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MATHEMATICAL MODELLING OF BUTT CURL DEFORMATION OF SHEET INGOTS. COMPARISON WITH EXPERIMENTAL RESULTS FOR DIFFERENT STARTER BLOCK SHAPES.

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Abstract

Temperatures during the start up phase of aluminium sheet ingot casting, calculated by a 3D-version of the model ALSIM, are used as input to the calculation of thermally induced strains and stresses in a 3D-version of the model ALSPEN.

Calculations, where experimental trials are simulated, are carried out for the traditional, recessed starting block type, and for a new type with a conical portion in the middle of the starting block.

It is focused on the butt curl deformation. A very good agreement between calculated results and measurements is obtained.

The calculated butt curl development is shown to be sensitive to the imposed boundary conditions in the thermal calculation.

Introduction

In the start up phase of direct chill casting of aluminium sheet ingots, a phenomenon called butt curl is known to cause problems. The butt curl is a deformation of the ingot butt characterized by a bowing-up of the shell formed against the starting block. The most pronounced effect occurs near the short sides of the ingot, as indicated in figure 1. The gap induced by this deformation may reach the magnitude of 150 mm, depending among other things on the ingot size.

After the start of curling there is only a small centre area of contact between the ingot and the starting block. The butt curl therefore leads to partially weak cooling on the bottom surface of the ingot, and occasionally a break-through of melt occurs. The butt curl also results in a bending of the narrow sides of the ingot, inwards away from the mould. This may induce melt-through/running over and other surface defects. A more complete review of defects and difficulties associated with the butt curl is given in [1].

The butt curl is usually seen to develop abruptly when the cooling water hits the ingot surface. The butt curl velocity, (the time derivative of the butt curl measure) has been observed to reach a maximum shortly after the water impingement and then to decay to a value close to zero after a few minutes of casting.

The butt curl development can be associated with large gradients in the thermal contractions induced by the direct water cooling. Efforts to reduce the surface heat flux by applying pulsed water cooling [2] or by using CO_2 dissolved in the cooling water [3], resulting in a more homogenous cooling and smaller thermal gradients, have been reported to successfully reduce the butt curl.

How casting speed and amount of cooling water can influence the butt curl development for some alloys, has previously been investigated by Droste and Schneider [4,1]. In a more recent series of experimental casts of an Al99.5 alloy (AA 1050), the starting block geometry has been varied [5], and the magnitude of the butt curl is seen to depend rather strongly on the design of the starting block.

The thermally induced deformations of sheet ingots can be calulated by using numerical models which solves the equations for conservation of energy, mass and momentum. In a recent paper, Hannart et al [6] have presented such a model. They focused on the cold cracking tendency, but among other results a calculated butt curl development corresponding well to a measured one was presented.

In our work, temperatures during the start up phase of sheet ingot casting, calculated by a 3D-version of the model ALSIM (ALSIM3), is used as input to the calculation of thermally induced strains and stresses in a 3D-version of the model ALSPEN (ALSPEN3). The thermal calculations by the ALSIM3, which is a thermal finite element model with the same functionalities as the 2D ALSIM model [7,8,9], are described in [10]. The 2D finite element model ALSPEN, which calculate thermally induced strains and stresses in cylindrical extrusion billets during casting, is described in [11]. Some capabilities of this model have been demonstrated in [12,13,14].

The main aim of this paper is to present the model ALSPEN3, and to verify its capabilities by the reproduction of measured butt curl developments for laboratory scale sheet ingots [5]. How the calculated butt curl development depends on some different choices of constitutive laws and assumed material properties is also shown. In addition, it is demonstrated how assumptions about the surface heat transfer may affect the calculated butt curl development. Finally, the butt curl mechanism is illustrated by analysing the results of ALSPEN3 calculations. By discussing how the starting block design influences the stresses and strain rates, we hope to contribute to the understanding of the butt curl phenomenon.

The varying thickness of the ingot near the butt, known as butt swell [3], may also be calculated by ALSPEN3, but this phenomenon calls for simulation of a much longer time period of casting than is necessary for calculation of the butt curl. This phenomenon is therefore left for future case studies.

The model ALSPEN3

In the stress calculations by ALSPEN3, the solution domain consists of the solidified part of the sheet ingot. During a simulation of the start up phase of DC casting, the solution domain accordingly increases in size. Due to presumed symmetric casting conditions, only a quarter of the ingot is considered, as shown in figure 1.

The stress distribution is governed by Cauchy's equations which are solved together with the compatibility and constitutive equations by a finite element technique. Apart from differences caused by the transition from cylindrical symmetry to a full 3D formulation, the mathematical formulations and numerical algorithms correspond to those applied in the 2D ALSPEN model [11].

Boundary conditions

The boundaries of the solution domain can be identified in figure 1, where also the system of coordinates is shown.

The upper boundary is defined by the isotherm corresponding to the coherency temperature T_c. This is the temperature above which the metal is considered as a liquid. At this boundary the metallostatic pressure is taken into account.

