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AN EMPIRICAL MODEL TO EXPLAIN CROSS-SECTION CHANGES OF D.C. SHEET INGOT DURING CASTING

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INTRODUCTION

Sheet ingots cast by the D.C. process are not regular in shape. If they are cast in a rectangular mould, the first part to solidify is rectangular but the rolling surfaces on the main body of the ingot are usually concave. To compensate for this, a convexity is designed into each of the long sides of the mould. A certain mould convexity (bow) can or will produce a flat ingot with a convex butt. The degree of mould convexity required to produce a flat ingot varies with the alloy being cast and increases with higher casting speed and greater ingot thickness. Contours for mould bow for the common alloys, speeds and thicknesses for each different sheet ingot product have been developed by trial and error.

Higher casting speeds and an increasing demand for larger ingot sizes necessitated a method for accurately predicting the required mould bow to produce flat ingots. By eliminating the trial and error, a considerable reduction in the time consuming mould readjustment period could be realized.

Considerable concern has arisen with problems caused by the rather large convex bottom butts that are common to D.C. cast sheet ingots. Although it is known that the butt bulge is related to the factors mentioned above, no clear understanding of the mechanism involved existed.

A simple model for sheet ingot cross section changes during solidification and cooling was proposed to account for the shape of the ingot produced at various stages of casting. The model was verified by casting trials and empirical equations that can be used for mould design were formulated.

DEVELOPMENT OF A MODEL

Defining the Shape Changes of a Rectangular (Sheet) Ingot during Casting

To produce an ingot of a given thickness, the mould opening required is considerably more than that accounted for by simple thermal contraction of the solidified ingot. This indicates that the solidified portion of the ingot adjacent to the molten sump contracts or is pulled away from the mould as solidification of the sump proceeds.

Figure 1 shows an exaggerated diagrammatic view of the mould shape that is required to produce a flat sheet ingot. During steady state casting conditions, the ingot initially forms in contact with the mould and then diminishes in thickness as it moves down and beyond the mould until the final cross-section, as shown in Figure 1, is reached with complete solidification.

In developing and testing the model, only the central portion of an ingot was considered since this area of constant contraction should be easiest to analyse. However, to be generally valid, the model that explains the central contraction must also be able to account for the change in contraction across the ingot width.

Figure 2 shows a diagrammatic, longitudinal short transverse cross-section of the centre of an ingot as it appears during steady-state casting. Approximated isotherms have been drawn in the solid ingot wall adjacent to the liquid sump. This type of temperature distribution causes the ingot rolling face to be drawn inwards as the wall cools. The suggested mechanism is outlined in Figure 3.

Referring to Figure 3, the incremental element, AB, that has just solidified at the solid-liquid interface has a temperature of approximately 660°C. As solidification proceeds, element AB passes through the isotherms shown in Figure 2 and cools to become element A'B'. Element A'B' is further cooled as it moves downward to position A"B". The element AB contracts as the ingot temperature descends to room temperature. To keep points B, B' and B" on the ingot centre line (and keep the ingot intact), point A must move towards the ingot centre as it takes the positions A' and A". This inward contraction of the ingot face is described as "rolling face pull-in" and appears as distance 'X' in Figure 3.

For an ingot of given thickness of any alloy, the rolling face pull-in and the sump depth are known to increase linearly with the casting speed. In order to create a general model based on specific data, a relationship between these variables is required. A simple way of relating these variables is to consider the mode of solidification during steady-state casting conditions.

Solidification Model

Figure 4 shows the steady-state sump profile at the ingot centre during casting. If one assumes that there is uniform heat extraction from the ingot faces in the sub-mould region then the sump should have a parabolic shape predicted by the equation for one-dimensional solidification with uniform heat removal.

$$t_L = K_1 \cdot \theta_L^{\frac{1}{2}} \qquad (1)$$

$$\text{But: } \theta_L = \frac{L}{V} \qquad (2)$$

$$\text{Therefore: } L = \frac{t_L^2 V}{K_1^2} \qquad (3)$$

$$\text{When: } L = D - \text{shell depth},\ t = \frac{T}{2} \qquad (4)$$

$$\text{Thus: } D - \text{shell depth} = K_2 T^2 V \qquad (5)$$

If this type of equation can be confirmed, and, if the relationship of $X \propto D \propto V$ for the specific case holds true in the general case, then measurements taken from a few ingots can be used to establish a predictive equation to determine X for other casting conditions.

