Chapter 22: Electric Fields and Gauss's Law

Concept Checks

22.1. b 22.2. b 22.3. a 22.4. c 22.5. c 22.6. e 22.7. c 22.8. c 22.9. e 22.10. e 22.11. a 22.12. a 22.13. c 22.14. d

Multiple-Choice Questions

22.1. e 22.2. d 22.3. a 22.4. a 22.5. d 22.6. c 22.7. c 22.8. c 22.9. a 22.10. a & d 22.11. a 22.12. a, d and e

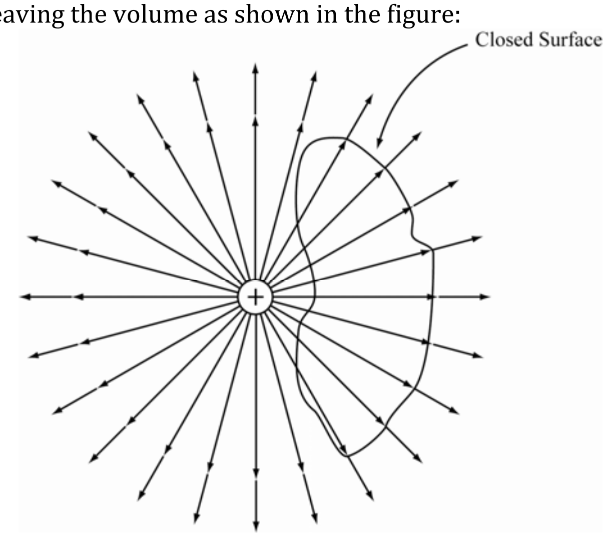
Conceptual Questions

22.13. The metal frame and sheet metal of the car form a Faraday cage, excluding the electric fields induced by the lightning. The current in the lightning strike flows around the outside of the car to ground. The passengers inside the car can be in contact with the inside of the car with no ill effects, but should not stick their hands out an open window.

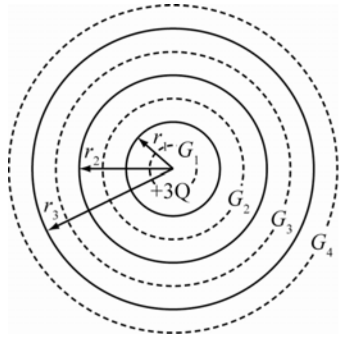
22.14. Since lightning can strike the tree and have the current flow through the wet tree, the current would jump to any object near the tree. To avoid lightning, go inside the house or a car. If I were outside, I would go to a low place and avoid trees or tall buildings. I should not lie down on the ground since the current can flow along the surface of the Earth.

22.15. If electric field lines crossed, there would be a charge at the crossing point. It is known that the electric field lines extend away from a positive change and the lines terminate at a negative charge. If in the vicinity of the crossing point there is no charge, then the lines cannot cross. Moreover, if we put a test charge on the crossing point, there would be two directions of the force; this is not possible; therefore the lines cannot cross.

22.16. The net flux through a closed surface is proportional to the net flux penetrating the surface, that is, the flux leaving the volume minus the flux entering the volume. This means that if there is a charge within a surface, the flux due to the charge will only exit through the surface creating a net flux no matter where the charge is located within the surface. If a charge moves just outside the surface, then the net flux crossing the surface would be zero since the flux entering the volume must be equal to the flux leaving the volume as shown in the figure:



22.17. Because of the spherical symmetry of this problem, Gauss's Law can be used to determine electric fields. The image below shows a cross-section of the nested spheres:



Gauss's Law is applied on four surfaces, G_1, G_2, G_3 and G_4 as shown in the figure.

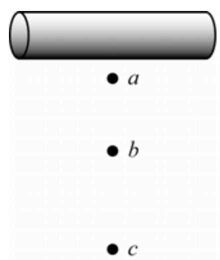
(a) In the region $r < r_1$, the electric field is zero because it is inside the conducting sphere.

(b) Applying Gauss's Law on the surface G_2 gives the electric field in the region $r_1 < r < r_2$, i.e., $E\left(4\pi r^2\right) = 3Q/\varepsilon_0$ or $E = 3Q/4\pi\varepsilon_0 r^2$.

(c) In the region $r_2 < r < r_3$, the electric field is zero since it is inside a conductor.

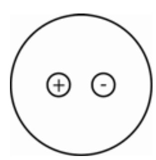
(d) In the region $r < r_3$, using Gauss's Law yields $E\left(4\pi r^2\right) = 3Q/\varepsilon_0$. Therefore, the electric field is $E = 3Q/4\pi\varepsilon_0 r^2$.

22.18.



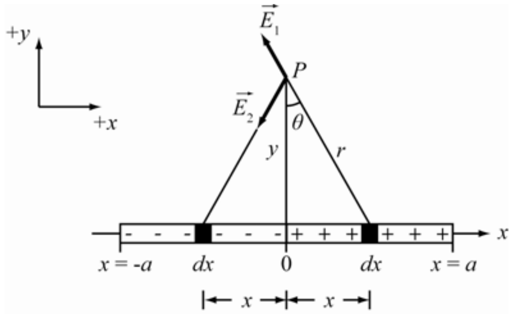
(a) If you are very close to the rod, the electric field can be approximated by the field produced by a very long rod. Then E is proportional to the linear charge density and to $1/r$.
(b) If you are a few centimeters away from the center, the rod's finite length becomes relevant and the rod can be treated as a line of charge with finite length, as in Example 22.3.
(c) If you are very far away, then the electric field behaves like that of a point charge. Therefore, the field is proportional to the total charge and to $1/r^2$.

22.19.



The total electric flux through a closed surface is equal to the net charge, q_{enc}, divided by the constant ε_0 or $\oint_{\text{net}} = q_{\text{enc}}/\varepsilon_0$. This is known as Gauss's Law. The strength of a dipole is $p = qd$. Because the dipole is completely enclosed by the spherical surface, the enclosed charge will be $q_{\text{enc}} = +q + (-q) = 0$. Thus the net flux through the closed surface will be zero.

22.20.



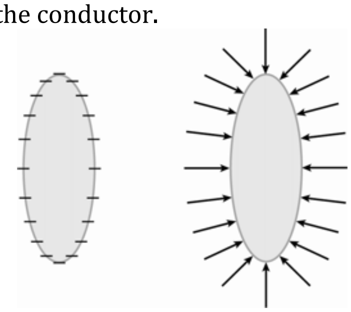
Consider two small elements dx at x and $-x$ as shown in the above figure. Due to the symmetry of the problem, it is found that the component of E_1 in the *y*-direction, E_{1y}, is equal in magnitude, but in the opposite direction, to the *y*-component of E_2. Therefore, only the *x*-components of electric fields contribute to the net field. Integrating over the length of wire yields $\vec{E}=\int_0^a \frac{2\sin\theta dq}{4\pi\varepsilon_0 r^2}(-\hat{x})$.

Using $dq=\lambda dx$, it simplifies to $\vec{E}=\left(\frac{-\hat{x}}{2\pi\varepsilon_0}\right)\int_0^a \frac{\sin\theta\lambda}{r^2}(dq)$. Substituting $r=\sqrt{x^2+y^2}$ and $\sin\theta = x/r$ yields $\vec{E}=\left(\frac{-\hat{x}\lambda}{2\pi\varepsilon_0}\right)\int_0^a \frac{xdx}{\left(x^2+y^2\right)^{3/2}}$. Using the substitution $z=x^2$ yields:

$$\vec{E}=\left(\frac{-\hat{x}\lambda}{2\pi\varepsilon_0}\right)\left(\frac{1}{2}\right)\int_0^{a^2}\frac{dz}{\left(z+y^2\right)}=\left(\frac{-\hat{x}\lambda}{2\pi\varepsilon_0}\right)\left[-2/\left(z+y^2\right)^{1/2}\right]_0^{a^2}=\left(\frac{-\hat{x}\lambda}{2\pi\varepsilon_0}\right)\left[-2/\left(z+y^2\right)^{1/2}\right]_0^{a^2}$$

$$=\left(-\hat{x}\right)\left(\frac{\lambda}{2\pi\varepsilon_0}\right)\left[\left(1/y\right)-1/\sqrt{a^2+y^2}\right].$$

22.21. Since the conductor has a negative charge, this means that the electric field lines are toward the conductor. Electrons inside the conductor can move freely and redistribute themselves such that the repulsion forces between electrons are minimized. As a consequence of this, the electrons are distributed on the surface of the conductor.



22.22. St. Elmo's Fire is a form of corona discharge; the same phenomenon whereby lightning rods bleed off accumulated ground charge to prevent lightening strokes. Lightning rods are not supposed to conduct a lightning strike to ground except as a last resort. In stormy weather, a ship or aircraft can become electrically charged by air friction. The charge will collect at the sharp edges or points on the structure of the ship or plane because the electric field is concentrated in areas of high curvature. Sufficiently large fields ionize the air at these areas, as the molecules of nitrogen and oxygen de-ionize they give off energy in the form of visible light. The ghostly glow known since the days of "wooden ships and iron men" is St. Elmo's Fire.

22.23. Consider the surface layer of charge to be divided into two component; a 'tile' in the vicinity of some point, and the 'rest' of the charge on the surface. Seen from close enough to the given point on the surface, the 'tile' appears as a flat plane of charge. Gauss's Law applied to the cylindrical surface pierced symmetrically by such a plane, implies that the 'tile' produces an electric field with the component $\sigma/2\varepsilon_0$ perpendicularly outward from the surface on the outside, inward on the inside. But Gauss's Law applied to a short cylinder ('pillbox') partially embedded in the conductor, implies that the entire charge layer produces an electric field with component σ/ε perpendicularly outward outside the surface, and zero inside. To yield this result, the 'rest' must produce electric field $\sigma/2\varepsilon_0$, outward, in the vicinity of the 'tile' inside and out. It is this electric field which exerts force on the 'tile', carries charge per unit area σ. Hence, every portion of the charge layer experiences outward force per unit area stress of magnitude $\Sigma = \sigma^2/2\varepsilon_0$. Note that the outward direction of the stress is independent of sign of σ.

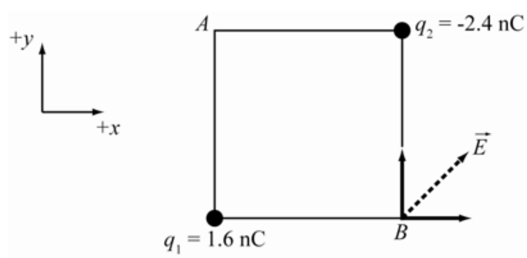
22.24. The net force on the dipole is zero, so there will be no translational motion of dipole. The net torque; however, is not zero, so the dipole will rotate. With the force on the positive charge to the right and the force on the negative charge to the left, the dipole will rotate counter-clockwise.

Exercises

22.25. The electric field produced by the charge is:

$$E = \frac{kq}{r^2} = \frac{\left(8.99\cdot 10^9 \text{ N m}^2/\text{C}^2\right)\left(4.00\cdot 10^{-9} \text{ C}\right)}{\left(0.250 \text{ m}\right)^2} = 575.36 \text{ N/C} \approx 575 \text{ N/C}.$$

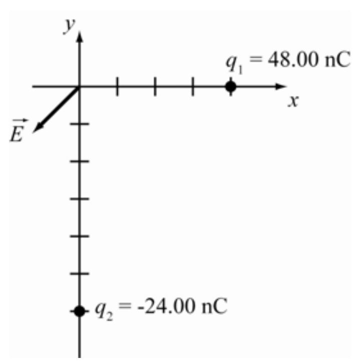
22.26.



The electric field vector will be $\vec{E} = \sum_i \vec{E}_i = \left(kq_1/r^2\right)\hat{x} + \left(kq_2/r^2\right)\hat{y} = k/r^2\left(q_1\hat{x} + q_2\hat{y}\right)$. The magnitude of the vector is:

$$\left|\vec{E}\right| = \sqrt{E_x^2 + E_y^2} = \frac{k}{r^2}\sqrt{q_x^2 + q_y^2} = \frac{8.99\cdot 10^9 \text{ N m}^2/\text{C}^2}{\left(1.0 \text{ m}\right)^2}\sqrt{\left(1.6\cdot 10^{-9} \text{ C}\right)^2 + \left(-2.410^{-9} \text{ C}\right)^2} = 25.931 \text{ N/C} = 26 \text{ N/C}.$$

22.27.



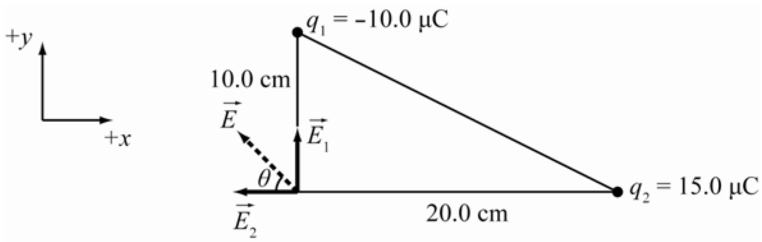
The electric field at the origin is $\vec{E} = \sum_i E_i = \left(k\left|q_1\right|/r_1^2\right)\left(\hat{x}\right) + \left(k\left|q_2\right|/r_2^2\right)\left(-\hat{y}\right)$. The direction is $\tan\theta = E_y/E_x$.

$$\theta_0 = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left[\frac{k|q_2|/r_2^2}{k|q_1|/r_1^2}\right] = \tan^{-1}\left[\frac{r_1^2|q_2|}{r_2^2|q_1|}\right] = \tan^{-1}\left[\frac{(4.000\text{ m})^2(24.00\text{ nC})}{(6.000\text{ m})^2(48.00\text{ nC})}\right] = 12.53°.$$

The electric field lies in the 3rd quadrant so $\theta = 180.00° + \theta_0 + 12.53° = 192.53°$. Rounding to four significant figures gives us $\theta = 192.5°$.

22.28. **THINK:** The electric field is the sum of the fields generated by the two charges of the corner triangle. The first charge is $q_1 = -1.0 \cdot 10^{-5}$ C and is located at $\vec{r}_1 = (0.10\text{ m})\hat{y}$. The second charge is $q_2 = 1.5 \cdot 10^{-5}$ C located at $\vec{r}_2 = (0.20\text{ m})\hat{x}$.

SKETCH:



RESEARCH: The electric field is given by the equation $\vec{E} = (kq/r^2)\hat{r}$.

SIMPLIFY: $\vec{E} = (kq_1/r_1^2)\hat{y} + (kq_2/r_2^2)\hat{x}$. The magnitude of the field is

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2} = k\sqrt{\left(\frac{q_1}{r_1^2}\right)^2 + \left(\frac{q_2}{r_2^2}\right)^2},$$

and has a direction $\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\left(\frac{q_1}{r_1^2}\right) \div \left(\frac{r_2^2}{q_2}\right)\right) = \tan^{-1}\left(\frac{r_2^2 q_1}{r_1^2 q_2}\right)$ where θ is in the second quadrant.

CALCULATE: $E = (8.99 \cdot 10^9\text{ N m}^2/\text{C}^2)\sqrt{\left(\frac{-1.0 \cdot 10^{-5}\text{ C}}{(0.100\text{ m})^2}\right)^2 + \left(\frac{1.5 \cdot 10^{-5}\text{ C}}{(0.200\text{ m})^2}\right)^2} = 9.6013 \cdot 10^6\text{ N/C}$

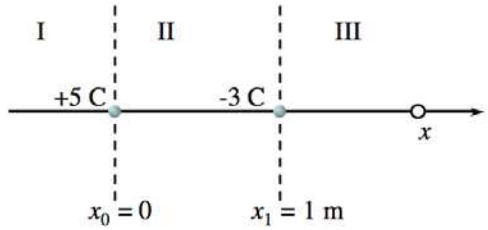
$$\theta = \tan^{-1}\frac{(0.200\text{ m})^2(1.0 \cdot 10^{-5}\text{ C})}{(0.100\text{ m})^2(1.5 \cdot 10^{-5}\text{ C})} = 69.444° \text{ or } \theta = 110.56°.$$

ROUND: The least precise value given in the question has two significant figures, so the answer should also be reported to two significant figures. The electric field produced at the corner is $E = 9.6 \cdot 10^6$ N/C at 110° from the *x*-axis.

DOUBLE-CHECK: Dimensional analysis confirms the answer is in the correct units.

22.29. **THINK:** We want to find out where the combined electric field from two point charges can be zero. Since the electric field falls off as the inverse second power of the distance to the charge, and since both charges are on the *x*-axis, only points on the same line have any chance of canceling the electric field from these two charges, resulting in a net zero electric field. The first charge, $q_1 = 5.0$ C, is at the origin. The second charge, $q_2 = -3.0$ C, is at $x = 1.0$ m. Consider where along the *x*-axis it is possible to have zero electric field. On the sketch we have marked three regions (I, II, and III). If we place a positive charge anywhere in region II, the 5 C will repel it and the −3 C will attract it, so that the positive charge moves to the right. If we place a negative charge in the same region, it will move to the left. So we know that the electric field cannot be zero anywhere in region II. Region I is closer to the 5 C charge. Since this is also the charge with the larger magnitude, its electric field will dominate region I, and thus there is no place in region I where the electric field is 0. This leaves region III, where the two electric fields from the point charges can cancel.

SKETCH:



RESEARCH: The electric field due to the charge at the origin is $E_0 = kq_0/x^2$. The other charge produces a field of $E_1 = kq_1/(x-x_1)^2$.

SIMPLIFY: The combined electric field is $E = kq_0/x^2 + kq_1/(x-x_1)^2$. Setting the electric field to zero, solve for x:

$$\frac{kq_0}{x^2}+\frac{kq_1}{(x-x_1)^2}=0 \;\Rightarrow\; \frac{kq_0}{x^2}=-\frac{kq_1}{(x-x_1)^2} \;\Rightarrow\; (x-x_1)^2 q_0 = -x^2 q_1 \;\Rightarrow\; (x-x_1)^2|q_0| = x^2|q_1|$$

We could now solve the resulting quadratic equation blindly and would obtain two solutions, each of which we would have to evaluate for validity. Instead, we can make use of the thinking we have done above. In the last step we used the fact that the charge at the origin is positive and the other is negative, replacing them with their absolute values. Now we can take the square root on both sides and choose the positive root, leaving us with

$$(x-x_1)\sqrt{|q_0|} = x\sqrt{|q_1|} \Rightarrow x = \frac{x_1\sqrt{|q_0|}}{\sqrt{|q_0|}-\sqrt{|q_1|}}$$

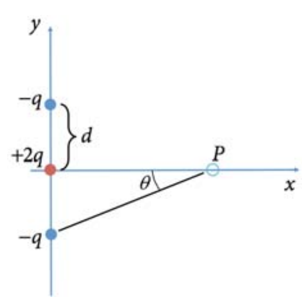
CALCULATE: $x = \dfrac{(1.00\text{ m})\sqrt{5.00\text{ C}}}{\sqrt{5.00\text{ C}}-\sqrt{3.00\text{ C}}} = 4.43649\text{ m}$

ROUND: The positions are reported to three significant figures. The electric field is zero at $x = 4.44$ m.

