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Analysis of the MHD Instabilities in Reduction Cells with Lyapunov Method

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Abstract

In this paper, MHD instability in reduction cells is analyzed with Lyapunov method. This paper shows that there are 3 types of status for fluids flow motion in reduction cells: asymptotic stability, stability, and instability. The vertical magnetic field was divided into the same dimensionless component with Taylor series, and the effect on the MHD instability of each component was being studied. New criteria equations are proposed for instabilities in aluminum reduction cells.

Introduction

Hall-Héroult molten salt electrolysis process has been used in primary aluminium smelting industry since 1886. Direct current supplies energy for aluminium reduction, and produces magnetic field. Strong Lorentz force is produced under the interaction among magnetic field and the current in electrolyte and liquid aluminium, which causes the fluid flow, interfacial deformation and fluctuation of electrolyte and liquid aluminium. This fluctuation will further cause secondary reaction of the metal in reduction cell, as well as current efficiency decrease. Meanwhile, fluctuation of metal interface may cause the instability of aluminium reduction cell production. Therefore, it's urgently required to reduce the interfacial deformation and fluctuation mentioned above.

The study of instabilities in reduction cells was first started from 1970's. A semi-dynamic mathematical model was built by Sele[1] to study the fluctuation of liquid aluminium-electrolyte interface when reduction cell is under the influence of disturbance current. The earliest criteria for instabilities in reduction cells, the Sele criteria, was also proposed by Sele[1]. However, it was later found by Urata[2] that the instability of molten fluid in reduction cells was actually the instability of a long wave, and corresponding wave equation was proposed. Soon afterwards, a lot of studies were done respectively by Sneyd and Wang[3], Bojarevics and Romerio[4], Davidson and Lindsay[5], Segatz and Droste[6]. Similar molten fluid wave equations, numerical methods or analytical solutions were respectively proposed, as well as their respective instability criteria. However, Sun[8] et al. deemed that the background flow of molten fluid should not be ignored also, and proposed nonlinear analysis method of instability.

Molten fluid interface fluctuations in reduction cells due to exterior disturbance are divided into two statuses by all the methods mentioned above: stability and instability. Under the ideal status, static interface is considered as a stability status; fluctuant interface with increasing amplitude is considered as instability status. Stability criteria are used to judge if status is stable or not.

It is difficult to explain actually the moving conditions of molten metal interface in reduction cells if interface status (stable or not) is judged only by if it is static or not, since the moving conditions of liquid bath-metal interface in reduction cells are quite complicated.

1 Molten fluid instability in reduction cells

Generally speaking, the so-called flow field we often say is the steady flow field, since its power source term (driving force) is steady Lorentz force (caused by both steady current density and steady magnetic field). The opposite of it is the transient Lorentz force, which is the key source of MHD instability (Magneto-Hydro-Dynamics instability).

Transient current (disturbance current) is produced when bath-metal interface is disturbed under equilibrium status. Transient Lorentz force comes from the interaction between transient electric and steady magnetic field. Based on the view of stability of motion, the stability of reduction cell is defined as the capacity of reduction cell to damp disturbance and to recover the original equilibrium status. For reduction cell, the stability is mainly MHD stability [9, 10]. There are other more complicated instabilities [8] of Kelvin-Helmholtz instability etc.

1.1 MHD model of reduction cells

MHD analysis model of reduction cells is shown in figure 1. In order to simplify the calculations, the dimensions of anodes and cathode in cells are considered as the same; transient analysis is mainly aimed at disturbance, all the current in molten fluid in the model under stable status is supposed as even, vertical and downward. j10 and j20 correspond respectively to the steady current at metal and bath layers respectively.

The current distribution in the molten fluid changes (as figure 2), when fluctuation or deformation happens on metal-bath interface (hereinafter referred to as metal interface); if it is decomposed as the total of steady current and transient current, the latter being the disturbance current. It can be seen that the transient disturbance current j1 in the metal is horizontal, but the transient current j2 in bath is vertical.

