Equations for Calculating Recovery of Soluble Values in a Countercurrent Decantation Washing System

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Abstract

Mud washing variables are equated in a manner which simplifies the calculation of multistage countercurrent systems. Imperfect mixing is accommodated. In a simplified case, the number of stages becomes the exponent in the term which sets the ratio of concentration differences at the terminals.

A convenient method of calculating multistage mud washing systems is described. This procedure is an outgrowth of early surveys of Bayer plant soda losses. These pointed to the mud washing system as one of the major exits, and a unit process, in theory amenable to considerable improvement. Numerical methods of approach available at the time (1948) were not only cumbersome but also inadequate to cover the observation that imperfect mixing occurs to some degree in an operating washer. On the latter point, the present treatment differs substantially from the several calculation methods now available in the literature.[1–4]

The need for improved computing procedures was met by adapting viewpoints and methods from somewhat analogous fractionation calculations.[5,6]

In the method to be taken up, a term for mixing efficiency is incorporated into the formulas in such manner that dealing with fractional stages or interpolating for concentrations is avoided. The equations carry a sufficient number of terms to represent the actual physical situation observed in an operating unit. The system has been extensively used both in economic studies of mud washing in the conventional Bayer process and in outlining numerous alternative arrays for the singular mud and dilution requirements of Jamaica bauxite operation.

Opportunity for further exploitation exists both in refining the constants and in extension of the understanding of the interplay of washing variables.

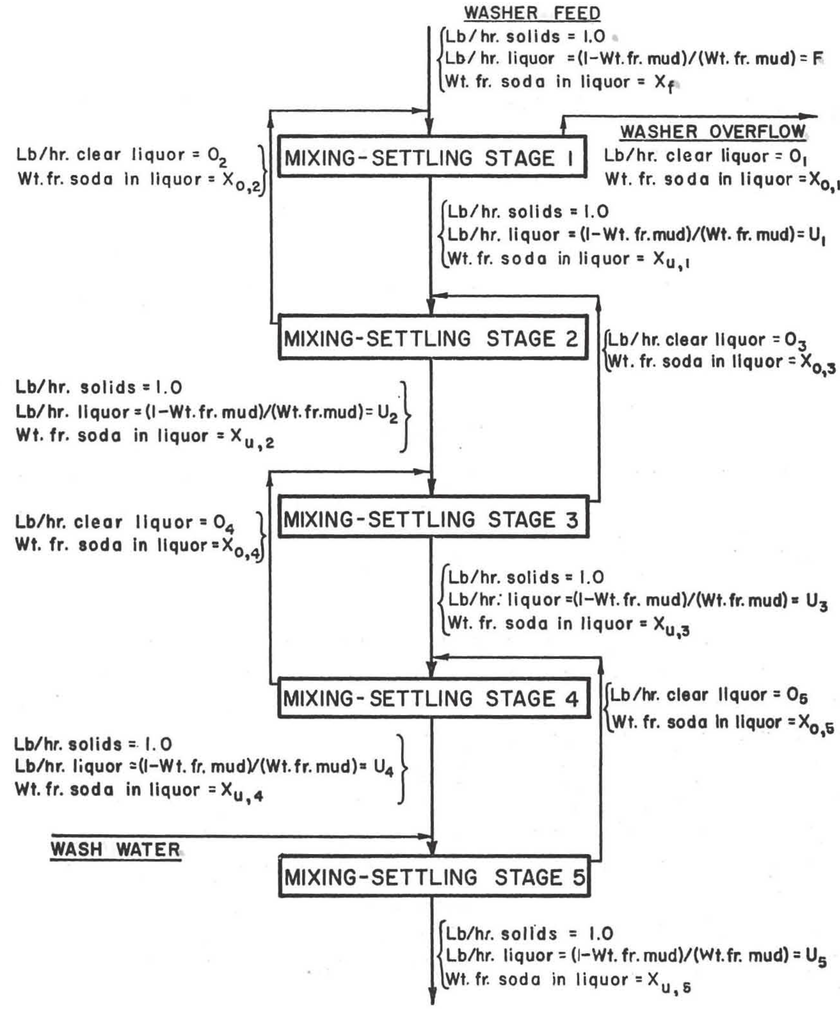

Fig. 1. Five-stage countercurrent washer.