DOUBLE-CHECK: This is a case where we can simply insert our result and verify that it does what it is supposed to: $E(x\text{=}4.4\text{ m}) = k(5\text{ C})/(4.4\text{ m})^2 + k(-3\text{ C})/(4.4\text{ m}-1\text{ m})^2 = 0$.

22.30. **THINK**: Let's fix the coordinate notation first. The charges are located at points (0,*d*), (0,0), and (0,-*d*) on the *y*-axis, and the point *P* is *P* = (*x*,0). In order to specify the electric field at a point in space, we need to specify the magnitude and the direction. Lets first think about the direction. The distribution of the charges is symmetric with respect to the *x*-axis. Thus if we flip the charge distribution upside down, we see the same picture. This means also that we can do this for the electric field generated by these charges. Right away this means that the electric field anywhere on the *x*-axis cannot have a *y*-component and can only have an *x*-component.

SKETCH:



RESEARCH: The electric field strength is given by $E = kQ/r^2$, and the electric fields from different charges add as vectors. We need to add the *x*-components of the electric fields from all charges. They are (from top to bottom along the *y*-axis):

$$E_1 = \frac{-kq}{d^2+x^2}$$

$$E_{1,x} = E\cos\theta = \frac{-kq}{d^2+x^2}\frac{x}{\sqrt{d^2+x^2}} = \frac{-kqx}{\left(d^2+x^2\right)^{3/2}}$$

$$E_{2,x} = \frac{2kq}{x^2}$$

$$E_3 = E_1 = \frac{-kq}{d^2+x^2}$$

$$E_{3,x} = E_{1,x} = \frac{-kqx}{\left(d^2+x^2\right)^{3/2}}$$

SIMPLIFY: All we have to do is add the individual *x*-components to find our expression for the *x*-component of the electric field along the *x*-axis:

$$E_x(x,0) = E_{1,x} + E_{2,x} + E_{3,x} = \frac{2kq}{x^2} - \frac{2kqx}{\left(d^2+x^2\right)^{3/2}} = 2kq\left(\frac{1}{x^2} - \frac{x}{\left(d^2+x^2\right)^{3/2}}\right)$$

(This is the expression for $x>0$; for $x<0$ it has the opposite sign so that it always points away from the origin.)

CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE-CHECK: For $x \to 0$ we see that the first term diverges as we get very close to the positive charge at the origin, which is as expected.

For large distances, $x \to \infty$, $d/x \to 0$, we expect at most a very weak electric field because the net charge of our configuration is 0. We can factor out the $1/x^2$ term to get

$$E_x(x,0) = \frac{2kq}{x^2}\left(1 - \frac{x^3}{\left(d^2+x^2\right)^{3/2}}\right) = \frac{2kq}{x^2}\left(1 - \frac{1}{\left(\left(\frac{d}{x}\right)^2+1\right)^{3/2}}\right) = \frac{2kq}{x^2}\left(1 - \left(\left(\frac{d}{x}\right)^2+1\right)^{-3/2}\right).$$ For

$\left(d^2/x^2\right) \ll 1$, the binomial expansion gives us

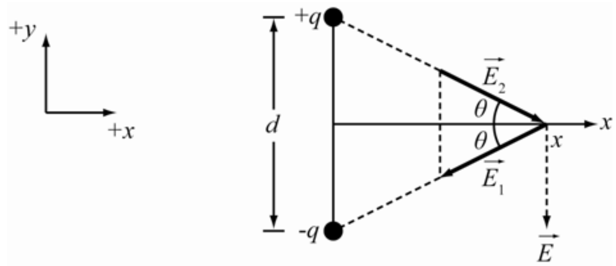
$$\left(\frac{d^2}{x^2}+1\right)^{-3/2} \approx 1 - \frac{3}{2}\frac{d^2}{x^2}.$$

The electric field then simplifies to

$$E_x(x \gg d,0) = \frac{2kq}{x^2}\left(1-\left(1-\frac{3}{2}\frac{d^2}{x^2}\right)\right) = \frac{2kq}{x^2}\frac{3}{2}\frac{d^2}{x^2} = \frac{3kqd^2}{x^4}.$$

Thus the electric field strength of this configuration, called a "quadrupole", falls with the inverse fourth power of the distance to the origin for large distances. (… as compared to the electric field from a dipole, which falls with the inverse third power).

22.31. The dipole is just two charges fixed together of opposite sign. The electric field at a point is the sum of the fields produced by each charge. The figure indicates that the electric field produced is created by the component of the field perpendicular to line *x*.



$$E = E_{1y} + E_{2y} = E_1 \sin\theta + E_2 \sin\theta = \frac{-kq}{\left(\left(\frac{d}{2}\right)^2 + x^2\right)}\sin\theta + \frac{-kq}{\left(\left(\frac{d}{2}\right)^2 + x^2\right)}\sin\theta = \frac{-2kq\sin\theta}{\left(\frac{d}{2}\right)^2 + x^2}$$

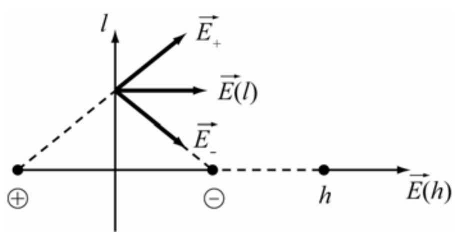
Note that $\sin\theta = \dfrac{d}{2\sqrt{(d/2)^2 + x^2}}$. This means the field is:

$$E = \frac{-2kqd}{2\left((d/2)^2 + x^2\right)^{3/2}} = \frac{-kqd}{\left((d/2)^2 + x^2\right)^{3/2}} = \frac{-kp}{\left((d/2)^2 + x^2\right)^{3/2}}.$$

If $x \gg d$ then $E = -kp/x^3$. The field along the axis of the dipole is $E = -2kp/x^3$, indicating that the field strength falls off more rapidly perpendicular to the dipole axis.

22.32. **THINK:** The field due to a dipole moment at a point *h* along the *x*-axis is $E(h) = k2qd/h^3$. I want to find the point perpendicular to the *x*-axis as measured from the origin (i.e., along the *y*-axis), where the electric field has this same value.

SKETCH:



RESEARCH: From the previous problem, the electric field along the *y*-axis is $E(l) = \dfrac{kqd}{\left(d^2/4 + l^2\right)^{3/2}}$.

Set $E(l) = E(h)$ and solve for l.

SIMPLIFY: $\dfrac{k(2qd)}{h^3} = \dfrac{kqd}{\left(\frac{d^2}{4} + l^2\right)^{3/2}} \Rightarrow \dfrac{2}{h^3} = \dfrac{1}{\left(\frac{d^2}{4} + l^2\right)^{3/2}} \Rightarrow 2\left(\frac{d^2}{4} + l^2\right)^{3/2} = h^3 \Rightarrow l = \sqrt{\left(\dfrac{h}{\sqrt[3]{2}}\right)^2 - \left(\dfrac{d}{2}\right)^2}$.

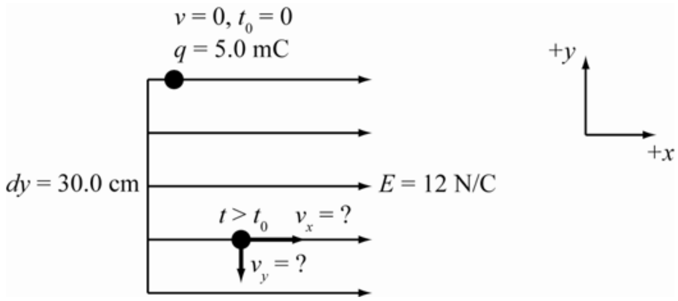
CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE-CHECK: According to this expression, *l* will always be less than *h*. This is consistent with the previous result that the electric field strength along a line perpendicular to the dipole axis falls off more rapidly than the field strength along the dipole axis.

22.33. **THINK:** As the $m = 4.0$ g ball falls the force of gravity acting on it will cause it to accelerate downwards. At the same time, the force due to the electric field acts on the ball causing it to accelerate towards the east. The forces act perpendicular to each other. The problem is solved by finding each component of the velocity. In order to find the velocity due to the electric field, the time required for the ball to travel 30.0 cm downwards is needed.

SKETCH:



RESEARCH: The velocity in the downward direction is found using $v_y^2 = v_{y0}^2 + 2gdy$. The time it takes to reach this velocity $t = v_y / g$. The acceleration eastward is calculated using $F = ma = qE$. The velocity is then $v_x = a_x t$.

SIMPLIFY: The *y*-component of the velocity is $v_y = \sqrt{2gdy}$ because the ball starts from rest. The time it takes for the ball to fall 30.0 cm is $t = \sqrt{2gdy} / g$. The acceleration eastward is $a = qE/m$. The velocity eastward is $v_x = a_x t \to v_x = \left(\frac{qE}{m}\right)\frac{\sqrt{2gdy}}{g} = \frac{qE}{m}\sqrt{\frac{2dy}{g}}$.

CALCULATE: $v_y = \sqrt{2\left(9.81 \text{ m/s}^2\right)\left(0.300 \text{ m}\right)} = 2.4261 \text{ m/s}$ downward

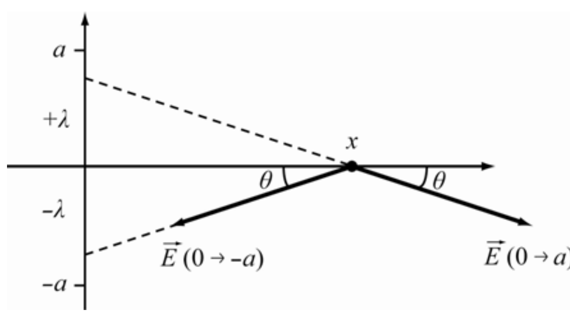
$$v_x = \left(5.0 \cdot 10^{-3} \text{ C}\frac{12 \text{ N/C}}{0.0040 \text{ kg}}\right)\sqrt{2\left(\frac{0.300 \text{ m}}{9.81 \text{ m/s}^2}\right)} = 3.7096 \text{ m/s} \text{ eastward}$$

ROUND: The velocity is report to three significant figures. The ball reaches a velocity of $(3.71 \text{ m/s})\hat{x} + (2.43 \text{ m/s})\hat{y}$.

DOUBLE-CHECK: This is a reasonable answer considering the size of the values given in the question.

22.34. **THINK:** A line of charge along the *y*-axis has linear charge density $+\lambda$ from $y = 0$ to $y = +a$, and $-\lambda$ from $y = 0$ to $y = -a$. I want to find an expression for the electric field at any point x along the *x*-axis. It is noted that the charge configuration is similar in structure to a dipole. By symmetry, the *x*-components of the field cancel out, and the net field is in the *y*-direction.

SKETCH:



RESEARCH: The electric field resulting from a charge distribution is the integral over the differential charge: $dE = kdq / r^2$. The *y*-component of the field is $dE_y = kdq\sin\theta / r^2$, where θ is the angle between the electric field produced by dq and the *y*-axis. Also, $r = \sqrt{x^2 + y^2}$, $\sin\theta = y / r$. From 0 to a, $dq = \lambda dy$, and from 0 to $-a$, $dq = -\lambda dy$.

SIMPLIFY: $dE_+ = dE_{y,+} = \frac{kdq}{r^2}\sin\theta = \left(\frac{k\lambda dy}{x^2 + y^2}\right)\left(\frac{y}{\sqrt{x^2 + y^2}}\right) = \frac{k\lambda y dy}{\left(x^2 + y^2\right)^{3/2}}$

$$dE_- = dE_{y,-} = \frac{kdq}{r^2}\sin(-\theta) = \left(\frac{-k\lambda dy}{x^2+y^2}\right)\left(-\frac{y}{\sqrt{x^2+y^2}}\right) = \frac{k\lambda ydy}{\left(x^2+y^2\right)^{3/2}}$$

The field due to the positive charge distribution is: $E_+ = \int_0^a \frac{k\lambda ydy}{\left(x^2+y^2\right)^{3/2}} = k\lambda\int_0^a \frac{ydy}{\left(x^2+y^2\right)^{3/2}}$. Similarly, the field due to the negative charge distribution is: $E_- = \int_0^{-a} \frac{k\lambda ydy}{\left(x^2+y^2\right)^{3/2}} = k\lambda\int_0^{-a} \frac{ydy}{\left(x^2+y^2\right)^{3/2}}$.

CALCULATE: Let $u = x^2+y^2$ then $du = 2ydy$ then:

$$E_+ = \frac{k\lambda}{2}\int_0^a \frac{du}{u^{3/2}} = \left(\frac{k\lambda}{2}\right)\left(\frac{-2}{u^{1/2}}\right)\Bigg|_0^a = -\frac{k\lambda}{u^{1/2}}\Bigg|_0^a = \left[\frac{-k\lambda}{\left(x^2+y^2\right)^{1/2}}\right]_0^a = -k\lambda\left[\frac{1}{x} - \frac{1}{\left(x^2+a^2\right)^{1/2}}\right],$$ and

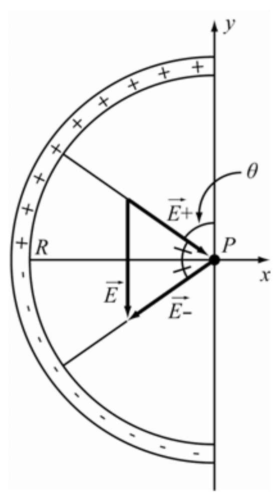
$$E_- = \frac{k\lambda}{2}\int_0^{-a} \frac{du}{u^{3/2}} = \left(\frac{k\lambda}{2}\right)\left(\frac{-2}{u^{1/2}}\right)\Bigg|_0^{-a} = -\frac{k\lambda}{u^{1/2}} = \left[-\frac{k\lambda}{\left(x^2+y^2\right)^{1/2}}\right]_0^{-a} = -k\lambda\left[\frac{1}{x} - \frac{1}{\left(x^2+(-a)^2\right)^{1/2}}\right].$$ The total electric field at x is: $E = E_+ + E_- = 2k\lambda\left[\frac{1}{\left(x^2+a^2\right)^{1/2}} - \frac{1}{x}\right]$.

ROUND: Not applicable.

DOUBLE CHECK: The electric field decreases inversely proportionally to the distance from the wire, as expected.

22.35. **THINK:** A semicircular rod carries a uniform charge of +Q along its upper half, and −Q along it's lower half. I want to determine the magnitude and direction of the electric field at the center of the semicircle. The rod has a length of $L = \pi R$. The charge density of the upper half of the rod is $\lambda = Q/L = Q/(1/2)\pi R = 2Q/\pi R$. Similarly, the lower half of the rod is $\lambda = -2Q/\pi R$.

SKETCH:



RESEARCH: From the symmetry of the semi-circle, the *x*-components of the field cancel, and the resulting electric field only has a *y*-component. The *y*-component of the electric field for the upper segment of the rod is given by

$$dE_{+y} = dE\cos\theta = \left(-kdq/R^2\right)\cos\theta = \left(-k\lambda dx/R^2\right)\cos\theta,$$

where $dx = Rd\theta$. Therefore, $dE_{+,y} = \left(-k\lambda Rd\theta/R^2\right)\cos\theta = -k\lambda\cos\theta d\theta/R$.

SIMPLIFY: Integrating both sides with respect to θ gives:

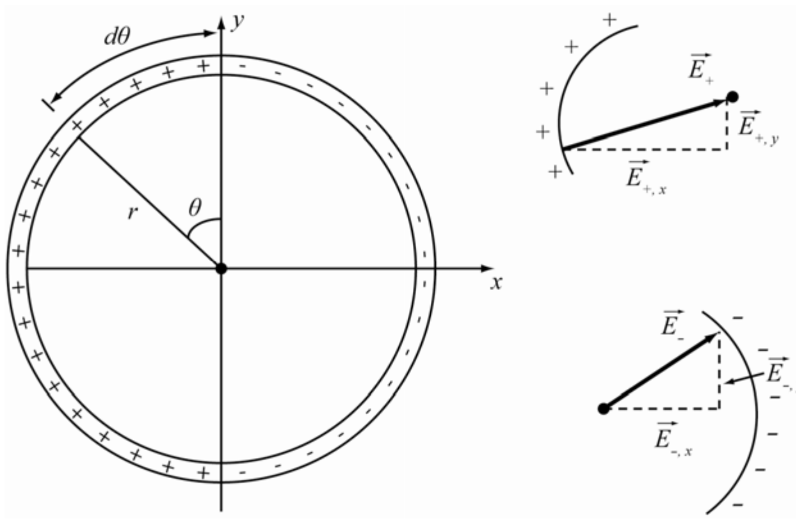
$$dE_{+y} = \left(-k\lambda / R\right)\int_0^{\pi/2} \cos\theta d\theta = \left(-k\lambda / R\right)\sin\theta\big|_0^{\pi/2} = \left(-k\lambda / R\right)\left(1-0\right) = \left(-k\lambda / R\right) = -k2Q / \pi R^2.$$

The lower half of the semicircle also contributes the same *y*-component. The total electric field at the origin is

$$\vec{E} = E_{+y}\hat{y} + E_{-y}\hat{y} = 2E_y\hat{y} = \left(\frac{-4kQ}{\pi R^2}\right)\hat{y} = \left(\frac{-4Q}{4\pi\varepsilon_0 \cdot \pi R^2}\right)\hat{y} = \left(\frac{-Q}{\pi^2\varepsilon_0 R^2}\right)\hat{y}.$$

CALCULATE: Not applicable.
ROUND: Not applicable.
DOUBLE CHECK: The resulting field points in the direction from the positive charge to the negative charge, as required.

22.36. **THINK:** Two semicircular rods, with uniformly distributed charges of $+1.00\ \mu\text{C}$ and $-1.00\ \mu\text{C}$, respectively, form a circle of radius $r = 10.0$ cm. I want to determine the magnitude and direction on the electric field at the center of the circle.
SKETCH:



RESEARCH: The charge densities of the positively charged and negatively charged rods are $+\lambda = Q / \pi R$ and $-\lambda = -Q / \pi R$, respectively. The differential element of the electric field is given by $dE = kdq / R^2$, where the differential element of charge along the line is $dq = \lambda dx = \lambda R d\theta$. It is also necessary to consider the *x*- and *y*-components of the differential elements.