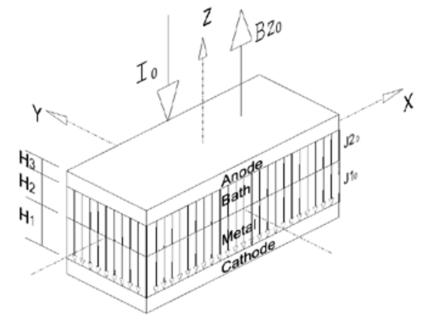


Figure 1 Current in molten fluid under equilibrium status

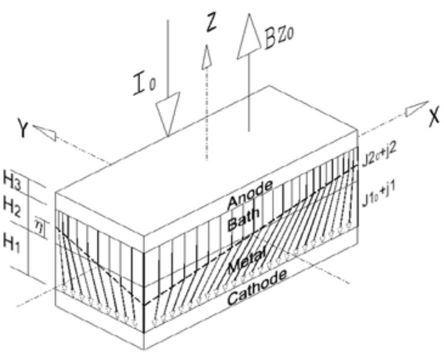


Figure 2 Current in molten fluid under disturbance

1.2 Transient fluctuation caused by transient Lorentz force

For the two layers of molten fluid, Navier-Stokes equation shall also be met:

$$\rho\left(\frac{\partial \boldsymbol{V}}{\partial t}+\boldsymbol{V}\cdot\nabla\boldsymbol{V}\right)=-\nabla P+\mu\nabla^2\boldsymbol{V}+\rho g\nabla H+\boldsymbol{F} \quad (1)$$

$$\nabla\cdot\boldsymbol{V}=0 \quad (2)$$

In which: ρ——Fluid density

V——Flow velocity

g——Gravity acceleration

P——Pressure

F——Lorentz force (steady force + transient force)

μ——Kinetic viscosity

H——Depth of fluid

If transient values are substituted to equation (1) to resolve divergences for both sides of this equation, the following equation can be deducted (see references [3]-[7] for details):

$$\left(\frac{\rho_1}{H_1}+\frac{\rho_2}{H_2}\right)\frac{\partial^2\eta}{\partial t^2}-g(\rho_1-\rho_2)\nabla^2\eta=-\nabla\cdot(f_1-f_2) \quad (3)$$

In which:

f_1, f_2——Disturbance Lorentz forces at metal and bath layers (N)

H1, H2——Depths of metal and bath layers (m)

η——Interface deformation height (m)

$f_1 >> f_2$, so

$$\left(\frac{\rho_1}{H_1}+\frac{\rho_2}{H_2}\right)\frac{\partial^2\eta}{\partial t^2}-g(\rho_1-\rho_2)\nabla^2\eta=\sigma\left(j_{1x}\frac{\partial B_z}{\partial y}-j_{1y}\frac{\partial B_z}{\partial x}\right) \quad (4)$$

On sidewall boundary:

$$\left(\frac{\rho_1}{H_1}+\frac{\rho_2}{H_2}\right)\frac{\partial^2\eta}{\partial t^2}-g(\rho_1-\rho_2)\nabla^2\eta=\sigma\left(\mathbf{n}_x j_{1x}-\mathbf{n}_y j_{1y}\right)B_z \quad (5)$$

Electric potential equation:

$$H_1H_2\nabla^2\varphi-\frac{\sigma_2}{\sigma_1}\varphi=-\frac{J_0}{\sigma_1}\eta \quad (6)$$

In which:

J_0——Steady current density of reduction cells (kA/m2)

σ_1, σ_2——Conductivities of metal, bath

φ——Electric potential (V)

The following equations are obtained, if equation (6) is put into (4) with consideration of boundary conditions (see references [3], [4], [5], [6] for details):

$$\ddot{\boldsymbol{\eta}}+\Omega_\gamma\boldsymbol{\eta}=\omega_B^2K\boldsymbol{\eta} \quad (7)$$

In which:

$$\omega_B^2=\frac{J_0B_z}{\tilde{\rho}H_1H_2} \quad (8)$$

$$\tilde{\rho}=\frac{\rho_1}{H_1}+\frac{\rho_2}{H_2} \quad (9)$$

In which:

Ω_g——Square of natural frequency

K——Interaction matrix of Lorentz forces

In which:

$$\boldsymbol{\eta}=\hat{\boldsymbol{\eta}}e^{i\omega t}$$

So, the eigenvalue equation becomes:

$$\omega^2\hat{\eta}=\left(\boldsymbol{\Omega}_g-\omega_B^2\boldsymbol{K}\right)\hat{\eta}=\left(\omega_g^2-\omega_B^2\boldsymbol{K}\right)\hat{\eta} \quad (10)$$

The transient solutions of interface deformation η and electric potential φ can be obtained by resolving eigenvalue equation (10) with numerical method and analytical solution. However, such a calculation amount is quite huge.