Due to symmetry, the displacements normal to the sides $X=0$ and $Y=0$ are prescribed to zero.

The effect of friction forces and of horizontal contact forces at the mould wall and at the starting block surface is generally neglected. Vertical contact forces from the starting block due to gravity is taken into account in a simplified way. This vertical force field, that counterbalances the gravity forces, is imposed on the bottom side of the ingot as a quickly decaying function of the calculated vertical displacement.

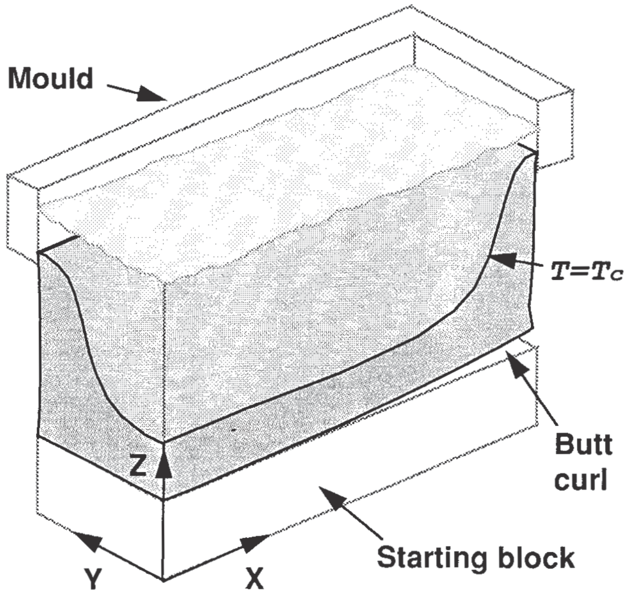

Figure 1: A quarter of an ingot in the start up phase of DC casting. The ALSPEN3 solution domain is outlined, and the system of coordinates is shown.

Constitutive modelling

The metal is described as an elastic-viscoplastic material for which plasticity and creep are treated in a unified manner. In recent years unified constitutive equations based on the concept of internal variables has been widely used. The use of such constitutive equations for stress prediction in aluminium casting, in particular the MATMOD equations by Miller [15], has been discussed in [16].

The numerical scheme for solving the constitutive equation in 2D ALSPEN given in [11], can easily be modified to handle some classes of internal variable constitutive equations. A modified version of the MATMOD equations has been implemented in ALSPEN3.

Experimental castings of the alloy AA 1050 is simulated in the case study presented here. However, no sets of AA 1050 material constants for constitutive equations implemented in ALSPEN3, were available prior to this work. A set of constitutive equations found in literature, by Lalli and DeArdo [17], was therefore implemented for modelling of the elastic-viscoplastic properties of commercial purity aluminium[1]. The material parameters for this model was developed from experiments with an alloy of purity 99.7 which had been taken through a process of deformation followed by heating and recrystallization. It is to be noted that the material constants were determined from experiments where strains and strain rates were considerable higher than values typical for DC-casting. This is a possible error source with respect to our prediction of stresses during DC casting.

Reported experimental results [1], however, indicate that the butt curl is not sensitive to modest variations in alloy composition, and the mechanical properties of annealed commercial purity aluminium is assumed to be little different from its properties during casting.

Calculated butt curl values have been reported to be strongly dependent on the values of material constants in the constitutive equation [6]. We therefore have done some work investigating the sensitivity of the calculated butt curl to alloy properties and choice of constitutive equation. For one of the simulated castings, results obtained by use of material constants for the alloy AA 6063, for the constitutive equations given in [11] and for the MATMOD equations, is compared to results obtained by use of the model by Lalli and DeArdo.

The mushy zone

The region where the material is in a partly solidified state, i. e. the mushy zone, represents a problem in our modelling work.

In mechanical testing of some alloys, the flow stress has been shown to depend strongly on the solid fraction at the final stage of solidification [18,19]. In addition, where the liquid feeding becomes insufficient, the solidification shrinkage should be taken into account together with the thermal contraction in the calculation of strains and stresses.

In spite of these facts, we have extrapolated the properties for the solid metal into the mushy zone in our calculations. This is done partly because of the limited knowledge of the mush properties of the alloy AA 1050, but also due to numerical limitations. The finite element discretization and the time step used do not offer sufficient numerical resolution for a detailed handling of the variations in con-

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[1] The initial value of the internal variable S given here, however, was higher than the steady state value for strain rates characteristic for the DC casting process for higher temperatures. Steady state creep was assumed in our calculations for such conditions.

traction rate and flow stress associated with the solidification range of AA 1050.

A coherency temperature of 649 °C, corresponding to a solid fraction of 0.9 was used in our calculations. (Due to the effect of discretization, this choice of T_c value resulted in temperatures at the upper boundary of the solution domain varying mostly in the range 620–640 °C.)

The handling of the mushy zone clearly represents an error source in our calculations. The effect of this error is, however, limited by the low stress values near the solidification front, and the solidification shrinkage is mainly assumed to generate local viscoplastic strains in the mushy zone due to the very low flow stress there.