SIMPLIFY: $dE_{+,x} = \dfrac{k\lambda dx \sin\theta}{R^2} = \dfrac{k\lambda d\theta \sin\theta}{R^2} = \dfrac{kQRd\theta\sin\theta}{\pi R^3} = \dfrac{kQ\sin\theta d\theta}{\pi R^2}$. Similarly, $dE_{+,y} = \dfrac{kQ\cos\theta d\theta}{\pi R^2}$; $dE_{-,x} = \dfrac{-kQ\sin\theta d\theta}{\pi R^2}$; $dE_{-,y} = \dfrac{-kQ\cos\theta d\theta}{\pi R^2}$. Integrating both sides of each expression gives:

$$E_{+,x} = \frac{kQ}{\pi R^2}\int_0^{\pi} \sin\theta d\theta = \frac{kQ}{\pi R^2}\left(-\cos\theta\right)\Big|_0^{\pi} = \frac{2kQ}{\pi R^2}$$

$$E_{+,y} = \frac{kQ}{\pi R^2}\int_0^{\pi} \cos\theta d\theta = \frac{kQ}{\pi R^2}\left(\sin\theta\right)\Big|_0^{\pi} = 0$$

$$E_{-,x} = -\frac{kQ}{\pi R^2}\int_{\pi}^{2\pi} \sin\theta d\theta = -\frac{kQ}{\pi R^2}\left(-\cos\theta\right)\Big|_{\pi}^{2\pi} = \frac{2kQ}{\pi R^2}$$

$$E_{-,y} = -\frac{kQ}{\pi R^2}\int_{\pi}^{2\pi} \cos\theta d\theta = -\frac{kQ}{\pi R^2}\left(\sin\theta\right)\Big|_{\pi}^{2\pi} = 0$$

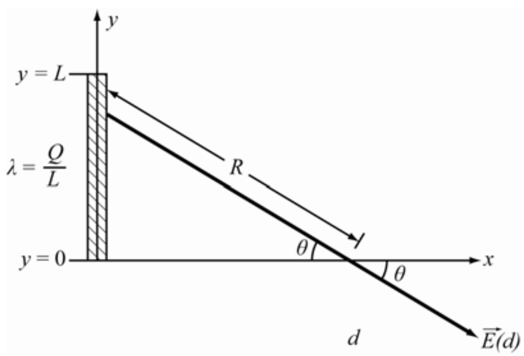
The total electric field at the center is given by: $E = E_{+,x} + E_{+,y} + E_{-,x} + E_{-,y} = \dfrac{2kQ}{\pi R^2} + 0 + \dfrac{2kQ}{\pi R^2} + 0 = \dfrac{4kQ}{\pi R^2}.$

CALCULATE: $E = \dfrac{4\left(8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2\right)\left(1.00 \cdot 10^{-6} \text{ C}\right)}{\pi\left(0.100 \text{ m}\right)^2} = 1.1446 \cdot 10^6 \text{ N/C}$

ROUND: The electric field is reported to three significant figures: $E = 1.14 \cdot 10^6$ N/C. Because all of the *y*- components are zero, the resultant field is in the positive *x*-direction.
DOUBLE-CHECK: Given the symmetry of the charge configuration, this is a reasonable result.

22.37. **THINK:** The charge Q is uniformly distributed along the rod of length L. The rod has linear charge density $\lambda = Q/L$. The electric field at a position $x = d$ can be calculated by integrating over the differential electric field due to the differential charge on the rod. The electric field differential $dE = kdq/r^2$, where the differential is along the *y*-axis, and $R = \sqrt{d^2 + y^2}$. The *x*- and *y*-components of the field must be considered individually. The *x*-component of the field differential is given by $dE_x = dE\cos\theta$, and the *y*-component is given by $dE_y = dE\sin\theta$.

SKETCH:



SIMPLIFY: $dE_x = \dfrac{kdQ}{R^2}\cos\theta = \dfrac{k\lambda dy}{R^2}\cos\theta = \dfrac{kQdy}{LR^2}\cos\theta = \dfrac{kQdy}{L\left(d^2 + y^2\right)^2}\cos\theta$

$$\cos\theta = \frac{d}{R} = \frac{d}{\sqrt{d^2 + y^2}} \Rightarrow dE_x = \left(\frac{kQdy}{L\left(d^2 + y^2\right)^2}\right)\left(\frac{d}{\sqrt{d^2 + y^2}}\right) = \frac{kdQdy}{L\left(d^2 + y^2\right)^{3/2}}$$

$$dE_y = \frac{kQdy}{L\left(d^2 + y^2\right)^2}\sin\theta;\ \sin\theta = \frac{y}{R} = \frac{y}{\sqrt{d^2 + y^2}} \Rightarrow dE_y = \frac{ykQdy}{L\left(d^2 + y^2\right)^{3/2}}$$

Integrate both expressions.

$$E_x = \int_0^L \frac{dkQdy}{L\left(d^2 + y^2\right)^{3/2}} = \frac{dkQ}{L}\int_0^L \frac{1}{\left(d^2 + y^2\right)^{3/2}}dy = \frac{kQd}{L}\left[\frac{y}{d^2\sqrt{d^2 + y^2}}\right]_0^L$$

$$E_y = \int_0^L \frac{ykQdy}{L\left(d^2 + y^2\right)^{3/2}} = \frac{kQ}{L}\int_0^L \frac{y}{\left(d^2 + y^2\right)^{3/2}}dy = \frac{kQ}{L}\left[\frac{-1}{\sqrt{d^2 + y^2}}\right]_0^L$$

$$\vec{E}(d) = E_x\hat{x} - E_y\hat{y}$$

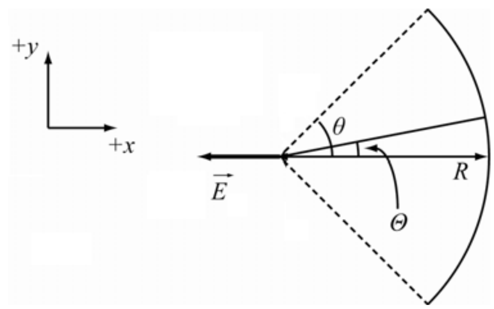
CALCULATE: $E_x = \dfrac{kQd}{L}\left[\dfrac{y}{d^2\sqrt{d^2 + y^2}}\right]_0^L = \dfrac{kQd}{L}\left(\dfrac{L}{d^2\sqrt{d^2 + L^2}} - 0\right) = \dfrac{kQ}{d\sqrt{d^2 + L^2}}$

$$E_y = \frac{kQ}{L}\left[\frac{-1}{\sqrt{d^2+y^2}}\right]_0^L = \frac{kQ}{L}\left(\frac{-1}{\sqrt{d^2+L^2}} - \frac{-1}{d}\right) = \frac{kQ}{dL} - \frac{kQ}{L\sqrt{d^2+L^2}}$$

$$\vec{E}(d) = \left(\frac{kQ}{d\sqrt{d^2+L^2}}\right)\hat{x} - \left(\frac{kQ}{dL} - \frac{kQ}{L\sqrt{d^2+L^2}}\right)\hat{y}$$

ROUND: Not applicable.
DOUBLE CHECK: The magnitude of the electric field decreases as d increases, as expected.

22.38. **THINK:** A wire bent into an arc of radius R and carrying a uniformly distributed charge Q will have a linear charge density of $\lambda = Q/2\theta R$. By the symmetry of the charge distribution, the y-components cancel, and only the x-component of the charge contributes to the electric field.
SKETCH:

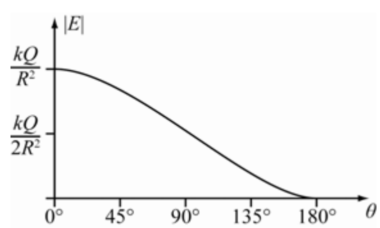


RESEARCH: An electric field produced by an infinitesimal segment of the arc is $dE = k\,dq/R^2 = k\lambda dx/R^2 = k\lambda R d\Theta/R^2 = k\lambda d\Theta/R$. The total electric field can be calculated by integrating over the differential elements of the field. Since the y-component of the field is zero, $E = E_x = \int_{-\theta}^{\theta} \frac{k\lambda d\Theta}{R}\cos\Theta$.
SIMPLIFY:

$$E = \int_{-\theta}^{\theta}\frac{k\lambda}{R}\cos\Theta d\Theta = \frac{k\lambda}{R}\int_{-\theta}^{\theta}\cos\Theta d\Theta = \frac{kQ}{2\theta R^2}\int_{-\theta}^{\theta}\cos\Theta d\Theta = \left[\frac{kQ}{2\theta R^2}\sin\Theta\right]_{-\theta}^{\theta} = \frac{kQ}{2\theta R^2}\left(\sin\theta - \sin\left(-\theta\right)\right)$$

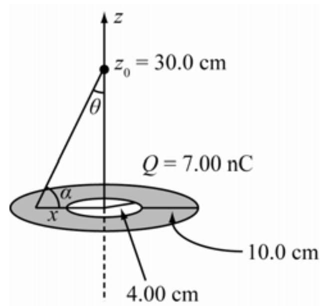
$$= \frac{kQ}{2\theta R^2}\left(\sin\theta + \sin\left(\theta\right)\right) = \frac{kQ\sin\theta}{\theta R^2}$$

CALCULATE:



ROUND: Not applicable.
DOUBLE CHECK: As $\theta \to 0$ the field is the same as that of a point charge, because $\lim_{\theta\to 0}\frac{kQ\sin\theta}{\theta R^2} = \frac{kQ}{R^2}\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = \frac{kQ}{R^2}$. The field becomes zero as the point is symmetrically enclosed by a ring of charge.

22.39. **THINK:** The washer will create an electric field that should be not to different from the electric field of the thin ring of charge we encountered in Solved Problem 22.1. The washer has a total charge $Q = 7.00$ nC, with inner and outer radius of the washer are $r_i = 2.00$ cm and $r_o = 5.0$ cm. The electric field at $z_o = 30.0$ cm away from the center of the washers is desired.
SKETCH:



RESEARCH: The surface density is $\sigma = Q/A$ where the area is $A = \pi\left(r_\text{o}^2 - r_\text{i}^2\right)$. The field will point in along the *z*-axis due to symmetry. The field due to a segment is $dE = kdq/R^2$. The distance from the segment of charges is $R = \sqrt{x^2 + z_\text{o}^2}$ and $\cos\theta = z_\text{o}/\sqrt{x^2+z_\text{o}^2}$.

SIMPLIFY: $E = \int \frac{kdq}{R^2}\cos\theta = \int_0^{2\pi}\int_{r_\text{i}}^{r_\text{o}} \frac{k\sigma dA z_\text{o}}{R^3} = \int_0^{2\pi}\int_{r_\text{i}}^{r_\text{o}} \frac{kQz_\text{o} x\,dx\,d\theta}{R^3\pi\left(r_\text{o}^2 - r_\text{i}^2\right)} = \frac{2\pi kQz_\text{o}}{\pi\left(r_\text{o}^2 - r_\text{i}^2\right)}\int_{r_\text{i}}^{r_\text{o}} \frac{x\,dx}{\left(x^2+z_\text{o}^2\right)^{3/2}}$. Evaluating the single integral gives:

$$E = \frac{2kQz_\text{o}}{\left(r_\text{o}^2 - r_\text{i}^2\right)}\left[\frac{-1}{\sqrt{x^2+z_\text{o}^2}}\right]_{r_\text{i}}^{r_\text{o}} = \frac{-2kQz_\text{o}}{\left(r_\text{o}^2 - r_\text{i}^2\right)}\left[\frac{1}{\left(r_\text{o}^2+z_\text{o}^2\right)^{1/2}} - \frac{1}{\left(r_\text{i}^2+z_\text{o}^2\right)^{1/2}}\right] = \frac{2kQz_\text{o}}{\left(r_\text{o}^2 - r_\text{i}^2\right)}\left[\frac{1}{\sqrt{r_\text{i}^2+z_\text{o}^2}} - \frac{1}{\sqrt{r_\text{o}^2+z_\text{o}^2}}\right].$$

CALCULATE:

$$E = \frac{2\left(8.99\cdot 10^9 \text{ N m}^2/\text{C}^2\right)\left(7.00\cdot 10^{-9}\text{ C}\right)\left(0.300\text{ m}\right)}{\left(0.0500\text{ m}\right)^2 - \left(0.0200\text{ m}\right)^2}\left(\frac{1}{\sqrt{\left(0.0200\right)^2+\left(0.300\right)^2}} - \frac{1}{\sqrt{\left(0.0500\right)^2+\left(0.300\right)^2}}\right)\frac{1}{\text{m}}$$

$$= 682.715 \text{ N/C}$$

ROUND: The values are given to three significant figures. The electric field is $E = 6.83\cdot 10^2$ N/C pointing towards the positive *z*-axis.

DOUBLE-CHECK: In Solved Problem 22.1 we found for the thin ring: $E = kQz_\text{o}/\left(r^2+z_0^2\right)^{3/2}$. Using the average of our outer and inner radius we then find from this formula:

$$E = \frac{\left(8.99\cdot 10^9\text{ N m}^2/\text{C}^2\right)\left(7.00\cdot 10^{-9}\text{ C}\right)\left(0.300\text{ m}\right)}{(\left(0.0350\text{ m}\right)^2 + \left(0.300\text{ m}\right)^2)^{3/2}} = 685.2\text{ N/C}$$

Since this is fairly close to our result for a ring with finite thickness, we have added confidence in our result.

22.40. The force on the particle is $F = qE$. The charge is $q = -2e$ so the force is

$$F = qE = -2eE = -2\left(1.60\cdot 10^{-19}\text{ C}\right)\left(10.0\cdot 10^3\text{ N/C}\right) = -3.20\cdot 10^{-15}\text{ N}.$$

22.41. The torque due to the field is

$$\left|\vec{\tau}\right| = \vec{p}\times\vec{E} = pE\sin\theta = qdE\sin\theta = \left(5.00\cdot 10^{-15}\text{ C}\right)\left(0.400\cdot 10^{-3}\text{ m}\right)\left(2.00\cdot 10^3\text{ N/C}\right)\left(\sin 60.0^\circ\right)$$
$$= 3.46\cdot 10^{-15}\text{ N m}.$$

22.42. The maximum torque occurs when the dipole is perpendicular to the field. The electric field is $\left|\tau\right| = \left|\vec{p}\times\vec{E}\right| = pE\sin\theta = \left(1.05\text{ D}\right)\left(3.34\cdot 10^{-30}\text{ C m/D}\right)\left(160.0\text{ N/C}\right)\left(\sin 90^\circ\right) = 5.61\cdot 10^{-28}\text{ N m}.$

22.43. The force acting on the electron is $F = ma = qE$. The acceleration is then $a = qE/m$. Assuming the electron is moving in the same direction as the electric field, the acceleration will oppose the motion. The velocity is given by $v^2 = v_0^2 + 2ax = v_0^2 + 2\left(\frac{qE}{m}\right)x = v_0^2 - \frac{2eEx}{m}$. Solving this equation for *x*: $\left(\frac{2eE}{m}\right)x = v_0^2 - v^2$ and $v = 0$, therefore $x = \frac{mv_0^2}{2eE}$. The distance traveled is

$$x = \frac{\left(9.109\cdot 10^{-31}\text{ kg}\right)\left(27.5\cdot 10^{6}\text{ m/s}\right)^2}{2\left(1.602\cdot 10^{-19}\text{ C}\right)\left(11{,}400\text{ N/C}\right)} = 0.1885\text{ m}.$$

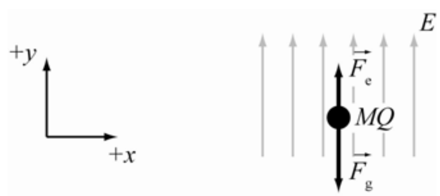
To three significant figures, the electron travels 0.189 m before it stops.

22.44. The dipole moment is $p = qd = ed = (1.602\cdot 10^{-19}\text{ C})(0.680\cdot 10^{-9}\text{ m}) = 1.089\cdot 10^{-28}\text{ C m} \approx 1.09\cdot 10^{-28}\text{ C m}$. The torque experienced by the dipole is

$$|\tau| = \left|\vec{p}\times\vec{\text{E}}\right| = p\text{E}\sin\theta = ed\text{E}\sin\theta = \left(1.089\cdot 10^{-28}\text{ C m}\right)\left(4.40\cdot 10^{3}\text{ N/C}\right)\left(\sin 45^\circ\right) = 3.39\cdot 10^{-25}\text{ N m}.$$

22.45. **THINK:** The net force on falling object in an electric field is the sum of the force due to gravity and the force due to the electric field. If the falling object carries a positive charge, then the force on the object due to the electric field acts in the direction opposite to the force of gravity.

SKETCH:



RESEARCH: The net upward force acting on the object is $F = F_e - F_g = QE - Mg = Ma$. This corresponds to a downward acceleration of $a = g - \frac{QE}{M}$. Recall that the speed of an object in free fall is given by $v_f^2 = v_0^2 + 2a\Delta y \Rightarrow v = \sqrt{2ah}$.

SIMPLIFY:

(a) $v = \sqrt{2ah} \;\Rightarrow\; a = \frac{v^2}{2h} = g - \frac{QE}{M} \;\Rightarrow\; v = \sqrt{2h\left(g - QE/M\right)}$

(b) If the value $g - QE/M$ is less than zero, then the argument of the square root is negative. This means the value is non-real and the body does not fall.

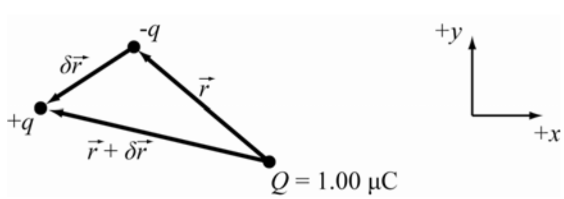
CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE CHECK: Dimensional analysis confirms that the units of the expression reduce to m/s, the correct units for velocity.

22.46. **THINK:** The force in between the charge and the dipole moment is equal to the force acting on each pole of the dipole. The dipole moment is $p = 6.20\cdot 10^{-30}$ C m and is $r = 1.00$ cm from the charge $Q = 1.00$ μC.

SKETCH:



RESEARCH: The force due to an electric field is $\vec{F} = q\vec{E}(r)$, where the electric field is $E(r) = \left(kQ/r^2\right)\hat{r}$.

SIMPLIFY: The total force is $\vec{F} = q\vec{E}(r+\delta r) - q\vec{E}(r)$. From the fundamental theorem of calculus,

$$\vec{F} = q\delta r \frac{d}{dr}E(r) = p\frac{d}{dr}\frac{kQ}{r^2}\hat{r} = pkQ\left(\frac{-2}{r^3}\right)\hat{r} = \frac{-2kpQ}{r^3}\hat{r}$$

CALCULATE: $F = \dfrac{-2\left(8.99\cdot 10^9 \text{ N m}^2/\text{C}\right)\left(6.2\cdot 10^{-30}\text{ C m}\right)\left(1.00\cdot 10^{-6}\text{ C}\right)}{\left(0.0100\text{ m}\right)^3}\hat{r} = 1.11476\cdot 10^{-19}\text{ N}$

ROUND: The force is reported to 3 significant figures.

