Solutions Manual

for

Digital Signal Processing using MATLAB - Second Edition

2007

Chapter 2

Discrete-Time Signals and Systems

P2.1 Generate the following sequences using the basic MATLAB signal functions and the basic MATLAB signal operations discussed in this chapter. Plot signal samples using the stem function.

1. $x_1(n) = 3\delta(n + 2) + 2\delta(n) - \delta(n - 3) + 5\delta(n - 7), -5 \leq n \leq 15$

```
% P0201a: x1(n) = 3*delta(n + 2) + 2*delta(n) - delta(n - 3) +
%                   5*delta(n - 7), -5 <= n <= 15.
clc; close all;

x1 = 3*impseq(-2,-5,15) + 2*impseq(0,-5,15) - impseq(3,-5,15) ...
    + 5*impseq(7,-5,15);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201a'); n1 = [-5:15];
Hs = stem(n1,x1,'filled'); set(Hs,'markersize',2);
axis([min(n1)-1,max(n1)+1,min(x1)-1,max(x1)+1]);
xlabel('n','FontSize',LFS); ylabel('x_1(n)','FontSize',LFS);
title('Sequence x_1(n)','FontSize',TFS);
set(gca,'XTickMode','manual','XTick',n1,'FontSize',8);
print -deps2 ../EPSFILES/P0201a;
```

The plots of $x_1(n)$ is shown in Figure 2.1.
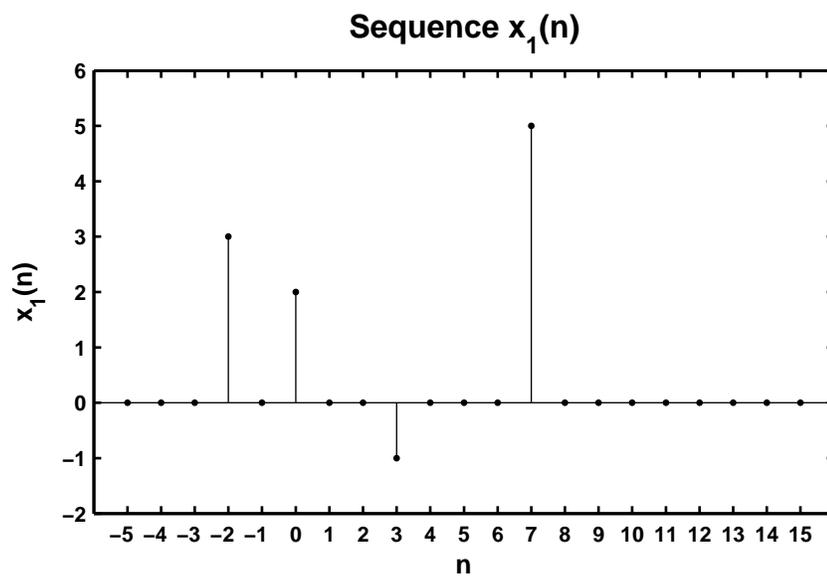
Sequence $x_1(n)$

Figure 2.1: Problem P2.1.1 sequence plot

2. $x_2(n) = \sum\limits_{k=-5}^{5} e^{-|k|}\delta(n-2k), -10 \le n \le 10.$

```
% P0201b: x2(n) = sum_{k = -5}^{5} e^{-|k|}*delta(n - 2k), -10 <= n <= 10
clc; close all;

n2 = [-10:10]; x2 = zeros(1,length(n2));
for k = -5:5
    x2 = x2 + exp(-abs(k))*impseq(2*k ,-10,10);
end
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201b');
Hs = stem(n2,x2,'filled'); set(Hs,'markersize',2);
axis([min(n2)-1,max(n2)+1,min(x2)-1,max(x2)+1]);
xlabel('n','FontSize',LFS); ylabel('x_2(n)','FontSize',LFS);
title('Sequence x_2(n)','FontSize',TFS);
set(gca,'XTickMode','manual','XTick',n2);
print -deps2 ../EPSFILES/P0201b;
```

The plots of $x_2(n)$ is shown in Figure 2.2.
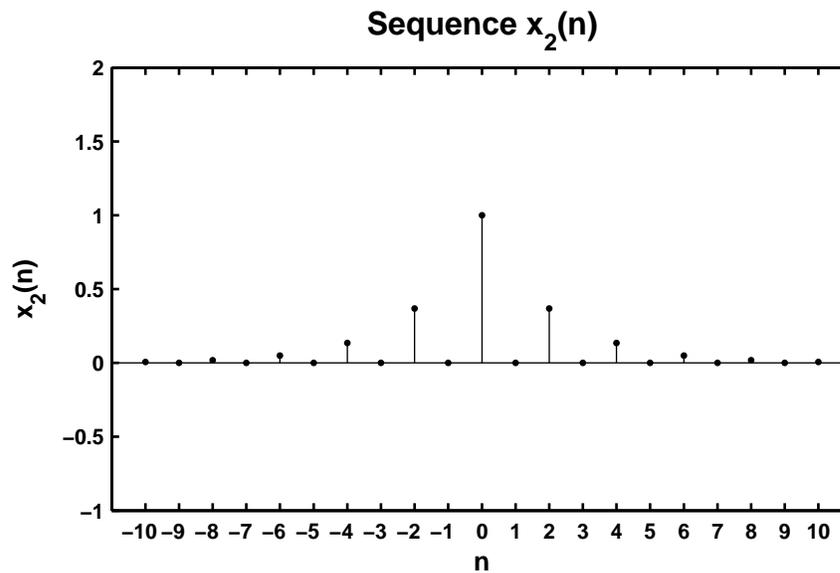
Figure 2.2: Problem P2.1.2 sequence plot

3. $x_3(n) = 10u(n) - 5u(n - 5) - 10u(n - 10) + 5u(n - 15)$.

```
% P0201c: x3(n) = 10u(n) - 5u(n - 5) + 10u(n - 10) + 5u(n - 15).
clc; close all;

x3 = 10*stepseq(0,0,20) - 5*stepseq(5,0,20) - 10*stepseq(10,0,20) ...
     + 5*stepseq(15,0,20);
n3 = [0:20];
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201c');
Hs = stem(n3,x3,'filled'); set(Hs,'markersize',2);
axis([min(n3)-1,max(n3)+1,min(x3)-1,max(x3)+2]);
ytick = [-6:2:12];
xlabel('n','FontSize',LFS); ylabel('x_3(n)','FontSize',LFS);
title('Sequence x_3(n)','FontSize',TFS);
set(gca,'XTickMode','manual','XTick',n3);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../EPSFILES/P0201c;
```

The plots of $x_3(n)$ is shown in Figure 2.3.
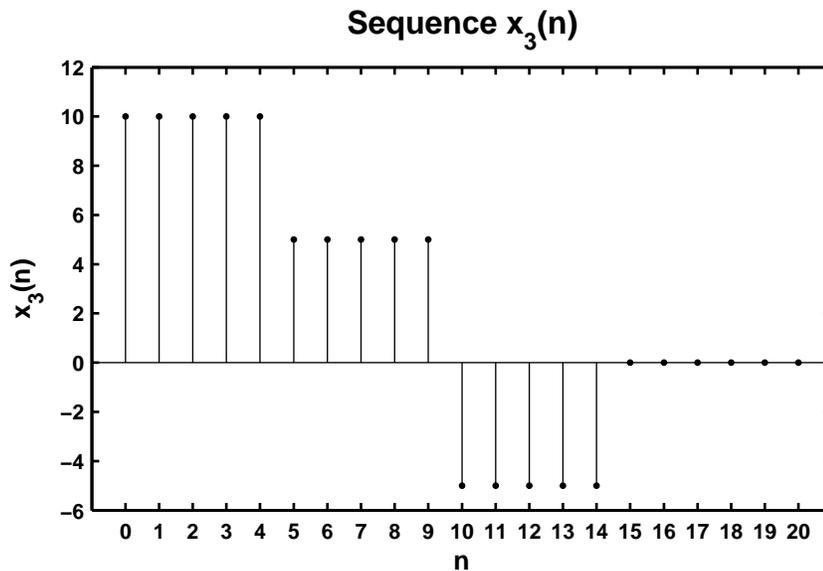
Figure 2.3: Problem P2.1.3 sequence plot

4. $x_4(n) = e^{0.1n}[u(n+20) - u(n-10)]$.

```
% P0201d: x4(n) = e ^ {0.1n} [u(n + 20) - u(n - 10)].
clc; close all;

n4 = [-25:15];
x4 = exp(0.1*n4).*(stepseq(-20,-25,15) - stepseq(10,-25,15));

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201d');
Hs = stem(n4,x4,'filled'); set(Hs,'markersize',2);
axis([min(n4)-2,max(n4)+2,min(x4)-1,max(x4)+1]);
xlabel('n','FontSize',LFS); ylabel('x_4(n)','FontSize',LFS);
title('Sequence x_4(n)','FontSize',TFS); ntick = [n4(1):5:n4(end)];
set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0201d; print -deps2 ../../Latex/P0201d;
```

The plots of $x_4(n)$ is shown in Figure 2.4.
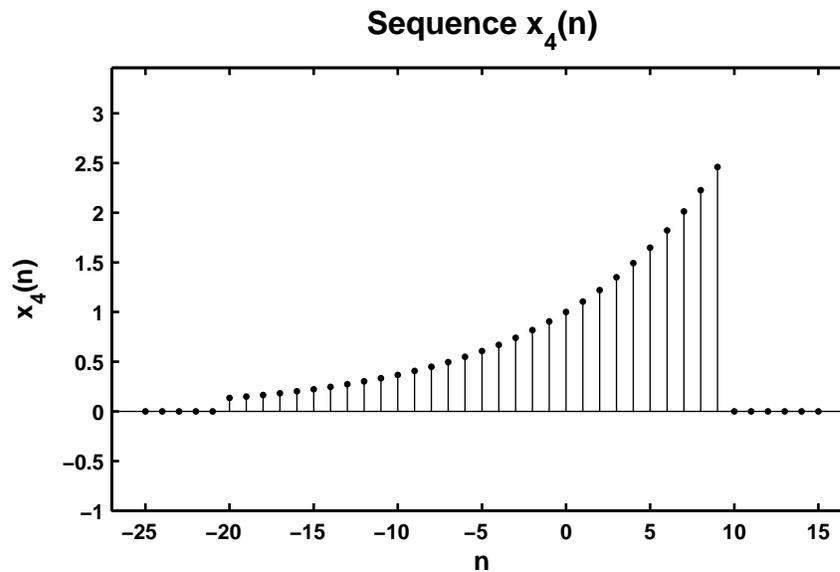
Figure 2.4: Problem P2.1.4 sequence plot

5. $x_5(n) = 5[\cos(0.49\pi n) + \cos(0.51\pi n)]$, $-200 \leq n \leq 200$. Comment on the waveform shape.

```
% P0201e: x5(n) = 5[cos(0.49*pi*n) + cos(0.51*pi*n)], -200 <= n <= 200.
clc; close all;

n5 = [-200:200]; x5  = 5*(cos(0.49*pi*n5) + cos(0.51*pi*n5));

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201e');
Hs = stem(n5,x5,'filled'); set(Hs,'markersize',2);
axis([min(n5)-10,max(n5)+10,min(x5)-2,max(x5)+2]);
xlabel('n','FontSize',LFS); ylabel('x_5(n)','FOntSize',LFS);
title('Sequence x_5(n)','FontSize',TFS);
ntick = [n5(1): 40:n5(end)]; ytick = [-12 -10:5:10 12];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0201e; print -deps2 ../../Latex/P0201e;
```

The plots of $x_5(n)$ is shown in Figure 2.5.
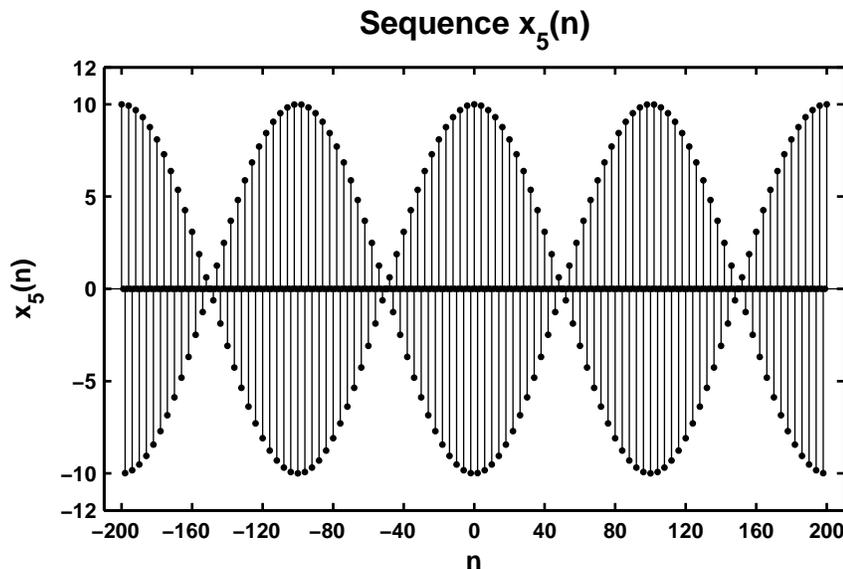

Figure 2.5: Problem P2.1.5 sequence plot

6. $x_6(n) = 2\sin(0.01\pi n)\cos(0.5\pi n)$, $-200 \le n \le 200$.

```
%P0201f: x6(n) = 2 sin(0.01*pi*n) cos(0.5*pi*n), -200 <= n <= 200.
clc; close all;

n6 = [-200:200]; x6 = 2*sin(0.01*pi*n6).*cos(0.5*pi*n6);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201f');
Hs = stem(n6,x6,'filled'); set(Hs,'markersize',2);
axis([min(n6)-10,max(n6)+10,min(x6)-1,max(x6)+1]);
xlabel('n','FontSize',LFS); ylabel('x_6(n)','FontSize',LFS);
title('Sequence x_6(n)','FontSize',TFS);
ntick = [n6(1): 40:n6(end)];
set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0201f; print -deps2 ../../Latex/P0201f;
```

The plots of $x_6(n)$ is shown in Figure 2.6.
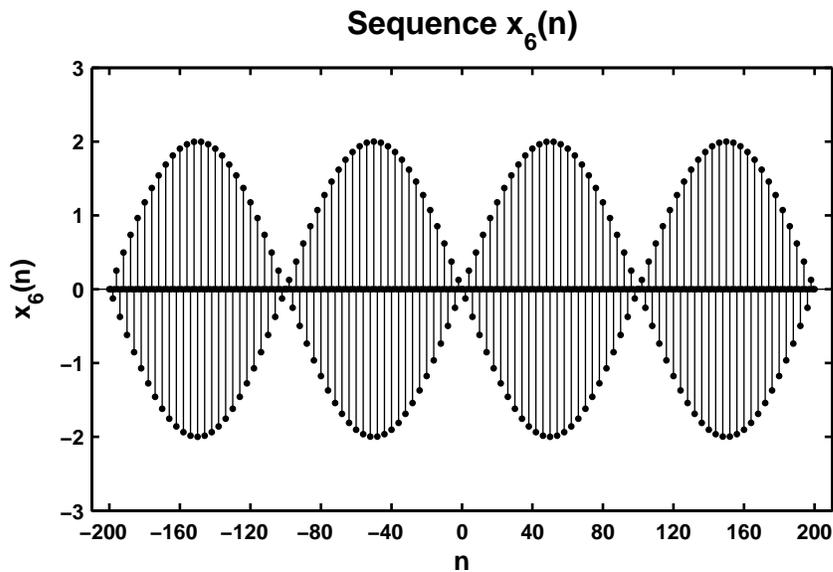

Figure 2.6: Problem P2.1.6 sequence plot

7. $x_7(n) = e^{-0.05n} \sin(0.1\pi n + \pi/3),\ 0 \le n \le 100$.

```
% P0201g: x7(n) = e ^ {-0.05*n}*sin(0.1*pi*n + pi/3), 0 <= n <=100.
clc; close all;

n7 = [0:100]; x7  = exp(-0.05*n7).*sin(0.1*pi*n7 + pi/3);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201g');
Hs = stem(n7,x7,'filled'); set(Hs,'markersize',2);
axis([min(n7)-5,max(n7)+5,min(x7)-1,max(x7)+1]);
xlabel('n','FontSize',LFS); ylabel('x_7(n)','FontSize',LFS);
title('Sequence x_7(n)','FontSize',TFS);
ntick = [n7(1): 10:n7(end)]; set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0201g;
```

The plots of $x_7(n)$ is shown in Figure 2.7.
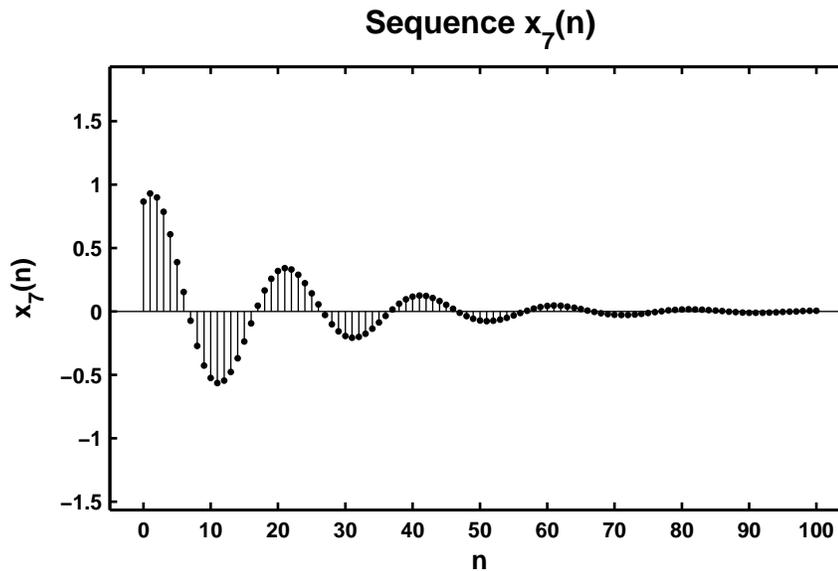

Figure 2.7: Problem P2.1.7 sequence plot

8. $x_8(n) = e^{0.01n} \sin(0.1\pi n),\ 0 \le n \le 100$.

```
% P0201h: x8(n) = e ^ {0.01*n}*sin(0.1*pi*n), 0 <= n <=100.
clc; close all;

n8 = [0:100]; x8  = exp(0.01*n8).*sin(0.1*pi*n8);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0201h');
Hs = stem(n8,x8,'filled'); set(Hs,'markersize',2);
axis([min(n8)-5,max(n8)+5,min(x8)-1,max(x8)+1]);
xlabel('n','FontSize',LFS); ylabel('x_8(n)','FontSize',LFS);
title('Sequence x_8(n)','FontSize',TFS);
ntick = [n8(1): 10:n8(end)]; set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0201h
```

The plots of $x_8(n)$ is shown in Figure 2.8.
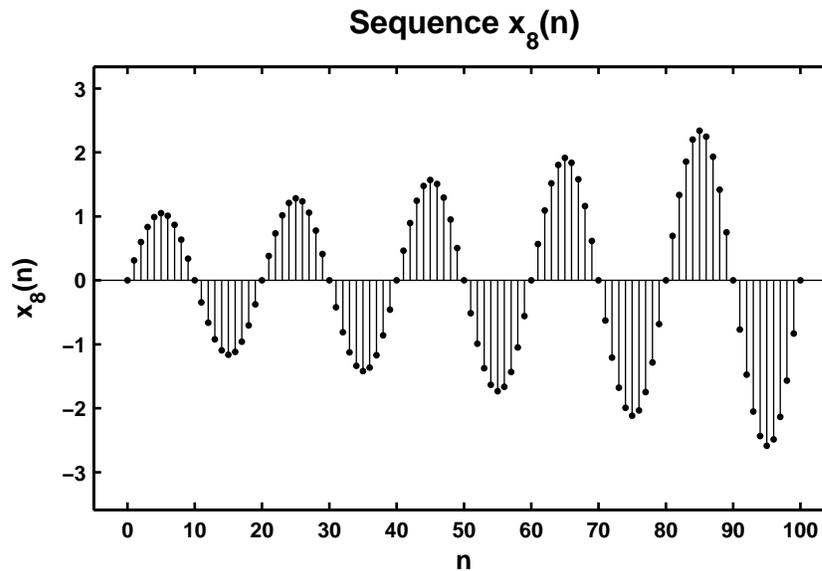
Figure 2.8: Problem P2.1.8 sequence plot

P2.2 Generate the following random sequences and obtain their histogram using the `hist` function with 100 bins. Use the `bar` function to plot each histogram.

1. $x_1(n)$ is a random sequence whose samples are independent and uniformly distributed over [0, 2] interval. Generate 100,000 samples.

```
% P0202a: x1(n) = uniform[0,2]
clc; close all;

n1 = [0:100000-1]; x1 = 2*rand(1,100000);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0202a');
[h1,x1out] = hist(x1,100); bar(x1out, h1);
axis([-0.1 2.1 0 1200]);
xlabel('interval','FontSize',LFS);
ylabel('number of elements','FontSize',LFS);
title('Histogram of sequence x_1(n) in 100 bins','FontSize',TFS);
print -deps2 ../CHAP2_EPSFILES/P0202a;
```

The plots of $x_1(n)$ is shown in Figure 2.9.
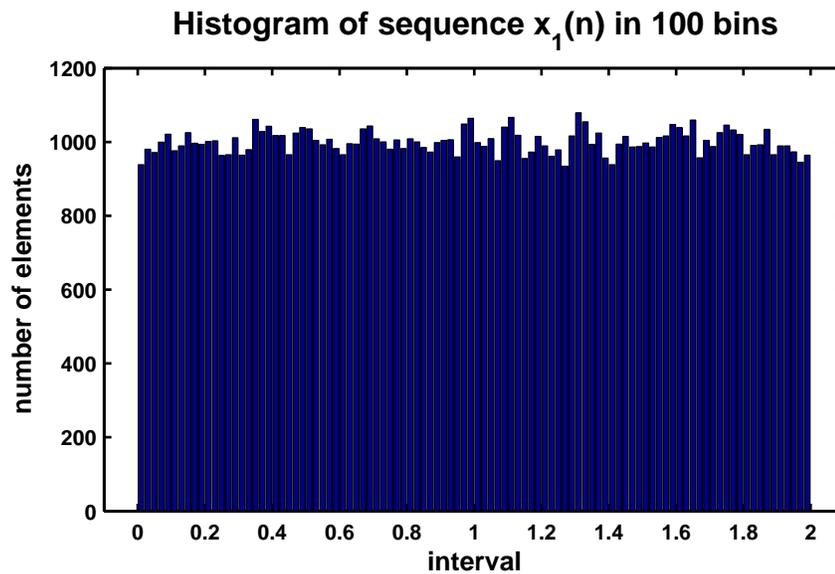
Figure 2.9: Problem P2.2.1 sequence plot

2. $x_2(n)$ is a Gaussian random sequence whose samples are independent with mean 10 and variance 10. Generate 10,000 samples.

```
% P0202b: x2(n) = gaussian{10,10}
clc; close all;

n2 = [1:10000]; x2 = 10 + sqrt(10)*randn(1,10000);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0202b');
[h2,x2out] = hist(x2,100); bar(x2out,h2);
xlabel('interval','FontSize',LFS);
ylabel('number of elements','FontSize',LFS);
title('Histogram of sequence x_2(n) in 100 bins','FontSize',TFS);
print -deps2 ../CHAP2_EPSFILES/P0202b;
```

The plots of $x_2(n)$ is shown in Figure 2.10.
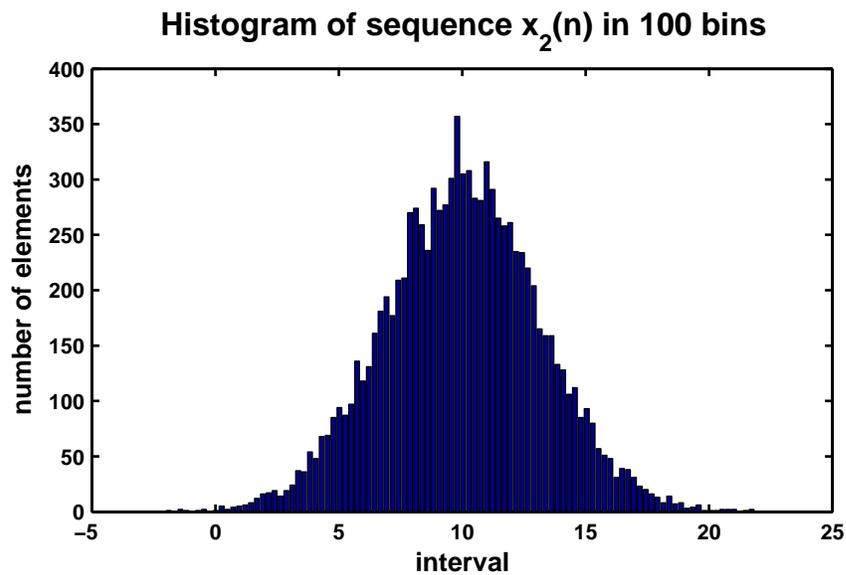
Figure 2.10: Problem P2.2.2 sequence plot

3. $x_3(n) = x_1(n) + x_1(n-1)$ where $x_1(n)$ is the random sequence given in part 1 above. Comment on the shape of this histogram and explain the shape.

```
% P0202c: x3(n) = x1(n) + x1(n - 1) where  x1(n) = uniform[0,2]
clc; close all;

n1 = [0:100000-1]; x1 = 2*rand(1,100000);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0202c');
[x11,n11] = sigshift(x1,n1,1);
[x3,n3] = sigadd(x1,n1,x11,n11);
[h3,x3out] = hist(x3,100);
bar(x3out,h3); axis([-0.5 4.5 0 2500]);
xlabel('interval','FontSize',LFS);
ylabel('number of elements','FontSize',LFS);
title('Histogram of sequence x_3(n) in 100 bins','FontSize',TFS);
print -deps2 ../CHAP2_EPSFILES/P0202c;
```

The plots of $x_3(n)$ is shown in Figure 2.11.
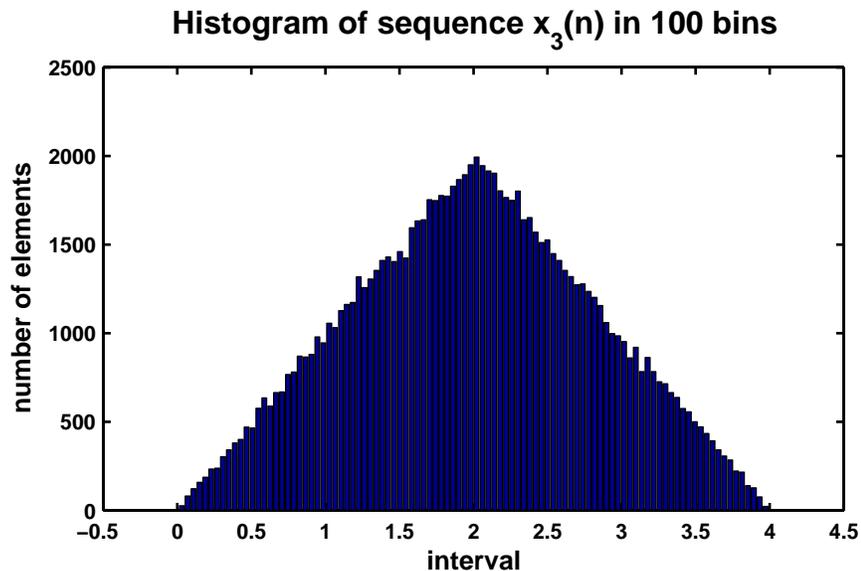
Figure 2.11: Problem P2.2.3 sequence plot

4. $x_4(n) = \sum_{k=1}^{4} y_k(n)$ where each random sequence $y_k(n)$ is independent of others with samples uniformly distributed over $[-0.5, 0.5]$. Comment on the shape of this histogram.

```
%P0202d: x4(n) = sum_{k=1} ^ {4} y_k(n), where each independent of others
%          with samples uniformly distributed over [-0.5,0.5];
clc; close all;

y1 = rand(1,100000) - 0.5; y2 = rand(1,100000) - 0.5;
y3 = rand(1,100000) - 0.5; y4 = rand(1,100000) - 0.5;
x4 = y1 + y2 + y3 + y4;

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0202d');
[h4,x4out] = hist(x4,100); bar(x4out,h4);
xlabel('interval','FontSize',LFS);
ylabel('number of elements','FontSize',LFS);
title('Histogram of sequence x_4(n) in 100 bins','FontSize',TFS);
print -deps2 ../CHAP2_EPSFILES/P0202d;
```

The plots of $x_4(n)$ is shown in Figure 2.12.
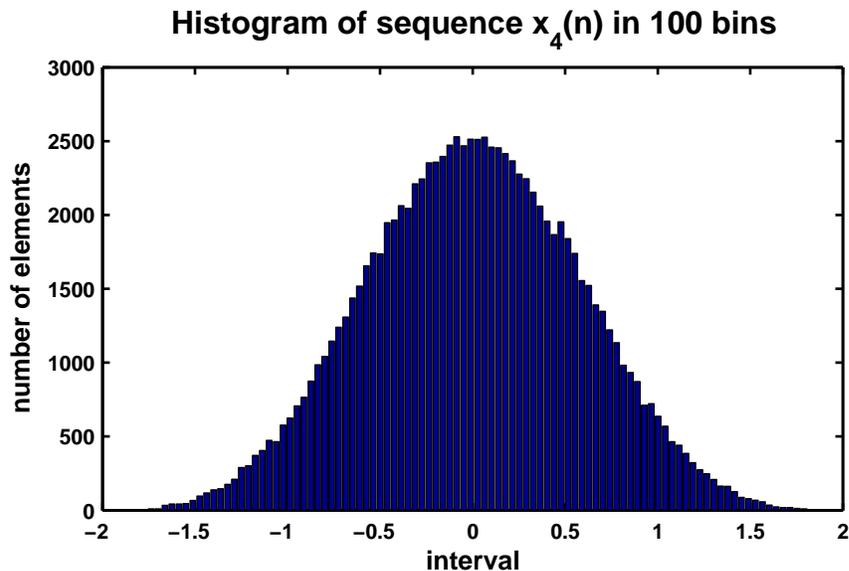
Figure 2.12: Problem P2.2.4 sequence plot

P2.3 Generate the following periodic sequences and plot their samples (using the `stem` function) over the indicated number of periods.

    1. $\tilde{x}_1(n) = \{\ldots, -2, -1, \underset{\uparrow}{0}, 1, 2, \ldots\}_{\text{periodic}}$. Plot 5 periods.

```
% P0203a: x1(n) = {...,-2,-1,0,1,2,-2,-1,0,1,2...} periodic. 5 periods
clc; close all;

n1 = [-12:12]; x1 = [-2,-1,0,1,2];
x1 = x1'*ones(1,5); x1 = (x1(:))';

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0203a');
Hs = stem(n1,x1,'filled'); set(Hs,'markersize',2);
axis([min(n1)-1,max(n1)+1,min(x1)-1,max(x1)+1]);
xlabel('n','FontSize',LFS); ylabel('x_1(n)','FontSize',LFS);
title('Sequence x_1(n)','FontSize',TFS);
ntick = [n1(1):2:n1(end)]; ytick = [min(x1) - 1:max(x1) + 1];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0203a
```

The plots of $\tilde{x}_1(n)$ is shown in Figure 2.13.
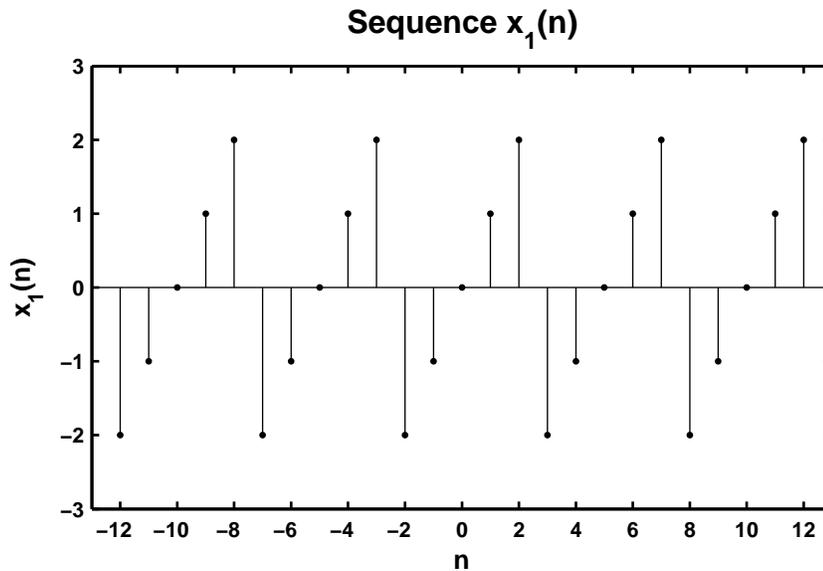
Figure 2.13: Problem P2.3.1 sequence plot

2. $\tilde{x}_2(n) = e^{0.1n}[u(n) - u(n - 20)]_{\text{periodic}}$. Plot 3 periods.

```
% P0203b: x2 = e ^ {0.1n} [u(n) - u(n-20)] periodic. 3 periods
clc; close all;

n2 = [0:21]; x2 = exp(0.1*n2).*(stepseq(0,0,21)-stepseq(20,0,21));
x2 = x2'*ones(1,3); x2 = (x2(:))'; n2 = [-22:43];

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0203b');
Hs = stem(n2,x2,'filled'); set(Hs,'markersize',2);
axis([min(n2)-2,max(n2)+4,min(x2)-1,max(x2)+1]);
xlabel('n','FontSize',LFS); ylabel('x_2(n)','FontSize',LFS);
title('Sequence x_2(n)','FontSize',TFS);
ntick = [n2(1):4:n2(end)-5 n2(end)];
set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../Chap2_EPSFILES/P0203b;
```

The plots of $\tilde{x}_2(n)$ is shown in Figure 2.14.
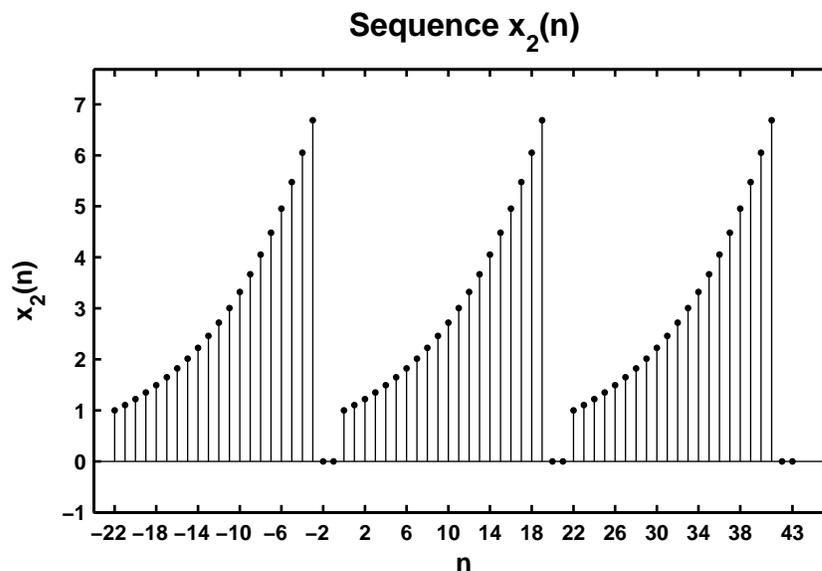
Figure 2.14: Problem P2.3.2 sequence plot

3. $\tilde{x}_3(n) = sin(0.1\pi n)[u(n) - u(n - 10)]$. Plot 4 periods.

```
% P0203c: x1(n) = {...,-2,-1,0,1,2,-2,-1,0,1,2...} periodic. 5 periods
clc; close all;

n3 = [0:11]; x3 = sin(0.1*pi*n3).*(stepseq(0,0,11)-stepseq(10,0,11));
x3 = x3'*ones(1,4); x3 = (x3(:))'; n3 = [-12:35];

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0203c');
Hs = stem(n3,x3,'filled'); set(Hs,'markersize',2);
axis([min(n3)-1,max(n3)+1,min(x3)-0.5,max(x3)+0.5]);
xlabel('n','FontSize',LFS); ylabel('x_3(n)','FontSize',LFS);
title('Sequence x_3(n)','FontSize',TFS);
ntick = [n3(1):4:n3(end)-3 n3(end)];
set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0203c;
```

The plots of $\tilde{x}_3(n)$ is shown in Figure 2.15.
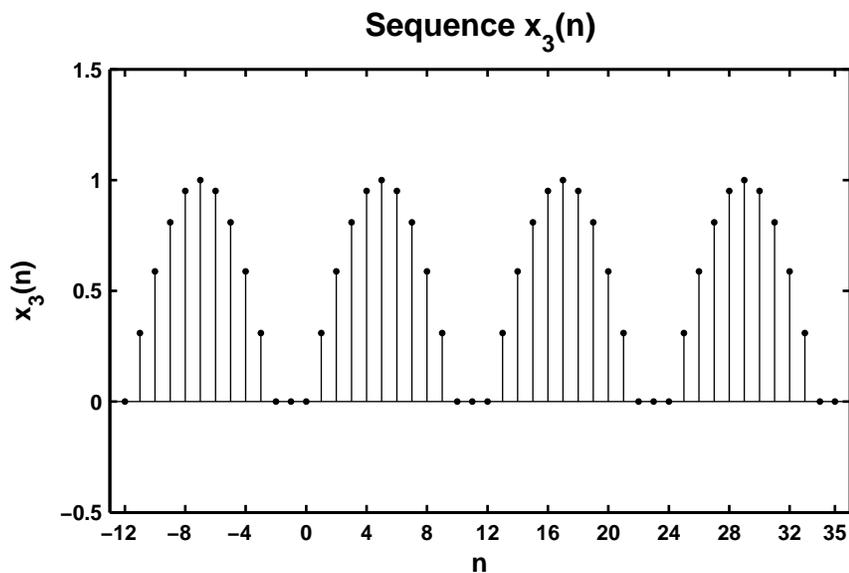
Figure 2.15: Problem P2.3.3 sequence plot

4. $\tilde{x}_4(n) = \{\ldots, \underset{\uparrow}{1}, 2, 3, \ldots\}_{\text{periodic}} + \{\ldots, \underset{\uparrow}{1}, 2, 3, 4, \ldots\}_{\text{periodic}}, \ 0 \leq n \leq 24$. What is the period of $\tilde{x}_4(n)$?

```
% P0203d x1(n) = {...,-2,-1,0,1,2,-2,-1,0,1,2...} periodic. 5 periods
clc; close all;

n4 = [0:24]; x4a = [1 2 3]; x4a = x4a'*ones(1,9); x4a = (x4a(:))';
x4b = [1 2 3 4]; x4b = x4b'*ones(1,7); x4b = (x4b(:))';
x4 = x4a(1:25) + x4b(1:25);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0203d');
Hs = stem(n4,x4,'filled'); set(Hs,'markersize',2);
axis([min(n4)-1,max(n4)+1,min(x4)-1,max(x4)+1]);
xlabel('n', 'FontSize', LFS); ylabel('x_4(n)','FontSize',LFS);
title('Sequence x_4(n):Period = 12','FontSize',TFS);
ntick = [n4(1) :2:n4(end)]; set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0203d;
```

The plots of $\tilde{x}_4(n)$ is shown in Figure 2.16. From the figure, the fundamental period of $\tilde{x}_4(n)$ is 12.
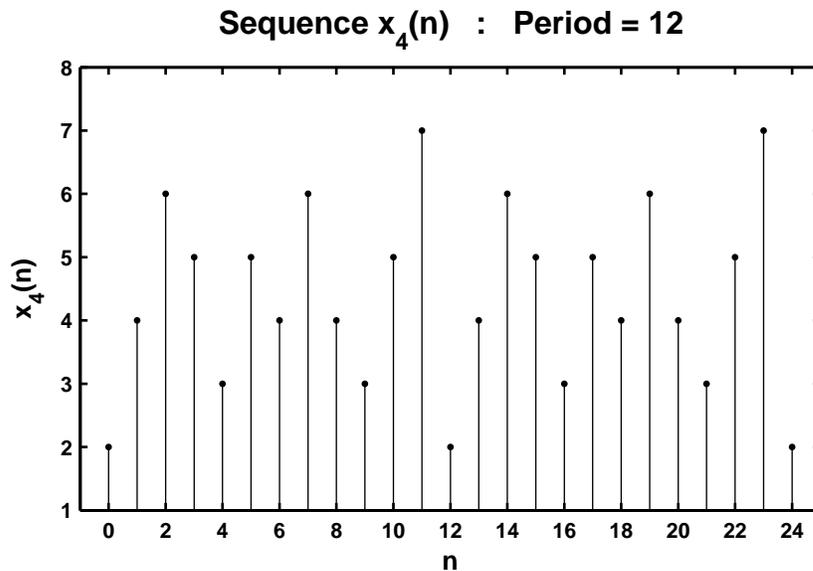
Figure 2.16: Problem P2.3.4 sequence plot

P2.4 Let $x(n) = \{2, 4, -3, \underset{\uparrow}{1}, -5, 4, 7\}$. Generate and plot the samples (use the `stem` function) of the following sequences.

1. $x_1(n) = 2x(n-3) + 3x(n+4) - x(n)$

```
% P0204a: x(n) = [2,4,-3,1,-5,4,7]; -3 <=n <= 3;
% x1(n) = 2x(n - 3) + 3x(n + 4) - x(n)
clc; close all;

n = [-3:3]; x = [2,4,-3,1,-5,4,7];
[x11,n11] = sigshift(x,n,3);                        % shift by 3
[x12,n12] = sigshift(x,n,-4);                       % shift by -4
[x13,n13] = sigadd(2*x11,n11,3*x12,n12); % add two sequences
[x1,n1] = sigadd(x13,n13,-x,n);             % add two sequences

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0204a');
Hs = stem(n1,x1,'filled'); set(Hs,'markersize',2);
axis([min(n1)-1,max(n1)+1,min(x1)-3,max(x1)+1]);
xlabel('n','FontSize',LFS);
ylabel('x_1(n)','FontSize',LFS);
title('Sequence x_1(n)','FontSize',TFS); ntick = n1;
ytick = [min(x1)-3:5:max(x1)+1];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0204a;
```

The plots of $x_1(n)$ is shown in Figure 2.17.
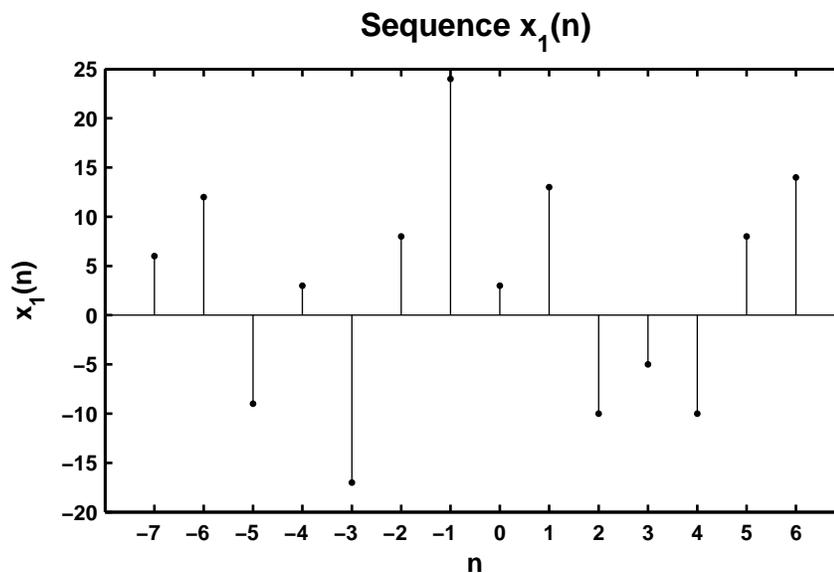
Figure 2.17: Problem P2.4.1 sequence plot

2. $x_2(n) = 4x(4+n) + 5x(n+5) + 2x(n)$

```
% P0204b: x(n) = [2,4,-3,1,-5,4,7]; -3 <=n <= 3;
% x2(n) = 4x(4 + n) + 5x(n + 5) + 2x(n)
clc; close all;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0204b');
n = [-3:3]; x = [2,4,-3,1,-5,4,7];

[x21,n21] = sigshift(x,n,-4);                        % shift by -4
[x22,n22] = sigshift(x,n,-5);                        % shift by -5
[x23,n23] = sigadd(4*x21,n21,5*x22,n22); % add two sequences
[x2,n2] = sigadd(x23,n23,2*x,n);          % add two sequences

Hs = stem(n2,x2,'filled'); set(Hs,'markersize',2);
axis([min(n2)-1,max(n2)+1,min(x2)-4,max(x2)+6]);
xlabel('n','FontSize',LFS); ylabel('x_2(n)','FontSize',LFS);
title('Sequence x_2(n)','FontSize',TFS); ntick = n2;
ytick = [-25 -20:10:60 65];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0204b;
```

The plots of $x_2(n)$ is shown in Figure 2.18.
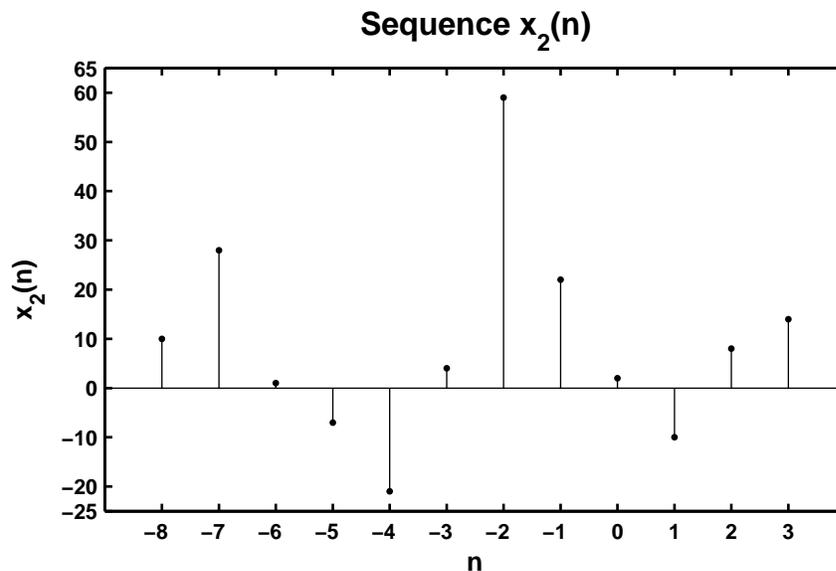
Figure 2.18: Problem P2.4.2 sequence plot

3. $x_3(n) = x(n+3)x(n-2) + x(1-n)x(n+1)$

```
% P0204c: x(n) = [2,4,-3,1,-5,4,7]; -3 <=n <= 3;
%         x3(n) = x(n + 3)x(n - 2) + x(1 - n)x(n + 1)
clc; close all;

n = [-3:3]; x = [2,4,-3,1,-5,4,7];    % given sequence x(n)
[x31,n31] = sigshift(x,n,-3);              % shift sequence by -3
[x32,n32] = sigshift(x,n,2);               % shift sequence by 2
[x33,n33] = sigmult(x31,n31,x32,n32);   % multiply 2 sequences
[x34,n34] = sigfold(x,n);                  % fold x(n)
[x34,n34] = sigshift(x34,n34,1);           % shift x(-n) by 1
[x35,n35] = sigshift(x,n,-1);              % shift x(n) by -1
[x36,n36] = sigmult(x34,n34,x35,n35);   % multiply 2 sequences
  [x3,n3] = sigadd(x33,n33,x36,n36);    % add 2 sequences

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0204c');
Hs = stem(n3,x3,'filled'); set(Hs,'markersize',2);
axis([min(n3)-1,max(n3)+1,min(x3)-10,max(x3)+10]);
xlabel('n','FontSize',LFS); ylabel('x_3(n)','FontSize',LFS);
title('Sequence x_3(n)','FontSize',TFS);
ntick = n3; ytick = [-30:10:60];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0204c;
```

The plots of $x_3(n)$ is shown in Figure 2.19.
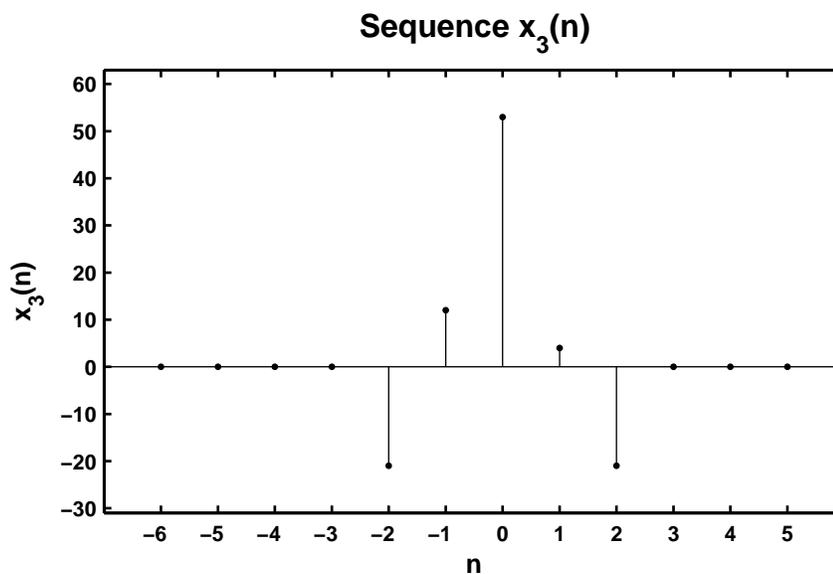
Figure 2.19: Problem P2.4.3 sequence plot

4. $x_4(n) = 2e^{0.5n}x(n) + \cos(0.1\pi n)x(n+2), \; -10 \le n \le 10$

```
% P0204d: x(n) = [2,4,-3,1,-5,4,7]; -3 <=n <= 3;
%          x4(n) = 2*e^{0.5n}*x(n)+cos(0.1*pi*n)*x(n+2), -10 <=n< =10
clc; close all;

n = [-3:3]; x = [2,4,-3,1,-5,4,7];          % given sequence x(n)
n4 = [-10:10]; x41 = 2*exp(0.5*n4); x412 = cos(0.1*pi*n4);
[x42,n42] = sigmult(x41,n4,x,n);
[x43,n43] = sigshift(x,n,-2);
[x44,n44] = sigmult(x412,n42,x43,n43);
  [x4,n4] = sigadd(x42,n42,x44,n44);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0204d');
Hs = stem(n4,x4,'filled'); set(Hs,'markersize',2);
axis([min(n4)-1,max(n4)+1,min(x4)-11,max(x4)+10]);
xlabel('n','FontSize',LFS); ylabel('x_4(n)','FontSize',LFS);
title('Sequence x_4(n)','FontSize',TFS);
ntick = n4; ytick = [ -20:10:70];
set(gca,'XTickMode','manual','XTick',ntick);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0204d;
```

The plot of $x_4(n)$ is shown in Figure 2.20.
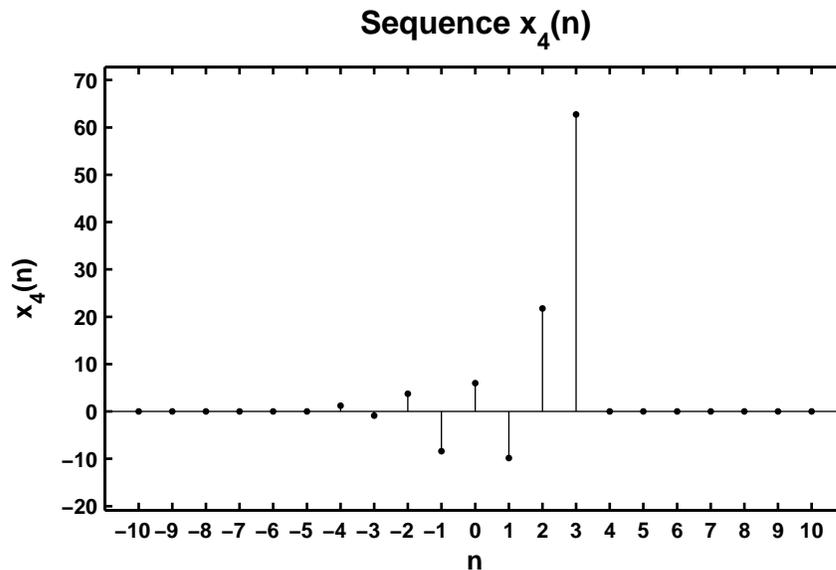
Figure 2.20: Problem P2.4.4 sequence plot

P2.5 The complex exponential sequence $e^{j\omega_0 n}$ or the sinusoidal sequence $\cos(\omega_0 n)$ are periodic if the *normalized* frequency $f_0 \triangleq \frac{\omega_0}{2\pi}$ is a rational number; that is, $f_0 = \frac{K}{N}$, where K and N are integers.

1. Analytical proof: The exponential sequence is periodic if

$$e^{j2\pi f_0(n+N)} = e^{j2\pi f_0 n} \text{ or } e^{j2\pi f_0 N} = 1 \Rightarrow f_0 N = K \text{ (an integer)}$$

which proves the result.

2. $x_1 = \exp(0.1\pi n), \ -100 \le n \le 100$.

```
% P0205b: x1(n) = e^{0.1*j*pi*n} -100 <=n <=100
clc; close all;

n1 = [-100:100]; x1 = exp(0.1*j*pi*n1);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0205b');
subplot(2,1,1); Hs1 = stem(n1,real(x1),'filled'); set(Hs1,'markersize',2);
axis([min(n1)-5,max(n1)+5,min(real(x1))-1,max(real(x1))+1]);
xlabel('n','FontSize',LFS); ylabel('Real(x_1(n))','FontSize',LFS);
title(['Real part of sequence x_1(n) = ' ...
        'exp(0.1 \times j \times pi \times n) ' char(10) ...
        ' Period = 20, K = 1, N = 20'],'FontSize',TFS);
ntick = [n1(1):20:n1(end)]; set(gca,'XTickMode','manual','XTick',ntick);

subplot(2,1,2); Hs2 = stem(n1,imag(x1),'filled'); set(Hs2,'markersize',2);
axis([min(n1)-5,max(n1)+5,min(real(x1))-1,max(real(x1))+1]);
xlabel('n','FontSize',LFS); ylabel('Imag(x_1(n))','FontSize',LFS);
title(['Imaginary part of sequence x_1(n) = ' ...
        'exp(0.1 \times j \times pi \times n) ' char(10) ...
        ' Period = 20, K = 1, N = 20'],'FontSize',TFS);
ntick = [n1(1):20:n1(end)]; set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0205b; print -deps2 ../../Latex/P0205b;
```

The plots of $x_1(n)$ is shown in Figure 2.21. Since $f_0 = 0.1/2 = 1/20$ the sequence is periodic. From the plot in Figure 2.21 we see that in one period of 20 samples $x_1(n)$ exhibits cycle. This is true whenever K and N are relatively prime.

3. $x_2 = \cos(0.1n), \ -20 \le n \le 20$.

```
% P0205c: x2(n) = cos(0.1n), -20 <= n <= 20
clc; close all;

n2 = [-20:20]; x2 = cos(0.1*n2);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0205c');
Hs = stem(n2,x2,'filled'); set(Hs,'markersize',2);
axis([min(n2)-1,max(n2)+1,min(x2)-1,max(x2)+1]);
xlabel('n','FontSize',LFS); ylabel('x_2(n)','FontSize',LFS);
title(['Sequence x_2(n) = cos(0.1 \times n)' char(10) ...
```

Sequence $x_2(n) = \cos(0.1 \times n)$

Not periodic since $f_0 = 0.1 / (2 \times \pi)$ is not a rational number
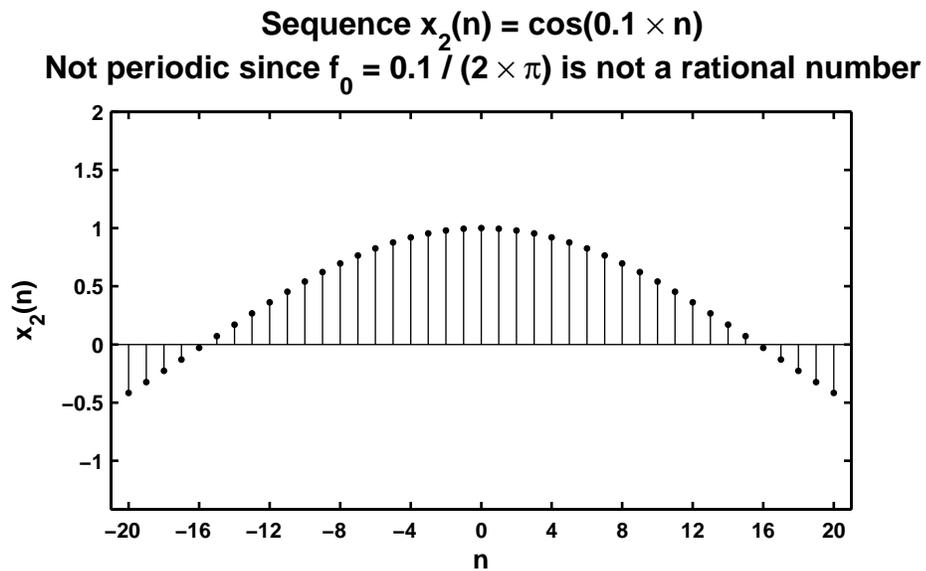
Figure 2.21: Problem P2.5.2 sequence plots

```
        'Not periodic since f_0 = 0.1 / (2 \times \pi)' ...
        ' is not a rational number'], 'FontSize',TFS);
ntick = [n2(1):4:n2(end)]; set(gca,'XTickMode','manual','XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0205c;
```

The plots of $x_1(n)$ is shown in Figure 2.22. In this case f_0 is not a rational number and hence the sequence $x_2(n)$ is not periodic. This can be clearly seen from the plot of $x_2(n)$ in Figure 2.22.

Sequence $x_2(n) = \cos(0.1 \times n)$
Not periodic since $f_0 = 0.1 / (2 \times \pi)$ is not a rational number
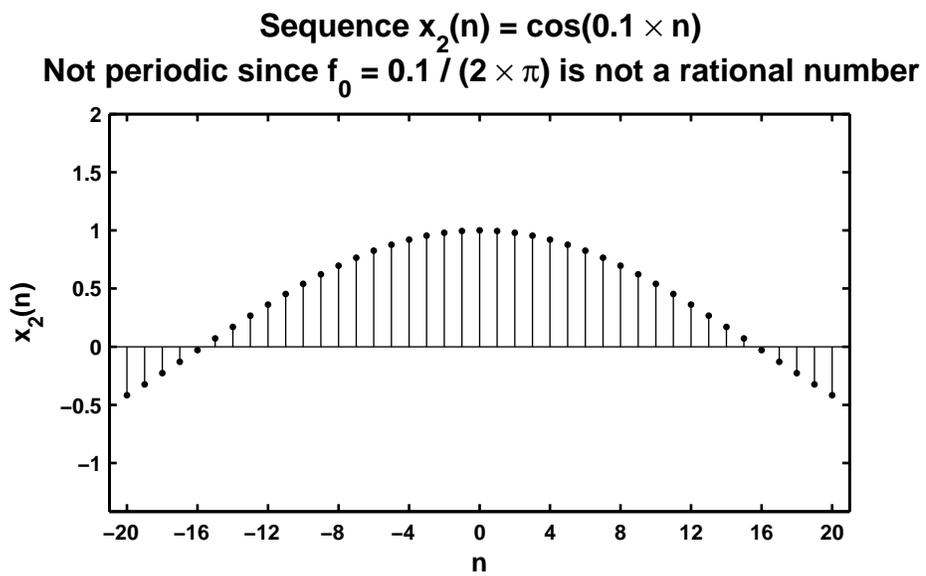
Figure 2.22: Problem P2.5.3 sequence plots

P2.6 Using the `evenodd` function decompose the following sequences into their even and odd components. Plot these components using the `stem` function.

1. $x_1(n) = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
   $\uparrow$

```
% P0206a: % Even odd decomposition of x1(n) = [0 1 2 3 4 5 6 7 8 9];
%                                          n   = 0:9;
clc; close all;

x1 = [0 1 2 3 4 5 6 7 8 9]; n1  = [0:9]; [xe1,xo1,m1] = evenodd(x1,n1);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0206a');
subplot(2,1,1); Hs = stem(m1,xe1,'filled'); set(Hs,'markersize',2);
axis([min(m1)-1,max(m1)+1,min(xe1)-1,max(xe1)+1]);
xlabel('n','FontSize',LFS); ylabel('x_e(n)','FontSize',LFS);
title('Even part of x_1(n)','FontSize',TFS);
ntick = [m1(1):m1(end)]; ytick = [-1:5];
set(gca,'XTick',ntick);set(gca,'YTick',ytick);
subplot(2,1,2); Hs = stem(m1,xo1,'filled'); set(Hs,'markersize',2);
axis([min(m1)-1,max(m1)+1,min(xo1)-2,max(xo1)+2]);
xlabel('n','FontSize',LFS); ylabel('x_o(n)','FontSize',LFS);
title('Odd part of x_1(n)','FontSize',TFS);
ntick = [m1(1):m1(end)]; ytick = [-6:2:6];
set(gca,'XTick',ntick);set(gca,'YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0206a; print -deps2 ../../Latex/P0206a;
```

The plots of $x_1(n)$ is shown in Figure 2.23.
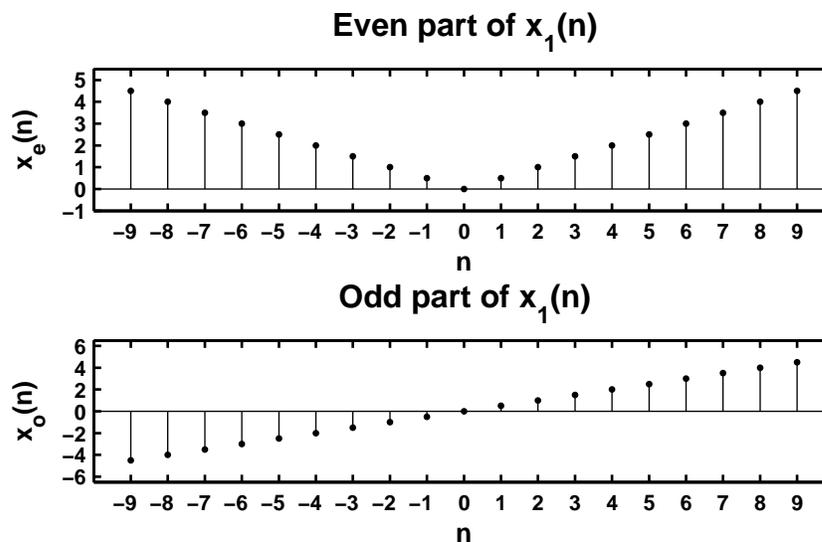
Figure 2.23: Problem P2.6.1 sequence plot

2. $x_2(n) = e^{0.1n}[u(n + 5) - u(n - 10)]$.

```
% P0206b: Even odd decomposition of x2(n) = e ^ {0.1n} [u(n + 5) - u(n - 10)];
clc; close all;

n2 = [-8:12]; x2 = exp(0.1*n2).*(stepseq(-5,-8,12) - stepseq(10,-8,12));
[xe2,xo2,m2] = evenodd(x2,n2);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0206b');
subplot(2,1,1); Hs = stem(m2,xe2,'filled'); set(Hs,'markersize',2);
axis([min(m2)-1,max(m2)+1,min(xe2)-1,max(xe2)+1]);
xlabel('n','FontSize',LFS); ylabel('x_e(n)','FontSize',LFS);
title('Even part of x_2(n) = exp(0.1n) [u(n + 5) - u(n - 10)]',...
      'FontSize',TFS);
ntick = [m2(1):2:m2(end)]; set(gca,'XTick',ntick);
subplot(2,1,2); Hs = stem(m2,xo2,'filled'); set(Hs,'markersize',2);
axis([min(m2)-1,max(m2)+1,min(xo2)-1,max(xo2)+1]);
xlabel('n','FontSize',LFS); ylabel('x_o(n)','FontSize',LFS);
title('Odd part of x_2(n) = exp(0.1n) [u(n + 5) - u(n - 10)]',...
      'FontSize',TFS);
ntick = [m2(1) :2:m2(end)]; set(gca,'XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0206b; print -deps2 ../../Latex/P0206b;
```

The plots of $x_2(n)$ is shown in Figure 2.24.
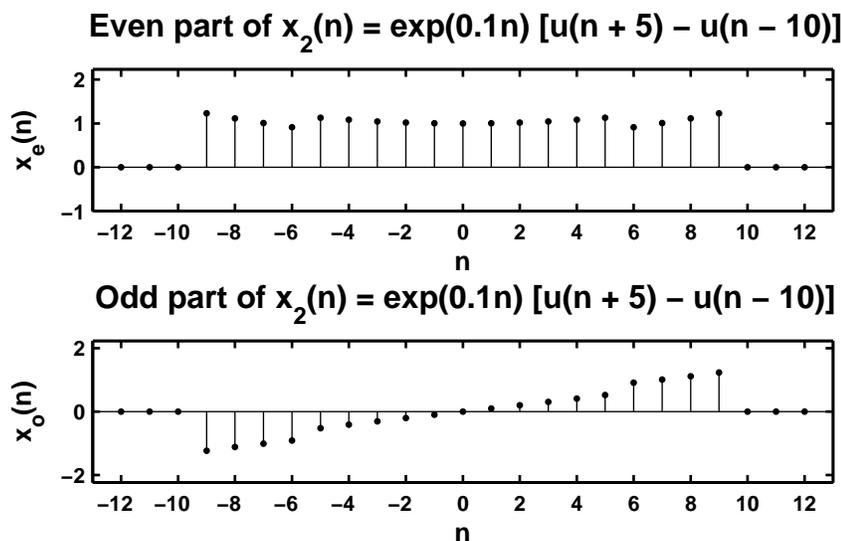
Figure 2.24: Problem P2.6.2 sequence plot

3. $x_3(n) = \cos(0.2\pi n + \pi/4)$, $-20 \le n \le 20$.

```
% P0206c: Even odd decomposition of x2(n) = cos(0.2*pi*n + pi/4);
%                                     -20 <= n <= 20;
clc; close all;

n3 = [-20:20]; x3 = cos(0.2*pi*n3 + pi/4);
[xe3,xo3,m3] = evenodd(x3,n3);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0206c');
subplot(2,1,1); Hs = stem(m3,xe3,'filled'); set(Hs,'markersize',2);
axis([min(m3)-2,max(m3)+2,min(xe3)-1,max(xe3)+1]);
xlabel('n','FontSize',LFS); ylabel('x_e(n)','FontSize',LFS);
title('Even part of x_3(n)  =  cos(0.2 \times \pi \times n + \pi/4)',...
        'FontSize',TFS);
ntick = [m3(1):4:m3(end)]; set(gca,'XTick',ntick);
subplot(2,1,2); Hs = stem(m3,xo3,'filled'); set(Hs,'markersize',2);
axis([min(m3)-2,max(m3)+2,min(xo3)-1,max(xo3)+1]);
xlabel('n','FontSize',LFS); ylabel('x_o(n)','FontSize',LFS);
title('Odd part of x_3(n)  =  cos(0.2 \times \pi \times n + \pi/4)',...
        'FontSize',TFS);
ntick = [m3(1):4 :m3(end)]; set(gca,'XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0206c; print -deps2 ../../Latex/P0206c;
```

The plots of $x_3(n)$ is shown in Figure 2.25.
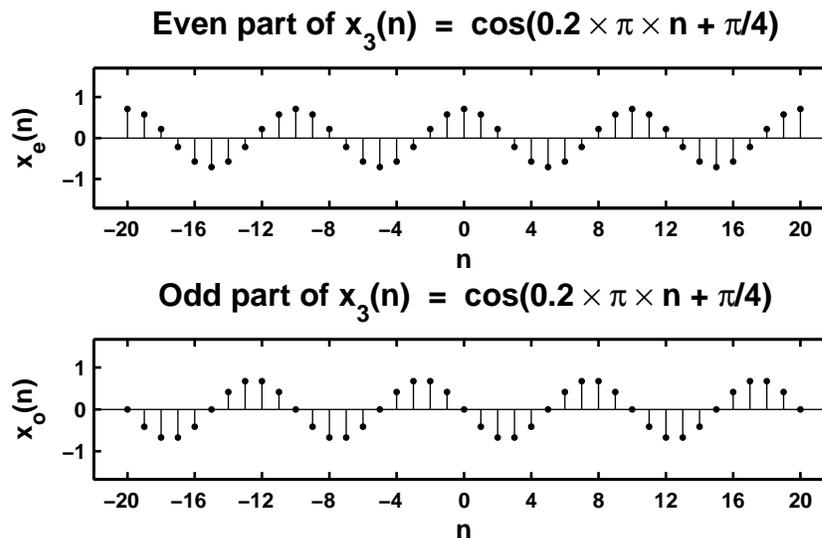
Figure 2.25: Problem P2.6.3 sequence plot

4. $x_4(n) = e^{-0.05n} \sin(0.1\pi n + \pi/3), \ 0 \le n \le 100$.

```
% P0206d: x4(n) = e ^ {-0.05*n}*sin(0.1*pi*n + pi/3), 0 <= n <= 100
clc; close all;

n4 = [0:100]; x4 = exp(-0.05*n4).*sin(0.1*pi*n4 + pi/3);
[xe4,xo4,m4] = evenodd(x4,n4);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0206d');
subplot(2,1,1); Hs = stem(m4,xe4,'filled'); set(Hs,'markersize',2);
axis([min(m4)-10,max(m4)+10,min(xe4)-1,max(xe4)+1]);
xlabel('n','FontSize',LFS); ylabel('x_e(n)','FontSize',LFS);
title(['Even part of x_4(n) = ' ...
       'exp(-0.05 \times n) \times sin(0.1 \times \pi \times n + ' ...
       '\pi/3)'],'FontSize',TFS);
ntick = [m4(1):20:m4(end)]; set(gca,'XTick',ntick);

subplot(2,1,2); Hs = stem(m4,xo4,'filled'); set(Hs,'markersize',2);
axis([min(m4)-10,max(m4)+10,min(xo4)-1,max(xo4)+1]);
xlabel('n','FontSize',LFS); ylabel('x_o(n)','FontSize',LFS);
title(['Odd part of x_4(n) = ' ...
       'exp(-0.05 \times n) \times sin(0.1 \times \pi \times n + ' ...
       '\pi/3)'],'FontSize',TFS);
ntick = [m4(1):20 :m4(end)]; set(gca,'XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0206d; print -deps2 ../../Latex/P0206d;
```

The plots of $x_1(n)$ are shown in Figure 2.26.
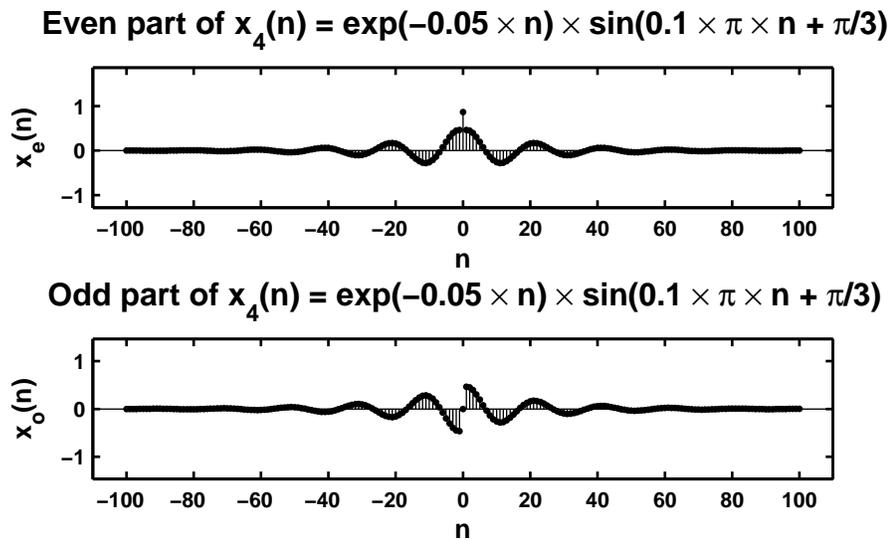
Figure 2.26: Problem P2.6.1 sequence plot

P2.7 A complex-valued sequence $x_e(n)$ is called *conjugate-symmetric* if $x_e(n) = x_e^*(-n)$ and a complex-valued sequence $x_o(n)$ is called *conjugate-antisymmetric* if $x_o(n) = -x_o^*(-n)$. Then any arbitrary complex-valued sequence $x(n)$ can be decomposed into $x(n) = x_e(n) + x_o(n)$ where $x_e(n)$ and $x_o(n)$ are given by

$$x_e(n) = \tfrac{1}{2}\left[x(n) + x^*(-n)\right] \qquad \text{and} \qquad x_o(n) = \tfrac{1}{2}\left[x(n) - x^*(-n)\right] \tag{2.1}$$

respectively.

1. Modify the evenodd function discussed in the text so that it accepts an arbitrary sequence and decomposes it into its conjugate-symmetric and conjugate-antisymmetric components by implementing (2.1).

2. $x(n) = 10\exp([-0.1 + j0.2\pi]n), \quad 0 \le n \le 10$

```
% P0207b: Decomposition of x(n) = 10*e ^ {(-0.1 + j*0.2*pi)*n},
%                                           0 < = n <= 10
% into its conjugate symmetric and conjugate antisymmetric parts.
clc; close all;

n = [0:10]; x = 10*(-0.1+j*0.2*pi)*n; [xe,xo,neo] = evenodd(x,n);
Re_xe = real(xe); Im_xe = imag(xe); Re_xo = real(xo); Im_xo = imag(xo);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0207b');
subplot(2,2,1); Hs = stem(neo,Re_xe); set(Hs,'markersize',2);
ylabel('Re[x_e(n)]','FontSize',LFS); xlabel('n','FontSize',LFS);
axis([min(neo)-1,max(neo)+1,min(Re_xe)-2,max(Re_xe)+2]);
ytick = [min(Re_xe)-2:2:max(Re_xe)+2]; set(gca,'YTick',ytick);
title(['Real part of' char(10) 'even sequence x_e(n)'],'FontSize',TFS);

subplot(2,2,3); Hs = stem(neo,Im_xe); set(Hs,'markersize',2);
ylabel('Im[x_e(n)]','FontSize',LFS); xlabel('n','FontSize',LFS);
axis([min(neo)-1,max(neo)+1,-40,40]);
ytick = [-40:20:40]; set(gca,'YTick',ytick);
title(['Imaginary part of' char(10) 'even sequence x_e(n)'],'FontSize',TFS);

subplot(2,2,2); Hs = stem(neo,Re_xo); set(Hs,'markersize',2);
ylabel('Re[x_o(n)]','FontSize',LFS); xlabel('n','FontSize',LFS);
axis([min(neo)-1,max(neo)+1,min(Re_xo)-1,max(Re_xo)+1]);
ytick = [-6:2:6]; set(gca,'YTick',ytick);
title(['Real part of' char(10) 'odd sequence x_o(n)'],'FontSize',TFS);

subplot(2,2,4); Hs = stem(neo,Im_xo); set(Hs,'markersize',2);
ylabel('Im[x_o(n)]','FontSize',LFS); xlabel('n','FontSize',LFS);
axis([min(neo)-1,max(neo)+1,min(Im_xo)-10,max(Im_xo)+10]);
ytick = [-10:10:40]; set(gca,'YTick',ytick);
title(['Imaginary part of' char(10) 'odd sequence x_o(n)'],'FontSize',TFS);
print -deps2 ../CHAP2_EPSFILES/P0207b;print -deps2 ../../Latex/P0207b;
```

The plots of $x(n)$ are shown in Figure 2.27.
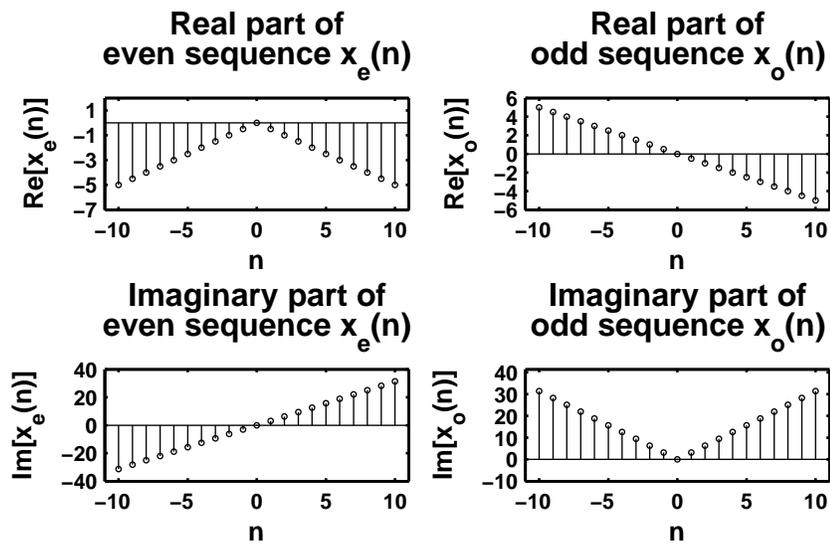
Figure 2.27: Problem P2.7.2 sequence plot

P2.8 The operation of *signal dilation* (or *decimation* or *down-sampling*) is defined by $y(n) = x(nM)$ in which the sequence $x(n)$ is down-sampled by an integer factor M.

1. MATLAB function:

2. $x_1(n) = \sin(0.125\pi n)$, $-50 \leq n \leq 50$. Decimation by a factor of 4.

```
% P0208b: x1(n) = sin(0.125*pi*n),-50 <= n <= 50
%          Decimate x(n) by a factor of 4 to obtain y(n)
clc; close all;

n1 = [-50:50]; x1 = sin(0.125*pi*n1); [y1,m1] = dnsample(x1,n1,4);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0208b');
subplot(2,1,1); Hs = stem(n1,x1); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS);
title('Original sequence x_1(n)','FontSize',TFS);
axis([min(n1)-5,max(n1)+5,min(x1)-0.5,max(x1)+0.5]);
ytick = [-1.5:0.5:1.5]; ntick = [n1(1):10:n1(end)];
set(gca,'XTick',ntick); set(gca,'YTick',ytick);
subplot(2,1,2); Hs = stem(m1,y1); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n) = x(4n)','FontSize',LFS);
title('y_1(n)  =  Original sequence x_1(n) decimated by a factor of 4',...
      'FontSize',TFS);
axis([min(m1)-2,max(m1)+2,min(y1)-0.5,max(y1)+0.5]);
ytick = [-1.5:0.5:1.5]; ntick = [m1(1):2:m1(end)];
set(gca,'XTick',ntick); set(gca,'YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0208b;
```

The plots of $x_1(n)$ and $y_1(n)$ are shown in Figure 2.28. Observe that the original signal $x_1(n)$ can be recovered.

3. $x(n) = \sin(0.5\pi n)$, $-50 \leq n \leq 50$. Decimation by a factor of 4.

```
% P0208c: x2(n) = sin(0.5*pi*n),-50 <= n <= 50
%          Decimate x2(n) by a factor of 4 to obtain y2(n)
clc; close all;

n2 = [-50:50]; x2 = sin(0.5*pi*n2); [y2,m2] = dnsample(x2,n2,4);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0208c');
subplot(2,1,1); Hs = stem(n2,x2); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS);
axis([min(n2)-5,max(n2)+5,min(x2)-0.5,max(x2)+0.5]);
title('Original sequence x_2(n)','FontSize',TFS);
ytick = [-1.5:0.5:1.5]; ntick = [n2(1):10:n2(end)];
set(gca,'XTick',ntick); set(gca,'YTick',ytick);
subplot(2,1,2); Hs = stem(m2,y2); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n) = x(4n)','FontSize',LFS);
axis([min(m2)-1,max(m2)+1,min(y2)-1,max(y2)+1]);
```
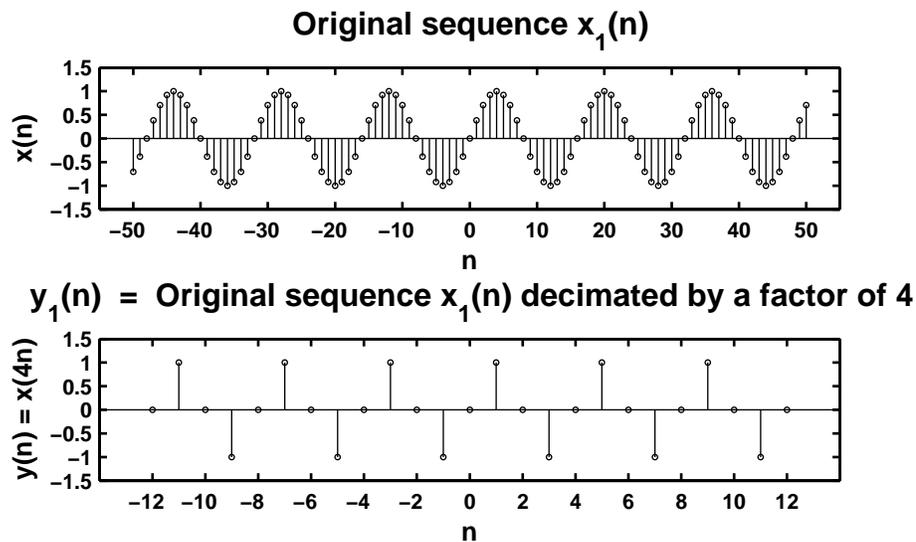
Figure 2.28: Problem P2.8.2 sequence plot

```
title('y_2(n) = Original sequence x_2(n) decimated by a factor of 4',...
     'FontSize',TFS);
ntick = [m2(1):2:m2(end)]; set(gca,'XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0208c; print -deps2 ../../Latex/P0208c;
```

The plots of $x_2(n)$ and $y_2(n)$ are shown in Figure 2.29. Observe that the downsampled signal is a signal with zero frequency. Thus the original signal $x_2(n)$ is lost.
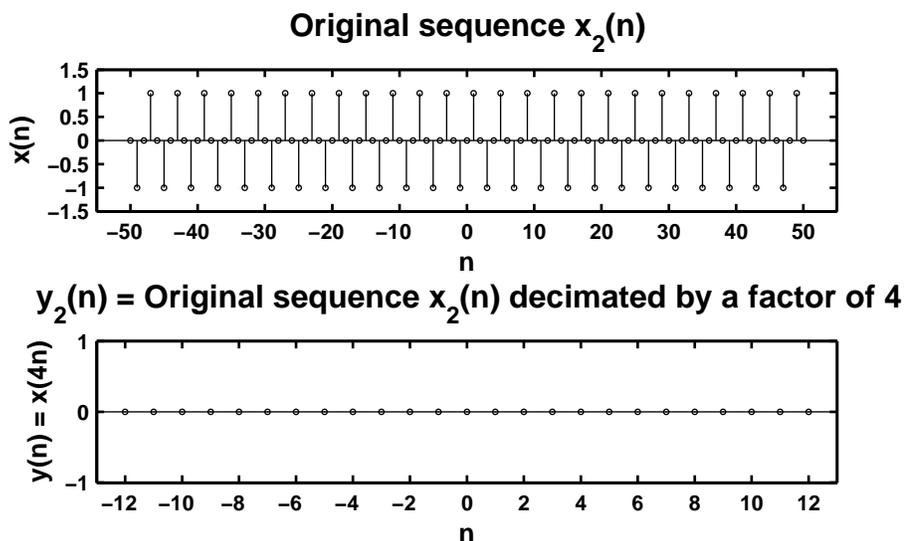
Figure 2.29: Problem P2.8.3 sequence plot

P2.9 The autocorrelation sequence $r_{xx}(\ell)$ and the crosscorrelation sequence $r_{xy}(\ell)$ for the sequences:

$$x(n) = (0.9)^n, \quad 0 \le n \le 20; \qquad y(n) = (0.8)^{-n}, \quad -20 \le n \le 0$$

```
% P0209a: autocorrelation of sequence x(n) = 0.9 ^ n,  0 <= n <= 20
% using the conv_m function
clc; close all;

nx = [0:20]; x = 0.9 .^ nx; [xf,nxf] = sigfold(x,nx);
[rxx,nrxx] = conv_m(x,nx,xf,nxf);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0209a');
Hs = stem(nrxx,rxx); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('r_x_x(n)','FontSize',LFS);
title('Autocorrelation of x(n)','FontSize',TFS);
axis([min(nrxx)-1,max(nrxx)+1,min(rxx),max(rxx)+1]);
ntick = [nrxx(1):4:nrxx(end)]; set(gca,'XTick',ntick);
print -deps2 ../CHAP2_EPSFILES/P0209a; print -deps2 ../../Latex/P0209a;
```

The plot of the autocorrelation is shown in Figure 2.30.
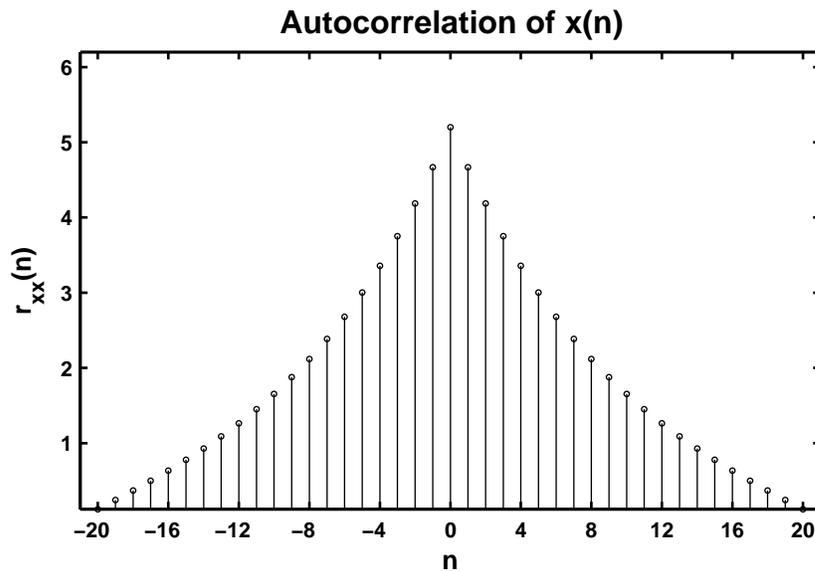
Figure 2.30: Problem P2.9 autocorrelation plot

```
% P0209b: crosscorrelation of sequence x(n) = 0.9 ^ n,  0 <= n <= 20
%         with sequence y = 0.8.^n,  -20 <=n <= 0 using the conv_m function
clc; close all;

nx = [0:20]; x = 0.9 .^ nx; ny = [-20:0]; y = 0.8 .^ ny;
[yf,nyf] = sigfold(y,ny); [rxy,nrxy] = conv_m(x,nx,yf,nyf);
```

```
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0209b');
Hs = stem(nrxy,rxy); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('r_x_y(n)','FontSize',LFS);
title('Crosscorrelation of x(n) and y(n)','FontSize',TFS);
axis([min(nrxy)-1,max(nrxy)+1,min(rxy)-1,max(rxy)+20]);
ytick = [0:50:300 320]; ntick = [nrxy(1):2:nrxy(end)];
set(gca,'XTick',ntick); set(gca,'YTick',ytick);
print -deps2 ../CHAP2_EPSFILES/P0209b;
```

The plot of the crosscorrelation is shown in Figure 2.31.
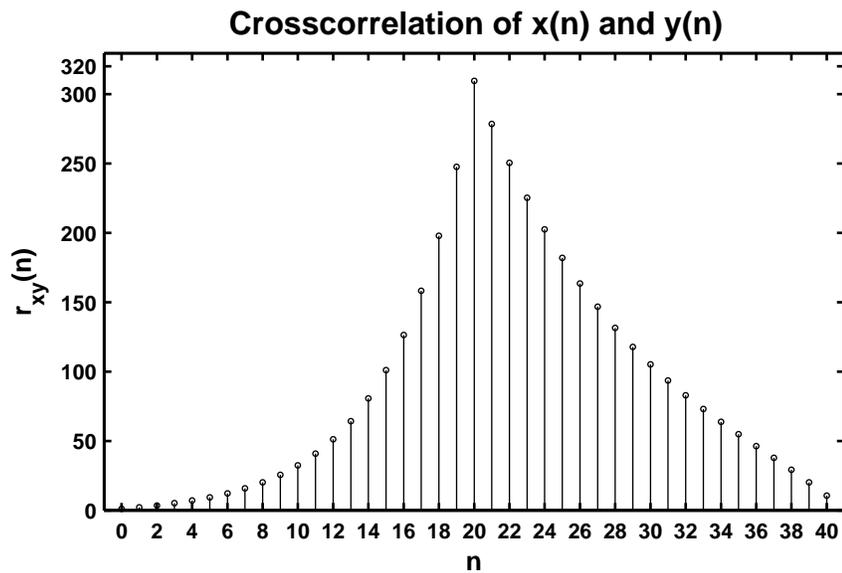
Figure 2.31: Problem P2.9 crosscorrelation plot

P2.10 In a certain concert hall, echoes of the original audio signal $x(n)$ are generated due to the reflections at the walls and ceiling. The audio signal experienced by the listener $y(n)$ is a combination of $x(n)$ and its echoes. Let $y(n) = x(n) + \alpha x(n - k)$ where k is the amount of delay in samples and α is its relative strength. We want to estimate the delay using the correlation analysis.

     1. Determine analytically the autocorrelation $r_{yy}(\ell)$ in terms of the autocorrelation $r_{xx}(\ell)$.

     2. Let $x(n) = \cos(0.2\pi n) + 0.5\cos(0.6\pi n)$, $\alpha = 0.1$, and $k = 50$. Generate 200 samples of $y(n)$ and determine its autocorrelation. Can you obtain α and k by observing $r_{yy}(\ell)$?

P2.11 Linearity of discrete-time systems.

System-1: $T_1[x(n)] = x(n)u(n)$

    1. Analytic determination of linearity:

$$T_1[a_1x_1(n) + a_2x_2(n)] = \{a_1x_1(n) + a_2x_2(n)\}u(n) = a_1x_1(n)u(n) + a_2x_2(n)u(n)$$
$$= a_1T_1[x_1(n)] + a_2T_1[x_2(n)]$$

    Hence the system $T_1[x(n)]$ is **linear**.

    2. MATLAB script:

```
% P0211a: To prove that the system T1[x(n)] = x(n)u(n) is linear
clear; clc; close all;

n = 0:100; x1 = rand(1,length(n));
x2 = sqrt(10)*randn(1,length(n)); u = stepseq(0,0,100);
y1 = x1.*u; y2 = x2.*u; y = (x1 + x2).*u;
diff = sum(abs(y - (y1 + y2)));
if (diff < 1e-5)
    disp(' *** System-1 is Linear *** ');
else
    disp(' *** System-1 is NonLinear *** ');
end
```

    MATLAB verification:

```
>> *** System-1 is Linear ***
```

System-2: $T_2[x(n)] = x(n) + n\,x(n+1)$

    1. Analytic determination of linearity:

$$T_2[a_1x_1(n) + a_2x_2(n)] = \{a_1x_1(n) + a_2x_2(n)\} + n\,\{a_1x_1(n+1) + a_2x_2(n+1)\}$$
$$= a_1\{x_1(n) + n\,x_1(n+1)\} + a_2\{x_2(n) + n\,x_2(n+1)\}$$
$$= a_1T_2[x_1(n)] + a_2T_2[x_2(n)]$$

    Hence the system is $T_2[x(n)]$ **linear**.

    2. MATLAB script:

```
% P0211b: To prove that the system T2[x(n)] = x(n) + n*x(n+1) is linear
clear; clc; close all;

n = 0:100; x1 = rand(1,length(n)); x2 = sqrt(10)*randn(1,length(n));
z  = n; [x11,nx11] = sigshift(x1,n,-1);
[x111,nx111] = sigmult(z,n,x11,nx11); [y1,ny1] = sigadd(x1,n,x111,nx111);

[x21,nx21] = sigshift(x2,n,-1); [x211,nx211] = sigmult(z,n,x21,nx21);
[y2,ny2] = sigadd(x2,n,x211,nx211);
xs = x1 + x2; [xs1,nxs1] = sigshift(xs,n,-1);
[xs11,nxs11] = sigmult(z,n,xs1,nxs1); [y,ny] = sigadd(xs,n,xs11,nxs11);
diff = sum(abs(y - (y1 + y2)));
if (diff < 1e-5)
    disp(' *** System-2 is Linear *** ');
```

```
    else
        disp(' *** System-2 is NonLinear *** ');
    end
```

MATLAB verification:

```
>> *** System-2 is Linear ***
```

System-3: $T_3[x(n)] = x(n) + \frac{1}{2}x(n-2) - \frac{1}{3}x(n-3)x(2n)$

1. Analytic determination of linearity:

$$
\begin{aligned}
T_3[a_1x_1(n) + a_2x_2(n)] &= a_1x_1(n) + a_2x_2(n) + \tfrac{1}{2}\{a_1x_1(n-2) + a_2x_2(n-2)\} \\
&\quad - \tfrac{1}{3}\{a_1x_1(n-3) + a_2x_2(n-3)\}\{a_1x_1(2n) + a_2x_2(2n)\} \\
&= a_1\left\{x_1(n) + \tfrac{1}{2}x_1(n-2) - \tfrac{1}{3}a_1x_1(n-3)x_1(2n)\right\} \\
&\quad + a_2\left\{x_2(n) + \tfrac{1}{2}x_2(n-2) - \tfrac{1}{3}a_2x_2(n-3)x_2(2n)\right\} \\
&\quad + \tfrac{1}{3}\{a_1x_1(n-3)a_2x_2(2n) + a_2x_2(n-3)a_1x_1(2n)\}
\end{aligned}
$$

which clearly is not equal to $a_1 T_3[x_1(n)] + a_2 T_3[x_2(n)]$. The product term in the input-output equation makes the system $T_3[x(n)]$ **nonlinear**.

2. MATLAB script:

```
% P0211c:  To prove that the system T3[x(n)] =   x(n) + 1/2*x(n - 2)
%                                              - 1/3*x(n - 3)*x(2n)
%           is linear
clear; clc; close all;

n = [0:100]; x1 = rand(1,length(n)); x2 = sqrt(10)*randn(1,length(n));
[x11,nx11] = sigshift(x1,n,2); x11 = 1/2*x11; [x12,nx12] = sigshift(x1,n,3);
x12 = -1/3*x12; [x13,nx13] = dnsample(x1,n,2);
[x14,nx14] = sigmult(x12,nx12,x13,nx13);
[x15,nx15] = sigadd(x11,nx11,x14,nx14);
[y1,ny1] = sigadd(x1,n,x15,nx15); [x21,nx21] = sigshift(x2,n,2);
x21 = 1/2*x21; [x22,nx22] = sigshift(x2,n,3);
x22 = -1/3*x22; [x23,nx23] = dnsample(x2,n,2);
[x24,nx24] = sigmult(x22,nx22,x23,nx23);
[x25,nx25] = sigadd(x21,nx21,x24,nx24); [y2,ny2] = sigadd(x2,n,x25,nx25);
xs = x1 + x2; [xs1,nxs1] = sigshift(xs,n,2);
xs1 = 1/2*xs1; [xs2,nxs2] = sigshift(xs,n,3); xs2 = -1/3*xs2;
[xs3,nxs3] = dnsample(xs,n,2); [xs4,nxs4] = sigmult(xs2,nxs2,xs3,nxs3);
[xs5,nxs5] = sigadd(xs1,nxs1,xs4,nxs4);
[y,ny] = sigadd(xs,n,xs5,nxs5); diff = sum(abs(y - (y1 + y2)));
if (diff < 1e-5)
    disp(' *** System-3 is Linear *** ');
else
    disp(' *** System-3 is NonLinear *** ');
end
```

MATLAB verification:

```
>> *** System-3 is NonLinear ***
```

System-4: $T_4[x(n)] = \sum_{k=-\infty}^{n+5} 2x(k)$

1. Analytic determination of linearity:

$$T_4[a_1x_1(n) + a_2x_2(n)] = \sum_{k=-\infty}^{n+5} 2\{a_1x_1(k) + a_2x_2(k)\} = a_1 \sum_{k=-\infty}^{n+5} 2x_1(k) + a_2 \sum_{k=-\infty}^{n+5} 2x_2(k)$$

$$= a_1 T_4[x_1(n)] + a_2 T_4[x_2(n)]$$

Hence the system $T_4[x(n)]$ is **linear**.

2. MATLAB script:

```
% P0211d: To prove that the system T4[x(n)] = sum_{k=-infinity}^{n+5}2*x(k)
%           is linear
clear; clc; close all;

n = [0:100]; x1 = rand(1,length(n)); x2 = sqrt(10)*randn(1,length(n));
y1 = cumsum(x1); ny1 = n - 5; y2 = cumsum(x2); ny2 = n - 5; xs = x1 + x2;
y = cumsum(xs); ny = n - 5; diff = sum(abs(y - (y1 + y2)));
if (diff < 1e-5)
    disp(' *** System-4 is Linear *** ');
else
    disp(' *** System-4 is NonLinear *** ');
end
```

MATLAB verification:
```
>> *** System-4 is Linear ***
```

System-5: $T_5[x(n)] = x(2n)$

1. Analytic determination of linearity:

$$T_5[a_1x_1(n) + a_2x_2(n)] = a_1x_1(2n) + a_2x_2(2n) = a_1 T_5[x_1(n)] + a_2 T_5[x_2(n)]$$

Hence the system $T_5[x(n)]$ is **linear**.

2. MATLAB script:

```
% P0211e: To prove that the system T5[x(n)] =   x(2n) is linear
clear; clc; close all;

n = 0:100; x1 = rand(1,length(n)); x2 = sqrt(10)*randn(1,length(n));
[y1,ny1] = dnsample(x1,n,2); [y2,ny2] = dnsample(x2,n,2); xs = x1 + x2;
[y,ny] = dnsample(xs,n,2); diff = sum(abs(y - (y1 + y2)));
if (diff < 1e-5)
    disp(' *** System-5 is Linear *** ');
else
    disp(' *** System-5 is NonLinear *** ');
end
```

MATLAB verification:
```
>> *** System-5 is Linear ***
```

System-6: $T_6[x(n)] = \mathrm{round}[x(n)]$

1. Analytic determination of linearity:

$$T_6[a_1 x_1(n) + a_2 x_2(n)] = \text{round}[a_1 x_1(n) + a_2 x_2(n)] \neq a_1 \text{round}[x_1(n)] + a_2 \text{round}[x_2(n)]$$

Hence the system $T_6[x(n)]$ is **nonlinear**.

2. MATLAB script:

```
% P0211f: To prove that the system T6[x(n)] =   round(x(n)) is linear
clear; clc; close all;

n = 0:100; x1 = rand(1,length(n)); x2 = sqrt(10)*randn(1,length(n));
y1 = round(x1); y2 = round(x2); xs = x1 + x2;
y  = round(xs); diff = sum(abs(y - (y1 + y2)));
if (diff < 1e-5)
    disp(' *** System-6 is Linear *** ');
else
    disp(' *** System-6 is NonLinear *** ');
end
```

MATLAB verification:

```
>> *** System-6 is NonLinear ***
```

P2.12 Time-invariance of discrete-time systems.

System-1: $T_1[x(n)] \triangleq y(n) = x(n)u(n)$

    1. Analytic determination of time-invariance:

$$T_1[x(n-k)] = x(n-k)u(n) \neq x(n-k)u(n-k) = y(n-k)$$

    Hence the system $T_1[x(n)]$ is **time-varying**.

    2. MATLAB script:

```
% P0212a: To determine whether T1[x(n)] = x(n)u(n) is time invariant
clear; clc; close all;

n = 0:100; x = sqrt(10)*randn(1,length(n)); u = stepseq(0,0,100);
y = x.*u; [y1,ny1] = sigshift(y,n,1); [x1,nx1] = sigshift(x,n,1);
[y2,ny2] = sigmult(x1,nx1,u,n); [diff,ndiff] = sigadd(y1,ny1,-y2,ny2);
diff = sum(abs(diff));
if (diff < 1e-5)
    disp(' *** System-1 is Time-Invariant *** ');
else
    disp(' *** System-1 is Time-Varying *** ');
end
```

    MATLAB verification:

```
>> *** System-1 is Time-Varying ***
```

System-2: $T_2[x(n)] \triangleq y(n) = x(n) + n\,x(n+1)$

    1. Analytic determination of time-invariance:

$$T_2[x(n-k)] = x(n-k) + n\,x(n-k+1) \neq x(n-k) + (n-k)\,x(n-k+1) = y(n-k)$$

    Hence the system is $T_2[x(n)]$ **time-varying**.

    2. MATLAB script:

```
% P0212b: To determine whether the system T2[x(n)] = x(n) + n*x(n + 1) is
%         time-invariant
clear; clc; close all;

n = 0:100; x = sqrt(10)*randn(1,length(n));
z  = n; [x1,nx1] = sigshift(x,n,-1);
[x11,nx11] = sigmult(z,n,x1,nx1); [y,ny] = sigadd(x,n,x11,nx11);
[y1,ny1] = sigshift(y,ny ,1); [xs,nxs]  = sigshift(x,n,1);
[xs1,nxs1] = sigshift(xs,nxs,-1); [xs11,nxs11] = sigmult(z,n,xs1,nxs1);
[y2,ny2] = sigadd(xs,nxs,xs11,nxs11); [diff,ndiff] = sigadd(y1,ny1,-y2,ny2);
diff = sum(abs(diff));
if (diff < 1e-5)
    disp(' *** System-2 is Time-Invariant *** ');
else
    disp(' *** System-2 is Time-Varying *** ');
end
```

MATLAB verification:
```
>> *** System-1 is Time-Varying ***
```

System-3: $T_3[x(n)] \triangleq y(n) = x(n) + \frac{1}{2}x(n-2) - \frac{1}{3}x(n-3)x(2n)$

1. Analytic determination of time-invariance:

$$T_3[x(n-k)] = x(n-k) + \frac{1}{2}x(n-k-2) - \frac{1}{3}x(n-k-3)x(2n-k)$$

$$\neq x(n-k) + \frac{1}{2}x(n-k-2) - \frac{1}{3}x(n-k-3)x(2n-2k) = y(n-k)$$

Hence the system is $T_3[x(n)]$ **time-varying**.

2. MATLAB script:
```
% P0212c: To find whether the system T3[x(n)] =   x(n) + 1/2*x(n - 2)
%                                       - 1/3*x(n - 3)*x(2n)
%                                          is time invariant
clear; clc; close all;

n = 0:100; x = sqrt(10)*randn(1,length(n)); [x1,nx1] = sigshift(x,n,2);
 x1 = 1/2*x1; [x2,nx2] = sigshift(x,n,3);   x2 = -1/3*x2;
[x3,nx3] = dnsample(x,n,2); [x4,nx4] = sigmult(x2,nx2,x3,nx3);
[x5,nx5] = sigadd(x1,nx1,x4,nx4); [y,ny]    = sigadd(x,n,x5,nx5);
[y1,ny1] = sigshift(y,ny,1); [xs,nxs] = sigshift(x,n,1);
[xs1,nxs1] = sigshift(xs,nxs,2); xs1 = 1/2*xs1;
[xs2,nxs2] = sigshift(xs,nxs,3); xs2 = -1/3*xs2;
[xs3,nxs3] = dnsample(xs,nxs,2); [xs4,nxs4] = sigmult(xs2,nxs2,xs3,nxs3);
[xs5,nxs5] = sigadd(xs1,nxs1,xs4,nxs4); [y2,ny2] = sigadd(xs,nxs,xs5,nxs5);
[diff,ndiff] = sigadd(y1,ny1,-y2,ny2); diff = sum(abs(diff));
if (diff < 1e-5)
    disp(' *** System-3 is Time-Invariant *** ');
else
    disp(' *** System-3 is Time-Varying *** ');
end
```
MATLAB verification:
```
>> *** System-3 is Time-Varying ***
```

System-4: $T_4[x(n)] \triangleq y(n) = \sum_{k=-\infty}^{n+5} 2x(k)$

1. Analytic determination of time-invariance:

$$T_4[x(n-\ell)] = \sum_{k=-\infty}^{n+5} 2x(k-\ell) = \sum_{k=-\infty}^{n-\ell+5} 2x(k) = y(n-\ell)$$

Hence the system $T_4[x(n)]$ is **time-invariant**.

2. MATLAB script:
```
% P0212d: To find whether the system T4[x(n)]=sum_{k=-infinity}^{n+5}2*x(k)
%          is time-invariant
clear; clc; close all;

n = 0:100; x = sqrt(10)*randn(1,length(n)); y = cumsum(x); ny = n - 5;
```

```
[y1,ny1] = sigshift(y,ny,-1); [xs,nxs] = sigshift(x,n,-1); y2 = cumsum(xs);
ny2 = nxs - 5; [diff,ndiff] = sigadd(y1,ny1,-y2,ny2); diff = sum(abs(diff));
if (diff < 1e-5)
    disp(' *** System-4 is Time-Invariant *** ');
else
    disp(' *** System-4 is Time-Varying *** ');
end
```

MATLAB verification:

```
>> *** System-4 is Time-Invariant ***
```

System-5: $T_5[x(n)] \triangleq y(n) = x(2n)$

1. Analytic determination of time-invariance:

$$T_5[x(n-k)] = x(2n-k) \neq x[2(n-k)] = y(n-k)$$

Hence the system $T_5[x(n)]$ is **time-varying**.

2. MATLAB script:

```
% P0212e: To determine whether the system T5[x(n)] = x(2n) is time-invariant
clear; clc; close all;

n = 0:100; x = sqrt(10)*randn(1,length(n)); [y,ny] = dnsample(x,n,2);
[y1,ny1] = sigshift(y,ny,1); [xs,nxs] = sigshift(x,n,1);
[y2,ny2] = dnsample(xs,nxs,2); [diff,ndiff] = sigadd(y1,ny1,-y2,ny2);
diff = sum(abs(diff));
if (diff < 1e-5)
    disp(' *** System-5 is Time-Invariant *** ');
else
    disp(' *** System-5 is Time-Varying *** ');
end
```

MATLAB verification:

```
>> *** System-5 is Time-Varying ***
```

System-6: $T_6[x(n)] \triangleq y(n) = \text{round}[x(n)]$

1. Analytic determination of time-invariance:

$$T_6[x(n-k)] = \text{round}[x(n-k)] = y(n-k)$$

Hence the system $T_6[x(n)]$ is **time-invariant**.

2. MATLAB script:

```
% P0212f: To determine if the system T6[x(n)]=round(x(n)) is time-invariant
clear; clc; close all;

n = 0:100; x = sqrt(10)*randn(1,length(n)); y = round(x); ny = n;
[y1,ny1] = sigshift(y,ny,1); [xs,nxs] = sigshift(x,n,1); y2  = round(xs);
ny2 = nxs; [diff,ndiff] = sigadd(y1,ny1,-y2,ny2); diff = sum(abs(diff));
if (diff < 1e-5)
    disp(' *** System-6 is Time-Invariant *** ');
else
    disp(' *** System-6 is Time-Varying *** ');
end
```

MATLAB verification:

```
>> *** System-6 is Time-Invariant ***
```

P2.13 Stability and Causality of Discrete-time Systems

System-1: $T_1[x(n)] \triangleq y(n) = x(n)u(n)$: This system is **stable** since $|y(n)| = |x(n)|$. It is also **causal** since the output depends only on the present value of the input.

System-2: $T_2[x(n)] \triangleq y(n) = x(n) + n x(n+1)$: This system is **unstable** since

$$|y(n)| \le |x(n)| + |n| \, |x(n+1)| \nearrow \infty \text{ as } n \nearrow \infty \text{ for } |x(n)| < \infty$$

It is also **noncausal** since the output $y(n)$ depends on the future input value $x(n+1)$ for $n > 0$.

System-3: $T_3[x(n)] \triangleq y(n) = x(n) + \frac{1}{2}x(n-2) - \frac{1}{3}x(n-3)x(2n)$: This system is **stable** since

$$|y(n)| \le |x(n)| + \frac{1}{2}|x(n-2)| + \frac{1}{3}|x(n-3)| \, |x(2n)| < \infty \text{ for } |x(n)| < \infty$$

It is however is **noncausal** since $y(1)$ needs $x(2)$ which is a future input value.

System-4: $T_4[x(n)] \triangleq y(n) = \sum_{k=-\infty}^{n+5} 2x(k)$: This system is **unstable** since

$$|y(n)| \le 2 \sum_{k=-\infty}^{n+5} |x(k)| \nearrow \infty \text{ as } n \nearrow \infty \text{ for } |x(n)| < \infty$$

It is also **noncausal** since the output $y(n)$ depends on the future input value $x(n+5)$ for $n > 0$.

System-5: $T_5[x(n)] \triangleq y(n) = x(2n)$: This system is **stable** since $|y(n)| = |x(2n)| < \infty$ for $|x(n)| < \infty$. It is however **noncausal** since $y(1)$ needs $x(2)$ which is a future input value.

System-6: $T_6[x(n)] \triangleq y(n) = \text{round}[x(n)]$: This system is **stable** and **causal**.

P2.14 Properties of linear convolution.

$$
\begin{aligned}
x_1(n) * x_2(n) &= x_2(n) * x_1(n) &&: \text{Commutation} \\
[x_1(n) * x_2(n)] * x_3(n) &= x_1(n) * [x_2(n) * x_3(n)] &&: \text{Association} \\
x_1(n) * [x_2(n) + x_3(n)] &= x_1(n) * x_2(n) + x_1(n) * x_3(n) &&: \text{Distribution} \\
x(n) * \delta(n - n_0) &= x(n - n_0) &&: \text{Identity}
\end{aligned}
$$

1. <u>Commutation</u>:

$$
\begin{aligned}
x_1(n) * x_2(n) &= \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(\underbrace{n-k}_{=m}) = \sum_{m=-\infty}^{\infty} x_1(n-m)\, x_2(m) \\
&= \sum_{m=-\infty}^{\infty} x_2(m)\, x_1(n-m) = x_2(n) * x_1(n)
\end{aligned}
$$

<u>Association</u>:

$$
\begin{aligned}
[x_1(n) * x_2(n)] * x_3(n) &= \left[\sum_{k=-\infty}^{\infty} x_1(k)\, x_2(n-k) \right] * x_3(n) \\
&= \sum_{m=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(m-k)\, x_3(n-m) \\
&= \sum_{k=-\infty}^{\infty} x_1(k) \left[\sum_{m=-\infty}^{\infty} x_2(\underbrace{m-k}_{=\ell})\, x_3(n-m) \right] \\
&= \sum_{k=-\infty}^{\infty} x_1(k) \left[\sum_{m=-\infty}^{\infty} x_2(\ell)\, x_3(n-k-\ell) \right] \\
&= \sum_{k=-\infty}^{\infty} x_1(k)\, [x_2(n-k) * x_3(n-k)] = x_1(n) * [x_2(n) * x_3(n)]
\end{aligned}
$$

<u>Distribution</u>:

$$
\begin{aligned}
x_1(n) * [x_2(n) + x_3(n)] &= \sum_{k=-\infty}^{\infty} x_1(k)\, [x_2(n-k) + x_3(n-k)] \\
&= \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(n-k) + \sum_{k=-\infty}^{\infty} x_1(k)\, x_3(n-k) \\
&= x_1(n) * x_2(n) + x_1(n) * x_3(n)
\end{aligned}
$$

<u>Identity</u>:

$$
x(n) * \delta(n - n_0) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n - n_0 - k) = x(n - n_0)
$$

since $\delta(n - n_0 - k) = 1$ for $k = n - n_0$ and zero elsewhere.

2. Verification using MATLAB:

Commutation MATLAB script:

```
% P0214a: To prove the Commutation property of convolution
%         i.e. conv(x1(n),x2(n)) = conv(x2(n), x1(n))
clear; clc; close all;

n1 = -10:30; n2 = 0:25; n3 = -10:10; x11 = cos(pi*n1 / 4);
n11 = n1; [x12,n12] = stepseq(-5,-10,30); [x13,n13] = stepseq(25,-10,30);
[x14,n14] = sigadd(x12,n12,-x13,n13); x1 = x11.*x14;
x21 = 0.9 .^ -n2; [x22,n22] = stepseq(0,0,25); [x23,n23] = stepseq(20,0,25);
x24 = x22 - x23; x2 = x21.*x24;
x3 = round((rand(1,21)*2 - 1)*5);

% Commutative property
[y1,ny1] = conv_m(x1,n1,x2,n2); [y2,ny2] = conv_m(x2,n2,x1,n1);
ydiff = max(abs(y1 - y2)), ndiff = max(abs(ny1 - ny2)),
```

MATLAB verification:

```
ydiff =
     0
ndiff =
     0
```

Association MATLAB script:

```
% P0214b: To prove the Association property of convolution
%         i.e. conv(conv(x1(n),x2(n)),x3(n)) = conv(x1(n),conv(x2(n),x3(n)))
clear; clc; close all;

n1 = -10:30; n2 = 0:25; n3 = -10:10; x11 = cos(pi*n1 / 4); n11 = n1;
[x12,n12] = stepseq(-5,-10,30); [x13,n13] = stepseq(25,-10,30);
[x14,n14] = sigadd(x12,n12,-x13,n13); x1 = x11.*x14;
x21 = 0.9 .^ -n2; [x22,n22] = stepseq(0,0,25); [x23,n23] = stepseq(20,0,25);
x24 = x22 - x23; x2 = x21.*x24; x3 = round((rand(1,21)*2 - 1)*5);

% Association property
[y1,ny1] = conv_m(x1,n1,x2,n2); [y1,ny1] = conv_m(y1,ny1,x3,n3);
[y2,ny2] = conv_m(x2,n2,x3,n3); [y2,ny2] = conv_m(x1,n1,y2,ny2);
ydiff = max(abs(y1 - y2)), ndiff = max(abs(ny1 - ny2)),
```

MATLAB verification:

```
ydiff =
     0
ndiff =
     0
```

Distribution MATLAB script:

```
% P0214c: To prove the Distribution property of convolution
%         i.e. conv(x1(n),(x2(n)+x3(n)))=conv(x1(n),x2(n))+conv(x1(n),x3(n))
clear; clc; close all;
```

```
n1 = -10:30; n2 = 0:25; n3 = -10:10; x11 = cos(pi*n1 / 4); n11 = n1;
[x12,n12] = stepseq(-5,-10,30); [x13,n13] = stepseq(25,-10,30);
[x14,n14] = sigadd(x12,n12,-x13,n13); x1 = x11.*x14; x21 = 0.9 .^ -n2;
[x22,n22] = stepseq(0,0,25); [x23,n23] = stepseq(20,0,25); x24 = x22 - x23;
x2 = x21.*x24; x3 = round((rand(1,21)*2 - 1)*5);

% Distributive property
[y1,ny1] = sigadd(x2,n2,x3,n3); [y1,ny1] = conv_m(x1,n1,y1,ny1);
[y2,ny2] = conv_m(x1,n1,x2,n2); [y3,ny3] = conv_m(x1,n1,x3,n3);
[y4,ny4] = sigadd(y2,ny2,y3,ny3); ydiff = max(abs(y1 - y4)),
ndiff = max(abs(ny1 - ny4)),
```

MATLAB verification:

```
ydiff =
     0
ndiff =
     0
```

Identity MATLAB script:

```
% P0214d: To prove the Identity property of convolution
%          i.e. conv(x(n),delta(n - n0))  = x(n - n0)
clear; clc; close all;

n1 = -10:30; n2 = 0:25; n3 = -10:10; x11 = cos(pi*n1 / 4); n11 = n1;
[x12,n12] = stepseq(-5,-10,30); [x13,n13] = stepseq(25,-10,30);
[x14,n14] = sigadd(x12,n12,-x13,n13); x1 = x11.*x14; x21 = 0.9 .^ -n2;
[x22,n22] = stepseq(0,0,25); [x23,n23] = stepseq(20,0,25); x24 = x22 - x23;
x2 = x21.*x24; x3 = round((rand(1,21)*2 - 1)*5);

% Identity property
n0 = fix(100*rand(1,1)-0.5); [dl,ndl] = impseq(n0,n0,n0);
[y11,ny11] = conv_m(x1,n1,dl,ndl); [y12,ny12] = sigshift(x1,n1,n0);
y1diff = max(abs(y11 - y12)), ny1diff = max(abs(ny11 - ny12)),

[y21,ny21] = conv_m(x2,n2,dl,ndl); [y22,ny22] = sigshift(x2,n2,n0);
y2diff = max(abs(y21 - y22)), ny2diff = max(abs(ny21 - ny22)),

[y31,ny31] = conv_m(x3,n3,dl,ndl); [y32,ny32] = sigshift(x3,n3,n0);
y3diff = max(abs(y31 - y32)), ny3diff = max(abs(ny31 - ny32)),
```

MATLAB verification:

```
ydiff =
     0
ndiff =
     0
```

P2.15 Convolutions using `conv_m` function.