Connection to thermal modelling

In modelling of the DC casting process, the thermal problem is in principle affected by the solution of the mechanical one through the dependency of heat transfer on the mechanical contact between the ingot and the mould, and between the ingot and the starting block.

By ALSIM3, however, the thermal problem is at present solved uncoupled from the mechanical one. Temperatures calculated in advance by ALSIM3 is used to calculate the thermally induced strains and stresses in ALSPEN3.

So far, we have considered it difficult to base the thermal boundary conditions on the displacements calculated by ALSPEN3. Air gaps smaller than assumed inaccuracies in the calculated displacements is believed to decrease the heat transfer considerably.

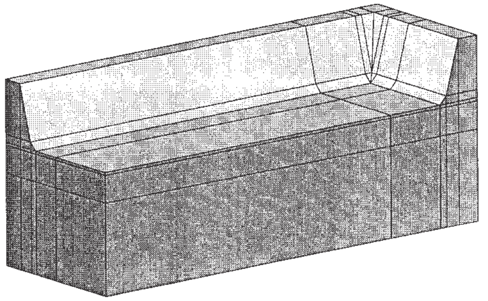
From the thermal modelling part of this case study [10], the heat transfer coefficient between the ingot and the starting block surface has turned out to be dependent on time, surface temperature and intrusion of cooling water into the gap. In addition, temperatures measured near the starting block surface for assumed identical casting conditions have been seen to display quite noticable variations and fluctuations.

The calculation cases

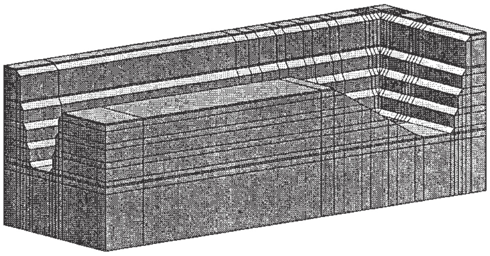
In all of the calculation cases, casting of laboratory scale AA 1050 sheet ingots of dimensions 600 mm × 200 mm were simulated. The casting velocity was 60 mm/sec.

The two basic calculation cases were for a 40 mm deep bowl shaped starting block and for a new type of starting block, 60 mm deep with a 40 mm cone. These starting block geometries, as used in the calculations, are shown in figure 2 (a) and (b) respectively. For more detailed information on these thermal calculation, we refer to [10].

In addition, castings with the use of bowl shaped starting blocks with depths 20 and 80 mm were simulated. For the case of the 40 mm deep bowl shaped starting block, the thermal boundary conditions were furthermore modified, in order to demonstrate the sensitivity of the butt curl to the prescribed heat transfer coefficients.



(a)



(b)

Figure 2: Geometries of quarter starting blocks as applied in calculations, (a) bowled shaped starting block, (b) starting block with a central cone.

The ALSPEN3 calculations simulated the casting process from a time typically a few seconds before the direct chill of the ingot started. Any deformations and stresses developed before this time, was neglected. This approach was found necessary, because the solidified material during the very first periode of casting forms a very thin shell, which occasionally is difficult to handle numerically.

Due to long calculation times associated with large solution domains, the ALSPEN3 calculations were stopped after simulation of 2–4 minutes of casting. Simulations of longer periods were considered to be of minor importance because of the diminishing butt curl velocity observed in experiments. The time step was varied through the ALSPEN calculations, from typically 1 sec. during periods of high butt curl velocity to approx. 3 sec. at the end of the simulations. The number of time steps was typically 75, and the number of elements increased gradually from about 2000 to 25000 during the longer simulations.

Results and discussion

Butt curl development

The calculated butt curl is the value of the computed vertical displacement at the bottom surface of the ingot a short distance from the centre of the small ingot side, corresponding to the measurements described in [5].

