Essential Readings in Light Metals: Cast Shop for Aluminum Production.
Edited by John F. Grandfield and Dmitry G. Eskin.

2. FURNACES, MELTING, FLUXING, AND ALLOYING

In this section we have grouped papers related to preparing the liquid metal for casting. It includes papers on sodium, lithium, and calcium pickup and removal and how alloy hardeners behave when added to the melt. Removal of inclusions via settling is also an important step in the process at this stage. The industry's move away from dangerous chlorine gas toward the addition of halide fluxes as a safer alternative that does not compromise on melt quality is also documented.

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OPTIMAL FUEL CONTROL OF A CASTING FURNACE

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Abstract

Starting with a 60-equation nonlinear dynamic model of the aluminum casting furnace, a model reduction is carried out to obtain a tenth-order model. Then variational calculus is applied to the reduced model to solve a fuel-optimal control problem. It is shown that for the casting furnace, optimal control is possible through the use of a reduced model and the application of appropriate optimization methods. A fuel economy close to 11% is obtained using the optimal fuel flowrate instead of the conventional constant fuel flowrate.

Introduction

The casting furnace is the centerpiece of the primary aluminum manufacturing process. It receives hot liquid metal coming from the electrolytic cells, brings it to a specified temperature and maintains it there while various preparations such as stirring, fluxing, alloying, skimming are made before casting takes place. While some solid charge can be simultaneously melted during the heating period, the main task is still liquid metal heating, holding and preparation.

The casting furnace consists of two parts, the combustion chamber on top where a burner provides the heat, and the metal below. It consumes large amounts of energy, and an optimal fuel control, even if it leads to only a few percentage points in fuel savings, will be worthwhile. Figure 1 describes a 72-ton casting furnace with a 4800 kW burner.

The analytic model

The best way to model a complex industrial process like the one taking place in the casting furnace is to write the equations describing the physical phenomena involved. We thus obtain the analytic model, also called the process model. The furnace is seen as made of three main components: the gas, the metal, and the refractories that form the roof of the combustion chamber. The gas is seen as well stirred; the metal and the refractories are treated as one-dimensional conducting media. Equations are

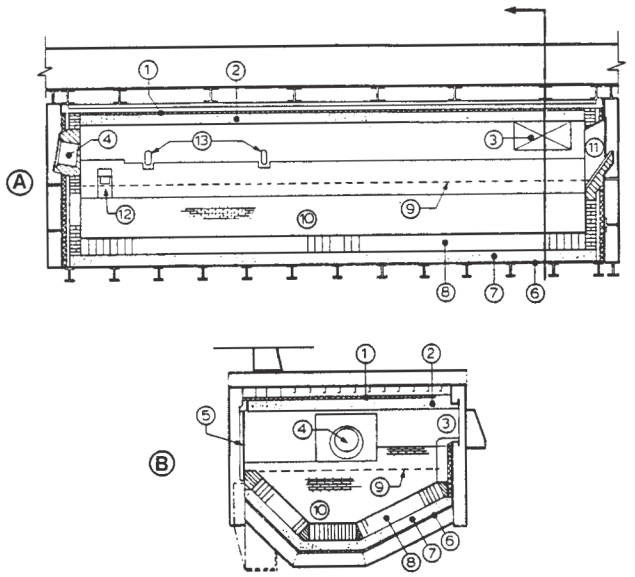

Figure 1 Cutaway views of the casting furnace
A- Lengthwise B- Across
(1,2) roof (3) stack (4) burner
(5) doors (6,7,8) floor (9,10) metal
(11) siphon (12) spout
(13) thermocouples

written for the conservation of energy, mass and chemical species. They are complemented with the heat conduction equations inside the metal and the refractories, and the radiative heat transfer equations between the gas, the metal and the refractories. To solve this model with good accuracy, 60 one-dimensional slices were used to discretize the system. This results in a 60th-order nonlinear system. This model, duly validated, provides a good representation of the real furnace. The model and simulation results are presented in [1].

For control purpose, clearly one cannot use this cumbersome system of equations, which, while ensuring good representativity, induces the designer to lose sight of the crucial elements of a control problem. A model reduction is necessary.