In order to establish the empirical relationship, accurate data describing D, T and V, for a given alloy was required.

CASTING TRIALS

Steady-State Trials

Conventional D.C. casting equipment was used for the experimental work. The alloy used was AA-1050. Four series of ingots were cast, each series was of a different size and each size was cast at several different speeds. The moulds were chosen to give nominal ingot thicknesses of 9, 12, 15 and 18 inches. To minimize the possibility of erroneous results at the ingot center due to end cooling effects, the moulds had a width to thickness ratio of at least 2.5.

For each ingot cast, the sump depth and sump profile were defined after steady-state casting conditions had been reached. A steel rod was used to measure sump depth during casting and molten zinc was added to trace the sump profile.

A longitudinal section was taken from each ingot at the center and etched to reveal the sump profile. Ingot thickness measurements were taken from each slice from a region below the point where the zinc additions were made.

Transient Start-up Trials

In order to follow the sump formation and define the ingot butt shape, sump depth and center line ingot thickness measurements were taken at various stages of the starting sequence for an 18-inch thick ingot cast at 4.2 inches per minute.

In addition, sump profiles were taken at regular intervals during the starting sequence to trace the evolution of an equilibrium sump.

RESULTS

Steady-State Trials

The data gathered in these casting trials are summarized in Table 1. The thickness measurements quoted are taken from ingots at room temperature. A sub-sump thermal contraction term is subtracted from the mould opening; the ingot thickness is then subtracted from this term to obtain the value of 2X.

In Figure 5, D - shell depth is plotted against T^2V. The overall linear relationship confirms that the central part of a rectangular ingot solidifies by one-dimensional uniform heat extraction. An equation for the relationship can be written as follows:

$$D - \text{shell depth} = 0.018\ T^2 V \qquad (6)$$

Figure 6 shows the zinc sump trace for a 15-inch thick ingot cast at 4.2 inches per minute. For a few ingots the value of K_1 was calculated, and using equation (1) a theoretical sump contour was plotted. The theoretical shape corresponded very well ($\pm \frac{1}{4}$ inch) with the actual shape.

Measurements of the sump depth made during casting indicated that the sump in the central part of the ingot was of constant depth. The end cooling effect did not extend more than one ingot thickness towards the center.

A linear relationship, as shown in Figure 7, is also obtained for 2X vs T^2V. There is some slight deviation from the straight line. However, for the range of sizes and casting speeds employed in the test, the following empirical equation can be written for rolling face pull-in for AA-1050 alloy:

$$2X = 0.0014\ T^2V \qquad (7)$$

From equations (6) and (7) it becomes obvious that X is proportional to D.

The central mould opening (MO) required to give a certain final ingot thickness may now be calculated as:

MO = Desired thickness + thermal contraction below sump + rolling face pull-in in sump zone.

$$MO = 1.0085\ T + 0.0014\ T^2V \qquad (8)$$

To check the accuracy of equation (8), the actual values of T and V were used to calculate the mould opening for each ingot. The average difference between the calculated and actual mould openings was 0.03 inches. The maximum difference was 0.09 inches.

Transient Start-up Trials

Table II contains a summary of the data from these trials. The change in sump depth with the length of ingot cast is shown in Figure 8. The sump initially grows at the same rate as the ingot length and then slows down to gradually approach the equilibrium sump depth. The sump depth slightly overshoots the equilibrium value before returning to it. Steady-state conditions are reached after twice the equilibrium sump depth has been cast.

Figure 9 illustrates the change in ingot thickness that occurs in the butt region. The thickness vs length cast curve is a mirror image of the sump depth vs length cast curve.

The evolution of the equilibrium sump is shown in Figure 10. Each line represents a sump contour after a further 5-inch increment of casting.

DISCUSSION

Equation (8) predicts, with reasonable accuracy, the central mould opening required to produce an ingot of a given required central thickness at a given casting speed for alloy AA-1050.

To expand this model to other alloy systems a new "pull-in" constant for equation (7) must be determined for each alloy. This is accomplished by measuring MO, T an V for one ingot of each alloy and calculating the constant by using equation (8).