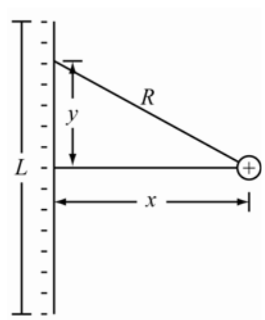
(a) The force between the dipole and the charge is $1.11\cdot 10^{-19}$ N.

(b) The molecule is attracted to the charge regardless of the sign of the charge. This occurs because the charge of opposite sign on the dipole will move closer to the charge creating an attractive force.

DOUBLE-CHECK: The mass of a water molecule is $3.01\cdot 10^{-26}$ kg, meaning the force is relatively large. To view a dipole attracted to a charge, place a charged rod or comb near running water from a faucet.

22.47. **THINK:** Assuming that the wire is made of a conducting material, the charges will be uniformly distributed over its length. The wire will produce an electric field. This field in turn produces a force on a proton, causing the proton to accelerate. The wire has a length of $L = 1.33$ m and a total charge of $Q = -3.05\cdot 10^6 e$. The proton is $x = 0.401$ m away from the center of the wire.

SKETCH:



RESEARCH: The linear density of the wire is $\lambda = Q/L$. Due to the symmetry around the center of the wire the field produced is only along the *x*-axis. The electric field due to a segment of charge is $dE = \left(kdq/R^2\right)\cos\theta$. The distance from the charge to the segment of the wire is $R = \sqrt{x^2+y^2}$. The force on the proton is $F = ma = qE(r)$.

SIMPLIFY: The electric field is:

$$|E| = \int \frac{k|dq|}{R^2}\cos\theta = \int_{-L/2}^{L/2} \frac{k|\lambda|dy}{R^2}\left(\frac{x}{R}\right) = k|\lambda|x\int_{-L/2}^{L/2}\frac{dy}{\left(x^2+y^2\right)^{3/2}} = k\lambda x\left[\frac{y}{x^2\sqrt{x^2+y^2}}\right]_{-L/2}^{L/2}$$

$$= \left(\frac{k|\lambda|}{x}\right)\left[\frac{L/2}{\left(x^2+L^2/4\right)^{1/2}} - \frac{-L/2}{\left(x^2+L^2/4\right)^{1/2}}\right] = \frac{k|\lambda|L}{x\left(x^2+L^2/4\right)^{1/2}} = \frac{k|Q|}{x\left(x^2+L^2/4\right)^{1/2}}$$

The acceleration of the proton is $a = \dfrac{|q||\text{E}|}{m} = \dfrac{k|q||Q|}{mx\left(x^2+L^2/4\right)^{1/2}}$.

CALCULATE: $|E| = \dfrac{\left(8.99\cdot 10^9 \text{ N m}^2/\text{C}\right)\left|\left(-3.05\cdot 10^6\right)\left(1.602\cdot 10^{-19}\text{ C}\right)\right|}{0.401\text{ m}\left[\left(0.401\text{ m}\right)^2 + \left(1.33\right)^2/4\right]^{1/2}} = 0.0141062$ N/C

$$a = \frac{1.602\cdot 10^{-19}\text{ C}}{1.672\cdot 10^{-27}\text{ kg}}\left(0.0141062\text{ N/C}\right) = 1{,}351{,}561\text{ m/s}^2$$

ROUND: The values are reported to 3 significant figures.
(a) The electric field produced by the wire at 0.401 m from its center is 0.0141 N/C.
(b) The acceleration of the proton is $1.35 \cdot 10^6 \text{ m/s}^2$.
(c) The force is attractive since the wire is negatively charged and the proton is positively charged. The force points towards the wire.
DOUBLE-CHECK: These are reasonable answers with appropriate units.

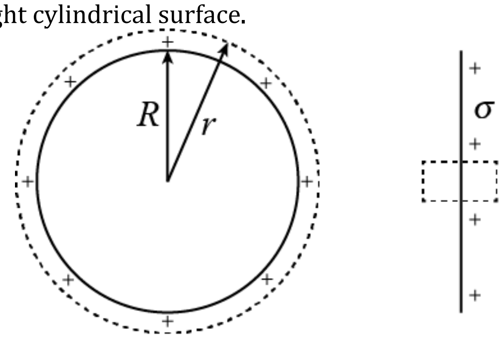
22.48. The flux through a Gaussian surface is the sum of the total charges within the surface divided by the permittivity of free space ε_0. $\Phi = \sum_\varepsilon Q_\text{i} = (3q)+(-q)+(2q)+(-7q)/\varepsilon_0 = -3q/\varepsilon_0$.

22.49. The sum of the flux through each surface is equal to the charge enclose divided by ε_0. $\sum_i \Phi_i = Q/\varepsilon_0$.

The charge is then

$$Q = \varepsilon_0 \sum_i \Phi_i = \left(8.85 \cdot 10^{-12} \text{ C}^2/\left(\text{N m}^2\right)\right)(-70.0-300.0-300.0+300.0-400.0-500.0) \text{ N m}^2$$
$$= -1.124 \cdot 10^{-8} \text{ C} \approx -1.12 \cdot 10^{-8} \text{ C}.$$

22.50. **THINK:** The first Gaussian surface is a sphere with radius $r = R +$ 0.00.0000010 m. This surface encloses all the charge on the sphere. The second Gaussian surface is a small, right cylinder, whose axis is perpendicular to the surface of the sphere and penetrates the surface. Taking the cylinder to be small compared to the sphere, we can consider the surface of the sphere to be locally flat. The charge density on the surface of the sphere will be the total charge divided by the surface area of the sphere. For this case, the electric field is constant outside the sphere and zero inside the sphere.
SKETCH: The sketch shows the two Gaussian surfaces.
(a) shows the spherical surface
(b) shows the small, right cylindrical surface.



(a) (b)

RESEARCH: For the spherical Gaussian surface, the electric field just outside the surface of the sphere is the same as a point charge, so the electric field is radial and perpendicular to the Gaussian surface. So we have $\Phi = \oiint \vec{E} \bullet d\vec{A} = E\left(4\pi r^2\right) = \dfrac{q}{\varepsilon_0}$. We choose a very small right cylinder so that the surface of the sphere is locally flat as show in the sketch. In this case, the electric field is perpendicular to surface. The charge density is $\sigma = \dfrac{q}{4\pi R^2}$. The electric field is parallel to the sides of the cylinder and perpendicular to the ends of the cylinder. So we have $\Phi = \oiint \vec{E} \bullet d\vec{A} = E_\text{inside} A + E_\text{outside} A = \dfrac{q}{\varepsilon_0} = \dfrac{\sigma A}{\varepsilon_0}$. The electric field inside the sphere is zero.

SIMPLIFY: For the spherical surface, the electric field is $E = \dfrac{q}{\varepsilon_0\left(4\pi r^2\right)} = k\dfrac{q}{r^2}.$

For the cylindrical surface, the electric field is $E_{\text{outside}} = \dfrac{\sigma}{\varepsilon_0} = \dfrac{\dfrac{q}{4\pi R^2}}{\varepsilon_0} = k\dfrac{q}{R^2}.$

CALCULATE: In this case, is very close to , so the answer for both cases is $E = k\dfrac{q}{R^2} = \left(8.99\cdot 10^9 \text{ N m}^2/\text{C}^2\right)\dfrac{6.1\cdot 10^{-6}\text{ C}}{\left(0.15\text{ m}\right)^2} = 2.44373\cdot 10^6 \text{ N/C}.$ The charge is positive so the field points outward from the surface of the sphere.

ROUND: We round the magnitude of the electric field to two significant figures $E = 2.4\cdot 10^6$ N/C.

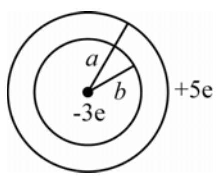
DOUBLE-CHECK: The units are correct for an electric field. The rather high magnitude results from the fact that field is calculated very close to the surface of the charged sphere. Our result for the small right cylindrical Gaussian surface in only correct very close to the surface of the sphere, so that the surface can considered locally flat.

22.51. The cube does not contain any charges, thus the total flux must be zero.
$A\left(E_A + E_B + E_C + E_D + E_E + E_F\right) = \sum_i \Phi_i = 0,$ and therefore,

$$\begin{aligned} E_F &= -\left(E_A + E_B + E_C + E_D + E_E\right) \\ &= -\left(-15.0\text{ N/C} + 20.0\text{ N/C} + 10.0\text{ N/C} + 25.0\text{ N/C} + 20.0\text{ N/C}\right) \\ &= -60\text{ N/C}. \end{aligned}$$

The field on the face F is 60.0 N/C into the face of the cube.

22.52.

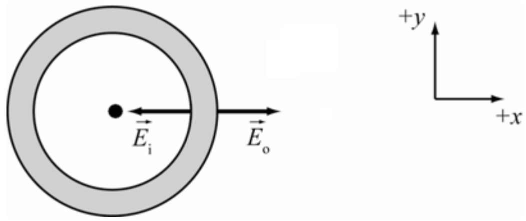


The charge inside the sphere induces a charge of $+3e$ on the inside surface of the sphere. The $+3e$ charge must come from somewhere. In this case the $+3e$ charge is removed from the outer surface charge. The outer surface charge is then $+2e$. The total charge within the material of the sphere is $+5e$.

22.53. Gauss's Law states that $\oiint E\cdot dA = \dfrac{q_{enc}}{\varepsilon_0}.$ The integral over the sphere gives $\oiint E\cdot dA = EA = E\left[4\pi R^2\right] = \dfrac{q_{\text{enc}}}{\varepsilon_0} \Rightarrow E = \dfrac{q_{enc}}{4\pi R^2\varepsilon_0}.$ The electric field outside a uniform distribution of charge is identical to the field created by a point charge of the same magnitude, located at the center of the distribution. Since the radius of the balloon never reaches R, the charge enclosed is constant and the electric field does not change.

22.54. **THINK:** The charges on the surface of the shell may be found using Gauss's Law. The inner and outer radii of the shell are $r_{\text{i}} = 8.00$ cm and $r_{\text{o}} = 10.0$ cm respectively. The electric field at the surface of the outer radius is 80.0 N/C pointing away from the center of the sphere. The electric field at the surface of the inner radius is 80.0 N/C and points towards the center of the sphere. Since the spherical shell does not produce any field in its interior, we can infer that there is a negative charge inside the hollow portion, equivalent to a point charge at the center.

SKETCH:



RESEARCH: Gauss's Law states that $\Phi_e = \frac{q_{enc}}{\varepsilon_0} = \oiint E \cdot dA$.

SIMPLIFY: For a spherically symmetric electric field, the charge enclosed within a Gaussian sphere of radius R is given by $\frac{q_{enc}}{\varepsilon_0} = \oiint E \cdot dA \Rightarrow q_{enc} = \varepsilon_0 E\left(4\pi R^2\right)$. This gives the (negative) charge at the center of the sphere. Since the field between the inner and outer surfaces of the shell is zero, this is also equal to the total (positive) surface charge at the inner radius of the conductor: $q_i = \varepsilon_o E_i\left(4\pi r_i^2\right)$. The Gaussian surface around the whole sphere contains the charge at the center and the charge of the shell. Since the charges at the center and on the inner surface are equal and opposite and therefore cancel, the field at the outer surface can be calculated as being due solely to the charge on the outer surface: $q_o = \varepsilon_o E_o\left(4\pi r_o^2\right)$.

CALCULATE: $q_i = \left(8.854 \cdot 10^{-12} \text{ C}^2/\text{N m}^2\right)\left(80.0 \text{ N/C}\right)4\pi\left(0.0800 \text{ m}\right)^2 = 5.6966 \cdot 10^{-11} \text{ C}$

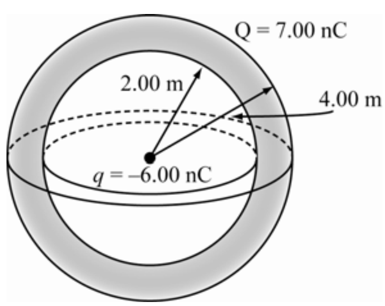
$$q_o = \left(8.854 \cdot 10^{-12} \text{ C}^2/\text{N m}^2\right)\left(80.0 \text{ N/C}\right)4\pi\left(0.100 \text{ m}\right)^2 = 8.9010 \cdot 10^{-11} \text{ C}$$

ROUND: Rounding to three significant figures, the inside and outside total charges over the surface of the sphere are $5.70 \cdot 10^{-11}$ C and $8.90 \cdot 10^{-11}$ C, respectively.

DOUBLE-CHECK: These are reasonable answers with appropriate units. As you would expect, given that the field strength is the same inside and out, the ratio of the charges is the ratio of the square of the radii: $8^2 : 10^2 = 5.70 : 8.90$.

22.55. **THINK:** The electric field at various points can be found using Gauss's Law. This law can also be used to find the charge on the outside surface of the conductor. There is a $q = -6.00$ nC charge at the center of the sphere. The shell has inner and outer radii of $r_i = 2.00$ m and $r_o = 4.00$ m respectively. The shell has a total charge of $Q = +7.00$ nC.

SKETCH:



RESEARCH: Gauss's Law states that $\oiint \vec{E} d\vec{A} = q_{enc}/\varepsilon_0$.

SIMPLIFY: The electric field of charges with spherical symmetry are given by Gauss' Law, where we take spherical Gaussian surfaces: $\oiint \vec{E} \cdot d\vec{A} = EA = E\left(4\pi r^2\right) = \frac{q_{enc}}{\varepsilon_0}$ or $\vec{E}(r) = \frac{q_{enc}}{4\pi\varepsilon_0 r^2}$. The electric field at $r_1 < r_i$ is $E(r_1) = \dfrac{q_{enc}}{4\pi\varepsilon_0 r_1^2}$. The electric field inside any conductor is always zero: $E(r_2) = 0$ where $r_i < r_2 < r_o$. The electric field outside of the conductor is $r_3 > r_o$. $E(r_3) = \dfrac{q_{enc}}{4\pi\varepsilon_0 r_3^2} = \dfrac{Q+q}{4\pi\varepsilon_0 r_3^2}$. Because the field inside the conductor must be zero, Gauss's Law indicates that the charge at the center of the shell is equal and oppposite to the charge on the inside of the shell: $E(r_2) = 0 = \dfrac{q_{enc}}{4\pi\varepsilon_0 r_2^2} = \dfrac{q+q_i}{4\pi\varepsilon_0 r_2^2}$ or $q = -q_i$. The charge on the sphere is equal to the sum of charges on the inner and outer surfaces of the shell $q_i + q_o = Q$. Thus, the outer surface charge is $\sigma = q_o / 4\pi r_o^2 = (Q - q_i)/4\pi r_o^2 = (Q+q)/4\pi r_o^2$.

CALCULATE:

(a) The electric field at $r_1 = 1.00$ m is $E_1 = \dfrac{-6.00 \cdot 10^{-9}\text{ C}}{4\pi\left(8.85 \cdot 10^{-12}\text{ C}^2/\left(\text{N m}^2\right)\right)(1.00\text{ m})^2} = -53.951$ N/C.

(b) The electric field at $r_2 = 3.00$ m is $E_2 = 0$ N/C.

(c) The electric field at $r_3 = 5.00$ m is $E = \dfrac{\left(7.00 \cdot 10^{-9}\text{ C} - 6.00 \cdot 10^{-9}\text{ C}\right)}{4\pi\left(8.85 \cdot 10^{-12}\text{ C}^2/\left(\text{N m}^2\right)\right)(5.00\text{ m})^2} = 0.3597$ N/C.

(d) The surface charge on the outside part of the shell is

$$\sigma = \frac{\left(7.00 \cdot 10^{-9}\text{ C} - 6.00 \cdot 10^{-9}\text{ C}\right)}{4\pi(4.00\text{ m})^2} = 4.974 \cdot 10^{-12}\text{ C/m}^2.$$

ROUND: All the values have an accuracy of three significant figures.

(a) The electric field at $r_1 = 1.00$ m is -54.0 N/C.

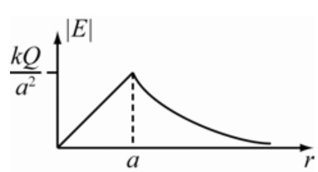
(b) The electric field at $r_2 = 3.00$ m is 0 N/C.

(c) The electric field at $r_3 = 5.00$ m is 0.360 N/C.

(d) The surface density on the outside surface is $4.97 \cdot 10^{-12}$ C/m^2.

DOUBLE-CHECK: These are reasonable results.

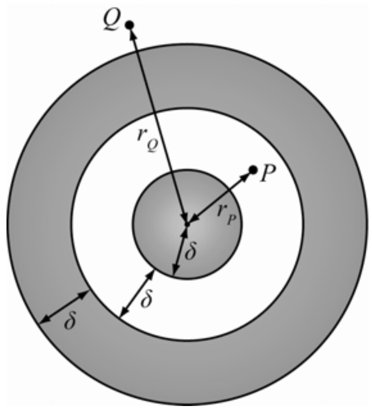
22.56. Inside the sphere of radius a, the charge density is $\rho = \dfrac{Q_{tot}}{V} = \dfrac{Q_{tot}}{(4/3)\pi a^3}$ and is zero anywhere else. Gauss's Law states $\oiint \vec{E}\, d\vec{A} = \dfrac{q_{enc}}{\varepsilon_0}$. The area of the Gaussian surface is always taken to be $A = 4\pi r^2$ and by spherical symmetry, the E-field points radially. Thus, $\oiint \vec{E} \cdot d\vec{A} = \dfrac{q_{enc}}{\varepsilon_0}$ gives $E = \left(\dfrac{q_{enc}}{A\varepsilon_0}\right)\hat{r} = \left(\dfrac{q_{enc}}{(4\pi\varepsilon_0)r^2}\right)\hat{r} = \left(\dfrac{kq_{enc}}{r^2}\right)\hat{r}$. If $r < a$, the enclosed charge is then $q_{enc} = \rho V = \dfrac{Q}{(4/3)\pi a^3}\left(\dfrac{4}{3}\pi r^3\right) = \dfrac{Qr^3}{a^3}$ and $E = \left(\dfrac{kq_{enc}}{r^2}\right) = \dfrac{kQr^3}{a^3 r^2} = \left(\dfrac{kQr}{a^3}\right)\hat{r}$. Otherwise, the surface encloses the whole charge Q. The electric field is then $E = \left(\dfrac{kQ}{r^2}\right)\hat{r}$ if $r > a$. Note that this behaves like a point charge, as would be expected once outside the radius. Below is a graph of $E(\vec{r})$.



