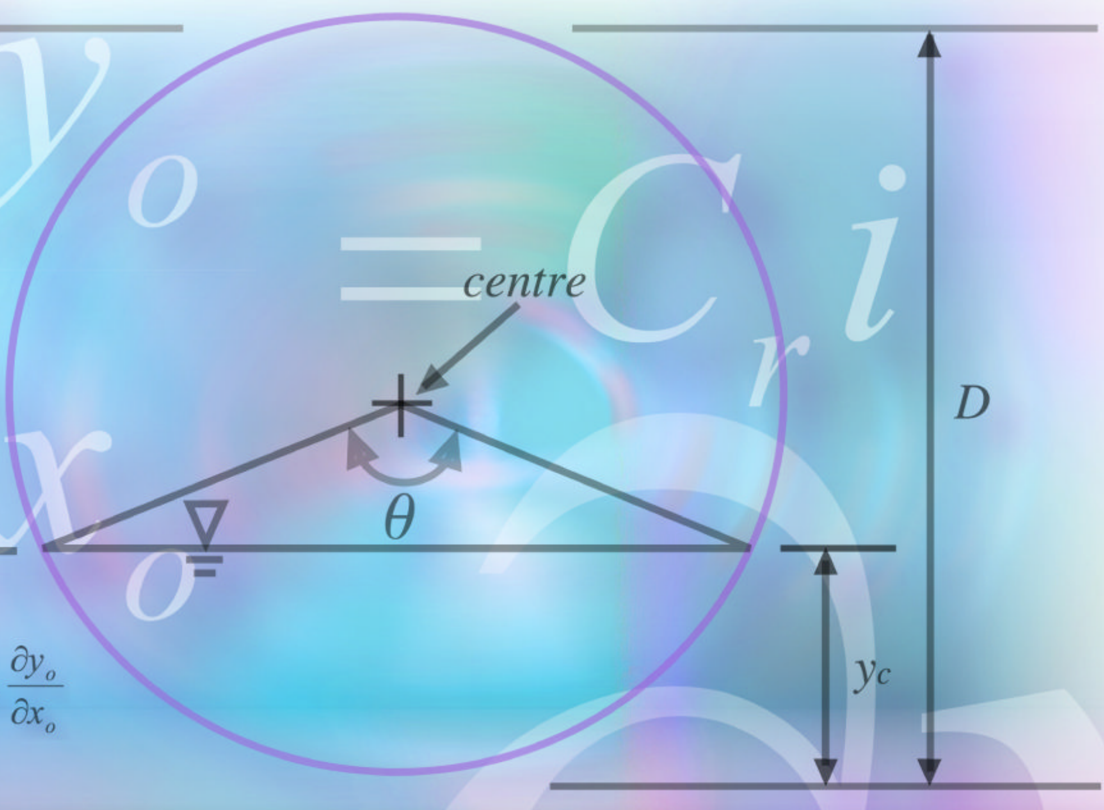
Hydrological Science and Engineering Series

Tommy S. W. Wong

Kinematic-Wave Rainfall-Runoff Formulas



$$\frac{dy_o}{dt} = \frac{\partial y_o}{\partial t} + c_k \frac{\partial y_o}{\partial x_o}$$

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KINEMATIC-WAVE RAINFALL-RUNOFF FORMULAS

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KINEMATIC-WAVE RAINFALL-RUNOFF FORMULAS

TOMMY S.W. WONG

Nova Science Publishers, Inc.
New York

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LIBRARY OF CONGRESS CATALOGING-IN-PUBLICATION DATA

Wong, Tommy S. W.
Kinematic-wave rainfall-runoff formulas / Tommy S.W. Wong.
p. cm.
Includes index.
ISBN 978-1-61470-182-8 (eBook)
1. Runoff--Mathematical models 2. Rain and rainfall--Mathematical models. 3. Hydrologic models. I. Title.
GB980.W66 2009
627'.042--dc22
2008051332

Published by Nova Science Publishers, Inc. ✢ New York

DEDICATED TO

my parents, Sze Fong Wong and En Yueh Woo

my parents-in-law, Chip Shing Sum and Luk Ying Ko

my darling wife, Christina

and my wonderful sons, Alston, Lester and Hanson

CONTENTS

PREFACE

This is not an ordinary book on rainfall and runoff. All the general and working formulas in this book are theoretically derived. The formulas are therefore globally and eternally applicable, as long as the situations under consideration are within the assumptions and limitations of the theory. This epitomizes the powerful nature of the physically-based approach in hydrology. This book covers formulas for flow depth; flow velocity; average flow velocity; wave celerity; average wave celerity; time of concentration; rising, equilibrium and falling phases of a hydrograph; forward characteristic; rising, equilibrium and falling phases of water surface profiles; duration of partial equilibrium discharge; and equilibrium detention storage for flow on an overland plane, and flow in nine different channel shapes, which are (i) circular, (ii) parabolic, (iii) rectangular (deep), (iv) rectangular (square), (v) rectangular (wide), (vi) trapezoidal with equal side slopes, (vii) trapezoidal with one side vertical, (viii) triangular, and (ix) vertical curb.

ACKNOWLEDGEMENTS

I would like to thank Bhagavan Sri Sathya Sai Baba for selecting me as his instrument in authoring this book for the benefit of mankind. I would also like to thank Emeritus Professor Charng Ning Chen for introducing the subject to me. Last but not least, I would like to thank the following who have contributed to this book in one way or another: Syed-Alwi-Bin-Sheikh-Bin-Hussien Alkaff, Jason Christopher Chan, Lee Ching Chung, Joel Ore Iwanaga, Ahmed Nusrath Bilal Izzath, Xiao Chi Koh, Heng Yein Kong, Yunjie Li, Jessie Su Hui Lim, Teik Peng Lim, Xiaoxie Liu, Teng Tsair Ma, Chia Li Ow, and Maichun Zhou.

List of Symbols

A flow area (m^2)
A_c flow area in channel (m^2)
A_c' parameter relating A_c to H for parabolic channel
A_e flow area in channel corresponding to equilibrium discharge (m^2)
A_{full} flow area in circular channel under full flow condition (m^2)
A_i flow area in channel at inflection point (m^2)
A_o area of overland plane (ha)
A_p flow area in channel corresponding to partial equilibrium discharge (m^2)
A_{Qmax} flow area in circular channel under maximum flow condition (m^2)
A_u flow area in channel corresponding to upstream discharge (m^2)
a parameter relating i to t_r
b parameter relating i to t_r
C_r runoff coefficient
c parameter relating i to t_r
c_{av} average kinematic wave celerity ($m \cdot s^{-1}$)
c_k kinematic wave celerity ($m \cdot s^{-1}$)
D diameter of circular channel (m)
D_{ec} equilibrium detention storage for a channel of length L_c (m^3)
D_{eo} equilibrium detention storage for an overland plane of length L_o ($m^3 \cdot m^{-1}$)
D_{eu} equilibrium detention storage for a plane or a channel of length L_u ($m^3 \cdot m^{-1}$ or m^3)
D_{euc} equilibrium detention storage for a channel of length ($L_u + L_c$) (m^3)
D_{euo} equilibrium detention storage for a plane of length ($L_u + L_o$) ($m^3 \cdot m^{-1}$)
F_e Froude number at the end of the plane at equilibrium
g acceleration due to gravity ($m \cdot s^{-2}$)
H height of focal point above parabolic channel invert (m)
i rainfall intensity ($mm \cdot h^{-1}$)
i_d design rainfall intensity for overland plane ($mm \cdot h^{-1}$)
K kinematic flow number
L_a arc length of parabola (m)
L_c length of channel (m)
L_d length of overland plane or channel contributing to duration of partial equilibrium discharge (m)

L_f length of overland plane or channel in which the flow equals to upstream inflow during falling phase (m)
L_o length of overland plane (m)
L_p length of overland plane or channel contributing to partial equilibrium discharge (m)
L_u length of upstream plane or channel (m)
n Manning's roughness coefficient ($s \cdot m^{-1/3}$)
n_c Manning's roughness coefficient for channel surface ($s \cdot m^{-1/3}$)
n_o Manning's roughness coefficient for overland surface ($s \cdot m^{-1/3}$)
P wetted perimeter (m)
P' parameter relating P to H for parabolic channel
Q discharge ($m^3 \cdot s^{-1}$)
Q_c discharge in channel ($m^3 \cdot s^{-1}$)
Q_d design discharge of overland plane ($m^3 \cdot s^{-1}$)
Q_e discharge at the end of channel at equilibrium ($m^3 \cdot s^{-1}$)
Q_{full} discharge in circular channel under full flow condition ($m^3 \cdot s^{-1}$)
Q_{max} discharge in circular channel under maximum flow condition ($m^3 \cdot s^{-1}$)
Q_o discharge on overland plane ($m^3 \cdot s^{-1}$)
Q_p discharge at the end of channel at partial equilibrium ($m^3 \cdot s^{-1}$)
Q_u upstream inflow to channel ($m^3 \cdot s^{-1}$)
q discharge per unit width of overland plane ($m^2 \cdot s^{-1}$)
q_e unit discharge at the end of overland plane at equilibrium ($m^2 \cdot s^{-1}$)
q_L lateral inflow per unit length of channel ($m^2 \cdot s^{-1}$)
q_p unit discharge at the end of overland plane at partial equilibrium ($m^2 \cdot s^{-1}$)
q_u unit upstream inflow to overland plane ($m^2 \cdot s^{-1}$)
R hydraulic radius (m)
S bed slope ($m \cdot m^{-1}$)
S_c slope of channel bed ($m \cdot m^{-1}$)
S_f friction slope ($m \cdot m^{-1}$)
S_o slope of overland plane ($m \cdot m^{-1}$)
T top width (m)
T_w wave period (min)
t time (min)
t_d duration of partial equilibrium discharge (min)
t_o time of concentration of overland flow (min)
t_q duration of lateral inflow (min)
t_r duration of rainfall (min)
t_t time of travel in channel (min)
t_u time of travel in upstream plane or channel (min)
v flow velocity ($m \cdot s^{-1}$)
v_{av} average flow velocity ($m \cdot s^{-1}$)
v_s steady-state, uniform, mean flow velocity in channel ($m \cdot s^{-1}$)
W base width of rectangular or trapezoidal channel (m)
w width of overland plane (m)
x semi-width of parabolic channel at height y (m)
x' parameter relating x to H for parabolic channel
x_c distance along a channel in the direction of flow (m)

x_i distance x_o or x_c of the inflection point (m)
x_o distance along an overland plane in the direction of flow (m)
y height above parabolic channel invert (m)
y_c flow depth in channel (m)
y_e flow depth at the end of overland plane at equilibrium (m)
y_i flow depth at inflection point (m)
y_o flow depth on overland plane (m)
y_p flow depth at the end of overland plane at partial equilibrium (m)
y_{Qmax} flow depth in circular channel under maximum flow condition (m)
y_s steady-state, uniform, flow depth in channel (m)
y_u flow depth on overland plane corresponding to upstream inflow (m)
Z parameter relating x to H for parabolic channel (m)
Z' parameter relating Z to H for parabolic channel
z reciprocal of channel side slope of trapezoidal, triangular, or vertical curb channel ($m \cdot m^{-1}$)
α_c parameter relating Q_c to A_c for open channel
α_o parameter relating q to y_o for overland plane
β_c parameter relating Q_c to A_c for open channel
β_o parameter relating q to y_o for overland plane
γ parameter relating A_{Qmax} to D for circular channel
μ parameter relating y_c to W
θ water surface angle for circular channel (rad)
τ dimensionless wave period
ψ parameter relating A_c to W

In: Kinematic-Wave Rainfall-Runoff Formulas ISBN: 978-1-60692-705-2
Tommy S.W. Wong, pp. 1-2 © 2009 Nova Science Publishers, Inc.

Chapter 1

1. INTRODUCTION

Ever since Lighthill and Whitham (1955) showed that the main body of a natural flood wave moves as the kinematic wave, there has been continual interest in the application of the kinematic wave theory to hydrologic engineering. The greatest strength in this application is the feasibility of obtaining physically-based analytical formulas. The values of this strength are two-fold:

1. It enables hydrologists and engineers to have a clear understanding of the contribution by each parameter in the physical process.
2. Without the need for any experimental data, it offers formulas that can be applied to practical situations, including ungauged catchments.

Further, these formulas have great advantages:

1. As the formulas are theoretically derived, the assumptions and limitations involved in the formulas can be clearly stated.
2. As the formulas are general in nature, they are globally and eternally applicable, as long as the situation under consideration is within the assumptions and limitations of the theory.
3. As the formulas are analytical, they can be used without the need for computer programming. Since the formulas are not hidden in some computer program, the steps leading to each answer can easily be traced.

To enable hydrologists and engineers to have ready access to the kinematic wave formulas, the objectives of this book are:

1. To show the derivation of the kinematic wave formulas for the rainfall-runoff process, and to highlight the assumptions and limitations in the derivations.
2. To present the kinematic wave formulas in a form that can be readily used by practitioners.

1.1. HOW TO USE THIS BOOK

The Chapters in this book are more or less self-contained; hence, they can be read fairly independently. The topics covered may be grouped under four phases of the rainfall-runoff process for an overland plane subject to uniform rainfall excess and with a constant upstream inflow, and for a channel subject to uniform lateral inflow and with a constant upstream inflow. For a catchment comprising a network of overland planes and channels, the outflow from the overland planes can become the lateral inflow to the channels. The four phases of the rainfall-runoff process are:

1. General phase covering (i) flow depth, (ii) flow velocity, (iii) average flow velocity, (iv) wave celerity, and (v) average wave celerity.
2. Rising phase covering (i) time of concentration or time of travel, (ii) rising phase of hydrograph, (iii) forward characteristic, and (iv) rising phase of water surface or flow area profile.
3. Equilibrium phase covering (i) design discharge of an overland plane, (ii) duration of partial equilibrium discharge, (iii) equilibrium phase of hydrograph, (iv) equilibrium phase of water surface or flow area profile, and (v) equilibrium detention storage.
4. Falling phase covering (i) falling phase of hydrograph, and (ii) falling phase of water surface or flow area profile.

This book may be read in the following ways:

1. Readers who are interested in the assumptions and background of the formulas may refer to Chapter 2 for flow on an overland plane, and Chapter 4 for flow in an open channel.
2. Readers who are interested in the working formulas may refer to Chapter 3 for flow on an overland plane, and Chapters 5-13 for flow in nine different channel shapes, which are (i) circular, (ii) parabolic, (iii) rectangular (deep), (iv) rectangular (square), (v) rectangular (wide), (vi) trapezoidal with equal side slopes, (vii) trapezoidal with one side vertical, (viii) triangular, and (ix) vertical curb.
3. Readers who are interested in the assumptions and background of the formulas and the working formulas may refer to all the Chapters.

For ease of reference, the applicability of the kinematic wave theory is summarized in Appendix A, the general formulas in Appendices B-C, the kinematic wave parameters in Appendix D, and the working formulas in Appendices E-S. The units for the working formulas are contained in the List of Symbols. Finally, the values for the runoff coefficient may be selected from the American Society of Civil Engineers (1992), the values for the Manning's roughness coefficient for overland surface may be selected from Engman (1986), and the values for the Manning's roughness coefficient for channel surface may be selected from Chow (1959) or Arcement and Schneider (1989).

In: Kinematic-Wave Rainfall-Runoff Formulas
Editor: Tommy S.W. Wong, pp. 3-28

ISBN: 978-1-60692-705-2
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Chapter 2

2. GENERAL FORMULAS FOR FLOW ON OVERLAND PLANE

In this Chapter, based on the kinematic wave theory; the general formulas for flow depth; flow velocity; average flow velocity; wave celerity; average wave celerity; time of concentration; design discharge; rising, equilibrium and falling phases of a hydrograph; forward characteristic; rising, equilibrium and falling phases of water surface profiles; duration of partial equilibrium discharge; and equilibrium detention storage for flow on an overland plane are derived.

2.1. FLOW CONDITIONS

Consider an overland plane of length L_o, subject to a uniform rainfall intensity i, and with a constant upstream inflow q_u, the unit discharge, q, along the equilibrium water surface profile for a unit width of the plane is:

$$q = q_u + C_r i x_o \tag{2.1}$$

where C_r = runoff coefficient, x_o = distance along the plane in the direction of flow. Substituting $x_o = L_o$ into Eq. (2.1) gives the discharge q_e at the end of the plane at equilibrium, i.e.

$$q_e = q_u + C_r i L_o \tag{2.2}$$

Further, the upstream inflow, q_u, can be considered to be contributed by an imaginary overland plane of length L_u, which is situated immediately upstream of the overland plane of length L_o. This imaginary upstream plane is also subject to a uniform rainfall intensity i, but with zero upstream inflow. At the outlet point of this upstream plane and at equilibrium, the discharge is q_u. Substituting $q_e = q_u$, $q_u = 0$ and $L_o = L_u$ into Eq. (2.2) gives the length of the upstream plane, L_u, in terms of the upstream inflow, q_u, as follows:

$$L_u = \frac{q_u}{C_r i} \tag{2.3}$$

Figure 2.1 shows the upstream plane, the overland plane, and the equilibrium water surface profile.

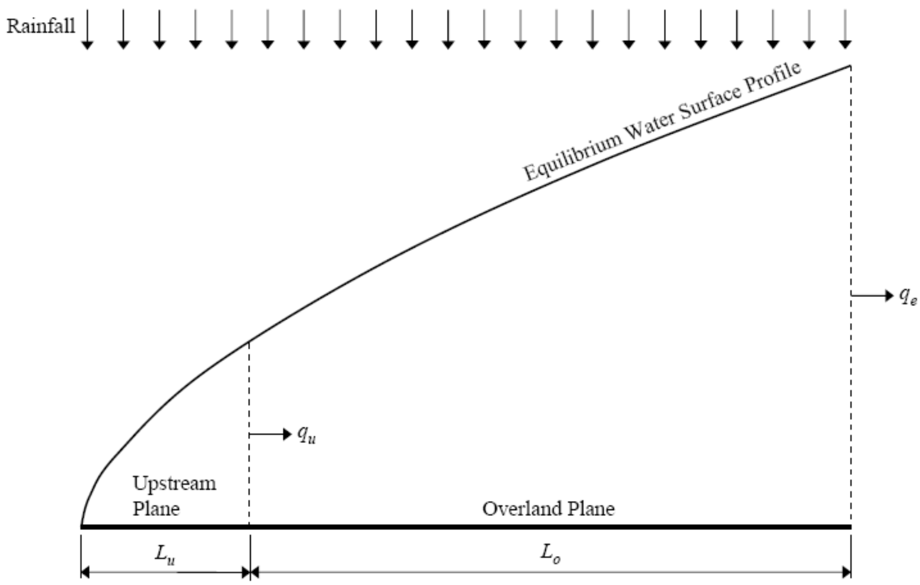

Figure 2.1. Upstream and Overland Planes with Equilibrium Water Surface Profile.

2.2. Dynamic Wave Equations

The mechanics of unsteady flow on an overland plane with a rainfall contribution can be expressed mathematically by the Saint Venant equation. Derived from the principles of continuity and momentum, the equations are (Chow et al 1988):

$$\frac{\partial y_o}{\partial t} + \frac{\partial q}{\partial x_o} = C_r i \tag{2.4}$$

$$\frac{1}{g}\frac{\partial v}{\partial t} + \frac{v}{g}\frac{\partial v}{\partial x_o} + \frac{\partial y_o}{\partial x_o} - \left(S_o - S_f\right) = 0 \tag{2.5}$$

where y_o = overland flow depth, t = time, g = acceleration due to gravity, v = flow velocity, S_o = overland slope, and S_f = friction slope. The assumptions inherent in Eqs. (2.4) and (2.5) are:

1. The flow is one dimensional (i.e. velocity varies in the longitudinal direction only). This implies that the velocity is constant and the water surface is horizontal across any section perpendicular to the longitudinal axis.
2. All flows are gradually varied with hydrostatic pressure prevailing at all points in the flow such that all vertical acceleration within the water column can be neglected.
3. The longitudinal axis of the overland plane can be approximated by a straight line (i.e. there is no secondary circulation).
4. The slope of the overland plane is small.
5. The overland plane is fixed (i.e. the effects of scour and deposition are negligible).
6. Resistance to flow can be described by empirical resistance formulas, such as Manning's equation.
7. The fluid is incompressible and homogeneous in density.
8. The momentum carried to the fluid from the rainfall is negligible.

The momentum equation (Eq. 2.5) consists of five terms; namely local acceleration, convective acceleration, pressure force, gravity force and friction force; each representing a physical process that governs the flow momentum described as follows:

1. The acceleration terms represent the effect of velocity change over time and space.
2. The pressure force term represents the effect of flow depth change.
3. The gravity force term S_o is proportional to the overland slope and accounts for the change in bed level.
4. The friction force term S_f is proportional to the friction slope and accounts for the friction loss for the flow on an overland plane.

2.3. KINEMATIC WAVE EQUATIONS

If the backwater effect is negligible and there is no rapid change in flow, the acceleration and pressure terms in Eq. (2.5) may be neglected (Stephenson 1981, Wong 1992), and the momentum equation reduces to:

$$S_o = S_f \tag{2.6}$$

Equations (2.4) and (2.6) are called the "kinematic wave equations". Equation (2.6) shows that the overland slope is parallel to the friction slope, which means that the kinematic wave is under the uniform flow condition. Thus, Eq. (2.6) can be replaced by the general uniform flow equation, which is:

$$q = \alpha_o y_o^{\beta_o} \tag{2.7}$$

where α_o and β_o = kinematic wave parameters relating q to y_o.

2.4. Flow Depth

Rearranging Eq. (2.7) gives the equation for the flow depth for a plane with and without upstream inflow:

$$y_o = \left(\frac{q}{\alpha_o}\right)^{1/\beta_o} \tag{2.8}$$

2.5. Flow Velocity

From continuity, the flow velocity, v, is related to the unit discharge, q, as follows:

$$v = \frac{q}{y_o} \tag{2.9}$$

Substituting Eq. (2.7) into Eq. (2.9) and the velocity, v, becomes (Wong 2003):

$$v = \alpha_o y_o^{\beta_o - 1} \tag{2.10}$$

Substituting Eq. (2.8) into Eq. (2.10) gives the equation for the velocity, v, in terms of unit discharge, q (Wong 2003):

$$v = \left(\alpha_o q^{\beta_o - 1}\right)^{1/\beta_o} \tag{2.11}$$

Substituting Eq. (2.1) into Eq. (2.11) gives the equation for the flow velocity along the equilibrium profile for a plane with upstream inflow:

$$v = \left[\alpha_o \left(q_u + C_r i x_o\right)^{\beta_o - 1}\right]^{1/\beta_o} \tag{2.12}$$

For a plane with zero upstream inflow (q_u= 0), Eq. (2.12) reduces to:

$$v = \left[\alpha_o \left(C_r i x_o\right)^{\beta_o - 1}\right]^{1/\beta_o} \tag{2.13}$$

2.6. Average Flow Velocity

Further, the average flow velocity, v_{av}, over the length of the plane, L_o, can be derived as follows (Wong 2003):

$$v_{av} = \frac{L_o}{\int_0^{L_o} \frac{1}{v} dx_o} \quad (2.14)$$

Substituting Eq. (2.12) into Eq. (2.14) and integrating (1/*v*) gives the equation for the average flow velocity for a plane with upstream inflow:

$$v_{av} = \frac{\alpha_o^{1/\beta_o} C_r i L_o}{\beta_o \left[(q_u + C_r i L_o)^{1/\beta_o} - q_u^{1/\beta_o} \right]} \quad (2.15)$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (2.15) reduces to:

$$v_{av} = \frac{1}{\beta_o} \left[\alpha_o (C_r i L_o)^{\beta_o - 1} \right]^{1/\beta_o} \quad (2.16)$$

2.7. KINEMATIC WAVE CELERITY

Differentiating Eq. (2.7) with respect to *t* gives:

$$\frac{\partial q}{\partial t} = \alpha_o \beta_o y_o^{\beta_o - 1} \left(\frac{\partial y_o}{\partial t} \right) \quad (2.17)$$

Rearranging Eq. (2.17) gives:

$$\frac{\partial y_o}{\partial t} = \frac{1}{\alpha_o \beta_o y_o^{\beta_o - 1}} \left(\frac{\partial q}{\partial t} \right) \quad (2.18)$$

Substituting Eq. (2.18) into Eq. (2.4), the continuity equation becomes:

$$\left[\frac{1}{\alpha_o \beta_o y_o^{\beta_o - 1}} \left(\frac{\partial q}{\partial t} \right) \right] + \left(\frac{\partial q}{\partial x_o} \right) = C_r i \quad (2.19)$$

Kinematic wave results in changes in *q,* which is dependent on both x_o and *t*, and the increment in flow rate *dq* can be written as:

$$dq = \frac{\partial q}{\partial t} dt + \frac{\partial q}{\partial x_o} dx_o \quad (2.20)$$

Dividing Eq. (2.20) by dx_o:

$$\frac{dq}{dx_o} = \left(\frac{\partial q}{\partial x_o}\right) + \left[\frac{\partial q}{\partial t}\left(\frac{dt}{dx_o}\right)\right] \tag{2.21}$$

If

$$C_r i = \frac{dq}{dx_o} \tag{2.22}$$

and

$$\frac{dt}{dx_o} = \frac{1}{\alpha_o \beta_o y_o^{\beta_o - 1}} \tag{2.23}$$

then Eq. (2.19) and Eq. (2.21) are identical. Differentiating Eq. (2.7) with respect to y_o:

$$\frac{dq}{dy_o} = \alpha_o \beta_o y_o^{\beta_o - 1} \tag{2.24}$$

Comparing Eq. (2.23) and Eq. (2.24) gives:

$$\frac{dq}{dy_o} = \frac{dx_o}{dt} \tag{2.25}$$

Since kinematic wave celerity, c_k, is:

$$c_k = \frac{dx_o}{dt} \tag{2.26}$$

Substituting Eq. (2.25) into Eq. (2.26) gives:

$$c_k = \frac{dx_o}{dt} = \frac{dq}{dy_o} = \alpha_o \beta_o y_o^{\beta_o - 1} \tag{2.27}$$

Substituting Eq. (2.8) into Eq. (2.27) gives:

$$c_k = \beta_o \left(\alpha_o q^{\beta_o - 1}\right)^{1/\beta_o} \tag{2.28}$$

Substituting Eq. (2.1) into Eq. (2.28) gives the equation for the wave celerity along the equilibrium profile for a plane with upstream inflow:

$$c_k = \beta_o\left[\alpha_o\left(q_u + C_r i x_o\right)^{\beta_o - 1}\right]^{1/\beta_o} \tag{2.29}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (2.29) reduces to:

$$c_k = \beta_o\left[\alpha_o\left(C_r i x_o\right)^{\beta_o - 1}\right]^{1/\beta_o} \tag{2.30}$$

2.8. AVERAGE WAVE CELERITY

The average wave celerity, c_{av}, over the length of the plane, L_o, can be derived as follows (Wong 1996):

$$c_{av} = \frac{L_o}{\int_0^{L_o} \frac{1}{c_k} dx_o} \tag{2.31}$$

Substituting Eq. (2.29) into Eq. (2.31) and integrating ($1/c_k$) gives the equation for the average wave celerity for a plane with upstream inflow:

$$c_{av} = \frac{\alpha_o^{1/\beta_o} C_r i L_o}{\left(q_u + C_r i L_o\right)^{1/\beta_o} - q_u^{1/\beta_o}} \tag{2.32}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (2.32) reduces to:

$$c_{av} = \left[\alpha_o\left(C_r i L_o\right)^{\beta_o - 1}\right]^{1/\beta_o} \tag{2.33}$$

2.9. TIME OF CONCENTRATION

The time of concentration for flow on an overland plane, t_o, can be obtained by dividing length of the plane, L_o, by the average wave celerity, c_{av}, as follows:

$$t_o = \frac{L_o}{c_{av}} \tag{2.34}$$

Substituting Eq. (2.32) into Eq. (2.34) gives the equation for the time of concentration for a plane with upstream inflow (Wong 1995):

$$t_o = \frac{1}{\alpha_o^{1/\beta_o}}\left[\frac{(q_u + C_r i L_o)^{1/\beta_o} - q_u^{1/\beta_o}}{C_r i}\right] \quad (2.35)$$

For a plane with zero upstream inflow (q_u = 0), Eq. (2.35) reduces to (Henderson and Wooding 1964, Wong 1995):

$$t_o = \left[\frac{L_o}{\alpha_o (C_r i)^{\beta_o - 1}}\right]^{1/\beta_o} \quad (2.36)$$

Further, for the upstream plane of length L_u, substituting $L_o = L_u$ into Eq. (2.36) gives the time of concentration, t_u, of the upstream plane:

$$t_u = \left[\frac{L_u}{\alpha_o (C_r i)^{\beta_o - 1}}\right]^{1/\beta_o} \quad (2.37)$$

2.10. DESIGN DISCHARGE

For estimating the design discharge of a desired recurrence interval, the rainfall intensity-duration curve of the same recurrence interval is used. For a given rainfall recurrence interval, the design concept is to choose a storm from the rainfall intensity-duration curve such that it produces the maximum peak discharge. This maximum discharge is the design discharge (Wong 2005a).

2.10.1. Rainfall Intensity-Duration Relationship

Analyses of the total rainfall curves show that for a given recurrence interval, the rainfall intensity varies inversely with the rainfall duration, and it can be mathematically described by (American Society of Civil Engineers 1992):

$$i = a/(c + t_r)^b \quad (2.38)$$

where t_r = rainfall duration, and a, b and c = constants. To facilitate the derivation of an explicit expression for the design discharge, Eq. (2.38) is reduced to (Wong 1995):

$$i = a t_r^{-b} \quad (2.39)$$

Although the use of Eq. (2.39) with a single set of a and b values cannot fit the entire rainfall intensity-duration curve, Chen and Evans (1977), and Wong (1992) showed that by dividing the rainfall curve into segments, it is possible to fit the entire rainfall curve with different values of a and b for each segment.

2.10.2. Design Discharge

For the purpose of estimating the design discharge, Wong (2005a) showed that the critical rainfall duration is the time of concentration. Eq. (2.36) and Eq. (2.39) are therefore solved simultaneously by equating $t_o = t_r$, resulting in an explicit expression for the design rainfall intensity, i_d, for a plane with zero upstream inflow:

$$i_d = \left[\frac{a^{1/b} C_r^{\frac{\beta_o - 1}{\beta_o}}}{\left(L_o / \alpha_o\right)^{1/\beta_o}} \right]^{\frac{b\beta_o}{b+\beta_o-b\beta_o}} \tag{2.40}$$

Figure 2.2 shows a graphical solution for obtaining i_d. The design discharge, Q_d, is related to the design rainfall intensity, i_d, and the area of the plane, A_o, as follows:

$$Q_d = C_r i_d A_o \tag{2.41}$$

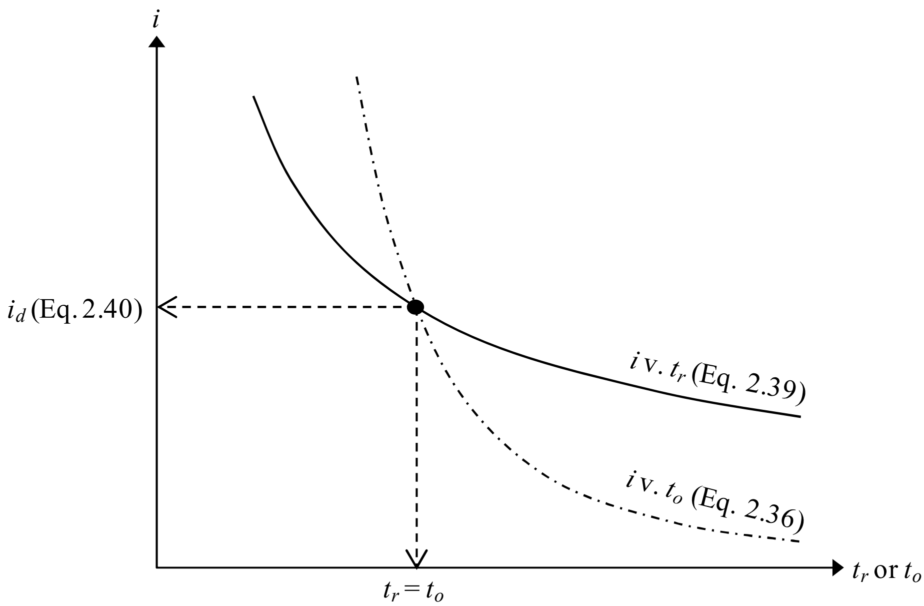

Figure 2.2. Design Rainfall Intensity for a Plane without Upstream Inflow.

For a rectangular plane, the area A_o, is related to the dimensions of the plane as:

$$A_o = L_o w \tag{2.42}$$

where w = width of the plane. Substituting Eq. (2.40) into Eq. (2.41) gives the equation for the peak discharge per unit area of the plane:

$$Q_d / A_o = \left[\frac{(aC_r)^{1/b}}{(L_o/\alpha_o)^{1/\beta_o}} \right]^{\frac{b\beta_o}{b+\beta_o-b\beta_o}} \tag{2.43}$$

2.11. HYDROGRAPH - RISING PHASE

Expanding the partial derivative, $(\partial q/\partial x_o)$, into total derivative results in:

$$\frac{\partial q}{\partial x_o} = \frac{dq}{dy_o}\left(\frac{\partial y_o}{\partial x_o}\right) \tag{2.44}$$

Substituting Eq. (2.27) into Eq. (2.44) gives:

$$\frac{\partial q}{\partial x_o} = c_k \frac{\partial y_o}{\partial x_o} \tag{2.45}$$

Substituting Eq. (2.45) into Eq. (2.4) gives:

$$\frac{\partial y_o}{\partial t} + c_k \frac{\partial y_o}{\partial x_o} = C_r i \tag{2.46}$$

Differentiating y_o with respect to x_o and t:

$$dy_o = \frac{\partial y_o}{\partial t} dt + \frac{\partial y_o}{\partial x_o} dx_o \tag{2.47}$$

Dividing Eq. (2.47) by dt and substituting Eq. (2.27) into it:

$$\frac{dy_o}{dt} = \frac{\partial y_o}{\partial t} + c_k \frac{\partial y_o}{\partial x_o} \tag{2.48}$$

Comparing Eqs. (2.46) and (2.48) gives:

$$\frac{dy_o}{dt} = C_r i \tag{2.49}$$

Integrating Eq. (2.49) from $\left(q_u/\alpha_o\right)^{1/\beta_o}$ to y_o for y_o and 0 to t (where $t \le t_o$) for t gives:

$$y_o = \left(\frac{q_u}{\alpha_o}\right)^{1/\beta_o} + C_r it \tag{2.50}$$

Substituting Eq. (2.50) into Eq. (2.7) gives the equation for the rising phase (rising limb) of the hydrograph for a plane with upstream inflow for $t \le t_o$:

$$q = \alpha_o \left[\left(\frac{q_u}{\alpha_o}\right)^{1/\beta_o} + C_r it\right]^{\beta_o} \tag{2.51}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (2.51) reduces to:

$$q = \alpha_o \left(C_r it\right)^{\beta_o} \tag{2.52}$$

Figures 2.3 and 2.4 show the rising phase (rising limb) of an equilibrium and a partial equilibrium runoff hydrographs for a plane without and for a plane with upstream inflow, respectively. If the hydrographs in figure 2.4 are shifted by a distance t_u to the right, they become the same as those in figure 2.3.

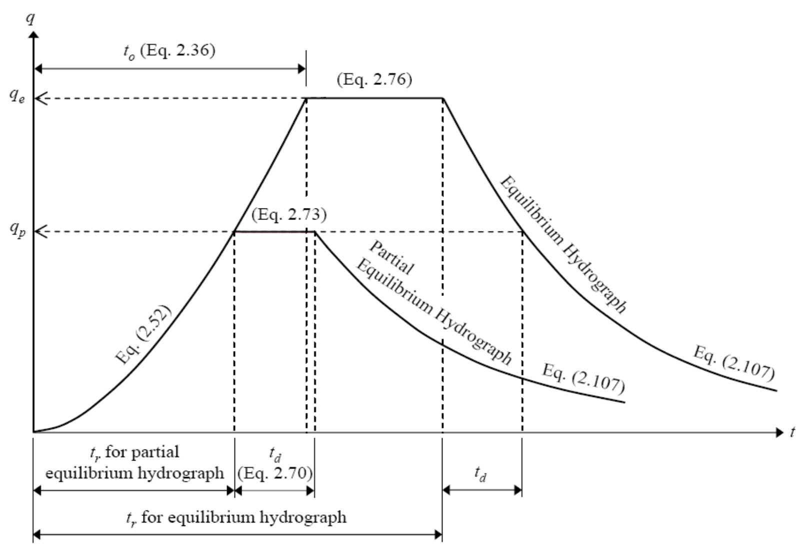

Figure 2.3. Equilibrium and Partial Equilibrium Runoff Hydrographs for a Plane without Upstream Inflow.

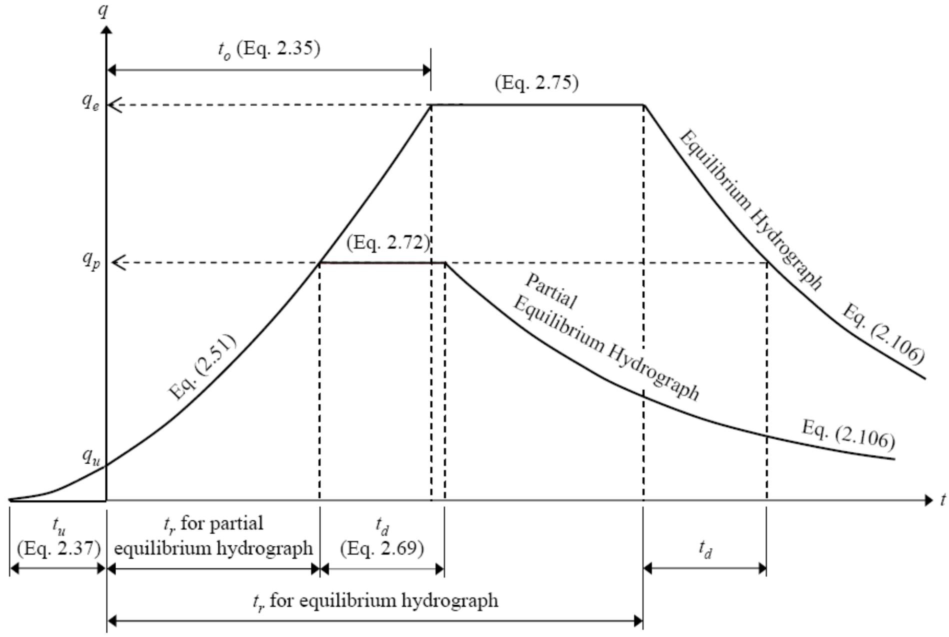

Figure 2.4. Equilibrium and Partial Equilibrium Runoff Hydrographs for a Plane with Upstream Inflow.

2.12. Forward Characteristic - Rising Phase

Substituting Eq. (2.1) into Eq. (2.51) gives the equation for the forward characteristic for a plane with upstream inflow:

$$t = \frac{1}{\alpha_o^{1/\beta_o}} \left[\frac{(q_u + C_r i x_o)^{1/\beta_o} - q_u^{1/\beta_o}}{C_r i} \right] \tag{2.53}$$

The forward characteristic traces the time it takes for the wave to travel downstream. With the kinematic wave equations (Eqs. 2.4 and 2.6), there is no backward characteristic, and this is why the kinematic wave approximation cannot simulate the backwater effect (Section 2.3).

For a plane with zero upstream inflow ($q_u = 0$), Eq. (2.53) reduces to:

$$t = \left[\frac{x_o}{\alpha_o (C_r i)^{\beta_o - 1}} \right]^{1/\beta_o} \tag{2.54}$$

As shown in figure 2.5, the forward characteristic commences at the upstream end of the overland plane ($x_o = 0$), the time it takes for the wave to travel the length of the plane, L_o, equals to the time of concentration, t_o (Eqs. 2.35 and 2.36).

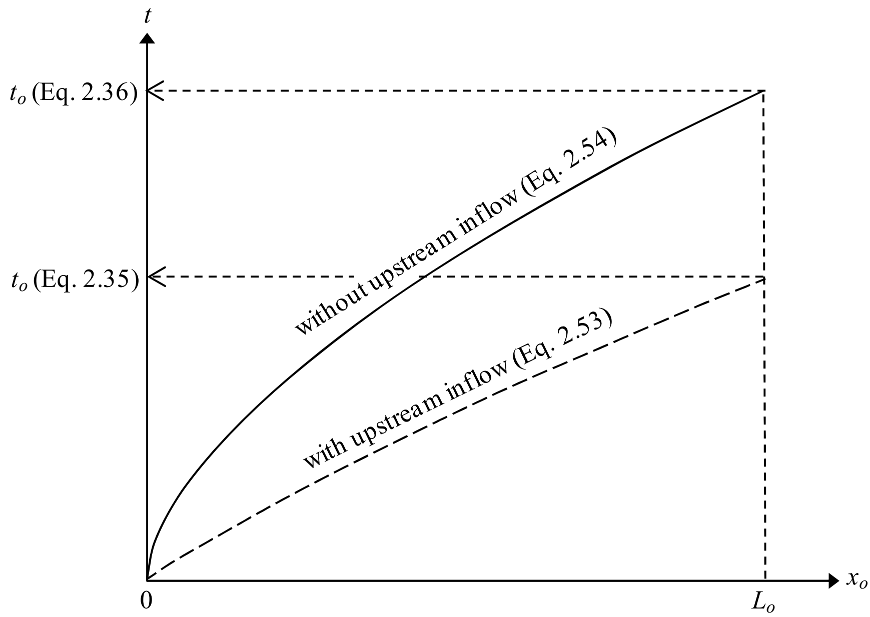

Figure 2.5. Forward Characteristics for Planes without and with Upstream Inflow.

2.13. WATER SURFACE PROFILE – RISING PHASE

Figure 2.6 shows the successive water surface profiles during the rising phase for a plane subject to a uniform rainfall intensity only, without upstream inflow. At $t = 0$, the profile is the line O-A, corresponding to $q = 0$ and $y_o = 0$. At time interval $0 < t < t_o$, the flow depth increases and the profile becomes the curve O-B-C. This is a partial equilibrium profile corresponding to the partial equilibrium discharge, q_p. The length, L_p, contributes to the discharge, q_p, which corresponds to the flow depth, y_p. Finally, at $t \geq t_o$, the flow depth increases even further and the profile reaches equilibrium. The equilibrium profile is the curve O-B-D. The length, L_o, contributes to the equilibrium discharge, q_e, which corresponds to the flow depth, y_e.

Figure 2.7 shows successive water surface profiles during the rising phase for a plane subject to a uniform rainfall intensity and with a constant upstream inflow. The upstream inflow, q_u, which corresponds to the flow depth, y_u, is considered to be contributed by an upstream plane of length, L_u. Hence at $t = 0$, the water surface profile is the curve O-O_u-A. At time interval $0 < t < t_o$, the flow depth increases and the profile becomes the curve O-O_u-B-C. This is a partial equilibrium profile corresponding to the partial equilibrium discharge, q_p. The length $(L_u + L_p)$ contributes to the discharge, q_p, which corresponds to the flow depth, y_p. Finally, at $t \geq t_o$, the flow depth increases even further and the profile reaches equilibrium. The equilibrium profile is the curve O-O_u-B-D. The length $(L_u + L_o)$ contributes to the equilibrium discharge, q_e, which corresponds to the flow depth, y_e.

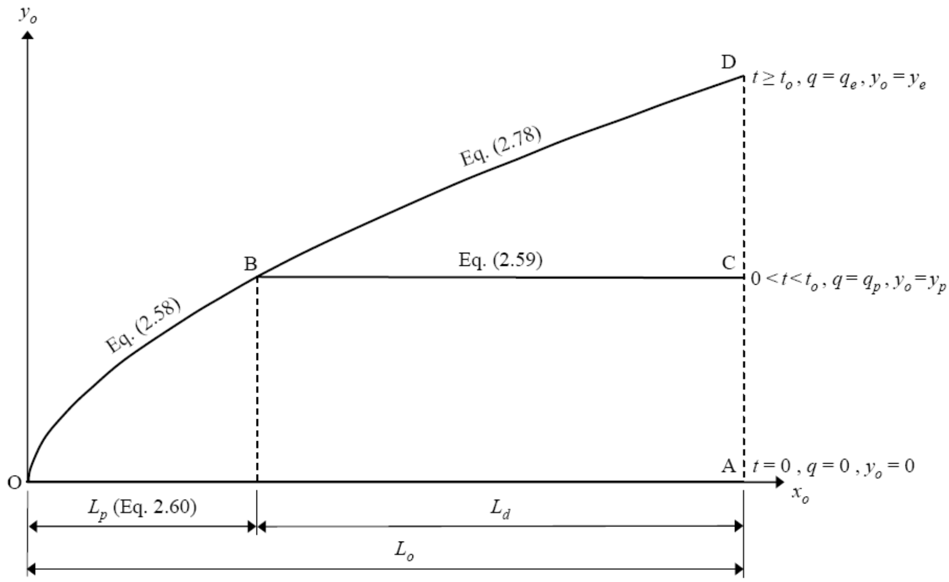

Figure 2.6. Successive Water Surface Profiles during Rising Phase for a Plane without Upstream Inflow.

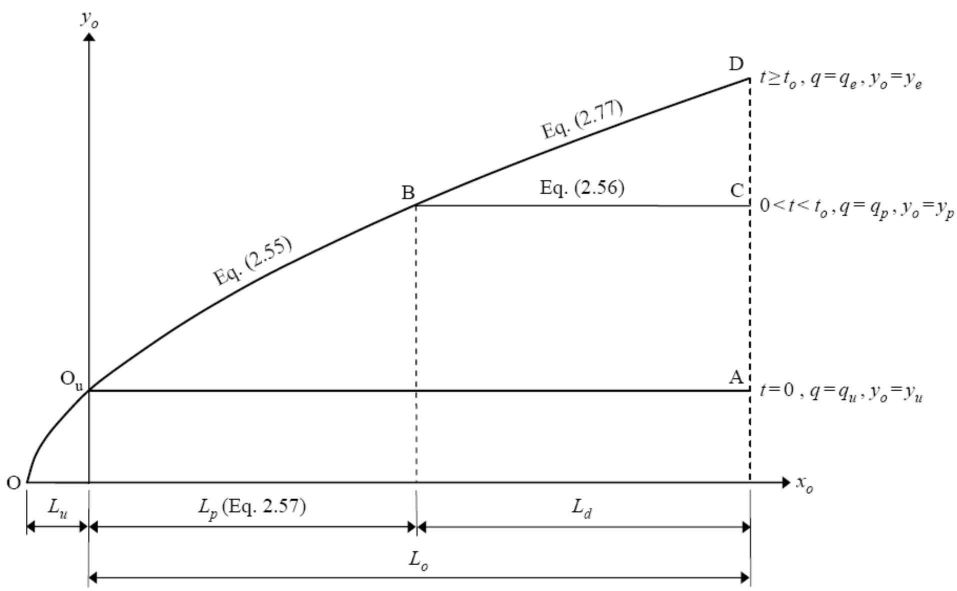

Figure 2.7. Successive Water Surface Profiles during Rising Phase for a Plane with Upstream Inflow.

From figure 2.7, it is apparent that the water surface profile (curve O-O_u-B) within the length, ($L_u + L_p$) is identical to the equilibrium water surface profile (curve O-O_u-B-D). Substituting Eq. (2.1) into Eq. (2.7), gives the equation for the profile between $-L_u \leq x_o \leq L_p$:

$$y_o = \left(\frac{q_u + C_r i x_o}{\alpha_o} \right)^{1/\beta_o} \tag{2.55}$$

Substituting $y_o = y_p$ and $x_o = L_p$ into Eq. (2.55) gives the equation for the profile between $L_p \le x_o \le L_o$:

$$y_p = \left(\frac{q_u + C_r i L_p}{\alpha_o} \right)^{1/\beta_o} \tag{2.56}$$

Substituting $q = q_p$ and $x_o = L_p$ into Eq. (2.1) and rearranging gives the distance L_p:

$$L_p = \frac{q_p - q_u}{C_r i} \tag{2.57}$$

If the profiles in figure 2.7 are shifted by a distance L_u to the right, they become the same as those in figure 2.6 which are for a plane with zero upstream inflow (q_u = 0). For such a case, Eqs. (2.55)-(2.57) reduce to:

$$y_o = \left(\frac{C_r i x_o}{\alpha_o} \right)^{1/\beta_o} \tag{2.58}$$

which is valid for $0 \le x_o \le L_p$,

$$y_p = \left(\frac{C_r i L_p}{\alpha_o} \right)^{1/\beta_o} \tag{2.59}$$

which is valid for $L_p \le x_o \le L_o$, and

$$L_p = \frac{q_p}{C_r i} \tag{2.60}$$

Equation (2.59) can also be derived by substituting $y_o = y_p$ and $x_o = L_p$ into Eq. (2.58).