Equation (7) gives the rolling face pull-in when the sump is fully developed as in Figure 4. When the sump is not fully developed, T/2 must be changed to the smaller ingot sump-wall thickness t_L. Equation (7) now becomes:

$$2X = 0.0056\ t_L^2 V \qquad (9)$$

for a partially developed sump. Equation (8) must similarily be modified to:

$$MO = 1.0085\ T + 0.0056\ t_L^2 V \qquad (10)$$

During steady-state casting the sump shape of a wide rectangular ingot is similar to that shown in Figure 11. From sections A-A, B-B and C-C it is seen that t_L increases from zero to T/2 as the section is taken closer to the ingot center. It follows from equation (10) that the mould opening required to produce a flat rolling face must also increase with distance from the end of the mould. Thus the model developed for the ingot center has at least qualitatively predicted the need for mould bow.

Wide sheet ingots have an almost constant sump depth in their central portion. The model predicts that the mould opening should be constant in this area to produce flat ingots. This is supported by practical experience.

The dependance of rolling face pull-in on the ingot sump-wall thickness can also be used to explain the changes in ingot thickness that occur during the initial stages of casting.

Equation (1) shows that t_L is zero at the start of a cast (since $\theta_L = 0$) and therefore there is no rolling face pull-in. The initial ingot thickness is equal to the mould opening minus the solid thermal contraction. Thus a convex mould initially produces a convex butt.

From equation (3), t_L^2 should increase linearly as the length of ingot cast increases, and as a result so should the rolling face pull-in. This should result in a gradual linear decrease in butt thickness with cast length. Figures 9 and 10 support these predictions.

The value of t_L increases until it reaches T/2, at which point the equilibrium sump has been reached and rolling face pull-in remains constant during the rest of the cast.

With the restrictions imposed by the solidification pattern for D.C. casting it is evident that the only way to eliminate the convex butt, and cast a flat ingot, is to change the mould bow during the start up period. To produce a flat ingot from end to end, the mould must start in a rectangular configuration and be slowly bowed outwards to compensate for the rolling face pull-in that increases with the thickening sump wall.

CONCLUSIONS

The objective of this work was to produce a mathematical model which would predict the cross-section changes of D.C. sheet ingots during casting. The model was developed and successfully tested. It expresses the relationship between rolling face pull-in, ingot thickness and casting speed. Accurate predictions of the mould opening required to produce a flat ingot of any desired thickness can be made. When high casting speeds are employed convex butts can only be avoided by changing the mould bow during the starting period of each cast.

NOMENCLATURE

D	Sump depth (in.)
T	Ingot thickness (in.)
t_L	Ingot sump-wall thickness at depth L (in.)
L	Distance below sub-mould cooling (in.)
θ	Time
V	Casting speed (in. min. $^{-1}$)
K_1, K_2	Constants that are a function of the alloy being cast
MO	Mould opening (in.)

TABLE I - STEADY-STATE CASTING TRIAL DATA FOR SHEET INGOT IN AA 1050 ALLOY

Nominal Ingot Thickness (In.)	Maximum Mould Opening MO (In.)	V Casting Speed (In./Min.)	D Sump Depth (In.)	T Actual Thickness (In.)	MO-Subsump Thermal Contraction (In.)	2X Pull-In Above Sump Bottom (In.)
9	9.88	3.6	7.25	9.37	9.80	0.43
		4.5	8.5	9.25	9.80	0.55
		5.5	10.25	9.13	9.80	0.67
		6.6	12.0	9.04	9.80	0.76
12	13.09	2.6	8.5	12.50	12.99	0.49
		3.5	10.5	12.25	12.99	0.74
		4.4	13.5	12.07	12.99	0.92
		5.1	15.0	11.93	12.99	1.06
15	16.25	2.8	13.0	15.25	16.12	0.87
		3.4	15.5	15.04	16.12	1.08
		4.2	18.5	14.82	16.12	1.30
		4.9	22.0	14.63	16.12	1.49
18	19.70	2.3	14.5	18.43	19.55	1.12
		3.1	19.25	18.07	19.55	1.48
		3.7	23.0	17.82	19.55	1.73
		4.6	27.5	17.56	19.55	1.99
		5.0	30.0	17.43	19.55	2.12

