Norman Schofield · Gonzalo Caballero · Daniel Kselman *Editors*

Advances in Political Economy

Institutions, Modelling and Empirical Analysis

This book presents latest research in the field of Political Economy, dealing with the integration of economics and politics and the way institutions affect social decisions. The focus is on innovative topics such as an institutional analysis based on case studies; the influence of activists on political decisions; new techniques for analyzing elections, involving game theory and empirical methods.

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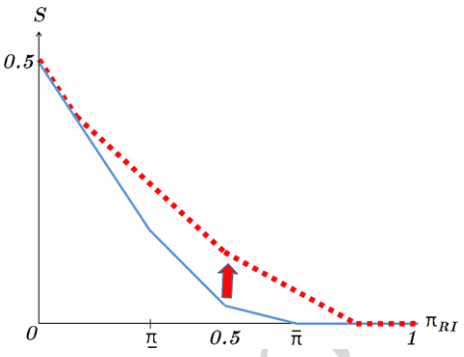

Fig. 4 The effect of increasing the quality of signals q on the primary skill bonus S

The result goes in the expected direction for moderate priors about the insider candidate's skill. For intermediate values of the prior π_{RI}, marginal increases in q will indeed increase S. The reason is that primary voters are unsure about the relative merits of the insider candidate compared to the unknown outsider that will join the race. They will pay close attention to the primary campaigns to nominate the candidate with a better performance. A higher quality of the information revealed will increase the probability of making the right nomination choice. Such an increasing effect is depicted in Fig. 4.

However, for other priors, the quality of a primary elections will bear no impact on its benefit. When the insider candidate is expected to be overwhelmingly competent in the general election, she will be nominated even if her performance in the primary is appalling. Primary voters will trust that her performance in the primary was due to bad luck. On the other hand, when the insider candidate is expected to be overwhelmingly unqualified, she will lose to the outsider candidate even if her performance was better. Primary voters will believe her performance was just a fluke that does not justify giving her a chance in the general election. In sum, for extremely high or extremely low values of π_{RI}, primary voters quickly make up their minds, either to nominate *RI* for sure or to nominate *RO* for sure, regardless of any campaign events that may occur. Improving the quality of primaries by marginally increasing q will have no effect on this decision.

In sum, primaries have two potential benefits: (1) allowing primary voters to replace the insider candidate with an outsider candidate whose prospect are believed to be superior; and (2) using new information revealed during the primary campaigns to discriminate between both candidates. As it turns out, whether those benefits actually occur depends crucially on the prior beliefs about the campaigning skill of the insider candidate. This finding is qualitatively summarized in Table 3.

To summarize this section, the benefit, when there is one, of primary elections is a larger probability of nominating a candidate with a high campaigning skill. I called that extra probability the primary skill bonus. Primaries might carry a cost however, in terms of the policy that candidates are induced to adopt. That cost is described in detail in the following section. As a consequence, the party leadership needs to carry out a cost-benefit analysis when choosing whether to hold a primary election or not.

Table 3 The two potential benefits of a primary election as a function of π_{RI}

Expectation that *RI* is high-skilled, π_{RI}	Benefit of primaries		
	Replacing *RI* with *RO*	Using the information revealed during the primary	Skill bonus of a primary *S*
Low	Yes, for sure	No, information ignored	High
Intermediate	Yes, probably	Yes, taken into account	Low
High	No, never	No, information ignored	Zero

6 The Cost of Primary Elections

As we just saw, the benefit to party leaders of adopting a competitive primary election is to increase the expected skill of their nominee. However, primaries might carry a cost in terms of the policy that candidates are induced to adopt. To be precise, a primary election has two differences with respect to an elite endorsement: first, the probability that R's nominee is high-skilled increases from π_{RI} to $\pi_{RI} + S$. And second, it would be *RM* and not *RE* that R's candidate would have made policy commitments to; and thus it would be the RAF's preferences rather than the leadership's preferences which would determine R's policy platform.

