A MATHEMATICAL MODEL FOR HOT CRACKING OF ALUMINIUM ALLOYS DURING D.C. CASTING

L. Katgerman

Aluminium Company of America
Alcoa Technical Center
Alcoa Center, PA 15069

A mathematical model to calculate the hot cracking tendencies during D.C. casting is described. The model combines a new simplified thermal model for D.C. casting with the concept of solidification shrinkage not eliminated by afterfeeding (Feurer) and the concept of the critical time interval during solidification (Clyne and Davies). This model is able to calculate hot cracking tendencies regarding the effects of composition, casting rate and ingot diameter. In spite of the absence of sufficient physical and solidification data it is shown that there is a satisfactory degree of correlation between prediction and practical casting knowledge.

INTRODUCTION

Hot cracking or centre cracking during D.C. casting of aluminium alloys frequently prevents the increase in casting speed to obtain a higher productivity. Measures to control hot cracking are mostly based on practical experience.
Existing theoretical models are only applied to special hot cracking tests or permanent mold and sand castings.

In our present model theoretical considerations of Feurer (1) and of Clyne and Davies (2) have been combined with D.C. casting of aluminium alloys. The Feurer approach emphasises the importance of liquid feeding in comparison with the stress-build up due to solidification shrinkage. This model has been applied to permanent mold and sand castings of binary alloys. In the Clyne-Davies analysis the time during which processes related to crack production may take place is considered and only hot cracking tendencies as a function of composition can be calculated.
The combination of both approaches with D.C. casting makes it possible to calculate hot cracking tendencies as a function of casting speed, ingot diameter and alloy composition (binary and commercial).

MODEL AND MATHEMATICAL FORMULATION

Hot cracks are generally defined as cracks which develop during solidification of an alloy above the non-equilibrium solidus temperature. Hot cracks nucleate at the bottom of the liquid metal pool during D.C. casting (3). According to Feurer (1) hot cracks are caused by the solidification shrinkage which cannot be eliminated by after flowing of the residual melt through the dendritic network. Because the resulting stress-build up will be dependent on the time interval during which feeding is inadequate both aspects have to be taken into account when hot cracking susceptibilities as a function of D.C. casting parameters are determined.

Feeding ability and solidification shrinkage.

By considering the dendritic network between liquidus and solidus temperature as a poreus medium Darcy's empirical law can be applied to the volume flow velocity q through the mushy zone (1, 4, 5) (see Nomenclature for notation)

$$\frac{dV}{dt} = \frac{KA}{\eta L} (P_o + P_M - P_c) \qquad (1)$$

Streat and Weinberg (4) have shown that the permeability K of a dendritic network can be written as:

$$K = f_L^2 \, d_s^2 / 8\pi\tau^3 \qquad (2)$$

From their experiments a tortuosity factor τ of 4.6 was determined implying that the flow channels are neither straight nor parallel.
The dendritic arm spacing for aluminium alloys is determined from data of Bower et al. (6) and Spear and Gardner (7) as a function of cooling rate or solidification time.
Taking for the capillary pressure $P_c = 4\,\gamma_{SL}/d_s$ the feeding ability in Eq. (1) can be calculated when combined with a heat-flow model for D.C. casting.

By considering the conservation of mass of a volume elemenent the rate of solidification shrinkage can be expressed as follows:

$$\frac{1}{v} \cdot \frac{dv}{dt} = \frac{1}{\bar{\rho}} \cdot \frac{d\bar{\rho}}{dt} = \frac{\rho_s - \rho_L}{(\rho_s - \rho_L)\, f_s + \rho_L} \cdot \frac{df_s}{dt} \qquad (3)$$

D.C. casting heat flow model.

For a numerical evaluation of the feeding and shrinkage behaviour it is necessary to know solidification time and fraction solid as function of the distance along the ingot. Therefore a simplified heatflow model was developed to determine these data.
The basic heat-flow equation for fixed cylindrical coordinates and steady state is:

$$\frac{\delta T}{\delta t} = 0 = \alpha \nabla^2 T - v_c \frac{\delta T}{\delta z} \qquad (4)$$

where z is the distance along the axis of the ingot. The convective cooling condition at the ingot surface is characterized by a heat-transfer coefficient h.

For a simple analysis it has been demonstrated (8) that with the assumption of a linear temperature profile in the solidified crust useful results can be obtained.
The further assumptions inherent in the mathematical description of the casting process are:
(a) the heat-flow is only in the radial direction
(b) the physical properties of liquid and solid are constant with the temperature
(c) the temperature field in the liquid region is constant
(d) the temperature at the solid-liquid interface is a function of the fraction solid.