TABLE II

TRANSIENT START-UP CASTING TRIAL DATA FOR AN 18-INCH THICK AA 1050 ALLOY INGOT CAST AT 4.2 INCHES PER MINUTE

Length Cast (in.)	Sump Depth (in.)	Centerline Ingot thickness (in.)
2		19.3
4		19.2
6		19.2
8	6.5	19.1
10		18.9
12		18.7
14	12.5	18.5
16		18.4
18		18.2
20	17.5	18.1
22		18.0
24	20.5	17.9
26		17.8
28	22.75	17.7
30		17.7
32	23.5	17.6
34		17.6
36	26.0	17.6
38		17.6
40	25.95	17.6
44	25.5	17.7
48	25.25	17.7
52	25.0	17.7
56	25.0	17.8
60	25.0	17.7

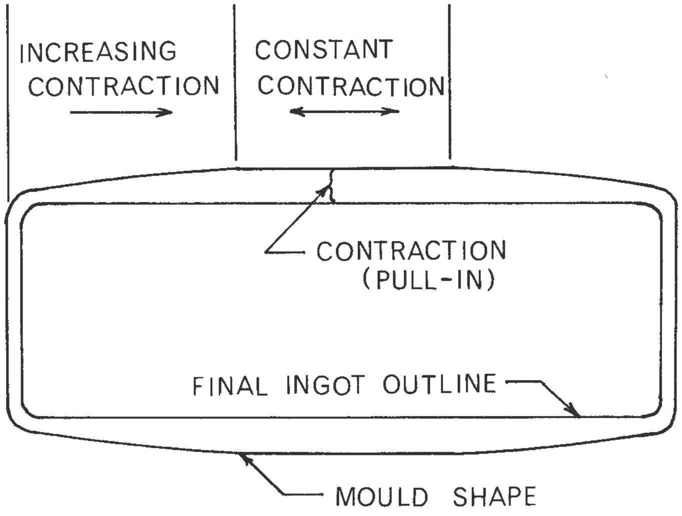

Figure 1. Typical mould shape used for sheet ingot casting.

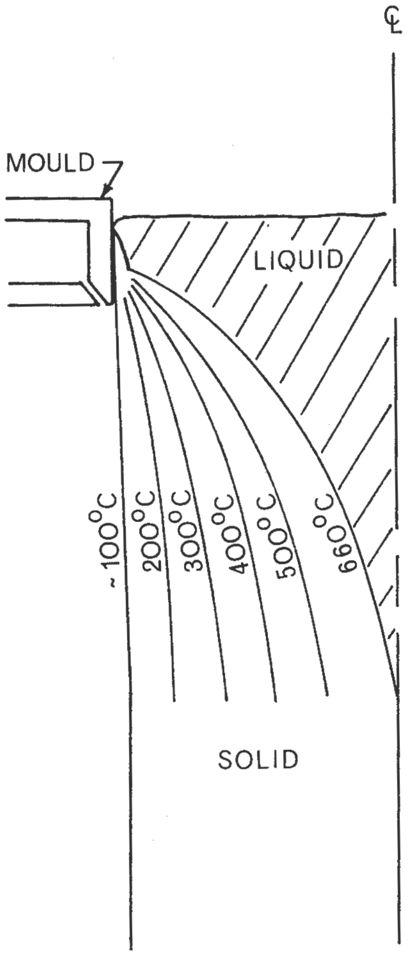

Fig. 2. Central ingot profile showing isotherms.

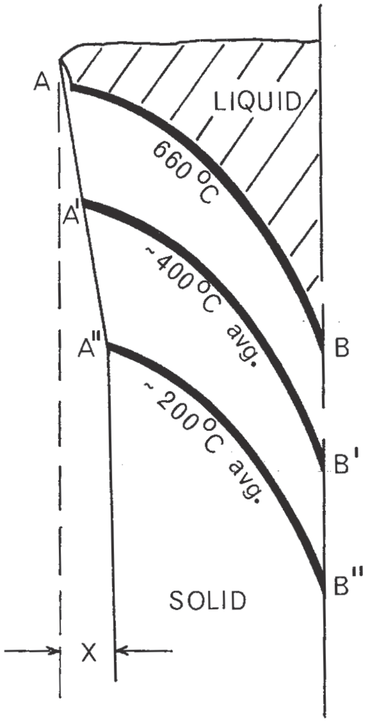

Fig. 3. Schematic representation of ingot rolling face pull-in mechanism.