By glancing at Table 4, we can readily see the trade-off that R's leadership faces in choosing a primary election over an elite endorsement. As a benefit, using a primary increases the probability of nominating a high-skilled candidate (due to the primary skill bonus S). As a cost, the payoff from having the highest skilled candidate decreases (due to the internal divergence $X_{RM} - X_{RE}$). Put differently, *a primary makes losing less likely but makes winning less attractive*.

The goal now is to find expressions for the expected utility of R's leadership by choosing either a primary election or an elite selection. I call $EU_{RE}(m_R)$ the expected utility of R's leadership from adopting m_R as its CSM. It can be derived from Theorem 1, which gives the outcomes of the election depending on the value $\Delta v \equiv v_R - v_L$. If L's candidate has a skill advantage, she will announce the platform X_L and she will win the election. If R's candidate has a skill advantage, she will announce the platform X_{RE} if she was nominated by an elite appointment or she will announce X_{RM} if she was nominated by a primary election; and either way she will win the election. If L's candidate and R's candidate have the same skill, they will both announce the platform 0 and they will tie in the election. These considerations lead to the following expressions for $EU_{RE}(m_R)$.

Table 4 The trade-off faced by party R's elite

	Probability that R wins the election	Utility of *RE* if R wins the election
Elite selection	π_{RI}	0
Primary election	$\pi_{RI} + S$	$-\lvert X_{RE} - X_{RM} \rvert$

Lemma 5 *The expected utility of R's leadership for each value of m_R is*

$$\begin{aligned} EU_{RE}(m_R = elite) = & -(X_{RE} - X_L)\pi_L(1 - \pi_{RI}) \\ & -(X_{RE} - 0)\big[\pi_L\pi_{RI} + (1 - \pi_{RI})\big] \\ & -(X_{RE} - X_{RE})(1 - \pi_L)\pi_{RI} \\ EU_{RE}(m_R = primary) = & -(X_{RE} - X_L)\pi_L\big(1 - (\pi_{RI} + S)\big) \\ & -(X_{RE} - 0)\big[\pi_L(\pi_{RI} + S) + (1 - \pi_L)\big(1 - (\pi_{RI} + S)\big)\big] \\ & -|X_{RE} - X_{RM}|(1 - \pi_L)(\pi_{RI} + S) \end{aligned}$$

Armed with these results, the leadership in party R can measure the consequences of choosing one CSM over the other.

7 The Optimal Selection of a CSM

The leadership in party R will choose the optimal rule m_R by comparing $EU_{RE}(m_R = elite)$ and $EU_{RE}(m_R = primary)$. It will choose the CSM that yields the highest expected utility, and if it is indifferent, I assume that it will choose an elite selection. A primary will be adopted if and only if $EU_{RE}(m_R = elite) < EU_{RE}(m_R = primary)$. That condition leads to the following result, recalling that $d_R \equiv |X_{RM} - X_{RE}|$.

Theorem 3 *The leadership of party R will adopt a primary election if and only if*

$$d_R < T$$

with $T \equiv \frac{S[X_{RE}(1-\pi_L) - X_L\pi_L]}{(1-\pi_L)(\pi_{RI}+S)}$.

The intuition behind this result is that R's leadership will delegate the nomination if and only if the RAF's ideology is close enough to its own. In other words, internal party democratization will only ensue from enough elite-mass congruence. How close do primary voters need to be to the party elite? It depends on a certain threshold, T, introduced in the theorem. If the preferences of the elite and the mass of party R are so incongruent that $T \leq d_R$ then the leadership will not adopt a primary election. This could happen for two reasons. On one hand, the RAF could be so far on the right of the leadership that $X_{RE} + T \leq X_{RM}$. In that case the leadership will not adopt a primary election because the primary voters are too *extremist*. On the other hand, the RAF could be so far on the left of the leadership that $X_{RM} \leq X_{RE} - T$. In that case the leadership will not adopt a primary election because the primary voters are too *centrist*.