1. $x(n) = \{2, -4, 5, \underset{\uparrow}{3}, -1, -2, 6\}$, $h(n) = \{1, -1, \underset{\uparrow}{1}, -1, 1\}$: MATLAB script:

```
n1  = -3:3; x = [2 -4 5 3 -1 -2 6]; n2 = -1:3; h = [1 -1 1 -1 1];
[y,n] = conv_m(x,n1,h,n2); y, n
y =
     2     -6     11     -8     7     -7     9     -4     7     -8     6
n =
    -4     -3     -2     -1      0      1      2      3     4      5     6
```

2. $x(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$, $h(n) = \{1, -2, \underset{\uparrow}{-3}, 4\}$: MATLAB script:

```
n1  = -3:3; x = [1 1 0 1 1]; n2 = -3:0; h = [1 -2  -3 4];
[y,n] = conv_m(x,n1,h,n2); y, n,
y =
     1     -1     -5     2     3     -5     1     4
n =
    -6     -5     -4     -3     -2     -1      0      1
```

3. $x(n) = (1/4)^{-n}[u(n+1) - u(n-4)]$, $h(n) = u(n) - u(n-5)$: MATLAB script:

```
n1 = -2:5; [x11,nx11] = stepseq(-1,-2,5); [x12,nx12] = stepseq(4,-2,5);
[x13,n13] = sigadd(x11,nx11,-x12,nx12); x14 = 0.25 .^ -n1; n14 = n1;
x = x14 .* x13;
n2 = 0:6; [h11,nh11] = stepseq(0,0,6); [h12,nh12] = stepseq(5,0,6); h=h11-h12;
[y,n] = conv_m(x,n1,h,n2); y, n,
y =  0      0.2500     1.2500     5.2500     21.2500     85.2500     85.0000     84.0000
     80.0000     64.0000          0          0          0          0
n = -2     -1      0      1      2      3      4      5      6      7      8      9      10
     11
```

4. $x(n) = n/4[u(n) - u(n-6)]$, $h(n) = 2[u(n+2) - u(n-3)]$: MATLAB script:

```
n1 = 0:7; [x11,nx11] = stepseq(0,0,7); [x12,nx12] = stepseq(6,0,7);
[x13,n13] = sigadd(x11,nx11,-x12,nx12); x14 = n1/4; n14 = n1; x = x14 .* x13;
n2 = -3:4; [h11,nh11] = stepseq(-2,-3,4); [h12,nh12] = stepseq(3,-3,4);
h = 2 * (h11 - h12); [y,n] = conv_m(x,n1,h,n2); y, n,
y = 0          0     0.5000     1.5000     3.0000     5.0000     7.5000     7.0000
    6.0000     4.5000     2.5000          0          0          0          0
n = -3     -2     -1      0      1      2      3      4      5      6      7      8      9
    10     11
```

P2.16 Let $x(n) = (0.8)^n u(n)$, $h(n) = (-0.9)^n u(n)$, and $y(n) = h(n) * x(n)$.

1. Convolution $y(n) = h(n) * x(n)$:

$$
\begin{aligned}
y(n) &= \sum_{k=-\infty}^{\infty} h(k)x(n-k) = \sum_{k=0}^{\infty}(-0.9)^k (0.8)^{n-k} u(n-k) \\
&= \left[\sum_{k=0}^{n}(-0.9)^k (0.8)^n (0.8)^{-k} \right] u(n) = (0.8)^n \left[\sum_{k=0}^{n} \left(-\frac{9}{8} \right)^k \right] u(n) \\
&= \frac{0.8^{n+1} - (-0.9)^{n+1}}{1.7}
\end{aligned}
$$

MATLAB script:

```
clc; close all;run defaultsettings;
n = [0:50]; x = 0.8.^n; h = (-0.9).^n;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0216');

% (a) Plot of the analytical convolution
y1 = ((0.8).^(n+1) - (-0.9).^(n+1))/(0.8+0.9);
subplot(1,3,1); Hs1 = stem(n,y1,'filled'); set(Hs1,'markersize',2);
title('Analytical'); xlabel('n'); ylabel('y(n)');
```

2. Computation using convolution of truncated sequences: MATLAB script

```
% (b) Plot using the conv function and truncated sequences
x2 = x(1:26); h2 = h(1:26); y2 = conv(h2,x2);
subplot(1,3,2); Hs2 = stem(n,y2,'filled'); set(Hs2,'markersize',2);
title('Using conv function'); xlabel('n'); %ylabel('y(n)');
```

3. To use the MATLAB's `filter` function we have to represent the $h(n)$ sequence by coefficients an equivalent difference equation. MATLAB script:

```
% (c) Plot of the convolution using the filter function
y3 = filter([1],[1,0.9],x);
subplot(1,3,3); Hs3 = stem(n,y3,'filled'); set(Hs3,'markersize',2);
title('Using filter function'); xlabel('n'); %ylabel('y(n)');
```

The plots of this solution are shown in Figure 2.32. The analytical solution to the convolution in 1 is the exact answer. In the `filter` function approach of 2, the infinite-duration sequence $x(n)$ is exactly represented by coefficients of an equivalent filter. Therefore, the filter solution should be exact except that it is evaluated up to the length of the input sequence. The truncated-sequence computation in 3 is correct up to the first 26 samples and then it degrades rapidly.
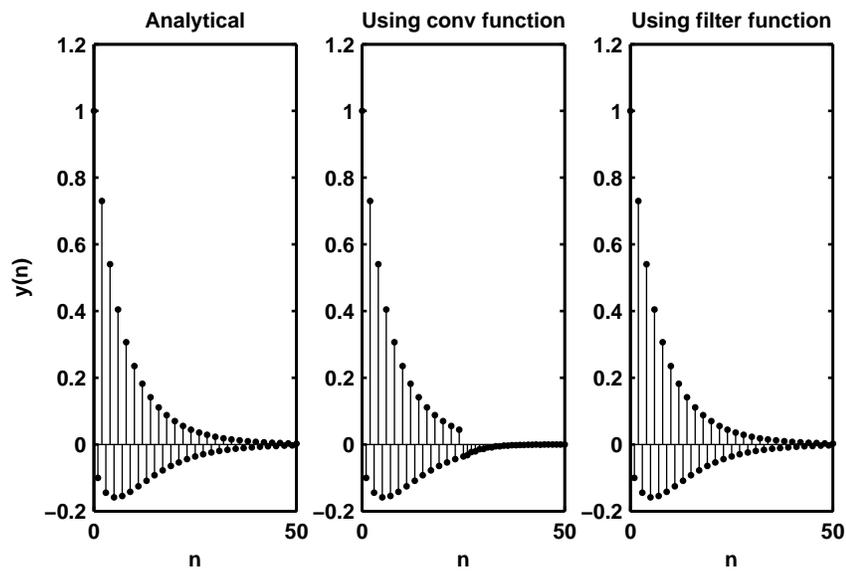
Figure 2.32: Problem P2.16 convolution plots

P2.17 Linear convolution as a matrix-vector multiplication. Consider the sequences

$$x(n) = \{1, 2, 3, 4, 5\} \text{ and } h(n) = \{6, 7, 8, 9\}$$

1. The linear convolution of the above two sequences is

$$y(n) = \{6, 19, 40, 70, 100, 94, 76, 45\}$$

2. The vector representation of the above operation is:

$$\underbrace{\begin{bmatrix} 6 \\ 19 \\ 40 \\ 70 \\ 100 \\ 94 \\ 76 \\ 45 \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} 6 & 0 & 0 & 0 & 0 \\ 7 & 6 & 0 & 0 & 0 \\ 8 & 7 & 6 & 0 & 0 \\ 9 & 8 & 7 & 6 & 0 \\ 0 & 9 & 8 & 7 & 6 \\ 0 & 0 & 9 & 8 & 7 \\ 0 & 0 & 0 & 9 & 8 \\ 0 & 0 & 0 & 0 & 9 \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}}_{\mathbf{x}}$$

(a) Note that the matrix $\mathbf{H}$ has an interesting structure. Each diagonal of $\mathbf{H}$ contains the same number. Such a matrix is called a Toeplitz matrix. It is characterized by the following property

$$[\mathbf{H}]_{i,j} = [\mathbf{H}]_{i-j}$$

which is similar to the definition of time-invariance.

(b) Note carefully that the first column of $\mathbf{H}$ contains the impulse response vector $h(n)$ followed by number of zeros equal to the number of $x(n)$ values minus one. The first row contains the first element of $h(n)$ followed by the same number of zeros as in the first column. Using this information and the above property we can generate the whole Topelitz matrix.

P2.18 Matlab function `conv_tp`:

(a) The Matlab function `conv_tp`:

```
function [y,H]=conv_tp(h,x)
% Linear Convolution using Toeplitz Matrix
% ---------------------------------------
% [y,H] = conv_tp(h,x)
% y = output sequence in column vector form
% H = Toeplitz matrix corresponding to sequence h so that y = Hx
% h = Impulse response sequence in column vector form
% x = input sequence in column vector form
%
Nx = length(x); Nh = length(h);
hc = [h; zeros(Nx-1, 1)]; hr = [h(1),zeros(1,Nx-1)];
H = toeplitz(hc,hr); y = H*x;
```

(b) Matlab verification:

```
x = [1,2,3,4,5]'; h = [6,7,8,9]';
[y,H] = conv_tp(h,x); y = y', H
y =
     6    19    40    70   100    94    76    45
H =
     6     0     0     0     0
     7     6     0     0     0
     8     7     6     0     0
     9     8     7     6     0
     0     9     8     7     6
     0     0     9     8     7
     0     0     0     9     8
     0     0     0     0     9
```

P2.19 A linear and time-invariant system is described by the difference equation

$$y(n) - 0.5y(n-1) + 0.25y(n-2) = x(n) + 2x(n-1) + x(n-3)$$

(a) Impulse response using the Using the `filter` function.

```
% P0219a: System response using the filter function
clc; close all;

b = [1 2 0 1]; a = [1 -0.5 0.25]; [delta,n] = impseq(0,0,100);
h = filter(b,a,delta);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0219a');
Hs = stem(n,h,'filled'); set(Hs,'markersize',2);
axis([min(n)-5,max(n)+5,min(h)-0.5,max(h)+0.5]);
xlabel('n','FontSize',LFS); ylabel('h(n)','FontSize',LFS);
title('Impulse response','FontSize',TFS);
print -deps2 ../EPSFILES/P0219a.eps;
```

The plots of the impulse response $h(n)$ is shown in Figure 2.33.
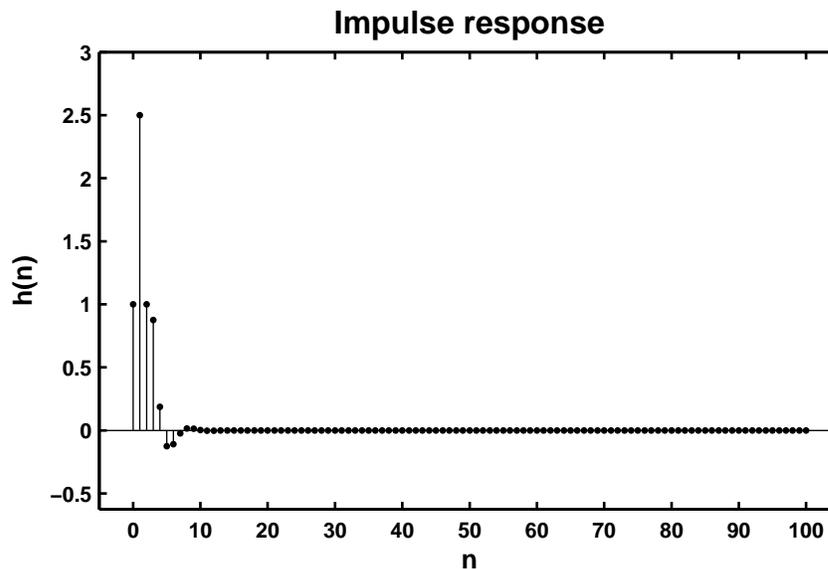

Figure 2.33: Problem P2.19.1 impulse response plot

(b) Clearly from Figure 2.33 the system is stable.

(c) Response $y(n)$ when the input is $x(n) = [5 + 3\cos(0.2\pi n) + 4\sin(0.6\pi n)]u(n)$:

```
% P0219c: Output response of a system using the filter function.
clc; close all;

b = [1 2 0 1]; a = [1 -0.5 0.25]; n = 0:200;
x = 5*ones(size(n))+3*cos(0.2*pi*n)+4*sin(0.6*pi*n); y = filter(b,a,x);
```

```
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0219c');
Hs = stem(n,y,'filled'); set(Hs,'markersize',2); axis([-10,210,0,50]);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title('Output response','FontSize',TFS);
print -deps2 ../EPSFILES/P0219c.eps;
```

The plots of the response $y(n)$ is shown in Figure 2.34.
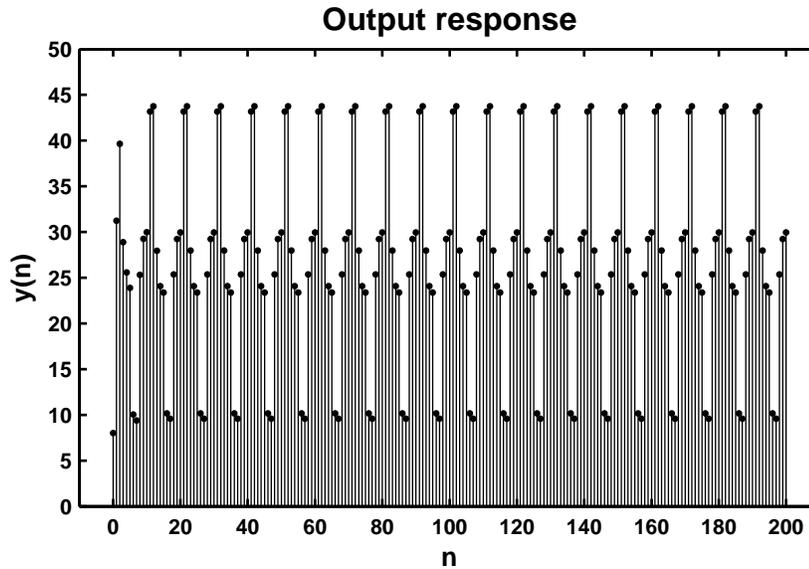
Figure 2.34: Problem P2.19.3 response plot

P2.20 A "simple" *digital differentiator*: $y(n) = x(n) - x(n-1)$

(a) Response to a rectangular pulse $x(n) = 5[u(n) - u(n-20)]$:

```
% P0220a: Simple Differentiator response to a rectangular pulse
clc; close all;

a = 1; b = [1 -1]; n1 = 0:22;
[x11,nx11] = stepseq(0,0,22); [x12,nx12] = stepseq(20,0,22);
x1 = 5*(x11 - x12); y1 = filter(b,a,x1);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0220a');
Hs = stem(n1,y1,'filled'); set(Hs,'markersize',2); axis([-1,23,-6,6]);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS); ytick = [-6:6];
title('Output response for rectangular pulse ','FontSize',TFS);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../EPSFILES/P0220a.eps;
```

The plots of the response $y(n)$ is shown in Figure 2.35.
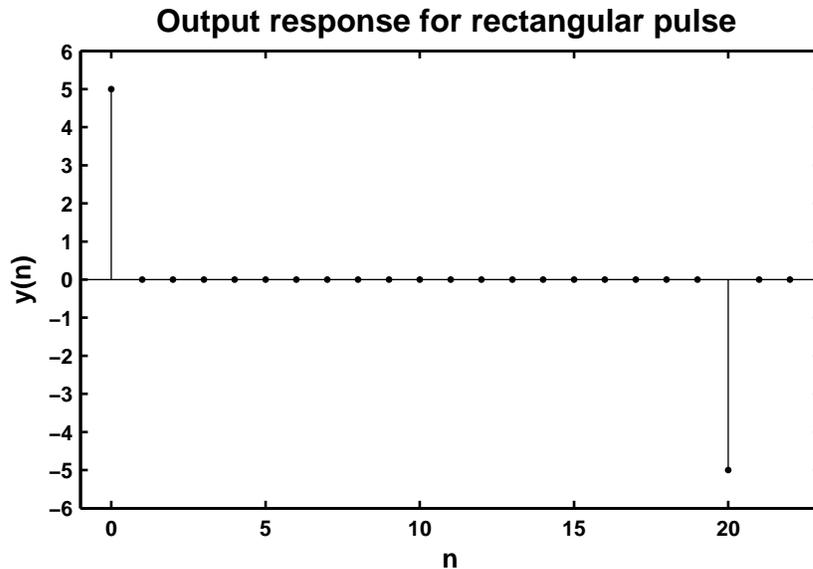
Figure 2.35: Problem P2.20.1 response plot

(b) Response to a triangular pulse $x(n) = n\,[u(n) - u(n - 10)] + (20 - n)\,[u(n - 10) - u(n - 20)]$:

```
% P0220b: Simple Differentiator response to a triangular pulse
clc; close all;

a = 1; b = [1 -1]; n2 = 0:21; [x11,nx11] = stepseq(0,0,21);
[x12,nx12] = stepseq(10,0,21); [x13,nx13] = stepseq(20,0,21);
x2  =  n2.*(x11 - x12) + (20 - n2).*(x12 - x13); y2 = filter(b,a,x2);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0220b');
Hs = stem(n2,y2,'filled'); set(Hs,'markersize',2);
axis([min(n2)-1,max(n2)+1,min(y2)-0.5,max(y2) + 0.5]);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title('Output response for triangular pulse','FontSize',TFS);
print -deps2 ../EPSFILES/P0220b.eps;
```

The plots of the response $y(n)$ is shown in Figure 2.36.
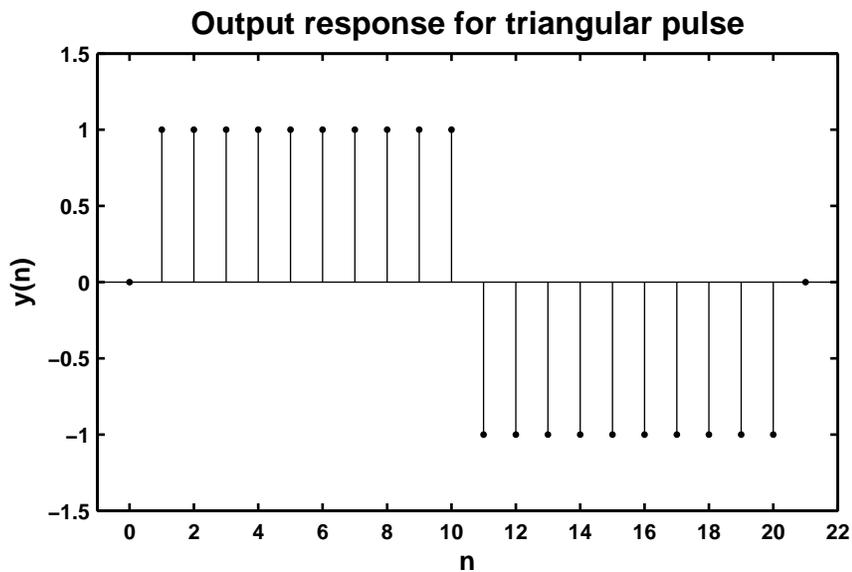
Figure 2.36: Problem P2.20.2 response plot

(c) Response to a sinusoidal pulse $x(n) = \sin\left(\dfrac{\pi n}{25}\right) [u(n) - u(n - 100)]$:

```
% P0220cSimple Differentiator response to a sinusoidal pulse
clc; close all;

a = 1; b = [1 -1]; n3 = 0:101; [x11,nx11] = stepseq(0,0,101);
[x12,nx12] = stepseq(100,0,101); x13 = x11-x12; x3 = sin(pi*n3/25).*x13;
y3 = filter(b,a,x3);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0220c');
Hs = stem(n3,y3,'filled'); set(Hs,'markersize',2);
axis([-5,105,-0.15,0.15]); ytick = [-0.15:0.05:0.15];
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title('Output response for sinusoidal pulse','FontSize',TFS);
set(gca,'YTickMode','manual','YTick',ytick);
print -deps2 ../EPSFILES/P0220c.eps;
```

The plots of the response $y(n)$ is shown in Figure 2.37.
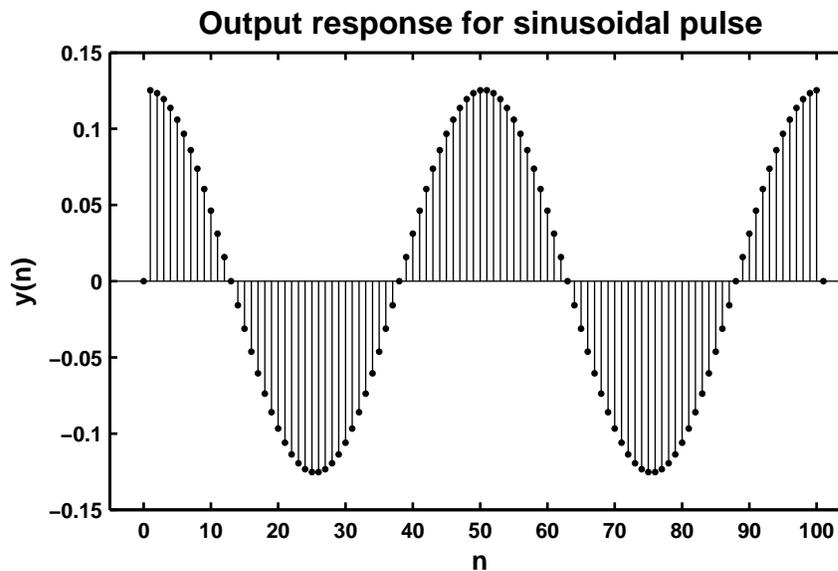
Figure 2.37: Problem P2.20.3 response plot

Chapter 3

Discrete-Time Fourier Transform

P3.1 Matlab Function [X] = dtft(x,n,w)

```
function [X] = dtft(x,n,w)
% Computes Discrete-time Fourier Transform
% [X] = dtft(x,n,w)
%
% X = DTFT values computed at w frequencies
% x = finite duration sequence over n (row vector)
% n = sample position row vector
% w = frequency row vector
X = x*exp(-j*n'*w);
```

1. $x(n) = (0.6)^{|n|} [u(n+10) - u(n-11)]$.

```
% P0301a: DTFT of x1(n) = 0.6 ^ |n|*(u(n+10)-u(n-11))
clc; close all;
%
[x11,n11] = stepseq(-10,-11,11); [x12,n12] = stepseq(11,-11,11);
[x13,n13] = sigadd(x11,n11,-x12,n12); n1 = n13; x1 = (0.6 .^ abs(n1)).*x13;
w1 = linspace(-pi,pi,201); X1 = dtft(x1,n1,w1);
magX1 = abs(X1); phaX1 = angle(X1);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0301a');
subplot(2,1,1); plot(w1/pi,magX1,'LineWidth',1.5);
axis([-1 1 0 4.5]); wtick = [-1:0.2:1]; magtick = [0:0.5:4.5];
xlabel('\omega/\pi',FNTSZ,LFS);
ylabel('|X|',FNTSZ,LFS);
title('Magnitude response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',magtick);
subplot(2,1,2); plot(w1/pi,phaX1*180/pi,'LineWidth',1.5);
axis([-1,1,-180,180]); phatick = [-180 0 180];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('Degrees',FNTSZ,LFS);
title('Phase Response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',phatick);
print -deps2 ../EPSFILES/P0301a;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.1.
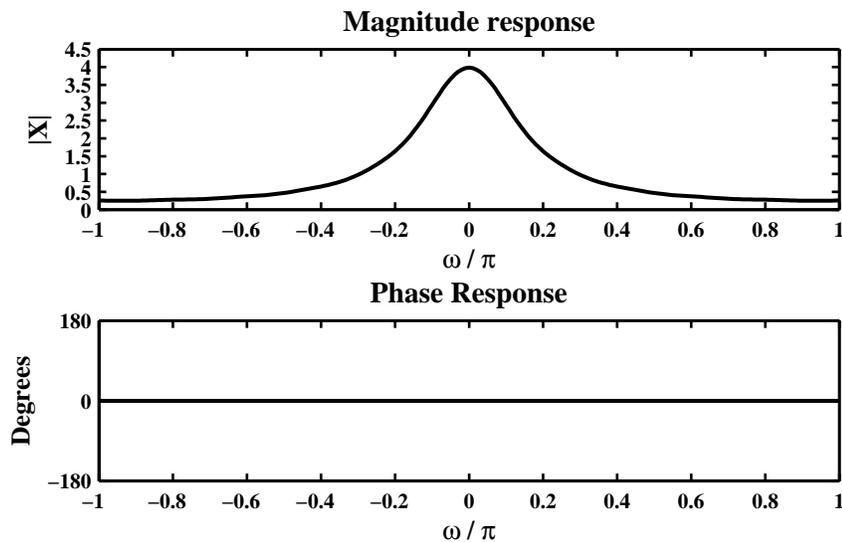
Figure 3.1: Problem P3.1.1 DTFT plots

2. $x(n) = n(0.9)^n [u(n) - u(n-21)]$.

```
% P0301b: % DTFT of x2(n) = n.*(0.9 ^ n) .*(u(n)-u(n-21))
clc; close all;
%
[x21,n21] = stepseq(0,0,22); [x22,n22] = stepseq(21,0,22);
[x23,n23] = sigadd(x21,n21,-x22,n22); n2 = n23; x2 = n2.*(0.9 .^ n2).*x23;
w2 = linspace(-pi,pi,201); X2 = dtft(x2,n2,w2);
magX2 = abs(X2); phaX2 = angle(X2);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0301b');
subplot(2,1,1); plot(w2/pi,magX2,'LineWidth',1.5);
wtick = [-1:0.2:1]; magtick = [0:10:60];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('|X|',FNTSZ,LFS);
title('Magnitude response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',magtick);
subplot(2,1,2); plot(w2/pi,phaX2*180/pi,'LineWidth',1.5);
axis([-1,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('Degrees',FNTSZ,LFS);
title('Phase Response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',phatick);
print -deps2 ../EPSFILES/P0301b;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.2.
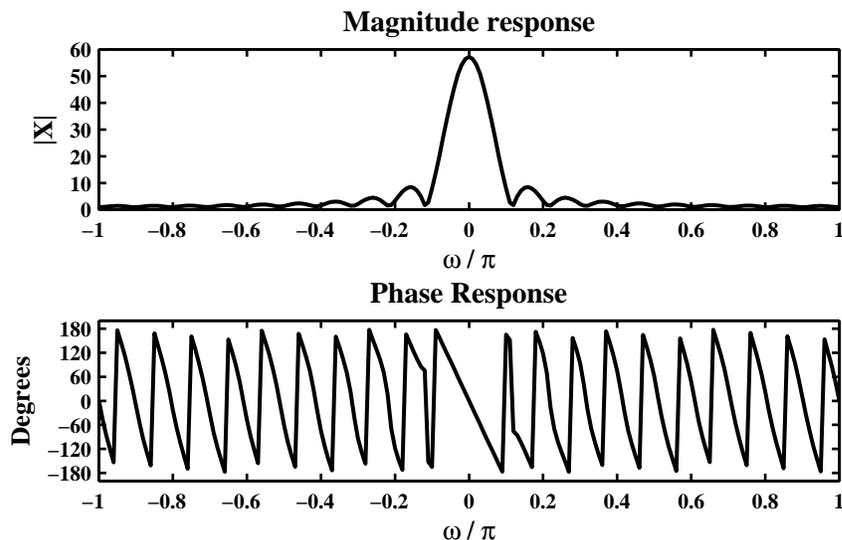
Figure 3.2: Problem P3.1.2 DTFT plots

3. $x(n) = [\cos(0.5\pi n) + j\sin(0.5\pi n)][u(n) - u(n - 51)]$.

```
% P0301c: % DTFT of x3(n) = (cos(0.5*pi*n)+j*sin(0.5*pi*n)).*(u(n)-u(n-51))
clc; close all;
%
[x31,n31] = stepseq(0,0,52); [x32,n32] = stepseq(51,0,52);
[x33,n33] = sigadd(x31,n31,-x32,n32); n3 = n33;
x3 = (cos(0.5*pi*n3)+j*sin(0.5*pi*n3)).*x33;
w3 = linspace(-pi,pi,201); X3 = dtft(x3,n3,w3);
magX3 = abs(X3); phaX3 = angle(X3);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0301c');
subplot(2,1,1); plot(w3/pi,magX3,'LineWidth',1.5);
wtick = [-1:0.2:1]; magtick = [0:10:60];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('|X|',FNTSZ,LFS);
title('Magnitude response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',magtick);
subplot(2,1,2,'LineWidth',1.5); plot(w3/pi,phaX3*180/pi);
axis([-1,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('Degrees',FNTSZ,LFS);
title('Phase Response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',phatick);
print -deps2 ../EPSFILES/P0301c;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.3.
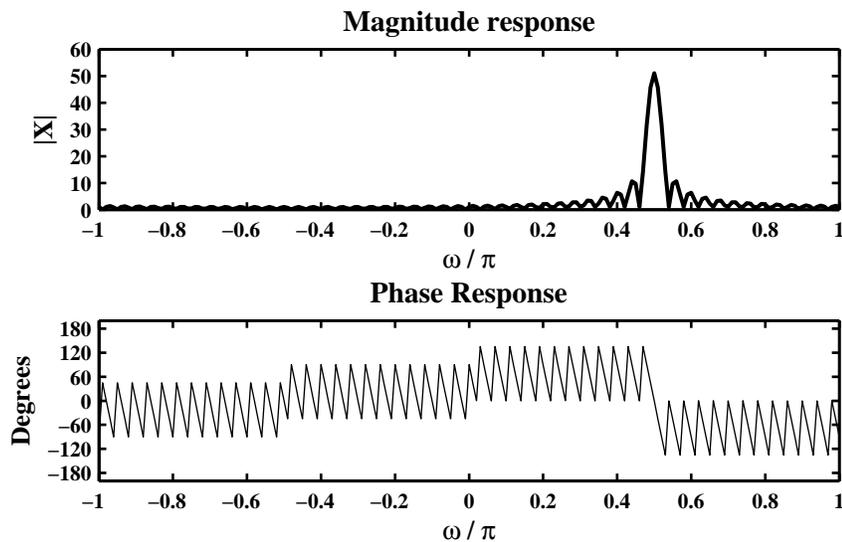

Figure 3.3: Problem P3.1.3 DTFT plots

4. $x(n) = \{4, 3, 2, 1, 1, 2, 3, 4\}$.

```
% P0301d: % DTFT of x4(n) = [4 3 2 1 1 2 3 4] ; n = 0:7;
clc; close all;
%
x4 = [4 3 2 1 1 2 3 4]; n4 = [0:7];
w4 = linspace(-pi,pi,201); X4 = dtft(x4,n4,w4);
magX4 = abs(X4); phaX4 = angle(X4);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0301d');
subplot(2,1,1); plot(w4/pi,magX4,'LineWidth',1.5);
axis([-1,1,0,25]); wtick = [-1:0.2:1]; magtick = [0:5:25];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('|X|',FNTSZ,LFS);
title('Magnitude response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',magtick);
subplot(2,1,2); plot(w4/pi,phaX4*180/pi,'LineWidth',1.5);
axis([-1,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('Degrees',FNTSZ,LFS);
title('Phase Response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',phatick);
print -deps2 ../EPSFILES/P0301d;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.4.
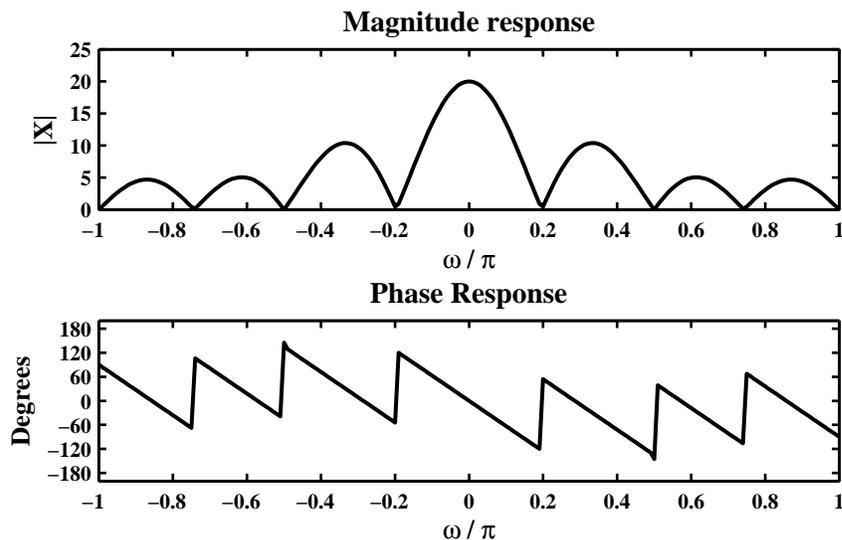

Figure 3.4: Problem P3.1.4 DTFT plots

5. $x(n) = \{4, 3, 2, 1, -1, -2, -3, -4\}$.

```
% P0301e: % DTFT of x5(n) = [4 3 2 1 -1 -2 -3 -4] ; n = 0:7;
clc; close all;
%
x5 = [4 3 2 1 -1 -2 -3 -4]; n5 = [0:7];
w5 = linspace(-pi,pi,201); X5 = dtft(x5,n5,w5);
magX5 = abs(X5); phaX5 = angle(X5);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0301e');
subplot(2,1,1); plot(w5/pi,magX5,'LineWidth',1.5);
wtick = [-1:0.2:1]; magtick = [0:5:20]; axis([-1 1 0 20]);
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('|X|',FNTSZ,LFS);
title('Magnitude response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',magtick);
subplot(2,1,2); plot(w5/pi,phaX5*180/pi,'LineWidth',1.5);
axis([-1,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('Degrees',FNTSZ,LFS);
title('Phase Response',FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',phatick);
print -deps2 ../EPSFILES/P0301e;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.5.
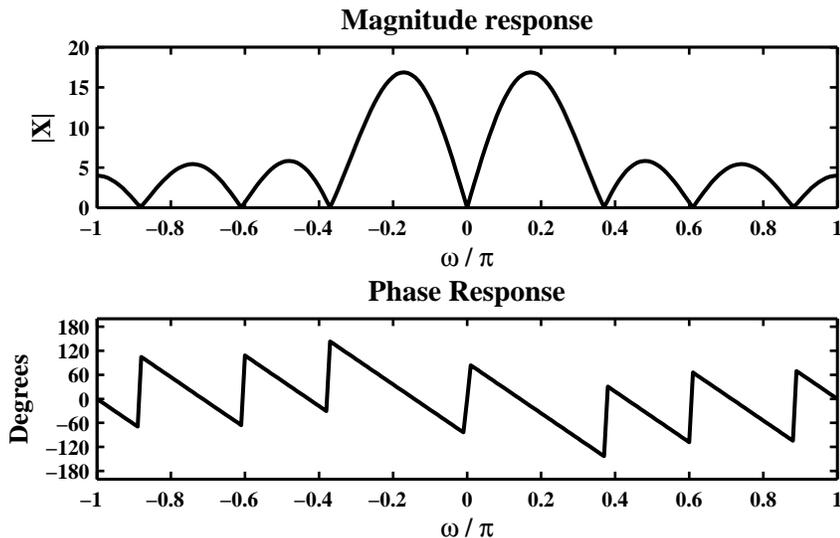
Figure 3.5: Problem P3.1.5 DTFT plots

P3.2 Let $x_1(n) = \{1, 2, 2, 1\}$. A new sequence $x_2(n)$ is formed using
$$\uparrow$$

$$x_2(n) = \begin{cases} x_1(n), & 0 \le n \le 3; \\ x_1(n-4), & 4 \le n \le 7; \\ 0, & \text{Otherwise.} \end{cases} \tag{3.1}$$

1. Clearly, $x_2(n) = x_1(n) + x_1(n-4)$. Hence

$$X_2(e^{j\omega}) = X_1(e^{j\omega}) + X_1(e^{j\omega})e^{-j4\omega} = 2e^{-j2\omega}\cos(2\omega)X_1(e^{j\omega})$$

Thus the magnitude $|X_1(e^{j\omega})|$ is scaled by 2 and changed by $|\cos(2\omega)|$ while the phase of $|X_1(e^{j\omega})|$ is changed by 2ω.

2. MATLAB Verification:

```
% P0302b: x1(n) = [1 2 2 1],n = [0:3];
%         x2(n) = x1(n)      ,n = [0:3];
%               = x1(n-4)   ,n = [4:7];
clc; close all;

n1 = [0:3]; x1 = [1 2 2 1]; n2 = [0:7]; x2 = [x1 x1];
w2 = linspace(-pi,pi,201); X1 = dtft(x1,n1,w2); X2 = dtft(x2,n2,w2);
magX1 = abs(X1); phaX1 = angle(X1); magX2 = abs(X2); phaX2 = angle(X2);
wtick = [-1:0.5:1]; phatick = [-180:60 :180];

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0302b');
subplot(2,2,1); plot(w2/pi,magX1,'LineWidth',1.5);
axis([-1 1 0 8]); magtick1 = [0:2:8];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('|X_1|',FNTSZ,LFS);
title(['Magnitude response' char(10) 'signal x_1'],FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',magtick1);
subplot(2,2,3); plot(w2/pi,phaX1*180/pi,'LineWidth',1.5); axis([-1 1 -200 200]);
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('Degrees',FNTSZ,LFS);
title(['Phase response' char(10) 'signal x_1'],FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',phatick);
subplot(2,2,2); plot(w2/pi,magX2,'LineWidth',1.5);
axis([-1 1 0 16]); magtick2 = [0:4:16];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('|X_2|',FNTSZ,LFS);
title(['Magnitude response' char(10) 'signal x_2'],FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',magtick2);
subplot(2,2,4); plot(w2/pi,phaX2*180/pi,'LineWidth',1.5); axis([-1 1 -200 200]);
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('Degrees',FNTSZ,LFS);
title(['Phase response' char(10) 'signal x_2'],FNTSZ,TFS);
set(gca,'XTick',wtick);
set(gca,'YTick',phatick);
print -deps2 ../EPSFILES/P0302b;
```

The magnitude and phase plots of $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$ are shown in Figure 3.6 which confirms the observation in part 1. above.
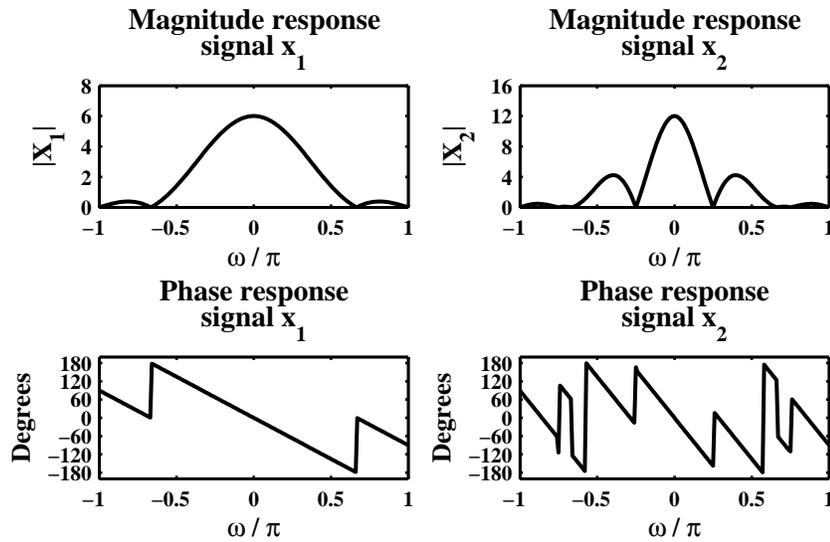

Figure 3.6: Problem P3.2.2 DTFT plots

P3.3 Analytical computation of the DTFTs and plotting of their magnitudes and angles.

1. $x(n) = 2\,(0.5)^n\,u(n+2)$.

$$X\!\left(e^{j\omega}\right) = 2\sum_{-\infty}^{\infty} 0.5^n u(n+2)e^{-jn\omega} = 2\sum_{-2}^{\infty} 0.5^n e^{-jn\omega} = 2(0.5)^{-2}e^{j2\omega}\sum_{0}^{\infty} 0.5^n e^{-jn\omega} = 8\frac{e^{j2\omega}}{1-0.5e^{-j\omega}}$$

MATLAB Verification:

```
% P0303a: DTFT of x1(n) = 2*((0.5)^n)*u(n+2) = 8*exp(j*2*w)/(1-0.5*exp(-j*w))
clc; close all;

w1 = linspace(0,pi,501); X1 = 8*exp(j*2*w1)./(1-0.5*exp(-j*w1));
magX1 = abs(X1); phaX1 = angle(X1);

Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0303a');
subplot(2,1,1); plot(w1/pi,magX1,'LineWidth',1.5);
wtick = [0:0.2:1]; magtick = [0:4:20]; axis([0,1,0,20]);
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('|X|',FNTSZ,LFS);
title('Magnitude response',FNTSZ,TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w1/pi,phaX1*180/pi,'LineWidth',1.5);
axis([0,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi',FNTSZ,LFS); ylabel('Degrees',FNTSZ,LFS);
title('Phase Response',FNTSZ,TFS);
set(gca,'XTick',wtick); set(gca,'YTick',phatick);
print -deps2 ../EPSFILES/P0303a;
```

The magnitude and phase plots of $X\!\left(e^{j\omega}\right)$ are shown in Figure 3.7.
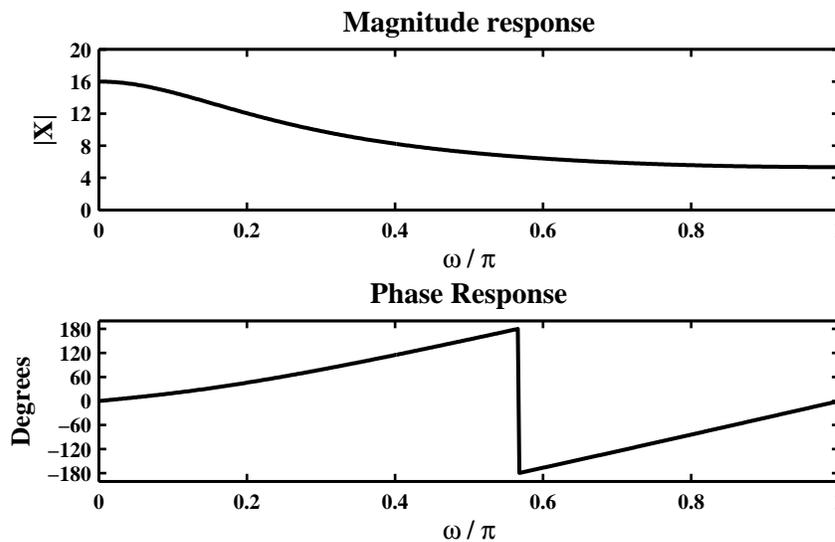
Figure 3.7: Problem P3.3.1 DTFT plots

2. $x(n) = (0.6)^{|n|} \left[u(n + 10) - u(n - 11) \right]$.

$$X\left(e^{j\omega}\right) = \sum_{-\infty}^{\infty} (0.6)^{|n|} \left[u(n + 10) - u(n - 11) \right] e^{-jn\omega} = \sum_{-10}^{10} 0.6^{|n|} e^{-jn\omega}$$

$$= \sum_{-10}^{0} 0.6^{-n} e^{-jn\omega} + \sum_{0}^{10} 0.6^{n} e^{-jn\omega} - 1 = \frac{0.64 - 2(0.6)^{11} \cos(11\omega) + 2(0.6)^{12} \cos(10\omega)}{1.36 - 1.2 \cos(\omega)}$$

MATLAB Verification:

```
% P0303b: DTFT of x2(n) = (0.6) ^ |n|*[u(n+10)-u(n-11)]
clc; close all;
w2 = linspace(0,pi,501);
X2 = (0.64-2*(0.6)^11*cos(11*w2)+2*(0.6)^12*cos(10*w2))./(1.36-1.2*cos(w2));
magX2 = abs(X2); phaX2 = angle(X2);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0303b');
subplot(2,1,1); plot(w2/pi,magX2,'LineWidth',1.5);
axis([0,1,0,5]); wtick = [0:0.2:1]; magtick = [0:1:5];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title('Magnitude response','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w2/pi,phaX2*180/pi,'LineWidth',1.5);
axis([0,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase Response','FontSize',TFS); set(gca,'XTick',wtick);
set(gca,'YTick',phatick); print -deps2 ../EPSFILES/P0303b;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.8.
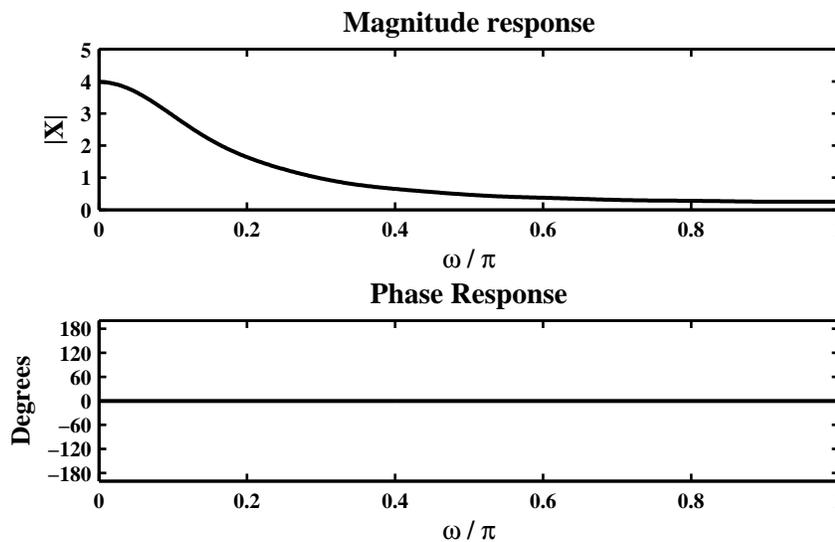
Figure 3.8: Problem P3.3.2 DTFT plots

3. $x(n) = n\,(0.9)^n\,u(n+3)$.

$$X\left(e^{j\omega}\right) = \sum_{-\infty}^{\infty} n\,(0.9)^n\,u(n+3)e^{-jn\omega} = \sum_{-3}^{\infty} n\,(0.9)^n\,e^{-jn\omega}$$

$$= -3(0.9)^{-3}e^{j3\omega} - 2(0.9)^{-2}e^{j2\omega} - (0.9)^{-1}e^{j1\omega} + \sum_{0}^{\infty} n\,(0.9)^n\,e^{-jn\omega}$$

$$= -4.1152e^{j3\omega} - 2.4691e^{j2\omega} - 1.1111e^{j\omega} + \frac{0.9e^{-j\omega}}{(1 - 0.9e^{-j\omega})^2} = \frac{-4.1151e^{j3\omega} + 4.9383e^{j2\omega}}{1 - 1.8e^{-j\omega} + 0.81e^{-j2\omega}}$$

MATLAB Verification:

```
% P0303c: DTFT of x3(n) = n*((0.9) ^ n)*u(n+3);
clc; close all;
w3 = linspace(0,pi,501); X3_num = (-4.1151*exp(j*3*w3)+4.9383*exp(j*2*w3));
X3_den = 1-1.8*exp(-j*w3)+0.81*exp(-j*2*w3); X3 = X3_num./X3_den;
magX3 = abs(X3); phaX3 = angle(X3);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0303c');
subplot(2,1,1); plot(w3/pi,magX3,'LineWidth',1.5);
axis([0,1,0,100]); wtick = [0:0.2:1]; magtick = [0:20:100];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title('Magnitude response','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w3/pi,phaX3*180/pi,'LineWidth',1.5);
axis([0,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase Response','FontSize',TFS); set(gca,'XTick',wtick);
set(gca,'YTick',phatick); print -deps2 ../EPSFILES/P0303c;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.9.
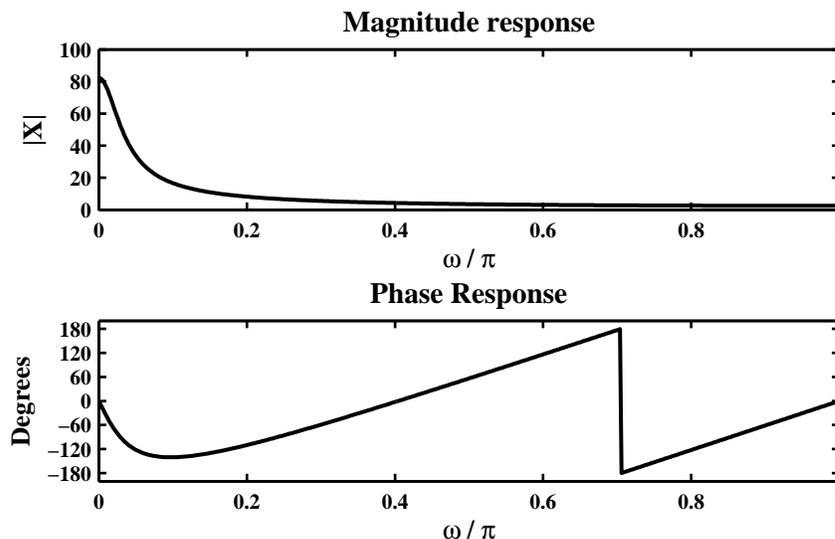
Figure 3.9: Problem P3.3.3 DTFT plots

4. $x(n) = \sum_{-\infty}^{\infty} (n+3)(0.8)^{n-1} u(n-2)$.

$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} (n+3)(0.8)^{n-1} u(n-2)e^{-jn\omega} = \sum_{-\infty}^{\infty} (n+5)(0.8)^{n+1} u(n)e^{-j(n+2)\omega}$$

$$= (0.8)e^{-j2\omega} \sum_{0}^{\infty} n(0.8)^n e^{-jn\omega} + 4e^{-j2\omega} \sum_{0}^{\infty} (0.8)^n e^{-jn\omega}$$

$$= \frac{0.64e^{-j3\omega}}{(1-0.8e^{-j\omega})^2} + \frac{4e^{-j2\omega}}{1-0.8e^{-j\omega}} = \frac{4e^{-j2\omega} - 2.56e^{-j3\omega}}{1-1.6e^{-j\omega} + 0.64e^{-j2\omega}}$$

MATLAB Verification:

```
% P0303d: DTFT of x4(n) = (n+3)*((0.8) ^ (n-1))*u(n-2);
clc; close all;
w4 = linspace(0,pi,501); X4_num = 4*exp(-2*j*w4)-2.56*exp(-3*j*w4);
X4_den = 1-1.6*exp(-1*j*w4)+0.64*exp(-2*j*w4); X4 = X4_num./X4_den;
magX4 = abs(X4); phaX4 = angle(X4);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0303d');
subplot(2,1,1); plot(w4/pi,magX4,'LineWidth',1.5);
axis([0 1 0 40]); wtick = [0:0.2:1]; magtick = [0:5:40];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title('Magnitude response','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w4/pi,phaX4*180/pi,'LineWidth',1.5);
axis([0,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase Response','FontSize',TFS); set(gca,'XTick',wtick);
set(gca,'YTick',phatick); print -deps2 ../EPSFILES/P0303d;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.10.
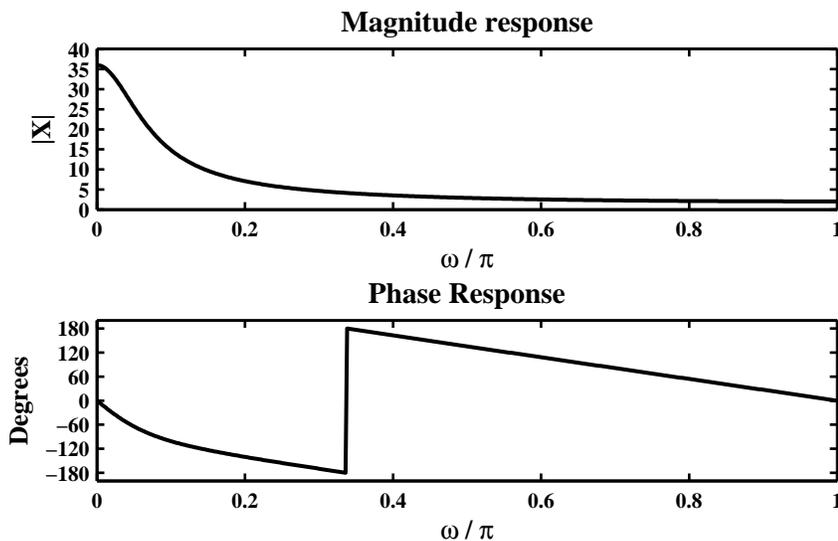
Figure 3.10: Problem P3.3.4 DTFT plots

5. $x(n) = 4\,(-0.7)^n \cos(0.25\pi n)u(n)$.

$$X\!\left(e^{j\omega}\right) = \sum_{-\infty}^{\infty} 4\,(-0.7)^n \cos(0.25\pi n)u(n)e^{-jn\omega} = 4\sum_{0}^{\infty}(-0.7)^n \cos(0.25\pi n)e^{-jn\omega}$$

$$= 4\frac{1 - (-0.7)\cos(0.25\pi)e^{-j\omega}}{1 - 2(-0.7)e^{-j\omega} + (-0.7)^2 e^{-j2\omega}} = 4\frac{1 + 0.495e^{-j\omega}}{1 + 1.4e^{-j\omega} + 0.49e^{-j2\omega}}$$

MATLAB Verification:

```
% P0303e: DTFT of x5(n) = 4*((-0.7) ^ n)*cos(0.25*pi*n)*u(n)
clc; close all;
w5 = [0:500]*pi/500; X51 = 4*(ones(size(w5))+0.7*cos(0.25*pi)*exp(-j*w5));
X52 = ones(size(w5))+1.4*cos(0.25*pi)*exp(-j*w5)+0.49*exp(-j*2*w5);
X5 = X51./X52; magX5 = abs(X5); phaX5 = angle(X5);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0303e');
subplot(2,1,1); plot(w5/pi,magX5,'LineWidth',1.5);
axis([0 1 0 10]); wtick = [0:0.2:1]; magtick = [0:2:10];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title('Magnitude response','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w5/pi,phaX5*180/pi,'LineWidth',1.5);
axis([0,1,-200,200]); phatick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase Response','FontSize',TFS); set(gca,'XTick',wtick);
set(gca,'YTick',phatick); print -deps2 ../EPSFILES/P0303e;
```

The magnitude and phase plots of $X(e^{j\omega})$ are shown in Figure 3.11.
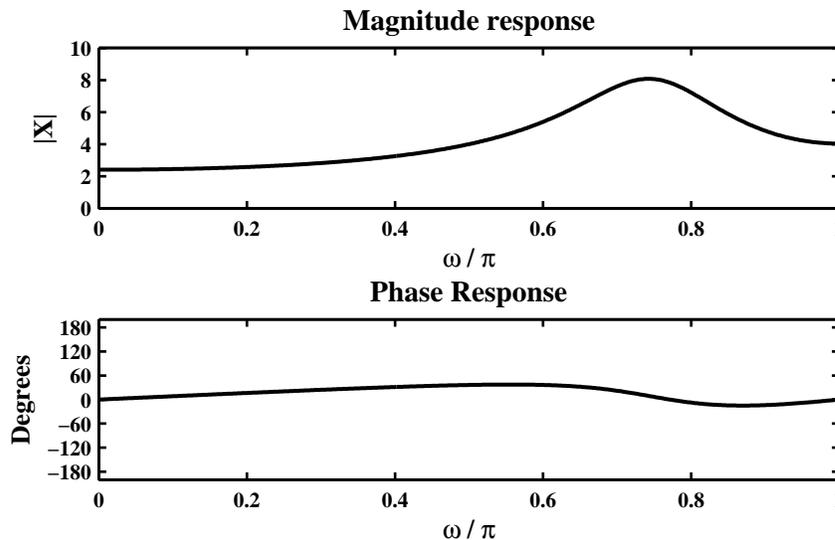
Figure 3.11: Problem P3.3.5 DTFT plots

P3.4 Window function DTFTs:

Rectangular Window: $\mathcal{R}_M(n) = u(n) - u(n - M)$

MATLAB script:

```
% P0304a: DTFT of a Rectangular Window, M = 10,25,50,101
clc; close all;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0304a');
w = linspace(-pi,pi,501); wtick = [-1:0.5:1]; magtick = [0:0.5:1.1];
% M = 10
M = 10; n = 0:M; x = ones(1,length(n));
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
ylabel('|X|','FontSize',LFS); title(['M = 10'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick);
% M = 25
M = 25; n = 0:M; x = ones(1,length(n));
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,2); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
title(['M = 25'],'FontSize',TFS); set(gca,'XTick',wtick,'YTick',magtick);
% M = 50
M = 50; n = 0:M; x = ones(1,length(n));
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,3); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['M = 50'],'FontSize',TFS); set(gca,'XTick',wtick,'YTick',magtick);
% M = 101
M = 101; n = 0:M; x = ones(1,length(n));
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,4); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
xlabel('\omega/\pi','FontSize',LFS); title(['M = 101'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0304a;
```

The magnitude plots of the DTFTs are shown in Figure 3.12.
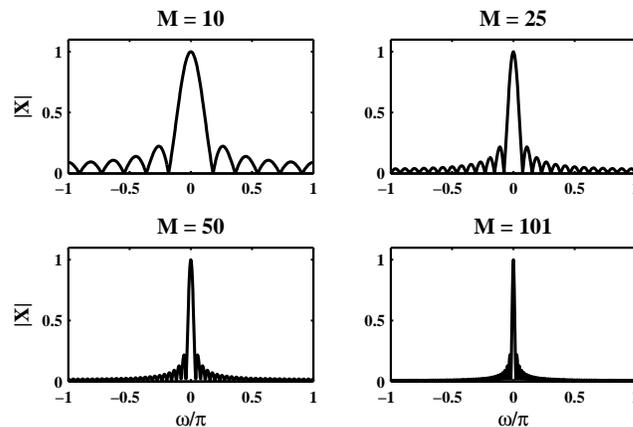
Figure 3.12: Problem P3.4 Rectangular window DTFT plots

Triangular Window: $\mathcal{T}_M(n) = \left[1 - \dfrac{|M - 1 - 2n|}{M - 1} \right] \mathcal{R}_M(n)$

MATLAB script:

```
% P0304b: DTFT of a Triangular Window,M = 10,25,50,101
clc; close all;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0304b');
w = linspace(-pi,pi,501); wtick = [-1:0.5:1]; magtick = [0:0.5:1.1];
% M = 10
M = 10; n = 0:M; x = (1-(abs( M-1-(2*n) )/(M+1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
ylabel('|X|','FontSize',LFS); title(['M = 10'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick);
% M = 25
M = 25; n = 0:M; x = (1-(abs( M-1-(2*n) )/(M+1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,2); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
title(['M = 25'],'FontSize',TFS); set(gca,'XTick',wtick,'YTick',magtick);
% M = 50
M = 50; n = 0:M; x = (1-(abs( M-1-(2*n) )/(M+1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,3); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['M = 50'],'FontSize',TFS); set(gca,'XTick',wtick,'YTick',magtick);
% M = 101
M = 101; n = 0:M; x = (1-(abs( M-1-(2*n) )/(M+1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,4); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
xlabel('\omega/\pi','FontSize',LFS); title(['M = 101'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0304b;
```

The magnitude plots of the DTFTs are shown in Figure 3.13.
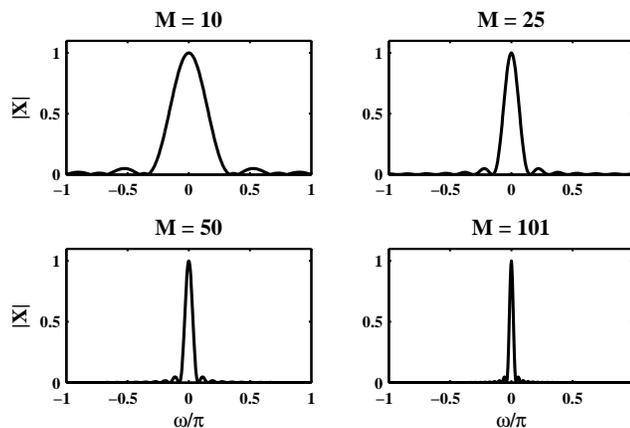
Figure 3.13: Problem P3.4 Triangular window DTFT plots

Hann Window: $\mathcal{C}_M(n) = 0.5 \left[1 - \cos \dfrac{2\pi n}{M-1} \right] \mathcal{R}_M(n)$

MATLAB script:

```
% P0304c: DTFT of a Hann Window,M = 10,25,50,101
clc; close all;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0304c');
w = linspace(-pi,pi,501); wtick = [-1:0.5:1]; magtick = [0:0.5:1.1];
% M = 10
M = 10; n = 0:M; x = 0.5*(1-cos((2*pi*n)/(M-1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
ylabel('|X|','FontSize',LFS); title(['M = 10'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick);
% M = 25
M = 25; n = 0:M; x = 0.5*(1-cos((2*pi*n)/(M-1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,2); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
title(['M = 25'],'FontSize',TFS); set(gca,'XTick',wtick,'YTick',magtick);
% M = 50
M = 50; n = 0:M; x = 0.5*(1-cos((2*pi*n)/(M-1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,3); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['M = 50'],'FontSize',TFS); set(gca,'XTick',wtick,'YTick',magtick);
% M = 101
M = 101; n = 0:M; x = 0.5*(1-cos((2*pi*n)/(M-1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,4); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
xlabel('\omega/\pi','FontSize',LFS); title(['M = 101'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0304c;
```

The magnitude plots of the DTFTs are shown in Figure 3.14.
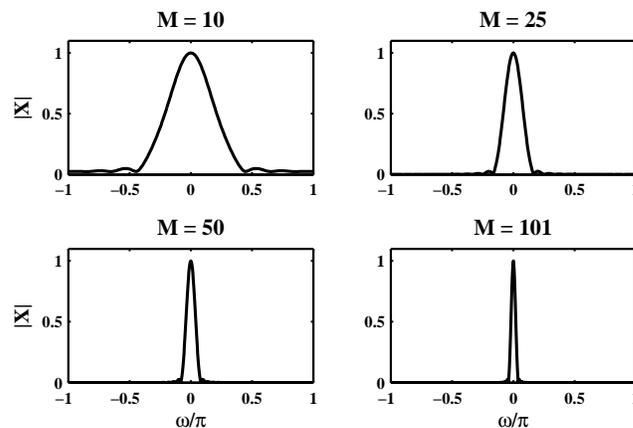
Figure 3.14: Problem P3.4 Hann window DTFT plots

Hamming Window: $\mathcal{H}_M(n) = \left[0.54 - 0.46\cos\dfrac{2\pi n}{M-1}\right]\mathcal{R}_M(n)$

MATLAB script:

```
% P0304d: DTFT of a Hamming Window,M = 10,25,50,101
clc; close all;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0304d');
w = linspace(-pi,pi,501); wtick = [-1:0.5:1]; magtick = [0:0.5:1.1];
% M = 10
M = 10; n = 0:M; x = (0.54-0.46*cos((2*pi*n)/(M-1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
ylabel('|X|','FontSize',LFS); title(['M = 10'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick);
% M = 25
M = 25; n = 0:M; x = (0.54-0.46*cos((2*pi*n)/(M-1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,2); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
title(['M = 25'],'FontSize',TFS); set(gca,'XTick',wtick,'YTick',magtick);
% M = 50
M = 50; n = 0:M; x = (0.54-0.46*cos((2*pi*n)/(M-1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,3); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['M = 50'],'FontSize',TFS); set(gca,'XTick',wtick,'YTick',magtick);
% M = 101
M = 101; n = 0:M; x = (0.54-0.46*cos((2*pi*n)/(M-1)) );
X = dtft(x,n,w); magX = abs(X); magX = magX/max(magX);
subplot(2,2,4); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.1]);
xlabel('\omega/\pi','FontSize',LFS); title(['M = 101'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0304d;
```

The magnitude plots of the DTFTs are shown in Figure 3.15.
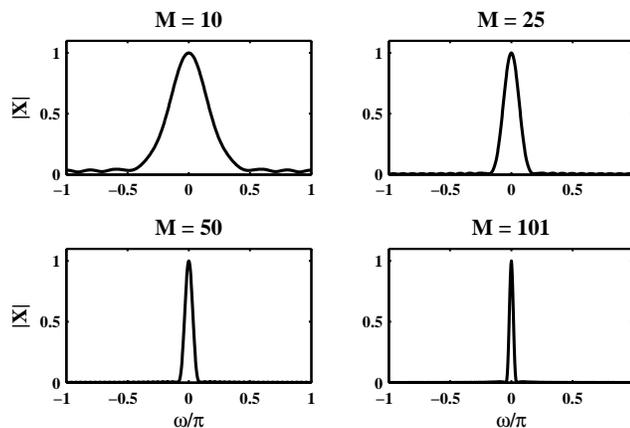
Figure 3.15: Problem P3.4 Hamming window DTFT plots

P3.5 Inverse DTFTs using the definition of the DTFT:

1. $X\left(e^{j\omega}\right) = 3 + 2\cos(\omega) + 4\cos(2\omega)$: Using the Euler identity

$$X\left(e^{j\omega}\right) = 3 + 2\frac{e^{j\omega} + e^{-j\omega}}{2} + 4\frac{e^{j2\omega} + e^{-j2\omega}}{2} = 2e^{j2\omega} + e^{j\omega} + 3 + e^{-j\omega} + 2e^{-j2\omega}$$

Hence $x(n) = \{2, 1, \underset{\uparrow}{3}, 1, 2\}$.

2. $X\left(e^{j\omega}\right) = [1 - 6\cos(3\omega) + 8\cos(5\omega)]e^{-j3\omega}$: Using the Euler identity

$$X\left(e^{j\omega}\right) = \left[1 - 6\frac{e^{j3\omega} + e^{-j3\omega}}{2} + 8\frac{e^{j5\omega} + e^{-j5\omega}}{2}\right]e^{-j3\omega}$$
$$= 4e^{j2\omega} - 3 + e^{-j3\omega} - 3e^{-j6\omega} + 4e^{-j8\omega}$$

Hence $x(n) = \{4, 0, \underset{\uparrow}{-3}, 0, 0, 1, 0, 0, -3, 0, 4\}$.

3. $X\left(e^{j\omega}\right) = 2 + j4\sin(2\omega) - 5\cos(4\omega)$: Using the Euler identity

$$X\left(e^{j\omega}\right) = 2 + j4\frac{e^{j2\omega} - e^{-j2\omega}}{2j} - 5\frac{e^{j4\omega} + e^{-j4\omega}}{2} = -\tfrac{5}{2}e^{j4\omega} + 2e^{j2\omega} + 2 - 2e^{-j2\omega} - \tfrac{5}{2}e^{-j4\omega}$$

Hence $x(n) = \{-\tfrac{5}{2}, 0, 2, 0, \underset{\uparrow}{2}, 0, -2, 0, -\tfrac{5}{2}\}$.

4. $X\left(e^{j\omega}\right) = [1 + 2\cos(\omega) + 3\cos(2\omega)]\cos(\omega/2)e^{-j5\omega/2}$: Using the Euler identity

$$X\left(e^{j\omega}\right) = \left[1 + 2\frac{e^{j\omega} + e^{-j\omega}}{2} + 3\frac{e^{j2\omega} + e^{-j2\omega}}{2}\right]\frac{e^{j\frac{1}{2}\omega} + e^{-j\frac{1}{2}\omega}}{2}e^{-j5\omega/2}$$
$$= \left[\tfrac{3}{2}e^{j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + \tfrac{3}{2}e^{-j2\omega}\right]\frac{e^{-j2\omega} + e^{-j3\omega}}{2}$$
$$= \tfrac{3}{4} + \tfrac{5}{4}e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} + \tfrac{5}{4}e^{-j4\omega} + \tfrac{3}{4}e^{-j5\omega}$$

Hence $x(n) = \{\underset{\uparrow}{\tfrac{3}{4}}, \tfrac{5}{4}, 1, 1, \tfrac{5}{4}, \tfrac{3}{4}\}$.

5. $X\left(e^{j\omega}\right) = j[3 + 2\cos(\omega) + 4\cos(2\omega)]\sin(\omega)e^{-j3\omega}$: Using the Euler identity

$$X\left(e^{j\omega}\right) = j\left[3 + 2\frac{e^{j\omega} + e^{-j\omega}}{2} + 4\frac{e^{j2\omega} + e^{-j2\omega}}{2}\right]\frac{e^{j\omega} - e^{-j\omega}}{2j}e^{-j3\omega}$$
$$= 2 + e^{-j\omega} + e^{-j2\omega} - e^{-j4\omega} - e^{-j5\omega} - 2e^{-j6\omega}$$

Hence $x(n) = \{\underset{\uparrow}{2}, 1, 1, 0, -1, -1, -2\}$.

P3.6 Inverse DTFTs using the definition of the IDTFT::

1. $X\left(e^{j\omega}\right) = \begin{cases} 1, & 0 \leq |\omega| \leq \pi/3; \\ 0, & \pi/3 < |\omega| \leq \pi. \end{cases}$

   Solution: Consider

   $$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X\left(e^{j\omega}\right) e^{jn\omega} \, d\omega = \frac{1}{2\pi} \int_{-\pi/3}^{\pi/3} e^{jn\omega} \, d\omega = \frac{e^{jn\omega}}{j2\pi n}\Bigg|_{-\pi/3}^{\pi/3} = \frac{\sin\left(\frac{\pi n}{3}\right)}{\pi n} = \frac{1}{3}\text{sinc}\left(\frac{n}{3}\right)$$

2. $X\left(e^{j\omega}\right) = \begin{cases} 0, & 0 \leq |\omega| \leq 3\pi/4; \\ 1, & 3\pi/4 < |\omega| \leq \pi. \end{cases}$

   Solution: Consider

   $$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X\left(e^{j\omega}\right) e^{jn\omega} \, d\omega = \frac{1}{2\pi} \int_{-\pi}^{-3\pi/4} e^{jn\omega} \, d\omega + \frac{1}{2\pi} \int_{3\pi/4}^{\pi} e^{jn\omega} \, d\omega$$

   $$= \frac{2}{2\pi} \int_{3\pi/4}^{\pi} \cos(n\omega) \, d\omega = \frac{1}{\pi} \frac{\sin(n\omega)}{n}\Bigg|_{3\pi/4}^{\pi} = \delta(n) - \frac{3}{4}\text{sinc}\left(\frac{3n}{4}\right)$$

3. $X\left(e^{j\omega}\right) = \begin{cases} 2, & 0 \leq |\omega| \leq \pi/8; \\ 1, & \pi/8 < |\omega| \leq 3\pi/4. \\ 0, & 3\pi/4 < |\omega| \leq \pi. \end{cases}$

   Solution: Consider

   $$x(n) = \frac{2}{2\pi} \left[\int_0^{\pi/8} 2\cos(n\omega) \, d\omega + \int_{\pi/8}^{3\pi/4} \cos(n\omega) \, d\omega \right] = \frac{1}{\pi} \left[2\frac{\sin(n\omega)}{n}\Bigg|_0^{\pi/8} + \frac{\sin(n\omega)}{n}\Bigg|_{\pi/8}^{3\pi/4} \right]$$

   $$= \frac{1}{n\pi} \left[2\sin\left(\frac{n\pi}{8}\right) + \sin\left(\frac{3n\pi}{4}\right) - \sin\left(\frac{n\pi}{8}\right) \right] = \frac{1}{n\pi} \left[\sin\left(\frac{n\pi}{8}\right) + \sin\left(\frac{3n\pi}{4}\right) \right]$$

   $$= \frac{1}{8}\text{sinc}\left(\frac{n}{8}\right) + \frac{3}{4}\text{sinc}\left(\frac{3n}{4}\right)$$

4. $X\left(e^{j\omega}\right) = \begin{cases} 0, & -\pi \leq \omega < \pi/4; \\ 1, & \pi/4 \leq |\omega| \leq 3\pi/4. \\ 0, & 3\pi/4 < |\omega| \leq \pi. \end{cases}$

   Solution: Consider

   $$x(n) = \frac{2}{2\pi} \int_{\pi/4}^{3\pi/4} \cos(n\omega) \, d\omega = \frac{\sin(n\omega)}{n\pi}\Bigg|_{\pi/4}^{3\pi/4} = \frac{\sin\left(\frac{3n\pi}{4}\right) - \sin\left(\frac{n\pi}{4}\right)}{n\pi} = \frac{3}{4}\text{sinc}\left(\frac{3n}{4}\right) - \frac{1}{4}\text{sinc}\left(\frac{n}{4}\right)$$

5. $X\left(e^{j\omega}\right) = \omega \, e^{j(\pi/2 - 10\omega)}$.

   Solution: Consider

   $$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \omega \, e^{j(\pi/2 - 10\omega)} e^{jn\omega} d\omega = \frac{j}{2\pi} \int_{-\pi}^{\pi} \omega \, e^{j(n-10)\omega} d\omega$$

   $$= \frac{j}{2\pi} \left[\frac{\omega e^{j(n-10)\omega}}{j(n-10)}\Bigg|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{e^{j(n-10)\omega}}{j(n-10)} \right] = \cos[(n-10)\pi] - \frac{\sin[(n-10)\pi]}{\pi(n-10)^2}$$

P3.7 A complex-valued sequence $x(n)$ can be decomposed into a conjugate symmetric part $x_e(n)$ and an conjugate anti-symmetric part $x_o(n)$ as

$$x_e(n) = \frac{1}{2}\left[x(n) + x^*(-n)\right]; \qquad x_o(n) = \frac{1}{2}\left[x(n) - x^*(-n)\right]$$

Consider

$$\mathcal{F}[x_e(n)] = \sum_{-\infty}^{\infty} x_e(n)e^{-jn\omega} = \sum_{-\infty}^{\infty}\frac{1}{2}\left[x(n) + x^*(-n)\right]e^{-jn\omega} = \frac{1}{2}\left[\sum_{-\infty}^{\infty}x(n)e^{-jn\omega} + \sum_{-\infty}^{\infty}x^*(-n)e^{-jn\omega}\right]$$

$$= \frac{1}{2}\left[X\left(e^{j\omega}\right) + X^*\left(e^{j\omega}\right)\right] = X_R\left(e^{j\omega}\right)$$

Similarly,

$$\mathcal{F}[x_o(n)] = \sum_{-\infty}^{\infty} x_o(n)e^{-jn\omega} = \sum_{-\infty}^{\infty}\frac{1}{2}\left[x(n) - x^*(-n)\right]e^{-jn\omega} = \frac{1}{2}\left[\sum_{-\infty}^{\infty}x(n)e^{-jn\omega} - \sum_{-\infty}^{\infty}x^*(-n)e^{-jn\omega}\right]$$

$$= \frac{1}{2}\left[X\left(e^{j\omega}\right) - X^*\left(e^{j\omega}\right)\right] = jX_I\left(e^{j\omega}\right)$$

MATLAB Verification using $x(n) = 2(0.9)^{-n}\left[\cos(0.1\pi n) + j\sin(0.9\pi n)\right]\left[u(n) - u(n - 10)\right]$:

```
% P0307: DTFT after even and odd part decomposition of x(n)
% x(n) = 2*(0.9)^(-n)*(cos(0.1*pi*n)+j*sin(0.9*pi*n))(u(n)-u(n-10))
clc; close all;
%
[x1,n1] = stepseq(0,0,10); [x2,n2] = stepseq(10,0,10);
[x3,n3] = sigadd(x1,n1,-x2,n2);
n = n3; x = 2*(0.9 .^ (-n)).*(cos(0.1*pi*n)+j*sin(0.9*pi *n)).*x3;
[xe,xo,m] = evenodd(x,n);
w = [-500:500]*pi/500; X = dtft(x,n,w); realX = real(X); imagX = imag(X);
Xe = dtft(xe,m,w); Xo = dtft(xo,m,w);
diff_e = max(abs(realX-Xe)); diff_o = max(abs(j*imagX-Xo));
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0307');
subplot(2,2,1); plot(w/pi,real(Xe),'LineWidth',1.5);
axis([-1 1 -30 20]); wtick = sort([-1:0.4:1 0]); magtick = [-30:10:20];
xlabel('\omega/\pi','FontSize',LFS); ylabel('X_e','FontSize',LFS);
title('DTFT of even part of x(n)','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,3); plot(w/pi,realX,'LineWidth',1.5); axis([-1 1 -30 20]);
wtick = sort([-1:0.4:1 0] ); magtick = [-30:10:20];
xlabel('\omega/\pi','FontSize',LFS); ylabel('X_R','FontSize',LFS);
title('Real part:DTFT of x(n)','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,2); plot(w/pi,imag(Xo),'LineWidth',1.5); axis([-1 1 -30 20]);
wtick = sort([-1:0.4:1 0]); magtick = [-30:10:20];
xlabel('\omega/\pi','FontSize',LFS); ylabel('X_o','FontSize',LFS);
title('DTFT of odd part of x(n)','FontSize',TFS);
```

```
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,4); plot(w/pi,imagX,'LineWidth',1.5); axis([-1 1 -30 20]);
wtick = sort([-1:0.4:1 0]); magtick = [-30:10:20];
xlabel('\omega/\pi','FontSize',LFS); ylabel('X_I','FontSize',LFS);
title('Imaginary part:DTFT of x(n)','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0307;
```

The magnitude plots of the DTFTs are shown in Figure 3.16.
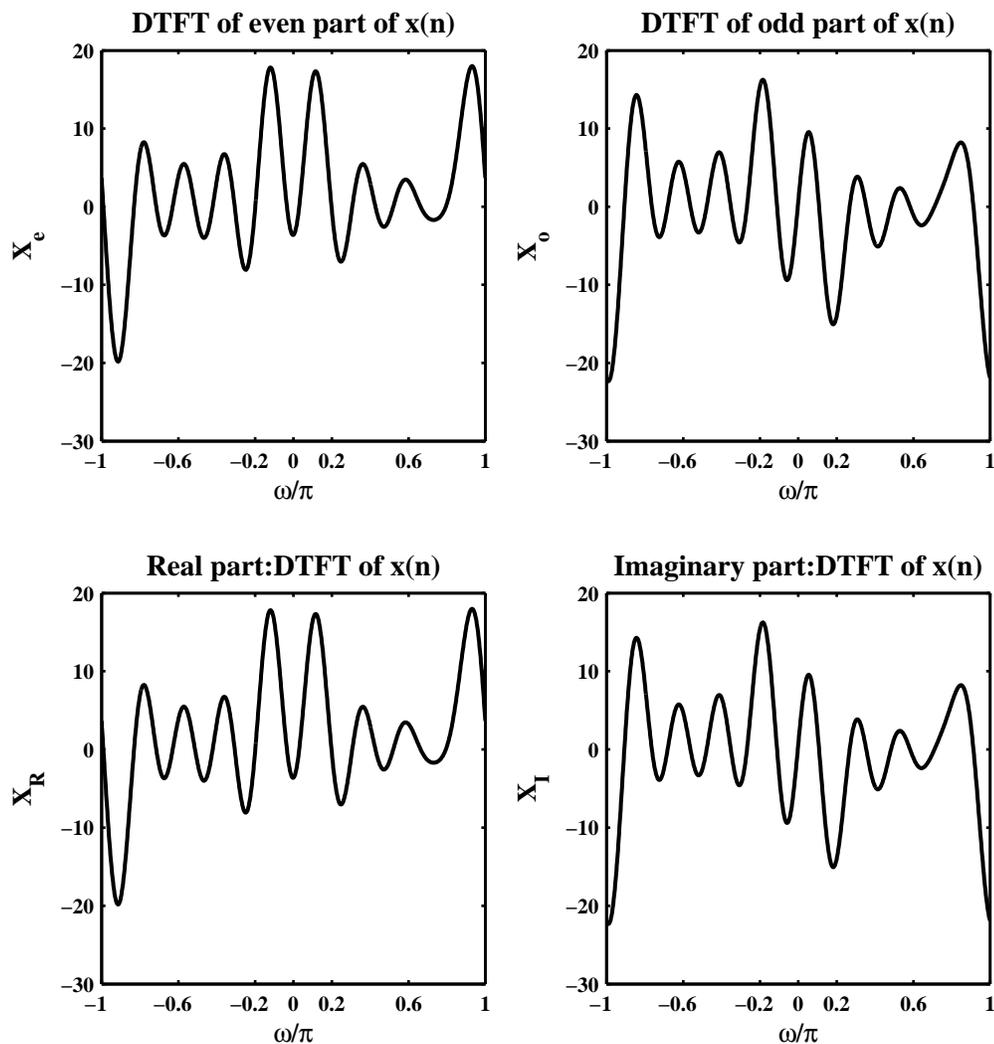
Figure 3.16: Problem P3.7 DTFT plots

P3.8 A complex-valued DTFT $X\left(e^{j\omega}\right)$ can be decomposed into its conjugate symmetric part $X_e\left(e^{j\omega}\right)$ and conjugate anti-symmetric part $X_o\left(e^{j\omega}\right)$, as

$$X\left(e^{j\omega}\right) = X_e\left(e^{j\omega}\right) + X_o\left(e^{j\omega}\right); \ \ X_e\left(e^{j\omega}\right) = \tfrac{1}{2}\left[X\left(e^{j\omega}\right) + X^*\left(e^{-j\omega}\right)\right], \ \ X_0\left(e^{j\omega}\right) = \tfrac{1}{2}\left[X\left(e^{j\omega}\right) - X^*\left(e^{-j\omega}\right)\right]$$

Consider

$$\mathcal{F}^{-1}\left[X_e\left(e^{j\omega}\right)\right] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_e\left(e^{j\omega}\right)e^{jn\omega}\,d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi}\tfrac{1}{2}\left[X\left(e^{j\omega}\right) + X^*\left(e^{-j\omega}\right)\right]e^{jn\omega}\,d\omega$$

$$= \tfrac{1}{2}[x(n) + x^*(-n)] = x_R(n)$$

Similarly,

$$\mathcal{F}^{-1}\left[X_o\left(e^{j\omega}\right)\right] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_o\left(e^{j\omega}\right)e^{jn\omega}\,d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi}\tfrac{1}{2}\left[X\left(e^{j\omega}\right) - X^*\left(e^{-j\omega}\right)\right]e^{jn\omega}\,d\omega$$

$$= \tfrac{1}{2}[x(n) - x^*(-n)] = jx_I(n)$$

MATLAB Verification using $x(n) = e^{j0.1\pi n}\left[u(n) - u(n - 20)\right]$:

```
% P0308: x(n) = exp(0.1*j*pi*n)*(u(n)-u(n-20));
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,6]);
%
[x1,n1] = stepseq(0,0,20); [x2,n2] = stepseq(20,0,20);
[x3,n3] = sigadd(x1,n1,-x2,n2); n = n3; x = exp(0.1*j*pi*n).*x3;
w1 = [-500:500]*pi/500; X = dtft(x,n,w1); [Xe,Xo,w2] = evenodd(X,[-500:500]);
w2 = w2*pi/500; xr = real(x); xi = imag(x); Xr = dtft(xr,n,w1);
Xi = dtft(j*xi,n,w1); diff_r = max(abs(Xr-Xe)); diff_i = max(abs(Xi-Xo));
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0308');
subplot(4,2,1); plot(w1/pi,abs(Xr),'LineWidth',1.5);
ylabel('|X_r|','FontSize',LFS);
title('Magnitude response of x_R','FontSize',TFS);
subplot(4,2,2); plot(w1/pi,angle(Xr)*180/pi,'LineWidth',1.5);
axis([-1 1 -200 200]); magtick = [-180:90:180];
ylabel('Degrees','FontSize',LFS);
title('Phase response of x_R','FontSize',TFS); set(gca,'YTick',magtick);
subplot(4,2,3); plot(w1/pi,abs(Xe),'LineWidth',1.5); axis([-1 1 0 15]);
ytick = [0:5:15]; ylabel('|X_e|','FontSize',LFS);
title([ 'Magnitude part of X_e' ],'FontSize',TFS); set(gca,'YTick',ytick);
subplot(4,2,4); plot(w1/pi,angle(Xe)*180/pi,'LineWidth',1.5);
axis([-1 1 -200 200]); magtick = [-180:90:180]; ylabel('Degrees','FontSize',LFS);
title([ 'Phase part of X_e' ],'FontSize',TFS); set(gca,'YTick',magtick);
subplot(4,2,5); plot(w1/pi,abs(Xi),'LineWidth',1.5);
ytick = [0:5:15]; axis([-1 1 0 15]); ylabel('|X_i|','FontSize',LFS);
title([ 'Magnitude response of j*x_I' ],'FontSize',TFS);
set(gca,'YTick',ytick);
subplot(4,2,6); plot(w1/pi,angle(Xi)*180/pi,'LineWidth',1.5);
axis([-1 1 -200 200]); magtick = [-180:90:180];
ylabel('Degrees','FontSize',LFS);
```

```
title([ 'Phase response of j*x_I' ],'FontSize',TFS); set(gca,'YTick',magtick);
subplot(4,2,7); plot(w1/pi,abs(Xo),'LineWidth',1.5);
ytick = [0:5:15]; axis([-1 1 0 15]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X_o|','FontSize',LFS);
title(['Magnitude part of X_o'],'FontSize',TFS); set(gca,'YTick',ytick);
subplot(4,2,8); plot(w1/pi,angle(Xo)*180/pi,'LineWidth',1.5);
axis([-1 1 -200 200]); magtick = [-180:90:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase part of X_o'],'FontSize',TFS); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0308;
```

The magnitude plots of the DTFTs are shown in Figure 3.17.
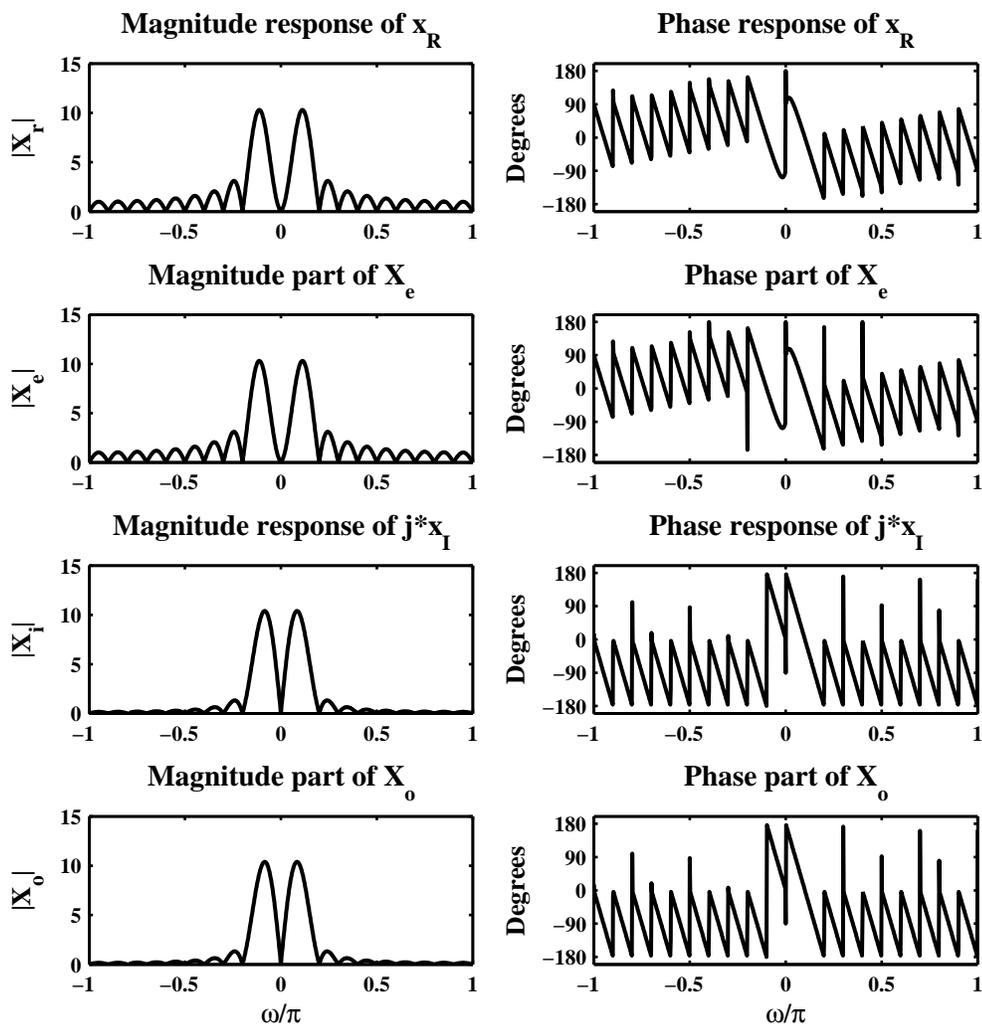
Figure 3.17: Problem P3.8 DTFT plots

P3.9 The real-part of the DTFT of a sinusoidal pulse $x(n) = (\cos \omega_0 n) \, \mathcal{R}_M(n)$:

First note that if the sequence $x(n)$ is a real-valued sequence, then the real part of its DTFT $X\left(e^{j\omega}\right)$ is given by

$$X_R\left(e^{j\omega}\right) = \sum_{-\infty}^{\infty} x(n) \cos(n\omega)$$

Hence for the given sinusoidal pulse, we have

$$X_R\left(e^{j\omega}\right) = \sum_0^{M-1} \cos(\omega_0 n) \cos(n\omega) = \frac{1}{2} \sum_0^{M-1} \cos[(\omega - \omega_0)n] + \frac{1}{2} \sum_0^{M-1} \cos[(\omega + \omega_0)n] \qquad (3.2)$$

Consider the first sum in (3.2),

$$\sum_0^{M-1} \cos[(\omega - \omega_0)n] = \frac{1}{2} \sum_0^{M-1} \left\{ e^{j(\omega - \omega_0)n} + e^{-j(\omega - \omega_0)n} \right\} = \frac{1}{2} \left\{ \frac{1 - e^{j(\omega - \omega_0)M}}{1 - e^{j(\omega - \omega_0)}} + \frac{1 - e^{-j(\omega - \omega_0)M}}{1 - e^{-j(\omega - \omega_0)}} \right\}$$

$$= \frac{1}{2} \left(\frac{1 - \cos(\omega - \omega_0) - \cos[(\omega - \omega_0)M] + \cos[(\omega - \omega_0)(M - 1)]}{1 - \cos(\omega - \omega_0)} \right)$$

$$= \frac{1}{2} \left(\frac{2 \sin^2[(\omega - \omega_0)/2] + 2 \sin[(\omega - \omega_0)/2] \sin[(\omega - \omega_0)(M - \frac{1}{2})]}{2 \sin^2[(\omega - \omega_0)/2]} \right)$$

$$= \frac{1}{2} \left(\frac{\sin[(\omega - \omega_0)/2] + \sin[(\omega - \omega_0)(M - \frac{1}{2})]}{\sin[(\omega - \omega_0)/2]} \right)$$

$$= \frac{\cos[(\omega - \omega_0)(M - 1)] \sin[(\omega - \omega_0)M/2]}{\sin[(\omega - \omega_0)/2]} \qquad (3.3)$$

Similarly,

$$\sum_0^{M-1} \cos[(\omega + \omega_0)n] = \sum_0^{M-1} \cos[\{\omega - (2\pi - \omega_0)\}n]$$

$$= \frac{\cos[\{\omega - (2\pi - \omega_0)\}(M - 1)] \sin[\{\omega - (2\pi - \omega_0)\}M/2]}{\sin[\{\omega - (2\pi - \omega_0)\}/2]} \qquad (3.4)$$

Substituting (3.3) and (3.4) in (3.2), we obtain the desired result.

Matlab Computation and plot of $X_R\left(e^{j\omega}\right)$ for $\omega_0 = \pi/2$ and $M = 5, \ 15, \ 25, \ 100$:

```
% P0309: DTFT of sinusoidal pulse for different values of M
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,9]);
%% M = 5
M = 5; w0 = pi/2; w = [-500:500]*pi/500;
X = 0.5*(exp(-j*(w-w0)*((M-1)/2)).*sin((w-w0)*M/2)./sin((w-w0+eps)/2)) + ...
    0.5* (exp(-j*(w+w0)*((M-1)/2)).*sin((w+w0)*M/2)./sin((w+w0+eps)/2));
magX = abs(X); phaX = angle(X);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0309');
subplot(4,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 4]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Magnitude Response M = 5'],'FontSize',TFS);
```

```
subplot(4,2,2); plot(w/pi,phaX*180/pi,'LineWidth',1.5); axis([-1 1 -200 200]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase Response M = 5'],'FontSize',TFS);
ytick = [-180 0 180]; set(gca,'YTickmode','manual','YTick',ytick);
%% M = 15
M = 15; w0 = pi/2; w = [-500:500]*pi/500;
X = 0.5*(exp(-j*(w-w0)*((M-1)/2)).*sin((w-w0)*M/2)./sin((w-w0+eps)/2)) + ...
    0.5* (exp(-j*(w+w0)*((M-1)/2)).*sin((w+w0)*M/2)./sin((w+w0+eps)/2));
magX = abs(X); phaX = angle(X);
%
subplot(4,2,3); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 10]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title([char(10) 'Magnitude Response M = 15' char(10)],'FontSize',TFS);
subplot(4,2,4); plot(w/pi,phaX*180/pi,'LineWidth',1.5); axis([-1 1 -200 200]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title([char(10) 'Phase Response M = 15' char(10)],'FontSize',TFS);
ytick = [-180 0 180]; set(gca,'YTickmode','manual','YTick',ytick);
%% M = 25
M = 25; w0 = pi/2; w = [-500:500]*pi/500;
X = 0.5*(exp(-j*(w-w0)*((M-1)/2)).*sin((w-w0)*M/2)./sin((w-w0+eps)/2)) + ...
    0.5* (exp(-j*(w+w0)*((M-1)/2)).*sin((w+w0)*M/2)./sin((w+w0+eps)/2));
magX = abs(X); phaX = angle(X);
%
subplot(4,2,5); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 15]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title([char(10) 'Magnitude Response M = 25' char(10)],'FontSize',TFS);
subplot(4,2,6); plot(w/pi,phaX*180/pi,'LineWidth',1.5); axis([-1 1 -200 200]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title([char(10) 'Phase Response M = 25' char(10)],'FontSize',TFS);
ytick = [-180 0 180];set(gca,'YTickmode','manual','YTick',ytick);
%%M = 101
M = 101; w0 = pi/2; w = [-500:500]*pi/500;
X = 0.5*(exp(-j*(w-w0)*((M-1)/2)).*sin((w-w0)*M/2)./sin((w-w0+eps)/2)) + ...
    0.5* (exp(-j*(w+w0)*((M-1)/2)).*sin((w+w0)*M/2)./sin((w+w0+eps)/2));
magX = abs(X); phaX = angle(X);

subplot(4,2,7); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 75]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title([char(10) 'Magnitude Response M = 101' char(10)],'FontSize',TFS);
ytick = [0 50 75];set(gca,'YTickmode','manual','YTick',ytick);
subplot(4,2,8); plot(w/pi,phaX*180/pi,'LineWidth',1.5); axis([-1 1 -200 200]);
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title([char(10) 'Phase Response M = 101' char(10)],'FontSize',TFS);
ytick = [-180 0 180];set(gca,'YTickmode','manual','YTick',ytick);
print -deps2 ../EPSFILES/P0309;
```

The plots of $X_{\mathrm{R}}\left(e^{j\omega}\right)$ are shown in Figure 3.18.
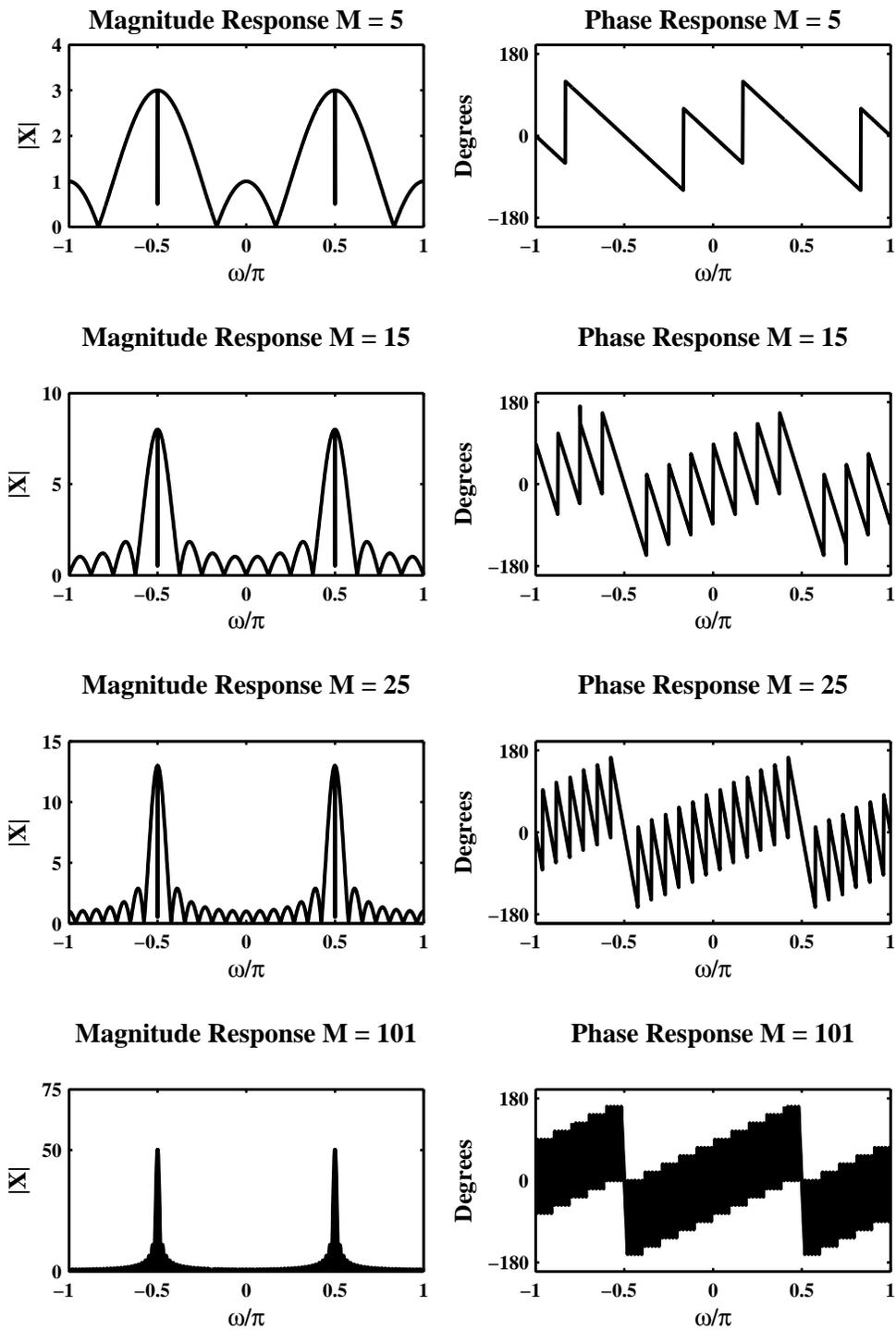
Figure 3.18: Problem P3.9 plots

P3.10 $x(n) = \mathcal{T}_{10}(n)$ is a triangular pulse given in Problem P4.3. DTFT calculations and plots using properties of the DTFT:

1. $x(n) = \mathcal{T}_{10}(-n)$:

$$X\left(e^{j\omega}\right) = \mathcal{F}[x(n)] = \mathcal{F}[\mathcal{T}_{10}(-n)] = \mathcal{F}[\mathcal{T}_{10}(n)]|_{\omega \to -\omega}$$
$$= \mathcal{F}[\mathcal{T}_{10}(n)]^* \; (\because \text{ real signal})$$

MATLAB script:

```
% P0310a: DTFT of x(n) = T_10(-n)
clc; close all; set(0,'defaultfigurepaperposition',[0,0,7,4]);
%
% Triangular Window T_10(n) & its DTFT
M = 10; n = 0:M; Tn = (1-(abs(M-1-2*n)/(M+1)));
w = linspace(-pi,pi,501); Tw = dtft(Tn,n,w); magTw = abs(Tw);
magTw = magTw/max(magTw); phaTw = angle(Tw)*180/pi;
% x(n) & its DTFT
[x,n] = sigfold(Tn,n); X = dtft(x,n,w); magX = abs(X);
magX = magX/max(magX); phaX = angle(X)*180/pi;
% DTFT of x(n) from the Property
Y = fliplr(Tw); magY = abs(Y)/max(abs(Y)); phaY = angle(Y)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0310a');
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,2); plot(w/pi,phaX,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.5:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,3); plot(w/pi,magY,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,4); plot(w/pi,phaY,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.4:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0310a;
```

The property verification using plots of $X\left(e^{j\omega}\right)$ is shown in Figure 3.19.
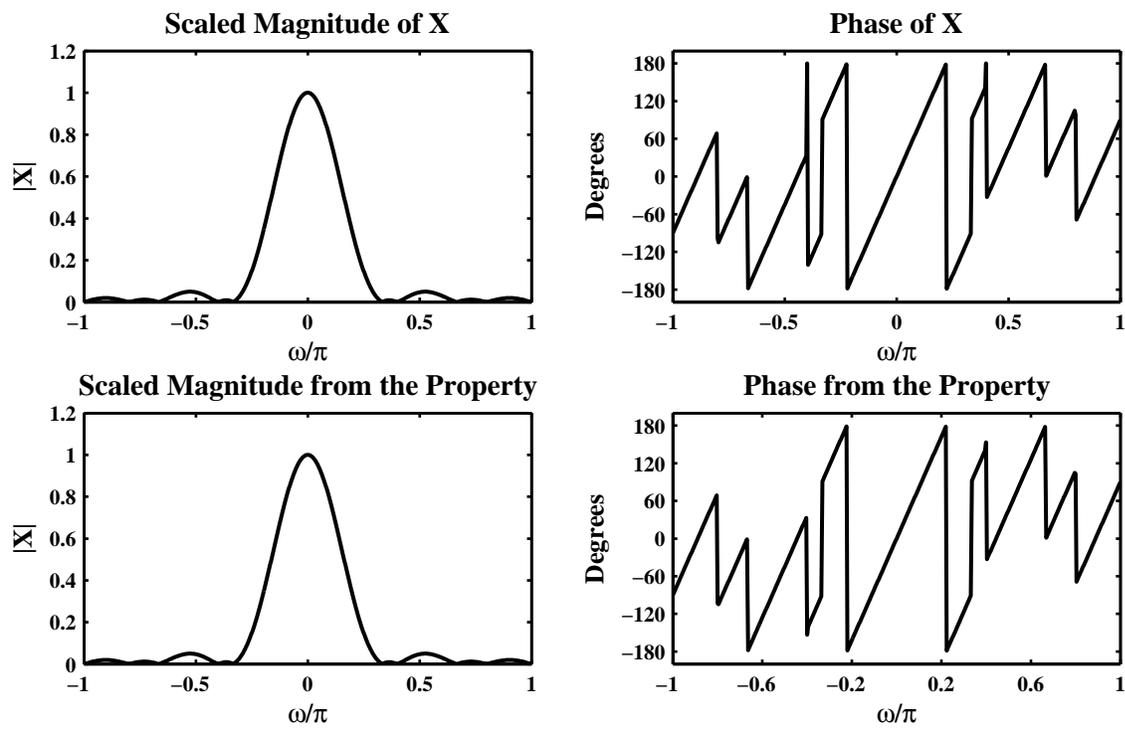
Figure 3.19: Problem P3.10a plots

2. $x(n) = \mathcal{T}_{10}(n) - \mathcal{T}_{10}(n - 10)$:

$$X\left(e^{j\omega}\right) = \mathcal{F}[x(n)] = \mathcal{F}[\mathcal{T}_{10}(n) - \mathcal{T}_{10}(n - 10)] = \mathcal{F}[\mathcal{T}_{10}(n)] - \mathcal{F}[\mathcal{T}_{10}(n)]e^{-j10\omega}$$
$$= \left(1 - e^{-j10\omega}\right)\mathcal{F}[\mathcal{T}_{10}(n)]$$

MATLAB script:

```
% P0310b: DTFT of T_10(n)-T_10(n-10);
%          T_10(n) = [1-(abs(M-1-2*n)/(M+1)) ]*R_M(n), M = 10
clc; close all; set(0,'defaultfigurepaperposition',[0,0,7,4]);
%
% Triangular Window T_10(n) & its DTFT
M = 10; n = 0:M; Tn = (1-(abs(M-1-2*n)/(M+1)));
w = linspace(-pi,pi,501); Tw = dtft(Tn,n,w); magTw = abs(Tw);
magTw = magTw/max(magTw); phaTw = angle(Tw)*180/pi;
% x(n) & its DTFT
[x1,n1] = sigshift(Tn,n,10); [x,n] = sigadd(Tn,n,-x1,n1);
X = dtft(x,n,w); magX = abs(X);
magX = magX/max(magX); phaX = angle(X)*180/pi;
% DTFT of x(n) from the Property
Y = Tw-exp(-j*w*10).*Tw; magY = abs(Y)/max(abs(Y)); phaY = angle(Y)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0310b');
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,2); plot(w/pi,phaX,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.5:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,3); plot(w/pi,magY,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,4); plot(w/pi,phaY,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.4:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0310b;
```

The property verification using plots of $X\left(e^{j\omega}\right)$ is shown in Figure 3.20.
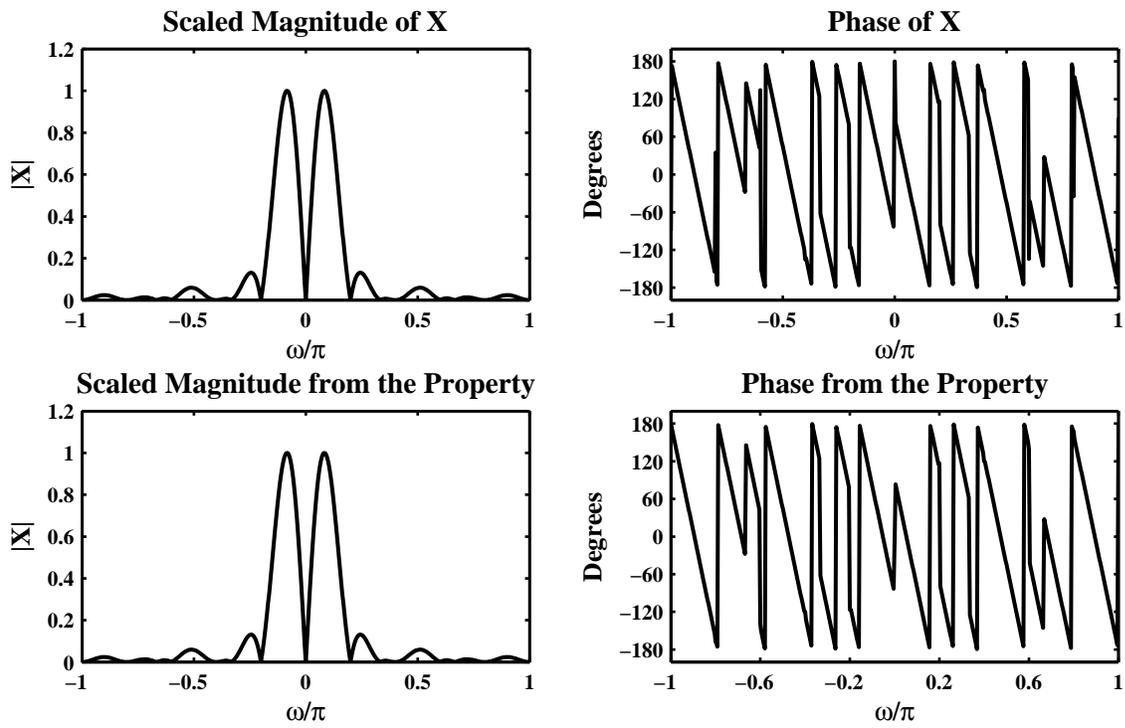
Figure 3.20: Problem P3.10b plots

3. $x(n) = \mathcal{T}_{10}(n) * \mathcal{T}_{10}(-n)$:

$$X\left(e^{j\omega}\right) = \mathcal{F}[x(n)] = \mathcal{F}[\mathcal{T}_{10}(n) * \mathcal{T}_{10}(-n)] = \mathcal{F}[\mathcal{T}_{10}(n)] * \mathcal{F}[\mathcal{T}_{10}(n)]^*$$
$$= |\mathcal{F}[\mathcal{T}_{10}(n)]|^2$$

MATLAB script:

```
% P0310c: DTFT of T_10(n) Conv T_10(-n)
%           T_10(n) = [1-(abs(M-1-2*n)/(M+1)) ]*R_M(n), M = 10
clc; close all; set(0,'defaultfigurepaperposition',[0,0,7,4]);
%
% Triangular Window T_10(n) & its DTFT
M = 10; n = 0:M; Tn = (1-(abs(M-1-2*n)/(M+1)));
w = linspace(-pi,pi,501); Tw = dtft(Tn,n,w); magTw = abs(Tw);
magTw = magTw/max(magTw); phaTw = angle(Tw)*180/pi;
% x(n) & its DTFT
[x1,n1] = sigfold(Tn,n); [x,n] = conv_m(Tn,n,x1,n1);
X = dtft(x,n,w); magX = abs(X);
magX = magX/max(magX); phaX = angle(X)*180/pi;
% DTFT of x(n) from the Property
Y = Tw.*fliplr(Tw); magY = abs(Y)/max(abs(Y)); phaY = angle(Y)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0310c');
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,2); plot(w/pi,phaX,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.5:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,3); plot(w/pi,magY,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,4); plot(w/pi,phaY,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.4:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0310c;
```

The property verification using plots of $X\left(e^{j\omega}\right)$ is shown in Figure 3.21.
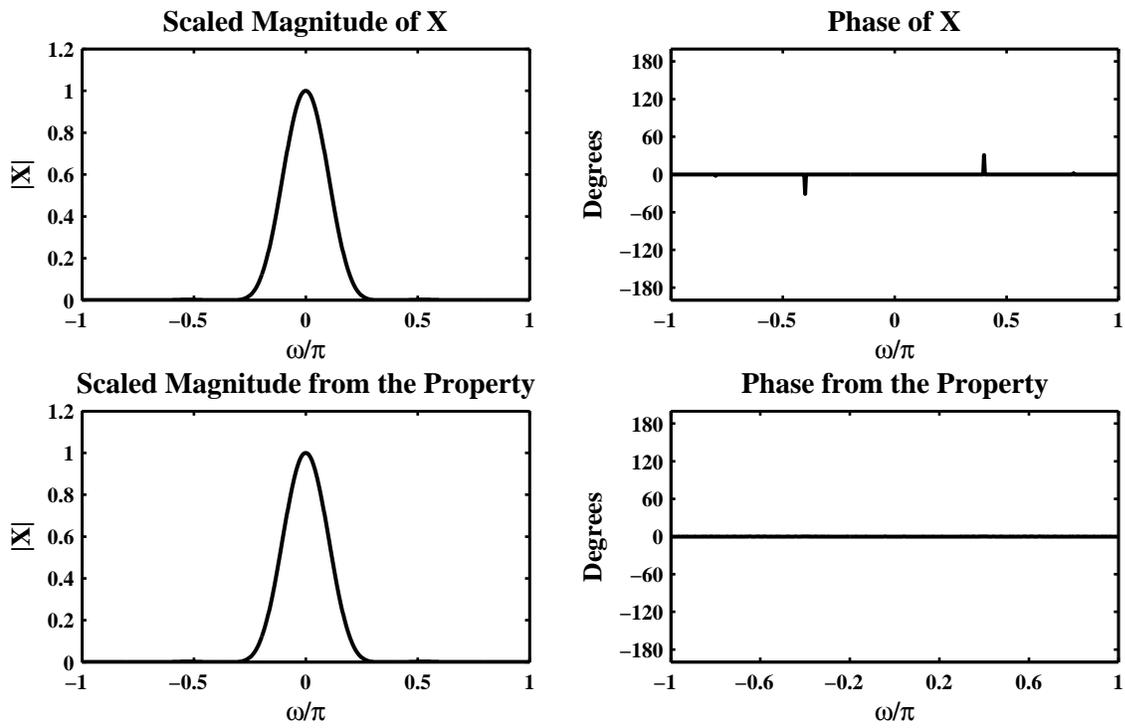
Figure 3.21: Problem P3.10c plots

4. $x(n) = \mathcal{T}_{10}(n)e^{j\pi n}$:

$$X\left(e^{j\omega}\right) = \mathcal{F}[x(n)] = \mathcal{F}[\mathcal{T}_{10}(n)e^{j\pi n}]$$
$$= \mathcal{F}[\mathcal{T}_{10}(n)]|_{\omega \to (\omega - \pi)}$$

MATLAB script:

```
% P0310d: DTFT of T_10(n)*exp(j*pi*n)
%          T_10(n) = [1-(abs(M-1-2*n)/(M+1)) ]*R_M(n), M = 10
clc; close all; set(0,'defaultfigurepaperposition',[0,0,7,4]);
%
% Triangular Window T_10(n) & its DTFT
M = 10; n = 0:M; Tn = (1-(abs(M-1-2*n)/(M+1)));
w = linspace(-pi,pi,501); Tw = dtft(Tn,n,w); magTw = abs(Tw);
magTw = magTw/max(magTw); phaTw = angle(Tw)*180/pi;
% x(n) & its DTFT
x = Tn.*exp(j*pi*n); X = dtft(x,n,w); magX = abs(X);
magX = magX/max(magX); phaX = angle(X)*180/pi;
% DTFT of x(n) from the Property
Y = [Tw(251:501),Tw(1:250)]; magY = abs(Y)/max(abs(Y)); phaY = angle(Y)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0310d');
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,2); plot(w/pi,phaX,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.5:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,3); plot(w/pi,magY,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,4); plot(w/pi,phaY,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.4:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0310d;
```

The property verification using plots of $X\left(e^{j\omega}\right)$ is shown in Figure 3.22.
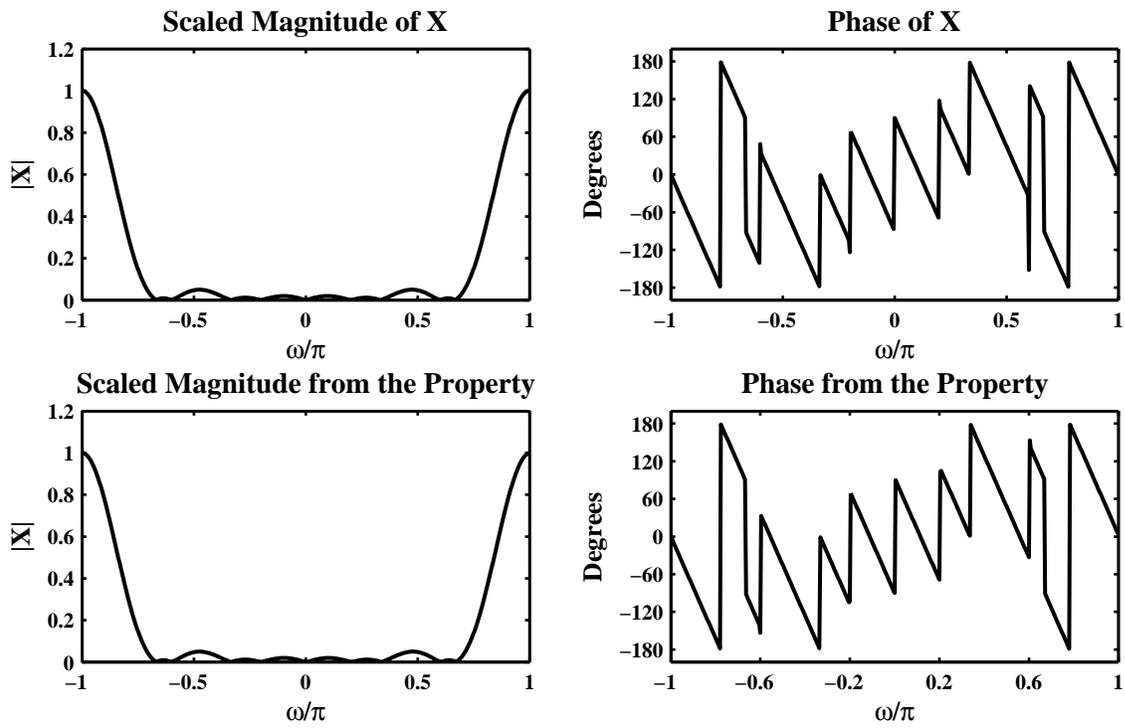
Figure 3.22: Problem P3.10d plots

5. $x(n) = \cos(0.1\pi n)\mathcal{T}_{10}(n)$:

$$X\left(e^{j\omega}\right) = \mathcal{F}[x(n)] = \mathcal{F}[\cos(0.1\pi n)\mathcal{T}_{10}(n)] = \mathcal{F}\left[\frac{1}{2}\left\{e^{j0.1\pi n} + e^{-j0.1\pi n}\right\}\mathcal{T}_{10}(n)\right]$$

$$= \frac{1}{2}\left.\mathcal{F}[\mathcal{T}_{10}(n)]\right|_{\omega\to(\omega-0.1\pi)} + \frac{1}{2}\left.\mathcal{F}[\mathcal{T}_{10}(n)]\right|_{\omega\to(\omega+0.1\pi)}$$

MATLAB script:

```
% P0310e: DTFT of x(n) = T_10(-n)
clc; close all; set(0,'defaultfigurepaperposition',[0,0,7,4]);
%
% Triangular Window T_10(n) & its DTFT
M = 10; n = 0:M; Tn = (1-(abs(M-1-2*n)/(M+1)));
w = linspace(-pi,pi,501); Tw = dtft(Tn,n,w); magTw = abs(Tw);
magTw = magTw/max(magTw); phaTw = angle(Tw)*180/pi;
% x(n) & its DTFT
x = cos(0.1*pi*n).*Tn; X = dtft(x,n,w); magX = abs(X);
magX = magX/max(magX); phaX = angle(X)*180/pi;
% DTFT of x(n) from the Property
Y = 0.5*([Tw(477:501),Tw(1:476)]+[Tw(26:501),Tw(1:25)]);
magY = abs(Y)/max(abs(Y)); phaY = angle(Y)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0310e');
subplot(2,2,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,2); plot(w/pi,phaX,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.5:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase of X'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,3); plot(w/pi,magY,'LineWidth',1.5); axis([-1 1 0 1.2]);
wtick = sort([-1:0.5:1]); magtick = [0:0.2:1.2];
xlabel('\omega/\pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title(['Scaled Magnitude from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,2,4); plot(w/pi,phaY,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = sort([-1:0.4:1]); magtick = [-180:60:180];
xlabel('\omega/\pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase from the Property'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0310e;
```

The property verification using plots of $X\left(e^{j\omega}\right)$ is shown in Figure 3.23.
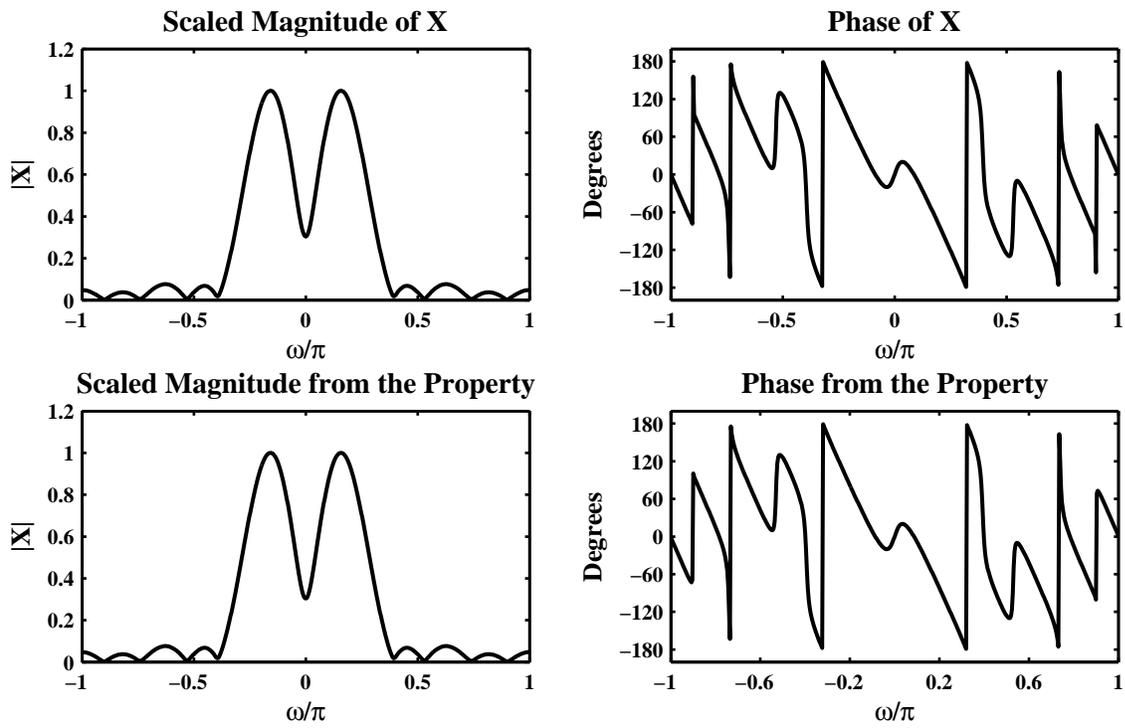
Figure 3.23: Problem P3.10e plots

P3.11 Determination and plots of the frequency response function $H\left(e^{j\omega}\right)$.