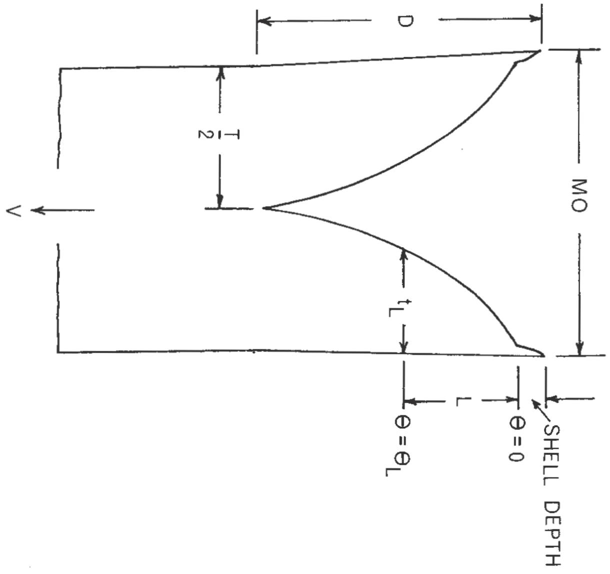

Fig. 4. Central sump shape formation by one-dimensional heat transfer.

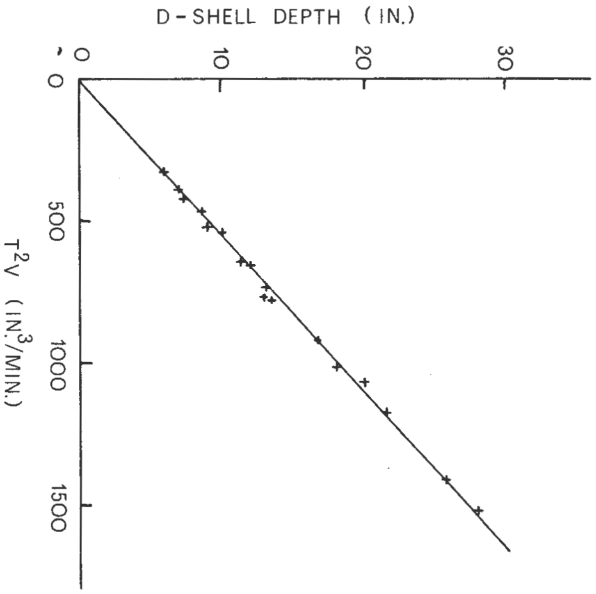

Fig. 5. D - shell depth vs T^2V from steady state trials.