Figure 1 shows a flow diagram of a five-stage countercurrent washer. Solids and liquid, in their settled proportions, move downward from each stage, meeting clarified and more dilute overflow from the indicated stage below. A numerical description of the situation is desired in a form readily adaptable to engineering purposes.

One can select any washing stage from a continuous system and take balance simultaneously on total liquid and total soluble material, to derive a general relationship which will apply to any number of stages in series.

The stages are numbered from top to bottom where n is the number of stages; $x_{u,n}$ is the weight fraction, soluble component in the underflow liquid leaving the nth stage; $X_{0,n+1}$ is the weight fraction, soluble component in the wash liquid to stage n from stage $n+1$; x_f is the weight fraction soluble component in the washer feed; E_n is the mixing efficiency of stage n; U_n is the lb liquid/lb dry mud feed in underflow of stage n; O_n is the lb liquid/lb dry mud feed in overflow from stage n; F is the lb liquid/lb dry mud in washer feed; and B_n is the value of bracketed term; material balance, any stage, use stage n.

Liquid:

$$U_{n-1} + O_{n+1} = U_n + O_n \qquad (1)$$

Soluble:

$$x_{n,n-1}\, U_{n-1} + X_{0,n+1}\, O_{n+1} = x_{u,n}\, U_n + X_{0,n} O_n \qquad (2)$$

If there were perfect mixing of the streams which enter stage n, the concentrations of the effluent streams U_n and O_n would be equal. But it can be observed that $x_{u,n}$ and $X_{0,n}$ are different. One can easily speculate that not every particle of solids meets its fair share of wash water or that there may be diffusion-controlled steps to cause this phenomenon. To describe the degree of completion of the mixing process, a counterpart of the Murphree fractionation plate efficiency is devised as:

$$E = (x_{u,n-1} - x_{u,n})/(x_{u,n-1} - X_{0,n}) \qquad (3)$$

Combine eqs. (A1), (A2), and (A3) in the Appendix to get

$$x_{u,n} - X_{0,n+1} = \left\{E_n\left(\frac{U_n}{O_{n+1}} - 1\right) + \frac{O_n}{O_{n+1}}\right\}(x_{u,n-1} - X_{0,n}) \qquad (4)$$

By similar procedures a balance on stage $n+1$ (the next stage below n), the subscripts will be increased by 1.

$$x_{u,n+1} - X_{0,n+2} = \left\{E_{n+1}\left(\frac{U_{n+1}}{O_{n+2}} - 1\right) + \frac{O_{n+1}}{O_{n+2}}\right\}\left(x_{u,n} - X_{0,n+1}\right)$$

It follows that

$$x_{u,n+1} - X_{0,n+2} = \left\{E_n\left(\frac{U_n}{O_{n+1}} - 1\right) + \frac{O_n}{O_{n+1}}\right\}\left\{E_{n+1}\left(\frac{U_{n+1}}{O_{n+2}} - 1\right) + \frac{O_{n+1}}{O_{n+2}}\right\}\left(x_{u,n-1} - X_{0,n}\right)$$

More to the point, the ratio of concentration differences at the terminals of any washing system is set by the product of bracketed terms of which there is one for each stage. Or,

$$\frac{\text{Final underflow concn.-wash concn.}}{\text{Feed concn.-overflow concn.}} = B_1\, B_2\, B_3 \ldots$$

where the B's are the bracketed terms.