1. $h(n) = (0.9)^{|n|}$

$$H\left(e^{j\omega}\right) = \sum_{h=-\infty}^{\infty} (n)e^{-jn\omega} = \sum_{0=-\infty}^{-1} .9^{-n}e^{-jn\omega} + \sum_{0=0}^{\infty} .9^{n}e^{-jn\omega} = \sum_{0=0}^{\infty} .9^{n}e^{jn\omega} - 1 + \sum_{0=0}^{\infty} .9^{n}e^{-jn\omega}$$

$$= \frac{1}{1-0.9e^{j\omega}} + \frac{1}{1-0.9e^{-j\omega}} - 1$$

$$= \frac{0.19}{1.81 - 1.8\cos(\omega)}$$

MATLAB script:

```
% P0311a: h(n) = (0.9)^|n|; H(w) = 0.19/(1.81-1.8*cos(w));
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
w = [-300:300]*pi/300; H = 0.19*ones(size(w))./(1.81-1.8*cos(w));
magH = abs(H); phaH = angle(H)*180/pi;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0311a');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 20]);
wtick = [-1:0.2:1]; magtick = [0:5:20];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title('Magnitude response of h(n) = (0.9)^{|n|}','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -180 180]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase response of h(n) = (0.9)^{|n|}','FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0311a;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.24.
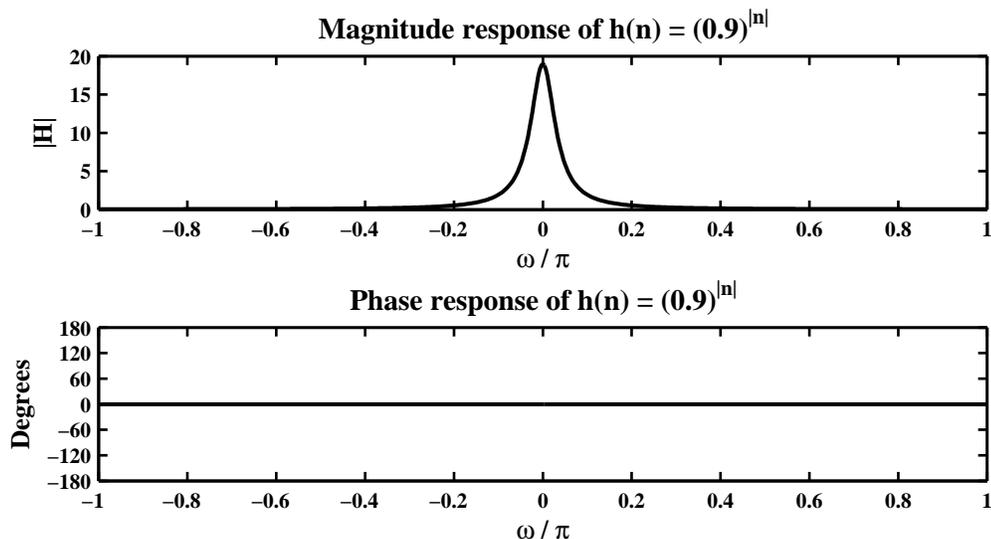
Figure 3.24: Problem P3.11a plots

2. $h(n) = \text{sinc}(0.2n)[u(n+20) - u(n-20)]$, where $\text{sinc}\,0 = 1$.

   MATLAB script:

```
% P0311b: h(n) = sinc(0.2*n)*[u(n+20)-u(n-20)]
clc; close all;set(0,'defaultfigurepaperposition',[0,0,6,3]);
[h1,n1] = stepseq(-20,-20,20); [h2,n2] = stepseq(20,-20,20);
[h3,n3] = sigadd(h1,n1,-h2,n2); n = n3; h = sinc(0.2*n).*h3;
w = [-300:300]*pi/300; H = dtft(h,n,w); magH = abs(H); phaH = angle(H)*180/pi;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0311b');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 6]);
wtick = [-1:0.2:1]; magtick = [0:1:6];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title(['Magnitude response of h(n) = sinc(0.2 \times n)\times' ...
    '[u(n+20)-u(n-20)]'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase response of h(n) = sinc(0.2 n)' ...
    '\times[u(n+20)-u(n-20)]'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0311b;
```

   The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.25.
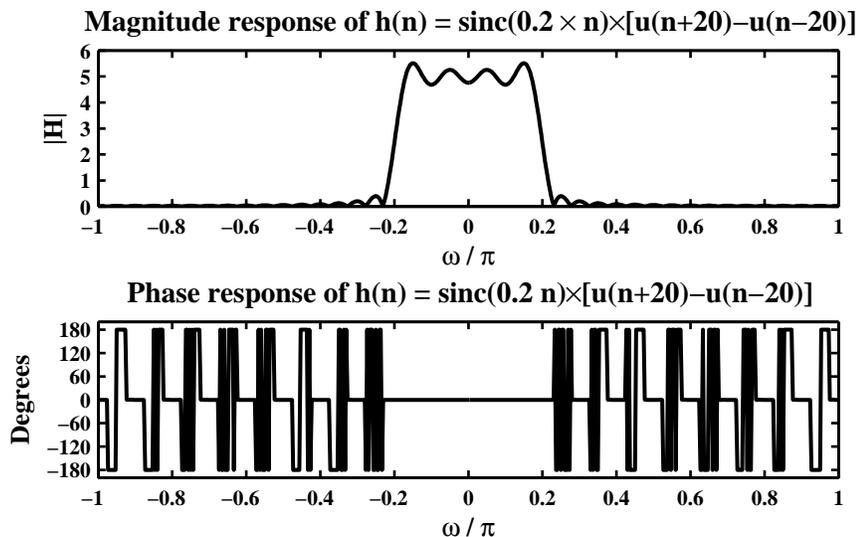

Figure 3.25: Problem P3.11b plots

3. $h(n) = \text{sinc}\,(0.2n)\,[u(n) - u\,(n - 40)]$

   MATLAB script:

```
% P0311c: h(n) = sinc(0.2*n)*[u(n)-u(n-40)]
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
[h1,n1] = stepseq(0,0,40); [h2,n2] = stepseq(40,0,40);
[h3,n3] = sigadd(h1,n1,-h2,n2); n = n3; h = sinc(0.2*n).*h3;
w = [-300:300]*pi/300; H = dtft(h,n,w); magH = abs(H); phaH = angle(H)*180/pi;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0311c');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 5]);
wtick = [-1:0.2:1]; magtick = [0:1:5];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title(['Magnitude response of h(n) = sinc(0.2n)\times' ...
    '[u(n)-u(n-40)]'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -180 180]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase response of h(n) = sinc(0.2)\times' ...
    '[u(n)-u(n-40)]'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0311c;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.26.
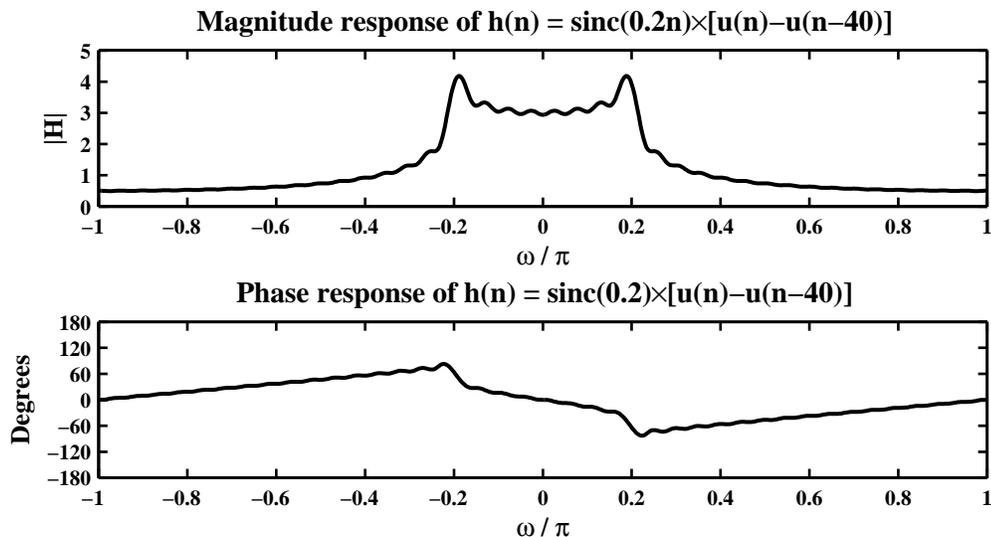
Figure 3.26: Problem P3.11c plots

4. $h(n) = \left[(0.5)^n + (0.4)^n\right] u(n)$

$$H\left(e^{j\omega}\right) = \sum_{h=-\infty}^{\infty} (n)e^{-jn\omega} = \sum_{0=0}^{\infty} .5^n e^{-jn\omega} + \sum_{0=0}^{\infty} .4^n e^{-jn\omega} = \frac{1}{1 - 0.5e^{-j\omega}} + \frac{1}{1 - 0.4e^{-j\omega}}$$

$$= \frac{2 - 0.9e^{-j\omega}}{1 - 0.9e^{-j\omega} + 0.2e^{-j2\omega}}$$

MATLAB script:

```
% P0311d: h(n) = ((0.5)^n+(0.4)^n) u(n);
%          H(w) = (2-0.9*exp(-j*w))./(1-0.9*exp(-j*w)+0.2*exp(-j*2*w))
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
w = [-300:300]*pi/300; H = (2-0.9*exp(-j*w))./(1-0.9*exp(-j*w)+0.2*exp(-j*2*w));
magH = abs(H); phaH = angle(H)*180/pi;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0311d');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 1 4]);
wtick = [-1:0.2:1]; magtick = [0:0.5:4];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title('Magnitude response:h(n) = [(0.5)^n+(0.4)^n] u(n)','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -180 180]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase response:h(n) = [(0.5)^n+(0.4)^n] u(n)','FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0311d;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.27.
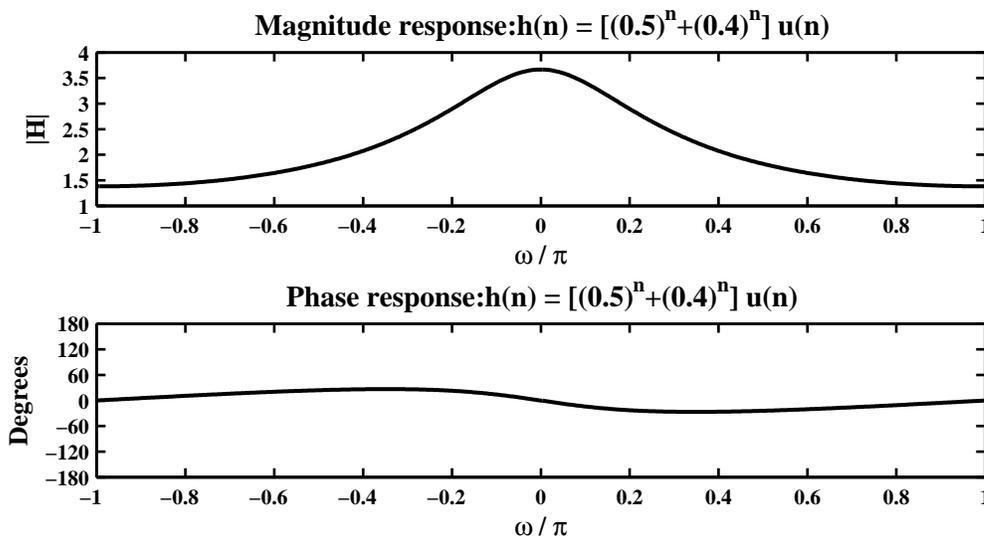

Figure 3.27: Problem P3.11d plots

5. $h(n) = (0.5)^{|n|} \cos(0.1\pi n) = \frac{1}{2} 0.5^{|n|} e^{j0.1\pi n} + \frac{1}{2} 0.5^{|n|} e^{-j0.1\pi n}$

$$H\left(e^{j\omega}\right) = \sum_{h=-\infty}^{\infty} (n) e^{-jn\omega} = \frac{1}{2} \left[\sum_{0=-\infty}^{\infty} .5^{|n|} e^{-j(\omega-0.1\pi)n} + \sum_{0=-\infty}^{\infty} .5^{|n|} e^{-j(\omega+0.1\pi)n} \right]$$

$$= \frac{0.5 \times 0.75}{1.25 - \cos(\omega - 0.1\pi)} + \frac{0.5 \times 0.75}{1.25 - \cos(\omega + 0.1\pi)}$$

MATLAB script:

```
% P0311e: h(n) = (0.5)^|n|*cos(0.1*pi*n);
%         H(w) = 0.5*0.75*ones(size(w)) ./(1.25-cos(w-(0.1*pi)))+
%               0.5*0.75*ones(size(w)) ./(1.25-cos(w+(0.1*pi)))
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
w = [-300:300]*pi/300; H = 0.5*0.75*ones(size(w))./(1.25-cos(w-(0.1*pi)))+...
        0.5*0.75*ones(size(w))./(1.25-cos(w+(0.1*pi)));
magH = abs(H); phaH = angle(H)*180/pi;
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0311e');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 3]);
wtick = [-1:0.2:1]; magtick = [0:0.5:3];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title(['Magnitude response'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -180 180]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase response'],'FontSize',TFS);
set(gca,'XTick',wtick,'YTick',magtick); print -deps2 ../EPSFILES/P0311e;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.28.
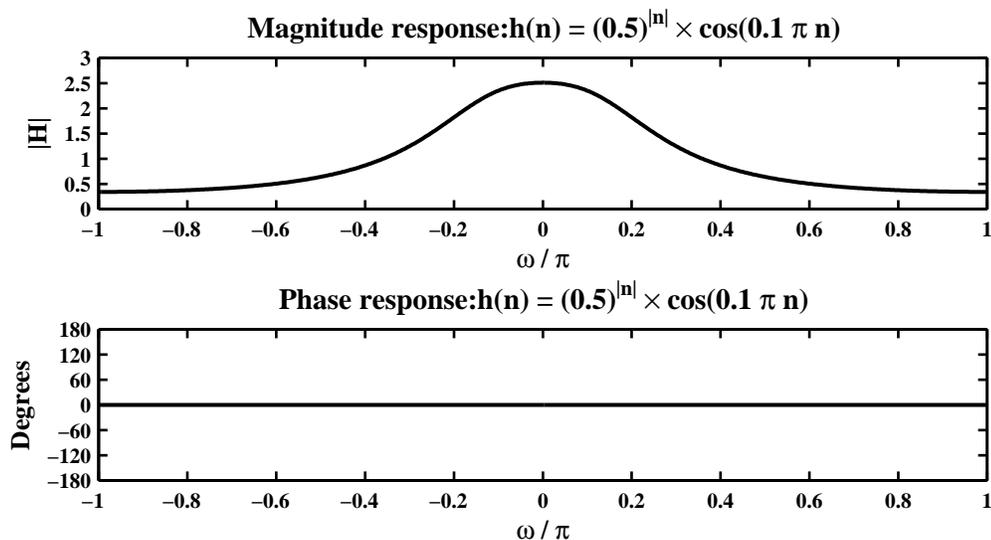

Figure 3.28: Problem P3.11e plots

P3.12 Let $x(n) = A\cos(\omega_0 n + \theta_0)$ be an input sequence to an LTI system described by the impulse response $h(n)$. Then the output sequence $y(n)$ is given by

$$y(n) = h(n) * x(n) = A \sum_{h=-\infty}^{\infty} (k) \cos[\omega_0(n-k) + \theta_0]$$

$$= \frac{A}{2} \sum_{h=-\infty}^{\infty} (k) \exp\left[j\omega_0(n-k) + \theta_0\right] + \frac{A}{2} \sum_{h=-\infty}^{\infty} (k) \exp\left[-j\omega_0(n-k) - \theta_0\right]$$

$$= \frac{A}{2} e^{j\theta_0} \left[\sum_{h=-\infty}^{\infty} (k) e^{-j\omega_0 k}\right] e^{j\omega_0 n} + \frac{A}{2} e^{-j\theta_0} \left[\sum_{h=-\infty}^{\infty} (k) e^{j\omega_0 k}\right] e^{-j\omega_0 n}$$

$$= \frac{A}{2} e^{j\theta_0} H\left(e^{j\omega_0}\right) e^{j\omega_0 n} + \frac{A}{2} e^{-j\theta_0} H^*\left(e^{j\omega_0}\right) e^{-j\omega_0 n}$$

$$= \frac{A}{2} e^{j\theta_0} |H\left(e^{j\omega_0}\right)| e^{j\angle H\left(e^{j\omega_0}\right)} e^{j\omega_0 n} + \frac{A}{2} e^{-j\theta_0} |H\left(e^{j\omega_0}\right)| e^{-j\angle H\left(e^{j\omega_0}\right)} e^{-j\omega_0 n}$$

$$= \frac{A}{2} |H\left(e^{j\omega_0}\right)| \left[\exp\left\{j\left[\omega_0 n + j\theta_0 + j\angle H\left(e^{j\omega_0}\right)\right]\right\} + \exp\left\{-j\left[\omega_0 n + j\theta_0 + j\angle H\left(e^{j\omega_0}\right)\right]\right\}\right]$$

$$= A|H\left(e^{j\omega_0}\right)| \cos\left[\omega_0 n + j\theta_0 + j\angle H\left(e^{j\omega_0}\right)\right]$$

P3.13 Sinusoidal steady-state responses

1. The input to the system $h(n) = (0.9)^{|n|}$ is $x(n) = 3\cos(0.5\pi n + 60°) + 2\sin(0.3\pi n)$. The steady-state response $y(n)$ is computed using MATLAB.

```
% P0313a: x(n) = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n); i.e.
%         x(n) = 3*cos(0.5*pi*n+pi/3)+2*cos(0.3*pi*n-pi/2)
%         h(n) = (0.9)^|n|
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
%
n = [-20:20]; x = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n);
w1 = 0.5*pi; H1 = 0.19*w1/(1.81-1.8*cos(w1));
w2 = 0.3*pi; H2 = 0.19*w2/(1.81-1.8*cos(w2));
magH1 = abs(H1); phaH1 = angle(H1); magH2 = abs(H2); phaH2 = angle(H2);
y = 3*magH1*cos(w1*n+pi/3+phaH1)+...
    2*magH2*cos(w2*n-pi/2+phaH2);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0313a');
subplot(2,1,1); Hs = stem(n,x,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS); axis([-22 22 -5 5]);
title(['Input sequence x(n): 3*cos(0.5{\pi}n + \pi/3) + 2sin(0.3{\pi}n)'],...
    'FontSize',TFS);
subplot(2,1,2); Hs = stem(n,y,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS); axis([-22 22 -5 5]);
title('Output sequence y(n) for h(n) = (0.9)^{|n|}','FontSize',TFS);
print -deps2 ../EPSFILES/P0313a;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.29.
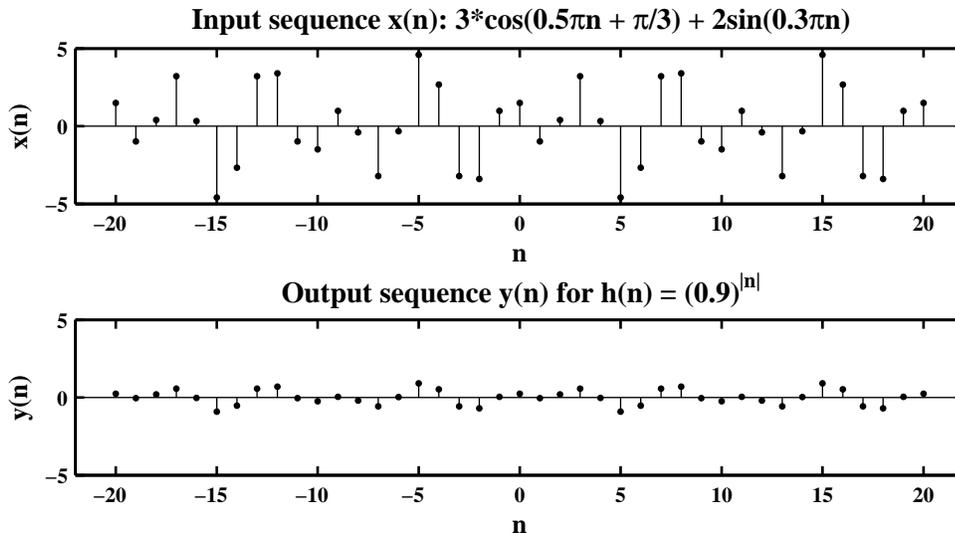

Figure 3.29: Problem P3.13a plots

2. The input to the system $h(n) = \text{sinc}(0.2n)[u(n+20) - u(n-20)]$, where $\text{sinc}\, 0 = 1$. The steady-state response $y(n)$ is computed using MATLAB.

```
% P0313b: x(n) = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n); i.e.
%          x(n) = 3*cos(0.5*pi*n+pi/3)+2*cos(0.3*pi*n-pi/2)
%          h(n) = sinc(0.2*n)*[u(n+20)-u(n-20)]
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
%
n = [-20:20]; x = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n);
[h1,n1] = stepseq(-20,-20,20); [h2,n2] = stepseq(20,-20,20);
[h3,n3] = sigadd(h1,n1,-h2,n2); n = n3; h = sinc(0.2*n).*h3;
w1 = 0.5*pi; H1 = dtft(h,n,w1); w2 = 0.3*pi; H2 = dtft(h,n,w2);
magH1 = abs(H1); phaH1 = angle(H1); magH2 = abs(H2); phaH2 = angle(H2);
y = 3*magH1*cos(w1*n+pi/3+phaH1)+2*magH2*cos(w2*n-pi/2+phaH2);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0313b');
subplot(2,1,1); Hs = stem(n,x,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS); axis([-22 22 -5 5]);
title(['Input sequence x(n): 3*cos(0.5{\pi}n + \pi/3) + 2sin(0.3{\pi}n)'],...
    'FontSize',TFS);
subplot(2,1,2); Hs = stem(n,y,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS); axis([-22 22 -5 5]);
title('Output sequence y(n) for h(n) = sinc(0.2 n)[u(n+20) - u(n-20)]',...
    'FontSize',TFS); print -deps2 ../EPSFILES/P0313b;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.30.
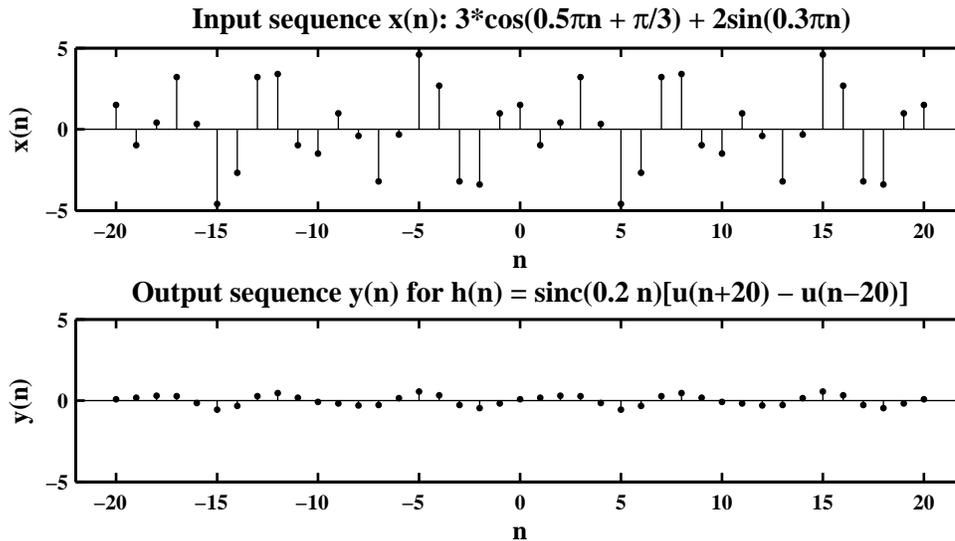

Figure 3.30: Problem P3.13b plots

3. The input to the system $h(n) = \text{sinc}\,(0.2n)\,[u(n) - u\,(n - 40)]$. The steady-state response $y(n)$ is computed using MATLAB.

```
% P0313c: x(n) = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n); i.e.
%          x(n) = 3*cos(0.5*pi*n+pi/3)+2*cos(0.3*pi*n-pi/2)
%          h(n) = sinc(0.2*n)*[u(n)-u(n-40)]
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
%
n = [-20:20]; x = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n);
[h1,n1] = stepseq(0,0,40); [h2,n2] = stepseq(40,0,40);
[h3,n3] = sigadd(h1,n1,-h2,n2); h = sinc(0.2*n3).*h3;
w1 = 0.5*pi; w2 = 0.3*pi; H1 = dtft(h,n3,w1); H2 = dtft(h,n3,w2);
magH1 = abs(H1); phaH1 = angle(H1); magH2 = abs(H2); phaH2 = angle(H2);
y = 3*magH1*cos(w1*n+pi/3+phaH1)+2*magH2*cos(w2*n-pi/2+phaH2);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0313c');
subplot(2,1,1); Hs = stem(n,x,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS); axis([-22 22 -5 5]);
title(['Input sequence x(n): 3*cos(0.5{\pi}n + \pi/3) + 2sin(0.3{\pi}n)'],...
    'FontSize',TFS);
subplot(2,1,2); Hs = stem(n,y,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS); axis([-22 22 -5 5]);
title('Output sequence y(n) for h(n) = sinc(0.2 n)[u(n) - u(n-40)]',...
    'FontSize',TFS); print -deps2 ../EPSFILES/P0313c;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.31.
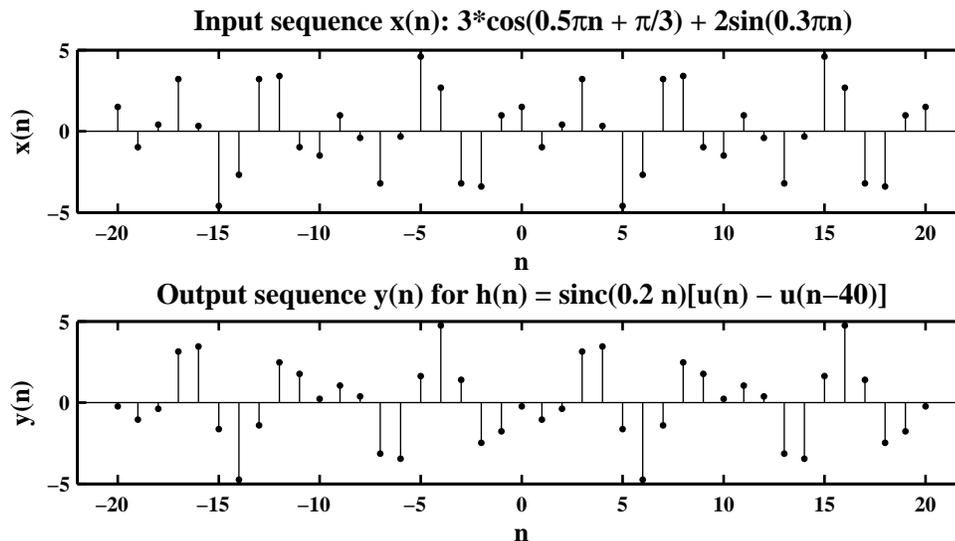

Figure 3.31: Problem P3.13c plots

4. The input to the system $h(n) = \left[(0.5)^n + (0.4)^n\right] u(n)$. The steady-state response $y(n)$ is computed using MATLAB.

```
% P0313d: x(n) = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n); i.e.
%          x(n) = 3*cos(0.5*pi*n+pi/3)+2*cos(0.3*pi*n-pi/2)
%          h(n) = ((0.5)^(n)+(0.4)^(n)).*u(n)
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
%
n = [-20:20]; x = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n);
w1 = 0.5*pi; H1 = (2-0.9*exp(-j*w1))./(1-0.9*exp(-j*w1)+0.2*exp(-j*2*w1));
w2 = 0.3*pi; H2 = (2-0.9*exp(-j*w2))./(1-0.9*exp(-j*w2)+0.2*exp(-j*2*w2));
magH1 = abs(H1); phaH1 = angle(H1); magH2 = abs(H2); phaH2 = angle(H2);
y = 3*magH1*cos(w1*n+pi/3+phaH1)+2*magH2*cos(w2*n-pi/2+phaH2);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0313d');
subplot(2,1,1); Hs = stem(n,x,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS); axis([-22 22 -10 10]);
title(['Input sequence x(n): 3*cos(0.5{\pi}n + \pi/3) + 2sin(0.3{\pi}n)'],...
    'FontSize',TFS);
subplot(2,1,2); Hs = stem(n,y,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS); axis([-22 22 -10 10]);
title('Output sequence y(n) for h(n) = [(0.5)^n+(0.4)^n] u(n)]',...
    'FontSize',TFS); print -deps2 ../EPSFILES/P0313d;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.32.
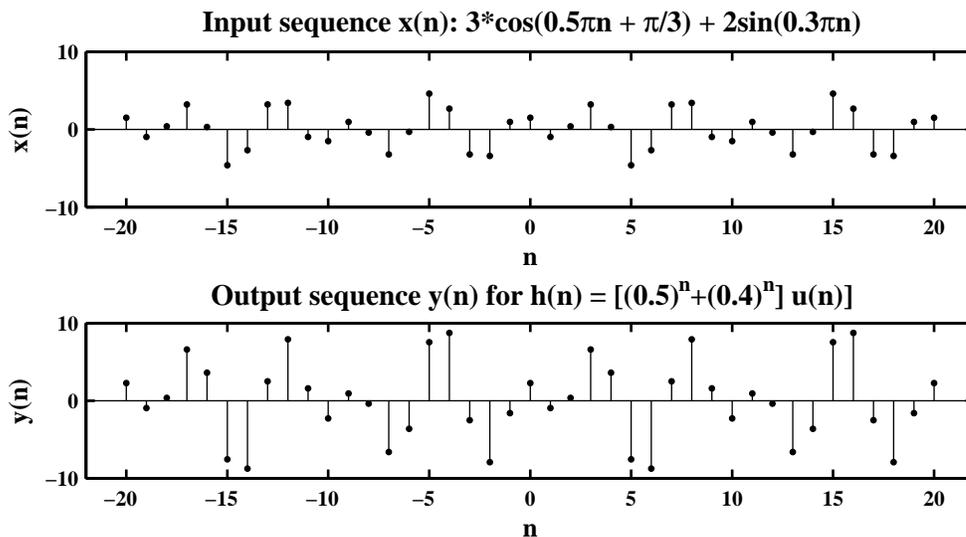

Figure 3.32: Problem P3.13d plots

5. The input to the system $h(n) = (0.5)^{|n|} \cos(0.1\pi n)$. The steady-state response $y(n)$ is computed using MATLAB.

```
% P0313e: x(n) = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n); i.e.
%          x(n) = 3*cos(0.5*pi*n+pi/3)+2*cos(0.3*pi*n-pi/2)
%          h(n) = (0.5)^|n|*cos(0.1*pi*n);
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
%
n = [-20:20]; x = 3*cos(0.5*pi*n+pi/3)+2*sin(0.3*pi*n);
w1 = 0.5*pi; H1 = 0.5*0.75*w1/(1.25-cos(w1-(0.1*pi)))+...
    0.5*0.75*w1/(1.25-cos(w1+(0.1*pi)));
w2 = 0.3*pi; H2 = 0.5*0.75*w2/(1.25-cos(w2-(0.1*pi)))+...
    0.5*0.75*w2/(1.25-cos(w2+(0.1*pi)));
magH1 = abs(H1); phaH1 = angle(H1); magH2 = abs(H2); phaH2 = angle(H2);
y = 3*magH1*cos(w1*n+pi/3+phaH1)+2*magH2*cos(w2*n-pi/2+phaH2);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0313e');
subplot(2,1,1); Hs = stem(n,x,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS); axis([-22 22 -6 6]);
title(['Input sequence x(n): 3*cos(0.5{\pi}n + \pi/3) + 2sin(0.3{\pi}n)'],...
    'FontSize',TFS);
subplot(2,1,2); Hs = stem(n,y,'filled'); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS); axis([-22 22 -6 6]);
title('Output sequence y(n) for h(n) = (0.5)^{|n|}cos(0.1{\pi}n)] u(n)]',...
    'FontSize',TFS); print -deps2 ../EPSFILES/P0313e;
```

The magnitude and phase response plots of $H\left(e^{j\omega}\right)$ are shown in Figure 3.33.
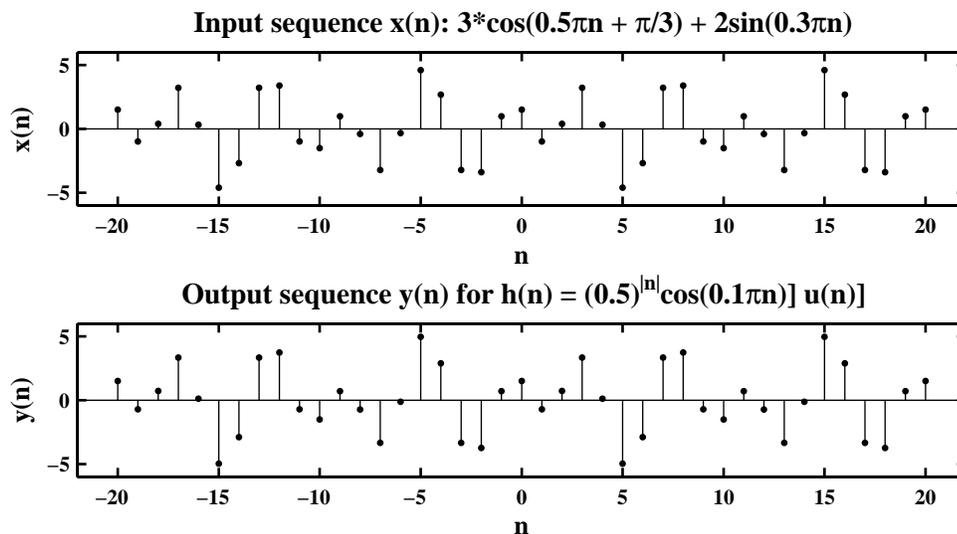

Figure 3.33: Problem P3.13e plots

P3.14 An ideal lowpass filter is described in the frequency-domain by

$$H_d(e^{j\omega}) = \begin{cases} 1 \cdot e^{-j\alpha\omega}, & |\omega| \leq \omega_c \\ 0, & \omega_c < |\omega| \leq \pi \end{cases}$$

where ω_c is called the cutoff frequency and α is called the phase delay.

1. The ideal impulse response $h_d(n)$ using the IDTFT relation (3.2):

$$h_d(n) = \mathcal{F}^{-1}\left[H_d(e^{j\omega})\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_d(e^{j\omega})e^{jn\omega}d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\alpha\omega}e^{jn\omega}d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j(n-\alpha)\omega}d\omega$$

$$= \frac{1}{2\pi}\left.\frac{e^{j(n-\alpha)\omega}}{j(n-\alpha)}\right|_{-\omega_c}^{\omega_c} = \frac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}$$

2. Plot of the truncated impulse response:

$$h(n) = \begin{cases} h_d(n), & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases} = \begin{cases} \dfrac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}, & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases}$$

for $N = 41$, $\alpha = 20$, and $\omega_c = 0.5\pi$. MATLAB script: MATLAB script:

```
% P0314b: Truncated Ideal Lowpass Filter; h(n) = h_d(n) ,0 <= n <= N-1
%                                              = 0       ,otherwise
clc; close all; set(0,'defaultfigurepaperposition',[0,0,5,2]);
%
n = [0:40]; alpha = 20; wc = 0.5*pi;
fc = wc/(2*pi); h = 2*fc*sinc(2*fc*(n-alpha));
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0314b');
Hs = stem(n,h,'filled'); set(Hs,'markersize',2); axis([-2 42 -0.2 0.6]);
xlabel('n','FontSize',LFS); ylabel('h(n)','FontSize',LFS);
title('Truncated Impulse Response h(n)','FontSize',TFS);
set(gca,'YTick',[-0.2:0.1:0.6]); print -deps2 ../EPSFILES/P0314b;
```

The truncated impulse response plot of $h_d(n)$ is shown in Figure 3.34.
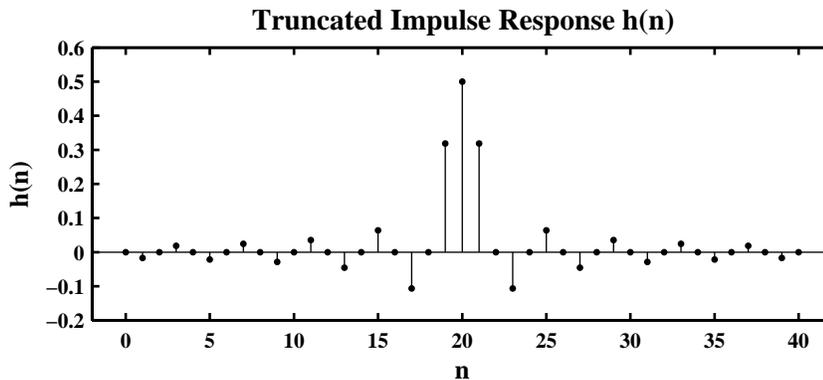
Figure 3.34: Problem P3.14b plot

3. Plot of the frequency response function $H\left(e^{j\omega}\right)$ and comparison with the ideal lowpass filter response $H_d\left(e^{j\omega}\right)$: MATLAB script:

```
% P0314c: Freq Resp of truncated and ideal impulse responses for lowpass filter
clc; close all;  set(0,'defaultfigurepaperposition',[0,0,6,3]);
%
K = 500; w = [-K:K]*pi/K; H = dtft(h,n,w); magH = abs(H); phaH = angle(H);
H_d = zeros(1,length(w)); H_d(K/2+1:3*K/2+1) = exp(-j*alpha*w(K/2+1:3*K/2+1));
magH_d = abs(H_d); phaH_d = angle(H_d); wtick = sort([-1:0.4:1 0]);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0314c');
subplot(2,2,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 1.2]);
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title('Magnitude of H(e^{j\omega})','FontSize',TFS); set(gca,'XTick',wtick);
subplot(2,2,2); plot(w/pi,phaH*180/pi,'LineWidth',1.5);
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase of H(e^{j\omega})','FontSize',TFS);
set(gca,'XTick',wtick); magtick = [-180:60:180];
set(gca,'YTick',magtick); set(gca,'XTick',wtick);
subplot(2,2,3); plot(w/pi,magH_d,'LineWidth',1.5); axis([-1 1 0 1.2]);
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H_d|','FontSize',LFS);
title('Magnitude of H_d(e^{j\omega})','FontSize',TFS);
set(gca,'XTick',wtick); ytick = [0:0.2:1.2]; set(gca,'YTick',ytick);
subplot(2,2,4); plot(w/pi,phaH_d*180/pi,'LineWidth',1.5);
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase of H_d(e^{j\omega})','FontSize',TFS);
set(gca,'XTick',wtick); magtick = [-180:60:180]; set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0314c;
```

The frequency responses are shown in Figure 3.35 from which we observe that the truncated response is a smeared or blurred version of the ideal response.
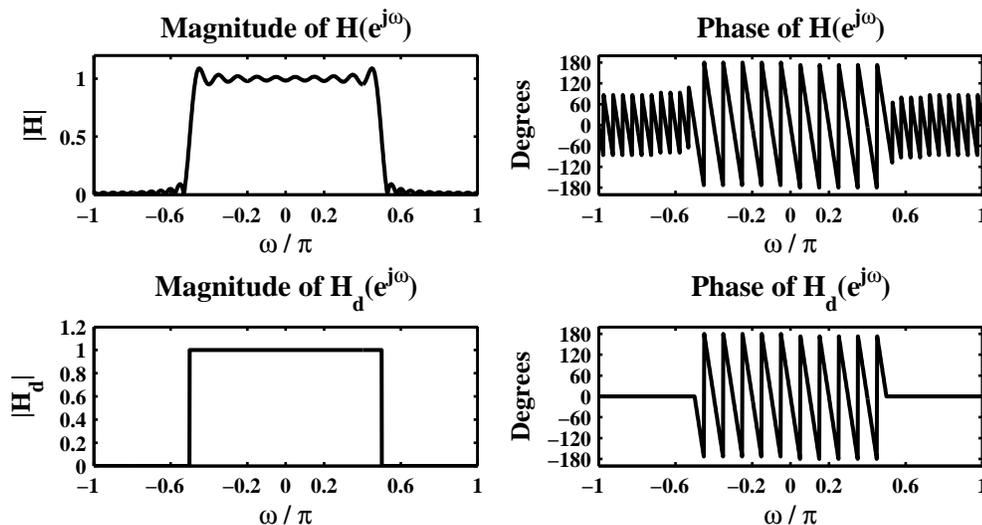

Figure 3.35: Problem P3.14c plots

P3.15 An ideal highpass filter is described in the frequency-domain by

$$H_d(e^{j\omega}) = \begin{cases} 1 \cdot e^{-j\alpha\omega}, & \omega_c < |\omega| \leq \pi \\ 0, & |\omega| \leq \omega_c \end{cases}$$

where ω_c is called the cutoff frequency and α is called the phase delay.

1. The ideal impulse response $h_d(n)$ using the IDTFT relation (3.2):

$$h_d(n) = \mathcal{F}^{-1}\left[H_d(e^{j\omega})\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_d(e^{j\omega})e^{jn\omega}d\omega = \frac{1}{2\pi}\int_{-\pi}^{-\omega_c} e^{-j\alpha\omega}e^{jn\omega}d\omega + \frac{1}{2\pi}\int_{\omega_c}^{\pi} e^{-j\alpha\omega}e^{jn\omega}d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(n-\alpha)\omega}d\omega - \frac{1}{2\pi}\int_{-\omega_c}^{\omega} e^{j(n-\alpha)\omega}d\omega = \frac{\sin[\pi(n-\alpha)]}{\pi(n-\alpha)} - \frac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}$$

2. Plot of the truncated impulse response:

$$h(n) = \begin{cases} h_d(n), & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases} = \begin{cases} \dfrac{\sin[\pi(n-\alpha)]}{\pi(n-\alpha)} - \dfrac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}, & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases}$$

for $N = 31$, $\alpha = 15$, and $\omega_c = 0.5\pi$. MATLAB script: MATLAB script:

```
% P0315b: Ideal Highpass Filter; h(n) = h_d(n) ,0 <= n <= N-1
%                                       = 0        ,otherwise
clc; close all; set(0,'defaultfigurepaperposition',[0,0,5,2]);
%
n = [0:40]; alpha = 20; wc = 0.5*pi; fc = wc/(2*pi);
h = sinc(n-alpha)-2*fc*sinc(2*fc*(n-alpha));
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0315b');
Hs = stem(n,h,'filled'); set(Hs,'markersize',2); axis([-2 42 -0.4 0.6]);
xlabel('n','FontSize',LFS); ylabel('h(n)','FontSize',LFS);
title('Truncated Impulse Response h(n)','FontSize',TFS);
set(gca,'YTick',[-0.4:0.1:0.6]); print -deps2 ../EPSFILES/P0315b;
```

The truncated impulse response plot of $h_d(n)$ is shown in Figure 3.36.
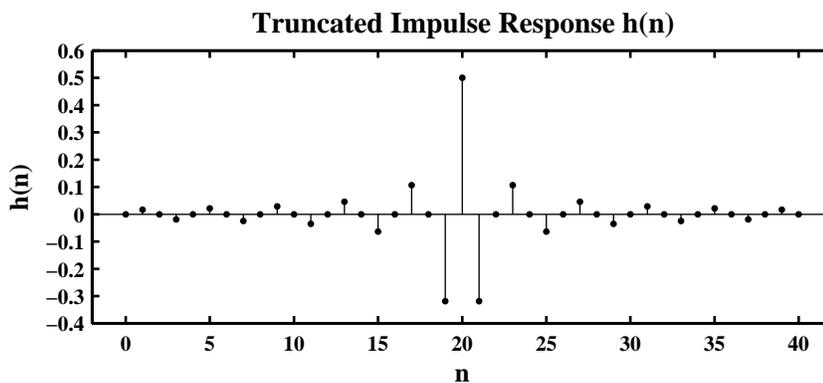

Figure 3.36: Problem P3.15b plot

3. Plot of the frequency response function $H\left(e^{j\omega}\right)$ and comparison with the ideal lowpass filter response $H_d\left(e^{j\omega}\right)$: MATLAB script:

```
% P0315c: Freq Resp of truncated and ideal impulse responses for highpass filter
clc; close all;  set(0,'defaultfigurepaperposition',[0,0,6,3]);
K = 500; w = [-K:K]*pi/K; H = dtft(h,n,w); magH = abs(H); phaH = angle(H);
H_d = zeros(1,length(w)); H_d(1:K/2+1) = exp(-j*alpha*w(1:K/2+1));
H_d(3*K/2+1:end) = exp(-j*alpha*w(3*K/2+1:end));
magH_d = abs(H_d); phaH_d = angle(H_d); wtick = sort([-1:0.4:1 0]);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0315c');
subplot(2,2,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 1.2]);
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title('Magnitude of H(e^{j\omega})','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'XTick',wtick);
subplot(2,2,2); plot(w/pi,phaH*180/pi,'LineWidth',1.5);
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase of H(e^{j\omega})','FontSize',TFS);
set(gca,'XTick',wtick); magtick = [-180:60:180]; set(gca,'YTick',magtick);
subplot(2,2,3); plot(w/pi,magH_d,'LineWidth',1.5); axis([ -1 1 0 1.2]);
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H_d|','FontSize',LFS);
title('Magnitude of H_d(e^{j\omega})','FontSize',TFS);
set(gca,'YTick',ytick); ytick = [0:0.2:1.2];set(gca,'XTick',wtick);
subplot(2,2,4); plot(w/pi,phaH_d*180/pi,'LineWidth',1.5);
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase of H_d(e^{j\omega})','FontSize',TFS);
set(gca,'XTick',wtick); magtick = [-180:60:180]; set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0315c;
```

The frequency responses are shown in Figure 3.37 from which we observe that the truncated response is a smeared or blurred version of the ideal response.
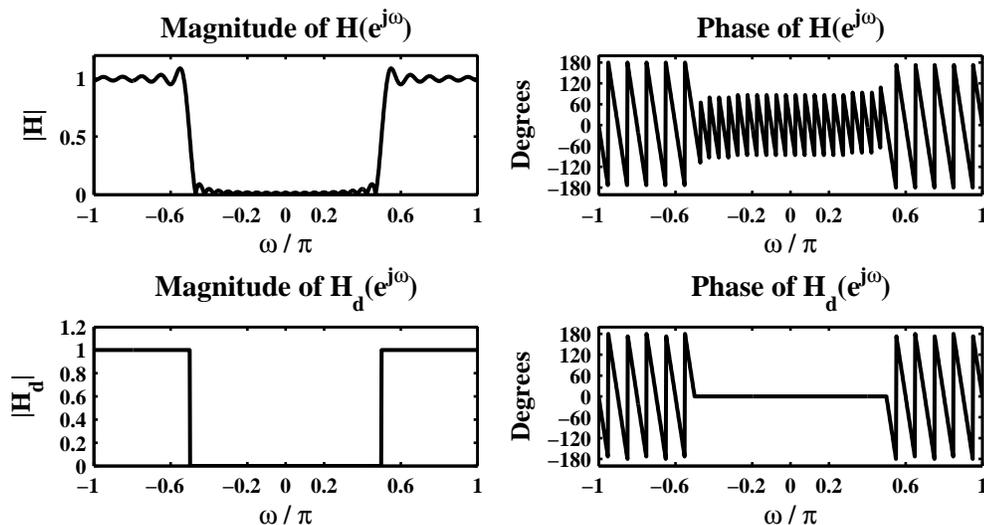

Figure 3.37: Problem P3.15c plots

P3.16 MATLAB function freqresp.

```
function [H] = freqresp(b,a,w)
% Frequency response function from difference equation
% [H] = freqresp(b,a,w)
%  H  = frequency response array evaluated at w frequencies
%  b  = numerator coefficient array
%  a  = denominator coeeficient array (a(1) = 1)
%  w  = frequency location array
%
b = reshape(b,1,length(b));
a = reshape(a,1,length(a));
w = reshape(w,1,length(w));
m = 0:length(b)-1; num = b*exp(-j*m'*w);
l = 0:length(a)-1; den = a*exp(-j*l'*w);
H = num./den;
```

P3.17 Computation and plot of the frequency response $H(e^{j\omega})$ using Matlab for each of the following systems:

1. $y(n) = \frac{1}{5}\sum_{m=0}^{4} x(n-m)$:

    Matlab script:

    ```
    % P0317a: y(n) = (1/5) sum_{0}^{4} x(n-m)
    clc; close all;
    %
    w = [-300:300]*pi/300; a = [1]; b = [0.2 0.2 0.2 0.2 0.2];
    [H] = freqresp(b,a,w); magH = abs(H); phaH = angle(H)*180/pi;
    %
    Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0317a');
    subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 1.2]);
    wtick = [-1:0.2:1]; magtick = [0:0.2:1.2];
    xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
    title(['Magnitude response'],'FontSize',TFS);
    set(gca,'XTick',wtick); set(gca,'YTick',magtick);
    subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -220 220]);
    wtick = [-1:0.2:1]; magtick = [-180:60:180];
    xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
    title('Phase Response ','FontSize',TFS);
    set(gca,'XTick',wtick); set(gca,'YTick',magtick);
    print -deps2 ../EPSFILES/P0317a;
    ```

   The frequency responses are shown in Figure 3.38
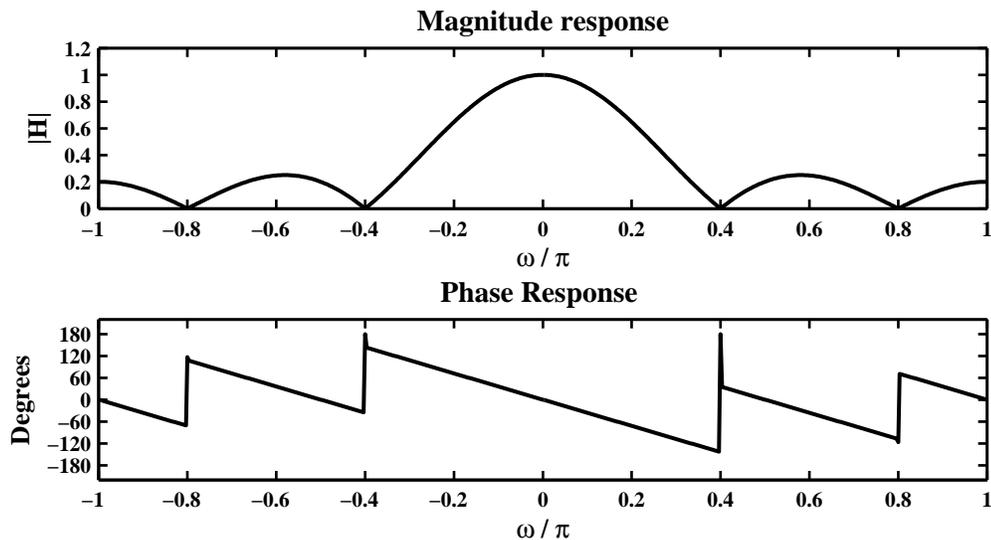
Figure 3.38: Frequency response plots in Problem P3.17a

2. $y(n) = x(n) - x(n-2) + 0.95y(n-1) - 0.9025y(n-2)$

MATLAB script:

```
% P0317b: y(n) = x(n)-x(n-2)+0.95*y(n-1)-0.9025*y(n-2)
clc; close all;
%
w = [-300:300]*pi/300; a = [1 -0.95 0.9025]; b = [1 0 -1];
[H] = freqresp(b,a,w); magH = abs(H); phaH = angle(H)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0317b');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 25]);
wtick = [-1:0.2:1]; magtick = [0:5:25];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title(['Magnitude response'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase response'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0317b;
```

The frequency responses are shown in Figure 3.39
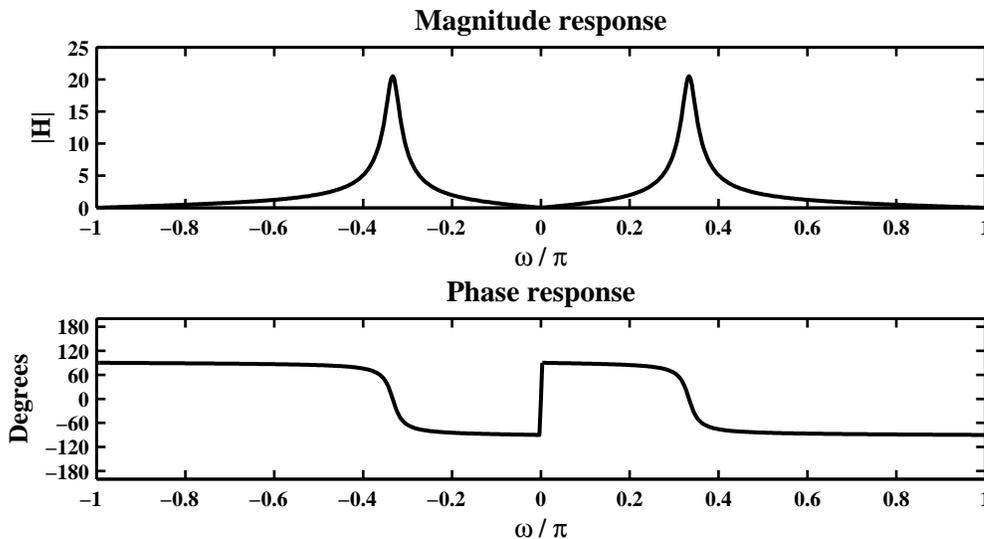
Figure 3.39: Frequency response plots in Problem P3.17b

3. $y(n) = x(n) - x(n-1) + x(n-2) + 0.95y(n-1) - 0.9025y(n-2)$

MATLAB script:

```
% P0317c: y(n) = x(n)-x(n-1)+x(n-2)+0.95*y(n-1)-0.9025*y(n-2)
clc; close all;
%
w = [-300:300]*pi/300; a = [1 -0.95 0.9025]; b = [1 -1 1];
[H] = freqresp(b,a,w); magH = abs(H); phaH = angle(H)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0317c');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 1.4]);
wtick = [-1:0.2:1]; magtick = [0:0.2:1.4];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title(['Magnitude response'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase response'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0317c;
```

The frequency responses are shown in Figure 3.40
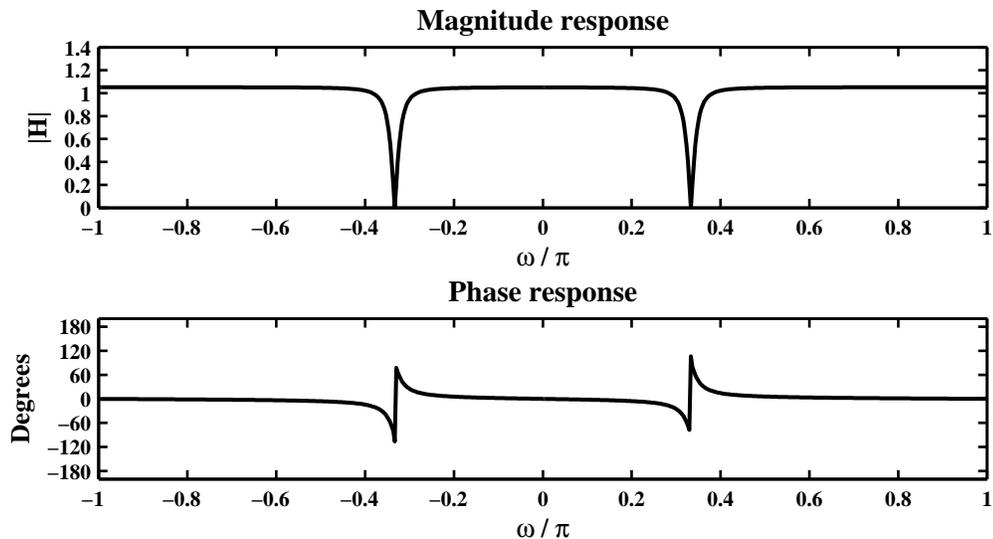
Figure 3.40: Frequency response plots in Problem P3.17c

4. $y(n) = x(n) - 1.7678x(n-1) + 1.5625x(n-2) + 1.1314y(n-1) - 0.64y(n-2)$

MATLAB script:

```
% P0317d: y(n) = x(n)-1.7678*x(n-1)+1.5625*x(n-2)+1.1314*y(n-1)-  0.64*y(n-2)
clc; close all;
%
w = [-300:300]*pi/300; a = [1 -1.1314 0.64]; b = [1 -1.7678 1.5625];
[H] = freqresp(b,a,w); magH = abs(H); phaH = angle(H)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0317d');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5);
wtick = [-1:0.2:1]; magtick = [1.5:0.02:1.6];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title(['Magnitude response'],'FontSize',TFS);
set(gca,'XTick',wtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -200 200]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase response','FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0317d;
```

The frequency responses are shown in Figure 3.41
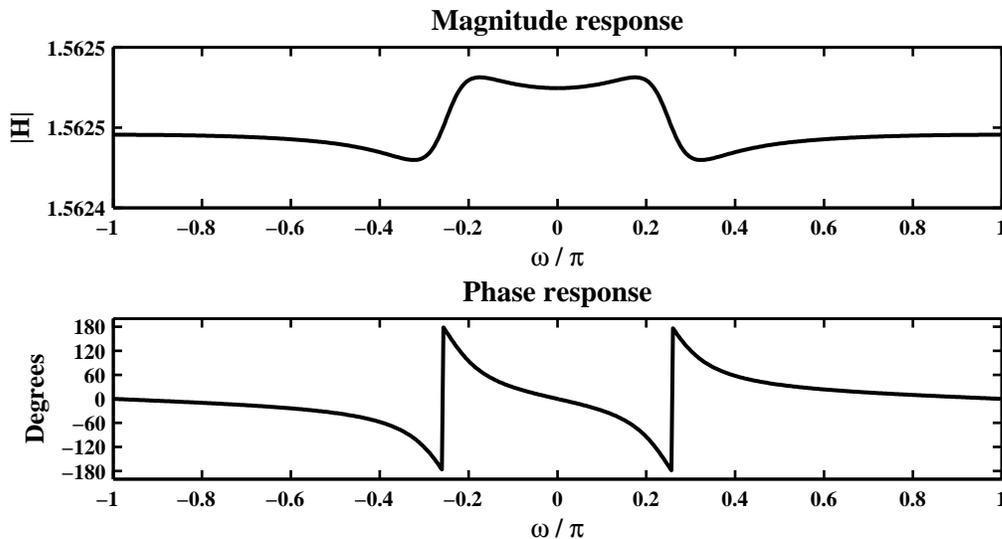
Figure 3.41: Frequency response plots in Problem P3.17d

5. $y(n) = x(n) - \sum_{\ell=1}^{5} (0.5)^{\ell} y(n - \ell)$

MATLAB script:

```
% P0317e: y(n) = x(n)-sum _ {l = 1} ^ {5} (0.5) ^ l*y(n-l);
clc; close all;
%
w = [-300:300]*pi/300; l = [0:5]; a = 0.5 .^ l; b = [1]; [H] = freqresp(b,a,w);
magH = abs(H); phaH = angle(H)*180/pi;
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0317e');
subplot(2,1,1); plot(w/pi,magH,'LineWidth',1.5); axis([-1 1 0 1.8]);
wtick = [-1:0.2:1]; magtick = [0:0.2:1.8];
xlabel('\omega / \pi','FontSize',LFS); ylabel('|H|','FontSize',LFS);
title(['Magnitude Response'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaH,'LineWidth',1.5); axis([-1 1 -180 180]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title(['Phase Response'],'FontSize',TFS);
set(gca,'XTick',wtick); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0317e;
```

The frequency responses are shown in Figure 3.42
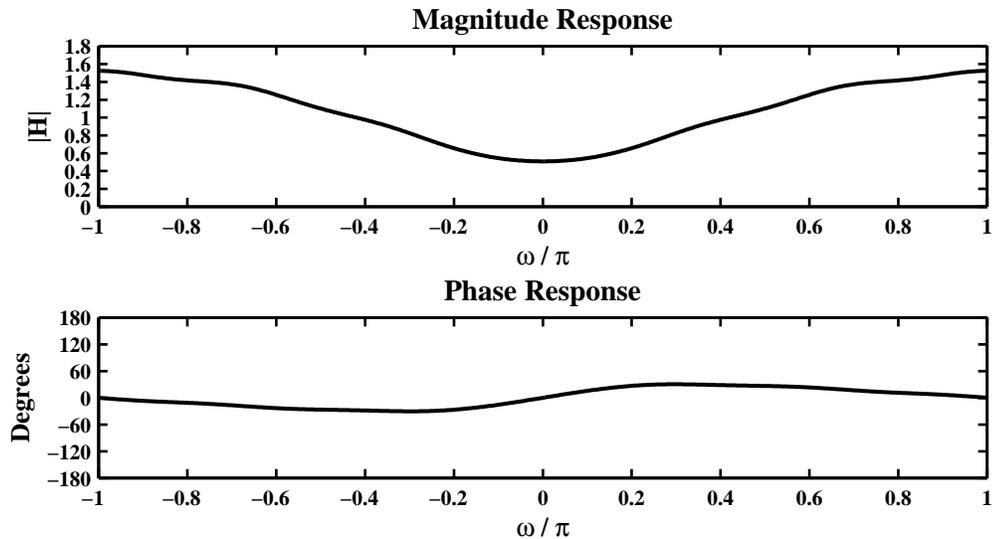
Figure 3.42: Frequency response plots in Problem P3.17e

P3.18 A linear, shift-invariant system is described by the difference equation

$$y(n) = \sum_{m=0}^{3} x(n - 2m) - \sum_{\ell=1}^{3} (0.81)^{\ell} y(n - 2\ell) \Rightarrow H\left(e^{j\omega}\right) = \frac{\sum_{m=0}^{3} e^{-j2m\omega}}{\sum_{\ell=0}^{3}(0.81)^{2\ell} e^{-j2\ell\omega}}$$

1. $x(n) = 5 + 10(-1)^n = 5 + 10\cos(n\pi)$: We need frequency responses at $\omega = 0$ and $\omega = \pi$.

$$H\left(e^{j0}\right) = \frac{\sum_{m=0}^{3} e^{-j0}}{\sum_{\ell=0}^{3}(0.81)^{2\ell} e^{-j0}} = 1.6885 \text{ and } H\left(e^{j\pi}\right) = \frac{\sum_{m=0}^{3} e^{-j2\pi\ell}}{\sum_{\ell=0}^{3}(0.81)^{2\ell} e^{-j2\pi\ell}} = 1.6885$$

Hence the steady-state response is $y(n) = 1.6885x(n) = 8.4424 + 16.8848(-1)^n$. Matlab script:

```
% P0318a: y(n) = sum_{m=0}^{3} x(n-2m)-sum_{l=1}^{3} (0.81)^l y(n-2l)
%         x(n) = 5+10(-1) ^ n;
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
n = [0:50]; a = [1  0  0.81^2  0  0.81^4  0  0.81^6]; b = [1 0 1 0 1 0 1];
w = [0 pi]; A = [5 10]; theta = [0 0]; [H] = freqresp(b,a,w);
magH = abs(H); phaH = angle(H); mag = A.*magH; pha = phaH+theta;
term1 = w'*n; term2 = pha'*ones(1,length(n)); cos_term = cos(term1+term2);
y1 = mag*cos_term; x = 5+10*(-1) .^ n; y2 = filter(b,a,x);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0318a');
subplot(2,1,1); Hs = stem(n,y1,'filled'); axis([-1 51 -10 30]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title('Steady state response y_{ss}(n) for x(n) = 5+10(-1)^{n}',...
      'FontSize',TFS); ytick = [-10:5:25]; set(gca,'YTick',ytick);
subplot(2,1,2); Hs = stem(n,y2,'filled'); axis([-1 51 -10 30]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title(['Output response y(n) using the filter function for x(n) = ' ...
      '5+10(-1)^{n}'],'FontSize',TFS); set(gca,'YTick',ytick);
print -deps2 ../EPSFILES/P0318a;
```
The steady-state responses are shown in Figure 3.43.
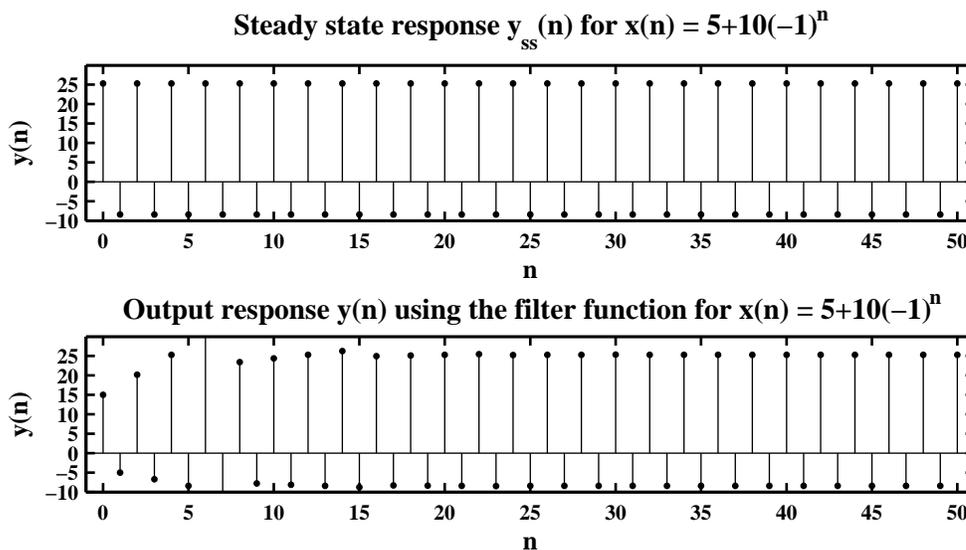
Figure 3.43: Steady-state response plots in Problem P3.18a

2. $x(n) = 1 + \cos(0.5\pi n + \pi/2)$: We need responses at $\omega = 0$ and $\omega = 0.5\pi$.

$$H\left(e^{j0}\right) = \frac{\sum_{m=0}^{3} e^{-j0}}{\sum_{\ell=0}^{3}(0.81)^{2\ell}e^{-j0}} = 1.6885 \text{ and } H\left(e^{j0.5\pi}\right) = \frac{\sum_{m=0}^{3} e^{-j\pi\ell}}{\sum_{\ell=0}^{3}(0.81)^{2\ell}e^{-j\pi\ell}} = 0$$

Hence the steady-state response is $y(n) = 1.6885$. MATLAB script:

```
% P0318b: y(n) = sum_{m=0}^{3} x(n-2m)-sum_{l=1}^{3} (0.81)^l y(n-2l)
%         x(n) = 1+cos(0.5*pi*n+pi/2);
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
n = [0:50]; a = [1  0  0.81^2  0  0.81^4  0  0.81^6]; b = [1 0 1 0 1 0 1];
w = [0 pi/2]; A = [1 1]; theta = [0 pi/2]; [H] = freqresp(b,a,w);
magH = abs(H); phaH = angle(H); mag = A.*magH; pha = phaH+theta;
term1 = w'*n; term2 = pha'*ones(1,length(n)); cos_term = cos(term1+term2);
y1 = mag*cos_term; x = 1+cos(0.5*pi*n+pi/2); y2 = filter(b,a,x);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0318b');
subplot(2,1,1); Hs = stem(n,y1,'filled'); axis([-1 51 0 2.5]);
ytick = [0:0.5:2.5]; set(Hs,'markersize',2); set(gca,'YTick',ytick);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title(['SS response y_{ss}(n): x(n) = 1+cos(0.5{\pi}n+\pi/2)'],'FontSize',TFS);
subplot(2,1,2); Hs = stem(n,y2,'filled'); set(Hs,'markersize',2);
axis([-1 51 0 2.5]); ytick = [0:0.5:2.5]; set(gca,'YTick',ytick);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title(['Output response y(n) using the filter function'],'FontSize',TFS);
print -deps2 ../EPSFILES/P0318b;
```

The steady-state responses are shown in Figure 3.44.
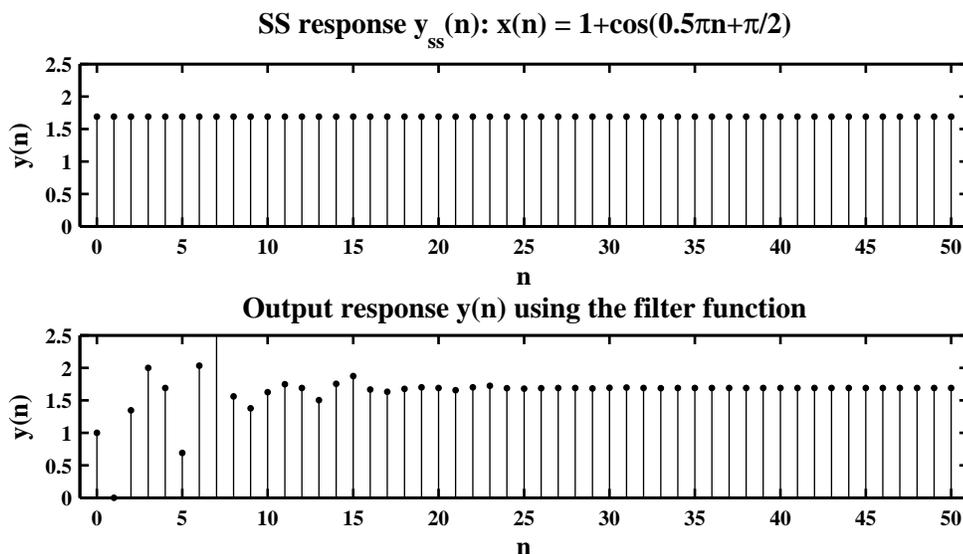
Figure 3.44: Steady-state response plots in Problem P3.18b

3. $x(n) = 2\sin(\pi n/4) + 3\cos(3\pi n/4)$: We need responses at $\omega = \pi/4$ and $\omega = 3\pi/4$.

$$H\left(e^{j0.25\pi}\right) = \frac{\sum_{m=0}^{3} e^{-j0.5\pi m}}{\sum_{\ell=0}^{3}(0.81)^{2\ell}e^{-j0.5\pi m}} = 0 \text{ and } H\left(e^{j0.75\pi}\right) = \frac{\sum_{m=0}^{3} e^{-j1.5\pi \ell}}{\sum_{\ell=0}^{3}(0.81)^{2\ell}e^{-j1.5\pi \ell}} = 0$$

Hence the steady-state response is $y(n) = 0$. MATLAB script:

```
% P0318c: y(n) = sum_{m=0}^{3} x(n-2m)-sum_{l=1}^{3} (0.81)^l y(n-2l)
%         x(n) = 2 * sin(\pi n/4)+3 * cos(3 \pi n/4);
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
n = [0:50]; a = [1  0  0.81^2  0  0.81^4  0  0.81^6]; b = [1 0 1 0 1 0 1];
w = [pi/4 3*pi/4]; A = [2 3]; theta = [-pi/2 0]; [H] = freqresp(b,a,w);
magH = abs(H); phaH = angle(H); mag = A.*magH; pha = phaH+theta;
term1 = w'*n; term2 = pha'*ones(1,length(n)); cos_term = cos(term1+term2);
y1 = mag*cos_term; x = 2*sin(pi*n/4)+3*cos(3*pi*n/4); y2 = filter(b,a,x);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0318c'); subplot(2,1,1);
Hs = stem(n,y1,'filled'); axis([-1 51 -3 4]); set(Hs,'markersize',2);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title(['SS response y_{ss}(n): x(n) = 2sin({\pi}n/4)+3cos(3 \pi n/4)'],...
       'FontSize',TFS);
subplot(2,1,2); Hs = stem(n,y2,'filled'); axis([-1 51 -3 4]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS); ylabel('y(n)',...
    'FontSize',LFS);
title(['Output response y(n) using the filter function'],'FontSize',TFS);
print -deps2 ../EPSFILES/P0318c;
```

The steady-state responses are shown in Figure 3.45.
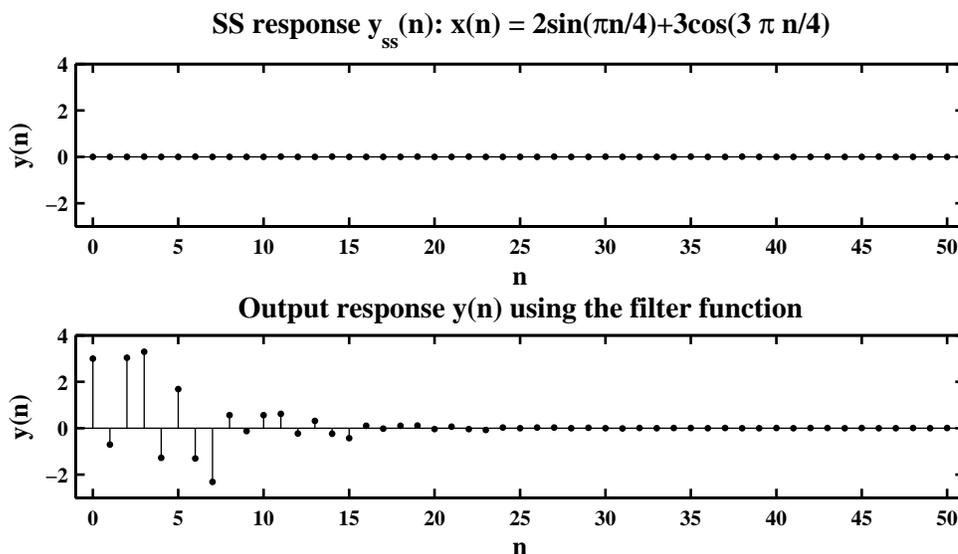
Figure 3.45: Steady-state response plots in Problem P3.18c

4. $x(n) = \sum_{k=0}^{5} (k+1) \cos(\pi k n / 4)$: We need responses at $\omega = k\pi/4$, $k = 0, 1, 2, 3, 4, 5$.

$$H\left(e^{j0}\right) = \frac{\sum_{m=0}^{3} e^{-j0}}{\sum_{\ell=0}^{3}(0.81)^{2\ell}e^{-j0}} = 1.6885 = H\left(e^{j\pi}\right) \text{ and}$$

$$H\left(e^{j0.25\pi}\right) = H\left(e^{j0.5\pi}\right) = H\left(e^{j0.75\pi}\right) = H\left(e^{j1.25\pi}\right) = 0$$

Hence the steady-state response is $y(n) = 1.6885 + 8.4425 \cos(n\pi)$. MATLAB script:

```
% P0318d: y(n) = sum_{m=0}^{3} x(n-2m)-sum_{l=1}^{3} (0.81)^l y(n-2l)
%          x(n) = sum_{k = 0}^{5} (k+1) cos(pi*k*n/4);
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
n = [0:50]; a = [1 0 0.81^2 0 0.81^4 0 0.81^6]; b = [1 0 1 0 1 0 1];
k = [0:5]; w = pi /4*k; A = (k+1); theta = zeros(1,length(k));
[H] = freqresp(b,a,w); magH = abs(H); phaH = angle(H); mag = A.*magH;
pha = phaH+theta; term1 = w'*n; term2 = pha'*ones(1,length(n)); cos_term = ...
    cos(term1+term2); y1 = mag*cos_term; x = A*cos(term1); y2 = filter(b,a,x);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0318d');
subplot(2,1,1); Hs = stem(n,y1,'filled'); axis([-1 51 -10 15]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS);
title(['SS response y_{ss}(n): x(n) = sum_{0}^{5} (k+1)cos({\pi}kn/4)'],...
        'FontSize',TFS);
ytick = [-20:5:30]; set(gca,'YTick',ytick); ylabel('y(n)','FontSize',LFS);
subplot(2,1,2); Hs = stem(n,y2,'filled'); axis([-1 51 -10 15]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS);
title(['Output response y(n) using the filter function'],'FontSize',TFS);
ytick = [-20:5:30]; set(gca,'YTick',ytick); ylabel('y(n)','FontSize',LFS);
print -deps2 ../EPSFILES/P0318d;
```

The steady-state responses are shown in Figure 3.46.
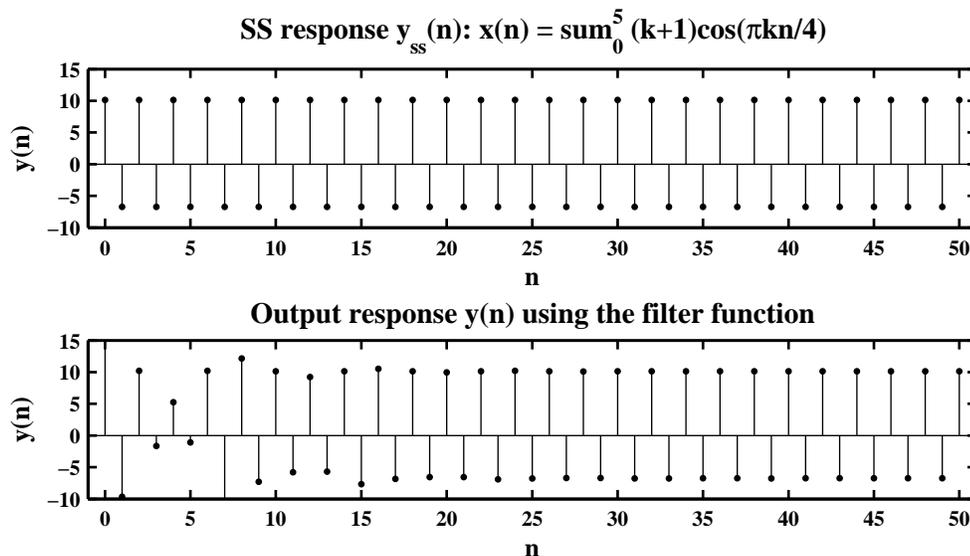
Figure 3.46: Steady-state response plots in Problem P3.18d

5. $x(n) = \cos(\pi n)$: We need response at $\omega = \pi$.

$$H\left(e^{j\pi}\right) = \frac{\sum_{m=0}^{3} e^{-j2\pi m}}{\sum_{\ell=0}^{3}(0.81)^{2\ell} e^{-j2\pi m}} = 1.6885$$

Hence the steady-state response is $y(n) = 1.6885 \cos(\pi n)$. MATLAB script:

```
% P0318e: y(n) = sum_{m=0}^{3} x(n-2m)-sum_{l=1}^{3} (0.81)^l y(n-2l)
% x(n) = cos(pi*n);
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,3]);
n = [0:50]; a = [1  0  0.81^2  0  0.81^4  0  0.81^6]; b = [1 0 1 0 1 0 1];
w = [pi]; A =  [1]; theta = [0]; [H] = freqresp(b,a,w); magH = abs(H);
phaH = angle(H); mag = A.*magH; pha = phaH+theta; term1 = w'*n;
term2 = pha'*ones(1,length(n)); cos_term = cos(term1+term2); y1 = mag*cos_term;
x = cos(pi*n); y2 = filter(b,a,x);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0318e');
subplot(2,1,1); Hs = stem(n,y1,'filled'); axis([-1 51 -2 2]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS);
title(['SS response y_{ss}(n) for x(n) = cos(\pi \times n)'],'FontSize',TFS);
ytick = [-2:0.5:2]; set(gca,'YTick',ytick); ylabel('y(n)','FontSize',LFS);
subplot(2,1,2); Hs = stem(n,y2,'filled'); axis([-1 51 -2 2]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS);
title(['Output response y(n) using the filter function'],'FontSize',TFS);
ytick = [-2:0.5:2]; set(gca,'YTick',ytick); ylabel('y(n)','FontSize',LFS);
print -deps2 ../EPSFILES/P0318e;
```

The steady-state responses are shown in Figure 3.47.
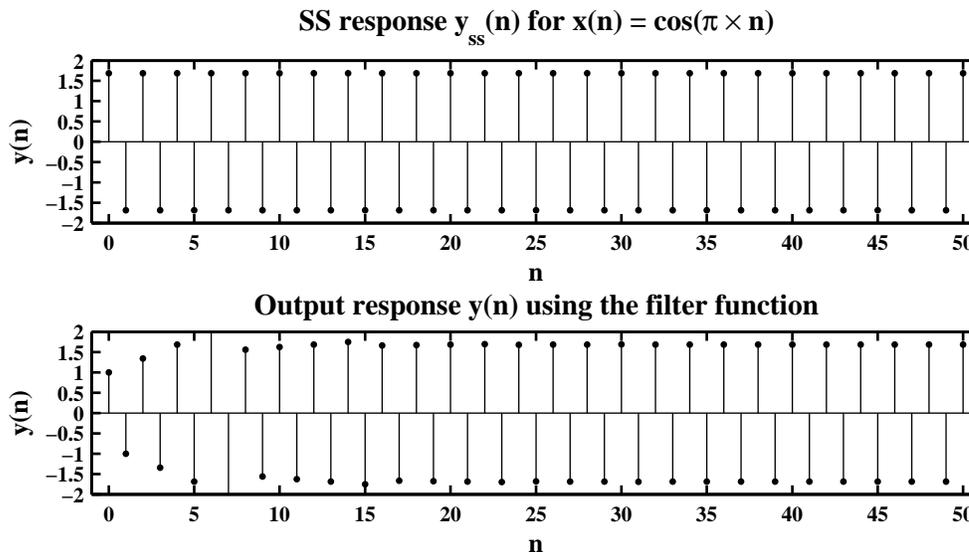
Figure 3.47: Steady-state response plots in Problem P3.18e

P3.19 An analog signal $x_a(t) = \sin(1000\pi t)$ is sampled using the following sampling intervals.

1. $T_s = 0.1$ ms: MATLAB script:

```
% P0319a: x_a(t) = sin(1000*pi*t); T_s = 0.1 ms;
clc; close all;
%
Ts = 0.0001; n = [-250:250]; x = sin(1000*pi*n*Ts); w = linspace(-pi,pi,501);
X = dtft(x,n,w); magX = abs(X); phaX = angle(X);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0319a');
subplot(2,1,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 0 300]);
wtick = [-1:0.2:1]; magtick = [0:100:300]; set(gca,'XTick',wtick);
xlabel('\omega / \pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title('Magnitude response x_1(n) = sin(1000 \pi n T_s), T_s = 0.1 msec'...
      ,'FontSize',TFS); set(gca,'YTick',magtick);
subplot(2,1,2); plot(w/pi,phaX*180/pi,'LineWidth',1.5); axis([-1 1 -180 180]);
wtick = [-1:0.2:1]; magtick = [-180:60:180];
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase response x_1(n) = sin(1000 \pi n T_s), T_s = 0.1 msec'...
      ,'FontSize',TFS); set(gca,'XTick',wtick); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0319a;
```

The spectra are shown in Figure 3.48.
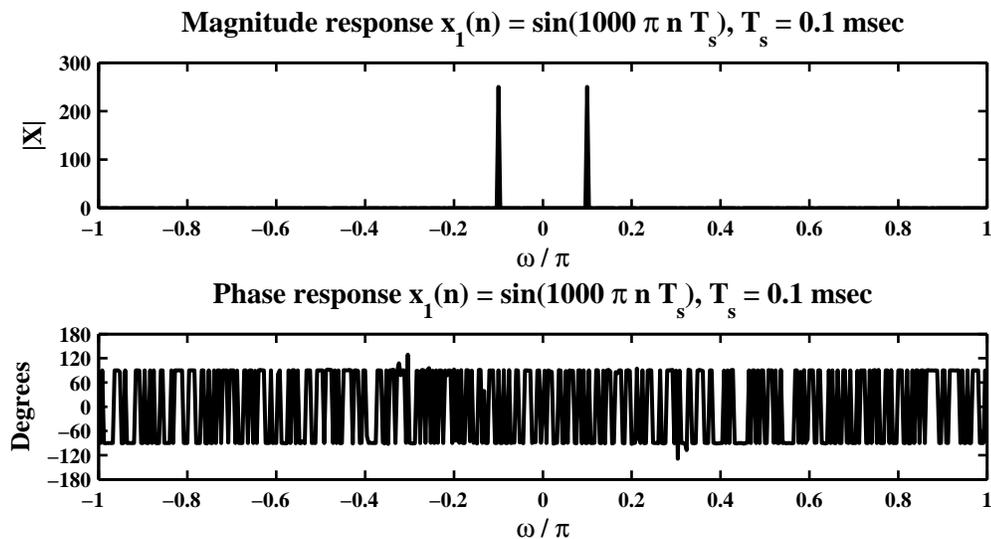
Figure 3.48: Spectrum plots in Problem P3.19a

2. $T_s = 1$ ms: MATLAB script:

```
% P0319b: x_a(t) = sin(1000*pi*t); T_s = 1 ms;
clc; close all;
%
Ts = 0.001; n = [-25:25]; x = sin(1000*pi*n*Ts); w = [-500:500]*pi/500;
X = dtft(x,n,w); magX = abs(X); phaX = angle(X);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0319b');
subplot(2,1,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 -1 1]);
xlabel('\omega / \pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title('Magnitude response x_1(n) = sin(1000 \pi n T_s), T_s = 1 msec'...
    ,'FontSize',TFS);
wtick = [-1:0.2:1]; set(gca,'XTick',wtick);
subplot(2,1,2); plot(w/pi,phaX*180/pi,'LineWidth',1.5); axis([-1 1 -180 180]);
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase response x_1(n) = sin(1000 \pi n T_s), T_s = 1 msec'...
    ,'FontSize',TFS); magtick = [-180:60:180];
wtick = [-1:0.2:1]; set(gca,'XTick',wtick); set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0319b;
```

The spectra are shown in Figure 3.49.
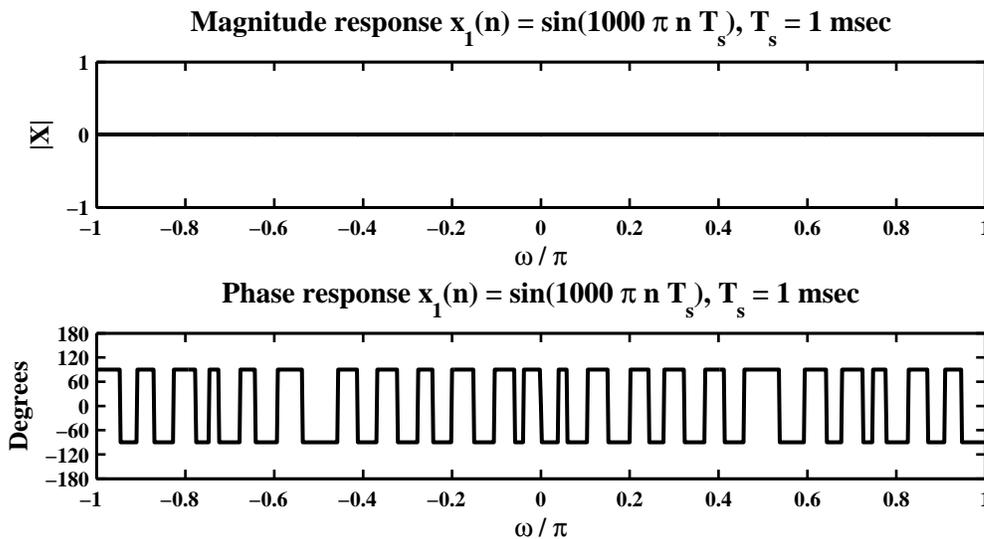
Figure 3.49: Spectrum plots in Problem P3.19b

3. $T_s = 0.01$ sec: MATLAB script:

```
% P0319c: x_a(t) = sin(1000*pi*t); T_s = 0.01 sec;
clc; close all;
%
Ts = 0.01; n = [-25:25]; x = sin(1000*pi*n*Ts); w = [-500:500]*pi/500;
X = dtft(x,n,w); magX = abs(X); phaX = angle(X);
%
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0319c');
subplot(2,1,1); plot(w/pi,magX,'LineWidth',1.5); axis([-1 1 -1 1]);
xlabel('\omega / \pi','FontSize',LFS); ylabel('|X|','FontSize',LFS);
title('Magnitude response x_1(n) = sin(1000 \pi n T_s),T_s = 0.01 sec'...
      ,'FontSize',TFS); wtick = [-1:0.2:1]; set(gca,'XTick',wtick);
subplot(2,1,2); plot(w/pi,phaX*180/pi,'LineWidth',1.5); axis([-1 1 -180 180]);
xlabel('\omega / \pi','FontSize',LFS); ylabel('Degrees','FontSize',LFS);
title('Phase response x_1(n) = sin(1000 \pi n T_s),T_s = 0.01 sec'...
      ,'FontSize',TFS); wtick = [-1:0.2:1]; set(gca,'XTick',wtick);
magtick = [-180:60:180]; set(gca,'YTick',magtick);
print -deps2 ../EPSFILES/P0319c;
```

The spectra are shown in Figure 3.50.
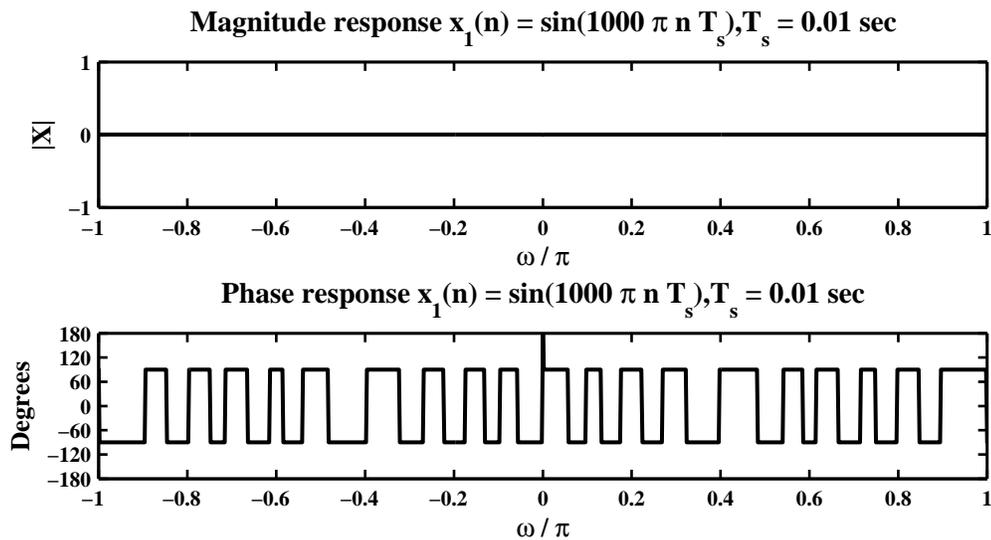
Figure 3.50: Spectrum plots in Problem P3.19c

P3.20 Sampling frequency $F_s = 8000$ sam/sec (or sampling interval $T_s = 0.125$ msec/sam) and impulse response $h(n) = (-0.9)^n u(n)$.

(a) $x_a(t) = 10\cos(10000\pi t)$. Hence $x(n) = x_a(nT_s) = 10\cos(10000\pi n0.000125) = 10\cos(1.25\pi n)$. Therefore, the digital frequency is $(1.25 - 2)\pi = -0.75\pi$ rad/sam.

(b) The steady-state response when $x(n) = 10\cos(-0.75\pi n) = 10\cos(0.75\pi n$: The frequency response is

$$H\left(e^{j\omega}\right) = \mathcal{F}[h(n)] = \mathcal{F}\left[(-0.9)^n u(n)\right] = \frac{1}{1 + 0.9e^{j\omega}}.$$

At $\omega = -0.75\pi$, the response is

$$H\left(e^{j0.75\pi}\right) = \frac{1}{1 + 0.9e^{j0.75\pi}} = 0.7329\left(\angle 1.0517^c\right).$$

Hence

$$y_{ss}(n) = 10(0.7329)\cos(0.75\pi n + 1.0517)$$

which after D/A conversion gives $y_{ss}(t)$ as

$$y_{ss,a}(t) = 7.329\cos(6000\pi t1.0517).$$

(c) The steady-state DC gain is obtained by setting $\omega = 0$ which is equal to $H\left(e^{j0}\right) = 1/(1+0.9) = 0.5263$. Hence $y_{ss}(n) = 10(0.5263) = y_{ss,a}(t) = 5.263$.

(d) Aliased frequencies of F_0 for the given sampling rate F_s are $F_0 + kF_s$. Now for $F_0 = 5$ KHz and $F_s = 8$ KHz, the aliased frequencies are $5 + 8k = \{13, 21, \ldots\}$ KHz. Therefore, two other $x_a(t)$'s are

$$10\cos(26000\pi t) \text{ and } 10\cos(42000\pi t).$$

(e) The prefilter should be a lowpass filter with the cutoff frequency of 4 KHz.

P3.21 Consider an analog signal $x_a(t) = \cos(20\pi t)$, $0 \le t \le 1$. It is sampled at $T_s = 0.01$, 0.05, and 0.1 sec intervals to obtain $x(n)$.

1. Plots of $x(n)$ for each T_s. MATLAB script:

```
% P0321a: plot x(n) for T_s = 0.01 sec,0.05 sec,0.1 sec
%          x_a(t) = cos(20*pi*t);
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,4]);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0321a');
T_s1 = 0.01; n1 = [0:100]; x1 = cos(20*pi*n1*T_s1);
subplot(3,1,1); Hs = stem(n1,x1,'filled'); axis([-5 105 -1.2 1.2]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS);
title(['x(n) = cos(20{\pi}nT_s) for T_s = 0.01 sec'],'FontSize',TFS);
ylabel('x(n)','FontSize',LFS);
T_s2 = 0.05; n2 = [0:20]; x2 = cos(20*pi*n2*T_s2);
subplot(3,1,2); Hs = stem(n2,x2,'filled'); set(Hs,'markersize',2);
set(gca,'XTick',[0:20]); axis([-2 22 -1.2 1.2]);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS);
title(['x(n) = cos(20{\pi}nT_s) for T_s = 0.05 sec'],'FontSize',TFS);
T_s3 = 0.1; n3 = [0:10]; x3 = cos(20*pi*n3*T_s3);
```

```
subplot(3,1,3); Hs = stem(n3,x3,'filled'); set(Hs,'markersize',2);
set(gca,'XTick',[0:10]); axis([-1 11 -1.2 1.2]);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS);
title(['x(n) = cos(20{\pi}nT_s) for T_s = 0.1 sec'],'FontSize',TFS);
print -deps2 ../EPSFILES/P0321a;
```

The plots are shown in Figure 3.51.
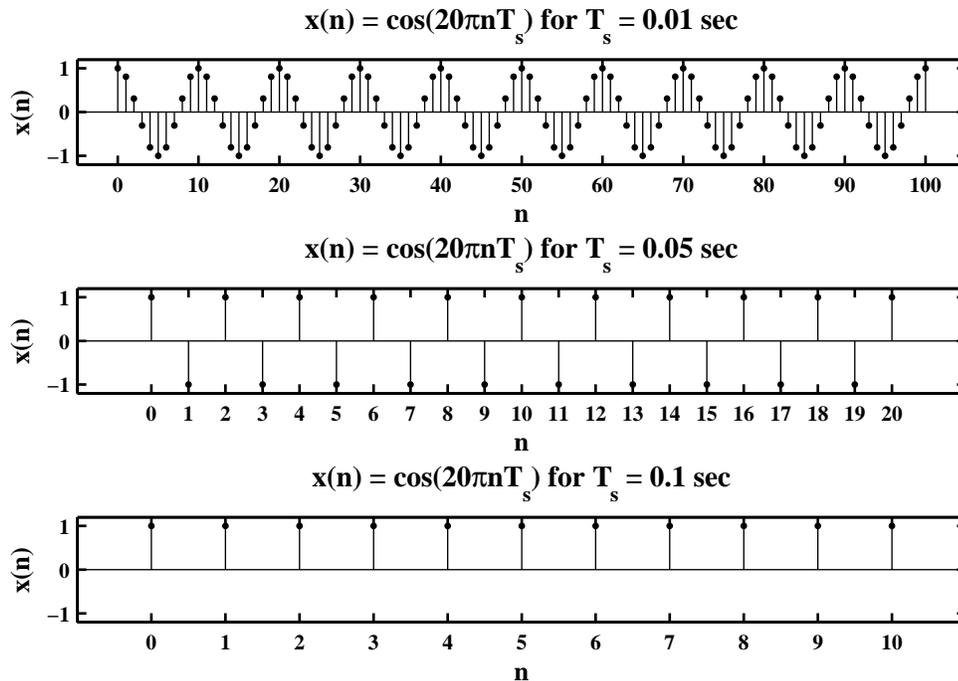
Figure 3.51: Plots of $x(n)$ for various T_s in Problem P3.21a.

2. Reconstruction from $x(n)$ using the sinc interpolation. MATLAB script:

```
% P0321a: plot x(n) for T_s = 0.01 sec,0.05 sec,0.1 sec
%         x_a(t) = cos(20*pi*t);
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,4]);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0321a');
T_s1 = 0.01; n1 = [0:100]; x1 = cos(20*pi*n1*T_s1);
subplot(3,1,1); Hs = stem(n1,x1,'filled'); axis([-5 105 -1.2 1.2]);
set(Hs,'markersize',2); xlabel('n','FontSize',LFS);
title(['x(n) = cos(20{\pi}nT_s) for T_s = 0.01 sec'],'FontSize',TFS);
ylabel('x(n)','FontSize',LFS);
T_s2 = 0.05; n2 = [0:20]; x2 = cos(20*pi*n2*T_s2);
subplot(3,1,2); Hs = stem(n2,x2,'filled'); set(Hs,'markersize',2);
set(gca,'XTick',[0:20]); axis([-2 22 -1.2 1.2]);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS);
title(['x(n) = cos(20{\pi}nT_s) for T_s = 0.05 sec'],'FontSize',TFS);
T_s3 = 0.1; n3 = [0:10]; x3 = cos(20*pi*n3*T_s3);
subplot(3,1,3); Hs = stem(n3,x3,'filled'); set(Hs,'markersize',2);
```

```
set(gca,'XTick',[0:10]); axis([-1 11 -1.2 1.2]);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS);
title(['x(n) = cos(20{\pi}nT_s) for T_s = 0.1 sec'],'FontSize',TFS);
print -deps2 ../EPSFILES/P0321a;
```

The reconstruction is shown in Figure 3.52.
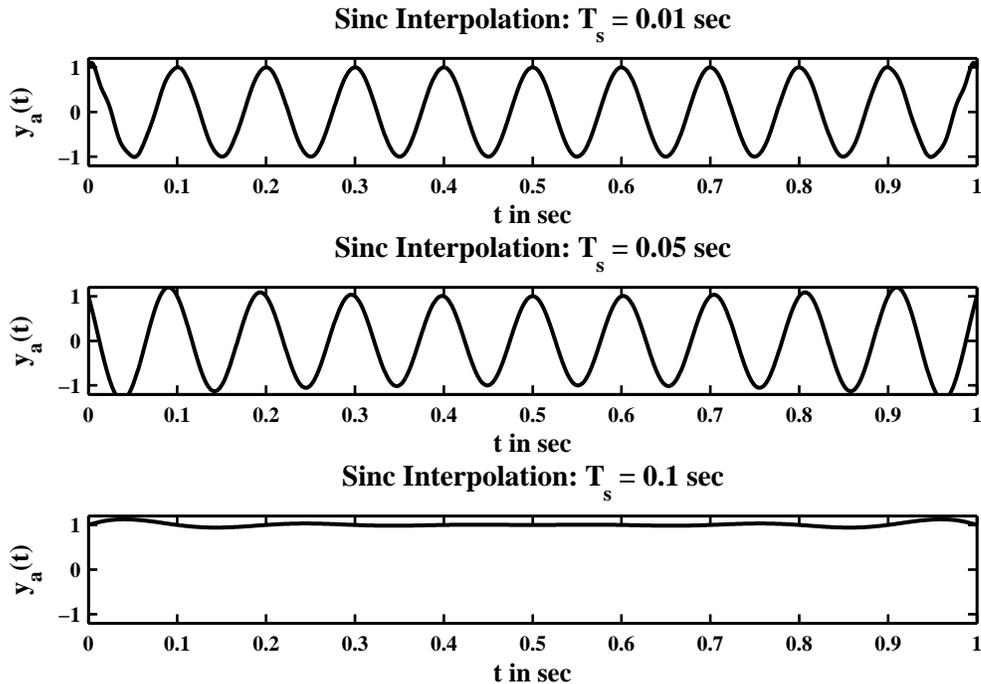
Figure 3.52: The sinc interpolation in Problem P3.21b.

3. Reconstruction from $x(n)$ using the spline interpolation. MATLAB script:

```
% P0321c Spline Interpolation: x_a(t) = cos(20*pi*t); 0 <= t <= 1;
%                              T_s = 0.01 sec,0.05 sec and 0.1 sec;
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,4]);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0321c');
%
Ts1 = 0.01; Fs1 = 1/Ts1; n1 = [0:100]; nTs1 = n1*Ts1;
x1 = cos(20*pi*nTs1); Dt = 0.001; t = 0:Dt:1; xa1 = spline(nTs1,x1,t);
subplot(3,1,1); plot(t,xa1,'LineWidth',1.5); axis([0 1 -1.2 1.2]);
xlabel('t in sec','FontSize',LFS); ylabel('y_a(t)','FontSize',LFS);
title(['Spline Interpolation: T_s = 0.01 sec'],'FontSize',TFS);
%
Ts2 = 0.05; Fs2 = 1/Ts2; n2 = [0:20]; nTs2 = n2*Ts2;
x2 = cos(20*pi*nTs2); Dt = 0.001; t = 0:Dt:1; xa2 = spline(nTs2,x2,t);
subplot(3,1,2); plot(t,xa2,'LineWidth',1.5); axis([0 1 -1.2 1.2]);
xlabel('t in sec','FontSize',LFS); ylabel('y_a(t)','FontSize',LFS);
title(['Spline Interpolation: T_s = 0.05 sec'],'FontSize',TFS); grid;
%
```

```
Ts3 = 0.1; Fs3 = 1/Ts3; n3 = [0:10]; nTs3 = n3*Ts3; x3 = cos(20*pi*nTs3);
Dt = 0.001; t = 0:Dt:1; xa3 = spline(nTs3,x3,t);
subplot(3,1,3); plot(t,xa3,'LineWidth',1.5); axis([0 1 -1.2 1.2]);
xlabel('t in sec','FontSize',LFS); ylabel('y_a(t)','FontSize',LFS);
title(['Spline Interpolation: T_s = 0.1 sec'],'FontSize',TFS); grid;
print -deps2 ../EPSFILES/P0321c;
```

The reconstruction is shown in Figure 3.53.
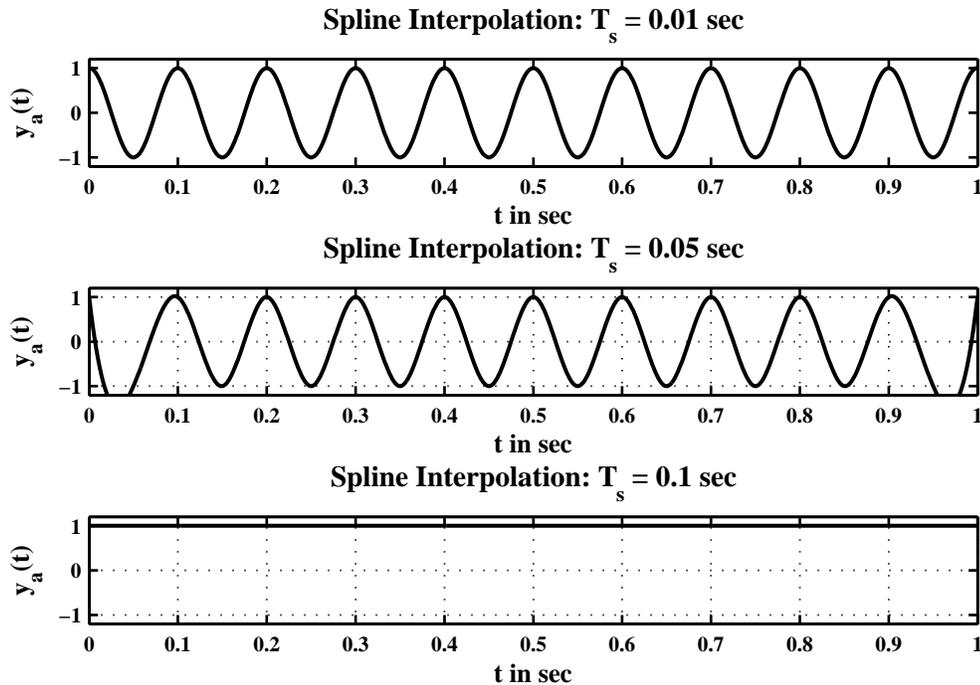
Figure 3.53: The sinc interpolation in Problem P3.21c.

4. Comments: From the plots in Figures it is clear that reconstructions from samples at $T_s = 0.01$ and 0.05 depict the original frequency (excluding end effects) but reconstructions for $T_s = 0.1$ show the original frequency aliased to zero. Furthermore, the cubic spline interpolation is a better reconstruction than the sinc interpolation, that is, the sinc interpolation is more susceptible to boundary effect.

P3.22 Consider the analog signal $x_a(t) = \cos(20\pi t + \theta)$, $0 \le t \le 1$. It is sampled at $T_s = 0.05$ sec intervals to obtain $x(n)$. Let $\theta = 0$, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$. For each of these θ values, perform the following.

(a) Plots of $x_a(t)$ and $x(n)$ for $\theta = 0$, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$. MATLAB script:

```
% P0322a: x_a(t) = cos(20*pi*t+theta); x(n) for theta = 0,pi/6,pi/4,pi/3, pi/2
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,7]);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0322a');
Ts = 0.05; Fs = 1/Ts; Dt = 0.001; t = 0:Dt:1; n = [0:20]; nTs = n*Ts;
theta1 = 0; x_a1 = cos(20*pi*t+theta1); x1 = cos(20*pi*nTs+theta1);
subplot(5,1,1); plot(t,x_a1,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x1,'o'); xlabel('t in sec','FontSize',LFS);
title('x_a(t) and x(n) for \theta = 0','FontSize',TFS);
```

```
ylabel('Amplitude','FontSize',LFS);
theta2 = pi/6; x_a2 = cos(20*pi*t+theta2); x2 = cos(20*pi*nTs+theta2);
subplot(5,1,2); plot(t,x_a2,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x2,'o'); xlabel('t in sec','FontSize',LFS);
title('x_a(t) and x(n) for \theta = \pi/6','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta3 = pi/4; x_a3 = cos(20*pi*t+theta3); x3 = cos(20*pi*nTs+theta3);
subplot(5,1,3); plot(t,x_a3,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x3,'o'); xlabel('t in sec','FontSize',LFS);
title('x_a(t) and x(n) for \theta = \pi/4','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta4 = pi/3; x_a4 = cos(20*pi*t+theta4); x4 = cos(20*pi*nTs+theta4);
subplot(5,1,4); plot(t,x_a4,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x4,'o'); xlabel('t in sec','FontSize',LFS);
title('x_a(t) and x(n) for \theta = \pi/3','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta5 = pi/2; x_a5 = cos(20*pi*t+theta5); x5 = cos(20*pi*nTs+theta5);
subplot(5,1,5); plot(t,x_a5,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x5,'o'); xlabel('t in sec','FontSize',LFS);
title('x_a(t) and x(n) for \theta = \pi/2','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS); print -deps2 ../EPSFILES/P0322a;
```

The reconstruction is shown in Figure 3.54.