The reduced model

To obtain a reduced model of the form $\dot{y} = Ay + Bu$, we use a statistical approach based on the data produced by simulations made with the analytic model. The elements of A and B will be obtained by a statistical approach based on least square approximation [2]. Two questions must be addressed. How many variables are needed for the reduced model, and can the reduced model be made linear?

The answer to the first question depends on the extent to which the temperatures vary inside each of the two conducting media, namely the metal and the refractories. The more they vary, the finer the discretization must be. This constitutes an important aspect of model reduction. Based on the temperature profiles inside each of the two conducting media given by the analytic model, it was determined that the metal would be discretized into 3 slices and the refractories into 6. This, in addition to the gas seen as a well stirred body, makes a total of 10 finite volumes, and the system is of tenth order. The state variables are the gas temperature, the 3 metal temperatures and the 6 refractory temperatures. The control variable is the fuel flowrate.

The second question requires more investigation. A full linearization was first tried in which the time derivative of each state variable was expressed as a linear combination of other state variables and/or control variable. The conduction equations inside the metal and the refractories are per se linear. But there are 4 nonlinear expressions of heat flowrates namely the heat generated by fuel combustion in the chamber, the heat transferred to the metal, the heat transferred to the refractories, and the heat loss from the roof to the ambient atmosphere. The last three are convective-radiative, causing considerable nonlinearities. All four heat flowrates were linearized as functions of temperatures. The heat generated by fuel combustion is also a linear function of fuel flow. The linear model thus obtained turned out to be representative of the analytic model in terms of temperature outputs. Among other things, the linearization showed that clearly the statistical approach was applicable to this model reduction problem.

However when the linear model is applied to fuel optimization, its representativity becomes inadequate. For the purpose of predicting the dynamic behavior of the furnace, a discrepancy of a few degrees in the output temperatures between analytic model and linear model is acceptable. For optimization purpose it is a different story. Here is an example. With an initial metal temperature of 685°C submitted to one hour of heating, the final temperature given by the analytic model is 701°C, and by the linear model is 699.2°C. This is a good approximation. Yet it represents a relative error of (701-699.2)/(701-685) = 11%. Since a fuel optimization is likely to yield an improvement of a few percentage points, there is a strong motivation to develop a reduced model that is even more representative. To do so it is necessary to keep the nonlinearities in the heat flow expressions instead of linearizing them as done previously. The worst nonlinear equation, that of the gas, is presented here for illustrative purpose.

$$M_g C_g \dot{T}_g = Q_f - Q_m - Q_r \qquad (1)$$

where

$$Q_f = (a_1 T_g^2 + a_2 T_g + a_3/T_g + A_4)u \qquad (2)$$

$$Q_m = A_m h_m (T_g - T_m) + \sigma \overrightarrow{GS}_m T_g^4 - \sigma(\overrightarrow{S_m G} + \overrightarrow{S_m S_r}) T_m^4 + \sigma \overrightarrow{S_r S_m} T_r^4 \qquad (3)$$

$$Q_r = A_r h_r (T_g - T_r) + \sigma \overrightarrow{GS}_r T_g^4 - \sigma(\overrightarrow{S_r G} + \overrightarrow{S_r S_m}) T_r^4 + \sigma \overrightarrow{S_m S_r} T_m^4 \qquad (4)$$

Instead of trying to linearize Equation (1) with respect to T_g, T_m, T_r and u, the expression for Q_f as given by Equation (2) is kept nonlinear as it is. In the expressions for Q_m and Q_r, the gas-to-surface directed interchange areas $\overrightarrow{GS}_m$ and $\overrightarrow{GS}_r$ are approximated by a third-power polynomial in T_g. The directed interchange areas, surface-to-gas ($\overrightarrow{S_m G}$, $\overrightarrow{S_r G}$) and surface-to-surface ($\overrightarrow{S_m S_r}$, $\overrightarrow{S_r S_m}$), are approximated by second-power polynomials in both temperatures involved. For example:

$$\overrightarrow{GS}_m = c_1 T_g^3 + c_2 T_g^2 + c_3 T_g + c_4 \qquad (5)$$

$$\overrightarrow{S_m G} = c_9 T_g^2 + c_{10} T_m^2 + c_{11} T_g T_m + c_{12} T_g + c_{13} T_m + c_{14} \qquad (6)$$

The coefficients c_1, c_2 ... are determined by the least square approximation method. The result is a set of ten nonlinear equations of the form:

$$\dot{T}_i = f(T_j, u) \qquad (7)$$

where T_j are the state variables and u the control variable (fuel flowrate).