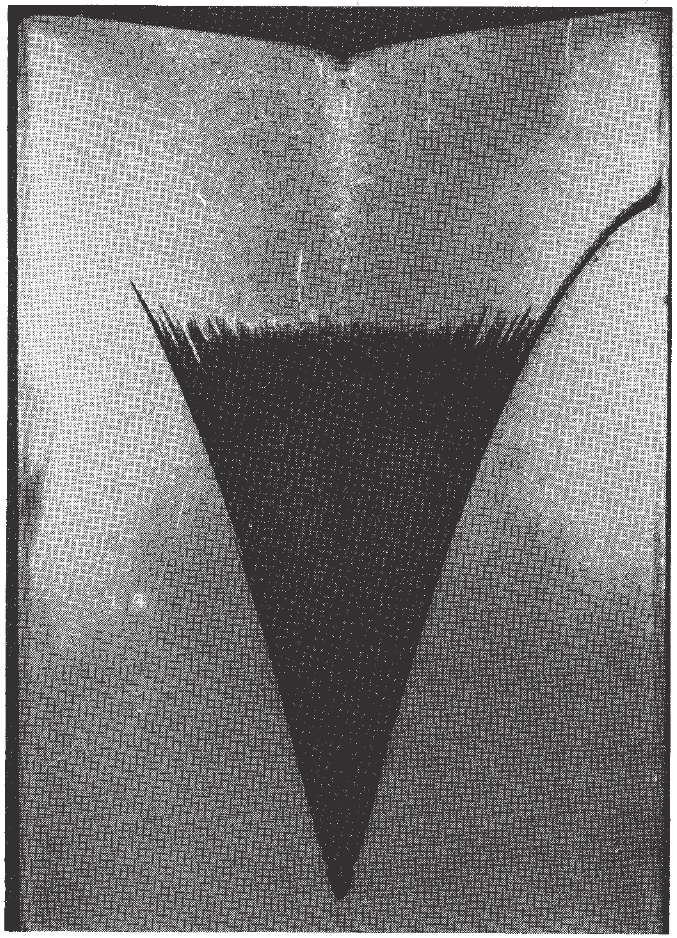

Figure 6. Zinc trace of ingot sump contour at steady state conditions.

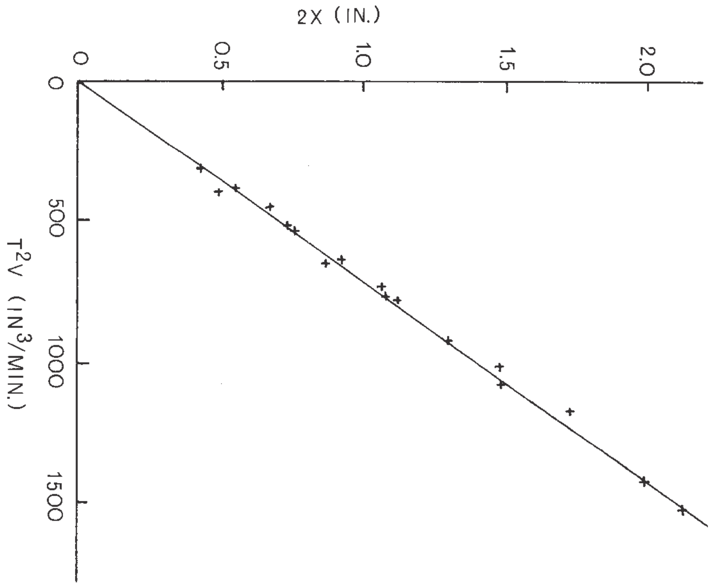

Fig. 7. 2X vs T^2V from steady state trials.

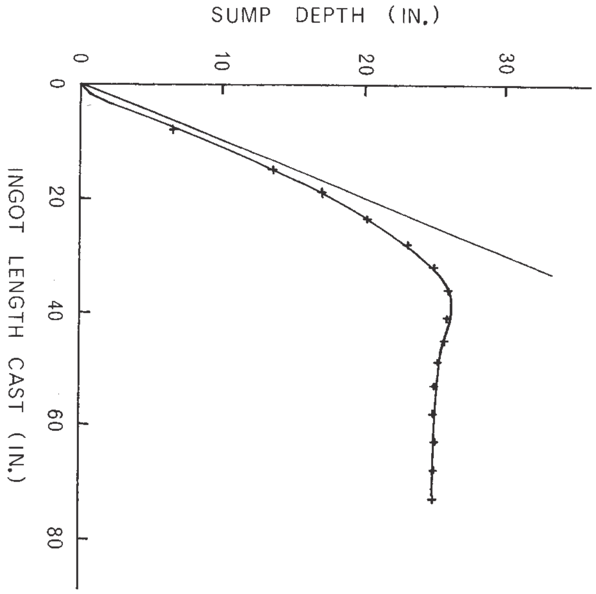

Fig.8 Sump depth vs cast length for an 18" thick ingot cast at 4.2" per minute

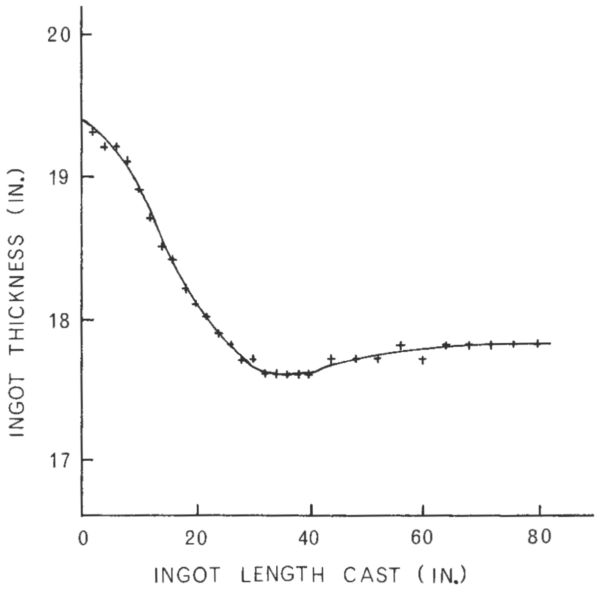

Figure 9. Ingot thickness vs cast length for an 18-inch thick ingot cast at 4.2 inches per minute.