2.14. DURATION OF PARTIAL EQUILIBRIUM DISCHARGE

In figure 2.7, the curve O-O_u-B-C is a partial equilibrium water surface profile corresponding to the partial equilibrium discharge, q_p. The duration of the partial equilibrium discharge, t_d, is the time taken for the water particle to travel from B to C, and is therefore related to the length, L_d, and the kinematic wave celerity, c_k , as follows:

$$t_d = \frac{L_d}{c_k} \tag{2.61}$$

As shown in figure 2.7, L_d is related to L_o as follows:

$$L_d = L_o - L_p \tag{2.62}$$

Substituting Eq. (2.62) into Eq. (2.61) gives:

$$t_d = \frac{L_o - L_p}{c_k} \tag{2.63}$$

Substituting Eq. (2.57) into Eq. (2.63) gives:

$$t_d = \frac{L_o - \left(\frac{q_p}{C_r i}\right) + \left(\frac{q_u}{C_r i}\right)}{c_k} \tag{2.64}$$

Substituting $q = q_p$ and $y_o = y_p$ into Eqs. (2.7) and (2.27) give:

$$q_p = \alpha_o y_p^{\beta_o} \tag{2.65}$$

and

$$c_k = \alpha_o \beta_o y_p^{\beta_o - 1} \tag{2.66}$$

Substituting Eqs. (2.65) and (2.66) into Eq. (2.64) gives:

$$t_d = \frac{L_o - \left(\frac{\alpha_o y_p^{\beta_o}}{C_r i}\right) + \left(\frac{q_u}{C_r i}\right)}{\alpha_o \beta_o y_p^{\beta_o - 1}} \tag{2.67}$$

Substituting $y_o = y_p$ and $t = t_r$ into Eq. (2.50) gives:

$$y_p = \left(\frac{q_u}{\alpha_o}\right)^{1/\beta_o} + C_r i t_r \tag{2.68}$$

Substituting Eq. (2.68) into Eq. (2.67) and rearranging gives the equation for the duration of partial equilibrium discharge for a plane with upstream inflow:

$$t_d = \frac{C_r i L_o + q_u - \alpha_o \left[\left(\frac{q_u}{\alpha_o}\right)^{1/\beta_o} + C_r i t_r\right]^{\beta_o}}{\alpha_o \beta_o C_r i \left[\left(\frac{q_u}{\alpha_o}\right)^{1/\beta_o} + C_r i t_r\right]^{\beta_o - 1}} \tag{2.69}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (2.69) reduces to:

$$t_d = \frac{L_o - \alpha_o (C_r i)^{\beta_o - 1} t_r^{\beta_o}}{\alpha_o \beta_o (C_r i t_r)^{\beta_o - 1}} \tag{2.70}$$

Substituting Eq. (2.36) into Eq. (2.70) gives t_d in terms of t_o:

$$t_d = \frac{t_o^{\beta_o} - t_r^{\beta_o}}{\beta_o t_r^{\beta_o - 1}} \tag{2.71}$$

The duration of partial equilibrium discharge, t_d, for a plane without and for a plane with upstream inflow are shown in figures 2.3 and 2.4, respectively.

2.15. Hydrograph – Equilibrium Phase

As shown in figures 2.3 and 2.4, the hydrograph may reach partial equilibrium or equilibrium depending on the duration of rainfall t_r. If $t_r < t_o$, the hydrograph reaches partial equilibrium with a constant discharge q_p. If $t_r \geq t_o$, the hydrograph reaches equilibrium with a constant discharge q_e.

2.15.1. Partial Equilibrium Discharge

Substituting $t = t_r$ (where $t_r < t_o$) into Eq. (2.51) gives the equation for the partial equilibrium discharge for a plane with upstream inflow:

$$q_p = \alpha_o \left[\left(\frac{q_u}{\alpha_o} \right)^{1/\beta_o} + C_r i t_r \right]^{\beta_o} \tag{2.72}$$

which is valid for $t_r \leq t \leq (t_r + t_d)$.

For a plane with zero upstream inflow ($q_u = 0$), Eq. (2.72) reduces to:

$$q_p = \alpha_o (C_r i t_r)^{\beta_o} \tag{2.73}$$

2.15.2. Equilibrium Discharge

Substituting $t = t_o$ into Eq. (2.51) gives the equilibrium discharge q_e:

$$q_e = \alpha_o \left[\left(\frac{q_u}{\alpha_o} \right)^{1/\beta_o} + C_r i t_o \right]^{\beta_o} \tag{2.74}$$

which is valid for $t_o \leq t \leq t_r$. Substituting Eq. (2.35) into Eq. (2.74) gives the equation for the equilibrium discharge for a plane with upstream inflow:

$$q_e = q_u + C_r i L_o \tag{2.75}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (2.75) reduces to:

$$q_e = C_r i L_o \tag{2.76}$$

2.16. Water Surface Profile – Equilibrium Phase

As shown in figure 2.7, the curve O-O_u-B-D is the equilibrium water surface profile. Substituting Eq. (2.1) into Eq. (2.7) gives the equation for the profile between $-L_u \leq x_o \leq L_o$:

$$y_o = \left(\frac{q_u + C_r i x_o}{\alpha_o} \right)^{1/\beta_o} \tag{2.77}$$

Equation (2.77) is identical to Eq. (2.55) because the equilibrium profile and the partial equilibrium profile are identical for $-L_u \leq x_o \leq L_p$ (figure 2.7).

For a plane with zero upstream inflow ($q_u = 0$), Eq. (2.77) reduces to:

$$y_o = \left(\frac{C_r i x_o}{\alpha_o} \right)^{1/\beta_o} \tag{2.78}$$

Equation (2.78) is the equation for the curve O-B-D in figure 2.6, which is valid for $0 \le x_o \le L_o$.

2.17. EQUILIBRIUM DETENTION STORAGE

The amount of water that is detained under the equilibrium condition is known as the equilibrium detention storage (Wong and Li 2000). As the equilibrium detention storage can be evaluated from a water surface profile or from a rising phase of a hydrograph, the general formula for the equilibrium detention storage of an overland plane with upstream inflow is derived using both approaches.

2.17.1. Water Surface Profile Approach

Rearranging Eq. (2.3) gives

$$q_u = C_r i L_u \tag{2.79}$$

Substituting Eq. (2.79) into Eq. (2.55) gives:

$$y_o = \left[\frac{C_r i (L_u + x_o)}{\alpha_o} \right]^{1/\beta_o} \tag{2.80}$$

As shown in figure 2.8, integrating Eq. (2.80) from $-L_u$ to L_o for x_o gives the equilibrium detention storage, D_{euo}, for an overland plane of length ($L_u + L_o$), which is the shaded areas *A* and *B*:

$$D_{euo} = \frac{\beta_o}{1+\beta_o} \left(\frac{C_r i}{\alpha_o} \right)^{1/\beta_o} (L_u + L_o)^{(1+\beta_o)/\beta_o} \tag{2.81}$$

Similarly, integrating Eq. (2.80) from $-L_u$ to 0 for x_o gives the equilibrium detention storage, D_{eu}, for an overland plane of length L_u, which is the shaded area *A* in figure 2.8:

$$D_{eu} = \frac{\beta_o}{1+\beta_o} \left(\frac{C_r i}{\alpha_o} \right)^{1/\beta_o} L_u^{(1+\beta_o)/\beta_o} \tag{2.82}$$

The difference between Eqs. (2.81) and (2.82) is the equilibrium detention storage, D_{eo}, for an overland plane of length L_o, which is the shaded area *B* in figure 2.8:

$$D_{eo} = \frac{\beta_o}{1+\beta_o}\left(\frac{C_r i}{\alpha_o}\right)^{1/\beta_o}\left[\left(L_u + L_o\right)^{(1+\beta_o)/\beta_o} - L_u^{(1+\beta_o)/\beta_o}\right] \tag{2.83}$$

Substituting Eq. (2.3) into Eq. (2.83) gives the equation for the equilibrium detention storage for a plane with upstream inflow (Wong and Li 2000):

$$D_{eo} = \frac{\beta_o}{\left(1+\beta_o\right)\alpha_o^{1/\beta_o} C_r i}\left[\left(q_u + C_r i L_o\right)^{(1+\beta_o)/\beta_o} - q_u^{(1+\beta_o)/\beta_o}\right] \tag{2.84}$$

For a plane with zero upstream inflow (i.e. $q_u = 0$), Eq. (2.84) reduces to:

$$D_{eo} = \frac{\beta_o}{1+\beta_o}\left(\frac{C_r i}{\alpha_o}\right)^{1/\beta_o} L_o^{(1+\beta_o)/\beta_o} \tag{2.85}$$

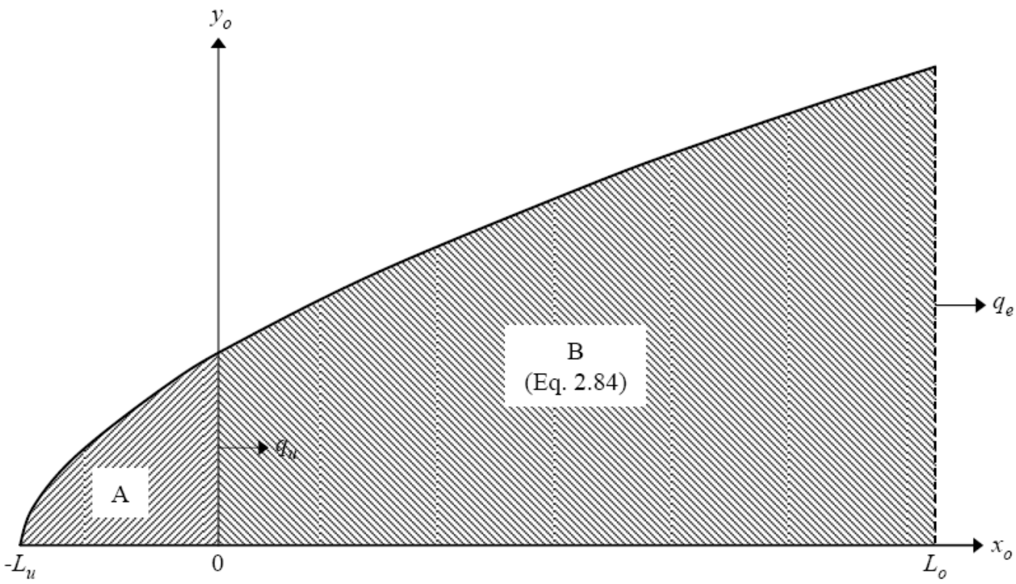

Figure 2.8. Determination of Equilibrium Detention Storage using Water Surface Profile Approach for a Plane with Upstream Inflow.

2.17.2. Hydrograph Approach

Similar to the derivation using the water surface profile, the upstream inflow, q_u, is considered to be produced by an upstream plane with a time of concentration, t_u, subject to rainfall intensity, i. Substituting $q = q_u$ and $t = t_u$ in Eq. (2.52) gives the upstream inflow, q_u, in terms of t_u as follows:

$$q_u = \alpha_o \left(C_r i\ t_u\right)^{\beta_o} \tag{2.86}$$

Rearranging Eq. (2.86) gives the time of concentration t_u in terms of q_u:

$$t_u = \frac{q_u^{1/\beta_o}}{C_r i \alpha_o^{1/\beta_o}} \tag{2.87}$$

Equation (2.87) can also be derived by substituting Eq. (2.3) into Eq. (2.37). As shown in figure 2.9, integrating (q_e - q) from -t_u to t_o for t gives the equilibrium detention storage, D_{euo}, for an overland plane of length (L_u + L_o), which is the shaded areas A and B:

$$D_{euo} = \int_{-t_u}^{t_o} \left(q_e - q\right) dt \tag{2.88}$$

Substituting Eqs. [(2.2), (2.35), (2.51) and (2.87)] into Eq. (2.88) and integrating gives:

$$D_{euo} = \frac{\beta_o}{1+\beta_o}\left(\frac{1}{C_r i \alpha_o^{1/\beta_o}}\right)\left(q_u + C_r i L_o\right)^{(1+\beta_o)/\beta_o} \tag{2.89}$$

Similarly, by integrating (q_u - q) from -t_u to 0 gives the equilibrium detention storage, D_{eu}, for an overland plane of length, L_u, which is the shaded area A in figure 2.9:

$$D_{eu} = \int_{-t_u}^{0} \left(q_u - q\right) dt \tag{2.90}$$

Substituting Eqs. [(2.51), (2.86) and (2.87)] into Eq. (2.90) and integrating gives:

$$D_{eu} = \frac{\beta_o}{1+\beta_o}\left(\frac{1}{C_r i \alpha_o^{1/\beta_o}}\right) q_u^{(1+\beta_o)/\beta_o} \tag{2.91}$$

The difference between Eqs. (2.89) and (2.91) is the equilibrium detention storage, D_{eo}, for an overland plane of length, L_o, which is Eq. (2.84). It is the shaded area B in figure 2.9.

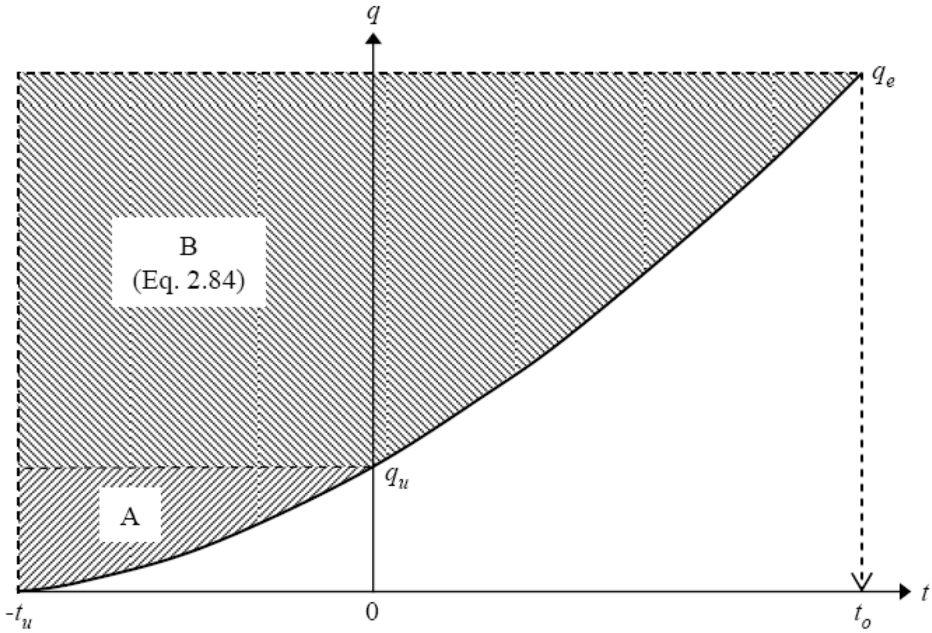

Figure 2.9. Determination of Equilibrium Detention Storage using Hydrograph Approach for a Plane with Upstream Inflow.

2.18. WATER SURFACE PROFILE - FALLING PHASE

During the falling phase, rainfall ceases (i.e. $i = 0$ for $0 \leq x_o \leq L_o$), Eq. (2.49) becomes (Henderson and Wooding 1964, Overton and Meadows 1976):

$$\frac{dy_o}{dt} = 0 \tag{2.92}$$

Integrating Eq. (2.92) gives:

$$y_o = \text{constant} \tag{2.93}$$

Equation (2.93) signifies that water flows out at constant depth. The celerity at which the water flows out is governed by the kinematic wave celerity, c_k (Eq. 2.27). Figure 2.10 shows the successive water surface profiles during the falling phase for a plane without upstream inflow. Curve O-D is the equilibrium profile at $t = t_r \geq t_o$, which is identical to the curve O-B-D in figure 2.6. After a time increment at $t = t_r + \Delta t$, the profile falls and becomes curve O-C. During the time increment Δt, the water particle a_1 travels a distance Δx_o to a_2 at constant flow depth. The distance, Δx_o, between points a_1 and a_2 can be derived from the kinematic wave celerity, c_k. Rearranging Eq. (2.27) gives:

$$\Delta x_o = \alpha_o \beta_o y_o^{\beta_o - 1} \Delta t \tag{2.94}$$

The distance between points b_1 and b_2 is also given by Eq. (2.94). Since the flow depth for the b points are larger than those for the a points, the corresponding wave celerity, c_k, is greater, and the corresponding distance Δx_o is therefore longer, as shown in figure 2.10. At $t > t_r + \Delta t$, the profile falls further and becomes curve O-B. Finally, at $t >> t_r + \Delta t$, when all the water flows out of the plane, the profile falls to the line O-A, which is identical to that in figure 2.6.

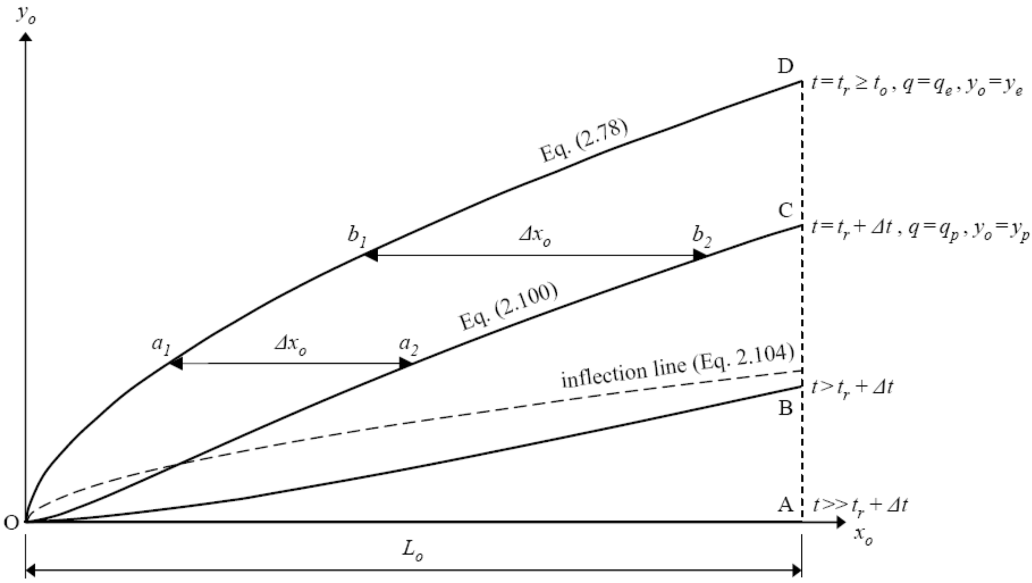

Figure 2.10. Successive Water Surface Profiles during Falling Phase for a Plane without Upstream Inflow.

Further, figure 2.11 shows the successive water surface profiles for a plane with a constant upstream inflow during the falling phase. The curve O-O_u-G-D is the equilibrium profile at time t_r, which is identical to curve O-O_u-B-D in figure 2.7. If the rainfall stops over the entire length ($L_u + L_o$), after a time interval Δt, the water surface profile falls and becomes curve O-E-C. However, since the upstream inflow is constant, the curve O-O_u is fixed. Hence, only the curve O_u-G-D falls. At time $t = t_r + \Delta t$, the water surface profile on the plane with a constant upstream inflow is the curve O_u-E-C, and the curve O-E does not exist. At time $t > t_r + \Delta t$, the water surface profile falls further and becomes the curve O_u-E-F-B. Finally, at time $t >> t_r + \Delta t$, the discharge reduces to the upstream discharge q_u. The water surface profile is the line O_u-E-F-A, which is identical to the line O_u-A in figure 2.7.

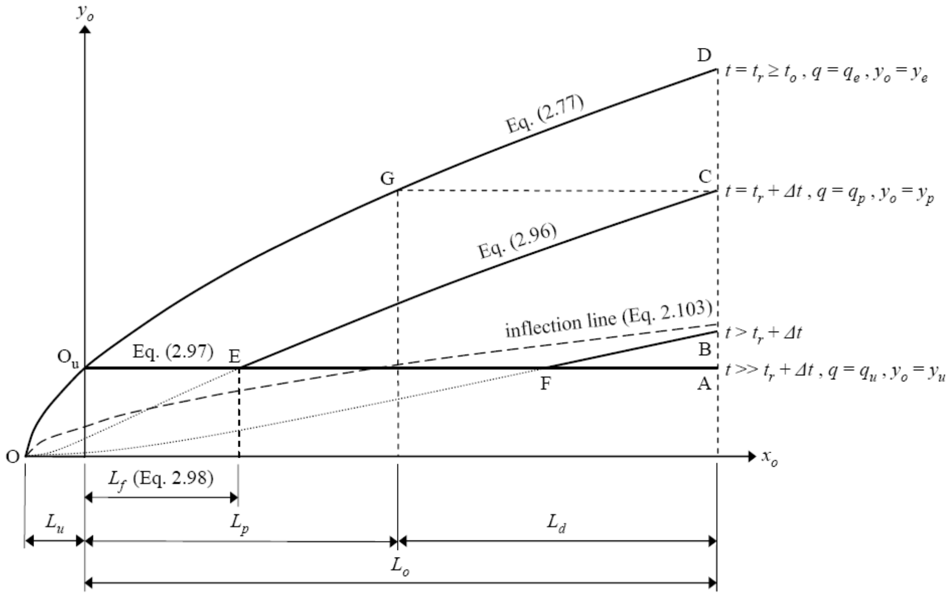

Figure 2.11. Successive Water Surface Profiles during Falling Phase for a Plane with Upstream Inflow.

As shown in figure 2.11, at time t_r, the distance x_o of any point on the equilibrium profile (curve O-O_u-G-D) can be expressed in terms of flow depth y_o by substituting Eq. (2.7) into Eq. (2.1):

$$x_o = \frac{\alpha_o y_o^{\beta_o} - q_u}{C_r i} \quad (2.95)$$

Integrating Eq. (2.27) from $(\alpha_o y_o^{\beta_o} - q_u)/C_r i$ (Eq. 2.95) to x_o for x_o and from t_r to t for t gives the equation for the curve O-E-C:

$$x_o = \alpha_o \beta_o y_o^{\beta_o - 1}(t - t_r) + \left(\frac{\alpha_o y_o^{\beta_o} - q_u}{C_r i} \right) \quad (2.96)$$

For a plane with a constant upstream inflow, Eq. (2.96) is only valid for $L_f \leq x_o \leq L_o$, where L_f = length of plane in which the flow is equal to upstream inflow during the falling phase. For the profile between $0 \leq x_o \leq L_f$, it is the line O_u-E, i.e.

$$y_o = y_u = \left(\frac{q_u}{\alpha_o} \right)^{1/\beta_o} \quad (2.97)$$

Substituting $y_o = y_u$ and $x_o = L_f$ into Eq. (2.96) gives the equation for L_f in terms of y_u:

$$L_f = \alpha_o \beta_o y_u^{\beta_o - 1}(t - t_r) \tag{2.98}$$

Substituting Eq. (2.97) into Eq. (2.98) gives the equation for L_f in terms of q_u:

$$L_f = \alpha_o^{1/\beta_o} \beta_o q_u^{(\beta_o - 1)/\beta_o}(t - t_r) \tag{2.99}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (2.96) reduces to (Wong 2008a):

$$x_o = \alpha_o \beta_o y_o^{\beta_o - 1}(t - t_r) + \left(\frac{\alpha_o y_o^{\beta_o}}{C_r i}\right) \tag{2.100}$$

which is valid for $0 \le x_o \le L_o$ (figure 2.10). Equations (2.96)-(2.100) are only valid for $t \ge t_r$.

2.18.1. Inflection Line

As shown in figure 2.10, the equilibrium water surface profiles (curve O-D) is concave downwards, while the water surface profile at time t > $t_r + \Delta t$ (curve O-B) is concave upwards. Similarly, in figure 2.11, the curve O-O_u-D is concave downwards, and the curve O-F-B is concave upwards. The equation for the inflection line can be derived by first obtaining the second derivative of Eq. (2.95) with respect to y_o:

$$\frac{d^2 x_o}{dy_o^2} = \alpha_o \beta_o (\beta_o - 1)(\beta_o - 2) y_o^{\beta_o - 3}(t - t_r) + \left[\frac{\alpha_o \beta_o (\beta_o - 1)}{C_r i} y_o^{\beta_o - 2}\right] \tag{2.101}$$

Next, by equating Eq. (2.101) to zero, and equating $y_o = y_i$ results in:

$$y_i = (2 - \beta_o)(t - t_r) C_r i \tag{2.102}$$

where y_i = flow depth of the inflection point. Substituting Eq. (2.102) into Eq. (2.96) and equating $x_o = x_i$ and $y_o = y_i$ gives the equation for the inflection line for a plane with upstream inflow:

$$x_i = \left(\frac{2}{2 - \beta_o}\right)\left(\frac{\alpha_o y_i^{\beta_o}}{C_r i}\right) - \left(\frac{q_u}{C_r i}\right) \tag{2.103}$$

where x_i = distance x_o of the inflection point.

For a plane with zero upstream inflow ($q_u = 0$), Eq. (2.103) reduces to (Wong 2008a):

$$x_i = \left(\frac{2}{2-\beta_o}\right)\left(\frac{\alpha_o y_i^{\beta_o}}{C_r i}\right) \tag{2.104}$$

Equations (2.103) and (2.104) have been superimposed respectively onto figures 2.10 and 2.11 as dashed lines which are labeled as an inflection line.

2.19. HYDROGRAPH - FALLING PHASE

As shown by Eq. (2.93), during the falling phase, water flows out at constant depth. Hence, the water particle at G flows out to C at constant depth (figure 2.11). The time required for the water particle to flow from G to C is in fact the same as the duration of partial equilibrium discharge, t_d, as shown in figure 2.4. Substituting $t_d = t - t_r$ and $q_p = q$ into Eq. (2.64) gives:

$$t - t_r = \frac{L_o - \left(\dfrac{q-q_u}{C_r i}\right)}{c_k} \tag{2.105}$$

Equation (2.105) may also be derived by integrating Eq. (2.26) from t_r to t (where $t \geq t_r$) for t and from $[(q\text{-}q_u)/C_r i]$ to L_o for x_o. Since the discharge on the overland plane cannot be less than upstream discharge, Eq. (2.105) is only valid for $q \geq q_u$. Substituting Eq. (2.28) into Eq. (2.105) gives the equation for the falling phase (falling limb) of the hydrograph, which is only valid for $q \geq q_u$:

$$t = \frac{L_o - \left(\dfrac{q-q_u}{C_r i}\right)}{\beta_o \alpha_o^{1/\beta_o} q^{[1-(1/\beta_o)]}} + t_r \tag{2.106}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (2.106) reduces to:

$$t = \frac{L_o - \left(\dfrac{q}{C_r i}\right)}{\beta_o \alpha_o^{1/\beta_o} q^{[1-(1/\beta_o)]}} + t_r \tag{2.107}$$

Figures 2.3 and 2.4 show the falling phase (falling limb) of an equilibrium and a partial equilibrium runoff hydrograph for a plane without and for a plane with upstream inflow, respectively.

In: Kinematic-Wave Rainfall-Runoff Formulas
Editor: Tommy S.W. Wong, pp. 29-38 ISBN: 978-1-60692-705-2
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Chapter 3

3. Working Formulas for Flow on Overland Plane

In this Chapter, based on the Manning's equation, the kinematic wave parameters for flow on an overland plane are derived. By applying these parameters to the general formulas in Chapter 2, working formulas for the flow depth, flow velocity, average flow velocity, wave celerity, average wave celerity, time of concentration, design discharge, rising and falling phases of hydrograph, forward characteristic, rising, equilibrium and falling phases of water surface profiles, duration of partial equilibrium discharge, and equilibrium detention storage are also derived.

3.1. Kinematic Wave Parameters

The Manning's equation is defined as:

$$Q = \frac{AR^{2/3}S^{1/2}}{n} \tag{3.1}$$

where Q = discharge, A = cross-sectional flow area, R = hydraulic radius, S = bed slope and n = Manning's roughness coefficient. The hydraulic radius, R, is related to the flow area, A, as follows:

$$R = \frac{A}{P} \tag{3.2}$$

where P = wetted perimeter.

By considering the overland plane as a rectangular channel, the flow area, A, and the wetted perimeter, P, are related to the flow depth, y_o, as follows:

$$A = wy_o \tag{3.3}$$

$$P = w + 2y_o \tag{3.4}$$

Substituting Eqs. (3.3) and (3.4) into Eq. (3.2) gives:

$$R = \frac{wy_o}{w + 2y_o} \tag{3.5}$$

Since overland flow depth is usually small as compared to the width of the overland plane, Eq. (3.5) reduces to:

$$R = \frac{y_o}{1 + \frac{2y_o}{w}} = y_o \tag{3.6}$$

Substituting Eqs. (3.3) and (3.6) and $Q = Q_o$, $S = S_o$, $n = n_o$ into Eq. (3.1) gives:

$$Q_o = \left(\frac{S_o^{1/2}}{n_o} \right) w y_o^{5/3} \tag{3.7}$$

where Q_o = discharge of the overland plane, and n_o = Manning's roughness coefficient of the overland surface. Dividing Eq. (3.7) by w gives the discharge per unit width of the overland plane, q:

$$q = \left(\frac{S_o^{1/2}}{n_o} \right) y_o^{5/3} \tag{3.8}$$

A comparison of Eq. (3.8) with Eq. (2.7) gives the kinematic wave parameters (Chen and Evans 1977):

$$\alpha_o = \frac{S_o^{1/2}}{n_o} \tag{3.9}$$

$$\beta_o = \frac{5}{3} \tag{3.10}$$

3.2. FLOW DEPTH

Rearranging Eq. (3.8) gives the equation for the flow depth for a plane with and without upstream inflow:

$$y_o = \left(\frac{n_o q}{S_o^{1/2}} \right)^{3/5} \tag{3.11}$$

3.3. Flow Velocity

Substituting Eqs. (3.9) and (3.10) into Eq. (2.12) gives the equation for the flow velocity along the equilibrium profile for a plane with upstream inflow:

$$v = 0.00238 \left(\frac{S_o^{1/2}}{n_o} \right)^{3/5} \left(3.6x10^6 q_u + C_r i x_o \right)^{2/5} \tag{3.12}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (3.12) reduces to:

$$v = 0.00238 \left(\frac{S_o^{1/2}}{n_o} \right)^{3/5} \left(C_r i x_o \right)^{2/5} \tag{3.13}$$

3.4. Average Flow Velocity

Substituting Eqs. (3.9) and (3.10) into Eq. (2.15) gives the equation for the average flow velocity for a plane with upstream inflow:

$$v_{av} = 0.00143 \left(\frac{S_o^{1/2}}{n_o} \right)^{3/5} \left[\frac{C_r i L_o}{\left(3.6 \times 10^6 q_u + C_r i L_o \right)^{3/5} - \left(3.6 \times 10^6 q_u \right)^{3/5}} \right] \tag{3.14}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (3.14) reduces to:

$$v_{av} = 0.00143 \left(\frac{S_o^{1/2}}{n_o} \right)^{3/5} \left(C_r i L_o \right)^{2/5} \tag{3.15}$$

3.5. Kinematic Wave Celerity

Substituting Eqs. (3.9) and (3.10) into Eq. (2.29) gives the equation for the kinematic wave celerity along the equilibrium profile for a plane with upstream inflow:

$$c_k = 0.00397\left(\frac{S_o^{1/2}}{n_o}\right)^{3/5}\left(3.6\times10^6 q_u + C_r i x_o\right)^{2/5} \tag{3.16}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (3.16) reduces to:

$$c_k = 0.00397\left(\frac{S_o^{1/2}}{n_o}\right)^{3/5}\left(C_r i x_o\right)^{2/5} \tag{3.17}$$

3.6. AVERAGE WAVE CELERITY

Substituting Eqs. (3.9) and (3.10) into Eq. (2.32) gives the equation for the average wave celerity for a plane with upstream inflow:

$$c_{av} = 0.00238\left(\frac{S_o^{1/2}}{n_o}\right)^{3/5}\left[\frac{C_r i L_o}{\left(3.6\times10^6 q_u + C_r i\ L_o\right)^{3/5} - \left(3.6\times10^6 q_u\right)^{3/5}}\right] \tag{3.18}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (3.18) reduces to:

$$c_{av} = 0.00238\left(\frac{S_o^{1/2}}{n_o}\right)^{3/5}\left(C_r i L_o\right)^{2/5} \tag{3.19}$$

3.7. TIME OF CONCENTRATION

Substituting Eqs. (3.9) and (3.10) into Eq. (2.35) gives the equation for the time of concentration for a plane with upstream inflow:

$$t_o = 6.988\left(\frac{n_o}{S_o^{1/2}}\right)^{3/5}\left[\frac{\left(3.6\times10^6 q_u + C_r i L_o\right)^{3/5} - \left(3.6\times10^6 q_u\right)^{3/5}}{C_r i}\right] \tag{3.20}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (3.20) reduces to (Woolhiser and Liggett 1967):

$$t_o = \frac{6.988}{\left(C_r i\right)^{2/5}}\left(\frac{n_o L_o}{S_o^{1/2}}\right)^{3/5} \tag{3.21}$$

3.8. DESIGN DISCHARGE

Substituting Eqs. (3.9) and (3.10) into Eq. (2.43) gives the equation for the design discharge per unit area of the plane for a plane with zero upstream inflow:

$$Q_d / A_o = \frac{1}{360}\left[\frac{(aC_r)^{1/b}}{6.988\left(\dfrac{n_o}{S_o^{1/2}}\right)^{3/5} L_o^{3/5}}\right]^{\frac{5b}{5-2b}} \tag{3.22}$$

3.9. HYDROGRAPH - RISING PHASE

Substituting Eqs. (3.9) and (3.10) into Eq. (2.51) gives the equation for the rising phase (rising limb) of a hydrograph for a plane with upstream inflow:

$$q = \frac{S_o^{1/2}}{n_o}\left[\left(\frac{n_o q_u}{S_o^{1/2}}\right)^{3/5} + \frac{C_r it}{60\times 10^3}\right]^{5/3} \tag{3.23}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (3.23) reduces to:

$$q = \frac{S_o^{1/2}}{n_o}\left(\frac{C_r it}{60\times 10^3}\right)^{5/3} \tag{3.24}$$

Equations (3.23) and (3.24) are valid for $t \le t_o$.

3.10. FORWARD CHARACTERISTIC - RISING PHASE

Substituting Eqs. (3.9) and (3.10) into Eq. (2.53) gives the equation for the forward characteristic for a plane with upstream inflow:

$$t = 6.988\left(\frac{n_o}{S_o^{1/2}}\right)^{3/5}\left[\frac{(3.6\times 10^6 q_u + C_r i x_o)^{3/5} - (3.6\times 10^6 q_u)^{3/5}}{C_r i}\right] \tag{3.25}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (3.25) reduces to:

$$t = \frac{6.988}{(C_r i)^{2/5}} \left(\frac{n_o x_o}{S_o^{1/2}} \right)^{3/5} \tag{3.26}$$

3.11. WATER SURFACE PROFILE - RISING PHASE

Substituting Eqs. (3.9) and (3.10) into Eq. (2.55) gives the equation for the rising phase of the water surface profile for a plane with upstream inflow, which is valid for $0 \le x_o \le L_p$:

$$y_o = 0.116 \times 10^{-3} \left[\left(\frac{n_o}{S_o^{1/2}} \right) \left(3.6 \times 10^6 q_u + C_r i x_o \right) \right]^{3/5} \tag{3.27}$$

Substituting Eqs. (3.7) and (3.8) into Eq. (2.56) gives the equation for the rising phase of the water surface profile for a plane with upstream inflow, which is valid for $L_p \le x_o \le L_o$.

$$y_p = 0.116 \times 10^{-3} \left[\left(\frac{n_o}{S_o^{1/2}} \right) \left(3.6 \times 10^6 q_u + C_r i L_p \right) \right]^{3/5} \tag{3.28}$$

From Eq. (2.57), the distance L_p is:

$$L_p = 3.6 \times 10^6 \left(\frac{q_p - q_u}{C_r i} \right) \tag{3.29}$$

For a plane with zero upstream inflow ($q_u = 0$), Eqs. (3.27)-(3.29) reduce to:

$$y_o = 0.116 \times 10^{-3} \left(\frac{n_o C_r i x_o}{S_o^{1/2}} \right)^{3/5} \tag{3.30}$$

which is valid for $0 \le x_o \le L_p$,

$$y_p = 0.116 \times 10^{-3} \left(\frac{n_o C_r i L_p}{S_o^{1/2}} \right)^{3/5} \tag{3.31}$$

which is valid for $L_p \le x_o \le L_o$, and

$$L_p = 3.6 \times 10^6 \left(\frac{q_p}{C_r i} \right) \tag{3.32}$$

3.12. Duration of Partial Equilibrium Discharge

Substituting Eqs. (3.9) and (3.10) into Eq. (2.69) gives the equation for the duration of partial equilibrium discharge for a plane with upstream inflow:

$$t_d = 36\times 10^3 \left\{ \frac{\dfrac{C_r i L_o}{3.6\times 10^6} + q_u - \dfrac{S_o^{1/2}}{n_o}\left[\left(\dfrac{n_o q_u}{S_o^{1/2}}\right)^{3/5} + \dfrac{C_r i t_r}{60\times 10^3}\right]^{5/3}}{\dfrac{S_o^{1/2} C_r i}{n_o}\left[\left(\dfrac{n_o q_u}{S_o^{1/2}}\right)^{3/5} + \dfrac{C_r i t_r}{60\times 10^3}\right]^{2/3}} \right\} \tag{3.33}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (3.33) reduces to:

$$t_d = 36\times 10^3 \left[\frac{\dfrac{C_r i L_o}{3.6\times 10^6} - \dfrac{S_o^{1/2}}{n_o}\left(\dfrac{C_r i t_r}{60\times 10^3}\right)^{5/3}}{\dfrac{S_o^{1/2} C_r i}{n_o}\left(\dfrac{C_r i t_r}{60\times 10^3}\right)^{2/3}} \right] \tag{3.34}$$

3.13. Hydrograph - Equilibrium Phase

As shown in figures 2.3 and 2.4, the hydrograph may reach partial equilibrium or equilibrium depending on the duration of rainfall t_r. If $t_r < t_o$, the hydrograph reaches partial equilibrium with a constant discharge q_p. If $t_r \geq t_o$, the hydrograph reaches equilibrium with a constant discharge q_e.

3.13.1. Partial Equilibrium Discharge

Substituting Eqs. (3.9) and (3.10) into Eq. (2.72) gives the equation for the partial equilibrium discharge for a plane with upstream inflow:

$$q_p = \frac{S_o^{1/2}}{n_o}\left[\left(\frac{n_o q_u}{S_o^{1/2}}\right)^{3/5} + \frac{C_r i t_r}{60\times 10^3}\right]^{5/3} \tag{3.35}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (3.35) reduces to:

$$q_p = \frac{S_o^{1/2}}{n_o}\left(\frac{C_r i t_r}{60\times 10^3}\right)^{5/3} \tag{3.36}$$

Equations (3.35) and (3.36) are valid for $t_r \le t \le (t_r + t_d)$.

3.13.2. Equilibrium Discharge

From Eq. (2.75), the equation for the equilibrium discharge for a plane with upstream inflow is:

$$q_e = q_u + \frac{C_r i L_o}{3.6\times10^6} \tag{3.37}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (3.37) reduces to:

$$q_e = \frac{C_r i L_o}{3.6\times10^6} \tag{3.38}$$

Equations (3.37) and (3.38) are valid for $t_o \le t \le t_r$.

3.14. Water Surface Profile - Equilibrium Phase

Substituting Eqs. (3.9) and (3.10) into Eq. (2.77) gives the equation for the equilibrium water surface profile for a plane with upstream inflow between $0 \le x_o \le L_o$:

$$y_o = 0.116\times10^{-3}\left[\left(\frac{n_o}{S_o^{1/2}}\right)\left(3.6\times10^6 q_u + C_r i x_o\right)\right]^{3/5} \tag{3.39}$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (3.39) reduces to:

$$y_o = 0.116\times10^{-3}\left(\frac{n_o C_r i x_o}{S_o^{1/2}}\right)^{3/5} \tag{3.40}$$

which is also valid for $0 \le x_o \le L_o$.

3.15. Equilibrium Detention Storage

Substituting Eqs. (3.9) and (3.10) into Eq. (2.84) gives the equation for the equilibrium detention storage for a plane with upstream inflow:

$$D_{eo} = \frac{72.8\times10^{-6}}{C_r i}\left(\frac{n_o}{S_o^{1/2}}\right)^{3/5}\left[\left(3.6\times10^6 q_u + C_r i L_o\right)^{8/5} - \left(3.6\times10^6 q_u\right)^{8/5}\right] \quad (3.41)$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (3.41) reduces to:

$$D_{eo} = 72.8\times10^{-6}\left(\frac{n_o}{S_o^{1/2}}\right)^{3/5}\left(C_r i\right)^{3/5} L_o^{\ 8/5} \quad (3.42)$$

3.16. WATER SURFACE PROFILE - FALLING PHASE

Substituting Eqs. (3.9) and (3.10) into Eq. (2.96) gives the equation for the falling phase of the water surface profile for a plane with upstream inflow, which is valid for $L_f \le x_o \le L_o$:

$$x_o = 100.0\left(\frac{S_o^{1/2} y_o^{\ 2/3}}{n_o}\right)\left(t - t_r\right) + \left[3.6\times10^6\left(\frac{\left(\frac{S_o^{1/2} y_o^{\ 5/3}}{n_o}\right) - q_u}{C_r i}\right)\right] \quad (3.43)$$

From Eq. (2.97), the equation for the profile between $0 \le x_o \le L_f$, is:

$$y_o = 0.116\times10^{-3}\left[\left(\frac{n_o}{S_o^{1/2}}\right)\left(3.6\times10^6 q_u\right)\right]^{3/5} \quad (3.44)$$

Substituting Eqs. (3.9) and (3.10) into Eq. (2.99) gives the equation for the distance L_f for a plane with upstream inflow, which is valid for $t \ge t_r$:

$$L_f = 0.238\left(\frac{S_o^{1/2}}{n_o}\right)^{3/5}\left(3.6\times10^6 q_u\right)^{2/5}\left(t - t_r\right) \quad (3.45)$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (3.43) reduces to:

$$x_o = \left(\frac{S_o^{1/2} y_o^{\ 2/3}}{n_o}\right)\left[100\left(t - t_r\right) + 3.6\times10^6\left(\frac{y_o}{C_r i}\right)\right] \quad (3.46)$$

which is valid for $0 \le x_o \le L_o$, and $t \ge t_r$.

3.17. HYDROGRAPH - FALLING PHASE

Substituting Eqs. (3.9) and (3.10) into Eq. (2.106) gives the equation for the falling phase (falling limb) of a hydrograph for a plane with upstream inflow.

$$t = 0.0100\left(\frac{n_o}{S_o^{1/2}}\right)^{3/5}\left[\frac{C_r i L_o - 3.6\times10^6\left(q - q_u\right)}{C_r i q^{2/5}}\right] + t_r \qquad (3.47)$$

For a plane with zero upstream inflow ($q_u = 0$), Eq. (3.47) reduces to:

$$t = 0.0100\left(\frac{n_o}{S_o^{1/2}}\right)^{3/5}\left(\frac{C_r i L_o - 3.6\times10^6 q}{C_r i q^{2/5}}\right) + t_r \qquad (3.48)$$

In: Kinematic-Wave Rainfall-Runoff Formulas
Editor: Tommy S.W. Wong, pp. 39-62
ISBN: 978-1-60692-705-2
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Chapter 4

4. General Formulas for Flow in Open Channel

In this Chapter, based on the kinematic wave theory, the general formulas for flow area; flow velocity; average flow velocity; wave celerity; average wave celerity; time of travel; rising, equilibrium and falling phases of hydrograph; forward characteristic; rising, equilibrium and falling phases of flow area profiles; duration of partial equilibrium discharge; and equilibrium detention storage for flow in an open channel are derived.

4.1. Flow Conditions

Consider an open channel of length L_c, subject to a uniformly distributed lateral inflow q_L, and with a constant upstream inflow Q_u, the discharge in channel, Q_c, along the equilibrium water surface profile of the channel is:

$$Q_c = Q_u + q_L x_c \tag{4.1}$$

where x_c = distance along the channel in the direction of flow. Substituting $x_c = L_c$ into Eq. (4.1) gives the discharge Q_e at the end of the channel at equilibrium, i.e.

$$Q_e = Q_u + q_L L_c \tag{4.2}$$

Further, the upstream inflow, Q_u, can be considered to be contributed by an imaginary channel of length L_u, which is situated immediately upstream of the channel of length L_c. This imaginary upstream channel is also subject to a uniformly distributed lateral inflow q_L, but with zero upstream inflow. At the outlet point of this upstream channel and at equilibrium, the discharge is Q_u. Substituting $Q_e = Q_u$, $Q_u = 0$ and $L_c = L_u$ into Eq. (4.2) gives the length of the upstream channel, L_u, in terms of the upstream inflow, Q_u, as follows:

$$L_u = \frac{Q_u}{q_L} \tag{4.3}$$

Figure 4.1 shows the upstream channel, the open channel, and the equilibrium water surface profile.

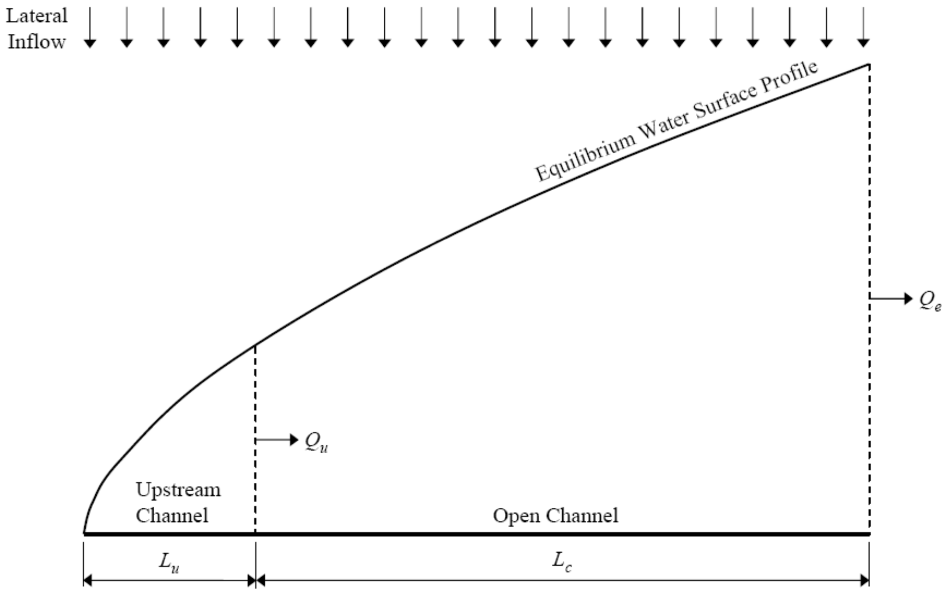

Figure 4.1. Upstream and Open Channels with Equilibrium Water Surface Profile.

4.2. Dynamic Wave Equations

The mechanics of unsteady open channel flow with a lateral inflow contribution can be expressed mathematically by the Saint Venant equation. Derived from the principles of continuity and momentum, the equations are (Chow et al 1988):

$$\frac{\partial A_c}{\partial t} + \frac{\partial Q_c}{\partial x_c} = q_L \tag{4.4}$$

$$\frac{1}{gA_c}\frac{\partial Q_c}{\partial t} + \frac{1}{gA_c}\frac{\partial}{\partial x_c}\left(\frac{Q_c^2}{A_c}\right) + \frac{\partial y_c}{\partial x_c} - \left(S_c - S_f\right) = 0 \tag{4.5}$$

where A_c = channel flow area, t = time, g = acceleration due to gravity, S_c = channel bed slope and S_f = friction slope. The assumptions inherent in Eqs. (4.4) and (4.5) are (DeVries and MacArthur 1979):

1. The flow is one dimensional (i.e. velocity varies in the longitudinal direction only). This implies that the velocity is constant and the water surface is horizontal across any section perpendicular to the longitudinal axis.
2. All flows are gradually varied with hydrostatic pressure prevailing at all points in the flow such that all vertical acceleration within the water column can be neglected.