As the reduced model is to be used for fuel optimization later, it must be valid not for a single value of u but for a range of u values. In other words the approximation must be done not around one operating point but over a whole range. The range chosen is $50 \leq u \leq 500$ m^3/h of natural gas. The upper limit is the nominal flowrate of the burner, the lower limit is dictated by flame stability. The data used in the model reduction is obtained by running the analytic model with different u values by steps of 50 m^3/h from 50 to 500.

Figures 2, 3, 4 respectively present the gas temperature, the central-node metal temperature and the refractories inner-surface temperature given by the analytic model and the reduced tenth-order model. The coincidence of the curves confirms the validity of the reduced model. The central-node metal temperature of Figure 3 plays a crucial role in furnace control. It is measured by a thermocouple and serves as controlled output variable for the optimization problem that follows.

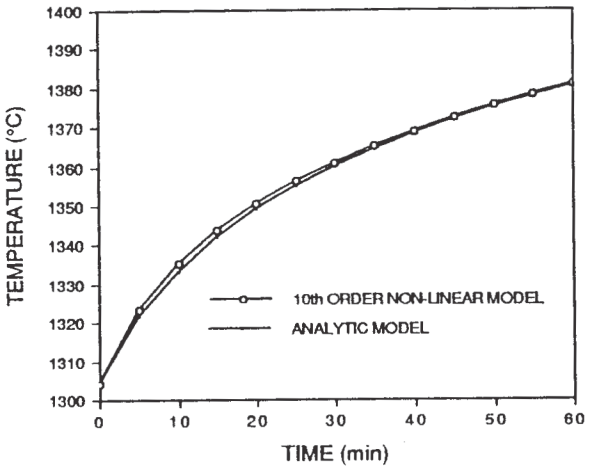

Figure 2 Gas temperature obtained from analytic model and from reduced tenth-order model. Constant fuel flowrate 450 m^3/h.

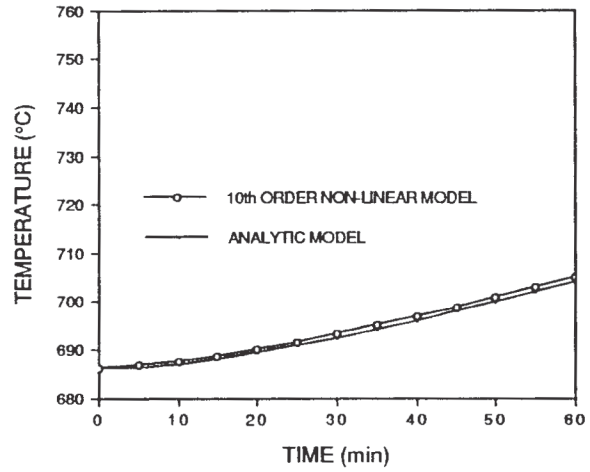

Figure 3 Central-node metal temperature obtained from analytic model and from reduced tenth-order model. Constant fuel flowrate 450m^3/h.

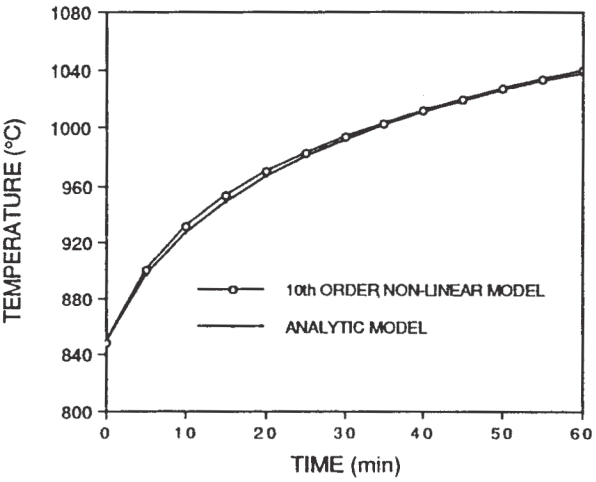

Figure 4 Refractories inner-surface temperature obtained from analytic model and from reduced tenth-order model. Constant fuel flowrate 450m^3/h.