A remaining consideration is that there must be an overall balance at the terminals which is expressed

$$Fx_f + O_{n+1}\, x_{n+1} = O_1 X_{0,1} + U_{n}\, x_{u,n}$$

The general case is covered by

$$x_{u,n} - X_{0,n+1} = \left[E_1\left(\frac{U_1}{O_2}\right) - 1\right) + \frac{O_1}{O_2}\right]\left[E_2\left(\frac{U_2}{O_3} - 1\right) + \frac{O_2}{O_3}\right] \ldots \qquad (5)$$

$$\left[E_n\left(\frac{U_n}{O_{n+1}} - 1\right) + \frac{O_n}{O_{n+1}}\right](x_f - X_{0,1})$$

$$Fx_f + O_{n+1}\, x_{n+1} = O_1\, X_{0,1} + U_n\, x_{u,n} \qquad (6)$$

In the special case where both mud density and the mixing efficiency are constant at all stages, the concentration relationship becomes:

$$x_{u,n} - X_{0,n+1} = \left\{E\left(\frac{U}{O} - 1\right) + 1\right\}^n\left(x_f - X_{0,1}\right) \qquad (5a)$$

If the mud density is constant and the mixing perfect, there is:

$$x_{u,n} - X_{0,n+1} = \left(\frac{U}{O}\right)^n\left(x_f - X_{0,1}\right) \qquad (5b)$$

The evaluation of the mixing efficiency of each stage has been difficult, and the results uncertain because of the necessity to get representative samples, particularly of per cent solids, from the washer boots. The boot is a relatively small diameter central well through which the solids descend from one deck of a stacked deck array to the deck below. A single effort in this direction will be reported later.

The washer feed and discharge however are easy to get at, and a workable approximation of an overall value is made by pretending that the per cent solids on each stage is equal to that of the bottom stage. A value of E, specific to these conditions, is solved for by using eqs. (5) and (6).

The data given in Table I were taken from a five-stage washing system, working near maximum capacity on mud residues from Surinam bauxite. The facilities consist of a stacked deck array, balanced hydrostatically. The mud phase meets the upcoming wash liquids in the boots and mixes without any mechanical stirring action.

Other operating periods, under less critical conditions have shown E values as high as 0.93, calculated in the same way as shown in Table I.

TABLE I

Five-Stage Washing System Data

n	$= 5$	
$x_{u,5}$	$= 0.02726$	wt fraction solubles in underflow
$X_{0,6}$	$= 0.01613$	wt fraction solubles in wash water
x_f	$= 0.19213$	wt fraction solubles in feed
X_{01}	$= 0.12323$	wt fraction solubles in overflow
$U_{1...5}$	$= 3.375$	washer underflow liquid per unit weight of solids
O_1	$= 6.199$	top stage overflow liquid per unit weight of solids
$O_{2...6}$	$= 5.589$	overflow stages 2...5, wash water per unit weight of solids

$$\frac{0.02726 - 0.01613}{0.19213 - 0.12323} = \left\{E\left(\frac{3.375}{5.589} - 1\right) + \frac{6.199}{5.589}\right\}\left\{E\left(\frac{3.375}{5.589} - 1\right) + 1\right\}^4$$

$E = 0.82$

During a period of experimental 10 stage washing, the figure dropped to 0.67.

After the revision of the plant to handle Jamaica bauxite, at much higher dilution ($O - U$), the E value was calculated at various times: 0.786, 0.80, 0.85, and 0.67.

This manipulation to arrive at a mixing term is artificial in that the per cent solids have been assumed equal at all stages. The E value is specific to that assumption, and it is conceivable that the intermediate decks suffer because of low per cent solids rather than poor mixing. This issue has not been particularly important in calculating losses since the products of the bracketed terms have turned out about as expected in an entirely new situation.

But the matter of discriminating between the action of E and per cent solids does become important in casting about for means of improving an existing system.