3. The longitudinal axis of the channel can be approximated by a straight line (i.e. there is no secondary circulation).
4. The slope of the channel bed is small.
5. The bed of the channel is fixed (i.e. the effects of scour and deposition are negligible).
6. Resistance to flow can be described by empirical resistance formulas, such as Manning's equation.
7. The fluid is incompressible and homogeneous in density.
8. The momentum carried to the fluid from the lateral inflow is negligible.

The momentum equation (Eq. 4.5) consists of five terms, namely local acceleration, convective acceleration, pressure force, gravity force and friction force, each representing a physical process that governs the flow momentum described as follows:

1. The acceleration terms represent the effect of velocity change over time and space.
2. The pressure force term represents the effect of flow depth change.
3. The gravity force term S_c is proportional to the channel bed slope and accounts for the change in bed level.
4. The friction force term S_f is proportional to the friction slope and accounts for the friction loss for the flow in an open channel.

4.3. Kinematic Wave Equations

If the backwater effect is negligible and there is no rapid change in flow, the acceleration and pressure terms in Eq. (4.5) may be neglected (Stephenson 1981, Wong 1992), and the momentum equation reduces to:

$$S_c = S_f \tag{4.6}$$

Equations (4.4) and (4.6) are called the "kinematic wave equations". Equation (4.6) shows that the channel bed slope is parallel to the friction slope, which means that the kinematic wave is under the uniform flow condition. Hence, Eq. (4.6) can be replaced by the general uniform flow equation, which is:

$$Q_c = \alpha_c A_c^{\beta_c} \tag{4.7}$$

where α_c and β_c = kinematic wave parameters relating Q_c to A_c.

4.4. Flow Area

Rearranging Eq. (4.7) gives the equation for the flow area, A_c , in terms of the discharge, Q_c , as follows:

$$A_c = \left(\frac{Q_c}{\alpha_c} \right)^{1/\beta_c} \tag{4.8}$$

4.5. Flow Velocity

From continuity, the flow velocity, v, is related to the channel discharge, Q_c, as follows:

$$v = \frac{Q_c}{A_c} \tag{4.9}$$

Substituting Eq. (4.7) into Eq. (4.9), the velocity, v, becomes:

$$v = \alpha_c A_c^{\beta_c - 1} \tag{4.10}$$

Substituting Eq. (4.8) into Eq. (4.10) gives the equation for the velocity, v, in terms of channel discharge, Q_c:

$$v = \left(\alpha_c Q_c^{\beta_c - 1} \right)^{1/\beta_c} \tag{4.11}$$

Substituting Eq. (4.1) into Eq. (4.11) gives the equation for the flow velocity along the equilibrium profile for a channel with upstream inflow:

$$v = \left[\alpha_c \left(Q_u + q_L x_c \right)^{\beta_c - 1} \right]^{1/\beta_c} \tag{4.12}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (4.12) reduces to:

$$v = \left[\alpha_c \left(q_L x_c \right)^{\beta_c - 1} \right]^{1/\beta_c} \tag{4.13}$$

4.6. Average Flow Velocity

Further, the average flow velocity, v_{av}, over the length of the open channel, L_c, can be derived as follows:

$$v_{av} = \frac{L_c}{\int_0^{L_c} \frac{1}{v} dx_c} \tag{4.14}$$

Substituting Eq. (4.12) into Eq. (4.14) and integrating ($1/v$) gives the equation for the average flow velocity for a channel with upstream inflow:

$$v_{av} = \frac{\alpha_c^{1/\beta_c} q_L L_c}{\beta_c \left[\left(Q_u + q_L L_c \right)^{1/\beta_c} - Q_u^{1/\beta_c} \right]} \tag{4.15}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (4.15) reduces to:

$$v_{av} = \frac{\alpha_c^{1/\beta_c} q_L L_c}{\beta_c \left[\left(q_L L_c \right)^{1/\beta_c} \right]} \tag{4.16}$$

4.7. Kinematic Wave Celerity

Differentiating Eq. (4.7) with respect to t gives:

$$\frac{\partial Q_c}{\partial t} = \alpha_c \beta_c A_c^{\beta_c - 1} \left(\frac{\partial A_c}{\partial t} \right) \tag{4.17}$$

Rearranging Eq. (4.17) gives:

$$\frac{\partial A_c}{\partial t} = \frac{1}{\alpha_c \beta_c A_c^{\beta_c - 1}} \left(\frac{\partial Q_c}{\partial t} \right) \tag{4.18}$$

Substituting Eq. (4.18) into Eq. (4.4), the continuity equation becomes:

$$\left[\frac{1}{\alpha_c \beta_c A_c^{\beta_c - 1}} \left(\frac{\partial Q_c}{\partial t} \right) \right] + \left(\frac{\partial Q_c}{\partial x_c} \right) = q_L \tag{4.19}$$

Kinematic waves result in changes in Q_c which is dependent on both x_c and t, and the increment in flow rate dQ_c can be written as:

$$dQ_c = \frac{\partial Q_c}{\partial t} dt + \frac{\partial Q_c}{\partial x_c} dx_c \tag{4.20}$$

Dividing Eq. (4.20) by dx_c:

$$\frac{dQ_c}{dx_c} = \left(\frac{\partial Q_c}{\partial x_c} \right) + \left[\frac{\partial Q_c}{\partial t} \left(\frac{dt}{dx_c} \right) \right] \tag{4.21}$$

If

$$q_L = \frac{dQ_c}{dx_c} \tag{4.22}$$

and

$$\frac{dt}{dx_c} = \frac{1}{\alpha_c \beta_c A_c^{\beta_c - 1}} \tag{4.23}$$

then Eq. (4.19) and Eq. (4.21) are identical. Differentiating Eq. (4.7) with respect to A_c:

$$\frac{dQ_c}{dA_c} = \alpha_c \beta_c A_c^{\beta_c - 1} \tag{4.24}$$

Comparing Eq. (4.23) and Eq. (4.24) gives:

$$\frac{dQ_c}{dA_c} = \frac{dx_c}{dt} \tag{4.25}$$

Since kinematic wave celerity, c_k, is:

$$c_k = \frac{dx_c}{dt} \tag{4.26}$$

Substituting Eq. (4.25) into Eq. (4.26) gives:

$$c_k = \frac{dx_c}{dt} = \frac{dQ_c}{dA_c} = \alpha_c \beta_c A_c^{\beta_c - 1} \tag{4.27}$$

Substituting Eq. (4.8) into Eq. (4.27) gives:

$$c_k = \beta_c \left(\alpha_c Q_c^{\beta_c - 1} \right)^{1/\beta_c} \tag{4.28}$$

Substituting Eq. (4.1) into Eq. (4.28) gives the equation for the wave celerity along the equilibrium profile for a channel with upstream inflow:

$$c_k = \beta_c \left[\alpha_c \left(Q_u + q_L x_c \right)^{\beta_c - 1} \right]^{1/\beta_c} \tag{4.29}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (4.29) reduces to:

$$c_k = \beta_c \left[\alpha_c \left(q_L x_c \right)^{\beta_c - 1} \right]^{1/\beta_c} \tag{4.30}$$

4.8. AVERAGE WAVE CELERITY

The average wave celerity, c_{av}, over the channel length, L_c, can be derived as follows:

$$c_{av} = \frac{L_c}{\int_0^{L_c} \frac{1}{c_k} dx_c} \tag{4.31}$$

Substituting Eq. (4.29) into Eq. (4.31) and integrating $(1/c_k)$ gives the equation for the average wave celerity for a channel with upstream inflow:

$$c_{av} = \frac{\alpha_c^{1/\beta_c} q_L L_c}{\left(Q_u + q_L L_c \right)^{1/\beta_c} - Q_u^{1/\beta_c}} \tag{4.32}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (4.32) reduces to:

$$c_{av} = \left[\alpha_c \left(q_L L_c \right)^{\beta_c - 1} \right]^{1/\beta_c} \tag{4.33}$$

4.9. TIME OF TRAVEL

The time of travel in channel, t_t, can be obtained by dividing channel length, L_c, by the average wave celerity, c_{av}, as follows:

$$t_t = \frac{L_c}{c_{av}} \tag{4.34}$$

Substituting Eq. (4.32) into Eq. (4.34) gives the equation for the time of travel for a channel with upstream inflow, (Wong 2001):

$$t_t = \frac{1}{\alpha_c^{1/\beta_c}} \left[\frac{\left(Q_u + q_L L_c \right)^{1/\beta_c} - Q_u^{1/\beta_c}}{q_L} \right] \tag{4.35}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (4.35) reduces to, (Wooding 1965, Wong and Chen 1989):

$$t_t = \left(\frac{L_c}{\alpha_c q_L^{\beta_c - 1}} \right)^{1/\beta_c} \tag{4.36}$$

Further, for the upstream channel of length L_u, substituting $L_c = L_u$ into Eq. (4.36) gives the time of travel, t_u, of the upstream channel:

$$t_u = \left(\frac{L_u}{\alpha_c q_L^{\beta_c - 1}} \right)^{1/\beta_c} \tag{4.37}$$

4.10. HYDROGRAPH - RISING PHASE

Expanding the partial derivative, $(\partial Q_c / \partial x_c)$, into total derivative results in:

$$\frac{\partial Q_c}{\partial x_c} = \frac{dQ_c}{dA_c} \left(\frac{\partial A_c}{\partial x_c} \right) \tag{4.38}$$

Substituting Eq. (4.27) into Eq. (4.38) gives:

$$\frac{\partial Q_c}{\partial x_c} = c_k \frac{\partial A_c}{\partial x_c} \tag{4.39}$$

Substituting Eq. (4.39) into Eq. (4.4) gives:

$$\frac{\partial A_c}{\partial t} + c_k \frac{\partial A_c}{\partial x_c} = q_L \tag{4.40}$$

Differentiating A_c with respect to x_c and t:

$$dA_c = \frac{\partial A_c}{\partial t} dt + \frac{\partial A_c}{\partial x_c} dx_c \tag{4.41}$$

Dividing Eq. (4.41) by dt and substituting Eq. (4.27) into it:

$$\frac{dA_c}{dt} = \frac{\partial A_c}{\partial t} + c_k \frac{\partial A_c}{\partial x_c} \tag{4.42}$$

Comparing Eqs. (4.40) and (4.42) gives:

$$\frac{dA_c}{dt} = q_L \tag{4.43}$$

Integrating Eq. (4.43) from $\left(Q_u/\alpha_c\right)^{1/\beta_c}$ to A_c for A_c and 0 to t (where $t \le t_t$) for t gives:

$$A_c = \left(\frac{Q_u}{\alpha_c}\right)^{1/\beta_c} + q_L t \tag{4.44}$$

Substituting Eq. (4.44) into Eq. (4.7) gives the equation for the rising phase (rising limb) of the hydrograph for a channel with upstream inflow for $t \le t_t$:

$$Q_c = \alpha_c \left[\left(\frac{Q_u}{\alpha_c}\right)^{1/\beta_c} + q_L t\right]^{\beta_c} \tag{4.45}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (4.45) reduces to:

$$Q_c = \alpha_c (q_L t)^{\beta_c} \tag{4.46}$$

Figures 4.2 and 4.3 show the rising phase (rising limb) of an equilibrium and a partial equilibrium runoff hydrographs for a channel without and for a channel with upstream inflow, respectively. If the hydrographs in figure 4.3 are shifted by a distance t_u to the right, they become the same as those in figure 4.2.

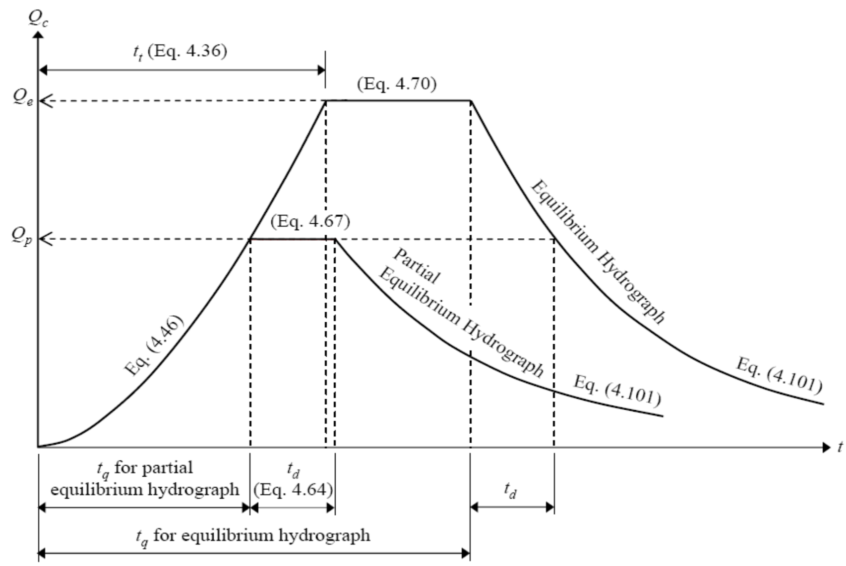

Figure 4.2. Equilibrium and Partial Equilibrium Runoff Hydrographs for a Channel without Upstream Inflow.

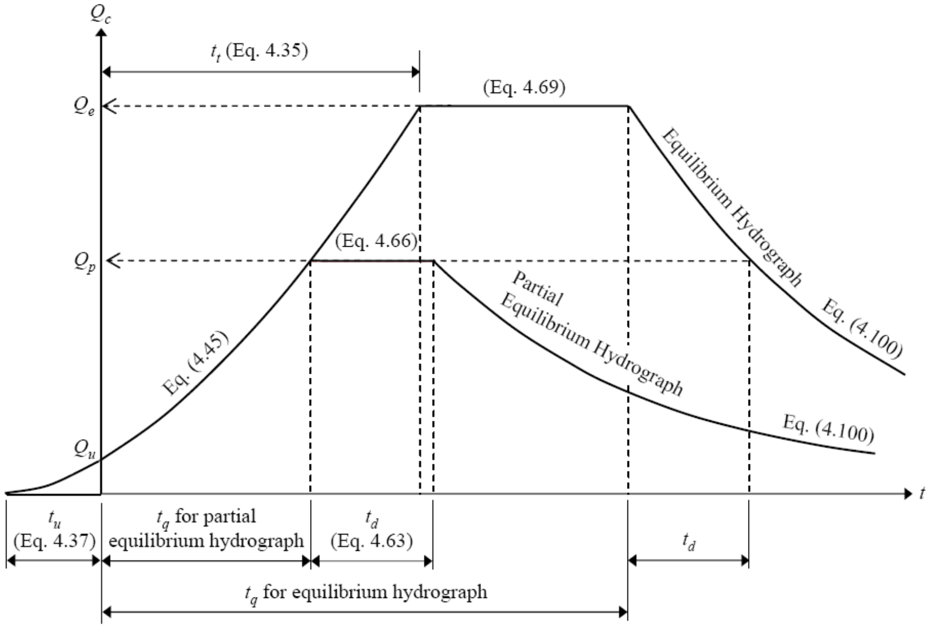

Figure 4.3. Equilibrium and Partial Equilibrium Runoff Hydrographs for a Channel with Upstream Inflow.

4.11. FORWARD CHARACTERISTIC - RISING PHASE

Substituting Eq. (4.1) into Eq. (4.45) gives the equation for the forward characteristic for a channel with upstream inflow:

$$t = \frac{1}{\alpha_c^{1/\beta_c}} \left[\frac{(Q_u + q_L x_c)^{1/\beta_c} - Q_u^{1/\beta_c}}{q_L} \right] \quad (4.47)$$

The forward characteristic traces the time it takes for the wave to travel downstream. With the kinematic wave equations (Eqs. 4.4 and 4.6), there is no backward characteristic, and this is why the kinematic wave approximation cannot simulate the backwater effect (Section 4.3).

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (4.47) reduces to:

$$t = \left(\frac{x_c}{\alpha_c q_L^{\beta_c - 1}} \right)^{1/\beta_c} \quad (4.48)$$

As shown in figure 4.4, the forward characteristic commences at the upstream end of the channel ($x_c = 0$), the time it takes for the wave to travel the length of the channel, L_c, equals to the time of travel in channel, t_t (Eqs. 4.35 and 4.36).

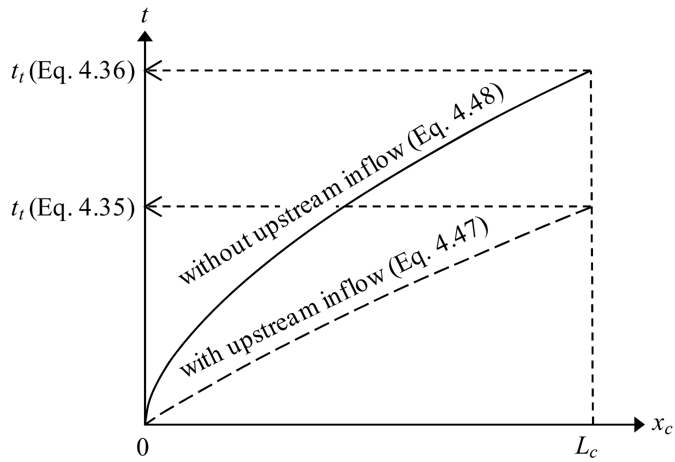

Figure 4.4. Forward Characteristics for Channels without and with Upstream Inflow.

4.12. Flow Area Profile - Rising Phase

Figure 4.5 shows the successive flow area profiles during the rising phase for a channel subject to a uniform lateral inflow only, without upstream inflow. At t = 0, the profile is the line O-A, corresponding to Q_c = 0 and A_c = 0. At time interval $0 < t < t_t$, the flow depth increases and the profile becomes the curve O-B-C. This is a partial equilibrium profile corresponding to the partial equilibrium discharge, Q_p. The length, L_p, contributes to the discharge, Q_p, which corresponds to the flow area, A_p. Finally, at $t \geq t_t$, the flow area increases even further and the profile reaches equilibrium. The equilibrium profile is the curve O-B-D. The length, L_c, contributes to the equilibrium discharge, Q_e, which corresponds to the flow area, A_e.

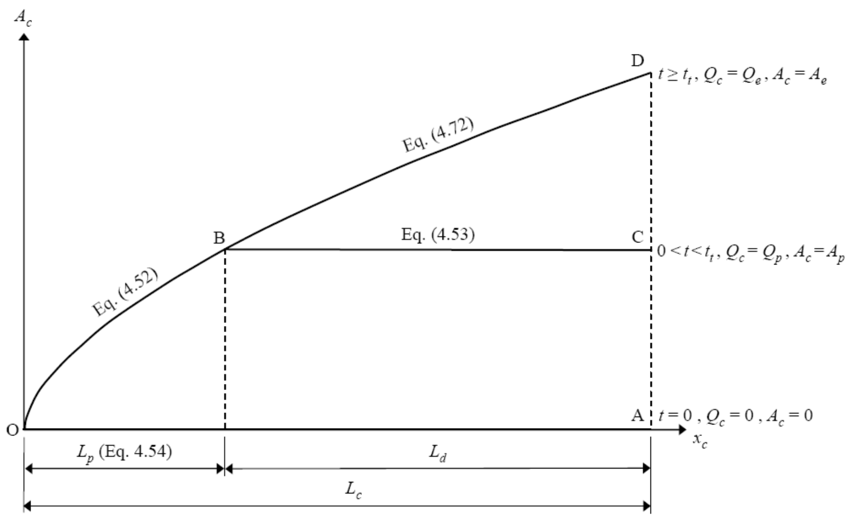

Figure 4.5. Successive Flow Area Profiles during Rising Phase for a Channel without Upstream Inflow.

Figure 4.6 shows successive flow area profiles during the rising phase for a channel subject to a uniform lateral inflow and with a constant upstream inflow. The upstream inflow, Q_u, which corresponds to the flow area, A_u, is considered to be contributed by an upstream channel of length, L_u. Hence at t = 0, the water area profile is the curve O-O$_u$-A. At time interval $0 < t < t_t$, the flow area increases and the profile becomes the curve O-O$_u$-B-C. This is a partial equilibrium profile corresponding to the partial equilibrium discharge, Q_p. The length ($L_u + L_p$) contributes to the discharge, Q_p, which corresponds to the flow area, A_p. Finally, at $t \geq t_t$, the flow area increases even further and the profile reaches equilibrium. The equilibrium profile is the curve O-O$_u$-B-D. The length ($L_u + L_c$) contributes to the equilibrium discharge, Q_e, which corresponds to the flow area, A_e.

From figure 4.6, it is apparent that the flow area profile (curve O-O$_u$-B) within the length, ($L_u + L_p$), is identical to the equilibrium flow area profile (curve O-O$_u$-B-D). Substituting Eq. (4.1) into Eq. (4.7) gives the equation for the profile between $-L_u \leq x_c \leq L_p$:

$$A_c = \left(\frac{Q_u + q_L x_c}{\alpha_c} \right)^{1/\beta_c} \tag{4.49}$$

Substituting $A_c = A_p$ and $x_c = L_p$ into Eq. (4.49) gives the equation for the profile between $L_p \leq x_c \leq L_c$:

$$A_p = \left(\frac{Q_u + q_L L_p}{\alpha_c} \right)^{1/\beta_c} \tag{4.50}$$

Substituting $Q_c = Q_p$ and $x_c = L_p$ into Eq. (4.1) gives the distance L_p:

$$L_p = \frac{Q_p - Q_u}{q_L} \tag{4.51}$$

If the profiles in figure 4.6 are shifted by a distance, L_u, to the right, they become the same as those in figure 4.5 which are for a channel with zero upstream inflow (Q_u = 0). For such a case, Eqs. (4.49)-(4.51) reduce to:

$$A_c = \left(\frac{q_L x_c}{\alpha_c} \right)^{1/\beta_c} \tag{4.52}$$

which is valid for $0 \leq x_c \leq L_p$,

$$A_p = \left(\frac{q_L L_p}{\alpha_c} \right)^{1/\beta_c} \tag{4.53}$$

which is valid for $L_p \leq x_c \leq L_c$, and

$$L_p = \frac{Q_p}{q_L} \tag{4.54}$$

Equation (4.53) can also be derived by substituting $A_c = A_p$ and $x_c = L_p$ into Eq. (4.52).

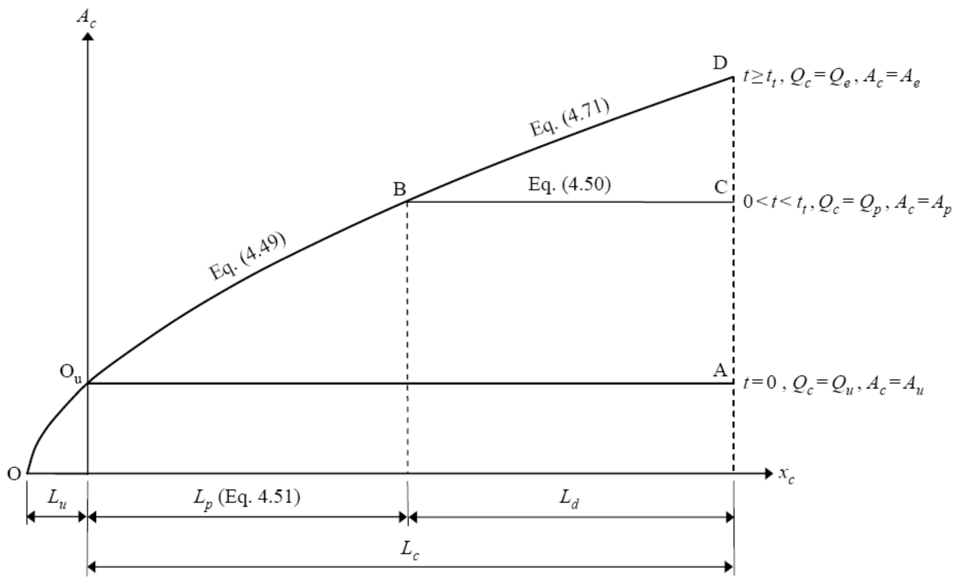

Figure 4.6. Successive Flow Area Profiles during Rising Phase for a Channel with Upstream Inflow.

4.13. DURATION OF PARTIAL EQUILIBRIUM DISCHARGE

In figure 4.6, the curve O-O_u-B-C is the partial equilibrium flow area profile corresponding to the partial equilibrium discharge, Q_p. The duration of the partial equilibrium discharge, t_d, is the time taken for the water particle to travel from B to C, and is therefore related to the length, L_d, and the kinematic wave celerity, c_k, as follows:

$$t_d = \frac{L_d}{c_k} \tag{4.55}$$

As shown in figure 4.6, L_d is related to L_c as follows:

$$L_d = L_c - L_p \tag{4.56}$$

Substituting Eq. (4.56) into Eq. (4.55) gives:

$$t_d = \frac{L_c - L_p}{c_k} \tag{4.57}$$

Substituting Eq. (4.51) into Eq. (4.57) gives:

$$t_d = \frac{L_c - \left(\frac{Q_p}{q_L}\right) + \left(\frac{Q_u}{q_L}\right)}{c_k} \tag{4.58}$$

Substituting $Q_c = Q_p$ and $A_c = A_p$ into Eqs. (4.7) and (4.27) gives:

$$Q_p = \alpha_c A_p^{\beta_c} \tag{4.59}$$

and

$$c_k = \alpha_c \beta_c A_p^{\beta_c - 1} \tag{4.60}$$

Substituting Eqs. (4.59) and (4.60) into Eq. (4.58) gives:

$$t_d = \frac{L_c - \left(\frac{\alpha_c A_p^{\beta_c}}{q_L}\right) + \left(\frac{Q_u}{q_L}\right)}{\alpha\beta A_p^{\beta_c - 1}} \tag{4.61}$$

Substituting $A_c = A_p$ and $t = t_q$ into Eq. (4.44) gives:

$$A_p = \left(\frac{Q_u}{\alpha_c}\right)^{1/\beta_c} + q_L t_q \tag{4.62}$$

where t_q = duration of lateral inflow. Substituting Eq. (4.62) into Eq. (4.61) and rearranging gives the duration of partial equilibrium discharge for a channel with upstream inflow:

$$t_d = \frac{L_c q_L + Q_u - \alpha_c \left[\left(\frac{Q_u}{\alpha_c}\right)^{1/\beta_c} + q_L t_q\right]^{\beta_c}}{\alpha_c \beta_c q_L \left[\left(\frac{Q_u}{\alpha_c}\right)^{1/\beta_c} + q_L t_q\right]^{\beta_c - 1}} \tag{4.63}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (4.63) reduces to:

$$t_d = \frac{L_c - \alpha_c q_L^{\beta_c - 1} t_q^{\beta_c}}{\alpha_c \beta_c (q_L t_q)^{\beta_c - 1}} \tag{4.64}$$

Substituting Eq. (4.36) into Eq. (4.64) gives t_d in terms of t_t:

$$t_d = \frac{t_t^{\beta_c} - t_q^{\beta_c}}{\beta_c t_q^{\beta_c - 1}} \tag{4.65}$$

The duration of partial equilibrium discharge, t_d, for a channel without and for a channel with upstream inflow are shown in figures 4.2 and 4.3, respectively.

4.14. HYDROGRAPH - EQUILIBRIUM PHASE

As shown in figures 4.2 and 4.3, the hydrograph may reach partial equilibrium, or equilibrium depending on the duration of lateral inflow, t_q. If $t_q < t_t$, the hydrograph reaches partial equilibrium with a constant discharge, Q_p. If $t_q \geq t_t$, the hydrograph reaches equilibrium with a constant discharge, Q_e.

4.14.1. Partial Equilibrium Discharge

Substituting $t = t_q$ (where $t_q < t_t$) into Eq. (4.45) gives the equation for the partial equilibrium discharge for a plane with upstream inflow:

$$Q_p = \alpha_c \left[\left(\frac{Q_u}{\alpha_c} \right)^{1/\beta_c} + q_L t_q \right]^{\beta_c} \tag{4.66}$$

which is valid for $t_q \leq t \leq (t_q + t_d)$.

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (4.66) reduces to:

$$Q_p = \alpha_c (q_L t_q)^{\beta_c} \tag{4.67}$$

4.14.2. Equilibrium Discharge

Substituting $t = t_t$ into Eq. (4.45) gives the equilibrium discharge Q_e:

$$Q_e = \alpha_c \left[\left(\frac{Q_u}{\alpha_c} \right)^{1/\beta_c} + q_L t_t \right]^{\beta_c} \tag{4.68}$$

which is valid for $t_t \leq t \leq t_q$. Substituting Eq. (4.35) into Eq. (4.68) gives the equation for the equilibrium discharge for a plane with upstream inflow:

$$Q_e = Q_u + q_L L_c \tag{4.69}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (4.69) reduces to:

$$Q_e = q_L L_c \tag{4.70}$$

4.15. FLOW AREA PROFILE - EQUILIBRIUM PHASE

As shown in figure 4.6, the curve O-O$_u$-B-D is the equilibrium water surface profile. Substituting Eq. (4.1) into Eq. (4.7) gives the equation for the profile between $-L_u \leq x_c \leq L_c$:

$$A_c = \left(\frac{Q_u + q_L x_c}{\alpha_c} \right)^{1/\beta_c} \tag{4.71}$$

Equation (4.70) is identical to Eq. (4.49) because the equilibrium profile and the partial equilibrium profile are identical for $-L_u \leq x_c \leq L_p$ (figure 4.6).

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (4.71) reduces to:

$$A_c = \left(\frac{q_L x_c}{\alpha_c} \right)^{1/\beta_c} \tag{4.72}$$

Equation (4.71) is the equation for the curve O-B-D in figure 4.5, which is valid for $0 \leq x_c \leq L_c$.

4.16. EQUILIBRIUM DETENTION STORAGE

The amount of water that is detained under the equilibrium condition is known as the equilibrium detention storage (Wong and Li 2000). As the equilibrium detention storage can be evaluated from a flow area profile or from a rising phase of a hydrograph, the general formula for the equilibrium detention storage of a channel with upstream inflow is derived using both approaches.

4.16.1. Flow Area Profile Approach

Rearranging Eq. (4.3) gives:

$$Q_u = q_L L_u \tag{4.73}$$

Substituting Eq. (4.73) into Eq. (4.49) gives:

$$A_c = \left[\frac{q_L(L_u + x_c)}{\alpha_c}\right]^{1/\beta_c} \tag{4.74}$$

As shown in figure 4.7, integrating Eq. (4.74) from -L_u to L_c for x_c gives the equilibrium detention storage for an open channel of length (L_u + L_c), which is the shaded areas *A* and *B*:

$$D_{euc} = \frac{\beta_c}{1+\beta_c}\left(\frac{q_L}{\alpha_c}\right)^{1/\beta_c}(L_u + L_c)^{(1+\beta_c)/\beta_c} \tag{4.75}$$

Similarly, integrating Eq. (4.73) from –L_u to 0 for x_c gives the equilibrium detention storage for an open channel of length, L_u, which is the shaded area *A* in figure 4.7:

$$D_{eu} = \frac{\beta_c}{1+\beta_c}\left(\frac{q_L}{\alpha_c}\right)^{1/\beta_c} L_u^{(1+\beta_c)/\beta_c} \tag{4.76}$$

The difference between Eqs. (4.75) and (4.76) is the equilibrium detention storage for an open channel of length, L_c, which is the shaded area *B* in figure 4.7:

$$D_{ec} = \frac{\beta_c}{1+\beta_c}\left(\frac{q_L}{\alpha_c}\right)^{1/\beta_c}\left[(L_u + L_c)^{(1+\beta_c)/\beta_c} - L_u^{(1+\beta_c)/\beta_c}\right] \tag{4.77}$$

Substituting Eq. (4.3) into Eq. (4.77) gives the equation for the equilibrium detention storage for a channel with upstream inflow:

$$D_{ec} = \frac{\beta_c}{(1+\beta_c)\alpha_c^{1/\beta_c}}\left[\frac{(Q_u + q_L L_c)^{(1+\beta_c)/\beta_c} - Q_u^{(1+\beta_c)/\beta_c}}{q_L}\right] \tag{4.78}$$

For a channel with zero upstream inflow (i.e. Q_u = 0), Eq. (4.78) reduces to:

$$D_{ec} = \frac{\beta_c}{1+\beta_c}\left(\frac{q_L}{\alpha_c}\right)^{1/\beta_c} L_c^{(1+\beta_c)/\beta_c} \tag{4.79}$$

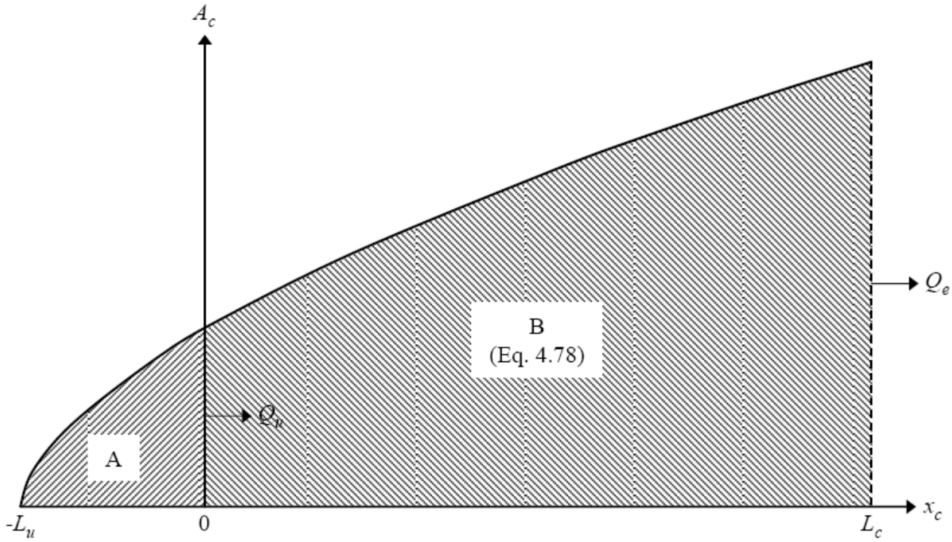

Figure 4.7. Determination of Equilibrium Detention Storage using Flow Area Profile Approach for a Channel with Upstream Inflow.

4.16.2. Hydrograph Approach

Similar to the derivation using the water surface profile, the upstream inflow, Q_u, is considered to be produced by an upstream channel with time of travel, t_t, subject to a uniform lateral inflow into the channel, q_L. The upstream inflow, Q_u, is then related to t_t as follows:

$$Q_u = \alpha_c \left(q_L t_t\right)^{\beta_c} \tag{4.80}$$

Substituting Eq. (4.3) into Eq. (4.37):

$$t_u = \frac{Q_u^{1/\beta_c}}{q_L \alpha_c^{1/\beta_c}} \tag{4.81}$$

At equilibrium ($x_c = L_c$), Eq. (4.1) becomes:

$$Q_e = Q_u + q_L L_c \tag{4.82}$$

where Q_e = equilibrium channel discharge. As shown in figure 4.8, integrating (Q_e - Q_c) from $-t_u$ to t_t for t gives the equilibrium detention storage for an open channel of length (L_u + L_c), which is the shaded areas A and B:

$$D_{euc} = \int_{-t_u}^{t_t} (Q_e - Q_c) dt \tag{4.83}$$

Substituting Eqs. [(4.35), (4.45), (4.81) and (4.82)] into Eq. (4.83) and integrating gives:

$$D_{euc} = \frac{\beta_c}{1+\beta_c}\left(\frac{1}{q_L \alpha_c^{1/\beta_c}}\right)(Q_u + q_L L_c)^{(1+\beta_c)/\beta_c} \tag{4.84}$$

Similarly, by integrating (Q_u - Q_c) from -t_u to 0 gives the equilibrium detention storage for an open channel of length L_u, which is the shaded area *A* in figure 4.8:

$$D_{eu} = \int_{-t_u}^{0} (Q_u - Q_c) dt \tag{4.85}$$

Substituting Eqs. [(4.45), (4.79) and (4.80)] into Eq. (4.85) and integrating gives:

$$D_{eu} = \frac{\beta_c}{1+\beta_c}\left(\frac{1}{q_L \alpha_c^{1/\beta_c}}\right) Q_u^{(1+\beta_c)/\beta_c} \tag{4.86}$$

The difference between Eqs. (4.84) and (4.86) is the equilibrium detention storage, D_{ec} for an open channel of length L_c, which is Eq. (4.78). It is the shaded area *B* in figure 4.8.

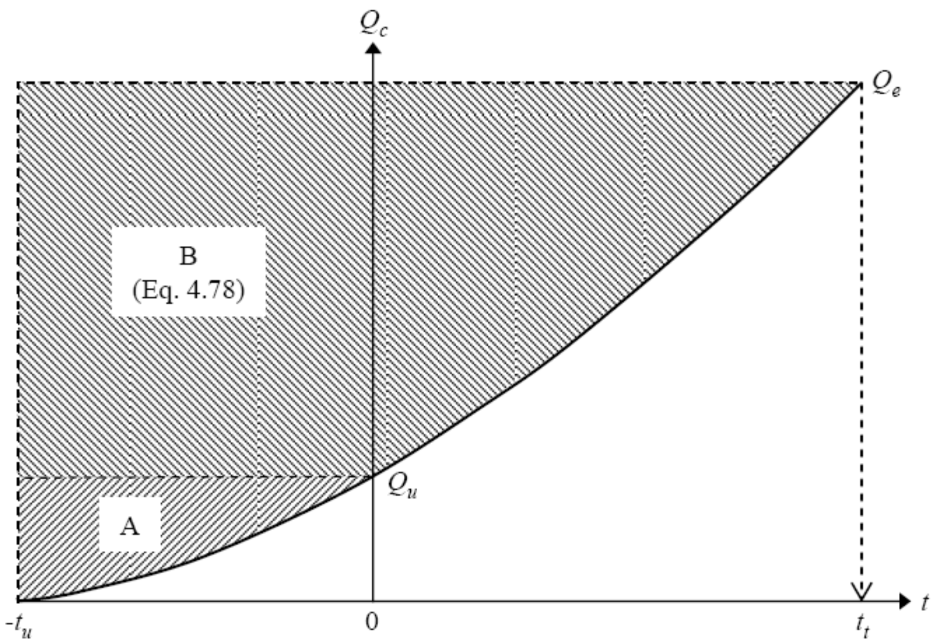

Figure 4.8. Determination of Equilibrium Detention Storage using Hydrograph Approach for a Channel with Upstream Inflow.

4.17. Flow Area Profile - Falling Phase

During the falling phase, lateral inflow ceases (i.e. $q_L = 0$ for $0 \leq x_c \leq L_c$), Eq. (4.43) becomes:

$$\frac{dA_c}{dt} = 0 \tag{4.87}$$

Integrating Eq. (4.87) gives:

$$A_c = \text{constant} \tag{4.88}$$

Equation (4.88) signifies that water flows out at constant flow area. The celerity at which the water flows out is governed by the kinematic wave celerity, c_k (Eq. 4.27). Figure 4.9 shows the successive flow area profiles during the falling phase for a channel without upstream inflow. Curve O-D is the equilibrium profile at $t = t_q \geq t_t$, which is identical to the curve O-B-D in figure 2.6. After a time increment at $t = t_q + \Delta t$, the profile falls and becomes curve O-C. During the time increment Δt, the water particle a_1 travels a distance Δx_o to a_2 at constant flow area. The distance, Δx_o, between points a_1 and a_2, can be derived from the kinematic wave celerity, c_k. Rearranging Eq. (4.27) gives:

$$\Delta x_c = \alpha_c \beta_c A_c^{\beta_c - 1} \Delta t \tag{4.89}$$

The distance between points b_1 and b_2 is also given by Eq. (4.89). Since the flow area for the b points are larger than those for the a points, the corresponding wave celerity, c_k, is greater, and the corresponding distance Δx_o is therefore longer, as shown in figure 4.9. At $t > t_q + \Delta t$, the profile falls further and becomes curve O-B. Finally, at $t >> t_q + \Delta t$, when all the water flows out of the channel, the profile falls to the line O-A, which is identical to that in figure 4.5.

Further, figure 4.10 shows the successive flow area profiles for a channel with a constant upstream inflow during the falling phase. The curve O-O_u-G-D is the equilibrium profile at time t_r, which is identical to curve O-O_u-B-D in figure 4.6. If the lateral inflow stops over the entire length ($L_u + L_o$), after a time interval Δt, the flow area profile falls and becomes curve O-E-C. However, since the upstream inflow is constant, the curve O-O_u is fixed. Hence, only the curve, O_u-G-D, falls. At time $t = t_q + \Delta t$, the flow area profile on the channel with a constant upstream inflow is the curve O_u-E-C, and the curve O-E does not exist. At time $t > t_q + \Delta t$, the flow area profile falls further and becomes the curve O_u-E-F-B. Finally, at time $t >> t_q + \Delta t$, the discharge reduces to the upstream discharge, Q_u. The flow area profile is the line O_u-E-F-A, which is identical to the line O_u-A in figure 4.6.

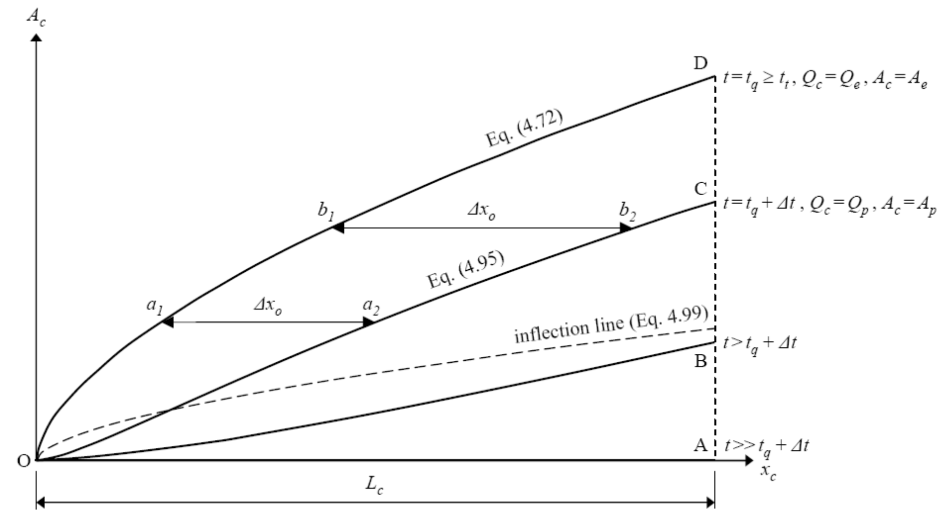

Figure 4.9. Successive Flow Area Profiles during Falling Phase for a Channel without Upstream Inflow.

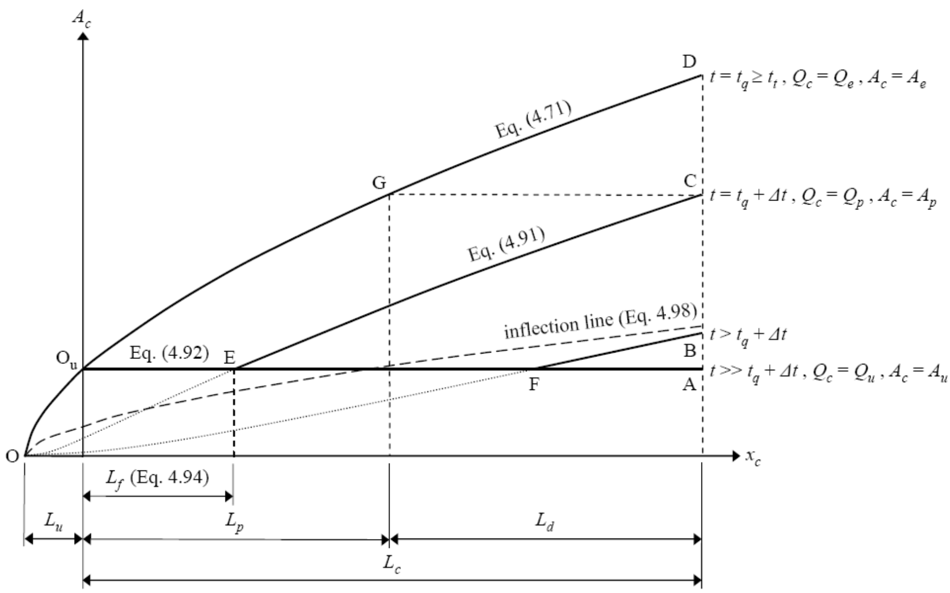

Figure 4.10. Successive Flow Area Profiles during Falling Phase for a Channel with Upstream Inflow.

As shown in figure 4.10, at time t_r, the distance x_c of any point on the equilibrium profile (curve O-O_u-G-D) can be expressed in terms of flow area A_c by substituting Eq. (4.7) into Eq. (4.1):

$$x_c = \frac{\alpha_c A_c^{\beta_c} - Q_u}{q_L} \tag{4.90}$$

Integrating Eq. (4.27) from $\left[\left(\alpha_c A^{\beta_c} - Q_u\right)/q_L\right]$ (Eq. 4.90) to x_c for x_c and from t_q to t (where $t \geq t_q$) for t gives the equation for the curve O-E-C:

$$x_c = \alpha_c \beta_c A_c^{\beta_c - 1}\left(t - t_q\right) + \left(\frac{\alpha_c A_c^{\beta_c} - Q_u}{q_L}\right) \tag{4.91}$$

For a channel with a constant upstream inflow, Eq. (4.91) is only valid for $L_f \leq x_c \leq L_c$, where L_f = length of channel in which the flow equals to upstream inflow during the falling phase. For the profile between $0 \leq x_c \leq L_f$, it is the line O_u-E, i.e.

$$A_c = A_u = \left(\frac{Q_u}{\alpha_c}\right)^{1/\beta_c} \tag{4.92}$$

Substituting $A_c = A_u$ and $x_c = L_f$ into Eq. (4.91) gives the equation for L_f in terms of A_u:

$$L_f = \alpha_c \beta_c A_u^{\beta_o - 1}\left(t - t_q\right) \tag{4.93}$$

Substituting Eq. (4.92) into Eq. (4.93) gives the equation for L_f in terms of Q_u:

$$L_f = \alpha_c^{1/\beta_c} \beta_c Q_u^{(\beta_c - 1)/\beta_c}\left(t - t_q\right) \tag{4.94}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (4.91) reduces to:

$$x_c = \alpha_c \beta_c A_c^{\beta_c - 1}\left(t - t_q\right) + \frac{\alpha_c A_c^{\beta_c}}{q_L} \tag{4.95}$$

which is valid for $0 \leq x_c \leq L_c$. Equations (4.91)-(4.95) are valid for $t \geq t_q$.

4.17.1. Inflection Line

As shown in figure 2.10, the equilibrium flow area profile (curve O-D) is concave downwards, while the flow area profile at time t > $t_q + \Delta t$ (curve O-B) is concave upwards. Similarly, in figure 4.10, the curve O-O_u-D is concave downwards, and the curve O-F-B is concave upwards. The equation for the inflection line can be derived by first obtaining the second derivative of Eq. (4.90), with respect to A_c:

$$\frac{d^2 x_c}{dA_c^2} = \alpha_c \beta_c (\beta_c - 1)(\beta_c - 2) A_c^{\beta_c - 3}\left(t - t_q\right) + \left[\frac{\alpha_c \beta_c (\beta_c - 1) A_c^{\beta_c - 2}}{q_L}\right] \tag{4.96}$$

Next, by equating Eq. (4.96) to zero and equating $A_c = A_i$ results in:

$$A_i = (2-\beta_c)(t-t_q)q_L \tag{4.97}$$

where A_i = flow area of the inflection point. Substituting Eq. (4.97) into Eq. (4.91) gives the equation for the inflection line for a channel with upstream inflow:

$$x_i = \left(\frac{2}{2-\beta_c}\right)\left(\frac{\alpha A_i^{\beta_c}}{q_L}\right) - \left(\frac{Q_u}{q_L}\right) \tag{4.98}$$

where x_i = distance x_c of the inflection point.

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (4.98) reduces to:

$$x_i = \left(\frac{2}{2-\beta_c}\right)\left(\frac{\alpha A_i^{\beta_c}}{q_L}\right) \tag{4.99}$$

Equations (4.98) and (4.99) have been superimposed respectively onto figures 4.9 and 4.10 as dashed lines, which are labeled as inflection lines.