As it turns out, the first reason (that primary voters might be too extreme) is frequently found in some way or another in scholarly comments about primary elections. Yet the second reason (that primary voters might be too moderate) is equally

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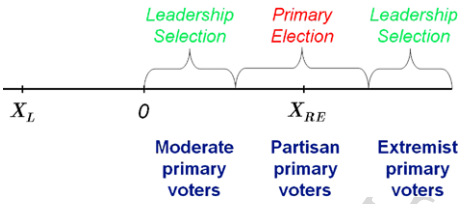

Fig. 5 The candidate-selection method as a function of the ideal point of the median primary voter, X_{RM}

intuitive but is seldom mentioned in the existing literature. The same intuition can be obtained from Fig. 5. For low values of X_{RM} (which I label "moderate primary voters") the party will endorse an insider candidate. For intermediate values of X_{RM} (which I label "partisan primary voters") the party will hold a competitive primary election. For high values of X_{RM} (which I label "extremist primary voters") the party will endorse an insider candidate. Consequently, the CSM has a non-monotonic relationship with the ideal point of the median primary voter.

From the results above it is clear that the threshold T determines how likely primary elections are. The interval $(X_{RE} - T, X_{RE} + T)$ corresponds to the values that X_{RM} should take for the nomination to be delegated to party members. Such interval can therefore be interpreted as the *likelihood that R will adopt a primary*. For a larger T it is more "likely" that the internal divergence between R's establishment and RAF will be lead to a primary. Then a way of phrasing the previous theorem is that *the likelihood of opening the CSM decreases with the internal divergence between the party's leadership and the primary voters*.

7.1 Comparative Statics

We would like to gain insight on what makes the adoption of primary elections more likely. According to the previous theorem, the likelihood of adopting a primary is given by T. Hence, I study how T changes with the parameters in the model. As it turns out, the results will crucially depend on the value of π_{RI}. To be specific, I need to divide two cases. The first case is $\pi_{RI} \in (0, \overline{\pi})$ corresponding to low and intermediate priors, and the second case is $\pi_{RI} \in [\overline{\pi}, 1)$ corresponding to high priors. Recall that $\underline{\pi}$ and $\overline{\pi}$ refer to two constants whose values are $\underline{\pi} \equiv \frac{(1-q)^2}{1-2q+2q^2}$ and $\overline{\pi} \equiv \frac{q^2}{1-2q+2q^2}$.

I start with low and intermediate prior beliefs about the skill of the insider candidate, which corresponds to the situation where primaries are most attractive.

Theorem 4 *Suppose the initial expectation that RI is high-skilled, π_{RI}, is such that $\pi_{RI} \in (0, \overline{\pi})$. Then the threshold T, which determines the likelihood of primaries, is*:

1. *Strictly positive*
2. *Strictly increasing with S*

3. *Strictly decreasing with* π_{RI}
4. *Strictly increasing with* q *if* $\pi_{RI} \in [\underline{\pi}, \overline{\pi})$, *and insensitive to* q *otherwise*
5. *Strictly increasing with* π_L
6. *Strictly decreasing with* X_L
7. *Strictly increasing with* X_{RE}.

The first two results of this theorem corroborate the benefit of primaries. First, I find that $T > 0$. Hence there will always exist a certain distance with the RAF that party leaders can tolerate for delegating it the nomination decision. Second, this threshold increases with the primary skill bonus. The larger the primary skill bonus S, the more likely it is that the elite will forgo appointing the insider in a smoke-filled room.

The third and fourth results decompose the effect of S in its two components, π_{RI} and q. The effect of the expected competence of the insider candidate is intuitive: the more competent the insider candidate is, the less likely that a primary will identify a better candidate, and hence the less attractive primaries are. This effect can be observed in Fig. 6 which depicts how the likelihood of adopting a primary decreases with the prior belief about the insider. The comes from Lemma 3 which established the negative effect of π_{RI} on S, and hence on T.