22.57. Using Gauss's Law $\oiint E\cdot dA = EA = E\left(4\pi r_{\text{E}}^2\right)=\frac{q_{\text{Earth}}}{\varepsilon_0}$. Solving for the charge gives

$$q_{\text{Earth}}=\varepsilon_0 E\left(4\pi r_{\text{Earth}}^2\right)=\left(8.85\cdot 10^{-12}\text{ C}^2/\text{N m}^2\right)\left(-150.\text{ N/C}\right)4\pi\left(6371\cdot 10^3\text{ m}\right)^2=-6.7711\cdot 10^5\text{ C}$$
$$\approx -6.77\cdot 10^5\text{ C}.$$

22.58. Let $\delta = 10.0\text{ cm}$ be the radius of the solid sphere, the distance between the solid sphere and the inner part of the hollow sphere, and the thickness of the hollow sphere. Let $r_P = 15.0\text{ cm}$ be the distance from the center to the point *P*, and let $r_Q = 35.0\text{ cm}$ be the distance from the center to the point *Q*.



(a) The Gaussian surface at r_p encloses the charge on the inner sphere. $E_1\left(4\pi r_p^2\right)=\frac{q_{\text{enc}}}{\varepsilon_0}$. The charge on the inner sphere is

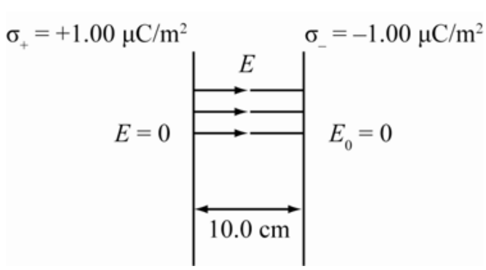
$$q=\varepsilon_0 4\pi E_1 r_p^2=\left(8.85\cdot 10^{-12}\text{ C}^2/\text{N m}^2\right)4\pi\left(-10000\text{ N/C}\right)\left(0.150\text{ m}\right)^2=-2.50\cdot 10^{-8}\text{ C}=-25.0\text{ nC}.$$

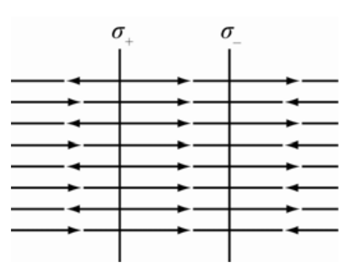
(b) For the electric field inside the shell to be zero, the charge on the inner surface of the shell must be equal to the negative of the charge on the inner sphere. $E=\frac{q_{\text{enc}}}{4\pi\varepsilon_0 r^2}=\frac{\left(q+q_{\text{i}}\right)}{4\pi\varepsilon_0 r^2}=0$ or $q_{\text{i}}=-q$. The charge on the inner surface of the shell is then $q_{\text{i}}=-q=-\left(-25.0\text{ nC}\right)=25.0\text{ nC}$.

(c) The Gaussian surface at $r_Q = 35.0\text{ cm}$ from the center encloses the inner charge and the charge on the shell: $E_2\left(4\pi r_Q^2\right)=\frac{q_{\text{enc}}}{\varepsilon_0}=\frac{\left(q+q_{\text{shell}}\right)}{\varepsilon_0}$ or $q+q_{\text{shell}}=\varepsilon_0 4\pi E_2 r_Q^2$. The charge on the shell is the sum of the charge on the inner and outer surfaces of the shell: $q_{\text{shell}}=q_{\text{i}}+q_{\text{o}}$. The charge on the outer surface of the shell is

$$q_{\text{o}}=q_{\text{shell}}-q_{\text{i}}=q_{shell}-(-q)=q_{shell}+q=\varepsilon_0 4\pi E_2 r_Q^2$$
$$=\left(8.85\cdot 10^{-12}\text{ C}^2/\text{N m}^2\right)4\pi\left(1.00\cdot 10^4\text{ N/C}\right)\left(0.350\text{ m}\right)^2=1.36\cdot 10^{-7}\text{ C}=0.136\,\mu\text{C}.$$

22.59.





The field due to either of the two sheets is found by taking a Gaussian cylinder with top-area A through either plane. Then $\oiint E\cdot dA = 2\cdot EA = \frac{Q_{enc}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$. For the positively charged plate the field points normally away from it. The negatively charged plate has field lines pointing towards it. Adding these fields together gives zero on the outside of the two plates, and $E = 2E_0 = 2\left(\frac{\sigma}{2\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0}$ within the two plates, directed towards the negative plate. The field is

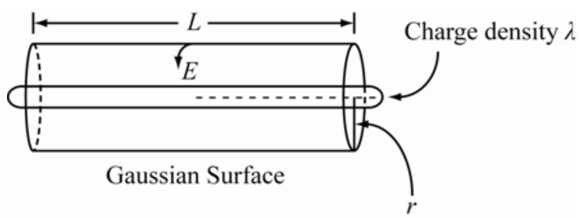
$$E = \frac{\left(1.00\cdot 10^{-6}\ \text{C}^2/\text{m}^2\right)}{\left(8.85\cdot 10^{-12}\ \text{C}^2/\left(\text{N m}^2\right)\right)} = 1.13\cdot 10^5\ \text{N/C},$$

and points from the positive plate to the negative plate. Therefore, the force an electron will experience between the two plates is given by

$$F = qE = eE = \left(1.602\cdot 10^{-19}\ \text{C}\right)\frac{\left(1.00\cdot 10^{-6}\ \text{C}^2/\text{m}^2\right)}{\left(8.85\cdot 10^{-12}\ \text{C}^2/\left(\text{N m}^2\right)\right)} = 1.8107\cdot 10^{-14}\ \text{N} \approx 1.81\cdot 10^{-14}\ \text{N}$$

Since the E-field outside the plates is 0, the electron will experience no force outside of the two plates.

22.60. The magnitude of an electric field is $1.23\cdot 10^3$ N/C at a distance 50.0 cm perpendicular to the wire. The direction of the electric field is pointing toward the wire.



Applying Gauss's Law on the surface shown above gives:

Noting that $\lambda = \frac{Q_{enc}}{L} \Rightarrow Q_{enc} = \lambda L$, $\oiint E\cdot dA = EA = E(2\pi rL) = \frac{\lambda L}{\varepsilon_0} \Rightarrow \lambda = 2\pi Er\varepsilon_0$. Observing the E-field's inward direction as negative, the charge density of the wire is

$$\lambda = 2\pi r\varepsilon_0 E = 2\pi\left(0.500\ \text{m}\right)\left(8.85\cdot 10^{-12}\ \text{C}^2/\left(\text{N m}^2\right)\right)\left(-1.23\cdot 10^3\ \text{N/C}\right)$$
$$= -3.4204\cdot 10^{-8}\ \text{C/m} \approx -3.42\cdot 10^{-8}\ \text{C/m}.$$

The number of electrons per meter is

$$N = \frac{\left(-3.42\cdot 10^{-8}\ \text{C/m}\right)}{\left(-1.602\cdot 10^{-19}\ \text{C}\right)} = 2.135\cdot 10^{11}\ \text{electrons/m}$$
$$N \approx 2.14\cdot 10^{11}\ \text{electrons per meter.}$$

22.61. **THINK:** A solid sphere of radius R has a non-uniform charge density $\rho = Ar^2$. Integrate the sphere.

SKETCH: Not required.

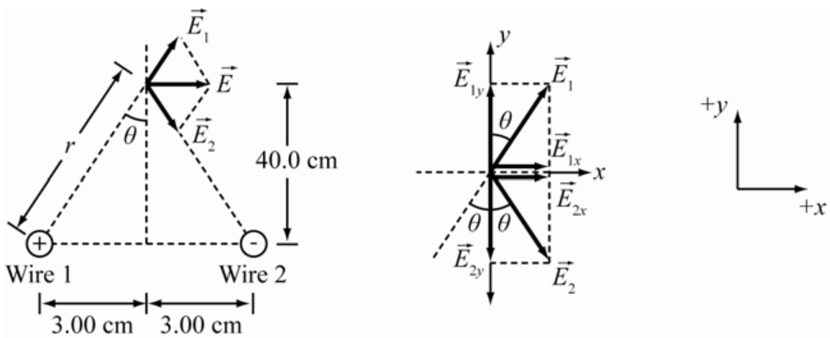
RESEARCH: The total charge is given by $Q = \int_{\text{Sphere}} \rho dV$.

SIMPLIFY: Integrating in the spherical polar coordinate yields:

$$Q=\int_0^R\int_0^{2\pi}\int_0^{\pi}\rho(r)r^2\sin\theta d\theta d\phi dr=\int_0^{2\pi}\int_0^{\pi}\sin\theta d\theta d\phi\int_0^R\left(Ar^2\right)r^2dr=4\pi A\int_0^R r^4dr$$
$$=4\pi A\left[\frac{r^5}{5}\right]_{r=o}^{r=R}=4\pi A\frac{R^5}{5}=\frac{4}{5}\pi AR^5.$$

CALCULATE: Not required.
ROUND: Not required.
DOUBLE-CHECK: One can check the result by single-variable integration, using spherical shells:
$dV=A_{shell}dr=\left(4\pi r^2\right)dr$
$Q=\int\rho dV=\int_0^R\left(Ar^2\right)\left(4\pi r^2\right)dr=4\pi A\int_0^R r^4dr=4\pi A\left[\frac{r}{5}\right]_0^R=\frac{4}{5}\pi AR^5$
Which agrees with the previous answer.

22.62. **THINK:** This is a superposition of two electric fields.
SKETCH:



RESEARCH: The magnitude of the electric field of a charged wire at a distance r from the wire is, by simple application of Gauss' Law, $E=\lambda/2\pi\varepsilon_0 r$, where λ is the linear charge density of the wire. The net electric field at P is given by $\vec{E}_{\text{net}}=\frac{\lambda}{2\pi\varepsilon_0 r}\left(\sin\theta\hat{x}+\cos\theta\hat{y}\right)+\frac{\lambda}{2\pi\varepsilon_0 r}\left(\sin\theta\hat{x}-\cos\theta\hat{y}\right)$

SIMPLIFY: By symmetry, $\vec{E}_{\text{net}}=\vec{E}_x=\left(\frac{\lambda}{2\pi\varepsilon_0 r}\right)\sin\theta\hat{x}.$

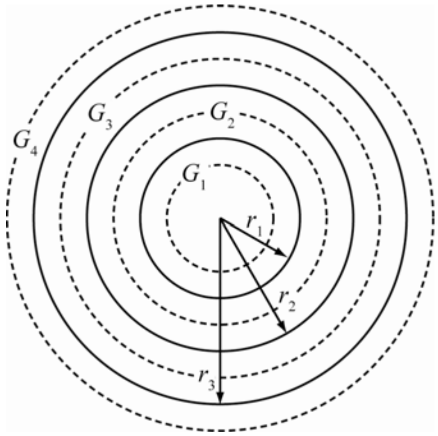
CALCULATE: $\lambda=1.00\ \mu\text{C/m}$, $r=\sqrt{3.00^2+40.0^2}\text{ cm}=40.11\text{ cm}$, $\sin\theta=\frac{3.00\text{ cm}}{40.11\text{ cm}}=0.07479$ and

$$\vec{E}_{\text{net}}=\frac{\left(1.00\cdot10^{-6}\text{ C/m}\right)\left(0.07479\right)}{2\pi\left(8.85\cdot10^{-12}\text{ C}^2/\left(\text{N m}^2\right)\right)\left(0.4011\text{ m}\right)}\hat{x}=\left(6707\text{ N/C}\right)\hat{x}.$$

ROUND: Keeping three significant figures yields $\vec{E}_{\text{net}}=\left(6.71\text{ kN/C}\right)\hat{x}.$

DOUBLE-CHECK: Since the vertical components cancel out, it makes sense that the answer is in the *x*-direction.

22.63. **THINK:** Since this problem has a spherical symmetry, it is possible to apply Gauss's Law.
SKETCH:



r_1 is the radius of a sphere with a charge density $\rho = 120 \text{ nC/cm}^3$. r_2 is the inner radius of a conducting shell. r_3 is the outer radius of the conducting shell. The shell has a net charge q_s.

RESEARCH: For this problem, four Gaussian surfaces, G_1 (within the sphere), G_2 (between the sphere and the shell), G_3 (within the shell), and G_4 (outside the shell) are used. By applying Gauss's Law on each surface, the electric field can be determined.

SIMPLIFY: For the Gaussian surface G_1, applying Gauss's Law gives

$$\oint \vec{E}d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \Rightarrow -E\oint dA = \frac{q_{\text{enc}}}{\varepsilon_0} = \frac{\rho V_{\text{enc}}}{\varepsilon_0}.$$

(a) Using $V_{\text{enc}} = \frac{4}{3}\pi r_a^3$, the electric field is $E_1\left(4\pi r_a^2\right) = \frac{\rho\left(\frac{4}{3}\pi r_a^3\right)}{\varepsilon_0}$, $E = \frac{\rho r_a}{3\varepsilon_0}$.

(b) For the Gaussian surface G_2, applying Gauss's Law yields:

$$\oiint E \cdot dA = \frac{q_{\text{enc}}}{\varepsilon_0} = \frac{\rho\left(\frac{4}{3}\pi r_1^3\right)}{\varepsilon_0} \rightarrow E\left(4\pi r_b^2\right) = \frac{4\rho\pi r_1^3}{3\varepsilon_0} \Rightarrow E = \frac{\rho r_1^3}{3\varepsilon_0 r_b^2}.$$

(c) For the Gaussian surface G_3, the electric field is zero since the surface is in a conductor.

(d) For the Gaussian surface G_4, applying Gauss's Law gives

$$\oiint E \cdot dA = \frac{q_{\text{enc}}}{\varepsilon_0} \Rightarrow E\left(4\pi r_d^2\right) = \frac{q_{sphere} + q_{shell}}{\varepsilon_0} = \frac{4\rho\pi r_1^3}{3\varepsilon_0} + \frac{q_{\text{shell}}}{\varepsilon_0} \Rightarrow E = \frac{\rho r_1^3}{3\varepsilon_0 r_d^2} + \frac{q_{shell}}{4\pi\varepsilon_0 r_d^2}$$

$$E = \frac{1}{r_d^2 \varepsilon_0}\left[\frac{\rho r_1^3}{3} + \frac{q_{shell}}{4\pi}\right].$$

CALCULATE: Substituting the numerical values, $\rho = 120 \text{ nC/cm}^3 = 0.12 \text{ C/m}^3$, $r_1 = 0.12$ m, $r_2 = 0.300$ m, $r_3 = 0.500$ m, $r_a = 0.100$ m, $r_b = 0.200$ m and $r_d = 0.800$ m yields the electric fields:

(a) $E = \frac{\rho r_a}{3\varepsilon_0} = \frac{\left(0.12 \text{ C/m}^3\right)\left(0.100 \text{ m}\right)}{3\left(8.85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2\right)} = 4.518 \cdot 10^8 \text{ N/C}$

(b) $E = \frac{\rho r_1^3}{3\varepsilon_0 r_b^2} = \frac{\left(0.12 \text{ C/m}^3\right)\left(0.12 \text{ m}\right)^3}{3\left(8.85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2\right)\left(0.200 \text{ m}\right)^2} = 1.953 \cdot 10^8 \text{ N/C}$

(c) $E = 0$ since it is in the conducting shell.

(d) $E=\frac{1}{r_d^2\varepsilon_0}\left[\frac{\rho r_1^3}{3}+\frac{q_{shell}}{4\pi}\right]=\frac{1}{\left(8.85\cdot 10^{-12}\text{ C}^2/\text{Nm}^2\right)\left(0.800\text{ m}\right)^2}\left[\frac{\left(0.12\text{ C/m}^3\right)\left(0.12\text{ m}\right)^3}{3}+\frac{-2.00\cdot 10^{-3}\text{ C}}{4\pi}\right]$

$=-1.589\cdot 10^7\text{ N/C}$

ROUND: Rounding to three significant figures:

(a) $E=4.52\cdot 10^8\text{ N/C}$

(b) $E=1.95\cdot 10^8\text{ N/C}$

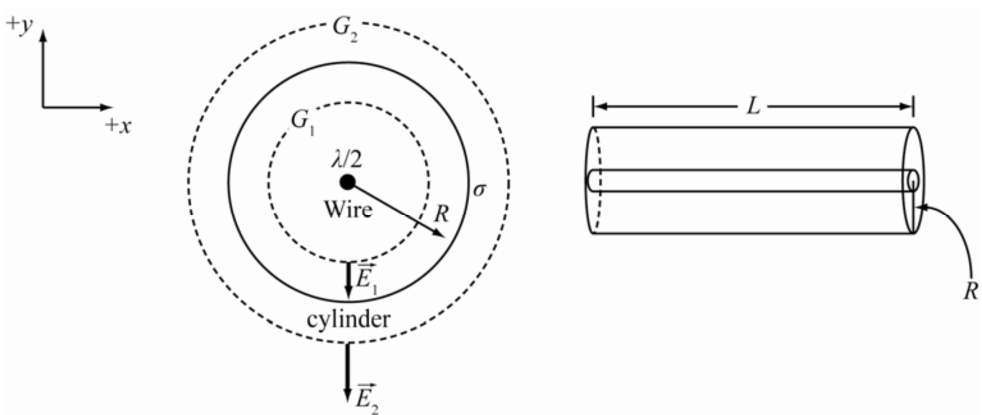
(c) $E=0$ since it is in the conducting shell.

(d) $E=-1.59\cdot 10^7\text{ N/C}$

DOUBLE-CHECK: The values of electric fields have the correct units and are of reasonable orders of magnitude.

22.64. **THINK:** Using the symmetry of a cylinder, Gauss's Law can be applied.

SKETCH:



Note that the Gaussian surfaces G_1 and G_2 are cylindrical surfaces with radii r_1 and r_2 and a length L.

RESEARCH: The electric field can be determined by applying Gauss's Law on the Gaussian surfaces G_1 and G_2.