The resolution is:

$$\varphi = \sum_{m,n} \varphi_{m,n} E_{m,n} e^{i\omega t} \tag{11}$$

$$\eta = \sum_{m,n} \eta_{m,n} E_{m,n} e^{i\omega t} \tag{12}$$

In which:

φ——Transient electric potential value

η——Transient interface deformation

$\varphi_{m,n}$——2-D Fourier coefficients of orders of transient electric potential

$\eta_{m,n}$——2-D Fourier coefficients of orders of transient displacement of the bath/metal interface

$E_{m,n}$——Eigen frequency

ω——Angular frequency

2 Stability classification of molten fluid in the reduction cell

The instability of the interface of molten fluid in the cell is analyzed with the motion stability method.

As per the definition of motion stability, the original equilibrium state of system will be destroyed once the system is disturbed; the system is stable if the system returns to the equilibrium state when the disturbance is removed; the system is not stable, if not. Besides some status which do not happen frequently, the following basic motion status will happen when the system is disturbed:

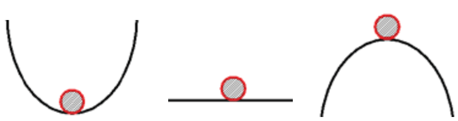
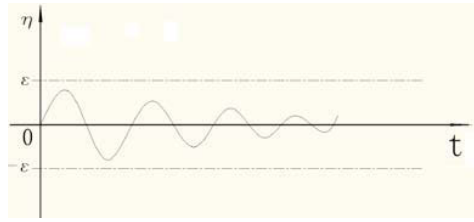


Figure 3 Sketch of Lyapunov motion stability

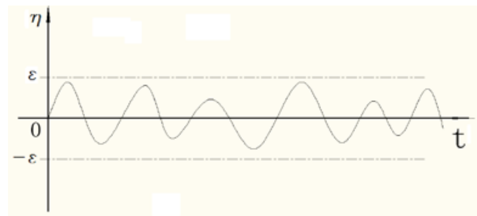
(a) Asymptotic stability; (b) stability; (c) instability

Assuming the 3 balls in figure 3 are in equilibrium state originally, if friction is ignored and the time goes to infinity, the ball in (a) shall return to original point, the ball in (b) shall keep uniform motion and the ball in (c) shall go faster and faster. Namely, (a) is called as asymptotic stability state, (b) as stability state and (c) as instability state.

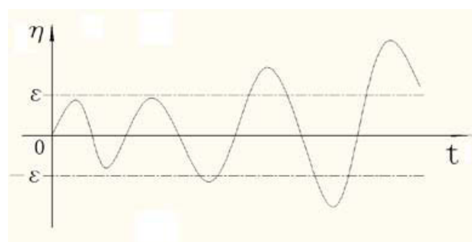
The instability of cell comes mainly from the instability of metal-bath interface, and the disturbance is mainly the interface/potential transient variation resulted by tapping, anode replacing, feeding, anode dusting and motion of bubbles etc. Similar to the motion of the balls in figure 3, there are 3 kinds of main motion states for the fluid interface in the cell after the disturbance disappears, shown as figure 4.



(a) Asymptotic stability



(b) Stability



(c) Instability

Figure 4 Sketch of motion stability of metal-bath interface variation in the cell during t→∞

As per the Lyapunov motion stability, we think the solution of the interface fluctuation at equilibrium position for the cell is: during t →∞, if $\|\eta(x,y,t)-\eta_0\| = \varepsilon$ (ε is the pre-set arbitrary value), it is stable; if $\|\eta(x,y,t)-\eta_0\| < \varepsilon$, it is asymptotically stable; if $\|\eta(x,y,t)-\eta_0\| > \varepsilon$, it is instable.