4.18. Hydrograph - Falling Phase

As shown by Eq. (4.88), during the falling phase, water flows out at constant flow area, hence, the water particle at G flows out to C at constant flow area (figure 4.10). The time required for the water particle to flow from G to C is in fact the same as the duration of partial equilibrium discharge, t_d, as shown in figure 4.3. Substituting $t_d = t - t_q$ and $Q_p = Q_c$ into Eq. (4.58) gives:

$$t - t_q = \frac{L_c - \left(\frac{Q_c - Q_u}{q_L}\right)}{c_k} \tag{4.100}$$

Equation (4.99) may also be derived by integrating Eq. (4.26) from t_q to t (where $t \geq t_q$) for t and from $[(Q_c-Q_u)/q_L]$ to L_c for x_c. Since the discharge in the channel cannot be less than upstream discharge, Eq. (4.100) is only valid for $Q_c \geq Q_u$. Substituting Eq. (4.28) into Eq. (4.100) gives the equation for the falling phase (falling limb) of the hydrograph, which is only valid for $Q_c \geq Q_u$:

$$t = \frac{L_c - \left(\frac{Q_c - Q_u}{q_L}\right)}{\alpha_c^{1/\beta_c}\beta_c Q_c^{(\beta_c-1)/\beta_c}} + t_q \tag{4.101}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (4.101) reduces to

$$t = \frac{L_c - \left(\dfrac{Q_c}{q_L}\right)}{\alpha_c^{1/\beta_c} \beta_c Q_c^{(\beta_c - 1)/\beta_c}} + t_q \qquad (4.102)$$

Figures 4.2 and 4.3 show the falling phase (falling limb) of an equilibrium and a partial equilibrium runoff hydrograph for a channel without and for a channel with upstream inflow, respectively.

In: Kinematic-Wave Rainfall-Runoff Formulas
Editor: Tommy S.W. Wong, pp. 63-76
ISBN: 978-1-60692-705-2
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Chapter 5

5. WORKING FORMULAS FOR FLOW IN CIRCULAR CHANNEL

In this Chapter, based on the Manning's equation, the kinematic wave parameters for flow in a circular channel are derived. By applying these parameters to the general formulas in Chapter 4, working formulas for the flow depth; flow velocity; average flow velocity; wave celerity; average wave celerity; time of travel; rising, equilibrium and falling phases of hydrograph; forward characteristic; rising, equilibrium and falling phases of water surface profiles; duration of partial equilibrium discharge; and equilibrium detention storage are also derived.

5.1. KINEMATIC WAVE PARAMETERS

For flow in a circular channel, the flow area A_c, and the wetted perimeter, *P,* are related to the diameter of circular channel *D,* and the water surface angle θ, as follows:

$$A_c = \frac{D^2}{8}(\theta - \sin\theta) \tag{5.1}$$

$$P = \frac{D\theta}{2} \tag{5.2}$$

Figure 5.1 shows the circular channel with diameter *D*, water surface angle θ, and flow depth y_c. Substituting Eqs. (5.1) and (5.2) and $Q = Q_c$, $S = S_c$, $A = A_c$, $n = n_c$ into Eq. (3.1) gives:

$$Q_c = 0.0496\left(\frac{S_c^{1/2} D^{8/3}}{n_c}\right)\left[\frac{(\theta - sin\,\theta)^{5/3}}{\theta^{2/3}}\right] \tag{5.3}$$

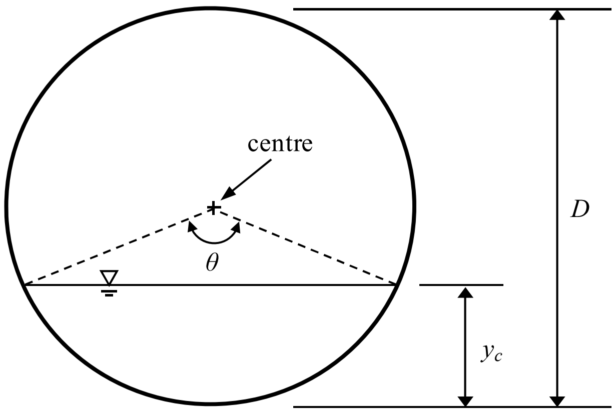

Figure 5.1. Cross-section of Circular Channel.

For full flow condition in a circular channel (i.e. $\theta = 2\pi$), Eqs. (5.1) and (5.3) become:

$$A_{full} = \frac{\pi D^2}{4} \tag{5.4}$$

$$Q_{full} = 0.312\left(\frac{S_c^{1/2} D^{8/3}}{n_c}\right) \tag{5.5}$$

where A_{full} = flow area under full flow condition, and Q_{full} = discharge under full flow condition. Dividing Eq. (5.3) by Eq. (5.5) and Eq. (5.1) by Eq. (5.4) give:

$$\frac{Q_c}{Q_{full}} = \frac{1}{2\pi}\left[\frac{(\theta - sin\,\theta)^{5/3}}{\theta^{2/3}}\right] \tag{5.6}$$

$$\frac{A_c}{A_{full}} = \frac{(\theta - sin\,\theta)}{2\pi} \tag{5.7}$$

Equations (5.6) and (5.7) are considered to be the true relationship between discharge and flow area for flow in a circular channel. This true relationship is shown in figure 5.2, and it is apparent that the discharge reaches a maximum under the partially full flow condition. Differentiating Q_c with respect to θ in Eq. (5.3) gives:

$$\frac{dQ_c}{d\theta} = 0.0165\left(\frac{S_c^{1/2} D^{8/3}}{n_c}\right)\left(\frac{\theta - sin\,\theta}{\theta}\right)^{2/3}\left(\frac{2\,sin\,\theta}{\theta} - 5\,cos\,\theta + 3\right) \tag{5.8}$$

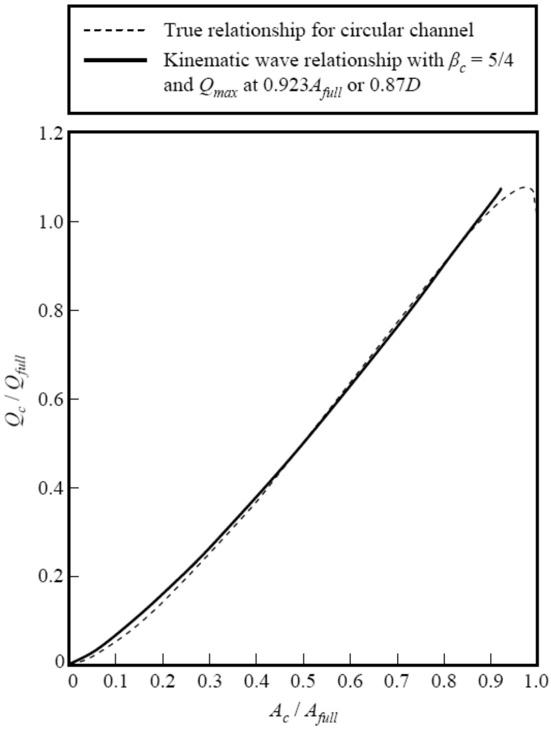

Figure 5.2. Comparison between True and Kinematic Wave Relationships for Flow in Circular Channel.

Equating Eq. (5.8) to zero shows that the maximum discharge, Q_{max}, occurs at θ = 5.278 rad (or 302.4°). Substituting θ = 5.278 rad into Eq. (5.6) gives:

$$\frac{Q_{max}}{Q_{full}} = 1.076 \tag{5.9}$$

Substituting Eq. (5.5) into Eq. (5.9) gives:

$$Q_{max} = 0.335 D^{8/3}\left(\frac{S_c^{1/2}}{n_c}\right) \tag{5.10}$$

To evaluate the kinematic wave parameters α_c and β_c using the same method that was used by Harley et al (1970), the parameters α_c and β_c are related to Q_{max} and A_{max} as follows:

$$Q_{\max} = \alpha_c A_{Q\max}^{\beta_c} \tag{5.11}$$

where A_{Qmax} = flow area under maximum discharge condition. Further, relating A_{Qmax} to D through a parameter γ:

$$\gamma = \frac{A_{Q\max}}{D^2} \tag{5.12}$$

Substituting D in Eq. (5.4) into Eq. (5.12) gives a relationship between γ and A_{full}:

$$\gamma = \frac{\pi A_{Qmax}}{4 A_{full}} \tag{5.13}$$

Substituting Eqs. (5.10) and (5.12) into Eq. (5.11) gives a relationship between α_c and γ:

$$\alpha_c = \left[\frac{0.335 D^{(8/3)-2\beta_c}}{\gamma^{\beta_c}} \right] \left(\frac{S_c^{1/2}}{n_c} \right) \tag{5.14}$$

Equation (5.14) shows that the value of α_c is dependent on the value of γ, which is dependent on the flow area A_{Qmax} (Eq. 5.16). To identify the values of α_c and β_c, Eq. (4.7) is divided by Eq. (5.11):

$$\frac{Q_c}{Q_{\max}} = \left(\frac{A_c}{A_{Q\max}} \right)^{\beta_c} \tag{5.15}$$

Substituting Eqs. (5.9) and (5.13) into (5.15) gives:

$$\frac{Q_c}{Q_{full}} = 1.076 \left(\frac{\pi}{4\gamma} \right)^{\beta_c} \left(\frac{A_c}{A_{full}} \right)^{\beta_c} \tag{5.16}$$

Wong and Zhou (2003) fitted the kinematic wave relationship (Eq. 5.19) to the true relationship (Eqs. 5.7 and 5.8), and found that the best fit occurs at $A_{Qmax}/A_{full} = 0.923$ (which corresponds to $y_{Qmax}/D = 0.87$ where y_{Qmax} = flow depth under maximum discharge condition), $\gamma = 0.725$, and $\beta_c = 5/4$, as shown in figure 5.2. Substituting $\beta_c = 5/4$ and $\gamma = 0.725$ into Eq. (5.14), gives the kinematic wave parameters, which are valid for $y_c \leq 0.87D$:

$$\alpha_c = 0.501 \left(\frac{S_c^{1/2} D^{1/6}}{n_c} \right) \tag{5.17}$$

$$\beta_c = \frac{5}{4} \tag{5.18}$$

5.2. Flow Depth

For flow in a circular channel, the flow depth, y_c, is related to D and θ, as follows:

$$y_c = \frac{D}{2}\left[1 - cos\left(\frac{\theta}{2}\right)\right] \tag{5.19}$$

Equating θ in Eq. (5.19) to that in Eq. (5.1), and by curve fitting results in the following equation relating A_c to y_c:

$$A_c = \frac{\pi D^2}{4}\left[-1.195\left(\frac{y_c}{D}\right)^3 + 1.801\left(\frac{y_c}{D}\right)^2 + 0.397\left(\frac{y_c}{D}\right)\right] \tag{5.20}$$

Substituting Eqs. (5.17) and (5.18) into Eq. (4.7) gives:

$$Q_c = 0.501\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right) A_c^{5/4} \tag{5.21}$$

Substituting Eqs. (5.20) into Eq. (5.21) gives:

$$Q_c = 0.370\left(\frac{S_c^{1/2} D^{8/3}}{n_c}\right)\left[-1.195\left(\frac{y_c}{D}\right)^3 + 1.801\left(\frac{y_c}{D}\right)^2 + 0.397\left(\frac{y_c}{D}\right)\right]^{5/4} \tag{5.22}$$

Rearranging Eq. (5.22) gives the equation for the flow depth for a channel with and without upstream inflow:

$$-1.195\left(\frac{y_c}{D}\right)^3 + 1.801\left(\frac{y_c}{D}\right)^2 + 0.397\left(\frac{y_c}{D}\right) = 2.213\left(\frac{n_c Q_c}{S_c^{1/2} D^{8/3}}\right)^{4/5} \tag{5.23}$$

5.3. Flow Velocity

Substituting Eqs. (5.17) and (5.18) into Eq. (4.12), gives the equation for the flow velocity along the equilibrium profile for a channel with upstream inflow:

$$v = 0.575\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)^{4/5} \left(Q_u + q_L x_c\right)^{1/5} \tag{5.24}$$

For a channel with zero upstream (Q_u = 0), Eq. (5.24) reduces to:

$$v = 0.575\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)^{4/5} \left(q_L x_c\right)^{1/5} \tag{5.25}$$

5.4. Average Flow Velocity

Substituting Eqs. (5.17) and (5.18) into Eq. (4.15) gives the equation for the average flow velocity for a channel with upstream inflow:

$$v_{av} = \frac{0.460\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)^{4/5} q_L L_c}{\left(Q_u + q_L L_c\right)^{4/5} - Q_u^{\;4/5}} \tag{5.26}$$

For a channel with zero upstream inflow (Q_u = 0), Eq. (5.26) reduces to:

$$v_{av} = 0.460\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)^{4/5} \left(q_L L_c\right)^{1/5} \tag{5.27}$$

5.5. Kinematic Wave Celerity

Substituting Eqs. (5.17) and (5.18) into Eq. (4.29) gives the equation for the wave celerity along the equilibrium profile for a channel with upstream inflow:

$$c_k = 0.719\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)^{4/5} \left(Q_u + q_L x_c\right)^{1/5} \tag{5.28}$$

For a channel with zero upstream inflow (Q_u = 0), Eq. (5.28) reduces to:

$$c_k = 0.719\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)^{4/5} \left(q_L x_c\right)^{1/5} \tag{5.29}$$

5.6. Average Wave Celerity

Substituting Eqs. (5.17) and (5.18) into Eq.(4.32) gives the equation for the wave celerity for a channel with upstream inflow:

$$c_{av} = \frac{0.575\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)^{4/5} q_L L_c}{\left(Q_u + q_L L_c\right)^{4/5} - Q_u^{\;4/5}} \tag{5.30}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (5.30) reduces to:

$$c_{av} = 0.575\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)^{4/5} \left(q_L L_c\right)^{1/5} \tag{5.31}$$

5.7. TIME OF TRAVEL

Substituting Eqs. (5.17) and (5.18) into Eq. (4.35) gives the equation for the time of travel for a channel with upstream inflow:

$$t_t = 0.0290\left(\frac{n_c}{S_c^{1/2} D^{1/6}}\right)^{4/5} \left[\frac{\left(Q_u + q_L L_c\right)^{4/5} - Q_u^{4/5}}{q_L}\right] \tag{5.32}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (5.32) reduces to:

$$t_t = \left(\frac{0.0290}{q_L^{1/5}}\right)\left(\frac{n_c L_c}{S_c^{1/2} D^{1/6}}\right)^{4/5} \tag{5.33}$$

5.8. HYDROGRAPH – RISING PHASE

Substituting Eqs. (5.17) and (5.18) into Eq. (4.45) gives the equation for the rising phase (rising limb) of the hydrograph for a channel with upstream inflow:

$$Q_c = 0.501\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)\left[1.738\left(\frac{n_c Q_u}{S_c^{1/2} D^{1/6}}\right)^{4/5} + 60 q_L t\right]^{5/4} \tag{5.34}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (5.34) reduces to:

$$Q_c = 83.66\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)\left(q_L t\right)^{5/4} \tag{5.35}$$

Equations (5.34) and (5.35) are valid for $t \le t_t$.

5.9. FORWARD CHARACTERISTIC - RISING PHASE

Substituting Eqs. (5.17) and (5.18) into Eq. (4.47) gives the equation for the forward characteristic of the rising phase for a channel with upstream inflow:

$$t = 0.0290\left(\frac{n_c}{S_c^{1/2}D^{1/6}}\right)^{4/5}\left[\frac{\left(Q_u + q_L x_c\right)^{4/5} - Q_u^{4/5}}{q_L}\right] \tag{5.36}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (5.36) reduces to:

$$t = \left(\frac{0.0290}{q_L^{1/5}}\right)\left(\frac{n_c x_c}{S_c^{1/2}D^{1/6}}\right)^{4/5} \tag{5.37}$$

5.10. WATER SURFACE PROFILE – RISING PHASE

Substituting Eqs. (5.17) and (5.18) into Eq. (4.49) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$A_c = 1.738\left[\left(\frac{n_c}{S_c^{1/2}D^{1/6}}\right)\left(Q_u + q_L x_c\right)\right]^{4/5} \tag{5.38}$$

Substituting Eqs. (5.20) into Eq. (5.38) gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$-1.195\left(\frac{y_c}{D}\right)^3 + 1.801\left(\frac{y_c}{D}\right)^2 + 0.397\left(\frac{y_c}{D}\right) = 2.213\left[\left(\frac{n_c}{S_c^{1/2}D^{8/3}}\right)\left(Q_u + q_L x_c\right)\right]^{4/5} \tag{5.39}$$

Substituting Eqs. (5.16) and (5.17) into Eq. (4.50) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $L_p \le x_c \le L_c$:

$$A_p = 1.738\left[\left(\frac{n_c}{S_c^{1/2}D^{1/6}}\right)\left(Q_u + q_L L_p\right)\right]^{4/5} \tag{5.40}$$

Substituting $A_c = A_p$, and $y_c = y_p$ into Eq. (5.20), and then substituting it into Eq. (5.40) gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $L_p \le x_c \le L_c$:

$$-1.195\left(\frac{y_p}{D}\right)^3 + 1.801\left(\frac{y_p}{D}\right)^2 + 0.397\left(\frac{y_p}{D}\right) = 2.213\left[\left(\frac{n_c}{S_c^{1/2}D^{8/3}}\right)\left(Q_u + q_L L_p\right)\right]^{4/5} \quad (5.41)$$

From Eq. (4.51), the distance L_p is:

$$L_p = \frac{Q_p - Q_u}{q_L} \quad (5.42)$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (5.38)-(5.42) reduce to:

$$A_c = 1.738\left(\frac{n_c q_L x_c}{S_c^{1/2}D^{1/6}}\right)^{4/5} \quad (5.43)$$

$$-1.195\left(\frac{y_c}{D}\right)^3 + 1.801\left(\frac{y_c}{D}\right)^2 + 0.397\left(\frac{y_c}{D}\right) = 2.213\left(\frac{n_c q_L x_c}{S_c^{1/2}D^{8/3}}\right)^{4/5} \quad (5.44)$$

which are valid for $0 \le x_c \le L_p$,

$$A_p = 1.738\left(\frac{n_c q_L L_p}{S_c^{1/2}D^{1/6}}\right)^{4/5} \quad (5.45)$$

$$-1.195\left(\frac{y_p}{D}\right)^3 + 1.801\left(\frac{y_p}{D}\right)^2 + 0.397\left(\frac{y_p}{D}\right) = 2.213\left(\frac{n_c q_L L_p}{S_c^{1/2}D^{8/3}}\right)^{4/5} \quad (5.46)$$

which are valid for $L_p \le x_c \le L_c$, and

$$L_p = \frac{Q_p}{q_L} \quad (5.47)$$

5.11. DURATION OF PARTIAL EQUILIBRIUM DISCHARGE

Substituting Eqs. (5.17) and (5.18) into Eq. (4.63) gives the equation for the duration of partial equilibrium discharge for a channel with upstream inflow:

$$t_d = \frac{L_c q_L + Q_u - 0.501\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)\left[1.738\left(\frac{n_c Q_u}{S_c^{1/2} D^{1/6}}\right)^{4/5} + 60 q_L t_q\right]^{5/4}}{37.58\left(\frac{S_c^{1/2} D^{1/6} q_L}{n_c}\right)\left[1.738\left(\frac{n_c Q_u}{S_c^{1/2} D^{1/6}}\right)^{4/5} + 60 q_L t_q\right]^{1/4}} \quad (5.48)$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (5.41) reduces to:

$$t_d = \frac{L_c - 83.66\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right) q_L^{1/4} t_q^{5/4}}{104.59\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)\left(q_L t_q\right)^{1/4}} \quad (5.49)$$

5.12. HYDROGRAPH - EQUILIBRIUM PHASE

As shown in figures 4.2 and 4.3, the hydrograph may reach partial equilibrium or equilibrium depending on the duration of lateral inflow t_q. If $t_q < t_t$, the hydrograph reaches partial equilibrium with a constant discharge Q_p. If $t_q \geq t_t$, the hydrograph reaches equilibrium with a constant discharge Q_e.

5.12.1. Partial Equilibrium Discharge

Substituting Eqs. (5.17) and (5.18) into Eq. (4.66) gives the equation for the partial equilibrium discharge for a channel with upstream inflow:

$$Q_p = 0.501\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)\left[1.738\left(\frac{n_c Q_u}{S_c^{1/2} D^{1/6}}\right)^{4/5} + 60 q_L t_q\right]^{5/4} \quad (5.50)$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (5.50) reduces to:

$$Q_p = 83.66\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)\left(q_L t_q\right)^{5/4} \quad (5.51)$$

Equations (5.50) and (5.51) are valid for $t_q \leq t \leq (t_q + t_d)$.

5.12.2. Equilibrium Discharge

From Eq. (4.69), the equation for the equilibrium discharge for a channel with upstream inflow is:

$$Q_e = Q_u + q_L L_c \tag{5.52}$$

For a channel with zero upstream inflow (Q_u = 0), Eq. (5.52) reduces to:

$$Q_e = q_L L_c \tag{5.53}$$

Equations (5.52) and (5.53) are valid for $t_t \le t \le t_q$.

5.13. WATER SURFACE PROFILE - EQUILIBRIUM PHASE

Substituting Eqs. (5.17) and (5.18) into Eq. (4.71) gives the equation for the equilibrium flow area profile for a channel with upstream inflow between $-L_u \le x_c \le L_c$:

$$A_c = 1.738\left[\left(\frac{n_c}{S_c^{1/2} D^{1/6}}\right)(Q_u + q_L x_c)\right]^{4/5} \tag{5.54}$$

Substituting Eqs. (5.20) into Eq. (5.54) gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $-L_u \le x_c \le L_c$:

$$-1.195\left(\frac{y_c}{D}\right)^3 + 1.801\left(\frac{y_c}{D}\right)^2 + 0.397\left(\frac{y_c}{D}\right) = 2.213\left[\left(\frac{n_c}{S_c^{1/2} D^{8/3}}\right)(Q_u + q_L x_c)\right]^{4/5} \tag{5.55}$$

For a channel with zero upstream inflow (Q_u = 0), Eqs. (5.54) and (5.55) reduce to:

$$A_c = 1.738\left(\frac{n_c q_L x_c}{S_c^{1/2} D^{1/6}}\right)^{4/5} \tag{5.56}$$

$$-1.195\left(\frac{y_c}{D}\right)^3 + 1.801\left(\frac{y_c}{D}\right)^2 + 0.397\left(\frac{y_c}{D}\right) = 2.213\left(\frac{n_c q_L x_c}{S_c^{1/2} D^{8/3}}\right)^{4/5} \tag{5.57}$$

which are valid for $0 \le x_c \le L_c$.

5.14. EQUILIBRIUM DETENTION STORAGE

Substituting Eqs. (5.17) and (5.18) into Eq. (4.78) gives the equation for the equilibrium detention storage for a channel with upstream inflow:

$$D_{ec} = 0.966\left(\frac{n_c}{S_c^{1/2}D^{1/6}}\right)^{4/5}\left[\frac{\left(Q_u + q_L L_c\right)^{9/5} - Q_u^{9/5}}{q_L}\right] \tag{5.58}$$

For a channel with zero upstream inflow (i.e. $Q_u = 0$), Eq. (5.58) reduces to:

$$D_{ec} = 0.966\left(\frac{n_c q_L L_c^{9/4}}{S_c^{1/2}D^{1/6}}\right)^{4/5} \tag{5.59}$$

5.15. WATER SURFACE PROFILE – FALLING PHASE

Substituting Eqs. (5.17) and (5.18) into Eq. (4.91) gives the equation for the falling phase of the flow area profile for a channel with upstream inflow, which is valid for $L_f \le x_c \le L_c$:

$$x_c = 37.58\left(\frac{S_c^{1/2}D^{1/6}A_c^{1/4}}{n_c}\right)\left(t - t_q\right) + \left\{\frac{\left[0.501\left(\frac{S_c^{1/2}D^{1/6}A_c^{5/4}}{n_c}\right)\right] - Q_u}{q_L}\right\} \tag{5.60}$$

Substituting Eq. (5.20) into Eq. (5.60) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $L_f \le x_c \le L_c$:

$$x_c = 35.38\left(\frac{S_c^{1/2}D^{2/3}}{n_c}\right)\left[-1.195\left(\frac{y_c}{D}\right)^3 + 1.801\left(\frac{y_c}{D}\right)^2 + 0.397\left(\frac{y_c}{D}\right)\right]^{1/4}\left(t - t_q\right)$$
$$+\left(\frac{\left\{0.370\left(\frac{S_c^{1/2}D^{8/3}}{n_c}\right)\left[-1.195\left(\frac{y_c}{D}\right)^3 + 1.801\left(\frac{y_c}{D}\right)^2 + 0.397\left(\frac{y_c}{D}\right)\right]^{5/4}\right\} - Q_u}{q_L}\right) \tag{5.61}$$

From Eq. (4.92), the equation for the flow area profile between $0 \le x_c \le L_f$ is:

$$A_c = 1.738\left(\frac{n_c Q_u}{S_c^{1/2} D^{1/6}}\right)^{4/5} \tag{5.62}$$

Substituting Eq. (5.20) into Eq. (5.62) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_f$:

$$-1.195\left(\frac{y_c}{D}\right)^3 + 1.801\left(\frac{y_c}{D}\right)^2 + 0.397\left(\frac{y_c}{D}\right) = 2.213\left(\frac{n_c Q_u}{S_c^{1/2} D^{8/3}}\right)^{4/5} \tag{5.63}$$

Substituting Eqs. (5.17) and (5.18) into Eq. (4.94) gives the equation for the distance L_f for a channel with upstream inflow, which is valid for $t \ge t_q$:

$$L_f = 43.15\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)^{4/5} Q_u^{1/5}\left(t - t_q\right) \tag{5.64}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (5.60) and (5.61) reduce to:

$$x_c = 37.58\left(\frac{S_c^{1/2} D^{1/6} A_c^{1/4}}{n_c}\right)\left(t - t_q\right) + \left[\frac{0.501\left(\frac{S_c^{1/2} D^{1/6} A_c^{5/4}}{n_c}\right)}{q_L}\right] \tag{5.65}$$

$$x_c = 35.38\left(\frac{S_c^{1/2} D^{2/3}}{n_c}\right)\left[-1.195\left(\frac{y_c}{D}\right)^3 + 1.801\left(\frac{y_c}{D}\right)^2 + 0.397\left(\frac{y_c}{D}\right)\right]^{1/4}\left(t - t_q\right)$$
$$+\left(\frac{\left\{0.370\left(\frac{S_c^{1/2} D^{8/3}}{n_c}\right)\left[-1.195\left(\frac{y_c}{D}\right)^3 + 1.801\left(\frac{y_c}{D}\right)^2 + 0.397\left(\frac{y_c}{D}\right)\right]^{5/4}\right\}}{q_L}\right) \tag{5.66}$$

which are valid for $0 \le x_c \le L_c$.

5.16. HYDROGRAPH - FALLING PHASE

Substituting Eqs. (5.17) and (5.18) into Eq. (4.101) gives the equation for the falling phase (falling limb) of a hydrograph for a channel with upstream inflow:

$$t = \left(\frac{0.0232}{Q_c^{1/5}} \right) \left(\frac{n_c}{S_c^{1/2} D^{1/6}} \right)^{4/5} \left[L_c - \left(\frac{Q_c - Q_u}{q_L} \right) \right] + t_q \tag{5.67}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (5.67) reduces to:

$$t = \left(\frac{0.0232}{Q_c^{1/5}} \right) \left(\frac{n_c}{S_c^{1/2} D^{1/6}} \right)^{4/5} \left[L_c - \left(\frac{Q_c}{q_L} \right) \right] + t_q \tag{5.68}$$

In: Kinematic-Wave Rainfall-Runoff Formulas
Editor: Tommy S.W. Wong, pp. 77-89
ISBN: 978-1-60692-705-2
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Chapter 6

6. WORKING FORMULAS FOR FLOW IN PARABOLIC CHANNEL

In this Chapter, based on the Manning's equation, the kinematic wave parameters for flow in a parabolic channel are derived. By applying these parameters to the general formulas in Chapter 4, working formulas for the flow depth; flow velocity; average flow velocity; wave celerity; average wave celerity; time of travel; rising, equilibrium and falling phases of hydrograph; forward characteristic; rising, equilibrium and falling phases of water surface profiles; duration of partial equilibrium discharge; and equilibrium detention storage are also derived.

6.1. KINEMATIC WAVE PARAMETERS

For the parabolic channel as shown in figure 6.1, the channel section can be described mathematically by:

$$y = \frac{x^2}{4H} \tag{6.1}$$

where y = height above the channel invert, x = semi-width at height y, and H = height of focal point above channel invert. From mathematics, the flow area A_c can be related to x and H, as follows:

$$A_c = \frac{x^3}{3H} \tag{6.2}$$

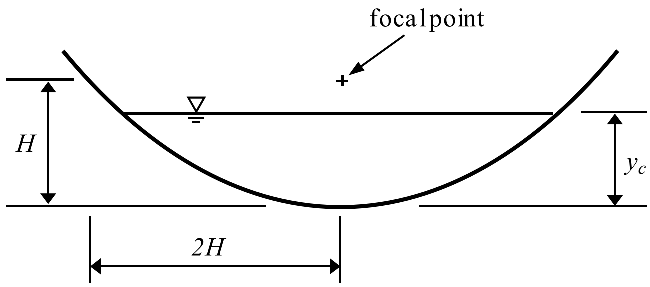

Figure 6.1. Cross-section of Parabolic Channel.

The arc length, L_a, of the parabola can be derived by integrating Eq. (6.1), as follows:

$$L_a = \int \left[dx^2 + dy^2 \right]^{1/2} = \int \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{1/2} dx \tag{6.3}$$

Upon integration, the arc length, L_a, of the parabola is:

$$L_a = \left[\frac{1}{2(2H)} \right] \left[xZ + (2H)^2 \ln(x' + Z') \right] \tag{6.4}$$

where

$$Z = \left[(2H)^2 + x^2 \right]^{1/2} \tag{6.5}$$

$$x' = \frac{x}{2H} \tag{6.6}$$

and

$$Z' = \frac{Z}{2H} = \left(1 + x'^2 \right)^{1/2} \tag{6.7}$$

In Eq. (6.4), the trigonometric equivalent of the logarithmic term is:

$$\ln(x' + Z') = \sinh^{-1}(x') \tag{6.8}$$

Equation (6.6) is defined as the dimensionless ratio of the flow semi-width to focal semi-width.

As derived from Eq. (6.4), the wetted perimeter P being twice the arc length L_a is:

$$P = \left(\frac{1}{2H}\right)\left[xZ + (2H)^2 \ln(x' + Z')\right] \quad (6.9)$$

Dividing Eq. (6.9) by (2*H*) gives the equation in a dimensionless form, as follows:

$$P' = \frac{P}{2H} = x'Z' + \ln(x' + Z') \quad (6.10)$$

To eliminate *x* in Eqs. (6.2)-(6.10), Eq. (6.2) is converted to a dimensionless form, as follows:

$$A_c' = \frac{A_c}{(2H)^2} = \frac{x^3}{3(2H)^2 H} = \frac{x'^3}{1.5} \quad (6.11)$$

Rearranging Eq. (6.11) gives:

$$x' = \left(\frac{3}{2}A_c'\right)^{1/3} = 1.145(A_c')^{1/3} \quad (6.12)$$

Substituting Eq. (6.12) into Eq. (6.7) gives:

$$Z' = \left[1 + 1.311(A_c')^{2/3}\right]^{1/2} \quad (6.13)$$

Substituting Eqs. (6.8), (6.12) and (6.13) into Eq. (6.10) gives:

$$P' = 1.145(A_c')^{1/3}\left[1 + 1.311(A_c')^{2/3}\right]^{1/2} + \sinh^{-1}\left[1.145(A_c')^{1/3}\right] \quad (6.14)$$

To expand Eq. (6.14) into a series, the following series expansions are used:

$$\left(1 + x'^2\right)^{1/2} = 1 + \frac{1}{2}x'^2 - \frac{1}{8}x'^4 + \frac{1}{16}x'^6 - \ldots\ldots \quad (6.15)$$

$$\sinh^{-1}(x') = x' - \frac{1}{6}x'^3 + \frac{3}{40}x'^5 - \frac{5}{112}x'^7 + \ldots\ldots \quad (6.16)$$

Equations (6.15) and (6.16) are only valid for $x' < 1$. Substituting Eqs. (6.15) and (6.16) into Eq. (6.14), and cancelling the higher order terms, the equation becomes:

$$P' \approx 2x' \quad (6.17)$$

Substituting Eq. (6.12) into Eq. (6.17) gives:

$$P' = 2.290(A_c')^{1/3} \tag{6.18}$$

Substituting Eqs. (6.10), and (6.11) into Eq. (6.18) gives a relationship between P and A_c, as follows:

$$P = 2.885(HA_c)^{1/3} \tag{6.19}$$

Brady (1983) showed that Eq. (6.19) is valid for

$$x' < 0.6 \tag{6.20}$$

As x is related to the top width, T, as:

$$x = \frac{T}{2} \tag{6.21}$$

and T is related to the flow depth, y_c, as:

$$T = 4(Hy_c)^{1/2} \tag{6.22}$$

Substituting Eqs. (6.20)-(6.22) into Eq. (6.6) gives:

$$y_c < 0.18B \tag{6.23}$$

Substituting Eq. (6.19) and $Q = Q_c$, $S = S_c$, $A = A_c$, $n = n_c$ into Eq. (3.1) gives:

$$Q_c = 0.493\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)A_c^{13/9} \tag{6.24}$$

A comparison of Eqs. (6.24) with Eq. (4.7) gives the kinematic wave parameters (Brady 1983), which are valid for $y_c < 0.18H$:

$$\alpha_c = 0.493\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right) \tag{6.25}$$

$$\beta_c = \frac{13}{9} \tag{6.26}$$

6.2. Flow Depth

For flow in a parabolic channel, the flow area A_c, is related to the flow depth y_c, and the parabola's focal height H, as follows (Jan 1979):

$$A_c = \frac{8H^{1/2}y_c^{3/2}}{3} \tag{6.27}$$

Substituting Eq. (6.27) into Eq. (6.24) gives:

$$Q_c = 2.033\left(\frac{S_c^{1/2}H^{1/2}}{n_c}\right)y_c^{13/6} \tag{6.28}$$

Rearranging Eq. (6.28) gives the equation for the flow depth for a channel with and without upstream inflow:

$$y_c = 0.721\left(\frac{n_c Q_c}{S_c^{1/2}H^{1/2}}\right)^{6/13} \tag{6.29}$$

6.3. Flow Velocity

Substituting Eqs. (6.25) and (6.26) into Eq. (4.12) gives the equation for the flow velocity along the equilibrium profile for a channel with upstream inflow:

$$v = 0.613\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)^{9/13}\left(Q_u + q_L x_c\right)^{4/13} \tag{6.30}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (6.30) reduces to:

$$v = 0.613\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)^{9/13}\left(q_L x_c\right)^{4/13} \tag{6.31}$$

6.4. Average Flow Velocity

Substituting Eqs. (6.25) and (6.26) into Eq. (4.15) gives the equation for the average flow velocity for a channel with upstream inflow:

$$v_{av} = \frac{0.424\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)^{9/13} q_L L_c}{\left(Q_u + q_L L_c\right)^{9/13} - Q_u^{9/13}} \tag{6.32}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (6.32) reduces to:

$$v_{av} = 0.424\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)^{9/13} \left(q_L L_c\right)^{4/13} \tag{6.33}$$

6.5. KINEMATIC WAVE CELERITY

Substituting Eqs. (6.25) and (6.26) into Eq. (4.29) gives the equation for the wave celerity along the equilibrium profile for a channel with upstream inflow:

$$c_k = 0.885\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)^{9/13} \left(Q_u + q_L x_c\right)^{4/13} \tag{6.34}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (6.34) reduces to:

$$c_k = 0.885\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)^{9/13} \left(q_L x_c\right)^{4/13} \tag{6.35}$$

6.6. AVERAGE WAVE CELERITY

Substituting Eqs. (6.25) and (6.26) into Eq. (4.32) gives the equation for the average wave celerity for a channel with upstream inflow:

$$c_{av} = \frac{0.613\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)^{9/13} q_L L_c}{\left(Q_u + q_L L_c\right)^{9/13} - Q_u^{9/13}} \tag{6.36}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (6.36) reduces to:

$$c_{av} = 0.613\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)^{9/13} \left(q_L L_c\right)^{4/13} \tag{6.37}$$

6.7. TIME OF TRAVEL

Substituting Eqs. (6.25) and (6.26) into Eq. (4.35) gives the equation for the time of travel for a channel with upstream inflow:

$$t_t = 0.0272\left(\frac{n_c H^{2/9}}{S_c^{1/2}}\right)^{9/13}\left[\frac{(Q_u + q_L L_c)^{9/13} - Q_u^{9/13}}{q_L}\right] \tag{6.38}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (6.38) reduces to:

$$t_t = \frac{0.0272}{q_L^{4/13}}\left(\frac{n_c H^{2/9} L_c}{S_c^{1/2}}\right)^{9/13} \tag{6.39}$$

6.8. HYDROGRAPH – RISING PHASE

Substituting Eqs. (6.25) and (6.26) into Eq. (4.45) gives the equation for the rising phase (rising limb) of the hydrograph for a channel with upstream inflow:

$$Q_c = 0.493\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)\left[1.639\left(\frac{n_c H^{2/9} Q_u}{S_c^{1/2}}\right)^{9/13} + 60 q_L t\right]^{13/9} \tag{6.40}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (6.41) reduces to:

$$Q_c = 182.5\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)(q_L t)^{13/9} \tag{6.41}$$

Equations (6.40) and (6.41) are valid for $t \le t_t$.

6.9. FORWARD CHARACTERISTIC - RISING PHASE

Substituting Eqs. (6.25) and (6.26) into Eq. (4.47) gives the equation for the forward characteristic of the rising phase for a channel with upstream inflow:

$$t = 0.0272\left(\frac{n_c H^{2/9}}{S_c^{1/2}}\right)^{9/13}\left[\frac{(q_L x_c + Q_u)^{9/13} - Q_u^{9/13}}{q_L}\right] \tag{6.42}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (6.42) reduces to:

$$t = \left(\frac{0.0272}{q_L^{4/13}}\right)\left(\frac{n_c H^{2/9} x_c}{S_c^{1/2}}\right)^{9/13} \tag{6.43}$$

6.10. WATER SURFACE PROFILE – RISING PHASE

Substituting Eqs. (6.25) and (6.26) into Eq. (4.49) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$A_c = 1.632\left[\left(\frac{n_c H^{2/9}}{S_c^{1/2}}\right)(Q_u + q_L x_c)\right]^{9/13} \tag{6.44}$$

Substituting Eq. (6.27) into Eq. (6.44) gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$y_c = 0.721\left[\left(\frac{n_c}{S_c^{1/2} H^{1/2}}\right)(Q_u + q_L x_c)\right]^{6/13} \tag{6.45}$$

Substituting Eqs. (6.25) and (6.26) into Eq. (4.50) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $L_p \le x_c \le L_c$:

$$A_p = 1.632\left[\left(\frac{n_c H^{2/9}}{S_c^{1/2}}\right)(Q_u + q_L L_p)\right]^{9/13} \tag{6.46}$$

Substituting $A_c = A_p$, and $y_c = y_p$ into Eq. (6.27) and then substituting it into Eq. (6.46) gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $L_p \le x_c \le L_c$:

$$y_c = 0.721\left[\left(\frac{n_c}{S_c^{1/2} H^{1/2}}\right)(Q_u + q_L L_p)\right]^{6/13} \tag{6.47}$$

From Eq. (4.51), the distance L_p is:

$$L_p = \frac{Q_p - Q_u}{q_L} \tag{6.48}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (6.44)-(6.48) reduce to:

$$A_c = 1.632\left(\frac{n_c H^{2/9} q_L x_c}{S_c^{1/2}}\right)^{9/13} \tag{6.49}$$

$$y_c = 0.721\left(\frac{n_c q_L x_c}{S_c^{1/2} H^{1/2}}\right)^{6/13} \tag{6.50}$$

which are valid for $0 \le x_o \le L_p$,

$$A_p = 1.632\left(\frac{n_c H^{2/9} q_L L_p}{S_c^{1/2}}\right)^{9/13} \tag{6.51}$$

$$y_c = 0.721\left(\frac{n_c q_L L_p}{S_c^{1/2} H^{1/2}}\right)^{6/13} \tag{6.52}$$

which are valid for $L_p \le x_c \le L_c$, and

$$L_p = \frac{Q_p}{q_L} \tag{6.53}$$

6.11. DURATION OF PARTIAL EQUILIBRIUM DISCHARGE

Substituting Eqs. (6.25) and (6.26) into Eq. (4.63) gives the equation for the duration of partial equilibrium discharge for a channel with upstream inflow:

$$t_d = \frac{L_c q_L + Q_u - 0.493\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)\left[1.632\left(\frac{n_c H^{2/9} Q_u}{S_c^{1/2}}\right)^{9/13} + 60 q_L t_q\right]^{13/9}}{42.73\left(\frac{S_c^{1/2} q_L}{n_c H^{2/9}}\right)\left[1.632\left(\frac{n_c H^{2/9} Q_u}{S_c^{1/2}}\right)^{9/13} + 60 q_L t_q\right]^{4/9}} \tag{6.54}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (6.54) reduces to:

$$t_d = \frac{L_c - 182.5\left(\dfrac{S_c^{1/2}}{n_c H^{2/9}}\right) q_L^{4/9} t_q^{13/9}}{263.6\left(\dfrac{S_c^{1/2}}{n_c H^{2/9}}\right)\left(q_L t_q\right)^{4/9}} \tag{6.55}$$

6.12. HYDROGRAPH - EQUILIBRIUM PHASE

As shown in figures 4.2 and 4.3, the hydrograph may reach partial equilibrium or equilibrium depending on the duration of lateral inflow t_q. If $t_q < t_t$, the hydrograph reaches partial equilibrium with a constant discharge Q_p. If $t_q \geq t_t$, the hydrograph reaches equilibrium with a constant discharge Q_e.

6.12.1. Partial Equilibrium Discharge

Substituting Eqs. (6.25) and (6.26) into Eq. (4.66) gives the equation for the partial equilibrium discharge for a channel with upstream inflow:

$$Q_p = 0.493\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)\left[1.639\left(\frac{n_c H^{2/9} Q_u}{S_c^{1/2}}\right)^{9/13} + 60 q_L t_q\right]^{13/9} \tag{6.56}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (5.56) reduces to:

$$Q_p = 182.5\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)\left(q_L t_q\right)^{13/9} \tag{6.57}$$

Equations (5.56) and (5.57) are valid for $t_q \leq t \leq (t_q + t_d)$.

6.12.2. Equilibrium Discharge

From Eq. (4.69), the equation for the equilibrium discharge for a channel with upstream inflow is:

$$Q_e = Q_u + q_L L_c \tag{5.58}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (5.58) reduces to:

$$Q_e = q_L L_c \tag{5.59}$$

Equations (5.58) and (5.59) are valid for $t_t \le t \le t_q$.

6.13. WATER SURFACE PROFILE - EQUILIBRIUM PHASE

Substituting Eqs. (6.25) and (6.26) into Eq. (4.71) gives the equation for the equilibrium flow area profile for a channel with upstream inflow between $-L_u \le x_c \le L_p$:

$$A_c = 1.632\left[\left(\frac{n_c H^{2/9}}{S_c^{1/2}}\right)(Q_u + q_L x_c)\right]^{9/13} \tag{6.60}$$

Substituting Eq. (6.27) into Eq. (6.60) gives the equation for the equilibrium water surface profile for a channel with upstream inflow, which is valid for $-L_u \le x_c \le L_p$:

$$y_c = 0.721\left[\left(\frac{n_c}{S_c^{1/2} H^{1/2}}\right)(Q_u + q_L x_c)\right]^{6/13} \tag{6.61}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (6.60) and (6.61) reduce to:

$$A_c = 1.632\left(\frac{n_c H^{2/9} q_L x_c}{S_c^{1/2}}\right)^{9/13} \tag{6.62}$$

$$y_c = 0.721\left[\left(\frac{n_c q_L x_c}{S_c^{1/2} H^{1/2}}\right)\right]^{6/13} \tag{6.63}$$

which are valid for $0 \le x_c \le L_c$.

6.14. EQUILIBRIUM DETENTION STORAGE

Substituting Eqs. (6.25) and (6.26) into Eq. (4.78) gives the equilibrium detention storage for a channel with upstream inflow:

$$D_{ec} = 0.964\left(\frac{n_c H^{2/9}}{S_c^{1/2}}\right)^{9/13}\left[\frac{(Q_u + q_L L_c)^{22/13} - Q_u^{22/13}}{q_L}\right] \tag{6.64}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (6.64) reduces to:

$$D_{ec} = 0.964\left(\frac{n_c H^{2/9} q_L}{S_c^{1/2}}\right)^{9/13} L_c^{22/13} \tag{6.65}$$

6.15. WATER SURFACE PROFILE – FALLING PHASE

Substituting Eqs. (6.25) and (6.26) into Eq. (4.91) gives the equation for the falling phase of the flow area profile for a channel with upstream inflow, which is valid for $L_f \le x_c \le L_c$.

$$x_c = 42.73\left(\frac{S_c^{1/2} A_c^{4/9}}{n_c H^{2/9}}\right)\left(t - t_q\right) + \left\{\frac{\left[0.493\left(\frac{S_c^{1/2} A_c^{13/9}}{n_c H^{2/9}}\right)\right] - Q_u}{q_L}\right\} \tag{6.66}$$

Substituting Eq. (6.27) into Eq. (6.66) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $L_f \le x_c \le L_c$:

$$x_c = 66.08\left(\frac{S_c^{1/2} y_c^{2/3}}{n_c}\right)\left(t - t_q\right) + \left\{\frac{\left[2.033\left(\frac{S_c^{1/2} H^{1/2} y_c^{13/6}}{n_c}\right)\right] - Q_u}{q_L}\right\} \tag{6.67}$$

From Eq. (4.92), the equation for the flow area profile between $0 \le x_c \le L_f$ is:

$$A_c = 1.632\left(\frac{n_c H^{2/9} Q_u}{S_c^{1/2}}\right)^{9/13} \tag{6.68}$$

Substituting Eq. (6.27) into Eq. (6.68) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_f$:

$$y_c = 0.721\left(\frac{n_c Q_u}{S_c^{1/2} H^{1/2}}\right)^{6/13} \tag{6.69}$$

Substituting Eqs. (6.25) and (6.26) into Eq. (4.94) gives the equation for the distance L_f for a channel with upstream inflow, which is valid for $t \ge t_q$:

$$L_f = 53.12\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)^{9/13} Q_u^{4/13}\left(t - t_q\right) \tag{6.70}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (6.66) and (6.67) reduce to:

$$x_c = 42.73\left(\frac{S_c^{1/2} A_c^{4/9}}{n_c H^{2/9}}\right)\left(t - t_q\right) + \left[\frac{0.493\left(\frac{S_c^{1/2} A_c^{13/9}}{n_c H^{2/9}}\right)}{q_L}\right] \tag{6.71}$$

$$x_c = 66.08\left(\frac{S_c^{1/2} y_c^{2/3}}{n_c}\right)\left(t - t_q\right) + \left[\frac{2.033\left(\frac{S_c^{1/2} H^{1/2} y_c^{13/6}}{n_c}\right)}{q_L}\right] \tag{6.72}$$

which are valid for $0 \le x_c \le L_c$.

6.16. Hydrograph - Falling Phase

Substituting Eqs. (6.25) and (6.26) into Eq. (4.101) gives the equation for the falling phase of a hydrograph for a channel with upstream inflow:

$$t = \frac{0.0188}{Q_c^{4/13}}\left(\frac{n_c H^{2/9}}{S_c^{1/2}}\right)^{9/13}\left[L_c - \left(\frac{Q_c - Q_u}{q_L}\right)\right] + t_q \tag{6.73}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (6.73) reduces to:

$$t = \frac{0.0188}{Q_c^{4/13}}\left(\frac{n_c H^{2/9}}{S_c^{1/2}}\right)^{9/13}\left[L_c - \left(\frac{Q_c}{q_L}\right)\right] + t_q \tag{6.74}$$

In: Kinematic-Wave Rainfall-Runoff Formulas
Editor: Tommy S.W. Wong, pp. 91-100
ISBN: 978-1-60692-705-2
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Chapter 7

7. WORKING FORMULAS FOR FLOW IN RECTANGULAR (DEEP) CHANNEL

In this Chapter, based on the Manning's equation, the kinematic wave parameters for flow in a rectangular (deep) channel are derived. By applying these parameters to the general formulas in Chapter 4, working formulas for the flow depth; flow velocity; average flow velocity; wave celerity; average wave celerity; time of travel; rising, equilibrium and falling of hydrograph; forward characteristic; rising, equilibrium and falling phases of water surface profiles; duration of partial equilibrium discharge; and equilibrium detention storage are also derived.