(b) Reconstruction of the analog signal $y_a(t)$ from the samples $x(n)$ using the sinc interpolation (for $\theta = 0$, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$. MATLAB script:

```
% P0322b: Sinc Interpolation for theta = 0,pi/6,pi/4,pi/3, pi/2
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,7]);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0322b');
Ts = 0.05; Fs = 1/Ts; Dt = 0.001; t = 0:Dt:1; n = [0:20]; nTs = n*Ts;
theta1 = 0; x1 = cos(20*pi*nTs+theta1);
y_a1 = x1*sinc(Fs*(ones(length(n),1)*t-nTs'*ones(1,length(t))));
subplot(5,1,1); plot(t,y_a1,'LineWidth',1.5); hold on;
plot(nTs,x1,'o'); axis([0 1 -1.2 1.2]); xlabel('t in sec','FontSize',LFS);
title('Sinc Interpolation for \theta = 0','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta2 = pi/6; x2 = cos(20*pi*nTs+theta2);
y_a2 = x2*sinc(Fs*(ones(length(n),1)*t-nTs'*ones(1,length(t))));
subplot(5,1,2); plot(t,y_a2,'LineWidth',1.5); hold on; axis([0 1 -1.2 1.2])
plot(nTs,x2,'o'); xlabel('t in sec','FontSize',LFS);
title('Sinc Interpolation for \theta = \pi/6','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta3 = pi/4; x3 = cos(20*pi*nTs+theta3);
y_a3 = x3*sinc(Fs*(ones(length(n),1)*t-nTs'*ones(1,length(t))));
subplot(5,1,3); plot(t,y_a3,'LineWidth',1.5); hold on; axis([0 1 -1.2 1.2])
plot(nTs,x3,'o'); xlabel('t in sec','FontSize',LFS);
title('Sinc Interpolation for \theta = \pi/4','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta4 = pi/3; x4 = cos(20*pi*nTs+theta4);
```
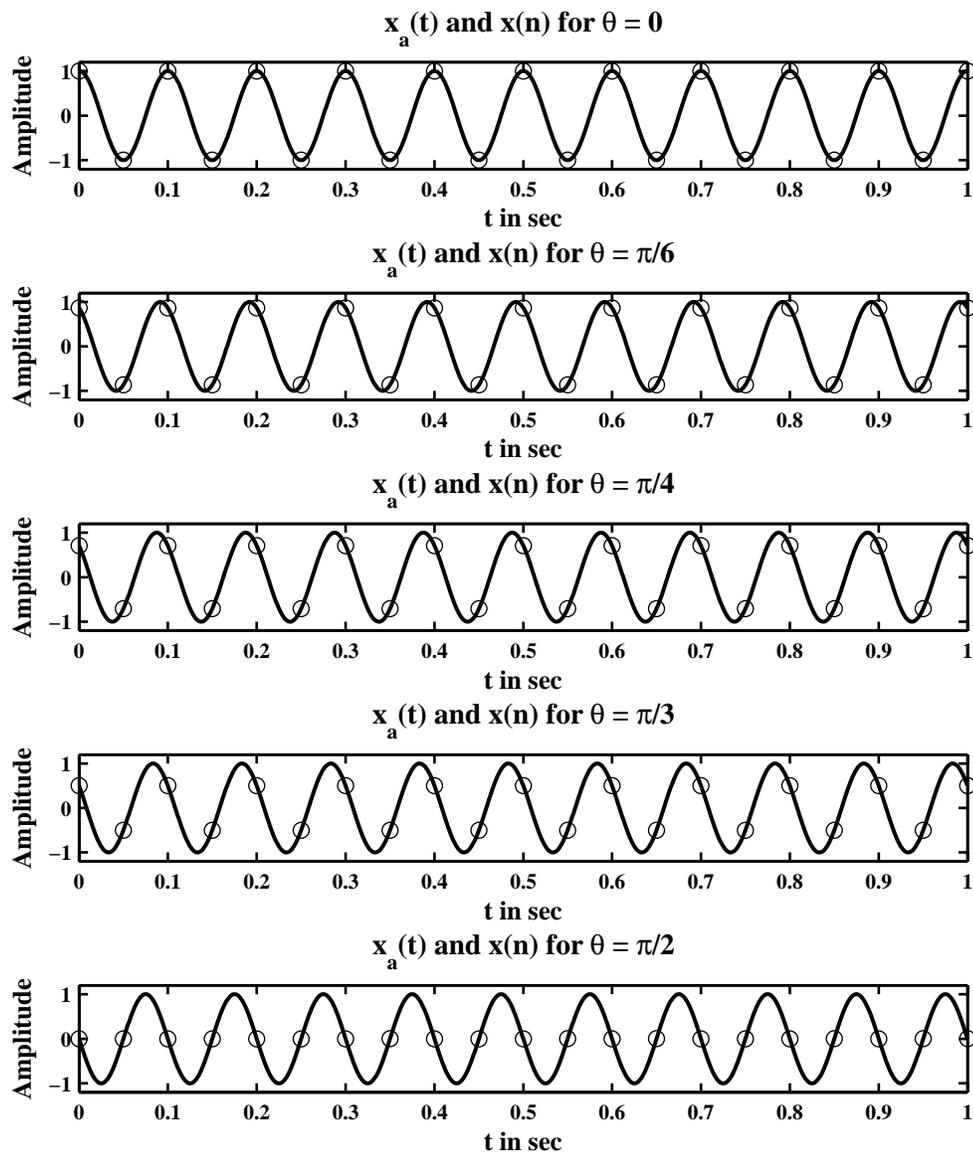
Figure 3.54: The sinc interpolation in Problem P3.22a.

```
y_a4 = x4*sinc(Fs*(ones(length(n),1)*t-nTs'*ones(1,length(t))));
subplot(5,1,4); plot(t,y_a4,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x4,'o'); xlabel('t in sec','FontSize',LFS);
title('Sinc Interpolation for \theta = \pi/3','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta5 = pi/2; x5 = cos(20*pi*nTs+theta5);
y_a5 = x5*sinc(Fs*(ones(length(n),1)*t-nTs'*ones(1,length(t))));
subplot(5,1,5); plot(t,y_a5,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x5,'o'); xlabel('t in sec','FontSize',LFS);
title('Sinc Interpolation for \theta = \pi/3','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS); print -deps2 ../EPSFILES/P0322b;
```

The reconstruction is shown in Figure 3.55.
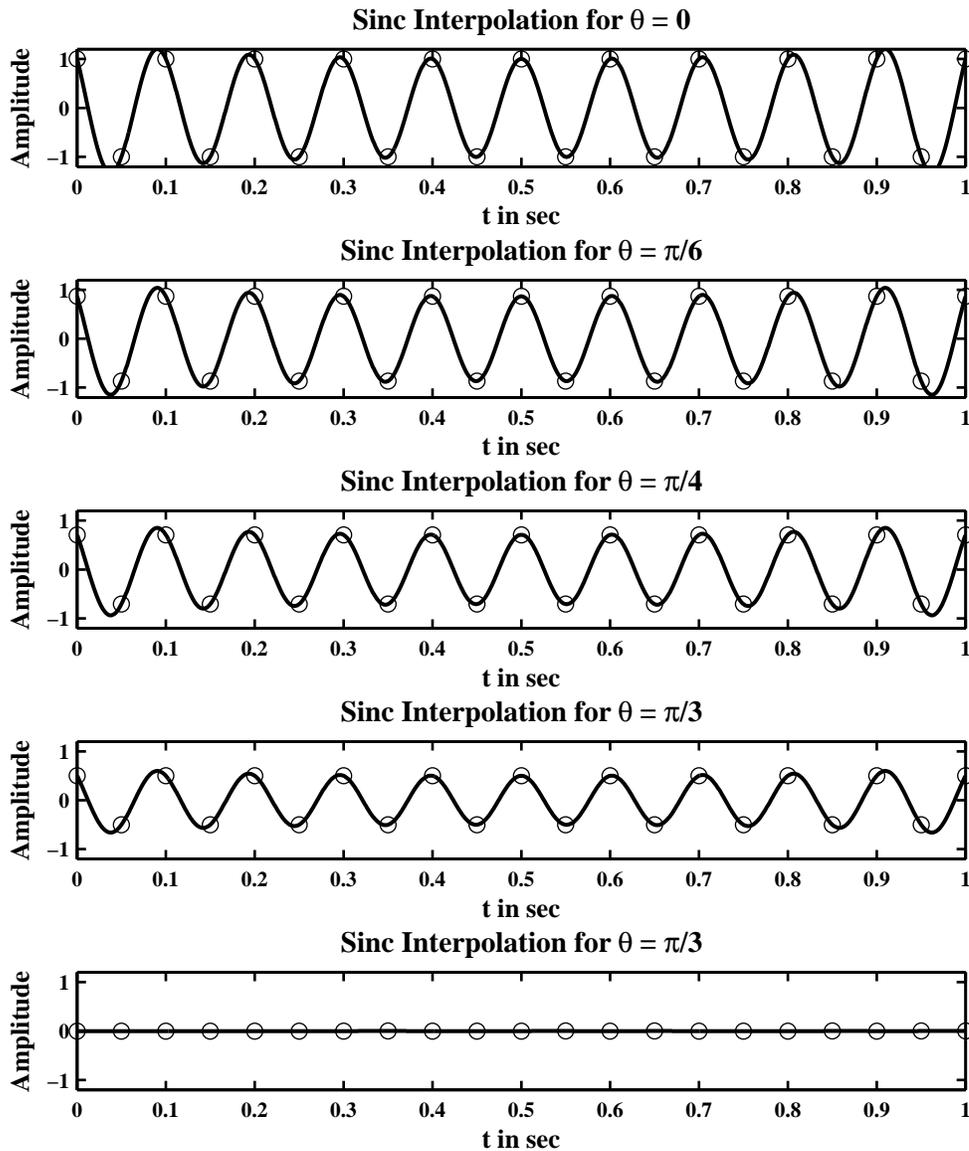
Figure 3.55: The sinc interpolation in Problem P3.22b.

(c) Reconstruction of the analog signal $y_a(t)$ from the samples $x(n)$ using the spline interpolation (for $\theta = 0$, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$. MATLAB script:

```
% P0322c: Spline Interpolation for theta = 0,pi/6,pi/4,pi/3, pi/2
clc; close all; set(0,'defaultfigurepaperposition',[0,0,6,7]);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0322c');
Ts = 0.05; Fs = 1/Ts; Dt = 0.001; t = 0:Dt:1; n = [0:20]; nTs = n*Ts;
theta1 = 0; x1 = cos(20*pi*nTs+theta1); y_a1 = spline(nTs,x1,t);
subplot(5,1,1); plot(t,y_a1,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x1,'o'); xlabel('t in sec','FontSize',LFS);
title('Spline Interpolation for theta = 0','FontSize',TFS);
```
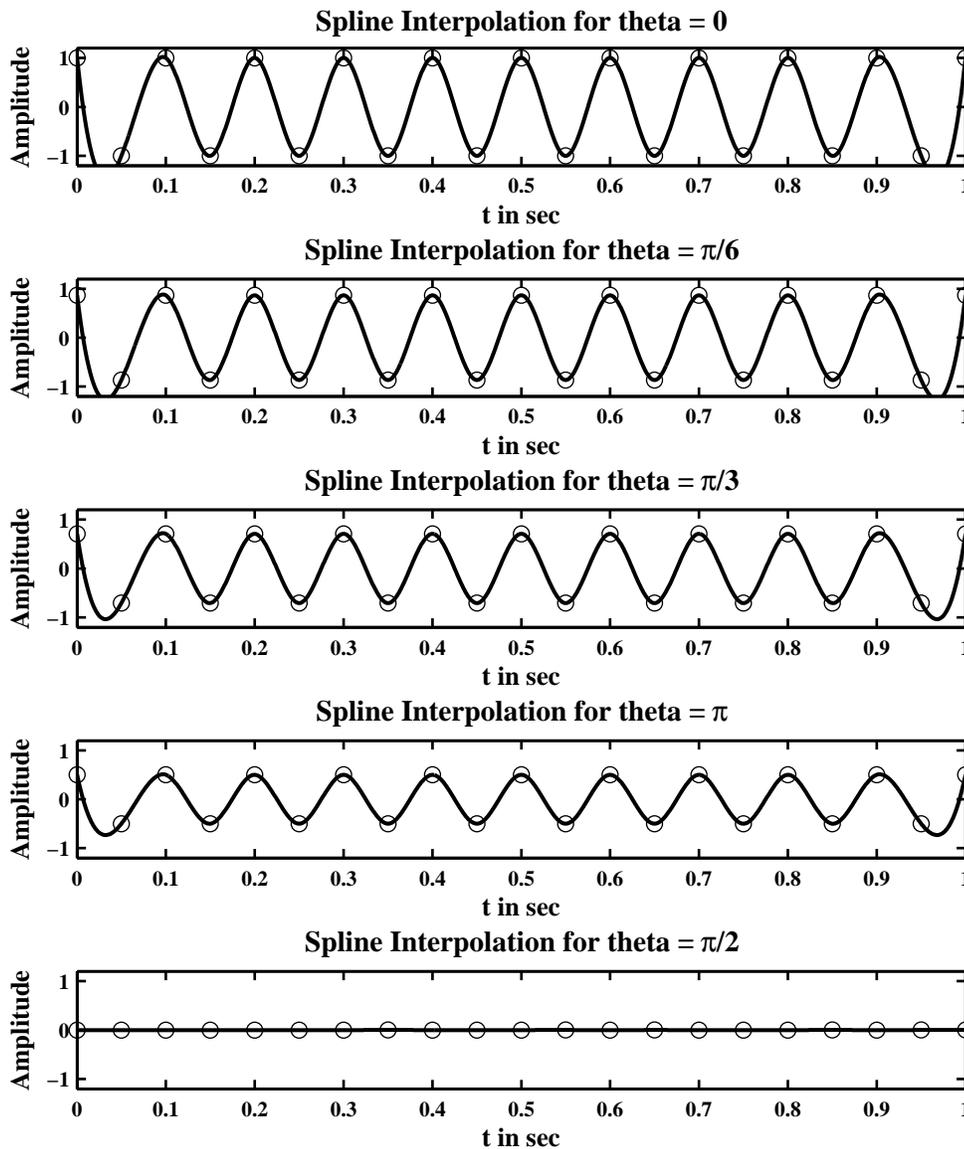
Figure 3.56: The sinc interpolation in Problem P3.22c.

```
ylabel('Amplitude','FontSize',LFS);
theta2 = pi/6; x2 = cos(20*pi*nTs+theta2); y_a2 = spline(nTs,x2,t);
subplot(5,1,2); plot(t,y_a2,'LineWidth',1.5); hold on; axis([0 1 -1.2 1.2]);
plot(nTs,x2,'o'); xlabel('t in sec','FontSize',LFS);
title('Spline Interpolation for theta = \pi/6','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta3 = pi/4; x3 = cos(20*pi*nTs+theta3); y_a3 = spline(nTs,x3,t);
subplot(5,1,3); plot(t,y_a3,'LineWidth',1.5); hold on; axis([0 1 -1.2 1.2]);
plot(nTs,x3,'o'); xlabel('t in sec','FontSize',LFS);
title('Spline Interpolation for theta = \pi/3','FontSize',TFS);
ylabel('Amplitude','FontSize',LFS);
theta4 = pi/3; x4 = cos(20*pi*nTs+theta4); y_a4 = spline(nTs,x4,t);
```

```
subplot(5,1,4); plot(t,y_a4,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x4,'o'); ylabel('Amplitude','FontSize',LFS);
title('Spline Interpolation for theta = \pi','FontSize',TFS);
xlabel('t in sec','FontSize',LFS);
theta5 = pi/2; x5 = cos(20*pi*nTs+theta5); y_a5 = spline(nTs,x5,t);
subplot(5,1,5); plot(t,y_a5,'LineWidth',1.5); axis([0 1 -1.2 1.2]); hold on;
plot(nTs,x5,'o'); ylabel('Amplitude','FontSize',LFS);
title('Spline Interpolation for theta = \pi/2','FontSize',TFS);
xlabel('t in sec','FontSize',LFS); print -deps2 ../EPSFILES/P0322c;
```

The reconstruction is shown in Figure 3.56.

(d) When a sinusoidal signal is sampled at $f = 2$ samples per cycle as is the case in this problem, then the resulting samples $x(n)$ has the amplitude that depends on the phase of the signal. In particular note that this amplitude is given by $\cos(\theta)$. Thus the amplitude of the reconstructed signal $y(t)$ is also equal to $\cos(\theta)$.

Chapter 4

The z-Transform

P4.1 The z-transform computation using definition (4.1).

1. $x(n) = \{3, 2, \underset{\uparrow}{1}, -2, -3\}$: Then $X(z) = 3z^2 + 2z + 1 - 2z^{-1} + 3z^{-2}, 0 < |z| < \infty$.

   MATLAB verification:
   ```
   % P0401a.m
   clc; close all;
   b1 = [0 2 3]; a1 = [1]; [delta,n] = impseq(0,0,4);
   xb1 = filter(b1,a1,delta); xb1 = fliplr(xb1); n1 = -fliplr(n);
   b2 = [1 -2 -3]; a2 = [1]; xb2 = filter(b2,a2,delta); n2 = n;
   [xa1,na1] = sigadd(xb1,n1,xb2,n2); xa2 = [0 0 3 2 1 -2 -3 0 0];
   error = max(abs(xa1-xa2))
   error =
        0
   ```

2. $x(n) = (0.8)^n u(n - 2)$: Then

$$X(z) = \sum_{n=2}^{\infty}(0.8)^n z^{-n} = (0.8)^2 z^{-2}\sum_{n=0}^{\infty}(0.8)^n z^{-n} = \frac{0.64z^{-2}}{1 - 0.8z^{-1}}; \ |z| > 0.8$$

   MATLAB verification:
   ```
   % P0401b.m
   clc; close all;
   b = [0 0 0.64]; a = [1 -0.8]; [delta,n] = impseq(0,0,10);
   xb1 = filter(b,a,delta);
   [u,n] = stepseq(2,0,10); xb2 = ((0.8).^n).*u;
   error =  max(abs(xb1-xb2))
   error =
      1.1102e-016
   ```

3. $x(n) = [(0.5)^n + (-0.8)^n]u(n)$: Then

$$X(z) = \mathcal{Z}\left[(0.5)^n u(n)\right] + \mathcal{Z}\left[(-0.8)^n u(n)\right] = \frac{1}{1 - 0.5z^{-1}} + \frac{1}{1 + 0.8z^{-1}}; \ \{|z| > 0.5\} \cap \{|z| > 0.8\}$$

$$= \frac{2 + 0.3z^{-1}}{1 + 0.3z^{-1} - 0.4z^{-2}}; \ |z| > 0.8$$

   MATLAB verification:

135

```
% P0401c.m
clc; close all;
b = [ 2 0.3]; a = [1 0.3 -0.4]; [delta,n] = impseq(0,0,7);
xb1 = filter(b,a,delta);
[u,n] = stepseq(0,0,7); xb2 = (((0.5).^n).*u)+(((-0.8).^n).*u);
error = max(abs(xb1-xb2))
error =
    1.1102e-016
```

4. $x(n) = 2^n \cos(0.4\pi n)u(-n)$: Consider

$$X(z) = \sum_{n=-\infty}^{\infty} 2^n \cos(0.4\pi n)u(-n)z^{-n} = \sum_{n=-\infty}^{0} 2^n \cos(0.4\pi n)z^{-n} = \sum_{n=0}^{\infty} 2^{-n} \cos(0.4\pi n)z^n$$

$$= \sum_{n=0}^{\infty} 2^{-n} \left(\frac{e^{j0.4\pi n} + e^{-j0.4\pi n}}{2} \right) z^n = \frac{1}{2} \sum_{n=0}^{\infty} \left(2^{-1}e^{j0.4\pi}z \right)^n + \frac{1}{2} \sum_{n=0}^{\infty} \left(2^{-1}e^{-j0.4\pi}z \right)^n$$

$$= \frac{1}{2} \left(\frac{1}{1 - \frac{1}{2}e^{j0.4\pi}z} \right) + \frac{1}{2} \left(\frac{1}{1 - \frac{1}{2}e^{-j0.4\pi}z} \right); \ |z| < 2$$

$$= \frac{1 - [0.5\cos(0.4\pi)]z}{1 - [\cos(0.4\pi)]z + 0.25z^2}; \ |z| < 2$$

Hence

$$X(z) = \frac{1 - [0.5\cos(0.4\pi)]z}{1 - 2[0.5\cos(0.4\pi)]z + 0.25z^2} = \frac{1 - [0.5\cos(0.4\pi)]z}{1 - [\cos(0.4\pi)]z + 0.25z^2}; \ |z| < 2$$

MATLAB verification:
```
% P0401d.m
clc; close all;
b = 0.5*[2 -cos(0.4*pi)]; a = [1 -cos(0.4*pi) 0.25];
[delta,n1] = impseq(0,0,7); xb1 = filter(b,a,delta); xb1 = fliplr(xb1);
[u,n2] = stepseq(-7,-7,0); xb2 = ((2.^n2).*cos(0.4*pi*n2)).*u;
error = max(abs(xb1-xb2))
error =
    2.7756e-017
```

5. $x(n) = (n+1)(3)^n u(n)$: Consider

$$x(n) = (n+1)(3)^n u(n) = n3^n u(n) + 3^n u(n)$$

Hence

$$X(z) = \mathcal{Z}\left[n3^n u(n)\right] + \mathcal{Z}\left[3^n u(n)\right] = -z\frac{d}{dz}\left(\frac{1}{1 - 3z^{-1}} \right) + \frac{1}{1 - 3z^{-1}}; \ |z| > 3$$

$$= \frac{3z^{-1}}{1 - 6z^{-1} + 9z^{-2}} + \frac{1}{1 - 3z^{-1}} = \frac{1 - 3z^{-1}}{1 - 9z^{-1} + 27z^{-2} - 27z^{-3}}; \ |z| > 3$$

MATLAB verification:
```
% P0401e.m
clc; close all;
b = [1 -3]; a = [1 -9 27 -27]; [delta,n1] = impseq(0,0,7);
xb1 = filter(b,a,delta);
[u,n2] = stepseq(0,0,7); xb2 = ((n2+1).*(3.^n2)).*u;
error = max(abs(xb1-xb2))
```

```
error =
  0
```

P4.2 Consider the sequence $x(n) = (0.9)^n \cos(\pi n/4) u(n)$. Let

$$
y(n) = \begin{cases} x(n/2), & n = 0, \pm 2, \pm 4, \cdots; \\ 0, & \text{otherwise.} \end{cases}
$$

1. The z-transform $Y(z)$ of $y(n)$ in terms of the z-transform $X(z)$ of $x(n)$: Consider

$$
Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n} = \sum_{n=-\infty}^{\infty} x(n/2) z^{-n}; \ n = 0, \pm 1, \pm 2, \ldots
$$

$$
= \sum_{m=-\infty}^{\infty} x(m) z^{-2m} = \sum_{m=-\infty}^{\infty} x(m) \left(z^2\right)^{-m} = X(z^2)
$$

2. The z-transform of $x(n)$ is given by

$$
X(z) = \mathcal{Z}\left[(0.9)^n \cos(\pi n/4) u(n)\right] = \frac{1 - [(0.9)\cos(\pi/4)] z^{-1}}{1 - 2[(0.9)\cos(\pi/4)] z^{-1} + (0.9)^2 z^{-2}}
$$

$$
= \frac{1 - 0.6364 z^{-1}}{1 - 1.2728 z^{-1} + 0.81 z^{-2}}; \ |z| > 0.9
$$

Hence

$$
Y(z) = \frac{1 - 0.6364 z^{-2}}{1 - 1.2728 z^{-2} + 0.81 z^{-4}}; \ |z| > \sqrt{0.9} = 0.9487
$$

3. MATLAB verification:

```
%  P0402c.m
clc; close all;
b = [1 0 -0.9*cos(pi/4)]; a = [1 0 2*-0.9*cos(pi/4) 0 0.81];
[delta,n1] = impseq(0,0,13); xb1 = filter(b,a,delta);
[u,n2] = stepseq(0,0,6); x1 = (((0.9).^n2).*cos(pi*n2/4)).*u;
xb2 = zeros(1,2*length(x1)); xb2(1:2:end) = x1;
error = max(abs(xb1-xb2))
error =
   1.2442e-016
```

P4.3 Computation of z-transform using properties and the z-transform table:

1. $x(n) = 2\delta(n-2) + 3u(n-3)$:

$$X(z) = 2z^{-2} + 3z^{-3}\frac{1}{1-z^{-1}} = \frac{2z^{-2}+z^{-3}}{1-z^{-1}}; \ |z| > 1$$

MATLAB script:
```
clc; close all;
b = [0 -8 0 -1.5 0 -1/16]; a = [1 0 3/16 0 3/256 0 1/(256*16)];
[delta,n1] = impseq(0,0,9); xb1 = filter(b,a,delta);
[u,n2] = stepseq(0,0,9);xb2 = (((n2-3).*((1/4).^(n2-2))).*cos((pi/2)*(n2-1))).*u;
error = max(abs(xb1-xb2))
error =
  0
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0403e');
[Hz,Hp,Hl] = zplane(b,a); set(Hz,'linewidth',1); set(Hp,'linewidth',1);
title('Pole-Zero plot','FontSize',TFS); print -deps2 ../epsfiles/P0403e;
```
The pole-zero plot is shown in Figure 4.1.
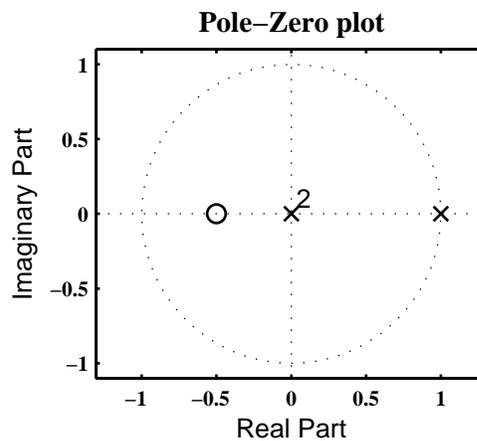
Figure 4.1: Problem P4.3.1 pole-zero plot

2. $x(n) = 3(0.75)^n \cos(0.3\pi n)u(n) + 4(0.75)^n \sin(0.3\pi n)u(n)$:

$$Z(z) = 3\frac{1 - [0.75\cos(0.3\pi)]z^{-1}}{1 - 2[0.75\cos(0.3\pi)]z^{-1} + (0.75)^2 z^{-2}} + 4\frac{[0.75\sin(0.3\pi)]z^{-1}}{1 - 2[0.75\cos(0.3\pi)]z^{-1} + (0.75)^2 z^{-2}}$$

$$= \frac{3 + [3\sin(0.3\pi) - 2.25\cos(0,3\pi)]z^{-1}}{1 - 1.5\cos(0,3\pi)z^{-1} + 0.5625z^{-2}} = \frac{3 + 1.1045z^{-1}}{1 - 0.8817z^{-1} + 0.5625z^{-2}}; \ |z| > 0.75$$

MATLAB script:
```
clc; close all;
b = [3 (3*sin(0.3*pi)-2.25*cos(0.3*pi))]; a = [1 -1.5*cos(0.3*pi) 0.5625];
[delta,n1] = impseq(0,0,7); xb1 = filter(b,a,delta); [u,n2] = stepseq(0,0, 7);
xb2 = 3*(((0.75).^n2).*cos(0.3*pi*n2)).*u+4*(((0.75).^n2).*sin(0.3*pi*n2)).*u ;
error = max(abs(xb1-xb2))
error =
  4.4409e-016
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0403b');
[Hz,Hp,Hl] = zplane(b,a); set(Hz,'linewidth',1); set(Hp,'linewidth',1);
title('Pole-Zero plot','FontSize',TFS); print -deps2 ../epsfiles/P0403b;
```
The pole-zero plot is shown in Figure 4.2.
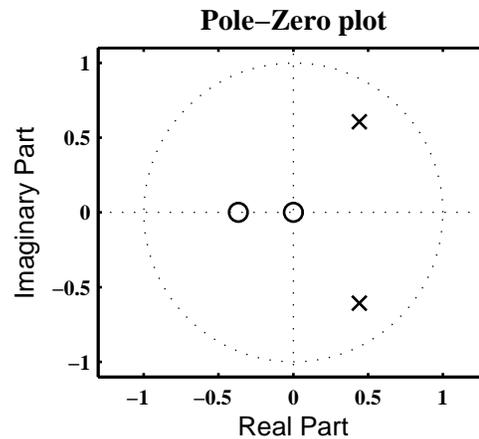
Figure 4.2: Problem P4.3.2 pole-zero plot

3. $x(n) = n \sin(\frac{\pi n}{3}) u(n) + (0.9)^n u(n-2)$: Consider

$$x(n) = n \sin\left(\frac{\pi n}{3}\right) u(n) + (0.9)^n u(n-2) = n \sin\left(\frac{\pi n}{3}\right) u(n) + 0.9^2 (0.9)^{n-2} u(n-2)$$

$$= n \sin\left(\frac{\pi n}{3}\right) u(n) + 0.81(0.9)^{n-2} u(n-2)$$

Hence

$$X(z) = \mathcal{Z}\left[n \sin\left(\frac{\pi n}{3}\right) u(n)\right] + \mathcal{Z}\left[0.81(0.9)^{n-2} u(n-2)\right]$$

$$= -z\frac{d}{dz}\left(\mathcal{Z}\left[\sin\left(\frac{\pi n}{3}\right) u(n)\right]\right) + 0.81 z^{-2} \mathcal{Z}\left[(0.9)^n u(n)\right]$$

$$= -z\frac{d}{dz}\left(\frac{\sin(\pi/3)z^{-1}}{1 - z^{-1} + z^{-2}}\right) + \frac{0.81 z^{-2}}{1 - 0.9 z^{-1}}; \ |z| > 1$$

$$= \frac{\sin(\pi/3)z^{-1} - \sin(\pi/3)z^{-3}}{1 - 2z^{-1} + 3z^{-2} - 2z^{-3} + z^{-4}} + \frac{0.81 z^{-2}}{1 - 0.9 z^{-1}}; \ |z| > 1$$

$$= \frac{0.866 z^{-1} + 0.0306 z^{-2} - 2.486 z^{-3} + 3.2094 z^{-4} - 1.62 z^{-5} + 0.81 z^{-6}}{1 - 2.9 z^{-1} + 4.8 z^{-2} - 4.7 z^{-3} + 2.8 z^{-4} - 0.9 z^{-5}}; \ |z| > 1$$

MATLAB script:
```
clc; close all;
b = [0 sin(pi/3) (0.81-0.9*sin(pi/3)) -(1.62+sin(pi/3)) ...
    (0.9*sin(pi/3)+2.43) -1.62 0.81];
a = [1 -2.9 4.8 -4.7 2.8 -0.9]; [delta,n1] = impseq(0,0,9);
xb1 = filter(b,a,delta);
[u2,n2] = stepseq(0,0,9); [u3,n3] = stepseq(2,0,9);
xb2 = (n2.*sin(pi/3*n2)).*u2+((0.9).^n3).*u3; error = max(abs(xb1-xb2))
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0403c');
[Hz,Hp,Hl] = zplane(b,a); set(Hz,'linewidth',1); set(Hp,'linewidth',1);
title('Pole-Zero plot','FontSize',TFS); print -deps2 ../epsfiles/P0403c;
error =
  2.1039e-014
```

The pole-zero plot is shown in Figure 4.3.
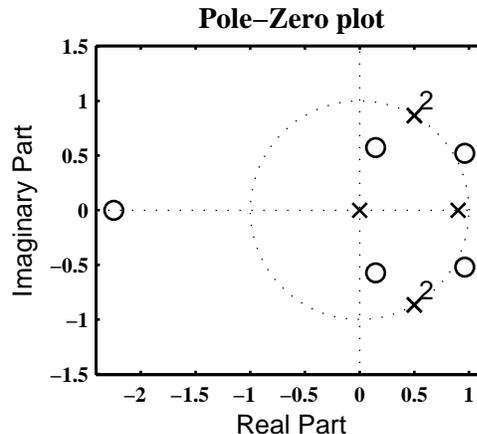


Figure 4.3: Problem P4.3.3 pole-zero plot

4. $x(n) = n^2(2/3)^{n-2}u(n-1)$: Consider

$$x(n) = n^2(2/3)^{n-2}u(n-1) = n^2(2/3)^{-1}(2/3)^{(n-1)}u(n-1) = \frac{3}{2}\left(n\left[n\left\{\left(\frac{2}{3}\right)^{(n-1)}u(n-1)\right\}\right]\right)$$

Let

$$x_1(n) = \left(\frac{2}{3}\right)^{(n-1)}u(n-1) \Rightarrow X_1(z) = \frac{z^{-1}}{1-\frac{2}{3}z^{-1}};\ |z| > \frac{2}{3}$$

Let

$$x_2(n) = n\,x_1(n) \Rightarrow X_2(z) = -z\frac{d}{dz}X_1(z) = \frac{z^{-1}}{1-\frac{4}{3}z^{-1}+\frac{4}{9}z^{-2}};\ |z| > \frac{2}{3}$$

Let

$$x_3(n) = n\,x_2(n) \Rightarrow X_3(z) = -z\frac{d}{dz}X_2(z) = \frac{z^{-1}-\frac{4}{9}z^{-3}}{1-\frac{8}{3}z^{-1}+\frac{8}{3}z^{-2}-\frac{32}{27}z^{-3}+\frac{16}{81}z^{-4}};\ |z| > \frac{2}{3}$$

Finally, $x(n) = \frac{3}{2}x_3(n)$. Hence

$$X(z) = \frac{3}{2}\left(\frac{z^{-1}-\frac{4}{9}z^{-3}}{1-\frac{8}{3}z^{-1}+\frac{8}{3}z^{-2}-\frac{32}{27}z^{-3}+\frac{16}{81}z^{-4}}\right) = \frac{\frac{3}{2}z^{-1}-\frac{2}{3}z^{-3}}{1-\frac{8}{3}z^{-1}+\frac{8}{3}z^{-2}-\frac{32}{27}z^{-3}+\frac{16}{81}z^{-4}};\ |z| > \frac{2}{3}$$

MATLAB script:
```
clc; close all; b = 3/2*[0 1 0 -4/9]; a = [1 -8/3 8/3 -32/27 16/81];
[delta,n1] = impseq(0,0,8); xb1 = filter(b,a,delta);
[u,n2] = stepseq(1,0,8); xb2 = ((n2.^2).*((2/3).^(n2-2))).*u;
error = max(abs(xb1-xb2))
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0403d');
[Hz,Hp,Hl] = zplane(b,a); set(Hz,'linewidth',1); set(Hp,'linewidth',1);
title('Pole-Zero plot','FontSize',TFS); print -deps2 ../epsfiles/P0403d;
error =
  9.7700e-015
```

The pole-zero plot is shown in Figure 4.4.
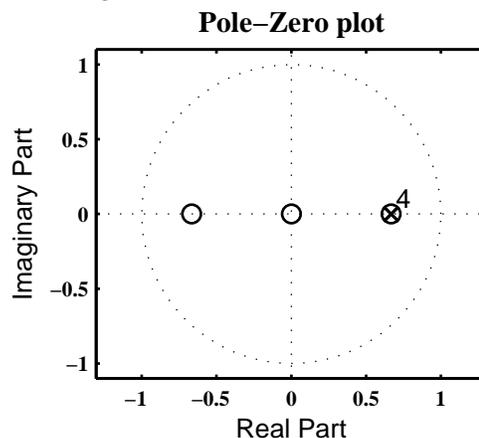


Figure 4.4: Problem P4.3.4 pole-zero plot

5. $x(n) = (n-3)\left(\frac{1}{4}\right)^{n-2}\cos\left\{\frac{\pi}{2}(n-1)\right\}u(n)$: Consider

$$x(n) = (n-3)\left(\frac{1}{4}\right)^{n-2}\cos\left\{\frac{\pi}{2}(n-1)\right\}u(n) = (n-3)\left(\frac{1}{4}\right)^{n-2}\sin\left\{\frac{\pi}{2}n\right\}u(n)$$

$$= (n-3)\left(\frac{1}{4}\right)^{-2}\left(\frac{1}{4}\right)^{n}\sin\left\{\frac{\pi}{2}n\right\}u(n) = 16(n-3)\left(\frac{1}{4}\right)^{n}\sin\left\{\frac{\pi}{2}n\right\}u(n)$$

$$= 16n\left(\frac{1}{4}\right)^{n}\sin\left\{\frac{\pi}{2}n\right\}u(n) - 48\left(\frac{1}{4}\right)^{n}\sin\left\{\frac{\pi}{2}n\right\}u(n)$$

Hence

$$X(z) = \mathcal{Z}\left[16n\left(\frac{1}{4}\right)^{n}\sin\left\{\frac{\pi}{2}n\right\}u(n)\right] - \mathcal{Z}\left[48\left(\frac{1}{4}\right)^{n}\sin\left\{\frac{\pi}{2}n\right\}u(n)\right]$$

$$= -z\frac{d}{dz}\left(\mathcal{Z}\left[\left(\frac{1}{4}\right)^{n}\sin\left\{\frac{\pi}{2}n\right\}u(n)\right]\right) - 48\frac{[\left(\frac{1}{4}\right)\sin(\pi/2)]z^{-1}}{1-2[\frac{1}{4}\cos(\pi/2)]z^{-1}+\left(\frac{1}{4}\right)^{2}z^{-2}};\ |z|>\frac{1}{4}$$

$$= -z\frac{d}{dz}\left(\frac{\frac{1}{4}z^{-1}}{1+\frac{1}{16}z^{-2}}\right) - \frac{12\,z^{-1}}{1+\frac{1}{16}z^{-2}} = \frac{4z^{-1}-\frac{1}{4}z^{-3}}{1+\frac{1}{8}z^{-2}+\frac{1}{256}z^{-4}} - \frac{12\,z^{-1}}{1+\frac{1}{16}z^{-2}};\ |z|>\frac{1}{4}$$

$$= \frac{-8z^{-1}-\frac{3}{2}z^{-3}-\frac{1}{16}z^{-5}}{1+\frac{3}{16}z^{-2}+\frac{3}{256}z^{-4}+\frac{1}{4096}z^{-6}};\ |z|>\frac{1}{4}$$

MATLAB script:
```
clc; close all; b = [0 -8 0 -1.5 0 -1/16]; a = [1 0 3/16 0 3/256 0 1/(256*16)];
[delta,n1] = impseq(0,0,9); xb1 = filter(b,a,delta);
[u,n2] = stepseq(0,0,9);xb2 = (((n2-3).*((1/4).^(n2-2))).*cos((pi/2)*(n2-1))).*u;
error = max(abs(xb1-xb2))
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0403e');
[Hz,Hp,Hl] = zplane(b,a); set(Hz,'linewidth',1); set(Hp,'linewidth',1);
title('Pole-Zero plot','FontSize',TFS); print -deps2 ../epsfiles/P0403e;
error =
   2.9392e-015
```

The pole-zero plot is shown in Figure 4.5.
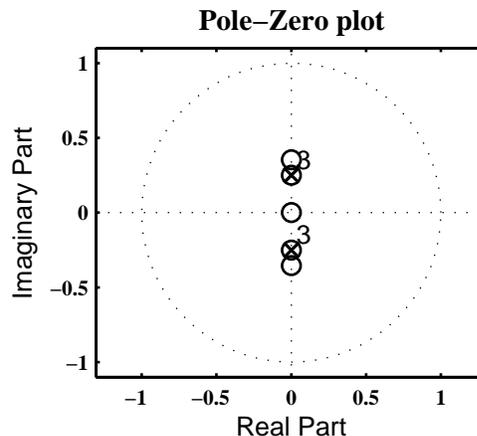


Figure 4.5: Problem P4.3.5 pole-zero plot

**P4.4** Let $x(n)$ be a complex-valued sequence with the real part $x_R(n)$ and the imaginary part $x_I(n)$.

1. The $z$-transform relations for real and imaginary parts.: Consider

$$X_R(z) \triangleq \mathcal{Z}[x_R(n)] = \mathcal{Z}\left[\frac{x(n) + x^*(n)}{2}\right] = \frac{\mathcal{Z}[x(n)] + \mathcal{Z}[x^*(n)]}{2}$$
$$= \frac{X(z) + X^*(z^*)}{2} \tag{4.1}$$

and

$$X_I(z) \triangleq \mathcal{Z}[x_I(n)] = \mathcal{Z}\left[\frac{x(n) - x^*(n)}{2}\right] = \frac{\mathcal{Z}[x(n)] - \mathcal{Z}[x^*(n)]}{2}$$
$$= \frac{X(z) - X^*(z^*)}{2} \tag{4.2}$$

2. Verification using $x(n) = \exp\{(-1 + j0.2\pi)n\}u(n)$: Consider

$$x(n) = \exp\{(-1 + j0.2\pi)n\}u(n) = e^{-n}e^{j0.2\pi n}u(n)$$
$$= e^{-n}\{\cos(0.2\pi n)u(n) + j\sin(0.2\pi n)u(n)\}$$

Hence the real and imaginary parts of $x(n)$, respectively, are

$$x_R(n) = e^{-n}\cos(0.2\pi n)u(n) = (1/e)^n\cos(0.2\pi n)u(n) \tag{4.3}$$
$$x_I(n) = e^{-n}\sin(0.2\pi n)u(n) = (1/e)^n\sin(0.2\pi n)u(n) \tag{4.4}$$

with $z$-transforms , respectively,

$$X_R(z) = \frac{1 - [(1/e)\cos(0.2\pi)]z^{-1}}{1 - [(2/e)\cos(0.2\pi)]z^{-1} + (1/e^2)z^{-2}} = \frac{1 - 0.2976z^{-1}}{1 - 0.5952z^{-1} + 0.1353z^{-2}}, \ |z| > 1/e \tag{4.5}$$

$$X_I(z) = \frac{[(1/e)\sin(0.2\pi)]z^{-1}}{1 - [(2/e)\cos(0.2\pi)]z^{-1} + (1/e^2)z^{-2}} = \frac{0.2162z^{-1}}{1 - 0.5952z^{-1} + 0.1353z^{-2}}, \ |z| > 1/e \tag{4.6}$$

The $z$-transform of $x(n)$ is

$$X(z) = \mathcal{Z}\left[\left(e^{-1+j0.2\pi}\right)^n u(n)\right] = \frac{1}{1 - e^{-1+j0.2\pi}z^{-1}}, \ |z| > 1/e \tag{4.7}$$

Substituting (4.7) in (4.1),

$$X_R(z) = \frac{1}{2}\left[\frac{1}{1 - e^{-1+j2\pi}z^{-1}} + \left(\frac{1}{1 - e^{-1+j2\pi}z^{-*}}\right)^*\right]$$
$$= \frac{1}{2}\left[\frac{1}{1 - e^{-1+j2\pi}z^{-1}} + \frac{1}{1 - e^{-1-j2\pi}z^{-1}}\right] = \frac{1}{2}\left[\frac{2 - 0.5952z^{-1}}{1 - 0.5952z^{-1} + 0.1353z^{-2}}\right]$$
$$= \frac{1 - 0.2976z^{-1}}{1 - 0.5952z^{-1} + 0.1353z^{-2}}, \ |z| > e^{-1} \tag{4.8}$$

as expected in (4.5). Similarly, Substituting (4.7) in (4.2),

$$X_I(z) = \frac{1}{2}\left[\frac{1}{1 - e^{-1+j2\pi}z^{-1}} - \left(\frac{1}{1 - e^{-1+j2\pi}z^{-*}}\right)^*\right]$$
$$= \frac{1}{2}\left[\frac{1}{1 - e^{-1+j2\pi}z^{-1}} - \frac{1}{1 - e^{-1-j2\pi}z^{-1}}\right] = \frac{1}{2}\left[\frac{0.2162z^{-1}}{1 - 0.5952z^{-1} + 0.1353z^{-2}}\right]$$
$$= \frac{0.2162z^{-1}}{1 - 0.5952z^{-1} + 0.1353z^{-2}}, \ |z| > e^{-1} \tag{4.9}$$

as expected in (4.6).

**P4.5** The $z$-transform of $x(n)$ is $X(z) = 1/(1 + 0.5z^{-1})$, $|z| \geq 0.5$.

1. The $z$-transforms of $x_1(n) = x(3-n) + x(n-3)$:

$$X_1(z) = \mathcal{Z}[x_1(n)] = \mathcal{Z}[x(3-n) + x(n-3)] = \mathcal{Z}[x\{-(n-3)\}] + \mathcal{Z}[x(n-3)]$$

$$= z^{-3}X(1/z) + z^{-3}X(z) = z^{-3}\left[\frac{1}{1+0.5z} + \frac{1}{1+0.5z^{-1}}\right], \quad 0.5 < |z| < 2$$

$$= \frac{0.5z^{-3} + 2z^{-4} + 0.5z^{-5}}{0.5 + 1.25z^{-1} + 0.5z^{-2}}, \quad 0.5 < |z| < 2$$

2. The $z$-transforms of $x_2(n) = (1 + n + n^2)x(n)$:

$$X_2(z) = \mathcal{Z}\left[\left(1 + n + n^2\right)x(n)\right] = \mathcal{Z}\left[x(n) + n\,x(n) + n^2x(n)\right] = X(z) - z\frac{d}{dz}X(z) + z^2\frac{d^2}{dz^2}X(z)$$

$$= \frac{1}{1+0.5z^{-1}} - \frac{0.5z^{-1}}{\left(1+0.5z^{-1}\right)^2} - \frac{z^{-1} + 0.5z^{-2}}{\left(1+0.5z^{-1}\right)^4} = \frac{1 - 0.25z^{-2}}{\left(1+0.5z^{-1}\right)^4}, \quad |z| > 0.5$$

3. The $z$-transforms of $x_3(n) = \left(\frac{1}{2}\right)^n x(n-2)$:

$$X_3(z) = \mathcal{Z}\left[\left(\frac{1}{2}\right)^n x(n-2)\right] = \mathcal{Z}[x(n-2)]\Big|_{\left(\frac{1}{2}\right)^{-1}z} = \mathcal{Z}[x(n-2)]\Big|_{2z}$$

$$= \left[z^{-2}X(z)\right]\Big|_{2z} = \left[\frac{z^{-2}}{1+0.5z^{-1}}, \; |z| > 0.5\right]\Big|_{2z} = \frac{0.25z^{-2}}{1+0.25z^{-1}}, \quad |z| > 0.25$$

4. The $z$-transforms of $x_4(n) = x(n+2) * x(n-2)$:

$$X_4(z) = \mathcal{Z}[x(n+2) * x(n-2)] = \left\{z^2X(z)\right\}\left\{z^{-2}X(z)\right\} = X^2(z)$$

$$= \frac{1}{\left(1+0.5z^{-1}\right)^2}, \quad |z| > 0.5$$

5. The $z$-transforms of $x_5(n) = \cos(\pi n/2)x^*(n)$:

$$X_5(z) = \mathcal{Z}\left[\cos(\pi n/2)x^*(n)\right] = \mathcal{Z}\left[\left(\frac{e^{j\pi n/2} + e^{-j\pi n/2}}{2}\right)x^*(n)\right]$$

$$= \frac{1}{2}\left(\mathcal{Z}\left[e^{j\pi n/2}x^*(n)\right] + \mathcal{Z}\left[e^{-j\pi n/2}x^*(n)\right]\right)$$

$$= \frac{1}{2}\left(\mathcal{Z}\left[x^*(n)\right]\Big|_{e^{-j\pi/2}z} + \mathcal{Z}\left[x^*(n)\right]\Big|_{e^{j\pi/2}z}\right)$$

$$= \frac{1}{2}\left[X^*\left(e^{j\pi/2}z^*\right) + X^*\left(e^{-j\pi/2}z^*\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{1+0.5e^{-j\pi/2}z^{-1}} + \frac{1}{1+0.5e^{j\pi/2}z^{-1}}\right]$$

$$= \frac{1}{1+0.25z^{-2}}, \quad |z| > 0.5$$

**P4.6** The $z$-transform of $x(n)$ is

$$X(z) = \frac{1 + z^{-1}}{1 + \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}; \quad |z| > \frac{1}{2}$$

1. The $z$-transforms of $x_1(n) = x(3 - n) + x(n - 3)$:

$$X_1(z) = \mathcal{Z}[x_1(n)] = \mathcal{Z}[x(3 - n) + x(n - 3)] = \mathcal{Z}[x\{-(n - 3)\}] + \mathcal{Z}[x(n - 3)]$$

$$= z^{-3}X(1/z) + z^{-3}X(z) = z^{-3}\left[\frac{1 + z}{1 + \frac{5}{6}z + \frac{1}{6}z^2} + \frac{1 + z^{-1}}{1 + \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}\right], \quad 0.5 < |z| < 2$$

$$= \frac{36z^{-1} + 72z^{-2} + 36z^{-3}}{6 + 35z^{-1} + 62z^{-2} + 35z^{-3} + 6z^{-4}}, \quad 0.5 < |z| < 2$$

2. The $z$-transforms of $x_2(n) = (1 + n + n^2)x(n)$:

$$X_2(z) = \mathcal{Z}\left[\left(1 + n + n^2\right)x(n)\right] = \mathcal{Z}\left[x(n) + n\,x(n) + n^2 x(n)\right] = X(z) - z\frac{d}{dz}X(z) + z^2\frac{d^2}{dz^2}X(z)$$

or $X_2(z) =$

$$\frac{1 + \frac{17}{3}z^{-1} + \frac{1}{2}z^{-2} + \frac{1429}{108}z^{-3} + \frac{2399}{286}z^{-4} + \frac{215}{72}z^{-5} + \frac{829}{1944}z^{-6} - \frac{167}{1944}z^{-7} - \frac{7}{144}z^{-8} - \frac{2}{243}z^{-9} - \frac{1}{1944}z^{-10}}{1 + 5z^{-1} + \frac{137}{12}z^{-2} + \frac{425}{27}z^{-3} + \frac{6305}{432}z^{-4} + \frac{2694}{281}z^{-5} + \frac{1711}{374}z^{-6} + \frac{449}{281}z^{-7} + \frac{1258}{3103}z^{-8} + \frac{425}{5832}z^{-9} + \frac{137}{15552}z^{-10} + \frac{5}{7776}z^{-11} + \frac{1}{46656}z^{-12}}, \quad |z| > 0.5$$

3. The $z$-transforms of $x_3(n) = \left(\frac{1}{2}\right)^n x(n - 2)$:

$$X_3(z) = \mathcal{Z}\left[\left(\frac{1}{2}\right)^n x(n - 2)\right] = \mathcal{Z}[x(n - 2)]\big|_{\left(\frac{1}{2}\right)^{-1}z} = \mathcal{Z}[x(n - 2)]\big|_{2z}$$

$$= \left[z^{-2}X(z)\right]\big|_{2z} = [, \ |z| > 0.5]\big|_{2z} = \frac{\frac{1}{4}z^{-2} + \frac{1}{8}z^{-3}}{1 + \frac{5}{12}z^{-1} + \frac{1}{24}z^{-2}}, \quad |z| > 0.25$$

4. The $z$-transforms of $x_4(n) = x(n + 2) * x(n - 2)$:

$$X_4(z) = \mathcal{Z}[x(n + 2) * x(n - 2)] = \left\{z^2X(z)\right\}\left\{z^{-2}X(z)\right\} = X^2(z)$$

$$= \frac{1 + 2z^{-1} + z^{-2}}{1 + \frac{5}{3}z^{-1} + \frac{37}{36}z^{-2} + \frac{5}{18}z^{-3} + \frac{1}{36}z^{-4}}, \quad |z| > 0.5$$

5. The $z$-transforms of $x_5(n) = \cos(\pi n/2)x^*(n)$:

$$X_5(z) = \mathcal{Z}\left[\cos(\pi n/2)x^*(n)\right] = \mathcal{Z}\left[\left(\frac{e^{j\pi n/2} + e^{-j\pi n/2}}{2}\right)x^*(n)\right]$$

$$= \frac{1}{2}\left(\mathcal{Z}\left[e^{j\pi n/2}x^*(n)\right] + \mathcal{Z}\left[e^{-j\pi n/2}x^*(n)\right]\right)$$

$$= \frac{1}{2}\left(\mathcal{Z}\left[x^*(n)\right]\big|_{e^{-j\pi/2}z} + \mathcal{Z}\left[x^*(n)\right]\big|_{e^{j\pi/2}z}\right)$$

$$= \frac{1}{2}\left[X^*\left(e^{j\pi/2}z^*\right) + X^*\left(e^{-j\pi/2}z^*\right)\right]$$

$$= \frac{1}{2}\left[\frac{1 - jz^{-1}}{1 - j\frac{5}{6}z^{-1} - \frac{1}{6}z^{-2}}\frac{1 + jz^{-1}}{1 + j\frac{5}{6}z^{-1} - \frac{1}{6}z^{-2}}\right]$$

$$= \frac{2 + \frac{4}{3}z^{-2}}{1 + \frac{13}{36}z^{-2} + \frac{1}{36}z^{-4}}, \quad |z| > 0.5$$

**P4.7** The inverse $z$-transform of $X(z)$ is $x(n) = (1/2)^n u(n)$. Sequence computation Using the $z$-transform properties:

1. $X_1(z) = \frac{z-1}{z}X(z)$: Consider

$$
x_1(n) = \mathcal{Z}^{-1}[X_1(z)] = \mathcal{Z}^{-1}\left[\left(1 - \frac{1}{z}\right)X(z)\right] = \mathcal{Z}^{-1}\left[X(z) - z^{-1}X(z)\right]
$$
$$
= x(n) - x(n-1) = 0.5^n u(n) - 0.5^{n-1}u(n-1) = 1 - 0.5^n u(n-1)
$$

2. $X_2(z) = zX(z^{-1})$: Consider

$$
x_2(n) = \mathcal{Z}^{-1}[X_2(z)] = \mathcal{Z}^{-1}\left[zX(z^{-1})\right] = \mathcal{Z}^{-1}\left[X(z^{-1})\right]\Big|_{n \to (n+1)} = \mathcal{Z}^{-1}[X(z)]\Big|_{n \to -(n+1)}
$$
$$
= (0.5)^{-(n+1)}u(-n-1) = 2^{n+1}u(-n-1)
$$

3. $X_3(z) = 2X(3z) + 3X(z/3)$: Consider

$$
x_3(n) = \mathcal{Z}^{-1}[X_3(z)] = \mathcal{Z}^{-1}[2X(3z) + 3X(z/3)] = 2(3^{-n})x(n) + 3(3^n)x(n)
$$
$$
= 2(3^{-n})(2^{-n})u(n) + 3(3^n)(2^{-n})u(n) = \left[2\left(\frac{1}{6}\right)^n + 3\left(\frac{3}{2}\right)^n\right]u(n)
$$

4. $X_4(z) = X(z)X(z^{-1})$: Consider

$$
x_4(n) = \mathcal{Z}^{-1}[X_4(z)] = \mathcal{Z}^{-1}\left[X(z)X(z^{-1})\right] = x(n) * x(-n)
$$
$$
= \left[0.5^n u(n)\right] * \left[2^n u(-n)\right] = \sum_{k=-\infty}^{\infty}(0.5)^k u(k)2^{n-k}u(-n+k)
$$
$$
= \begin{cases} 2^n \sum_{k=0}^{\infty}(0.5)^k 2^{-k}, & n < 0; \\ 2^n \sum_{k=n}^{\infty}(0.5)^k 2^{-k}, & n \geq 0. \end{cases} = \begin{cases} 2^n \sum_{k=-\infty}^{\infty}(0.25)^k, & n < 0; \\ 2^n 2^{-2n} \sum_{k=-\infty}^{\infty}(0.25)^k, & n \geq 0. \end{cases}
$$
$$
= \frac{4}{3}2^{|n|}
$$

5. $X_5(z) = z^2 \frac{dX(z)}{dz}$: Consider

$$
x_5(n) = \mathcal{Z}^{-1}[X_5(z)] = \mathcal{Z}^{-1}\left[z^2 \frac{dX(z)}{dz}\right] = n^2 x(n)
$$
$$
= n^2(1/2)^n u(n)
$$

**P4.8** If sequences $x_1(n)$, $x_2(n)$, and $x_3(n)$ are related by $x_3(n) = x_1(n) * x_2(n)$ then

$$\sum_{n=-\infty}^{\infty} x_3(n) = \left( \sum_{n=-\infty}^{\infty} x_1(n) \right) \left( \sum_{n=-\infty}^{\infty} x_2(n) \right)$$

1. Proof using the definition of convolution:

$$\sum_{n=-\infty}^{\infty} x_3(n) = \sum_{n=-\infty}^{\infty} x_1(n) * x_2(n) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x_1(k) x_2(n-k) = \left( \sum_{k=-\infty}^{\infty} x_1(k) \right) \sum_{n=-\infty}^{\infty} x_2(n-k)$$

$$= \left( \sum_{n=-\infty}^{\infty} x_1(n) \right) \left( \sum_{n=-\infty}^{\infty} x_2(n) \right)$$

as expected.

2. Proof using the convolution property:

$$\mathcal{Z}[x_3(n)] = \mathcal{Z}[x_1(n) * x_2(n)] = \mathcal{Z}[x_1(n)] \, \mathcal{Z}[x_2(n)]$$

$$\left( \sum_{n=-\infty}^{\infty} x_3(n) z^{-n} \right)\Bigg|_{z=1} = \left( \sum_{n=-\infty}^{\infty} x_1(n) z^{-n} \right)\Bigg|_{z=1} \left( \sum_{n=-\infty}^{\infty} x_2(n) z^{-n} \right)\Bigg|_{z=1}$$

$$\sum_{n=-\infty}^{\infty} x_3(n) = \left( \sum_{n=-\infty}^{\infty} x_1(n) \right) \left( \sum_{n=-\infty}^{\infty} x_2(n) \right)$$

as expected.

3. MATLAB verification:

```
clc; close all;
N = 1000; n1 = [0:N]; x1 = rand(1,length(n1));
n2 = [0:N]; x2 = rand(1,length(n2)); [x3,n3] = conv_m(x1,n1,x2,n2);
sumx1 = sum(x1); sumx2 = sum(x2); sumx3 = sum(x3);
error = max(abs(sumx3-sumx1*sumx2))
error =
  2.9104e-011
```

P4.9 Polynomial operations using MATLAB:

1. $X_1(z) = (1 - 2z^{-1} + 3z^{-2} - 4z^{-3})(4 + 3z^{-1} - 2z^{-2} + z^{-3})$

```
% P0409a.m
clc; close all;
n1 = [0:3]; y1 = [1 -2 3 -4]; n2 = [0:3]; y2 = [4 3 -2 1];
[x1,n] = conv_m(y1,n1,y2,n2)
x1 =
    4    -5     4    -2   -20    11    -4
n =
    0     1     2     3     4     5     6
```

Hence

$$X_1(z) = 4 - 5z^{-1} + 4z^{-2} - 2z^{-3} - 20z^{-4}11 + z^{-5} - 4z^{-6}$$

2. $X_2(z) = (z^2 - 2z + 3 + 2z^{-1} + z^{-2})(z^3 - z^{-3})$

```
% P0409b.m
clc; close all;
n1 = [-2:2]; y1 = [1 -2 3 2 1]; n2 = [-3:3]; y2 = [1 0 0 0 0 0 1];
[x2,n] = conv_m(y1,n1,y2,n2)
x2 =
    1    -2     3     2     1     0     1    -2     3     2     1
n =
   -5    -4    -3    -2    -1     0     1     2     3     4     5
```

Hence

$$X_2(z) = z^5 - 2z^4 + 3z^3 + 2z^4 + z + z^{-1} - 2z^{-2} + 3z^{-3} + 2z^{-4} + z^{-5}$$

3. $X_3(z) = (1 + z^{-1} + z^{-2})^3$

```
% P0409c.m
clc; close all;
n1 = [0 1 2]; y1 = [1 1 1]; [y2,n2] = conv_m(y1,n1,y1,n1);
[x3,n] = conv_m(y1,n1,y2,n2)
x3 =
    1     3     6     7     6     3     1
n =
    0     1     2     3     4     5     6
```

Hence

$$X_3(z) = 1 + 3z^{-1} + 6z^{-2} + 7z^{-3} + 6z^{-4} + 3z^{-5} + z^{-6}$$

4. $X_4(z) = X_1(z)X_2(z) + X_3(z)$

```
% P0409d.m
clc; close all;
n11 = [0:3]; y11 = [1 -2 3 -4]; n12 = [0:3]; y12 = [4 3 -2 1];
[y13,n13] = conv_m(y11,n11,y12,n12);
n21 = [-2:2]; y21 = [1 -2 3 2 1]; n22 = [-3:3]; y22 = [1 0 0 0 0 0 1];
[y23,n23] = conv_m(y21,n21,y22,n22);
n31 = [0 1 2]; y31 = [1 1 1];
[y32,n32] = conv_m(y31,n31,y31,n31); [y33,n33] = conv_m(y31,n31,y32,n32);
[y41,n41] = conv_m(y13,n13,y23,n23); [x4,n] = sigadd(y41,n41,y33,n33)
x4 =
  Columns 1 through 12
      4    -13    26    -17    -10     49    -79     -8     23     -8    -11     49
  Columns 13 through 17
    -86     -1    -10      3     -4
n =
  Columns 1 through 12
     -5     -4     -3     -2     -1      0      1      2      3      4      5      6
  Columns 13 through 17
      7      8      9     10     11
```

Hence

$$X_4(z) = 4z^5 - 13z^4 + 26z^3 - 17z^2 - 10z^1 + 49 - 79z^{-1} - 8z^{-2} + 23z^{-3} - 8z^{-4} - 11z^{-5}$$
$$+ 49z^{-6} - 86z^{-7} - z^{-8} - 10z^{-9} + 3z^{-10} - 4z^{-11}$$

5. $X_5(z) = (z^{-1} - 3z^{-3} + 2z^{-5} + 5z^{-7} - z^{-9})(z + 3z^2 + 2z^3 + 4z^4)$

```
% P0409e.m
clc; close all;
n1 = [0:9]; y1 = [0 1 0 -3 0 2 0 5 0 -1]; n2 = [-4:0]; y2 = [4 2 3 1 0];
[x5,n] = conv_m(y1,n1,y2,n2)
x5 =
  Columns 1 through 12
      0      4      2     -9     -5     -1      1     26     12     11      3     -3
  Columns 13 through 14
     -1      0
n =
  Columns 1 through 12
     -4     -3     -2     -1      0      1      2      3      4      5      6      7
  Columns 13 through 14
      8      9
```

Hence

$$X_5(z) = 4z^3 + 2z^2 - 9z^1 - 5 - z^{-1} + z^{-2} + 26z^{-3} + 12z^{-4} + 11z^{-5} + 3z^{-6} - 3z^{-7} - z^{-8}$$

P4.10 The MATLAB function `deconv_m`:

```
function [p,np,r,nr] = deconv_m(b,nb,a,na)
% Modified deconvolution routine for noncausal sequences
% function [p,np,r,nr] = deconv_m(b,nb,a,na)
%
%   p = polynomial part of support np1 <= n <= np2
% np = [np1, np2]
%   r = remainder part of support nr1 <= n <= nr2
% nr = [nr1, nr2]
%   b = numerator polynomial of support nb1 <= n <= nb2
% nb = [nb1, nb2]
%   a = denominator polynomial of support na1 <= n <= na2
% na = [na1, na2]
%
[p,r] = deconv(b,a);
np1 = nb(1) - na(1); np2 = np1 + length(p)-1; np = [np1:np2];
nr1 = nb(1); nr2 = nr1 + length(r)-1; nr = [nr1:nr2];
```

MATLAB verification:

```
% P0410
clc; close all;
nb = [-2:3]; b = [1 1 1 1 1 1]; na = [-1:1]; a = [1 2 1];
[p,np,r,nr] = deconv_m(b,nb,a,na)
p =
     1    -1     2    -2
np =
    -1     0     1     2
r =
     0     0     0     0     3     3
nr =
    -2    -1     0     1     2     3
```

Hence

$$\frac{z^2 + z + 1 + z^{-1} + z^{-2} + z^{-3}}{z + 2 + z^{-1}} = (z - 1 + 2z^{-1} - 2z^{-2}) + \frac{3z^{-2} + 3z^{-3}}{z + 2 + z^{-1}}$$

P4.11 Inverse z-transforms using the partial fraction expansion method:

1. $X_1(z) = (1 - z^{-1} - 4z^{-2} + 4z^{-3})/(1 - \frac{11}{4}z^{-1} + \frac{13}{8}z^{-2} - \frac{1}{4}z^{-3})$. The sequence is right-sided.
   MATLAB script:

```
% P0611a: Inverse z-Transform of X1(z)
clc; close all;
b1 = [1,-1,-4,4]; a1 = [1,-11/4,13/8,-1/4];
[R,p,k] = residuez(b1,a1)
 R =
      0.0000
    -10.0000
     27.0000
 p =
      2.0000
      0.5000
      0.2500
 k =
     -16
```

   or

   $$X_1(z) = \frac{1 - z^{-1} - 4z^{-2} + 4z^{-3}}{1 - \frac{11}{4}z^{-1} + \frac{13}{8}z^{-2} - \frac{1}{4}z^{-3}} = -16 + \frac{0}{1 - 2z^{-1}} - \frac{10}{1 - 0.5z^{-1}} + \frac{27}{1 - 0.25z^{-1}}, \ |z| > 0.5$$

   Note that from the second term on the right, there is a pole-zero cancellation. Hence

   $$x_1(n) = -16\delta(n) - 10(0.5)^n u(n) + 27(0.25)^n u(n)$$

2. $X_2(z) = (1 + z^{-1} - 4z^{-2} + 4z^{-3})/(1 - \frac{11}{4}z^{-1} + \frac{13}{8}z^{-2} - \frac{1}{4}z^{-3})$. The sequence is absolutely summable.
   MATLAB script:

```
% P0611b: Inverse z-Transform of X2(z)
clc; close all;
b2 = [1,1,-4,4]; a2 = [1,-11/4,13/8,-1/4];
[R,p,k] = residuez(b2,a2)
 R =
      1.5238
    -12.6667
     28.1429
 p =
      2.0000
      0.5000
      0.2500
 k =
     -16
```

   or

   $$X_2(z) = \frac{1 - z^{-1} - 4z^{-2} + 4z^{-3}}{1 - \frac{11}{4}z^{-1} + \frac{13}{8}z^{-2} - \frac{1}{4}z^{-3}} = -16 + \frac{1.5238}{1 - 2z^{-1}} - \frac{12.6667}{1 - 0.5z^{-1}} + \frac{28.1429}{1 - 0.25z^{-1}}, \ 0.5 < |z| > 2$$

Hence

$$x_2(n) = -16\delta(n) - 1.5238(2)^n u(-n-1) - 12.6667(0.5)^n u(n) + 28.1429(0.25)^n u(n)$$

3. $X_3(z) = (z^3 - 3z^2 + 4z + 1)/(z^3 - 4z^2 + z - 0.16)$. The sequence is left-sided. Consider

$$X_3(z) = \frac{z^3 - 3z^2 + 4z + 1}{z^3 - 4z^2 + z - 0.16} = \frac{1 - 3z^{-1} + 4z^{-2} + z^{-3}}{1 - 4z^{-1} + z^{-2} - 0.16z^{-3}}$$

MATLAB script for the PFE:

```
% P0611c: Inverse z-Transform of X3(z)
clc; close all;
b3 = [1,-3,4,1]; a3 = [1,-4,1,-0.16];
[R,p,k] = residuez(b3,a3)
% R =
%     0.5383
%     3.3559 + 5.7659i
%     3.3559 - 5.7659i
% p =
%     3.7443
%     0.1278 + 0.1625i
%     0.1278 - 0.1625i
% k =
%    -6.2500

r = abs(p(2))
% r =
%     0.2067

[b,a] = residuez(R(2:3),p(2:3),[])
% b =
%     6.7117    -2.7313
% a =
%     1.0000    -0.2557     0.0427
```

or

$$X_3(z) = -6.25 + \frac{0.5383}{1 - 3.7443z^{-1}} + \frac{3.3559 + j5.7659}{1 - (0.1278 + j0.1625)z^{-1}}$$
$$+ \frac{3.3559 - j5.7659}{1 - (0.1278 - j0.1625)z^{-1}}, \quad |z| < 0.2067$$

Hence

$$x_3(n) = -6.25\delta(n) - 0.5383(3.7443)^n u(-n-1)$$
$$- (3.3559 + j5.7659)(0.1278 + j0.1625)^n u(-n-1)$$
$$- (3.3559 - j5.7659)(0.1278 - j0.1625)^n u(-n-1)$$

4. $X_4(z) = z/(z^3 + 2z^2 + 1.25z + 0.25)$, $|z| > 1$. Consider

$$X_4(z) = \frac{z}{z^3 + 2z^2 + 1.25z + 0.25} = \frac{z^{-2}}{1 + 2z^{-1} + 1.25z^{-2} + 0.25z^{-3}}$$

MATLAB script for the PFE:

```
% P0611d: Inverse z-Transform of X4(z)
clc; close all;
b4 = [0,0,1]; a4 = [1,2,1.25,0.25];
[R,p,k] = residuez(b4,a4)
 R =
    4.0000
   -0.0000 + 0.0000i
   -4.0000 - 0.0000i
 p =
   -1.0000
   -0.5000 + 0.0000i
   -0.5000 - 0.0000i
 k =
    []
```

or

$$X_4(z) = \frac{4}{1 + z^{-1}} - \frac{4}{\left(1 + 0.5z^{-1}\right)^2} = \frac{4}{1 + z^{-1}} - 8z\frac{0.5z^{-1}}{\left(1 + 0.5z^{-1}\right)^2}, \quad |z| > 1$$

Hence

$$x_4(n) = 4(-1)^n u(n) - 8(n+1)(0.5)^{n+1} u(n+1)$$

5. $X_5(z) = z/(z^2 - 0.25)^2$, $|z| < 0.5$. Consider

$$X_5(z) = \frac{z}{(z^2 - 0.25)^2} = \frac{z^{-3}}{(1 - 0.25z^{-2})^2}, \quad |z| < 0.5$$

MATLAB script for PFE:

```
% P0611e: Inverse z-Transform of X5(z)
clc; close all;
b5 = [0,0,0,1]; a5 = conv([1,0,-0.25],[1,0,-0.25]);
[R,p,k] = residuez(b5,a5)
 R =
    4.0000 + 0.0000i
   -2.0000
   -4.0000 - 0.0000i
    2.0000 + 0.0000i
 p =
   -0.5000
   -0.5000
    0.5000 + 0.0000i
    0.5000 - 0.0000i
 k =
    []
```

or

$$X_5(z) = \frac{4}{1 + 0.5z^{-1}} - 2\frac{1}{(1 + 0.5z^{-1})^2} - 4\frac{1}{1 - 0.5z^{-1}} + 2\frac{1}{(1 - 0.5z^{-1})^2}, \; |z| < 0.5$$

$$= \frac{4}{1 - (-0.5)z^{-1}} + 4z\frac{(-0.5)z^{-1}}{[1 - (-0.5)z^{-1}]^2} - 4\frac{1}{1 - 0.5z^{-1}} + 4z\frac{0.5z^{-1}}{(1 - 0.5z^{-1})^2}, \; |z| < 0.5$$

Hence

$$x_5(n) = -4(-0.5)^n u(-n-1) - 4(n+1)(-0.5)^n u[-(n+1)-1] + 4(0.5)^n u(-n-1)$$
$$- 4(n+1)(0.5)^n u[-(n+1)-1]$$
$$= -4(-0.5)^n u(-n-1) - 4(n+1)(-0.5)^n u[-n-2] + 4(0.5)^n u(-n-1)$$
$$- 4(n+1)(0.5)^n u[-n-2]$$
$$= 4(0.5)^n \left[1 - (-1)^n\right] u(-n-1) - 4(n+1)(0.5)^n \left[1 + (-1)^n\right] u(-n-2)$$

P4.12 Consider the sequence given below:

$$x(n) = A_c(r)^n \cos(\pi v_0 n) u(n) + A_s(r)^n \sin(\pi v_0 n) u(n) \tag{4.10}$$

1. The z-transform of $x(n)$ in (4.10): Taking z-transform of (4.10),

$$X(z) = A_c \frac{1 - r \cos(\pi v_0) z^{-1}}{1 - 2r \cos(\pi v_0) z^{-1} + r^2 z^{-2}} + A_s \frac{r \sin(\pi v_0) z^{-1}}{1 - 2r \cos(\pi v_0) z^{-1} + r^2 z^{-2}}$$

$$= \frac{A_c + r [A_s \sin(\pi v_0) - A_c \cos(\pi v_0)] z^{-1}}{1 - 2r \cos(\pi v_0) z^{-1} + r^2 z^{-2}} \triangleq \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}; \quad |z| > |r|$$

Comparing the last step above, we have

$$b_0 = A_c; \ b_1 = r[A_s \sin(\pi v_0) - A_c \cos(\pi v_0)]; \ a_1 = -2r \cos(\pi v_0); \ a_2 = r^2 \tag{4.11}$$

2. The signal parameters A_c, A_s, r, and v_0 in terms of the rational function parameters b_0, b_1, a_1, and a_2: Using (4.11 and solving parameters in the following order: A_c, then r, then v_0, and finally A_s, we obtain

$$A_c = b_0; \quad r = \sqrt{a_2}; \quad v_0 = \frac{\arccos(-a_1/2r)}{\pi}; \quad A_s = \frac{2b_1 - a_1 b_0}{\sqrt{4a_2 - a_1^2}}$$

3. MATLAB function invCCPP:

```
function [Ac,As,r,v0] = inv_CC_PP(b0,b1,a1,a2)
% [Ac,As,r,v0] = inv_CC_PP(b0,b1,a1,a2)
Ac = b0;
r = sqrt(a2);
w0 = acos(-a1/(2*r));
As = (b1/r+Ac*cos(w0))/sin(w0);
v0 = w0/(pi);
```

P4.13 Suppose $X(z)$ is given as follows:

$$X(z) = \frac{2 + 3z^{-1}}{1 - z^{-1} + 0.81z^{-2}}, \quad |z| > 0.9$$

1. The signal $x(n)$ in a form that contains no complex numbers: MATLAB script:

```
% P0413.m
clc; close all;
b0 = 2; b1 = 3; a1 = -1; a2 = 0.81;
% (a): Computation of sequence parameters
[Ac,As,r,v0] = invCCPP(b0,b1,a1,a2);
disp(sprintf('\nx(n) = %1i*(%3.1f)^n*cos(%5.4f*pi*n)u(n) ',Ac,r,v0));
disp(sprintf('       + %5.4f*(%3.1f)^n*sin(%5.4f*pi*n)u(n)\n',As,r,v0));

x(n) = 2*(0.9)^n*cos(0.3125*pi*n)u(n)
       + 5.3452*(0.9)^n*sin(0.3125*pi*n)u(n)
```

Hence

$$x(n) = 2(0.9)^n \cos(0.3125\pi n)u(n) + 5.3452(0.9)^n \sin(0.3125\pi n)u(n)$$

2. MATLAB verification:

```
% (b) Matlab Verification
n = 0:20; x = Ac*(r.^n).*cos(v0*pi*n) + As*(r.^n).*sin(v0*pi*n);
y = filter([b0,b1],[1,a1,a2],impseq(0,0,20));
error = abs(max(x-y))
 error =
   1.7764e-015
```

P4.14 The z-transform of a causal sequence is given as:

$$X(z) = \frac{-2 + 5.65z^{-1} - 2.88z^{-2}}{1 - 0.1z^{-1} + 0.09z^{-2} + 0.648z^{-3}} \tag{4.12}$$

which contains a complex-conjugate pole pair as well as a real-valued pole.

1. Rearrangement of $X(z)$ into a first- and second-order sections: MATLAB Script:

```
% P0414
clc; close all;
b = [-2 5.65 -2.88]; a = [1 -0.1 .09 0.648]; [R,p,k] = residuez(b,a)
 R =
    1.0000 - 0.8660i
    1.0000 + 0.8660i
   -4.0000
 p =
    0.4500 + 0.7794i
    0.4500 - 0.7794i
   -0.8000
 k =
      []
[b1,a1] = residuez(R(1:2),p(1:2),k)
 b1 =
    2.0000    0.4500
 a1 =
    1.0000   -0.9000    0.8100
```

Hence

$$X(z) = \frac{(2) + (0.45)z^{-1}}{1 + (-0.9)z^{-1} + (0.81)z^{-2}} + \frac{(-4)}{1 - (-0.8)z^{-1}}$$

2. Computation of the causal sequence $x(n)$ from the $X(z)$ so that it contains no complex numbers: MATLAB Script:

```
[Ac,As,r,v0] = invCCPP(b1(1),b1(2),a1(2),a1(3));
disp(sprintf('\nx1(n) = %2.0f*(%3.1f)^n*cos(%5.4f*pi*n)u(n) ',Ac,r,v0));
disp(sprintf('        + %5.4f*(%3.1f)^n*sin(%5.4f*pi*n)u(n)\n',As,r,v0));
 x1(n) =   2*(0.9)^n*cos(0.3333*pi*n)u(n)
         + 1.7321*(0.9)^n*sin(0.3333*pi*n)u(n)
```

Hence the sequence $x(n)$ is:

$$x(n) = 2(0.9)^n \cos(\pi n/3)u(n) + \sqrt{3}(0.9)^n \sin(\pi n/3)u(n) - 4(-0.8)^n u(n)$$

P4.15 System representations and input/output calculations:

1. $h(n) = 5(1/4)^n u(n)$

    i. The system function: Taking the z-transform of $h(n)$,

    $$H(z) = \mathcal{Z}[h(n)] = \mathcal{Z}\left[5(1/4)^n u(n)\right] = \frac{5}{1 - 0.25z^{-1}}, \ |z| > 0.5$$

    ii. The difference equation representation: From $H(z)$ above,

    $$y(n) = 5x(n) + 0.25y(n-1)$$

    iii. The pole-zero plot is shown in Figure 4.6.
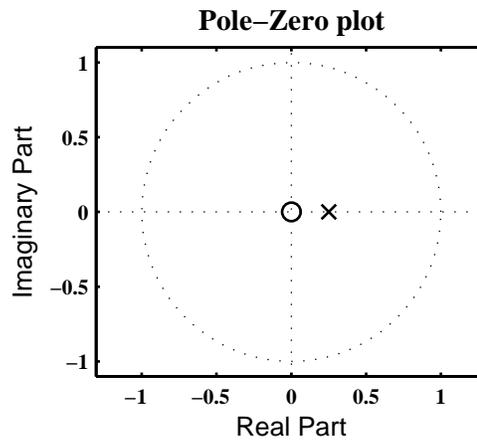

Figure 4.6: Problem P4.15.1 pole-zero plot

    iv. The output $y(n)$ for the input $x(n) = \left(\frac{1}{4}\right)^n u(n)$: Taking the z-transform of $x(n)$,

    $$X(z) = \mathcal{Z}\left[(1/4)^n u(n)\right] = \frac{1}{1 - 0.25z^{-1}}, \ |z| > 0.25$$

Now the z-transform of $y(n)$ is

$$Y(z) = H(z)X(z) = \left(\frac{5}{1 - 0.25z^{-1}}\right)\left(\frac{1}{1 - 0.25z^{-1}}\right) = \frac{5}{\left(1 - 0.25z^{-1}\right)^2}, \ |z| > 0.25$$

$$= 20z\frac{0.25z^{-1}}{\left(1 - 0.25z^{-1}\right)^2}, \ |z| > 0.25$$

Hence

$$y(n) = 20(n+1)(0.25)^{n+1}u(n+1)$$

2. $h(n) = n(1/3)^n u(n) + (-1/4)^n u(n)$

   i. The system function: Taking the z-transform of $h(n)$

$$H(z) = \mathcal{Z}[h(n)] = \mathcal{Z}\left[n(1/3)^n u(n) + (-1/4)^n u(n)\right]$$

$$= \frac{(1/3)z^{-1}}{\left[1 - (1/3)z^{-1}\right]^2} + \frac{1}{1 + (1/4)z^{-1}}, \ |z| > (1/3)$$

$$= \frac{1 - \frac{1}{3}z^{-1} + \frac{7}{36}z^{-2}}{1 - \frac{5}{12}z^{-1} - \frac{1}{18}z^{-2} + \frac{1}{36}z^{-3}}, \ |z| > (1/3)$$

   ii. The difference equation representation: From $H(z)$ above,

$$y(n) = x(n) - \frac{1}{3}x(n-1) + \frac{7}{36}x(n-2) + \frac{5}{12}y(n-1) + \frac{1}{18}y(n-2) - \frac{1}{36}y(n-3)$$

   iii. The pole-zero plot is shown in Figure 4.7.
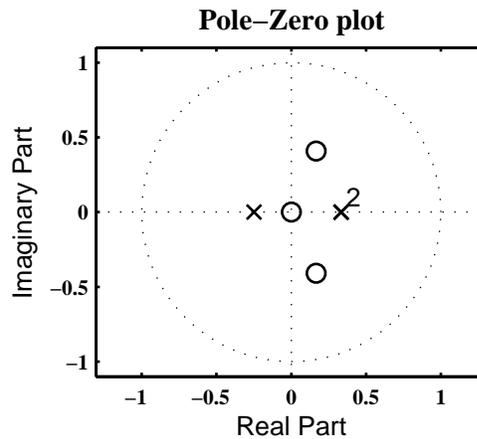
Figure 4.7: Problem P4.15.2 pole-zero plot

   iv. The output $y(n)$ for the input $x(n) = \left(\frac{1}{4}\right)^n u(n)$: Taking the z-transform of $x(n)$,

$$X(z) = \mathcal{Z}\left[(1/4)^n u(n)\right] = \frac{1}{1 - \frac{1}{4}z^{-1}}, \ |z| > \frac{1}{4}$$

Now the z-transform of $y(n)$ is

$$Y(z) = H(z)X(z) = \left(\frac{1 - \frac{1}{3}z^{-1} + \frac{7}{36}z^{-2}}{1 - \frac{5}{12}z^{-1} - \frac{1}{18}z^{-2} + \frac{1}{36}z^{-3}}\right)\left(\frac{1}{1 - 0.25z^{-1}}\right), \ |z| > \frac{1}{3}$$

$$= \frac{-16}{1 - \frac{1}{3}z^{-1}} + \frac{4}{\left(1 - \frac{1}{3}z^{-1}\right)^2} + \frac{\frac{1}{2}}{1 + \frac{1}{4}z^{-1}} + \frac{\frac{25}{2}}{1 - \frac{1}{4}z^{-1}}, \ |z| > \frac{1}{3}$$

$$= \frac{-16}{1 - \frac{1}{3}z^{-1}} + 12z\frac{\frac{1}{3}z^{-1}}{\left(1 - \frac{1}{3}z^{-1}\right)^2} + \frac{\frac{1}{2}}{1 + \frac{1}{4}z^{-1}} + \frac{\frac{25}{2}}{1 - \frac{1}{4}z^{-1}}, \ |z| > \frac{1}{3}$$

Hence

$$y(n) = -16(\tfrac{1}{3})^n u(n) + 12(n+1)(\tfrac{1}{3})^{n+1}u(n+1) + \frac{1}{2}(-\tfrac{1}{4})^n u(n) + \frac{25}{2}(\tfrac{1}{4})^n u(n)$$

3. $h(n) = 3(0.9)^n \cos(\pi n/4 + \pi/3)u(n+1)$: Consider

$$
h(n) = \frac{10}{3}\left[(0.9)^{n+1}\cos\left\{\frac{\pi(n+1)}{4} + \frac{\pi}{12}\right\}u(n+1)\right]
$$

$$
= \left[\frac{10}{3}\cos\left(\frac{\pi}{12}\right)\right](0.9)^{n+1}\cos\left[\frac{\pi}{4}(n+1)\right]u(n+1) - \left[\frac{10}{3}\sin\left(\frac{\pi}{12}\right)\right](0.9)^{n+1}\sin\left[\frac{\pi}{4}(n+1)\right]u(n+1)
$$

$$
= 3.2198(0.9)^{n+1}\cos\left[\frac{\pi}{4}(n+1)\right]u(n+1) - 0.8627(0.9)^{n+1}\sin\left[\frac{\pi}{4}(n+1)\right]u(n+1)
$$

i. The system function: Taking the z-transform of $h(n)$

$$
H(z) = \mathcal{Z}[h(n)] = z\left(3.2198\frac{1 - 0.6364z^{-1}}{1 - 1.2728z^{-1} + 0.81z^{-2}} - 0.8627\frac{0.6364z^{-1}}{1 - 1.2728z^{-1} + 0.81z^{-2}}\right)
$$

$$
= \frac{3.2198z - 2.5981}{1 - 1.2728z^{-1} + 0.81z^{-2}}, \quad |z| > 0.9
$$

ii. The difference equation representation: From $H(z)$ above,

$$
y(n) = 3.2198x(n+1) - 2.5981x(n) + 1.2728y(n-1) - 0.81y(n-2)
$$

iii. The pole-zero plot is shown in Figure 4.8.
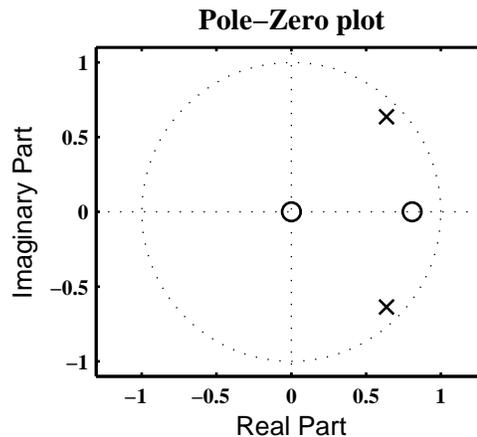

Pole–Zero plot

Figure 4.8: Problem P4.15.3 pole-zero plot

iv. The output $y(n)$ for the input $x(n) = \left(\frac{1}{4}\right)^n u(n)$: The z-transform of $x(n)$ is $X(z) = \frac{1}{1 - 0.25z^{-1}}$, $|z| > 0.25$. Now the z-transform of $y(n)$ is

$$
Y(z) = H(z)X(z) = \left(\frac{3.2198z - 2.5981}{1 - 1.2728z^{-1} + 0.81z^{-2}}\right)\left(\frac{1}{1 - 0.25z^{-1}}\right), \quad |z| > 0.9
$$

$$
= z\left(\frac{4.0285 - 2.6203z^{-1}}{1.0000 - 1.2728z^{-1} + 0.81z^{-2}} - \frac{0.8087}{1 - \frac{1}{4}z^{-1}}\right), \quad |z| > 0.9
$$

Hence

$$
y(n) = \left\{4.0285(0.9)^{n+1}\cos\left[\frac{\pi(n+1)}{4}\right] - 0.0889(0.9)^{n+1}\sin\left[\frac{\pi(n+1)}{4}\right] - 0.8087(\tfrac{1}{4})^{n+1}\right\}u(n+1)
$$

4. $h(n) = \dfrac{(0.5)^n \sin[(n+1)\pi/3]}{\sin(\pi/3)} u(n)$: Consider

$$
\begin{aligned}
h(n) &= \frac{1}{\sin(\frac{\pi}{3})} \left[(0.5)^n \sin\left\{\frac{\pi n}{3} + \frac{\pi}{3}\right\} u(n) \right] \\
&= \left[\frac{1}{\sin(\frac{\pi}{3})} \sin\left(\frac{\pi}{3}\right) \right] (0.5)^n \sin\left[\frac{\pi}{3}n\right] u(n) + \left[\frac{1}{\sin(\frac{\pi}{3})} \cos\left(\frac{\pi}{3}\right) \right] (0.5)^n \cos\left[\frac{\pi}{3}n\right] u(n) \\
&= (0.5)^n \sin\left[\frac{\pi}{3}n\right] u(n) + 0.5774(0.5)^n \cos\left[\frac{\pi}{3}n\right] u(n)
\end{aligned}
$$

i. The system function: Taking the z-transform of $h(n)$

$$
\begin{aligned}
H(z) = \mathcal{Z}[h(n)] &= \frac{0.4330z^{-1}}{1 - 0.5z^{-1} + 0.25z^{-2}} + 0.5774\frac{1 - 0.25z^{-1}}{1 - 0.5z^{-1} + 0.25z^{-2}} \\
&= \frac{0.5774 + 0.2887z^{-1}}{1 - 0.5z^{-1} + 0.25z^{-2}}, \quad |z| > 0.5
\end{aligned}
$$

ii. The difference equation representation: From $H(z)$ above,

$$
y(n) = 0.5774x(n) + 0.2887x(n-1) + 0.5y(n-1) - 0.25y(n-2)
$$

iii. The pole-zero plot is shown in Figure 4.9.
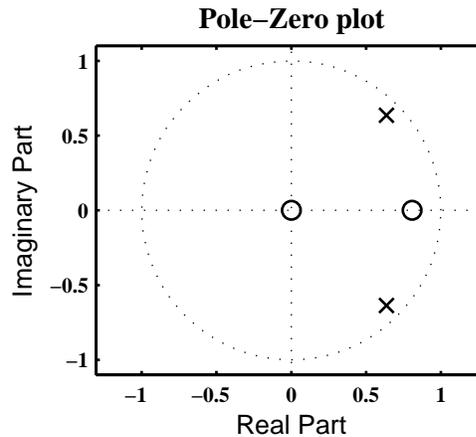
Pole–Zero plot

Figure 4.9: Problem P4.15.4 pole-zero plot

iv. The output $y(n)$ for the input $x(n) = \left(\frac{1}{4}\right)^n u(n)$: The z-transform of $x(n)$ is $X(z) = \dfrac{1}{1 - 0.25z^{-1}}$, $|z| > 0.25$. Now the z-transform of $y(n)$ is

$$
\begin{aligned}
Y(z) = H(z)X(z) &= \left(\frac{0.5774 + 0.2887z^{-1}}{1 - 0.5z^{-1} + 0.25z^{-2}} \right) \left(\frac{1}{1 - 0.25z^{-1}} \right), \quad |z| > 0.5 \\
&= \frac{0.5774z^{-1}}{1.0000 - 0.5z^{-1} + 0.25z^{-2}} - \frac{0.5774}{1 - \frac{1}{4}z^{-1}}, \quad |z| > 0.5
\end{aligned}
$$

Hence

$$
y(n) = \frac{4}{3}(0.5)^n \sin(\tfrac{\pi}{3}n)u(n) + 0.5774(\tfrac{1}{4})^n u(n)
$$

5. $h(n) = [2 - \sin(\pi n)]u(n) = 2u(n)$

    i. The system function: $H(z) = \mathcal{Z}[h(n)] = \mathcal{Z}[2u(n)] = \dfrac{2}{1 - z^{-1}}$, $|z| > 1$.

    ii. The difference equation representation: $y(n) = 2x(n) + y(n-1)$

   iii. The pole-zero plot is shown in Figure 4.10.
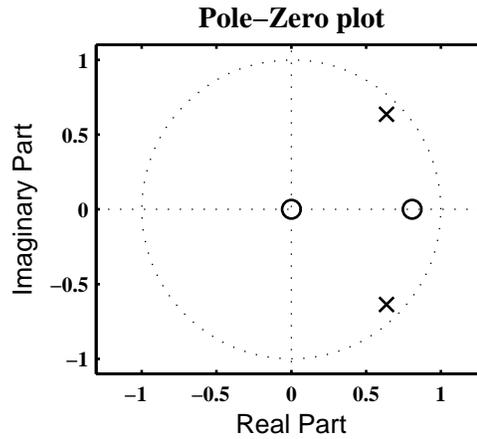
Figure 4.10: Problem P4.15.5 pole-zero plot

    iv. The output $y(n)$ for the input $x(n) = \left(\frac{1}{4}\right)^n u(n)$: Taking the z-transform of $x(n)$,

$$X(z) = \mathcal{Z}\left[(1/4)^n u(n)\right] = \frac{1}{1 - 0.25z^{-1}}, \quad |z| > 0.25$$

Now the z-transform of $y(n)$ is

$$Y(z) = H(z)X(z) = \left(\frac{2}{1 - z^{-1}}\right)\left(\frac{1}{1 - 0.25z^{-1}}\right), \quad |z| > 1$$

$$= \frac{8/3}{1 - z^{-1}} - \frac{2/3}{1 - \frac{1}{4}z^{-1}}, \quad |z| > 1$$

Hence

$$y(n) = \frac{8}{3}u(n) - \frac{2}{3}\left(\frac{1}{4}\right)^n u(n)$$

P4.16 Consider the system shown below.
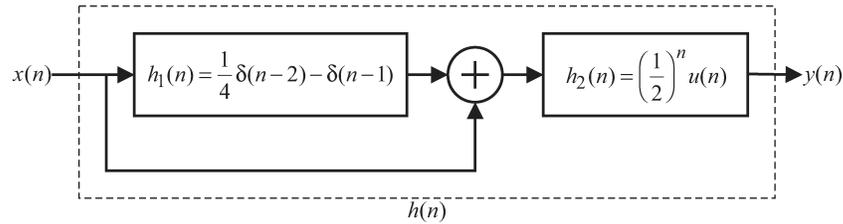
1. The overall system impulse response, $h(n)$, using the z-transform approach: The above system is given by

$$H(z) = H_2(z)\left[1 + H_1(z)\right] = \frac{1}{1 - 0.5z^{-1}}\left[1 + 0.25z^{-2} - z^{-1}\right]$$

$$= \frac{\left(1 - 0.5z^{-1}\right)^2}{1 - 0.5z^{-1}} = 1 - 0.5z^{-1}, \ |z| \neq 0$$

Hence after taking inverse z-transform , we obtain

$$h(n) = \delta(n) - \frac{1}{2}\delta(n - 1)$$

2. Difference equation representation of the overall system: From the overall system function $H(z)$,

$$H(z) = \frac{Y(z)}{X(z)} = 1 - 0.5z^{-1} \Rightarrow y(n) = x(n) - 0.5x(n - 1)$$

3. Causality and stability: Since $h(n) = 0$ for $n < 0$, the system is causal. Since $h(n)$ is of finite duration (only two samples), $h(n)$ is absolutely summable. Hence BIBO stable.

4. Frequency response $H\left(e^{j\omega}\right)$ of the overall system.

$$H\left(e^{j\omega}\right) = \mathcal{F}\left[h(n)\right] = \mathcal{F}\left[\delta(n) - \frac{1}{2}\delta(n - 1)\right] = 1 - \frac{1}{2}e^{-j\omega}$$

5. Frequency response over $0 \leq \omega \leq \pi$ is shown in Figure 4.11.
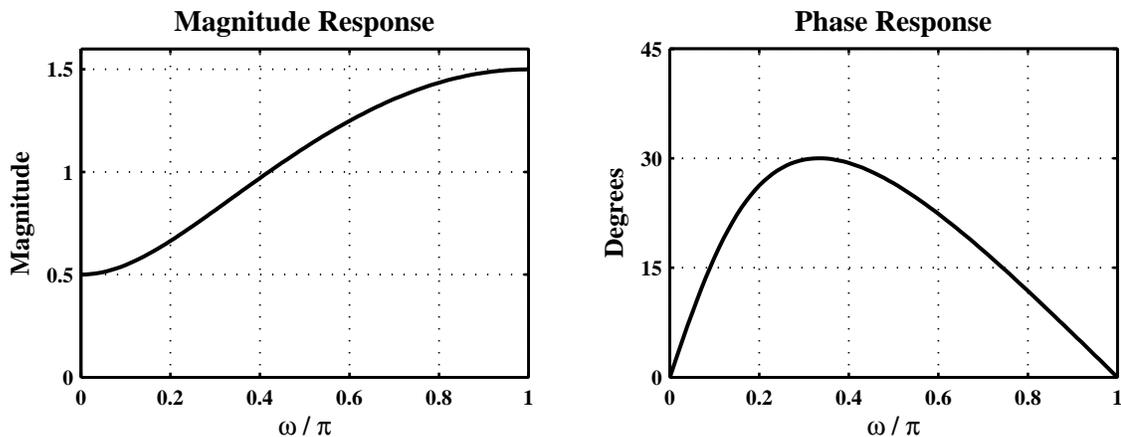

Figure 4.11: Problem P4.16 frequency-response plot

P4.17 System representations and input/output calculations:

1. $H(z) = (z + 1)/(z - 0.5)$, causal system. Consider

$$H(z) = \frac{z + 1}{z - 0.5} = \frac{1 + z^{-1}}{1 - 0.5z^{-1}} = \frac{1}{1 - 0.5z^{-1}} + \frac{z^{-1}}{1 - 0.5z^{-1}}, \quad |z| > 0.5$$

   i. The impulse response: Taking the inverse z-transform of $H(z)$,

$$h(n) = \mathcal{Z}^{-1}[H(z)] = (0.5)^n u(n) + (0.5)^{n-1} u(n - 1)$$

   ii. The difference equation representation: From $H(z)$ above,

$$y(n) = x(n) + x(n - 1) + 0.5y(n - 1)$$

   iii. The pole-zero plot is shown in Figure 4.12.
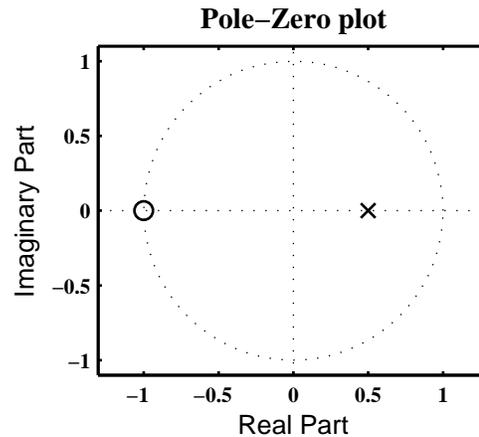
Figure 4.12: Problem P4.17.1 pole-zero plot

   iv. The output $y(n)$ for the input $x(n) = 3\cos(\pi n/3)u(n)$: Taking the z-transform of $x(n)$,

$$X(z) = \mathcal{Z}[3\cos(\pi n/3)u(n)] = 3\frac{1 - [\cos(\pi/3)]z^{-1}}{1 - [2\cos(\pi/3)]z^{-1} + z^{-2}} = 3\frac{1 - 0.5z^{-1}}{1 - z^{-1} + z^{-2}}, \quad |z| > 1$$

Now the z-transform of $y(n)$ is

$$Y(z) = H(z)X(z) = \left(\frac{1 + z^{-1}}{1 - 0.5z^{-1}}\right)\left(3\frac{1 - 0.5z^{-1}}{1 - z^{-1} + z^{-2}}\right) = 3\frac{1 + z^{-1}}{1 - z^{-1} + z^{-2}}, \quad |z| > 1$$

$$= 3\frac{1 - 0.5z^{-1} + 1.5z^{-1}}{1 - z^{-1} + z^{-2}} = 3\frac{1 - 0.5z^{-1}}{1 - z^{-1} + z^{-2}} + 3\sqrt{3}\frac{\frac{\sqrt{3}}{2}z^{-1}}{1 - z^{-1} + z^{-2}}, \quad |z| > 1$$

Hence

$$y(n) = 3\cos(\pi n/3)u(n) + 3\sqrt{3}\sin(\pi n/3)u(n)$$

2. $H(z) = (1 + z^{-1} + z^{-2})/(1 + 0.5z^{-1} - 0.25z^{-2})$, stable system. Consider

$$H(z) = \frac{1 + z^{-1} + z^{-2}}{1 + 0.5z^{-1} - 0.25z^{-2}} = -4 + \frac{0.9348}{1 + 0.809z^{-1}} + \frac{4.0652}{1 - 0.309z^{-1}}, \ |z| > 0.809$$

i. The impulse response: Taking the inverse z-transform of $H(z)$,

$$h(n) = \mathcal{Z}^{-1}[H(z)] = -4\delta(n) + 0.9348(-0.809)^n u(n) + 4.0652(0.309)^n u(n)$$

ii. The difference equation representation: From $H(z)$ above,

$$y(n) = x(n) + x(n-1) + x(n-2) - 0.5y(n-1) + 0.25y(n-2)$$

iii. The pole-zero plot is shown in Figure 4.13.
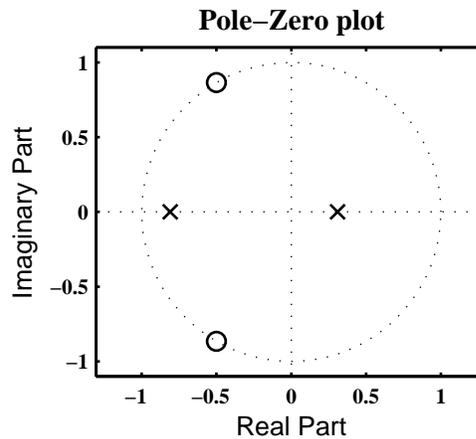
Figure 4.13: Problem P4.17.2 pole-zero plot

iv. The output $y(n)$ for the input $x(n) = 3\cos(\pi n/3)u(n)$: Taking the z-transform of $x(n)$,

$$X(z) = \mathcal{Z}[3\cos(\pi n/3)u(n)] = 3\frac{1 - [\cos(\pi/3)]z^{-1}}{1 - [2\cos(\pi/3)]z^{-1} + z^{-2}} = 3\frac{1 - 0.5z^{-1}}{1 - z^{-1} + z^{-2}}, \ |z| > 1$$

Now the z-transform of $y(n)$ is

$$Y(z) = H(z)X(z) = \left(\frac{1 + z^{-1} + z^{-2}}{1 + 0.5z^{-1} - 0.25z^{-2}}\right)\left(3\frac{1 - 0.5z^{-1}}{1 - z^{-1} + z^{-2}}\right), \ |z| > 1$$

$$= \frac{1.2055}{1 + 0.809z^{-1}} - \frac{0.9152}{1 - 0.309z^{-1}} + \frac{2.7097\left(1 - \frac{1}{2}z^{-1}\right)}{1 - z^{-1} + z^{-2}} + \frac{3.3524\left(\frac{\sqrt{3}}{2}z^{-1}\right)}{1 - z^{-1} + z^{-2}}, \ |z| > 1$$

Hence

$$y(n) = 1.2055(-0.809)^n u(n) - 0.9152(0.309)^n u(n) + 2.7097\cos(\pi n/3)u(n)$$
$$+ 3.3524\sin(\pi n/3)u(n)$$

3. $H(z) = (z^2 - 1)/(z - 3)^2$, anti-causal system. Consider

$$H(z) = \frac{z^2 - 1}{(z - 3)^2} = \frac{1 - z^{-2}}{1 - 6z^{-1} + 9z^{-2}} = -\frac{1}{9} + \frac{2/9}{1 - 3z^{-1}} + \frac{8z}{27}\frac{3z^{-1}}{\left(1 - 3z^{-1}\right)^2}, \ |z| < 3$$

   i. The impulse response: Taking the inverse z-transform of $H(z)$,

$$h(n) = \mathcal{Z}^{-1}[H(z)] = -\frac{1}{9}\delta(n) - \frac{2}{9}3^n u(-n - 1) - \frac{8}{27}(n + 1)3^{n+1}u(-n - 2)$$

   ii. The difference equation representation: From $H(z)$ above,

$$y(n) = x(n) - x(n - 2) + 6y(n - 1) - 9y(n - 2)$$

   iii. The pole-zero plot is shown in Figure 4.14.
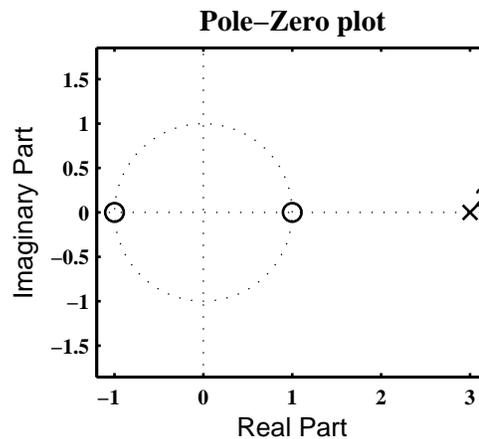
Pole–Zero plot

Figure 4.14: Problem P4.17.3 pole-zero plot

   iv. The output $y(n)$ for the input $x(n) = 3\cos(\pi n/3)u(n)$: Taking the z-transform of $x(n)$,

$$X(z) = \mathcal{Z}[3\cos(\pi n/3)u(n)] = 3\frac{1 - [\cos(\pi/3)]z^{-1}}{1 - [2\cos(\pi/3)]z^{-1} + z^{-2}} = 3\frac{1 - 0.5z^{-1}}{1 - z^{-1} + z^{-2}}, \ |z| > 1$$

Now the z-transform of $y(n)$ is

$$Y(z) = H(z)X(z) = \left(\frac{1 - z^{-2}}{1 - 6z^{-1} + 9z^{-2}}\right)\left(3\frac{1 - 0.5z^{-1}}{1 - z^{-1} + z^{-2}}\right), \ 1 < |z| < 3$$

$$= \frac{43/49}{1 - 3z^{-1}} + \frac{20z}{21}\frac{3z^{-1}}{\left(1 - 3z^{-1}\right)^2} + \frac{\frac{-36}{49}\left(1 - \frac{1}{2}z^{-1}\right)}{1 - z^{-1} + z^{-2}} + \frac{\frac{193}{1820}\left(\frac{\sqrt{3}}{2}z^{-1}\right)}{1 - z^{-1} + z^{-2}}, \ |z| > 1$$

Hence

$$y(n) = -\frac{43}{49}3^n u(-n - 1) - \frac{20}{21}(n + 1)3^{n+1}u(-n - 2) - \frac{36}{49}\cos(\pi n/3)u(n)$$
$$+ \frac{193}{1820}\sin(\pi n/3)u(n)$$

4. $H(z) = \dfrac{z}{z - 0.25} + \dfrac{1 - 0.5z^{-1}}{1 + 2z^{-1}}$, stable system. Consider

$$H(z) = \frac{1}{1 - 0.25z^{-1}} + \frac{1 - 0.5z^{-1}}{1 + 2z^{-1}} = \frac{2 + \frac{5}{4}z^{-1} + \frac{1}{8}z^{-2}}{1 + \frac{7}{4}z^{-1} - \frac{1}{2}z^{-2}}$$

$$= -\frac{1}{4} + \frac{1}{1 - 0.25z^{-1}} + \frac{5/4}{1 + 2z^{-1}}, \quad 0.25 < |z| < 2$$

i. The impulse response: Taking the inverse z-transform of $H(z)$,

$$h(n) = \mathcal{Z}^{-1}[H(z)] = -\frac{1}{4}\delta(n) + \left(\frac{1}{4}\right)^n u(n) - \frac{5}{4}2^n u(-n - 1)$$

ii. The difference equation representation: From $H(z)$ above,

$$y(n) = 2x(n) + \frac{5}{4}x(n - 1) - \frac{1}{8}x(n - 2) - \frac{7}{4}y(n - 1) + \frac{1}{2}y(n - 2)$$

iii. The pole-zero plot is shown in Figure 4.15.
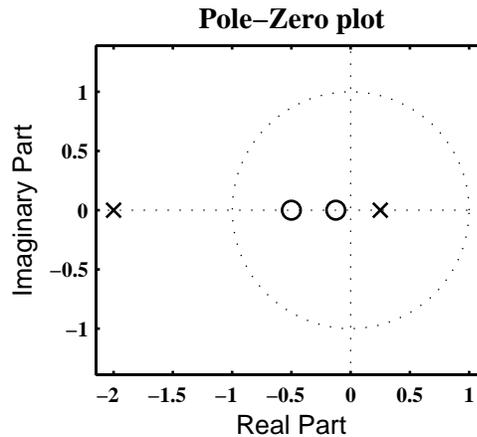
Pole–Zero plot

Figure 4.15: Problem P4.17.4 pole-zero plot

iv. The output $y(n)$ for the input $x(n) = 3\cos(\pi n/3)u(n)$: Taking the z-transform of $x(n)$,

$$X(z) = \mathcal{Z}[3\cos(\pi n/3)u(n)] = 3\frac{1 - [\cos(\pi/3)]z^{-1}}{1 - [2\cos(\pi/3)]z^{-1} + z^{-2}} = 3\frac{1 - 0.5z^{-1}}{1 - z^{-1} + z^{-2}}, \quad |z| > 1$$

Now the z-transform of $y(n)$ is

$$Y(z) = H(z)X(z) = \left(\frac{2 + \frac{5}{4}z^{-1} + \frac{1}{8}z^{-2}}{1 + \frac{7}{4}z^{-1} - \frac{1}{2}z^{-2}}\right)\left(3\frac{1 - 0.5z^{-1}}{1 - z^{-1} + z^{-2}}\right), \quad 1 < |z| < 3$$

$$= \frac{75/28}{1 + 2z^{-1}} - \frac{3/13}{1 - \frac{1}{4}z^{-1}} + \frac{\frac{1293}{364}\left(1 - \frac{1}{2}z^{-1}\right)}{1 - z^{-1} + z^{-2}} - \frac{\frac{323}{2553}\left(\frac{\sqrt{3}}{2}z^{-1}\right)}{1 - z^{-1} + z^{-2}}, \quad |z| > 1$$

Hence

$$y(n) = -\frac{75}{28}2^n u(-n - 1) - \frac{3}{13}\left(\frac{1}{4}\right)^n u(n) + \frac{1293}{364}\cos(\pi n/3)u(n)$$

$$- \frac{323}{2553}\sin(\pi n/3)u(n)$$

5. $H(z) = (1 + z^{-1} + z^{-2})^2$. Consider

$$H(z) = (1 + z^{-1} + z^{-2})^2 = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}, \ |z| > 0$$

i. The impulse response: Taking the inverse z-transform of $H(z)$,

$$h(n) = \mathcal{Z}^{-1}[H(z)] = \{\underset{\uparrow}{1}, 2, 3, 2, 1\}$$

ii. The difference equation representation: From $H(z)$ above,

$$y(n) = x(n) + 2x(n-1) + 3x(n-2) + 2x(n-3) + x(n-4)$$

iii. The pole-zero plot is shown in Figure 4.16.
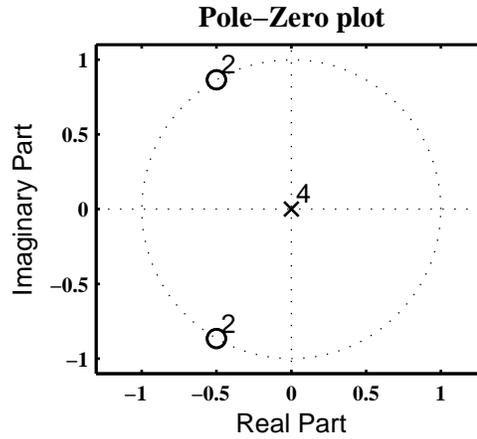
Figure 4.16: Problem P4.17.5 pole-zero plot

iv. The output $y(n)$ for the input $x(n) = 3\cos(\pi n/3)u(n)$: Taking the z-transform of $x(n)$,

$$X(z) = \mathcal{Z}[3\cos(\pi n/3)u(n)] = 3\frac{1 - [\cos(\pi/3)]z^{-1}}{1 - [2\cos(\pi/3)]z^{-1} + z^{-2}} = 3\frac{1 - 0.5z^{-1}}{1 - z^{-1} + z^{-2}}, \ |z| > 1$$

Now the z-transform of $y(n)$ is

$$Y(z) = H(z)X(z) = 3\left[\frac{(1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4})(1 - 0.5z^{-1})}{1 - z^{-1} + z^{-2}}\right], \ |z| > 1$$

$$= 9 + \frac{3}{2}z^{-1} - \frac{3}{2}z^{-2} - \frac{3}{2}z^{-3} + \frac{\frac{1293}{364}\left(1 - \frac{1}{2}z^{-1}\right)}{1 - z^{-1} + z^{-2}} - \frac{\frac{328}{2553}\left(\frac{\sqrt{3}}{2}z^{-1}\right)}{1 - z^{-1} + z^{-2}}, \ |z| > 1$$

Hence

$$y(n) = 9\delta(n) + \frac{3}{2}\delta(n-1) - \frac{3}{2}\delta(n-2) - \frac{3}{2}\delta(n-3) + \frac{1293}{364}\cos(\pi n/3)u(n)$$
$$- \frac{328}{2553}\sin(\pi n/3)u(n)$$

P4.18 System representations and input/output calculations:

1. $y(n) = [x(n) + 2x(n-1) + x(n-3)]/4$

    i. The system function representation: Taking the z-transform of the above difference equation,

$$Y(z) = \frac{1}{4}[X(z) + 2z^{-1}X(z) + z^{-3}X(z)] \Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1} + z^{-3}}{4}$$

    ii. The impulse response: Taking the inverse z-transform of $H(z)$,

$$h(n) = \frac{1}{4}[\delta(n) + 2\delta(n-1) + \delta(n-3)]$$

    iii. The pole-zero plot is shown in Figure 4.17.
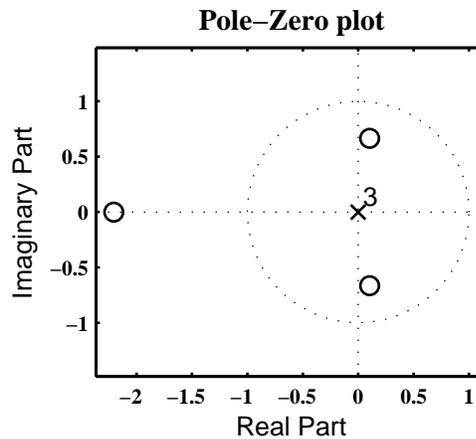
Figure 4.17: Problem P4.18.1 pole-zero plot

    iv. The output $y(n)$ for the input $x(n) = 2(0.9)^n u(n)$: Taking the z-transform of $x(n)$,

$$X(z) = \mathcal{Z}\left[2(0.9)^n u(n)\right] = \frac{2}{1 - 0.9z^{-1}}, \ |z| > 0.9$$

Now the z-transform of $y(n)$ is

$$Y(z) = H(z)X(z) = \left(\frac{1 + 2z^{-1} + z^{-3}}{4}\right)\left(\frac{2}{1 - 0.9z^{-1}}\right), \ |z| > 0.9$$

$$= -\frac{1310}{729} - \frac{50}{81}z^{-1} - \frac{5}{9}z^{-2} + \frac{\frac{990}{431}}{1 - 0.9z^{-1}}, \ |z| > 0.9$$

Hence

$$y(n) = -\frac{1310}{729}\delta(n) - \frac{50}{81}\delta(n-1) - \frac{5}{9}\delta(n-2) + \frac{990}{431}(0.9)^n u(n)$$

2. $y(n) = x(n) + 0.5x(n-1) - 0.5y(n-1) + 0.25y(n-2)$

   i. The system function representation: Taking the z-transform of the above difference equation,

$$Y(z) = X(z) + 0.5z^{-1}X(z) - 0.5z^{-1}Y(z) + 0.25z^{-2}Y(z)$$

or

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 0.5z^{-1}}{1 + 0.5z^{-1} - 0.25z^{-2}} = \frac{1 + 0.5z^{-1}}{\left(1 + 0.809z^{-1}\right)\left(1 - 0.309z^{-1}\right)}, \ |z| > 0.809$$

   ii. The impulse response: Taking the inverse z-transform of $H(z)$,

$$h(n) = \mathcal{Z}^{-1}\left[\frac{1 + 0.5z^{-1}}{\left(1 + 0.809z^{-1}\right)\left(1 - 0.309z^{-1}\right)}\right] = \mathcal{Z}^{-1}\left[\frac{0.2764}{1 + 0.809z^{-1}} + \frac{0.7236}{1 - 0.309z^{-1}}\right]$$

$$= 0.2764(-0.809)^n u(n) + 0.7236(0.308)^n u(n)$$

   iii. The pole-zero plot is shown in Figure 4.18.
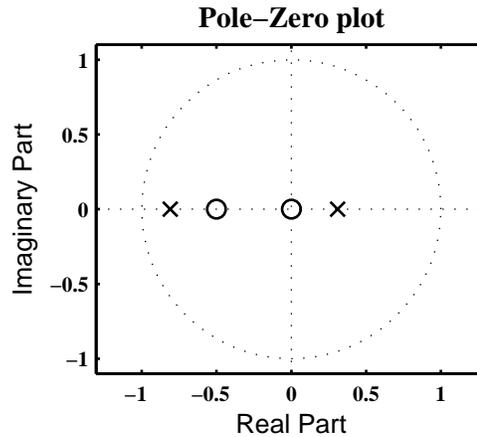

Figure 4.18: Problem P4.18.2 pole-zero plot

   iv. The output $y(n)$ for the input $x(n) = 2(0.9)^n u(n)$: Taking the z-transform of $x(n)$,

$$X(z) = \mathcal{Z}\left[2(0.9)^n u(n)\right] = \frac{2}{1 - 0.9z^{-1}}, \ |z| > 0.9$$

Now the z-transform of $y(n)$ is

$$Y(z) = H(z)X(z) = \left(\frac{1 + 0.5z^{-1}}{1 + 0.5z^{-1} - 0.25z^{-2}}\right)\left(\frac{2}{1 - 0.9z^{-1}}\right), \ |z| > 0.9$$

$$= \frac{0.2617}{1 + 0.809z^{-1}} - \frac{0.7567}{1 - 0.309z^{-1}} + \frac{2.495}{1 - 0.9z^{-1}}, \ |z| > 0.9$$

Hence

$$y(n) = 0.2617(-0.809)^n u(n) - 0.7567(0.309)^n u(n) + 2.495(0.9)^n u(n)$$

3. $y(n) = 2x(n) + 0.9y(n-1)$

    i. The system function representation: Taking the z-transform of the above difference equation,

$$Y(z) = 2X(z) + 0.9z^{-1}Y(z)$$

or
$$H(z) = \frac{Y(z)}{X(z)} = \frac{2}{1 - 0.9z^{-1}}, \ |z| > 0.9$$

    ii. The impulse response: Taking the inverse z-transform of $H(z)$,

$$h(n) = 2(0.9)^n u(n)$$

   iii. The pole-zero plot is shown in Figure 4.19.
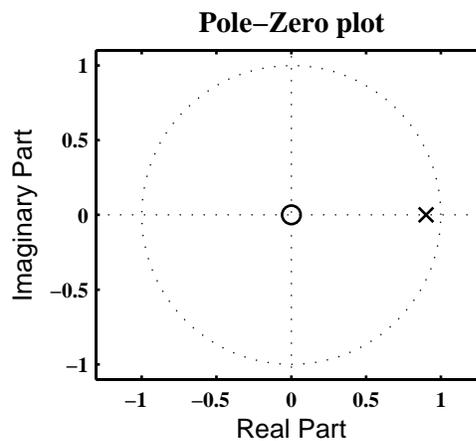
Figure 4.19: Problem P4.18.3 pole-zero plot

    iv. The output $y(n)$ for the input $x(n) = 2(0.9)^n u(n)$: Taking the z-transform of $x(n)$,

$$X(z) = \mathcal{Z}\left[2(0.9)^n u(n)\right] = \frac{2}{1 - 0.9z^{-1}}, \ |z| > 0.9$$

Now the z-transform of $y(n)$ is

$$Y(z) = H(z)X(z) = \left(\frac{2}{1 - 0.9z^{-1}}\right)\left(\frac{2}{1 - 0.9z^{-1}}\right), \ |z| > 0.9$$

$$= \frac{40}{9}z\frac{0.9z^{-1}}{\left(1 - 0.9z^{-1}\right)^2}, \ |z| > 0.9$$

Hence

$$y(n) = \frac{40}{9}(n+1)(0.9)^{n+1}u(n+1)$$

4. $y(n) = -0.45x(n) - 0.4x(n-1) + x(n-2) + 0.4y(n-1) + 0.45y(n-2)$

   i. The system function representation: Taking the z-transform of the above difference equation,

   $$Y(z) = -0.45X(z) - 0.4z^{-1}X(z) + z^{-2}X(z) + 0.4z^{-1}Y(z) + 0.45z^{-2}Y(z)$$

   or $$H(z) = \frac{Y(z)}{X(z)} = \frac{-0.45 - 0.4z^{-1} + z^{-2}}{1 - 0.4z^{-1} - 0.45z^{-2}} = \frac{-0.45 - 0.4z^{-1} + z^{-2}}{\left(1 + 0.5z^{-1}\right)\left(1 - 0.9z^{-1}\right)}, \; |z| > 0.9$$

   ii. The impulse response: Taking the inverse z-transform of $H(z)$,

   $$h(n) = \mathcal{Z}^{-1}\left[\frac{-0.45 - 0.4z^{-1} + z^{-2}}{\left(1 + 0.5z^{-1}\right)\left(1 - 0.9z^{-1}\right)}\right] = \mathcal{Z}^{-1}\left[-\frac{20}{9} + \frac{1.5536}{1 + 0.5z^{-1}} + \frac{0.2187}{1 - 0.9z^{-1}}\right]$$

   $$= -\frac{20}{9}\delta(n) + 1.5536(-0.5)^n u(n) + 0.2187(0.9)^n u(n)$$

   iii. The pole-zero plot is shown in Figure 4.20.
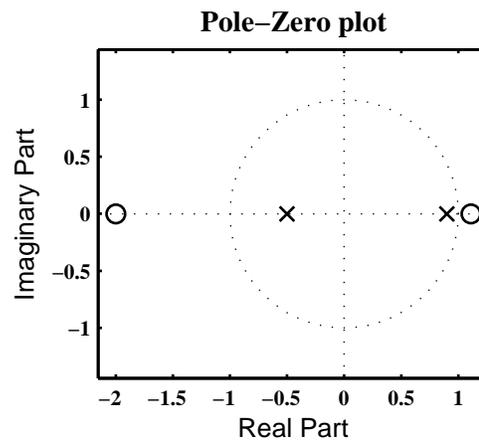
Figure 4.20: Problem P4.18.4 pole-zero plot

   iv. The output $y(n)$ for the input $x(n) = 2(0.9)^n u(n)$: Taking the z-transform of $x(n)$,

   $$X(z) = \mathcal{Z}\left[2(0.9)^n u(n)\right] = \frac{2}{1 - 0.9z^{-1}}, \; |z| > 0.9$$

   Now the z-transform of $y(n)$ is

   $$Y(z) = H(z)X(z) = \left(\frac{-0.45 - 0.4z^{-1} + z^{-2}}{1 - 0.4z^{-1} - 0.45z^{-2}}\right)\left(\frac{2}{1 - 0.9z^{-1}}\right), \; |z| > 0.9$$

   $$= \frac{1.1097}{1 + 0.5z^{-1}} - \frac{2.4470}{1 - 0.9z^{-1}} + 0.4859z\frac{0.9z^{-1}}{\left(1 - 0.9z^{-1}\right)^2}, \; |z| > 0.9$$

   Hence

   $$y(n) = 1.1097(-0.5)^n u(n) - 2.4470(0.9)^n u(n) + 0.4859(n+1)(0.9)^{n+1} u(n+1)$$

5. $y(n) = \sum_{m=0}^{4}(0.8)^m x(n-m) - \sum_{\ell=1}^{4}(0.9)^\ell y(n-\ell)$

   i. The system function representation: Taking the z-transform of the above difference equation,

$$Y(z) = \sum_{m=0}^{4}(0.8)^m z^{-m} X(z) - \sum_{\ell=1}^{4}(0.9)^\ell z^{-\ell} Y(z)$$

   or

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{m=0}^{4}(0.8)^m z^{-m}}{1 + \sum_{\ell=1}^{4}(0.9)^\ell z^{-\ell}}$$

$$= \frac{1 + 0.8z^{-1} + 0.64 + z^{-2} + 0.512z^{-3} + 0.4096z^{-4}}{\left(1 - 0.5562z^{-1} + 0.81z^{-2}\right)\left(1 + 1.4562z^{-1} + 0.81z^{-2}\right)}, \; |z| > 0.9$$

   ii. The impulse response: Taking the inverse z-transform of $H(z)$,

$$h(n) = \mathcal{Z}^{-1}\left[0.6243 + \frac{0.1873 + 0.0651z^{-1}}{1 - 0.5562z^{-1} + 0.81z^{-2}} + \frac{0.1884 + 0.1353z^{-1}}{1 + 1.4562z^{-1} + 0.81z^{-2}}\right]$$

$$= 0.1884\delta(n) + \left[0.1879(0.9)^n \cos(0.4\pi n + 4.63°) + 0.1885(0.9)^n \cos(0.8\pi n + 1.1°)\right]u(n)$$

   iii. The pole-zero plot is shown in Figure 4.21.
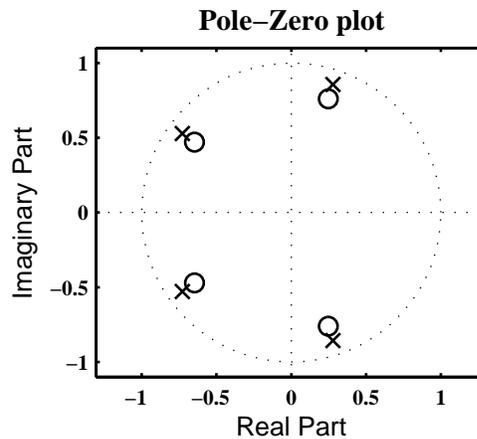
Figure 4.21: Problem P4.18.5 pole-zero plot

   iv. The output $y(n)$ for the input $x(n) = 2(0.9)^n u(n)$: Taking the z-transform of $x(n)$,

$$X(z) = \mathcal{Z}\left[2(0.9)^n u(n)\right] = \frac{2}{1 - 0.9z^{-1}}, \; |z| > 0.9$$

   Now the z-transform of $y(n)$ is

$$Y(z) = H(z)X(z) = \left(\frac{1 + 0.8z^{-1} + 0.64 + z^{-2} + 0.512z^{-3} + 0.4096z^{-4}}{1 + 0.9z^{-1} + 0.81 + z^{-2} + 0.279z^{-3} + 0.6561z^{-4}}\right)\left(\frac{2}{1 - 0.9z^{-1}}\right)$$

$$= \frac{0.2081 + 0.1498z^{-1}}{1 - 0.5562z^{-1} + 0.81z^{-2}} + \frac{0.1896 + 0.1685z^{-1}}{1 + 1.4562z^{-1} + 0.81z^{-2}} + \frac{1.6023}{1 - 0.9z^{-1}}, \; |z| > 0.9$$

   Hence

$$y(n) = 0.3197(0.9)^n \cos(0.4\pi n - 49.37°)u(n) + 0.1982(0.9)^n \cos(0.8\pi n - 16.9°)u(n)$$
$$+ 1.6023(0.9)^n u(n)$$

P4.19 Separation of the total response $y(n)$ into (i) the homogeneous part, (ii) the particular part, (iii) the transient response, and (iv) the steady-state response for each of the systems given in Problem P4.18.

