°.

الزاوية القائمة هي زاوية قياسها 90 درجة بالضبط.

right triangle A triangle having one right angle.

المثلث القائم الزاوية هو مثلث إحدى زواياه قائمة.

Ss

sample A randomly selected group chosen for the purpose of collecting data.

العينة هي مجموعة مختارة بشكل عشوائي بهدف تجميع البيانات.

scale The set of all possible values of a given measurement, including the least and greatest numbers in the set, separated by the intervals used.

المقياس هو مجموعة من جميع القيم الممكنة لقياس معين، بما في ذلك أصغر وأكبر عدد في المجموعة، وتفصل بالفترات المستخدمة.

المقياس يعطي المقياس نسبة تعمل على مقارنة مقاييس رسم

scale The scale gives the ratio that compares the measurements of a drawing or model to the measurements of the real object.

أو نموذج لقياس الشيء الحقيقي.

scale drawing A drawing that is used to represent objects that are too large or too small to be drawn at actual size.

الرسم المقياسي هو الرسم المستخدم لتمثيل الأشياء الكبيرة للغاية أو الصغيرة للغاية التي لا يمكن رسمها بالحجم الفعلي.

scalene triangle A triangle having no congruent sides.

المثلث مختلف الأضلاع هو مثلث أضلاعه غير متطابقة.

scaling To multiply or divide two related quantities by the same number.

التدريج هو ضرب أو قسمة كميتين مرتبطتين بنفس العدد.

sequence A list of numbers in a specific order, such as 0, 1, 2, 3, or 2, 4, 6, 8.

التسلسل هو قائمة بالأرقام بترتيب معين، مثل 0, 1, 2, 3 أو 2, 4, 6, 8.

similar figures Figures that have the same shape but not necessarily the same size.

الأشكال المتشابهة هي أشكال لها نفس الشكل ولكن ليست بالقياس ذاته بالضرورة.

slant height The height of each lateral face.

الارتفاع الجانبي هو ارتفاع كل وجه جانبي.

solution The value of a variable that makes an equation true. The solution of $12 = x + 7$ is 5.

الحل هو قيمة متغير تنتج معادلة صحيحة. الحل الخاص بـ $12 = x + 7$ هو 5.

solve To replace a variable with a value that results in a true sentence.

الحل هو استبدال متغير بقيمة تؤدي إلى عبارة صحيحة.

square A rectangle having four right angles and four congruent sides.

المربع هو مستطيل له أربع زوايا قائمة وأربعة أضلاع متطابقة.

square root The factors multiplied to form perfect squares.

الجذر التربيعي هو عوامل مضاعفة لتشكيل مربعات كاملة.

statistical question A question that anticipates and accounts for a variety of answers.

السؤال الإحصائي هو سؤال يتوقع ويمثل مجموعة متنوعة من الإجابات.

statistics Collecting, organizing, and interpreting data.

الإحصائيات هي جمع البيانات وتنظيمها وتفسيرها.

stem-and-leaf plot A system where data are organized from least to greatest. The digits of the least place value usually form the leaves, and the next place-value digits form the stems.

Stem	Leaf
1	2 4 5
2	
3	1 2 3 3 9
4	0 4 6 7

4 | 7 = 47

مخطط الساق والأوراق هو نظام يتم فيه تنظيم البيانات من الأصغر إلى الأكبر. تشكل الأرقام ذات القيمة المكانية الأقل عادة الأوراق، وتشكل الأرقام ذات القيمة المكانية الأعلى الساق.

الورقة	الساق
2 4 5	1
	2
1 2 3 3 9	3
0 4 6 7	4

4 | 7 = 47

stems The digits of the greatest place value of data in a stem-and-leaf plot.

السيقان هي الأرقام الأ على قيمة مكانية للبيانات في مخطط الساق والأوراق.

straight angle An angle that measures exactly 180°.

الزاوية المستقيمة هي زاوية قياسها 180 درجة بالضبط.

Subtraction Property of Equality If you subtract the same number from each side of an equation, the two sides remain equal.

خاصية الطرح في المعادلة إذا طرحت العدد نفسه من كلا طرفي المعادلة، يظل الطرفان متساويين.

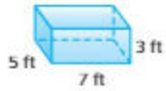
supplementary angles Two angles are supplementary if the sum of their measures is 180°.

∠1 and ∠2 are supplementary angles.

زاويتان متكاملتان تكون الزاويتان متكاملتين إذا كان مجموع قياسهما يساوي 180 درجة.

