$\boxed{6}$ $\int \sec 2t \tan 2t \, dt$

$= \frac{1}{2} \sec 2t + c$

$\boxed{7}$ $\int (x^2+4)^2 \, dt$

$= \int (x^4 + 8x^2 + 16) \, dt$

$= \frac{x^5}{5} + \frac{8}{3} x^3 + 16x + c$

9] $\int \frac{3}{16+x^2} \, dx = \frac{3}{4} \tan^{-1} \frac{x}{4} + c$

10] $\int \frac{2}{4+4x^2} \, dx = \frac{1}{2} \tan^{-1} x + c$

11 $\displaystyle\int \frac{1}{\sqrt{3-2x-x^2}}\,dx$

$$= \int \frac{1}{\sqrt{3-(x^2+2x)}}\,dx$$

$$= \int \frac{1}{\sqrt{3-(x^2+2x+1)+1}}\,dx$$

$$= \int \frac{1}{\sqrt{3-(x+1)^2+1}}\,dx$$

$$= \int \frac{1}{\sqrt{4-(x+1)^2}}\,dx$$

$$= \int \frac{1}{\sqrt{4\left[1-\left(\frac{x+1}{2}\right)^2\right]}}\,dx$$

4P

سؤال 11 ب

$$\frac{1}{2}\int \frac{1}{\sqrt{1-\left(\frac{x+1}{2}\right)^2}}\,dx$$

$$u=\frac{x+1}{2} \Rightarrow 2du = dx$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\cdot 2du$$