Fuel-optimal control

In the foregoing, a reduced nonlinear tenth-order model was obtained by applying the least square approximation technique to the simulated data produced by the analytic model of the furnace. We now use the reduced model to solve the fuel-optimal control problem. The problem is to find a time-varying fuel flowrate to bring the mass of liquid metal from an initial temperature to a prescribed final temperature in a given time with minimum fuel. The state variables, from now on denoted by y(t), are the ten temperatures in the system: one in the gas, three in the metal and six in the roof refractories. The control variable u(t) is the fuel flowrate. The controlled output variable is the central-node metal temperature. Pontryagin's maximum principle will be used to formulate the optimization as a two-point boundary value problem.

Essentially it is a Lagrange problem where the equality constraints on the state variables y(t) are provided by the state equations forming the reduced model:

$$\dot{y}(t) = f[y(t), u(t), t] \qquad (8)$$

An inequality constraint is imposed on the control variable as an upper limit $u_{max} = 500\ m^3/h$ and a lower limit $u_{min} = 50\ m^3/h$:

$$50 \leq u(t) \leq 500 \qquad \text{for } 0 \leq t \leq t_f \qquad (9)$$

Such inequality constraint is equivalent to this equality constraint where γ is a slack variable:

$$[u(t) - u_{min}]\,[u_{max} - u(t)] - \gamma^2(t) = 0 \qquad (10)$$

which clearly forces u(t) to stay in the admissible range.

The cost function to be minimized is

$$J = \int_0^{t_f} u^2\, dt \qquad (11)$$

where u is the fuel flowrate and t_f the final time. The choice of u^2 instead of u as the integrand is to avoid the linearity of u in the expression of J; such linearity would yield as optimal solution a bang-bang control, an uninteresting option for the problem at hand.

The initial conditions are given by the known gas, metal and refractory temperature values at the beginning of the one hour heating period, which is the object of the optimization problem at hand. The final condition of interest is the average metal temperature at the end of the one hour heating period. In practice however, metal temperature is measured by a thermocouple positioned at mid-depth of the metal which corresponds to the central node in the metal. So this central-node metal temperature is taken as controlled output variable. In this work the final condition is defined as a central-node metal temperature of 705°C.

To the problem thus formulated, we apply the Euler-Lagrange equation to obtain the conditions for an optimum. The Euler-Lagrange equation is written for each component of y(t), u(t) and γ(t), and the terminal transversality conditions are applied to account for the final condition of the problem. The optimal control problem then becomes a two-point boundary value problem made of 23 algebraic and differential equations, namely the 10 state equations, the 10 Euler-Lagrange equations, the optimality condition, and the two algebraic equations resulting from the inequality constraint on y(t).

The problem was discretized into 73 grid points and solved by the relaxation method on a VAX-11/785. After several attempts to reduce computing time, the number of iterations was decreased from 156 to 8, and the required CPU time shrunk from 74 to 2 minutes. Work is still continuing to further improve speed and accuracy [3].

Results

Figure 5 shows the optimal time-varying fuel flowrate $u_{opt}(t)$ obtained, as compared with the

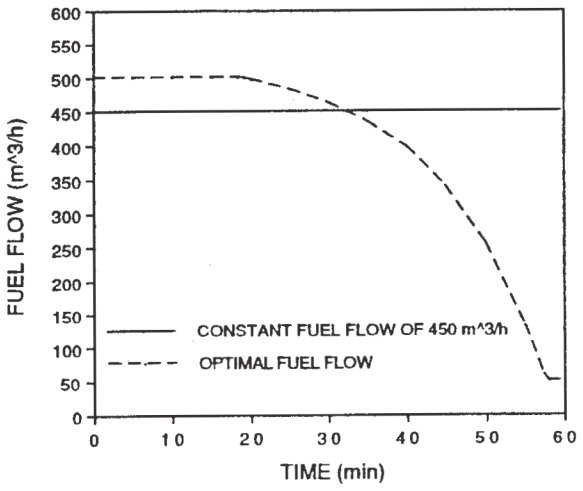

Figure 5 Optimal fuel flowrate calculated by the model. Straight line represents the conventional constant fuel flowrate of 450m^3/h.

conventional nonoptimal constant fuel flowrate of 450 m^3/h. Optimality calls for a maximum fuel flowrate of 500 m^3/m during the first 20 minutes or so, followed by a decreasing fuel until time t_f. The gain on fuel cost, calculated as the percent decrease of the area under the u(t) curve is 10.9%. Total fuel consumption is reduced from 450 m^3 to 401m^3 for the one-hour heating period.