1. $y(n) = [x(n) + 2x(n-1) + x(n-3)]/4$: The total response is

$$y(n) = -\frac{1310}{729}\delta(n) - \frac{50}{81}\delta(n-1) - \frac{5}{9}\delta(n-2) + \frac{990}{431}(0.9)^n u(n)$$

 i. Homogeneous part: Since the system is an FIR filter, the homogeneous equation is $y(n) = 0$. Thus $y_h(n) = 0$.

 ii. Particular part: Hence the total response is the particular part, or

$$y_p(n) = -\frac{1310}{729}\delta(n) - \frac{50}{81}\delta(n-1) - \frac{5}{9}\delta(n-2) + \frac{990}{431}(0.9)^n u(n)$$

 iii. Transient response: Since the entire response decays to zero,

$$y_{tr}(n) = -\frac{1310}{729}\delta(n) - \frac{50}{81}\delta(n-1) - \frac{5}{9}\delta(n-2) + \frac{990}{431}(0.9)^n u(n)$$

 iv. Steady-state response: Clearly, $y_{ss}(n) = 0$.

2. $y(n) = x(n) + 0.5x(n-1) - 0.5y(n-1) + 0.25y(n-2)$: The total response is

$$y(n) = 0.2617(-0.809)^n u(n) - 0.7567(0.309)^n u(n) + 2.495(0.9)^n u(n)$$

 i. Homogeneous part: The first two terms in $y(n)$ are due to the system poles, hence

$$y_h(n) = 0.2617(-0.809)^n u(n) - 0.7567(0.309)^n u(n)$$

 ii. Particular part: The last term in $y(n)$ is due to the input pole, hence

$$y_p(n) = 2.495(0.9)^n u(n)$$

 iii. Transient response: Since all poles of $Y(z)$ are inside the unit circle,

$$y_{tr}(n) = 0.2617(-0.809)^n u(n) - 0.7567(0.309)^n u(n) + 2.495(0.9)^n u(n)$$

 iv. Steady-state response: Clearly, $y_{ss}(n) = 0$.

3. $y(n) = 2x(n) + 0.9y(n-1)$: The total response is

$$y(n) = \frac{40}{9}(n+1)(0.9)^{n+1} u(n+1)$$

 i. Homogeneous part: Since the system pole and the input pole are the same and hence are indistinguishable. Therefore, the total response can be equally divided into two parts or

$$y_h(n) = \frac{20}{9}(n+1)(0.9)^{n+1} u(n+1)$$

 ii. Particular part: Since the system pole and the input pole are the same and hence are indistinguishable. Therefore, the total response can be equally divided into two parts or

$$y_p(n) = \frac{20}{9}(n+1)(0.9)^{n+1} u(n+1)$$

iii. Transient response: Since all poles of $Y(z)$ are inside the unit circle,

$$y_{\text{tr}}(n) = \frac{40}{9}(n+1)(0.9)^{n+1}u(n+1)$$

iv. Steady-state response: Clearly, $y_{\text{ss}}(n) = 0$.

4. $y(n) = -0.45x(n) - 0.4x(n-1) + x(n-2) + 0.4y(n-1) + 0.45y(n-2)$: The total response is

$$y(n) = 1.1097(-0.5)^n u(n) - 2.4470(0.9)^n u(n) + 0.4859(n+1)(0.9)^{n+1}u(n+1)$$

i. Homogeneous part: There are two system poles, $p_1 = -0.5$ and $p_2 = 0.9$. Clearly, p_2 is also an input pole. Hence the response due to p_2 has to be divided to include in both parts. Hence

$$y_{\text{h}}(n) = 1.1097(-0.5)^n u(n) - 1.1135(0.9)^n u(n) + 0.24295(n+1)(0.9)^{n+1}u(n+1)$$

ii. Particular part: from above,

$$y_{\text{p}}(n) = -1.1135(0.9)^n u(n) + 0.24295(n+1)(0.9)^{n+1}u(n+1)$$

iii. Transient response: Since all poles of $Y(z)$ are inside the unit circle,

$$y_{\text{tr}}(n) = 1.1097(-0.5)^n u(n) - 2.4470(0.9)^n u(n) + 0.4859(n+1)(0.9)^{n+1}u(n+1)$$

iv. Steady-state response: Clearly, $y_{\text{ss}}(n) = 0$.

5. $y(n) = \sum_{m=0}^{4}(0.8)^m x(n-m) - \sum_{\ell=1}^{4}(0.9)^\ell y(n-\ell)$: The total response is

$$y(n) = 0.3197(0.9)^n \cos(0.4\pi n - 49.37°)u(n) + 0.1982(0.9)^n \cos(0.8\pi n - 16.9°)u(n)$$
$$+ 1.6023(0.9)^n u(n)$$

i. Homogeneous part: The first two terms in $y(n)$ are due to the system poles, hence

$$y_{\text{h}}(n) = 0.3197(0.9)^n \cos(0.4\pi n - 49.37°)u(n) + 0.1982(0.9)^n \cos(0.8\pi n - 16.9°)u(n)$$

ii. Particular part: The last term in $y(n)$ is due to the input pole, hence

$$y_{\text{p}}(n) = 1.6023(0.9)^n u(n)$$

iii. Transient response: Since all poles of $Y(z)$ are inside the unit circle,

$$y_{\text{tr}}(n) = 0.3197(0.9)^n \cos(0.4\pi n - 49.37°)u(n) + 0.1982(0.9)^n \cos(0.8\pi n - 16.9°)u(n)$$
$$+ 1.6023(0.9)^n u(n)$$

iv. Steady-state response: Clearly, $y_{\text{ss}}(n) = 0$.

P4.20 A stable system has the following pole-zero locations:

$$\text{zeros: } \pm 1, \, \pm j1 \qquad \text{Poles: } \pm 0.9, \, \pm j0.9$$

It is also known that $H\left(e^{j\pi/4}\right) = 1$.

1. The system function $H(z)$ and its region of convergence: Consider

$$H(z) = K\frac{(z-1)(z+1)(z-j)(z+j)}{(z-0.9)(z+0.9)(z-j0.9)(z+j0/9)} = K\frac{1-z^{-4}}{1-0.6561z^{-4}}, \quad |z| > 0.9$$

Now at $z = e^{j\pi/4}$, we have $H\left(e^{j\pi/4}\right) = 1$. Hence

$$1 = H\left(e^{j\pi/4}\right) = K\frac{1-e^{j\pi}}{1-0.6561e^{j\pi}} = K \times 1.2077 \Rightarrow K = 0.8281$$

or

$$H(z) = \frac{0.8281\left(1-z^{-4}\right)}{1-0.6561z^{-4}}, \quad |z| > 0.9$$

2. The difference equation representation: From

$$H(z) = \frac{0.8281\left(1-z^{-4}\right)}{1-0.6561z^{-4}} = \frac{Y(z)}{X(z)}$$

we have

$$y(n) = 0.8281x(n) - 0.8281x(n-4) + 0.6561y(n-4)$$

3. The steady-state response $y_{ss}(n)$ for the input $x(n) = \cos(\pi n/4)u(n)$: From the z-transform table,

$$X(z) = \frac{1-[\cos(\pi/4)]z^{-1}}{1-[2\cos(\pi/4)]z^{-1}+z^{-2}} = \frac{1-\frac{1}{\sqrt{2}}z^{-1}}{1-\sqrt{2}z^{-1}+z^{-2}}$$

Hence

$$Y(z) = H(z)X(z) = \left[\frac{0.8281\left(1-z^{-4}\right)}{1-0.6561z^{-4}}\right]\left(\frac{1-\frac{1}{\sqrt{2}}z^{-1}}{1-\sqrt{2}z^{-1}+z^{-2}}\right)$$

$$= \frac{1-\frac{1}{\sqrt{2}}z^{-1}}{1-\sqrt{2}z^{-1}+z^{-2}} - \frac{0.0351}{1-0.9z^{-1}} - \frac{0.0509}{1+0.9z^{-1}} + \frac{-0.0860-0.1358z^{-1}}{1-0.81z^{-2}}, \quad |z| > 1$$

The first term above has poles on the unit circle and hence gives the steady-state response

$$y_{ss}(n) = \cos(\pi n/4)$$

4. The transient response $y_{tr}(n)$ for the input $x(n) = \cos(\pi n/4)u(n)$: The remaining terms in $y(n)$ are the transient response terms. Using the `inv_CC_PP` function we have

$$Y_{tr}(z) = -\frac{0.0351}{1-0.9z^{-1}} - \frac{0.0509}{1+0.9z^{-1}} - 0.0860\frac{1}{1-0.81z^{-2}} - 0.1509\frac{0.9z^{-1}}{1-0.81z^{-2}}, \quad |z| > 1$$

Hence

$$y_{tr}(n) = -0.0351(0.9)^n u(n) - 0.0509(-0.9)^n u(n) - 0.086(0.9)^n \cos(\pi n/2)u(n)$$
$$- 0.1509(0.9)^n \sin(\pi n/2)u(n)$$

P4.21 A digital filter is described by the frequency response function

$$H(e^{j\omega}) = [1 + 2\cos(\omega) + 3\cos(2\omega)]\cos(\omega/2)e^{-j5\omega/2}$$

which can be written as

$$H(e^{j\omega}) = \left[1 + 2\left(\frac{e^{j\omega} + e^{-j\omega}}{2}\right) + 3\left(\frac{e^{j2\omega} + e^{-j2\omega}}{2}\right)\right]\frac{e^{j\frac{1}{2}\omega} + e^{-j\frac{1}{2}\omega}}{2}e^{-j\frac{5}{2}\omega}$$

$$= \frac{3}{4} + \frac{5}{4}e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} + \frac{5}{4}e^{-j4\omega} + \frac{3}{4}e^{-j5\omega}$$

or after substituting $e^{-j\omega} = z^{-1}$, we obtain

$$H(z) = \frac{3}{4} + \frac{5}{4}z^{-1} + z^{-2} + z^{-3} + \frac{5}{4}z^{-4} + \frac{3}{4}z^{-5}$$

1. The difference equation representation: From $H(z)$ above

$$y(n) = \frac{3}{4}x(n) + \frac{5}{4}x(n-1) + x(n-2) + x(n-3) + \frac{5}{4}x(n-4) + \frac{3}{4}x(n-5)$$

2. The magnitude and phase response plots are shown in Figure 4.22.
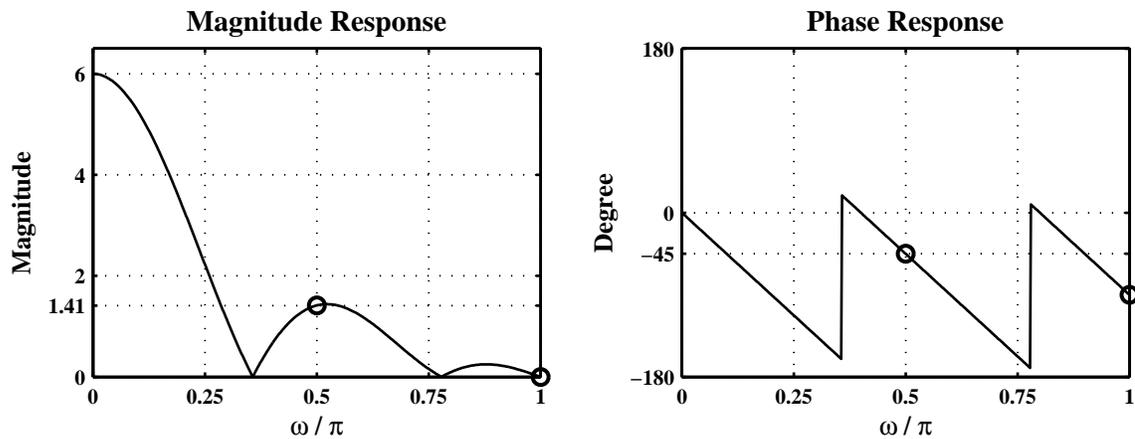
Figure 4.22: Problem P4.21.2 frequency-response plots

The magnitude and phase at $\omega = \pi/2$ are $\sqrt{2}$ and $-45°$, respectively. The magnitude at $\omega = \pi$ is zero.

3. The output sequence $y(n)$ for the input $x(n) = \sin(\pi n/2) + 5\cos(\pi n)$: MATLAB script:

```
clc; close all;  set(0,'defaultfigurepaperposition',[0,0,7,5]);
b = [3/4 5/4 1 1 5/4 3/4]; a = [1 0];
n = 0:200; x = sin(pi*n/2)+5*cos(pi*n); y = filter(b,a,x);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0421c');
subplot(2,1,1); Hs = stem(n,x); set(Hs,'markersize',2); axis([-2 202 -7 6]);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS);
title('x(n) = sin(\pi \times n / 2)+5 \times cos(\pi \times n)',...
        'FontSize',TFS);
subplot(2,1,2); Hs = stem(n,y); set(Hs,'markersize',2); axis([-2 202 -2 4]);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title('Output sequence after filtering','FontSize',TFS);
print -deps2 ../epsfiles/P0421c;
```

The input and output sequence plots are shown in Figure 4.23. It shows that the sinusoidal sequence with the input frequency $\omega = \pi$ is completely suppressed in the steady-state output. The steady-state response of $x(n) = sin(\pi n/2)$ should be (using the magnitude and phase at $\omega = \pi/2$ computed in part 2. above)

$$y_{ss}(n) = \sqrt{2}\sin(\pi n/2 - 45°) = \sqrt{2}\cos(45°)\sin(\pi n/2) - \sqrt{2}\sin(45°)\cos(\pi n/2)$$
$$= sin(\pi n/2) - cos(\pi n/2) = \{\ldots, \underset{\uparrow}{-1}, 1, 1, -1, -1, \ldots\}$$

as verified in the bottom plot of Figure 4.23.
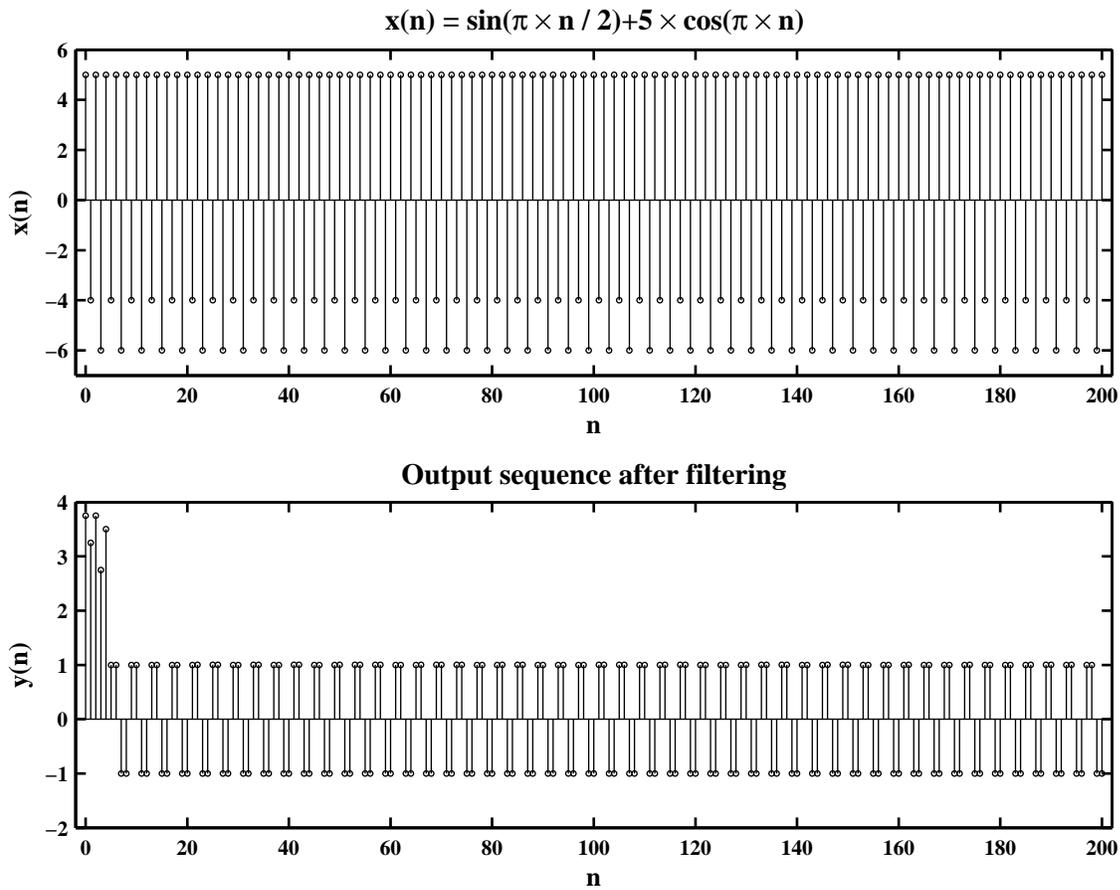
Figure 4.23: Problem P4.21.3 input and output sequence plots

P4.22 A digital filter is described by the frequency response function

$$H(e^{j\omega}) = \frac{1 + e^{-j4\omega}}{1 - 0.8145e^{-j4\omega}}$$

which after substituting $e^{-j\omega} = z^{-1}$ becomes

$$H(z) = \frac{1 + z^{-4}}{1 - 0.8145z^{-4}}$$

1. The difference equation representation: From $H(z)$ above

$$y(n) = x(n) + x(n-4) + 0.8145y(n-4)$$

2. The magnitude and phase response plots are shown in Figure 4.24.
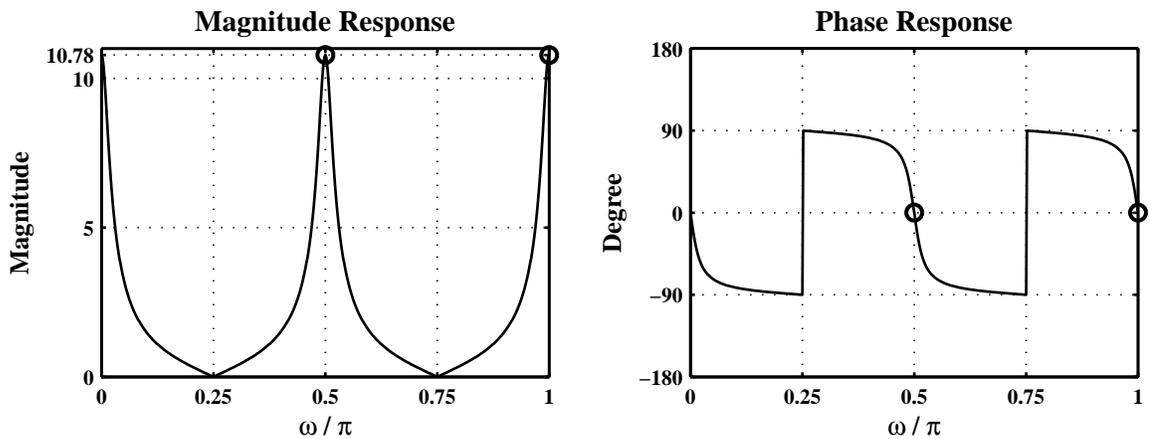
Figure 4.24: Problem P4.22.2 frequency-response plots

The magnitudes and phases at both $\omega = \pi/2$ and $\omega = \pi$ are 10.78 and 0°, respectively.

3. The output sequence $y(n)$ for the input $x(n) = \sin(\pi n/2) + 5\cos(\pi n)$: MATLAB script:

```
clc; close all;  set(0,'defaultfigurepaperposition',[0,0,7,5]);
b = [1 0 0 0 1]; a = [1 0 0 0 -0.8145];
n = 0:200; x = sin(pi*n/2)+5*cos(pi*n); y = filter(b,a,x);
Hf_1 = figure; set(Hf_1,'NumberTitle','off','Name','P0422c');
subplot(2,1,1); Hs = stem(n,x); set(Hs,'markersize',2); axis([-2 202 -7 7]);
xlabel('n','FontSize',LFS); ylabel('x(n)','FontSize',LFS);
title('x(n) = sin(\pi \times n / 2)+5 \times cos(\pi \times n)',...
      'FontSize',TFS);
subplot(2,1,2); Hs = stem(n,y); set(Hs,'markersize',2); axis([-2 202 -70 70]);
xlabel('n','FontSize',LFS); ylabel('y(n)','FontSize',LFS);
title('Output sequence after filtering','FontSize',TFS);
print -deps2 ../epsfiles/P0422c;
```

The input and output sequence plots are shown in Figure 4.25.The steady-state response of $x(n)$ should be (using the magnitude and phase at $\omega = \pi/2$ computed in part 2. above)

$$y_{ss}(n) = 10.78\sin(\pi n/2) + 10.78 \times 5\cos(\pi n)$$
$$= 10.78sin(\pi n/2) + 53.91cos(\pi n/2)$$

The bottom plot of Figure 4.25 shows that both sinusoidal sequences have the scaling of 10.78 and no delay distortion.

$$x(n) = \sin(\pi \times n / 2) + 5 \times \cos(\pi \times n)$$

Output sequence after filtering
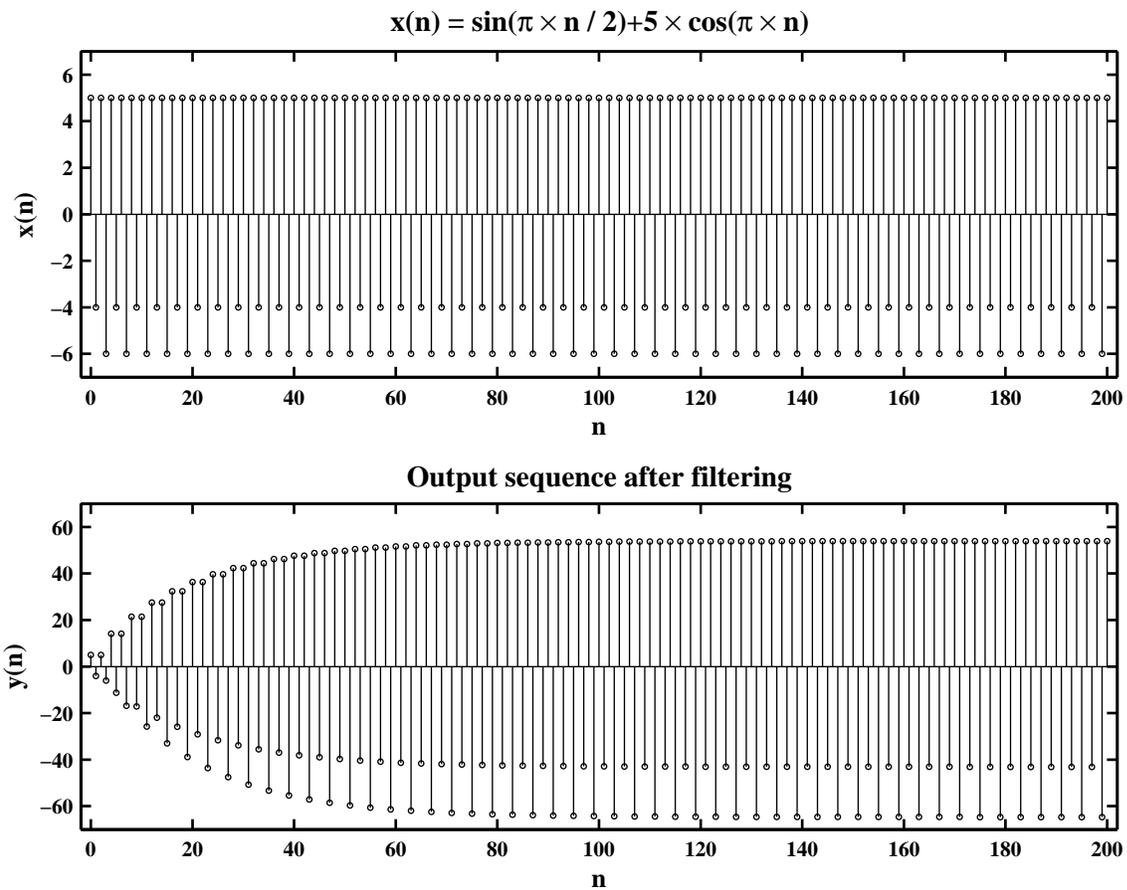
Figure 4.25: Problem P4.22.3 input and output sequence plots

P4.23 Difference equation solution using the one-sided z-transform approach.

$$\begin{aligned} y(n) &= 0.81 y(n-2) + x(n) - x(n-1), \ n \geq 0; \quad y(-1) = 2, \ y(-2) = 2 \\ x(n) &= (0.7)^n u(n+1) \end{aligned}$$

Notice that

$$x(n) = (0.7)^n u(n+1) = \begin{cases} (0.7)^{-1}, & n = -1; \\ (0.7)^n u(n), & n \geq 0. \end{cases}$$

After taking the one-sided z-transform of the above difference equation, we obtain

$$\begin{aligned} Y^+(z) &= 0.81 \left[y(-2) + y(-1)z^{-1} + z^{-2} Y^+(z) \right] + X^+(z) - \left[x(-1) + z^{-1} X^+(z) \right] \\ &= 0.81 z^{-2} Y^+(z) + \left[1 - z^{-1} \right] X^+(z) + [0.81 y(-2) - x(-1)] + 0.81 y(-1) z^{-1} \end{aligned}$$

or

$$Y^+(z) = \frac{1 - z^{-1}}{1 - 0.81 z^{-1}} X^+(z) + \frac{[0.81 y(-2) - x(-1)] + 0.81 y(-1) z^{-1}}{1 - 0.81 z^{-1}}$$

After substituting the initial conditions and $X^+(z) = \mathcal{Z}[0.7^n u(n)] = \dfrac{1}{1 - 0.7 z^{-1}}$, we obtain

$$\begin{aligned} Y^+(z) &= \left(\frac{1 - z^{-1}}{1 - 0.81 z^{-1}} \right) \left(\frac{1}{1 - 0.7 z^{-1}} \right) + \frac{0.1914 + 1.62 z^{-1}}{1 - 0.81 z^{-1}} \\ &= \frac{1.1914 + 0.4860 z^{-1} - 1.1340 z^{-2}}{\left(1 - 0.81 z^{-1} \right) \left(1 - 0.7 z^{-1} \right)} = 2 + \frac{0.4642}{1 - 0.81 z^{-1}} + \frac{2.7273}{1 - 0.7 z^{-1}} \end{aligned}$$

Hence upon inverse transformation

$$y(n) = 2\delta(n) + 0.4642(0.81)^n u(n) + 2.7273(0.7)^n u(n)$$

MATLAB verification:

```
clc; close all;
b1 = [1 -1]; nb1 = [0 1]; a11 = [1 0 -0.81]; na11 = [0 1 2]; a12 = [1 -0.7];
na12 = [0 1]; [a1,na1] = conv_m(a11,na11,a12,na12);
b2 = [0.1914 1.62]; nb2 = [0 1]; a2 = [1 0 -0.81]; na2 = [0 1 2];
[bnr1,nbnr1] = conv_m(b1,nb1,a2,na2); [bnr2,nbnr2] = conv_m(b2,nb2,a1,na1);
[b,nb] = sigadd(bnr1,nbnr1,bnr2,nbnr2); [a,na] = conv_m(a1,na1,a2,na2);
[R,p,k] = residuez(b,a);

 R =

    -0.2106-0.0000i
     0.0000
     0.7457+0.0000i
     0.0000-0.0000i
     0.6562

 p =
```

```
    -0.9000
    -0.9000
     0.9000+0.0000i
     0.9000-0.0000i
     0.7000

 k = []

n = [0:20]; x = 0.7.^n; xic = [0.1914 1.62];
yb1 = filter(b1,a11,x,xic);
yb2 = R(1)*((p(1)).^n)+R(3)*((p(3)).^n)+R(5)*((p(5)).^n);
error = max(abs(yb1-yb2))
error =
  6.2150e-008
```

P4.24 Difference equation solution for $y(n)$, $n \geq 0$: Given

$$y(n) - 0.4y(n-1) - 0.45y(n-2) = 0.45x(n) + 0.4x(n-1) - x(n-2)$$

driven by the input $x(n) = 2 + \left(\frac{1}{2}\right)^n u(n)$ and subject to $y(-1) = 0$, $y(-2) = 3$; $x(-1) = x(-2) = 2$. Taking the one-sided z-transform of the difference equation, we obtain

$$Y^+(z) - 0.4y(-1) - 0.4z^{-1}Y^+(z) - 0.45y(-2) - 0.45y(-1)z^{-1} - 0.45z^{-2}Y^+(z)$$
$$= 0.45X^+(z) + 0.4x(-1) + 0.4z^{-1}X^+(z) - x(-2) - x(-1)z^{-1} - z^{-2}X^+(z)$$

or
$$Y^+(z) = \frac{0.45 + 0.4z^{-1} - z^{-2}}{1 - 0.4z^{-1} - 0.45z^{-2}}X^+(z)$$
$$+ \frac{[0.4y(-1) + 0.45y(-2) + 0.4x(-1) - x(-2)] + [0.45y(-1) - x(-1)]z^{-1}}{1 - 0.4z^{-1} - 0.45z^{-2}}$$

After substituting the initial conditions and $X^+(z) = \mathcal{Z}\left[[2 + \left(\frac{1}{2}\right)^n]u(n)\right] = \frac{2}{1 - z^{-1}} + \frac{1}{1 - 0.5z^{-1}}$, we obtain

$$Y^+(z) = \left(\frac{0.45 + 0.4z^{-1} - z^{-2}}{1 - 0.4z^{-1} - 0.45z^{-2}}\right)\left(\frac{2}{1 - z^{-1}} + \frac{1}{1 - 0.5z^{-1}}\right) + \frac{0.15 - 2z^{-1}}{1 - 0.4z^{-1} - 0.45z^{-2}}$$
$$= \frac{1.35 + 0.3z^{-1} - 3.8z^{-2} + 2z^{-3}}{\left(1 - 0.9z^{-1}\right)\left(1 + 0.5z^{-1}\right)\left(1 - z^{-1}\right)\left(1 - 0.5z^{-1}\right)} + \frac{0.15 - 2z^{-1}}{1 - 0.4z^{-1} - 0.45z^{-2}} \qquad (4.13)$$
$$= \frac{1.5 - 1.925z^{-1} - 0.725z^{-2} + z^{-3}}{1 - 1.9z^{-1} + 0.65z^{-2} + 0.475z^{-3} - 0.225z^{-4}}$$
$$= \frac{-2}{1 - z^{-1}} + \frac{2.1116}{1 - 0.9z^{-1}} + \frac{1.7188}{1 - 0.5z^{-1}} - \frac{0.3304}{1 + 0.5z^{-1}}$$

Hence after inverse transformation

$$y(n) = \left[-2 + 2.1116(0.9)^n + 1.7188(0.5)^n - 0.3303(-0.5)^n\right]u(n)$$

(a) Transient response: This response is due to the poles inside the unit circle or equivalently, the part of $y(n)$ that decays to zero as $n \nearrow \infty$. Thus

$$y_{\text{tr}}(n) = \left[2.1116(0.9)^n + 1.7188(0.5)^n - 0.3303(-0.5)^n\right]u(n)$$

(b) Steady-state response: This response is due to poles on the unit circle. Hence $y_{\text{ss}}(n) = -2$.

(c) Zero input response: In (4.13), the last term on the right corresponds to the initial condition or zero-input response. Hence

$$Y_{\text{ZI}}^+(z) = \frac{0.15 - 2z^{-1}}{1 - 0.4z^{-1} - 0.45z^{-2}} = \frac{-1.3321}{1 - 0.9z^{-1}} + \frac{1.4821}{1 + 0.5z^{-1}}$$

or
$$y_{\text{ZI}}(n) = \left[-1.3321(0.9)^n + 1.4821(-0.5)^n\right]u(n)$$

(d) Zero-state response: In (4.13), the first term on the right corresponds to the input excitation or is the zero-state response. Hence

$$Y_{\text{ZS}}^+(z) = \frac{1.35 + 0.3z^{-1} - 3.8z^{-2} + 2z^{-3}}{\left(1 - 0.9z^{-1}\right)\left(1 + 0.5z^{-1}\right)\left(1 - z^{-1}\right)\left(1 - 0.5z^{-1}\right)}$$
$$= \frac{-2}{1 - z^{-1}} + \frac{3.4438}{1 - 0.9z^{-1}} + \frac{1.7187}{1 - 0.5z^{-1}} - \frac{1.8125}{1 + 0.5z^{-1}}$$

or
$$y_{\text{ZS}}(n) = \left[-2 + 3.4438(0.9)^n + 1.7187(0.5)^n - 1.8125(-0.5)^n\right]u(n)$$

P4.25 A stable, linear and time-invariant system is given by the following system function

$$H(z) = \frac{4z^2 - 2\sqrt{2}z + 1}{z^2 - 2\sqrt{2}z + 4} = \frac{4 - 2\sqrt{2}z^{-1} + z^{-2}}{1 - 2\sqrt{2}z^{-1} + 4z^{-2}} \tag{4.14}$$

$$= \frac{\left(1 - 0.5e^{j45°}z^{-1}\right)\left(1 - 0.5e^{-j45°}z^{-1}\right)}{\left(1 - 2e^{j45°}z^{-1}\right)\left(1 - 2e^{-j45°}z^{-1}\right)}, \ |z| < 2 \text{ (for stability)} \tag{4.15}$$

$$= 0.25 + \frac{2.1866e^{-j30.96°}}{1 - 2e^{j45°}z^{-1}} + \frac{2.1866e^{j30.96°}}{1 - 2e^{-j45°}z^{-1}}, \ |z| < 2 \tag{4.16}$$

which is an anti-causal system.

1. The difference equation representation: From (4.14) above,

$$y(n) - 2\sqrt{2}y(n-1) + 4y(n-2) = 4x(n) - 2\sqrt{2}x(n-1) + x(n-2)$$

   Hence for anti-causal implementation

$$y(n) = \frac{1}{4}x(n) - \frac{1}{\sqrt{2}}x(n+1) + x(n+2) + \frac{1}{\sqrt{2}}y(n+1) - \frac{1}{4}y(n+2)$$

2. The pole-zero plot: From (4.15), the zeros are at $0.5e^{\pm 45°}$ and poles are at $2e^{\pm 45°}$. The pole-zero plot is shown in Figure 4.26.
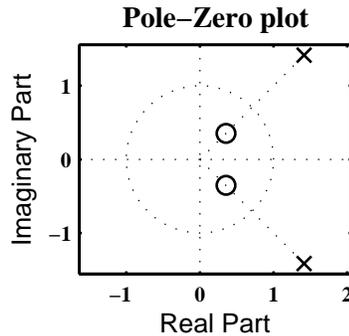
Figure 4.26: Problem P4.25 pole-zero plot

3. The unit sample response $h(n)$: From (4.16),

$$h(n) = 0.25\delta(n) - \left[2.1866e^{-j30.96°}\left(2e^{j45°}\right)^n + 2.1866e^{j30.96°}\left(2e^{-j45°}\right)^n\right]u(-n-1)$$
$$= 0.25\delta(n) - 2.1866(2)^n\left[e^{-j30.96°}e^{j\pi/4n} + e^{j30.96°}e^{-j\pi/4n}\right]u(-n-1)$$
$$= 0.25\delta(n) - 2.1866(2)^n\cos(\pi n/4 - 30.96°)u(-n-1)$$

4. The system is anti-causal. The causal (but not stable) unit-sample response is given by the system function

$$H(z) = 0.25 + \frac{2.1866e^{-j30.96°}}{1 - 2e^{j45°}z^{-1}} + \frac{2.1866e^{j30.96°}}{1 - 2e^{-j45°}z^{-1}}, \ |z| > 2$$

   Hence
$$h(n) = 0.25\delta(n) + 2.1866(2)^n\cos(\pi n/4 - 30.96°)u(n)$$

P4.26 The zero-input, zero-state, and steady-state responses of the system

$$y(n) = 0.9801y(n-2) + x(n) + 2x(n-1) + x(n-2), \ n \geq 0; \quad y(-2) = 1, \ y(-1) = 0$$

to the input $x(n) = 5(-1)^n u(n)$: After taking the one-sided z-transform of the above difference equation, we obtain

$$Y^+(z) = 0.9801\left[y(-2) + y(-1)z^{-1} + z^{-2}Y^+(z)\right] + X^+(z) + 2\left[x(-1) + z^{-1}X^+(z)\right]$$
$$+ \left[x(-2) + x(-1)z^{-1} + z^{-2}X^+(z)\right]$$

After substituting the initial conditions and $X(z) = \mathcal{Z}\left[5(-1)^n u(n)\right] = \dfrac{5}{1+z^{-1}}$, we obtain

$$Y^+(z) = \frac{1+2z^{-1}+z^{-2}}{1-0.9801z^{-2}}X^+(z) + \frac{[2x(-1)+x(-2)+0.9801y(-2)] + [2+x(-1)]z^{-1}}{1-0.9801z^{-2}}$$

$$= \left(\frac{1+2z^{-1}+z^{-2}}{1-0.9801z^{-2}}\right)\left(\frac{5}{1+z^{-1}}\right) + \frac{0.9801+2z^{-1}}{1-0.9801z^{-2}} \tag{4.17}$$

$$= \frac{5+10z^{-1}+5z^{-2}}{1+z^{-1}-0.9801z^{-2}-0.9801z^{-3}} + \frac{0.9801+2z^{-1}}{1-0.9801z^{-2}} \tag{4.18}$$

$$= \frac{5.9801+12.9801z^{-1}+7z^{-2}}{1+z^{-1}-0.9801z^{-2}-0.9801z^{-3}}$$

$$= \frac{-0.5453}{1+0.99z^{-1}} + \frac{6.5254}{1-0.99z^{-1}} \tag{4.19}$$

Hence

$$y(n) = -0.5453(-0.99)^n u(n) + 6.5254(0.99)^n u(n)$$

(a) Zero-input response: From (4.17), we have

$$H_{\text{ZI}}(z) = \frac{0.9801+2z^{-1}}{1-0.9801z^{-2}} = \frac{-0.5201}{1+0.99z^{-1}} + \frac{1.5002}{1-0.99z^{-1}}$$

Hence

$$y_{\text{ZI}}(n) = -0.5201(-0.99)^n u(n) + 1.5002(0.99)^n u(n)$$

(b) Zero-state response: From (4.18), we have

$$H_{\text{ZS}}(z) = \frac{5+10z^{-1}+5z^{-2}}{1+z^{-1}-0.9801z^{-2}-0.9801z^{-3}} = \frac{-0.0253}{1+0.99z^{-1}} + \frac{5.0253}{1-0.99z^{-1}}$$

Hence

$$y_{\text{ZS}}(n) = -0.0253(-0.99)^n u(n) + 5.0253(0.99)^n u(n)$$

(c) Steady-state response: Since the total response $y(n)$ goes to zero as $n \nearrow \infty$, there is no steady-state response.

Chapter 5

The Discrete-Time Fourier Transform

P5.1 Compute the DFS coefficients of the following periodic sequences using the DFS definition and then verify your answers using MATLAB .

1. $\tilde{x}_1(n) = \{4, 1, -1, 1\}$, $N = 4$

    ```
    xtilde1 = [4,1,-1,1]; N = 4; Xtilde1 = dfs(xtilde1,N)
    Xtilde1 =
        5.0000              5.0000 + 0.0000i   1.0000 - 0.0000i   5.0000 + 0.0000i
    ```

2. $\tilde{x}_2(n) = \{2, 0, 0, 0, -1, 0, 0, 0\}$, $N = 8$

    ```
    xtilde2 = [2,0,0,0,-1,0,0,0]; N = 8; Xtilde2 = dfs(xtilde2,N)
    Xtilde2 =
      Columns 1 through 4
        1.0000              3.0000 + 0.0000i   1.0000 - 0.0000i   3.0000 + 0.0000i
      Columns 5 through 8
        1.0000 - 0.0000i   3.0000 + 0.0000i   1.0000 - 0.0000i   3.0000 + 0.0000i
    ```

3. $\tilde{x}_3(n) = \{1, 0, -1, -1, 0\}$, $N = 5$

    ```
    xtilde3 = [1,0,-1,-1,0]; N = 5; Xtilde3 = dfs(xtilde3,N)
    Xtilde3 =
      Columns 1 through 4
       -1.0000              2.6180 + 0.0000i   0.3820 - 0.0000i   0.3820
      Column 5
        2.6180 + 0.0000i
    ```

4. $\tilde{x}_4(n) = \{0, 0, 2j, 0, 2j, 0\}$, $N = 6$

    ```
    xtilde4 = [0,0,2j,0,2j,0]; N = 6; Xtilde4 = dfs(xtilde4,N)
    Xtilde4 =
      Columns 1 through 4
            0 + 4.0000i   0.0000 - 2.0000i   0.0000 - 2.0000i  -0.0000 + 4.0000i
      Columns 5 through 6
        0.0000 - 2.0000i   0.0000 - 2.0000i
    ```

5. $\tilde{x}_5(n) = \{3, 2, 1\}$, $N = 3$

```
xtilde5 = [3,2,1]; N = 3; Xtilde5 = dfs(xtilde5,N)
Xtilde5 =
    6.0000              1.5000 - 0.8660i   1.5000 + 0.8660i
```

P5.2 Determine the periodic sequences given the following periodic DFS coefficients. First use the IDFS definition and then verify your answers using MATLAB .

1. $\tilde{X}_1(k) = \{4, 3j, -3j\}$, $N = 3$

   ```
   Xtilde1 = [4,j*3,-j*3]; N = 3; xtilde1 = idfs(Xtilde1,N)

   xtilde1 =
       1.3333              -0.3987 + 0.0000i   3.0654 - 0.0000i
   ```

2. $\tilde{X}_2(k) = \{j, 2j, 3j, 4j\}$, $N = 4$

   ```
   Xtilde2 = [j,j*2,j*3,j*4]; N = 4; xtilde2 = idfs(Xtilde2,N)

   xtilde2 =
           0 + 2.5000i   0.5000 - 0.5000i  -0.0000 - 0.5000i  -0.5000 - 0.5000i
   ```

3. $\tilde{X}_3(k) = \{1, 2 + 3j, 4, 2 - 3j\}$, $N = 4$

   ```
   Xtilde3 = [1,2+j*3,4,2-j*3]; N = 4; xtilde3 = idfs(Xtilde3,N)

   xtilde3 =
       2.2500              -2.2500 + 0.0000i   0.2500              0.7500 - 0.0000i
   ```

4. $\tilde{X}_4(k) = \{0, 0, 2, 0\}$, $N = 5$

   ```
   Xtilde4 = [0,0,2,0,0]; N = 5; xtilde4 = idfs(Xtilde4,N)

   xtilde4 =
     Columns 1 through 4
       0.4000              -0.3236 + 0.2351i   0.1236 - 0.3804i   0.1236 + 0.3804i
     Column 5
      -0.3236 - 0.2351i
   ```

5. $\tilde{X}_5(k) = \{3, 0, 0, 0, -3, 0, 0, 0\}$, $N = 8$

   ```
   Xtilde5 = [3,0,0,0,-3,0,0,0]; N = 8; xtilde5 = idfs(Xtilde5,N)

   xtilde5 =
     Columns 1 through 4
            0              0.7500 - 0.0000i        0 + 0.0000i   0.7500 - 0.0000i
     Columns 5 through 8
            0 + 0.0000i   0.7500 - 0.0000i        0 + 0.0000i   0.7500 - 0.0000i
   ```

P5.3 Let $\tilde{x}_1(n)$ be periodic with fundamental period $N = 40$ where one period is given by

$$\tilde{x}_1(n) = \begin{cases} 5\sin(0.1\pi n), & 0 \le n \le 19 \\ 0, & 20 \le n \le 39 \end{cases}$$

and let $\tilde{x}_2(n)$ be periodic with fundamental period $N = 80$, where one period is given by

$$\tilde{x}_2(n) = \begin{cases} 5\sin(0.1\pi n), & 0 \le n \le 19 \\ 0, & 20 \le n \le 79 \end{cases}$$

These two periodic sequences differ in their periodicity but otherwise have the same non-zero samples.

1. Computation of $\tilde{X}_1(k)$ using MATLAB :

```
n1 = [0:39]; xtilde1 = [5*sin(0.1*pi*[0:19]),zeros(1,20)]; N1 = length(n1);
[Xtilde1] = dft(xtilde1,N1); k1 = n1;
mag_Xtilde1 = abs(Xtilde1); pha_Xtilde1 = angle(Xtilde1)*180/pi;
zei = find(mag_Xtilde1 < 1000*eps);
pha_Xtilde1(zei) = zeros(1,length(zei));
Hf_1 = figure('Units','normalized','position',[0.1,0.1,0.8,0.8],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,5]);
set(Hf_1,'NumberTitle','off','Name','P5.3.1');
subplot(3,1,1); H_s1 = stem(n1,xtilde1,'filled'); set(H_s1,'markersize',3);
axis([-1,N1,-6,6]);
title('One period of the periodic sequence xtilde_1(n)','fontsize',10);
ntick = [n1(1):2:n1(N1),N1]'; ylabel('Amplitude');
set(gca,'XTickMode','manual','XTick',ntick,'FontSize',8);
subplot(3,1,2); H_s2 = stem(k1,mag_Xtilde1,'filled'); set(H_s2,'markersize',3);
axis([-1,N1,0,max(mag_Xtilde1)+10]);
title('Magnitude of Xtilde_1(k)','fontsize',10); ylabel('Magnitude');
ktick = [k1(1):2:k1(N1),N1]';
set(gca,'XTickMode','manual','XTick',ktick,'FontSize',8)
subplot(3,1,3); H_s3 = stem(k1,pha_Xtilde1,'filled'); set(H_s3,'markersize',3);
title('Phase of Xtilde_1(k)','fontsize',10); xlabel('k'); ylabel('Degrees');
ktick = [k1(1):2:k1(N1),N1]'; axis([-1,N1,-200,200]);
set(gca,'XTickMode','manual','XTick',ktick,'FontSize',8);
set(gca,'YTickMode','manual','YTick',[-180;-90;0;90;180]);
```

Plots of $\tilde{x}_1(n)$ and $\tilde{X}_1(k)$ are shown in Figure 5.1.

2. Computation of $\tilde{X}_2(k)$ using MATLAB :

```
n2 = [0:79]; xtilde2 = [xtilde1, zeros(1,40)]; N2 = length(n2);
[Xtilde2] = dft(xtilde2,N2); k2 = n2;
mag_Xtilde2 = abs(Xtilde2); pha_Xtilde2 = angle(Xtilde2)*180/pi;
zei = find(mag_Xtilde2 < 1000*eps);
pha_Xtilde2(zei) = zeros(1,length(zei));
Hf_2 = figure('Units','normalized','position',[0.1,0.1,0.8,0.8],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,5]);
set(Hf_2,'NumberTitle','off','Name','P5.3.2');
subplot(3,1,1); H_s1 = stem(n2,xtilde2,'filled'); set(H_s1,'markersize',3);
title('One period of the periodic sequence xtilde2(n)','fontsize',10);
```

One period of the periodic sequence xtilde₁(n)

Magnitude of Xtilde₁(k)

Phase of Xtilde₁(k)
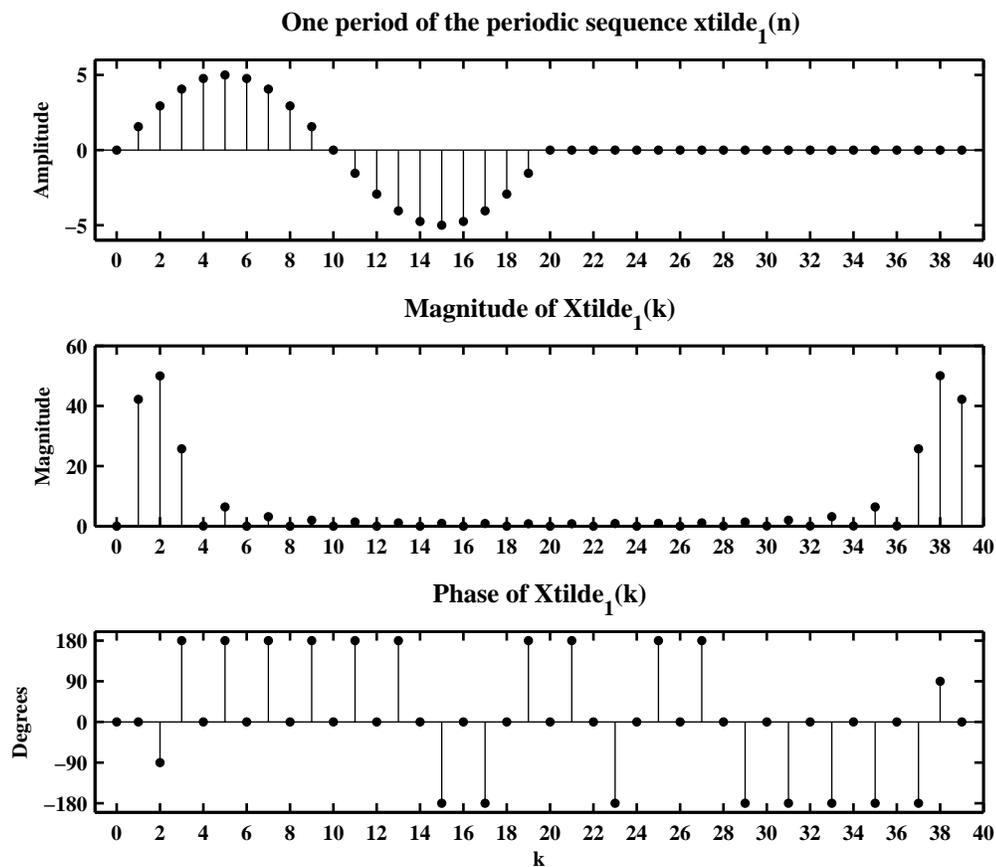
Figure 5.1: Plots of $\tilde{x}_1(n)$ and $\tilde{X}_1(k)$ in Problem 5.3a

```
ntick = [n2(1):5:n2(N2),N2]'; ylabel('xtilde2'); axis([-1,N2,-6,6]);
set(gca,'XTickMode','manual','XTick',ntick)
subplot(3,1,2); H_s2 = stem(k2,mag_Xtilde2,'filled'); set(H_s2,'markersize',3);
axis([-1,N2,0,60]);
title('Magnitude of Xtilde2(k)','fontsize',10); ylabel('|Xtilde2|')
ktick = [k2(1):5:k2(N2),N2]';
set(gca,'XTickMode','manual','XTick',ktick)
subplot(3,1,3); H_s3 = stem(k2,pha_Xtilde2,'filled'); set(H_s3,'markersize',3);
title('Phase of Xtilde2(k)','fontsize',10); xlabel('k'); ylabel('Degrees')
ktick = [k2(1):5:k2(N2),N2]'; axis([-1,N2,-200,200]);
set(gca,'XTickMode','manual','XTick',ktick)
set(gca,'YTickMode','manual','YTick',[-180;-90;0;90;180])
```

Plots of $\tilde{x}_2(n)$ and $\tilde{X}_2(k)$ are shown in Figure 5.2.

3. Changing the period from $N = 40$ to $N = 80$ resulted in a lower frequency sampling interval (higher frequency resolution) ω_1, i.e., in (5) $\omega_1 = \pi/20$ and in (5) $\omega_2 = \pi/40$. Hence there are more terms in the DFS expansion of $\tilde{x}_2(n)$. The shape of the DTFT begins to fill in with $N = 80$.
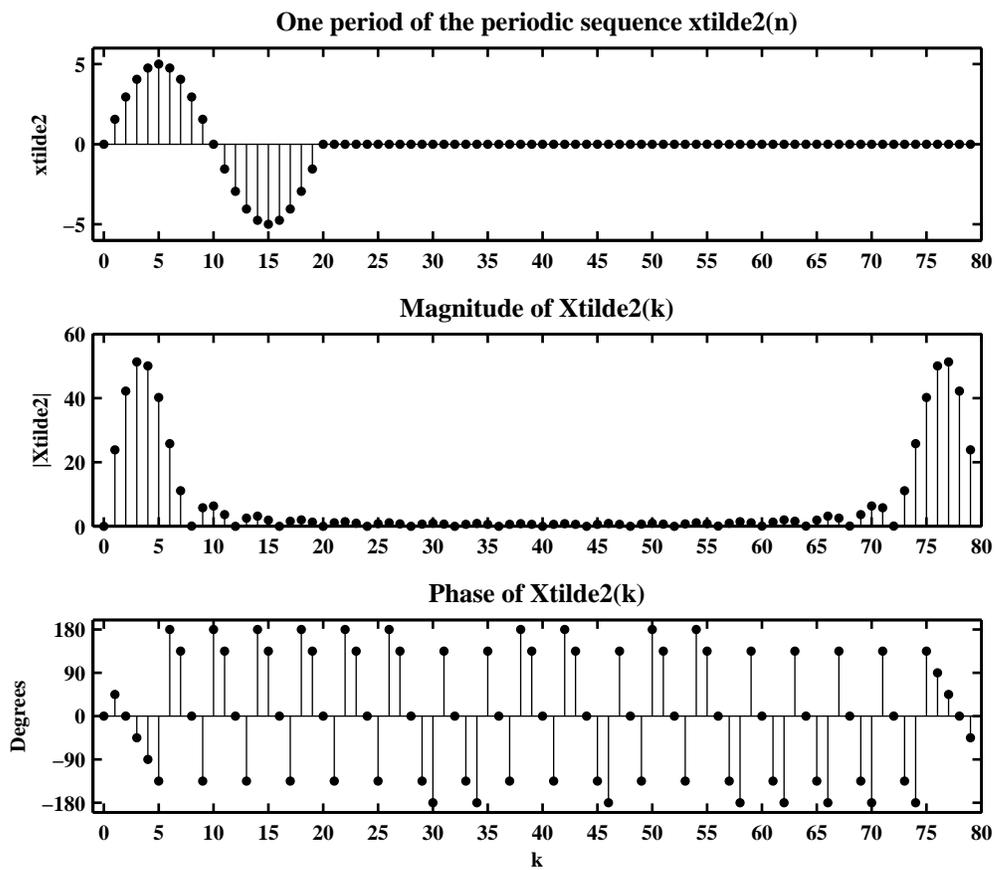
Figure 5.2: Plots of Magnitude and Phase of $\tilde{X}_2(k)$ in Problem 5.3b

P5.4 Consider the periodic sequence $\tilde{x}_1(n)$ given in Problem 5.3. Let $\tilde{x}_2(n)$ be periodic with fundamental period $N = 40$, where one period is given by

$$\tilde{x}_2(n) = \begin{cases} \tilde{x}_1(n), & 0 \le n \le 19 \\ -\tilde{x}_1(n - 20), & 20 \le n \le 39 \end{cases}$$

1. Determine analytically the DFS $\tilde{X}_2(k)$ in terms of $\tilde{X}_1(k)$.
2. Computation of the DFS $\tilde{X}_2(k)$ using MATLAB :

```
n1 = [0:19]; xtilde1 = [5*sin(0.1*pi*n1)];
n2 = [0:39]; xtilde2 = [xtilde1, -xtilde1]; N2 = length(n2);
[Xtilde2] = dft(xtilde2,N2); k2 = n2;
mag_Xtilde2 = abs(Xtilde2); pha_Xtilde2 = angle(Xtilde2)*180/pi;
zei = find(mag_Xtilde2 < 1000*eps);
pha_Xtilde2(zei) = zeros(1,length(zei));
Hf_1 = figure('Units','normalized','position',[0.1,0.1,0.8,0.8],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P5.4.2');
subplot(3,1,1); H_s1 = stem(n2,xtilde2,'filled'); set(H_s1,'markersize',3);
axis([-1,N2,-6,6]);
title('One period of the periodic sequence xtilde_2(n)','fontsize',10);
ntick = [n2(1):5:n2(N2),N2]'; ylabel('Amplitude');
set(gca,'XTickMode','manual','XTick',ntick)
subplot(3,1,2); H_s2 = stem(k2,mag_Xtilde2,'filled'); set(H_s2,'markersize',3);
axis([-1,N2,0,100]);
title('Magnitude of Xtilde2(k)','fontsize',10); ylabel('Magnitude')
ktick = [k2(1):5:k2(N2),N2]';
set(gca,'XTickMode','manual','XTick',ktick)
subplot(3,1,3); H_s3 = stem(k2,pha_Xtilde2,'filled'); set(H_s3,'markersize',3);
title('Phase of Xtilde2(k)','fontsize',10); xlabel('k'); ylabel('Degrees')
ktick = [k2(1):5:k2(N2),N2]'; axis([-1,N2,-200,200]);
set(gca,'XTickMode','manual','XTick',ktick)
set(gca,'YTickMode','manual','YTick',[-180;-90;0;90;180])
```

Plots of $\tilde{x}_2(n)$ and $\tilde{X}_2(k)$ are shown in Figure 5.3.

3. Verify your answer in part 1 above using the plots of $\tilde{X}_1(k)$ and $\tilde{X}_2(k)$?
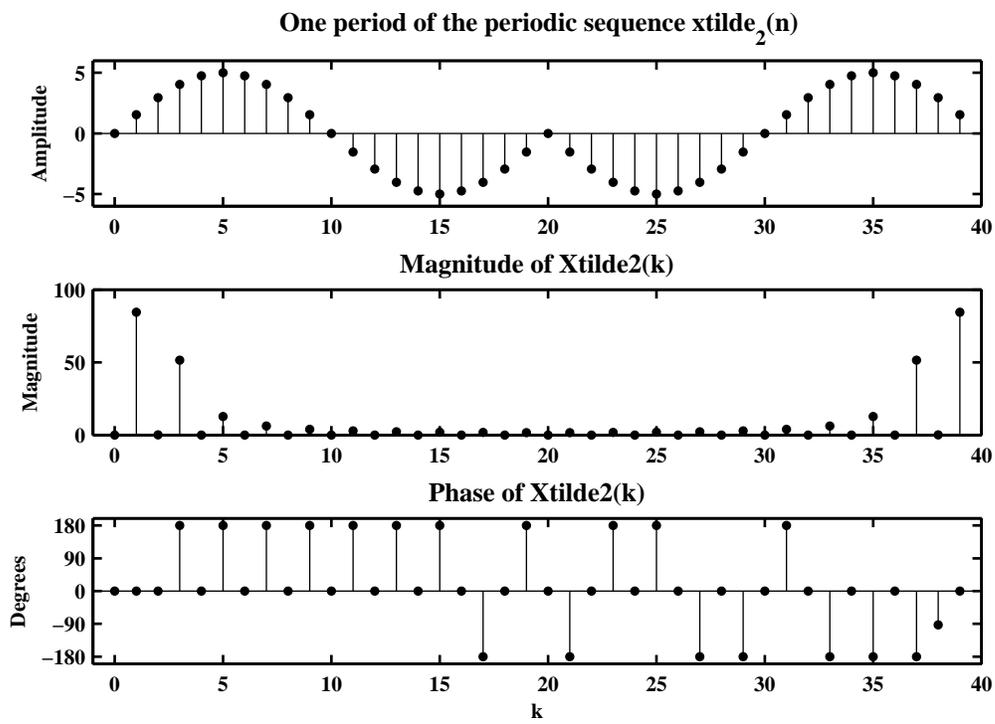
Figure 5.3: Plots of Magnitude and Phase of $\tilde{X}_2(k)$ in Problem 5.4b

P5.5 Consider the periodic sequence $\tilde{x}_1(n)$ given in Problem 5.3. Let $\tilde{x}_3(n)$ be periodic with period 80, obtained by concatenating two periods of $\tilde{x}_1(n)$, i.e.,

$$\tilde{x}_3(n) = \big[\tilde{x}_1(n), \tilde{x}_1(n)\big]_{\text{PERIODIC}}$$

Clearly, $\tilde{x}_3(n)$ is different from $\tilde{x}_2(n)$ of Problem 5.3 even though both of them are periodic with period 80.

1. Computation and plot of the DFS $\tilde{X}_3(k)$ using MATLAB :

```
n1 = [0:39]; xtilde1 = [5*sin(0.1*pi*[0:19]),zeros(1,20)];
n3 = [0:79]; xtilde3 = [xtilde1, xtilde1]; N3 = length(n3);
[Xtilde3] = dft(xtilde3,N3); k3 = n3;
mag_Xtilde3 = abs(Xtilde3); pha_Xtilde3 = angle(Xtilde3)*180/pi;
zei = find(mag_Xtilde3 < 0.00001);
pha_Xtilde3(zei) = zeros(1,length(zei));
Hf_1 = figure('Units','normalized','position',[0.1,0.1,0.8,0.8],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,5]);
set(Hf_1,'NumberTitle','off','Name','P5.5.1');
subplot(3,1,1); H_s1 = stem(n3,xtilde3,'filled'); set(H_s1,'markersize',3);
title('One period of the periodic sequence xtilde_3(n)','fontsize',10);
ylabel('Amplitude'); ntick = [n3(1):5:n3(N3),N3]';axis([-1,N3,-6,6]);
set(gca,'XTickMode','manual','XTick',ntick,'fontsize',8)
subplot(3,1,2); H_s2 = stem(k3,mag_Xtilde3,'filled'); set(H_s2,'markersize',3);
axis([-1,N3,min(mag_Xtilde3),max(mag_Xtilde3)]);
title('Magnitude of Xtilde_3(k)','fontsize',10); ylabel('Magnitude')
ktick = [k3(1):5:k3(N3),N3]'; set(gca,'XTickMode','manual','XTick',ktick)
subplot(3,1,3); H_s3 = stem(k3,pha_Xtilde3,'filled'); set(H_s3,'markersize',3);
title('Phase of Xtilde3(k)','fontsize',10); xlabel('k'); ylabel('Degrees');
ktick = [k3(1):5:k3(N3),N3]';axis([-1,N3,-180,180]);
set(gca,'XTickMode','manual','XTick',ktick)
set(gca,'YTickMode','manual','YTick',[-180;-90;0;90;180])
```

Plots of $\tilde{x}_3(n)$ and $\tilde{X}_3(k)$ are shown in Figure 5.4.

2. Comparing the magnitude plot above with that of $\tilde{X}_1(k)$ in Problem (5), we observe that these plots are essentially similar. Plots of $\tilde{X}_3(k)$ have one zero between every sample of $\tilde{X}_1(k)$. (In general, for phase plots, we do get non-zero phase values when the magnitudes are zero. Clearly these phase values have no meaning and should be ignored. This happens because of a particular algorithm used by MATLAB. We avoided this problem by using the `find` function.) This makes sense because sequences $\tilde{x}_1(n)$ and $\tilde{x}_3(n)$, when viewed over $-\infty < n < \infty$ interval, look exactly same. The effect of periodicity doubling is in the doubling of magnitude of each sample.

1. We can now generalize this argument. If

$$\tilde{x}_M(n) = \left\{ \underbrace{\tilde{x}_1(n), \tilde{x}_1(n), \ldots, \tilde{x}_1(n)}_{M \text{ times}} \right\}_{\text{PERIODIC}}$$

then there will be $(M-1)$ zeros between samples of $\tilde{X}_M(k)$. The magnitudes of non-zero samples of $\tilde{X}_M(k)$ will be M times the magnitudes of the samples of $\tilde{X}_1(k)$, i.e.,

$$\begin{aligned} \tilde{X}_M(Mk) &= M\tilde{X}_1(k) &, k = 0, 1, \ldots, N-1 \\ \tilde{X}_M(k) &= 0 &, k \neq 0, 1, \ldots, MN \end{aligned}$$
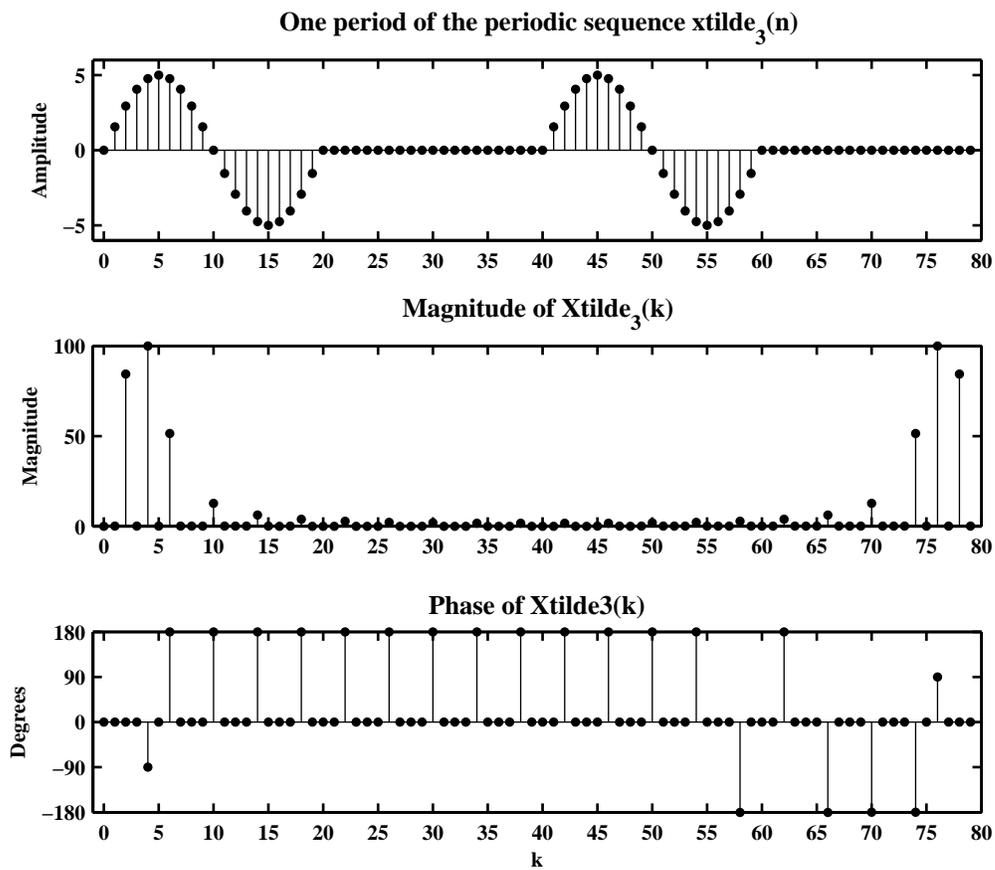
Figure 5.4: Plots of $\tilde{x}_3(n)$ and $\tilde{X}_3(k)$ in Problem 5.5a

P5.6 Let $X(e^{j\omega})$ be the DTFT of a finite-length sequence

$$x(n) = \begin{cases} n+1, & 0 \le n \le 49; \\ 100-n, & 50 \le n \le 99; \\ 0, & \text{otherwise.} \end{cases}$$

1. Let

$$y_1(n) = \overset{\text{10-point}}{\text{IDFS}} \left[X(e^{j0}), X(e^{j2\pi/10}), X(e^{j4\pi/10}), \ldots, X(e^{j18\pi/10}) \right]$$

which is a 10-point IDFS of ten samples of $X\left(e^{j\omega}\right)$ on the unit circle. Thus

$$y_1(n) = \sum_{r=-\infty}^{\infty} x(n-10r) = \{1 + 11 + \cdots + 41 + 50 + 40 + \cdots + 10,$$

$$2 + 12 + \cdots + 42 + 49 + \cdots + 9, \cdots \}_{\text{periodic}}$$

$$= \{255, 255, \ldots, 255\}_{\text{periodic}}$$

MATLAB verification:

```
n = 0:99; x = [n(1:50)+1,100-n(51:100)];
N1 = 10; k1 = 0:N1-1; w1 = 2*pi*k1/N1;
Y1 = dtft(x,n,w1); y1 = real(idfs(Y1,N1));
```

See the `stem` plot of $y_1(n)$ in Figure 5.5.

2. Let

$$y_2(n) = \overset{\text{200-point}}{\text{IDFS}} \left[X(e^{j0}), X(e^{j2\pi/200}), X(e^{j4\pi/200}), \ldots, X(e^{j398\pi/200}) \right]$$

which is a 200-point IDFS of 200 samples of $X\left(e^{j\omega}\right)$ on the unit circle. Thus

$$y_2(n) = \left. \begin{cases} x(n), & 0 \le n \le 49; \\ 0, & 50 \le n \le 100. \end{cases} \right\}_{\text{periodic}}$$

MATLAB verification:

```
n = 0:99; x = [n(1:50)+1,100-n(51:100)];
N2 = 200; k2 = 0:N2-1; w2 = 2*pi*k2/N2;
Y2 = dtft(x,n,w2); y2 = real(idfs(Y2,N2));
```

See the `stem` plot of $y_1(n)$ in Figure 5.5.

.

3. The sequence $y_1(n)$ is a 10-point aliasing version on $x(n)$ while $y_2(n)$ is a zero-padded version of $x(n)$.

Original 100−point sequence x(n)

Aliased sequence y_1(n)

Unaliased sequence y_2(n)
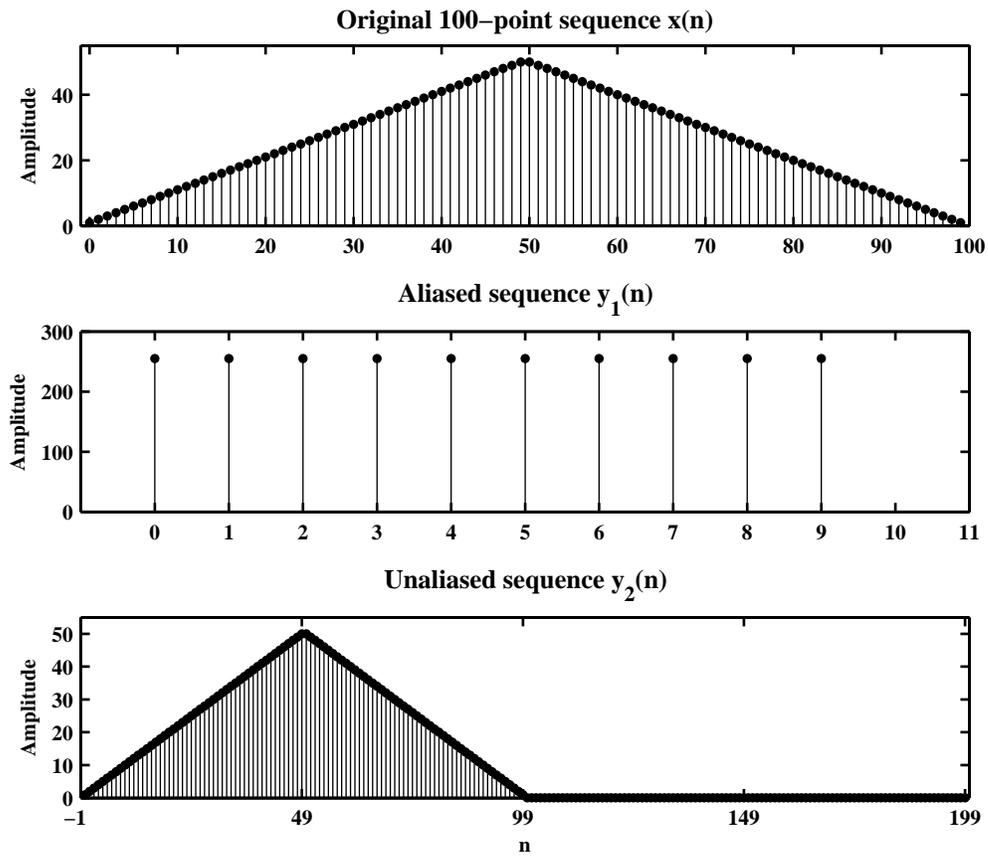
Figure 5.5: Plots of $y_1(n)$ and $y_2(k)$ in Problem 5.6

P5.7 Let $\tilde{x}(n)$ be a periodic sequence with period N and let

$$\tilde{y}(n) \stackrel{\triangle}{=} \tilde{x}(-n) = \tilde{x}(N-n)$$

that is, $\tilde{y}(n)$ is a periodically folded version of $\tilde{x}(n)$. Let $\tilde{X}(k)$ and $\tilde{Y}(k)$ be the DFS sequences.

1. Consider

$$\tilde{Y}(k) = \text{DFS}\left[\tilde{y}(n)\right] = \sum_{n=0}^{N-1} \tilde{y}(n) W_N^{nk} = \sum_{n=0}^{N-1} \tilde{x}(-n) W_N^{nk} = \sum_{n=0}^{N-1} \tilde{x}(N-n) W_N^{nk}$$

$$= \sum_{\ell=1}^{N} \tilde{x}(\ell) W_N^{(N-\ell)k} = \sum_{\ell=0}^{N-1} \tilde{x}(\ell) W_N^{Nk} W_N^{-\ell k} = \sum_{\ell=0}^{N-1} \tilde{x}(\ell) W_N^{-\ell k} \quad (\because \text{ periodic})$$

$$= \tilde{X}(-k) = \tilde{X}(N-k)$$

2. Let $\tilde{x}(n) = \{2, 4, 6, 1, 3, 5\}_{\text{PERIODIC}}$ with $N = 6$.
   $\quad\uparrow$

   (a) Sketch of $\tilde{y}(n)$ for $0 \le n \le 5$:

One period of the periodic sequence xtilde(n)
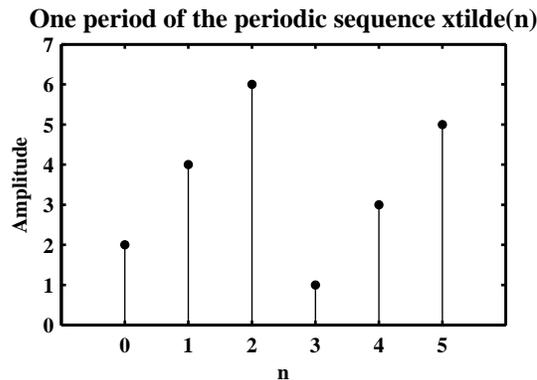
   (b) Computation of $\tilde{X}(k)$ for $0 \le k \le 5$:

```
X_tilde = dft(x_tilde,N)
X_tilde =
  Columns 1 through 4
   21.0000              1.0000 - 1.7321i  -6.0000 + 3.4641i   1.0000 - 0.0000i
  Columns 5 through 6
   -6.0000 - 3.4641i   1.0000 + 1.7321i
```

   (c) Computation of $\tilde{Y}(k)$ for $0 \le k \le 5$:

```
y_tilde = [x_tilde(1),fliplr(x_tilde(2:end))];
Y_tilde = dft(y_tilde,N)
Y_tilde =
  Columns 1 through 4
   21.0000              1.0000 + 1.7321i  -6.0000 - 3.4641i   1.0000 + 0.0000i
  Columns 5 through 6
   -6.0000 + 3.4641i   1.0000 - 1.7321i
```

   (d) MATLAB verification:

```
W_tilde = [X_tilde(1),fliplr(X_tilde(2:end))];
error = max(abs(Y_tilde - W_tilde))
error =
   2.5434e-014
```

P5.8 Consider the finite-length sequence given below.

$$x(n) = \begin{cases} \text{sinc}^2\{(n-50)/2\}, & 0 \le n \le 100; \\ 0, & \text{else.} \end{cases}$$

1. DFT $X(k)$:

```
n = 0:100; xn = sinc((n-50)/2).^2; N = length(xn); % given signal x(n)
Xk = dft(xn,N); k = 0:N-1;                          % DFT of x(n)
mag_Xk = abs(Xk); pha_Xk = angle(Xk)*180/pi;        % Mag and Phase of X(k)
zei = find(mag_Xk < 0.00001);                       % Set phase values to
pha_Xk(zei) = zeros(1,length(zei));                 % zero when mag is zero

Hf_1 = figure('Units','normalized','position',[0.1,0.1,0.8,0.8],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,5]);
set(Hf_1,'NumberTitle','off','Name','P5.8');
subplot(2,1,1); H_s1 = stem(k,mag_Xk,'filled'); set(H_s1,'markersize',3);
set(gca,'XTick',[0:20:N],'fontsize',8); axis([0,N,0,2.5])
set(gca,'YTick',[0:0.5:2.5],'fontsize',8); ylabel('Magnitude');
title('Magnitude plots of DFT and DTFT','fontsize',10); hold on
subplot(2,1,2); H_s2 = stem(k,pha_Xk,'filled'); set(H_s2,'markersize',3);
set(gca,'XTick',[0:20:N],'fontsize',8); axis([0,N,-200,200])
set(gca,'YTick',[-180;-90;0;90;180],'fontsize',8);
xlabel('k'); ylabel('Degrees');
title('Phase plots of DFT and DTFT','fontsize',10); hold on
```

The `stem` plot of $X(k)$ is shown in 5.6.

2. DTFT $X\left(e^{j\omega}\right)$:

```
[X,w] = freqz(xn,1,1000,'whole');                   % DTFT of xn
mag_X = abs(X); pha_X = angle(X)*180/pi;            % mag and phase of DTFT
Dw = (2*pi)/N;                                       % frequency resolution
subplot(2,1,1); plot(w/Dw,mag_X); grid
hold off
subplot(2,1,2); plot(w/Dw,pha_X); grid
hold off
```

The continuous plot of $X\left(e^{j\omega}\right)$ is also shown in Figure 5.6.

3. Clearly, the DFT in part 1. is the sampled version of $X\left(e^{j\omega}\right)$.

4. It is possible to reconstruct the DTFT from the DFT if length of the DFT is larger than or equal to the length of sequence $x(n)$. We can reconstruct using the complex interpolation formula

$$X\left(e^{j\omega}\right) = \sum_{k=0}^{N-1} X(k)\,\phi\left(\omega - \frac{2\pi k}{N}\right), \quad \text{where} \quad \phi(\omega) = e^{-j\omega(N-1)/2}\frac{\sin(\omega N/2)}{N\sin(\omega/2)}$$

For $N = 101$, we have

$$X\left(e^{j\omega}\right) = \sum_{k=0}^{100} X(k)\,e^{-j(50)\omega}\frac{\sin(50.5\omega)}{101\sin(\omega/2)}$$
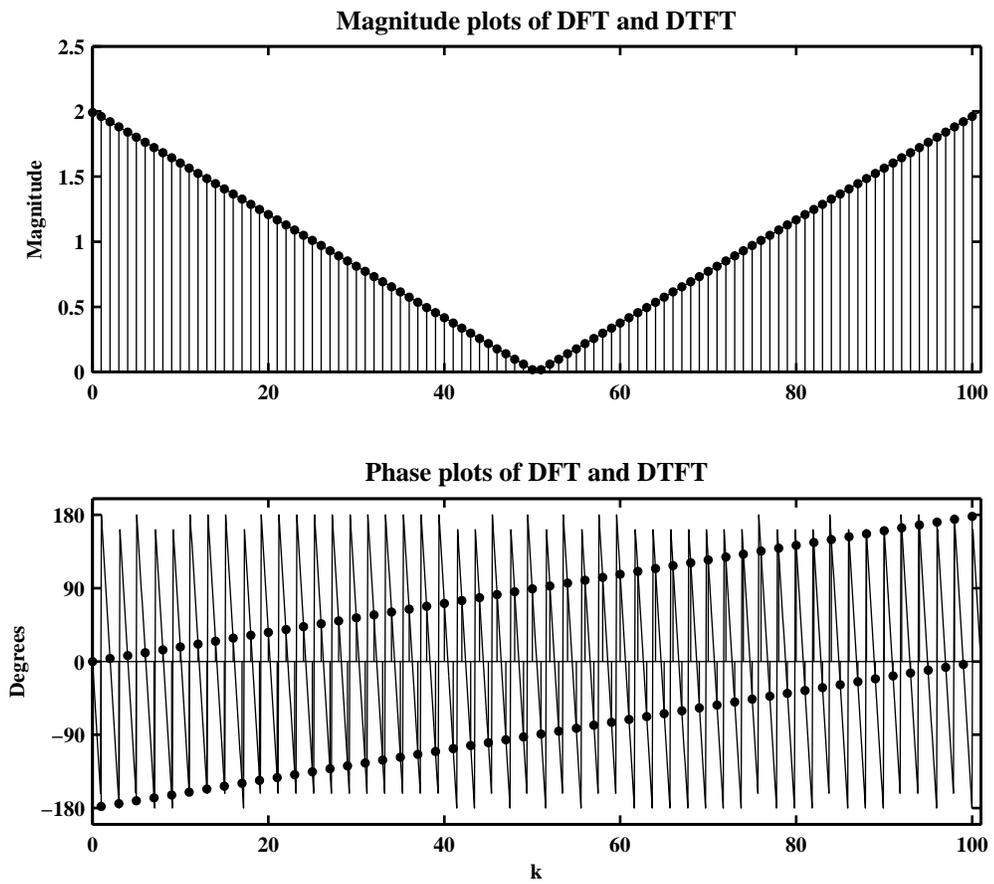
Figure 5.6: Plots of DTFT and DFT of signal in Problem 5.8

P5.9 The DTFT $X(e^{j\omega})$ of the following finite-length sequence using DFT as a computation tool

$$x(n) = \begin{cases} 2e^{-0.9|n|}, & -5 \leq n \leq 5; \\ 0, & \text{otherwise.} \end{cases}$$

MATLAB script:

```
clc; close all;
n = -5:5; xn = 2*exp(-0.9*abs(n)); N1 = length(xn);
N = 201; xn = [xn,zeros(1,N-N1)]; Xk = dft(xn,N); Xk = real(Xk);
w = linspace(-pi,pi,N); Xk = fftshift(Xk);
Hf_1 = figure('Units','normalized','position',[0.1,0.1,0.8,0.8],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,3]);
set(Hf_1,'NumberTitle','off','Name','P5.9');
plot(w/pi,Xk,'g','linewidth',1.5); axis([-1,1,-4,5]); hold on;
plot([-1,1],[0,0],'w',[0,0],[-4,5],'w','linewidth',0.5);
title('DTFT of x(n) = 2e^{-0.9|n|}, -5\leq n\leq 5','fontsize',10);
xlabel('\omega/\pi','fontsize',10); ylabel('Amplitude','fontsize',10);
```

The plot of the DTFT $X(e^{j\omega})$ is shown in 5.7.
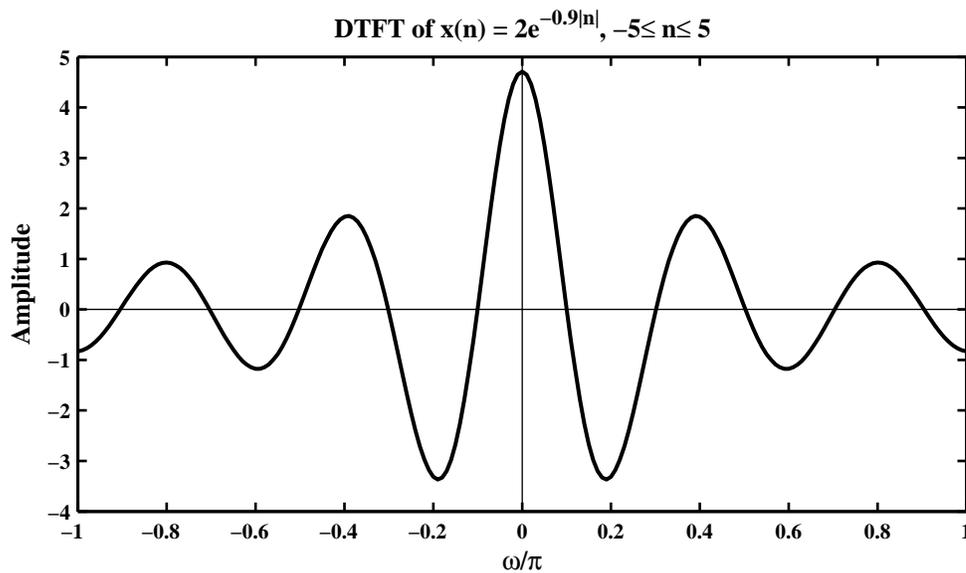
Figure 5.7: Plots of DTFT and DFT of signal in Problem 5.9

P5.10 Plot of the DTFT magnitude and angle of each of the following sequences using the DFT as a computation tool.

1. $x_1(n) = (0.6)^{|n|}[u(n + 10) - u(n - 10)]$. MATLAB script:

```
n1 = [-10:10]; x1 = (0.6).^abs(n1); N1 = length(n1); N = 200;    % Length of DFT
x1 = [x1(11:end), zeros(1,N-N1), x1(1:10)]; % Assemble x1
[X1] = fft(x1,N); w = (0:N/2)*2*pi/N;
mag_X1 = abs(X1(1:N/2+1)); pha_X1 = angle(X1(1:N/2+1))*180/pi;
Hf_1 = figure('Units','inches','position',[1,1,6,4],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P5.10.1');
subplot(2,1,1); plot(w/pi,mag_X1,'g','linewidth',1); axis([0,1,0,11]);
title('Magnitude of DTFT X_1(e^{j\omega})'); ylabel('Magnitude');
subplot(2,1,2); plot(w/pi,pha_X1,'g','linewidth',1); axis([0,1,-200,200]);
title('Angle of DTFT X_1(e^{j\omega})'); ylabel('Degrees'); xlabel('\omega/\pi');
print -deps2 ../EPSFILES/P0510a
```

The plot of the DTFT $X_1(e^{j\omega})$ is shown in 5.8.
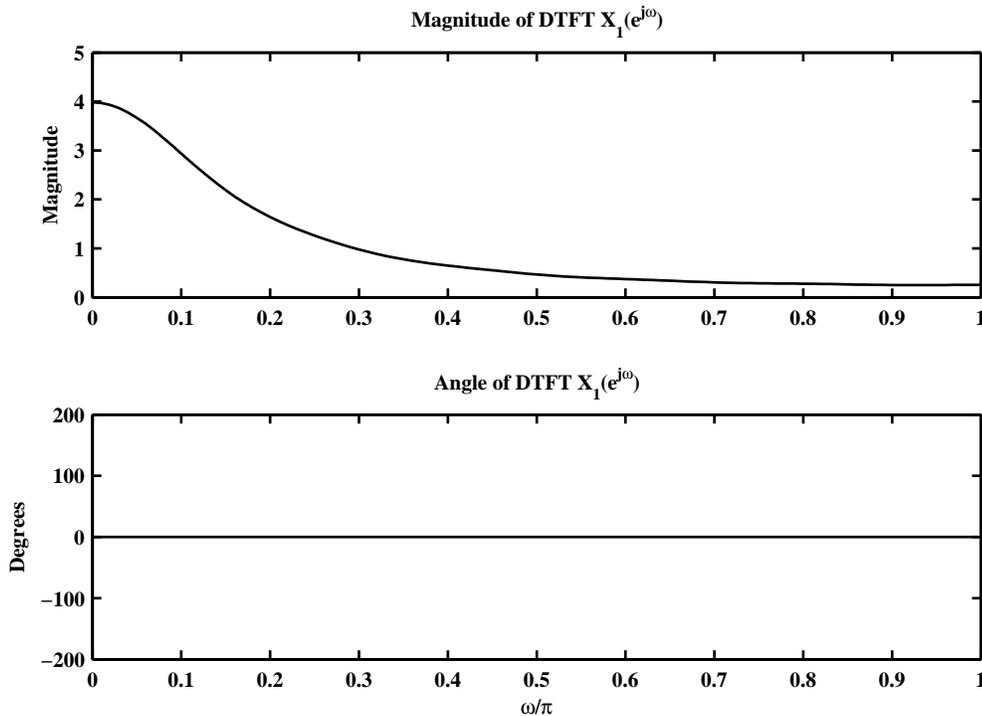
Figure 5.8: Plots of DTFT magnitude and phase in Problem 5.10.1

2. $x_2(n) = n(0.9)^n, 0 \le n \le 20$. MATLAB script:

```
n2 = [0:20]; x2 = n2.*(0.9).^n2; N2 = length(n2); N = 400;   % Length of DFT
x2 = [x2,zeros(1,N-N2)]; % Assemble x2
[X2] = fft(x2,N); w = (0:N/2)*2*pi/N;
mag_X2 = abs(X2(1:N/2+1)); pha_X2 = angle(X2(1:N/2+1))*180/pi;
Hf_2 = figure('Units','inches','position',[1,1,6,4],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_2,'NumberTitle','off','Name','P5.10.2');
subplot(2,1,1); plot(w/pi,mag_X2,'g','linewidth',1); %axis([0,1,0,5]);
title('Magnitude of DTFT X_2(e^{j\omega})'); ylabel('Magnitude');
subplot(2,1,2); plot(w/pi,pha_X2,'g','linewidth',1); axis([0,1,-200,200]);
title('Angle of DTFT X_2(e^{j\omega})'); ylabel('Degrees'); xlabel('\omega/\pi');
print -deps2 ../EPSFILES/P0510b
```

The plot of the DTFT $X_2(e^{j\omega})$ is shown in 5.9.
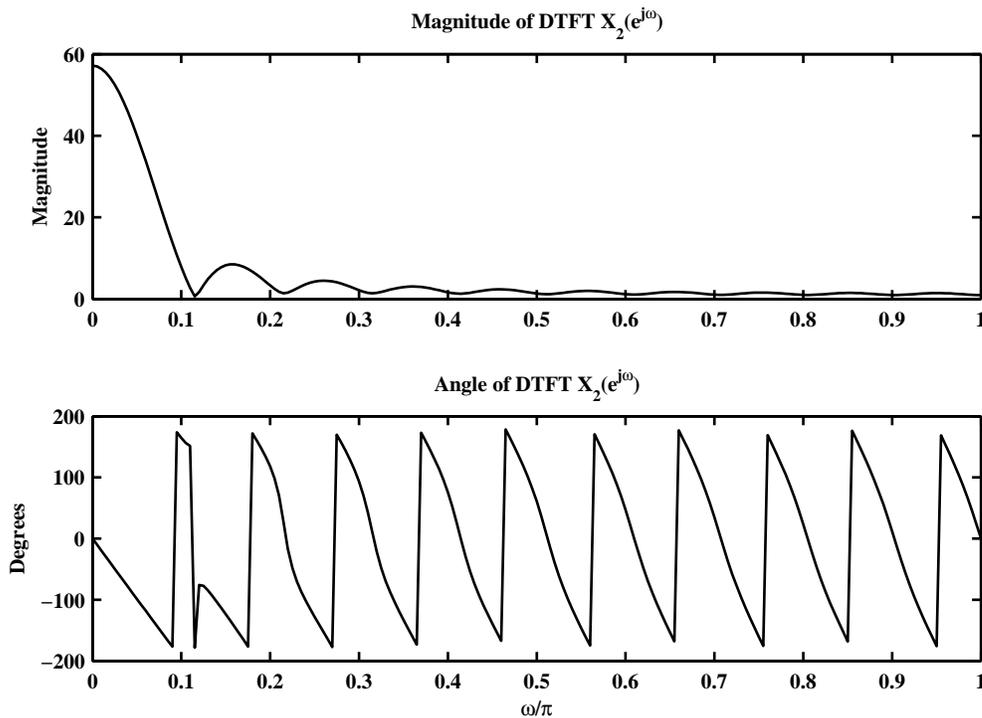
Figure 5.9: Plots of DTFT magnitude and phase in Problem 5.10.2

3. $x_3(n) = \cos(0.5\pi n) + j\sin(0.5\pi n), 0 \le n \le 50$. MATLAB script:

```
n3 = [0:50]; x3 = cos(0.5*pi*n3)+j*sin(0.5*pi*n3); N = 500;% Length of DFT
N3 = length(n3); x3 = [x3,zeros(1,N-N3)]; % Assemble x3
[X3] = dft(x3,N); w = (0:N/2)*2*pi/N;
mag_X3 = abs(X3(1:N/2+1)); pha_X3 = angle(X3(1:N/2+1))*180/pi;
Hf_3 = figure('Units','inches','position',[1,1,6,4],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_3,'NumberTitle','off','Name','P5.10.3');
subplot(2,1,1); plot(w/pi,mag_X3,'g','linewidth',1); %axis([0,1,0,7000]);
title('Magnitude of DTFT X_3(e^{j\omega})'); ylabel('Magnitude');
subplot(2,1,2); plot(w/pi,pha_X3,'g','linewidth',1); axis([0,1,-200,200]);
title('Angle of DTFT X_3(e^{j\omega})'); ylabel('Degrees'); xlabel('\omega/\pi');
print -deps2 ../EPSFILES/P0510c
```

The plot of the DTFT $X_3(e^{j\omega})$ is shown in 5.10.
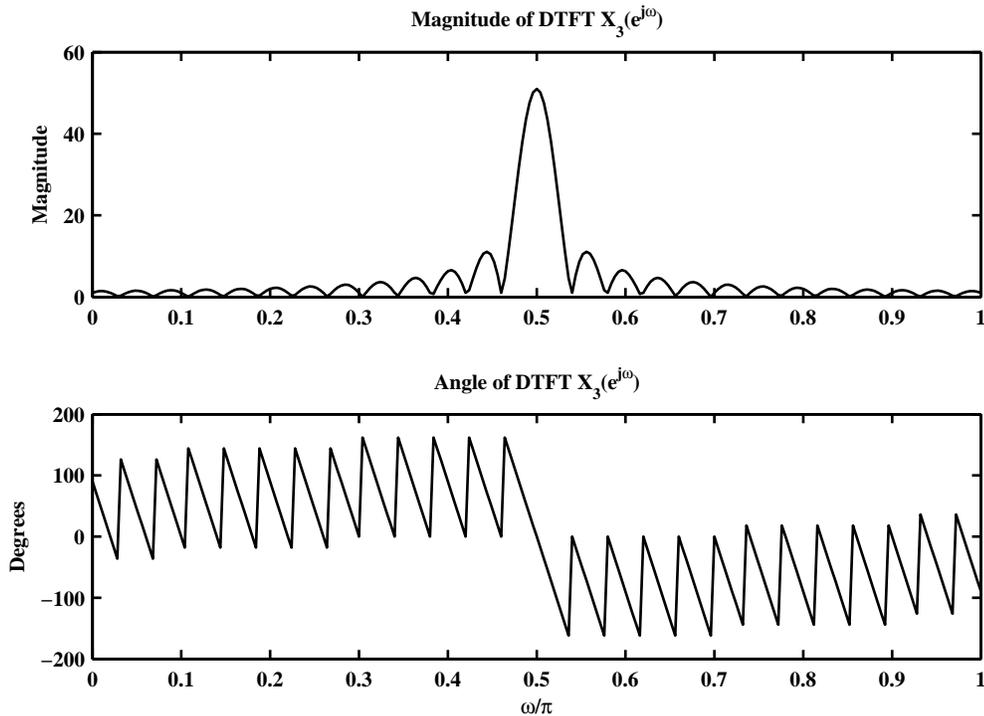
Figure 5.10: Plots of DTFT magnitude and phase in Problem 5.10.3

4. $x(n) = \{1, 2, 3, 4, 3, 2, 1\}$. MATLAB script:

```
n4 = [-3:3]; x4 = [1,2,3,4,3,2,1]; N4 = length(n4); N = 100;    % Length of DFT
[X4] = dft([x4, zeros(1,N-N4)],N); w = (0:N/2)*2*pi/N;
mag_X4 = abs(X4(1:N/2+1)); pha_X4 = angle(X4(1:N/2+1))*180/pi;
Hf_4 = figure('Units','inches','position',[1,1,6,4],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_4,'NumberTitle','off','Name','P5.10.4');
subplot(2,1,1); plot(w/pi,mag_X4,'g','linewidth',1); axis([0,1,0,20]);
title('Magnitude of DTFT X_4(e^{j\omega})'); ylabel('Magnitude');
subplot(2,1,2); plot(w/pi,pha_X4,'g','linewidth',1); axis([0,1,-200,200]);
title('Angle of DTFT X_4(e^{j\omega})'); ylabel('Degrees'); xlabel('\omega/\pi');
print -deps2 ../EPSFILES/P0510d
```

The plot of the DTFT $X_4(e^{j\omega})$ is shown in 5.11.
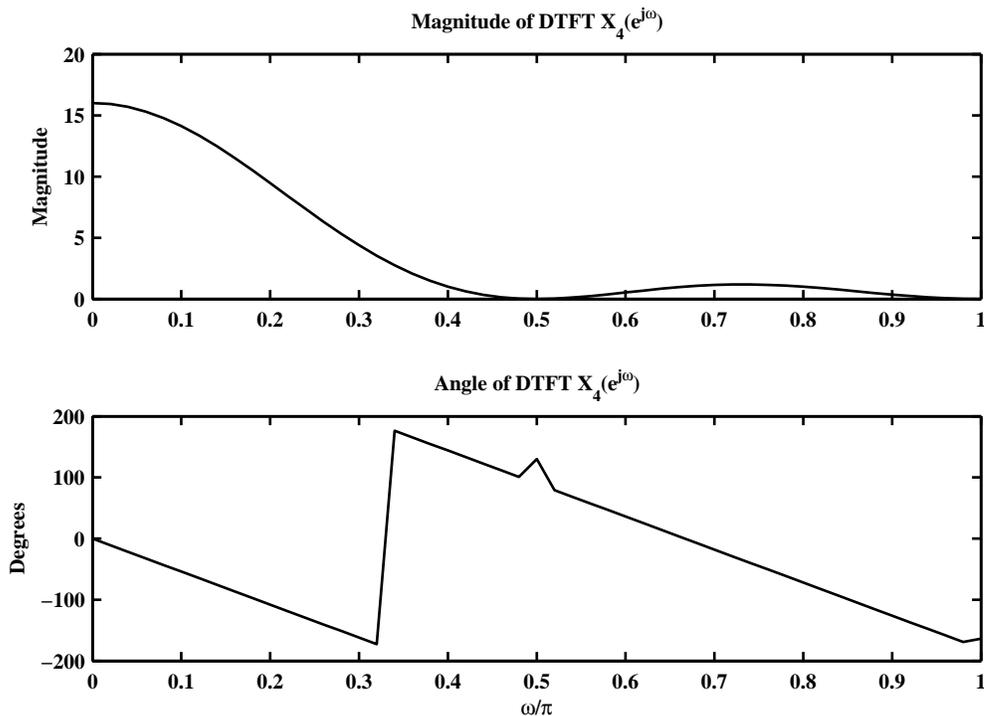
Figure 5.11: Plots of DTFT magnitude and phase in Problem 5.10.4

5. $x(n) = \{-1, -2, -3, \underset{\uparrow}{0}, 3, 2, 1\}$. MATLAB script:

```
n5 = [-3:3]; x5 = [-1,-2,-3,0,3,2,1]; N5 = length(n5); N = 100;    % Length of DFT
[X5] = dft([x5, zeros(1,N-N5)],N); w = (0:N/2)*2*pi/N;
mag_X5 = abs(X5(1:N/2+1)); pha_X5 = angle(X5(1:N/2+1))*180/pi;
Hf_5 = figure('Units','inches','position',[1,1,6,4],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_5,'NumberTitle','off','Name','P5.10.5');
subplot(2,1,1); plot(w/pi,mag_X5,'g','linewidth',1); axis([0,1,0,20]);
title('Magnitude of DTFT X_5(e^{j\omega})'); ylabel('Magnitude');
subplot(2,1,2); plot(w/pi,pha_X5,'g','linewidth',1); axis([0,1,-200,200]);
title('Angle of DTFT X_5(e^{j\omega})'); ylabel('Degrees'); xlabel('\omega/\pi');
print -deps2 ../EPSFILES/P0510e
```

The plot of the DTFT $X_5(e^{j\omega})$ is shown in 5.12.
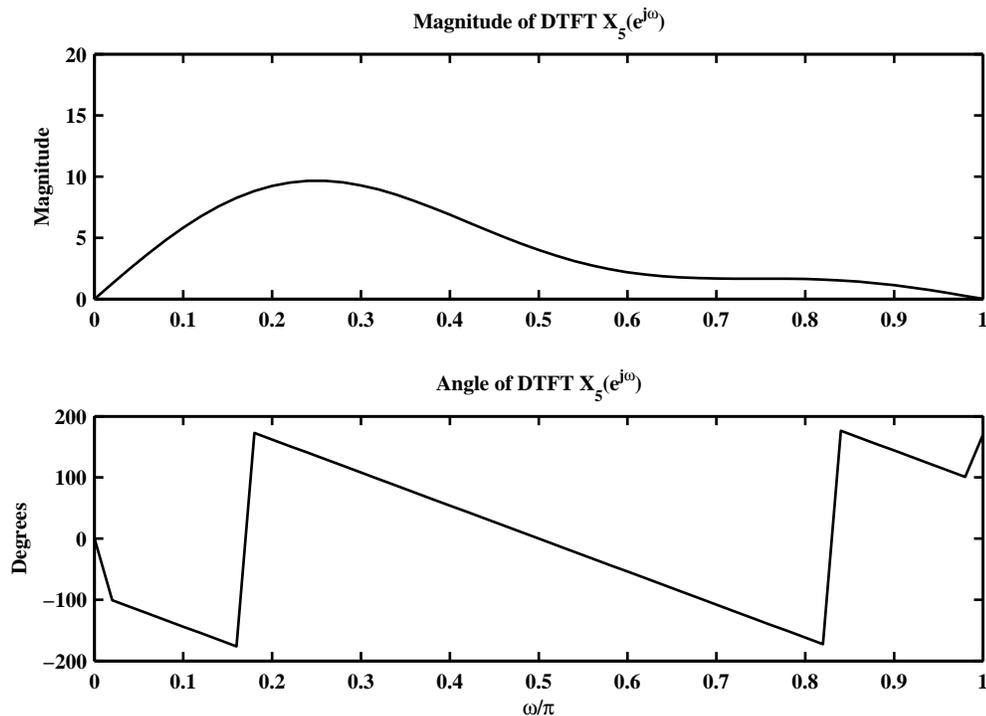
Figure 5.12: Plots of DTFT magnitude and phase in Problem 5.10.5

P5.11 Let $H(e^{j\omega})$ be the frequency response of a real, causal impulse response $h(n)$.

1. It is known that $\text{Re}\left\{H\left(e^{j\omega}\right)\right\} = \sum_{k=0}^{5} (0.9)^k \cos\omega k$. Consider

$$\text{Re}\left\{H\left(e^{j\omega}\right)\right\} = \text{Re}\left\{\sum_{k=0}^{\infty} h(k)e^{-j\omega k}\right\} = \sum_{k=0}^{\infty} h(k)\,\text{Re}\left\{e^{-j\omega k}\right\} = \sum_{k=0}^{\infty} h(k)\cos\omega k$$

Comparing with the given expression, we obtain

$$h(n) = \begin{cases} (0.9)^n, & 0 \le n \le 5 \\ 0, & \text{else} \end{cases}$$

MATLAB verification:

```
n = 0:5; h = (0.9).^n; N1 = length(h); N = 100; h = [h,zeros(1,N-N1)];
H = dft(h,N); Hr = real(H);
k = [0:5]; w = linspace(0,2*pi,N+1);
Hr_check = (0.9.^k)*cos(k'*w(1:end-1));
error = max(abs(Hr-Hr_check))
error =
   4.0856e-014
```

2. It is known that $\text{Im}\left\{H\left(e^{j\omega}\right)\right\} = \sum_{\ell=0}^{5} 2\ell\sin\omega\ell$ and $\int_{-\pi}^{\pi} H\left(e^{j\omega}\right) d\omega = 0$. From the second condition

$$\int_{-\pi}^{\pi} H\left(e^{j\omega}\right) d\omega = h(0) = 0$$

Consider

$$\text{Im}\left\{H\left(e^{j\omega}\right)\right\} = \text{Im}\left\{\sum_{\ell=0}^{\infty} h(\ell)e^{-j\omega\ell}\right\} = \sum_{\ell=0}^{\infty} h(\ell)\,\text{Im}\left\{e^{-j\omega\ell}\right\} = -\sum_{\ell=0}^{\infty} h(\ell)\sin\omega\ell$$

Comparing with the given expression, we obtain

$$h(n) = \begin{cases} -2n, & 0 \le n \le 5 \\ 0, & \text{else} \end{cases}$$

MATLAB verification:

```
n = 0:5; h = -2*n; N1 = length(h); N = 100; h = [h,zeros(1,N-N1)];
H = dft(h,N); Hi = imag(H);
l = [0:5]; w = linspace(0,2*pi,N+1);
Hi_check = 2*l*sin(l'*w(1:end-1));
error = max(abs(Hi-Hi_check))
error =
   3.8014e-013
```

P5.12 Let $X(k)$ denote the N-point DFT of an N-point sequence $x(n)$. The DFT $X(k)$ itself is an N-point sequence.

1. The N-point DFT of $x(n)$: $X(k) = \sum_{m=0}^{N-1} x(m) W_N^{mk}$. The N-point DFT of $X(k)$: $y(n) = \sum_{k=0}^{N-1} X(k) W_N^{kn}$.
   Hence,

$$
\begin{aligned}
y(n) &= \sum_{k=0}^{N-1} \left\{ \sum_{m=0}^{N-1} x(m) W_N^{mk} \right\} W_N^{kn} = \sum_{m=0}^{N-1} x(m) \sum_{k=0}^{N-1} W_N^{mk} W_N^{kn}, \ 0 \le n \le N-1 \\
&= \sum_{m=0}^{N-1} x(m) \sum_{k=0}^{N-1} W_N^{(m+n)k} = \sum_{m=0}^{N-1} x(m) \sum_{r=-\infty}^{\infty} N\delta(m+n-rN), 0 \le n \le N-1 \\
&= N \sum_{r=-\infty}^{\infty} x(-n+rN) = Nx((-n))_N, 0 \le n \le N-1
\end{aligned}
$$

This means that $y(n)$ is a "circularly folded and amplified (by N)" version of $x(n)$. Continuing further, if we take two more DFTs of $x(n)$ then

$$x(n) \longrightarrow \boxed{\begin{array}{c} N\text{-point} \\ \text{DFT} \end{array}} \longrightarrow \boxed{\begin{array}{c} N\text{-point} \\ \text{DFT} \end{array}} \longrightarrow \boxed{\begin{array}{c} N\text{-point} \\ \text{DFT} \end{array}} \longrightarrow \boxed{\begin{array}{c} N\text{-point} \\ \text{DFT} \end{array}} \longrightarrow N^2 x(n)$$
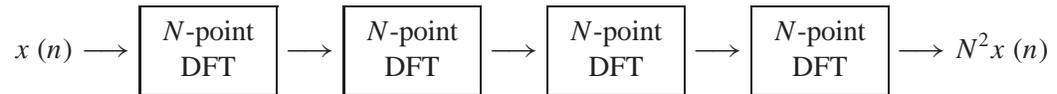
Therefore, if a given DFT function is working correctly then four successive applications of this function on any arbitrary signal will produce the same signal (multiplied by N^2). This approach can be used to verify a DFT function.

2. MATLAB function for circular folding:

```
function x2 = circfold(x1,N)
% Circular folding with respect to N
% --------------------------------
% function x2 = circfold(x1,N)
%        x2(n) = x1((-n) mod N)
%
x2 = real(dft(dft(x1,N),N))/N;
```

3. MATLAB verification:

```
x = [1,3,5,7,9,-7,-5,-3,-1], N = length(x);

x =
    1    3    5    7    9   -7   -5   -3   -1


Y = circfold(x,N)

Y =
    1.000   -1.000   -3.000   -5.000   -7.000    9.000    7.000    5.000    3.000
```

P5.13 Let $X(k)$ be an N-point DFT of an N-point sequence $x(n)$. Let N be an even integer.

1. Given that $x(n) = x(n + N/2)$ for all n, consider

$$
\begin{aligned}
X(k) &= \sum_{n=0}^{N-1} x(n) W_N^{nk} = \sum_{n=0}^{N/2-1} x(n) W_N^{nk} + \sum_{n=N/2}^{N-1} x(n) W_N^{nk} \\
&= \sum_{n=0}^{N/2-1} x(n) W_N^{nk} + \sum_{n=0}^{N/2-1} x(n + N/2) W_N^{(n+N/2)k} \quad [\because n \to n + N/2] \\
&= \sum_{n=0}^{N/2-1} x(n) W_N^{nk} + \sum_{n=0}^{N/2-1} x(n) W_N^{nk} W_N^{Nk/2} \quad [\because x(n) = x(n + N/2)] \\
&= \sum_{n=0}^{N/2-1} x(n) \left\{ 1 + (-1)^k \right\} W_N^{nk} \quad [\because W_N^{N/2} = -1] = \begin{cases} 0, & k \text{ odd;} \\ \text{Non-zero,} & k \text{ even.} \end{cases}
\end{aligned}
$$

Verification using $x(n) = \{1, 2, -3, 4, 5, 1, 2, -3, 4, 5\}$:

```
x = [1,2,-3,4,5,1,2,-3,4,5]; N = length(x); X = dft(x,N)
X =
   18.0000            -0.0000 + 0.0000i   4.7082 +13.9353i  -0.0000 + 0.0000i
   -8.7082 - 9.7881i   0.0000 - 0.0000i  -8.7082 + 9.7881i  -0.0000 - 0.0000i
    4.7082 -13.9353i   0.0000 - 0.0000i
```

2. Given that $x(n) = -x(n + N/2)$ for all n, consider

$$
\begin{aligned}
X(k) &= \sum_{n=0}^{N-1} x(n) W_N^{nk} = \sum_{n=0}^{N/2-1} x(n) W_N^{nk} + \sum_{n=N/2}^{N-1} x(n) W_N^{nk} \\
&= \sum_{n=0}^{N/2-1} x(n) W_N^{nk} + \sum_{n=0}^{N/2-1} x(n + N/2) W_N^{(n+N/2)k} \quad [\because n \to n + N/2] \\
&= \sum_{n=0}^{N/2-1} x(n) W_N^{nk} - \sum_{n=0}^{N/2-1} x(n) W_N^{nk} W_N^{Nk/2} \quad [\because x(n) = -x(n + N/2)] \\
&= \sum_{n=0}^{N/2-1} x(n) \left\{ 1 - (-1)^k \right\} W_N^{nk} \quad [\because W_N^{N/2} = -1] \\
&= \begin{cases} 0, & k \text{ even;} \\ \text{Non-zero,} & k \text{ odd.} \end{cases}
\end{aligned}
$$

Verification using $x(n) = \{1, 2, -3, 4, 5, -1, -2, 3, -4, -5\}$.

```
x = [1,2,-3,4,5,-1,-2,3,-4,-5]; N = length(x); X = dft(x,N)
X =
        0            -7.1803 -10.1311i  -0.0000 - 0.0000i  15.1803 -12.1392i
   0.0000 + 0.0000i  -6.0000 - 0.0000i   0.0000 - 0.0000i  15.1803 +12.1392i
  -0.0000 + 0.0000i  -7.1803 +10.1311i
```

P5.14 Let $X(k)$ be an N-point DFT of an N-point sequence $x(n)$. Let $N = 4\nu$ where ν is an integer.