SIMPLIFY: For the Gaussian surface G_1, applying Gauss's Law produces

$$\oint \vec{E}_1 d\vec{A}=\frac{q_{\text{enc}}}{\varepsilon_0}=\frac{(\lambda/2)L}{\varepsilon_0}\Rightarrow \vec{E}_1\left(2\pi r_1 L\right)=\frac{(\lambda/2)L}{\varepsilon_0}\hat{r}\Rightarrow E_1=\frac{\lambda}{4\pi\varepsilon_0 r_1}\hat{r}.$$

Similarly for the Gaussian surface G_2, using Gauss's Law gives

$$\oint \vec{E}_2 d\vec{A}=\frac{q_{\text{enc}}}{\varepsilon_0}=\frac{(\lambda/2)L+\sigma(2\pi RL)}{\varepsilon_0}\Rightarrow E_2\left(2\pi r_2 L\right)=\frac{(\lambda/2)L+\sigma(2\pi RL)}{\varepsilon_0}\hat{r}\Rightarrow \vec{E}_2=\frac{\lambda+4\pi R\sigma}{4\pi\varepsilon_0 r_2}.$$

Therefore, the expressions of the electric fields are:

(a) For $r\le R$, the electric field is $\vec{E}=\frac{\lambda}{4\pi\varepsilon_0 r}\hat{r}.$

(b) For $r\ge R$, the electric field is $\vec{E}=\frac{\lambda+4\pi R\sigma}{4\pi\varepsilon_0 r_2}\hat{r}.$

CALCULATE: Not required.

ROUND: Not required.

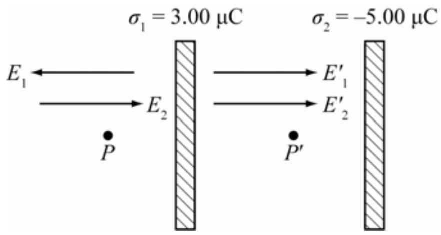
DOUBLE-CHECK: Since the metal cylinder is a conductor, all its charge resides on its outer surface. This means that the field inside the cylinder is not affected by the charge on the cylinder. Therefore, for $r\le R$, the electric field is only due to the wire. For $r\ge R$, the charge on the cylinder produces an electric field as if all its charge was concentrated in the center of the cylinder. Therefore, the electric field can be found by replacing $\lambda/2$ with $(\lambda/2)+2\pi R\sigma$ as the new linear density of a wire.

22.65. **THINK:** Use the values from the question: $\sigma_1 = 3.00\ \mu\text{C/m}^2$, and $\sigma_2 = -5.00\ \mu\text{C/m}^2$.

(a) The total field can be determined by superposition of the fields from both plates. The field contributions from the two charged sheets are opposing each other at point *P*, to the left of the first sheet.

(b) The situation is similar to a) except that the fields due to both charged sheets point in the same direction at point P'.

SKETCH:



RESEARCH:

(a) At point P, the field due to sheet #1 is given by $E_1 = -(\sigma_1/2\varepsilon_0)\hat{x}$, and the field due to sheet #2 is given by $E_2 = -(\sigma_2/2\varepsilon_0)\hat{x}$. Note that $E_{\text{total}} = E_1 + E_2$.

(b) At point P', the field due to sheet #1 is given by $E_1' = (\sigma_1/2\varepsilon_0)\hat{x}$, and the field due to sheet #2 is given by $E_2' = -(\sigma_2/2\varepsilon_0)\hat{x}$. Again, $E_{\text{total}}' = E_1' + E_2'$.

SIMPLIFY:

(a) $E = \left(\frac{-\sigma_1}{2\varepsilon_0}\right)\hat{x} + \left(\frac{-\sigma_2}{2\varepsilon_0}\right)\hat{x} = \frac{-(\sigma_1+\sigma_2)}{2\varepsilon_0}$

(b) $E' = \left(\frac{\sigma_1}{2\varepsilon_0}\right)\hat{x} + \left(\frac{-\sigma_2}{2\varepsilon_0}\right)\hat{x} = \frac{(\sigma_1-\sigma_2)}{2\varepsilon_0}$

CALCULATE:

(a) $E_{\text{total}} = \frac{-(3.00-5.00)\cdot 10^{-6}\ \text{C/m}^2}{2\left(8.85\cdot 10^{-12}\ \text{C}^2/\left(\text{N m}^2\right)\right)}\hat{x} = \left(1.130\cdot 10^5\ \text{N/C}\right)\hat{x}$

(b) $E_{\text{total}} = \frac{\left(3.00-(-5.00)\right)\cdot 10^{-6}\ \text{N/C}}{2\left(8.85\cdot 10^{-12}\ \text{C}^2/\left(\text{N m}^2\right)\right)}\hat{x} = \left(4.520\cdot 10^5\ \text{N/C}\right)\hat{x}$

ROUND:

(a) $E_{\text{total}} = 1.13\cdot 10^5\ \text{N/C}$ in the positive *x*-direction

(b) $E_{\text{total}} = 4.52\cdot 10^5\ \text{N/C}$ in the positive *x*-direction

DOUBLE-CHECK: The results are reasonable because the answer in (b) is four times larger than that found in (a) since in (a) the fields are opposing each other and in (b) the fields are in same direction.

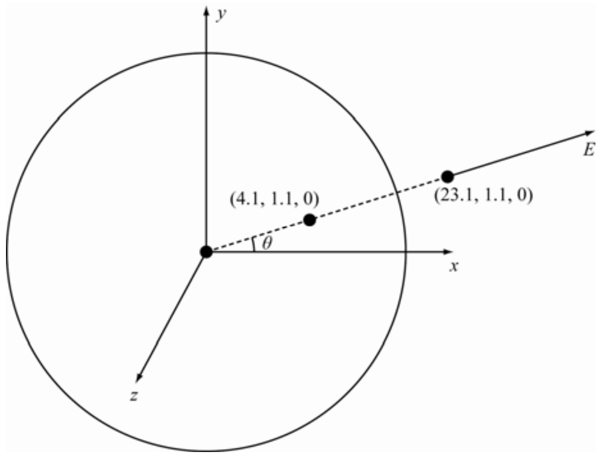
22.66. **THINK:**

(a) The field due to a charged sphere outside the radius of the sphere is equivalent to the field due to a point charge of equal magnitude at the center of the sphere.

(b) The electric field radiates outward, perpendicular to the surface of the sphere.

(c) The field inside a conductor is zero.

SKETCH:



RESEARCH:

(a) The field is given by: $q = 0.271\text{ nC}$ and $r^2 = (23.1\text{ cm})^2 + (1.10\text{ cm})^2 + (0\text{ cm})^2$.

(b) The angle is given by $\tan\theta = (1.10\text{ cm})/(23.1\text{ cm})$ or $\theta = \tan^{-1}(1.10\text{ cm})/(23.1\text{ cm})$.

(c) The field is zero inside a conductor.

SIMPLIFY: Not required.

CALCULATE:

(a) $E = \dfrac{(0.271\text{ nC})(10^{-9}\text{ C/nC})}{4\pi(8.85\cdot 10^{-12}\text{ C}^2/\text{N m}^2)\left((0.231)^2 + (0.0110)^2\right)\text{ N/C}} = 45.56\text{ N/C}$

(b) $\theta = \tan^{-1}\left(\dfrac{1.10\text{ cm}}{23.1\text{ cm}}\right) = 2.7263°$

(c) 0 N/C

ROUND:

Rounding to three significant figures:

(a) $E \approx 45.6\text{ N/C}$

(b) $\theta \approx 2.73°$

(c) 0 N/C

DOUBLE-CHECK:

(a) Not required.

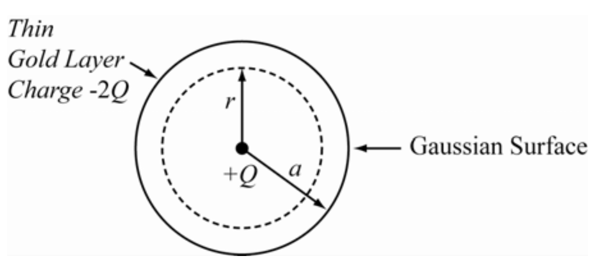
(b) Since the y-component is much less than the x-component I expected the angle to be small, which it is.

(c) Not required.

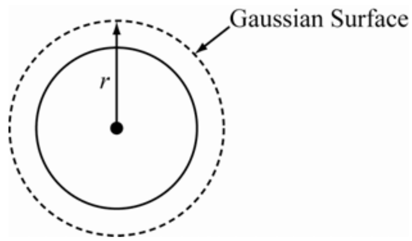
22.67. **THINK:** The spherical symmetry of the charged object allows the use of Gauss's Law to calculate the electric field. To do this, separate Gaussian surfaces must be considered for $r < a$ and $r > a$.

SKETCH:

(a)



(b)



RESEARCH:

(a) The total charge inside the Gaussian surface is given by $q=\int_0^r \rho_0 4\pi r^2 dr^1$. The charge density is $\rho_0 = Q/V_{\text{sphere}}$, and the volume is $V_{\text{sphere}} = (4/3)\pi a^3$.

(b) The total charge is simply the charge of the non-conducting layer and the gold layer: $q = \text{Total Charge} = Q - 2Q = -Q$.

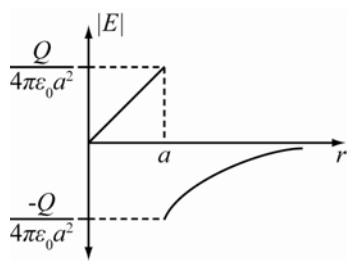
Gauss's Law states $\oint \vec{E}\cdot d\vec{A} = q_{\text{enc}}/\varepsilon_0$. Since the Gaussian surface in this case is a sphere, Gauss's Law simplifies to $E(4\pi r^2) = q/\varepsilon_0$.

SIMPLIFY:

(a) $q=\int_0^r \rho_0 4\pi r^2 dr^1 = \rho_0\int_0^r 4\pi r^2 dr^1 = \rho_0 (4/3)\pi r^3 = \left(\frac{Q}{(4/3)\pi a^3}\right)\left(\frac{2}{3}\pi r^3\right)$. Substituting $\rho_0 = \frac{Q}{(4/3)\pi a^3}$, $q=\frac{Qr^3}{a^3}$. $E(4\pi r^2) = \frac{q_{enc}}{\varepsilon_0} = \frac{Qr^3}{a^3\varepsilon_0} \Rightarrow \vec{E}(\vec{r}) = \left(\frac{Qr}{4\pi a^3\varepsilon_0}\right)\hat{r}$, $r<a$. The direction is radially outward.

(b) $E(4\pi r^2) = \frac{q_{enc}}{\varepsilon_0} = \frac{Q-2Q}{\varepsilon_0} = \frac{-Q}{\varepsilon_0} \Rightarrow E = -\left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)$ for $r>a \Rightarrow \vec{E} = -\left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)\hat{r}$. The direction is towards the center of the sphere.

(c)



The discontinuity at $r=a$ is due to the surface charge density of the gold. The charge on the gold layer causes a sudden spike in the total charge resulting in a discontinuity in the electric fields.

CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE-CHECK:

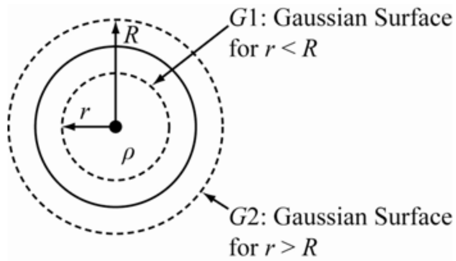
(a) The electric field increases r gets larger since the charge inside the Gaussian surface increases as a function of r^3 while the area increases as a function of r^2. Since the increase of the area decreases the field by a function of r^2 and the charge increases the field by r^3 it is reasonable that the field increases, as a function of r.

(b) The sphere acts like a point source is as expected.

(c) There is a discontinuity in the E v. r graph due to the presence of a surface charge density on the gold layer, which is expected.

22.68. **THINK:** By constructing Gaussian surfaces in both regions $r<R$ and $r>R$, the electric field can be calculated using Gauss's Law.

SKETCH:



RESEARCH: The total charge inside the Gaussian surface is given by $q=\int_0^r \rho_0 4\pi r^2 dr'$. The charge density is given by $\rho(r)=(\beta/r)\sin(\pi r/2R)$. For the Gaussian surface outside the sphere $(r>R)$, the total charge is given by $q=\int_0^r \rho_0 4\pi r^2 dr'$. The electric field can be calculate using Gauss's Law: $\oint \vec{E}\, d\vec{A}=q_{\text{enc}}/\varepsilon_0$, which for a spherical Gaussian surface is $E=(4\pi r^2)=q/\varepsilon_0$.

SIMPLIFY: For the case $r<R$, $q=\int_0^r\left(\frac{\beta}{r'}\right)\sin\left(\frac{\pi r'}{2R}\right)\left(4\pi r'^2\right)dr'=4\pi\beta\int_0^r r'\sin\left(\frac{\pi r'}{2R}\right)dr'$

$$=4\pi\beta\left[\left(\frac{-2Rr}{\pi}\right)\cos\left(\frac{\pi r}{2R}\right)\Big|_0^r+\left(\frac{2R}{\pi}\right)\int_0^r\cos\left(\frac{\pi r'}{2R}\right)dr'\right]$$

Integration by parts: $q=4\pi\beta\left[\left(\frac{-2Rr}{\pi}\right)\cos\left(\frac{\pi r}{2R}\right)+\left(\frac{2R}{\pi}\right)^2\sin\left(\frac{\pi r'}{2R}\right)\right]_0^r$

$$=\frac{-8\beta R}{\pi}\left[\pi r\cdot\cos\left(\frac{\pi r}{2R}\right)-2R\sin\left(\frac{\pi r}{2R}\right)\right].$$

For the case $r>R$, q is given by

$$q=\int_0^R\rho(r'')\left(4\pi r'^2\right)dr'=4\pi\beta\left[\left(\frac{-2R(R)}{\pi}\right)\cos\left(\frac{\pi R}{2R}\right)+\left(\frac{2R}{\pi}\right)^2\sin\left(\frac{\pi R}{2R}\right)\right]_0^R$$

$$=4\pi\beta\left[\left(\frac{-2R^2}{\pi}\right)(0)+\left(\frac{4R^2}{\pi^2}\right)\sin\left(\frac{\pi}{2}\right)\right]=4\pi\beta\left[\left(\frac{-2R^2}{\pi}\right)(0)+\left(\frac{4R^2}{\pi^2}\right)(1)\right]=\frac{16\beta R^2}{\pi}.$$

The electric field is given by $E(4\pi r^2)=\frac{q}{\varepsilon_0}\Rightarrow E=\frac{q}{4\pi r^2\varepsilon_0}$.

For the case $r<R$,

$$E=\left(\frac{1}{4\pi r^2\varepsilon_0}\right)\cdot\frac{-8\beta R}{\pi}\left[\pi r\cdot\cos\left(\frac{\pi r}{2R}\right)-2R\sin\left(\frac{\pi r}{2R}\right)\right]=\frac{-8\beta Rk}{\pi r^2}\left[\pi r\cdot\cos\left(\frac{\pi r}{2R}\right)-2R\sin\left(\frac{\pi r}{2R}\right)\right]\qquad(1)$$

For $r>R$,

$$q=\left(\frac{16\beta\rho^2}{\pi}\right)\left(\frac{1}{4\pi r^2\varepsilon_0}\right)=\left(\frac{4\beta R^2}{\pi^2 r^2\varepsilon_0}\right).\qquad(2)$$

For $r=R$,

$$(1)=\frac{-8\beta Rk}{\pi R^2}\left[\pi R\cdot\cos\left(\frac{\pi R}{2R}\right)-2R\sin\left(\frac{\pi R}{2R}\right)\right]=\frac{-8\beta k}{\pi}\left[\pi\cdot\cos\left(\frac{\pi}{2}\right)-2\sin\left(\frac{\pi}{2}\right)\right]=\frac{-8\beta k}{\pi}\left[0-2(1)\right]=\frac{16\beta k}{\pi}$$

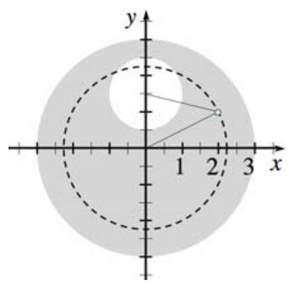
$$(2)=\left(\frac{4\beta R^2}{\pi^2\varepsilon_0 R}\right)=\left(\frac{4\beta}{\pi^2\varepsilon_0}\right)=\frac{16\beta k}{\pi}$$

$$\therefore(1)=(2)$$

The expressions are equal when $r=R$.

CALCULATE: Not applicable.
ROUND: Not applicable.
DOUBLE-CHECK: The two expressions are equal at $r = R$, which should be the case since there are no surface charge densities present cause discontinuities for $r > R$, the objects act like a point source, which is expected from a charged sphere.