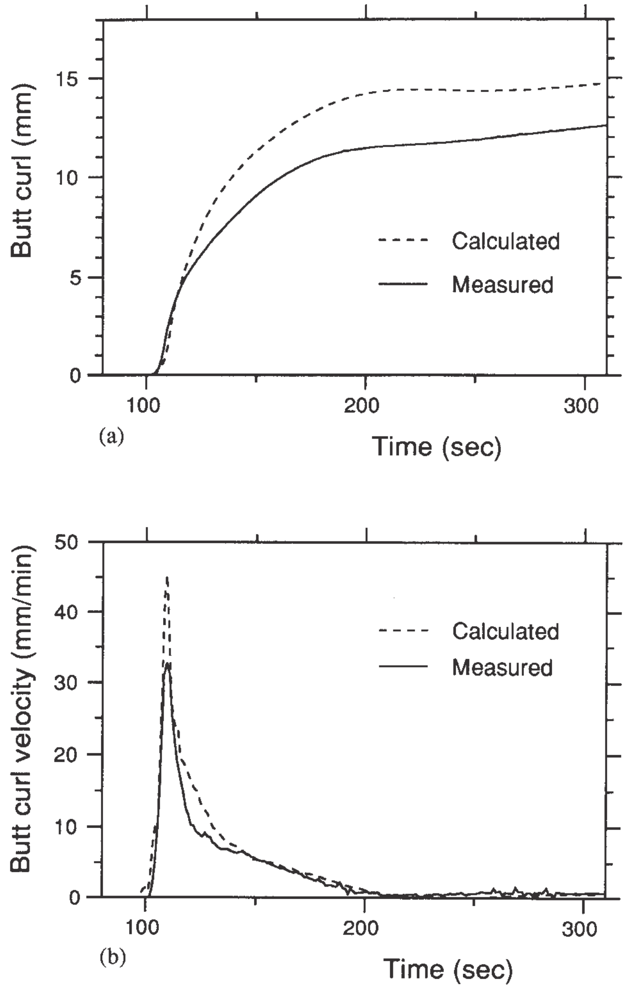


Figure 3: Comparison of measurements and calculated results for the bowl shaped starting block, (a) butt curl and (b) corresponding butt curl velocity.

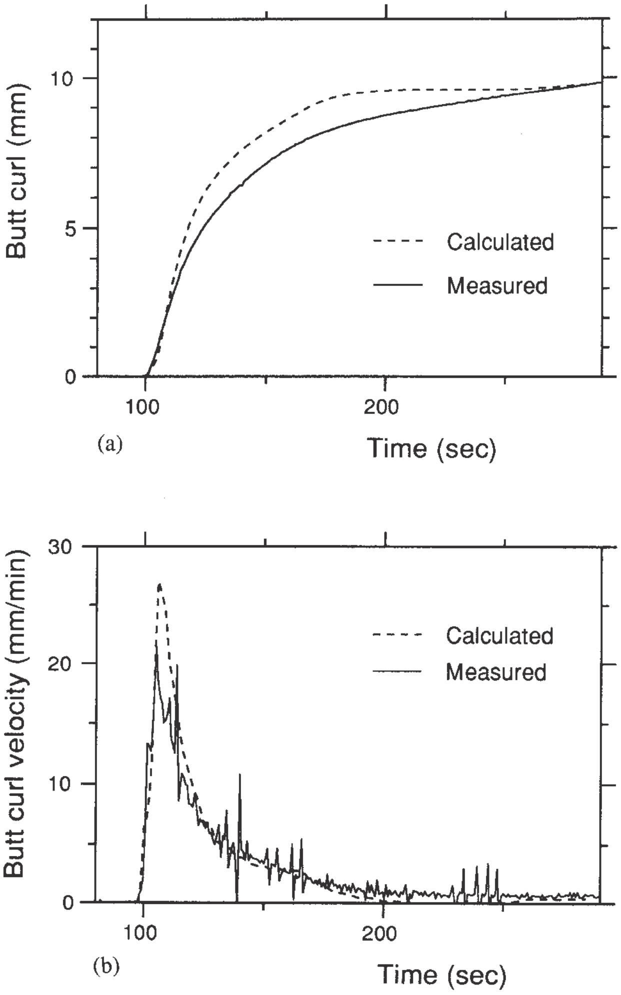


Figure 4: Comparison of measurments and calculated results for the starting block with cone, (a) butt curl and (b) corresponding butt curl velocity.

In figure 3 (a) the butt curl development calculated by ALSPEN3 is compared to a measured one [20] for the case of a bowl shaped starting block of depth 40 mm. The corresponding values of the butt curl velocity are shown in figure 4 (b). The same quantities for the new starting block with a cone is shown in figure 4 (a) and (b) respectively.

An additional 2–3 mm increase in butt curl was measured in the experiments [20] from the shown value at time 300 sec. to the end of casting.

Overall a very good agreement between calculated and measured butt curl values is seen in the figures. The reduction of butt curl by introduction of the new starting block with a cone in comparison with the use of the traditional starting block, is clearly reproduced in the calculations.

The discrepancies however seen, between the measured and the calculated values may reflect some of the problems associatied with the modelling of the early development of the butt curl. Asymmetric measured butt curl development, which may be interpreted as a ”rocking” of the ingot butt, is evident during the first seconds after the impact of direct water chill [20]. Some effect of possible sticking and friction must be considered.

The calculated early development of butt curl have shown some sensitivity to the prescribed coherency temperature.
Some uncertainty associated with the material description must also be considered, as a large portion of the solution domain in this periode have temperatures close to the solidus temperature.

Sensitivity to material description

Figure 5 (a) and (b) shows the early development of butt curl and butt curl velocity, calculated for the case of the 40 mm deep bowl shaped starting block. Three different constitutive descriptions are used here. As seen from the figure, quite similar butt curl results are obtained by using the constitutive model for commercial purity aluminium [17], the constitutive model for AA 6063 from [11] and the MATMOD equations for AA 6063.