The special set of data given in Table II show an effort to evaluate the per cent solids from direct measurements of E. A mechanical sampling device was contrived to reach into the boot and cut a core from the downflowing solids. At the same time samples of the clear overflows were taken for analysis. In this case it is quite probable that the liquor samples are good but doubtful that the solid samples are representative.

TABLE II

Evaluation of Solids from Direct Measurements of E[a]

Underflows	Overflows	E	Wt. fraction solids
x_f 0.20445			0.131
	$X_{0,1}$ 0.13487	$E_1 = 0.932$	
$x_{u,1}$ 0.13956			*(0.195)*
	$X_{0,2}$ 0.07766	$E_1 = 0.929$	
$x_{u,2}$ 0.082063			*(0.140)*
	$X_{0,3}$ 0.05437	$E_3 = 0.948$	
$x_{u,3}$ 0.055802			*(0.175)*
	$X_{0,4}$ 0.03484	$E_4 = 0.872$	
$x_{u,4}$ 0.03753			*(0.171)*
	$x_{0,5}$ 0.02467	$E_5 = 0.977$	
$x_{u,5}$ 0.02496			0.232
	$X_{0,6}$ 0.01585		

[a] Values for E come from applying eq. (3), and values for weight fraction of solids in parentheses are derived via material balance requirements of eqs. (5) and (6).

Evidence indicates that some stages in a series may be faulty either in per cent solids or in poor mixing and the nature of the cure will depend on which dominates.

Use of the equations will be illustrated by computing losses from a six-stage washing system with the following conditions: Mud rate, 1; mud feed enters at 12% solids; mud density 15% solids on upper five decks; mud density 20% solids on bottom (6th) deck, wash water—underflow liquid = 6 lb/lb mud; $x_f = 0.18$; $X_{0,7} = 0$; and $E = 0.82$.

Liquor flows:

$$F = 1\ (1\text{–}0.12)/0.12 = 7.333$$

$$O_1 = 7.333 + 6 = 13.333$$

$$U_1 = U_2 = U_3 = U_4 = U_5 = 1\ (1\text{–}0.15)/0.15 = 5.666$$

$$O_2 = O_3 = O_4 = O_5 = O_6 = 5.666 + 6 = 11.666$$

$$U_6 = 1\ (1\text{–}0.20)/0.20 = 4$$

$$O_7 = 4 + 6 = 10 = \text{wash water}$$

Terminal concentrations via eq. (5):

$$x_{u,6} - 0 = (B_1B_2B_3B_4B_5B_6)(0.18 - X_{0,1})$$

$$B_1 = E_1\left(\frac{U_1}{O_2} - 1\right) + \frac{O_1}{O_2} = 0.82\left(\frac{5.666}{11.666} - 1\right) + \frac{13.333}{11.666} = 0.72115$$

$$B_{2,5} = E_2\left(\frac{U_2}{O_3} - 1\right) + \frac{O_2}{O_3} = 0.82\left(\frac{5.666}{11.666} - 1\right) + \frac{11.666}{11.666} = 0.57826$$

$$B_6 = E_6\left(\frac{U_6}{O_7} - 1\right) + \frac{O_6}{O_7} = 0.82\left(\frac{4}{10} - 1\right) + \frac{11.666}{10} = 0.67460$$

$$x_{u,6} - 0 = B_1\,(B_2)^4\,B_6\,(0.18 - X_{0,1})$$

$$x_{u,6} - 0 = 0.05439\,(0.18 - X_{0,1})$$

Overall balance, soluble component:

$$Fx_f + O_7X_{0,7} = O_1X_{0,1} + U_6x_{u,6}$$

$$7.33\,(0.18) + 0 = 13.33\,X_{0,1} + 4.0\,x_{u,6}$$

Solve both functions of $X_{0,1}$ and $x_{u,6}$ to get

$$X_{0,1} = 0.09766$$

$$x_{u,6} = 0.004480$$

Loss per lb mud $= U_6\,x_{u,6} - 0 = 4.0\,(0.004480) = 0.01791.$

If there are four stages instead of six, $(B_2)^4$ in the foregoing example is replaced by $(B_2)^2$, and