22.69. **THINK:** The principle of superposition can be used to find the electric field at the specified point. The electric field at the point $(2.00,1.00)$ is modeled as the sum of a positively charged cylindrical rod with no hole and a negatively charged cylindrical rod whose size and location are identical to those of the cavity. Let's first think about the case of the positively charged cylindrical rod without a hole. Since the point of interest is inside the rod, the entire charge distribution of the rod cannot contribute. Instead we draw our Gaussian surface as a cylinder with our point of interest on its rim (see sketch below, where the dashed circle in the cross-sectional view represents the Gaussian cylinder).
SKETCH:



RESEARCH: In section 22.9 of the textbook it was shown that for cylindrical symmetry of the charge distribution the electric field outside the charge distribution can be written as $E = 2k\lambda / r$, where *r* is the distance to the central axis of the charge distribution and λ is the charge per unit length.
In the problem here the charge was initially uniformly distributed over the entire cross-sectional area, which means that the value of λ for the Gaussian surface and for the hole are proportional to their cross-sectional area: $\lambda_{\text{Gauss}} = \lambda_{\text{rod}}(r/R)^2$, and $\lambda_{\text{hole}} = -\lambda_{\text{rod}}(r_{\text{hole}}/R)^2$.
Now we have the tools to calculate the magnitudes of the individual electric fields of the rod and of the hole. What is left is to add the two, which is a vector addition. So we have to determine the *x*- and *y*-components of the fields individually and the combine them.
If E_1 is the field from the dashed cylinder and E_2 is that of the cavity then from considering the geometry the relation are given by: $E_{1x} = E_1 2/\left(2^2+1^2\right)^{1/2}$, $E_{2x} = E_2 2/\left(2^2+1^2\right)^{1/2}$, $E_{1y} = E_1/\left(2^2+1^2\right)^{1/2}$ and $E_{2y} = 0.5E_2/\left(2^2+0.5^2\right)^{1/2}$.
The net electric field is given by the following relations $E_x = E_{1x} + E_{2x}$ and $E_y = E_{1y} + E_{2y}$.
SIMPLIFY:
$E_1 = 2k\lambda_{\text{Gauss}}/r = 2k\lambda_{\text{rod}}(r/R)^2/r = 2k\lambda_{\text{rod}}r/R^2$
$E_2 = 2k\lambda_{\text{hole}}/r_2 = -2k\lambda_{\text{rod}}(r_{\text{hole}}/R)^2/r_2$
where r_2 is the distance between our point of interest and the center of the hole.

$$E_x = E_1\frac{2}{\left(2^2+1^2\right)^{1/2}} + E_2\frac{2}{\left(2^2+0.5^2\right)^{1/2}}, \text{ and } E_y = E_1\frac{1}{\left(2^2+1^2\right)} + E_2\frac{0.5}{\left(2^2+0.5^2\right)^{1/2}}.$$

CALCULATE: $r = \left(\left(0.01\text{ m}\right)^2 + \left(0.0200\text{ m}\right)^2\right)^{1/2} = 0.02236\text{ m}$

$r_2 = \left(\left(0.00500\text{ m}\right)^2 + \left(0.0200\text{ m}\right)^2\right)^{1/2} = 0.02062\text{ m}$

$$E_1 = \frac{2(8.99 \cdot 10^9 \text{ Nm}^2/\text{C}^2)(6.00 \cdot 10^{-7} \text{ C/m})(0.02236 \text{ m})}{(0.0300 \text{ m})^2} = 2.680 \cdot 10^5 \text{ N/C}$$

$$E_{21} = -\frac{2(8.99 \cdot 10^9 \text{ Nm}^2/\text{C}^2)(6.00 \cdot 10^{-7} \text{ C/m})}{(0.02062 \text{ m})}\left(\frac{0.0100 \text{ m}}{0.0300 \text{ m}}\right)^2 = -0.581 \cdot 10^5 \text{ N/C}$$

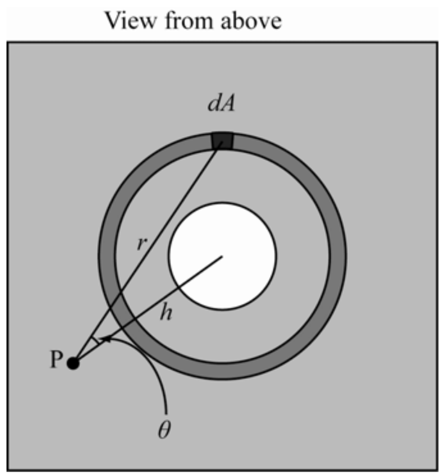
$E_x = 1.833 \cdot 10^5$ N/C, $E_y = 1.339 \cdot 10^5$ N/C

ROUND: $E_x = 183$ kN/C, $E_y = 134$ kN/C

DOUBLE-CHECK: We can calculate the magnitude and direction of the combined electric field and find: $E = \sqrt{E_x^2 + E_y^2} = 227$ kN/C, and $\theta = \tan^{-1}(E_y / E_x) = 36.1°$. If the hole would not have been drilled, the magnitude would have been the magnitude we calculated above for E_1, $E_1 = 268$ kN/C, and it would have pointed along the $\vec{r}$ vector with an angle of 26.6°. This means that our result states that the magnitude of the electric field is weakened due to the presence of the hole, and that it does not point radial outward any more, but further away from the *x*-axis. Both of these results are in accordance with expectations and add confidence to our result: the hole modifies the electric field somewhat, but does not do so radically.

22.70. **THINK:** Use the principle of superposition and model the problem as a positive infinite plane and a negative circular disc.

SKETCH:



RESEARCH: The electric field contributed by the plane is given by: $E_{\text{plane}} = \sigma / 2\varepsilon_0$. One can find the electric field of a disc by adding up the contributions from each small area. From the symmetry one many conclude that the field points vertically. The contribution of each small area to the field in the *y*-direction is given by: $dE = (-\sigma dA / 4\pi\varepsilon_0)(\cos\theta / r^2)$, $\cos\theta = h / r$, $r^2 = \rho^2 + h^2$, $E_{\text{disc}} = \int dE$. $E_{\text{total}} = E_{\text{plane}} + E_{\text{disc}}$.

$h = 0.200$ m, $R = 0.050$ m, $\sigma = 1.3 \text{ C/m}^2$.

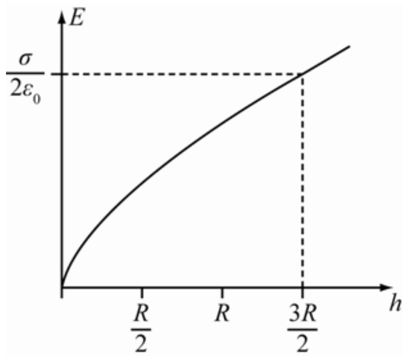
SIMPLIFY: $dE = -\frac{\sigma dA}{4\pi\varepsilon_0}\left(\frac{\cos\theta}{r^2}\right) = \frac{-\sigma(\rho d\varepsilon de)}{4\pi\varepsilon_0}\left(\frac{h}{(\rho^2 + h^2)^{3/2}}\right)$

$$E_{\text{disc}} = \left(-\frac{\sigma h}{4\pi\varepsilon_0}\right)\int_0^{2\pi} d\theta \int_0^R d\rho \frac{\rho}{(\rho^2 + h^2)^{3/2}} = \left(-\frac{\sigma h}{4\pi\varepsilon_0}\right)(2\pi)\left[(\rho^2 - h^2)^{-1/2}\right]_0^R = \frac{\sigma h}{2\varepsilon_0}\left[\frac{1}{(h^2 + R^2)^{1/2}} - \frac{1}{h}\right]$$

$$E_{\text{total}} = E_{\text{disc}} + E_{\text{plane}} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma h}{2\varepsilon_0}\left[\frac{1}{h} - \frac{1}{(h^2 + R^2)^{1/2}}\right] = \left(\frac{\sigma}{2\varepsilon_0}\right)\frac{h}{(h^2 + R^2)^{1/2}}$$

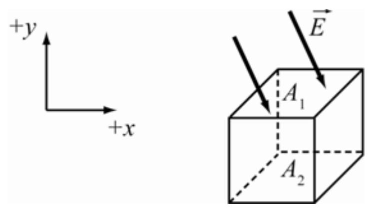
CALCULATE: $E_{\text{total}} = \dfrac{1.30\text{ C/m}^2}{2\left(8.85\cdot 10^{-12}\text{ C}^2/\text{ N m}^2\right)} \cdot \dfrac{0.200\text{ m}}{\left(\left(0.200\text{ m}\right)^2 + \left(0.0500\text{ m}\right)^2\right)^{1/2}} = 7.125\cdot 10^{10}\text{ N/C}$

ROUND: $E_{\text{total}} \approx 7.13\cdot 10^{10}\text{ N/C}$



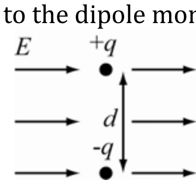
DOUBLE-CHECK: The plot shows that for large h the result is the same as that of an infinite plane without a hole as one would expect.

22.71. Regardless of what orientation the cube is in, we can always enclose it in a Gaussian surface that just covers the cube. Gauss's Law states that $\oiint \vec{E} d\vec{A} = q_{\text{enc}} / \varepsilon_0$.



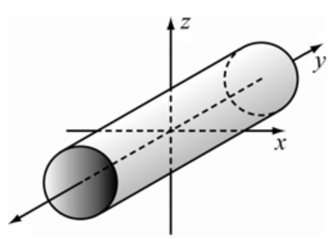
Now consider the flux through one particular face given by $\vec{E}\vec{A}_1$. There exists a flux through the opposite face given by $\vec{E}\vec{A}_2$ with the relation $\vec{E}\vec{A}_1 = -\vec{E}\vec{A}_2$ since $\vec{A}_1$ and $\vec{A}_2$ point the opposite way. The sum of the flux contributed between the two opposite sides is $\vec{E}\vec{A}_1 + \vec{E}\vec{A}_2 = 0$. If this calculation is done for each side then the total flux is 0 and hence the total charge must be 0 by Gauss's Law.

22.72. The dipole moment is given by $p = qd$ where d is the distance between the charges. The maximum torque is when the field is perpendicular to the dipole moment.



The torque is then $\tau = qEd = pE = \left(8.0\cdot 10^{-30}\text{ C m}\right)\left(500.0\text{ N/C}\right) = 4.0\cdot 10^{-27}\text{ N m}$.

22.73.

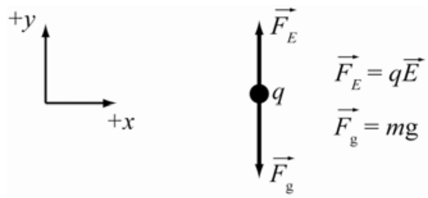


Consider a cylindrical Gaussian surface with a radius of 4.00 cm. By Gauss's Law, $\oiint \vec{E} d\vec{A} = q_{\text{enc}} / \varepsilon_0$. The charge inside the cylinder is $q = \rho \pi r^2 l$, so the field is given by

$$E(2\pi r l) = \frac{\rho \pi r^2 l}{\varepsilon_0} \Rightarrow E = \frac{\rho r}{2\varepsilon_0} = \frac{(6.40 \cdot 10^{-8} \text{ C/m}^3)(0.0400 \text{ m})}{2(8.854 \cdot 10^{-12} \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2)} = 1.45 \cdot 10^2 \text{ N/C}$$

away from the *y*-axis. The information concerning the radius of the cylinder is irrelevant.

22.74. The electric force and the gravitational force must balance.

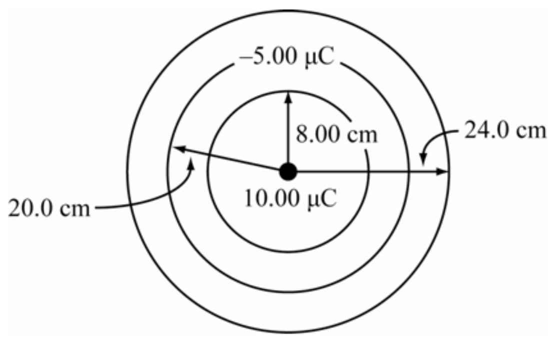


$qE - mg = 0 \Rightarrow E = mg/q, \quad g = 9.81 \text{ m/s}^2$

(a) $m_{\text{electron}} = 9.109 \cdot 10^{-31} \text{ kg}, \; q = -1.602 \cdot 10^{-19} \text{ C}, \; E = \dfrac{(9.109 \cdot 10^{-31} \text{ kg})(9.81 \text{ m/s}^2)}{-1.602 \cdot 10^{-19} \text{ C}} = -5.58 \cdot 10^{-11} \text{ N/C}$ with the field directed down.

(b) $m_{\text{proton}} = 1.672 \cdot 10^{-27} \text{ kg}, \; q = 1.602 \cdot 10^{-19} \text{ C}, \; E = \dfrac{(1.672 \cdot 10^{-27} \text{ kg})(9.81 \text{ m/s}^2)}{1.602 \cdot 10^{-19} \text{ C}} = 1.02 \cdot 10^{-7} \text{ N/C}$ with the field directed up.

22.75.

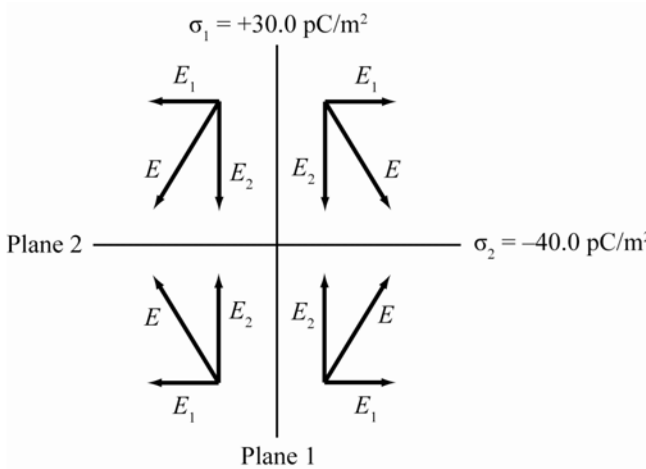


(a) Construct a Gaussian surface (spherical) with radius between 20.0 cm and 24.0 cm. Gauss's Law states that the total flux is equal to q/ε_0, since the electric field inside the last metallic shell is zero, the flux must be zero and hence the total charge must be zero. Since the total charge to be zero:

$$q_{\text{inside wall}} + 10.00 \text{ μC} - 5.00 \text{ μC} = 0 \Rightarrow q_{\text{inside wall}} = -5.00 \text{ μC}.$$

(b) Constructing a Gaussian sphere that contains all the shells, it can be determined that since the electric field is zero, outside the largest shell the flux is also zero and hence the total charge must be zero. $q_{\text{outside wall}} + q_{\text{inside wall}} + 10.00 \text{ μC} - 5.00 \text{ μC} = 0 \Rightarrow q_{\text{outside wall}} - 5.00 \text{ μC} - 5.00 \text{ μC} + 10.00 \text{ μC} = 0$, which then implies $q_{\text{outside wall}} = 0$.

22.76.



The fields from both plates are always perpendicular to each other. The field E_1 from plane 1 always points away from plane 1. The field E_2 from plane 2 always points toward plane 2. The combined field E points in different directions depending on where you measure it, but the magnitude of the field is the same everywhere.

$$E=\sqrt{E_1^2+E_2^2}=\sqrt{\left(\frac{\sigma_1}{2\varepsilon_0}\right)^2+\left(\frac{\sigma_2}{2\varepsilon_0}\right)^2}$$

$$E=\frac{\sqrt{\sigma_1^2+\sigma_2^2}}{2\varepsilon_0}=\frac{\sqrt{\left(30.0\text{ pC/m}^2\right)^2+\left(-40.0\text{ pC/m}^2\right)^2}}{2\left(8.85\cdot10^{-12}\text{ m}^{-3}\text{ kg}^{-1}\text{ s}^4\text{ A}^2\right)}=2.82\text{ N/C}$$

22.77. The sum of the forces on the electron is given by $F_{\text{total}}=F_{\text{gravity}}+F_{\text{coulomb}}=-mg+qE.$ $E=-150.\text{ N/C},$ $q=-1.602\cdot10^{-19}\text{ C},$

$m=9.11\cdot10^{-31}\text{ kg}.$ Thus, $F_{net}=qE-mg=ma\Rightarrow a_e=\dfrac{eE}{m_e}-g.$

$$a_e=\frac{\left(1.602\cdot10^{-19}\text{ C}\right)\left(150.\text{ N/C}\right)}{\left(9.11\cdot10^{-31}\text{ kg}\right)}-\left(9.81\text{ m/s}^2\right)=2.64\cdot10^{13}\text{ m/s}^2.$$

22.78. This problem can be solved using Gauss's Law. $\text{Flux}=\dfrac{q_{\text{total}}}{\varepsilon_0}=\oiint\vec{E}\cdot d\vec{a}=\oiint\vec{E}_n\cdot d\vec{a}=10\text{ N m}^2/\text{C}.$ Since

$Eda=E_n\,da,$ $q_{\text{total}}=\varepsilon_0\left(10.0\text{ N m}^2/\text{C}\right)=\left(8.85\cdot10^{-12}\text{ C}^2/\left(\text{N m}^2\right)\right)\left(10.0\text{ N m}^2/\text{C}\right)=8.85\cdot10^{-11}\text{ C}.$

22.79. This problem can be solved using Gauss's Law. $\text{Flux}=q_{\text{total}}/\varepsilon_0.$ The approximation can be made that the flux leaving the ends of the rod are negligible, so $\text{Flux}=q_{\text{total}}/\varepsilon_0=\lambda l/\varepsilon_0$ where l is the length of the rod.

$$\lambda=\frac{\varepsilon_0\Phi}{l}=\frac{\left(8.85\cdot10^{-12}\right)\left(1.46\cdot10^6\text{ N m}^2/\text{C}\right)}{0.300\text{ m}}=4.31\cdot10^{-5}\text{ C/m}$$

22.80. **THINK:** I first need to find the relationship between the first wire and the second wire.
SKETCH: Not required.

RESEARCH: The field due to the first wire is given by: $E_1=\dfrac{2k\lambda}{r}=2.73\text{ N/C}.$ The field due to the second wire is given by $E_2=2k\left(0.81\lambda\right)/\left(6.5r\right).$

SIMPLIFY: $E_2 = \frac{2k(0.81\lambda)}{6.5r} = \left(\frac{0.81}{6.5}\right)\frac{2k\lambda}{r} = \left(\frac{0.81}{6.5}\right)E_1$

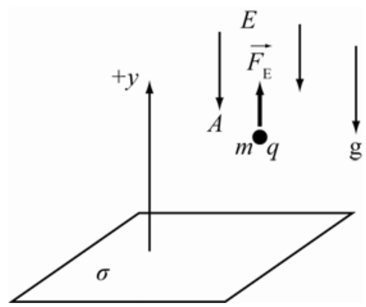
CALCULATE: $E_2 = \left(\frac{0.81}{6.5}\right)E_1 = \left(\frac{0.81}{6.5}\right)(2.73\text{ N/C}) = 0.3402\text{ N/C}$

ROUND: 0.340 N/C

DOUBLE-CHECK: The answer is comparable to the electric field of the original wire which makes it reasonable.

22.81. **THINK:** I want to find the charge, q_1 needed to balance out the force of gravity. After finding q, I can determine the number of electrons based on the charge of a single electron.

SKETCH:



RESEARCH: The net force on the object must equal zero in order for the object to remain motionless. $F_{\text{total}} = F_{\text{gravity}} + F_{\text{coulomb}} = 0$, $F_{\text{gravity}} = -mg$, $F_{\text{coulomb}} = Eq$, $E = \sigma/2\varepsilon_0$ for an infinite plane. The number of electrons is q/q_{electron}.

SIMPLIFY: $F_{\text{total}} = F_{\text{gravity}} + F_{\text{coulomb}} = 0 \Rightarrow F_{\text{total}} = -mg + Eq = 0,\ Eq = mg \Rightarrow \frac{\sigma}{2\varepsilon_0}q = mg \Rightarrow q = \frac{2mg\varepsilon_0}{\sigma}$.

Number of electrons $= \frac{2mg\varepsilon_0}{\sigma q_{\text{electron}}}$. $g = 9.81\text{ m/s}^2$, $\sigma = -3.50\cdot 10^{-5}\text{ C/m}^2$, $m = 1.00$ g.

CALCULATE:

Number of electrons $= \frac{2(1.00\cdot 10^{-3}\text{ kg})(9.81\text{ m/s}^2)(8.85\cdot 10^{-12}\text{ m}^{-3}\text{ kg}^{-1}\text{ s}^4\text{ A}^2)}{(-3.50\cdot 10^{-5}\text{ C/m}^2)(-1.602\cdot 10^{-19}\text{ C})} = 3.097\cdot 10^{10}$ electrons

ROUND: $3.10\cdot 10^{10}$ electrons

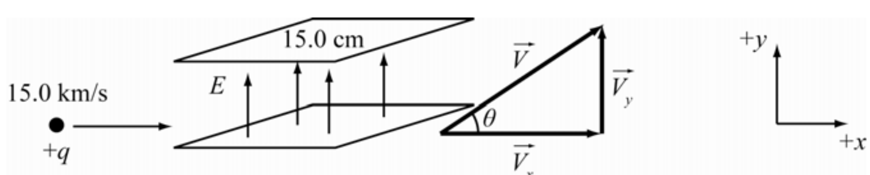
DOUBLE-CHECK: This number, though large, is reasonable since the amount of charge on each electron is tiny.