People have imagined that the system is stable only if it is at the equilibrium position (namely there is no interface fluctuation). However, as per the viewpoint of motion stability, if the fluctuation after disturbance gradually returns to the equilibrium position, or the fluctuation remains in a small range, this could be seen as that the system is stable. The value of ε must not affect the current efficiency, it is selected as about 1~2cm for reduction cells based on our experience.

3 Analytical method of Lyapunov motion stability

As per the first method of Lyapunov, the system stability is judged with the following steps:

1) The system is linearized with Taylor decomposition;

2) The eigenvalue of the system is judged. If the real part of eigenvalue is not positive (0, or negative), the system is stable;

4 Analysis on stability of cell with Lyapunov motion stability

The equation (7) is changed to the following[5]:

$$\ddot{\boldsymbol{\eta}} + \Omega_g \boldsymbol{\eta} = \frac{J_0 B_z}{\tilde{\rho} H_1 H_2} K \boldsymbol{\eta} \tag{13}$$

The steady current density of cell is vertical and downward, so $\frac{J_0 B_z}{\tilde{\rho} H_1 H_2}$ in (13) mainly depends on the distribution of vertical magnetic field B_z, namely, if the current J_0, molten fluid density ρ and depth H are determined, the distribution of vertical magnetic field B_z is the determinant of MHD stability.

4.1 Expansion of vertical magnetic field Bz with Taylor decomposition

The distribution of vertical magnetic field is quite complicated in practice, but with Taylor series decomposition method, it can be expanded to [7]:

$$B_Z = C_0 + C_X X + C_Y Y + C_{XY} XY + Rn \tag{14}$$

In which:

C_0——value of equivalent uniform vertical magnetic field

C_X——equivalent slope (gradient) in direction X

C_Y——equivalent slope (gradient) in direction Y

C_{XY}——value of horizontal tensor

Rn——high order Lagrange remainder term

By decomposition, the distribution of arbitrarily complicated vertical magnetic field can be expanded to the sum of distribution of simple magnetic field with Taylor method. Equation (14) is substituted in equation (13):

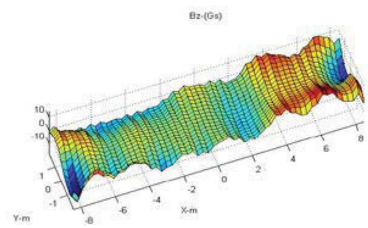
$$\ddot{\boldsymbol{\eta}} + \Omega_g \boldsymbol{\eta} = \frac{J_0 K}{\tilde{\rho} H_1 H_2}\left(C_0 + C_X X + C_Y Y + C_{XY} XY + Rn\right)\boldsymbol{\eta} \tag{15}$$

As per the characteristics of wave equation, the solution of equation (15) should be the sum of calculation results of expanded terms of B_Z.

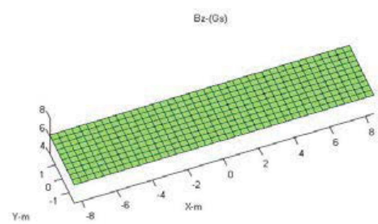
4.2 Common distribution of vertical magnetic field

In actual condition, any complicated vertical magnetic field can be decomposed to limited number of components using the equation (14). Meanwhile, it can be seen that *Rn* doesn't affect the MHD instability from §4.3 mentioned below. The common distribution of vertical magnetic field B_Z includes the following cases:

1) B_Z, basically vertically upward (or downward), is mainly composed of C_0+Rn, shown in figure 5. ($C_X \approx 0$, $C_Y \approx 0$, $C_{XY} \approx 0$)



(a) Example of B_Z, basically upward



(b) Distribution of C_0

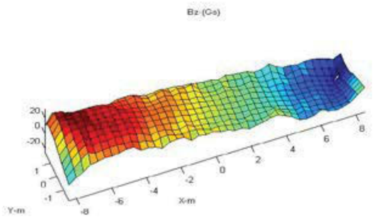
Figure 5 Distribution of B_Z (a) and C_0 (b)