The effect of q is also intuitive though more complex. As I mentioned, an increase in q can be interpreted as an improvement in the information-revelation feature of primaries. For intermediate values of π_{RI}, an increase in q will increase S as we know from Lemma 3, which in turn will increase T. In other words, a primary election is more attractive for party leaders when its ability to reveal information is larger. This effect can be observed in Fig. 7 which depicts how the likelihood of adopting a primary increases when the quality of primaries increase.

This result contradicts a certain view of primaries in the literature. It is sometimes advised that primary elections should be short and smooth to avoid candidates draining their energy and resources (see for example Ezra (2001)). The theorem above provides a different perspective. A party can actually benefit from having long and challenging primaries, as this would increase the amount of information revealed about pre-candidates (namely q). This result is new in the literature about

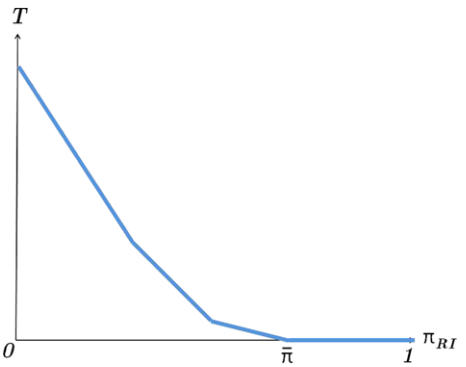

Fig. 6 The likelihood of adopting a primary as a function of the insider's probability of being high-skilled π_{RI} (all things equal)

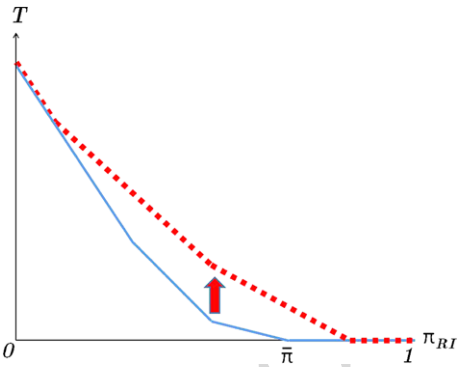

Fig. 7 The effect of increasing the quality of signals q (all things equal) on the likelihood to adopt a primary

primaries, as it could only be obtained by making the realistic assumption that primaries can only reveal information *partially* rather than *fully*.

The last part of the result is more surprising. For low values of π_{RI}, an increase in q will not have any effect on T. The reason is that candidates' performances in the primary would actually being ignored. Primary voters have already made up their minds in favor of an outsider candidates irrespective of her eventual performance in the primary. So increasing or decreasing the amount of information will not alter the nomination decision and consequently will not make primaries more or less attractive.

The fourth, fifth and sixth results broadly indicate that disadvantaged parties are more likely to adopt primaries than advantaged parties. They were all previously found in Serra (2011) so I do not elaborate on them here. Rather I focus on the importance of π_{RI} which is a new contribution.

In particular, the following result departs from previous research as it provides conditions for an insider candidate to avoid a primary challenge. As it turns out, an insider might have a good enough reputation that party leaders will *inevitably* nominate her by not opening the competition to outsiders under *any* circumstance.

Theorem 5 *Suppose the initial expectation that RI is high-skilled, π_{RI}, is such that $\pi_{RI} \in [\overline{\pi}, 1)$. Then the threshold T, which determines the likelihood of primaries, is zero and primaries will never be adopted under any value of the other parameters.*

In other words, the insider's reputation could be so good that leaders will inexorably appoint her. This type or reputation could be enjoyed, for example, by an incumbent who has already won a previous election. Strikingly, a primary election will be eschewed even if primaries reveal a maximum amount of information; even is there is perfect congruence between the elite and the membership of the party; and even if party R has important weaknesses with respect to L. There exists a threshold above which π_{RI} will prevent the use of primary elections for all values of q, X_{RM}, X_{RE}, X_L and π_L.