7.1. KINEMATIC WAVE PARAMETERS

For flow in a rectangular channel, the flow area A_c, and the wetted perimeter P, are related to the channel width W, and the flow depth y_c, as follows:

$$A_c = Wy_c \tag{7.1}$$

$$P = W + 2y_c \tag{7.2}$$

Substituting Eqs. (7.1) and (7.2) and $A = A_c$ into Eq. (3.2) gives:

$$R = \frac{Wy_c}{W + 2y_c} \tag{7.3}$$

Rearranging Eq. (7.3) gives:

$$R = \frac{W}{\dfrac{W}{y_c} + 2} \tag{7.4}$$

For a rectangular deep channel, as shown in figure 7.1, $y_c >> W$ and Eq. (7.4) reduces to:

$$R \approx \frac{W}{2} \tag{7.5}$$

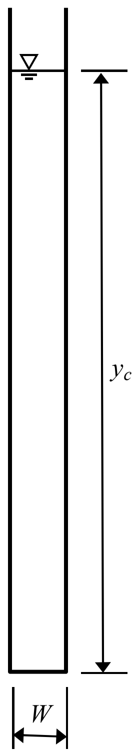

Figure 7.1. Cross-section of Rectangular (Deep) Channel.

Substituting Eq. (7.5) and $Q = Q_c$, $S = S_c$, $A = A_c$, $n = n_c$ into Eq. (3.1) gives:

$$Q_c = 0.630\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right)A_c \tag{7.6}$$

A comparison of Eq. (7.6) with Eq. (4.7) gives the kinematic wave parameters (Wong 2002, Wong and Zhou 2006):

$$\alpha_c = 0.630\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right) \tag{7.7}$$

$$\beta_c = 1 \tag{7.8}$$

7.2. FLOW DEPTH

Substituting Eqs. (7.1) into Eq. (7.6) gives:

$$Q_c = 0.630\left(\frac{S_c^{1/2}W^{5/3}y_c}{n_c}\right) \tag{7.9}$$

Rearranging Eq. (7.9) gives the equation for the flow depth for a channel with and without upstream inflow:

$$y_c = 1.587\left(\frac{n_c Q_c}{S_c^{1/2}W^{5/3}}\right) \tag{7.10}$$

7.3. FLOW VELOCITY

Substituting Eqs. (7.7) and (7.8) into Eq. (4.12) gives the equation for the flow velocity along the equilibrium profile for a channel with and without upstream inflow:

$$v = 0.630\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right) \tag{7.11}$$

7.4. AVERAGE FLOW VELOCITY

Substituting Eqs. (7.7) and (7.8) into Eq. (4.15) gives the equation for the average flow velocity for a channel with and without upstream inflow:

$$v_{av} = 0.630\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right) \tag{7.12}$$

7.5. KINEMATIC WAVE CELERITY

Substituting Eqs. (7.7) and (7.8) into Eq. (4.29) gives the working equation for the wave celerity along the equilibrium profile for a channel with and without upstream inflow:

$$c_k = 0.630\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right) \tag{7.13}$$

7.6. AVERAGE WAVE CELERITY

Substituting Eqs. (7.7) and (7.8) into Eq. (4.32) gives the working equation for the average wave celerity for a channel with and without upstream inflow:

$$c_{av} = 0.630\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right) \tag{7.14}$$

7.7. TIME OF TRAVEL

Substituting Eqs. (7.7) and (7.8) into Eq. (4.35) gives the formula for the time of travel for a channel with and without upstream inflow:

$$t_t = 0.0265\left(\frac{n_c L_c}{S_c^{1/2}W^{2/3}}\right) \tag{7.15}$$

7.8. HYDROGRAPH – RISING PHASE

Substituting Eqs. (7.7) and (7.8) into Eq. (4.45) gives the equation for the rising phase of the hydrograph for a channel with upstream inflow:

$$Q_c = 0.630\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right)\left[1.587\left(\frac{n_c Q_u}{S_c^{1/2}W^{2/3}}\right) + 60 q_L t\right] \tag{7.16}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (7.16) reduces to:

$$Q_c = 37.80\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right) q_L t \tag{7.17}$$

Equations (7.16) and (7.17) are valid for $t \le t_t$.

7.9. FORWARD CHARACTERISTIC - RISING PHASE

Substituting Eqs. (7.7) and (7.8) into Eq. (4.47) gives the equation for the forward characteristic of the rising phase for a channel with and without upstream inflow:

$$t = 0.0265\left(\frac{n_c x_c}{S_c^{1/2} W^{2/3}}\right) \tag{7.18}$$

7.10. WATER SURFACE PROFILE – RISING PHASE

Substituting Eqs. (7.7) and (7.8) into Eq. (4.49) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$A_c = 1.587\left(\frac{n_c}{S_c^{1/2} W^{2/3}}\right)\left(Q_u + q_L x_c\right) \tag{7.19}$$

Substituting Eq. (7.1) into Eq. (7.19) gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$y_c = 1.587\left(\frac{n_c}{S_c^{1/2} W^{5/3}}\right)\left(Q_u + q_L x_c\right) \tag{7.20}$$

Substituting Eqs. (7.7) and (7.8) into Eq. (4.50) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $L_p \le x_c \le L_c$:

$$A_p = 1.587\left(\frac{n_c}{S_c^{1/2} W^{2/3}}\right)\left(Q_u + q_L L_p\right) \tag{7.21}$$

Substituting $A_c = A_p$, and $y_c = y_p$ into Eq. (7.1) and then substituting it into Eq. (7.21) gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $L_p \le x_c \le L_c$:

$$y_p = 1.587\left(\frac{n_c}{S_c^{1/2} W^{5/3}}\right)\left(Q_u + q_L L_p\right) \tag{7.22}$$

From Eq. (4.51), the distance L_p is:

$$L_p = \frac{Q_p - Q_u}{q_L} \tag{7.23}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (7.19)-(7.23) reduce to:

$$A_c = 1.587\left(\frac{n_c}{S_c^{1/2}W^{2/3}}\right)q_L x_c \tag{7.24}$$

$$y_c = 1.587\left(\frac{n_c}{S_c^{1/2}W^{5/3}}\right)q_L x_c \tag{7.25}$$

which are valid for $0 \le x_o \le L_p$, and

$$A_p = 1.587\left(\frac{n_c}{S_c^{1/2}W^{2/3}}\right)q_L L_p \tag{7.26}$$

$$y_p = 1.587\left(\frac{n_c}{S_c^{1/2}W^{5/3}}\right)q_L L_p \tag{7.27}$$

which are valid for $L_p \le x_c \le L_c$, and

$$L_p = \frac{Q_p}{q_L} \tag{7.28}$$

7.11. DURATION OF PARTIAL EQUILIBRIUM DISCHARGE

Substituting Eqs. (7.7) and (7.8) into Eq. (4.63) gives the equation for the duration of partial equilibrium discharge for a channel with upstream inflow:

$$t_d = \frac{L_c q_L + Q_u - 0.630\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right)\left[1.587\left(\frac{n_c Q_u}{S_c^{1/2}W^{2/3}}\right) + 60 q_L t_q\right]}{37.80\left(\frac{S_c^{1/2}W^{2/3}q_L}{n_c}\right)} \tag{7.29}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (7.29) reduces to:

$$t_d = \frac{L_c - 37.80\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right)t_q}{37.80\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right)} \tag{7.30}$$

7.12. HYDROGRAPH - EQUILIBRIUM PHASE

As shown in figures 4.2 and 4.3, the hydrograph may reach partial equilibrium or equilibrium depending on the duration of lateral inflow t_q. If $t_q < t_t$, the hydrograph reaches partial equilibrium with a constant discharge Q_p. If $t_q \geq t_t$, the hydrograph reaches equilibrium with a constant discharge Q_e.

7.12.1. Partial Equilibrium Discharge

Substituting Eqs. (7.7) and (7.8) into Eq. (4.66) gives the equation for the partial equilibrium discharge for a channel with upstream inflow:

$$Q_p = 0.630\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right)\left[1.587\left(\frac{n_c Q_u}{S_c^{1/2}W^{2/3}}\right) + 60 q_L t_q\right] \tag{7.31}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (7.31) reduces to:

$$Q_p = 37.80\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right) q_L t_q \tag{7.32}$$

Equations (7.31) and (7.32) are valid for $t_q \leq t \leq (t_q + t_d)$.

7.12.2. Equilibrium Discharge

From Eq. (4.69), the equation for the equilibrium discharge for a channel with upstream inflow is:

$$Q_e = Q_u + q_L L_c \tag{7.33}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (7.33) reduces to:

$$Q_e = q_L L_c \tag{7.34}$$

Equations (7.33) and (7.34) are valid for $t_t \leq t \leq t_q$.

7.13. WATER SURFACE PROFILE - EQUILIBRIUM PHASE

Substituting Eqs. (7.7) and (7.8) into Eq. (4.71) gives the equation for the equilibrium flow area profile for a channel with upstream inflow between $-L_u \leq x_c \leq L_p$:

$$A_c = 1.587\left(\frac{n_c}{S_c^{1/2}W^{2/3}}\right)(Q_u + q_L x_c) \qquad (7.35)$$

Substituting Eq. (7.1) into Eq. (7.35) gives the equation for the equilibrium water surface profile for a channel with upstream inflow, which is valid for $-L_u \leq x_c \leq L_p$:

$$y_c = 1.587\left(\frac{n_c}{S_c^{1/2}W^{5/3}}\right)(Q_u + q_L x_c) \qquad (7.36)$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (7.35) and (7.36) reduce to:

$$A_c = 1.587\left(\frac{n_c}{S_c^{1/2}W^{2/3}}\right)q_L x_c \qquad (7.37)$$

$$y_c = 1.587\left(\frac{n_c}{S_c^{1/2}W^{5/3}}\right)q_L x_c \qquad (7.38)$$

which are valid for $0 \leq x_c \leq L_c$.

7.14. Equilibrium Detention Storage

Substituting Eqs. (7.7) and (7.8) into Eq. (4.78) gives the equilibrium detention storage for a channel with upstream inflow:

$$D_{ec} = 0.794\left(\frac{n_c}{S_c^{1/2}W^{2/3}}\right)\left[\frac{(Q_u + q_L L_c)^2 - Q_u^2}{q_L}\right] \qquad (7.39)$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (7.39) reduces to:

$$D_{ec} = 0.794\left(\frac{n_c}{S_c^{1/2}W^{2/3}}\right)q_L L_c^2 \qquad (7.40)$$

7.15. Water Surface Profile – Falling Phase

Substituting Eqs. (7.7) and (7.8) into Eq. (4.91) gives the equation for the falling phase of the flow area profile for a channel with upstream inflow, which is valid for $L_f \leq x_c \leq L_c$.

$$x_c = 37.80\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right)\left(t-t_q\right)+\left\{\frac{\left[0.630\left(\frac{S_c^{1/2}W^{2/3}A_c}{n_c}\right)\right]-Q_u}{q_L}\right\} \tag{7.41}$$

Substituting Eq. (7.1) into Eq. (7.41) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $L_f \le x_c \le L_c$:

$$x_c = 37.80\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right)\left(t-t_q\right)+\left\{\frac{\left[0.630\left(\frac{S_c^{1/2}W^{5/3}y_c}{n_c}\right)\right]-Q_u}{q_L}\right\} \tag{7.42}$$

From Eq. (4.92), the equation for the flow area profile between $0 \le x_c \le L_f$ is:

$$A_c = 1.587\left(\frac{n_c Q_u}{S_c^{1/2}W^{2/3}}\right) \tag{7.43}$$

Substituting Eq. (7.1) into Eq. (7.43) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_f$:

$$y_c = 1.587\left(\frac{n_c Q_u}{S_c^{1/2}W^{5/3}}\right) \tag{7.44}$$

Substituting Eqs. (7.7) and (7.8) into Eq. (4.94) gives the equation for the distance L_f for a channel with upstream inflow, which is valid for $t \ge t_q$:

$$L_f = 37.80\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right)\left(t-t_q\right) \tag{7.45}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (7.41) and (7.42) reduce to:

$$x_c = 37.80\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right)\left(t-t_q\right)+\left[\frac{0.630\left(\frac{S_c^{1/2}W^{2/3}A_c}{n_c}\right)}{q_L}\right] \tag{7.46}$$

$$x_c = 37.80\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right)\left(t - t_q\right) + \left[\frac{0.630\left(\frac{S_c^{1/2}W^{5/3}y_c}{n_c}\right)}{q_L}\right] \qquad (7.47)$$

which are valid for $0 \le x_c \le L_c$.

7.16. HYDROGRAPH - FALLING PHASE

Substituting Eqs. (7.7) and (7.8) into Eq. (4.101) gives the equation for the falling phase (falling limb) of a hydrograph for a channel with upstream inflow:

$$t = 0.0265\left(\frac{n_c}{S_c^{1/2}W^{2/3}}\right)\left[L_c - \left(\frac{Q_c - Q_u}{q_L}\right)\right] + t_q \qquad (7.48)$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (7.48) reduces to:

$$t = 0.0265\left(\frac{n_c}{S_c^{1/2}W^{2/3}}\right)\left[L_c - \left(\frac{Q_c}{q_L}\right)\right] + t_q \qquad (7.49)$$

In: Kinematic-Wave Rainfall-Runoff Formulas
Editor: Tommy S.W. Wong, pp. 101-111
ISBN: 978-1-60692-705-2
© 2009 Nova Science Publishers, Inc.

Chapter 8

8. Working Formulas for Flow in Rectangular (Square) Channel

In this Chapter, based on the Manning's equation, the kinematic wave parameters for flow in a rectangular (square) channel are derived. By applying these parameters to the general formulas in Chapter 4, working formulas for the flow depth; flow velocity; average flow velocity; wave celerity; average wave celerity; time of travel; rising, equilibrium and falling phases of hydrograph; forward characteristic; rising, equilibrium and falling phases of water surface profiles; duration of partial equilibrium discharge; and equilibrium detention storage are also derived.

8.1. Kinematic Wave Parameters

For a rectangular square channel, as shown in figure 8.1, $y_c = W$ and Eq. (7.3) reduces to:

$$R = \frac{y_c}{3} \tag{8.1}$$

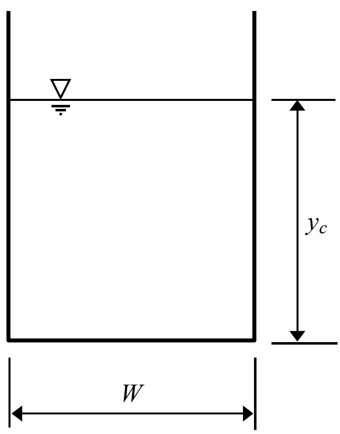

Figure 8.1. Cross-section of Rectangular (Square) Channel.

Substituting Eq. (7.1) into Eq. (8.1) gives:

$$R = \frac{A}{3W} \tag{8.2}$$

Substituting Eq. (8.2) and $Q = Q_c$, $S = S_c$, $A = A_c$, $n = n_c$ into (3.1) gives:

$$Q_c = 0.481\left(\frac{S_c^{1/2}}{n_c}\right)A_c^{4/3} \tag{8.3}$$

A comparison of Eq. (8.3) with Eq. (4.7) gives the kinematic wave parameters (Wong 2002, Wong and Zhou 2006):

$$\alpha_c = 0.481\left(\frac{S_c^{\ 1/2}}{n_c}\right) \tag{8.4}$$

$$\beta_c = \frac{4}{3} \tag{8.5}$$

8.2. FLOW DEPTH

Substituting W = y_c into Eq. (7.1) gives:

$$A_c = y_c^2 \tag{8.6}$$

Substituting Eq. (8.6) into Eq. (8.3) and rearranging gives the equation for the flow depth for a channel with and without upstream inflow:

$$y_c = 1.316\left(\frac{n_c Q_c}{S_c^{1/2}}\right)^{3/8} \tag{8.7}$$

8.3. FLOW VELOCITY

Substituting Eqs. (8.4) and (8.5) into Eq. (4.11) gives the equation for the flow velocity along the equilibrium profile for a channel with upstream inflow:

$$v = 0.578\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left(Q_u + q_L x_c\right)^{1/4} \tag{8.8}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (8.8) reduces to:

$$v = 0.578\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}(q_L x_c)^{1/4} \tag{8.9}$$

8.4. AVERAGE FLOW VELOCITY

Substituting Eqs. (8.4) and (8.5) into Eq. (4.15) gives the equation for the average flow velocity for a channel with upstream inflow:

$$v_{av} = \frac{0.433\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4} q_L L_c}{(Q_u + q_L L_c)^{3/4} - Q_u^{3/4}} \tag{8.10}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (8.10) reduces to:

$$v_{av} = 0.433\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}(q_L L_c)^{1/4} \tag{8.11}$$

8.5. KINEMATIC WAVE CELERITY

Substituting Eqs. (8.4) and (8.5) into Eq. (4.29) gives the equation for the wave celerity along the equilibrium profile for a channel with upstream inflow:

$$c_k = 0.770\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}(Q_u + q_L x_c)^{1/4} \tag{8.12}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (8.12) reduces to:

$$c_k = 0.770\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}(q_L x_c)^{1/4} \tag{8.13}$$

8.6. AVERAGE WAVE CELERITY

Substituting Eqs. (8.4) and (8.5) into Eq. (4.32) gives the equation for the wave celerity for a channel with upstream inflow:

$$c_{av} = \frac{0.578\left(\dfrac{S_c^{1/2}}{n_c}\right)^{3/4} q_L L_c}{\left(Q_u + q_L L_c\right)^{3/4} - Q_u^{\ 3/4}} \tag{8.14}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (8.14) reduces to:

$$c_{av} = 0.578\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4} \left(q_L L_c\right)^{1/4} \tag{8.15}$$

8.7. TIME OF TRAVEL

Substituting Eqs. (8.4) and (8.5) into Eq. (4.35) gives the equation for the time of travel for a channel with upstream inflow:

$$t_t = 0.0289\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4} \left[\frac{\left(Q_u + q_L L_c\right)^{3/4} - Q_u^{3/4}}{q_L}\right] \tag{8.16}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (8.16) reduces to:

$$t_t = \frac{0.0289}{q_L^{1/4}}\left(\frac{n_c L_c}{S_c^{1/2}}\right)^{3/4} \tag{8.17}$$

8.8. HYDROGRAPH – RISING PHASE

Substituting Eqs. (8.4) and (8.5) into Eq. (4.44) gives the equation for the rising phase of the hydrograph for a channel with upstream inflow:

$$Q_c = 0.481\left(\frac{S_c^{1/2}}{n_c}\right)\left[1.731\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4} + 60 q_L t\right]^{4/3} \tag{8.18}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (8.18) reduces to:

$$Q_c = 113.0\left(\frac{S_c^{1/2}}{n_c}\right)\left(q_L t\right)^{4/3} \tag{8.19}$$

Equations (8.18) and (8.19) are valid for $t \le t_t$.

8.9. FORWARD CHARACTERISTIC - RISING PHASE

Substituting Eqs. (8.4) and (8.5) into Eq. (4.47) gives the equation for the forward characteristic of the rising phase for a channel with upstream inflow:

$$t = 0.0289\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left[\frac{(Q_u + q_L x_c)^{3/4} - Q_u^{3/4}}{q_L}\right] \tag{8.20}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (8.20) reduces to:

$$t = \frac{0.0289}{q_L^{1/4}}\left(\frac{n_c x_c}{S_c^{1/2}}\right)^{3/4} \tag{8.21}$$

8.10. WATER SURFACE PROFILE – RISING PHASE

Substituting Eqs. (8.4) and (8.5) into Eq. (4.49) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$A_c = 1.731\left[\left(\frac{n_c}{S_c^{1/2}}\right)(Q_u + q_L x_c)\right]^{3/4} \tag{8.22}$$

Substituting Eq. (8.6) into Eq. (8.22) gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$y_c = 1.316\left[\left(\frac{n_c}{S_c^{1/2}}\right)(Q_u + q_L x_c)\right]^{3/8} \tag{8.23}$$

Substituting Eqs. (8.4) and (8.5) into Eq. (4.50) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $L_p \le x_c \le L_c$.

$$A_p = 1.731\left[\left(\frac{n_c}{S_c^{1/2}}\right)(Q_u + q_L L_p)\right]^{3/4} \tag{8.24}$$

Substituting $A_c = A_p$, and $y_c = y_p$ into Eq. (8.6) and then substituting it into Eq. (8.24) gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$y_p = 1.316\left[\left(\frac{n_c}{S_c^{1/2}}\right)\left(Q_u + q_L L_p\right)\right]^{3/8} \tag{8.25}$$

From Eq. (4.51), the distance L_p is:

$$L_p = \frac{Q_p - Q_u}{q_L} \tag{8.26}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (8.22)-(8.26) reduce to:

$$A_c = 1.731\left[\left(\frac{n_c}{S_c^{1/2}}\right)q_L x_c\right]^{3/4} \tag{8.27}$$

$$y_c = 1.316\left[\left(\frac{n_c}{S_c^{1/2}}\right)q_L x_c\right]^{3/8} \tag{8.28}$$

which are valid for $0 \leq x_c \leq L_p$,

$$A_p = 1.731\left[\left(\frac{n_c}{S_c^{1/2}}\right)q_L L_p\right]^{3/4} \tag{8.29}$$

$$y_p = 1.316\left[\left(\frac{n_c}{S_c^{1/2}}\right)q_L L_p\right]^{3/8} \tag{8.30}$$

which are valid for $L_p \leq x_c \leq L_c$, and

$$L_p = \frac{Q_p}{q_L} \tag{8.31}$$

8.11. DURATION OF PARTIAL EQUILIBRIUM DISCHARGE

Substituting Eqs. (8.4) and (8.5) into Eq. (4.63) gives the equation for the duration of partial equilibrium discharge for a channel with upstream inflow:

$$t_d = \frac{L_c q_L + Q_u - 0.481\left(\frac{S_c^{1/2}}{n_c}\right)\left[1.731\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4} + 60 q_L t_q\right]^{4/3}}{38.48\left(\frac{S_c^{1/2} q_L}{n_c}\right)\left[1.731\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4} + 60 q_L t_q\right]^{1/3}} \tag{8.32}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (8.32) reduces to:

$$t_d = \frac{L_c - 0.481\left(\frac{S_c^{1/2}}{n_c}\right) q_L^{1/3} t_q^{4/3}}{38.48\left(\frac{S_c^{1/2}}{n_c}\right)\left(q_L t_q\right)^{1/3}} \tag{8.33}$$

8.12. HYDROGRAPH - EQUILIBRIUM PHASE

As shown in figures 4.2 and 4.3, the hydrograph may reach partial equilibrium or equilibrium depending on the duration of lateral inflow t_q. If $t_q < t_t$, the hydrograph reaches partial equilibrium with a constant discharge Q_p. If $t_q \geq t_t$, the hydrograph reaches equilibrium with a constant discharge Q_e.

8.12.1. Partial Equilibrium Discharge

Substituting Eqs. (8.4) and (8.5) into Eq. (4.66) gives the equation for the partial equilibrium discharge for a channel with upstream inflow:

$$Q_p = 0.481\left(\frac{S_c^{1/2}}{n_c}\right)\left[1.731\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4} + 60 q_L t_q\right]^{4/3} \tag{8.34}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (8.34) reduces to:

$$Q_p = 113.0\left(\frac{S_c^{1/2}}{n_c}\right)\left(q_L t_q\right)^{4/3} \tag{8.35}$$

Equations (8.34) and (8.35) are valid for $t_q \leq t \leq (t_q + t_d)$.

8.12.2. Equilibrium Discharge

From Eq. (4.69), the equation for the equilibrium discharge for a channel with upstream inflow is:

$$Q_e = Q_u + q_L L_c \tag{8.36}$$

For a channel with zero upstream inflow (Q_u = 0), Eq. (8.36) reduces to:

$$Q_e = q_L L_c \tag{8.37}$$

Equations (8.36) and (8.37) are valid for $t_t \leq t \leq t_q$.

8.13. WATER SURFACE PROFILE - EQUILIBRIUM PHASE

Substituting Eqs. (8.4) and (8.5) into Eq. (4.71) gives the equation for the equilibrium flow area profile for a channel with upstream inflow between $-L_u \leq x_c \leq L_c$.

$$A_c = 1.731\left[\left(\frac{n_c}{S_c^{1/2}}\right)(Q_u + q_L x_c)\right]^{3/4} \tag{8.38}$$

Substituting Eq. (8.6) into Eq. (8.38) gives the equation for the equilibrium water surface profile for a channel with upstream inflow, which is valid for $-L_u \leq x_c \leq L_c$:

$$y_c = 1.316\left[\left(\frac{n_c}{S_c^{1/2}}\right)(Q_u + q_L x_c)\right]^{3/8} \tag{8.39}$$

For a channel with zero upstream inflow (Q_u = 0), Eqs. (8.38) and (8.39) reduce to:

$$A_c = 1.731\left[\left(\frac{n_c}{S_c^{1/2}}\right)q_L x_c\right]^{3/4} \tag{8.40}$$

$$y_c = 1.316\left[\left(\frac{n_c}{S_c^{1/2}}\right)q_L x_c\right]^{3/8} \tag{8.41}$$

which are valid for $0 \leq x_c \leq L_c$.

8.14. EQUILIBRIUM DETENTION STORAGE

Substituting Eqs. (8.4) and (8.5) into Eq. (4.78) gives the equilibrium detention storage for a channel with upstream inflow:

$$D_{ec} = 0.989\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left[\frac{\left(Q_u + q_L L_c\right)^{7/4} - Q_u^{7/4}}{q_L}\right] \tag{8.42}$$

For a channel with zero upstream inflow (i.e. $Q_u = 0$), Eq. (8.42) reduces to:

$$D_{ec} = 0.989\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4} q_L^{3/4} L_c^{\ 7/4} \tag{8.43}$$

8.15. WATER SURFACE PROFILE – FALLING PHASE

Substituting Eqs. (8.4) and (8.5) into Eq. (4.91) gives the equation for the falling phase of the flow area profile for a channel with upstream inflow, which is valid for $L_f \le x_c \le L_c$:

$$x_c = 38.48\left(\frac{S_c^{1/2} A_c^{1/3}}{n_c}\right)\left(t - t_q\right) + \left\{\frac{\left[0.481\left(\frac{S_c^{1/2} A_c^{4/3}}{n_c}\right)\right] - Q_u}{q_L}\right\} \tag{8.44}$$

Substituting Eq. (8.6) into Eq. (8.44) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $L_f \le x_c \le L_c$:

$$x_c = 38.48\left(\frac{S_c^{1/2} y_c^{\ 2/3}}{n_c}\right)\left(t - t_q\right) + \left\{\frac{\left[0.481\left(\frac{S_c^{1/2} y_c^{\ 8/3}}{n_c}\right)\right] - Q_u}{q_L}\right\} \tag{8.45}$$

From Eq. (4.92), the equation for the flow area profile between $0 \le x_c \le L_f$ is:

$$A_c = 1.731\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4} \tag{8.46}$$

Substituting Eq. (8.6) into Eq. (8.46) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_f$:

$$y_c = 1.316\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/8} \tag{8.47}$$

Substituting Eqs. (8.4) and (8.5) into Eq. (4.94) gives the equation for the distance L_f for a channel with upstream inflow, which is valid for $t \ge t_q$:

$$L_f = 46.21\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4} Q_u^{1/4}\left(t - t_q\right) \tag{8.48}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (8.44) and (8.45) reduce to:

$$x_c = 38.48\left(\frac{S_c^{1/2} A_c^{1/3}}{n_c}\right)\left(t - t_q\right) + \left[\frac{0.481\left(\frac{S_c^{1/2} A_c^{4/3}}{n_c}\right)}{q_L}\right] \tag{8.49}$$

$$x_c = 38.48\left(\frac{S_c^{1/2} {y_c}^{2/3}}{n_c}\right)\left(t - t_q\right) + \left[\frac{0.481\left(\frac{S_c^{1/2} {y_c}^{8/3}}{n_c}\right)}{q_L}\right] \tag{8.50}$$

which are valid for $0 \le x_c \le L_c$.

8.16. HYDROGRAPH - FALLING PHASE

Substituting Eqs. (8.4) and (8.5) into Eq. (4.101) gives the equation for the falling phase (falling limb) of a hydrograph for a channel with upstream inflow:

$$t = \left(\frac{0.0216}{Q_c^{1/4}}\right)\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left[L_c - \left(\frac{Q_c - Q_u}{q_L}\right)\right] + t_q \tag{8.51}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (8.51) reduces to:

$$t = \left(\frac{0.0216}{Q_c^{1/4}} \right) \left(\frac{n_c}{S_c^{1/2}} \right)^{3/4} \left[L_c - \left(\frac{Q_c}{q_L} \right) \right] + t_q \tag{8.52}$$

In: Kinematic-Wave Rainfall-Runoff Formulas
Editor: Tommy S.W. Wong, pp. 113-123
ISBN: 978-1-60692-705-2
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Chapter 9

9. WORKING FORMULAS FOR FLOW IN RECTANGULAR (WIDE) CHANNEL

In this Chapter, based on the Manning's equation, the kinematic wave parameters for flow in a rectangular (wide) channel are derived. By applying these parameters to the general formulas in Chapter 4, working formulas for the flow depth; flow velocity; average flow velocity; wave celerity; average wave celerity; time of travel; rising, equilibrium and falling phases of hydrograph; forward characteristic; rising, equilibrium and falling phases of water surface profiles; duration of partial equilibrium discharge; and equilibrium detention storage are also derived.

9.1. KINEMATIC WAVE PARAMETERS

Rearranging Eq. (7.3) gives:

$$R = \frac{y_c}{1 + \frac{2y_c}{W}} \qquad (9.1)$$

For a rectangular wide channel, as shown in figure 9.1, $y_c << W$ and Eq. (9.1) reduces to:

$$R \approx y_c \qquad (9.2)$$

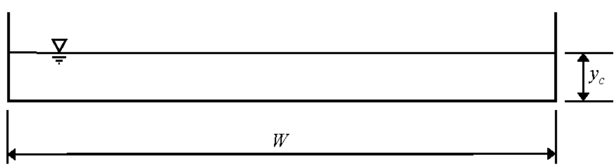

Figure 9.1. Cross-section of Rectangular (Wide) Channel.

Substituting Eq. (7.1) into Eq. (9.2) gives:

$$R = \frac{A}{W} \tag{9.3}$$

Substituting Eq. (9.3) and $Q = Q_c$, $S = S_c$, $A = A_c$, $n = n_c$ into (3.1) gives:

$$Q_c = \left(\frac{S_c^{1/2}}{n_c W^{2/3}} \right) A_c^{5/3} \tag{9.4}$$

A comparison of Eq. (9.4) with Eq. (4.7) gives the kinematic wave parameters (Wong 2002, Wong and Zhou 2006):

$$\alpha_c = \left(\frac{S_c^{1/2}}{n_c W^{2/3}} \right) \tag{9.5}$$

$$\beta_c = \frac{5}{3} \tag{9.6}$$

9.2. Flow Depth

Substituting Eqs. (7.1) into Eq. (9.4) gives:

$$Q_c = \left(\frac{S_c^{1/2}}{n_c} \right) W y_c^{5/3} \tag{9.7}$$

Rearranging Eq. (9.7) gives the equation for the flow depth for a channel with and without upstream inflow:

$$y_c = \left(\frac{n_c Q_c}{S_c^{1/2} W} \right)^{3/5} \tag{9.8}$$

9.3. Flow Velocity

Substituting Eqs. (9.5) and (9.6) into Eq. (4.12) gives the equation for the flow velocity along the equilibrium profile for a channel with upstream inflow:

$$v = \left(\frac{S_c^{1/2}}{n_c W^{2/3}} \right)^{3/5} \left(Q_u + q_L x_c \right)^{2/5} \tag{9.9}$$

For a channel with zero upstream inflow (Q_u = 0), Eq. (9.9) reduces to:

$$v = \left(\frac{S_c^{1/2}}{n_c W^{2/3}} \right)^{3/5} \left(q_L x_c \right)^{2/5} \tag{9.10}$$

9.4. Average Flow Velocity

Substituting Eqs. (9.5) and (9.6) into Eq.(4.15) gives the equation for the average flow velocity for a channel with upstream inflow:

$$v_{av} = \frac{0.600 \left(\frac{S_c^{1/2}}{n W^{2/3}} \right)^{3/5} q_L L_c}{\left(Q_u + q_L L_c \right)^{3/5} - Q_u^{3/5}} \tag{9.11}$$

For a channel with zero upstream inflow (Q_u = 0), Eq. (9.11) reduces to:

$$v_{av} = 0.600 \left(\frac{S_c^{1/2}}{n W^{2/3}} \right)^{3/5} \left(q_L L_c \right)^{2/5} \tag{9.12}$$

9.5. Kinematic Wave Celerity

Substituting Eqs. (9.5) and (9.6) into Eq. (4.29) gives the equation for the wave celerity along the equilibrium profile for a channel with upstream inflow:

$$c_k = 1.667 \left(\frac{S_c^{1/2}}{n_c W^{2/3}} \right)^{3/5} \left(Q_u + q_L x_c \right)^{2/5} \tag{9.13}$$

For a channel with zero upstream inflow (Q_u = 0), Eq. (9.13) reduces to:

$$c_k = 1.667 \left(\frac{S_c^{1/2}}{n_c W^{2/3}} \right)^{3/5} \left(q_L x_c \right)^{2/5} \tag{9.14}$$

9.6. AVERAGE WAVE CELERITY

Substituting Eqs. (9.5) and (9.6) into Eq. (4.32) gives the equation for the wave celerity for a channel with upstream inflow:

$$c_{av} = \frac{\left(\dfrac{S_c^{1/2}}{n_c W^{2/3}} \right)^{3/5} q_L L_c}{\left(Q_u + q_L L_c \right)^{3/5} - Q_u^{\ 3/5}} \tag{9.15}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (9.15) reduces to:

$$c_{av} = \left(\frac{S_c^{1/2}}{n_c W^{2/3}} \right)^{3/5} \left(q_L L_c \right)^{2/5} \tag{9.16}$$

9.7. TIME OF TRAVEL

Substituting Eqs. (9.5) and (9.6) into Eq. (4.35) gives the equation for the time of travel for a channel with upstream inflow:

$$t_t = 0.0167 \left(\frac{n_c W^{2/3}}{S_c^{1/2}} \right)^{3/5} \left[\frac{\left(Q_u + q_L L_c \right)^{3/5} - Q_u^{3/5}}{q_L} \right] \tag{9.17}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (9.17) reduces to:

$$t_t = \frac{0.0167}{q_L^{2/5}} \left(\frac{n_c W^{2/3} L_c}{S_c^{1/2}} \right)^{3/5} \tag{9.18}$$

9.8. HYDROGRAPH – RISING PHASE

Substituting Eqs. (9.5) and (9.6) into Eq. (4.45) gives the equation for the rising phase (rising limb) of the hydrograph for a channel with upstream inflow:

$$Q_c = \left(\frac{S_c^{1/2}}{n_c W^{2/3}} \right) \left[\left(\frac{n W^{2/3} Q_u}{S_c^{\ 1/2}} \right)^{3/5} + 60 q_L t \right]^{5/3} \tag{9.19}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (9.19) reduces to:

$$Q_c = 919.6\left(\frac{S_c^{1/2}}{n_c W^{2/3}}\right)(q_L t)^{5/3} \tag{9.20}$$

Equations (9.19) and (9.20) are valid for $t \le t_t$.

9.9. FORWARD CHARACTERISTIC - RISING PHASE

Substituting Eqs. (9.5) and (9.6) into Eq.(4.47) gives the equation for the forward characteristic of the rising phase for a channel with upstream inflow:

$$t = 0.0167\left(\frac{n_c W^{2/3}}{S_c^{1/2}}\right)^{3/5}\left[\frac{(Q_u + q_L x_c)^{3/5} - Q_u^{3/5}}{q_L}\right] \tag{9.21}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (9.21) reduces to:

$$t = \frac{0.0167}{q_L^{\;2/5}}\left(\frac{n_c W^{2/3} x_c}{S_c^{1/2}}\right)^{3/5} \tag{9.22}$$

9.10. WATER SURFACE PROFILE – RISING PHASE

Substituting Eqs. (9.5) and (9.6) into Eq. (4.49) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$A_c = \left[\left(\frac{n_c W^{2/3}}{S_c^{1/2}}\right)(Q_u + q_L x_c)\right]^{3/5} \tag{9.23}$$

Substituting Eq. (7.1) into Eq. (9.23) gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$y_c = \left[\left(\frac{n_c}{S_c^{1/2} W}\right)(Q_u + q_L x_c)\right]^{3/5} \tag{9.24}$$

Substituting Eqs. (9.5) and (9.6) into Eq. (4.50) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $L_p \le x_c \le L_c$.

$$A_p = \left[\left(\frac{n_c W^{2/3}}{S_c^{1/2}} \right) \left(Q_u + q_L L_p \right) \right]^{3/5} \tag{9.25}$$

Substituting $A_c = A_p$, and $y_c = y_p$ into Eq. (7.1) and then substituting it into Eq. (9.25) gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$y_p = \left[\left(\frac{n_c}{S_c^{1/2} W} \right) \left(Q_u + q_L L_p \right) \right]^{3/5} \tag{9.26}$$

From Eq. (4.51), the distance L_p is:

$$L_p = \frac{Q_p - Q_u}{q_L} \tag{9.27}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (9.23)-(9.27) reduce to:

$$A_c = \left[\left(\frac{n_c W^{2/3}}{S_c^{1/2}} \right) q_L x_c \right]^{3/5} \tag{9.28}$$

$$y_c = \left[\left(\frac{n_c}{S_c^{1/2} W} \right) q_L x_c \right]^{3/5} \tag{9.29}$$

which are valid for $0 \le x_c \le L_p$, and

$$A_p = \left[\left(\frac{n_c W^{2/3}}{S_c^{1/2}} \right) q_L L_p \right]^{3/5} \tag{9.30}$$

$$y_p = \left[\left(\frac{n_c}{S_c^{1/2} W} \right) q_L L_p \right]^{3/5} \tag{9.31}$$

which are valid for $L_p \le x_c \le L_c$, and

$$L_p = \frac{Q_p}{q_L} \tag{9.32}$$

9.11. DURATION OF PARTIAL EQUILIBRIUM DISCHARGE

Substituting Eqs. (9.5) and (9.6) into Eq. (4.63) gives the equation for the duration of partial equilibrium discharge for a channel with upstream inflow:

$$t_d = \frac{L_c q_L + Q_u - \left(\dfrac{S_c^{1/2}}{n_c W^{2/3}}\right)\left[\left(\dfrac{n_c W^{2/3} Q_u}{S_c^{1/2}}\right)^{3/5} + 60 q_L t_q\right]^{5/3}}{100.0\left(\dfrac{S_c^{1/2} q_L}{n_c W^{2/3}}\right)\left[\left(\dfrac{n_c W^{2/3} Q_u}{S_c^{1/2}}\right)^{3/5} + 60 q_L t_q\right]^{2/3}} \tag{9.33}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (9.29) reduces to:

$$t_d = \frac{L_c - 919.6\left(\dfrac{S_c^{1/2}}{n_c W^{2/3}}\right) q_L^{2/3} t_q^{5/3}}{1532.6\left(\dfrac{S_c^{1/2}}{n_c W^{2/3}}\right)\left(q_L t_q\right)^{2/3}} \tag{9.34}$$

9.12. HYDROGRAPH - EQUILIBRIUM PHASE

As shown in figures 4.2 and 4.3, the hydrograph may reach partial equilibrium or equilibrium depending on the duration of lateral inflow t_q. If $t_q < t_t$, the hydrograph reaches partial equilibrium with a constant discharge Q_p. If $t_q \geq t_t$, the hydrograph reaches equilibrium with a constant discharge Q_e.

9.12.1. Partial Equilibrium Discharge

Substituting Eqs. (9.5) and (9.6) into Eq. (4.66) gives the equation for the partial equilibrium discharge for a channel with upstream inflow:

$$Q_p = \left(\frac{S_c^{1/2}}{n_c W^{2/3}}\right)\left[\left(\frac{n W^{2/3} Q_u}{S_c^{1/2}}\right)^{3/5} + 60 q_L t_q\right]^{5/3} \tag{9.35}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (9.35) reduces to:

$$Q_p = 919.6\left(\frac{S_c^{1/2}}{n_c W^{2/3}}\right)\left(q_L t_q\right)^{5/3} \tag{9.36}$$

Equations (9.35) and (9.36) are valid for $t_q \le t \le (t_q + t_d)$.

9.12.2. Equilibrium Discharge

From Eq. (4.69), the equation for the equilibrium discharge for a channel with upstream inflow is:

$$Q_e = Q_u + q_L L_c \tag{9.37}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (9.37) reduces to:

$$Q_e = q_L L_c \tag{9.38}$$

Equations (9.37) and (9.38) are valid for $t_t \le t \le t_q$.

9.13. Water Surface Profile - Equilibrium Phase

Substituting Eqs. (9.5) and (9.6) into Eq. (4.71) gives the equation for the equilibrium flow area profile for a channel with upstream inflow between $-L_u \le x_c \le L_c$:

$$A_c = \left[\left(\frac{n_c W^{2/3}}{S_c^{1/2}}\right)\left(Q_u + q_L x_c\right)\right]^{3/5} \tag{9.39}$$

Substituting Eq. (7.1) into Eq. (9.39) gives the equation for the equilibrium water surface profile for a channel with upstream inflow, which is valid for $-L_u \le x_c \le L_c$:

$$y_c = \left[\left(\frac{n_c}{S_c^{1/2} W}\right)\left(Q_u + q_L x_c\right)\right]^{3/5} \tag{9.40}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (9.39) and (9.40) reduce to:

$$A_c = \left[\left(\frac{n_c W^{2/3}}{S_c^{1/2}}\right) q_L x_c\right]^{3/5} \tag{9.41}$$

$$y_c = \left[\left(\frac{n_c}{S_c^{1/2} W}\right) q_L x_c\right]^{3/5} \tag{9.42}$$

which are valid for $0 \le x_c \le L_c$.

9.14. EQUILIBRIUM DETENTION STORAGE

Substituting Eqs. (9.5) and (9.6) into Eq. (4.78) gives the equilibrium detention storage for a channel with upstream inflow:

$$D_{ec} = 0.625\left(\frac{n_c W^{2/3}}{S_c^{1/2}}\right)^{3/5}\left[\frac{(Q_u + q_L L_c)^{8/5} - Q_u^{8/5}}{q_L}\right] \tag{9.43}$$

For a channel with zero upstream inflow (i.e. $Q_u = 0$), Eq. (9.43) reduces to:

$$D_{ec} = 0.625\left(\frac{n_c W^{2/3}}{S_c^{1/2}}\right)^{3/5} q_L^{3/5} L_c^{8/5} \tag{9.44}$$

9.15. WATER SURFACE PROFILE – FALLING PHASE

Substituting Eqs. (9.5) and (9.6) into Eq. (4.91) gives the equation for the falling phase of the flow area profile for a channel with upstream inflow, which is valid for $L_f \le x_c \le L_c$:

$$x_c = 100.0\left(\frac{S_c^{1/2} A_c^{2/3}}{n_c W^{2/3}}\right)(t - t_q) + \left[\frac{\left(\frac{S_c^{1/2} A_c^{5/3}}{n_c W^{2/3}}\right) - Q_u}{q_L}\right] \tag{9.45}$$

Substituting Eq. (7.1) into Eq. (9.45) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $L_f \le x_c \le L_c$:

$$x_c = 100.0\left(\frac{S_c^{1/2} y_c^{2/3}}{n_c}\right)(t - t_q) + \left[\frac{\left(\frac{S_c^{1/2} W y_c^{5/3}}{n_c}\right) - Q_u}{q_L}\right] \tag{9.46}$$

From Eq. (4.92), the equation for the flow area profile between $0 \le x_c \le L_f$ is:

$$A_c = \left(\frac{n_c W^{2/3} Q_u}{S_c^{1/2}} \right)^{3/5} \tag{9.47}$$

Substituting Eq. (7.1) into Eq. (9.47) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_f$:

$$y_c = \left(\frac{n_c Q_u}{S_c^{1/2} W} \right)^{3/5} \tag{9.48}$$

Substituting Eqs. (9.5) and (9.6) into Eq. (4.94) gives the equation for the distance L_f for a channel with upstream inflow, which is valid for $t \ge t_q$:

$$L_f = 100.0 \left(\frac{S_c^{1/2}}{n_c W^{2/3}} \right)^{3/5} Q_u^{\;2/5} \left(t - t_q \right) \tag{9.49}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (9.45) and (9.46) reduce to:

$$x_c = 100.0 \left(\frac{S_c^{1/2} A_c^{2/3}}{n_c W^{2/3}} \right) \left(t - t_q \right) + \left[\frac{\left(\dfrac{S_c^{1/2} A_c^{5/3}}{n_c W^{2/3}} \right)}{q_L} \right] \tag{9.50}$$

$$x_c = 100.0 \left(\frac{S_c^{1/2} y_c^{2/3}}{n_c} \right) \left(t - t_q \right) + \left\{ \frac{\left[\left(\dfrac{S_c^{1/2} W y_c^{5/3}}{n_c} \right) \right]}{q_L} \right\} \tag{9.51}$$

which are valid for $0 \le x_c \le L_{c.}$

9.16. HYDROGRAPH - FALLING PHASE

Substituting Eqs. (9.5) and (9.6) into Eq. (4.101) gives the equation for the falling phase (falling limb) of a hydrograph for a channel with upstream inflow:

$$t = \left(\frac{0.0100}{Q_c^{2/5}} \right) \left(\frac{n_c W^{2/3}}{S_c^{1/2}} \right)^{3/5} \left[L_c - \left(\frac{Q_c - Q_u}{q_L} \right) \right] + t_q \tag{9.52}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (9.52) reduces to:

$$t = \left(\frac{0.0100}{Q_c^{2/5}}\right)\left(\frac{n_c W^{2/3}}{S_c^{1/2}}\right)^{3/5}\left[L_c - \left(\frac{Q_c}{q_L}\right)\right] + t_q \qquad (9.53)$$

In: Kinematic-Wave Rainfall-Runoff Formulas ISBN: 978-1-60692-705-2
Editor: Tommy S.W. Wong, pp. 125-136 © 2009 Nova Science Publishers, Inc.

Chapter 10

10. Working Formulas for Flow in Trapezoidal Channel with Equal Side Slopes

In this Chapter, based on the Manning's equation, the kinematic wave parameters for flow in a trapezoidal channel with equal side slopes are derived. By applying these parameters to the general formulas in Chapter 4, working formulas for the flow depth; flow velocity; average flow velocity; wave celerity; average wave celerity; time of travel; rising, equilibrium and falling phases of hydrograph; forward characteristic; rising, equilibrium and falling phases of water surface profiles; duration of partial equilibrium discharge; and equilibrium detention storage are also derived.

10.1. Kinematic Wave Parameters

For flow in a trapezoidal channel with equal side slopes, the flow area A_c, and the wetted perimeter P, are related to the base width W, reciprocal of channel side slope z, and the flow depth y_c, as follows:

$$A_c = zy_c^2 + Wy_c \tag{10.1}$$

$$P = 2y_c\left(1+z^2\right)^{1/2} + W \tag{10.2}$$

Figure 10.1 shows the trapezoidal channel with base width W, reciprocal of channel side slope z, and flow depth y_c. Next, defining two dimensionless variables, ψ and μ, as:

$$\psi = A_c / W^2 \tag{10.3}$$

$$\mu = y_c / W \tag{10.4}$$

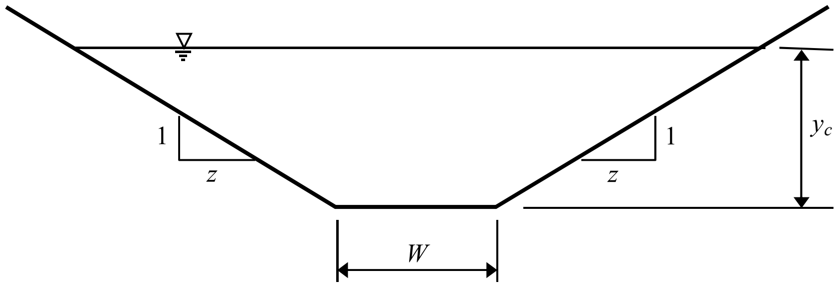

Figure 10.1. Cross-section of Trapezoidal Channel with Equal Side Slopes.