$$x_{u,4} - 0 = B_1B_2^2\,B_4\,(0.18 - X_{0,1})$$

$$x_{u,4} - 0 = 0.16267\,(0.18 - X_{0,1})$$

$$7.33\,(0.18) + 0 = 13.33\,X_{0,1} + 4.0\,x_{u,4}$$

$$X_{0,1} = 0.094842$$

$$x_{u,4} = 0.013853$$

Loss per lb mud $= 4(0.013853) = 0.05541.$

In the Bayer operation, it is common to have a more or less dilute side stream to dispose of. This can be fed to the washer as a secondary feed and its effect can be computed by applying the equations first to the group of stages above its point of entry, then to the group below, bridging the two sections by an appropriate material balance. For example,

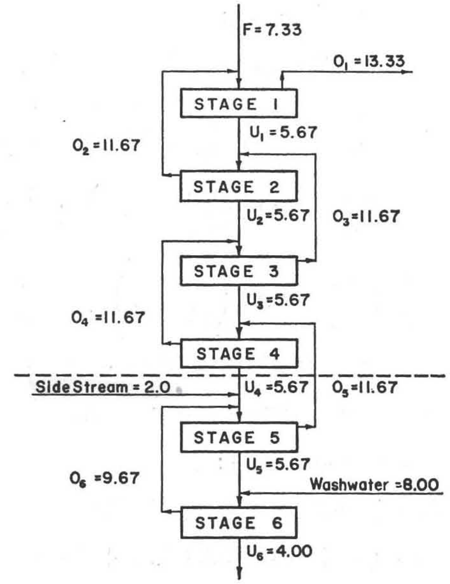

Fig. 2. Six-stage countercurrent washer with side stream.

ssume that in the six-stage system, a side stream of 2 lb of liquid at 0.01 oncentration is mixed with the underflow from stage 4, as in Figure 2.

The complication of the additional stream is handled by calling the oncentration x_a, in the mixture of the stage 4 underflow with the side tream.

Table III gives the liquid flows

TABLE III
Liquid Flows

F	7.333		
		O_1	13.33
U_1	5.666		
		O_2	11.666
U_2	5.666		
		O_3	11.666
U_3	5.666		
		O_4	11.666
U_4	5.666		
		O_5	11.666
U_4 + side stream	= 5.666 + 2		
U_5	5.666		
		O_6	9.666
U_6	4		
		O_7	8 (washwater)

Stages One through Four

Equation (5):

$$x_{u,4} - X_{0,5} = \left\{0.82\left(\frac{5.666}{11.666} - 1\right) + \frac{13.333}{11.666}\right\}\left\{0.82\left(\frac{5.666}{11.666} - 1\right) + \frac{11.666}{11.666}\right\}^3\left\{0.18 - X_{0,1}\right\} = 0.13944(0.18 - X_{0,1})$$

Equation (6):

$$7.33(0.18) + 11.666X_{0,5} = 5.666x_{u,4} + 13.33X_{0,1}$$

$$(a) \qquad x_{u_4} = 1.9510X_{0,1} - 0.17119$$

Stages Five through Six

Equation (5):

$$x_{u_6} - 0 = \left\{0.82\left(\frac{5.666}{9.666} - 1\right) + \frac{11.666}{9.666}\right\}\left\{0.82\left(\frac{4}{8} - 1\right) + \frac{9.666}{8}\right\}(x_a - X_{0,5}) = 0.69254\ (x_a - X_{0_5})$$