1. It is given that $x(n) = x(n + \nu)$ for all n. Let $n = m + p\nu$; $0 \le m \le \nu - 1$, $0 \le p \le 3$, then

$$x(n) = x(n + \nu) \Rightarrow x(m + p\nu) = x(m), \quad 0 \le m \le \nu - 1 \tag{5.1}$$

Now the DFT $X(k)$ can be written as

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} = \sum_{p=0}^{3} \sum_{m=0}^{\nu-1} x(m + p\nu) W_N^{(m+p\nu)k} = \sum_{p=0}^{3} \sum_{m=0}^{\nu-1} x(m) W_N^{mk} W_N^{p\nu k} \quad [\because (5.1)]$$

$$= \sum_{m=0}^{\nu-1} x(m) W_N^{mk} \sum_{p=0}^{3} W_N^{p\nu k} = \sum_{m=0}^{\nu-1} x(m) W_N^{mk} \sum_{p=0}^{3} (W_N^{\nu k})^p = \sum_{m=0}^{\nu-1} x(m) W_N^{mk} \left[\frac{1 - W_N^{Nk}}{1 - W_N^{\nu k}} \right]$$

$$= \sum_{m=0}^{\nu-1} x(m) W_N^{mk} \left[\frac{1 - W_N^{Nk}}{1 - W_N^{N(k/4\ell)}} \right] = \begin{cases} \text{Non-zero,} & k = 4\ell \text{ for } 0 \le \ell \le \nu - 1; \\ 0, & k \ne 4\ell \text{ for } 0 \le \ell \le \nu - 1. \end{cases}$$

Verification for $x(n) = \{1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3\}$.

```
x = [1,2,3,1,2,3,1,2,3,1,2,3]; N = length(x); X = dft(x,N)
X =
   24.0000                 -0.0000 - 0.0000i  -0.0000 - 0.0000i  -0.0000 + 0.0000i
   -6.0000 + 3.4641i        0.0000 - 0.0000i  -0.0000 - 0.0000i  -0.0000 - 0.0000i
   -6.0000 - 3.4641i        0.0000 - 0.0000i   0.0000 - 0.0000i   0.0000 - 0.0000i
```

2. It is given that $x(n) = -x(n + \nu)$ for all n. Let $n = m + p\nu$; $0 \le m \le \nu - 1$, $0 \le p \le 3$, then

$$x(n) = -x(n + \nu) \Rightarrow x(m + p\nu) = (-1)^p x(m), \quad 0 \le m \le \nu - 1, \ 0 \le p \le 3 \tag{5.2}$$

Now the DFT $X(k)$ can be written as

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} = \sum_{p=0}^{3} \sum_{m=0}^{\nu-1} x(m + p\nu) W_N^{(m+p\nu)k} = \sum_{p=0}^{3} \sum_{m=0}^{\nu-1} (-1)^p x(m) W_N^{mk} W_N^{p\nu k} \quad [\because (5.2)]$$

$$= \sum_{m=0}^{\nu-1} x(m) W_N^{mk} \sum_{p=0}^{3} (-1)^p W_N^{p\nu k} = \sum_{m=0}^{\nu-1} x(m) W_N^{mk} \sum_{p=0}^{3} \left(W_N^{-N/2} W_N^{\nu k} \right)^p \quad [\because -1 = W_N^{-N/2}]$$

$$= \sum_{m=0}^{\nu-1} x(m) W_N^{mk} \left[\frac{1 - W_N^{N(k-2)}}{1 - W_N^{\nu(k-2)}} \right] = \begin{cases} \text{Non-zero,} & k = 4\ell + 2 \text{ for } 0 \le \ell \le \nu - 1; \\ 0, & k \ne 4\ell + 2 \text{ for } 0 \le \ell \le \nu - 1. \end{cases}$$

Verification for $x(n) = \{1, 2, 3, -1, -2, -3, 1, 2, 3, -1, -2, -3\}$.

```
x = [1,2,3,-1,-2,-3,1,2,3,-1,-2,-3]; N = length(x); X = dft(x,N)
X =
        0              0         2.0000 -17.3205i   0.0000 + 0.0000i
        0              0         8.0000 + 0.0000i  -0.0000 + 0.0000i
        0              0         2.0000 +17.3205i  -0.0000 - 0.0000i
```

P5.15 Let $X(k)$ be an N-point DFT of an N-point sequence $x(n)$. Let $N = 2\mu\nu$ where μ and ν are integers.

1. It is given that $x(n) = x(n + \nu)$ for all n. Let $n = m + p\nu$; $0 \leq m \leq \nu - 1, 0 \leq p \leq (2\mu - 1)$, then

$$x(n) = x(n + \nu) \Rightarrow x(m + p\nu) = x(m), \quad 0 \leq m \leq \nu - 1, \ 0 \leq p \leq (2\mu - 1) \qquad (5.3)$$

Now the DFT $X(k)$ can be written as

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} = \sum_{p=0}^{2\mu-1} \sum_{m=0}^{\nu-1} x(m + p\nu) W_N^{(m+p\nu)k} = \sum_{p=0}^{2\mu-1} \sum_{m=0}^{\nu-1} x(m) \, W_N^{mk} \, W_N^{p\nu k} \qquad [\because (5.3)]$$

$$= \sum_{m=0}^{\nu-1} x(m) \, W_N^{mk} \sum_{p=0}^{2\mu-1} W_N^{p\nu k} = \sum_{m=0}^{\nu-1} x(m) \, W_N^{mk} \sum_{p=0}^{2\mu-1} \left(W_N^{\nu k} \right)^p = \sum_{m=0}^{\nu-1} x(m) \, W_N^{mk} \left[\frac{1 - W_N^{Nk}}{1 - W_N^{\nu k}} \right]$$

$$= \sum_{m=0}^{\nu-1} x(m) \, W_N^{mk} \left[\frac{1 - W_N^{Nk}}{1 - W_N^{N(k/2\mu)}} \right] = \begin{cases} \text{Non-zero}, & k = (2\mu)\ell \text{ for } 0 \leq \ell \leq \nu - 1; \\ 0, & k \neq (2\mu)\ell \text{ for } 0 \leq \ell \leq \nu - 1. \end{cases}$$

Verification for $x(n) = \{1, -2, 3, 1, -2, 3, 1, -2, 3, 1, -2, 3, 1, -2, 3, 1, -2, 3\}$.

```
x = [1,-2,3,1,-2,3,1,-2,3,1,-2,3,1,-2,3,1,-2,3]; N = length(x); X = dft(x,N)
X =
   12.0000                 0.0000 - 0.0000i   -0.0000 + 0.0000i    0.0000 + 0.0000i
    0.0000                 0.0000 + 0.0000i    3.0000 +25.9808i   -0.0000 - 0.0000i
   -0.0000 + 0.0000i      -0.0000 - 0.0000i   -0.0000 - 0.0000i   -0.0000 - 0.0000i
    3.0000 -25.9808i       0.0000 + 0.0000i    0.0000             0.0000 - 0.0000i
    0.0000 - 0.0000i       0.0000 - 0.0000i
```

2. It is given that $x(n) = -x(n + \nu)$ for all n. Let $n = m + p\nu$; $0 \leq m \leq \nu - 1, 0 \leq p \leq (2\mu - 1)$, then

$$x(n) = -x(n + \nu) \Rightarrow x(m + p\nu) = (-1)^p x(m), \quad 0 \leq m \leq \nu - 1, \ 0 \leq p \leq (2\mu - 1) \qquad (5.4)$$

Now the DFT $X(k)$ can be written as

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} = \sum_{p=0}^{2\mu-1} \sum_{m=0}^{\nu-1} x(m + p\nu) W_N^{(m+p\nu)k} = \sum_{p=0}^{2\mu-1} \sum_{m=0}^{\nu-1} (-1)^p x(m) \, W_N^{mk} \, W_N^{p\nu k} \qquad [\because (5.4)]$$

$$= \sum_{m=0}^{\nu-1} x(m) \, W_N^{mk} \sum_{p=0}^{2\mu-1} (-1)^p W_N^{p\nu k} = \sum_{m=0}^{\nu-1} x(m) \, W_N^{mk} \sum_{p=0}^{2\mu-1} \left(W_N^{-N/2} W_N^{\nu k} \right)^p \qquad [\because -1 = W_N^{-N/2}]$$

$$= \sum_{m=0}^{\nu-1} x(m) \, W_N^{mk} \left[\frac{1 - W_N^{N(k-2)}}{1 - W_N^{N(k-2)/(2\mu)}} \right] = \begin{cases} \neq 0, & k = 2\mu\ell + 2 = 2(\mu\ell + 1), 0 \leq \ell \leq \nu - 1; \\ 0, & k \neq 2\mu\ell + 2 = 2(\mu\ell + 1), 0 \leq \ell \leq \nu - 1. \end{cases}$$

Verification for $x(n) = \{1, -2, 3, -1, 2, -3, 1, -2, 3, -1, 2, -3, 1, -2, 3, -1, 2, -3\}$.

```
x = [1,-2,3,-1,2,-3,1,-2,3,-1,2,-3,1,-2,3,-1,2,-3]; N = length(x); X = dft(x,N)
X =
        0                -0.0000                 -0.0000 + 0.0000i   -9.0000 - 5.1962i
   0.0000 - 0.0000i       0.0000 - 0.0000i       -0.0000 - 0.0000i   -0.0000 - 0.0000i
   0.0000 - 0.0000i      36.0000 + 0.0000i       -0.0000 + 0.0000i   -0.0000 + 0.0000i
  -0.0000 + 0.0000i      -0.0000 + 0.0000i       -0.0000 + 0.0000i   -9.0000 + 5.1962i
  -0.0000 - 0.0000i      -0.0000 - 0.0000i
```

P5.16 Let $X(k)$ and $Y(k)$ be 10-point DFTs of two 10-point sequences $x(n)$ and $y(n)$, respectively where

$$X(k) = \exp(j0.2\pi k), \quad 0 \leq k \leq 9$$

Computation of $Y(k)$ properties of the DFT.

1. $y(n) = x((n-5)))_{10}$: Circular shift by 5. MATLAB script:

```
k = 0:9; X = exp(j*0.2*pi*k); N = length(X);
m = 5; WN = exp(-j*2*pi/N); Y = X.*WN.^(m*k)
% verification
x = real(idft(X,N)); y = cirshftt(x,m,N); Y1 = dft(y,N)
difference = abs(max(Y-Y1))


Y =
  Columns 1 through 4
    1.0000                -0.8090 - 0.5878i    0.3090 + 0.9511i    0.3090 - 0.9511i
  Columns 5 through 8
   -0.8090 + 0.5878i    1.0000 + 0.0000i   -0.8090 - 0.5878i    0.3090 + 0.9511i
  Columns 9 through 10
    0.3090 - 0.9511i   -0.8090 + 0.5878i
Y1 =
  Columns 1 through 4
    1.0000                -0.8090 - 0.5878i    0.3090 + 0.9511i    0.3090 - 0.9511i
  Columns 5 through 8
   -0.8090 + 0.5878i    1.0000 + 0.0000i   -0.8090 - 0.5878i    0.3090 + 0.9511i
  Columns 9 through 10
    0.3090 - 0.9511i   -0.8090 + 0.5878i
difference =
  2.7756e-015
```

2. $y(n) = x((n+4)))_{10}$: Circular shift by -4. MATLAB script:

```
k = 0:9; X = exp(j*0.2*pi*k); N = length(X);
m = -4; WN = exp(-j*2*pi/N); Y = X.*WN.^(m*k)
% verification
x = real(idft(X,N)); y = cirshftt(x,m,N); Y1 = dft(y,N)
difference = abs(max(Y-Y1))


Y =
  Columns 1 through 4
    1.0000                -1.0000 + 0.0000i    1.0000 - 0.0000i   -1.0000 + 0.0000i
  Columns 5 through 8
    1.0000 - 0.0000i   -1.0000 + 0.0000i    1.0000 - 0.0000i   -1.0000 + 0.0000i
  Columns 9 through 10
    1.0000 - 0.0000i   -1.0000 + 0.0000i
Y1 =
  Columns 1 through 4
    1.0000                -1.0000 - 0.0000i    1.0000 + 0.0000i   -1.0000 - 0.0000i
```

```
     Columns 5 through 8
       1.0000 - 0.0000i  -1.0000 - 0.0000i   1.0000 + 0.0000i  -1.0000 - 0.0000i
     Columns 9 through 10
       1.0000 - 0.0000i  -1.0000 + 0.0000i
   difference =
     2.2249e-015
```

3. $y(n) = x((3 - n)))_1 0$: Circular-fold and circular-shift by 3. MATLAB script:

```
   k = 0:9; X = exp(j*0.2*pi*k); N = length(X);
   Y = circfold(X,N); m = 3; WN = exp(-j*2*pi/N); Y = X.*WN.^(m*k)
   % verification
   x = real(idft(X,N)); y = circfold(x,N); y = cirshftt(x,m,N); Y1 = dft(y,N)
   difference = abs(max(Y-Y1))


   Y =
     Columns 1 through 4
       1.0000              0.3090 - 0.9511i  -0.8090 - 0.5878i  -0.8090 + 0.5878i
     Columns 5 through 8
       0.3090 + 0.9511i   1.0000 + 0.0000i   0.3090 - 0.9511i  -0.8090 - 0.5878i
     Columns 9 through 10
      -0.8090 + 0.5878i   0.3090 + 0.9511i
   Y1 =
     Columns 1 through 4
       1.0000              0.3090 - 0.9511i  -0.8090 - 0.5878i  -0.8090 + 0.5878i
     Columns 5 through 8
       0.3090 + 0.9511i   1.0000 + 0.0000i   0.3090 - 0.9511i  -0.8090 - 0.5878i
     Columns 9 through 10
      -0.8090 + 0.5878i   0.3090 + 0.9511i
   difference =
     2.6790e-015
```

4. $y(n) = x(n)e^{j3\pi n/5}$: Circular shift in the freq-domain by 3. MATLAB script:

```
   k = 0:9; X = exp(j*0.2*pi*k); N = length(X); l = 3;
   Y = cirshftt(X,l,N)
   % verification
   x = real(idft(X,N)); n = 0:9; WN = exp(-j*2*pi/N);
   y = x.*WN.^(-l*n); Y1 = dft(y,N)
   difference = abs(max(Y-Y1))


   Y =
     Columns 1 through 4
      -0.3090 - 0.9511i   0.3090 - 0.9511i   0.8090 - 0.5878i   1.0000
     Columns 5 through 8
       0.8090 + 0.5878i   0.3090 + 0.9511i  -0.3090 + 0.9511i  -0.8090 + 0.5878i
     Columns 9 through 10
      -1.0000 + 0.0000i  -0.8090 - 0.5878i
```

```
Y1 =
  Columns 1 through 4
  -0.3090 - 0.9511i    0.3090 - 0.9511i    0.8090 - 0.5878i    1.0000 + 0.0000i
  Columns 5 through 8
   0.8090 + 0.5878i    0.3090 + 0.9511i   -0.3090 + 0.9511i   -0.8090 + 0.5878i
  Columns 9 through 10
  -1.0000             -0.8090 - 0.5878i
difference =
  3.3880e-015
```

5. $y(n) = x(n)(10)x((-n))_1 0$: Circular convolution with circularly-folded sequence. MATLAB script:

```
k = 0:9; X = exp(j*0.2*pi*k); N = length(X);
Y = circfold(X,N); Y = X.*Y
% verification
x = real(idft(X,N)); y = circfold(x,N); y = circonvt(x,y,N); Y1 = dft(y,N)
difference = abs(max(Y-Y1))


Y =
  Columns 1 through 4
   1.0000 - 0.0000i    1.0000 - 0.0000i    1.0000 + 0.0000i    1.0000 - 0.0000i
  Columns 5 through 8
   1.0000 - 0.0000i    1.0000 - 0.0000i    1.0000 - 0.0000i    1.0000 + 0.0000i
  Columns 9 through 10
   1.0000              1.0000 + 0.0000i
Y1 =
  Columns 1 through 4
   1.0000 + 0.0000i    1.0000 - 0.0000i    1.0000 - 0.0000i    1.0000 - 0.0000i
  Columns 5 through 8
   1.0000 + 0.0000i    1.0000 + 0.0000i    1.0000 - 0.0000i    1.0000 - 0.0000i
  Columns 9 through 10
   1.0000 - 0.0000i    1.0000 - 0.0000i
difference =
  4.7761e-015
```

P5.17 The first six values of the 10-point DFT of a real-valued sequence $x(n)$ are given by

$$\{10, -2 + j3, 3 + j4, 2 - j3, 4 + j5, 12\}$$

DFT computations using DFT properties:

1. $x_1(n) = x((2 - n))_{10}$: Circular-folding followed by circ-shifting by 2.
   MATLAB script:

```
N = 10; X = [10,-2+j*3,3+j*4,2-j*3,4+j*5,12];
X = [X,conj(X(5:-1:2))]; x = real(idft(X,N))
WN = exp(-j*2*pi/N); k = 0:N-1; m = 2;
X1 = circfold(X,N); X1 = (WN.^(m*k)).*X1
% Matlab Verification
x1 = circfold(x,N); x1 = cirshftt(x1,m,N); X12 = dft(x1,N)
difference = max(abs(X1-X12))


x =
  Columns 1 through 7
    3.6000    -2.2397     1.0721    -1.3951     3.7520     1.2000     0.6188
  Column 8 through 10
    0.0217     1.4132     1.9571
X1 =
  Columns 1 through 4
  10.0000 + 0.0000i  -3.4712 + 0.9751i  -4.7782 + 1.4727i  -3.3814 - 1.2515i
  Columns 5 through 8
   5.9914 + 2.2591i  12.0000 + 0.0000i   5.9914 - 2.2591i  -3.3814 + 1.2515i
  Columns 9 through 10
  -4.7782 - 1.4727i  -3.4712 - 0.9751i
X12 =
  Columns 1 through 4
  10.0000 + 0.0000i  -3.4712 + 0.9751i  -4.7782 + 1.4727i  -3.3814 - 1.2515i
  Columns 5 through 8
   5.9914 + 2.2591i  12.0000 + 0.0000i   5.9914 - 2.2591i  -3.3814 + 1.2515i
  Columns 9 through 10
  -4.7782 - 1.4727i  -3.4712 - 0.9751i
difference =
  1.2462e-014
```

2. $x_2(n) = x((n+5))_{10}$: 10-point circular shifting by -5.

MATLAB script:

```
N = 10; X = [10,-2+j*3,3+j*4,2-j*3,4+j*5,12];
X = [X,conj(X(5:-1:2))]; x = real(idft(X,N))
WN = exp(-j*2*pi/N); k = 0:N-1; m = -5;
X2 = (WN.^(m*k)).*dft(x,N)
% Matlab verification
x2 = cirshftt(x,m,N); X22 = dft(x2,N)
difference = max(abs(X2-X22))

x =
  Columns 1 through 7
    3.6000    -2.2397    1.0721    -1.3951    3.7520    1.2000    0.6188
  Column 8 through 10
    0.0217    1.4132    1.9571
X2 =
  Columns 1 through 4
  10.0000              2.0000 - 3.0000i   3.0000 + 4.0000i  -2.0000 + 3.0000i
  Columns 5 through 8
   4.0000 + 5.0000i -12.0000 + 0.0000i   4.0000 - 5.0000i  -2.0000 - 3.0000i
  Columns 9 through 10
   3.0000 - 4.0000i   2.0000 + 3.0000i
X22 =
  Columns 1 through 4
  10.0000              2.0000 - 3.0000i   3.0000 + 4.0000i  -2.0000 + 3.0000i
  Columns 5 through 8
   4.0000 + 5.0000i -12.0000 - 0.0000i   4.0000 - 5.0000i  -2.0000 - 3.0000i
  Columns 9 through 10
   3.0000 - 4.0000i   2.0000 + 3.0000i
difference =
  3.2150e-014
```

3. $x_3(n) = x(n)x((-n))_{10}$: Multiplication by circularly-folded sequence.

   MATLAB script:

```
N = 10; X = [10,-2+j*3,3+j*4,2-j*3,4+j*5,12];
X = [X,conj(X(5:-1:2))]; x = real(idft(X,N))
X3 = circfold(X,N), X3 = circonvf(X,X3,N)/N
% Matlab verification
x3 = circfold(x,N); x3 = x.*x3; X32 = dft(x3,N)
difference = max(abs(X3-X32))

x =
  Columns 1 through 7
    3.6000    -2.2397     1.0721    -1.3951     3.7520     1.2000     0.6188
  Column 8 through 10
    0.0217     1.4132     1.9571
X3 =
  Columns 1 through 4
  10.0000 + 0.0000i  -2.0000 - 3.0000i   3.0000 - 4.0000i   2.0000 + 3.0000i
  Columns 5 through 8
   4.0000 - 5.0000i  12.0000 + 0.0000i   4.0000 + 5.0000i   2.0000 - 3.0000i
  Columns 9 through 10
   3.0000 + 4.0000i  -2.0000 + 3.0000i
X3 =
  Columns 1 through 4
  19.2000 + 0.0000i  10.2000 - 0.0000i  15.6000 + 0.0000i   6.4000 + 0.0000i
  Columns 5 through 8
  10.8000 - 0.0000i  24.4000 + 0.0000i  10.8000 + 0.0000i   6.4000 - 0.0000i
  Columns 9 through 10
  15.6000 + 0.0000i  10.2000 + 0.0000i
X32 =
  Columns 1 through 4
  19.2000 + 0.0000i  10.2000 - 0.0000i  15.6000 - 0.0000i   6.4000 + 0.0000i
  Columns 5 through 8
  10.8000 - 0.0000i  24.4000 - 0.0000i  10.8000 + 0.0000i   6.4000 - 0.0000i
  Columns 9 through 10
  15.6000 - 0.0000i  10.2000 - 0.0000i
difference =
  2.4416e-014
```

4. $x_4(n) = x(n) \ \textcircled{10} \ x((-n))_{10}$: 10-point circular convolution with a circularly-folded sequence.
   MATLAB script:

```
N = 10; n = [0:N-1]; X = [10,-2+j*3,3+j*4,2-j*3,4+j*5,12];
X = [X,conj(X(5:-1:2))]; x = real(idft(X,N))
X0 = X(mod(-k,N)+1); X4 = X .* X0
% Verification
x4 = circonvt(x,x(mod(-n,N)+1),N); X42 = dft(x4,N)
difference = max(abs(X4-X42))

x =
  Columns 1 through 7
    3.6000    -2.2397     1.0721    -1.3951     3.7520     1.2000     0.6188
  Column 8 through 10
    0.0217     1.4132     1.9571
X4 =
   100    13    25    13    41   144    41    13    25    13
X42 =
  1.0e+002 *
  Columns 1 through 4
   1.0000               0.1300 + 0.0000i   0.2500 + 0.0000i   0.1300 - 0.0000i
  Columns 5 through 8
   0.4100 - 0.0000i   1.4400 + 0.0000i   0.4100 + 0.0000i   0.1300 + 0.0000i
  Columns 9 through 10
   0.2500 - 0.0000i   0.1300 - 0.0000i
difference =
  1.0378e-013
```

5. $x_5(n) = x(n)e^{-j4\pi n/5}$: Circular-shifting by -4 in the frequency-domain.

MATLAB script:

```
N = 10; n = [0:N-1]; X = [10,-2+j*3,3+j*4,2-j*3,4+j*5,12]; m = 4;
X = [X,conj(X(5:-1:2))]; x = real(idft(X,N))
X5 = [X(m+1:end),X(1:m)]
% Verification
WN = exp(-j*2*pi/N); x5 = x.*(WN.^(m*n)); X51 = dft(x5,N)
difference = max(abs(X5-X51))

x =
  Columns 1 through 7
    3.6000   -2.2397    1.0721   -1.3951    3.7520    1.2000    0.6188
  Column 8 through 10
    0.0217    1.4132    1.9571
X5 =
  Columns 1 through 4
   4.0000 + 5.0000i  12.0000            4.0000 - 5.0000i   2.0000 + 3.0000i
  Columns 5 through 8
   3.0000 - 4.0000i  -2.0000 - 3.0000i  10.0000           -2.0000 + 3.0000i
  Columns 9 through 10
   3.0000 + 4.0000i   2.0000 - 3.0000i
X51 =
  Columns 1 through 4
   4.0000 + 5.0000i  12.0000 + 0.0000i   4.0000 - 5.0000i   2.0000 + 3.0000i
  Columns 5 through 8
   3.0000 - 4.0000i  -2.0000 - 3.0000i  10.0000 + 0.0000i  -2.0000 + 3.0000i
  Columns 9 through 10
   3.0000 + 4.0000i   2.0000 - 3.0000i
difference =
  2.4895e-014
```

P5.18 Complex-valued N-point sequence $x(n)$ can be decomposed into N-point circular-conjugate-symmetric and circular-conjugate-antisymmetric sequences using the following relations

$$x_{\mathrm{ccs}}(n) \triangleq \frac{1}{2}\left[x(n) + x^*((-n))_N\right]$$

$$x_{\mathrm{cca}}(n) \triangleq \frac{1}{2}\left[x(n) - x^*((-n))_N\right]$$

If $X_{\mathrm{R}}(k)$ and $X_{\mathrm{I}}(k)$ are the real and imaginary parts of the N-point DFT of $x(n)$, then

$$\mathrm{DFT}\,[x_{\mathrm{ccs}}(n)] = X_{\mathrm{R}}(k) \quad \text{and} \quad \mathrm{DFT}\,[x_{\mathrm{cca}}(n)] = j\,X_{\mathrm{I}}(k)$$

1. Using the DFT properties of conjugation and circular folding, we obtain

$$\mathrm{DFT}\,[x_{\mathrm{ccs}}(n)] = \frac{1}{2}\left\{\mathrm{DFT}\,[x(n)] + \mathrm{DFT}\left[x^*((-n))_N\right]\right\}$$

$$= \frac{1}{2}\left\{X(k) + \hat{X}^*((-k))_N\right\}, \quad \text{where } \hat{X}(k) = \mathrm{DFT}\,[x((-n))_N]$$

$$= \frac{1}{2}\left\{X(k) + X^*(k)\right\} = \mathrm{Re}\,[X(k)] = X_{\mathrm{R}}(k)$$

similarly, we can show that

$$\mathrm{DFT}\,[x_{\mathrm{cca}}(n)] = j\,\mathrm{Im}\,[X(k)] = j\,X_{\mathrm{I}}(k)$$

2. The modified `circevod` function:

```
function [xccs, xcca] = circevod(x)
% Complex-valued signal decomposition into circular-even and circular-odd parts
% ---------------------------------------------------------------------------
% [xccs, xcca] = circecod(x)
%
N = length(x); n = 0:(N-1);
xccs = 0.5*(x + conj(x(mod(-n,N)+1)));
xcca = 0.5*(x - conj(x(mod(-n,N)+1)));
```

3. Let $X(k) = [3\cos(0.2\pi k) + j4\sin(0.1\pi k)][u(k) - u(k - 20)]$ be a 20-point DFT. MATLAB verification:

```
N = 20; k = 0:N-1; X = 3*cos(0.2*pi*k) + j*sin(0.1*pi*k);
n = 0:N-1; x = idft(X,N); [xccs, xcca] = circevod(x);
Xccs = dft(xccs,N); Xcca = dft(xcca,N);
Hf_1 = figure('Units','inches','position',[1,1,6,4],...
    'paperunits','inches','paperposition',[0,0,6,4],'color',[0,0,0]);
set(Hf_1,'NumberTitle','off','Name','P5.18.3');
subplot(2,2,1); H_s1 = stem(n,real(X),'filled'); set(H_s1,'markersize',3);
title('X_R(k)'); ylabel('Amplitude'); axis([-0.5,20.5,-4,4]);
subplot(2,2,3); H_s2 = stem(n,real(Xccs),'filled'); set(H_s2,'markersize',3);
title('X_{ccs}(k)'); ylabel('Amplitude'); xlabel('k'); axis([-0.5,20.5,-4,4]);
subplot(2,2,2); H_s3 = stem(n,imag(X),'filled'); set(H_s3,'markersize',3);
title('X_I(k)'); ylabel('Amplitude'); axis([-0.5,20.5,-1.1,1.1]);
subplot(2,2,4); H_s4 = stem(n,imag(Xcca),'filled'); set(H_s4,'markersize',3);
title('X_{cca}(k)'); ylabel('Amplitude'); xlabel('k'); axis([-0.5,20.5,-1.1,1.1]);
```

The plots are shown in Figure 5.13.
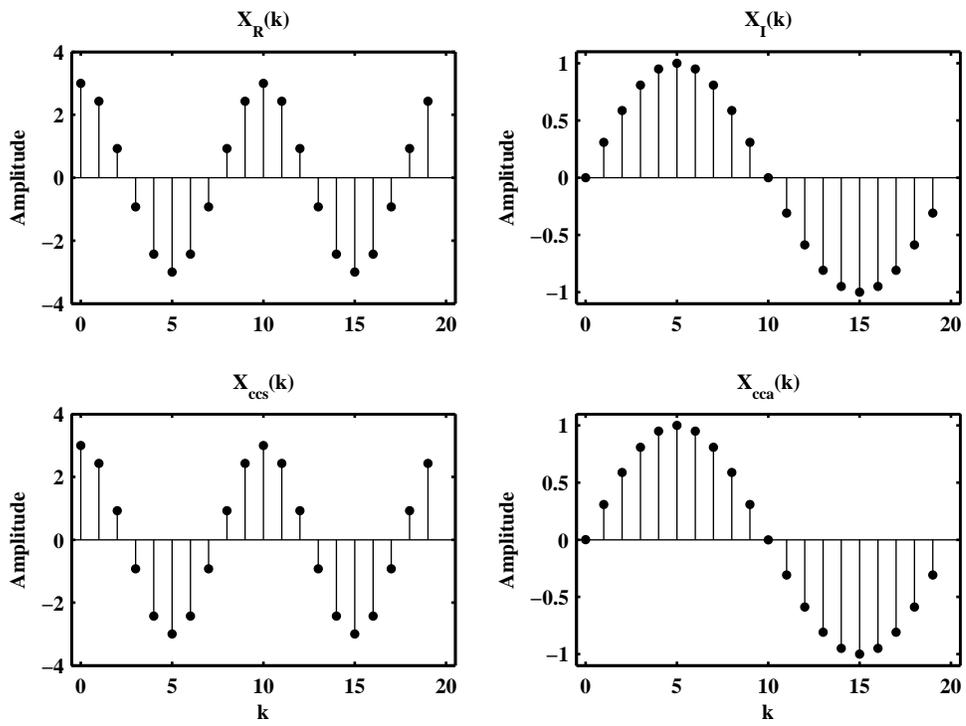
Figure 5.13: Plots in Problem P5.18.3

P5.19 (Two real DFTs using one complex DFT) If $X(k)$ is the N-point DFT of an N-point complex-valued sequence

$$x(n) = x_{\mathrm{R}}(n) + jx_{\mathrm{I}}(n)$$

where $x_{\mathrm{R}}(n)$ and $x_{\mathrm{I}}(n)$ are the real and imaginary parts of $x(n)$, then

$$\mathrm{DFT}\,[x_{\mathrm{R}}(n)] = X_{\mathrm{ccs}}(k) \quad \text{and} \quad \mathrm{DFT}\,[jx_{\mathrm{I}}(n)] = X_{\mathrm{cca}}(k)$$

where $X_{\mathrm{ccs}}(k)$ and $X_{\mathrm{cca}}(k)$ are the circular-even and circular-odd components of $X(k)$ as defined in Problem 5.18.

1. Analytical proof: Consider

$$X_{\mathrm{R}}(k) \triangleq \mathrm{DFT}\,[x_{\mathrm{R}}(n)] = \frac{1}{2}\left\{\mathrm{DFT}\,[x(n)] + \mathrm{DFT}\left[x^*(n)\right]\right\}$$

$$= \frac{1}{2}\left\{X(k) + X^*((-k))_N\right\} \triangleq X_{\mathrm{ccs}}$$

Similarly

$$\left.\begin{aligned} jX_{\mathrm{I}}(k) &\triangleq \mathrm{DFT}\,[jx_{\mathrm{I}}(n)] = \frac{1}{2}\left\{\mathrm{DFT}\,[x(n)] - \mathrm{DFT}\,[x^*(n)]\right\} \\ &= \frac{1}{2}\left\{X(k) - X^*((-k))_N\right\} \triangleq X_{\mathrm{cca}} \end{aligned}\right\} \Rightarrow X_{\mathrm{I}}(k) = \frac{X_{\mathrm{cca}}(k)}{j} = -jX_{\mathrm{cca}}(k)$$

2. This property can be used to compute the DFTs of two real-valued N-point sequences using one N-point DFT operation. Specifically, let $x_1(n)$ and $x_2(n)$ be two N-point sequences. Then we can form a complex-valued sequence

$$x(n) = x_1(n) + jx_2(n)$$

and use the above property. MATLAB function `real2dft`:

```
function [X1,X2] = real2dft(x1,x2,N)
% DFTs of two real sequences
% [X1,X2] = real2dft(x1,x2,N)
%   X1 = N-point DFT of x1
%   X2 = N-point DFT of x2
%   x1 = real-valued sequence of length <= N
%   x2 = real-valued sequence of length <= N
%    N = length of DFT
%
% Check for length of x1 and x2
if length(x1) > N
        error('*** N must be >= the length of x1 ***')
end
if length(x2) > N
        error('*** N must be >= the length of x2 ***')
end
N1 = length(x1); x1 = [x1 zeros(1,N-N1)];
N2 = length(x2); x2 = [x2 zeros(1,N-N2)];
x = x1 + j*x2;
X = dft(x,N);
[X1, X2] = circevod(X); X2 = X2/j;
```

We will also need the `circevod` function for complex sequences (see Problem P5.18). This can be obtained from the one given in the text by two simple changes.

```
function [xccs, xcca] = circevod(x)
% Complex signal decomposition into circular-even and circular-odd parts
% -------------------------------------------------------------------
% [xccs, xcca] = circecod(x)
%
N = length(x); n = 0:(N-1);
xccs = 0.5*(x + conj(x(mod(-n,N)+1)));
xcca = 0.5*(x - conj(x(mod(-n,N)+1)));
```

3. Compute and plot the DFTs of the following two sequences using the above function

$$x_1(n) = \cos(0.1\pi n), \ x_2(n) = \sin(0.2\pi n); \quad 0 \le n \le 39$$

MATLAB verification:

```
N = 40; n = 0:N-1; x1 = cos(0.1*pi*n); x2 = sin(0.2*pi*n);
[X1,X2] = real2dft(x1,x2,N);
X11 = dft(x1,N); X21 = dft(x2,N);
difference = max(abs(X1-X11))
difference = max(abs(X2-X21))

difference =
   3.6876e-013
difference =
   3.6564e-013
```

P5.20 Circular shifting: The MATLAB routine `cirshftf.m` to implement circular shift is written using the frequency-domain property

$$y(n) \triangleq x((n - m))_N = \text{IDFT}\left[X(k)W_N^{mk}\right]$$

This routine will be used in the next problem to generate a circulant matrix and has the following features. If m is a scaler then $y(n)$ is circularly shifted sequence (or array). If m is a vector then $y(n)$ is a matrix, each row of which is a circular shift in $x(n)$ corresponding to entries in the vector m.

```
function y = cirshftf(x,m,N)
%  Circular shift of m samples wrt size N in sequence x: (freq domain)
%  ----------------------------------------------------------------
% function y=cirshift(x,m,N)
%       y : output sequence containing the circular shift
%       x : input sequence of length <= N
%       m : sample shift
%       N : size of circular buffer
%
%  Method: y(n) = idft(dft(x(n))*WN^(mk))
%
%    If m is a scalar then y is a sequence (row vector)
%    If m is a vector then y is a matrix, each row is a circular shift
%        in x corresponding to entries in vecor m
%    M and x should not be matrices
%
% Check whether m is scalar, vector, or matrix
[Rm,Cm] = size(m);
if Rm > Cm
    m = m'; % make sure that m is a row vector
end
[Rm,Cm] = size(m);
if Rm > 1
    error('*** m must be a vector ***') % stop if m is a matrix
end
% Check whether x is scalar, vector, or matrix
[Rx,Cx] = size(x);
if Rx > Cx
    x = x'; % make sure that x is a row vector
end
[Rx,Cx] = size(x);
if Rx > 1
    error('*** x must be a vector ***') % stop if x is a matrix
end
% Check for length of x
if length(x) > N
        error('N must be >= the length of x')
end
x=[x zeros(1,N-length(x))];
X=dft(x,N);
```

```
X=ones(Cm,1)*X;
WN=exp(-2*j*pi/N);
k=[0:1:N-1];
Y=(WN.^(m' * k)).*X;
y=real(conj(dfs(conj(Y),N)))/N;
```

Matlab verification:

(a) $x(n) = \{5, 4, 3, 2, 1, 0, 0, 1, 2, 3, 4\}, 0 \le n \le 10; m = -5, N = 11$
    ↑

```
   x = [5,4,3,2,1,0,0,1,2,3,4,5];
   m = -5; N = 12;
   y = cirshftf(x,m,N); y = real(y)

   y =
        0     0     1     2     3     4     5     5     4     3     2     1
```

(b) $x(n) = \{5, 4, 3, 2, 1, 0, 0, 1, 2, 3, 4\}, 0 \le n \le 10; m = 8, N = 15$
    ↑

```
   x = [5,4,3,2,1,0,0,1,2,3,4,5];
   m = 8; N = 15;
   y = cirshftf(x,m,N); y = real(y)

   y =
        1     2     3     4     5     0     0     0     5     4     3     2
        1     0     0
```

P5.21 Parseval's relation for the DFT:

$$
\sum_{n=0}^{N-1} |x(n)|^2 = \sum_{n=0}^{N-1} x(n)\,x^*(n) = \sum_{n=0}^{N-1} \left\{ \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, W_N^{-nk} \right\} x^*(n)
$$

$$
= \frac{1}{N} \sum_{k=0}^{N-1} X(k) \left\{ \sum_{n=0}^{N-1} x^*(n)\, W_N^{-nk} \right\} = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \left\{ \sum_{n=0}^{N-1} x(n)\, W_N^{nk} \right\}^*
$$

Therefore,

$$
\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, X^*(k) = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2
$$

MATLAB verification:

```
x = [5,4,3,2,1,0,0,1,2,3,4,5]; N = length(x);
% power of x(n) in the time-domain
power_x = sum(x.*conj(x))


power_x =
        110


% Power in the frequency-domain
X = dft(x,N); power_X = (1/N)*sum(X.*conj(X))


power_X =
        110
```

P5.22 A 512-point DFT $X(k)$ of a real-valued sequence $x(n)$ has the following DFT values:

$$X(0) = 20 + j\alpha; \quad X(5) = 20 + j30; \quad X(k_1) = -10 + j15; \quad X(152) = 17 + j23;$$

$$X(k_2) = 20 - j30; \quad X(k_3) = 17 - j23; \quad X(480) = -10 - j15; \quad X(256) = 30 + j\beta$$

and all other values are known to be zero.

1. The real-valued coefficients α and β: Since the sequence $x(n)$ is real-valued, $X(k)$ is conjugate symmetric which means that $X(0)$ and $X(N/2)$ are also real-valued. Since $N = 512$, $X(0)$ and $X(256)$ are real-valued. Hence $\alpha = \beta = 0$.

2. The values of the integers k_1, k_2, and k_3: Again using the conjugate symmetry property of the DFT, we have $X(k) = X^*(N - k)$. Thus

$$X(5) = 20 + j30 = X^*(512 - 5) = X^*(507) \Rightarrow X(507) = 20 - j30 \Rightarrow k_2 = 507$$

$$X(480) = -10 - j15 = X^*(512 - 480) = X^*(32) \Rightarrow X(32) = -10 + j15 \Rightarrow k_1 = 32$$

$$X(152) = 17 + j23 = X^*(512 - 152) = X^*(360) \Rightarrow X(360) = 17 - j23 \Rightarrow k_3 = 360$$

3. The energy of the signal $x(n)$: Using Parseval's relation,

$$\mathcal{E}_x = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{N} \sum_{k=-\infty}^{\infty} |X(k)|^2$$

$$= \frac{1}{512} \left[|X(0)|^2 + 2|X(5)|^2 + 2|X(32)|^2 + 2|X(152)|^2 + |X(256)|^2 \right] = 12.082$$

4. Sequence $x(n)$ in a closed form: The time-domain sequence $x(n)$ is a linear combination of the harmonically related complex exponential. Hence

$$x(n) = \frac{1}{512} \left[X(0) + X(5)e^{-2\pi 5n/512} + X^*(5)e^{2\pi 5n/512} + X(32)e^{-2\pi 32n/512} + X^*(32)e^{2\pi 32n/512} \right.$$

$$\left. + X(152)e^{-2\pi 152n/512} + X^*(152)e^{2\pi 152n/512} + X(256)e^{-2\pi 256n/512} \right]$$

$$= \frac{1}{512} \left[X(0) + 2\text{Re}\left\{ X(5)e^{-2\pi 5n/512} \right\} + 2\text{Re}\left\{ X(32)e^{-2\pi 32n/512} \right\} + 2\text{Re}\left\{ X(152)e^{-2\pi 152n/512} \right\} \right.$$

$$\left. + X(256)(-1)^n \right]$$

$$= \frac{1}{512} \left[20 + 72.111\cos(0.019531\pi n - 56.32°) + 36.056\cos(0.125\pi n - 123.69°) \right.$$

$$\left. + 57.201\cos(0.59375\pi n - 53.531°) + 30(-1)^n \right]$$

P5.23 Let $x(n)$ be a finite length sequence given by

$$x(n) = \left\{ \ldots, 0, 0, 0, \underset{\uparrow}{1}, 2, -3, 4, -5, 0, \ldots \right\}$$

Then the sequence

$$x((-8 - n))_7 \mathcal{R}_7(n) = x((-[n + 8]))_7 \mathcal{R}_7(n) = x((-[n + 8 - 7]))_7 \mathcal{R}_7(n)$$
$$= x((-[n + 1]))_7 \mathcal{R}_7(n)$$

where

$$\mathcal{R}_7(n) = \begin{cases} 1, & 0 \leq n \leq 6 \\ 0, & \text{else} \end{cases}$$

is a circularly folded and circularly shifted-by-(-1) version of the 7-point sequence $\{1, 2, -3, 4, -5, 0, 0\}$. Hence

$$x((-8 - n))_7 \mathcal{R}_7(n) = \{0, 0, -5, 4, -3, 2, 1\}$$

P5.24 Circular convolution using circulant matrix operation.

$$x_1(n) = \{1, 2, 2\}, \quad x_2(n) = \{1, 2, 3, 4\}, \quad x_3(n) \triangleq x_1(n) \textcircled{4} x_2(n)$$

1. Using the results from Example 5.13, we can express the above signals as

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{X}_2 = \begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

The matrix $\mathbf{X}_2$ has the property that its every row or column can be obtained from the previous row or column using circular shift. Such a matrix is called a *circulant* matrix. It is completely described by the first column or the row.

2. Circular convolution:

$$\mathbf{x}_3 = \mathbf{X}_2 \mathbf{x}_1 = \begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 15 \\ 12 \\ 9 \\ 14 \end{bmatrix}$$

P5.25 MATLAB function `circulnt`:

```
function C = circulnt(x,N)
%  Circulant matrix generation using vector data values
%  ---------------------------------------------------
%  function C = circulnt(h,N)
%
%   C : Circulant matrix
%   x : input sequence of length <= N
%   N : size of the circular buffer
%   Method: C = h((n-m) mod N);

  Mx = length(x);                       % length of x
  x = [x, zeros(N-Mx,1)];               % zero-pad x
  C = zeros(N,N);                       % establish size of C
  m = 0:N-1;                            % indices n and m
  x = circfold(x,N);                    % Circular folding
  C = cirshift(x,m,N);                  % Circular shifting
```

MATLAB verification on sequences in Problem 5.24:

```
N = 4; x1 = [1,2,2,0]; x2 = [1,2,3,4]; X2 = circulnt(x2,N)

X2 =
    1.0000    4.0000    3.0000    2.0000
    2.0000    1.0000    4.0000    3.0000
    3.0000    2.0000    1.0000    4.0000
    4.0000    3.0000    2.0000    1.0000


x3 = X2*x1'; x3 = x3'

x3 =
   15.0000   12.0000    9.0000   14.0000
```

P5.26 MATLAB function circonvf:

```
function y = circonvf(x1,x2,N)
%
%function y=circonvf(x1,x2,N)
%
%  N-point circular convolution between x1 and x2: (freq domain)
%  -----------------------------------------------------------
%       y  : output sequence containing the circular convolution
%       x1 : input sequence of length N1 <= N
%       x2 : input sequence of length N2 <= N
%       N  : size of circular buffer
%
%  Method: y(n) = idft(dft(x1)*dft(x2))

% Check for length of x1
if length(x1) > N
        error('N must be >= the length of x1')
end

% Check for length of x2
if length(x2) > N
        error('N must be >= the length of x2')
end

x1=[x1 zeros(1,N-length(x1))];
x2=[x2 zeros(1,N-length(x2))];

X1=fft(x1); X2=fft(x2);
y=real(ifft(X1.*X2));
```

Circular convolution $\{4, 3, 2, 1\} \circledS{4} \{1, 2, 3, 4\}$:

```
x1 = [4,3,2,1]; x2 = [1,2,3,4];
x3 = circonvf(x1,x2,4)

x3 =
    24    22    24    30
```

P5.27 The following four sequences are given:

$$x_1(n) = \{\underset{\uparrow}{1}, 3, 2, -1\}; \quad x_2(n) = \{\underset{\uparrow}{2}, 1, 0, -1\}; \quad x_3(n) = x_1(n) * x_2(n); \quad x_4(n) = x_1(n) \circledS x_2(n)$$

1. Linear convolution $x_3(n)$:

$$x_3(n) = x_1(n) * x_2(n) = \{\underset{\uparrow}{2}, 7, 7, -1, -4, -2, 1\}$$

2. Computation of $x_4(n)$ using $x_3(n)$ alone: The error in the two convolutions is given by

$$e(n) \triangleq x_4(n) - x_3(n) = x_3(n + N)$$

we have, for $N = 5$,

$$
\begin{aligned}
e(0) &= x_4(0) - x_3(0) = x_3(5) \Rightarrow x_4(0) = x_3(0) + x_3(5) = 2 - 2 = 0 \\
e(1) &= x_4(1) - x_3(1) = x_3(6) \Rightarrow x_4(1) = x_3(1) + x_3(6) = 7 + 1 = 8 \\
e(2) &= x_4(2) - x_3(2) = x_3(7) \Rightarrow x_4(2) = x_3(2) + x_3(7) = 7 + 0 = 7 \\
e(3) &= x_4(3) - x_3(3) = x_3(8) \Rightarrow x_4(3) = x_3(3) + x_3(8) = -1 + 0 = -1 \\
e(4) &= x_4(4) - x_3(4) = x_3(9) \Rightarrow x_4(4) = x_3(4) + x_3(9) = -4 + 0 = -4
\end{aligned}
$$

P5.28 Computation and plotting of the N-point circular convolutions between two finite-length sequences.

1. $x_1(n) = \sin(\pi n/3)\mathcal{R}_6(n)$, $x_2(n) = \cos(\pi n/4)\mathcal{R}_8(n)$; $N = 10$: MATLAB script:

```
N = 10; n = 0:N-1;n1 = 0:5; x1 = sin(pi*n1/3);
n2 = 0:7; x2 = cos(pi*n2/4); x3 = circonvt(x1,x2,N);
Hf_1 = figure('Units','inches','position',[1,1,5,2],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,5,2]);
set(Hf_1,'NumberTitle','off','Name','P5.28.1');
H_s1 = stem(n,x3,'filled'); set(H_s1,'markersize',3);
title('Circular Convolution {\itx}_3({\itn})','fontsize',10);
ylabel('Amplitude'); xlabel('{\itn}'); axis([-1,N,min(x3)-1,max(x3)+1]);
```

The sample plot is shown in Figure 5.14.
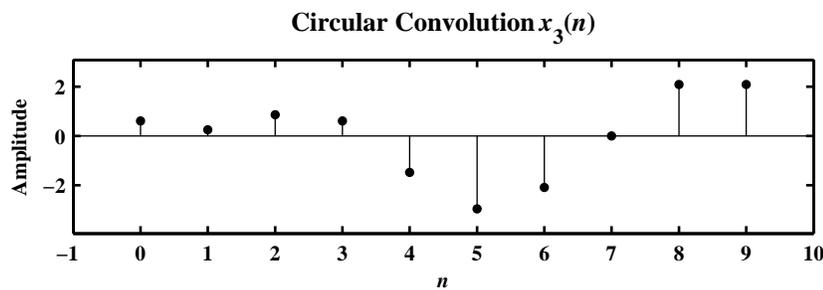
Figure 5.14: The sample plot in Problem P5.28.1

2. $x_1(n) = \cos\left(2\pi n/N\right)\mathcal{R}_N(n)$, $x_2(n) = \sin\left(2\pi n/N\right)\mathcal{R}_N(n)$; $N = 32$: MATLAB script:

```
N = 32; n = 0:N-1;
x1 = cos(2*pi*n/N); x2 = sin(2*pi*n/N); x3 = circonvt(x1,x2,N);
Hf_2 = figure('Units','inches','position',[1,1,5,1.5],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,5,1.5]);
set(Hf_2,'NumberTitle','off','Name','P5.28.2');
H_s2 = stem(n,x3,'filled'); set(H_s2,'markersize',3);
title('Circular Convolution {\itx}_3({\itn})','fontsize',10);
ylabel('Amplitude'); xlabel('{\itn}'); axis([-1,N,min(x3)-1,max(x3)+1]);
```

The sample plot is shown in Figure 5.15.
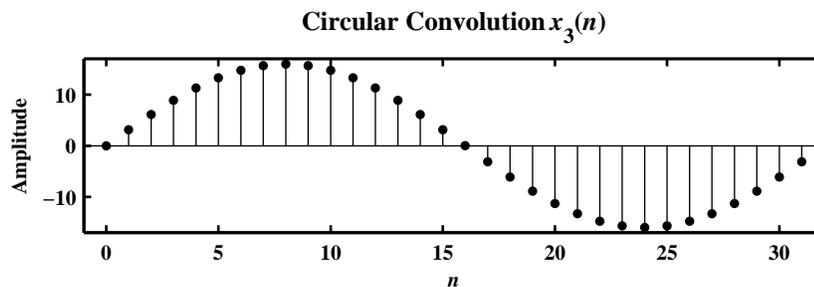
Figure 5.15: The sample plot in Problem P5.28.2

3. $x_1(n) = (0.8)^n \mathcal{R}_N(n), \quad x_2(n) = (-0.8)^n \mathcal{R}_N(n); \quad N = 20$: MATLAB script:

```
N = 20; n = 0:N-1;
x1 = (0.8).^n; x2 = (-0.8).^n; x3 = circonvt(x1,x2,N);
Hf_3 = figure('Units','inches','position',[1,1,5,1.5],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,5,1.5]);
set(Hf_3,'NumberTitle','off','Name','P5.28.3');
H_s3 = stem(n,x3,'filled'); set(H_s3,'markersize',3);
title('Circular Convolution {\itx}_3({\itn})','fontsize',10);
ylabel('Amplitude'); xlabel('{\itn}'); axis([-1,N,min(x3)-0.5,max(x3)+0.5]);
```

The sample plot is shown in Figure 5.16.
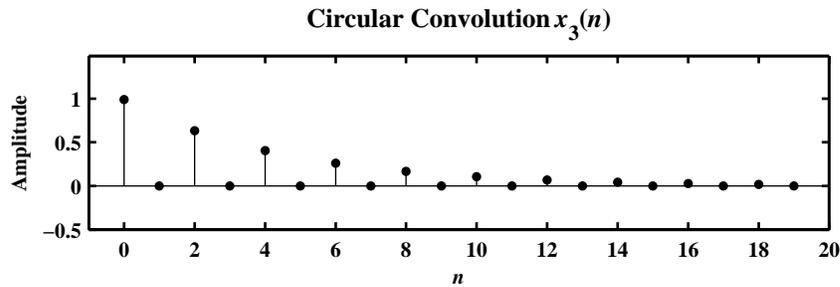
Figure 5.16: The sample plot in Problem P5.28.3

4. $x_1(n) = n\mathcal{R}_N(n), \quad x_2(n) = (N-n)\mathcal{R}_N(n); \quad N = 10$: MATLAB script:

```
N = 10; n = 0:N-1;
x1 = n; x2 = (N-n); x3 = circonvt(x1,x2,N);
Hf_4 = figure('Units','inches','position',[1,1,5,1.5],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,5,1.5]);
set(Hf_4,'NumberTitle','off','Name','P5.28.4');
H_s4 = stem(n,x3,'filled'); set(H_s4,'markersize',3);
title('Circular Convolution {\itx}_3({\itn})','fontsize',10);
ylabel('Amplitude'); xlabel('{\itn}'); axis([-1,N,min(x3)-10,max(x3)+10]);
```

The sample plot is shown in Figure 5.17.
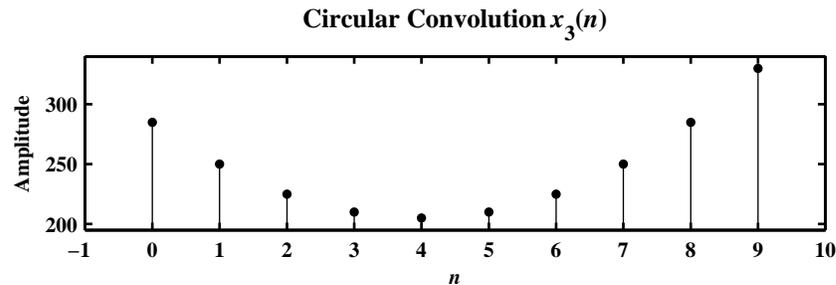
Figure 5.17: The sample plot in Problem P5.28.4

5. $x_1(n) = (0.8)^n R_{20}, x_2(n) = u(n) - u(n-40); N = 50$: MATLAB script:

```
N = 50; n = 0:N-1; n1 = 0:19; x1 = (0.8).^n1;
n2 = 0:39; x2 = ones(1,40); x3 = circonvt(x1,x2,N);
Hf_5 = figure('Units','inches','position',[1,1,5,1.5],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,5,1.5]);
set(Hf_5,'NumberTitle','off','Name','P5.28.5');
H_s5 = stem(n,x3,'filled'); set(H_s5,'markersize',3);
title('Circular Convolution {\itx}_3({\itn})','fontsize',10);
ylabel('Amplitude'); xlabel('{\itn}'); axis([-1,N,min(x3)-1,max(x3)+1]);
```

The sample plot is shown in Figure 5.18.
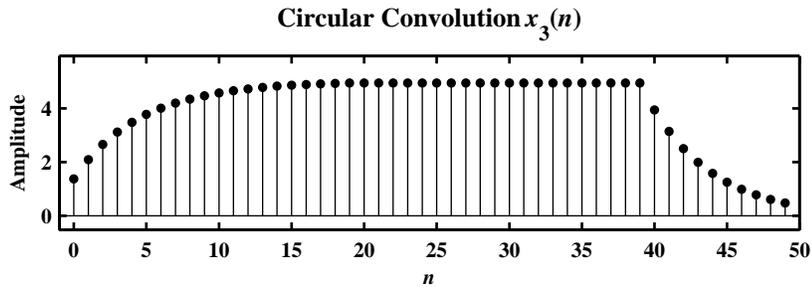
Figure 5.18: The sample plot in Problem P5.28.5

P5.29 Let $x_1(n)$ and $x_2(n)$ be two N-point sequences.

1. Since $y(n) = x_1(n) \textcircled{N} x_2(n) = \sum_{k=0}^{N-1} x_1(k) x_2((n-k))_N$ we have

$$
\begin{aligned}
\sum_{n=0}^{N-1} y(n) &= \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} x_1(k) x_2((n-k))_N = \sum_{k=0}^{N-1} x_1(k) \sum_{n=0}^{N-1} x_2((n-k))_N \\
&= \sum_{n=0}^{N-1} x_1(n) \left[\sum_{n=0}^{k-1} x_2(n-k+N) + \sum_{n=k}^{N-1} x_2(n-k) \right] \\
&= \sum_{n=0}^{N-1} x_1(n) \left[\sum_{n=N-k}^{N-1} x_2(n) + \sum_{n=0}^{N-1-k} x_2(n) \right] = \sum_{n=0}^{N-1} x_1(n) \left[\sum_{n=0}^{N-k-1} x_2(n) + \sum_{n=N-k}^{N-1} x_2(n) \right] \\
&= \left(\sum_{n=0}^{N-1} x_1(n) \right) \left(\sum_{n=0}^{N-1} x_2(n) \right)
\end{aligned}
$$

2. Verification using the following sequences:

$$
x_1(n) = \{9, 4, -1, 4, -4, -1, 8, 3\}; \quad x_2(n) = \{-5, 6, 2, -7, -5, 2, 2, -2\}
$$

Consider

$$
x_1(n) = \{9, 4, -1, 4, -4, -1, 8, 3\} \Rightarrow \sum_{n=0}^{7} x_1(n) = 22
$$

$$
x_2(n) = \{-5, 6, 2, -7, -5, 2, 2, -2\} \Rightarrow \sum_{n=0}^{7} x_2(n) = -7
$$

$$
y(n) = x_1(n) \textcircled{8} x_2(n) = \{14, -9, -32, -74, -7, -16, -57, 27\} \Rightarrow \sum_{n=0}^{7} y(n) = -154
$$

Hence

$$
\sum_{n=0}^{7} y(n) = -154 = (22) \times (-7) = \left(\sum_{n=0}^{7} x_1(n) \right) \left(\sum_{n=0}^{7} x_2(n) \right)
$$

P5.30 Let $X(k)$ be the 8-point DFT of a 3-point sequence $x(n) = \{5, \overset{\uparrow}{-4}, 3\}$. Let $Y(k)$ be the 8-point DFT of a sequence $y(n)$ where $Y(k) = W_8^{5k} X((-k))_8$. Then using the circular folding and the circular shifting properties of the DFT, we have

$$
\begin{aligned}
y(n) &= \text{IDFT}\left[W_8^{5k} X((-k))_8 \right] = \text{IDFT}\left[X((-k))_8 \right]_{n \to (n-5)} \\
&= \left[x((-n))_8 \big|_{n \to (n-5)} \right] \mathcal{R}_8(n) = x((5-n))_8 \mathcal{R}_8(n) = \{0, 0, 0, 3, -4, \overset{\uparrow}{5}, 0, 0\}
\end{aligned}
$$

P5.31 Computation of (i) the N-point circular convolution $x_3(n) = x_1(n) \circledN x_2(n)$, (ii) the linear convolution $x_4(n) = x_1(n) * x_2(n)$, and (iii) the error sequence $e(n) = x_3(n) - x_4(n)$ for the following sequences:

1. $x_1(n) = \{1, 1, 1, 1\}$, $x_2(n) = \cos(\pi n/4)\mathcal{R}_6(n)$; $N = 8$:

```
x1 = [1,1,1,1]; x2 = cos(pi*[0:5]/4); N = 8; n = 0:N-1;
x3 = circonvt(x1,x2,N);
x4 = conv(x1,x2); n4 = 0:length(x4)-1;
e1 = x3 - x4(1:N); e2 = x4(N+1:end);
```

The plots of various signals are shown in Figure 5.19.
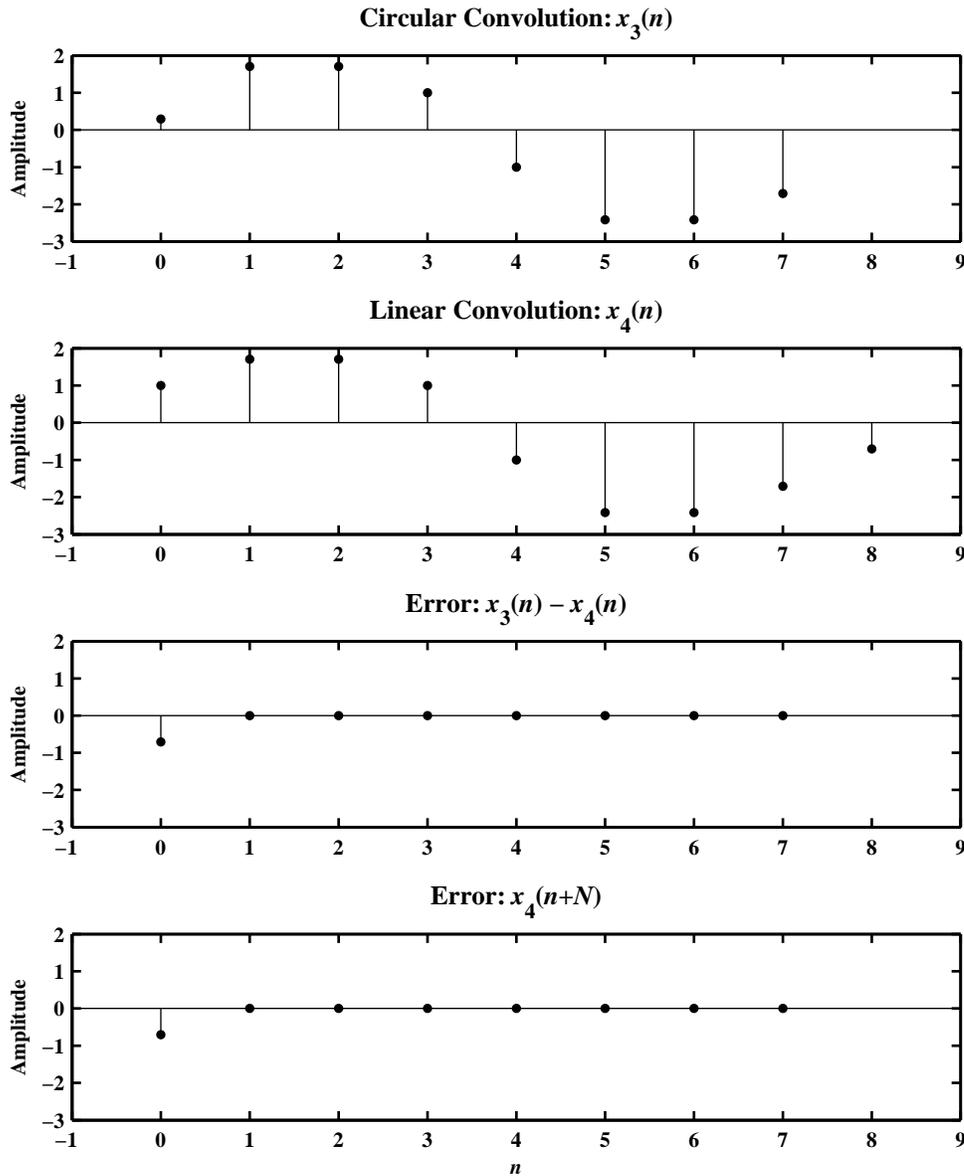
Figure 5.19: The sample plot of various signals in Problem P5.31.1

2. $x_1(n) = \cos(2\pi n/N)\mathcal{R}_{16}(n)$, $x_2(n) = \sin(2\pi n/N)\mathcal{R}_{16}(n)$; $N = 32$:

```
N = 32; x1 = cos(2*pi*[0:15]/N); x2 = sin(2*pi*[0:15]/N);
x3 = circonvt(x1,x2,N); n3 = 0:N-1;
x4 = conv(x1,x2); n4 = 0:length(x4)-1;
e1 = x3 - [x4,0];
e2 = x4(N+1:end);
```

The plots of various signals are shown in Figure 5.20.
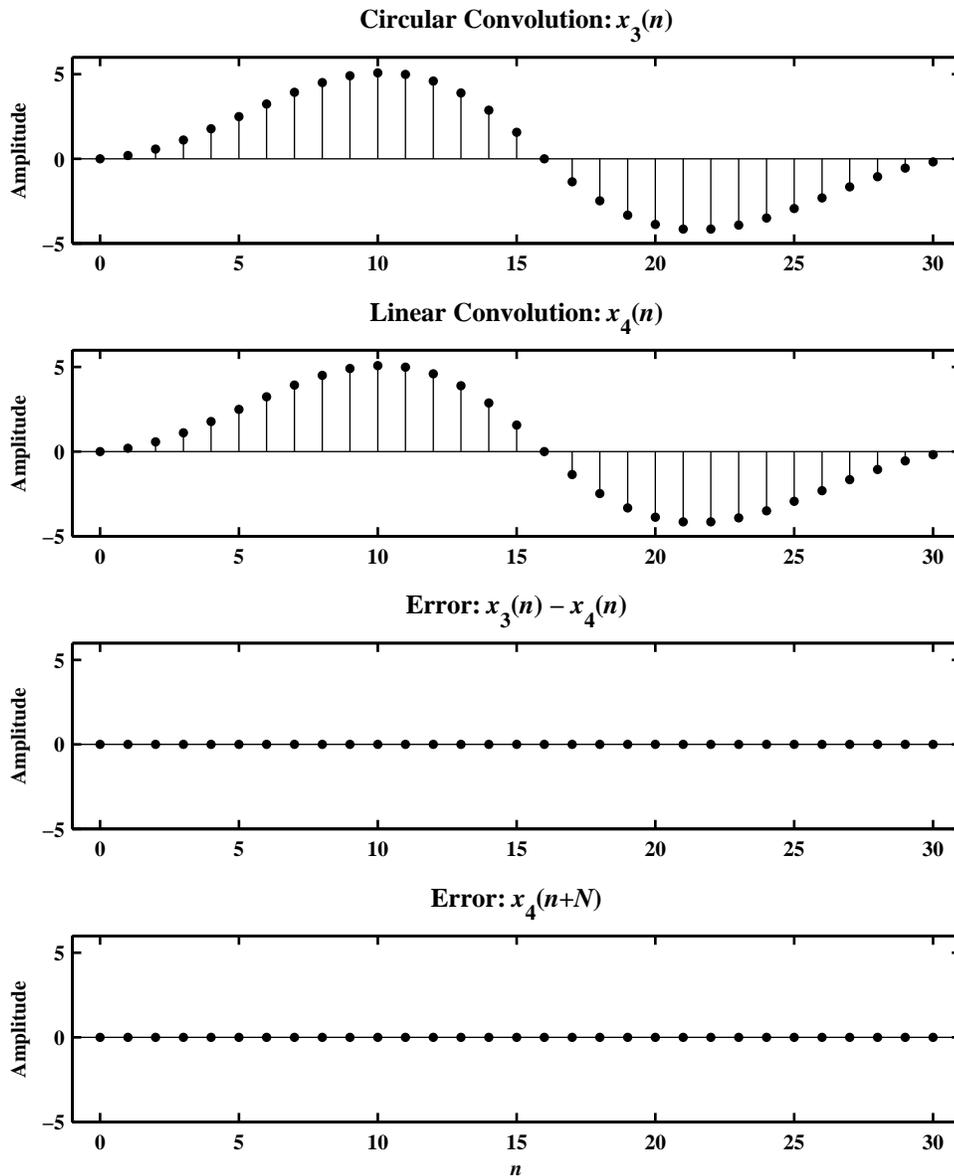
Figure 5.20: The sample plot of various signals in Problem P5.31.2

3. $x_1(n) = (0.8)^n \mathcal{R}_{10}(n)$, $x_2(n) = (-0.8)^n \mathcal{R}_{10}(n)$; $N = 15$:

```
N = 15; x1 = (0.8).^[0:9]; x2 = (-0.8).^[0:9];
x3 = circonvt(x1,x2,N); n3 = 0:N-1;
x4 = conv(x1,x2); n4 = 0:length(x4)-1;
e1 = x3 - x4(1:N);
e2 = x4(N+1:end); Ne2 = length(e2);
```

The plots of various signals are shown in Figure 5.21.
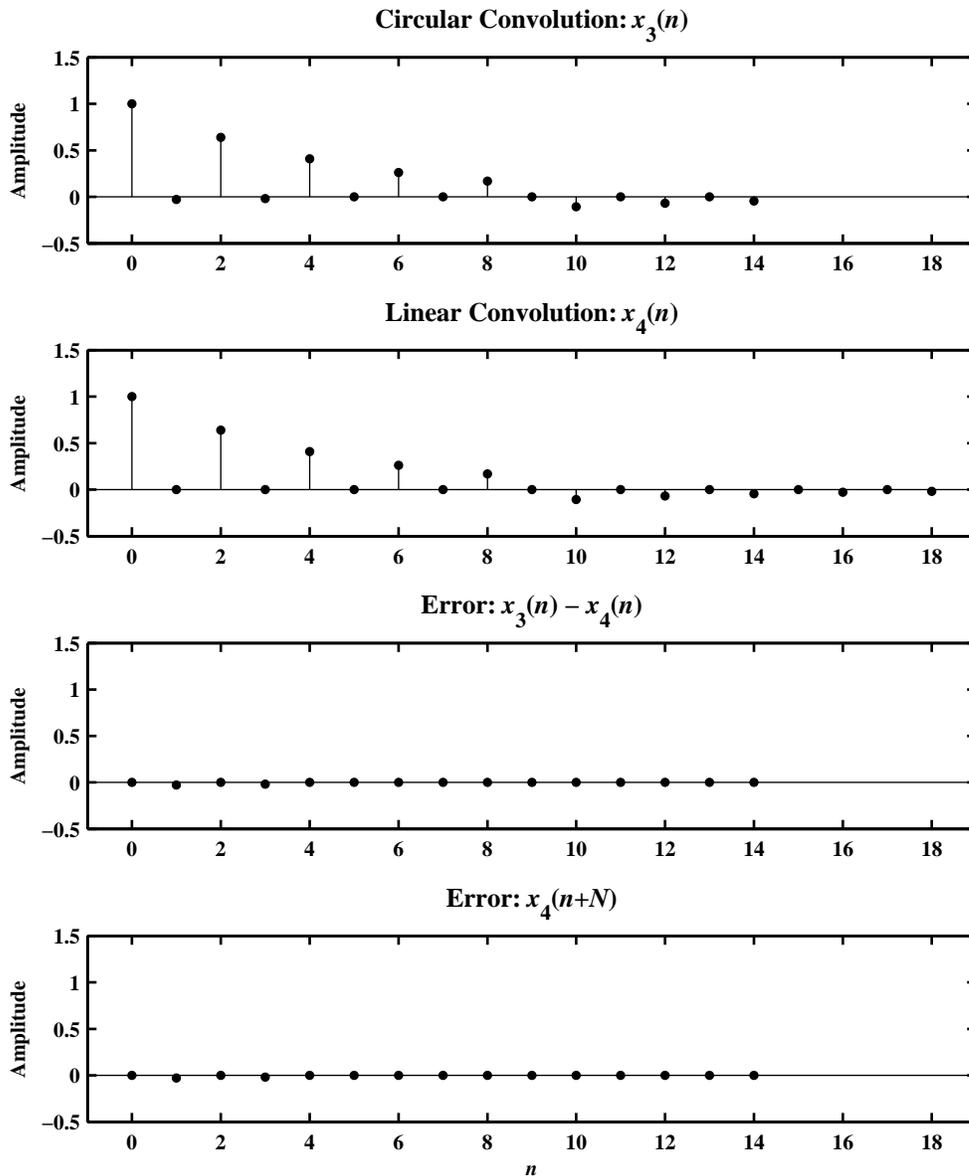
Figure 5.21: The sample plot of various signals in Problem P5.31.3

4. $x_1(n) = n\mathcal{R}_{10}(n)$, $x_2(n) = (N - n)\mathcal{R}_{10}(n)$; $N = 10$:

```
N = 10; n = 0:N-1; x1 = n; x2 = N-n;
x3 = circonvt(x1,x2,N); n3 = 0:N-1;
x4 = conv(x1,x2); n4 = 0:length(x4)-1;
e1 = x3 - x4(1:N);
e2 = x4(N+1:end); Ne2 = length(e2);
```

The plots of various signals are shown in Figure 5.22.
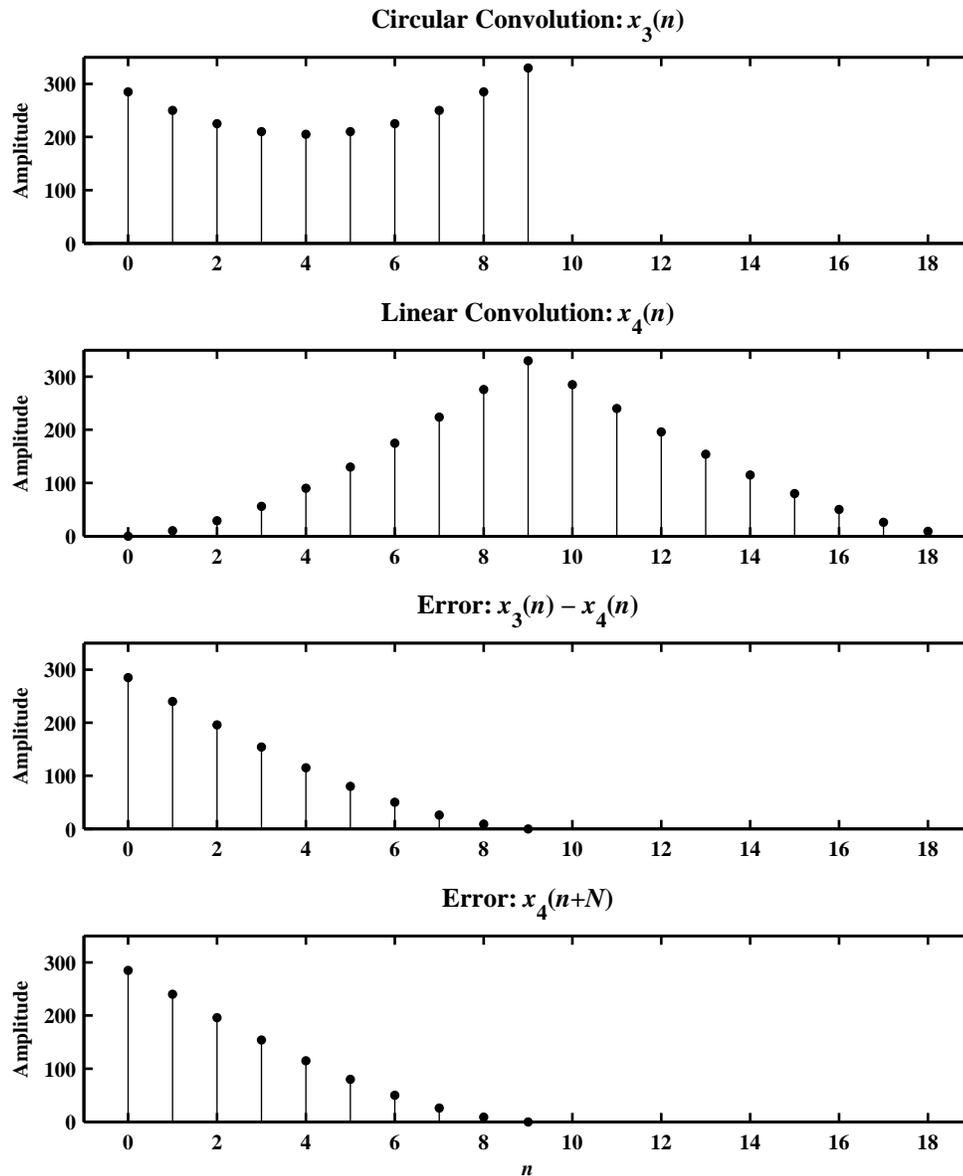
Figure 5.22: The sample plot of various signals in Problem P5.31.4

5. $x_1(n) = \{1, -1, 1, -1\}$, $x_2(n) = \{1, 0, -1, 0\}$; $N = 5$:

```
N = 5; n = 0:N-1; x1 = [1,-1,1,-1]; x2 = [1,0,-1,0];
x3 = circonvt(x1,x2,N); n3 = 0:N-1;
x4 = conv(x1,x2); n4 = 0:length(x4)-1;
e1 = x3 - x4(1:N);
e2 = x4(N+1:end); Ne2 = length(e2);
```

The plots of various signals are shown in Figure 5.23.
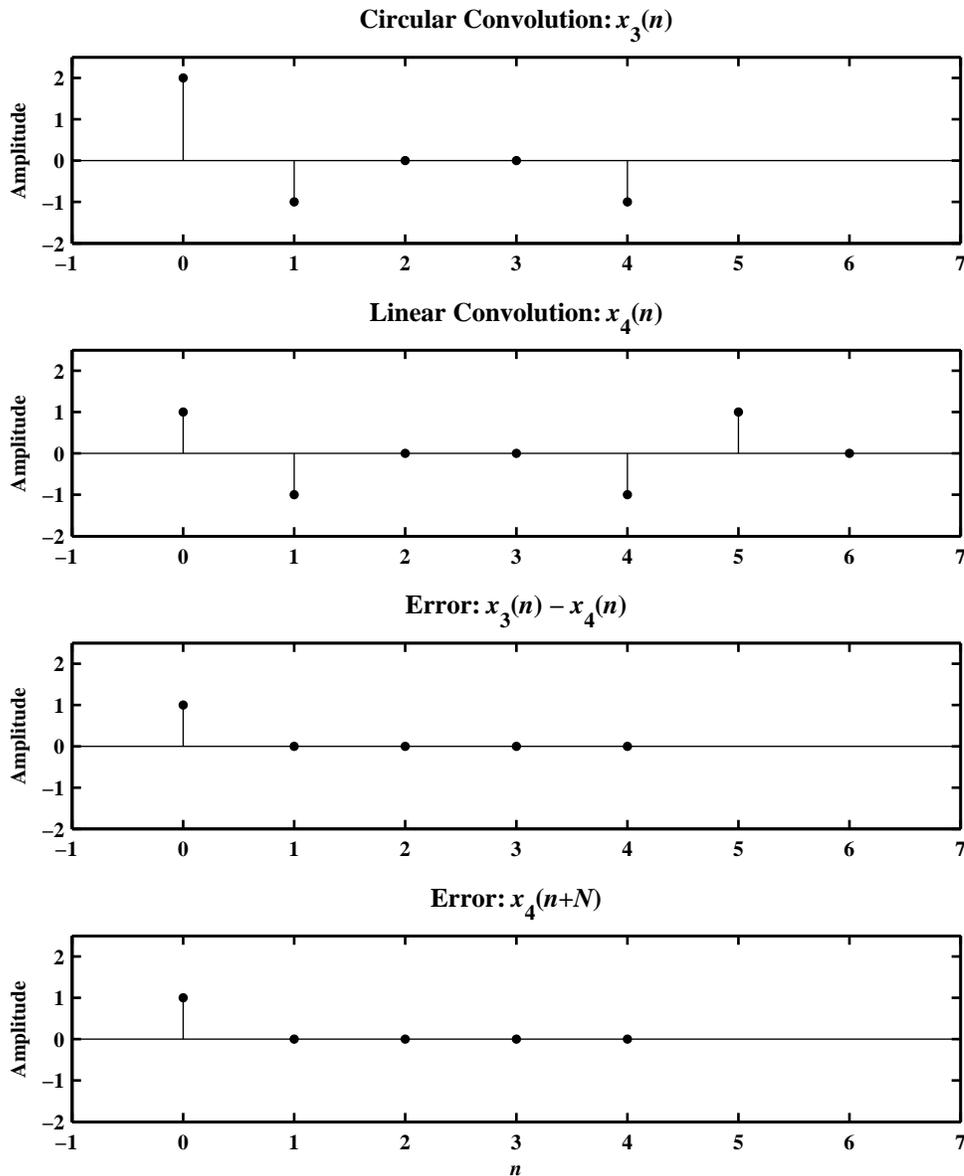
Figure 5.23: The sample plot of various signals in Problem P5.31.5

P5.32 The overlap-add method of block convolution is an alternative to the overlap-save method. Let $x(n)$ be a long sequence of length ML where $M, L \gg 1$. Divide $x(n)$ into M segments $\{x_m(n), m = 1, \ldots, M\}$ each of length L

$$x_m(n) = \begin{cases} x(n), & mL \le n \le (m+1)L - 1 \\ 0, & \text{elsewhere} \end{cases} \quad \text{so that} \quad x(n) = \sum_{m=0}^{M-1} x_m(n)$$

Let $h(n)$ be an L-point impulse response, then

$$y(n) = x(n) * h(n) = \sum_{m=0}^{M-1} x_m(n) * h(n) = \sum_{m=0}^{M-1} y_m(n); \quad y_m(n) \triangleq x_m(n) * h(n)$$

Clearly, $y_m(n)$ is a $(2L - 1)$-point sequence. In this method we have to save the intermediate convolution results and then properly overlap these before adding to form the final result $y(n)$. To use DFT for this operation we have to choose $N \ge (2L - 1)$.

1. MATLAB function to implement the overlap-add method using the circular convolution operation:

```
function [y] = ovrlpadd(x,h,N)
% Overlap-Add method of block convolution
% ----------------------------------------
% [y] = ovrlpadd(x,h,N)
% y = output sequence
% x = input sequence
% h = impulse response
% N = DFT length >= 2*length(h)-1
%
Lx = length(x); L = length(h); L1 = L-1;
h = [h zeros(1,N-L)];
%
M = ceil(Lx/L);              % Number of blocks
x = [x, zeros(1,M*L-Lx)]; % append (M*N-Lx) zeros
Y = zeros(M,N);              % Initialize Y matrix
%
% convolution with succesive blocks
for m = 0:M-1
    xm = [x(m*L+1:(m+1)*L),zeros(1,N-L)];
    Y(m+1,:) = circonvt(xm,h,N);
end
%
% Overlap and Add
Y = [Y,zeros(M,1)]; Y = [Y;zeros(1,N+1)];
Y1 = Y(:,1:L); Y1 = Y1'; y1 = Y1(:);
Y2 = [zeros(1,L);Y(1:M,L+1:2*L)]; Y2 = Y2'; y2 = Y2(:);
y = y1+y2; y = y'; y = removetrailzeros(y);
```

2. The radix-2 FFT implementation for high-speed block convolution:

```
function [y] = hsolpadd(x,h)
% High-Speed Overlap-Add method of block convolution
```

```
% ----------------------------------------------------
% [y] = hsolpadd(x,h)
% y = output sequence (real-valued)
% x = input sequence (real-valued)
% h = impulse response (real-valued)
%

Lx = length(x); L = length(h); N = 2^ceil(log2(2*L-1));
H = fft(h,N);
%
M = ceil(Lx/L);              % Number of blocks
x = [x, zeros(1,M*L-Lx)]; % append (M*N-Lx) zeros
Y = zeros(M,N);              % Initialize Y matrix
%
% convolution with succesive blocks
for m = 0:M-1
    xm = [x(m*L+1:(m+1)*L),zeros(1,N-L)];
    Y(m+1,:) = real(ifft(fft(xm,N).*H,N));
end
%
% Overlap and Add
Y = [Y,zeros(M,1)]; Y = [Y;zeros(1,N+1)];
Y1 = Y(:,1:L); Y1 = Y1'; y1 = Y1(:);
Y2 = [zeros(1,L);Y(1:M,L+1:2*L)]; Y2 = Y2'; y2 = Y2(:);
y = y1+y2; y = y'; y = removetrailzeros(y);
```

3. Verification using the following two sequences

$$x(n) = \cos\left(\pi n/500\right) \mathcal{R}_{4000}(n), \quad h(n) = \{1, -1, 1, -1\}$$

```
n = 0:4000-1; x = cos(pi*n/500); h = [1,-1,1,-1];
y1 = ovrlpadd(x,h,7);
y2 = hsolpadd(x,h);
y3 = conv(x,h);
e1 = max(abs(y1-y3))
e2 = max(abs(y1-y2(1:end-1)))

e1 =
  2.7756e-017
e2 =
  3.6342e-016
```

P5.33 Given the sequences $x_1(n)$ and $x_2(n)$ shown below:

$$x_1(n) = \{2, 1, 1, 2\}, \quad x_2(n) = \{1, -1, -1, 1\}$$

1. Circular convolutions $x_1(n) \textcircled{N} x_2(n)$:

   $N = 4 \; : \; x_1(n) \textcircled{4} x_2(n) = \{0, 0, 0, 0\}$

   $N = 7 \; : \; x_1(n) \textcircled{7} x_2(n) = \{2, -1, 2, 0, -2, 1, -2\}$

   $N = 8 \; : \; x_1(n) \textcircled{8} x_2(n) = \{2, -1, 2, 0, -2, 1, -2, 0\}$

2. The linear convolution: $x_1(n) * x_2(n) = \{2, -1, 2, 0, -2, 1, -2\}$.

3. From the results of the above two parts, the minimum value of N to make the circular convolution equal to the linear convolution is 7.

4. If we make N equal to the length of the linear convolution which is equal to the length of $x_1(n)$ plus the length of $x_2(n)$ minus one, then the desired result can be achieved. In this case then $N = 4 + 4 - 1 = 7$, as expected.

P5.34 Let

$$x(n) = \begin{cases} A\cos(2\pi\ell n/N), & 0 \le n \le N-1 \\ 0, & \text{elsewhere} \end{cases} = A\cos(2\pi\ell n/N)\mathcal{R}_N(n)$$

where ℓ is an integer. Notice that $x(n)$ contains *exactly* ℓ periods (or cycles) of the cosine waveform in N samples. This is a windowed cosine sequence containing *no leakage*.

1. Consider the DFT $X(k)$ of $x(n)$ which is given by

$$
\begin{aligned}
X(k) &= \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{N-1} A\cos\left(\frac{2\pi\ell n}{N}\right) e^{-j\frac{2\pi}{N}kn}, \quad 0 \le k \le N-1 \\
&= \frac{A}{2}\sum_{n=0}^{N-1}\left\{ e^{j\frac{2\pi}{N}\ell n} + e^{-j\frac{2\pi}{N}\ell n}\right\} e^{-j\frac{2\pi}{N}kn}, \quad 0 \le k \le N-1 \\
&= \frac{A}{2}\sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}(k-\ell)n} + \frac{A}{2}\sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}(k+\ell)n}, \quad 0 \le k \le N-1 \\
&= \frac{AN}{2}\delta(k-\ell) + \frac{AN}{2}\delta(k-N+\ell); \quad 0 \le k \le (N-1), \quad 0 < \ell < N
\end{aligned}
$$

   which is a real-valued sequence.

2. If $\ell = 0$, then the DFT $X(k)$ is given by

$$
\begin{aligned}
X(k) &= \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{N-1} A\, e^{-j\frac{2\pi}{N}kn}, \quad 0 \le k \le N-1 \\
&= AN\delta(k); \quad 0 \le k \le (N-1)
\end{aligned}
$$

3. If $\ell < 0$ or $\ell > N$, then we must replace it by $((\ell))_N$ in the result of part 1., i.e.

$$X(k) = \frac{AN}{2}\delta[k - ((\ell))_N] + \frac{AN}{2}\delta[k - N + ((\ell))_N]; \quad 0 \le k \le (N-1)$$

4. Verification of the results of parts 1., 2., and 3. above using MATLAB and the following sequences:

(a) $x_1(n) = 3\cos(0.04\pi n)\mathcal{R}_{200}(n)$:

```
N = 200; n = 0:N-1; x1 = 3*cos(0.04*pi*n); l = 4;
k = 0:N-1; X1 = real(fft(x1,N));
Hf_1 = figure('Units','inches','position',[1,1,6,4],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P5.34.4(a)');
subplot(2,1,1); H_s1 = stem(n,x1,'g','filled'); set(H_s1,'markersize',1);
title('Sequence: {\itx}_1({\itn})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-4,4]); xlabel('\itn');
subplot(2,1,2); H_s2 = stem(n,X1,'r','filled'); set(H_s2,'markersize',2);
title('DFT: {\itX}_1({\itk})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-10,310]); xlabel('\itk');
set(gca,'xtick',[0,l,N-l],'ytick',[0,300])
```

The sequence $x_1(n)$ and its DFT $X_1(k)$ are shown in Figure 5.24.
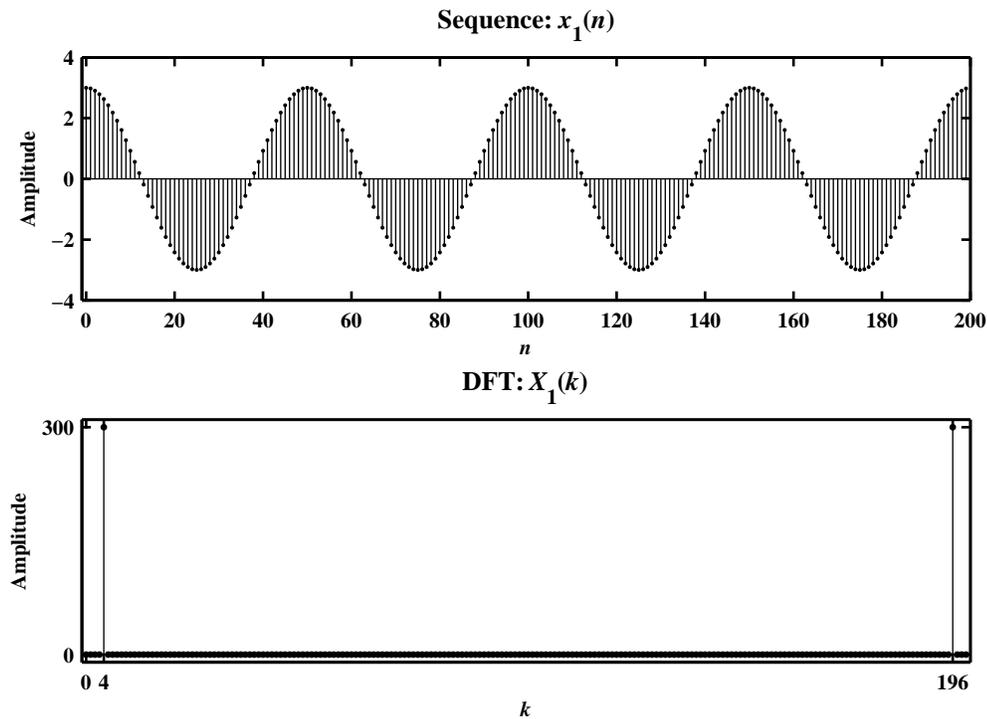
Figure 5.24: The signal $x_1(n)$ and its DFT $X_1(k)$ in Problem P5.34.4(a)

(b) $x_2(n) = 5\mathcal{R}_{50}(n)$:

```
N = 50; n = 0:N-1; x2 = 5*cos(0*pi*n); l = 0;
k = 0:N-1; X2 = real(fft(x2,N));
Hf_2 = figure('Units','inches','position',[1,1,6,4],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_2,'NumberTitle','off','Name','P5.34.4(b)');
subplot(2,1,1); H_s1 = stem(n,x2,'g','filled'); set(H_s1,'markersize',2);
title('Sequence: {\itx}_2({\itn})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-1,6]); xlabel('\itn');
subplot(2,1,2); H_s2 = stem(n,X2,'r','filled'); set(H_s2,'markersize',2);
title('DFT: {\itX}_2({\itk})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-10,260]); xlabel('\itk');
set(gca,'xtick',[0,N-1],'ytick',[0,250])
```

The sequence $x_2(n)$ and its DFT $X_2(k)$ are shown in Figure 5.25.
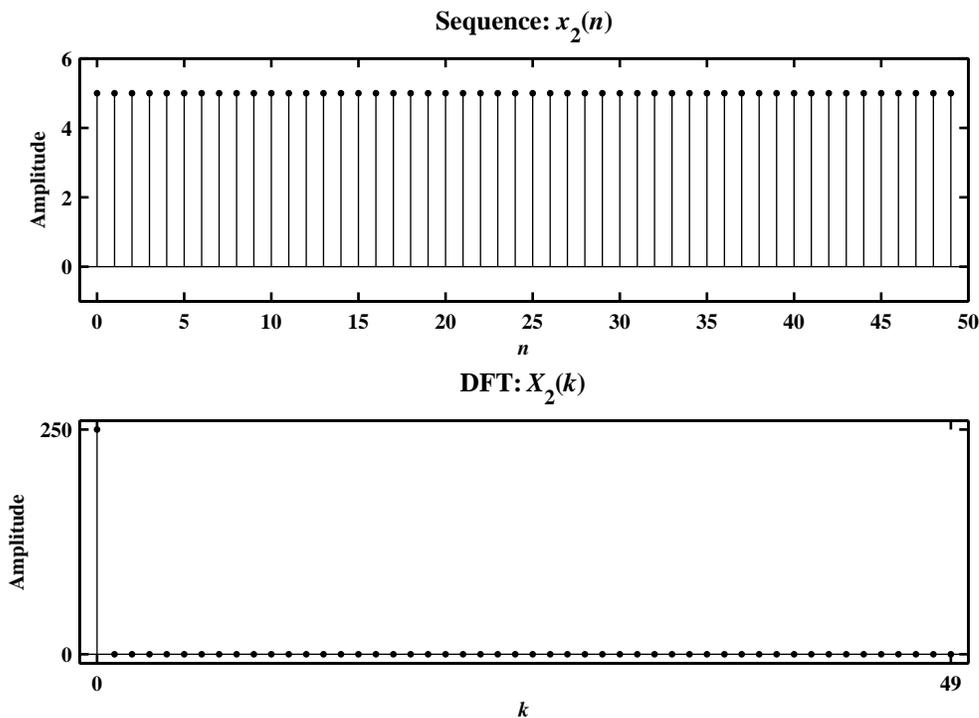
Figure 5.25: The signal $x_2(n)$ and its DFT $X_2(k)$ in Problem P5.34.4(b)

(c) $x_3(n) = [1 + 2\cos(0.5\pi n) + \cos(\pi n)]\mathcal{R}_{100}(n)$:

```
N = 100; n = 0:N-1; x3 = 1+2*cos(0.5*pi*n)+cos(pi*n); l1 = 0; l2 = 25; l3 = 50;
k = 0:N-1; X3 = real(fft(x3,N));
Hf_3 = figure('Units','inches','position',[1,1,6,4],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_3,'NumberTitle','off','Name','P5.34.4(c)');
subplot(2,1,1); H_s1 = stem(n,x3,'g','filled'); set(H_s1,'markersize',2);
title('Sequence: {\itx}_3({\itn})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-1,5]); xlabel('\itn');
subplot(2,1,2); H_s2 = stem(n,X3,'r','filled'); set(H_s2,'markersize',2);
title('DFT: {\itX}_3({\itk})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-10,110]); xlabel('\itk');
set(gca,'xtick',[l1,l2,l3,N-l2,N-1],'ytick',[0,100])
```

The sequence $x_3(n)$ and its DFT $X_3(k)$ are shown in Figure 5.26.
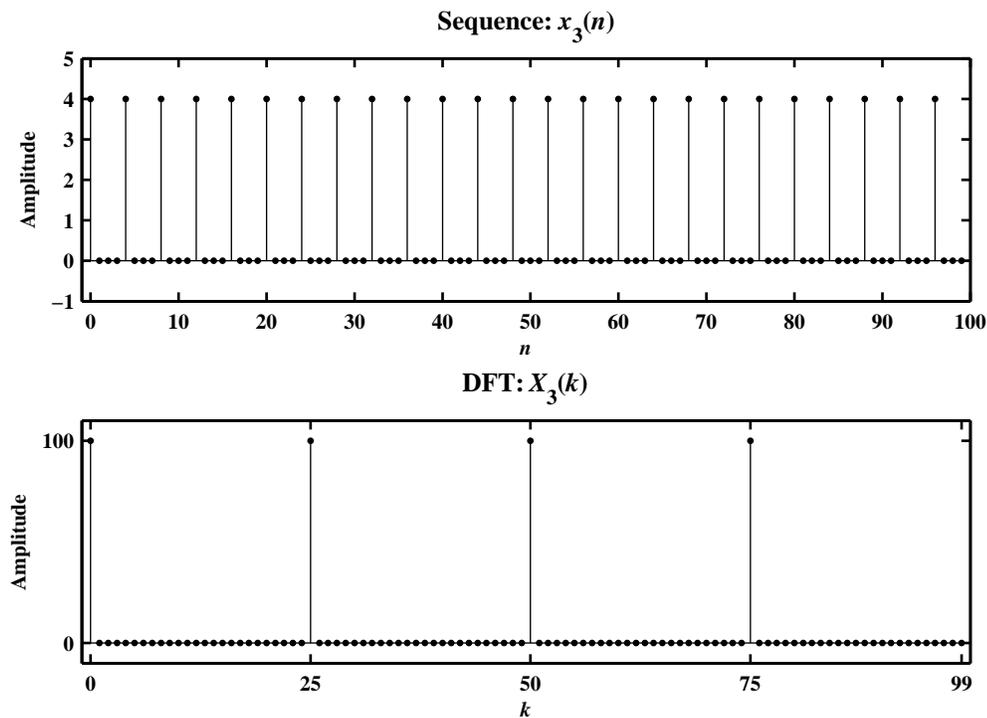
Figure 5.26: The signal $x_3(n)$ and its DFT $X_3(k)$ in Problem P5.34.4(c)

(d) $x_4(n) = \cos(25\pi n/16)\mathcal{R}_{64}(n)$:

```
N = 64; n = 0:N-1; x4 = cos(25*pi*n/16); l = 50;
k = 0:N-1; X4 = real(fft(x4,N));
Hf_4 = figure('Units','inches','position',[1,1,6,4],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_4,'NumberTitle','off','Name','P5.34.4(d)');
subplot(2,1,1); H_s1 = stem(n,x4,'g','filled'); set(H_s1,'markersize',2);
title('Sequence: {\itx}_4({\itn})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-1.1,1.1]); xlabel('\itn');
subplot(2,1,2); H_s2 = stem(n,X4,'r','filled'); set(H_s2,'markersize',2);
title('DFT: {\itX}_4({\itk})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-5,35]); xlabel('\itk');
set(gca,'xtick',[0,N-l,l,N-1],'ytick',[0,32])
```

The sequence $x_4(n)$ and its DFT $X_4(k)$ are shown in Figure 5.27.
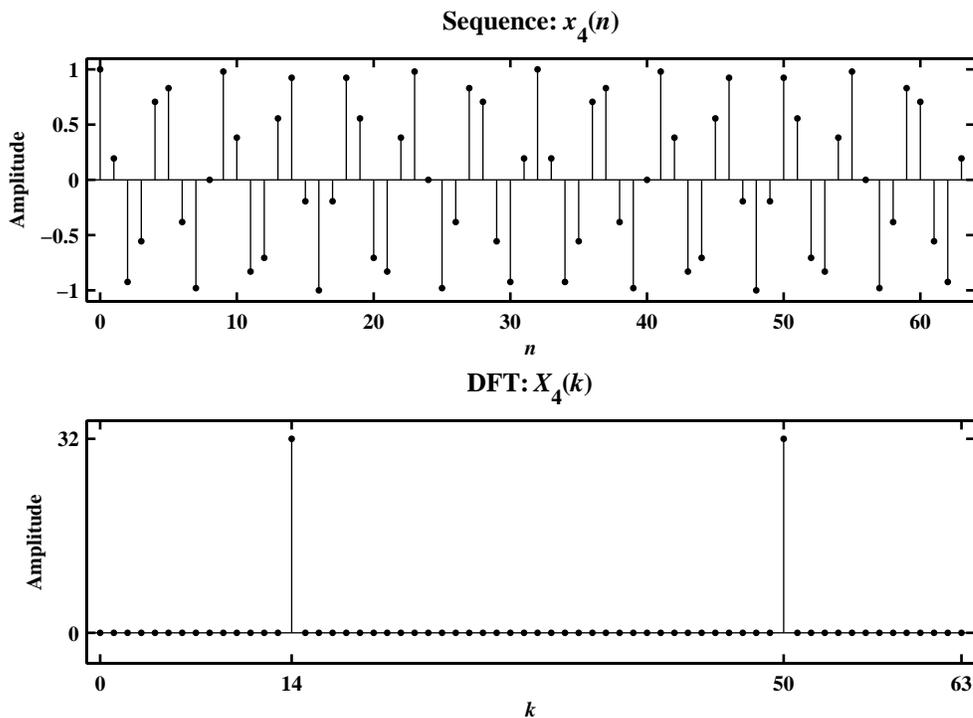
Figure 5.27: The signal $x_4(n)$ and its DFT $X_4(k)$ in Problem P5.34.4(d)

(e) $x_5(n) = [\cos(0.1\pi n) - 3\cos(1.9\pi n)]\mathcal{R}_{40}(n)$:

```
N = 40; n = 0:N-1; x5 = 4*cos(0.1*pi*n)-3*cos(1.9*pi*n); l1 = 2; l2 = 38;
k = 0:N-1; X5 = real(fft(x5,N));
Hf_5 = figure('Units','inches','position',[1,1,6,4],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_5,'NumberTitle','off','Name','P5.34.4(e)');
subplot(2,1,1); H_s1 = stem(n,x5,'g','filled'); set(H_s1,'markersize',2);
title('Sequence: {\itx}_5({\itn})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-1.1,1.1]); xlabel('\itn');
subplot(2,1,2); H_s2 = stem(n,X5,'r','filled'); set(H_s2,'markersize',2);
title('DFT: {\itX}_5({\itk})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-5,25]); xlabel('\itk');
set(gca,'xtick',[0,l1,l2,N],'ytick',[0,20])
```

The sequence $x_5(n)$ and its DFT $X_5(k)$ are shown in Figure 5.28.
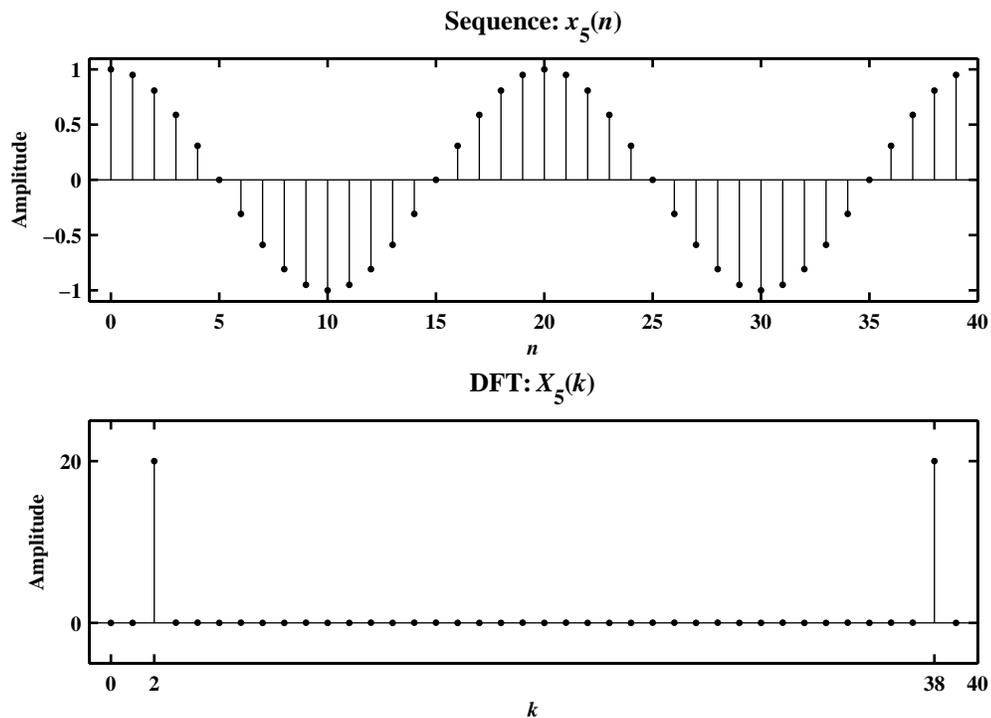
Figure 5.28: The sample plot of various signals in Problem P5.34.4(e)

P5.35 Let $x(n) = A\cos(\omega_0 n)\mathcal{R}_N(n)$, where ω_0 is a real number.

1. Consider

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\left(\frac{2\pi}{N}\right)nk} = A\sum_{n=0}^{N-1}\cos(\omega_0 n)\left\{\cos\left(\frac{2\pi}{N}nk\right) - j\sin\left(\frac{2\pi}{N}nk\right)\right\}$$

$$X_R(k) + jX_I(k) = A\sum_{n=0}^{N-1}\cos(\omega_0 n)\cos\left(\frac{2\pi}{N}nk\right) - jA\sum_{n=0}^{N-1}\cos(\omega_0 n)\sin\left(\frac{2\pi}{N}nk\right)$$

Hence

$$X_R(k) = A\sum_{n=0}^{N-1}\cos(\omega_0 n)\cos\left(\frac{2\pi}{N}nk\right) \tag{5.5}$$

$$X_I(k) = -A\sum_{n=0}^{N-1}\cos(\omega_0 n)\sin\left(\frac{2\pi}{N}nk\right) \tag{5.6}$$

Consider the real-part in (5.5),

$$X_R(k) = A\sum_{n=0}^{N-1}\cos(\omega_0 n)\cos\left(\frac{2\pi}{N}nk\right) = \frac{A}{2}\sum_{n=0}^{N-1}\left\{\cos\left(\omega_0 n - \frac{2\pi}{N}nk\right) + \cos\left(\omega_0 n + \frac{2\pi}{N}nk\right)\right\}$$

$$= \frac{A}{2}\sum_{n=0}^{N-1}\left\{\cos\left(2\pi f_0 n - \frac{2\pi}{N}nk\right) + \cos\left(2\pi f_0 n + \frac{2\pi}{N}nk\right)\right\} \qquad [\because \omega_0 = 2\pi f_0]$$

$$= \frac{A}{2}\sum_{n=0}^{N-1}\left\{\cos\left[\frac{2\pi}{N}(f_0 N - k)n\right] + \cos\left[\frac{2\pi}{N}(f_0 N + k)n\right]\right\}$$

$$= \frac{A}{2}\sum_{n=0}^{N-1}\left\{\cos\left[\frac{2\pi}{N}(k - f_0 N)n\right] + \cos\left[\frac{2\pi}{N}\{k - (N - f_0 N)\}n\right]\right\}, \quad 0 \le k < N \tag{5.7}$$

To reduce the sum-of-cosine terms in (5.7), consider

$$\sum_{n=0}^{N-1}\cos\left(\frac{2\pi}{N}vn\right) = \frac{1}{2}\sum_{n=0}^{N-1}e^{j\left(\frac{2\pi}{N}\right)vn} + \frac{1}{2}\sum_{n=0}^{N-1}e^{-j\left(\frac{2\pi}{N}\right)vn} = \frac{1}{2}\left(\frac{1 - e^{j2\pi v}}{1 - e^{j\frac{2\pi}{N}v}}\right) + \frac{1}{2}\left(\frac{1 - e^{-j2\pi v}}{1 - e^{-j\frac{2\pi}{N}v}}\right)$$

$$= \frac{1}{2}e^{-j\pi v\left(\frac{N-1}{N}\right)}\frac{\sin(\pi v)}{\sin(\pi v/N)} + \frac{1}{2}e^{j\pi n\left(\frac{N-1}{N}\right)}\frac{\sin(\pi v)}{\sin(\pi v/N)}$$

$$= \cos\left\{\frac{\pi v(N-1)}{N}\right\}\frac{\sin(\pi n)}{\sin(\pi v/N)} \tag{5.8}$$

Now substituting (5.8) in the first term of (5.7) with $v = (k - f_0 N)$ and in the second term of (5.7) with $v = (k - [N - f_0 N])$, we obtain the desired result

$$X_R(k) = \frac{A}{2}\cos\left\{\frac{\pi(N-1)}{N}(k - f_0 N)\right\}\frac{\sin[\pi(f_0 N - k)]}{\sin[\frac{\pi}{N}(f_0 N - k)]}$$

$$+ \frac{A}{2}\cos\left\{\frac{\pi(N-1)}{N}(k - [N - f_0 N])\right\}\frac{\sin\{\pi(k - [N - f_0 N])\}}{\sin[\frac{\pi}{N}(f_0 N - k)]} \tag{5.9}$$

Similarly, we can show that

$$
\begin{aligned}
X_I(k) = &-\frac{A}{2} \sin\left\{\tfrac{\pi(N-1)}{N}(k - f_0 N)\right\} \frac{\sin[\pi(f_0 N - k)]}{\sin[\frac{\pi}{N}(f_0 N - k)]} \\
&- \frac{A}{2} \sin\left\{\tfrac{\pi(N-1)}{N}(k - [N - f_0 N])\right\} \frac{\sin\{\pi(k - [N - f_0 N])\}}{\sin[\frac{\pi}{N}(f_0 N - k)]}
\end{aligned}
\tag{5.10}
$$

2. The above result implies that the original frequency ω_0 of the cosine waveform has *leaked* into other frequencies that form the harmonics of the time-limited sequence and hence it is called the leakage property of cosines. It is a natural result due to the fact that bandlimited periodic cosines are sampled over noninteger periods. Due to this fact, the periodic extension of $x(n)$ does not result in a continuation of the cosine waveform but has a jump at every N interval. This jump results in the leakage of one frequency into the abducent frequencies and hence the result of the Problem P5.34.1 do not apply.

3. Verification of the leakage property using $x(n) = \cos(5\pi n/99)\, \mathcal{R}_{200}(n)$: The sequence $x(n)$, the real-part of its DFT $X_R(k)$, and the imaginary part of its DFT $X_I(k)$ are shown in Figure 5.29.
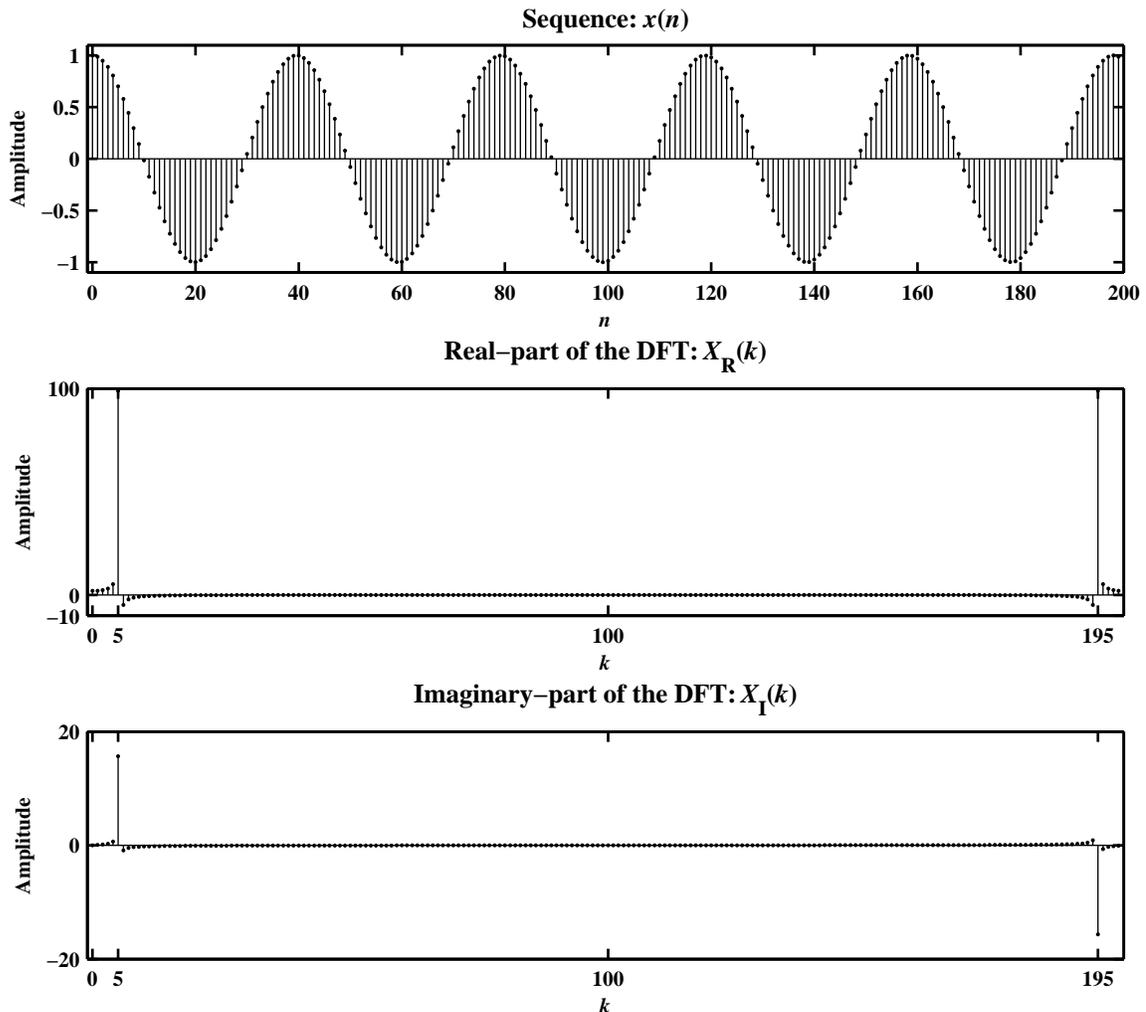
Figure 5.29: The leakage property of a cosine signal in Problem P5.35.3

P5.36 Let

$$x(n) = \begin{cases} A \sin(2\pi \ell n/N), & 0 \le n \le N - 1 \\ 0, & \text{elsewhere} \end{cases} = A \sin(2\pi \ell n/N)\mathcal{R}_N(n)$$

where ℓ is an integer. Notice that $x(n)$ contains *exactly* ℓ periods (or cycles) of the sine waveform in N samples. This is a windowed sine sequence containing *no leakage*.

1. Consider the DFT $X(k)$ of $x(n)$ which is given by

$$
\begin{aligned}
X(k) &= \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{N-1} A \sin\left(\frac{2\pi \ell n}{N}\right) e^{-j\frac{2\pi}{N}kn}, \quad 0 \le k \le N-1 \\
&= \frac{A}{j2} \sum_{n=0}^{N-1} \left\{ e^{j\frac{2\pi}{N}\ell n} - e^{-j\frac{2\pi}{N}\ell n} \right\} e^{-j\frac{2\pi}{N}kn}, \quad 0 \le k \le N-1 \\
&= \frac{A}{j2} \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}(k-\ell)n} - \frac{A}{2} \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}(k+\ell)n}, \quad 0 \le k \le N-1 \\
&= \frac{AN}{j2}\delta\,(k - \ell) - \frac{AN}{j2}\delta(k - N + \ell); \quad 0 \le k \le (N-1), \quad 0 < \ell < N
\end{aligned}
$$

which is a purely imaginary-valued sequence.