The resulting fuel savings can be explained as follows. While the constant fuel causes all temperatures in the system to increase continually from time 0 to t_f, the optimal fuel flowrate causes the temperatures to increase for about half of the one hour heating period. Subsequently the fuel flow decreases, and the gas and refractory inner-surface temperatures also decrease. But even during this second half period, the gas and the refractories keep sending energy, mainly by radiation, to the metal thus continuing to raise its temperature. However the energy lost through the stack gas decreases considerably due to the lower gas temperature. To verify the above arguments, a comparison was made between the two energy balances, one obtained with constant fuel u = 450 m^3/h, the other with $u_{opt}(t)$.

Table I: Partial reproduction of the energy balances obtained with constant u and with $u_{opt}(t)$

ENERGY (MJ)	u=450	$u=u_{opt}(t)$	COMPARISON
Absorbed by metal	1918.7	1825.2	-4.8%
Absorbed by refractories	3572.3	3156.7	-11.6%
Lost through stack	10436.7	9229.8	-11.6%
Sum of the above	15927.7	14211.7	-10.8%

Table 1 shows that with $u_{opt}(t)$, the energy to be provided by the fuel decreases substantially, the biggest chunk in terms of megajoules is the decrease in the stack loss. The energy absorbed by the metal decreases somewhat, but the decrease in the energy absorbed by the refractories is more important. This illustrates the contribution of the refractories to raise the metal temperature in the second half of the heating period during which direct heating by the fuel is reduced.

Note that the difference between the 10.8% savings appearing in Table I and the 10.9% total savings comes from the fact that Table I is only a partial reproduction of the energy balances where other items (e.g. the refractories heat loss to atmosphere) do not appear.

Figures 6, 7 and 8 show respectively the gas temperature, the central-node metal temperature and the refractory inner-surface temperature. Each of these temperatures is calculated first with constant u = 450 then with $u_{opt}(t)$, to show the effect of optimization. The most eloquent effects are found on Figures 6 and 8 showing the gas and refractory temperatures decreasing drastically toward the end of the heating period.

Each of the figures from 6 to 8 also presents the temperatures obtained by running the analytic model with $u_{opt}(t)$ as input. The purpose is to show that the optimal solution obtained by the reduced model also gives the same results when applied to the analytic model.

The optimal solution suggested makes sense economically. Based on a cost of $0.17 per cubic meter of natural gas, and a daily average of 3 batches per furnace, a 10.9% fuel reduction yields a yearly savings of 31 000$ per furnace. Also worth mentioning are the impact on the environment (less stack discharge) and the increase in the life expectancy

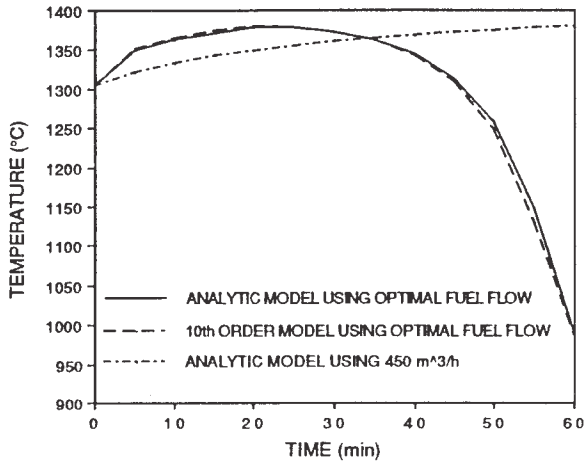

Figure 6 Gas temperature obtained with optimal fuel and constant fuel, showing the effect of optimization.