Hence this result provides an explanation for the empirical observation that many incumbents get re-nominated in their parties without a primary challenge. The rea-

son is that for sufficiently high expectations about the insider candidate's skill, primaries do not bring any advantage at all: both the RAF and the elite are sure to nominate the same candidate. This comes from Lemma 2. Given that primaries do not bring a benefit, any amount of elite-mass incongruence is enough to deter party democratization. S is equal to zero and hence T is equal to zero, which means that any value of d_R is intolerable for party leaders.

8 Conclusions and Discussion

When can an incumbent or any well-known insider feel safe against a challenge for the nomination of a future election? When can he or she be confident that party leaders will directly appoint her rather than holding a competitive primary election? Primary elections are a frequent method used by political parties around the world to select their candidates—and increasingly so. The premise in this paper is that primary elections can serve as a mechanism to reveal information about the candidates' personal appeal to voters. In particular, by forcing candidates to run a primary campaign before the general election campaign, the candidates reveal their campaigning skills and the primary voters can select them accordingly.

An implication of those two features is that a primary election will increase the expected valence of the party's nominee. Such benefit has been modeled previously, for example in Adams and Merrill (2008), Serra (2011), Snyder and Ting (2011), and indeed the findings in this paper corroborates some of the findings in that previous literature (for example that primaries are most beneficial to the weakest parties as found by Adams and Merrill (2008), Serra (2011)).

However those models assume that primaries reveal information fully, meaning that candidates' performance in the primary are a perfect forecast of their performance in the general election. In contrast, this paper assumes that primaries only reveal information partially, meaning that candidate's performance in the primary are a noisy and imperfect forecast of their performance in the general election.

Making this realistic assumption led to new insights. The prior reputation of the party insider (the parameter π_{RI}) turns out to play a crucial role in deterring the use of primaries. Primaries are less appealing to party leaders the better the insider candidate is believed to be. In fact, if the party insider has a good enough reputation for winning votes, for example by virtue of being an incumbent who won a previous election, then a primary election will be eschewed altogether. The paper thus provides an explanation for the empirical fact that many incumbents get re-nominated by their parties without a primary challenge.

This new setup also allowed studying the behavior of primary voters more precisely. As expected, primary voters may use the information provided by primary campaigns to select the pre-candidate with a most impressive performance. However, as it turns out they will only do so for moderate expectation about the ability of the insider candidate. If, on the other hand, the insider is believed to be extremely

competent or extremely incompetent, primary voters will actually ignore the contenders' performance in the primary campaigns and vote exclusively according to their preexisting priors. In other words, primary voters will completely disregard the information provided to them.

I finish with a prescriptive note. If we believe that democratization should occur in any representative institution, we should care about when and why political parties become internally democratic. A question for reformers, then, is how to make competitive primary elections more prevalent. This paper provides several suggestions, but the most direct one is to improve the revelation of information during the primary cycle (the parameter q). Political parties and the general public can benefit from improving the design of primaries to test the pre-candidates' campaigning skills thoroughly enough. For example, parties could include more debates, make campaigns longer, and allow tough critiques among contenders. In other words, the more challenging primaries are, the more information they will reveal about the pre-candidates. A recent example is the competition between Hillary Clinton and Barack Obama during the Democratic primary election. Several Democratic supporters complained that the competition between Clinton and Obama was too long and too severe. Those Democrats worried about the possible costs to their party's prospects in the general election. I do not deny that such costs existed: the potential drawbacks of a competitive primary election include division and resentment among the party base, among other possible costs. But this paper points to a benefit that was seldom mentioned during the 2008 primary. Observers claimed that too much information was being revealed about Clinton and Obama—information which could later be misused by the Republicans. My premise, however, is that such information would have been revealed anyway in the course of the general-election campaign. As a consequence, it was beneficial for the Democratic sympathizers to acquire that information beforehand to help them select their nominee wisely. According to this paper, the length and intensity of the primary campaign are not necessarily a curse for the party, but could actually be a blessing.

Appendix with the Proofs

A.1 Proof of Theorem 1

Table 1 here is a particular case of Table 1 in Theorem 1 of Serra (2011).