$$=\int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1}(u)+c$$

$$=\sin^{-1}\left(\frac{x+1}{2}\right)+c$$

سؤال 12

$$\int \frac{x+1}{\sqrt{3-2x-x^2}}\,dx$$

$u = 3-2x-x^2$

$du = (-2-2x)\,dx$

$du = -2(x+1)\,dx$

$dx = \frac{du}{-2(x+1)}$

$$\int \frac{\cancel{x+1}}{\sqrt{u}} \cdot \frac{du}{-2(\cancel{x+1})}$$

$$\int \frac{u^{-1/2}}{-2}\,du$$

$$= \frac{-1}{2}\,\frac{u^{1/2}}{1/2} + c$$

$$= -\sqrt{u} + c$$

$$= -\sqrt{3-2x-x^2} + c$$

$$= -\sqrt{3-(x^2+2x+1)+1} + c$$

$$= -\sqrt{4-(x+1)^2} + c$$

$\frac{13}{489}$] $\int \frac{4}{5+2x+x^2}\,dx$

$= \int \frac{4}{5+(x^2+2x+1)-1}\,dx$

$= \int \frac{4}{4+(x+1)^2}\,dx$

$= \int \frac{4}{4\left(1+\left(\frac{x+1}{2}\right)^2\right)}\,dx$

$u = \frac{x+1}{2} \Rightarrow du = \frac{1}{2}dx$

$= \int \frac{4}{4(1+u^2)} \cdot 2\,du$

$= 2\tan^{-1} u + c$

$= 2\tan^{-1}\left(\frac{x+1}{2}\right) + c$

14) $$\int \frac{4x+4}{5+2x+x^2}\,dx$$

$$= 2\int \frac{2(x+1)}{4+(x+1)^2}\,dx$$

$$= 2\ln\left|4+(x+1)^2\right| + c$$

15) $$\int \frac{4t}{5+2t+t^2}\,dt$$

$$= \int \frac{4t+4-4}{5+2t+t^2}\,dt$$

$$= \int \frac{4t+4}{5+2t+t^2}\,dt - \int \frac{4}{5+2t+t^2}\,dt$$

$$= 2\ln\left|4+(t+1)^2\right| - 2\tan^{-1}\left(\frac{t+1}{2}\right) + c$$

16) $\int \frac{t+1}{t^2+2t+4}\,dt$

$$\int \frac{t+1}{(t^2+2t+1)+3}\,dt$$

$$\int \frac{t+1}{(t+1)^2+3}\,dt$$

$$= \frac{1}{2}\int \frac{2(t+1)}{(t+1)^2+3}\,dt$$

$$= \frac{1}{2}\ln\left|(t+1)^2+3\right| + c$$

17) $\int e^{3-2x}\,dx = \frac{-1}{2}e^{3-2x}\,dx$

18) $\int 3e^{-6x}\,dx = \frac{-1}{2}e^{-6x} + c$

19) $I=\int \frac{4}{x^{1/3}(1+x^{2/3})} dx$

$u = 1 + x^{2/3} \Rightarrow du = \frac{2}{3} x^{-1/3} dx$

$3du = 2x^{-1/3} dx \Rightarrow dx = \frac{3du}{2x^{-1/3}}$

$I = \int \frac{4}{x^{1/3} u} \cdot \frac{3du}{2 \cdot x^{-1/3}}$

$= \int \frac{6\,du}{u} = 6\ln|u| + c$

$= 6\ln|(1+x^{2/3})| + c$

20) $\int \frac{2}{x^{1/4}+x} dx$

$\int \frac{2}{x^{1/4}(1+x^{3/4})} dx$

P(10)

let $u = 1 + x^{3/4} \Rightarrow du = \frac{3}{4} x^{-1/4} dx$

$4du = 3x^{-1/4} dx \Rightarrow dx = \dfrac{4du}{3x^{-1/4}}$

$$I = \int \frac{2}{x^{1/4}(u)} \cdot \frac{4\,du}{3x^{-1/4}}$$

$$= \frac{8}{3}\int \frac{du}{u} = \frac{8}{3}\ln|u| + c$$

$$= \frac{8}{3}\ln|(1+x^{3/4})| + c$$

21) $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

$u = \sqrt{x}$

$du = \dfrac{dx}{2\sqrt{x}}$

$dx = 2\sqrt{x}\,du$

$$I = \int \frac{\sin u}{\sqrt{x}} \cdot 2\sqrt{x}\,du$$

$$= -2\cos\sqrt{x} + c$$

(21)

22 $\int \frac{\cos\left(\frac{1}{x}\right)}{x^2}\,dx$

$u = \frac{1}{x} \Rightarrow du = \frac{-1}{x^2}\,dx$

$dx = -x^2\,du$

$I = \int \frac{\cos u}{x^2} \cdot -x^2\,du$

$= \int -\cos u\,du = -\sin\frac{1}{x} + c$

23) $\int_0^{\pi} \cos x\, e^{\sin x}\,dx$

$= e^{\sin x}\Big]_0^{\pi} = e^{\sin \pi} - 1$

$= e^0 - 1$

$= 0$

(P12)

25) $\displaystyle\int_{-\pi/4}^{0} \frac{\sin t}{\cos^2 t}\, dt = \int_{-\frac{\pi}{4}}^{0} \sec t \tan t \, dt$

$\displaystyle = \sec t \Big]_{-\pi/4}^{0} = 1 - \sqrt{2}$

27) $\displaystyle\int \frac{x^2}{1+x^6}\, dx$

$u = x^3 \Rightarrow du = 3x^2\, dx$

$dx = \dfrac{du}{3x^2}$

$\displaystyle I = \int \frac{x^2}{1+u^2} \cdot \frac{du}{3x^2}$

$\displaystyle = \frac{1}{3}\int \frac{du}{1+u^2} = \frac{1}{3}\tan^{-1} x^3 + c$

28) $\int \frac{x^5}{1+x^6}\,dx$

$= \frac{1}{6}\int \frac{6x^5}{1+x^6}\,dx$

$= \frac{1}{6}\ln|1+x^6|+c$

30) $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

$u = e^x \Rightarrow du = e^x\,dx$

$\Rightarrow dx = \frac{du}{e^x}$

$I = \int \frac{e^x}{\sqrt{1-u^2}}\cdot\frac{du}{e^x}$

$= \int \frac{du}{\sqrt{1-u^2}}\,du = \sin^{-1}(e^x)+c$

31 $\int \frac{x}{\sqrt{1-x^4}}\,dx$

$u = x^2 \Rightarrow du = 2x\,dx$

$dx = \frac{du}{2x}$

$I = \int \frac{x}{\sqrt{1-u^2}} \cdot \frac{du}{2x}$

$= \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\sin^{-1}(x^3) + c$

32 $\int \frac{2x^3}{\sqrt{1-x^4}}\,dx$

$u = 1-x^4 \Rightarrow du = -4x^3\,dx$

$I = \frac{-1}{2}\int u^{-1/2}\,du = -(1-x^4)^{1/2} + c$

33) $\int \frac{1+x}{1+x^2}\,dx$

$= \int \frac{1}{1+x^2}\,dx + \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$