Substituting Eqs. (10.3) and (10.4) into Eq. (10.1) gives the following relationship between ψ and μ:

$$z\mu^2 + \mu - \psi = 0 \tag{10.5}$$

For $z \neq 0$, the positive solution for Eq. (10.5) is:

$$\mu = \frac{(1+4z\psi)^{1/2} - 1}{2z} \tag{10.6}$$

Substituting Eq. (10.4) into Eq. (10.6) to eliminate μ results:

$$y_c = \left[\frac{(1+4z\psi)^{1/2} - 1}{2z}\right]W \tag{10.7}$$

Substituting Eq. (10.7) into Eq. (10.2) gives an expression for P:

$$P = \left\{1 + \frac{(1+z^2)^{1/2}\left[(1+4z\psi)^{1/2} - 1\right]}{z}\right\}W \tag{10.8}$$

Substituting Eqs. (10.3) and (10.8) into Eq. (3.1) results in a dimensionless equation in terms of ψ, which can be considered as the "true" relationship between Q_c and A_c for a trapezoidal channel of equal side slopes:

$$\frac{n_c Q_c}{S_c^{1/2} W^{8/3}} = \left\{\frac{z}{z + (1+z^2)^{1/2}\left[(1+4z\psi)^{1/2} - 1\right]}\right\}^{2/3} \psi^{5/3} \tag{10.9}$$

As shown in figure 10.2, by mathematical fitting to the true relationships for $0.1 \le z \le 5.0$, Wong and Zhou (2006) obtained the following kinematic wave parameters:

$$\alpha_c = 0.340\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right) \tag{10.10}$$

$$\beta_c = 1.379 \tag{10.11}$$

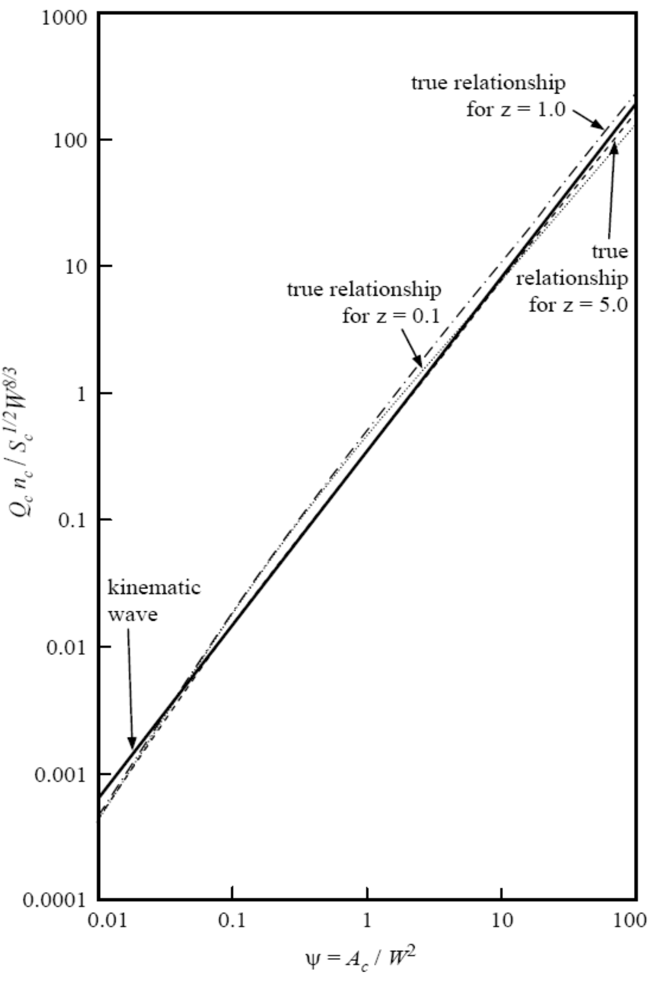

Figure 10.2. Comparison between True and Kinematic Wave Relationships for Flow in Trapezoidal Channel with Equal Side Slopes.

10.2. FLOW DEPTH

Substituting Eqs. (10.10) and (10.11) into Eq. (4.7) gives:

$$Q_c = 0.340\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)A_c^{1.379} \tag{10.12}$$

Substituting Eqs. (10.1) into Eq. (10.12) gives:

$$Q_c = 0.340\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)\left(zy_c^2 + Wy_c\right)^{1.379} \tag{10.13}$$

Rearranging Eq. (10.13) gives:

$$zy_c^2 + Wy_c - 2.187\left(\frac{n_c W^{0.0909} Q_c}{S_c^{1/2}}\right)^{0.725} = 0 \tag{10.14}$$

Solving Eq. (10.14) gives the equation for the flow depth for a channel with and without upstream inflow:

$$y_c = \frac{-W + \left[W^2 + 8.748z\left(\frac{n_c W^{0.0909} Q_c}{S_c^{1/2}}\right)^{0.725}\right]^{1/2}}{2z} \tag{10.15}$$

10.3. FLOW VELOCITY

Substituting Eqs. (10.10) and (10.11) into Eq. (4.12) gives the equation of flow velocity for a channel with upstream inflow:

$$v = \left[0.340\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)\left(Q_u + q_L x_c\right)^{0.379}\right]^{0.725} \tag{10.16}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (10.16) reduces to:

$$v = \left[0.340\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)\left(q_L x_c\right)^{0.379}\right]^{0.725} \tag{10.17}$$

10.4. AVERAGE FLOW VELOCITY

Substituting Eqs. (10.10) and (10.11) into Eq. (4.15) gives the equation of average flow velocity along the equilibrium profile for a channel with upstream inflow:

$$v_{av} = \frac{0.332\left(\dfrac{S_c^{1/2}}{n_c W^{0.0909}}\right)^{0.725} q_L L_c}{\left(Q_u + q_L L_c\right)^{0.725} - Q_u^{\ 0.725}} \tag{10.18}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (10.18) reduces to:

$$v_{av} = 0.332\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)^{0.725} \left(q_L L_c\right)^{0.275} \tag{10.19}$$

10.5. KINEMATIC WAVE CELERITY

Substituting Eqs. (10.10) and (10.11) into Eq. (4.29) gives the equation of wave celerity along the equilibrium profile for a channel with upstream inflow:

$$c_k = 0.630\left[\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)\left(Q_u + q_L x_c\right)^{0.379}\right]^{0.725} \tag{10.20}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (10.20) reduces to:

$$c_k = 0.630\left[\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)\left(q_L x_c\right)^{0.379}\right]^{0.725} \tag{10.21}$$

10.6. AVERAGE WAVE CELERITY

Substituting Eqs. (10.10) and (10.11) into Eq. (4.32) gives the equation of wave celerity for a channel with upstream inflow:

$$c_{av} = \frac{0.457\left(\dfrac{S_c^{1/2}}{n_c W^{0.0909}}\right)^{0.725} q_L L_c}{\left(Q_u + q_L L_c\right)^{0.725} - Q_u^{\ 0.725}} \tag{10.22}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (10.22) reduces to:

$$c_{av} = 0.457\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)^{0.725} \left(q_L L_c\right)^{0.275} \tag{10.23}$$

10.7. Time of Travel

Substituting Eqs. (10.10) and (10.11) into Eq. (4.35) gives the equation of time of travel for a channel with upstream inflow:

$$t_t = 0.0364\left(\frac{n_c W^{0.0909}}{S_c^{1/2}}\right)^{0.725}\left[\frac{(Q_u + q_L L_c)^{0.725} - Q_u^{0.725}}{q_L}\right] \tag{10.24}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (10.24) reduces to:

$$t_t = \left(\frac{0.0364}{q_L^{0.275}}\right)\left(\frac{n_c W^{0.0909}}{S_c^{1/2}}\right)^{0.725} \tag{10.25}$$

10.8. Hydrograph – Rising Phase

Substituting Eqs. (10.10) and (10.11) into Eq. (4.45) gives the equation for the rising phase of the hydrograph for a channel with upstream inflow:

$$Q_c = 0.340\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)\left[2.186\left(\frac{n_c W^{0.0909} Q_u}{S_c^{1/2}}\right)^{0.725} + 60 q_L t\right]^{1.379} \tag{10.26}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (10.26) reduces to:

$$Q_c = 96.28\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)(q_L t)^{1.379} \tag{10.27}$$

Equations (10.26) and (10.27) are valid for $t \le t_t$.

10.9. Forward Characteristic - Rising Phase

Substituting Eqs. (10.10) and (10.11) into Eq. (4.47) gives the equation for the forward characteristic of the rising phase for a channel with upstream inflow:

$$t = 0.0364\left(\frac{n_c W^{0.0909}}{S_c^{1/2}}\right)^{0.725}\left[\frac{(Q_u + q_L x_c)^{0.725} - Q_u^{0.725}}{q_L}\right] \tag{10.28}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (10.28) reduces to:

$$t = \frac{0.0364}{q_L^{0.275}} \left(\frac{n_c W^{0.0909} x_c}{S_c^{1/2}} \right)^{0.725} \tag{10.29}$$

10.10. WATER SURFACE PROFILE – RISING PHASE

Substituting Eqs. (10.10) and (10.11) into Eq. (4.49) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$A_c = 2.186 \left[\left(\frac{n_c W^{0.0909}}{S_c^{1/2}} \right) (q_L x_c + Q_u) \right]^{0.725} \tag{10.30}$$

Substituting Eq. (10.1) into Eq. (10.30), and solving it gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$y_c = \frac{-W + \left\{ W^2 + 8.748 z \left[\dfrac{n_c W^{0.0909} (q_L x_c + Q_u)}{S_c^{1/2}} \right]^{0.725} \right\}^{1/2}}{2z} \tag{10.31}$$

Substituting Eqs. (10.10) and (10.11) into Eq. (4.50) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $L_p \le x_c \le L_c$:

$$A_p = 2.186 \left[\left(\frac{n_c W^{0.0909}}{S_c^{1/2}} \right) (q_L L_p + Q_u) \right]^{0.725} \tag{10.32}$$

Substituting $A_c = A_p$, and $y_c = y_p$ into Eq. (10.1), and then substituting it into Eq. (10.32), and solving it gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $L_p \le x_c \le L_c$:

$$y_p = \frac{-W + \left\{ W^2 + 8.748 z \left[\dfrac{n_c W^{0.0909} (q_L L_p + Q_u)}{S_c^{1/2}} \right]^{0.725} \right\}^{1/2}}{2z} \tag{10.33}$$

From Eq. (4.51), the distance L_p is:

$$L_p = \frac{Q_p - Q_u}{q_L} \tag{10.34}$$

For a channel with zero upstream inflow (Q_u = 0), Eqs. (10.30)-(10.34) reduce to:

$$A_c = 2.186\left(\frac{n_c W^{0.0909} q_L x_c}{S_c^{1/2}}\right)^{0.725} \tag{10.35}$$

$$y_c = \frac{-W + \left[W^2 + 8.748z\left(\frac{n_c W^{0.0909} q_L x_c}{S_c^{1/2}}\right)^{0.725}\right]^{1/2}}{2z} \tag{10.36}$$

which are valid for $0 \le x_c \le L_p$,

$$A_p = 2.186\left(\frac{n_c W^{0.0909} q_L L_p}{S_c^{1/2}}\right)^{0.725} \tag{10.37}$$

$$y_p = \frac{-W + \left[W^2 + 8.748z\left(\frac{n_c W^{0.0909} q_L L_p}{S_c^{1/2}}\right)^{0.725}\right]^{1/2}}{2z} \tag{10.38}$$

which are valid for $L_p \le x_c \le L_c$, and

$$L_p = \frac{Q_p}{q_L} \tag{10.39}$$

10.11. DURATION OF PARTIAL EQUILIBRIUM DISCHARGE

Substituting Eqs. (10.10) and (10.11) into Eq. (4.63) gives the equation for the duration of partial equilibrium discharge for a channel with upstream inflow:

$$t_d = \frac{L_c q_L + Q_u - 0.340\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)\left[2.186\left(\frac{n_c W^{0.0909} Q_u}{S_c^{1/2}}\right)^{0.725} + 60 q_L t_q\right]^{1.379}}{28.13\left(\frac{S_c^{1/2} q_L}{n_c W^{0.0909}}\right)\left[2.186\left(\frac{n_c W^{0.0909} Q_u}{S_c^{1/2}}\right)^{0.725} + 60 q_L t_q\right]^{0.379}} \tag{10.40}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (10.40) reduces to:

$$t_d = \frac{L_c - 96.41\left(\dfrac{S_c^{1/2}}{n_c W^{0.0909}}\right) q_L^{0.379} t_q^{1.379}}{133.0\left(\dfrac{S_c^{1/2}}{n_c W^{0.0909}}\right)\left(q_L t_q\right)^{0.379}} \qquad (10.41)$$

10.12. Hydrograph - Equilibrium Phase

As shown in figures 4.2 and 4.3, the hydrograph may reach partial equilibrium or equilibrium depending on the duration of lateral inflow t_q. If $t_q < t_t$, the hydrograph reaches partial equilibrium with a constant discharge Q_p. If $t_q \geq t_t$, the hydrograph reaches equilibrium with a constant discharge Q_e.

10.12.1. Partial Equilibrium Discharge

Substituting Eqs. (10.10) and (10.11) into Eq. (4.66) gives the equation for the partial equilibrium discharge for a channel with upstream inflow:

$$Q_p = 0.340\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)\left[2.186\left(\frac{n_c W^{0.0909} Q_u}{S_c^{1/2}}\right)^{0.725} + 60 q_L t_q\right]^{1.379} \qquad (10.42)$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (10.42) reduces to:

$$Q_p = 96.28\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)\left(q_L t_q\right)^{1.379} \qquad (10.43)$$

Equations (10.42) and (10.43) are valid for $t_q \leq t \leq (t_q + t_d)$.

10.12.2. Equilibrium Discharge

From Eq. (4.69), the equation for the equilibrium discharge for a channel with upstream inflow is:

$$Q_e = Q_u + q_L L_c \qquad (10.44)$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (10.44) reduces to:

$$Q_e = q_L L_c \tag{10.45}$$

Equations (10.44) and (10.45) are valid for $t_t \le t \le t_q$.

10.13. WATER SURFACE PROFILE - EQUILIBRIUM PHASE

Substituting Eqs. (10.10) and (10.11) into Eq. (4.71) gives the equation for the equilibrium flow area profile for a channel with upstream inflow between $-L_u \le x_c \le L_p$:

$$A_c = 2.186\left[\left(\frac{n_c W^{0.0909}}{S_c^{1/2}}\right)(q_L x_c + Q_u)\right]^{0.725} \tag{10.46}$$

Substituting Eq. (10.1) into Eq. (10.40), and solving it gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $-L_u \le x_c \le L_p$:

$$y_c = \frac{-W + \left\{W^2 + 8.748z\left[\frac{n_c W^{0.0909}(q_L x_c + Q_u)}{S_c^{1/2}}\right]^{0.725}\right\}^{1/2}}{2z} \tag{10.47}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (10.46) and (10.47) reduce to:

$$A_c = 2.186\left(\frac{n_c W^{0.0909} q_L x_c}{S_c^{1/2}}\right)^{0.725} \tag{10.48}$$

$$y_c = \frac{-W + \left[W^2 + 8.748z\left(\frac{n_c W^{0.0909} q_L x_c}{S_c^{1/2}}\right)^{0.725}\right]^{1/2}}{2z} \tag{10.49}$$

which are valid for $0 \le x_c \le L_c$.

10.14. EQUILIBRIUM DETENTION STORAGE

Substituting Eqs. (10.10) and (10.11) into Eq. (4.78) gives the equilibrium detention storage for a channel with upstream inflow:

$$D_{ec} = 1.268\left(\frac{n_c W^{0.0909}}{S_c^{1/2}}\right)^{0.725}\left[\frac{\left(Q_u + q_L L_c\right)^{1.725} - Q_u^{1.725}}{q_L}\right] \tag{10.50}$$

For a channel with zero upstream inflow (i.e. $Q_u = 0$), Eq. (10.50) reduces to:

$$D_{ec} = 1.268\left(\frac{n_c W^{0.0909} q_L}{S_c^{1/2}}\right)^{0.725} L_c^{1.725} \tag{10.51}$$

10.15. WATER SURFACE PROFILE – FALLING PHASE

Substituting Eqs. (10.10) and (10.11) into Eq. (4.91) gives the equation for the falling phase of the flow area profile for a channel with upstream inflow, which is valid for $L_f \leq x_c \leq L_c$.

$$x_c = 28.14\left(\frac{S_c^{1/2} A_c^{0.379}}{n_c W^{0.0909}}\right)\left(t - t_q\right) + \left[\frac{0.340\left(\frac{S_c^{1/2} A_c^{1.379}}{n_c W^{0.0909}}\right) - Q_u}{q_L}\right] \tag{10.52}$$

Substituting Eq. (10.1) into Eq. (10.46) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $L_f \leq x_c \leq L_c$:

$$x_c = 28.14\left[\frac{S_c^{1/2}\left(zy_c^2 + Wy_c\right)^{0.379}}{n_c W^{0.0909}}\right]\left(t - t_q\right) + \left\{\frac{0.340\left[\frac{S_c^{1/2}\left(zy_c^2 + Wy_c\right)^{1.379}}{n_c W^{0.0909}}\right] - Q_u}{q_L}\right\} \tag{10.53}$$

From Eq. (4.92), the equation for the flow area profile between $0 \leq x_c \leq L_f$ is:

$$A_c = 2.186\left(\frac{n_c W^{0.0909} Q_u}{S_c^{1/2}}\right)^{0.725} \tag{10.54}$$

Substituting Eq. (10.1) into Eq. (10.50) and solving it gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \leq x_c \leq L_f$.

$$y_c = \frac{-W + \left[W^2 + 8.748z \left(\frac{n_c W^{0.0909} Q_u}{S_c^{1/2}} \right)^{0.725} \right]^{1/2}}{2z} \tag{10.55}$$

Substituting Eqs. (10.10) and (10.11) into Eq. (4.94) gives the equation for the distance L_f for a channel with upstream inflow, which is valid for $t \geq t_q$:

$$L_f = 27.84 \left(\frac{S_c^{1/2}}{n_c W^{0.0909}} \right)^{0.725} Q_u^{0.275} \left(t - t_q \right) \tag{10.56}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (10.52) and (10.53) reduce to:

$$x_c = 28.14 \left(\frac{S_c^{1/2} A_c^{0.379}}{n_c W^{0.0909}} \right) \left(t - t_q \right) + \left[\frac{0.340 \left(\frac{S_c^{1/2} A_c^{1.379}}{n_c W^{0.0909}} \right)}{q_L} \right] \tag{10.57}$$

$$x_c = 28.14 \left[\frac{S_c^{1/2} \left(z y_c^2 + W y_c \right)^{0.379}}{n_c W^{0.0909}} \right] \left(t - t_q \right) + \left\{ \frac{0.340 \left[\frac{S_c^{1/2} \left(z y_c^2 + W y_c \right)^{1.379}}{n_c W^{0.0909}} \right]}{q_L} \right\} \tag{10.58}$$

which are valid for $0 \leq x_c \leq L_c$.

10.16. HYDROGRAPH - FALLING PHASE

Substituting Eqs. (10.10) and (10.11) into Eq. (4.101) gives the equation for the falling phase of a hydrograph for a channel with upstream inflow:

$$t = \frac{0.0264}{Q_c^{0.275}} \left(\frac{n_c W^{0.0909}}{S_c^{1/2}} \right)^{0.725} \left[L_c - \left(\frac{Q_c - Q_u}{q_L} \right) \right] + t_q \tag{10.59}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (10.59) reduces to:

$$t = \frac{0.0264}{Q_c^{0.275}} \left(\frac{n_c W^{0.0909}}{S_c^{1/2}} \right)^{0.725} \left[L_c - \left(\frac{Q_c}{q_L} \right) \right] + t_q \tag{10.60}$$

In: Kinematic-Wave Rainfall-Runoff Formulas
Editor: Tommy S.W. Wong, pp. 137-149
ISBN: 978-1-60692-705-2
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Chapter 11

11. WORKING FORMULAS FOR FLOW IN TRAPEZOIDAL CHANNEL WITH ONE SIDE VERTICAL

In this Chapter, based on the Manning's equation, the kinematic wave parameters for flow in a trapezoidal channel with one side vertical are derived. By applying these parameters to the general formulas in Chapter 4, working formulas for the flow depth; flow velocity; average flow velocity; wave celerity; average wave celerity; time of travel; rising, equilibrium and falling phases of hydrograph; forward characteristic; rising, equilibrium and falling phases of water surface profiles; duration of partial equilibrium discharge; and equilibrium detention storage are also derived.

11.1. KINEMATIC WAVE PARAMETERS

For flow in a trapezoidal channel with one side vertical, the flow area A_c, and the wetted perimeter P, are related to the flow depth y_c, and the reciprocal of channel side slope z, as follows:

$$A_c = 0.5zy_c^2 + Wy_c \tag{11.1}$$

$$P = \left[1 + \left(1 + z^2\right)^{1/2}\right]y_c + W \tag{11.2}$$

Figure 11.1 shows the trapezoidal channel with base width W, reciprocal of channel side slope z, and flow depth y_c. Next, defining two dimensionless variables, ψ and μ, as:

$$\psi = A_c / W^2 \tag{11.3}$$

$$\mu = y_c / W \tag{11.4}$$

Substituting Eqs. (11.3) and (11.4) into Eq. (11.1) gives the following relationship between ψ and μ:

$$0.5z\mu^2 + \mu - \psi = 0 \tag{11.5}$$

For z ≠ 0, the positive solution for Eq. (11.5) is:

$$\mu = \frac{(1+2z\psi)^{1/2} - 1}{z} \tag{11.6}$$

Substituting Eq. (11.4) into Eq. (11.6) to eliminate μ results in:

$$y_c = \left[\frac{(1+2z\psi)^{1/2} - 1}{z}\right]W \tag{11.7}$$

Substituting Eq. (11) into Eq. (6) gives an expression for P:

$$P = \left\{1 + \frac{\left[1+(1+z^2)^{1/2}\right]\left[(1+2z\psi)^{1/2} - 1\right]}{z}\right\}W \tag{11.8}$$

Substituting Eqs. (11.3) and (11.8) into Eq. (3.1) results in a dimensionless equation in terms of ψ, which can be considered as the "true" relationship Q_c and A_c for a trapezoidal channel with one side vertical:

$$\frac{n_c Q_c}{S_c^{1/2} W^{8/3}} = \left\{\frac{z}{z + \left[1+(1+z^2)^{1/2}\right]\left[(1+2z\psi)^{1/2} - 1\right]}\right\}^{2/3} \psi^{5/3} \tag{11.9}$$

As shown in figure 11.2, by mathematical fitting to the true relationships for $0.1 \le z \le 5.0$, Wong and Zhou (2006) obtained the following kinematic wave parameters:

$$\alpha_c = 0.323\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right) \tag{11.10}$$

$$\beta_c = 1.360 \tag{11.11}$$

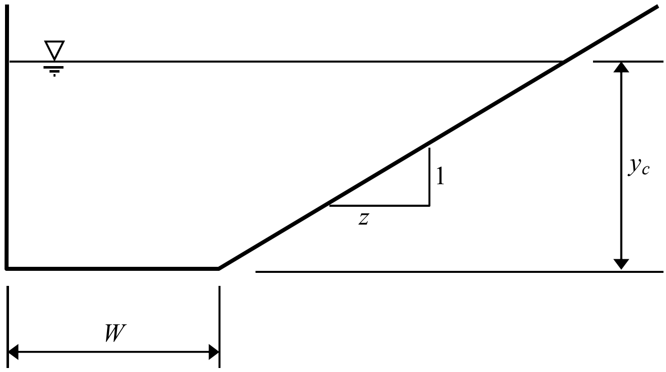

Figure 11.1. Cross-section of Trapezoidal Channel with One Side Vertical.

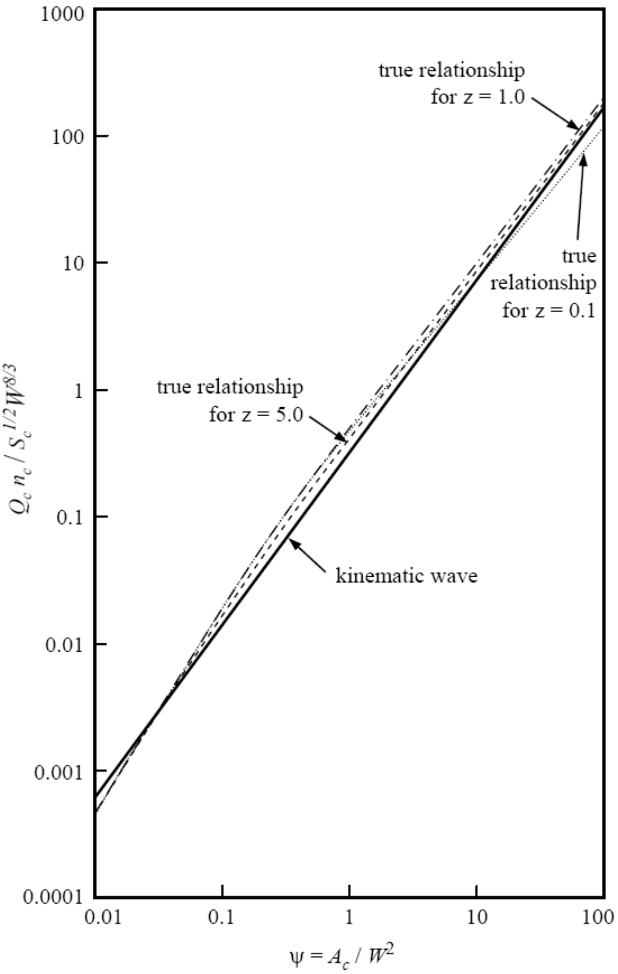

Figure 11.2. Comparison between True and Kinematic Wave Relationships for Flow in Trapezoidal Channel with One Side Vertical.

11.2. Flow Depth

Substituting Eqs. (11.10) and (11.11) into Eq. (4.7) gives:

$$Q_c = 0.323\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)A_c^{1.360} \tag{11.12}$$

Substituting Eqs. (11.1) into Eq. (11.12) gives:

$$Q_c = 0.323\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)\left(0.5zy_c^2 + Wy_c\right)^{1.360} \tag{11.13}$$

Rearranging Eq. (11.13) gives:

$$0.5zy_c^2 + Wy_c - 2.296\left(\frac{n_c W^{0.0526} Q_c}{S_c^{1/2}}\right)^{0.735} = 0 \tag{11.14}$$

Solving Eq. (11.14) gives the equation for the flow depth for a channel with and without upstream inflow:

$$y_c = \frac{-W + \left[W^2 + 4.592z\left(\frac{n_c W^{0.0526} Q_c}{S_c^{1/2}}\right)^{0.735}\right]^{1/2}}{z} \tag{11.15}$$

11.3. Flow Velocity

Substituting Eqs. (11.10) and (11.11) into Eq. (4.12) gives the equation of flow velocity along the equilibrium profile for a channel with upstream inflow:

$$v = 0.436\left[\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)\left(Q_u + q_L x_c\right)^{0.360}\right]^{0.735} \tag{11.16}$$

For a channel with zero upstream inflow zero upstream inflow (Q_u = 0), Eq. (11.16) reduces to:

$$v = 0.436\left[\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)\left(q_L x_c\right)^{0.360}\right]^{0.735} \tag{11.17}$$

11.4. AVERAGE FLOW VELOCITY

Substituting Eqs. (11.10) and (11.11) into Eq. (4.15) gives the equation of average flow velocity for a channel with upstream inflow:

$$v_{av} = \frac{0.321\left(\dfrac{S_c^{1/2}}{n_c W^{0.0526}}\right)^{0.735} q_L L_c}{\left(Q_u + q_L L_c\right)^{0.735} - Q_u^{\ 0.735}} \tag{11.18}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (11.18) reduces to:

$$v_{av} = 0.321\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)^{0.735} \left(q_L L_c\right)^{0.265} \tag{11.19}$$

11.5. KINEMATIC WAVE CELERITY

Substituting Eqs. (11.10) and (11.11) into Eq. (4.29) gives the equation of wave celerity along the equilibrium profile for a channel with upstream inflow:

$$c_k = 0.593\left[\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)\left(Q_u + q_L x_c\right)^{0.360}\right]^{0.735} \tag{11.20}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (11.20) reduces to:

$$c_k = 0.593\left[\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)\left(q_L x_c\right)^{0.360}\right]^{0.735} \tag{11.21}$$

11.6. AVERAGE WAVE CELERITY

Substituting Eqs. (11.10) and (11.11) into Eq. (4.32) gives the equation of wave celerity for a channel with upstream inflow:

$$c_{av} = \frac{0.436\left(\dfrac{S_c^{1/2}}{n_c W^{0.0526}}\right)^{0.735} q_L L_c}{\left(Q_u + q_L L_c\right)^{0.735} - Q_u^{\ 0.735}} \tag{11.22}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (11.22) reduces to:

$$c_{av} = 0.436\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)^{0.735} (q_L L_c)^{0.265} \tag{11.23}$$

11.7. TIME OF TRAVEL

Substituting Eqs. (11.10) and (11.11)) into Eq. (4.35) gives the equation of time of travel for a channel with upstream inflow:

$$t_t = 0.0382\left(\frac{n_c W^{0.0526}}{S_c^{1/2}}\right)^{0.735}\left[\frac{(Q_u + q_L L_c)^{0.735} - Q_u^{0.735}}{q_L}\right] \tag{11.24}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (11.24) reduces to:

$$t_t = \left(\frac{0.0382}{q_L^{0.265}}\right)\left(\frac{n_c W^{0.0526} L_c}{S_c^{1/2}}\right)^{0.735} \tag{11.25}$$

11.8. HYDROGRAPH – RISING PHASE

Substituting Eqs. (11.10) and (11.11) into Eq. (4.45) gives the equation for the rising phase of the hydrograph for a channel with upstream inflow:

$$Q_c = 0.323\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)\left[2.295\left(\frac{n_c W^{0.0526} Q_u}{S_c^{1/2}}\right)^{0.735} + 60 q_L t\right]^{1.360} \tag{11.26}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (11.26) reduces to:

$$Q_c = 84.62\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)(q_L t)^{1.360} \tag{11.27}$$

Equations (11.26) and (11.27) are valid for $t \le t_t$.

11.9. FORWARD CHARACTERISTIC - RISING PHASE

Substituting Eqs. (11.10) and (11.11) into Eq. (4.47) gives the equation for the forward characteristic of the rising phase for a channel with upstream inflow:

$$t = 0.0383\left(\frac{n_c W^{0.0526}}{S_c^{1/2}}\right)^{0.735}\left[\frac{(Q_u + q_L x_c)^{0.735} - Q_u^{0.735}}{q_L}\right] \tag{11.28}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (11.28) reduces to:

$$t = \frac{0.0383}{q_L^{0.265}}\left(\frac{n_c W^{0.0526} x_c}{S_c^{1/2}}\right)^{0.735} \tag{11.29}$$

11.10. WATER SURFACE PROFILE – RISING PHASE

Substituting Eqs. (11.10) and (11.11) into Eq. (4.49) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$A_c = 2.295\left[\left(\frac{n_c W^{0.0526}}{S_c^{1/2}}\right)(q_L x_c + Q_u)\right]^{0.735} \tag{11.30}$$

Substituting Eq. (11.1) into Eq. (11.28), and solving it gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$y_c = \frac{-W + \left\{W^2 + 4.592z\left[\frac{n_c W^{0.0526}(q_L x_c + Q_u)}{S_c^{1/2}}\right]^{0.735}\right\}^{1/2}}{z} \tag{11.31}$$

Substituting Eqs. (11.10) and (11.11) into Eq. (4.50) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $L_p \le x_c \le L_c$:

$$A_p = 2.295\left[\left(\frac{n_c W^{0.0526}}{S_c^{1/2}}\right)(q_L L_p + Q_u)\right]^{0.735} \tag{11.32}$$

Substituting $A_c = A_p$, and $y_c = y_p$ into Eq. (11.1), and then substituting it into Eq. (11.32), and solving it gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $L_p \le x_c \le L_c$:

$$y_p = \frac{-W + \left\{W^2 + 4.592z\left[\frac{n_c W^{0.0526}\left(q_L L_p + Q_u\right)}{S_c^{1/2}}\right]^{0.735}\right\}^{1/2}}{z} \tag{11.33}$$

from Eq. (4.51), the distance L_p is:

$$L_p = \frac{Q_p - Q_u}{q_L} \tag{11.34}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (11.30)-(11.34) reduce to:

$$A_c = 2.295\left(\frac{n_c W^{0.0526} q_L x_c}{S_c^{1/2}}\right)^{0.735} \tag{11.35}$$

$$y_c = \frac{-W + \left[W^2 + 4.592z\left(\frac{n_c W^{0.0526} q_L x_c}{S_c^{1/2}}\right)^{0.735}\right]^{1/2}}{z} \tag{11.36}$$

which are valid for $0 \le x_c \le L_p$,

$$A_p = 2.295\left(\frac{n_c W^{0.0526} q_L L_p}{S_c^{1/2}}\right)^{0.735} \tag{11.37}$$

$$y_p = \frac{-W + \left[W^2 + 4.592z\left(\frac{n_c W^{0.0526} q_L L_p}{S_c^{1/2}}\right)^{0.735}\right]^{1/2}}{z} \tag{11.38}$$

which are valid for $L_p \le x_c \le L_c$, and

$$L_p = \frac{Q_p}{q_L} \tag{11.39}$$

11.11. Duration of Partial Equilibrium Discharge

Substituting Eqs. (11.10) and (11.11) into Eq. (4.63) gives the equation for the duration of partial equilibrium discharge for a channel with upstream inflow:

$$t_d = \frac{L_c q_L + Q_u - 0.323\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)\left[2.295\left(\frac{n_c W^{0.0526} Q_u}{S_c^{1/2}}\right)^{0.735} + 60 q_L t_q\right]^{1.360}}{26.54\left(\frac{S_c^{1/2} q_L}{n_c W^{0.0526}}\right)\left[2.295\left(\frac{n_c W^{0.0526} Q_u}{S_c^{1/2}}\right)^{0.735} + 60 q_L t_q\right]^{0.360}} \tag{11.40}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (11.38) reduces to:

$$t_d = \frac{L_c - 84.62\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right) q_L^{0.360} t_q^{1.360}}{115.1\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)\left(q_L t_q\right)^{0.360}} \tag{11.41}$$

11.12. Hydrograph - Equilibrium Phase

As shown in figures 4.2 and 4.3, the hydrograph may reach partial equilibrium or equilibrium depending on the duration of lateral inflow t_q. If $t_q < t_t$, the hydrograph reaches partial equilibrium with a constant discharge Q_p. If $t_q \geq t_t$, the hydrograph reaches equilibrium with a constant discharge Q_e.

11.12.1. Partial Equilibrium Discharge

Substituting Eqs. (11.10) and (11.11) into Eq. (4.66) gives the equation for the partial equilibrium discharge for a channel with upstream inflow:

$$Q_p = 0.323\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)\left[2.295\left(\frac{n_c W^{0.0526} Q_u}{S_c^{1/2}}\right)^{0.735} + 60 q_L t_q\right]^{1.360} \tag{11.42}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (11.42) reduces to:

$$Q_p = 84.62\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)(q_L t_q)^{1.360} \tag{11.43}$$

Equations (11.42) and (11.43) are valid for $t_q \leq t \leq (t_q + t_d)$.

11.12.2. Equilibrium Discharge

From Eq. (4.69), the equation for the equilibrium discharge for a channel with upstream inflow is:

$$Q_e = Q_u + q_L L_c \tag{11.44}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (11.44) reduces to:

$$Q_e = q_L L_c \tag{11.45}$$

Equations (11.44) and (11.45) are valid for $t_t \leq t \leq t_q$.

11.13. Water Surface Profile - Equilibrium Phase

Substituting Eqs. (11.10) and (11.11) into Eq. (4.71) gives the equation for the equilibrium flow area profile for a channel with upstream inflow between $-L_u \leq x_c \leq L_p$.

$$A_c = 2.295\left[\left(\frac{n_c W^{0.0526}}{S_c^{1/2}}\right)(q_L x_c + Q_u)\right]^{0.735} \tag{11.46}$$

Substituting Eq. (11.1) into Eq. (11.46), and solving it gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $-L_u \leq x_c \leq L_p$:

$$y_c = \frac{-W + \left\{W^2 + 4.592z\left[\frac{n_c W^{0.0526}(q_L x_c + Q_u)}{S_c^{1/2}}\right]^{0.735}\right\}^{1/2}}{z} \tag{11.47}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (11.46) and (11.47) reduce to:

$$A_c = 2.295\left(\frac{n_c W^{0.0526} q_L x_c}{S_c^{1/2}}\right)^{0.735} \tag{11.48}$$

$$y_c = \frac{-W + \left[W^2 + 4.592z\left(\frac{n_c W^{0.0526} q_L x_c}{S_c^{1/2}}\right)^{0.735}\right]^{1/2}}{z} \tag{11.49}$$

which are valid for $0 \leq x_c \leq L_c$.

11.14. EQUILIBRIUM DETENTION STORAGE

Substituting Eqs. (11.10) and (11.11) into Eq. (4.78) gives the equilibrium detention storage for a channel with upstream inflow:

$$D_{ec} = 1.322\left(\frac{n_c W^{0.0526}}{S_c^{1/2}}\right)^{0.735}\left[\frac{(Q_u + q_L L_c)^{1.735} - Q_u^{1.735}}{q_L}\right] \tag{11.50}$$

For a channel with zero upstream inflow (i.e. $Q_u = 0$), Eq. (11.50) reduces to:

$$D_{ec} = 1.322\left(\frac{n_c W^{0.0526} q_L}{S_c^{1/2}}\right)^{0.735} L_c^{1.735} \tag{11.51}$$

11.15. WATER SURFACE PROFILE – FALLING PHASE

Substituting Eqs. (11.10) and (11.11) into Eq. (4.91) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $L_f \leq x_c \leq L_c$.

$$x_c = 26.36\left(\frac{S_c^{1/2} A_c^{0.360}}{n_c W^{0.0526}}\right)\left(t - t_q\right) + \left[\frac{0.323\left(\frac{S_c^{1/2} A_c^{1.360}}{n_c W^{0.0526}}\right) - Q_u}{q_L}\right] \tag{11.52}$$

Substituting Eq. (11.1) into Eq. (11.52) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $L_f \leq x_c \leq L_c$:

$$x_c = 26.36\left[\frac{S_c^{1/2}\left(0.5zy_c^2 + Wy_c\right)^{0.360}}{n_c W^{0.0526}}\right]\left(t - t_q\right) + \left\{\frac{0.323\left[\frac{S_c^{1/2}\left(0.5zy_c^2 + Wy_c\right)^{1.360}}{n_c W^{0.0526}}\right] - Q_u}{q_L}\right\} \quad (11.53)$$

From Eq. (4.92), the equation for the flow area profile between $0 \leq x_c \leq L_f$ is:

$$A_c = 2.295\left(\frac{n_c W^{0.0526} Q_u}{S_c^{1/2}}\right)^{0.735} \quad (11.54)$$

Substituting Eq. (11.1) into Eq. (11.54) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \leq x_c \leq L_f$:

$$y_c = \frac{-W + \left[W^2 + 4.592z\left(\frac{n_c W^{0.0526} Q_u}{S_c^{1/2}}\right)^{0.735}\right]^{1/2}}{z} \quad (11.55)$$

Substituting Eqs. (11.10) and (11.11) into Eq. (4.94) gives the equation for the distance L_f for a channel with upstream inflow, which is valid for $t \geq t_q$:

$$L_f = 35.56\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)^{0.735} Q_u^{0.265}\left(t - t_q\right) \quad (11.56)$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (11.52) and (11.53) reduce to:

$$x_c = 26.357\left(\frac{S_c^{1/2} A_c^{0.360}}{n_c W^{0.0526}}\right)\left(t - t_q\right) + \left[\frac{0.323\left(\frac{S_c^{1/2} A_c^{1.360}}{n_c W^{0.0526}}\right)}{q_L}\right] \quad (11.57)$$

$$x_c = 26.357\left[\frac{S_c^{1/2}\left(0.5zy_c^2 + Wy_c\right)^{0.360}}{n_c W^{0.0526}}\right]\left(t - t_q\right) + \left\{\frac{0.323\left[\frac{S_c^{1/2}\left(0.5zy_c^2 + Wy_c\right)^{1.360}}{n_c W^{0.0526}}\right]}{q_L}\right\} \quad (11.58)$$

which are valid for $0 \le x_c \le L_c$.

11.16. HYDROGRAPH - FALLING PHASE

Substituting Eqs. (11.10) and (11.11) into Eq. (4.101) gives the equation for the falling phase of a hydrograph for a channel with upstream inflow:

$$t = \frac{0.0281}{Q_c^{0.265}} \left(\frac{n_c W^{0.0526}}{S_c^{1/2}} \right)^{0.735} \left[L_c - \left(\frac{Q_c - Q_u}{q_L} \right) \right] + t_q \tag{11.59}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (11.59) reduces to:

$$t = \frac{0.0281}{Q_c^{0.265}} \left(\frac{n_c W^{0.0526}}{S_c^{1/2}} \right)^{0.735} \left[L_c - \left(\frac{Q_c}{q_L} \right) \right] + t_q \tag{11.60}$$

In: Kinematic-Wave Rainfall-Runoff Formulas ISBN: 978-1-60692-705-2
Editor: Tommy S.W. Wong, pp. 151-161 © 2009 Nova Science Publishers, Inc.

Chapter 12

12. WORKING FORMULAS FOR FLOW IN TRIANGULAR CHANNEL

In this Chapter, based on the Manning's equation, the kinematic wave parameters for flow in a triangular channel are derived. By applying these parameters to the general formulas in Chapter 4, working formulas for the flow depth; flow velocity; average flow velocity; wave celerity; average wave celerity; time of travel; rising, equilibrium and falling phases of hydrograph; forward characteristic; rising, equilibrium and falling phases of water surface profiles; duration of partial equilibrium discharge; and equilibrium detention storage are also derived.

12.1. KINEMATIC WAVE PARAMETERS

For flow in a triangular channel as shown in figure 12.1, the flow area A_c, and the wetted perimeter P, are related to the flow depth, y_c and the reciprocal of channel side slope, z as follows:

$$A_c = zy_c^2 \tag{12.1}$$

$$P = 2y_c\left(1+z^2\right)^{1/2} \tag{12.2}$$

Substituting Eqs. (12.1) and (12.2) and $A = A_c$ into Eq. (3.2) gives:

$$R = \frac{zy_c}{2\left(1+z^2\right)^{1/2}} \tag{12.3}$$

Substituting Eq. (12.1) into Eq. (12.3) gives:

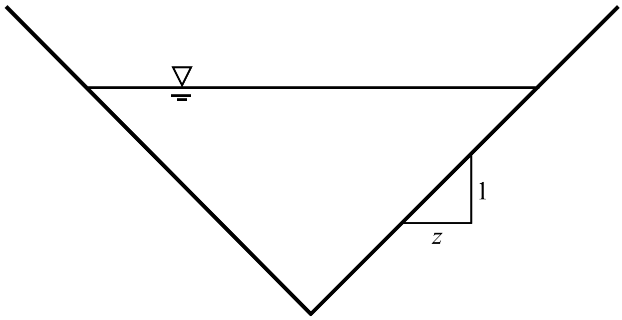

Figure 12.1. Cross-section of Triangular Channel.

$$R = \frac{1}{2}\left(\frac{zA_c}{1+z^2}\right)^{1/2} \tag{12.4}$$

Substituting Eq. (12.4) and $Q = Q_c$, $S = S_c$, $A = A_c$, $n = n_c$ into Eq. (3.1) gives:

$$Q_c = 0.630\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3} A_c^{4/3} \tag{12.5}$$

A comparison of Eq. (12.5) with Eq. (4.7) gives the kinematic wave parameters (Wong 2008b):

$$\alpha_c = 0.630\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3} \tag{12.6}$$

$$\beta_c = \frac{4}{3} \tag{12.7}$$

12.2. FLOW DEPTH

Substituting Eq. (12.1) into Eq. (12.5) gives:

$$Q_c = 0.630\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z^5}{1+z^2}\right)^{1/3} y_c^{8/3} \tag{12.8}$$

Rearranging Eq. (12.8) gives the equation for the flow depth for a channel with and without upstream inflow:

$$y_c = 1.190 \left(\frac{n_c Q_c}{S_c^{1/2}} \right)^{3/8} \left(\frac{1+z^2}{z^5} \right)^{1/8} \tag{12.9}$$

12.3. Flow Velocity

Substituting Eqs. (12.6) and (12.7) into Eq. (4.12) gives the equation of flow velocity along the equilibrium profile for a channel with upstream inflow:

$$v = 0.707 \left(\frac{S_c^{1/2}}{n_c} \right)^{3/4} \left[\frac{z(Q_u + q_L x_c)}{1+z^2} \right]^{1/4} \tag{12.10}$$

For a channel with zero upstream inflow zero upstream inflow (Q_u = 0), Eq. (12.10) reduces to:

$$v = 0.707 \left(\frac{S_c^{1/2}}{n_c} \right)^{3/4} \left(\frac{z q_L x_c}{1+z^2} \right)^{1/4} \tag{12.11}$$

12.4. Average Flow Velocity

Substituting Eqs. (12.6) and (12.7) into Eq. (4.15) gives the equation of average flow velocity for a channel with upstream inflow:

$$v_{av} = \frac{0.530 \left(\frac{S_c^{1/2}}{n_c} \right)^{3/4} \left(\frac{z}{1+z^2} \right)^{1/4} q_L L_c}{(Q_u + q_L L_c)^{3/4} - Q_u^{\ 3/4}} \tag{12.12}$$

For a channel with zero upstream inflow (Q_u = 0), Eq. (12.12) reduces to:

$$v_{av} = 0.530 \left(\frac{S_c^{1/2}}{n_c} \right)^{3/4} \left(\frac{z q_L L_c}{1+z^2} \right)^{1/4} \tag{12.13}$$

12.5. Kinematic Wave Celerity

Substituting Eqs. (12.6) and (12.7) into Eq. (4.29) gives the equation of wave celerity along the equilibrium profile for a channel with upstream inflow:

$$c_k = 0.943\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left[\frac{z(Q_u + q_L x_c)}{1+z^2}\right]^{1/4} \tag{12.14}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (12.14) reduces to:

$$c_k = 0.943\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left[\frac{z q_L x_c}{1+z^2}\right]^{1/4} \tag{12.15}$$

12.6. Average Wave Celerity

Substituting Eqs. (12.6) and (12.7) into Eq. (4.32) gives the equation of wave celerity for a channel with upstream inflow:

$$c_{av} = \frac{0.707\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left(\frac{z}{1+z^2}\right)^{1/4} q_L L_c}{(Q_u + q_L L_c)^{3/4} - Q_u^{3/4}} \tag{12.16}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (12.16) reduces to:

$$c_{av} = 0.707\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left(\frac{z q_L L_c}{1+z^2}\right)^{1/4} \tag{12.17}$$

12.7. Time of Travel

Substituting Eqs. (12.6) and (12.7) into Eq. (4.35) gives the equation of time of travel for a channel with upstream inflow:

$$t_t = 0.0236\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z}\right)^{1/4}\left[\frac{(Q_u + q_L L_c)^{3/4} - Q_u^{3/4}}{q_L}\right] \tag{12.18}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (12.18) reduces to:

$$t_t = 0.0236\left(\frac{n_c L_c}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z q_L}\right)^{1/4} \tag{12.19}$$

12.8. HYDROGRAPH – RISING PHASE

Substituting Eqs. (12.6) and (12.7) into Eq. (4.45) gives the equation for the rising phase of the hydrograph for a channel with upstream inflow:

$$Q_c = 0.630\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3}\left[1.414\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z}\right)^{1/4} + 60 q_L t\right]^{4/3} \qquad (12.20)$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (12.20) reduces to:

$$Q_c = 148.0\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3}(q_L t)^{4/3} \qquad (12.21)$$

Equations (12.20) and (12.21) are valid for $t \le t_t$.