Under these conditions the heat-flow equation (4) taking into account the conservation of energy at the solid-liquid interface can be written as:

$$v_c \rho_s L_f \frac{dfs}{dz} - \tfrac{1}{2} \rho_s c_{ps} v_c \frac{d}{dz} (fs\,(T_i + T_o))$$
$$- \rho_L c_{pL} v_c \frac{d}{dz} ((1 - fs)\, T_i) = \frac{2h}{R} (T_o - T_w) \qquad (5)$$

For binary alloys the temperature T_i at the solid-liquid interface can be obtained from the Scheil-equation and for commercial alloys from experimental data.
The ingot surface temperature T_o can be calculated with the application of the linear temperature profile from:

$$h\,(T_o - T_w) = \frac{\lambda_s}{s} (T_i - T_o) \qquad (6)$$

When the fraction solid f_{se} at the beginning of the eutectic temperature is reached it is assumed that the eutectic reaction occurs linearly in the temperature range of 0.1°C (9).

$$\frac{d_{fs}}{dT_i} = \frac{1 - f_{se}}{0.1} \qquad (7)$$

Hot cracking criteria

In the case of inadequate afterfeeding at a distance from the meniscus there is a volume deficit that can be so large that the resulting stresses cause cracking. The distance or time during which afterfeeding is inadequate can be used as an indication for hot cracking.
Clyne and Davies (2) define a hot cracking index as the ratio of the time interval during which interdendritic separation occurs ($0.01 < f_L < 0.1$) to the time interval during which stress relaxation can take place ($0.1 < f_L < 0.6$). Applied to the present heat-flow model of D.C. casting this hot cracking index can be expressed as

$$\text{H.C.} = {}^{(z_{99} - z_{90})}/(z_{90} - z_{40}) \qquad (8)$$

where z_{99}, z_{90} and z_{40} are the distances along the ingot axis where f_s = 0.99, 0.90 and 0.40 respectively.
When afterfeeding is considered the vulnerable time proportionality changes from $z_{99} - z_{90}$ to $z_{99} - z_{cr}$, and a hot cracking index can be defined as

$$\text{H.C.} = {}^{(z_{99} - z_{cr})}/(z_{cr} - z_{40}) \qquad (9)$$

in which z_{cr} is the distance from where afterfeeding is inadequate.
In this form the hot cracking index is a function of casting parameters.

PHYSICAL AND CASTING PARAMETERS

The solid density of all alloys was taken as 2.55 10^3 kg/m^3. Liquid densities were calculated as a function of alloy composition from limited binary solidification shrinkage data (10).
The other physical data were taken as follows,

L_f = 3.93 J/kg, c_{ps} = 8.9 10^2 J/kg°C, c_{pL} = 1.08 10^2 J/kg°C,

λ_s = 209 W/m °C, λ_L = 92 W/m °C, γ_{sL} = 0.121 N/m

and η = 1.3 10^{-3} Ns/m^2

The heat-transfer coefficient h during casting was taken as 1.2 10^4 W/m^2 K. In general industrial casting practice the cooling water flow rate per unit length is taken constant for all ingot sizes (11), which implies a constant heat-transfer coefficient in first approximation for all ingot sizes and casting rates. Because non-equilibrium solidification is assumed the fraction solid as a function of temperature for binary alloys was calculated from the Scheil equation. For commercial alloys experimental data from a differential thermal analyses were used (12).

RESULTS AND DISCUSSION

Effect of casting rate and ingot diameter on hot cracking

Table 1 shows the effect of casting rate for a binary Al-4.5 wt% Mg alloy.

TABLE 1

Effect of casting rate on hot cracking of Al-4.5 wt% Mg. (Ingot diameter 0.381 m (15")).

Casting rate	$z_{99} - z_{cr}$	H.C. indices	
m/s	10^{-2} m	Eq.(8)	Eq.(9)
$7.4\ 10^{-4}$	0.6	0.43	0.60
$1.5\ 10^{-3}$	1.6	0.43	0.80
$2.2\ 10^{-3}$	2.7	0.43	0.84
$3.0\ 10^{-3}$	3.5	0.43	0.88

The indexes based on the feeding behaviour show an increasing cracking tendency for increasing casting rates, which is confirmed by industrial experience (3,11). The hot cracking index as proposed by Clyne and Davies is unable to explain this effect.

In Table 2 the calculated results for two ingot diameters are shown.

TABLE 2

Effect of ingot diameter on hot cracking of Al-4.5 wt% Mg. (Casting rate $1.5\ 10^{-3}$ m (3.5 ipm)).

Ingot diameter	$z_{99} - z_{cr}$	H.C. indices	
m	10^{-2} m	Eq.(8)	Eq.(9)
0.381	1.6	0.43	0.80
0.762	2.9	0.43	0.60

The increase in the calculated inadequate feeding distance ($z_{99} - z_{cr}$) is in agreement with casting experience. However both other indices fail to explain this effect. A cracking index proposed by Bryson (3) based on the difference between centre and surface cooling rates was also unable to explain the effect of ingot diameter on hot cracking (13).