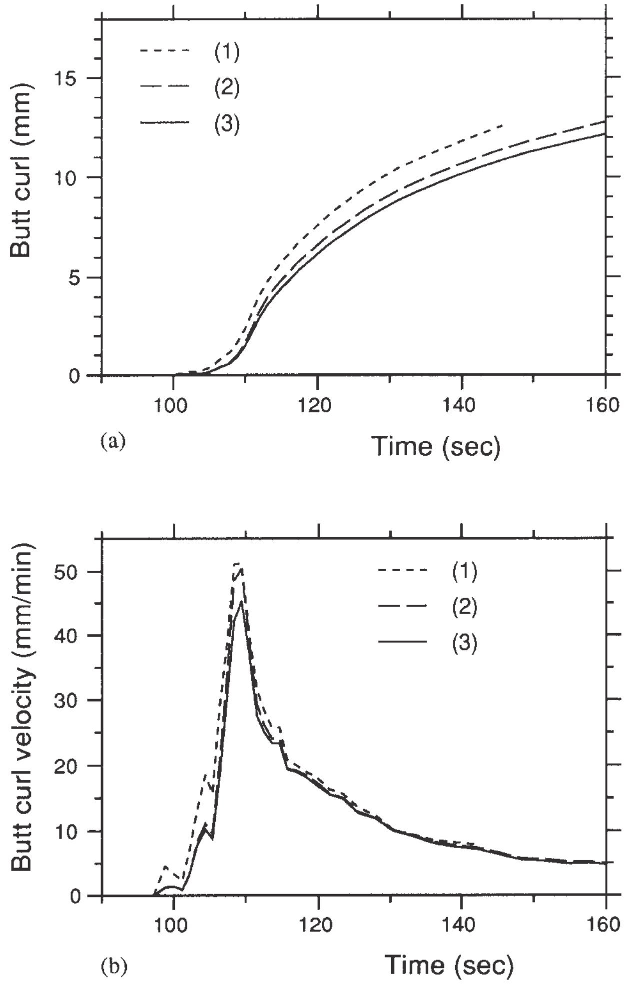


Figure 5: Calculated (a) butt curl and (b) corresponding butt curl velocity by use of (1) constitutive equation for AA 6063 [11], (2) MATMOD equations with AA 6063 data, (3) constitutive equation for commercial purity Al [17]

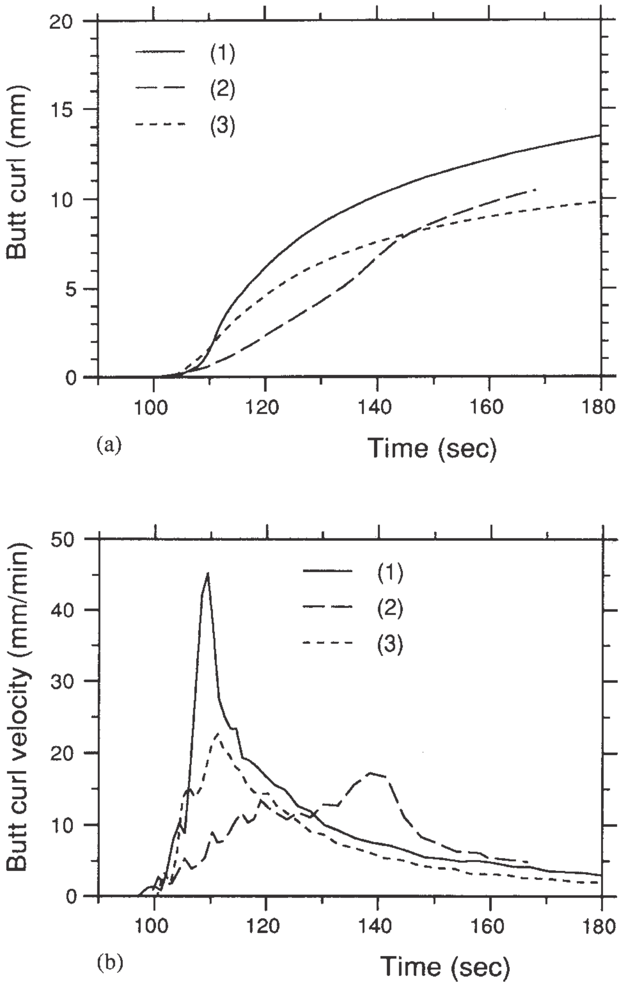


Figure 6: Calculated (a) butt curl (b) corresponding butt curl velocity, (1) as shown in figure 3, (2) with reduced heat transfer in direct water cooling zone, (3) with enhanced heat transfer to starting block.

As stress values calculated for AA 6063 have about twice the magnitude of those for commercial purity aluminium, this indicates that the butt curl is not necessarily dependent on the strenght of the alloy.

In other tests, however, the ratio of flow stress at low temperatures compared to flow stress at high temperatures has shown to have some influence on the calculated butt curl development. The discrepancy between the results obtained with the two constitutive models for AA 6063, indicates that the choice of the constitutive equations to which the experimental data is fitted have some significance.

From these calculations and other experience, although no extensive sensitivity analysis of constitutive behavior has been executed to date, we consider the uncertainty of mechanical material properties not to be a severe obstacle for reasonable calculations of the butt curl phenomenon.