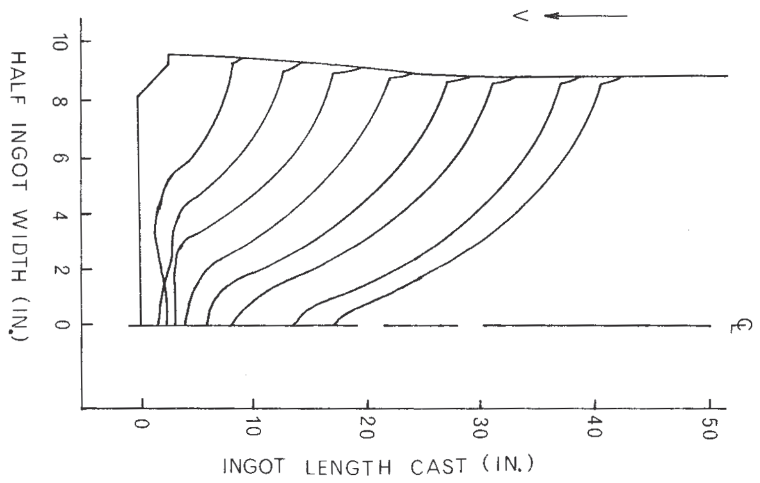

Figure 10. Evolution of the equilibrium sump for an 18-inch thick ingot cast at 4.2 inches per minute.

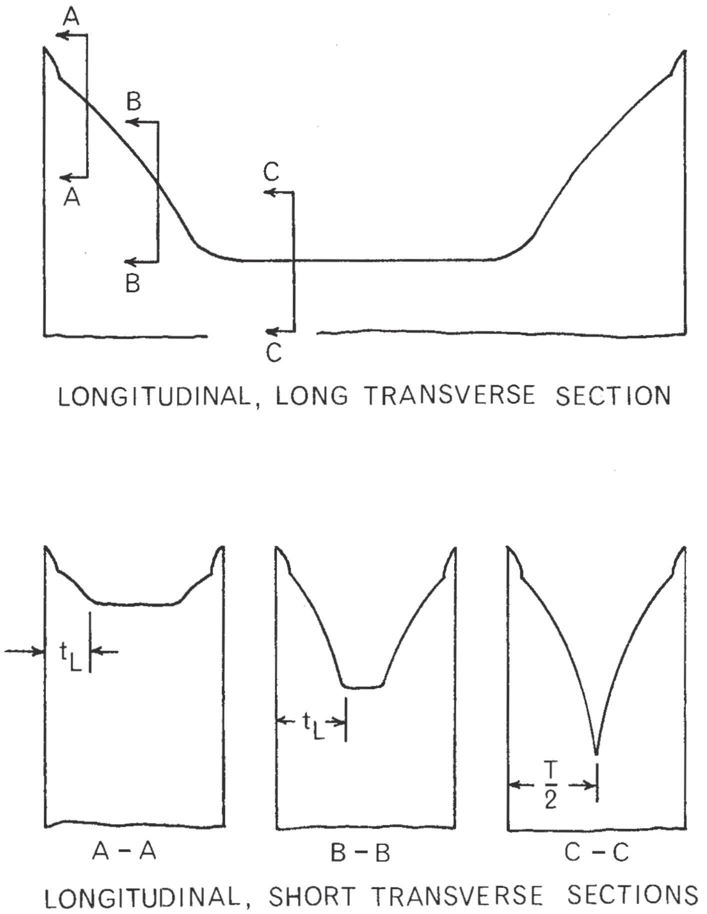

Figure 11. Schematic representations of equilibrium sheet ingot sumps.