12.9. FORWARD CHARACTERISTIC - RISING PHASE

Substituting Eqs. (12.6) and (12.7) into Eq. (4.47) gives the equation for the forward characteristic of the rising phase for a channel with upstream inflow:

$$t = 0.0236\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z}\right)^{1/4}\left[\frac{(Q_u + q_L x_c)^{3/4} - Q_u^{3/4}}{q_L}\right] \qquad (12.22)$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (12.22) reduces to:

$$t = 0.0236\left(\frac{n_c x_c}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z q_L}\right)^{1/4} \qquad (12.23)$$

12.10. WATER SURFACE PROFILE – RISING PHASE

Substituting Eqs. (12.6) and (12.7) into Eq. (4.49) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$A_c = 1.414\left[\left(\frac{n_c}{S_c^{1/2}}\right)\left(\frac{1+z^2}{z}\right)^{1/3}(Q_u + q_L x_c)\right]^{3/4} \qquad (12.24)$$

Substituting Eq. (12.1) into Eq. (12.24) gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$y_c = 1.189\left[\left(\frac{n_c}{S_c^{1/2}}\right)\left(\frac{1+z^2}{z^5}\right)^{1/3}\left(Q_u + q_L x_c\right)\right]^{3/8} \tag{12.25}$$

Substituting Eqs. (12.6) and (12.7) into Eq. (4.50) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $L_p \le x_c \le L_c$:

$$A_p = 1.414\left[\left(\frac{n_c}{S_c^{1/2}}\right)\left(\frac{1+z^2}{z}\right)^{1/3}\left(Q_u + q_L L_p\right)\right]^{3/4} \tag{12.26}$$

Substituting $A_c = A_p$, and $y_c = y_p$ into Eq. (12.1) and then substituting it into Eq. (12.26) gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $L_p \le x_c \le L_c$:

$$y_p = 1.189\left[\left(\frac{n_c}{S_c^{1/2}}\right)\left(\frac{1+z^2}{z^5}\right)^{1/3}\left(Q_u + q_L L_p\right)\right]^{3/8} \tag{12.27}$$

From Eq. (4.51), the distance L_p is:

$$L_p = \frac{Q_p - Q_u}{q_L} \tag{12.28}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (12.24)-(12.28) reduce to:

$$A_c = 1.414\left[\frac{n_c q_L x_c}{S_c^{1/2}}\left(\frac{1+z^2}{z}\right)^{1/3}\right]^{3/4} \tag{12.29}$$

$$y_c = 1.189\left[\frac{n_c q_L x_c}{S_c^{1/2}}\left(\frac{1+z^2}{z^5}\right)^{1/3}\right]^{3/8} \tag{12.30}$$

which are valid for $0 \le x_c \le L_p$,

$$A_p = 1.414\left[\frac{n_c q_L L_p}{S_c^{1/2}}\left(\frac{1+z^2}{z}\right)^{1/3}\right]^{3/4} \tag{12.31}$$

$$y_p = 1.189\left[\frac{n_c q_L L_p}{S_c^{1/2}}\left(\frac{1+z^2}{z^5}\right)^{1/3}\right]^{3/8} \tag{12.32}$$

which are valid for $L_p \leq x_c \leq L_c$, and

$$L_p = \frac{Q_p}{q_L} \tag{12.33}$$

12.11. Duration of Partial Equilibrium Discharge

Substituting Eqs. (12.6) and (12.7) into Eq. (4.63) gives the equation for the duration of partial equilibrium discharge for a channel with upstream inflow:

$$t_d = \frac{L_c q_L + Q_u - 0.630\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3}\left[1.414\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z}\right)^{1/4} + 60 q_L t_q\right]^{4/3}}{50.40\left(\frac{S_c^{1/2} q_L}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3}\left[1.414\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z}\right)^{1/4} + 60 q_L t_q\right]^{1/3}} \tag{12.34}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (12.34) reduces to:

$$t_d = \frac{L_c - 148.0\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3} q_L^{1/3} t_q^{4/3}}{197.3\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3}\left(q_L t_q\right)^{1/3}} \tag{12.35}$$

12.12. Hydrograph - Equilibrium Phase

As shown in figures 4.2 and 4.3, the hydrograph may reach partial equilibrium or equilibrium depending on the duration of lateral inflow t_q. If $t_q < t_t$, the hydrograph reaches partial equilibrium with a constant discharge Q_p. If $t_q \geq t_t$, the hydrograph reaches equilibrium with a constant discharge Q_e.

12.12.1. Partial Equilibrium Discharge

Substituting Eqs. (12.6) and (12.7) into Eq. (4.66) gives the equation for the partial equilibrium discharge for a channel with upstream inflow:

$$Q_p = 0.630\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3}\left[1.414\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z}\right)^{1/4} + 60 q_L t_q\right]^{4/3} \quad (12.36)$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (12.36) reduces to:

$$Q_p = 148.0\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3}\left(q_L t_q\right)^{4/3} \quad (12.37)$$

Equations (12.36) and (12.37) are valid for $t_q \le t \le (t_q + t_d)$.

12.12.2. Equilibrium Discharge

From Eq. (4.69), the equation for the equilibrium discharge for a channel with upstream inflow is:

$$Q_e = Q_u + q_L L_c \quad (12.38)$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (12.38) reduces to:

$$Q_e = q_L L_c \quad (12.39)$$

Equations (12.38) and (12.39) are valid for $t_t \le t \le t_q$.

12.13. Water Surface Profile - Equilibrium Phase

Substituting Eqs. (12.6) and (12.7) into Eq. (4.71) gives the equation for the equilibrium flow area profile for a channel with upstream inflow between $-L_u \le x_c \le L_p$.

$$A_c = 1.414\left[\left(\frac{n_c}{S_c^{1/2}}\right)\left(\frac{1+z^2}{z}\right)^{1/3}\left(Q_u + q_L x_c\right)\right]^{3/4} \quad (12.40)$$

Substituting Eq. (12.1) into Eq. (12.40) gives the equation for the equilibrium water surface profile for a channel with upstream inflow, which is valid for $-L_u \le x_c \le L_p$:

$$y_c = 1.189\left[\left(\frac{n_c}{S_c^{1/2}}\right)\left(\frac{1+z^2}{z^5}\right)^{1/3}(Q_u + q_L x_c)\right]^{3/8} \quad (12.41)$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (12.40) and (12.41) reduce to:

$$A_c = 1.414\left[\frac{n_c q_L x_c}{S_c^{1/2}}\left(\frac{1+z^2}{z}\right)^{1/3}\right]^{3/4} \quad (12.42)$$

$$y_c = 1.189\left[\frac{n_c q_L x_c}{S_c^{1/2}}\left(\frac{1+z^2}{z^5}\right)^{1/3}\right]^{3/8} \quad (12.43)$$

which are valid for $0 \le x_c \le L_c$.

12.14. EQUILIBRIUM DETENTION STORAGE

Substituting Eqs. (12.6) and (12.7) into Eq. (4.78) gives the equilibrium detention storage for a channel with upstream inflow:

$$D_{ec} = 0.808\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z}\right)^{1/4}\left[\frac{(Q_u + q_L L_c)^{7/4} - Q_u^{7/4}}{q_L}\right] \quad (12.44)$$

For a channel with zero upstream inflow (i.e. $Q_u = 0$), Eq. (12.44) reduces to:

$$D_{ec} = 0.808\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z}\right)^{1/4} q_L^{3/4} L^{7/4} \quad (12.45)$$

12.15. WATER SURFACE PROFILE – FALLING PHASE

Substituting Eqs. (12.6) and (12.7) into Eq. (4.91) gives the equation for the falling phase of the flow area profile for a channel with upstream inflow, which is valid for $L_f \le x_c \le L_c$:

$$x_c = 50.40\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{zA_c}{1+z^2}\right)^{1/3}\left(t-t_q\right)+\left[\frac{0.630\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{zA_c^4}{1+z^2}\right)^{1/3}-Q_u}{q_L}\right] \qquad (12.46)$$

Substituting Eq. (12.1) into Eq. (12.46) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $L_f \le x_c \le L_c$:

$$x_c = 50.40\left(\frac{S_c^{1/2}}{n_c}\right)\left[\frac{\left(zy_c\right)^2}{1+z^2}\right]^{1/3}\left(t-t_q\right)+\left[\frac{0.630\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z^5y_c^8}{1+z^2}\right)^{1/3}-Q_u}{q_L}\right] \qquad (12.47)$$

From Eq. (4.92), the equation for the flow area profile between $0 \le x_c \le L_f$ is:

$$A_c = 1.414\left(\frac{n_cQ_u}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z}\right)^{1/4} \qquad (12.48)$$

Substituting Eq. (12.1) into Eq. (12.48) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_f$:

$$y_c = 1.190\left(\frac{n_cQ_u}{S_c^{1/2}}\right)^{3/8}\left(\frac{1+z^2}{z^5}\right)^{1/8} \qquad (12.49)$$

Substituting Eqs. (12.6) and (12.7) into Eq. (4.94) gives the equation for the distance L_f for a channel with upstream inflow, which is valid for $t \ge t_q$:

$$L_f = 56.57\left[\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3}\right]^{3/4} Q_u^{1/4}\left(t-t_q\right) \qquad (12.50)$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (12.46) and (12.47) reduce to:

$$x_c = 50.40\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{zA_c}{1+z^2}\right)^{1/3}\left(t-t_q\right)+\left[\frac{0.630\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{zA_c^4}{1+z^2}\right)^{1/3}}{q_L}\right] \tag{12.51}$$

$$x_c = 50.40\left(\frac{S_c^{1/2}}{n_c}\right)\left[\frac{(zy_c)^2}{1+z^2}\right]^{1/3}\left(t-t_q\right)+\left[\frac{0.630\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z^5 y_c^8}{1+z^2}\right)^{1/3}}{q_L}\right] \tag{12.52}$$

which are valid for $0 \le x_c \le L_c$.

12.16. HYDROGRAPH - FALLING PHASE

Substituting Eqs. (12.6) and (12.7) into Eq. (4.101) gives the equation for the falling phase of a hydrograph for a channel with upstream inflow:

$$t = 0.0177\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{zQ_c}\right)^{1/4}\left[L_c - \left(\frac{Q_c - Q_u}{q_L}\right)\right]+t_q \tag{12.53}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (12.53) reduces to:

$$t = 0.0177\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{zQ_c}\right)^{1/4}\left[L_c - \left(\frac{Q_c}{q_L}\right)\right]+t_q \tag{12.54}$$

In: Kinematic-Wave Rainfall-Runoff Formulas
Editor: Tommy S.W. Wong, pp. 163-174
ISBN: 978-1-60692-705-2
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Chapter 13

13. WORKING FORMULAS FOR FLOW IN VERTICAL CURB CHANNEL

In this Chapter, based on the Manning's equation, the kinematic wave parameters for flow in a vertical curb channel are derived. By applying these parameters to the general formulas in Chapter 4, working formulas for the flow depth; flow velocity; average flow velocity; wave celerity; average wave celerity; time of travel; rising, equilibrium and falling phases of hydrograph; forward characteristic; rising, equilibrium and falling phases of water surface profiles; duration of partial equilibrium discharge; and equilibrium detention storage are also derived.

13.1. KINEMATIC WAVE PARAMETERS

For flow in a vertical curb channel as shown in figure 13.1, the flow area A_c, and the wetted perimeter P, are related to the flow depth, y_c and the reciprocal of channel side slope, z as follows:

$$A_c = 0.5zy_c^2 \qquad (13.1)$$

$$P = y_c\left[1 + \left(1 + z^2\right)^{1/2}\right] \qquad (13.2)$$

Substituting Eqs. (13.1) and (13.2) and $A = A_c$ into Eq. (3.2) gives:

$$R = \frac{0.5zy_c}{1 + \left(1 + z^2\right)^{1/2}} \qquad (13.3)$$

Substituting Eq. (13.1) into Eq. (13.3) gives:

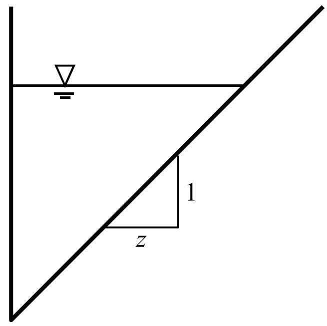

Figure 13.1. Cross-section of Vertical Curb Channel.

$$R = \frac{0.707 z^{1/2} A_c^{1/2}}{1 + \left(1 + z^2\right)^{1/2}} \tag{13.4}$$

Substituting Eq. (13.3) and $Q = Q_c$, $S = S_c$, $A = A_c$, $n = n_c$ into Eq. (3.1) gives:

$$Q_c = 0.794 \left(\frac{S_c^{1/2}}{n_c} \right) \left\{ \frac{z}{\left[1 + \left(1 + z^2\right)^{1/2} \right]^2} \right\}^{1/3} A_c^{4/3} \tag{13.5}$$

A comparison of Eq. (13.5) with Eq. (4.7), gives the kinematic wave parameters (Wong 2008b):

$$\alpha_c = 0.794 \left(\frac{S_c^{1/2}}{n_c} \right) \left\{ \frac{z}{\left[1 + \left(1 + z^2\right)^{1/2} \right]^2} \right\}^{1/3} \tag{13.6}$$

$$\beta_c = \frac{4}{3} \tag{13.7}$$

13.2. FLOW DEPTH

Substituting Eqs. (13.1) into Eq. (13.5) gives:

$$Q_c = 0.315 \left(\frac{S_c^{1/2}}{n_c} \right) \left\{ \frac{z^5}{\left[1 + \left(1 + z^2\right)^{1/2} \right]^2} \right\}^{1/3} y_c^{8/3} \tag{13.8}$$

Rearranging Eq. (13.8) gives the equation for the flow depth for a channel with and without upstream inflow:

$$y_c = 1.542\left(\frac{n_c Q_c}{S_c^{1/2}}\right)^{3/8}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z^5}\right\}^{1/8} \tag{13.9}$$

13.3. FLOW VELOCITY

Substituting Eqs. (13.6) and (13.7) into Eq. (4.12) gives the equation of flow velocity along the equilibrium profile for a channel with upstream inflow:

$$v = 0.841\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/4}\left(Q_u + q_L x_c\right)^{1/4} \tag{13.10}$$

For a channel with zero upstream inflow zero upstream inflow (Q_u = 0), Eq. (13.10) reduces to:

$$v = 0.841\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left\{\frac{z q_L x_c}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/4} \tag{13.11}$$

13.4. AVERAGE FLOW VELOCITY

Substituting Eqs. (13.6) and (13.7) into Eq. (4.15) gives the equation of average flow velocity for a channel with upstream inflow:

$$v_{av} = \frac{0.631\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/4} q_L L_c}{\left(Q_u + q_L L_c\right)^{3/4} - Q_u^{3/4}} \tag{13.12}$$

For a channel with zero upstream inflow (Q_u = 0), Eq. (13.12) reduces to:

$$v_{av} = 0.631\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left\{\frac{z q_L L_c}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/4} \tag{13.13}$$

13.5. Kinematic Wave Celerity

Substituting Eqs. (13.6) and (13.7) into Eq. (4.29) gives the equation of wave celerity along the equilibrium profile for a channel with upstream inflow:

$$c_k = 1.122\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/4}\left(Q_u + q_L x_c\right)^{1/4} \tag{13.14}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (13.14) reduces to:

$$c_k = 1.122\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left\{\frac{z q_L x_c}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/4} \tag{13.15}$$

13.6. Average Wave Celerity

Substituting Eqs. (13.6) and (13.7) into Eq. (4.32) gives the equation of wave celerity for a channel with upstream inflow:

$$c_{av} = \frac{0.841\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/4} q_L L_c}{\left(Q_u + q_L L_c\right)^{3/4} - Q_u^{3/4}} \tag{13.16}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (13.16) reduces to:

$$c_{av} = 0.841\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left\{\frac{z q_L L_c}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/4} \tag{13.17}$$

13.7. Time of Travel

Substituting Eqs. (13.6) and (13.7) into Eq. (4.35) gives the equation of time of travel for a channel with upstream inflow:

$$t_t = 0.0198\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/4}\left[\frac{\left(Q_u + q_L L_c\right)^{3/4} - Q_u^{3/4}}{q_L}\right] \tag{13.18}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (13.18) reduces to:

$$t_t = 0.0198\left(\frac{n_c L_c}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z q_L}\right\}^{1/4} \tag{13.19}$$

13.8. HYDROGRAPH – RISING PHASE

Substituting Eqs. (13.6) and (13.7) into Eq. (4.45) gives the equation for the rising phase of the hydrograph for a channel with upstream inflow:

$$Q_c = 0.794\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}\left(1.189\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/4} + 60 q_L t\right)^{4/3} \tag{13.20}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (13.20) reduces to:

$$Q_c = 186.5\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}\left(q_L t\right)^{4/3} \tag{13.21}$$

Equations (13.20) and (13.21) are valid for $t \le t_t$.

13.9. FORWARD CHARACTERISTIC - RISING PHASE

Substituting Eqs. (13.6) and (13.7) into Eq.(4.47) gives the equation for the forward characteristic of the rising phase for a channel with upstream inflow:

$$t = 0.0198\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/4}\left[\frac{\left(Q_u + q_L x_c\right)^{3/4} - Q_u^{3/4}}{q_L}\right] \tag{13.22}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (13.22) reduces to:

$$t = 0.0198\left(\frac{n_c x_c}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/4} \tag{13.23}$$

13.10. WATER SURFACE PROFILE – RISING PHASE

Substituting Eqs. (13.6) and (13.7) into Eq. (4.49) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$A_c = 1.189\left(\left(\frac{n_c}{S_c^{1/2}}\right)\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/3}\left(Q_u + q_L x_c\right)\right)^{3/4} \tag{13.24}$$

Substituting Eq. (13.1) into Eq. (13.24) gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_p$:

$$y_c = 1.542\left(\left(\frac{n_c}{S_c^{1/2}}\right)\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z^5}\right\}^{1/3}\left(Q_u + q_L x_c\right)\right)^{3/8} \tag{13.25}$$

Substituting Eqs. (13.6) and (13.7) into Eq. (4.50) gives the equation for the rising phase of the flow area profile for a channel with upstream inflow, which is valid for $L_p \le x_c \le L_c$:

$$A_p = 1.189\left(\left(\frac{n_c}{S_c^{1/2}}\right)\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/3}\left(Q_u + q_L L_p\right)\right)^{3/4} \tag{13.26}$$

Substituting $A_c = A_p$, and $y_c = y_p$ into Eq. (13.1) and then substituting it into Eq. (13.26) gives the equation for the rising phase of the water surface profile for a channel with upstream inflow, which is valid for $L_p \le x_c \le L_c$:

$$y_p = 1.542\left(\left(\frac{n_c}{S_c^{1/2}}\right)\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z^5}\right\}^{1/3}\left(Q_u + q_L L_p\right)\right)^{3/8} \tag{13.27}$$

From Eq. (4.51), the distance L_p is:

$$L_p = \frac{Q_p - Q_u}{q_L} \tag{13.28}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (13.24)-(13.28) reduce to:

$$A_c = 1.189\left(\frac{n_c q_L x_c}{S_c^{1/2}}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/3}\right)^{3/4} \tag{13.29}$$

$$y_c = 1.542\left(\frac{n_c q_L x_c}{S_c^{1/2}}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z^5}\right\}^{1/3}\right)^{3/8} \tag{13.30}$$

which are valid for $0 \le x_c \le L_p$,

$$A_p = 1.189\left(\frac{n_c q_L L_p}{S_c^{1/2}}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/3}\right)^{3/4} \tag{13.31}$$

$$y_p = 1.542\left(\frac{n_c q_L L_p}{S_c^{1/2}}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z^5}\right\}^{1/3}\right)^{3/8} \tag{13.32}$$

which are valid for $L_p \le x_c \le L_c$, and

$$L_p = \frac{Q_p}{q_L} \tag{13.33}$$

13.11. Duration of Partial Equilibrium Discharge

Substituting Eqs. (13.6) and (13.7) into Eq. (4.63) gives the equation for the duration of partial equilibrium discharge for a channel with upstream inflow:

$$t_d = \frac{L_c q_L + Q_u - 0.794\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}\left(1.189\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/4} + 60 q_L t_q\right)^{4/3}}{63.52\left(\frac{S_c^{1/2} q_L}{n_c}\right)\left[\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}\left(1.189\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/4} + 60 q_L t_q\right)\right]^{1/3}} \tag{13.34}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (13.34) reduces to:

$$t_d = \frac{L_c - 186.5\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3} q_L^{1/3} t_q^{4/3}}{248.7\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{q_L t_q z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}} \tag{13.35}$$

13.12. HYDROGRAPH - EQUILIBRIUM PHASE

As shown in figures 4.2 and 4.3, the hydrograph may reach partial equilibrium or equilibrium depending on the duration of lateral inflow t_q. If $t_q < t_t$, the hydrograph reaches partial equilibrium with a constant discharge Q_p. If $t_q \geq t_t$, the hydrograph reaches equilibrium with a constant discharge Q_e.

13.12.1. Partial Equilibrium Discharge

Substituting Eqs. (13.6) and (13.7) into Eq. (4.66) gives the equation for the partial equilibrium discharge for a channel with upstream inflow:

$$Q_p = 0.794\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}\left(1.189\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/4} + 60 q_L t_q\right)^{4/3} \tag{13.36}$$

For a channel with zero upstream inflow (Q_u = 0), Eq. (13.36) reduces to:

$$Q_p = 186.5\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}\left(q_L t_q\right)^{4/3} \tag{13.37}$$

Equations (13.36) and (13.37) are valid for $t_q \leq t \leq (t_q + t_d)$.

13.12.2. Equilibrium Discharge

From Eq. (4.69), the equation for the equilibrium discharge for a channel with upstream inflow is:

$$Q_e = Q_u + q_L L_c \tag{13.38}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eq. (13.38) reduces to:

$$Q_e = q_L L_c \tag{13.39}$$

Equations (13.38) and (13.39) are valid for $t_t \le t \le t_q$.

13.13. WATER SURFACE PROFILE - EQUILIBRIUM PHASE

Substituting Eqs. (13.6) and (13.7) into Eq. (4.71) gives the equation for the equilibrium water surface profile for a channel with upstream inflow between $-L_u \le x_c \le L_p$.

$$A_c = 1.189\left(\left(\frac{n_c}{S_c^{1/2}}\right)\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/3}\left(Q_u + q_L x_c\right)\right)^{3/4} \tag{13.40}$$

Substituting Eq. (13.1) into Eq. (13.40) gives the equation for the equilibrium water surface profile for a channel with upstream inflow, which is valid for $-L_u \le x_c \le L_p$:

$$y_c = 1.542\left(\left(\frac{n_c}{S_c^{1/2}}\right)\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z^5}\right\}^{1/3}\left(Q_u + q_L x_c\right)\right)^{3/8} \tag{13.41}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (13.40) and (13.41) reduce to:

$$A_c = 1.189\left(\frac{n_c q_L x_c}{S_c^{1/2}}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/3}\right)^{3/4} \tag{13.42}$$

$$y_c = 1.542\left(\frac{n_c q_L x_c}{S_c^{1/2}}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z^5}\right\}^{1/3}\right)^{3/8} \tag{13.43}$$

which are valid for $0 \le x_c \le L_c$.

13.14. Equilibrium Detention Storage

Substituting Eqs. (13.6) and (13.7) into Eq. (4.78) gives the equilibrium detention storage for a channel with upstream inflow:

$$D_{ec} = 0.679\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/4}\left[\frac{\left(Q_u+q_L L_c\right)^{7/4}-Q_u^{7/4}}{q_L}\right] \qquad (13.44)$$

For a channel with zero upstream inflow (i.e. $Q_u = 0$), Eq. (13.44) reduces to:

$$D_{ec} = 0.679\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/4} q_L^{3/4} L_c^{7/4} \qquad (13.45)$$

13.15. Water Surface Profile – Falling Phase

Substituting Eqs. (13.6) and (13.7) into Eq. (4.91) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $L_f \le x_c \le L_c$.

$$x_c = 63.52\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{zA_c}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}\left(t-t_q\right)+\left(\frac{0.794\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{zA_c^4}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}-Q_u}{q_L}\right) \qquad (13.46)$$

Substituting Eq. (13.1) into Eq. (13.46) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $L_f \le x_c \le L_c$:

$$x_c = 50.42\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{zy_c}{1+\left(1+z^2\right)^{1/2}}\right)^{2/3}\left(t-t_q\right)+\left(\frac{0.315\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{z^5 y_c^8}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}-Q_u}{q_L}\right) \qquad (13.47)$$

From Eq. (4.92), the equation for the flow area profile between $0 \le x_c \le L_f$ is:

$$A_c = 1.189\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/4} \tag{13.48}$$

Substituting Eq. (13.1) into Eq. (13.48) gives the equation for the falling phase of the water surface profile for a channel with upstream inflow, which is valid for $0 \le x_c \le L_f$:

$$y_c = 1.542\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/8}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z^5}\right\}^{1/8} \tag{13.49}$$

Substituting Eqs. (13.6) and (13.7) into Eq. (4.94) gives the equation for the distance L_f for a channel with upstream inflow, which is valid for $t \ge t_q$:

$$L_f = 67.29\left(\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}\right)^{3/4} Q_u^{1/4}\left(t-t_q\right) \tag{13.50}$$

For a channel with zero upstream inflow ($Q_u = 0$), Eqs. (13.46) and (13.47) reduce to:

$$x_c = 63.52\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{zA_c}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}\left(t-t_q\right)+\left(\frac{0.794\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{zA_c^4}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}}{q_L}\right)^{1/3} \tag{13.51}$$

$$x_c = 50.42\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{zy_c}{1+\left(1+z^2\right)^{1/2}}\right)^{2/3}\left(t-t_q\right)+\left(\frac{0.315\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{z^5 y_c^8}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}}{q_L}\right)^{1/3} \tag{13.52}$$

which are valid for $0 \le x_c \le L_c$.

13.16. HYDROGRAPH - FALLING PHASE

Substituting Eqs. (13.6) and (13.7) into Eq. (4.101) gives the equation for the falling phase of a hydrograph for a channel with upstream inflow:

$$t = 0.0149\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{zQ_c}\right\}^{1/4}\left[L_c-\left(\frac{Q_c-Q_u}{q_L}\right)\right]+t_q \qquad (13.53)$$

For a channel with zero upstream inflow (Q_u = 0), Eq. (13.53) reduces to:

$$t = 0.0149\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{zQ_c}\right\}^{1/4}\left[L_c-\left(\frac{Q_c}{q_L}\right)\right]+t_q \qquad (13.54)$$

In: Kinematic-Wave Rainfall-Runoff Formulas
Editor: Tommy S.W. Wong, pp. 175-231
ISBN: 978-1-60692-705-2
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APPENDICES

APPENDIX A. APPLICABILITY OF KINEMATIC WAVE THEORY

A.1. Flow on Overland Plane

The applicability of the kinematic wave theory to overland flow situations with sufficient accuracy as compared to the solution from the Saint Venant equations have been investigated by several researchers (Woolhiser and Liggett 1967, Ponce et al. 1978, Morris and Woolhiser 1980). For overland flow, the applicability of the theory can be defined by the Morris and Woolhiser (1980) criterion:

$$KF_e^2 \geq 5 \tag{A.1}$$

where K = kinematic flow number, and F_e = Froude number at the end of the plane at equilibrium. The parameter KF_e can be related to the physical characteristics of an overland plane and the rainfall intensity as follows (Wong 2005b):

$$KF_e^2 = 8586\left[\frac{S_o^{1.3}L_o^{0.4}}{n_o^{\ 0.6}(C_r i)^{0.6}}\right] \tag{A.2}$$

where S_o = slope of the overland plane, L_o = length of the overland plane, and n_o = Manning's roughness coefficient of the overland surface, C_r = runoff coefficient, and i = rainfall intensity. Substituting Equation (A.2) into Equation (A.1) gives:

$$\left[\frac{S_o^{1.3}L_o^{0.4}}{n_o^{\ 0.6}(C_r i)^{0.6}}\right] \geq 0.000582 \tag{A.3}$$

In general, the theory is applicable to overland flow situations where the backwater effect is not significant (Overton and Meadows 1976).

A.2. Flow in Open Channel

The applicability of the kinematic wave theory to open channel flow situations with sufficient accuracy as compared to the solution from the diffusive wave equations can be defined by the Ponce et al. (1978) criterion:

$$\tau = \frac{T_w S_c v_s}{y_s} > 1.383 \tag{A.4}$$

where τ = dimensionless wave period. T_w = wave period that can be taken as twice the time-of-rise of the flood wave (Ponce 1991), S_c = channel bed slope, v_s = steady-state, uniform, mean flow velocity in the channel, and y_s = steady-state, uniform, flow depth in the channel.

As a rule of thumb, the American Society of Civil Engineers (1996, 1997) simplified the criterion to:

$$S_c > 0.002 \tag{A.5}$$

In general, the theory is applicable to most open channel flow situations where backwater effect is not significant (Overton and Meadows 1976).

APPENDIX B. GENERAL FORMULAS FOR FLOW ON OVERLAND PLANE

B.1. Flow Depth

$$y_o = \left(\frac{q}{\alpha_o} \right)^{1/\beta_o}$$

B.2. Flow Velocity

$$v = \left[\alpha_o \left(q_u + C_r i x_o \right)^{\beta_o - 1} \right]^{1/\beta_o}$$

B.3. Average Flow Velocity

$$v_{av} = \frac{\alpha_o^{1/\beta_o} C_r i L_o}{\beta_o \left[\left(q_u + C_r i L_o \right)^{1/\beta_o} - q_u^{1/\beta_o} \right]}$$

B.4. Kinematic Wave Celerity

$$c_k = \beta_o \left[\alpha_o \left(q_u + C_r i x_o \right)^{\beta_o - 1} \right]^{1/\beta_o}$$

B.5. Average Wave Celerity

$$c_{av} = \frac{\alpha_o^{1/\beta_o} C_r i L_o}{\left(q_u + C_r i L_o \right)^{1/\beta_o} - q_u^{1/\beta_o}}$$

B.6. Time of Concentration

$$t_o = \frac{1}{\alpha_o^{1/\beta_o}} \left[\frac{\left(q_u + C_r i L_o \right)^{1/\beta_o} - q_u^{1/\beta_o}}{C_r i} \right]$$

B.7. Design Discharge

$$Q_d / A_o = \left[\frac{(aC_r)^{1/b}}{(L_o/\alpha_o)^{1/\beta_o}} \right]^{\frac{b\beta_o}{b+\beta_o-b\beta_o}}$$

B.8. Hydrograph – Rising Phase

$$q = \alpha_o \left[\left(\frac{q_u}{\alpha_o} \right)^{1/\beta_o} + C_r it \right]^{\beta_o}$$

for $t \le t_o$

B.9. Forward Characteristic – Rising Phase

$$t = \frac{1}{\alpha_o^{1/\beta_o}} \left[\frac{(q_u + C_r i x_o)^{1/\beta_o} - q_u^{1/\beta_o}}{C_r i} \right]$$

B.10. Water Surface Profile – Rising Phase

$$y_o = \left(\frac{q_u + C_r i x_o}{\alpha_o} \right)^{1/\beta_o}$$

for $0 \le x_o \le L_p$

$$y_p = \left(\frac{q_u + C_r i L_p}{\alpha_o} \right)^{1/\beta_o}$$

for $L_p \le x_o \le L_o$

$$L_p = \frac{q_p - q_u}{C_r i}$$

B.11. Duration of Partial Equilibrium Discharge

$$t_d = \frac{C_r i L_o + q_u - \alpha_o \left[\left(\frac{q_u}{\alpha_o} \right)^{1/\beta_o} + C_r i t_r \right]^{\beta_o}}{\alpha_o \beta_o C_r i \left[\left(\frac{q_u}{\alpha_o} \right)^{1/\beta_o} + C_r i t_r \right]^{\beta_o - 1}}$$

B.12. Hydrograph – Equilibrium Phase

B.12.1. Partial Equilibrium Discharge

$$q_p = \alpha_o \left[\left(\frac{q_u}{\alpha_o} \right)^{1/\beta_o} + C_r i t_r \right]^{\beta_o}$$

for $t_r \leq t \leq (t_r + t_d)$

B.12.1. Equilibrium Discharge

$$q_e = q_u + C_r i L_o$$

for $t_o \leq t \leq t_r$

B.13. Water Surface Profile – Equilibrium Phase

$$y_o = \left(\frac{q_u + C_r i x_o}{\alpha_o} \right)^{1/\beta_o}$$

for $0 \leq x_o \leq L_o$

B.14. Equilibrium Detention Storage

$$D_{eo} = \frac{\beta_o}{(1+\beta_o)\alpha_o^{1/\beta_o} C_r i} \left[(q_u + C_r i L_o)^{(1+\beta_o)/\beta_o} - q_u^{(1+\beta_o)/\beta_o} \right]$$

B.15. Water Surface Profile – Falling Phase

$$y_o = \left(\frac{q_u}{\alpha_o} \right)^{1/\beta_o}$$

for $0 \le x_o \le L_f$

$$x_o = \alpha_o \beta_o y_o^{\beta_o - 1}(t - t_r) + \left(\frac{\alpha_o y_o^{\beta_o} - q_u}{C_r i} \right)$$

for $L_f \le x_o \le L_o$

$$L_f = \alpha_o^{1/\beta_o} \beta_o q_u^{(\beta_o - 1)/\beta_o} (t - t_r)$$

B.16. Hydrograph – Falling Phase

$$t = \frac{L_o - \left(\frac{q - q_u}{C_r i} \right)}{\beta_o \alpha_o^{1/\beta_o} q^{[1-(1/\beta_o)]}} + t_r$$

for $t \ge t_r$

APPENDIX C. GENERAL FORMULAS FOR FLOW IN OPEN CHANNEL

C.1. Flow Area

$$A_c = \left(\frac{Q_c}{\alpha_c} \right)^{1/\beta_c}$$

C.2. Flow Velocity

$$v = \left[\alpha_c \left(Q_u + q_L x_c \right)^{\beta_c - 1} \right]^{1/\beta_c}$$

C.3. Average Flow Velocity

$$v_{av} = \frac{\alpha_c^{\ 1/\beta_c} q_L L_c}{\beta_c \left[\left(Q_u + q_L L_c \right)^{1/\beta_c} - Q_u^{\ 1/\beta_c} \right]}$$

C.4. Kinematic Wave Celerity

$$c_k = \beta_c \left[\alpha_c \left(Q_u + q_L x_c \right)^{\beta_c - 1} \right]^{1/\beta_c}$$

C.5. Average Wave Celerity

$$c_{av} = \frac{\alpha_c^{\ 1/\beta_c} q_L L_c}{\left(Q_u + q_L L_c \right)^{1/\beta_c} - Q_u^{\ 1/\beta_c}}$$

C.6. Time of Travel

$$t_t = \frac{1}{\alpha_c^{\ 1/\beta_c}} \left[\frac{\left(Q_u + q_L L_c \right)^{1/\beta_c} - Q_u^{\ 1/\beta_c}}{q_L} \right]$$

C.7. Hydrograph – Rising Phase

$$Q_c = \alpha_c \left[\left(\frac{Q_u}{\alpha_c} \right)^{1/\beta_c} + q_L t \right]^{\beta_c}$$

for $t \le t_t$

C.8. Forward Characteristic – Rising Phase

$$t = \frac{1}{\alpha_c{}^{1/\beta_c}} \left[\frac{\left(Q_u + q_L x_c\right)^{1/\beta_c} - Q_u{}^{1/\beta_c}}{q_L} \right]$$

C.9. Flow Area Profile – Rising Phase

$$A_c = \left(\frac{Q_u + q_L x_c}{\alpha_c} \right)^{1/\beta_c}$$

for $0 \le x_c \le L_p$

$$A_p = \left(\frac{Q_u + q_L L_p}{\alpha_c} \right)^{1/\beta_c}$$

for $L_p \le x_c \le L_c$

$$L_p = \frac{Q_p - Q_u}{q_L}$$

C.10. Duration of Partial Equilibrium Discharge

$$t_d = \frac{L_c q_L + Q_u - \alpha_c \left[\left(\frac{Q_u}{\alpha_c} \right)^{1/\beta_c} + q_L t_q \right]^{\beta_c}}{\alpha_c \beta_c q_L \left[\left(\frac{Q_u}{\alpha_c} \right)^{1/\beta_c} + q_L t_q \right]^{\beta_c - 1}}$$

C.11. Hydrograph – Equilibrium Phase

C.11.1. Partial Equilibrium Discharge

$$Q_p = \alpha_c \left[\left(\frac{Q_u}{\alpha_c} \right)^{1/\beta_c} + q_L t_q \right]^{\beta_c}$$

for $t_q \le t \le (t_q + t_d)$

C.11.2. Equilibrium Discharge

$$Q_e = Q_u + q_L L_c$$

for $t_t \le t \le t_q$

C.12. Flow Area Profile – Equilibrium Phase

$$A_c = \left(\frac{Q_u + q_L x_c}{\alpha_c} \right)^{1/\beta_c}$$

for $0 \le x_c \le L_c$

C.13. Equilibrium Detention Storage

$$D_{ec} = \frac{\beta_c}{(1+\beta_c)\alpha_c^{1/\beta_c}} \left[\frac{(Q_u + q_L L_c)^{(1+\beta_c)/\beta_c} - Q_u^{(1+\beta_c)/\beta_c}}{q_L} \right]$$

C.14. Flow Area Profile – Falling Phase

$$A_c = \left(\frac{Q_u}{\alpha_c} \right)^{1/\beta_c}$$

for $0 \le x_c \le L_f$

$$x_c = \left[\alpha_c \beta_c A_c^{\beta_c - 1} (t - t_q) \right] + \left(\frac{\alpha_c A_c^{\beta_c} - Q_u}{q_L} \right)$$

for $L_f \le x_c \le L_c$

$$L_f = \alpha_c^{1/\beta_c} \beta_c Q_u^{(\beta_c - 1)/\beta_c} \left(t - t_q\right)$$

C.15. Hydrograph – Falling Phase

$$t = \frac{L_c - \left(\dfrac{Q_c - Q_u}{q_L}\right)}{\alpha_c^{1/\beta_c} \beta_c Q_c^{(\beta_c - 1)/\beta_c}} + t_q$$

for $t \geq t_q$

APPENDIX D. KINEMATIC WAVE PARAMETERS

D.1. Overland Plane

$$\alpha_o = \frac{S_o^{1/2}}{n_o}$$

$$\beta_o = \frac{5}{3}$$

D.2. Circular Channel

$$\alpha_c = 0.501\left(\frac{S_c^{1/2}D^{1/6}}{n_c}\right)$$

$$\beta_c = \frac{5}{4}$$

D.3. Parabolic Channel

$$\alpha_c = 0.493\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)$$

$$\beta_c = \frac{13}{9}$$

D.4. Rectangular (Deep) Channel

$$\alpha_c = 0.630\left(\frac{S_c^{1/2}W^{2/3}}{n_c}\right)$$

$$\beta_c = 1$$

D.5. Rectangular (Square) Channel

$$\alpha_c = 0.481\left(\frac{S_c^{\ 1/2}}{n_c}\right)$$

$$\beta_c = \frac{4}{3}$$

D.6. Rectangular (Wide) Channel

$$\alpha_c = \left(\frac{S_c^{1/2}}{n_c W^{2/3}}\right)$$

$$\beta_c = \frac{5}{3}$$

D.7. Trapezoidal Channel with Equal Side Slopes

$$\alpha_c = 0.340\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)$$

$$\beta_c = 1.379$$

D.8. Trapezoidal Channel with One Side Vertical

$$\alpha_c = 0.323\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)$$

$$\beta_c = 1.360$$

D.9. Triangular Channel

$$\alpha_c = 0.630\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3}$$

$$\beta_c = \frac{4}{3}$$

D.10. Vertical Curb Channel

$$\alpha_c = 0.794\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}$$

$$\beta_c = \frac{4}{3}$$

APPENDIX E. WORKING FORMULAS FOR FLOW DEPTH

E.1. Overland Plane

$$y_o = \left(\frac{n_o q}{S_o^{1/2}} \right)^{3/5}$$

E.2. Circular Channel

$$-1.195\left(\frac{y_c}{D} \right)^3 + 1.801\left(\frac{y_c}{D} \right)^2 + 0.397\left(\frac{y_c}{D} \right) = 2.213\left(\frac{n_c Q_c}{S_c^{1/2} D^{8/3}} \right)^{4/5}$$

E.3. Parabolic Channel

$$y_c = 0.721\left(\frac{n_c Q_c}{S_c^{1/2} H^{1/2}} \right)^{6/13}$$

E.4. Rectangular (Deep) Channel

$$y_c = 1.587\left(\frac{n_c Q_c}{S_c^{1/2} W^{5/3}} \right)$$

E.5. Rectangular (Square) Channel

$$y_c = 1.316\left(\frac{n_c Q_c}{S_c^{1/2}} \right)^{3/8}$$

E.6. Rectangular (Wide) Channel

$$y_c = \left(\frac{n_c Q_c}{S_c^{1/2} W} \right)^{3/5}$$

E.7. Trapezoidal Channel with Equal Side Slopes

$$y_c = \frac{-W + \left[W^2 + 8.748z \left(\frac{n_c W^{0.0909} Q_c}{S_c^{1/2}} \right)^{0.725} \right]^{1/2}}{2z}$$

E.8. Trapezoidal Channel with One Side Vertical

$$y_c = \frac{-W + \left[W^2 + 4.592z \left(\frac{n_c W^{0.0526} Q_c}{S_c^{1/2}} \right)^{0.735} \right]^{1/2}}{z}$$

E.9. Triangular Channel

$$y_c = 1.190 \left(\frac{n_c Q_c}{S_c^{1/2}} \right)^{3/8} \left(\frac{1 + z^2}{z^5} \right)^{1/8}$$

E.10. Vertical Curb Channel

$$y_c = 1.542 \left(\frac{n_c Q_c}{S_c^{1/2}} \right)^{3/8} \left\{ \frac{\left[1 + \left(1 + z^2 \right)^{1/2} \right]^2}{z^5} \right\}^{1/8}$$

APPENDIX F. WORKING FORMULAS FOR FLOW VELOCITY

F.1. Overland Plane

$$v = 0.00238\left(\frac{S_o^{1/2}}{n_o}\right)^{3/5}\left(3.6x10^6 q_u + C_r i x_o\right)^{2/5}$$

F.2. Circular Channel

$$v = 0.575\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)^{4/5}\left(Q_u + q_L x_c\right)^{1/5}$$

F.3. Parabolic Channel

$$v = 0.613\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)^{9/13}\left(Q_u + q_L x_c\right)^{4/13}$$

F.4. Rectangular (Deep) Channel

$$v = 0.630\left(\frac{S_c^{1/2} W^{2/3}}{n_c}\right)$$

F.5. Rectangular (Square) Channel

$$v = 0.578\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left(Q_u + q_L x_c\right)^{1/4}$$

F.6. Rectangular (Wide) Channel

$$v = \left(\frac{S_c^{1/2}}{n_c W^{2/3}}\right)^{3/5}\left(Q_u + q_L x_c\right)^{2/5}$$

F.7. Trapezoidal Channel with Equal Side Slopes

$$v = \left[0.340 \left(\frac{S_c^{1/2}}{n_c W^{0.0909}} \right) \left(Q_u + q_L x_c \right)^{0.379} \right]^{0.725}$$

F.8. Trapezoidal Channel with One Side Vertical

$$v = 0.436 \left[\left(\frac{S_c^{1/2}}{n_c W^{0.0526}} \right) \left(Q_u + q_L x_c \right)^{0.360} \right]^{0.735}$$

F.9. Triangular Channel

$$v = 0.707 \left(\frac{S_c^{1/2}}{n_c} \right)^{3/4} \left[\frac{z \left(Q_u + q_L x_c \right)}{1 + z^2} \right]^{1/4}$$

F.10. Vertical Curb Channel

$$v = 0.841 \left(\frac{S_c^{1/2}}{n_c} \right)^{3/4} \left\{ \frac{z}{\left[1 + \left(1 + z^2 \right)^{1/2} \right]^2} \right\}^{1/4} \left(Q_u + q_L x_c \right)^{1/4}$$

Appendix G. Working Formulas for Average Flow Velocity

G.1. Overland Plane

$$v_{av} = 0.00143\left(\frac{S_o^{1/2}}{n_o}\right)^{3/5}\left[\frac{C_r i L_o}{\left(3.6\times 10^6 q_u + C_r i L_o\right)^{3/5} - \left(3.6\times 10^6 q_u\right)^{3/5}}\right]$$

G.2. Circular Channel

$$v_{av} = \frac{0.460\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)^{4/5} q_L L_c}{\left(Q_u + q_L L_c\right)^{4/5} - Q_u^{\ 4/5}}$$

G.3. Parabolic Channel

$$v_{av} = \frac{0.424\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)^{9/13} q_L L_c}{\left(Q_u + q_L L_c\right)^{9/13} - Q_u^{9/13}}$$

G.4. Rectangular (Deep) Channel

$$v_{av} = 0.630\left(\frac{S_c^{1/2} W^{2/3}}{n_c}\right)$$

G.5. Rectangular (Square) Channel

$$v_{av} = \frac{0.433\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4} q_L L_c}{\left(Q_u + q_L L_c\right)^{3/4} - Q_u^{\ 3/4}}$$

G.6. Rectangular (Wide) Channel

$$v_{av} = \frac{0.600\left(\dfrac{S_c^{1/2}}{nW^{2/3}}\right)^{3/5} q_L L_c}{\left(Q_u + q_L L_c\right)^{3/5} - Q_u^{\ 3/5}}$$

G.7. Trapezoidal Channel with Equal Side Slopes

$$v_{av} = \frac{0.332\left(\dfrac{S_c^{1/2}}{n_c W^{0.0909}}\right)^{0.725} q_L L_c}{\left(Q_u + q_L L_c\right)^{0.725} - Q_u^{\ 0.725}}$$

G.8. Trapezoidal Channel with One Side Vertical

$$v_{av} = \frac{0.321\left(\dfrac{S_c^{1/2}}{n_c W^{0.0526}}\right)^{0.735} q_L L_c}{\left(Q_u + q_L L_c\right)^{0.735} - Q_u^{\ 0.735}}$$

G.9. Triangular Channel

$$v_{av} = \frac{0.530\left(\dfrac{S_c^{1/2}}{n_c}\right)^{3/4}\left(\dfrac{z}{1+z^2}\right)^{1/4} q_L L_c}{\left(Q_u + q_L L_c\right)^{3/4} - Q_u^{\ 3/4}}$$

G.10. Vertical Curb Channel

$$v_{av} = \frac{0.631\left(\dfrac{S_c^{1/2}}{n_c}\right)^{3/4}\left\{\dfrac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/4} q_L L_c}{\left(Q_u + q_L L_c\right)^{3/4} - Q_u^{\ 3/4}}$$

Appendix H. Working Formulas for Kinematic Wave Celerity

H.1. Overland Plane

$$c_k = 0.00397\left(\frac{S_o^{1/2}}{n_o}\right)^{3/5}\left(3.6\times10^6 q_u + C_r i x_o\right)^{2/5}$$

H.2. Circular Channel

$$c_k = 0.719\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)^{4/5}\left(Q_u + q_L x_c\right)^{1/5}$$

H.3. Parabolic Channel

$$c_k = 0.885\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)^{9/13}\left(Q_u + q_L x_c\right)^{4/13}$$

H.4. Rectangular (Deep) Channel

$$c_k = 0.630\left(\frac{S_c^{1/2} W^{2/3}}{n_c}\right)$$

H.5. Rectangular (Square) Channel

$$c_k = 0.770\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left(Q_u + q_L x_c\right)^{1/4}$$

H.6. Rectangular (Wide) Channel

$$c_k = 1.667\left(\frac{S_c^{1/2}}{n_c W^{2/3}}\right)^{3/5}\left(Q_u + q_L x_c\right)^{2/5}$$

H.7. Trapezoidal Channel with Equal Side Slopes

$$c_k = 0.630\left[\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)\left(Q_u + q_L x_c\right)^{0.379}\right]^{0.725}$$

H.8. Trapezoidal Channel with One Side Vertical

$$c_k = 0.593\left[\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)\left(Q_u + q_L x_c\right)^{0.360}\right]^{0.735}$$

H.9. Triangular Channel

$$c_k = 0.943\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left[\frac{z\left(Q_u + q_L x_c\right)}{1+z^2}\right]^{1/4}$$

H.10. Vertical Curb Channel

$$c_k = 1.122\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4}\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/4}\left(Q_u + q_L x_c\right)^{1/4}$$