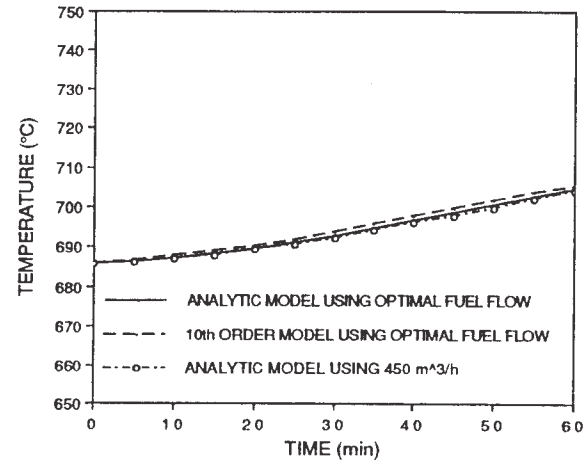

Figure 7 Central-node metal temperature obtained with optimal fuel and constant fuel.

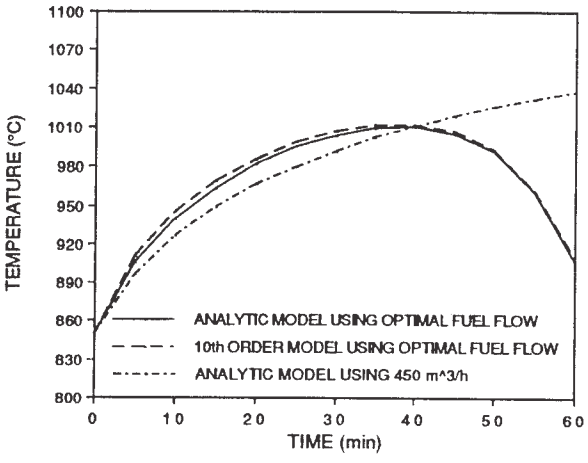

Figure 8 Refractories inner-surface temperature obtained with optimal fuel and constant fuel, showing the effect of optimization

of the refractories, submitted to less fuel combustion.

Conclusion

It has been shown that by applying variational calculus and Pontryagin's maximum principle to the reduced tenth-order nonlinear model of the aluminum casting furnace, one can solve the optimal fuel problem. The conclusion to be retained is that it is possible to solve the optimal control problems for this class of industrial processes, characterized by a complex, high-ordered analytic model reduced to a relatively low-ordered nonlinear model. Since the fuel cost for these processes is high, even an improvement of a few percentage points is worth the effort. In this paper, optimization was performed on the principal energy-consuming operation, namely the first one-hour heating period of the batch. With some additional work to model the other operations in the sequence (stirring, fluxing, alloying, skimming) it is possible to perform the optimization over the complete batch. This suggests a direction for further development based on this groundwork.

Acknowledgements

This study is an extension of a project on the mathematical modelling of the aluminum casting furnace, jointly undertaken by the Université du Québec à Chicoutimi (UQAC) and Alcan International Ltd with the collaboration of Alcan Smelters and Chemicals Ltd, both of Jonquière, Quebec, Canada. The analytic model was part of the results of this joint project. The optimal control reported here is the object of the project extension. We thank Alcan for authorizing this publication. We acknowledge the financial support of the NSERC of Canada.

Nomenclature

A_m	=	area of metal surface	[m^2]
A_r	=	area of refractory inner surface	[m^2]
a_i, c_i	=	coefficients of the approximating polynomials	
C_g	=	specific heat of gas	[J/kg•J]
h_m	=	convective heat transfer coefficient gas-to-metal	[$W/m^2 \cdot K$]
h_r	=	convective heat transfer coefficient gas-to-refractories	[$W/m^2 \cdot K$]
J	=	cost function	
M_g	=	mass of gas	[kg]
Q_f	=	heat generated by fuel combustion	[W]
Q_m	=	heat flowrate to metal	[W]
Q_r	=	heat flowrate to refractories	[W]
T_g	=	gas temperature	[K]
T_m	=	metal surface temperature	[K]

T_r = refractory inner surface temperature [K]
T_i = temperature of node i [K]
t = time [s]
t_f = final time [s]
u = fuel flowrate [m^3/h]
y = state variables [K]
$\overrightarrow{GS_i}$, $\overrightarrow{S_iS_j}$, $\overrightarrow{S_iG}$ = directed interchange areas [m^2]
σ = Stefan-Boltzmann constant [$W/m^2 \cdot K^4$]

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