2) B_Z, symmetrically distributed along Y axis, is mainly composed of $C_X X+Rn$, shown in figure 6. ($C_0 \approx 0$, $C_Y \approx 0$, $C_{XY} \approx 0$)

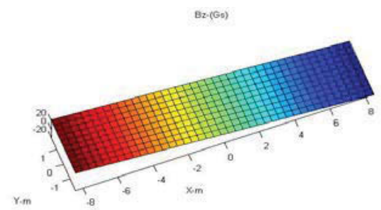
3) B_Z, symmetrically distributed along X axis, is mainly composed of $C_Y Y+Rn$, shown in figure 7. ($C_0 \approx 0$, $C_X \approx 0$, $C_{XY} \approx 0$)

As per the statistics, there is little vertical magnetic field distributed in such way in existing cells all over the world.

4) B_Z, anti-symmetrically distributed along X and Y axes, is mainly composed of $C_{XY} XY+Rn$, shown in figure 8. ($C_0 \approx 0$, $C_X \approx 0$, $C_Y \approx 0$)

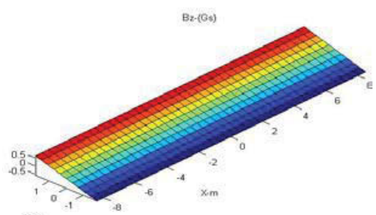


(a) Example of B_Z, symmetrically distributed along Y axis



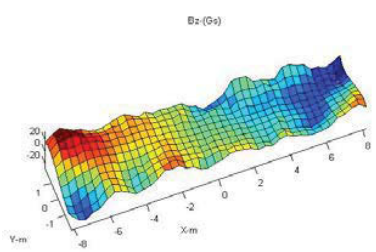
(b) Distribution of $C_X X$

Figure 6 Distribution of B_Z (a) and $C_X X$ (b)

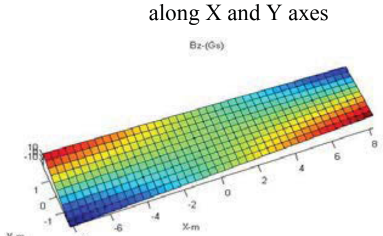


Distribution of $C_Y Y$

Figure 7 Vertical magnetic field B_Z, symmetrically distributed along X axis



(a) Example of B_Z, anti-symmetrically distributed along X and Y axes



(b) Distribution of C_{XY}

Figure 8 Distribution of B_Z (a) and $C_{XY}XY$ (b)

4.3 Analysis of Lyapunov motion stability regarding distribution of typical vertical magnetic field

By Taylor decomposition, the fluctuation resulting from the complicated vertical magnetic field B_Z can be decomposed into the fluctuation resulting from each simple component. Due to the characteristics of long-wave fluctuation in the cell, only finite components can cause the instability with action of disturbance. So only the analysis on such simple components can determine which component cause the most obvious instability, which can simplify the calculation greatly.

Through analysis, we found that C_{XY} will cause short wavelength disturbances. So most of researchers think such component with the action of damping of friction in the molten fluid is stable, without self-oscillation[2]. For the same reason, the high order component Rn of vertical magnetic field may not result in self-oscillation.

Moreover, due to the characteristics of cell (width<length of cell, symmetric arrangement of upstream/downstream busbar and conductor etc.), the symmetric distribution along X axis of vertical magnetic field does not happen generally, namely $C_Y \approx 0$.

The specific threshold values that may result in instability can be calculated with analytical method or numerical method. Generally, for conditions of C_0 (shown as figure 6), the stability criterions by Sele[1], Bojarevics[4] etc. are accurate enough; for the conditions of C_X (shown as figure 7), the stability criterions by Segatz[6], Urata[7] etc. can be considered.

5 Criterions of stability

As per the above analysis, the judgment of cell interface stability can be divided into 2 steps; for the derivation of its formula of criterion, see literature [7] for details.