22.82. **THINK:**

(a) The necessary electric field strength can be determined by finding the acceleration required to achieve the desired deflection. The final speed of the proton can be found through the relation between the proton's initial velocity and its angle of deflection.

(b) The electric field strength required to give the protons a specific acceleration will impart a different acceleration to the kaons due to difference in mass.

SKETCH:



RESEARCH:
(a,b) Initially the velocity in the *y*-direction, v_y, is zero. The only part of the velocity affected by the electric field is v_y, v_x is the same before and after the deflection. $v_y = at$, t is the time the proton spends in between the plates. $F = m_0 a = Eq$, $\tan\theta = v_y / v_x$, $v^2 = v_x^2 + v_y^2$, where v is the new speed. $t = l / v_x$, l is the distance the proton has to traverse between the plates. $\theta = 1.50 \cdot 10^{-3}$ rad, $l = 15.0$ cm, $v_x = 15.0$ km/s.
(c) The mass of a proton $= 1.67 \cdot 10^{-27}$ kg. The mass of a kaon is $= 8.81 \cdot 10^{-28}$ kg. The speed of the kaon is given by setting the momentum of a kaon equal to the momentum of a proton:
$m_{\text{kaon}} v_{\text{kaon}} = m_{\text{proton}} v_{\text{proton}}$.
SIMPLIFY:
(a,b) $v_y = v_x \tan\theta = at \;\Rightarrow\; v_y / t = a$

$$Eq = ma = \frac{mv_y}{t} = \frac{mv_x \tan\theta}{t} = \frac{mv_x \tan\theta}{l / v_x} = \frac{mv_x^2 \tan\theta}{l} \;\Rightarrow\; E = \frac{mv_x^2 \tan\theta}{lq}$$

$$v^2 = v_x^2 + v_y^2 = v_x^2 + \left(v_x \tan\theta\right)^2 = v_x^2\left(1 + \tan^2\theta\right)$$

(c) Take the result from part (a) to find θ.

$$E = \frac{mv_x^2 \tan\theta}{lq} \;\Rightarrow\; \tan\theta = \frac{qEl}{mv_x^2} \;\Rightarrow\; \theta = \tan^{-1}\left(\frac{qEl}{mv_x^2}\right)$$

CALCULATE:
(a) $$E = \frac{\left(1.67 \cdot 10^{-27} \text{ kg}\right)\left(15.0 \cdot 10^3 \text{ m/s}\right)^2 \tan\left(1.50 \cdot 10^{-3} \text{ rad}\right)}{\left(0.150 \text{ m}\right)\left(1.602 \cdot 10^{-19} \text{ C}\right)} = 0.023455 \text{ N/C}$$

(b) $$v = \left(15.0 \cdot 10^3 \text{ m/s}\right)\left[1 + \tan^2\left(1.50 \cdot 10^{-3} \text{ rad}\right)\right]^{1/2} = 15.000017 \text{ km/s}$$

(c) $$v_{\text{kaon}} = \frac{1.67 \cdot 10^{-27} \text{ kg}}{8.81 \cdot 10^{-28} \text{ kg}}\left(15.0 \cdot 10^3 \text{ m/s}\right) = 28434 \text{ m/s}$$

With the results from part (a), the electric field is $E = mv_x^2 \tan\theta / (lq)$.

$$\theta = 1.50 \cdot 10^{-3} \text{rad}, \quad E = \frac{\left(1.67 \cdot 10^{-27} \text{ kg}\right)\left(15.0 \cdot 10^3 \text{ m/s}\right)^2 \tan\left(1.50 \cdot 10^{-3} \text{ rad}\right)}{\left(0.150 \text{ m}\right)\left(1.602 \cdot 10^{-19} \text{ C}\right)} = 0.02345507 \text{ N/C}$$

$$\theta = \tan^{-1}\left(\frac{qEl}{mv_x^2}\right) = \tan^{-1}\left[\frac{\left(1.602 \cdot 10^{-19} \text{C}\right)\left(0.02345507 \text{ N/C}\right)\left(0.150 \text{ m}\right)}{\left(8.81 \cdot 10^{-28} \text{kg}\right)\left(28434 \text{ m/s}\right)^2}\right] = 7.91295 \cdot 10^{-4} \text{rad}$$

ROUND:
(a) $E \approx 0.0235$ N/C
(b) $v \approx 1.50 \cdot 10^4$ km/s
(c) $\theta \approx 7.91 \cdot 10^{-4}$ rad
DOUBLE-CHECK:
The change in speed is small compared to the magnitude of the speed, which is expected since the deflection was also small. The deflection of the kaon is less that the deflection of a proton with the same momentum because the kaon has a higher speed.

22.83. THINK: Using the charge density, Gauss's Law can be used to find the electric field as a function of the radius.
SKETCH: Not required.

RESEARCH: The charge inside a spherical Gaussian surface is given by $q=\rho V_{\text{sphere}}$. $V_{\text{sphere}}=(4/3)\pi r^3$, $\rho=3.57\cdot10^{-6}\text{ C/m}^3$ and $r=0.530$ m. Gauss's Law gives the field $\oiint\vec{E}d\vec{A}=E(4\pi r^2)=q/\varepsilon_0$.

SIMPLIFY: $E(4\pi r^2)=\dfrac{q}{\varepsilon_0}\Rightarrow E=\dfrac{1}{4\pi r^2}\left(\dfrac{q}{\varepsilon_0}\right)=\dfrac{1}{4\pi r^2}\left(\dfrac{\rho V}{\varepsilon_0}\right)=\dfrac{1}{4\pi r^2}\left(\dfrac{\rho(4/3)\pi r^3}{\varepsilon_0}\right)=\dfrac{\rho r}{3\varepsilon_0}$

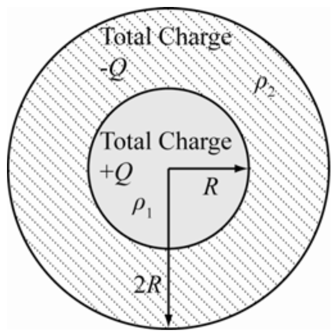
CALCULATE: $E=\dfrac{(3.57\cdot10^{-6})(0.530)}{3(8.85\cdot10^{-12})}\text{ N/C}=7.127\cdot10^4\text{ N/C}$

ROUND: $E=7.13\cdot10^4$ N/C

DOUBLE-CHECK: The result was independent of the actual radius of the sphere as it should be.

22.84. **THINK:** Gauss's Law can be used to determine the electric field as a function of radius for the three cases $r<R$, $R\le r\le 2R$ and $r>2R$.

SKETCH:



RESEARCH: The electric field through the surface of a sphere of radius r is given by Gauss's Law:

$$\oiint\vec{E}\cdot d\vec{A}=E(4\pi r^2)=q/\varepsilon_0.$$

For $r<R$, the enclosed charge is given by:

$$q_1=\int_0^r\rho_1(4\pi r'^2)dr',$$

where

$$\rho_1=\frac{Q}{(4/3)\pi R^3}.$$

For $R\le r\le 2R$, the enclosed charge is given by:

$$q_2=Q+\int_R^r\rho_2(4\pi r'^2)dr',$$

where

$$\rho_2=\frac{-Q}{(4/3)\pi\left((2R)^3-R^3\right)}.$$

For $r>2R$, the enclosed charge is $q_3=Q-Q=0$.

SIMPLIFY:

For $r<R$:

$$q_1=\int_0^r\frac{3Q}{4\pi R^3}(4\pi r'^2)dr'=\frac{3Q}{R^3}\int_0^r r'^2dr'=\frac{3Q}{R^3}\left(\frac{r^3}{3}\right)=\left(\frac{Q}{R^3}\right)r^3$$

$$E_{r<R}(4\pi r^2)=\frac{q_1}{\varepsilon_0}=\left(\frac{Q}{\varepsilon_0R^3}\right)r^3\Rightarrow E_{r<R}=\frac{Qr}{4\pi\varepsilon_0R^3}$$

For $R\le r\le 2R$,

$$q_2 = Q + \int_R^r \rho_2 \left(4\pi r'^2\right) dr' = Q + \int_R^r \left(\frac{-3Q}{28\pi R^3}\right)\left(4\pi r'^2\right) dr' = Q - \frac{3Q}{7R^3}\int_R^r r'^2 dr' = Q - \frac{Q}{7R^3}\left(r^3 - R^3\right)$$

$$E_{R<r<2R} = \left(\frac{1}{4\pi\varepsilon_0 r^2}\right)\left(Q - \frac{Q}{7R^3}\left(r^3 - R^3\right)\right) = \left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)\left(1 - \left(\frac{r^3}{7R^3} - \frac{1}{7}\right)\right) = \left(\frac{Q}{28\pi\varepsilon_0}\right)\left(\frac{8}{r^2} - \frac{r}{R^3}\right)$$

For $r > 2R$: Since the total charge is zero, by Gauss's Law $E_{r>2R} = 0$.

CALCULATE: Not applicable.

ROUND: Not applicable.

DOUBLE-CHECK: It is expected that the expression for $r < R$ and $R < r < 2R$ are equal at $r = R$ and the expressions for $r > 2R$ and $R \le r \le 2R$, are equal at $r = 2R$. For $r = R$:

$$E_{r<R} = \frac{Qr}{4\pi\varepsilon_0 R^3} = \frac{Q}{4\pi\varepsilon_0 R^2}$$

$$E_{R<r<2R} = \left(\frac{Q}{28\pi\varepsilon_0}\right)\left(\frac{8}{r^2} - \frac{r}{R^3}\right) = \left(\frac{Q}{28\pi\varepsilon_0}\right)\left(\frac{8}{R^2} - \frac{R}{R^3}\right) = \frac{Q}{4\pi\varepsilon_0 R^2} = E_{r<R}$$

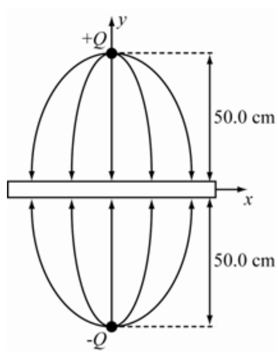
For $r = 2R$:

$$E_{R<r<2R} = \left(\frac{Q}{28\pi\varepsilon_0}\right)\left(\frac{8}{r^2} - \frac{r}{R^3}\right) = \left(\frac{Q}{28\pi\varepsilon_0}\right)\left(\frac{8}{4R^2} - \frac{2R}{R^3}\right) = \left(\frac{Q}{28\pi\varepsilon_0}\right)\left(\frac{2}{R^2} - \frac{2}{R^2}\right) = 0 = E_{r>2R}$$

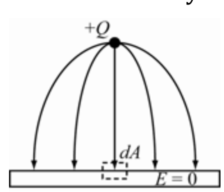
The expressions are equal, so the solution is reasonable.

22.85. **THINK:** The electric field due to the charge induces a charge distribution on the floor below it. As a result, the charge experiences a force directed toward the floor. Since the charge and its 'mirror image" describe a dipole, the electric field lines are perpendicular to the floor. I want to determine the force acting on the charge, the electric field just above the floor, the surface charge density and the total surface charge induced on the floor.

SKETCH:



A Gaussian pill box may be drawn along an infinitesimally small area as follows:



RESEARCH: The electric field due to the charge is given by $E = kq/r^2$, where q is the magnitude of the charge and r is the distance from the charge to the floor. The force experienced by the charge is given by Coulomb's law; $F = \left(1/4\pi\varepsilon_0\right)\left(q_1 q_2 / r^2\right)$. Since the electric field points in the negative y-direction, only the y-contribution from each charge need be found. The y-contribution is given by

$$E=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r^2}\right)\cos\theta,\ \cos\theta=\frac{a}{r}\text{, and } r=\left(a^2+\rho^2\right)^{1/2}.$$

Since the *y*-component from both charges is the same (i.e. since the charges are equal in magnitude), the total electric field is then: $E_{\text{total}}=\frac{2}{4\pi\varepsilon_0}\left(\frac{q}{r^2}\right)\cos\theta$. Using Gauss's Law on the pillbox, $EdA=dq/\varepsilon_0 \ \Rightarrow\ dq=\sigma dA$. The total charge is given by $q=\int_{\substack{\text{infinite}\\\text{plane}}}\sigma dA$.

SIMPLIFY:

(b) $F=\left(\frac{q_1q_2}{4\pi\varepsilon_0\left(2a\right)^2}\right)$

(c) $E_{\text{total}}=\frac{2}{4\pi\varepsilon_0}\left(\frac{Q}{r^2}\right)\cos\theta=\frac{2}{4\pi\varepsilon_0}\left(\frac{Qa}{r^3}\right)=\frac{1}{2\pi\varepsilon_0}\left(\frac{Qa}{\left(a^2+\rho^2\right)^{3/2}}\right)$

(d) $EdA=\frac{dq}{\varepsilon_0}=\frac{\sigma dA}{\varepsilon_0},\ \ E=\frac{\sigma}{\varepsilon_0}$ and $\sigma=E\varepsilon_0=\frac{1}{2\pi}\left(\frac{Qa}{\left(a^2+\rho^2\right)^{3/2}}\right)$

(e) $q=\int\sigma dA\int_0^\infty\left(\frac{\rho}{2\pi}\right)\left(\frac{Qa}{\left(a^2+\rho^2\right)^{3/2}}\right)dp=2\pi\int_0^\infty\left(\frac{\rho}{2\pi}\right)\left(\frac{Qa}{\left(a^2+\rho^2\right)^{3/2}}\right)d\rho=\frac{aQ}{2}\left(-2\right)\left[\left(a^2+\rho^2\right)^{-1/2}\right]_0^\infty=Q$

CALCULATE:

(b) $F=\frac{\left(1.00\cdot10^{-6}\text{ C}\right)\left(-1.00\cdot10^{-6}\text{ C}\right)}{4\pi\left(8.85\cdot10^{-12}\text{C}^2/\text{N m}^2\right)\left(1.00\text{ m}\right)^2}=-8.9918\cdot10^{-3}\text{ N}$

(c) Not applicable.
(d) Not applicable.
(e) Not applicable.

ROUND:

To three significant figures:

(b) $F=8.99\cdot10^{-3}$ N downward

DOUBLE-CHECK:

(a) The sketch is symmetric as it should be.
(b) The force is downward as it should be since the positive charge is attracted to the negative charge.
(c) The field gets weaker as ρ gets larger as expected since the source is farther away with increasing ρ.
(d) The surface charge density gets smaller as ρ gets larger since the source is farther away with increasing ρ.
(e) Since all the field lines coming from the charge go onto the top of the slab it is not unreasonable that the total charge induced is equal to the charge in magnitude.

Multi-Version Exercises

Exercises 22.86–22.88 The electric field a distance *d* from the wire is $E=\frac{2k\lambda}{d}$. The force is then $F=qE=\frac{e2k\lambda}{d}$. From Newton's Second Law we have $F=ma=\frac{e2k\lambda}{d}$. So the acceleration is $a=\frac{e2k\lambda}{md}$.

22.86. $$a=\frac{e2k\lambda}{md}=\frac{2\left(1.602\cdot 10^{-19}\text{ C}\right)\left(8.99\cdot 10^{9}\text{ N m}^2/\text{C}^2\right)\left(2.849\cdot 10^{-12}\text{ C/m}\right)}{\left(1.673\cdot 10^{-27}\text{ kg}\right)\left(0.6815\text{ m}\right)}=7.198\cdot 10^{6}\text{ m/s}^2$$

22.87. $$a=\frac{e2k\lambda}{md}$$
$$\lambda=\frac{amd}{2ek}=\frac{\left(1.111\cdot 10^{7}\text{ m/s}^2\right)\left(1.673\cdot 10^{-27}\text{ kg}\right)\left(0.6897\text{ m}\right)}{2\left(1.602\cdot 10^{-19}\text{ C}\right)\left(8.99\cdot 10^{9}\text{ N m}^2/\text{C}^2\right)}=4.451\cdot 10^{-12}\text{ C/m}$$

22.88. $$a=\frac{e2k\lambda}{md}$$
$$d=\frac{2ek\lambda}{ma}=\frac{2\left(1.602\cdot 10^{-19}\text{ C}\right)\left(8.99\cdot 10^{9}\text{ N m}^2/\text{C}^2\right)\left(6.055\cdot 10^{-12}\text{ C/m}\right)}{\left(1.673\cdot 10^{-27}\text{ kg}\right)\left(1.494\cdot 10^{7}\text{ m/s}^2\right)}=0.6978\text{ m.}$$

Exercises 22.89–22.91 The magnitude of the electric field at the center due to a differential element $d\ell$ is $dE=\frac{k\lambda d\ell}{R^2}$. The *x*-components add to zero, leaving only a field in the *y*-direction. The *y*-component is $dE_y=\frac{k\lambda d\ell}{R^2}\sin\theta$. Taking $d\ell=Rd\theta$ we have $dE_y=\frac{k\lambda R}{R^2}\sin\theta d\theta=\frac{k\lambda}{R}\sin\theta d\theta$. We integrate from 0 to π to get the magnitude of the electric field:
$$\int_0^{\pi}\frac{k\lambda}{R}\sin\theta d\theta=-\frac{k\lambda}{R}\left[\cos\theta\right]_0^{\pi}=2\frac{k\lambda}{R}=\frac{2\pi k\lambda}{L}.$$
So $E=\frac{2\pi k\lambda}{L}$.

22.89. $$E=\frac{2\pi k\lambda}{L}=\frac{2\pi\left(8.99\cdot 10^{9}\text{ N m}^2/\text{C}^2\right)\left(5.635\cdot 10^{-8}\text{ C/m}\right)}{\left(0.2213\text{ m}\right)}=1.438\cdot 10^{4}\text{ N/C}$$

22.90. $$E=\frac{2\pi k\lambda}{L}$$
$$\lambda=\frac{EL}{2\pi k}=\frac{\left(3.117\cdot 10^{4}\text{ N/C}\right)\left(0.1055\text{ m}\right)}{2\pi\left(8.99\cdot 10^{9}\text{ N m}^2/\text{C}^2\right)}=5.822\cdot 10^{-8}\text{ C/m}$$

22.91. $$E=\frac{2\pi k\lambda}{L}$$
$$L=\frac{2\pi k\lambda}{E}=\frac{2\pi\left(8.99\cdot 10^{9}\text{ N m}^2/\text{C}^2\right)\left(6.005\cdot 10^{-8}\text{ C/m}\right)}{2.425\cdot 10^{4}\text{ N/C}}=0.1399\text{ m}=13.99\text{ cm}$$