Influence of thermal boundary conditions

In thermal calculations simulating the start up phase of DC casting of sheet ingots, the temperature development depends strongly on the thermal boundary conditions. The thermal boundary conditions therefore influence the ALSPEN3 results both through the thermal contractions, through the temperature dependence of the elastic-viscoplastic properties and by the effect on the T_c isotherm position defining the geometry of the solution domain.

The sensitivity of the butt curl development to the thermal boundary conditions is clearly illustrated by figure 6. Here the butt curl and butt curl velocity calculated for the bowl shaped starting block is compared with results based on thermal calculations where boundary conditions were modified.

In one calculation the surface heat transfer coefficient was limited to the value of 10000 W/(m^2·K) for surface temperatures above 300 °C in the direct water cooling zone. In this calculation the ingot surface temperature in the water cooling zone remained quite high for approximately 30 seconds, corresponding to a delay of the efficient bubble boiling mechanism. This delay is reflected in figure 6 (b) by the displacement of the peak in the maximum butt curl velocity.

In another calculation, the heat transfer coefficient between the ingot and the starting block was kept at a constant value of 1500 W/(m^2·K) on the entire boundary prior to the impact of the direct water cooling. Thereafter, the heat transfer coefficient was reduced to a value of 400 W/(m^2·K) for X-values greater than 5 cm, in order to reflect the effect of the air gap,

The first example illustrates that a modification of the heat transfer in the direct water cooling zone may lead to a qualitative different development of the butt curl. The latter example illustrates the effect of the stiffness of the solidified shell, formed prior to water impingement, on the quantitative response to the butt curl inducing forces.

These calculations clearly demonstrate the importance of accurate thermal modelling in attempts to predict the butt curl development. These results also illustrate the possibility of controlling the butt curl development, given a possible control of the heat transfer.

Variation of starting block depth

In [5], the effect on the butt curl of the depth of bowl shaped starting blocks is emphasized. In figure 7, calculated values of butt curl at time 2 minutes after the water impingement is compared to measured values [5] at the end of casting. Expecting the final evolution of the butt curl to be rather independent of the bowl depth, the calculated results are seen to correspond quite well to the experimental values.

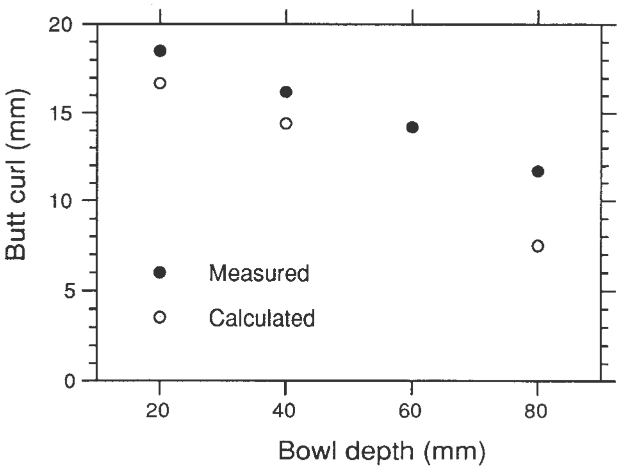


Figure 7: Final measured butt curl, from [5], compared to calculated values 2 minutes after start of direct water chill.

The butt curl mechanism

The characteristic feature of the butt curl deformation can be described as a contraction/compression in X-direction increasing with Z-value, in the absence of shear. A schematic illustration is given in figure 8.

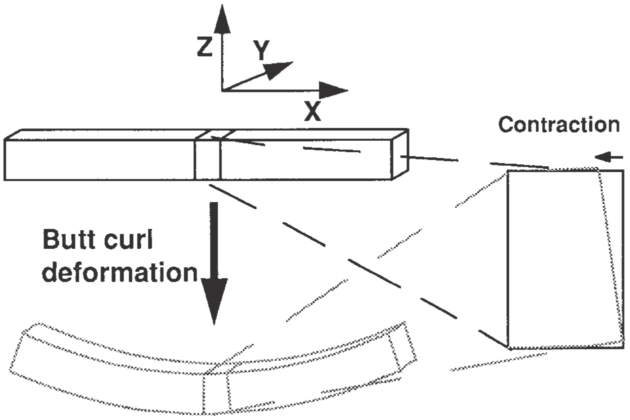


Figure 8: Illustration of the deformation connected to the butt curl. Portions of the central parts of the schematic drawn ingot butts are magnified.

From a leverage kind of argument, the deformation (curl) in the central part of the ingot can be shown to have the major impact on the magnitude of the butt curl gap near the short sides.