2. If $\ell = 0$, then the DFT $X(k)$ is given by

$$
\begin{aligned}
X(k) &= \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{N-1} 0\, e^{-j\frac{2\pi}{N}kn}, \quad 0 \le k \le (N-1) \\
&= 0; \quad 0 \le k \le (N-1)
\end{aligned}
$$

3. If $\ell < 0$ or $\ell > N$, then we must replace it by $((\ell))_N$ in the result of part 1., i.e.

$$X(k) = \frac{AN}{j2}\delta\,[k - ((\ell))_N] - \frac{AN}{j2}\delta\,[k - N + ((\ell))_N]; \quad 0 \le k \le (N-1)$$

4. Verification of the results of parts 1., 2., and 3. above using Matlab and the following sequences:

(a) $x_1(n) = 3\sin(0.04\pi n)\mathcal{R}_{200}(n)$:

```
N = 200; n = 0:N-1; x1 = 3*sin(0.04*pi*n); l = 4;
k = 0:N-1; X1 = imag(fft(x1,N));
Hf_1 = figure('Units','inches','position',[1,1,6,4],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P5.34.4(a)');
subplot(2,1,1); H_s1 = stem(n,x1,'g','filled'); set(H_s1,'markersize',1);
title('Sequence: {\itx}_1({\itn})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-4,4]); xlabel('\itn');
subplot(2,1,2); H_s2 = stem(n,X1,'r','filled'); set(H_s2,'markersize',2);
title('DFT: {\itX}_1({\itk})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-350,350]); xlabel('\itk');
set(gca,'xtick',[0,l,N-l],'ytick',[-300,0,300])
```

The sequence $x_1(n)$ and its DFT $X_1(k)$ are shown in Figure 5.30.
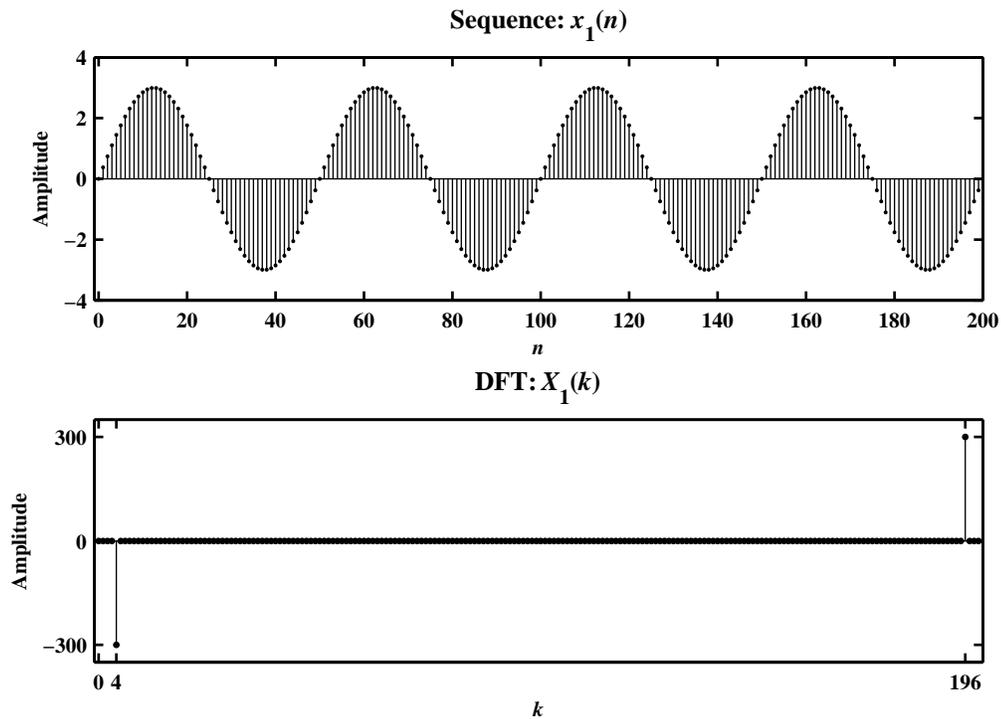
Figure 5.30: The signal $x_1(n)$ and its DFT $X_1(k)$ in Problem P5.36.6(a)

(b) $x_2(n) = 5\sin(10\pi n)\mathcal{R}_{50}(n)$:

```
N = 50; n = 0:N-1; x2 = 5*sin(10*pi*n); l = 0;
k = 0:N-1; X2 = imag(fft(x2,N));
Hf_2 = figure('Units','inches','position',[1,1,6,4],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_2,'NumberTitle','off','Name','P5.34.4(b)');
subplot(2,1,1); H_s1 = stem(n,x2,'g','filled'); set(H_s1,'markersize',2);
title('Sequence: {\itx}_2({\itn})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-1,1]); xlabel('\itn');
subplot(2,1,2); H_s2 = stem(n,X2,'r','filled'); set(H_s2,'markersize',2);
title('DFT: {\itX}_2({\itk})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-1,1]); xlabel('\itk');
set(gca,'xtick',[0,N-1],'ytick',[-1,0,1])
```

The sequence $x_1(n)$ and its DFT $X_1(k)$ are shown in Figure 5.31.
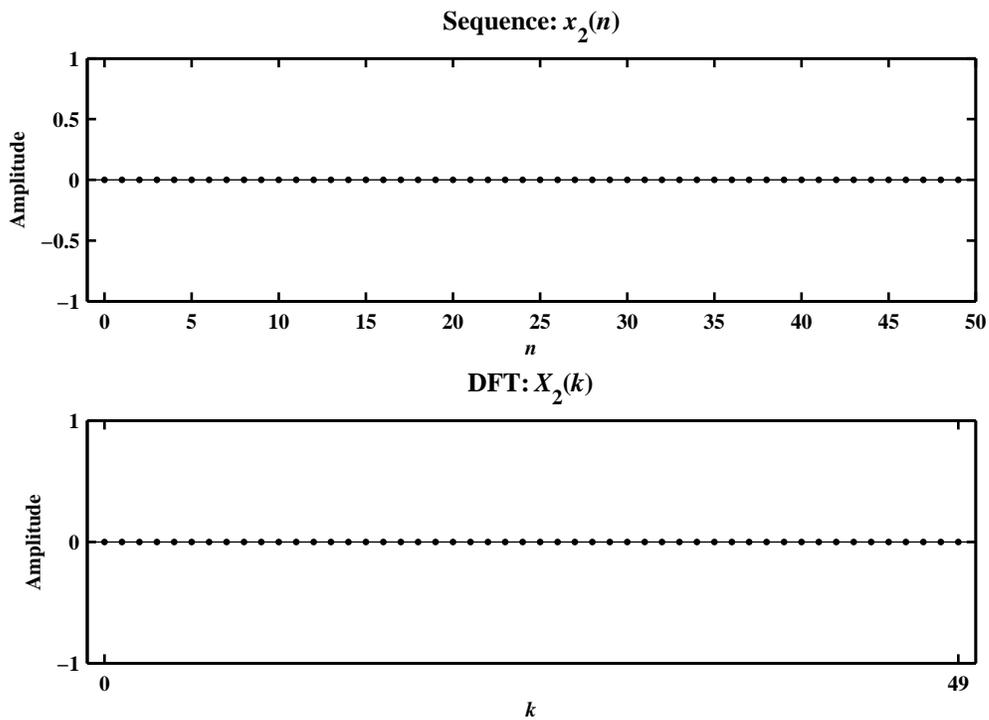
Figure 5.31: The signal $x_1(n)$ and its DFT $X_1(k)$ in Problem P5.34.6(b)

(c) $x_3(n) = [2\sin(0.5\pi n) + \sin(\pi n)]\mathcal{R}_{100}(n)$:

```
N = 100; n = 0:N-1; x3 = 2*sin(0.5*pi*n)+0*sin(pi*n); l1 = 0; l2 = 25; l3 = 50;
k = 0:N-1; X3 = imag(fft(x3,N));
Hf_3 = figure('Units','inches','position',[1,1,6,4],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_3,'NumberTitle','off','Name','P5.34.4(c)');
subplot(2,1,1); H_s1 = stem(n,x3,'g','filled'); set(H_s1,'markersize',2);
title('Sequence: {\itx}_3({\itn})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-3,3]); xlabel('\itn');
subplot(2,1,2); H_s2 = stem(n,X3,'r','filled'); set(H_s2,'markersize',2);
title('DFT: {\itX}_3({\itk})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-120,120]); xlabel('\itk');
set(gca,'xtick',[l1,l2,l3,N-l2,N-1],'ytick',[-100,0,100])
```

The sequence $x_1(n)$ and its DFT $X_1(k)$ are shown in Figure 5.32.
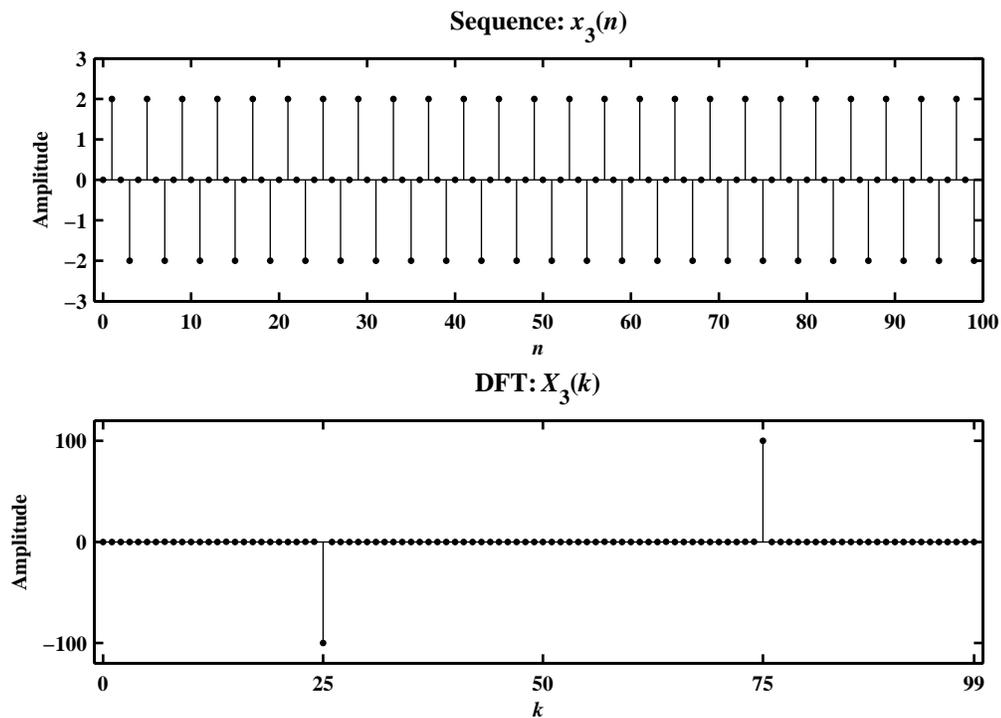
Figure 5.32: The signal $x_1(n)$ and its DFT $X_1(k)$ in Problem P5.34.6(c)

(d) $x_4(n) = \sin(25\pi n/16)\mathcal{R}_{64}(n)$:

```
N = 64; n = 0:N-1; x4 = sin(25*pi*n/16); l = 50;
k = 0:N-1; X4 = imag(fft(x4,N));
Hf_4 = figure('Units','inches','position',[1,1,6,4],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_4,'NumberTitle','off','Name','P5.34.4(d)');
subplot(2,1,1); H_s1 = stem(n,x4,'g','filled'); set(H_s1,'markersize',2);
title('Sequence: {\itx}_4({\itn})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-1.1,1.1]); xlabel('\itn');
subplot(2,1,2); H_s2 = stem(n,X4,'r','filled'); set(H_s2,'markersize',2);
title('DFT: {\itX}_4({\itk})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-40,40]); xlabel('\itk');
set(gca,'xtick',[0,N-l,l,N-1],'ytick',[-32,0,32])
```

The sequence $x_1(n)$ and its DFT $X_1(k)$ are shown in Figure 5.33.
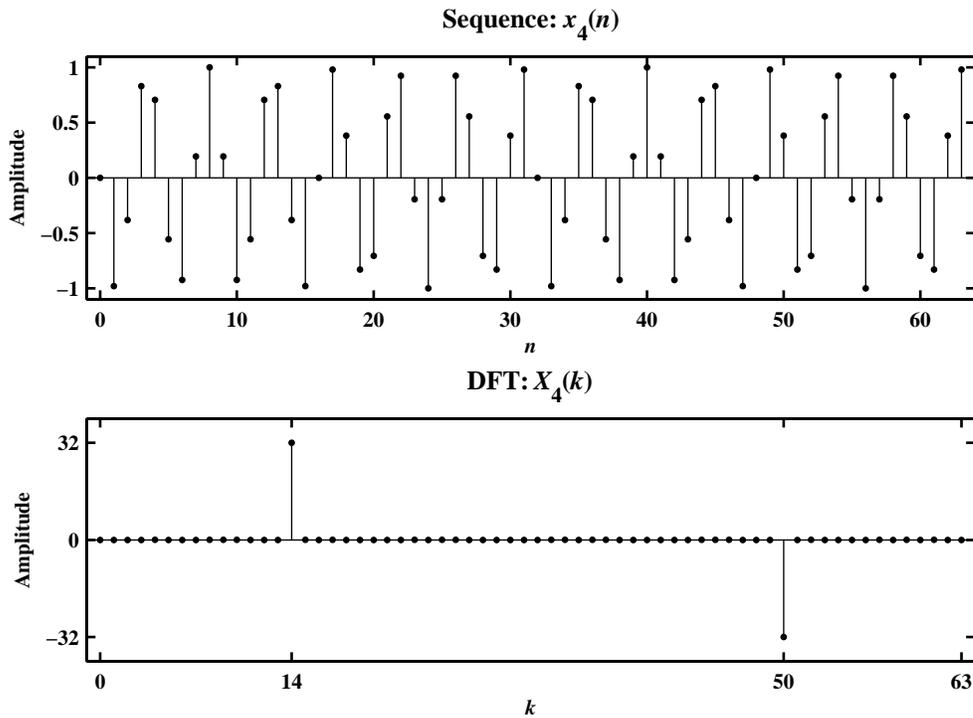
Figure 5.33: The signal $x_1(n)$ and its DFT $X_1(k)$ in Problem P5.34.6(d)

(e) $x_5(n) = [4\sin(0.1\pi n) - 3\sin(1.9\pi n)]\mathcal{R}_{20}(n)$:

```
N = 20; n = 0:N-1; x5 = 4*sin(0.1*pi*n)-3*sin(1.9*pi*n); l1 = 1; l2 = 19;
k = 0:N-1; X5 = imag(fft(x5,N));
Hf_5 = figure('Units','inches','position',[1,1,6,4],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_5,'NumberTitle','off','Name','P5.34.4(e)');
subplot(2,1,1); H_s1 = stem(n,x5,'g','filled'); set(H_s1,'markersize',2);
title('Sequence: {\itx}_5({\itn})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-10,10]); xlabel('\itn');
subplot(2,1,2); H_s2 = stem(n,X5,'r','filled'); set(H_s2,'markersize',2);
title('DFT: {\itX}_5({\itk})','fontsize',10);
ylabel('Amplitude'); axis([-1,N,-80,80]); xlabel('\itk');
set(gca,'xtick',[0,l1,l2,N],'ytick',[-70,0,70])
```

The sequence $x_1(n)$ and its DFT $X_1(k)$ are shown in Figure 5.34.
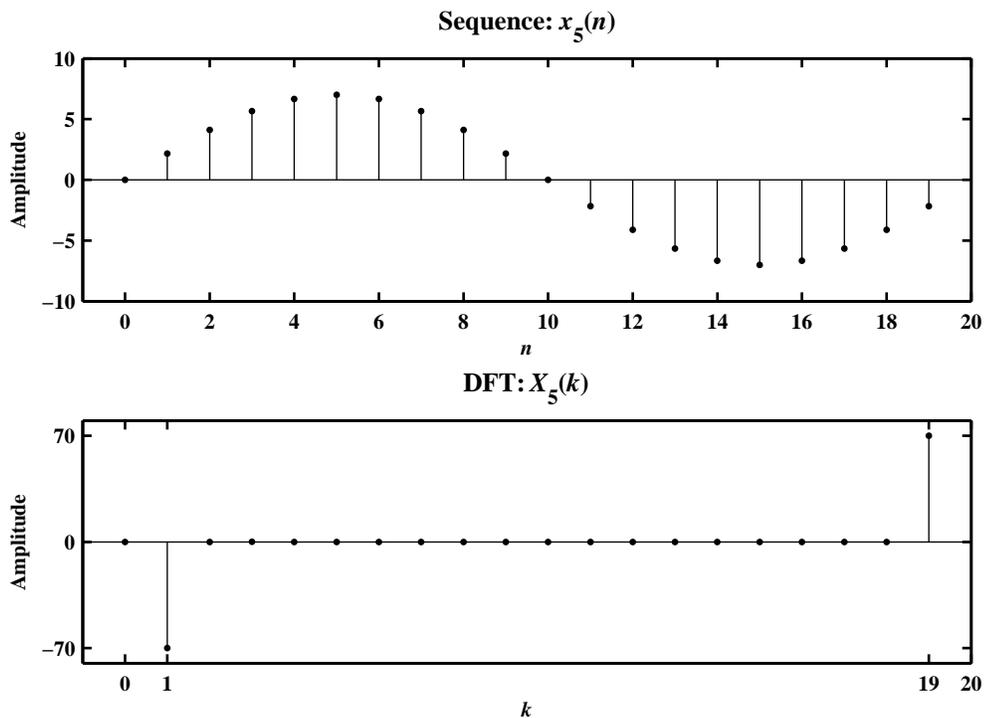
Figure 5.34: The signal $x_1(n)$ and its DFT $X_1(k)$ in Problem P5.34.6(e)

P5.37 Let $x(n) = A \sin(\omega_0 n) \mathcal{R}_N(n)$, where ω_0 is a real number.

1. Consider

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\left(\frac{2\pi}{N}\right)nk} = A \sum_{n=0}^{N-1} \sin(\omega_0 n) \left\{ \cos\left(\frac{2\pi}{N}nk\right) - j \sin\left(\frac{2\pi}{N}nk\right) \right\}$$

$$X_R(k) + j X_I(k) = A \sum_{n=0}^{N-1} \sin(\omega_0 n) \cos\left(\frac{2\pi}{N}nk\right) - j A \sum_{n=0}^{N-1} \sin(\omega_0 n) \sin\left(\frac{2\pi}{N}nk\right)$$

Hence

$$X_R(k) = A \sum_{n=0}^{N-1} \sin(\omega_0 n) \cos\left(\frac{2\pi}{N}nk\right) \tag{5.11}$$

$$X_I(k) = -A \sum_{n=0}^{N-1} \sin(\omega_0 n) \sin\left(\frac{2\pi}{N}nk\right) \tag{5.12}$$

Consider the real-part in (5.11),

$$X_R(k) = A \sum_{n=0}^{N-1} \sin(\omega_0 n) \cos\left(\frac{2\pi}{N}nk\right) = \frac{A}{2} \sum_{n=0}^{N-1} \left\{ \sin\left(\omega_0 n - \frac{2\pi}{N}nk\right) + \sin\left(\omega_0 n + \frac{2\pi}{N}nk\right) \right\}$$

$$= \frac{A}{2} \sum_{n=0}^{N-1} \left\{ \sin\left(2\pi f_0 n - \frac{2\pi}{N}nk\right) + \sin\left(2\pi f_0 n + \frac{2\pi}{N}nk\right) \right\} \qquad [\because \omega_0 = 2\pi f_0]$$

$$= \frac{A}{2} \sum_{n=0}^{N-1} \left\{ \sin\left[\frac{2\pi}{N}(f_0 N - k)n\right] + \sin\left[\frac{2\pi}{N}(f_0 N + k)n\right] \right\}$$

$$= \frac{A}{2} \sum_{n=0}^{N-1} \left\{ -\sin\left[\frac{2\pi}{N}(k - f_0 N)n\right] + \sin\left[\frac{2\pi}{N}\{k - (N - f_0 N)\}n\right] \right\}, \quad 0 \le k < N \tag{5.13}$$

To reduce the sum-of-sine terms in (5.13), consider

$$\sum_{n=0}^{N-1} \sin\left(\frac{2\pi}{N}vn\right) = \frac{1}{j2} \sum_{n=0}^{N-1} e^{j\left(\frac{2\pi}{N}\right)vn} - \frac{1}{j2} \sum_{n=0}^{N-1} e^{-j\left(\frac{2\pi}{N}\right)vn} = \frac{1}{j2}\left(\frac{1 - e^{j2\pi v}}{1 - e^{j\frac{2\pi}{N}v}}\right) - \frac{1}{j2}\left(\frac{1 - e^{-j2\pi v}}{1 - e^{-j\frac{2\pi}{N}v}}\right)$$

$$= \frac{1}{j2} e^{j\pi v\left(\frac{N-1}{N}\right)} \frac{\sin(\pi v)}{\sin(\pi v/N)} - \frac{1}{j2} e^{-j\pi n\left(\frac{N-1}{N}\right)} \frac{\sin(\pi v)}{\sin(\pi v/N)}$$

$$= \sin\left\{\frac{\pi v(N-1)}{N}\right\} \frac{\sin(\pi v)}{\sin(\pi v/N)} \tag{5.14}$$

Now substituting (5.14) in the first term of (5.13) with $v = (k - f_0 N)$ and in the second term of (5.13) with $v = (k - [N - f_0 N])$, we obtain the desired result

$$X_R(k) = -\frac{A}{2} \sin\left\{\frac{\pi(N-1)}{N}(k - f_0 N)\right\} \frac{\sin[\pi(f_0 N - k)]}{\sin[\frac{\pi}{N}(f_0 N - k)]}$$

$$+ \frac{A}{2} \sin\left\{\frac{\pi(N-1)}{N}(k - [N - f_0 N])\right\} \frac{\sin\{\pi(k - [N - f_0 N])\}}{\sin\{\frac{\pi}{N}(f_0 N - k)\}} \tag{5.15}$$

Similarly, we can show that

$$
\begin{aligned}
X_I(k) = & -\frac{A}{2} \sin\left\{\frac{\pi(N-1)}{N}(k - f_0 N)\right\} \frac{\sin[\pi(f_0 N - k)]}{\sin[\frac{\pi}{N}(f_0 N - k)]} \\
& + \frac{A}{2} \sin\left\{\frac{\pi(N-1)}{N}(k - [N - f_0 N])\right\} \frac{\sin\{\pi(k - [N - f_0 N])\}}{\sin[\frac{\pi}{N}(f_0 N - k)]}
\end{aligned}
\tag{5.16}
$$

2. The above result is the leakage property of sines. It implies that the original frequency ω_0 of the sine waveform has *leaked* into other frequencies that form the harmonics of the time-limited sequence. It is a natural result due to the fact that bandlimited periodic sines are sampled over noninteger periods. Due to this fact, the periodic extension of $x(n)$ does not result in a continuation of the sine waveform but has a jump at every N interval. This jump results in the leakage of one frequency into the abducent frequencies and hence the result of the Problem P5.36.1 do not apply.

3. Verification of the leakage property using $x(n) = \sin(5\pi n/99)\mathcal{R}_{100}(n)$: The sequence $x(n)$, the real-part of its DFT $X_R(k)$, and the imaginary part of its DFT $X_I(k)$ are shown in Figure 5.35.
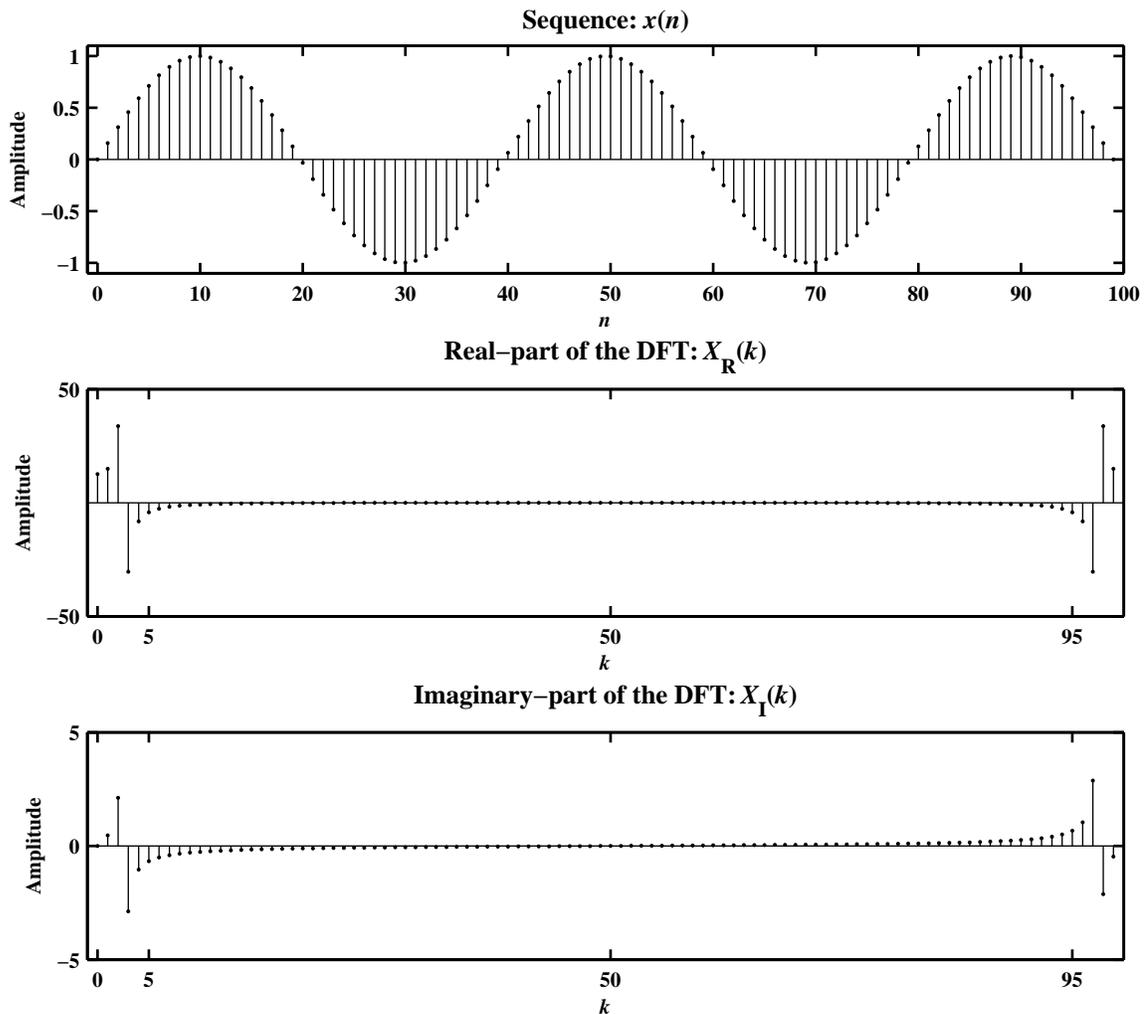
Figure 5.35: The leakage property of a sine signal in Problem P5.37.3

P5.38 An analog signal $x_a(t) = 2\sin(4\pi t) + 5\cos(8\pi t)$ is sampled at $t = 0.01n$ for $n = 0, 1, \ldots, N-1$ to obtain an N-point sequence $x(n)$. An N-point DFT is used to obtain an estimate of the magnitude spectrum of $x_a(t)$.

1. Out of the given three values, $N = 50$ provides complete cycles of both the sine and the cosine components. Thus $N = 50$ provides the most accurate estimate as shown in Figure 5.36.
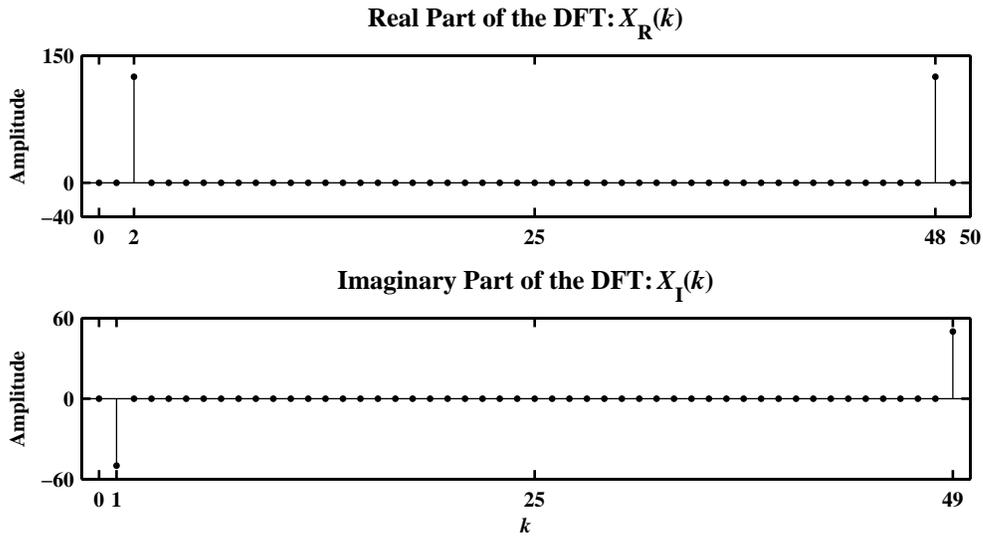
Figure 5.36: The accurate spectrum of the signal in Problem P5.38.1

2. Out of the given three values, $N = 99$ provides almost complete cycles of both the sine and the cosine components. Thus $N = 99$ provides the least amount of leakage as shown in Figure 5.37.
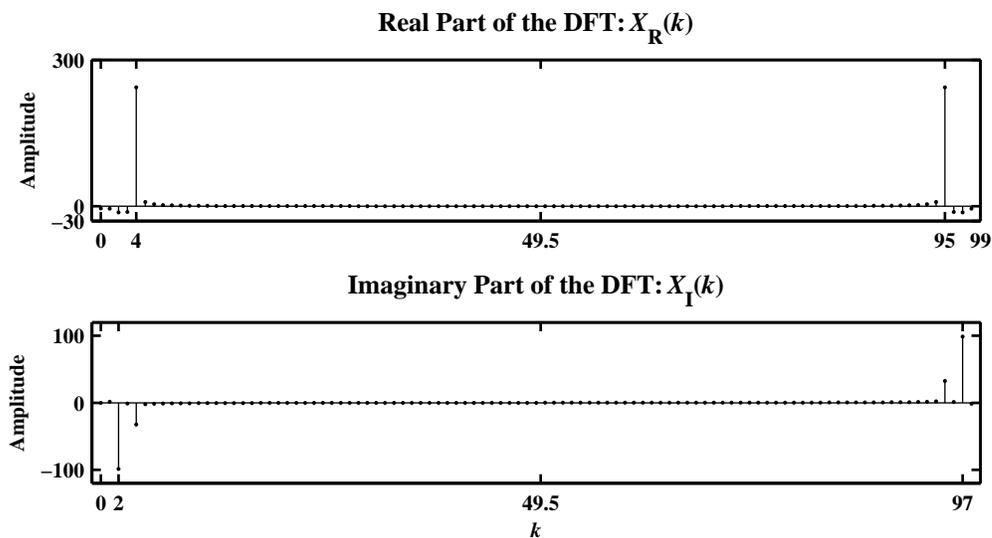
Figure 5.37: The least amount of leakage in the spectrum of the signal in Problem P5.38.2

P5.39 Using (5.49), determine and draw the signal flow graph for the $N = 8$ point, radix-2 decimation-in-frequency FFT algorithm. Using this flow graph, determine the DFT of the sequence

$$x(n) = \cos(\pi n/2), \quad 0 \le n \le 7.$$

P5.40 Using (5.49), determine and draw the signal flow graph for the $N = 16$ point, radix-4 decimation-in-time FFT algorithm. Using this flow graph determine the DFT of the sequence

$$x(n) = \cos(\pi n/2), \quad 0 \le n \le 15.$$

P5.41 Let $x(n)$ be a uniformly distributed random number between $[-1, 1]$ for $0 \le n \le 10^6$. Let

$$h(n) = \sin(0.4\pi n), \quad 0 \le n \le 100$$

1. Using the conv function, determine the output sequence $y(n) = x(n) * h(n)$.

2. Consider the overlap-and-save method of block convolution along with the FFT algorithm to implement high-speed block convolution. Using this approach, determine $y(n)$ with FFT sizes of 1024, 2048, and 4096.

3. Compare the above approaches in terms of the convolution results and their execution times.

Chapter 6

Digital Filter Structures

P6.1 Direct form I block diagrams.

    1. $y(n) = x(n) + 2x(n-1) + 3x(n-2)$:
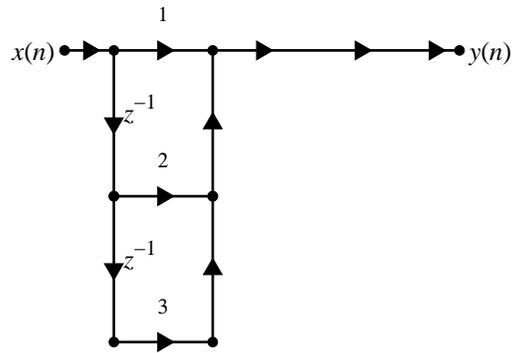
    2. $H(z) = \dfrac{1}{1 - 1.7z^{-1} + 1.53z^{-2} - 0.648z^{-3}}$:
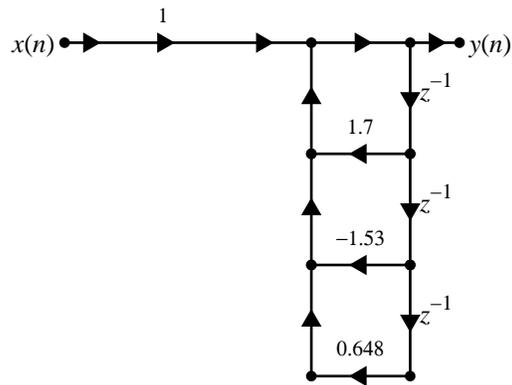
3. $y(n) = 1.7y(n-1) - 1.36y(n-2) + 0.576y(n-3) + x(n)$:
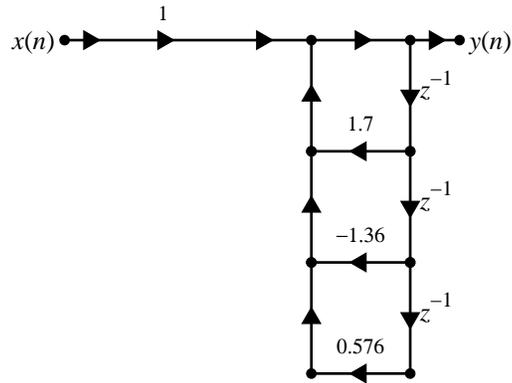
4. $y(n) = 1.6y(n-1) + 0.64y(n-2) + x(n) + 2x(n-1) + x(n-2)$:
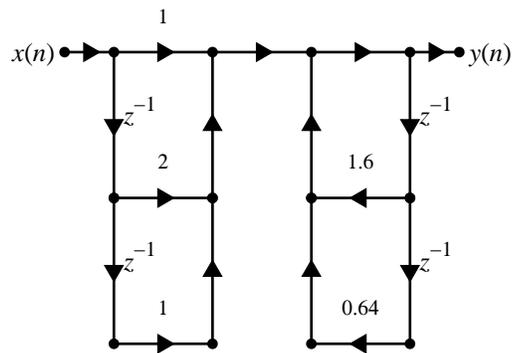
5. $H(z) = \dfrac{1 - 3z^{-1} + 3z^{-2} + z^{-3}}{1 + 0.2z^{-1} - 0.14z^{-2} + 0.44z^{-3}}$:
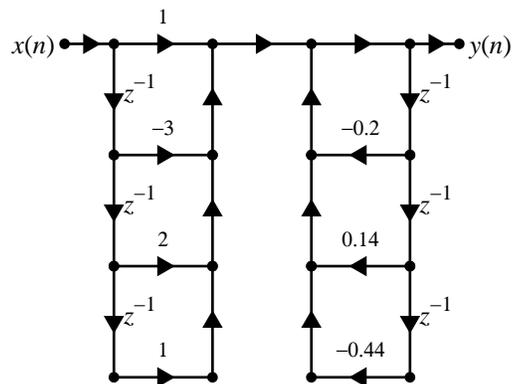
P6.2 Consider the following two block diagrams:

(i)   (ii)
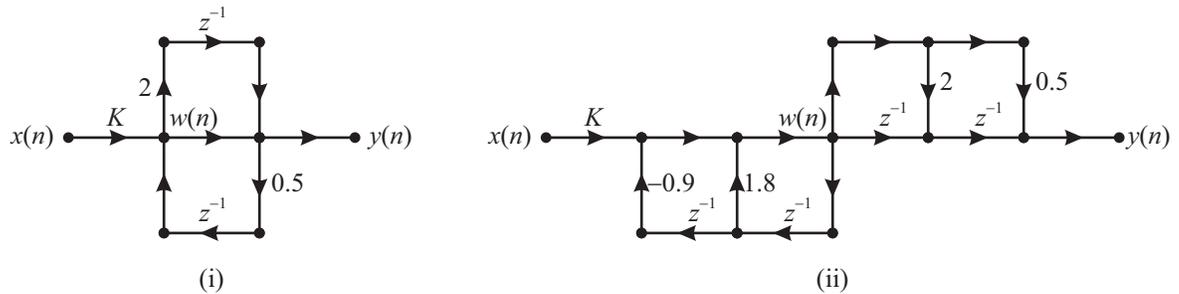
1. The system function $H(z) = Y(z)/X(z)$:

   (i) Referring to signal nodes in the above figure (i):

   $$w(n) = K\,x(n) + \tfrac{1}{2}y(n-1)$$
   $$y(n) = w(n) + 2w(n-1) = K\,x(n) + \tfrac{1}{2}y(n-1) + 2K\,x(n-1) + 2\tfrac{1}{2}y(n-2)$$
   $$= K\,x(n) + 2K\,x(n-1) + \tfrac{1}{2}y(n-1) + y(n-2)$$

   Hence
   $$H(z) = K\frac{1 + 2z^{-1}}{1 - \tfrac{1}{2}z^{-1} - z^{-2}} \tag{6.1}$$

   (ii) Referring to signal nodes in the above figure (ii):

   $$w(n) = K\,x(n) + 1.8w(n-1) - 0.9w(n-2) \Rightarrow W(z) = K\frac{X(z)}{1 - 0.8z^{-1} + 0.2z^{-2}}$$
   $$y(n) = 0.5w(n) + 2w(n-1) + w(n-2) \Rightarrow Y(z) = \left(0.5 + 2z^{-1} + z^{-2}\right)W(z)$$

   Hence
   $$H(z) = \frac{Y(z)}{X(z)} = K\frac{0.5 + 2z^{-1} + z^{-2}}{1 - 0.8z^{-1} + 0.2z^{-2}} \tag{6.2}$$

2. Canonical structure:

   (i) The given structure is canonical

   (ii) The given structure is not canonical. The canonical structure is
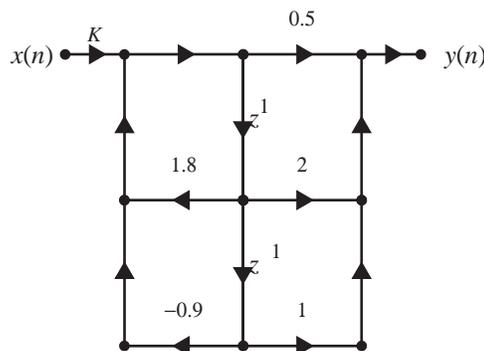
3. Value of K so that $H(e^{j0}) = 1$:

   (i) From (6.1), $H(1) = 1 = K\frac{1+2}{1-\frac{1}{2}-1} = -6K \Rightarrow K = -\frac{1}{6}$.

(ii) From (6.2), $H(1) = 1 = K\frac{0.5+2+1}{1-1.8+0.9} = 35K \Rightarrow K = -\frac{1}{35}$.

P6.3 Consider the LTI system described by

$$y(n) = a\,y(n-1) + b\,x(n) \tag{6.3}$$

1. Block diagram of the above system with input node $x(n)$ and output node $y(n)$ is shown below.

(a)
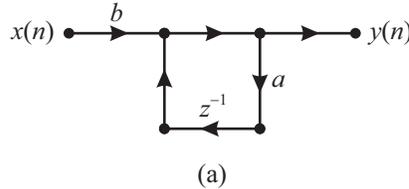
2. *Transposed* block diagram: The block diagram due to steps (i) and (ii) is shown below.

(i) and (ii)
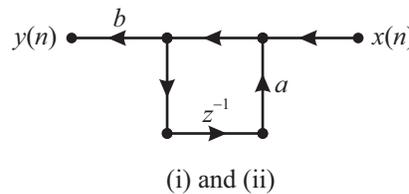
The final block diagram after redrawing is shown below.

(b)
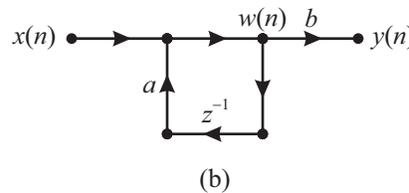
3. Difference equation representation of the transposed structure in part 2 above: Referring to the nodes in the transposed structure above

$$w(n) = x(n) + a\,w(n-1) \Rightarrow W(z) = \frac{1}{1-az^{-1}}\,X(z)$$

and

$$y(n) = b\,w(n) \Rightarrow Y(z) = b\,W(z)$$

or

$$Y(z)\left(1 - az^{-1}\right) = b\,X(z) \Rightarrow y(n) = b\,x(n) + a\,y(n-1)$$

which is the same equation as (6.3).

P6.4 Consider the LTI system given by

$$H(z) = \frac{1 - 2.818z^{-1} + 3.97z^{-2} - 2.8180z^{-3} + z^{-4}}{1 - 2.536z^{-1} + 3.215z^{-2} - 2.054z^{-3} + 0.6560z^{-4}} \tag{6.4}$$

1. The normal direct form I structure block diagram is shown below on the left.
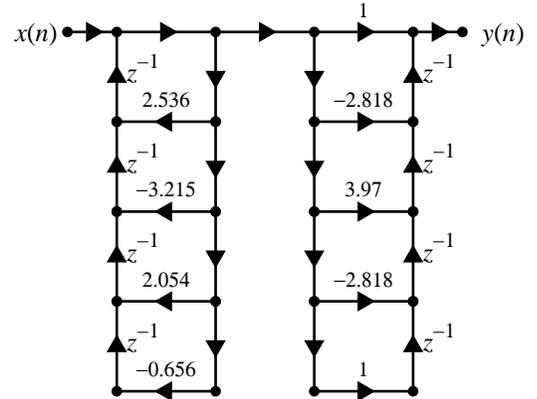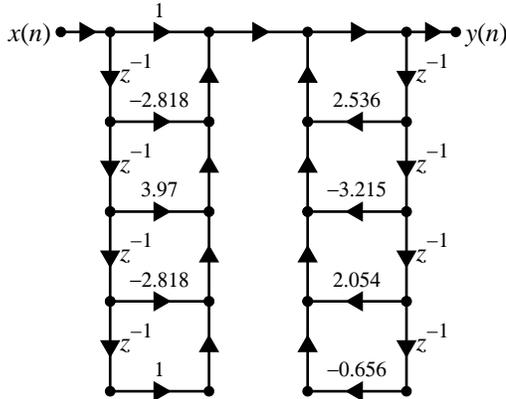
2. The transposed direct form I structure block diagram is shown above on the right.

3. The normal direct form II structure block diagram is shown below on the left.
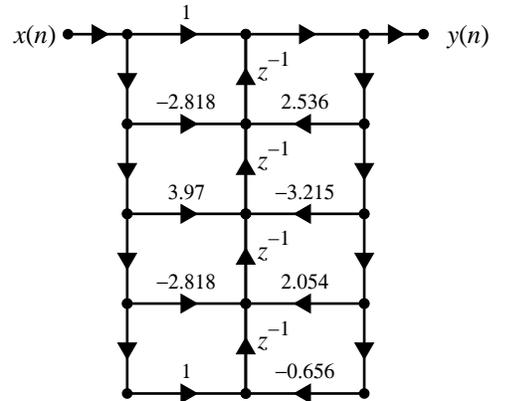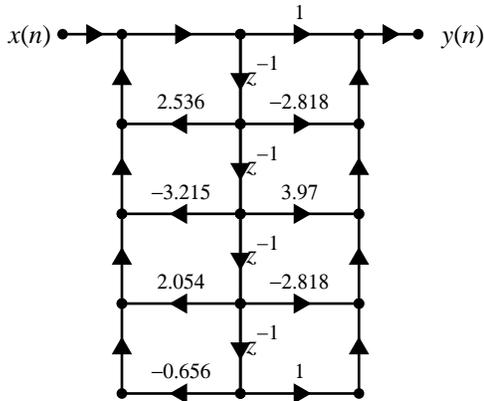
   Clearly it looks similar to that in part 2.

4. The transposed direct form II structure block diagram is shown above on the right. Clearly it looks similar to that in part 1.

P6.5 Consider the LTI system given by

$$H(z) = \frac{1 - 2.818z^{-1} + 3.97z^{-2} - 2.8180z^{-3} + z^{-4}}{1 - 2.536z^{-1} + 3.215z^{-2} - 2.054z^{-3} + 0.6560z^{-4}}$$

1. A cascade structure containing second-order normal direct from II sections: MATLAB script:

```
b = [1,-2.818,3.97,-2.818,1]; a = [1,-2.536,3.215,-2.054,0.656];
[b0,B,A] = dir2cas(b,a)
b0 =
    1
B =
    1.0000   -1.2854    1.0000
    1.0000   -1.5326    1.0000
A =
    1.0000   -1.1553    0.8099
    1.0000   -1.3807    0.8100
```
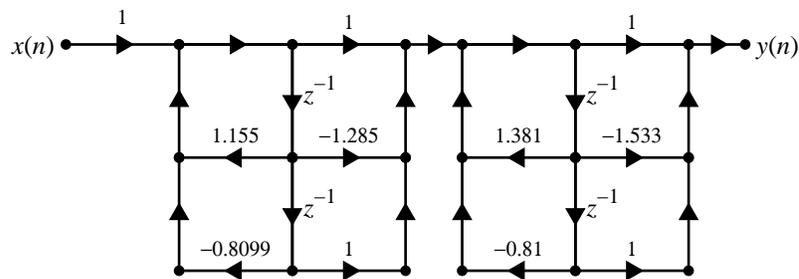

2. A cascade structure containing second-order transposed direct from II sections: MATLAB script:

```
b = [1,-2.818,3.97,-2.818,1]; a = [1,-2.536,3.215,-2.054,0.656];
[b0,B,A] = dir2cas(b,a)
b0 =
    1
B =
    1.0000   -1.2854    1.0000
    1.0000   -1.5326    1.0000
A =
    1.0000   -1.1553    0.8099
    1.0000   -1.3807    0.8100
```
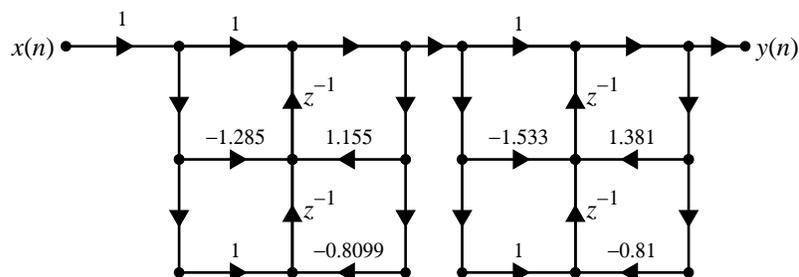
3. A parallel structure containing second-order normal direct from II sections:

MATLAB script:

```
b = [1,-2.818,3.97,-2.818,1];
a = [1,-2.536,3.215,-2.054,0.656];
[C,B,A] = dir2par(b,a)
C =
    1.5244
B =
   -0.2460    0.2644
   -0.2783    0.1222
A =
    1.0000   -1.1553    0.8099
    1.0000   -1.3807    0.8100
```
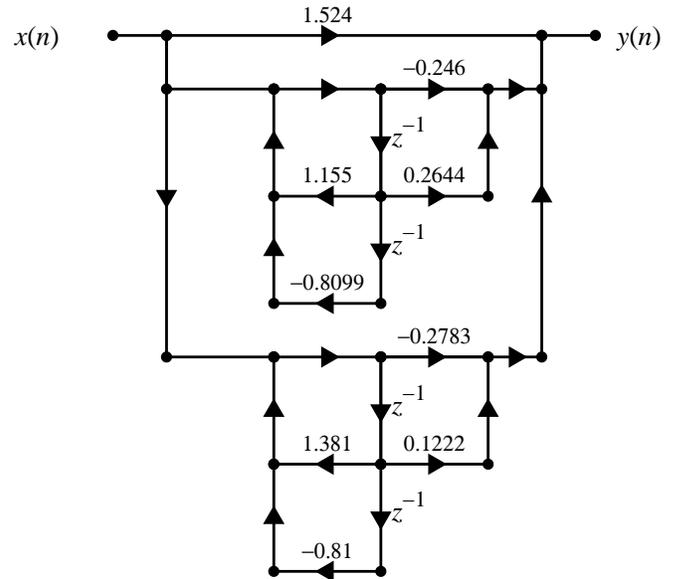

4. A parallel structure containing second-order transposed direct from II sections:

MATLAB script:

```
b = [1,-2.818,3.97,-2.818,1];
a = [1,-2.536,3.215,-2.054,0.656];
[C,B,A] = dir2par(b,a)
C =
    1.5244
B =
   -0.2460    0.2644
   -0.2783    0.1222
A =
    1.0000   -1.1553    0.8099
    1.0000   -1.3807    0.8100
```
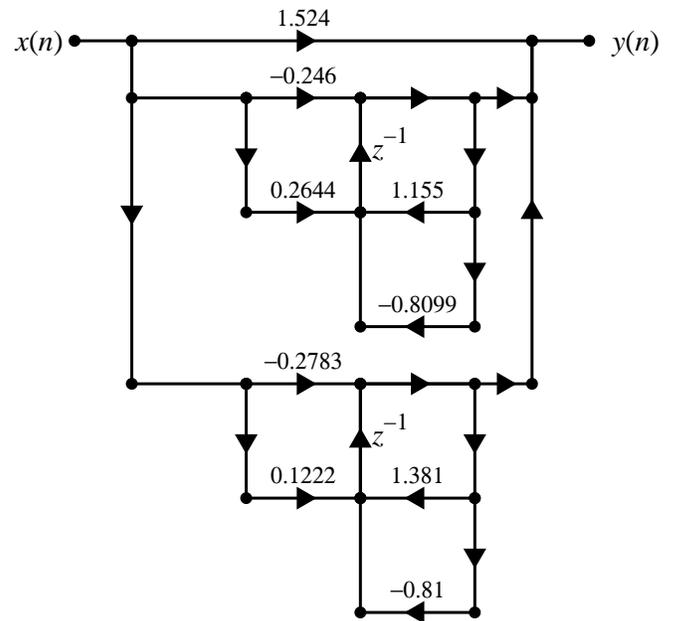
P6.6 A causal linear time-invariant system is described by

$$y(n) = \sum_{k=0}^{4} \cos(0.1\pi k)x(n-k) - \sum_{k=1}^{5}(0.8)^k \sin(0.1\pi k)y(n-k)$$

1. Normal direct form I:

   MATLAB script:

```
b = cos(0.1*pi*[0:4]); a = [1,((0.8).^[1:5]).*sin(0.1*pi*[1:5])];
```
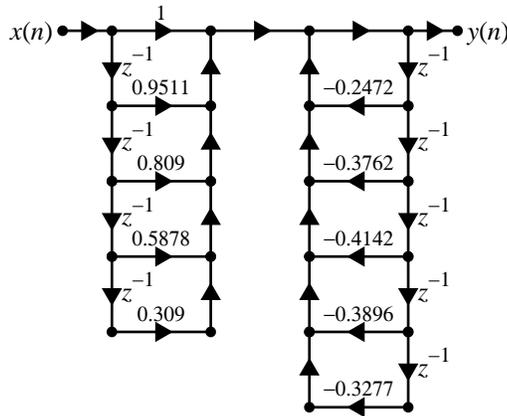

Response of the system to

$$x(n) = \left[1 + 2(-1)^n\right], \quad 0 \le n \le 50$$

MATLAB script:

```
n = 0:50; x = 1 + 2*(-1).^n; y = filter(b,a,x);
H_stem = stem(n,y,'g','filled'); set(H_stem,'markersize',3); axis([-1,51,-1,4]);
title('Output Sequence {\ity}({\itn})','fontname','times','fontweight','normal')
```
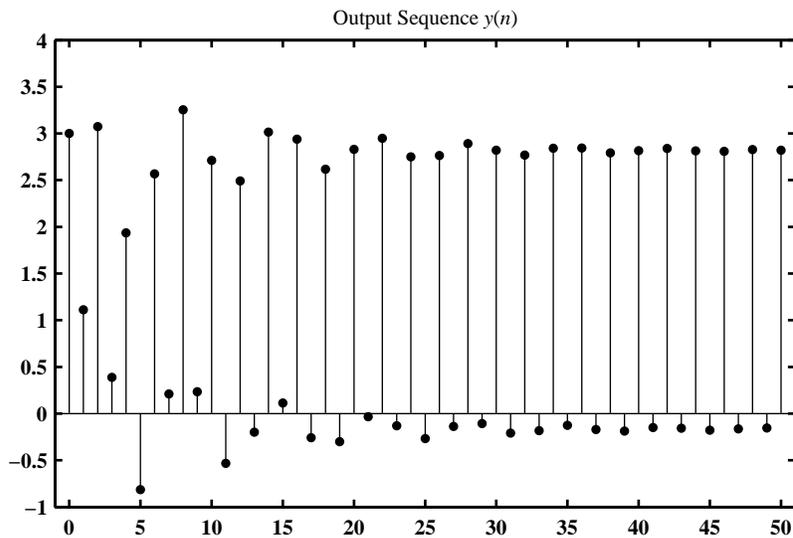
2. Transposed direct form II:

   MATLAB script:

   ```
   b = cos(0.1*pi*[0:4]); a = [1,((0.8).^[1:5]).*sin(0.1*pi*[1:5])];
   ```
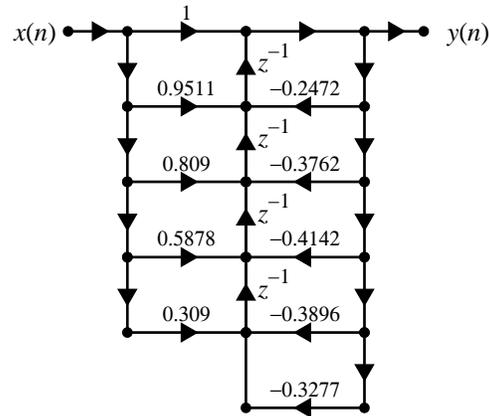

Response of the system to

$$x(n) = \left[1 + 2(-1)^n\right], \quad 0 \leq n \leq 50$$

MATLAB script:

```
n = 0:50; x = 1 + 2*(-1).^n; y = filter(b,a,x);
H_stem = stem(n,y,'g','filled'); set(H_stem,'markersize',3); axis([-1,51,-1,4]);
title('Output Sequence {\ity}({\itn})','fontname','times','fontweight','normal')
```
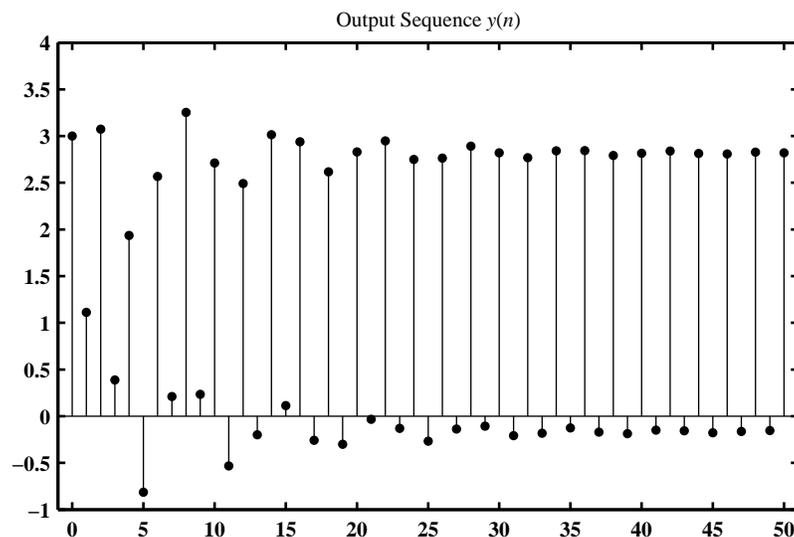
3. Cascade form containing second-order normal direct form II sections:

MATLAB script:

```
b = cos(0.1*pi*[0:4]); a = [1,((0.8).^[1:5]).*sin(0.1*pi*[1:5])];
[b0,B,A] = dir2cas(b,a)
b0 =
     1
B =
    1.0000    1.2047    0.5177
    1.0000   -0.2537    0.5969
    1.0000        0         0
A =
    1.0000    0.6158    0.5556
    1.0000   -1.0844    0.8238
    1.0000    0.7159        0
```
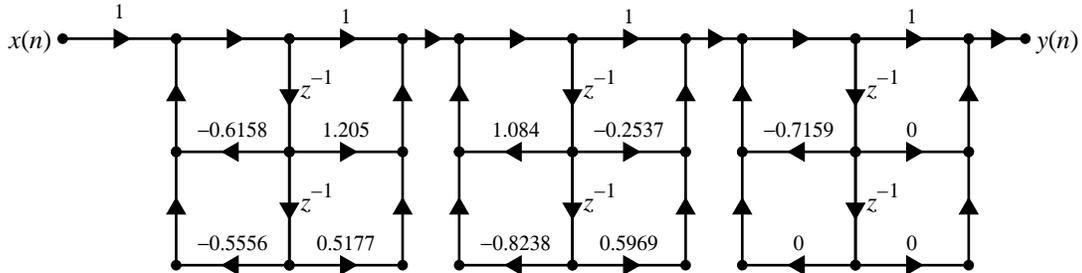

Response of the system to

$$x(n) = \left[1 + 2(-1)^n\right], \quad 0 \le n \le 50$$

MATLAB script:

```
n = 0:50; x = 1 + 2*(-1).^n; y = casfiltr(b0,B,A,x);
H_stem = stem(n,y,'g','filled'); set(H_stem,'markersize',3); axis([-1,51,-1,4]);
title('Output Sequence {\ity}({\itn})','fontname','times','fontweight','normal')
```


Output Sequence $y(n)$
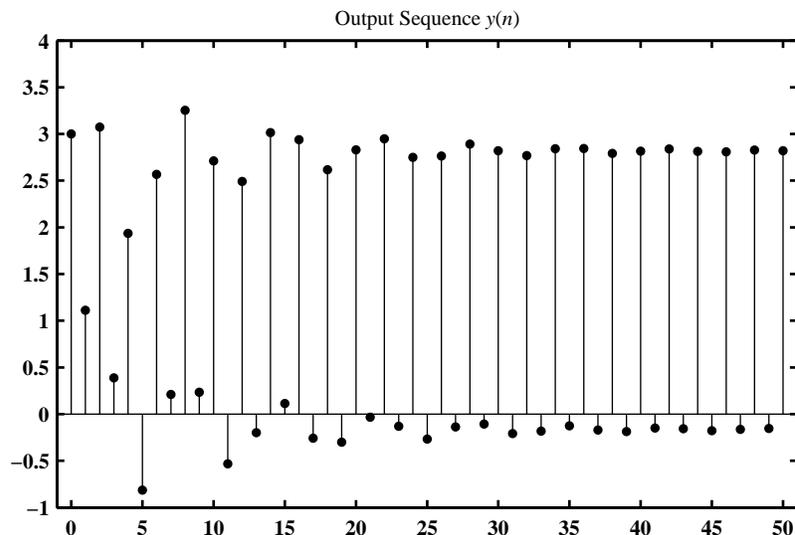
4. Parallel form containing second-order transposed direct form II sections:

MATLAB script:

```
b = cos(0.1*pi*[0:4]);
a = [1,((0.8).^[1:5]).*...
     sin(0.1*pi*[1:5])];
[C,B,A] = dir2par(b,a)
C =
       []
B =
    0.3221    0.2644
    0.5145    0.1969
    0.1634         0
A =
    1.0000    0.6158    0.5556
    1.0000   -1.0844    0.8238
    1.0000    0.7159         0
```

Response of the system to

$$x(n) = \left[1 + 2(-1)^n\right], \quad 0 \le n \le 50$$
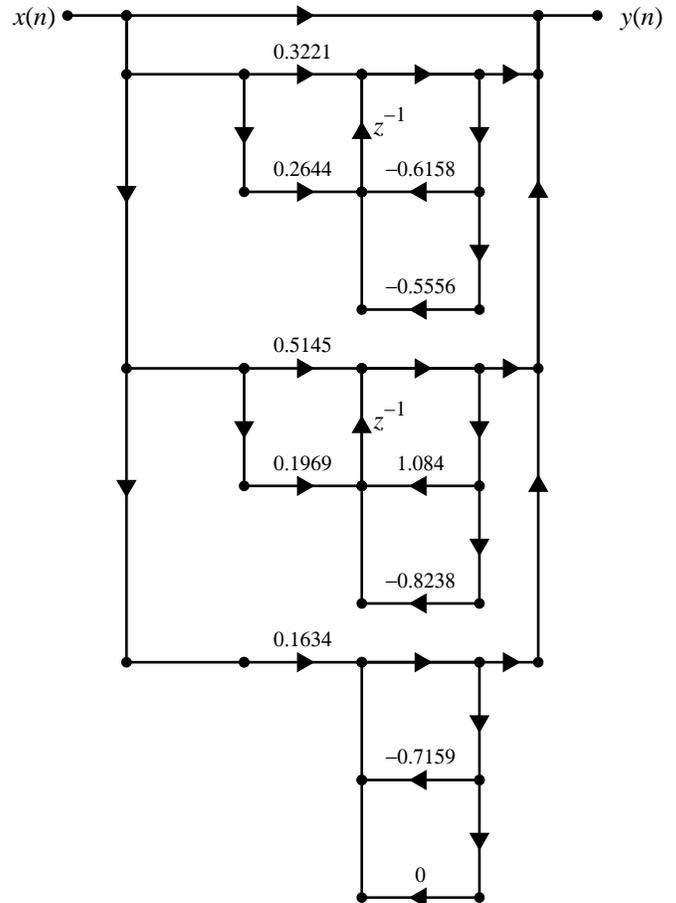
MATLAB script:

```
n = 0:50; x = 1 + 2*(-1).^n; y = parfiltr(C,B,A,x);
H_stem = stem(n,y,'g','filled'); set(H_stem,'markersize',3); axis([-1,51,-1,4]);
title('Output Sequence {\ity}({\itn})','fontname','times','fontweight','normal')
```

Output Sequence $y(n)$

5. Lattice-ladder form:

MATLAB script:

```
b = cos(0.1*pi*[0:4]); a = [1,((0.8).^[1:5]).*sin(0.1*pi*[1:5])];
[K,C] = dir2ladr(b,a)
K =
   -0.0361    0.2141    0.3176    0.3457    0.3277
C =
    0.5950    0.7722    0.7126    0.5464    0.3090         0
```


Response of the system to

$$x(n) = \left[1 + 2(-1)^n\right], \quad 0 \le n \le 50$$

MATLAB script:

```
n = 0:50; x = 1 + 2*(-1).^n; [y] = ladrfilt(K,C,x);
H_stem = stem(n,y,'g','filled'); set(H_stem,'markersize',3); axis([-1,51,-1,4]);
title('Output Sequence {\ity}({\itn})','fontname','times','fontweight','normal')
```


Output Sequence $y(n)$
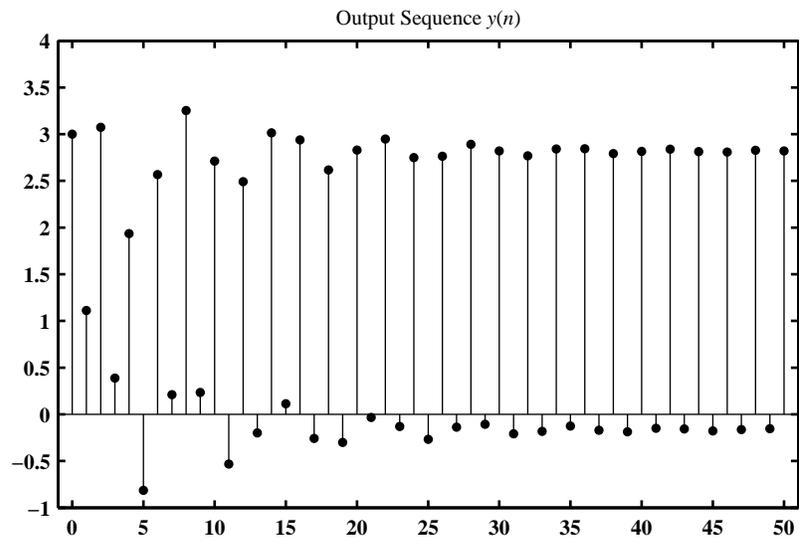
P6.7 An IIR filter is described by the following system function

$$H(z) = 2\left(\frac{1 + 0z^{-1} + z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}\right) + \left(\frac{2 - z^{-1}}{1 - 0.75z^{-1}}\right) + \left(\frac{1 + 2z^{-1} + z^{-2}}{1 + 0.81z^{-2}}\right)$$

1. Transposed direct form I:

   MATLAB script:

```
%% (a) Transposed Direct form-I
% Given H(z)
b1 = [2,0,1]; a1 = [1,-0.8,0.64]; [R1,p1,k1] = residuez(b1,a1);
b2 = [2,-1]; a2 = [1,-0.75]; [R2,p2,k2] = residuez(b2,a2);
b3 = [1,2,1]; a3 = [1,0,0.81]; [R3,p3,k3] = residuez(b3,a3);
R = [R1;R2;R3]; p = [p1;p2;p3]; k = k1+k2+k3; [b,a] = residuez(R,p,k)
b =
    5.0000   -3.6500    5.4600   -4.2610    2.7748   -1.6059
a =
    1.0000   -1.5500    2.0500   -1.7355    1.0044   -0.3888
```

2. Normal direct form II:

   MATLAB script:

```
%% (a) Transposed Direct form-I
% Given H(z)
b1 = [2,0,1]; a1 = [1,-0.8,0.64]; [R1,p1,k1] = residuez(b1,a1);
b2 = [2,-1]; a2 = [1,-0.75]; [R2,p2,k2] = residuez(b2,a2);
b3 = [1,2,1]; a3 = [1,0,0.81]; [R3,p3,k3] = residuez(b3,a3);
R = [R1;R2;R3]; p = [p1;p2;p3]; k = k1+k2+k3; [b,a] = residuez(R,p,k)
b =
    5.0000   -3.6500    5.4600   -4.2610    2.7748   -1.6059
a =
    1.0000   -1.5500    2.0500   -1.7355    1.0044   -0.3888
```
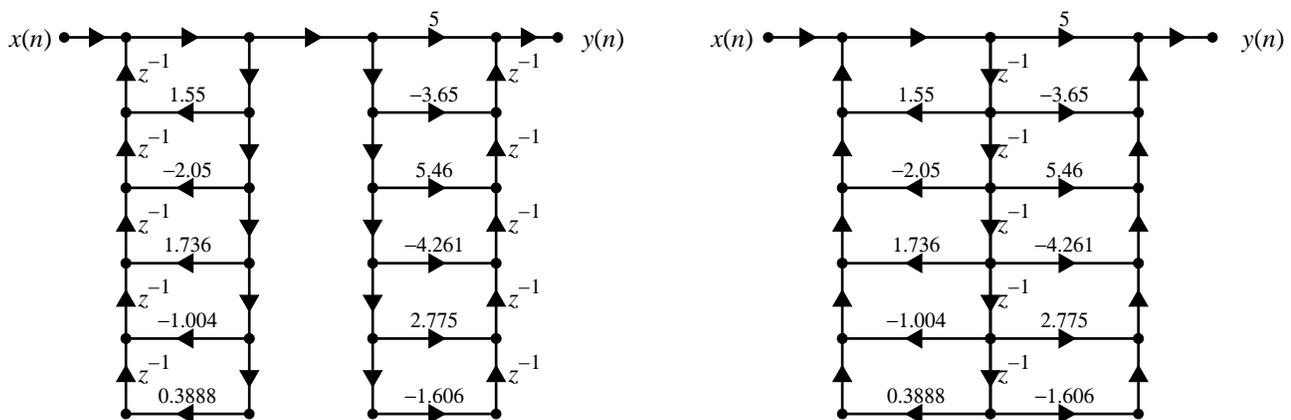
3. Cascade form containing transposed second-order direct form II sections:

MATLAB script:

```
%% (c) Transposed Cascade form
% Given H(z)
b1 = [2,0,1]; a1 = [1,-0.8,0.64]; [R1,p1,k1] = residuez(b1,a1);
b2 = [2,-1]; a2 = [1,-0.75]; [R2,p2,k2] = residuez(b2,a2);
b3 = [1,2,1]; a3 = [1,0,0.81]; [R3,p3,k3] = residuez(b3,a3);
R = [R1;R2;R3]; p = [p1;p2;p3]; k = k1+k2+k3; [b,a] = residuez(R,p,k)

b =
    5.0000   -3.6500    5.4600   -4.2610    2.7748   -1.6059
a =
    1.0000   -1.5500    2.0500   -1.7355    1.0044   -0.3888
[b0,B,A] = dir2cas(b,a)
b0 =
     5
B =
    1.0000    0.5424    0.7734
    1.0000   -0.5876    0.6064
    1.0000   -0.6848         0
A =
    1.0000   -0.0000    0.8100
    1.0000   -0.8000    0.6400
    1.0000   -0.7500         0
```
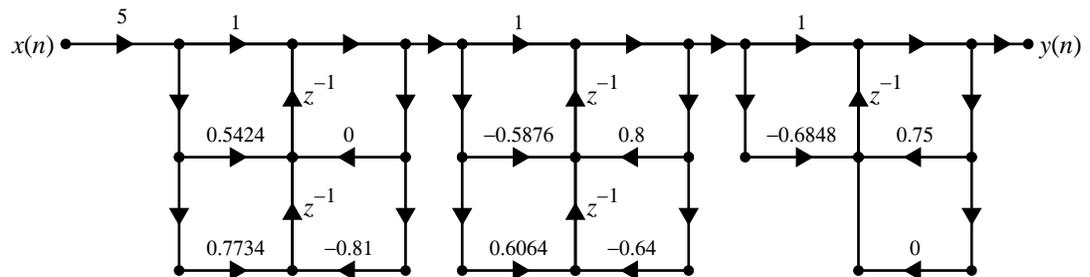
4. Parallel form containing normal second-order direct form II sections:

MATLAB script:

```
%% (d) Normal Parallel form
% Given H(z)
b1 = [2,0,1]; a1 = [1,-0.8,0.64]; [R1,p1,k1] = residuez(b1,a1);
b2 = [2,-1]; a2 = [1,-0.75]; [R2,p2,k2] = residuez(b2,a2);
b3 = [1,2,1]; a3 = [1,0,0.81]; [R3,p3,k3] = residuez(b3,a3);
R = [R1;R2;R3]; p = [p1;p2;p3]; k = k1+k2+k3; [b,a] = residuez(R,p,k)

b =
    5.0000   -3.6500    5.4600   -4.2610    2.7748   -1.6059
a =
    1.0000   -1.5500    2.0500   -1.7355    1.0044   -0.3888
```

```
[C,B,A] = dir2par(b,a),

C =
    4.1304
B =
   -0.2346    2.0000
    0.4375    1.2500
    0.6667         0
A =
    1.0000   -0.0000    0.8100
    1.0000   -0.8000    0.6400
    1.0000   -0.7500         0
```
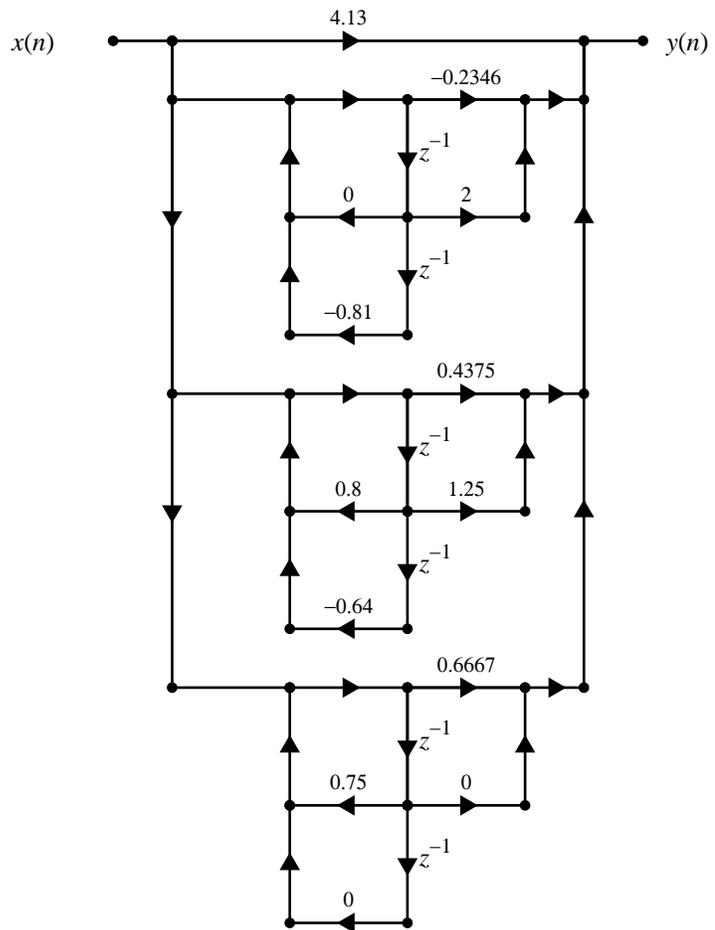
5. Lattice-ladder form:

MATLAB script:

```
%% Lattice-Ladder form
% Given H(z)
b1 = [2,0,1]; a1 = [1,-0.8,0.64]; [R1,p1,k1] = residuez(b1,a1);
b2 = [2,-1]; a2 = [1,-0.75]; [R2,p2,k2] = residuez(b2,a2);
b3 = [1,2,1]; a3 = [1,0,0.81]; [R3,p3,k3] = residuez(b3,a3);
R = [R1;R2;R3]; p = [p1;p2;p3]; k = k1+k2+k3; [b,a] = residuez(R,p,k);

[K,C] = dir2ladr(b,a)
K =
   -0.3933    0.7034   -0.5916    0.4733   -0.3888
C =
    2.7756   -0.0253    1.5817   -0.5787    0.2857   -1.6059
```
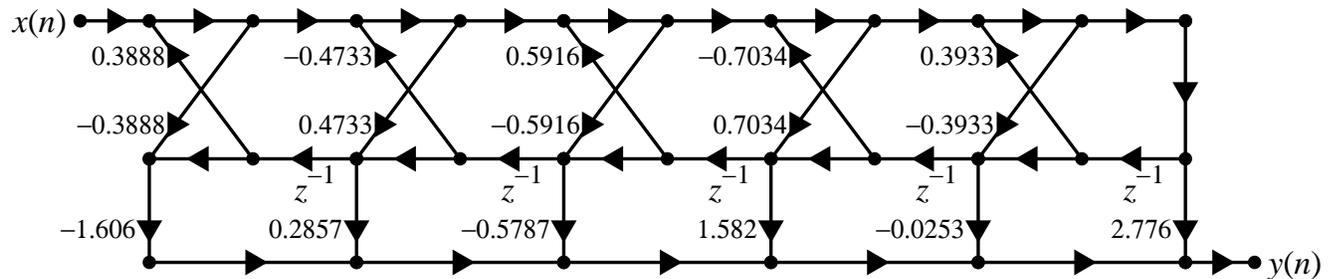
P6.8 An IIR filter is described by the following system function

$$H(z) = \left(\frac{-14.75 - 12.9z^{-1}}{1 - \frac{7}{8}z^{-1} + \frac{3}{32}z^{-2}}\right) + \left(\frac{24.5 + 26.82z^{-1}}{1 - z^{-1} + \frac{1}{2}z^{-2}}\right)\left(\frac{1 + 2z^{-1} + z^{-2}}{1 + 0.81z^{-2}}\right)$$

1. Normal direct form I:

   MATLAB script:

```
%% (a) Normal Direct form-I
% Given H(z)
b1 = [-14.75,-12.9]; a1 = [1,-7/8,3/32];
b2 = [24.5,26.82,0]; a2 = [1,-1,1/2];
b3 = [1,2,1]; a3 = [1,0,0.81];
[b4,a4] = cas2dir(1,[b2;b3],[a2;a3]);
[C,B,A] = dir2par(b4,a4);
B = [B;b1]; A = [A;a1];
[b,a] = par2dir(C,B,A)

b =
    9.7500   56.2325    7.6719  -39.3959  -11.6666   -2.7101        0
a =
    1.0000   -1.8750    2.2787   -2.0500    1.2366   -0.4303    0.0380
```
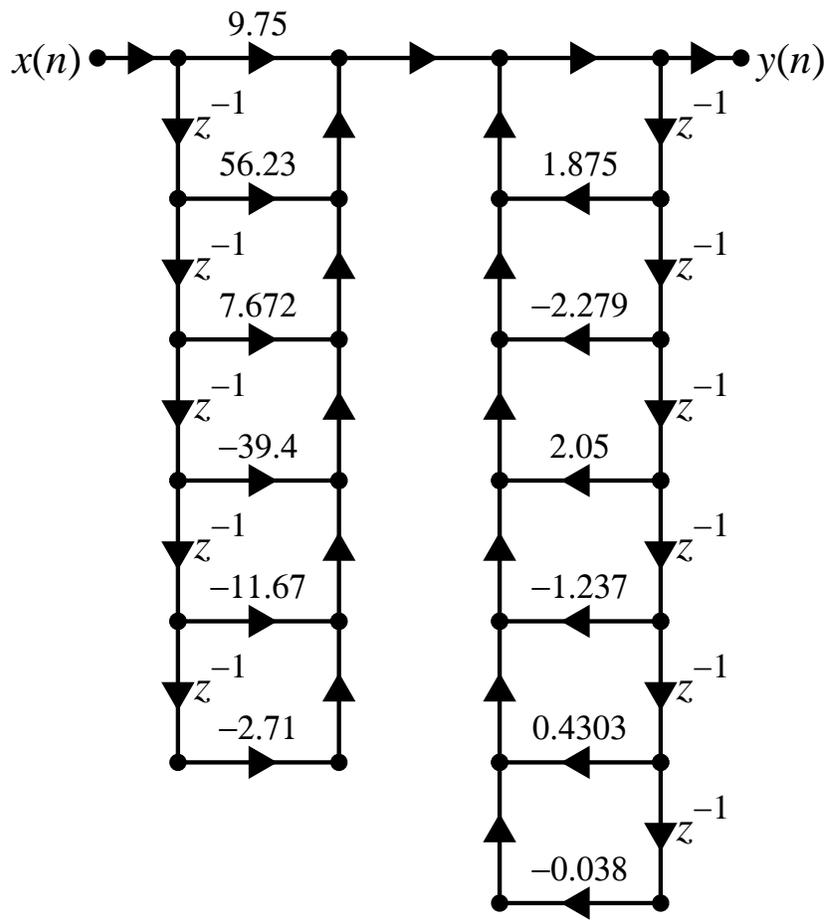
2. Normal direct form II:

   MATLAB script:

```
%% (b) Normal Direct form-II
% Given H(z)
b1 = [-14.75,-12.9]; a1 = [1,-7/8,3/32];
b2 = [24.5,26.82,0]; a2 = [1,-1,1/2];
b3 = [1,2,1]; a3 = [1,0,0.81];
[b4,a4] = cas2dir(1,[b2;b3],[a2;a3]);
[C,B,A] = dir2par(b4,a4);
B = [B;b1]; A = [A;a1];
[b,a] = par2dir(C,B,A)

b =
    9.7500   56.2325    7.6719  -39.3959  -11.6666   -2.7101         0
a =
    1.0000   -1.8750    2.2787   -2.0500    1.2366   -0.4303    0.0380
```
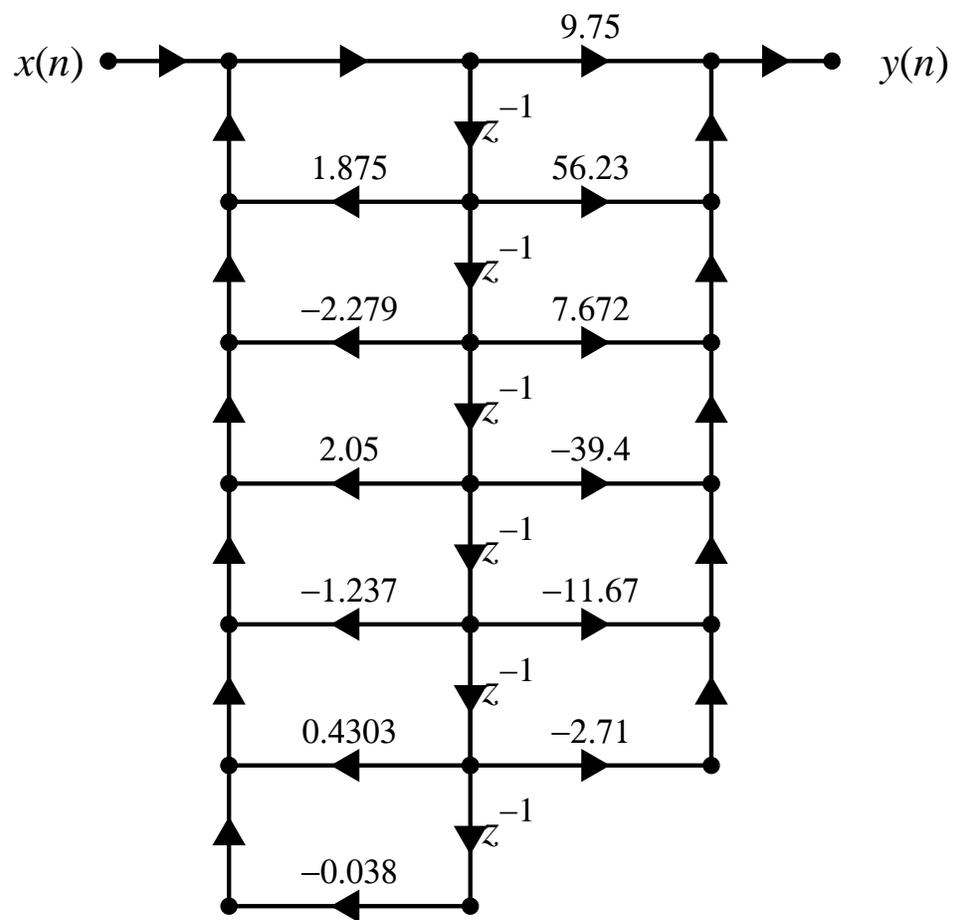
3. Cascade form containing transposed second-order direct form II sections:

MATLAB script:

```
%% (c) Transposed Cascade form
% Given H(z)
b1 = [-14.75,-12.9]; a1 = [1,-7/8,3/32];
b2 = [24.5,26.82,0]; a2 = [1,-1,1/2];
b3 = [1,2,1]; a3 = [1,0,0.81];
[b4,a4] = cas2dir(1,[b2;b3],[a2;a3]);
[C,B,A] = dir2par(b4,a4);
B = [B;b1]; A = [A;a1];
[b,a] = par2dir(C,B,A);
[b0,B,A] = dir2cas(b,a)


b0 =
    9.7500
B =
    1.0000     0.2731     0.0663
    1.0000     6.3691     4.7918
    1.0000    -0.8748          0
A =
    1.0000     0.0000     0.8100
    1.0000    -1.0000     0.5000
    1.0000    -0.8750     0.0937
```
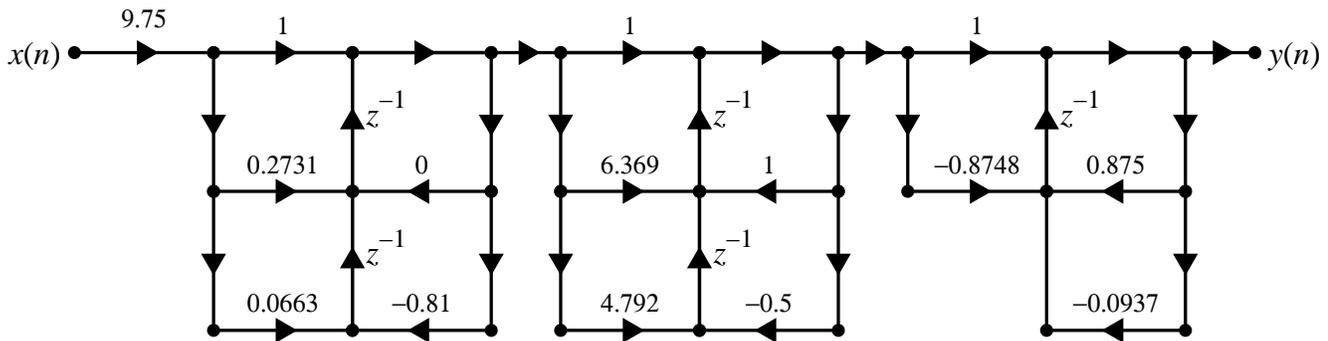
4. Parallel form containing transposed second-order direct form II sections:

MATLAB script:

```
%% (d) Transposed Parallel form
% Given H(z)
b1 = [-14.75,-12.9]; a1 = [1,-7/8,3/32];
b2 = [24.5,26.82,0]; a2 = [1,-1,1/2];
b3 = [1,2,1]; a3 = [1,0,0.81];
[b4,a4] = cas2dir(1,[b2;b3],[a2;a3]);
[C,B,A] = dir2par(b4,a4);
B = [B;b1]; A = [A;a1];
[b,a] = par2dir(C,B,A);
[C,B,A] = dir2par(b,a)
C =
     0
B =
  -58.1234   -40.2767
   82.6234    57.9733
  -14.7500   -12.9000
A =
    1.0000    0.0000    0.8100
    1.0000   -1.0000    0.5000
    1.0000   -0.8750    0.0937
```
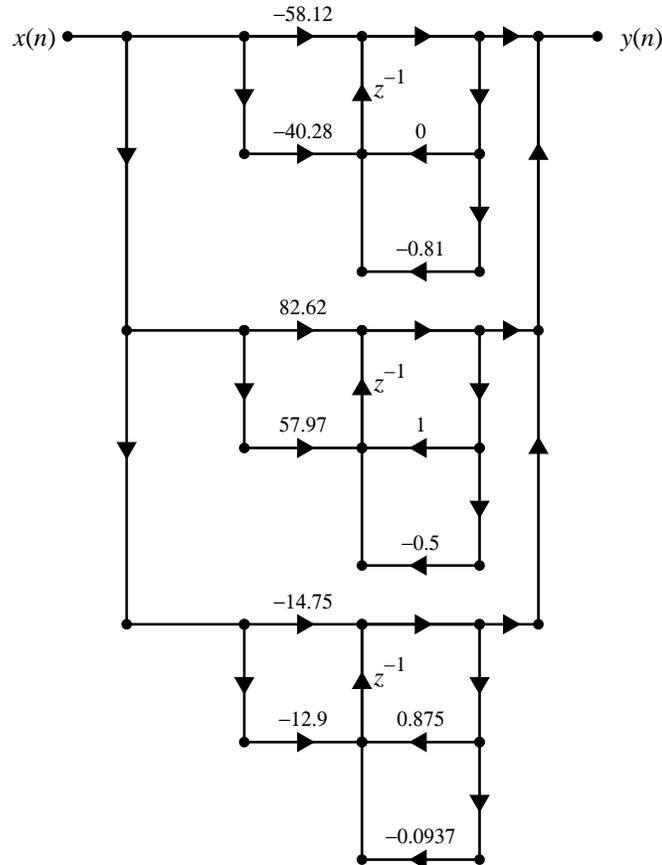
5. Lattice-ladder form:

   MATLAB script:

```
%% Lattice-Ladder form
% Given H(z)
b1 = [-14.75,-12.9]; a1 = [1,-7/8,3/32];
b2 = [24.5,26.82,0]; a2 = [1,-1,1/2];
b3 = [1,2,1]; a3 = [1,0,0.81];
[b4,a4] = cas2dir(1,[b2;b3],[a2;a3]);
[C,B,A] = dir2par(b4,a4);
B = [B;b1]; A = [A;a1];
[b,a] = par2dir(C,B,A);
[K,C] = dir2ladr(b,a)
K =
   -0.6719     0.5069    -0.6124     0.5539    -0.3596     0.0380
C =
   43.1171    55.8710   -49.2642   -61.1147   -16.7111    -2.7101          0
```
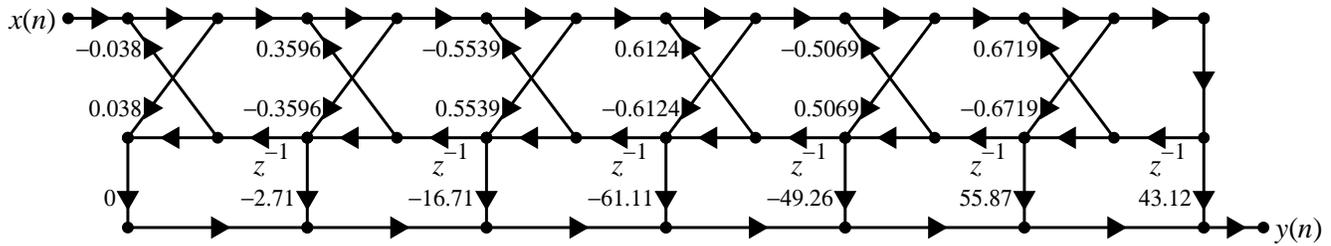
P6.9 The causal linear time-invariant system describes the lattice/ladder structure.

1. Direct form I: Given Lattice-Ladder

```
K = [-0.56,2/3,1/2]; C = [0,0,0,1]; [b,a] = ladr2dir(K,C)
b =
    0.5000     0.2000    -0.6000     1.0000
a =
    1.0000    -0.6000     0.2000     0.5000
```
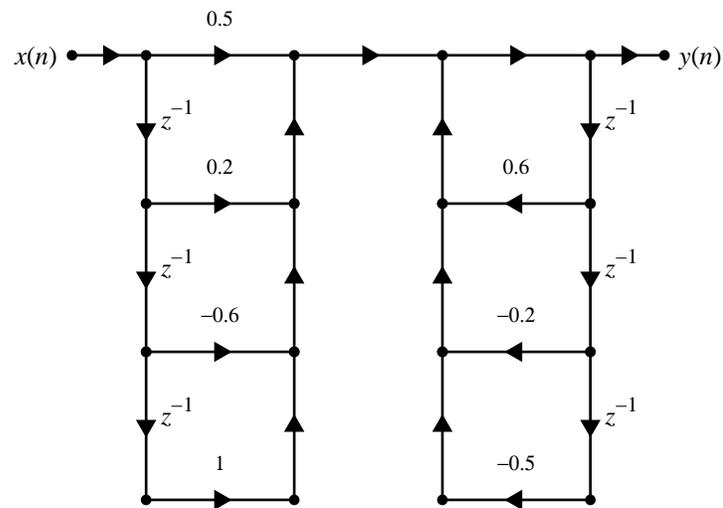

2. Direct form II:Given Lattice-Ladder

```
K = [-0.56,2/3,1/2]; C = [0,0,0,1]; [b,a] = ladr2dir(K,C)
b =
    0.5000     0.2000    -0.6000     1.0000
a =
    1.0000    -0.6000     0.2000     0.5000
```
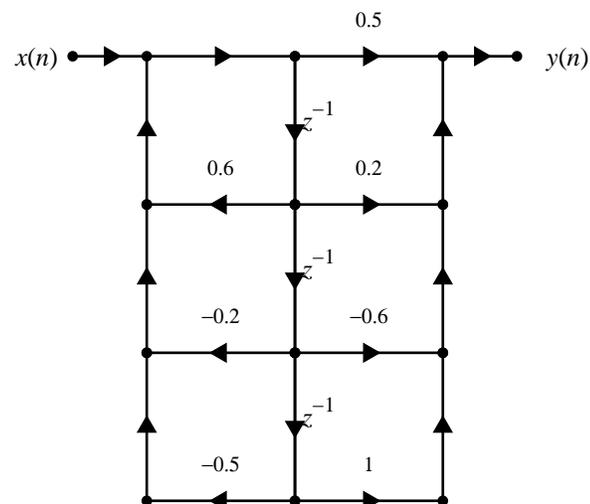
3. Cascade form containing second-order direct form II sections: Given Lattice-Ladder

```
K = [-0.56,2/3,1/2]; C = [0,0,0,1]; [b,a] = ladr2dir(K,C);
[b0,B,A] = dir2cas(b,a),
b0 =
    0.5000
B =
    1.0000   -1.3448    1.1463
    1.0000    1.7448         0
A =
    1.0000   -1.1731    0.8724
    1.0000    0.5731         0
```
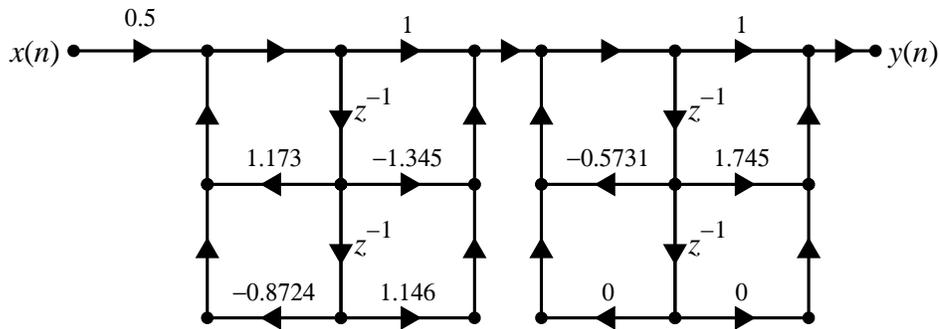
4. Parallel form containing second-order direct form II sections: Given Lattice-Ladder

```
K = [-0.56,2/3,1/2]; C = [0,0,0,1]; [b,a] = ladr2dir(K,C);
[C,B,A] = dir2par(b,a)
C =
     2
B =
   -0.2748     0.1201
   -1.2252          0
A =
    1.0000    -1.1731     0.8724
    1.0000     0.5731          0
```
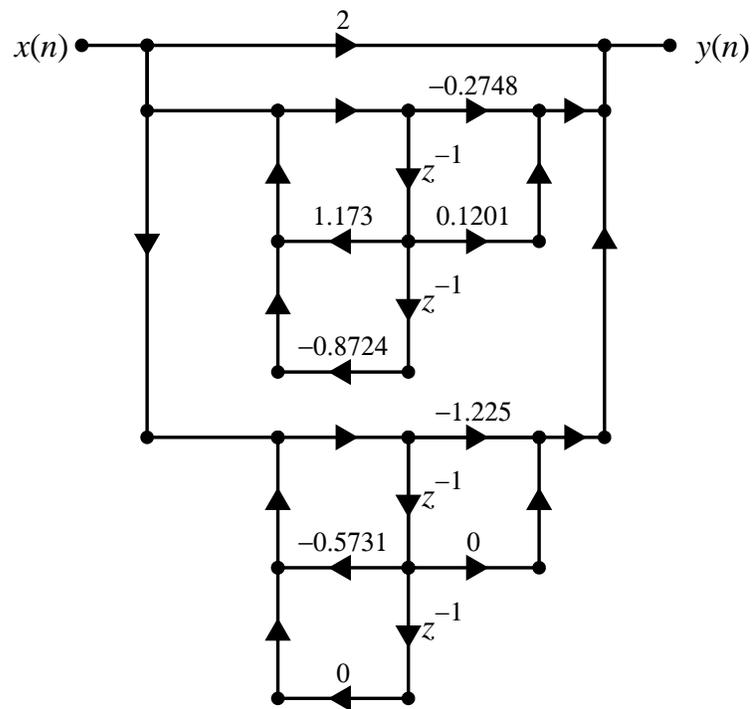
**P6.10** Given linear time-invariant system with system function:

$$H(z) = \frac{0.05 - 0.01z^{-1} - 0.13z^{-2} + 0.13z^{-4} + 0.01z^{-5} - 0.05z^{-6}}{1 - 0.77z^{-1} + 1.59z^{-2} - 0.88z^{-3} + 1.2z^{-4} - 0.35z^{-5} + 0.31z^{-6}}$$

1. The required form is the cascade form. MATLAB script:

```
b = [0.05,-0.01,-0.13,0.13,0.01,-0.05];
a = [1,-0.77,1.59,-0.88,1.2,-0.35,0.31];
[b0,B,A] = dir2cas(b,a)
b0 =
    0.0500
B =
    1.0000   -1.5900    1.0000
    1.0000    2.3900    1.0000
    1.0000   -1.0000         0
A =
    1.0000    0.5732    0.7751
    1.0000   -0.2611    0.4961
    1.0000   -1.0821    0.8062
```

Block diagram:
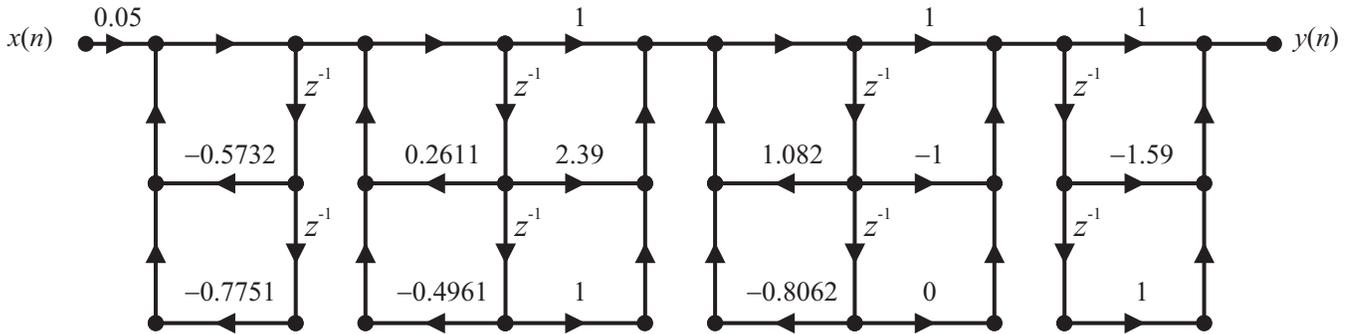
2. This solution is not unique since numerator and denominator biquads can be grouped differently.

**P6.11** Given linear time-invariant system with system function:

$$H(z) = \frac{0.051 + 0.088z^{-1} + 0.06z^{-2} - 0.029z^{-3} - 0.069z^{-4} - 0.046z^{-5}}{1 - 1.34z^{-1} + 1.478z^{-2} - 0.789z^{-3} + 0.232z^{-4}}$$

The given flow graph is a parallel structure. MATLAB script:

```
b = [0.051,0.088,0.06,-0.029,-0.069,-0.046];
a = [1,-1.34,1.478,-0.789,0.232];
[C,B,A] = dir2par(b,a)

C =
   -0.9717   -0.1983
B =
   -0.3654    0.0465
    1.3881   -0.6876
A =
    1.0000   -0.4929    0.7519
    1.0000   -0.8471    0.3086
```

Block diagram:
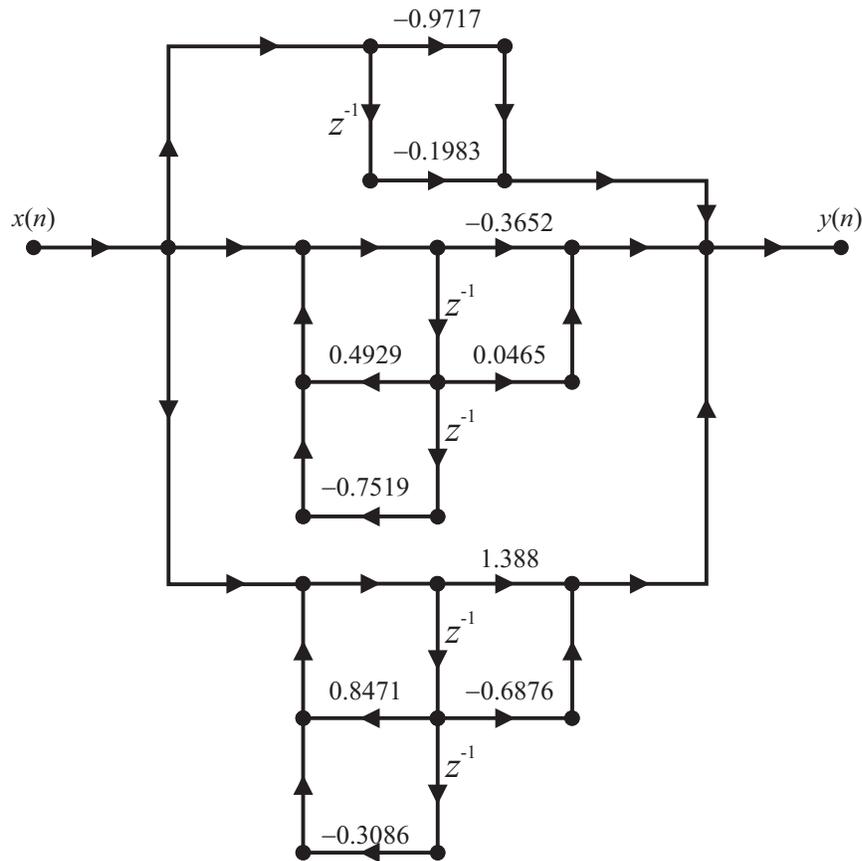
This solution is unique.

**P6.12** The linear time-invariant system function:

$$H(z) = \frac{0.05 - 0.01z^{-1} - 0.13z^{-2} + 0.13z^{-4} + 0.01z^{-5} - 0.05z^{-6}}{1 - 0.77z^{-1} + 1.59z^{-2} - 0.88z^{-3} + 1.2z^{-4} - 0.35z^{-5} + 0.31z^{-6}}$$

1. The given signal flow graph is a parallel connection containing one second-order cascade branch.
   MATLAB script:

```
% Given Direct form
b = [0.05,-0.01,-0.13,0,0.13,0.01,-0.05];
a = [1,-0.77,1.59,-0.88,1.2,-0.35,0.31];
% Convert to a parallel form
[C,B,A] = dir2par(b,a)
C =
   -0.1613
B =
   -0.2654   -0.0295
    0.7206   -0.1148
   -0.2439    0.0964
A =
    1.0000    0.5732    0.7751
    1.0000   -0.2611    0.4961
    1.0000   -1.0821    0.8062
% Convert the first two biquads into direct form
[b1,a1] = par2dir(C,B(1:2,:),A(1:2,:)); b1 = real(b1), a1 = real(a1)
b1 =
    0.2939    0.2878    0.1879   -0.1168   -0.0620
a1 =
    1.0000    0.3121    1.1215    0.0820    0.3845
% Convert the resulting direct into cascade form
[b0,B1,A1] = dir2cas(b1,a1)
b0 =
    0.2939
B1 =
    1.0000    1.1383    1.0262
    1.0000   -0.1592   -0.2056
A1 =
    1.0000    0.5732    0.7751
    1.0000   -0.2611    0.4961
```

Block diagram:
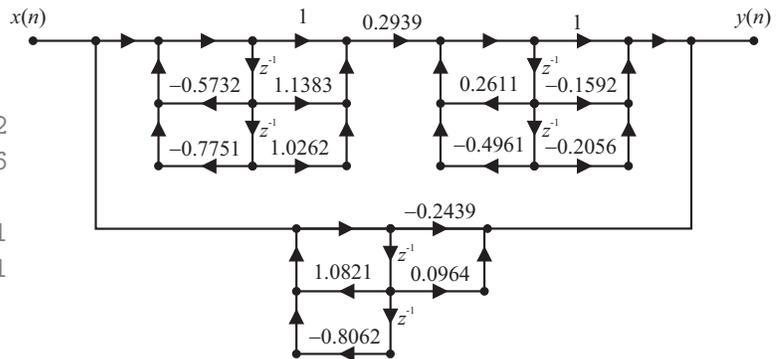
2. The solution is not unique since any two out of three parallel biquads can be used to construct a cascade branch.

P6.13 The filter structure contains a parallel connection of cascade sections.

1. Direct form (normal) structure: MATLAB script:

```
%% (a) Normal Direct form-I
% Given Structure
%  Upper parallel branch
[b1,a1] = cas2dir(1,[0.5,2,1.5;1,3,0],[1,1,0.9;1,-1,0.8]);
b1 = removetrailzeros(b1); [C1,B1,A1] = dir2par(b1,a1);
% Lower parallel branch
[b0,B2,A2] = dir2cas([1,2,1],[1,0.5,0.5,-0.4]);
[b2,a2] = cas2dir(b0,[3,-0.5,2;B2],[1,0.4,0.4;A2]);
b2 = removetrailzeros(b2); a2 = removetrailzeros(a2);
[C2,B2,A2] = dir2par(b2,a2);
% Overall parallel
C = C1+C2; B = [B1;B2]; A = [A1;A2];
% Overall direct
[b,a] = par2dir(C,B,A); b = real(b), a = real(a)
b =
Columns 1 through 9
3.5000    9.4500   17.3000   22.1500   18.7300   11.0200    3.6700    1.3000    0.7200
a =
Columns 1 through 10
1.0000    0.9000    1.8000    0.5300    1.4400    0.3780    0.8200   -0.1160    0.0448
```

Block diagram:
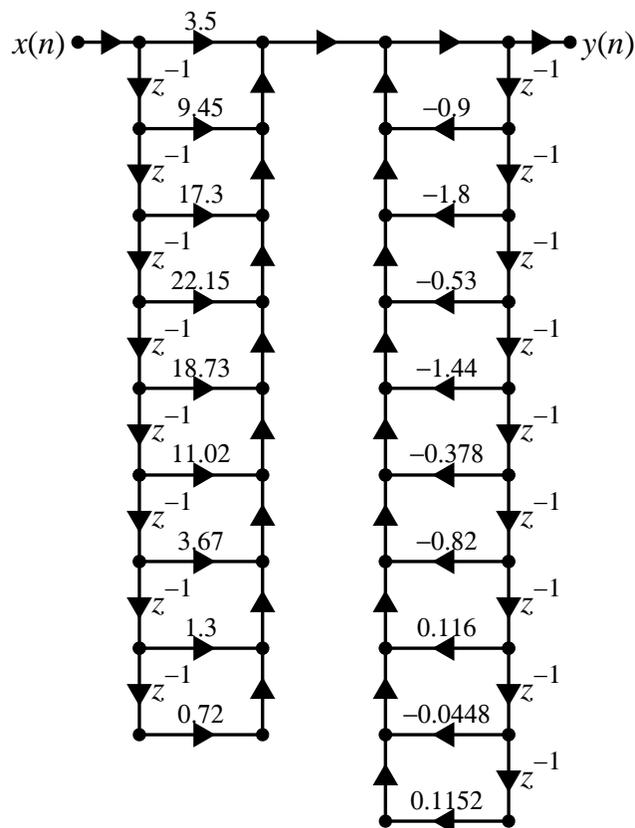
2. Direct form (transposed) structure: MATLAB script:

```
%% (a) Transposed Direct form-II
% Given Structure
%  Upper parallel branch
[b1,a1] = cas2dir(1,[0.5,2,1.5;1,3,0],[1,1,0.9;1,-1,0.8]);
b1 = removetrailzeros(b1); [C1,B1,A1] = dir2par(b1,a1);
% Lower parallel branch
[b0,B2,A2] = dir2cas([1,2,1],[1,0.5,0.5,-0.4]);
[b2,a2] = cas2dir(b0,[3,-0.5,2;B2],[1,0.4,0.4;A2]);
b2 = removetrailzeros(b2); a2 = removetrailzeros(a2);
[C2,B2,A2] = dir2par(b2,a2);
% Overall parallel
C = C1+C2; B = [B1;B2]; A = [A1;A2];
% Overall direct
[b,a] = par2dir(C,B,A); b = real(b), a = real(a)
b =
Columns 1 through 9
3.5000    9.4500   17.3000   22.1500   18.7300   11.0200    3.6700    1.3000    0.7200
a =
Columns 1 through 10
1.0000    0.9000    1.8000    0.5300    1.4400    0.3780    0.8200   -0.1160    0.0448   -0.
```

Block diagram:
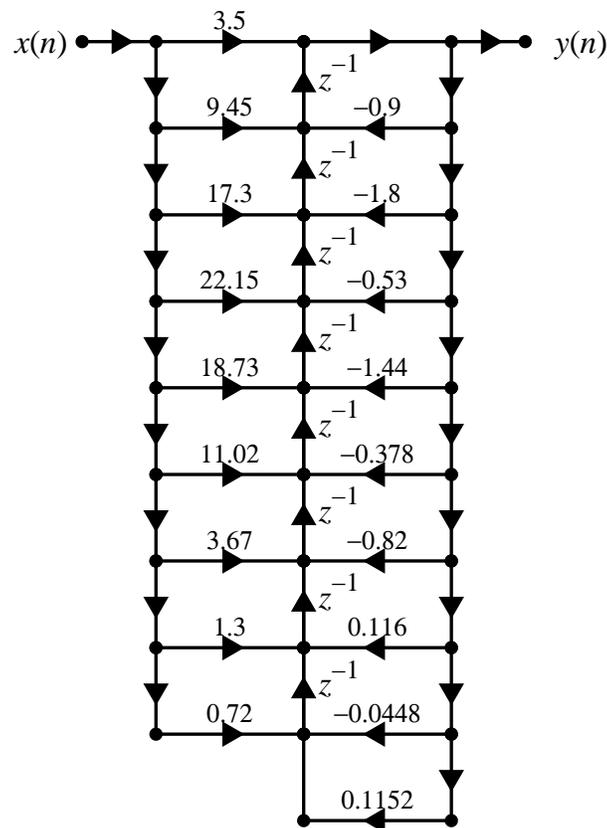
3. Cascade form structure containing second-order sections: MATLAB script:

```
%% (c) Normal Cascade form
% Given Structure
%  Upper parallel branch
[b1,a1] = cas2dir(1,[0.5,2,1.5;1,3,0],[1,1,0.9;1,-1,0.8]);
[C1,B1,A1] = dir2par(b1,a1);
% Lower parallel branch
[b0,B2,A2] = dir2cas([1,2,1],[1,0.5,0.5,-0.4]);
[b2,a2] = cas2dir(b0,[3,-0.5,2;B2],[1,0.4,0.4;A2]);
b2 = removetrailzeros(b2); a2 = removetrailzeros(a2);
[C2,B2,A2] = dir2par(b2,a2);
% Overall parallel
C = C1+C2; B = [B1;B2]; A = [A1;A2];
% Overall direct
[b,a] = par2dir(C,B,A); b = real(b); a = real(a);
[b0,Bc,Ac] = dir2cas(b,a)
b0 =
    3.5000
Bc =
    1.0000    1.0414    0.9377
    1.0000    0.2423    1.5819
    1.0000   -0.3323    0.1852
    1.0000    1.7486    0.7486
    1.0000    0.0000         0
Ac =
    1.0000    1.0000    0.9000
    1.0000    0.9387    0.9118
    1.0000    0.4000    0.4000
    1.0000   -1.0000    0.8000
    1.0000   -0.4387    0.0000
```

Block diagram:
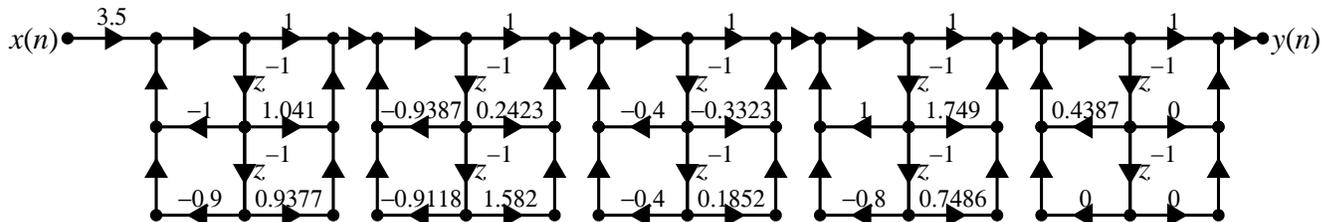
4. Parallel form structure containing second-order sections:

```
%% (d) Normal Parallel form
% Given Structure
%  Upper parallel branch
[b1,a1] = cas2dir(1,[0.5,2,1.5;1,3,0],[1,1,0.9;1,-1,0.8]);
[C1,B1,A1] = dir2par(b1,a1);
% Lower parallel branch
[b0,B2,A2] = dir2cas([1,2,1],[1,0.5,0.5,-0.4]);
[b2,a2] = cas2dir(b0,[3,-0.5,2;B2],[1,0.4,0.4;A2]);
b2 = removetrailzeros(b2); a2 = removetrailzeros(a2);
[C2,B2,A2] = dir2par(b2,a2);
% Overall parallel
C = C1+C2, B = [B1;B2], A = [A1;A2],
C =
     []
B =
    0.9370   -1.1349
   -0.4370    6.0088
    2.1502    5.0236
   -3.3424   -3.3813
    4.1923         0
A =
    1.0000    1.0000    0.9000
    1.0000   -1.0000    0.8000
    1.0000    0.9387    0.9118
    1.0000    0.4000    0.4000
    1.0000   -0.4387         0
```

Block diagram:
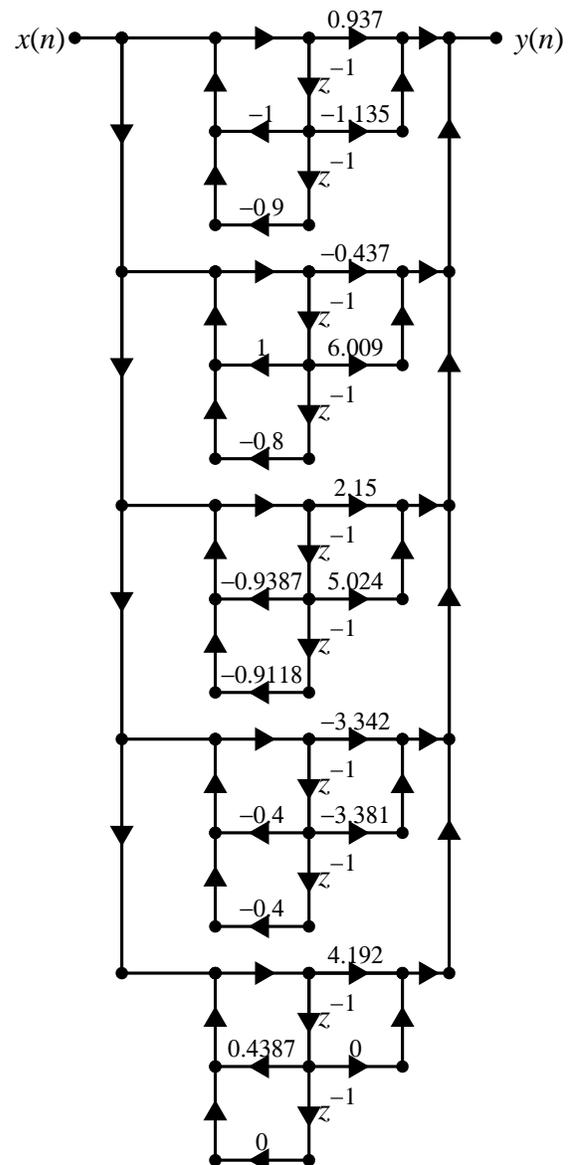
P6.14 In filter structure shown the systems $H_1(z)$ and $H_2(z)$ are in parallel where

$$H_1(z) = 2 + \frac{0.2 - 0.3z^{-1}}{1 + 0.9z^{-1} + 0.9z^{-2}} + \frac{0.4 + 0.5z^{-1}}{1 + -0.8z^{-1} + 0.8z^{-2}}$$

and

$$H_2(z) = \left(\frac{2 + z^{-1} - z^{-2}}{1 + 1.7z^{-1} + 0.72z^{-2}} \right) \left(\frac{3 + 4z^{-1} + 5z^{-2}}{1 - 1.5z^{-1} + 0.56z^{-2}} \right)$$

1. $H(z)$ as a rational function: MATLAB script:

```
% Given Structure
%  H1(z) in parallel form: Leave as is
C1 = 2; B1 = [0.2,-0.3;0.4,0.5]; A1 = [1,0.9,0.9;1,-0.8,0.8];
% H2(z) in cascade form: Convert to parallel form
b0 = 1; B2 = [2,1,-1;3,4,5]; A2 = [1,1.7,0.72;1,-1.5,0.56];
[b2,a2] = cas2dir(b0,B2,A2); [C2,B2,A2] = dir2par(b2,a2);
% Combine two parallel forms
C = C1+C2; B = [B1;B2]; A = [A1;A2];
% (a) Rational function H(z)
[b,a] = par2dir(C,B,A); b = real(b), a = real(a)
b =
    8.6000   12.7200   17.9680   12.6292    8.6276    8.2575    2.4425
    0.6204   -3.0194
a =
    1.0000    0.3000   -0.2700   -0.0590   -0.1342    0.0589   -0.5193
   -0.0922    0.2903
```

2. Cascade form structure:

```
[b0,Bc,Ac] = dir2cas(b,a)
b0 =
    8.6000
Bc =
    1.0000    0.9999    0.7846
    1.0000    0.8038    1.2272
    1.0000   -0.8268    0.7566
    1.0000    0.5022   -0.4820
Ac =
    1.0000    0.9000    0.9000
    1.0000   -0.8000    0.8000
    1.0000    1.7000    0.7200
    1.0000   -1.5000    0.5600
```
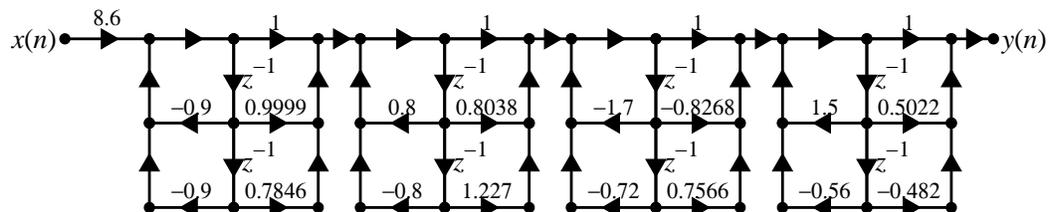
3. Parallel form structure:

```
[C,B,A] = dir2par(b,a)
C =
  -10.4008
B =
     0.2000   -0.3000
     0.4000    0.5000
     5.6943    5.5629
    12.7065   -5.1424
A =
     1.0000    0.9000    0.9000
     1.0000   -0.8000    0.8000
     1.0000    1.7000    0.7200
     1.0000   -1.5000    0.5600
```
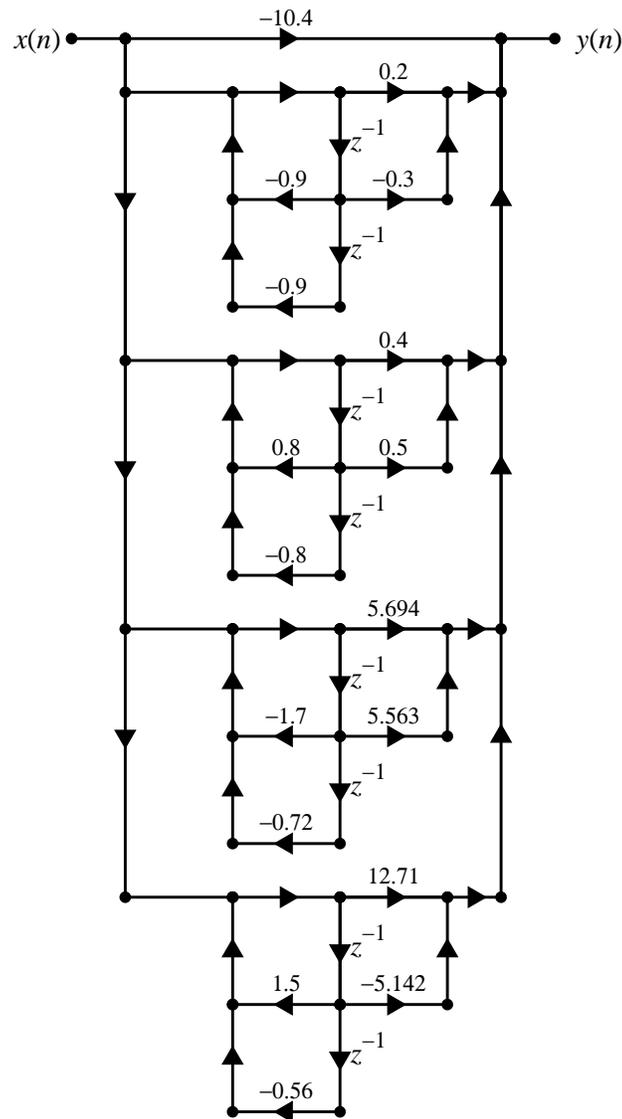
P6.15 The digital filter structure shown is a cascade of two parallel sections and corresponds to a 10th-order IIR digital filter system function

$$H(z) = \frac{1 - 2.2z^{-2} + 1.6368z^{-4} - 0.48928z^{-6} + 5395456 \times 10^{-8}z^{-8} - 147456 \times 10^{-8}z^{-10}}{1 - 1.65z^{-2} + 0.8778z^{-4} - 0.17281z^{-6} + 1057221 \times 10^{-8}z^{-8} - 893025 \times 10^{-10}z^{-10}}$$

1. Due to a mistake in labeling, two of the multiplier coefficients in this structure have *incorrect values* (rounded to 4 decimals). To locate these two multipliers and determine their correct values we will investigate their pole-zero structure and combine the appropriate pairs. MATLAB Script:

```
clc; format short;
% Given Rational Function
b = [1,0,-2.2,0,1.6368,0,-0.48928,0,5395456e-8,0,-147456e-8];
a = [1,0,-1.65,0,0.8778,0,-0.17281,0,1057221e-8,0,-893025e-10];
% Poles/Zeros of the System function
zs = roots(b); zs = sort(zs); ps = roots(a); ps = sort(ps);
% Poles and zeros chosen from each half to create two parallel sections

%  Parallel Section-1
b3 = poly(zs([1:5])); a3 = poly(ps([1:5])); [C3,B3,A3] = dir2par(b3,a3)

C3 =
    4.0635
B3 =
   -0.0973   -0.0815
   -0.5052   -0.2245
   -2.4609        0
A3 =
    1.0000    1.6000    0.6300
    1.0000    0.8000    0.1500
    1.0000    0.1000        0

%  Parallel Section-2
b4 = poly(zs([6:10])); a4 = poly(ps([6:10])); [C4,B4,A4] = dir2par(b4,a4)

C4 =
    4.0635
B4 =
   -2.8255    0.7747
   -0.2076    0.1319
   -0.0304        0
A4 =
    1.0000   -0.4000    0.0300
    1.0000   -1.2000    0.3500
    1.0000   -0.9000        0
```

Hence from inspection, the two incorrect values are -0.5502 in the first parallel section and -0.9 in the second parallel section. The correct block diagram is:
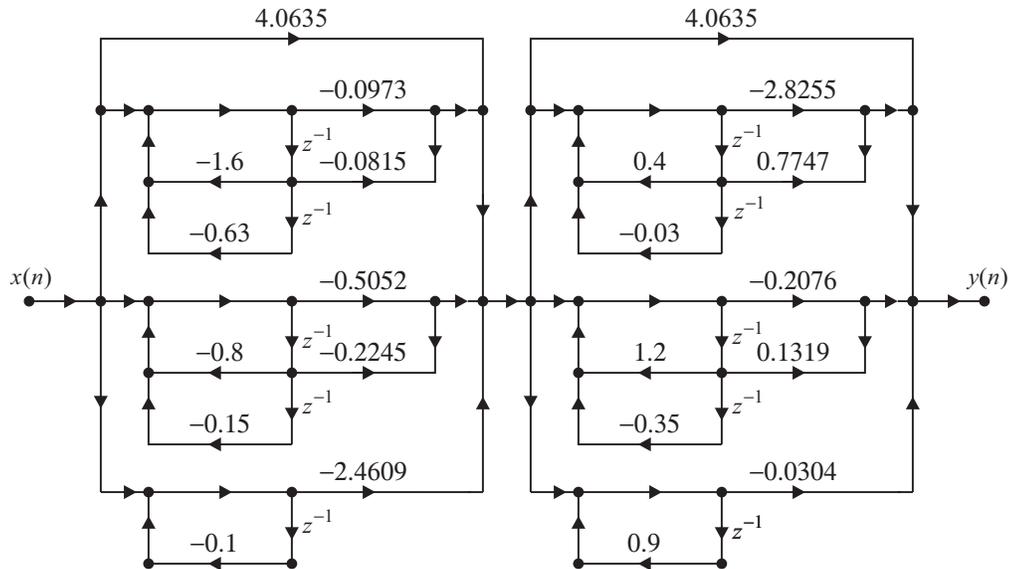
2. An overall cascade structure containing second-order section and which contains the least number of multipliers: This can be obtained by combining pole or zero pairs with the same magnitude but differing signs. MATLAB script:

```
%% (b) Overall Cascade structure containing the least number of multipliers
%      Combine the poles and zeros with the same magnitude but +- signs.
b0 = 1;
B = [poly(zs([1,end]));poly(zs([2,end-1]));poly(zs([3,end-2]));...
     poly(zs([4,end-3]));poly(zs([5,end-4]))]
A = [poly(ps([1,end]));poly(ps([2,end-1]));poly(ps([3,end-2]));...
     poly(ps([4,end-3]))]
B =
     1.0000    0.0000   -1.0000
     1.0000   -0.0000   -0.6400
     1.0000    0.0000   -0.3600
     1.0000   -0.0000   -0.1600
     1.0000   -0.0000   -0.0400
A =
     1.0000    0.0000   -0.8100
     1.0000   -0.0000   -0.4900
     1.0000    0.0000   -0.2500
     1.0000   -0.0000   -0.0900
```

The following block diagram has only 7 multipliers (not counting multiplication by 0, 1, or -1).
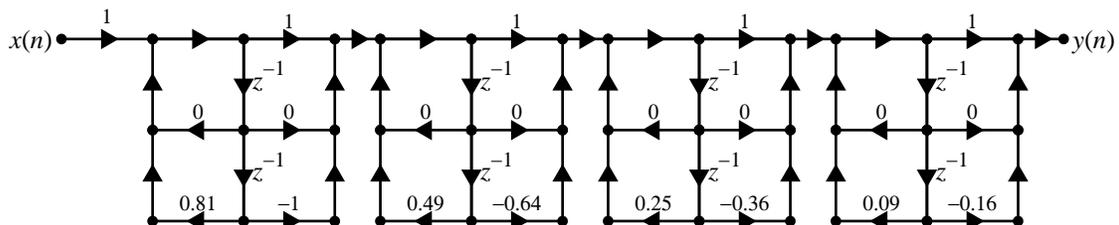
P6.16 A linear-phase FIR filter is obtained by requiring certain symmetry conditions on its impulse responses.