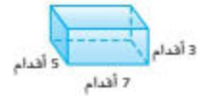
∠1 و∠2 هما زاويتان متكاملتان

surface area The sum of the areas of all the surfaces (faces) of a three-dimensional figure.
$S.A. = 2\ell h + 2\ell w + 2hw$



$S.A. = 2(7 \times 3) + 2(7 \times 5) + 2(3 \times 5)$
$= 142$ square feet

مساحة السطح هي مجموع مساحات أسطح (أوجه) الشكل ثلاثي الأبعاد.
$S.A. = 2\ell h + 2\ell w + 2hw$



$S.A. = 2(7 \times 3) + 2(\quad \times 5) + 2(3 \times 5)$
قدمًا مربعًا 142 =

survey A question or set of questions designed to collect data about a specific group of people, or population.

الدراسة الاستقصائية هي عبارة عن سؤال أو مجموعة أسئلة تهدف إلى جمع البيانات حول مجموعة معينة من البشر أو مجتمع إحصائي.

symmetric distribution Data that are evenly distributed.

التوزيع المتماثل هو بيانات يتم توزيعها بالتساوي.

Tt

term Each number in a sequence.

الحد هو كل عدد في تسلسل.

term Each part of an algebraic expression separated by a plus or minus sign.

الحد هو كل جزء من التعبير الجبري الذي تفصله علامة الجمع أو الطرح.

terminating decimal A decimal is called terminating if its repeating digit is 0.

الكسر العشري المنتهي يُطلق على الكسر العشري أنه منتهٍ إذا كان الرقم المتكرر هو 0.

third quartile For a data set with median M, the third quartile is the median of the data values greater than M.

الرُبيع الثالث بالنسبة لمجموعة البيانات ذات الوسيط M، يكون الربيع الثالث هو وسيط قيم البيانات التي تكون أكبر من قيمة M.

three-dimensional figure A figure with length, width, and height.

الشكل الثلاثي الأبعاد هو شكل له طول وعرض وارتفاع.

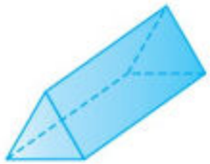
trapezoid A quadrilateral with one pair of parallel sides.

شبه المنحرف هو شكل رباعي الأضلاع مكون من زوج من الأضلاع المتوازية.

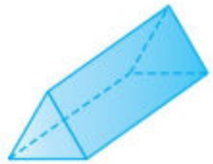
triangle A figure with three sides and three angles.

المثلث هو شكل له ثلاثة أضلاع وثلاث زوايا.

triangular prism A prism that has triangular bases.



المنشور الثلاثي هو منشور يحتوي على قواعد مثلثة الشكل.



Uu

unit price The cost per unit.

سعر الوحدة هو تكلفة كل وحدة.

unit rate A rate that is simplified so that it has a denominator of 1.

معدل الوحدة هو معدل تم تبسيطه بحيث تصبح قيمة المقام 1.

unit ratio A unit rate where the denominator is one unit.

نسبة الوحدة هي معدل الوحدة بحيث يكون المقام وحدة واحدة.

Vv

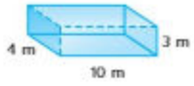
variable A symbol, usually a letter, used to represent a number.

المتغير هو رمز وعادةً ما يكون حرفًا، ويُستخدم في تمثيل عدد ما.

vertex The point where three or more faces intersect.

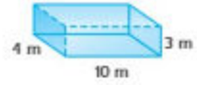
الرأس هو نقطة تتقاطع فيها ثلاثة وجوه فأكثر.

volume The amount of space inside a three-dimensional figure. Volume is measured in cubic units.



$V = 10 \times 4 \times 3 = 120$ cubic meters

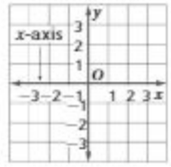
الحجم هو مقدار الفراغ داخل شكل ثلاثي الأبعاد. ويتم قياس الحجم بوحدات مكعبة.



الحجم = $10 \times 4 \times 3 = 120$ مترًا مكعبًا

Xx

x-axis The horizontal line of the two perpendicular number lines in a coordinate plane.



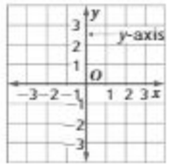
المحور الأفقي x هو الخط المستقيم الأفقي في خطي الأعداد المتعامدين داخل المستوى الإحداثي.

x-coordinate The first number of an ordered pair. The x-coordinate corresponds to a number on the x-axis.

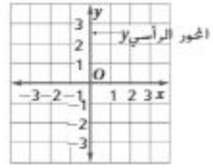
الإحداثي x هو العدد الأول في الزوج المرتب. ويطابق الإحداثي x العدد الموجود في المحور الأفقي x.

Yy

y-axis The vertical line of the two perpendicular number lines in a coordinate plane.



المحور الرأسي y هو الخط المستقيم الرأسي في خطي الأعداد المتعامدين داخل المستوى الإحداثي.



y-coordinate The second number of an ordered pair. The y-coordinate corresponds to a number on the y-axis.

الإحداثي y هو العدد الثاني في الزوج المرتب. ويطابق الإحداثي y العدد الموجود في المحور الرأسي y.