APPENDIX I. WORKING FORMULAS FOR AVERAGE WAVE CELERITY

I.1. Overland Plane

$$c_{av} = 0.00238\left(\frac{S_o^{1/2}}{n_o}\right)^{3/5}\left[\frac{C_r i L_o}{\left(3.6\times 10^6 q_u + C_r i\ L_o\right)^{3/5} - \left(3.6\times 10^6 q_u\right)^{3/5}}\right]$$

I.2. Circular Channel

$$c_{av} = \frac{0.575\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)^{4/5} q_L L_c}{\left(Q_u + q_L L_c\right)^{4/5} - Q_u^{\ 4/5}}$$

I.3. Parabolic Channel

$$c_{av} = \frac{0.613\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)^{9/13} q_L L_c}{\left(Q_u + q_L L_c\right)^{9/13} - Q_u^{9/13}}$$

I.4. Rectangular (Deep) Channel

$$c_{av} = 0.630\left(\frac{S_c^{1/2} W^{2/3}}{n_c}\right)$$

I.5. Rectangular (Square) Channel

$$c_{av} = \frac{0.578\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4} q_L L_c}{\left(Q_u + q_L L_c\right)^{3/4} - Q_u^{\ 3/4}}$$

I.6. Rectangular (Wide) Channel

$$c_{av} = \frac{\left(\dfrac{S_c^{1/2}}{n_c W^{2/3}}\right)^{3/5} q_L L_c}{\left(Q_u + q_L L_c\right)^{3/5} - Q_u^{\ 3/5}}$$

I.7. Trapezoidal Channel with Equal Side Slopes

$$c_{av} = \frac{0.457\left(\dfrac{S_c^{1/2}}{n_c W^{0.0909}}\right)^{0.725} q_L L_c}{\left(Q_u + q_L L_c\right)^{0.725} - Q_u^{\ 0.725}}$$

I.8. Trapezoidal Channel with One Side Vertical

$$c_{av} = \frac{0.436\left(\dfrac{S_c^{1/2}}{n_c W^{0.0526}}\right)^{0.735} q_L L_c}{\left(Q_u + q_L L_c\right)^{0.735} - Q_u^{\ 0.735}}$$

I.9. Triangular Channel

$$c_{av} = \frac{0.707\left(\dfrac{S_c^{1/2}}{n_c}\right)^{3/4}\left(\dfrac{z}{1+z^2}\right)^{1/4} q_L L_c}{\left(Q_u + q_L L_c\right)^{3/4} - Q_u^{\ 3/4}}$$

I.10. Vertical Curb Channel

$$c_{av} = \frac{0.841\left(\dfrac{S_c^{1/2}}{n_c}\right)^{3/4}\left\{\dfrac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/4} q_L L_c}{\left(Q_u + q_L L_c\right)^{3/4} - Q_u^{\ 3/4}}$$

APPENDIX J. WORKING FORMULAS FOR TIME OF CONCENTRATION AND TIME OF TRAVEL

J.1. Overland Plane

$$t_o = 6.988\left(\frac{n_o}{S_o^{1/2}}\right)^{3/5}\left[\frac{\left(3.6\times10^6 q_u + C_r i L_o\right)^{3/5} - \left(3.6\times10^6 q_u\right)^{3/5}}{C_r i}\right]$$

J.2. Circular Channel

$$t_t = 0.0290\left(\frac{n_c}{S_c^{1/2} D^{1/6}}\right)^{4/5}\left[\frac{\left(Q_u + q_L L_c\right)^{4/5} - Q_u^{4/5}}{q_L}\right]$$

J.3. Parabolic Channel

$$t_t = 0.0272\left(\frac{n_c H^{2/9}}{S_c^{1/2}}\right)^{9/13}\left[\frac{\left(Q_u + q_L L_c\right)^{9/13} - Q_u^{9/13}}{q_L}\right]$$

J.4. Rectangular (Deep) Channel

$$t_t = 0.0265\left(\frac{n_c L_c}{S_c^{1/2} W^{2/3}}\right)$$

J.5. Rectangular (Square) Channel

$$t_t = 0.0289\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left[\frac{\left(Q_u + q_L L_c\right)^{3/4} - Q_u^{3/4}}{q_L}\right]$$

J.6. Rectangular (Wide) Channel

$$t_t = 0.0167\left(\frac{n_c W^{2/3}}{S_c^{1/2}}\right)^{3/5}\left[\frac{\left(Q_u + q_L L_c\right)^{3/5} - Q_u^{3/5}}{q_L}\right]$$

J.7. Trapezoidal Channel with Equal Side Slopes

$$t_t = 0.0364\left(\frac{n_c W^{0.0909}}{S_c^{1/2}}\right)^{0.725}\left[\frac{(Q_u + q_L L_c)^{0.725} - Q_u^{0.725}}{q_L}\right]$$

J.8. Trapezoidal Channel with One Side Vertical

$$t_t = 0.0382\left(\frac{n_c W^{0.0526}}{S_c^{1/2}}\right)^{0.735}\left[\frac{(Q_u + q_L L_c)^{0.735} - Q_u^{0.735}}{q_L}\right]$$

J.9. Triangular Channel

$$t_t = 0.0236\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z}\right)^{1/4}\left[\frac{(Q_u + q_L L_c)^{3/4} - Q_u^{3/4}}{q_L}\right]$$

J.10. Vertical Curb Channel

$$t_t = 0.0198\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+(1+z^2)^{1/2}\right]^2}{z}\right\}^{1/4}\left[\frac{(Q_u + q_L L_c)^{3/4} - Q_u^{3/4}}{q_L}\right]$$

APPENDIX K. WORKING FORMULAS FOR HYDROGRAPH – RISING PHASE

K.1. Overland Plane

$$q = \frac{S_o^{1/2}}{n_o}\left[\left(\frac{n_o q_u}{S_o^{1/2}}\right)^{3/5} + \frac{C_r i t}{60 \times 10^3}\right]^{5/3}$$

for $t \leq t_o$

K.2. Circular Channel

$$Q_c = 0.501\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)\left[1.738\left(\frac{n_c Q_u}{S_c^{1/2} D^{1/6}}\right)^{4/5} + 60 q_L t\right]^{5/4}$$

for $t \leq t_t$

K.3. Parabolic Channel

$$Q_c = 0.493\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)\left[1.639\left(\frac{n_c H^{2/9} Q_u}{S_c^{1/2}}\right)^{9/13} + 60 q_L t\right]^{13/9}$$

for $t \leq t_t$

K.4. Rectangular (Deep) Channel

$$Q_c = 0.630\left(\frac{S_c^{1/2} W^{2/3}}{n_c}\right)\left[1.587\left(\frac{n_c Q_u}{S_c^{1/2} W^{2/3}}\right) + 60 q_L t\right]$$

for $t \leq t_t$

K.5. Rectangular (Square) Channel

$$Q_c = 0.481\left(\frac{S_c^{1/2}}{n_c}\right)\left[1.731\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4} + 60 q_L t\right]^{4/3}$$

for $t \le t_t$

K.6. Rectangular (Wide) Channel

$$Q_c = \left(\frac{S_c^{1/2}}{n_c W^{2/3}}\right)\left[\left(\frac{n W^{2/3} Q_u}{S_c^{\ 1/2}}\right)^{3/5} + 60 q_L t\right]^{5/3}$$

for $t \le t_t$

K.7. Trapezoidal Channel with Equal Side Slopes

$$Q_c = 0.340\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)\left[2.186\left(\frac{n_c W^{0.0909} Q_u}{S_c^{1/2}}\right)^{0.725} + 60 q_L t\right]^{1.379}$$

for $t \le t_t$

K.8. Trapezoidal Channel with One Side Vertical

$$Q_c = 0.323\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)\left[2.295\left(\frac{n_c W^{0.0526} Q_u}{S_c^{1/2}}\right)^{0.735} + 60 q_L t\right]^{1.360}$$

for $t \le t_t$

K.9. Triangular Channel

$$Q_c = 0.630\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3}\left[1.414\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z}\right)^{1/4} + 60 q_L t\right]^{4/3}$$

for $t \le t_t$

K.10. Vertical Curb Channel

$$Q_c = 0.794\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}\left(1.189\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/4} + 60 q_L t\right)^{4/3}$$

for $t \le t_t$

APPENDIX L. WORKING FORMULA FOR FORWARD CHARACTERISTIC – RISING PHASE

L.1. Overland Plane

$$t = 6.988\left(\frac{n_o}{S_o^{1/2}}\right)^{3/5}\left[\frac{\left(3.6\times10^6 q_u + C_r i x_o\right)^{3/5} - \left(3.6\times10^6 q_u\right)^{3/5}}{C_r i}\right]$$

L.2. Circular Channel

$$t = 0.0290\left(\frac{n_c}{S_c^{1/2} D^{1/6}}\right)^{4/5}\left[\frac{\left(Q_u + q_L x_c\right)^{4/5} - Q_u^{4/5}}{q_L}\right]$$

L.3. Parabolic Channel

$$t = 0.0272\left(\frac{n_c H^{2/9}}{S_c^{1/2}}\right)^{9/13}\left[\frac{\left(q_L x_c + Q_u\right)^{9/13} - Q_u^{9/13}}{q_L}\right]$$

L.4. Rectangular (Deep) Channel

$$t = 0.0265\left(\frac{n_c x_c}{S_c^{1/2} W^{2/3}}\right)$$

L.5. Rectangular (Square) Channel

$$t = 0.0289\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left[\frac{\left(Q_u + q_L x_c\right)^{3/4} - Q_u^{3/4}}{q_L}\right]$$

L.6. Rectangular (Wide) Channel

$$t = 0.0167\left(\frac{n_c W^{2/3}}{S_c^{1/2}}\right)^{3/5}\left[\frac{\left(Q_u + q_L x_c\right)^{3/5} - Q_u^{3/5}}{q_L}\right]$$

L.7. Trapezoidal Channel with Equal Side Slopes

$$t = 0.0364\left(\frac{n_c W^{0.0909}}{S_c^{1/2}}\right)^{0.725}\left[\frac{\left(Q_u + q_L x_c\right)^{0.725} - Q_u^{0.725}}{q_L}\right]$$

L.8. Trapezoidal Channel with One Side Vertical

$$t = 0.0383\left(\frac{n_c W^{0.0526}}{S_c^{1/2}}\right)^{0.735}\left[\frac{\left(Q_u + q_L x_c\right)^{0.735} - Q_u^{0.735}}{q_L}\right]$$

L.9. Triangular Channel

$$t = 0.0236\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z}\right)^{1/4}\left[\frac{\left(Q_u + q_L x_c\right)^{3/4} - Q_u^{3/4}}{q_L}\right]$$

L.10. Vertical Curb Channel

$$t = 0.0198\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/4}\left[\frac{\left(Q_u + q_L x_c\right)^{3/4} - Q_u^{3/4}}{q_L}\right]$$

APPENDIX M. WORKING FORMULAS FOR WATER SURFACE PROFILE – RISING PHASE

M.1. Overland Plane

$$y_o = 0.116\times10^{-3}\left[\left(\frac{n_o}{S_o^{1/2}}\right)\left(3.6\times10^6 q_u + C_r i x_o\right)\right]^{3/5}$$

for $0 \le x_o \le L_p$

$$y_p = 0.116\times10^{-3}\left[\left(\frac{n_o}{S_o^{1/2}}\right)\left(3.6\times10^6 q_u + C_r i L_p\right)\right]^{3/5}$$

for $L_p \le x_o \le L_o$

$$L_p = 3.6\times10^6\left(\frac{q_p - q_u}{C_r i}\right)$$

M.2. Circular Channel

$$-1.195\left(\frac{y_c}{D}\right)^3 + 1.801\left(\frac{y_c}{D}\right)^2 + 0.397\left(\frac{y_c}{D}\right) = 2.213\left[\left(\frac{n_c}{S_c^{1/2}D^{8/3}}\right)\left(Q_u + q_L x_c\right)\right]^{4/5}$$

for $0 \le x_c \le L_p$

$$-1.195\left(\frac{y_p}{D}\right)^3 + 1.801\left(\frac{y_p}{D}\right)^2 + 0.397\left(\frac{y_p}{D}\right) = 2.213\left[\left(\frac{n_c}{S_c^{1/2}D^{8/3}}\right)\left(Q_u + q_L L_p\right)\right]^{4/5}$$

for $L_p \le x_c \le L_c$

$$L_p = \frac{Q_p - Q_u}{q_L}$$

M.3. Parabolic Channel

$$y_c = 0.721\left[\left(\frac{n_c}{S_c^{1/2} H^{1/2}}\right)\left(Q_u + q_L x_c\right)\right]^{6/13}$$

for $0 \leq x_c \leq L_p$

$$y_c = 0.721\left[\left(\frac{n_c}{S_c^{1/2} H^{1/2}}\right)\left(Q_u + q_L L_p\right)\right]^{6/13}$$

for $L_p \leq x_c \leq L_c$

$$L_p = \frac{Q_p - Q_u}{q_L}$$

M.4. Rectangular (Deep) Channel

$$y_c = 1.587\left(\frac{n_c}{S_c^{1/2} W^{5/3}}\right)\left(Q_u + q_L x_c\right)$$

for $0 \leq x_c \leq L_p$

$$y_p = 1.587\left(\frac{n_c}{S_c^{1/2} W^{5/3}}\right)\left(Q_u + q_L L_p\right)$$

for $L_p \leq x_c \leq L_c$

$$L_p = \frac{Q_p - Q_u}{q_L}$$

M.5. Rectangular (Square) Channel

$$y_c = 1.316\left[\left(\frac{n_c}{S_c^{1/2}}\right)(Q_u + q_L x_c)\right]^{3/8}$$

for $0 \le x_c \le L_p$

$$y_p = 1.316\left[\left(\frac{n_c}{S_c^{1/2}}\right)(Q_u + q_L L_p)\right]^{3/8}$$

for $L_p \le x_c \le L_c$

$$L_p = \frac{Q_p - Q_u}{q_L}$$

M.6. Rectangular (Wide) Channel

$$y_c = \left[\left(\frac{n_c}{S_c^{1/2} W}\right)(Q_u + q_L x_c)\right]^{3/5}$$

for $0 \le x_c \le L_p$

$$y_p = \left[\left(\frac{n_c}{S_c^{1/2} W}\right)(Q_u + q_L L_p)\right]^{3/5}$$

for $L_p \le x_c \le L_c$

$$L_p = \frac{Q_p - Q_u}{q_L}$$

M.7. Trapezoidal Channel with Equal Side Slopes

$$y_c = \frac{-W + \left\{ W^2 + 8.748z \left[\frac{n_c W^{0.0909} \left(q_L x_c + Q_u \right)}{S_c^{1/2}} \right]^{0.725} \right\}^{1/2}}{2z}$$

for $0 \le x_c \le L_p$

$$y_p = \frac{-W + \left\{ W^2 + 8.748z \left[\frac{n_c W^{0.0909} \left(q_L L_p + Q_u \right)}{S_c^{1/2}} \right]^{0.725} \right\}^{1/2}}{2z}$$

for $L_p \le x_c \le L_c$

$$L_p = \frac{Q_p - Q_u}{q_L}$$

M.8. Trapezoidal Channel with One Side Vertical

$$y_c = \frac{-W + \left\{ W^2 + 4.592z \left[\frac{n_c W^{0.0526} \left(q_L x_c + Q_u \right)}{S_c^{1/2}} \right]^{0.735} \right\}^{1/2}}{z}$$

for $0 \le x_c \le L_p$

$$y_p = \frac{-W + \left\{ W^2 + 4.592z \left[\frac{n_c W^{0.0526} \left(q_L L_p + Q_u \right)}{S_c^{1/2}} \right]^{0.735} \right\}^{1/2}}{z}$$

for $L_p \le x_c \le L_c$

$$L_p = \frac{Q_p - Q_u}{q_L}$$

M.9. Triangular Channel

$$y_c = 1.189\left[\left(\frac{n_c}{S_c^{1/2}}\right)\left(\frac{1+z^2}{z^5}\right)^{1/3}\left(Q_u + q_L x_c\right)\right]^{3/8}$$

for $0 \le x_c \le L_p$

$$y_p = 1.189\left[\left(\frac{n_c}{S_c^{1/2}}\right)\left(\frac{1+z^2}{z^5}\right)^{1/3}\left(Q_u + q_L L_p\right)\right]^{3/8}$$

for $L_p \le x_c \le L_c$

$$L_p = \frac{Q_p - Q_u}{q_L}$$

M.10. Vertical Curb Channel

$$y_c = 1.542\left(\left(\frac{n_c}{S_c^{1/2}}\right)\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z^5}\right\}^{1/3}\left(Q_u + q_L x_c\right)\right)^{3/8}$$

for $0 \le x_c \le L_p$

$$y_p = 1.542\left(\left(\frac{n_c}{S_c^{1/2}}\right)\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z^5}\right\}^{1/3}\left(Q_u + q_L L_p\right)\right)^{3/8}$$

for $L_p \le x_c \le L_c$

$$L_p = \frac{Q_p - Q_u}{q_L}$$

APPENDIX N. WORKING FORMULAS FOR DURATION OF PARTIAL EQUILIBRIUM DISCHARGE

N.1. Overland Plane

$$t_d = 36\times10^3\left\{\frac{\dfrac{C_r i L_o}{3.6\times10^6}+q_u-\dfrac{S_o^{1/2}}{n_o}\left[\left(\dfrac{n_o q_u}{S_o^{1/2}}\right)^{3/5}+\dfrac{C_r i t_r}{60\times10^3}\right]^{5/3}}{\dfrac{S_o^{1/2}C_r i}{n_o}\left[\left(\dfrac{n_o q_u}{S_o^{1/2}}\right)^{3/5}+\dfrac{C_r i t_r}{60\times10^3}\right]^{2/3}}\right\}$$

N.2. Circular Channel

$$t_d = \frac{L_c q_L + Q_u - 0.501\left(\dfrac{S_c^{1/2}D^{1/6}}{n_c}\right)\left[1.738\left(\dfrac{n_c Q_u}{S_c^{1/2}D^{1/6}}\right)^{4/5}+60 q_L t_q\right]^{5/4}}{37.58\left(\dfrac{S_c^{1/2}D^{1/6}q_L}{n_c}\right)\left[1.738\left(\dfrac{n_c Q_u}{S_c^{1/2}D^{1/6}}\right)^{4/5}+60 q_L t_q\right]^{1/4}}$$

N.3. Parabolic Channel

$$t_d = \frac{L_c q_L + Q_u - 0.493\left(\dfrac{S_c^{1/2}}{n_c H^{2/9}}\right)\left[1.632\left(\dfrac{n_c H^{2/9} Q_u}{S_c^{1/2}}\right)^{9/13}+60 q_L t_q\right]^{13/9}}{42.73\left(\dfrac{S_c^{1/2}q_L}{n_c H^{2/9}}\right)\left[1.632\left(\dfrac{n_c H^{2/9} Q_u}{S_c^{1/2}}\right)^{9/13}+60 q_L t_q\right]^{4/9}}$$

N.4. Rectangular (Deep) Channel

$$t_d = \frac{L_c q_L + Q_u - 0.630\left(\dfrac{S_c^{1/2}W^{2/3}}{n_c}\right)\left[1.587\left(\dfrac{n_c Q_u}{S_c^{1/2}W^{2/3}}\right)+60 q_L t_q\right]}{37.80\left(\dfrac{S_c^{1/2}W^{2/3}q_L}{n_c}\right)}$$

N.5. Rectangular (Square) Channel

$$t_d = \frac{L_c q_L + Q_u - 0.481\left(\frac{S_c^{1/2}}{n_c}\right)\left[1.731\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4} + 60 q_L t_q\right]^{4/3}}{38.48\left(\frac{S_c^{1/2} q_L}{n_c}\right)\left[1.731\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4} + 60 q_L t_q\right]^{1/3}}$$

N.6. Rectangular (Wide) Channel

$$t_d = \frac{L_c q_L + Q_u - \left(\frac{S_c^{1/2}}{n_c W^{2/3}}\right)\left[\left(\frac{n_c W^{2/3} Q_u}{S_c^{1/2}}\right)^{3/5} + 60 q_L t_q\right]^{5/3}}{100.0\left(\frac{S_c^{1/2} q_L}{n_c W^{2/3}}\right)\left[\left(\frac{n_c W^{2/3} Q_u}{S_c^{1/2}}\right)^{3/5} + 60 q_L t_q\right]^{2/3}}$$

N.7. Trapezoidal Channel with Equal Side Slopes

$$t_d = \frac{L_c q_L + Q_u - 0.340\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)\left[2.186\left(\frac{n_c W^{0.0909} Q_u}{S_c^{1/2}}\right)^{0.725} + 60 q_L t_q\right]^{1.379}}{28.13\left(\frac{S_c^{1/2} q_L}{n_c W^{0.0909}}\right)\left[2.186\left(\frac{n_c W^{0.0909} Q_u}{S_c^{1/2}}\right)^{0.725} + 60 q_L t_q\right]^{0.379}}$$

N.8. Trapezoidal Channel with One Side Vertical

$$t_d = \frac{L_c q_L + Q_u - 0.323\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)\left[2.295\left(\frac{n_c W^{0.0526} Q_u}{S_c^{1/2}}\right)^{0.735} + 60 q_L t_q\right]^{1.360}}{26.54\left(\frac{S_c^{1/2} q_L}{n_c W^{0.0526}}\right)\left[2.295\left(\frac{n_c W^{0.0526} Q_u}{S_c^{1/2}}\right)^{0.735} + 60 q_L t_q\right]^{0.360}}$$

N.9. Triangular Channel

$$t_d = \frac{L_c q_L + Q_u - 0.630\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3}\left[1.414\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z}\right)^{1/4} + 60 q_L t_q\right]^{4/3}}{50.40\left(\frac{S_c^{1/2} q_L}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3}\left[1.414\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z}\right)^{1/4} + 60 q_L t_q\right]^{1/3}}$$

N.10. Vertical Curb Channel

$$t_d = \frac{L_c q_L + Q_u - 0.794\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}\left(1.189\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/4} + 60 q_L t_q\right)^{4/3}}{63.52\left(\frac{S_c^{1/2} q_L}{n_c}\right)\left[\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}\left(1.189\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/4} + 60 q_L t_q\right)\right]^{1/3}}$$

APPENDIX O. WORKING FORMULAS FOR HYDROGRAPH – EQUILIBRIUM PHASE

O.1. Overland Plane

O.1.1. Partial Equilibrium Discharge

$$q_p = \frac{S_o^{1/2}}{n_o}\left[\left(\frac{n_o q_u}{S_o^{1/2}}\right)^{3/5} + \frac{C_r i t_r}{60\times10^3}\right]^{5/3}$$

for $t_r \le t \le (t_r + t_d)$

O.1.2. Equilibrium Discharge

$$q_e = q_u + \frac{C_r i L_o}{3.6\times10^6}$$

for $t_o \le t \le t_r$

O.2.Circular Channel

O.2.1. Partial Equilibrium Discharge

$$Q_p = 0.501\left(\frac{S_c^{1/2} D^{1/6}}{n_c}\right)\left[1.738\left(\frac{n_c Q_u}{S_c^{1/2} D^{1/6}}\right)^{4/5} + 60 q_L t_q\right]^{5/4}$$

for $t_q \le t \le (t_q + t_d)$

O.2.2. Equilibrium Discharge

$$Q_e = Q_u + q_L L_c$$

for $t_t \le t \le t_q$

O.3. Parabolic Channel

O.3.1. Partial Equilibrium Discharge

$$Q_p = 0.493\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)\left[1.639\left(\frac{n_c H^{2/9} Q_u}{S_c^{1/2}}\right)^{9/13} + 60 q_L t_q\right]^{13/9}$$

for $t_q \le t \le (t_q + t_d)$

O.3.2. Equilibrium Discharge

$$Q_e = Q_u + q_L L_c$$

for $t_t \le t \le t_q$

O.4. Rectangular (Deep) Channel

O.4.1. Partial Equilibrium Discharge

$$Q_p = 0.630\left(\frac{S_c^{1/2} W^{2/3}}{n_c}\right)\left[1.587\left(\frac{n_c Q_u}{S_c^{1/2} W^{2/3}}\right) + 60 q_L t_q\right]$$

for $t_q \le t \le (t_q + t_d)$

O.4.2. Equilibrium Discharge

$$Q_e = Q_u + q_L L_c$$

for $t_t \le t \le t_q$

O.5. Rectangular (Square) Channel

O.5.1. Partial Equilibrium Discharge

$$Q_p = 0.481\left(\frac{S_c^{1/2}}{n_c}\right)\left[1.731\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4} + 60 q_L t_q\right]^{4/3}$$

for $t_q \leq t \leq (t_q + t_d)$

O.5.2. Equilibrium Discharge

$$Q_e = Q_u + q_L L_c$$

for $t_t \leq t \leq t_q$

O.6. Rectangular (Wide) Channel

O.6.1. Partial Equilibrium Discharge

$$Q_p = \left(\frac{S_c^{1/2}}{n_c W^{2/3}}\right)\left[\left(\frac{n W^{2/3} Q_u}{S_c^{1/2}}\right)^{3/5} + 60 q_L t_q\right]^{5/3}$$

for $t_q \leq t \leq (t_q + t_d)$

O.6.2. Equilibrium Discharge

$$Q_e = Q_u + q_L L_c$$

for $t_t \leq t \leq t_q$

O.7. Trapezoidal Channel with Equal Side Slopes

O.7.1. Partial Equilibrium Discharge

$$Q_p = 0.340\left(\frac{S_c^{1/2}}{n_c W^{0.0909}}\right)\left[2.186\left(\frac{n_c W^{0.0909} Q_u}{S_c^{1/2}}\right)^{0.725} + 60 q_L t_q\right]^{1.379}$$

for $t_q \le t \le (t_q + t_d)$

O.7.2. Equilibrium Discharge

$$Q_e = Q_u + q_L L_c$$

for $t_t \le t \le t_q$

O.8. Trapezoidal Channel with One Side Vertical

O.8.1. Partial Equilibrium Discharge

$$Q_p = 0.323\left(\frac{S_c^{1/2}}{n_c W^{0.0526}}\right)\left[2.295\left(\frac{n_c W^{0.0526} Q_u}{S_c^{1/2}}\right)^{0.735} + 60 q_L t_q\right]^{1.360}$$

for $t_q \le t \le (t_q + t_d)$

O.8.2. Equilibrium Discharge

$$Q_e = Q_u + q_L L_c$$

for $t_t \le t \le t_q$

O.9. Triangular Channel

O.9.1. Partial Equilibrium Discharge

$$Q_p = 0.630\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3}\left[1.414\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z}\right)^{1/4} + 60 q_L t_q\right]^{4/3}$$

for $t_q \leq t \leq (t_q + t_d)$

O.9.2. Equilibrium Discharge

$$Q_e = Q_u + q_L L_c$$

for $t_t \leq t \leq t_q$

O.10. Vertical Curb Channel

O.10.1. Partial Equilibrium Discharge

$$Q_p = 0.794\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}\left(1.189\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/4} + 60 q_L t_q\right)^{4/3}$$

for $t_q \leq t \leq (t_q + t_d)$

O.10.2. Equilibrium Discharge

$$Q_e = Q_u + q_L L_c$$

for $t_t \leq t \leq t_q$

APPENDIX P. WORKING FORMULAS FOR WATER SURFACE PROFILE – EQUILIBRIUM PHASE

P.1. Overland Plane

$$y_o = 0.116 \times 10^{-3} \left[\left(\frac{n_o}{S_o^{1/2}} \right) \left(3.6 \times 10^6 q_u + C_r i x_o \right) \right]^{3/5}$$

for $0 \le x_o \le L_o$

P.2. Circular Channel

$$-1.195 \left(\frac{y_c}{D} \right)^3 + 1.801 \left(\frac{y_c}{D} \right)^2 + 0.397 \left(\frac{y_c}{D} \right) = 2.213 \left[\left(\frac{n_c}{S_c^{1/2} D^{8/3}} \right) \left(Q_u + q_L x_c \right) \right]^{4/5}$$

for $0 \le x_c \le L_c$

P.3. Parabolic Channel

$$y_c = 0.721 \left[\left(\frac{n_c}{S_c^{1/2} H^{1/2}} \right) \left(Q_u + q_L x_c \right) \right]^{6/13}$$

for $0 \le x_c \le L_c$

P.4. Rectangular (Deep) Channel

$$y_c = 1.587 \left(\frac{n_c}{S_c^{1/2} W^{5/3}} \right) \left(Q_u + q_L x_c \right)$$

for $0 \le x_c \le L_c$

P.5. Rectangular (Square) Channel

$$y_c = 1.316\left[\left(\frac{n_c}{S_c^{1/2}}\right)(Q_u + q_L x_c)\right]^{3/8}$$

for $0 \le x_c \le L_c$

P.6. Rectangular (Wide) Channel

$$y_c = \left[\left(\frac{n_c}{S_c^{1/2} W}\right)(Q_u + q_L x_c)\right]^{3/5}$$

for $0 \le x_c \le L_c$

P.7. Trapezoidal Channel with Equal Side Slopes

$$y_c = \frac{-W + \left\{W^2 + 8.748z\left[\frac{n_c W^{0.0909}(q_L x_c + Q_u)}{S_c^{1/2}}\right]^{0.725}\right\}^{1/2}}{2z}$$

for $0 \le x_c \le L_c$

P.8. Trapezoidal Channel with One Side Vertical

$$y_c = \frac{-W + \left\{W^2 + 4.592z\left[\frac{n_c W^{0.0526}(q_L x_c + Q_u)}{S_c^{1/2}}\right]^{0.735}\right\}^{1/2}}{z}$$

for $0 \le x_c \le L_c$

P.9. Triangular Channel

$$y_c = 1.189\left[\left(\frac{n_c}{S_c^{1/2}}\right)\left(\frac{1+z^2}{z^5}\right)^{1/3}\left(Q_u + q_L x_c\right)\right]^{3/8}$$

for $0 \leq x_c \leq L_c$

P.10. Vertical Curb Channel

$$y_c = 1.542\left(\left(\frac{n_c}{S_c^{1/2}}\right)\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z^5}\right\}^{1/3}\left(Q_u + q_L x_c\right)\right)^{3/8}$$

for $0 \leq x_c \leq L_c$

APPENDIX Q. WORKING FORMULAS FOR EQUILIBRIUM DETENTION STORAGE

Q.1. Overland Plane

$$D_{eo} = \frac{72.8 \times 10^{-6}}{C_r i}\left(\frac{n_o}{S_o^{1/2}}\right)^{3/5}\left[\left(3.6 \times 10^6 q_u + C_r i L_o\right)^{8/5} - \left(3.6 \times 10^6 q_u\right)^{8/5}\right]$$

Q.2. Circular Channel

$$D_{ec} = 0.966\left(\frac{n_c}{S_c^{1/2} D^{1/6}}\right)^{4/5}\left[\frac{\left(Q_u + q_L L_c\right)^{9/5} - Q_u^{9/5}}{q_L}\right]$$

Q.3. Parabolic Channel

$$D_{ec} = 0.964\left(\frac{n_c H^{2/9}}{S_c^{1/2}}\right)^{9/13}\left[\frac{\left(Q_u + q_L L_c\right)^{22/13} - Q_u^{22/13}}{q_L}\right]$$

Q.4. Rectangular (Deep) Channel

$$D_{ec} = 0.794\left(\frac{n_c}{S_c^{1/2} W^{2/3}}\right)\left[\frac{\left(Q_u + q_L L_c\right)^{2} - Q_u^{2}}{q_L}\right]$$

Q.5. Rectangular (Square) Channel

$$D_{ec} = 0.989\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left[\frac{\left(Q_u + q_L L_c\right)^{7/4} - Q_u^{7/4}}{q_L}\right]$$

Q.6. Rectangular (Wide) Channel

$$D_{ec} = 0.625\left(\frac{n_c W^{2/3}}{S_c^{1/2}}\right)^{3/5}\left[\frac{\left(Q_u + q_L L_c\right)^{8/5} - Q_u^{8/5}}{q_L}\right]$$

Q.7. Trapezoidal Channel with Equal Side Slopes

$$D_{ec} = 1.268\left(\frac{n_c W^{0.0909}}{S_c^{1/2}}\right)^{0.725}\left[\frac{\left(Q_u + q_L L_c\right)^{1.725} - Q_u^{1.725}}{q_L}\right]$$

Q.8. Trapezoidal Channel with One Side Vertical

$$D_{ec} = 1.322\left(\frac{n_c W^{0.0526}}{S_c^{1/2}}\right)^{0.735}\left[\frac{\left(Q_u + q_L L_c\right)^{1.735} - Q_u^{1.735}}{q_L}\right]$$

Q.9. Triangular Channel

$$D_{ec} = 0.808\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left(\frac{1+z^2}{z}\right)^{1/4}\left[\frac{\left(Q_u + q_L L_c\right)^{7/4} - Q_u^{7/4}}{q_L}\right]$$

Q.10. Vertical Curb Channel

$$D_{ec} = 0.679\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z}\right\}^{1/4}\left[\frac{\left(Q_u + q_L L_c\right)^{7/4} - Q_u^{7/4}}{q_L}\right]$$

APPENDIX R. WORKING FORMULA FOR WATER SURFACE PROFILE – FALLING PHASE

R.1. Overland Plane

$$y_o = 0.116\times 10^{-3}\left[\left(\frac{n_o}{S_o^{1/2}}\right)\left(3.6\times 10^6 q_u\right)\right]^{3/5}$$

for $0 \le x_o \le L_f$

$$x_o = 100.0\left(\frac{S_o^{1/2} {y_o}^{2/3}}{n_o}\right)(t - t_r) + \left[3.6\times 10^6\left(\frac{\left(\frac{S_o^{1/2} {y_o}^{5/3}}{n_o}\right) - q_u}{C_r i}\right)\right]$$

for $L_f \le x_o \le L_o$

$$L_f = 0.238\left(\frac{S_o^{1/2}}{n_o}\right)^{3/5}\left(3.6\times 10^6 q_u\right)^{2/5}(t - t_r)$$

R.2. Circular Channel

$$-1.195\left(\frac{y_c}{D}\right)^3 + 1.801\left(\frac{y_c}{D}\right)^2 + 0.397\left(\frac{y_c}{D}\right) = 2.213\left(\frac{n_c Q_u}{S_c^{1/2} D^{8/3}}\right)^{4/5}$$

for $0 \le x_c \le L_f$

$$x_c = 35.38\left(\frac{S_c^{1/2}D^{2/3}}{n_c}\right)\left[-1.195\left(\frac{y_c}{D}\right)^3 + 1.801\left(\frac{y_c}{D}\right)^2 + 0.397\left(\frac{y_c}{D}\right)\right]^{1/4}\left(t - t_q\right)$$

$$+\left(\frac{\left\{0.370\left(\frac{S_c^{1/2}D^{8/3}}{n_c}\right)\left[-1.195\left(\frac{y_c}{D}\right)^3 + 1.801\left(\frac{y_c}{D}\right)^2 + 0.397\left(\frac{y_c}{D}\right)\right]^{5/4}\right\} - Q_u}{q_L}\right)$$

for $L_f \le x_c \le L_c$

$$L_f = 43.15\left(\frac{S_c^{1/2}D^{1/6}}{n_c}\right)^{4/5} Q_u^{1/5}\left(t - t_q\right)$$

R.3. Parabolic Channel

$$y_c = 0.721\left(\frac{n_c Q_u}{S_c^{1/2}H^{1/2}}\right)^{6/13}$$

for $0 \le x_c \le L_f$

$$x_c = 66.08\left(\frac{S_c^{1/2}y_c^{2/3}}{n_c}\right)\left(t - t_q\right) + \left\{\frac{\left[2.033\left(\frac{S_c^{1/2}H^{1/2}y_c^{13/6}}{n_c}\right)\right] - Q_u}{q_L}\right\}$$

for $L_f \le x_c \le L_c$

$$L_f = 53.12\left(\frac{S_c^{1/2}}{n_c H^{2/9}}\right)^{9/13} Q_u^{4/13}\left(t - t_q\right)$$

R.4. Rectangular (Deep) Channel

$$y_c = 1.587\left(\frac{n_c Q_u}{S_c^{1/2} W^{5/3}}\right)$$

for $0 \le x_c \le L_f$

$$x_c = 37.80\left(\frac{S_c^{1/2} W^{2/3}}{n_c}\right)\left(t - t_q\right) + \left\{\frac{\left[0.630\left(\frac{S_c^{1/2} W^{5/3} y_c}{n_c}\right)\right] - Q_u}{q_L}\right\}$$

for $L_f \le x_c \le L_c$

$$L_f = 37.80\left(\frac{S_c^{1/2} W^{2/3}}{n_c}\right)\left(t - t_q\right)$$

R.5. Rectangular (Square) Channel

$$y_c = 1.316\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/8}$$

for $0 \le x_c \le L_f$

$$x_c = 38.48\left(\frac{S_c^{1/2} y_c^{2/3}}{n_c}\right)\left(t - t_q\right) + \left\{\frac{\left[0.481\left(\frac{S_c^{1/2} y_c^{8/3}}{n_c}\right)\right] - Q_u}{q_L}\right\}$$

for $L_f \le x_c \le L_c$

$$L_f = 46.21\left(\frac{S_c^{1/2}}{n_c}\right)^{3/4} Q_u^{1/4}\left(t - t_q\right)$$

R.6. Rectangular (Wide) Channel

$$y_c = \left(\frac{n_c Q_u}{S_c^{1/2} W} \right)^{3/5}$$

for $0 \le x_c \le L_f$

$$x_c = 100.0 \left(\frac{S_c^{1/2} y_c^{2/3}}{n_c} \right) \left(t - t_q \right) + \left[\frac{\left(\frac{S_c^{1/2} W y_c^{5/3}}{n_c} \right) - Q_u}{q_L} \right]$$

for $L_f \le x_c \le L_c$

$$L_f = 100.0 \left(\frac{S_c^{1/2}}{n_c W^{2/3}} \right)^{3/5} Q_u^{\ 2/5} \left(t - t_q \right)$$

R.7. Trapezoidal Channel with Equal Side Slopes

$$y_c = \frac{-W + \left[W^2 + 8.748 z \left(\frac{n_c W^{0.0909} Q_u}{S_c^{1/2}} \right)^{0.725} \right]^{1/2}}{2z}$$

for $0 \le x_c \le L_f$

$$x_c = 28.14 \left[\frac{S_c^{1/2} \left(z y_c^2 + W y_c \right)^{0.379}}{n_c W^{0.0909}} \right] \left(t - t_q \right) + \left\{ \frac{0.340 \left[\frac{S_c^{1/2} \left(z y_c^2 + W y_c \right)^{1.379}}{n_c W^{0.0909}} \right] - Q_u}{q_L} \right\}$$

for $L_f \le x_c \le L_c$

$$L_f = 27.84 \left(\frac{S_c^{1/2}}{n_c W^{0.0909}} \right)^{0.725} Q_u^{0.275} \left(t - t_q \right)$$

R.8. Trapezoidal Channel with One Side Vertical

$$y_c = \frac{-W + \left[W^2 + 4.592 z \left(\frac{n_c W^{0.0526} Q_u}{S_c^{1/2}} \right)^{0.735} \right]^{1/2}}{z}$$

for $0 \le x_c \le L_f$

$$x_c = 26.36 \left[\frac{S_c^{1/2} \left(0.5 z y_c^2 + W y_c \right)^{0.360}}{n_c W^{0.0526}} \right] \left(t - t_q \right) + \left\{ \frac{0.323 \left[\frac{S_c^{1/2} \left(0.5 z y_c^2 + W y_c \right)^{1.360}}{n_c W^{0.0526}} \right] - Q_u}{q_L} \right\}$$

for $L_f \le x_c \le L_c$

$$L_f = 35.56 \left(\frac{S_c^{1/2}}{n_c W^{0.0526}} \right)^{0.735} Q_u^{0.265} \left(t - t_q \right)$$

R.9. Triangular Channel

$$y_c = 1.190 \left(\frac{n_c Q_u}{S_c^{1/2}} \right)^{3/8} \left(\frac{1 + z^2}{z^5} \right)^{1/8}$$

for $0 \le x_c \le L_f$

$$x_c = 50.40 \left(\frac{S_c^{1/2}}{n_c} \right) \left[\frac{\left(z y_c \right)^2}{1 + z^2} \right]^{1/3} \left(t - t_q \right) + \left[\frac{0.630 \left(\frac{S_c^{1/2}}{n_c} \right) \left(\frac{z^5 y_c^8}{1 + z^2} \right)^{1/3} - Q_u}{q_L} \right]$$

for $L_f \le x_c \le L_c$

$$L_f = 56.57\left[\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{z}{1+z^2}\right)^{1/3}\right]^{3/4} Q_u^{1/4}\left(t-t_q\right)$$

R.10. Vertical Curb Channel

$$y_c = 1.542\left(\frac{n_c Q_u}{S_c^{1/2}}\right)^{3/8}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{z^5}\right\}^{1/8}$$

for $0 \le x_c \le L_f$

$$x_c = 50.42\left(\frac{S_c^{1/2}}{n_c}\right)\left(\frac{zy_c}{1+\left(1+z^2\right)^{1/2}}\right)^{2/3}\left(t-t_q\right)+\left(\frac{0.315\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{z^5 y_c^8}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}-Q_u}{q_L}\right)$$

for $L_f \le x_c \le L_c$

$$L_f = 67.29\left(\left(\frac{S_c^{1/2}}{n_c}\right)\left\{\frac{z}{\left[1+\left(1+z^2\right)^{1/2}\right]^2}\right\}^{1/3}\right)^{3/4} Q_u^{1/4}\left(t-t_q\right)$$

APPENDIX S. WORKING FORMULA FOR HYDROGRAPH – FALLING PHASE

S.1. Overland Plane

$$t = 0.0100\left(\frac{n_o}{S_o^{1/2}}\right)^{3/5}\left[\frac{C_r i L_o - 3.6\times10^6(q-q_u)}{C_r i q^{2/5}}\right] + t_r$$

for $t \geq t_r$

S.2. Circular Channel

$$t = \left(\frac{0.0232}{Q_c^{1/5}}\right)\left(\frac{n_c}{S_c^{1/2}D^{1/6}}\right)^{4/5}\left[L_c - \left(\frac{Q_c - Q_u}{q_L}\right)\right] + t_q$$

for $t \geq t_q$

S.3. Parabolic Channel

$$t = \frac{0.0188}{Q_c^{4/13}}\left(\frac{n_c H^{2/9}}{S_c^{1/2}}\right)^{9/13}\left[L_c - \left(\frac{Q_c - Q_u}{q_L}\right)\right] + t_q$$

for $t \geq t_q$

S.4. Rectangular (Deep) Channel

$$t = 0.0265\left(\frac{n_c}{S_c^{1/2}W^{2/3}}\right)\left[L_c - \left(\frac{Q_c - Q_u}{q_L}\right)\right] + t_q$$

for $t \geq t_q$

S.5. Rectangular (Square) Channel

$$t = \left(\frac{0.0216}{Q_c^{1/4}}\right)\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left[L_c - \left(\frac{Q_c - Q_u}{q_L}\right)\right] + t_q$$

for $t \geq t_q$

S.6. Rectangular (Wide) Channel

$$t = \left(\frac{0.0100}{Q_c^{2/5}}\right)\left(\frac{n_c W^{2/3}}{S_c^{1/2}}\right)^{3/5}\left[L_c - \left(\frac{Q_c - Q_u}{q_L}\right)\right] + t_q$$

for $t \geq t_q$

S.7. Trapezoidal Channel with Equal Side Slopes

$$t = \frac{0.0264}{Q_c^{0.275}}\left(\frac{n_c W^{0.0909}}{S_c^{1/2}}\right)^{0.725}\left[L_c - \left(\frac{Q_c - Q_u}{q_L}\right)\right] + t_q$$

for $t \geq t_q$

S.8. Trapezoidal Channel with One Side Vertical

$$t = \frac{0.0281}{Q_c^{0.265}}\left(\frac{n_c W^{0.0526}}{S_c^{1/2}}\right)^{0.735}\left[L_c - \left(\frac{Q_c - Q_u}{q_L}\right)\right] + t_q$$

for $t \geq t_q$

S.9. Triangular Channel

$$t = 0.0177\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left(\frac{1 + z^2}{zQ_c}\right)^{1/4}\left[L_c - \left(\frac{Q_c - Q_u}{q_L}\right)\right] + t_q$$

for $t \geq t_q$

S.10. Vertical Curb Channel

$$t = 0.0149\left(\frac{n_c}{S_c^{1/2}}\right)^{3/4}\left\{\frac{\left[1+\left(1+z^2\right)^{1/2}\right]^2}{zQ_c}\right\}^{1/4}\left[L_c - \left(\frac{Q_c - Q_u}{q_L}\right)\right] + t_q$$

for $t \geq t_q$

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Rainfall-Runoff Formulas

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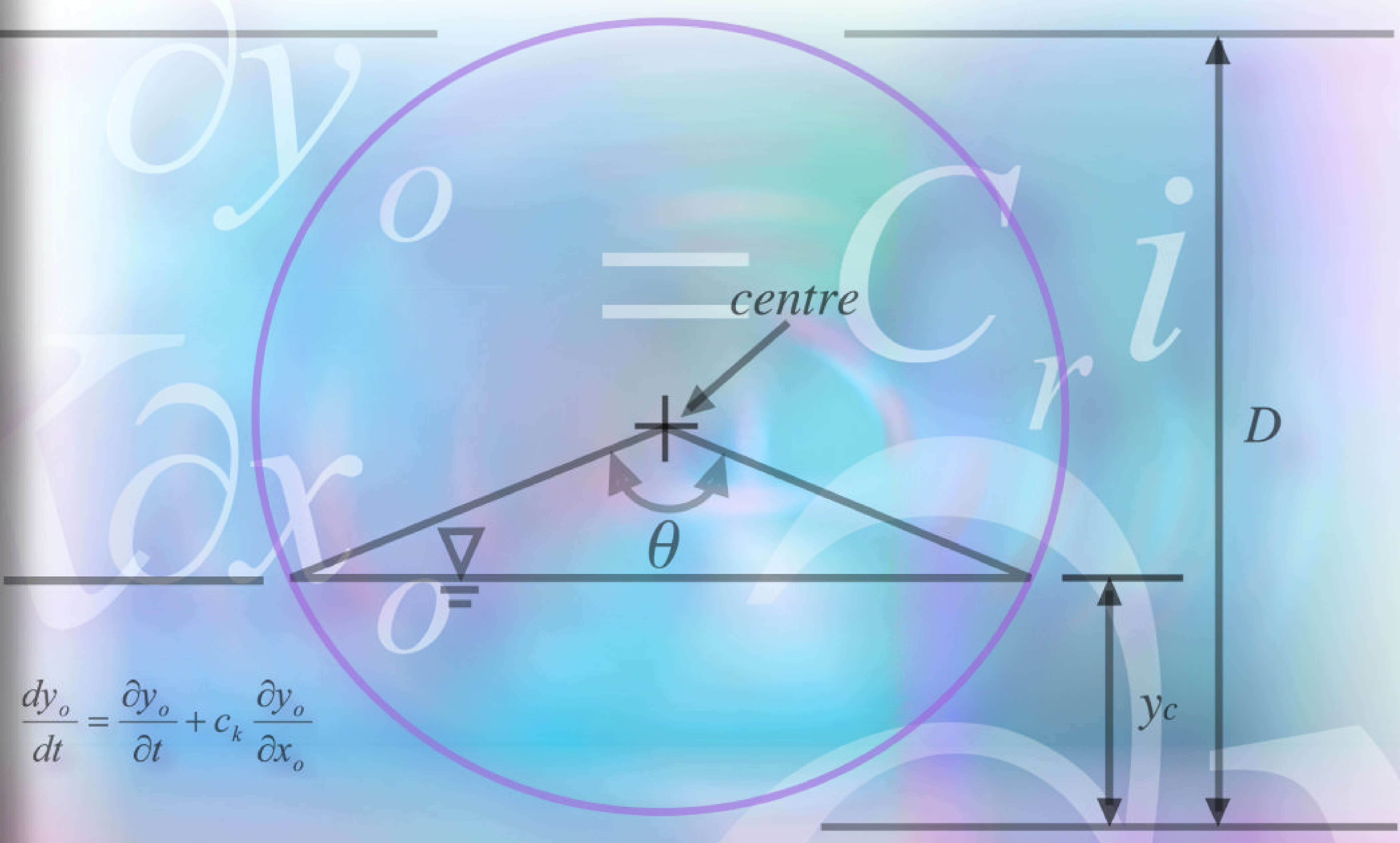
Tommy S. W. Wong

Kinematic-Wave Rainfall-Runoff Formulas



NOVA

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