Effect of alloy composition

Fig. 1 shows the effect of Mg-content on hot cracking tendencies for a binary Al-Mg alloy. The experimental data points are taken from ref. (2). It is clear from Fig. 1 that the hot cracking tendencies according Eq. (9) and the inadequate feeding distance are in better agreement with the experimental data than Eq.(8).

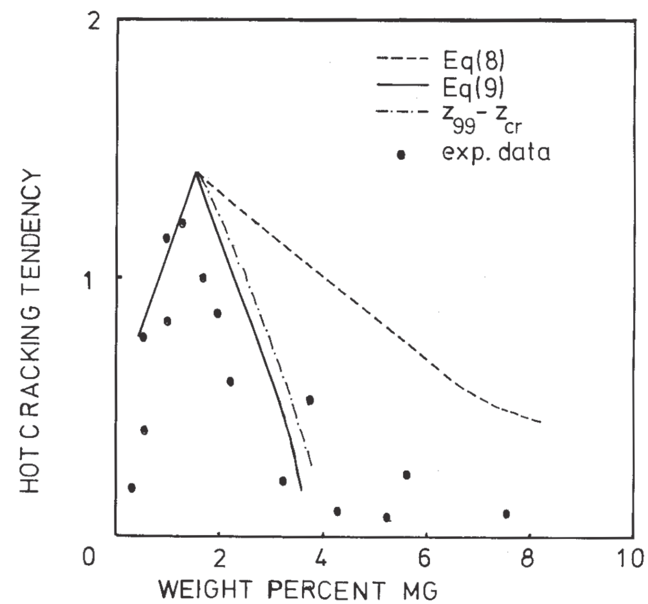

Fig. 1. Comparison of calculated and experimental hot cracking tendencies for binary Al-Mg alloys.

Table 3 shows the results for 11 commercial alloy compositions. In order to compare the calculated results with D.C. casting practice the 11 alloys in consideration have been given a relative hot cracking rate on a scale from 1 to 10 according to existing casting existing casting experience (11). All calculated values have been converted to the same scale.

The calculated results show only a satisfactory agreement within the same alloy group. The agreement with experience is somewhat improved when the coherency temperature is taken into account.
The coherency temperature is defined as the temperature where the dendritic network starts to form and can be determined from viscosity experiments (12). The hot cracking index then can be written as

$$H.C. = (z_{99} - z_{cr})/(z_{cr} - z_{coh}) \quad (10)$$

The better results with the coherency temperature are an indication that more information is needed for a commercial alloy comparison. Therefore high temperature mechanical data especially in the as-cast condition are necessary. These data are very limited at the moment.
In combination with a more sophisticated model then the effect of the resulting stress from inadequate feeding can be considered.

TABLE 3

Experimental and calculated relative hot cracking tendencies for commercial aluminium alloys.

Alloy	Experimental	Inadequate feeding	H.C. indices Eq.(8)	Eq.(9)	Eq.(10)
1100	2	5.1	4	5.4	5.1
2014	3	1	1	1	1
2024	2	9.6	1.1	8.7	7.7
3003	2	4.5	4.2	4.7	4
3004	5	9.2	10	9.4	7.6
5052	3	8.8	6.9	9	7.9
5182	5	10	9.1	10	10
6061	10	9.8	8.6	9.9	9.4
6063	9	6.9	4.8	7.3	7.3
7050	7	2.7	1.7	2.5	1
7075	1	1	1.2	1	1.3

CONCLUSIONS

1. The present model is able to predict hot cracking tendencies as a function of casting rate and ingot diameter in agreement with existing casting experience.

2. For binary Al-Mg alloys the hot cracking index based on inadequate afterfeeding is in better agreement with experimental data. For a commercial alloy comparison the model is too simple to give correct predictions. Better results can be expected when high temperature mechanical characteristics in as-cast condition are considered.

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NOMENCLATURE

List of symbols

A	= ingot cross section (m^2)
c_{pL}, c_{ps}	= specific heat liquid, solid ($^J/kg$ K)
d_s	= dendritic arm spacing (m)
f_L, f_s	= volume fraction liquid, solid
h	= heat-transfer coefficient ($^W/m^2$ K)
K	= permeability poreus medium (m^2)
L	= length poreus zone (m)
L_f	= latent heat of solidification ($^J/kg$)
p_o, p_c, p_M	= atmosferic, capillary, metallostatic pressure ($^N/m^2$)
R	= ingot radius (m)
L	= volume flow rate ($^{m^3}/s$)
s	= thickness of solidified layer (m)
T	= temperature (^oC)
T_i, T_o	= solid-liquid interface, surface temperature (^oC)
T_W	= cooling water temperature (^oC)
V	= volume (m^3)
v_c	= casting rate ($^m/s$)
z	= distance along ingot axis (m)
γ_{sL}	= solid-liquid interfacial energy ($^N/m$)
η	= viscosity ($^Ns/m^2$)
$\bar{\rho}$, ρ_L, ρ_s	= average, liquid and solid density
τ	= tortuosity factor
λ_s	= thermal conductivity solid ($^W/m$ K)

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