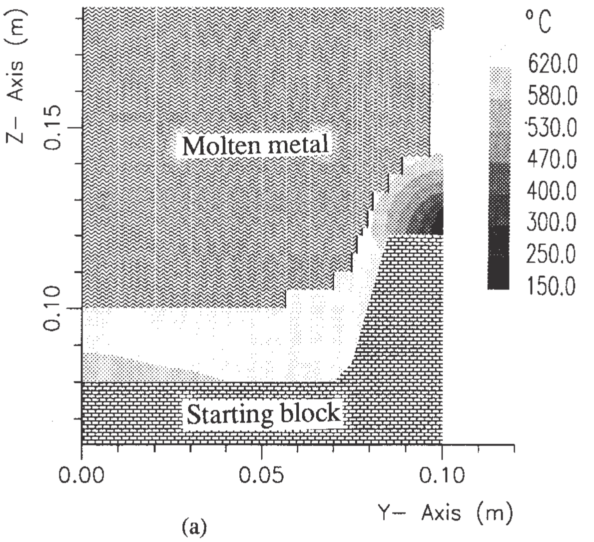
For a sheet ingot with a width to thickness ratio of 3, as in our case, a two dimensional thermal calculation in a cross section normal to the X-axis is often considered adequate for simulating the temperatures in a large central portion of the ingot. These thermal conditions are reflected in only minor variations with X-position for ALSPEN3 results in this central part of the ingot, especially during the early phase of the casting.

From these arguments, we propose the following kinematic quantity to be associated with the butt curl velocity: $-\frac{\partial \dot{\varepsilon}_{xx}}{\partial Z}$, i. e. the sign reversed Z-derivative of the (total) strain rate in the X-direction, typical for the central part of the ingot.

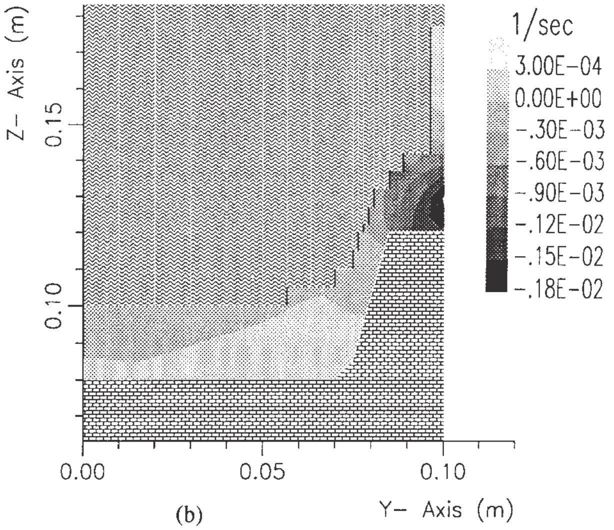
The total strain rate is composed of a thermal, an elastic and a viscoplastic part. The thermal strain rate is the driving force of deformation. The material response to the spatial variations in thermal strain rates appear as viscoplastic and elastic strain rates and as stresses. The elastic strain rates are in our case, however, almost negligible.

The thermal contractions have generally their highest values in the uppermost solidified part of the ingot, near the water impingement point. This contributes to the quantity $-\frac{\partial \dot{\varepsilon}_{xx}}{\partial Z}$. After a quite short periode of casting, however, the colder part of the ingot resists additional butt curl. We will here focus on the most important and easily explained mechanisms in the very first period of the butt curl development.

From the arguments above, we find it suitable to present quantities important to the butt curl phenomenon in contour plots for cross sections normal to the X-axis. Figure 9 (a), (b), (c) and (d) show respectively the temperatures, the thermal strain rates and the xx-component of stresses and viscoplastic strain rates in the cross section $X = 0$ approximately at the time of maximum butt curl velocity for the bowl shaped starter block. This situation occurs only about 7 sec. after the start of direct water cooling of the ingot.



(a)



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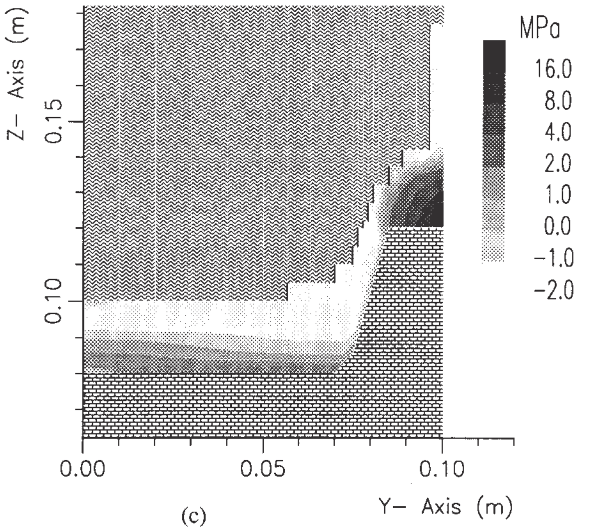