$= \tan^{-1}x + \frac{1}{2}\ln|x^2+1| + c$

34) $\int \frac{1}{\sqrt{x}+x}\,dx$

$= \int \frac{1}{\sqrt{x}\,(1+\sqrt{x})}\,dx$

$= 2\int \frac{\frac{1}{2}x^{-1/2}}{1+x^{1/2}}\,dx$

$2\ln|1+x^{1/2}| + c$

35) $\int \frac{\ln x^2}{x}\,dx = \int \frac{2\ln x}{x}\,dx$

$= 2\int \ln x \left(\frac{1}{x}\right) dx$

$u = \ln x \qquad du = \frac{1}{x}dx$

$dx = x\,du$

$2\int u\,du = \cancel{2}\left(\frac{u^2}{\cancel{2}}\right) + c$

$= \frac{\cancel{2}(\ln x)^2}{\cancel{2}} + c$

36) $\int_1^3 e^{2\ln x}\,dx = \int_1^3 e^{\ln x^2}\,dx$

$= \int_1^3 x^2\,dx = \left.\frac{x^3}{3}\right]_1^3$

$= 26/3$

37 $\displaystyle\int_3^4 x\sqrt{x-3}\,dx$

$$= \int_3^4 (x-3+3)\sqrt{x-3}\,dx$$

$$= \int_3^4 (x-3)\sqrt{x-3}\,dx$$

$$+ \int 3\,(x-3)^{1/2}\,dx$$

$$= \int_3^4 (x-3)^{3/2}\,dx + 3\int (x-3)^{1/2}\,dx$$

$$\frac{2}{5}(x-3)^{5/2}\Big]_3^4 + \frac{(3)(2)}{3}(x-3)^{3/2}\Big]_3^4$$

$$= \frac{12}{5}$$

38) $\int_0^1 x(x-3)^2\,dx$

$= \int_0^1 (x^3 - 6x^2 + 9x)\,dx$

$= \frac{11}{4}$

39) $\int_1^4 \frac{x^2+1}{\sqrt{x}}\,dx$

$= \int_1^4 x^{3/2}\,dx + \int_1^4 x^{-1/2}\,dx$

$= \frac{72}{5}$

$* \int \frac{5}{3+x^3}\,dx$ لا يحل

$\boxed{42}$ $\int \sin^3 x\,dx$

$= \int \sin^2 x \cdot \sin x\,dx$

$= \int (1-\cos^2 x)\sin x\,dx$

let $u = \cos x \Rightarrow du = -\sin x\,dx$

$I = \int (1-u^2) \cdot \sin x \cdot \frac{du}{-\sin x}$

$= -\left[u - \frac{u^3}{3}\right] + c$

$= -\left[\cos x - \frac{\cos^3 x}{3}\right] + c$

41 $\int \frac{5}{3+x^2}\,dx$

$$= \int \frac{5}{3\left(1+\left(\frac{x}{\sqrt{3}}\right)^2\right)}\,dx$$

$$u = \frac{x}{\sqrt{3}} \Rightarrow \sqrt{3}\,du = dx$$

$$I = \int \frac{5}{3(1+u^2)}\sqrt{3}\,du$$

$$= \frac{5\sqrt{3}}{3}\tan^{-1} u + c$$

$$= \frac{5\sqrt{3}}{3}\tan^{-1}\frac{x}{\sqrt{3}} + c$$

يكمل

43 $\int \ln x \, dx$

* $\int \frac{\ln x}{2x} dx$, $u = \ln x$

$= \frac{1}{4} \ln^2 x + c$

44) $\int \frac{x^3}{1+x^8} dx$ $u = x^4$

$= \frac{1}{4} \tan^{-1} x^4 + c$

* $\int \frac{x^4}{1+x^8} dx$ يكمل

45) $\int e^{-x^2} dx$ يكمل

$\int x e^{-x^2} dx = \frac{-1}{2} e^{-x^2} + c$