A.2 Proof of Lemma 1

If there is a primary election, Party R's RAF will vote for the candidate that it believes to have highest probability of being high-skilled. The beliefs it holds about each candidate's skill depend on two pieces of information: its prior beliefs, and

the information acquired throughout the primary campaign. Given that the RAF members are rational, they will update their prior beliefs based on the performances s_{RI} and s_{RO} to form a couple of posterior beliefs about the probabilities that *RI* and *RO* are high-skilled. If the RAF uses Bayes Rule to update its prior beliefs after receiving a given estimate, its posterior beliefs will be given by

$$P(v_{RI}=1|s_{RI}=low)=\frac{(1-q)\pi_{RI}}{(1-q)\pi_{RI}+q(1-\pi_{RI})}$$

$$P(v_{RI}=1|s_{RI}=high)=\frac{q\pi_{RI}}{q\pi_{RI}+(1-q)(1-\pi_{RI})}$$

$$P(v_{RO}=1|s_{RO}=low)=1-q$$

$$P(v_{RO}=1|s_{RO}=high)=q$$

There are four couple of performances (s_{RI}, s_{RO}) that the RAF could observe, which are $(0,0)$, $(1,1)$, $(0,1)$ and $(1,0)$, I study each of them in turn, along with the decision that the RAF makes upon receiving those couples of estimates.

- If the RAF observes $s_{RI}=low$ and $s_{RO}=low$:

The RAF will vote for *RI* if $P(v_{RO}=1|s_{RO}=low)<P(v_{RI}=1|s_{RI}=low)$ which is equivalent (after some algebra) to $\frac{1}{2}<\pi_{RI}$. Then, given my indifference assumption, the RAF will vote for *RO* if $\pi_{RI}<\frac{1}{2}$, will vote for *RI* if $\frac{1}{2}<\pi_{RI}$, and will randomize equally if $\pi_{RI}=\frac{1}{2}$.

- If the RAF observes $s_{RI}=high$ and $s_{RO}=high$:

The RAF will vote for *RI* if $P(v_{RO}=1|s_{RO}=high)<P(v_{RI}=1|s_{RI}=high)$ which is equivalent (after some algebra) to $\frac{1}{2}<\pi_{RI}$. Then, given my indifference assumption, the RAF will vote for *RO* if $\pi_{RI}<\frac{1}{2}$, will vote for *RI* if $\frac{1}{2}<\pi_{RI}$, and will randomize equally if $\pi_{RI}=\frac{1}{2}$.

- If the RAF observes $s_{RI}=low$ and $s_{RO}=high$:

The RAF will vote for *RI* (in other words, disregard the candidates' performance) if $P(v_{RO}=1|s_{RO}=high)<P(v_{RI}=1|s_{RI}=low)$ which is equivalent (after some algebra, and noting that $1-2q+2q^2>0$) to $\frac{q^2}{1-2q+2q^2}<\pi_{RI}$. Then, given my indifference assumption (and noting that $\frac{1}{2}<\frac{q^2}{1-2q+2q^2}$), the RAF will vote for *RI* if and only $\overline{\pi}\leq\pi_{RI}$, with $\overline{\pi}\equiv\frac{q^2}{1-2q+2q^2}$.

- If the RAF observes $s_{RI}=high$ and $s_{RO}=low$:

The RAF will vote for *RO* (in other words, disregard the candidates' performance) if $P(v_{RO}=1|s_{RO}=low)<P(v_{RI}=1|s_{RI}=high)$ which is equivalent (after some algebra, and noting that $1-2q+2q^2>0$) to $\pi_{RI}<\frac{(1-q)^2}{1-2q+2q^2}$. Then, given