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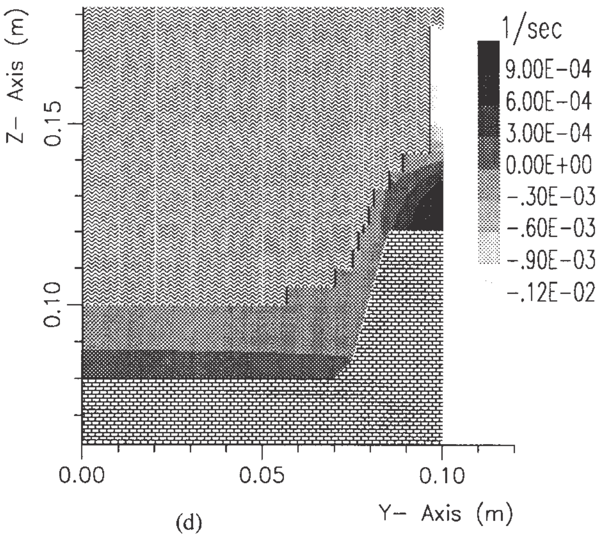

(d)

Figure 9: Contour plots of calculated results in the central cross section normal to the X-axis for the case of a 40 mm deep starting block. (a) temperatures, (b) thermal strain rate, (c) xx-component of stress tensor, (d) xx-component of viscoplastic strain rate tensor.

The high negative thermal strain rate values near the water impingement point, compared to the very low ones near the bottom of the starter block bowl, is considered to contribute to a large value of the quantity $-\frac{\partial \dot{\epsilon}_{xx}}{\partial Z}$.

The largest calculated stresses are found in the colder areas near the water impingement point where the thermal contraction rate is high. These stresses induce tensile viscoplastic strain rates, which are smaller in magnitude than the thermal ones, resulting in an overall contraction. At the bottom of the starting block bowl, both the thermal and viscoplastic strain rates are small. The viscoplastic strain rate is seen to change from tension to compression in the middle of the solidified sole.

A large magnitude of the final butt curl is generally connected with a large butt curl velocity in the early phase of the butt curl [1,5]. The strong dependency of the butt curl on the bowl depth can from these observations be explained in the following way. For a given thermally induced contraction near the water impingement point, together with negligible contractions in the thin solidified shell at the bottom of the starting block bowl, the quantity $-\frac{\partial \dot{\epsilon}_{xx}}{\partial Z}$ will be approx. inversely proportional to the bowl depth. We assume this argument to have a reasonable validity in the early course of the butt curl, and for bowl depths larger than the thickness of the solidified shell.

The additional reduction of butt curl associated with the introduction of a central cone in the starting block, calls for another kind of explanation. Figure 10 shows the xx-component of stresses for the starting block with a cone a few seconds after the start of butt curling. The solidified metal in the depression of the starting block constitutes a stiffening frame opposing the butt curl deformation.

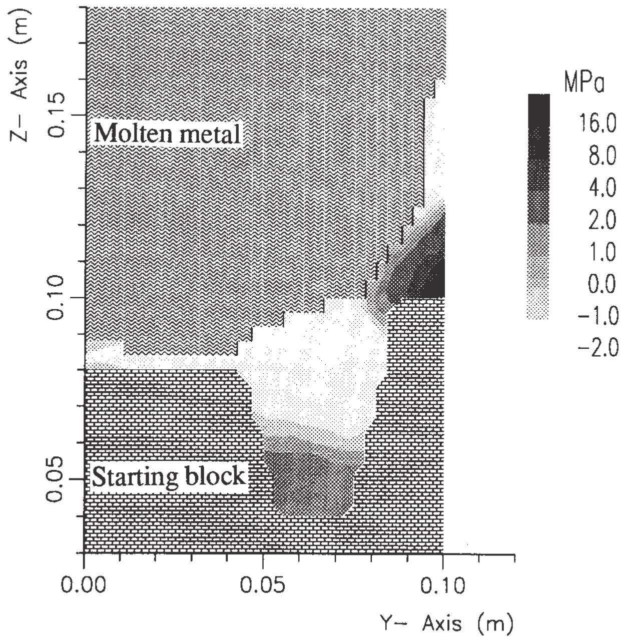

Figure 10: Contour plot of calculated xx-component of stress tensor in cross section normal to the X-axis for the case of starting block with central cone.

Compared to the configuration in figure 9, a larger torque is here assumed necessary to bend the "beam" of solidified metal, situated on the edge, in order to induce a butt curl — even if the flow stress values here are low. This torque must be caused by tensile stresses near the water cooling zone, and these stresses are in turn assumed to induce viscoplastic strains, reducing the effective contractions, and thereby reducing the butt curl.

Concluding remarks

The butt curl development of experimental casts with different types of starting blocks has been successfully reproduced by simulations with the model ALSPEN3.

The strong dependency of the butt curl on the bowl depth of starting blocks observed in experiments, is also appearent in the calculation results. This effect has been explained by a quantity relating the butt curl to the variation of the thermally induced contraction with Z-position.

The sensitivity of the calculated butt curl on thermal boundary conditions emphasizes the need of accurate thermal modelling. Lack of material data and uncertainties in the constitutive modelling are also factors making accurate predictions by modelling difficult.

The model ALSPEN3 seems, however, to be a useful tool by which case studies can give valuable insight in phenomena and mechanisms important to the process of DC casting of sheet ingots.

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