1) Equivalent value C_0 of vertical magnetic field results in instability or not, which is judged by:

$$\left(\frac{\pi}{2}\right)^4 \left(\frac{1}{L_y{}^2} - \frac{1}{L_x{}^2}\right) > \frac{J_0 C_O}{\Delta\rho g h_1 h_2} \tag{16}$$

In which,

L_X, L_Y——dimension of cell in direction X and Y (m)

If the criterion is met, go to the next step;

If not, the system is instable.

2) Equivalent gradient Cx and Cy of vertical magnetic field result in instability, which is judged by:

$$(\frac{\pi}{2})^4(\frac{1}{L_y^{\ 2}}-\frac{1}{L_x^{\ 2}}) > \frac{J_0 C_X X}{\Delta\rho g h_1 h_2} \tag{17}$$

And

$$(\frac{\pi}{2})^4(\frac{1}{L_y^{\ 2}}-\frac{1}{L_x^{\ 2}}) > \frac{J_0 C_Y Y}{\Delta\rho g h_1 h_2} \tag{18}$$

If the criterion is met, the system is stable;

If not, the system is instable.

The high order remainder term may not result in the instability.

In summary, it is required to judge in 2 steps. The equivalent value C_0 of vertical magnetic field is required to be judged firstly because it is the most important. Even though C_0 is stable, the next step is required. The system can be judged as stable only if both the two steps of judgment on C_X and C_Y meet stability criteria.

6 Discussions

Under normal conditions, the metal interface status (stable or not) in most of cells depends on the equivalent value C_0 and equivalent gradient in direction X. The vertical magnetic field for cell stability should be distributed anti-symmetrically along axis X and Y. Therefore, the design of cell bus should ensure:

1) Equivalent value C_0 of vertical magnetic field B_Z is small;

2) Equivalent value C_X in direction X of vertical magnetic field B_Z is small;

3) The vertical magnetic field B_Z should be distributed anti-symmetrically along axle X and Y, with small equivalent value C_{XY}.

The above qualitative requirements are presented in some articles [4, 10] regarding busbar design by some researchers, but mostly without reasons presented.

This analysis method can be suitable for the judgment of stability of most of cells; but cannot be suitable for some newly-emerged structures of cell, such as cell with irregular cathode, drained cell etc.

7 Acknowledgements

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References

[1] SELE T. Instabilities of the metal surface in electrolytic alumina reduction cells[J]. Metallurgical and Materials Transctions B. 1977, vol.8(4):613-618

[2] URATA N. Magnetics and metal pad instability[J]. Light Metals 1985:581-591

[3] SNEYD D, WANG A. Interfacial instability due to MHD mode coupling in aluminium reduction cells[J]. Journal of Fluid Mechanics 1994, vol. 263:343-360

[4] BOJAREVICS V, ROMERIO M. Long waves instability of liquid metal-electrolyte interface in aluminium electrolysis cells: A generalization of Sele's criterion[J]. European Journal of Mechanics, B/Fluids 1994, vol. 13(1):33-56

[5] DAVIDSON P, LINDSAY R. Stability of interfacial waves in aluminium reduction cells[J]. Journal of Fluid Mechanics 1998, vol.362:273-295

[6] SEGATZ M, DROSTE C. Analysis of Magnetohydrodynamic instabilities in aluminum reduction cells[J]. Light Metals 1994, 313-322

[7] URATA N. Wave mode coupling and instability in the internal wave in aluminum reduction cells[J]. Light Metals 2005, 455-460

[8] SUN H, ZIKANOV O, ZIEGLER D. Non-linear two-dimensional model of melt flows and interface instability in aluminum reduction cells[J]. Fluid Dynamics Research, 2004, Vol. 35: 255–274

[9] SHIN D, SNEYD A. Metal pad instabilities in reduction cells[J]. Light Metals, 2000

[10] ZIEGLER D. Stability of metal-electrolyte interface in HalI-Heroult cells: effect of the steady velocity[J]. Metallurgical Transactions B, 1993, Vol. 24B:899-906

[11] DORF R. BISHOP R. Modern Control Systems[M]. 8th. Addison-Wesley Longman Publishing Co., Inc. Boston, MA, USA