Table A.1 The primary vote as a function of the signals

	$s_{RI} = low$ $s_{RO} = low$	$s_{RI} = high$ $s_{RO} = high$	$s_{RI} = low$ $s_{RO} = high$	$s_{RI} = high$ $s_{RO} = low$
if $\pi_{RI} \in (0, \underline{\pi}]$	Vote for *RO*	Vote for *RO*	Vote for *RO*	Vote for *RO*
if $\pi_{RI} \in (\underline{\pi}, \frac{1}{2})$	Vote for *RO*	Vote for *RO*	Vote for *RO*	Vote for *RI*
if $\pi_{RI} = \frac{1}{2}$	Randomize	Randomize	Vote for *RO*	Vote for *RI*
if $\pi_{RI} \in (\frac{1}{2}, \overline{\pi})$	Vote for *RI*	Vote for *RI*	Vote for *RO*	Vote for *RI*
if $\pi_{RI} \in [\overline{\pi}, 1)$	Vote for *RI*	Vote for *RI*	Vote for *RI*	Vote for *RI*

my indifference assumption (and noting that $\frac{(1-q)^2}{1-2q+2q^2} < \frac{1}{2}$), the RAF will vote for *RO* if and only $\pi_{RI} \leq \underline{\pi}$, with $\underline{\pi} \equiv \frac{(1-q)^2}{1-2q+2q^2}$.

Table A.1 summarizes these results. Which is what the lemma claims.

A.3 Proof of Theorem 2

This conclusion comes directly from two observations: (1) With an elite selection, the party will directly appoint *RI*, and thus $P(v_R = V | m_R = elite) = \pi_{RI}$. And (2) with a primary election the probability of nominating a high-skilled candidate will increase by S by definition, such that $P(v_R = V | m_R = primary) = \pi_{RI} + S$.

A.4 Proof of Lemma 2

I start by calculating the exact value of S. All its properties are derived from this value. We can use the RAF's behavior described in the previous lemma. For that, I first need to calculate $P(v_R = V | primary)$. We can do so by noting that

$$P(v_R = V | primary) = \sum_{v_{RI}, v_{RO}} \sum_{s_{RI}, s_{RO}} P(v_R = V | primary, s_{RI}, s_{RO}; v_{RI}, v_{RO}) \cdot P(s_{RI}, s_{RO} | v_{RI}, v_{RO}) \cdot P(v_{RI}, v_{RO})$$

which uses the definition of conditional probability twice.

Each summand in that expression is straightforward to calculate. $P(v_{RI}, v_{RO})$ depends only on the prior probabilities that v_{RI} and v_{RO} are high-skilled, which are π_{RI} for the insider and $\frac{1}{2}$ for the outsider. $P(s_{RI}, s_{RO} | v_{RI}, v_{RO})$ depends only on the accuracy of the signals, which is q. And $P(v_R = V | primary; s_{RI}, s_{RO}; v_{RI}, v_{RO})$ depends on how the RAF will vote given the candidates' performances, which I just computed in the table above. Multiplying and adding those probabilities is easy but

too long to develop here (the detailed calculations are reported in previous versions of this paper). With the appropriate algebra we find that

$$P(v_R = V|primary) = \begin{cases} \frac{1}{2} & \text{if } \pi_{RI} \in (0, \underline{\pi}] \\ \pi_{RI}q^2 + q - \frac{1}{2}q^2 - \pi_{RI}q + \frac{1}{2}\pi_{RI} & \text{if } \pi_{RI} \in (\underline{\pi}, \frac{1}{2}) \\ \frac{1}{2}q + \frac{1}{4} & \text{if } \pi_{RI} = \frac{1}{2} \\ \pi_{RI}q - \pi_{RI}q^2 + \frac{1}{2}q^2 + \frac{1}{2}\pi_{RI} & \text{if } \pi_{RI} \in (\frac{1}{2}, \overline{\pi}) \\ \pi_{RI} & \text{if } \pi_{RI} \in [\overline{\pi}, 1) \end{cases}$$