Equations (6):

$$7.666\ x_a = 11.666X_{0_5} + 4.0x_{u_6}$$

$$(b) \qquad x_a = 3.2113x_{u_6}$$

Overall balance on stages 4 and 5:

$$5.666\ x_{u_4} + 2(0.01) = x_a(5.666 + 2)$$

$$(c) \qquad x_a = 0.73910\ x_{u_4} + 0.002608$$

Eliminate x_a; (*b*) and (*c*):

$$(d) \qquad 0.7391\ x_{u,4} + 0.002608 = 3.2113\ x_{u_6}$$

Eliminate $x_{u,4}$; (*d*) and (*a*):

$$(e) \qquad X_{0_1} = 2.2270\ x_{u_6} + 0.085938$$

Balance total soluble matter into and out of the system:

$$7.33(0.18) + (2.0)(0.01) = 13.33X_{0,1} + 4.0x_{u,6}$$

$$(f) \qquad X_{01} = 2.2270\ x_{u6} + 0.085938$$

$$x_{u,6} = 0.005762$$

Loss, lb/lb mud $= 4(0.00576) = 0.0230$.

$$X_{0,1} = 0.098769$$

$$x_a = 0.018502$$

$$X_{0,5} = 0.01018$$

$$x_{u,4} = 0.02151$$

Numerous special calculations of this type have indicated that the minimum total loss will occur when the side stream is fed with an overflow of equal concentration.

Additional uses for this calculation system suggest themselves.

One of these of mathematical interest has been in investigating the unsteady state induced by operating with a mud lake. In this case the rainfall may largely govern concentration of the final stage and the liquid inventory may swing at the same time.

Numerical forms are such that the calculus of this situation may nevertheless be handled. One may readily infer that the necessary assumption of rainfall rate has a perceptible leverage on such calculations. While it is obvious without any figuring that there is an optimum lake area for a particular mud rate-rainfall situation, it would be difficult to approximate a formal solution without the aid of arithmetic short cuts.

The major application has been in tracing out quantitatively the partials of dilution $(O-U)$, number of stages, mud density, and mixing efficiency as a portion of the problem of Bayer plant expansion.

Appendix

Derivation of eq. (4).

Liquid:

$$U_{n-1} + O_{n+1} = U_n + O_n \qquad \text{(A1)}$$

Soluble:

$$x_{u,n-1}\ U_{n-1} + X_{0,n+1}O_{n+1} = x_{u,n}U_n + X_{0,n}O_n \qquad \text{(A2)}$$

Mixing efficiency:

$$E_n = (x_{u,n-1} - x_{u,n})/(x_{u,n-1} - X_{0,n}) \qquad \text{(A3)}$$

In eq. (A2), substitute $U_{n-1} = U_n + O_n - O_{n+1}$ and divide both sides by O_{n+1}.

$$\frac{U_n}{O_{n+1}}(x_{u,n-1} - x_{u,n}) + \frac{O_n}{O_{n+1}}(x_{u,n-1} - X_{0,n}) = x_{u,n-1} - X_{0,n+1}$$

Substitute $E_n(x_{u,n-1} - X_{0,n})$ for $(x_{u,n-1} - x_{u,n})$

$$\frac{U_n}{O_{n+1}} E_n(x_{u,n-1} - X_{0,n}) + \frac{O_n}{O_{n+1}}(x_{u,n-1} - X_{0,n}) = x_{u,n-1} - X_{0,n+1}$$

Subtract $E_n(x_{u,n-1} - X_{0,n})$ from the left side and subtract $(x_{u,n-1} - x_{u,n})$ from the right side to get

$$\frac{U_n}{O_{n+1}} E_n(x_{u,n-1} - X_{0,n}) + \frac{O_n}{O_{n+1}}(x_{u,n-1} - X_{0,n}) - E_n(x_{u,n-1} - X_{0,n}) = x_{u,n-1} - X_{0,n+1} - (x_{u,n-1} - x_{u,n})$$

or

$$x_{u,n} - X_{0,n+1} = \left\{E_n\left(\frac{U_n}{O_{n+1}} - 1\right) + \frac{O_n}{O_{n+1}}\right\}(x_{u,n-1} - X_{0,n}) \quad \text{(A4)}$$

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