I can now calculate the value of interest, S. The values above are used to calculate $S \equiv P(v_R = V|primary) - P(v_R = V|leadership)$, remembering that $P(v_R = V|leadership) = \pi_{RI}$. With some algebra and noting the continuity of S at $\pi_{RI} = \underline{\pi}$, $\pi_{RI} = \frac{1}{2}$ and $\pi_{RI} = \overline{\pi}$, we find that

$$S = \begin{cases} \frac{1}{2} - \pi_{RI} & \text{for } \pi_{RI} \in (0, \underline{\pi}] \\ \pi_{RI}q^2 - \pi_{RI}q - \frac{1}{2}q^2 - \frac{1}{2}\pi_{RI} + q & \text{for } \pi_{RI} \in [\underline{\pi}, \frac{1}{2}] \\ -\pi_{RI}q^2 + \pi_{RI}q + \frac{1}{2}q^2 - \frac{1}{2}\pi_{RI} & \text{for } \pi_{RI} \in [\frac{1}{2}, \overline{\pi}] \\ 0 & \text{for } \pi_{RI} \in [\overline{\pi}, 1) \end{cases}$$

which are the values we were looking for.

Now we need to analyze the sign of S. If $\pi_{RI} \in (0, \underline{\pi}]$ we have that $S = \frac{1}{2} - \pi_{RI} > 0 \Leftrightarrow \pi_{RI} < \frac{1}{2}$, but that is satisfied because $\pi_{RI} \leq \underline{\pi}$ and I have already noted that $\underline{\pi} < \frac{1}{2}$. If $\pi_{RI} \in [\underline{\pi}, \frac{1}{2}]$ we have that $S = \pi_{RI}q^2 - \pi_{RI}q - \frac{1}{2}q^2 - \frac{1}{2}\pi_{RI} + q > 0 \Leftrightarrow \pi_{RI} < \frac{2q-q^2}{1+2q-2q^2}$ (noting that $1 + 2q - 2q^2 > 0$) which is satisfied because $\frac{1}{2} < \frac{2q-q^2}{1+2q-2q^2}$. If $\pi_{RI} \in [\frac{1}{2}, \overline{\pi})$ we have that $S = -\pi_{RI}q^2 + \pi_{RI}q + \frac{1}{2}q^2 - \frac{1}{2}\pi_{RI} > 0 \Leftrightarrow \pi_{RI} < \frac{q^2}{1-2q+2q^2}$ which is satisfied because $\overline{\pi} = \frac{q^2}{1-2q+2q^2}$. And finally if $\pi_{RI} \in [\overline{\pi}, 1)$ we have $S = 0$. So we have indeed $S > 0$ for $\pi_{RI} \in (0, \underline{\pi}] \cup [\underline{\pi}, \frac{1}{2}] \cup [\frac{1}{2}, \overline{\pi})$ and $S = 0$ for $\pi_{RI} \in [\overline{\pi}, 1)$, as the lemma claims.

A.5 Proof of Lemma 3

I calculate the differential of S with respect to π_{RI} and check its sign. If $\pi_{RI} \in (0, \underline{\pi})$, $\frac{\partial S}{\partial \pi_{RI}} = -1$ which is strictly negative. If $\pi_{RI} \in (\underline{\pi}, \frac{1}{2})$, $\frac{\partial S}{\partial \pi_{RI}} = q^2 - q - \frac{1}{2}$ which is strictly negative for $q \in (\frac{1}{2}, 1)$. If $\pi_{RI} \in (\frac{1}{2}, \overline{\pi})$, $\frac{\partial S}{\partial \pi_{RI}} = -q^2 + 2q - 1$ which is strictly negative for $q \in (\frac{1}{2}, 1)$. So S is decreasing with π_{RI} in all those intervals. S is non-differentiable at $\pi_{RI} = \underline{\pi}$ and $\pi_{RI} = \frac{1}{2}$, but is continuous at both points, and is therefore decreasing just like their neighboring points. Hence S decreases with π_{RI} when $\pi_{RI} \in (0, \underline{\pi}) \cup \{\underline{\pi}\} \cup (\underline{\pi}, \frac{1}{2}) \cup \{\frac{1}{2}\} \cup (\frac{1}{2}, \overline{\pi})$.