1. In the case of symmetrical impulse response, we have $h(n) = h(M - 1 - n)$, $0 \leq n \leq M - 1$. If M is an odd number then

$$
H(e^{j\omega}) = \sum_{0}^{M-1} h(n)\, e^{-j\omega n} = \sum_{0}^{\frac{M-3}{2}} h(n)\, e^{-j\omega n} + h\left(\frac{M-1}{2}\right) e^{-j\omega\left(\frac{M-1}{2}\right)} + \sum_{\frac{M+1}{2}}^{M-1} h(n)\, e^{-j\omega n}
$$

$$
= \sum_{0}^{\frac{M-3}{2}} h(n)\, e^{-j\omega n} + h\left(\frac{M-1}{2}\right) e^{-j\omega\left(\frac{M-1}{2}\right)} + \sum_{\frac{M+1}{2}}^{M-1} h(M - 1 - n)\, e^{-j\omega n}
$$

$$
= \sum_{0}^{\frac{M-3}{2}} h(n)\, e^{-j\omega n} + h\left(\frac{M-1}{2}\right) e^{-j\omega\left(\frac{M-1}{2}\right)} + \sum_{0}^{\frac{M-3}{2}} h(n)\, e^{-j\omega(M-1-n)}
$$

$$
= e^{-j\omega\left(\frac{M-1}{2}\right)} \left[\sum_{0}^{\frac{M-3}{2}} h(n)\, e^{-j\omega\left(n - \frac{M-1}{2}\right)} + h\left(\frac{M-1}{2}\right) + \sum_{0}^{\frac{M-3}{2}} h(n)\, e^{j\omega\left(n - \frac{M-1}{2}\right)} \right]
$$

$$
= e^{-j\omega\left(\frac{M-1}{2}\right)} \left[h\left(\frac{M-1}{2}\right) + \sum_{0}^{\frac{M-3}{2}} 2h(n) \cos\left[\left(n - \frac{M-1}{2}\right)\right] \right] \tag{6.5}
$$

Since the term in the bracket on the right in (6.5) is real-valued, the resulting phase response is given by the first term. It is linear in ω and is given by

$$
\angle H\left(e^{j\omega}\right) = -\left(\frac{M-1}{2}\right)\omega, \quad -\pi < \omega \leq \pi \tag{6.6}
$$

Similarly, if M is an even number then the term $h\left(\frac{M-1}{2}\right) = 0$ in (6.5) and instead of the term $(M - 3)/2$, we get the term $(M/2 - 1)$ in (6.5). Hence the phase response is still linear and is given by (6.6).

2. Linear-phase structures for the symmetric form when $M = 5$ and $M = 6$:

$M = 5$ $M = 6$
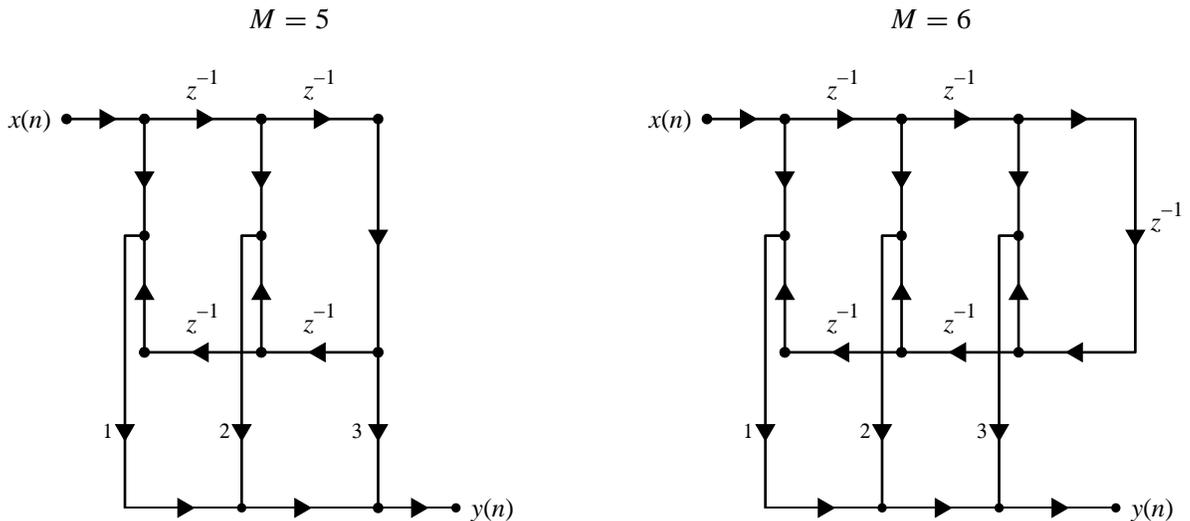
3. In the case of antisymmetrical impulse response, we have $h(n) = -h(M-1-n), 0 \leq n \leq M-1$. If M is an odd number then $h\left(\frac{M-1}{2}\right) = 0$. Hence

$$H(e^{j\omega}) = \sum_{0}^{M-1} h(n)\, e^{-j\omega n} = \sum_{0}^{\frac{M-3}{2}} h(n)\, e^{-j\omega n} + \sum_{\frac{M+1}{2}}^{M-1} h(n)\, e^{-j\omega n}$$

$$= \sum_{0}^{\frac{M-3}{2}} h(n)\, e^{-j\omega n} - \sum_{\frac{M+1}{2}}^{M-1} h(M-1-n)\, e^{-j\omega n} = \sum_{0}^{\frac{M-3}{2}} h(n)\, e^{-j\omega n} - \sum_{0}^{\frac{M-3}{2}} h(n)\, e^{-j\omega(M-1-n)}$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)} \left[\sum_{0}^{\frac{M-3}{2}} h(n)\, e^{-j\omega\left(n-\frac{M-1}{2}\right)} - \sum_{0}^{\frac{M-3}{2}} h(n)\, e^{j\omega\left(n-\frac{M-1}{2}\right)} \right]$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)} \left[-j \sum_{0}^{\frac{M-3}{2}} 2h(n) \sin\left\{ \left(n - \frac{M-1}{2}\right)\right\} \right]$$

$$= e^{j\left[\pm\frac{\pi}{2} - \left(\frac{M-1}{2}\right)\omega\right]} \sum_{0}^{\frac{M-3}{2}} 2h(n) \sin\left\{ \left(n - \frac{M-1}{2}\right)\right\} \tag{6.7}$$

Again the term in the bracket on the right in (6.7) is real-valued and hence the resulting phase response is given by the first term. It is a linear equation in ω and is given by

$$\angle H\left(e^{j\omega}\right) = \pm\frac{\pi}{2} - \left(\frac{M-1}{2}\right)\omega, \quad -\pi < \omega \leq \pi \tag{6.8}$$

Similarly, if M is an even number then instead of the term $(M-3)/2$, we get the term $(M/2-1)$ in (6.7). Hence the phase response is still linear and is given by (6.8).

4. Linear-phase structures for the antisymmetric form when $M = 5$ and $M = 6$:
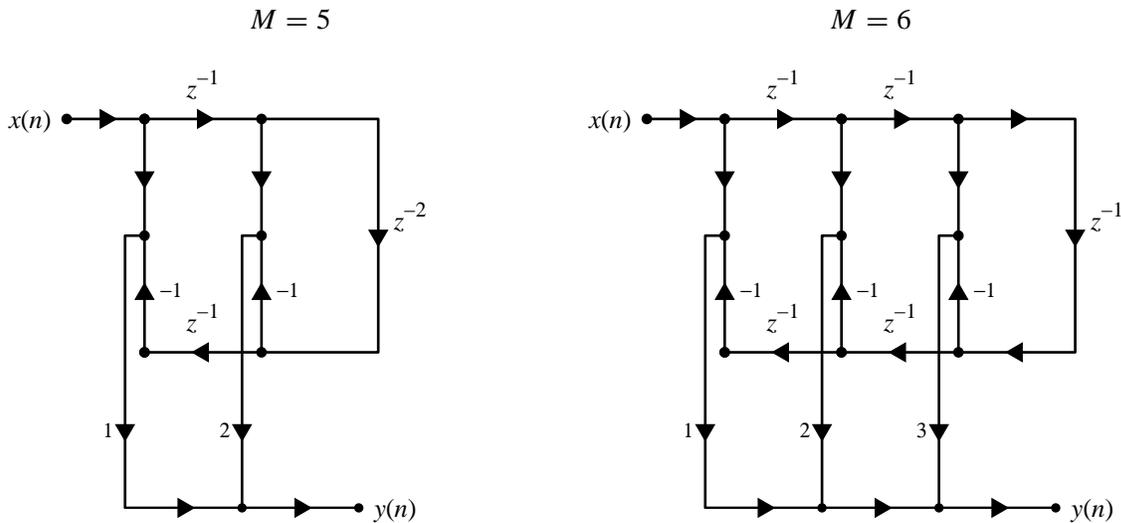
P6.17 An FIR filter is described by the difference equation

$$y(n) = \sum_{k=0}^{6} e^{-0.9|k-3|} x(n-k)$$

1. Direct form: MATLAB script:

```
% (a) Direct form
% Given FIR filter coefficients
k = [0:6]; b = exp(-0.9*abs(k-3))
b =
    0.0672    0.1653    0.4066    1.0000    0.4066    0.1653    0.0672
```

The block diagram is:
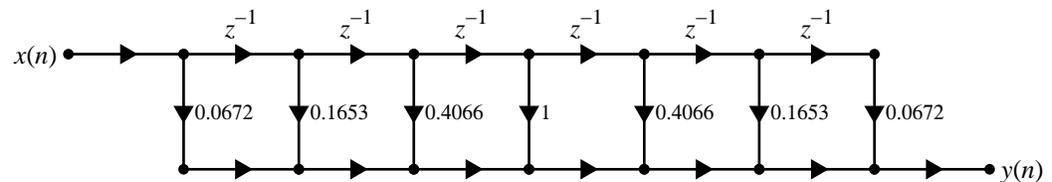
2. Linear-phase form: MATLAB script:

```
% (b) Linear-phase form
% Given FIR filter coefficients
k = [0:6]; b = exp(-0.9*abs(k-3))
b =
    0.0672    0.1653    0.4066    1.0000    0.4066    0.1653    0.0672
```

The block diagram is:
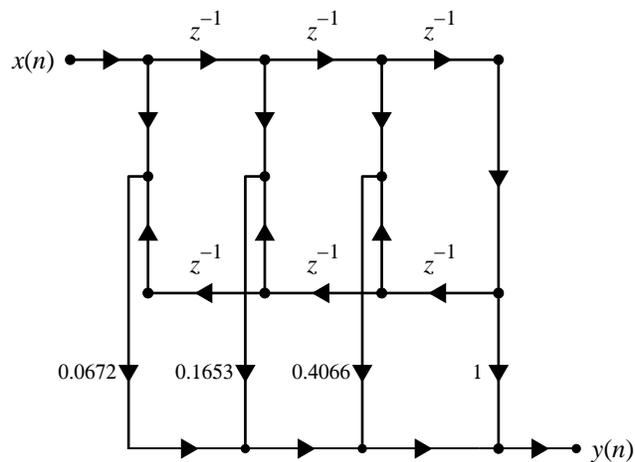
3. Cascade form: MATLAB script:

```
% (c) Cascade Form
% Given FIR filter coefficients
k = [0:6]; b = exp(-0.9*abs(k-3)); a = 1;
[b0,B,A] = dir2cas(b,a)
b0 =
    0.0672
B =
    1.0000   -0.0366    0.1817
    1.0000   -0.2015    5.5030
    1.0000    2.6978    1.0000
```

The block diagram is:
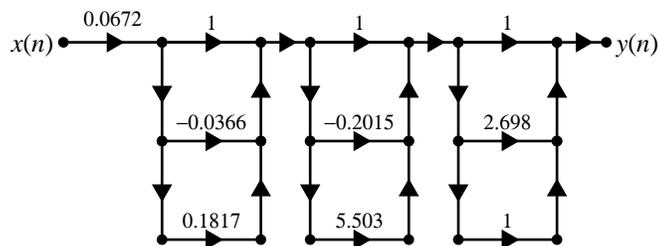
4. Frequency sampling form: MATLAB script:

```
% (d) Frequency-sampling form
% Given FIR filter coefficients
k = [0:6]; b = exp(-0.9*abs(k-3));
[C,B,A] = dir2fs(b)
C =
    2.6246
    1.2100
    0.8872
    2.2781
B =
   -0.9010    0.9010
    0.6235   -0.6235
   -0.2225    0.2225
A =
    1.0000   -1.2470    1.0000
    1.0000    0.4450    1.0000
    1.0000    1.8019    1.0000
    1.0000   -1.0000         0
```

The block diagram is:
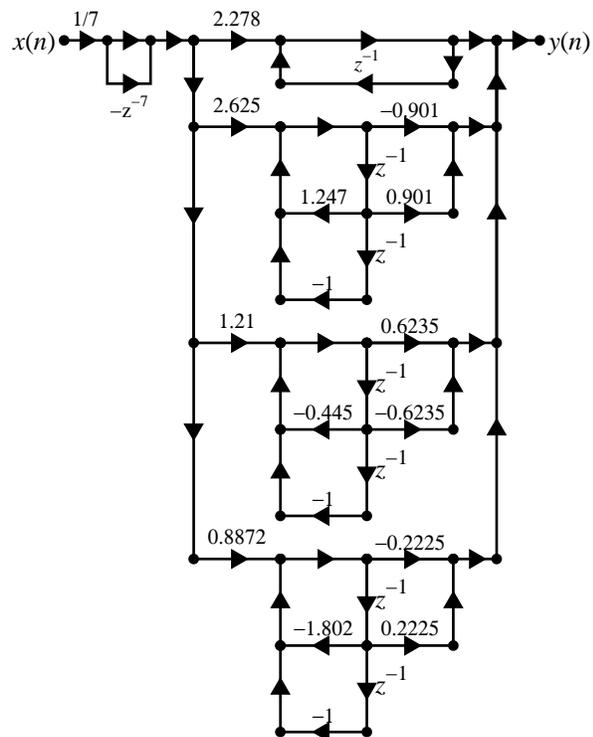
P6.18 A linear time-invariant system is given by the system function

$$H(z) = 2 + 3z^{-1} + 5z^{-2} - 3z^{-3} + 4z^{-5} + 8z^{-7} - 7z^{-8} + 4z^{-9}$$

1. Direct form: MATLAB script:

```
% (a) Direct form
% Given FIR filter coefficients
b = [2,3,5,-3,0,4,0,8,-7,4]; a = 1;
```

The block diagram is:
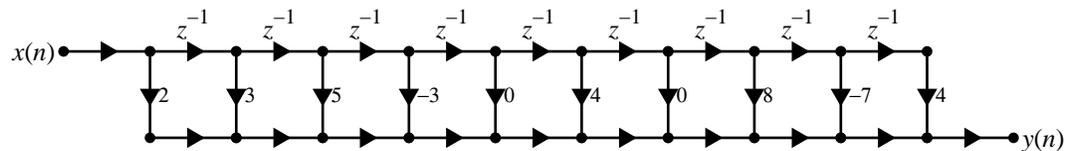
2. Cascade form: MATLAB script:

```
% (b) Cascade form
% Given FIR filter coefficients
b = [2,3,5,-3,0,4,0,8,-7,4]; a = 1;
[b0,B,A] = dir2cas(b,a)
b0 =
     2
B =
    1.0000    1.9987    3.5650
    1.0000    0.6721    1.1062
    1.0000   -0.8648    0.4364
    1.0000   -1.5766    0.9146
    1.0000    1.2706         0
```

The block diagram is:
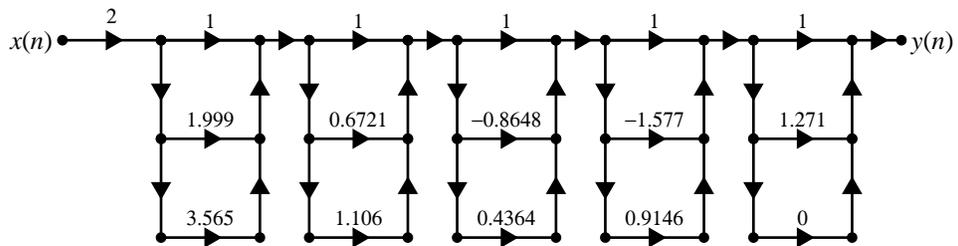
3. Lattice form: MATLAB script:

```
% (c) Lattice form
% Given FIR filter coefficients
b = [2,3,5,-3,0,4,0,8,-7,4]; a = 1;
[K] = dir2latc(b)
K =
    2.0000   -1.8759   -0.9002   -6.5739    1.0170    1.4501   -1.4653
   -1.7519    2.1667    2.0000
```

The block diagram is:
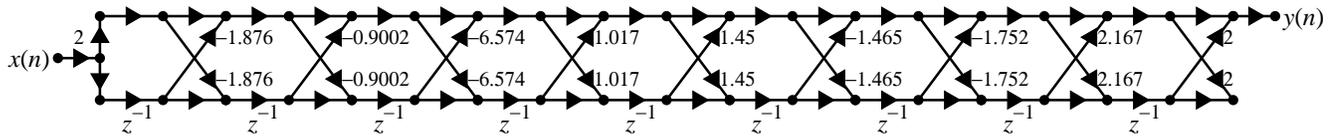
4. Frequency sampling form: MATLAB script:

```
% (d) Frequency-sampling form
% Given FIR filter coefficients
b = [2,3,5,-3,0,4,0,8,-7,4]; a = 1;
[C,B,A] = dir2fs(b)
C =
    3.0867
   27.6302
    4.2979
   44.9952
   16.0000
  -16.0000
B =
    0.9719   -0.6479
    0.4152    0.7369
    0.6980   -0.4653
    0.0562   -0.5414
A =
    1.0000   -1.6180    1.0000
    1.0000   -0.6180    1.0000
    1.0000    0.6180    1.0000
    1.0000    1.6180    1.0000
    1.0000   -1.0000        0
    1.0000    1.0000        0
```
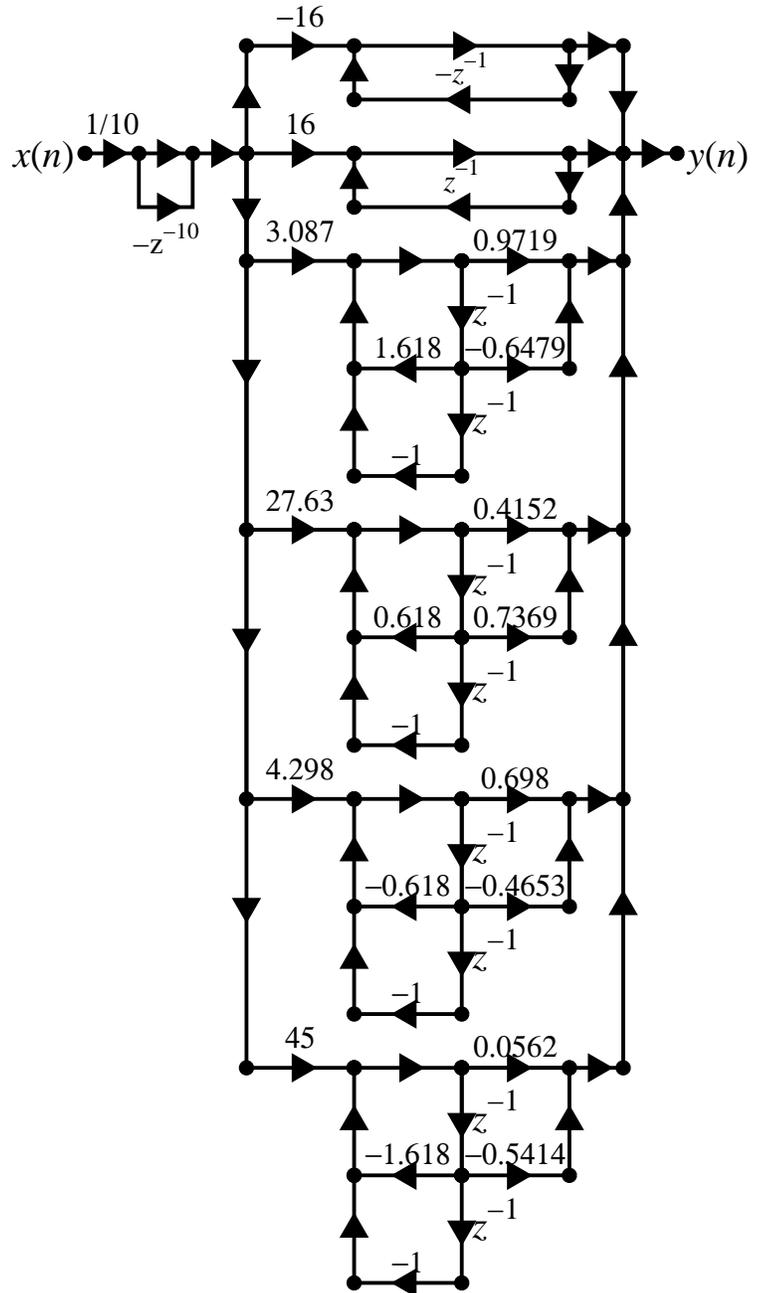
The block diagram is:

P6.19 For real-valued FIR filters, the DFT, $\tilde{H}(k)$, is conjugate symmetric the DFT

$$\tilde{H}(k) = \begin{cases} \tilde{H}(0), & k = 0 \\ \tilde{H}^*(M - k) = \tilde{H}^*(-k), & k = 1, \ldots, M - 1 \end{cases} \tag{6.9}$$

and the W_M^{-k} factor is also conjugate symmetric

$$W_M^{-k} = W_M^{M-k} = \left(W_M^k\right)^* \tag{6.10}$$

Then

$$H(z) = \left(\frac{1 - z^{-M}}{M}\right) \sum_{k=0}^{M-1} \frac{\tilde{H}(k)}{1 - W_M^{-k} z^{-1}} \tag{6.11}$$

can be put in the form

$$H(z) = \frac{1 - z^{-M}}{M} \left\{ \sum_{k=1}^{L} 2 \left|\tilde{H}(k)\right| H_k(z) + \frac{\tilde{H}(0)}{1 - z^{-1}} + \frac{\tilde{H}(M/2)}{1 + z^{-1}} \right\} \tag{6.12}$$

where $L = \frac{M-1}{2}$ for M odd, $L = \frac{M}{2} - 1$ for M even, and

$$H_k(z) = \frac{\cos\left[\angle\tilde{H}(k)\right] - z^{-1}\cos\left[\angle\tilde{H}(k) - \frac{2\pi k}{M}\right]}{1 - 2z^{-1}\cos\left(\frac{2\pi k}{M}\right) + z^{-2}} \tag{6.13}$$

Proof. The sum in (6.11) can be expressed as (assuming M even)

$$H(z) = \left(\frac{1 - z^{-M}}{M}\right) \left[\frac{\tilde{H}(0)}{1 - z^{-1}} + \sum_{k=1}^{M/2-1} \frac{\tilde{H}(k)}{1 - W_M^{-k} z^{-1}} + \frac{\tilde{H}(M/2)}{1 + z^{-1}} + \sum_{k=M/2+1}^{M-1} \frac{\tilde{H}(k)}{1 - W_M^{-k} z^{-1}} \right]$$

$$= \left(\frac{1 - z^{-M}}{M}\right) \left[\sum_{k=1}^{M/2-1} \frac{\tilde{H}(k)}{1 - W_M^{-k} z^{-1}} + \sum_{k=M/2+1}^{M-1} \frac{\tilde{H}^*(M-k)}{1 - \left(W_M^k\right)^* z^{-1}} + \frac{\tilde{H}(0)}{1 - z^{-1}} + \frac{\tilde{H}(M/2)}{1 + z^{-1}} \right]$$

$$= \left(\frac{1 - z^{-M}}{M}\right) \left[\sum_{k=1}^{M/2-1} \frac{\tilde{H}(k)}{1 - W_M^{-k} z^{-1}} + \sum_{k=1}^{M/2-1} \frac{\tilde{H}^*(k)}{1 - \left(W_M^{-k}\right)^* z^{-1}} + \frac{\tilde{H}(0)}{1 - z^{-1}} + \frac{\tilde{H}(M/2)}{1 + z^{-1}} \right]$$

$$= \left(\frac{1 - z^{-M}}{M}\right) \left[\sum_{k=1}^{M/2-1} \frac{\tilde{H}(k) - \tilde{H}(k)\left(W_M^{-k}\right)^* z^{-1} + \tilde{H}^*(k) - \tilde{H}^*(k) W_M^{-k} z^{-1}}{[1 - W_M^{-k} z^{-1}][1 - \left(W_M^{-k}\right)^* z^{-1}]} + \frac{\tilde{H}(0)}{1 - z^{-1}} + \frac{\tilde{H}(M/2)}{1 + z^{-1}} \right]$$

Consider

$$\frac{\tilde{H}(k) - \tilde{H}(k)\left(W_M^{-k}\right)^* z^{-1} + \tilde{H}^*(k) - \tilde{H}^*(k) W_M^{-k} z^{-1}}{[1 - W_M^{-k} z^{-1}][1 - \left(W_M^{-k}\right)^* z^{-1}]}$$

$$= \frac{\tilde{H}(k) + \tilde{H}^*(k) - \tilde{H}(k)\left(W_M^{-k}\right)^* z^{-1} - \tilde{H}^*(k) W_M^{-k} z^{-1}}{1 - 2z^{-1}[W_M^{-k} + \left(W_M^{-k}\right)^*] + z^{-2}}$$

$$= \frac{2\left|\tilde{H}(k)\right| \cos\left[\angle\tilde{H}(k)\right] - z^{-1} 2\left|\tilde{H}(k)\right| \cos\left[\angle\tilde{H}(k) - \frac{2\pi k}{M}\right]}{1 - 2z^{-1}\cos\left(\frac{2\pi k}{M}\right) + z^{-2}}$$

$$= 2\left|\tilde{H}(k)\right| H_k(z)$$

which completes the proof.

P6.20 To avoid poles on the unit circle in the frequency sampling structure, $H(z)$ is sampled at $z_k = re^{j2\pi k/M}$, $k = 0, \ldots, M-1$ where $r \approx 1$(but < 1).

1. Revised frequency-sampling structure: Replacing

$$z \to rz, \quad W_M^{-k} \to rW_M^{-k}, \quad \text{and} \quad \tilde{H}(k) \to H\left(re^{j2\pi k/M}\right) \approx \tilde{H}(k)$$

in (6.11), we obtain

$$\begin{aligned} H(z) \cong H(rz) &\cong \left(\frac{1-(rz)^{-M}}{M}\right)\sum_{k=0}^{M-1}\frac{\tilde{H}(k)}{1-rW_M^{-k}z^{-1}} \\ &= \left(\frac{1-(rz)^{-M}}{M}\right)\sum_{k=0}^{M-1}\frac{\tilde{H}(k)}{1-W_M^{-k}\left(rz^{-1}\right)} \end{aligned} \qquad (6.14)$$

Following steps similar to those in P6.19, we obtain

$$H(z) = \frac{1-(rz)^{-M}}{M}\left\{\sum_{k=1}^{L}2\left|\tilde{H}(k)\right|H_k(z) + \frac{\tilde{H}(0)}{1-rz^{-1}} + \frac{\tilde{H}(M/2)}{1+rz^{-1}}\right\}$$

where

$$H_k(z) = \frac{\cos\left[\angle\tilde{H}(k)\right]-rz^{-1}\cos\left[\angle\tilde{H}(k)-\frac{2\pi k}{M}\right]}{1-2rz^{-1}\cos\left(\frac{2\pi k}{M}\right)+r^2z^{-2}}, \quad k=1,\ldots,L$$

and M is even.

2. New MATLAB function `dir2fs`:

```
function [C,B,A,rM] = dir2fs(h,r)
% Direct form to Frequency Sampling form conversion
% -------------------------------------------------
% [C,B,A,rM] = dir2fs(h,r)
%  C = Row vector containing gains for parallel sections
%  B = Matrix containing numerator coefficients arranged in rows
%  A = Matrix containing denominator coefficients arranged in rows
% rM = r^M factor needed in the feedforward loop
%  h = impulse response vector of an FIR filter
%  r = radius of the circle over which samples are taken (r<1)
%

M = length(h);
if nargin == 1
    r = 1; rM = 1;
elseif nargin == 2
    rM = r^M;
end
H = fft(h,M); magH = abs(H); phaH = angle(H)';
% check even or odd M
if (M == 2*floor(M/2))
        L = M/2-1;   % M is even
```

```
        A1 = [1,-r,0;1,r,0];
        C1 = [real(H(1)),real(H(L+2))];
    else
         L = (M-1)/2; % M is odd
        A1 = [1,-r,0];
        C1 = [real(H(1))];
    end
    k = [1:L]';
    % initialize B and A arrays
    B = zeros(L,2); A = ones(L,3);
    % compute denominator coefficients
    A(1:L,2) = [-2*r*cos(2*pi*k/M)]; A(1:L,3) = [r*r]; A = [A;A1];
    % compute numerator coefficients
    B(1:L,1) = cos(phaH(2:L+1));
    B(1:L,2) = -r*cos(phaH(2:L+1)-(2*pi*k/M));
    % compute gain coefficients
    C = [2*magH(2:L+1),C1]';
```

3. The frequency sampling structure for the impulse response given in Example 6.6 using the above function:
   MATLAB script:

```
% 3. Example 6.6 impulse response
h = [1,2,3,2,1]/9;
[C,B,A,rM] = dir2fs(h,0.99)
C =
    0.5818
    0.0849
    1.0000
B =
   -0.8090    0.8009
    0.3090   -0.3059
A =
    1.0000   -0.6119    0.9801
    1.0000    1.6019    0.9801
    1.0000   -0.9900         0
rM =
    0.9510
```

Block diagram:
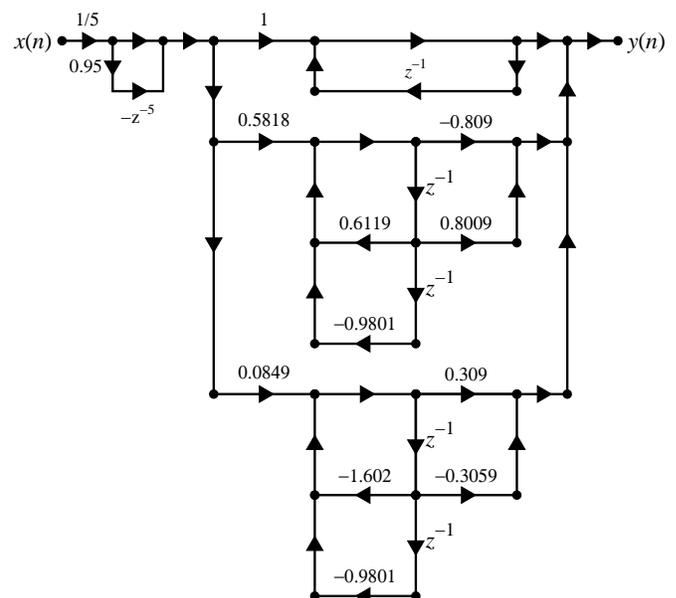
P6.21 Impulse response of an FIR filter with lattice parameters

$$K_0 = 2, \quad K_1 = 0.6, \quad K_2 = 0.3, \quad K_3 = 0.5, \quad K_4 = 0.9$$

```
% Direct form
% Given Lattice Structure
K = [2,0.6,0.3,0.5,0.9];
b = latc2dir(K)

b =
    2.0000    2.7600    2.6220    2.6740    1.8000
```

1. The block diagram of the direct form is
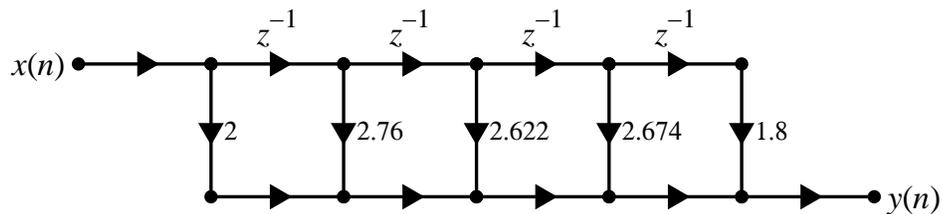
2. The block diagram of the lattice form is
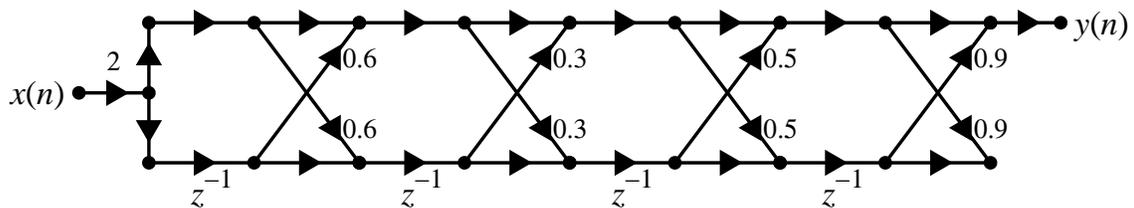
P6.22 Consider the following system function of an FIR filter

$$H(z) = 1 - 4z^{-1} + 6.4z^{-2} - 5.12z^{-3} + 2.048z^{-4} - 0.32768z^{-5}$$

1. Block diagram structures in the following form:

   (a) Normal direct form:
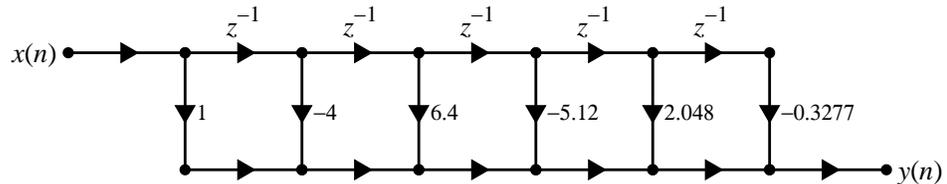
   Transposed direct form:
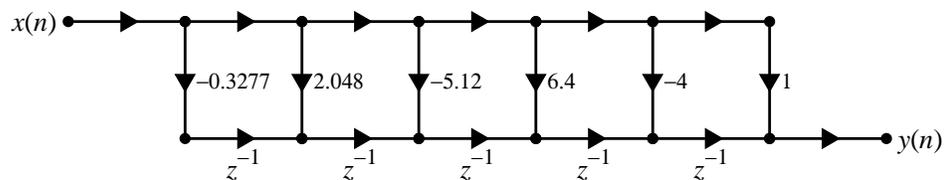
   (b) Cascade of five first-order sections: MATLAB script:

```
b = [1,-4,6.4,-5.12,2.048,-0.32768];
broots = round(real(roots(b))*10)/10; broots = broots'
broots =
     0.8000    0.8000    0.8000    0.8000    0.8000
```

   Block diagram:
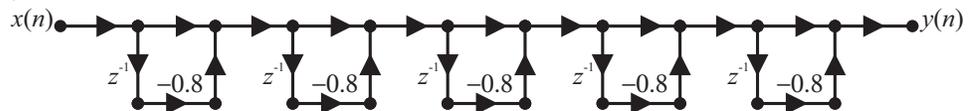
   (c) Cascade of one first-order section and two second-order sections: MATLAB script:

```
% Given FIR filter
b = [1,-4,6.4,-5.12,2.048,-0.32768]; a = 1;
[b0,B,A] = dir2cas(b,a)
b0 =
     1
B =
    1.0000   -1.5995    0.6396
    1.0000   -1.6013    0.6411
    1.0000   -0.7992         0
```

   Block diagram:
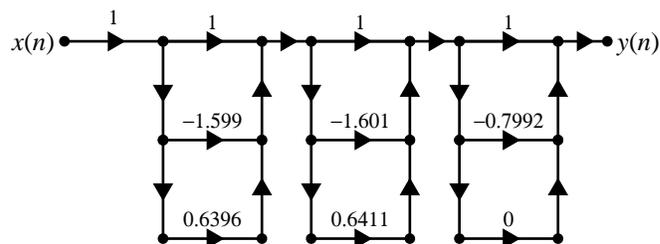
(d) Cascade of one second-order section and one third-order section: MATLAB script:

```
% Given FIR filter
b = [1,-4,6.4,-5.12,2.048,-0.32768];
broots = round(real(roots(b))*10)/10; broots = broots';
b1 = poly(broots(1:2)), b2 = poly(broots(3:5))
b1 =
    1.0000   -1.6000    0.6400
b2 =
    1.0000   -2.4000    1.9200   -0.5120
```

Block diagram:
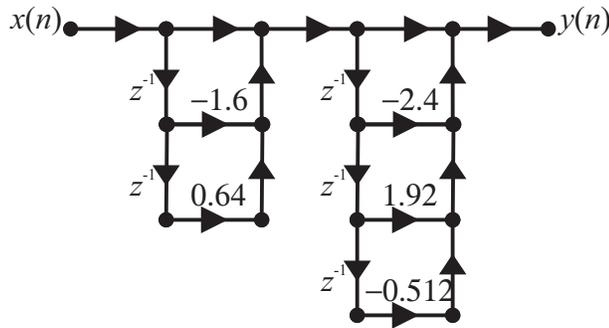
(e) Frequency-sampling structure with real coefficients: MATLAB script:

```
% Given FIR filter
b = [1,-4,6.4,-5.12,2.048,-0.32768]
[C,B,A] = dir2fs(b)
C =
    1.2934
   18.5996
    0.0003
   18.8957
B =
   -0.4142    0.9953
   -0.6645   -0.9794
A =
    1.0000   -1.0000    1.0000
    1.0000    1.0000    1.0000
    1.0000   -1.0000         0
    1.0000    1.0000         0
```

Block diagram:
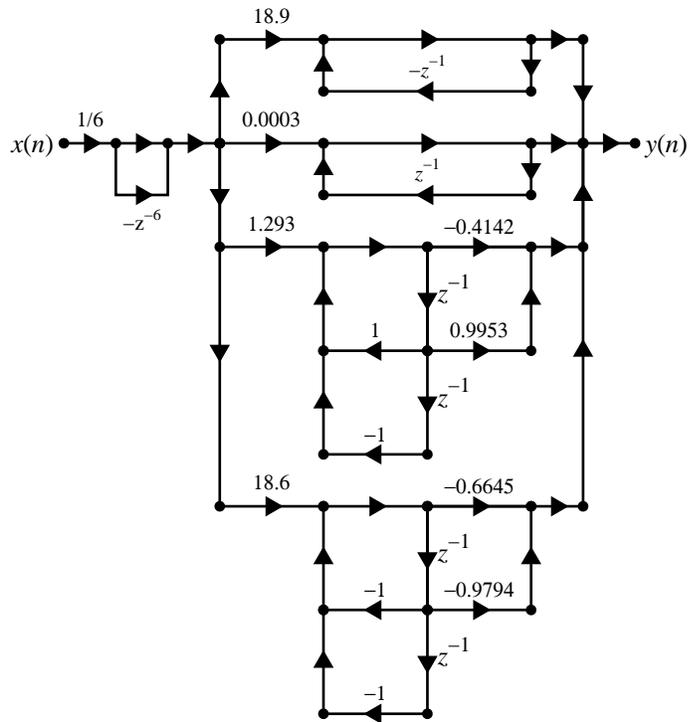
2. The computational complexity:

Structure	# of Mults	# of Adds
i. Direct	5	5
ii. Cascade-1	5	5
iii. Cascade-2	5	5
iv. Cascade-3	5	5
v. Freq. Samp	9	12

P6.23 A causal digital filter is described by the following zeros:

$$z_1 = 0.5\,e^{j60°}, \quad z_2 = 0.5\,e^{-j60°}, \quad z_3 = 2\,e^{j60°}, \quad z_4 = 2\,e^{-j60°},$$

$$z_5 = 0.25\,e^{j30°}, \quad z_6 = 0.25\,e^{-j30°}, \quad z_7 = 4\,e^{j30°}, \quad z_8 = 4\,e^{-j30°},$$

and poles: $\{p_i\}_{i=1}^8 = 0$.

1. Phase response of the filter:

```
% Pole-zero description
r1 = 0.5; theta1 = 30; r2 = 0.25; theta2 = 60;
z1 = r1*exp( j*theta1*pi/180); z2 = r1*exp(-j*theta1*pi/180);
z3 = 1/r1*exp( j*theta1*pi/180); z4 = 1/r1*exp(-j*theta1*pi/180);
z5 = r2*exp( j*theta2*pi/180); z6 = r2*exp(-j*theta2*pi/180);
z7 = 1/r2*exp( j*theta2*pi/180); z8 = 1/r2*exp(-j*theta2*pi/180);
z = [z1;z2;z3;z4;z5;z6;z7;z8]; b = poly(z)
b =
    1.0000   -8.5801   42.7155 -113.2754  162.5092 -113.2754   42.7155
   -8.5801    1.0000
```

Hence

$$H(z) = 1 - 8.5801z^{-1} + 42.7155z^{-2} - 113.2754z^{-3} + 162.5092z^{-4} - 113.2754z^{-5} + 42.7155z^{-6} - 8.5801z^{-7} + z^{-8}$$

Due to symmetry in the coefficients, the phase response is $\angle H(e^{j\omega}) = -4\omega$, which is linear.

2. Impulse response of the filter:

$$h(n) = \{1, -8.5801, 42.7155, -113.2754, 162.5092, -113.2754, 42.7155, -8.5801, 1\}$$

3. Direct form structure:
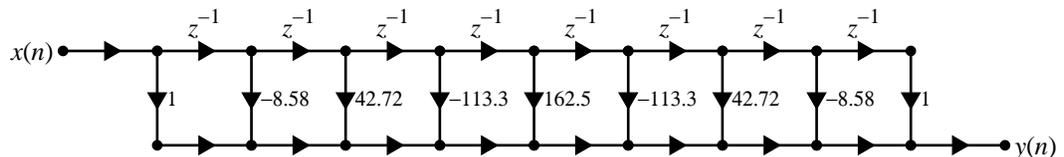
4. Linear-phase form structure:
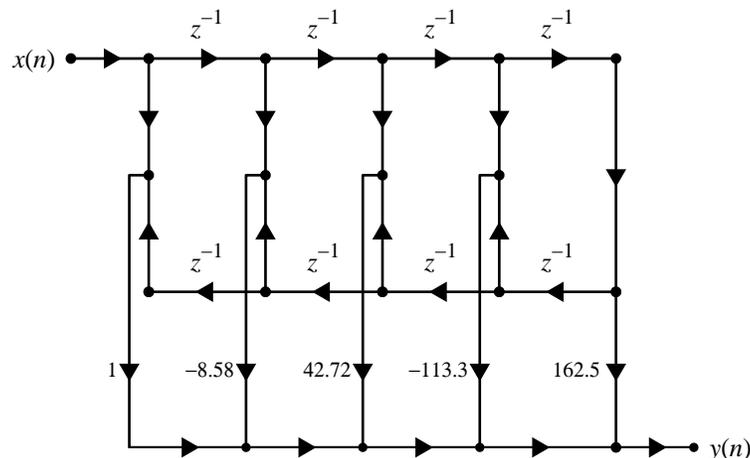
Chapter 7

FIR Filter Design

P7.1 The absolute and relative (dB) specifications for a lowpass filter are related by (7.1) and (7.2).

(a)
```
function [Rp,As] = delta2db(delta1,delta2)
% Conversion from Absolute delta specs to Relative dB specs
% [Rp,As] = delta2db(delta1,delta2)
% Rp = Passband ripple
% As = Stopband attenuation
% d1 = Passband tolerance
% d2 = Stopband tolerance
Rp = -20*log10((1-delta1)/(1+delta1));
As = -20*log10(delta2/(1+delta1));
```
MATLAB Verification:

```
delta1 = 0.01; delta2 = 0.001;
[Rp,As] = delta2db(delta1,delta2)
Rp =
    0.1737
As =
   60.0864
```

(b)
```
function [d1,d2] = db2delta(Rp,As)
% Conversion from Relative dB specs to Absolute delta specs.
% [d1,d2] = db2delta(Rp,As)
% d1 = Passband tolerance
% d2 = Stopband tolerance
% Rp = Passband ripple
% As = Stopband attenuation
K = 10^(Rp/20);
d1 = (K-1)/(K+1); d2 = (1+d1)*(10^(-As/20));
```
MATLAB Verification:

```
[delta1,delta2] = db2delta(Rp,As)
delta1 =
    0.0144
delta2 =
    0.0032
```

P7.2 The Type-1 linear-phase FIR filter is characterized by a symmetric $h(n)$ and M-odd. Hence

$$h(n) = h(M - 1 - n)), \quad 0 \le n \le M - 1, \quad \alpha = \frac{M - 1}{2} \text{ is an integer}$$

Consider the frequency response $H(e^{j\omega})$ given by

$$H(e^{j\omega}) = \sum_{n=0}^{M-1} h(n)e^{-j\omega n} = \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega n} + h\left(\frac{M-1}{2}\right)e^{-j\omega\left(\frac{M-1}{2}\right)} + \sum_{n=\frac{M+1}{2}}^{M-1} h(n)e^{-j\omega n}$$

Using change of variables in the third sum:

$$n \to M - 1 - n \Rightarrow \frac{M+1}{2} \to \frac{M-3}{2}, \ M - 1 \to 0, \text{ and } h(M - 1 - n) \to h(n)$$

we obtain

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega n} + h\left(\frac{M-1}{2}\right)e^{-j\omega\left(\frac{M-1}{2}\right)} + \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega(M-1-n)}$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)}\left\{\sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega n + j\omega\left(\frac{M-1}{2}\right)} + h\left(\frac{M-1}{2}\right) + \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega(M-1-n)+j\omega\left(\frac{M-1}{2}\right)}\right\}$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)}\left\{h\left(\frac{M-1}{2}\right) + \sum_{n=0}^{\frac{M-3}{2}} h(n)\left[e^{+j\omega\left(\frac{M-1}{2}-n\right)} + e^{-j\omega\left(\frac{M-1}{2}-n\right)}\right]\right\}$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)}\left\{h\left(\frac{M-1}{2}\right) + \sum_{n=0}^{\frac{M-3}{2}} 2h(n)\cos\left[\omega\left(\frac{M-1}{2}-n\right)\right]\right\}$$

Hence

$$H_r(\omega) = \left\{h\left(\frac{M-1}{2}\right) + \sum_{n=0}^{\frac{M-3}{2}} 2h(n)\cos\left[\omega\left(\frac{M-1}{2}-n\right)\right]\right\}$$

Define $a(n) = 2h\left(\frac{M-1}{2} - n\right), n = 1, 2, \ldots, (M-1)/2$ and $a(0) = h[(M-1)/2]$. Then

$$H_r(\omega) = \left\{a(0) + \sum_{n=1}^{\frac{M-1}{2}} a(n)\cos(\omega n)\right\} = \sum_{n=0}^{\frac{M-1}{2}} a(n)\cos(\omega n)$$

$$= \sum_{n=0}^{L} a(n)\cos(\omega n); \quad L = \frac{M-1}{2}$$

P7.3 The Type-2 linear-phase FIR filter is characterized by symmetric $h(n)$ and M-even, i.e.,

$$h(n) = h(M - 1 - n), \ 0 \le n \le M - 1; \quad \alpha = \frac{M-1}{2} \text{ is not an integer}$$

(a) Consider,

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{M-1} h(n) e^{-j\omega n} = \sum_{n=0}^{\frac{M}{2}-1} h(n) e^{-j\omega n} + \sum_{\frac{M}{2}}^{M-1} h(n) e^{-j\omega n}$$

Change of variable in the second sum above:

$$n \to M - 1 - n \Rightarrow \frac{M}{2} \to \frac{M}{2} - 1, \ M - 1 \to 0, \text{ and } h(n) \to h(n)$$

Hence,

$$\begin{aligned}
H\left(e^{j\omega}\right) &= \sum_{n=0}^{\frac{M}{2}-1} h(n) e^{-j\omega n} + \sum_{n=0}^{\frac{M}{2}-1} h(n) e^{-j\omega(M-1-n)} \\
&= e^{-j\omega\left(\frac{M-1}{2}\right)} \sum_{n=0}^{\frac{M}{2}-1} h(n) \left\{ e^{-j\omega n + j\omega\left(\frac{M-1}{2}\right)} + e^{-j\omega(M-1-n)+j\omega\left(\frac{M-1}{2}\right)} \right\} \\
&= e^{-j\omega\left(\frac{M-1}{2}\right)} \sum_{n=0}^{\frac{M}{2}-1} h(n) \left\{ e^{+j\omega\left(\frac{M-1}{2}-n\right)} + e^{-j\omega\left(\frac{M-1}{2}-n\right)} \right\} \\
&= e^{-j\omega\left(\frac{M-1}{2}\right)} \sum_{n=0}^{\frac{M}{2}-1} 2h(n) \cos\left[\left(\frac{M-1}{2}-n\right)\omega\right]
\end{aligned}$$

Change of variable:

$$\frac{M}{2} - n \to n \Rightarrow n = 0 \to n = \frac{M}{2}, \ n = \frac{M}{2} - 1 \to n = 1$$

and

$$\cos\left[\left(\frac{M-1}{2}-n\right)\omega\right] \to \cos\left[\omega\left(n-\frac{1}{2}\right)\right]$$

Hence,

$$H\left(e^{j\omega}\right) = e^{-j\omega\left(\frac{M-1}{2}\right)} \sum_{n=1}^{\frac{M}{2}} 2h\left(\frac{M}{2}-n\right) \cos\left[\omega\left(n-\frac{1}{2}\right)\right]$$

Define $b(n) = 2h\left(\frac{M}{2}-n\right)$. Then,

$$H\left(e^{j\omega}\right) = e^{-j\omega\left(\frac{M-1}{2}\right)} \sum_{n=1}^{\frac{M}{2}} b(n) \cos\left[\omega\left(n-\frac{1}{2}\right)\right] \Rightarrow H_r(\omega) = \sum_{n=1}^{\frac{M}{2}} b(n) \cos\left[\omega\left(n-\frac{1}{2}\right)\right]$$

(b) Now $\cos\left[\omega\left(n-\frac{1}{2}\right)\right]$ can be recursively computed using the trigonometric identity:

$$\cos\left[(2n-1)\frac{\omega}{2}\right] = 2\cos\left[(n-1)\omega\right]\cos\left(\frac{\omega}{2}\right) - \cos\left[(2n-3)\frac{\omega}{2}\right], \ n = 2, 3, \ldots$$

and hence can be written as a linear combination of higher harmonics in $\cos\omega$ multiplied by $\cos\left(\frac{\omega}{2}\right)$, i.e.,

$$\begin{aligned}
\cos\left(\frac{1\omega}{2}\right) &= \cos\frac{\omega}{2}\{\cos 0\omega\} \\
\cos\left(\frac{3\omega}{2}\right) &= \cos\frac{\omega}{2}\{2\cos\omega - 1\} \\
\cos\left(\frac{5\omega}{2}\right) &= \cos\frac{\omega}{2}\{\cos 0\omega - \cos\omega + \cos 2\omega\}
\end{aligned}$$

etc. Note that the lowest harmonic frequency is zero and the highest harmonic frequency is $(n - 1)\,\omega$ in the $\cos\omega\left(n - \frac{1}{2}\right)$ expansion. Hence,

$$
H_r(\omega) = \sum_{n=1}^{\frac{M}{2}} b(n) \cos\left[\omega\left(n - \frac{1}{2}\right)\right] = \cos\frac{\omega}{2} \sum_{0}^{\frac{M}{2}-1} \tilde{b}(n) \cos(\omega n)
$$

where $\tilde{b}(n)$ are related to $b(n)$ through the above trigonometric identities.

P7.4 The Type-3 linear-phase FIR filter is characterized by antisymmetric $h(n)$ and M odd, i.e.,

$$h(n) = -h(M-1-n), \ 0 \le n \le M-1; \ h\left(\frac{M-1}{2}\right) = 0; \quad \alpha = \frac{M-1}{2} \text{ is an integer}$$

(a) Consider,

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{M-1} h(n) e^{-j\omega n} = \sum_{n=0}^{\frac{M-3}{2}} h(n) e^{-j\omega n} + \sum_{\frac{M+1}{2}}^{M-1} h(n) e^{-j\omega n}$$

Change of variable in the second sum above:

$$n \to M-1-n \Rightarrow \frac{M+1}{2} \to \frac{M-3}{2}, \ M-1 \to 0, \ \text{and} \ h(n) \to -h(n)$$

Hence,

$$
\begin{aligned}
H\left(e^{j\omega}\right) &= \sum_{n=0}^{\frac{M-3}{2}} h(n) e^{-j\omega n} - \sum_{n=0}^{\frac{M-3}{2}} h(n) e^{-j\omega(M-1-n)} \\
&= e^{-j\left(\frac{M-1}{2}\right)} \sum_{n=0}^{\frac{M-3}{2}} h(n) \left\{ e^{-j\omega n + j\omega\left(\frac{M-1}{2}\right)} - e^{-j\omega(M-1-n)+j\omega\left(\frac{M-1}{2}\right)} \right\} \\
&= e^{-j\left(\frac{M-1}{2}\right)} \sum_{n=0}^{\frac{M-3}{2}} h(n) \left\{ e^{+j\omega\left(\frac{M-1}{2}-n\right)} - e^{-j\omega\left(\frac{M-1}{2}-n\right)} \right\} \\
&= je^{-j\left(\frac{M-1}{2}\right)} \sum_{n=0}^{\frac{M-3}{2}} 2h(n) \sin\left[\left(\frac{M-1}{2}-n\right)\omega\right]
\end{aligned}
$$

Change of variable:

$$\frac{M-1}{2} - n \to n \Rightarrow n = 0 \to n = \frac{M-1}{2}, \ n = \frac{M-3}{2} \to n = 1$$

and

$$\sin\left[\left(\frac{M-1}{2}-n\right)\omega\right] \to \sin(\omega n)$$

Hence,

$$H\left(e^{j\omega}\right) = je^{-j\omega\left(\frac{M-1}{2}\right)} \sum_{n=1}^{\frac{M-1}{2}} 2h\left(\frac{M-1}{2}-n\right) \sin(\omega n)$$

Define $c(n) = 2h\left(\frac{M-1}{2}-n\right)$. Then,

$$H\left(e^{j\omega}\right) = je^{-j\omega\left(\frac{M-1}{2}\right)} \sum_{n=1}^{\frac{M-1}{2}} c(n) \sin(\omega n) \Rightarrow H_r(\omega) = \sum_{n=1}^{\frac{M-1}{2}} c(n) \sin(\omega n)$$

(b) Now $\sin(\omega n)$ can be expressed as $\sin(\omega)$ times a polynomial in $\cos(\omega)$ as

$$\sin(\omega n) = \sin(\omega) U_{n-1}[\cos(\omega)]$$

where U_n is a Chebyshev polynomial of the second kind. Thus $\sin(\omega n)$ can be written as a linear combination of higher harmonics in $\cos(\omega)$ multiplied by $\sin(\omega)$, i.e.,

$$
\begin{aligned}
\sin(\omega) &= \sin(\omega)\{\cos(0\omega)\} \\
\sin(2\omega) &= \sin(\omega)\{2\cos(\omega)\} \\
\sin(3\omega) &= \sin(\omega)\{\cos 0\omega + 2\cos 2\omega\}
\end{aligned}
$$

etc. Note that the lowest harmonic frequency is zero and the highest harmonic frequency is $(n-1)\,\omega$ in the $\sin \omega n$ expansion. Hence,

$$
H_r(\omega) = \sum_{n=1}^{\frac{M-1}{2}} c(n) \sin(\omega n) = \sin \omega \sum_{0}^{\frac{M-3}{2}} \tilde{c}(n) \cos(\omega n)
$$

where $\tilde{c}(n)$ are related to $c(n)$ through the above trigonometric identities.

P7.5 The Type-4 linear-phase FIR filter is characterized by antisymmetric $h(n)$ and M-even, i.e.,

$$h(n) = -h(M-1-n),\ 0 \le n \le M-1; \quad \alpha = \frac{M-1}{2} \text{ is not an integer}$$

(a) Consider,

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{M-1} h\left(n\right) e^{-j\omega n} = \sum_{n=0}^{\frac{M}{2}-1} h\left(n\right) e^{-j\omega n} + \sum_{\frac{M}{2}}^{M-1} h\left(n\right) e^{-j\omega n}$$

Change of variable in the second sum above:

$$n \to M - 1 - n \Rightarrow \frac{M}{2} \to \frac{M}{2} - 1,\ M - 1 \to 0,\ \text{and } h(M-1-n) \to -h(n)$$

Hence,

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{\frac{M}{2}-1} h\left(n\right) e^{-j\omega n} - \sum_{n=0}^{\frac{M}{2}-1} h\left(n\right) e^{-j\omega(M-1-n)}$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)} \sum_{n=0}^{\frac{M}{2}-1} h(n) \left\{ e^{-j\omega n + j\omega\left(\frac{M-1}{2}\right)} - e^{-j\omega(M-1-n)+j\omega\left(\frac{M-1}{2}\right)} \right\}$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)} \sum_{n=0}^{\frac{M}{2}-1} h(n) \left\{ e^{+j\omega\left(\frac{M-1}{2}-n\right)} - e^{-j\omega\left(\frac{M-1}{2}-n\right)} \right\}$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)} \sum_{n=0}^{\frac{M}{2}-1} 2jh(n) \sin\left[\left(\frac{M-1}{2}-n\right)\omega\right]$$

Change of variable:

$$\frac{M}{2} - n \to n \Rightarrow n = 0 \to n = \frac{M}{2},\ n = \frac{M}{2} - 1 \to n = 1$$

and

$$\sin\left[\left(\frac{M-1}{2}-n\right)\omega\right] \to \sin\left[\omega\left(n-\frac{1}{2}\right)\right]$$

Hence,

$$H\left(e^{j\omega}\right) = e^{-j\omega\left(\frac{\pi}{2}-\frac{M-1}{2}\right)} \sum_{n=1}^{\frac{M}{2}} 2h\left(\frac{M}{2}-n\right) \sin\left[\omega\left(n-\frac{1}{2}\right)\right]$$

Define $d(n) = 2h\left(\frac{M}{2}-n\right)$. Then,

$$H\left(e^{j\omega}\right) = e^{-j\omega\left(\frac{\pi}{2}\frac{M-1}{2}\right)} \sum_{n=1}^{\frac{M}{2}} d\left(n\right) \sin\left[\omega\left(n-\frac{1}{2}\right)\right] \Rightarrow H_r\left(\omega\right) = \sum_{n=1}^{\frac{M}{2}} d\left(n\right) \sin\left[\omega\left(n-\frac{1}{2}\right)\right]$$

(b) Now $\sin\left[\omega\left(n-\frac{1}{2}\right)\right]$ can be written as a linear combination of higher harmonics in $\cos\omega$ multiplied by $\sin\left(\frac{\omega}{2}\right)$, i.e.,

$$\sin\left(\frac{1\omega}{2}\right) = \sin\frac{\omega}{2}\{\cos 0\omega\}$$

$$\sin\left(\frac{3\omega}{2}\right) = \sin\frac{\omega}{2}\{1 + 2\cos\omega\}$$

$$\sin\left(\frac{5\omega}{2}\right) = \sin\frac{\omega}{2}\{\cos 0\omega + 2\cos\omega + 2\cos 2\omega\}$$

etc. Note that the lowest harmonic frequency is zero and the highest harmonic frequency is $(n-1)\,\omega$ in the $\sin\omega\left(n-\frac{1}{2}\right)$ expansion. Hence,

$$H_r(\omega) = \sum_{n=1}^{\frac{M}{2}} d(n) \sin\left[\omega\left(n-\frac{1}{2}\right)\right] = \sin\frac{\omega}{2} \sum_{0}^{\frac{M}{2}-1} \tilde{d}(n)\cos(\omega n)$$

where $\tilde{d}(n)$ are related to $d(n)$ through the above trigonometric identities.

P7.6 MATLAB Function:

```
function [Hr,w,P,L] = ampl_res(h);
%
% function [Hr,w,P,L] = ampl_res(h)
%
% Computes Amplitude response Hr(w) and its polynomial P of order L,
%   given a linear-phase FIR filter impulse response h.
% The type of filter is determined automatically by the subroutine.
%
% Hr = Amplitude Response
%  w = frequencies between [0 pi] over which Hr is computed
%  P = Polynomial coefficients
%  L = Order of P
%  h = Linear Phase filter impulse response
%

 M = length(h);
 L = floor(M/2);
if fix(abs(h(1:1:L))*10^10) ~= fix(abs(h(M:-1:M-L+1))*10^10)
    error('Not a linear-phase impulse response')
end

if 2*L ~= M
    if fix(h(1:1:L)*10^10) == fix(h(M:-1:M-L+1)*10^10)
        disp('*** Type-1 Linear-Phase Filter ***')
        [Hr,w,P,L] = hr_type1(h);
    elseif fix(h(1:1:L)*10^10) == -fix(h(M:-1:M-L+1)*10^10)
        disp('*** Type-3 Linear-Phase Filter ***')
        h(L+1) = 0;
        [Hr,w,P,L] = hr_type3(h);
    end
else
    if fix(h(1:1:L)*10^10) == fix(h(M:-1:M-L+1)*10^10)
        disp('*** Type-2 Linear-Phase Filter ***')
        [Hr,w,P,L] = hr_type2(h);
    elseif fix(h(1:1:L)*10^10) == -fix(h(M:-1:M-L+1)*10^10)
        disp('*** Type-4 Linear-Phase Filter ***')
        [Hr,w,P,L] = hr_type4(h);
    end
end
```

MATLAB Verification:

```
%% P7.6: Amplitude Response Function
clc; close all;

%% 1. h_I(n)
```

```
n = 0:10; h_I = (0.9).^abs(n-5).*cos(pi*(n-5)/12);
[Hr,w,P,L] = ampl_res(h_I); P, L
*** Type-1 Linear-Phase Filter ***
P =
    1.0000    1.7387    1.4030    1.0310    0.6561    0.3057
L =
     5

%% 2. h_II(n)
n = 0:9; h_II = (0.9).^abs(n-4.5).*cos(pi*(n-4.5)/11);
[Hr,w,P,L] = ampl_res(h_II); P, L
*** Type-2 Linear-Phase Filter ***
P =
    1.8781    1.5533    1.1615    0.7478    0.3507
L =
     5

%% 3. h_III(n)
n = 0:10; h_III = (0.9).^abs(n-5).*sin(pi*(n-5)/12);
[Hr,w,P,L] = ampl_res(h_III); P, L
*** Type-3 Linear-Phase Filter ***
P =
         0   -0.4659   -0.8100   -1.0310   -1.1364   -1.1407
L =
     5

%% 4. h_IV(n)
n = 0:9; h_IV = (0.9).^abs(n-4.5).*sin(pi*(n-4.5)/11);
[Hr,w,P,L] = ampl_res(h_IV); P, L
*** Type-4 Linear-Phase Filter ***
P =
   -0.2700   -0.7094   -1.0064   -1.1636   -1.1944
L =
     5

%% 5. h(n)
n = 0:9; h_IV = (0.9).^n.*cos(pi*(n-5)/12);
[Hr,w,P,L] = ampl_res(h_IV); P, L
??? Error using ==> ampl_res
Not a linear-phase impulse response
```

P7.7 Properties of linear-phase FIR filters:

1. The filter $H(z)$ has the following four zeros

$$z_1 = re^{j\theta}, \quad z_2 = \frac{1}{r}e^{j\theta}, \quad z_3 = re^{-j\theta}, \quad z_4 = \frac{1}{r}e^{-j\theta}$$

The system function can be written as

$$
\begin{aligned}
H(z) &= \left(1 - z_1 z^{-1}\right)\left(1 - z_2 z^{-1}\right)\left(1 - z_3 z^{-1}\right)\left(1 - z_4 z^{-1}\right) \\
&= \left(1 - re^{j\theta} z^{-1}\right)\left(1 - \frac{1}{r}e^{j\theta} z^{-1}\right)\left(1 - re^{-j\theta} z^{-1}\right)\left(1 - \frac{1}{r}e^{-j\theta} z^{-1}\right) \\
&= \left\{1 - (2r\cos\theta)\,z^{-1} + r^2 z^{-2}\right\}\left\{1 - \left(2r^{-1}\cos\theta\right) z^{-1} + r^{-2} z^{-2}\right\} \\
&= 1 - 2\cos\theta\left(r + r^{-1}\right) z^{-1} + \left(r^2 + r^{-2} + 4\cos^2\theta\right) z^{-2} - 2\cos\theta\left(r + r^{-1}\right) z^{-3} + z^{-4}
\end{aligned}
$$

Hence the impulse response of the filter is

$$h(n) = \left\{\underset{\uparrow}{1}, -2\cos\theta\left(r + r^{-1}\right), \left(r^2 + r^{-2} + 4\cos^2\theta\right), -2\cos\theta\left(r + r^{-1}\right), 1\right\}$$

which is a finite-duration symmetric impulse response. This implies that the filter is a linear-phase FIR filter.

2. The filter $H(z)$ has the following two zeros

$$z_1 = e^{j\theta} \quad \text{and} \quad z_2 = e^{-j\theta}$$

The system function can be written as

$$
\begin{aligned}
H(z) &= \left(1 - z_1 z^{-1}\right)\left(1 - z_2 z^{-1}\right) = \left(1 - e^{j\theta} z^{-1}\right)\left(1 - e^{-j\theta} z^{-1}\right) \\
&= \left\{1 - (2\cos\theta)\,z^{-1} + z^{-2}\right\}
\end{aligned}
$$

Hence the impulse response of the filter is

$$h(n) = \left\{\underset{\uparrow}{1}, -2\cos\theta, 1\right\}$$

which is a finite-duration symmetric impulse response. This implies that the filter is a linear-phase FIR filter.

3. The filter $H(z)$ has the following two zeros

$$z_1 = r \quad \text{and } z_2 = \frac{1}{r}$$

The system function can be written as

$$
\begin{aligned}
H(z) &= \left(1 - z_1 z^{-1}\right)\left(1 - z_2 z^{-1}\right) = \left(1 - rz^{-1}\right)\left(1 - \frac{1}{r}z^{-1}\right) \\
&= 1 - \left(r + r^{-1}\right) z^{-1} + z^{-2}
\end{aligned}
$$

Hence the impulse response of the filter is

$$h(n) = \left\{\underset{\uparrow}{1}, -\left(r + r^{-1}\right), 1\right\}$$

which is a finite-duration symmetric impulse response. This implies that the filter is a linear-phase FIR filter.

4. If $H(z)$ has a zero at $z_1 = 1$ or a zero at $z_1 = -1$ then $H(z)$ can be written as

$$H(z) = (1 - z^{-1}) \text{ or } H(z) = (1 + z^{-1})$$

with impulse responses

$$h(n) = \left\{ \underset{\uparrow}{1}, 1 \right\} \text{ or } h(n) = \left\{ \underset{\uparrow}{1}, -1 \right\}$$

both of which are finite-duration with symmetric and antisymmetric impulse responses, respectively. This implies that the filter is a linear-phase FIR filter.

5. Zero plots using MATLAB :

```
%% P7.7: Pole-Zero Plots of Linear-Phase Filters
clc; close all;
%% 1. h_I(n)
n = 0:10; h_I = (0.9).^abs(n-5).*cos(pi*(n-5)/12);
Hf_1 = figure('Units','inches','position',[1,1,3,3],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,3,3]);
set(Hf_1,'NumberTitle','off','Name','P7.7.1');
zplane(h_I,1); title('{\ith}_{I}({\itn})')
%% 2. h_II(n)
n = 0:9; h_II = (0.9).^abs(n-4.5).*cos(pi*(n-4.5)/11);
Hf_2 = figure('Units','inches','position',[1,1,3,3],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,3,3]);
set(Hf_2,'NumberTitle','off','Name','P7.7.2');
zplane(h_II,1); title('{\ith}_{II}({\itn})')
%% 3. h_III(n)
n = 0:10; h_III = (0.9).^abs(n-5).*sin(pi*(n-5)/12);
Hf_3 = figure('Units','inches','position',[1,1,3,3],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,3,3]);
set(Hf_3,'NumberTitle','off','Name','P7.7.3');
zplane(h_III,1); title('{\ith}_{III}({\itn})')
%% 4. h_IV(n)
n = 0:9; h_IV = (0.9).^abs(n-4.5).*sin(pi*(n-4.5)/11);
Hf_4 = figure('Units','inches','position',[1,1,3,3],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,3,3]);
set(Hf_4,'NumberTitle','off','Name','P7.7.4');
zplane(h_IV,1); title('{\ith}_{IV}({\itn})')
%% 5. h(n)
n = 0:9; h = (0.9).^n.*cos(pi*(n-5)/12);
Hf_5 = figure('Units','inches','position',[1,1,3,3],...
    'color',[0,0,0],'paperunits','inches','paperposition',[0,0,3,3]);
set(Hf_5,'NumberTitle','off','Name','P7.7.5');
zplane(h,1); title('{\ith}({\itn})')
```

The zero-plots are shown in Figure 7.1. Clearly the first four plots satisfy the zero-placement requirements and hence the corresponding filters are linear-phase filters.
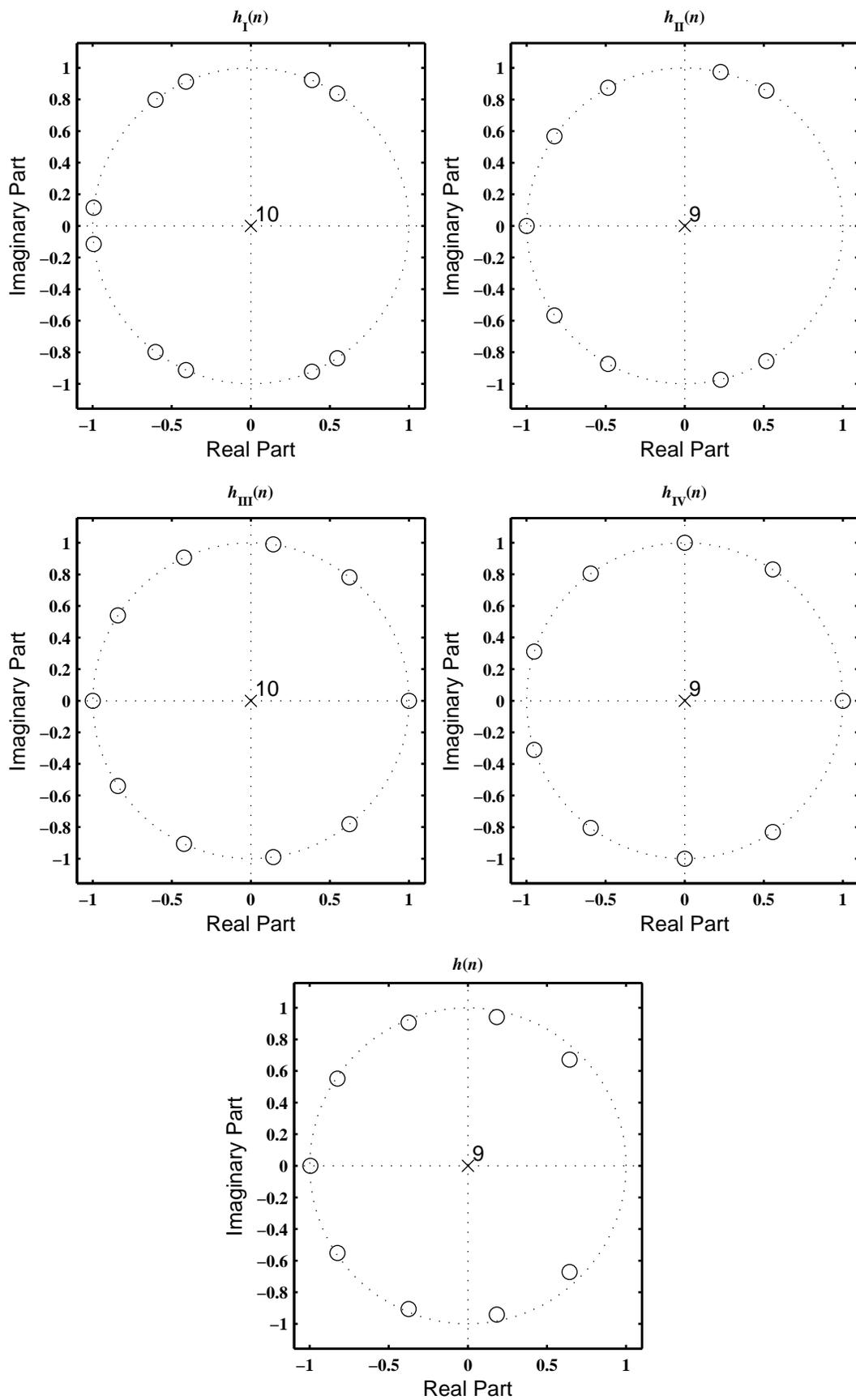
Figure 7.1: Plots of zeros in Problem 7.7

P7.8 A notch filter is an LTI system which is used to eliminate an arbitrary frequency $\omega = \omega_0$. The ideal linear-phase notch filter frequency response is given by

$$H_d\left(e^{j\omega}\right) = \begin{cases} 0, & |\omega| = \omega_0; \\ 1 \cdot e^{-j\alpha\omega}, & \text{otherwise.} \end{cases} \qquad (\alpha \text{ is a delay in samples})$$

(a) Determine the ideal impulse response, $h_d(n)$, of the ideal notch filter.

(b) Using $h_d(n)$, design a linear-phase FIR notch filter using a length 51 rectangular window to eliminate the frequency $\omega_0 = \pi/2$ rad/sample. Plot amplitude the response of resulting filter.

(c) Repeat the above part using a length 51 Hamming window. Compare your results.

P7.9 Design of a linear-phase bandpass filter using the Hann window design technique with specifications:

$$
\begin{aligned}
\text{lower stopband edge:} & \quad 0.2\pi \\
\text{upper stopband edge:} & \quad 0.75\pi \\
\text{lower passband edge:} & \quad 0.35\pi \\
\text{upper passband edge:} & \quad 0.55\pi
\end{aligned}
\qquad
\begin{aligned}
A_s &= 40 \text{ dB} \\
\\
R_p &= 0.25 \text{ dB}
\end{aligned}
$$

```
clc; close all;
%% Specifications:
ws1 = 0.2*pi;  % lower stopband edge
wp1 = 0.35*pi; % lower passband edge
wp2 = 0.55*pi; % upper passband edge
ws2 = 0.75*pi; % upper stopband edge
 Rp = 0.25;    % passband ripple
 As = 40;      % stopband attenuation
%
% Select the min(delta1,delta2) since delta1=delta2 in windodow design
[delta1,delta2] = db2delta(Rp,As);
if (delta1 < delta2)
   delta2 = delta1; disp('Delta1 is smaller than delta2')
   [Rp,As] = delta2db(delta1,delta2)
end
%
tr_width = min((wp1-ws1),(ws2-wp2));
M = ceil(6.2*pi/tr_width); M = 2*floor(M/2)+1, % choose odd M
M =
    43
n = 0:M-1; w_han = (hann(M))';
wc1 = (ws1+wp1)/2; wc2 = (ws2+wp2)/2;
hd = ideal_lp(wc2,M)-ideal_lp(wc1,M); h = hd .* w_han;
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
Rpd = -min(db((wp1/delta_w)+1:(wp2/delta_w)+1)), % Actual passband ripple
Rpd =
    0.1030
Asd = floor(-max(db(1:(ws1/delta_w)+1))),        % Actual Attn
Asd =
    44
%
%% Filter Response Plots
Hf_1 = figure('Units','inches','position',[1,1,6,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P7.9');
subplot(2,2,1); Hs_1= stem(n,hd,'filled'); set(Hs_1,'markersize',3);
title('Ideal Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h_d(n)')
subplot(2,2,2); Hs_1 = stem(n,w_han,'filled'); set(Hs_1,'markersize',3);
axis([-1,M,-0.1,1.1]); xlabel('n'); ylabel('w_{han}(n)'); title('Hann Window');
set(gca,'XTick',[0;M-1],'fontsize',8); set(gca,'YTick',[0;1],'fontsize',8)
```

```
subplot(2,2,3); Hs_1 = stem(n,h,'filled'); set(Hs_1,'markersize',3);
title('Actual Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h(n)')
subplot(2,2,4); plot(w/pi,db,'linewidth',1); title('Magnitude Response in dB');
axis([0,1,-As-30,5]); xlabel('\omega/\pi'); ylabel('Decibels')
set(gca,'XTick',[0;0.2;0.35;0.55;0.75;1])
set(gca,'XTickLabel',[' 0  ';'0.2 ';'0.35';'0.55';'0.75';' 1  '],'fontsize',8)
set(gca,'YTick',[-40;0]); set(gca,'YTickLabel',[' 40';' 0 ']);grid
```

The filter response plots are shown in Figure 7.2.

Figure 7.2: Filter design plots in Problem 7.9

P7.10 Design of a bandstop filter using the Hamming window design technique with specifications:

$$
\begin{array}{llll}
\text{lower stopband edge:} & 0.4\pi & & \\
\text{upper stopband edge:} & 0.6\pi & A_s = 50 \text{ dB} \\
\text{lower passband edge:} & 0.3\pi & & \\
\text{upper passband edge:} & 0.7\pi & R_p = 0.2 \text{ dB}
\end{array}
$$

```
clc; close all;
%% Specifications:
wp1 = 0.3*pi;  % lower passband edge
ws1 = 0.4*pi;  % lower stopband edge
ws2 = 0.6*pi;  % upper stopband edge
wp2 = 0.7*pi;  % upper passband edge
 Rp = 0.2;       % passband ripple
 As = 50;        % stopband attenuation
%
% Select the min(delta1,delta2) since delta1=delta2 in windodow design
[delta1,delta2] = db2delta(Rp,As);
if (delta1 < delta2)
   delta2 = delta1; disp('Delta1 is smaller than delta2')
   [Rp,As] = delta2db(delta1,delta2)
end
%
tr_width = abs(min((wp1-ws1),(ws2-wp2)));
M = ceil(6.6*pi/tr_width); M = 2*floor(M/2)+1, % choose odd M
M =
    67
n = 0:M-1; w_ham = (hamming(M))';
wc1 = (ws1+wp1)/2; wc2 = (ws2+wp2)/2;
hd = ideal_lp(pi,M)+ideal_lp(wc1,M)-ideal_lp(wc2,M); h = hd .* w_ham;
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
Asd = floor(-max(db(ceil(ws1/delta_w)+1:floor(ws2/delta_w)+1))), % Actual Attn
Asd =
    50
Rpd = -min(db(1:floor(wp1/delta_w)+1)),              % Actual passband ripple
Rpd =
    0.0435
%
%% Filter Response Plots
Hf_1 = figure('Units','inches','position',[1,1,6,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P7.10');
subplot(2,2,1); Hs_1= stem(n,hd,'filled'); set(Hs_1,'markersize',3);
title('Ideal Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h_d(n)')
subplot(2,2,2); Hs_1 = stem(n,w_ham,'filled'); set(Hs_1,'markersize',3);
axis([-1,M,-0.1,1.1]); xlabel('n'); ylabel('w_{ham}(n)'); title('Hamming Window');
```

```
set(gca,'XTick',[0;M-1],'fontsize',8); set(gca,'YTick',[0;1],'fontsize',8)
subplot(2,2,3); Hs_1 = stem(n,h,'filled'); set(Hs_1,'markersize',3);
title('Actual Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h(n)')
subplot(2,2,4); plot(w/pi,db,'linewidth',1); title('Magnitude Response in dB');
axis([0,1,-As-30,5]); xlabel('\omega/\pi'); ylabel('Decibels')
set(gca,'XTick',[0;0.3;0.4;0.6;0.7;1])
set(gca,'XTickLabel',[' 0 ';'0.3';'0.4';'0.6';'0.7';' 1 '],'fontsize',8)
set(gca,'YTick',[-50;0]); set(gca,'YTickLabel',[' 50';' 0 ']);grid
```

The filter response plots are shown in Figure 7.3.

Figure 7.3: Filter design plots in Problem 7.10

P7.11 Design of a bandpass filter using the Hamming window design technique with The specifications:

$$
\begin{array}{ll}
\text{lower stopband edge:} & 0.3\pi \\
\text{upper stopband edge:} & 0.6\pi \\
\text{lower passband edge:} & 0.4\pi \\
\text{upper passband edge:} & 0.5\pi
\end{array}
\qquad
\begin{array}{l}
A_s = 50 \text{ dB} \\
\\
R_p = 0.5 \text{ dB}
\end{array}
$$

```
clc; close all;
%% Specifications:
ws1 = 0.3*pi;  % lower stopband edge
wp1 = 0.4*pi;  % lower passband edge
wp2 = 0.5*pi;  % upper passband edge
ws2 = 0.6*pi;  % upper stopband edge
 Rp = 0.5;      % passband ripple
 As = 50;       % stopband attenuation
%
% Select the min(delta1,delta2) since delta1=delta2 in windodow design
[delta1,delta2] = db2delta(Rp,As);
if (delta1 < delta2)
   delta2 = delta1; disp('Delta1 is smaller than delta2')
   [Rp,As] = delta2db(delta1,delta2)
end
%
tr_width = abs(min((wp1-ws1),(ws2-wp2)));
M = ceil(6.6*pi/tr_width); M = 2*floor(M/2)+1, % choose odd M
M =
    67
n = 0:M-1; w_ham = (hamming(M))';
wc1 = (ws1+wp1)/2; wc2 = (ws2+wp2)/2;
hd = ideal_lp(wc2,M)-ideal_lp(wc1,M); h = hd .* w_ham;
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
Rpd = -min(db(ceil(wp1/delta_w)+1:floor(wp2/delta_w)+1)), % Actual passband ripple
Rpd =
    0.0488
Asd = floor(-max(db(1:(ws1/delta_w)+1))),        % Actual Attn
Asd =
    51
%
%% Filter Response Plots
Hf_1 = figure('Units','inches','position',[1,1,6,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P7.11');
subplot(2,2,1); Hs_1= stem(n,hd,'filled'); set(Hs_1,'markersize',3);
title('Ideal Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h_d(n)')
subplot(2,2,2); Hs_1 = stem(n,w_ham,'filled'); set(Hs_1,'markersize',3);
axis([-1,M,-0.1,1.1]); xlabel('n'); ylabel('w_{ham}(n)'); title('Hamming Window');
```

```
set(gca,'XTick',[0;M-1],'fontsize',8); set(gca,'YTick',[0;1],'fontsize',8)
subplot(2,2,3); Hs_1 = stem(n,h,'filled'); set(Hs_1,'markersize',3);
title('Actual Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h(n)')
subplot(2,2,4); plot(w/pi,db,'linewidth',1); title('Magnitude Response in dB');
axis([0,1,-As-30,5]); xlabel('\omega/\pi'); ylabel('Decibels')
set(gca,'XTick',[0;0.3;0.4;0.5;0.6;1])
set(gca,'XTickLabel',[' 0 ';'0.3';'0.4';'0.5';'0.6';' 1 '],'fontsize',8)
set(gca,'YTick',[-50;0]); set(gca,'YTickLabel',[' 50';' 0 ']);grid
```

The filter response plots are shown in Figure 7.4.

Figure 7.4: Filter design plots in Problem 7.11

P7.12 Design of a highpass filter using the Blackman window design with specifications:

$$\text{stopband edge:} \quad 0.4\pi, \quad A_s = 50 \text{ dB}$$
$$\text{passband edge:} \quad 0.6\pi, \quad R_p = 0.001 \text{ dB}$$

```
clc; close all;
%% Specifications:
ws = 0.4*pi;  % stopband edge
wp = 0.6*pi;  % passband edge
Rp = 0.004;   % passband ripple
As = 50;      % stopband attenuation
%
% Select the min(delta1,delta2) since delta1=delta2 in windodow design
[delta1,delta2] = db2delta(Rp,As);
if (delta1 < delta2)
   delta2 = delta1; disp('Delta1 is smaller than delta2')
Delta1 is smaller than delta2
   [Rp,As] = delta2db(delta1,delta2)
Rp =
    0.0040
As =
    72.7577
end
%
tr_width = abs(wp-ws);
M = ceil(11*pi/tr_width); M = 2*floor(M/2)+1, % choose odd M
M =
    55
n = 0:M-1; w_blk = (blackman(M))';
wc = (ws+wp)/2;
hd = ideal_lp(pi,M)-ideal_lp(wc,M); h = hd .* w_blk;
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
Rpd = -min(db(ceil(wp/delta_w)+1:floor(pi/delta_w)+1)), % Actual passband ripple
Rpd =
    0.0039
Asd = floor(-max(db(1:(ws/delta_w)+1))),               % Actual Attn
Asd =
    71
%
%% Zoomed Filter Response Plot
Hf_1 = figure('Units','inches','position',[1,1,5,3],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,5,3]);
set(Hf_1,'NumberTitle','off','Name','P7.12');
plot(w(301:501)/pi,db(301:501),'linewidth',1); title('Zoomed Magnitude Response in dB');
axis([0.6,1,-0.005,0.001]); xlabel('\omega/\pi'); ylabel('Decibels')
set(gca,'XTick',[0.6;1])
set(gca,'XTickLabel',['0.6';' 1 '],'fontsize',8)
set(gca,'YTick',[-0.004;0]); set(gca,'YTickLabel',['-0.004';'  0   ']);grid
```

The zoomed magnitude filter response plot is shown in Figure 7.5.

Zoomed Magnitude Response in dB
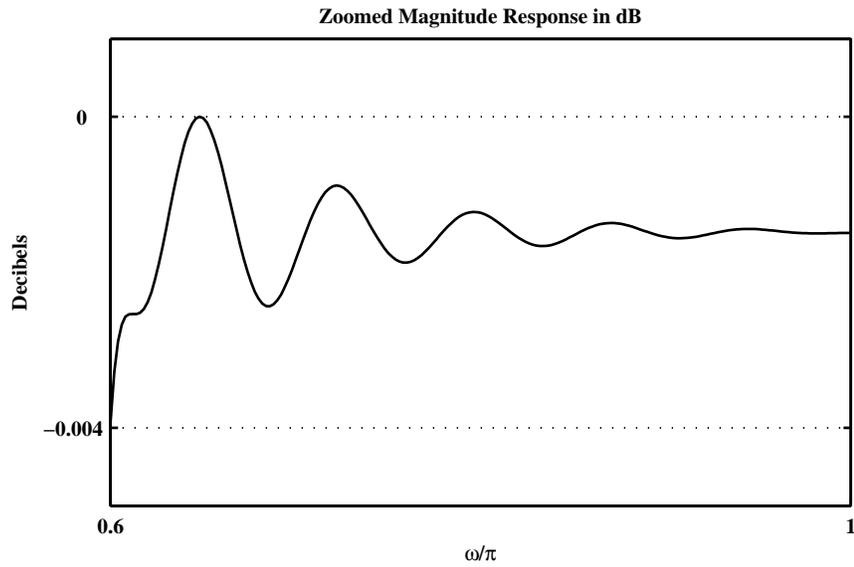
Figure 7.5: Filter design plots in Problem 7.12

P7.13 Design of a linear-phase FIR digital filter Using the Kaiser window method that meets the following specifications:

$$
\begin{array}{rcccccccl}
0.975 & \leq & |H(e^{j\omega})| & \leq & 1.025, & 0 & \leq & \omega & \leq & 0.25\pi \\
0 & \leq & |H(e^{j\omega})| & \leq & 0.005, & 0.35\pi & \leq & \omega & \leq & 0.65\pi \\
0.975 & \leq & |H(e^{j\omega})| & \leq & 1.025, & 0.75\pi & \leq & \omega & \leq & \pi
\end{array}
$$

```
clc; close all;
%% Specifications:
wp1 = 0.25*pi;   % lower passband edge
ws1 = 0.35*pi;   % lower stopband edge
ws2 = 0.65*pi;   % upper stopband edge
wp2 = 0.75*pi;   % upper passband edge
delta1 = 0.025;  % passband ripple
delta2 = 0.005;  % stopband ripple
%
% Convert to decibels
[Rp,As] = delta2db(delta1,delta2)
Rp =
    0.4344
As =
   46.2351
%
tr_width = abs(min((wp1-ws1),(ws2-wp2))); M = ceil((As-7.95)/(2.285*tr_width)+1)+1;
M = 2*floor(M/2)+1, % choose odd M
M =
    57
n = [0:1:M-1]; beta = 0.1102*(As-8.7); w_kai = (kaiser(M,beta))'; % Kaiser Window
wc1 = (ws1+wp1)/2; wc2 = (ws2+wp2)/2;
hd = ideal_lp(pi,M)+ideal_lp(wc1,M)-ideal_lp(wc2,M);               % Ideal HP Filter
h = hd .* w_kai;                                                   % Window design
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
Asd = floor(-max(db(ceil(ws1/delta_w)+1:floor(ws2/delta_w)+1))), % Actual Attn
Asd =
    49
Rpd = -min(db(1:floor(wp1/delta_w)+1)),                          % Actual passband ripple
Rpd =
    0.0492
[Hr,w,P,L] = Ampl_res(h);
%
%% Filter Response Plots
Hf_1 = figure('Units','inches','position',[1,1,7,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,7,4]);
set(Hf_1,'NumberTitle','off','Name','P7.13');
subplot('position',[0.08,0.6,0.25,0.35]);
Hs_1= stem(n,hd,'filled'); set(Hs_1,'markersize',3);
title('Ideal Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h_d(n)');
```

```
subplot('position',[0.41,0.6,0.25,0.35]);
Hs_2 = stem(n,w_kai,'filled'); set(Hs_2,'markersize',3);
axis([-1,M,-0.1,1.1]); xlabel('n'); ylabel('w_{ham}(n)'); title('Kaiser Window');
set(gca,'XTick',[0;M-1],'fontsize',8); set(gca,'YTick',[0;1],'fontsize',8);
subplot('position',[0.74,0.6,0.25,0.35]);
Hs_3 = stem(n,h,'filled'); set(Hs_3,'markersize',3);
title('Actual Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8);
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h(n)');
subplot('position',[0.09,0.1,0.4,0.35]);
plot(w/pi,db,'linewidth',1); title('Magnitude Response in dB');
axis([0,1,-As-30,5]); xlabel('\omega/\pi'); ylabel('Decibels');
set(gca,'XTick',[0;0.25;0.35;0.65;0.75;1])
set(gca,'XTickLabel',[' 0  ';'0.25';'0.35';'0.65';'0.75';' 1  '],'fontsize',8)
set(gca,'YTick',[-Asd;0]); set(gca,'YTickLabel',[' 49';' 0 ']);grid;
subplot('position',[0.59,0.1,0.4,0.35]);
plot(w/pi,Hr,'linewidth',1); title('Apmlitude Response');
axis([0,1,-0.05,1.05]); xlabel('\omega/\pi'); ylabel('Amplitude');
set(gca,'XTick',[0;0.25;0.35;0.65;0.75;1]);
set(gca,'XTickLabel',[' 0  ';'0.25';'0.35';'0.65';'0.75';' 1  '],'fontsize',8)
set(gca,'YTick',[0;1]); grid;
```

The filter response plots are shown in Figure 7.6.

Figure 7.6: Filter design plots in Problem 7.13

P7.14 Design a linear-phase FIR digital filter using the Kaiser window method to meet the following specifications:

$$
\begin{aligned}
0 &\leq |H(e^{j\omega})| \leq 0.01, & 0 &\leq \omega \leq 0.25\pi \\
0.95 &\leq |H(e^{j\omega})| \leq 1.05, & 0.35\pi &\leq \omega \leq 0.65\pi \\
0 &\leq |H(e^{j\omega})| \leq 0.01, & 0.75\pi &\leq \omega \leq \pi
\end{aligned}
$$

```
clc; close all;
%% Specifications:
ws1 = 0.25*pi;   % lower stopband edge
wp1 = 0.35*pi;   % lower passband edge
wp2 = 0.65*pi;   % upper passband edge
ws2 = 0.75*pi;   % upper stopband edge
delta1 = 0.05;   % passband ripple
delta2 = 0.01;   % stopband ripple
%
% Convert to decibels
[Rp,As] = delta2db(delta1,delta2)
Rp =
    0.8693
As =
    40.4238
%
tr_width = abs(min((wp1-ws1),(ws2-wp2)));
M = ceil((As-7.95)/(2.285*tr_width)+1)+1; M = 2*floor(M/2)+1, % choose odd M
M =
      49
n = [0:1:M-1]; beta = 0.1102*(As-8.7); w_kai = (kaiser(M,beta))'; % Kaiser Window
wc1 = (ws1+wp1)/2; wc2 = (ws2+wp2)/2;
hd = ideal_lp(wc2,M)-ideal_lp(wc1,M);
% Determine the Window Design Impulse Response and Frequency Response
h = hd .* w_kai; [db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
Rpd = -min(db(ceil(wp1/delta_w)+1:floor(wp2/delta_w))),   % Actual passband ripple
Rpd =
    0.1033
Asd = floor(-max(db(1:floor(ws1/delta_w)+1))),            % Actual Attn
Asd =
      42
[Hr,w,P,L] = Ampl_res(h);
%
%% Filter Response Plots
Hf_1 = figure('Units','inches','position',[1,1,7,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,7,4]);
set(Hf_1,'NumberTitle','off','Name','P7.14');
subplot('position',[0.08,0.6,0.25,0.35]);
Hs_1= stem(n,hd,'filled'); set(Hs_1,'markersize',3);
title('Ideal Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h_d(n)');
```

```
subplot('position',[0.41,0.6,0.25,0.35]);
Hs_2 = stem(n,w_kai,'filled'); set(Hs_2,'markersize',3);
axis([-1,M,-0.1,1.1]); xlabel('n'); ylabel('w_{ham}(n)'); title('Kaiser Window');
set(gca,'XTick',[0;M-1],'fontsize',8); set(gca,'YTick',[0;1],'fontsize',8);
subplot('position',[0.74,0.6,0.25,0.35]);
Hs_3 = stem(n,h,'filled'); set(Hs_3,'markersize',3);
title('Actual Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8);
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h(n)');
subplot('position',[0.09,0.1,0.4,0.35]);
plot(w/pi,db,'linewidth',1); title('Magnitude Response in dB');
axis([0,1,-As-30,5]); xlabel('\omega/\pi'); ylabel('Decibels');
set(gca,'XTick',[0;0.25;0.35;0.65;0.75;1])
set(gca,'XTickLabel',[' 0  ';'0.25';'0.35';'0.65';'0.75';' 1  '],'fontsize',8)
set(gca,'YTick',[-Asd;0]); set(gca,'YTickLabel',[' 42';' 0 ']);grid;
subplot('position',[0.59,0.1,0.4,0.35]);
plot(w/pi,Hr,'linewidth',1); title('Apmlitude Response');
axis([0,1,-0.05,1.05]); xlabel('\omega/\pi'); ylabel('Amplitude');
set(gca,'XTick',[0;0.25;0.35;0.65;0.75;1]);
set(gca,'XTickLabel',[' 0  ';'0.25';'0.35';'0.65';'0.75';' 1  '],'fontsize',8)
set(gca,'YTick',[0;1]); grid;
```

The filter response plots are shown in Figure 7.7.

Figure 7.7: Filter design plots in Problem 7.14

P7.15 Design of the staircase filter of Example 7.26 using the Kaiser window approach with the specifications:

$$
\begin{array}{llllll}
\text{Band-1:} & 0 & \le & \omega & \le & 0.3\pi, & \text{Ideal gain} = 1, & \delta_1 = 0.01 \\
\text{Band-2:} & 0.4\pi & \le & \omega & \le & 0.7\pi, & \text{Ideal gain} = 0.5, & \delta_2 = 0.005 \\
\text{Band-3:} & 0.8\pi & \le & \omega & \le & \pi, & \text{Ideal gain} = 0, & \delta_3 = 0.001
\end{array}
$$

```
clc; close all;
%% Specifications:
w1 = 0.0*pi;      % lower Band-1 edge
w2 = 0.3*pi;      % upper Band-1 edge
w3 = 0.4*pi;      % lower Band-2 edge
w4 = 0.7*pi;      % upper Band-2 edge
w5 = 0.8*pi;      % lower Band-3 edge
w6 = 1.0*pi;      % upper Band-3 edge
delta1 = 0.01;    % Band-1 ripple
delta2 = 0.005;   % Band-2 ripple
delta3 = 0.001;   % Band-3 ripple
%
% Determine Kaiser Window Parameters
delta = min([delta1,delta2,delta3]); tr_width = min([w3-w2,w5-w4]);
[Rp,As] = delta2db(delta1,delta3);
M = ceil((As-7.95)/(2.285*tr_width)+1)+1; M = 2*floor(M/2)+1, % choose odd M
M =
    75
n = [0:1:M-1]; beta = 0.1102*(As-8.7); w_kai = (kaiser(M,beta))'; % Kaiser Window
% Determine Ideal Impulse Response
wc1 = (w2+w3)/2; wc2 = (w4+w5)/2;
hd = ideal_lp(wc1,M)+0.5*(ideal_lp(wc2,M)-ideal_lp(wc1,M));
% Determine the Window Design Impulse Response and Frequency Response
h = hd .* w_kai; [db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
Asd = floor(-max(db(ceil(w5/delta_w)+1:501))),            % Actual Attn
Asd =
    65
[Hr,w,P,L] = Ampl_res(h);
%
%% Filter Response Plots
Hf_1 = figure('Units','inches','position',[1,1,7,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,7,4]);
set(Hf_1,'NumberTitle','off','Name','P7.15');

subplot('position',[0.08,0.6,0.25,0.35]);
Hs_1= stem(n,hd,'filled'); set(Hs_1,'markersize',3);
title('Ideal Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h_d(n)');

subplot('position',[0.41,0.6,0.25,0.35]);
Hs_2 = stem(n,w_kai,'filled'); set(Hs_2,'markersize',3);
```

```
axis([-1,M,-0.1,1.1]); xlabel('n'); ylabel('w_{ham}(n)'); title('Kaiser Window');
set(gca,'XTick',[0;M-1],'fontsize',8); set(gca,'YTick',[0;1],'fontsize',8);

subplot('position',[0.74,0.6,0.25,0.35]);
Hs_3 = stem(n,h,'filled'); set(Hs_3,'markersize',3);
title('Actual Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8);
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h(n)');

subplot('position',[0.09,0.1,0.4,0.35]);
plot(w/pi,db,'linewidth',1); title('Magnitude Response in dB');
axis([0,1,-As-30,5]); xlabel('\omega/\pi'); ylabel('Decibels');
set(gca,'XTick',[0;0.3;0.4;0.7;0.8;1]);
set(gca,'YTick',[-Asd;0]); set(gca,'YTickLabel',[' 65';' 0 ']);grid;

subplot('position',[0.59,0.1,0.4,0.35]);
plot(w/pi,Hr,'linewidth',1); title('Apmlitude Response');
axis([0,1,-0.05,1.05]); xlabel('\omega/\pi'); ylabel('Amplitude');
set(gca,'XTick',[0;0.3;0.4;0.7;0.8;1]);
set(gca,'YTick',[0;0.5;1]); grid;
```

The window-designed filter has the length of 75 while the one designed in Example 7.26 has the length of 49. The filter response plots are shown in Figure 7.8.
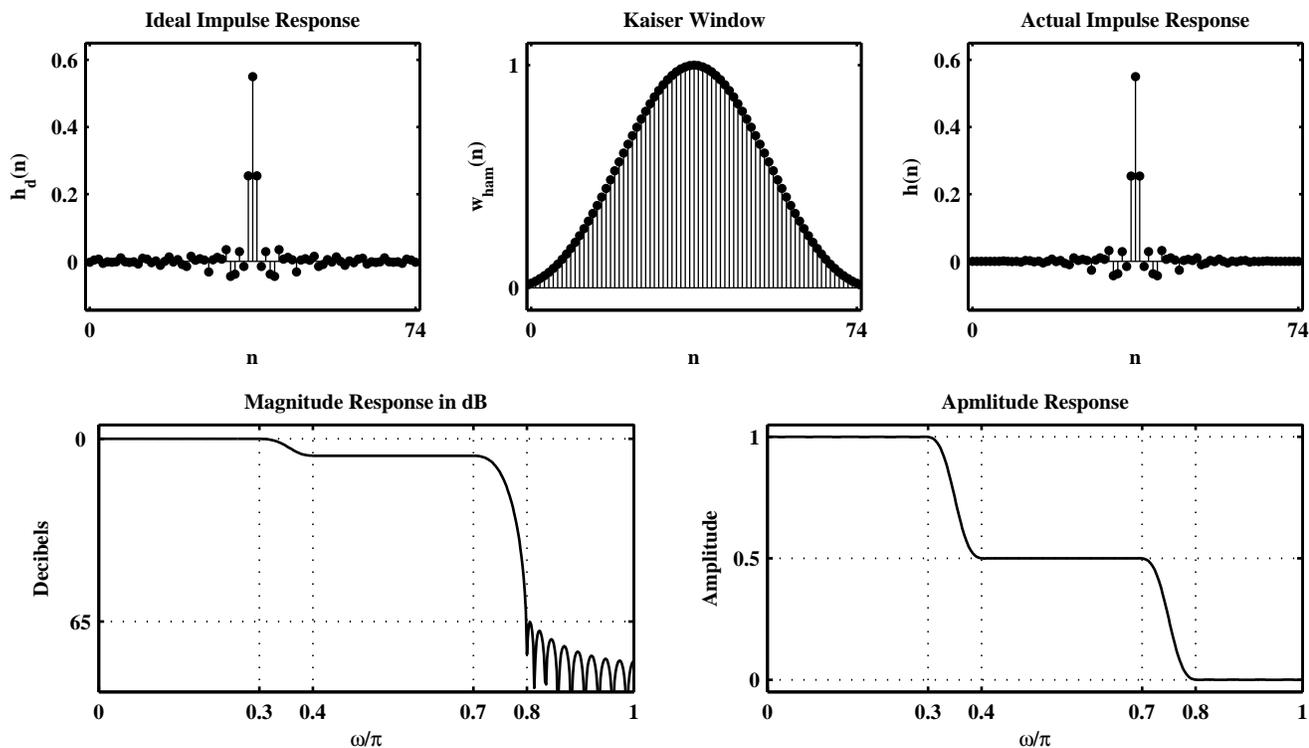

Figure 7.8: Filter design plots in Problem 7.15

P7.16 Design of a bandpass filter using the Kaiser window design technique that has the minimum length and that satisfies the following specifications:

$$\left.\begin{array}{l} \text{lower stopband edge} = 0.3\pi \\ \text{upper stopband edge} = 0.6\pi \end{array}\right\} \quad A_s \quad = \quad 40 \text{ dB}$$

$$\left.\begin{array}{l} \text{lower passband edge} = 0.4\pi \\ \text{upper passband edge} = 0.5\pi \end{array}\right\} \quad R_p \quad = \quad 0.5 \text{ dB.}$$

```
clc; close all;
%% Specifications:
ws1 = 0.3*pi;    % lower stopband edge
wp1 = 0.4*pi;    % lower passband edge
wp2 = 0.5*pi;    % upper passband edge
ws2 = 0.6*pi;    % upper stopband edge
 Rp = 0.5;       % passband ripple
 As = 40;        % stopband attenuation
%
% Select the min(delta1,delta2) since delta1=delta2 in windodow design
[delta1,delta2] = db2delta(Rp,As);
if (delta1 < delta2)
   delta2 = delta1; disp('Delta1 is smaller than delta2')
   [Rp,As] = delta2db(delta1,delta2)
end
% Determine Kaiser Window Parameters
tr_width = abs(min((wp1-ws1),(ws2-wp2)));
M = ceil((As-7.95)/(2.285*tr_width)+1)+1; M = 2*floor(M/2)+3, % choose odd M
M =
    49
n = [0:1:M-1]; beta = 0.1102*(As-8.7); w_kai = (kaiser(M,beta))'; % Kaiser Window
% Determine Ideal Impulse Response
wc1 = (ws1+wp1)/2; wc2 = (ws2+wp2)/2;
hd = ideal_lp(wc2,M)-ideal_lp(wc1,M);
% Determine the Window Design Impulse Response and Frequency Response
h = hd .* w_kai; [db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
Rpd = -min(db(ceil(wp1/delta_w)+1:floor(wp2/delta_w))),   % Actual passband ripple
Rpd =
    0.1872
Asd = floor(-max(db(1:floor(ws1/delta_w)))),              % Actual Attn
Asd =
    40
[Hr,w,P,L] = Ampl_res(h);
%
%% Filter Response Plots
Hf_1 = figure('Units','inches','position',[1,1,7,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,7,4]);
set(Hf_1,'NumberTitle','off','Name','P7.16');
subplot('position',[0.08,0.6,0.25,0.35]);
```

```
Hs_1= stem(n,hd,'filled'); set(Hs_1,'markersize',3);
title('Ideal Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h_d(n)');
subplot('position',[0.41,0.6,0.25,0.35]);
Hs_2 = stem(n,w_kai,'filled'); set(Hs_2,'markersize',3);
axis([-1,M,-0.1,1.1]); xlabel('n'); ylabel('w_{ham}(n)'); title('Kaiser Window');
set(gca,'XTick',[0;M-1],'fontsize',8); set(gca,'YTick',[0;1],'fontsize',8);
subplot('position',[0.74,0.6,0.25,0.35]);
Hs_3 = stem(n,h,'filled'); set(Hs_3,'markersize',3);
title('Actual Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8);
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h(n)');
subplot('position',[0.09,0.1,0.4,0.35]);
plot(w/pi,db,'linewidth',1); title('Magnitude Response in dB');
axis([0,1,-As-30,5]); xlabel('\omega/\pi'); ylabel('Decibels');
set(gca,'XTick',[0;0.3;0.4;0.5;0.6;1])
set(gca,'YTick',[-Asd;0]); set(gca,'YTickLabel',[' 40';' 0 ']);grid;
subplot('position',[0.59,0.1,0.4,0.35]);
plot(w/pi,Hr,'linewidth',1); title('Apmlitude Response');
axis([0,1,-0.05,1.05]); xlabel('\omega/\pi'); ylabel('Amplitude');
set(gca,'XTick',[0;0.3;0.4;0.5;0.6;1]); set(gca,'YTick',[0;1]); grid;
```

The filter response plots are shown in Figure 7.9.
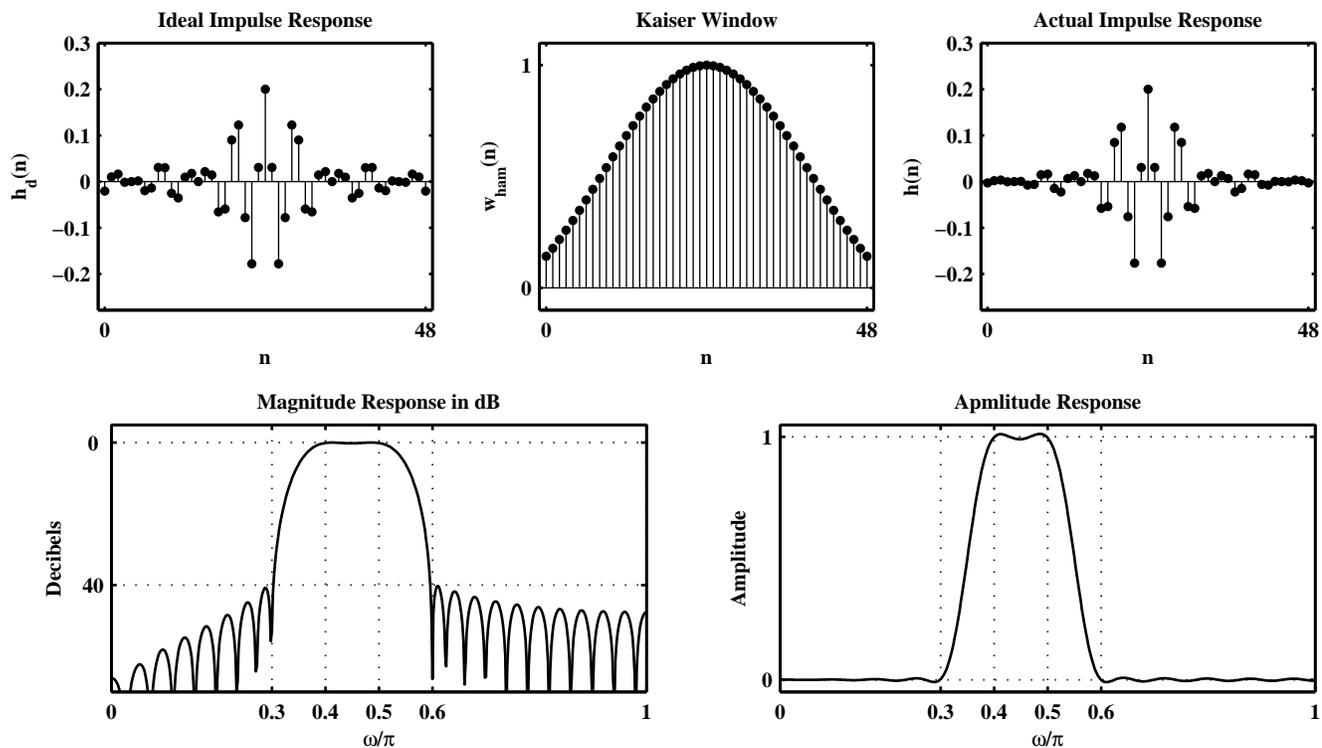
Figure 7.9: Filter design plots in Problem 7.16

P7.17 Repeat of the Problem P7.9 using the `fir1` function:

```
clc; close all;
%% Specifications:
ws1 = 0.2*pi;  % lower stopband edge
wp1 = 0.35*pi; % lower passband edge
wp2 = 0.55*pi; % upper passband edge
ws2 = 0.75*pi; % upper stopband edge
 Rp = 0.25;    % passband ripple
 As = 40;      % stopband attenuation
%
% Select the min(delta1,delta2) since delta1=delta2 in windodow design
[delta1,delta2] = db2delta(Rp,As);
if (delta1 < delta2)
   delta2 = delta1; disp('Delta1 is smaller than delta2')
   [Rp,As] = delta2db(delta1,delta2)
end
%
tr_width = min((wp1-ws1),(ws2-wp2));
M = ceil(6.2*pi/tr_width); M = 2*floor(M/2)+1, % choose odd M
M =
    43
n = 0:M-1; w_han = (hann(M))';
wc1 = (ws1+wp1)/2; wc2 = (ws2+wp2)/2; hd = ideal_lp(wc2,M)-ideal_lp(wc1,M);
h = fir1(M-1,[wc1,wc2]/pi,'bandpass',w_han);
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
Rpd = -min(db((wp1/delta_w)+1:(wp2/delta_w)+1)), % Actual passband ripple
Rpd =
    0.1030
Asd = floor(-max(db(1:(ws1/delta_w)+1))),        % Actual Attn
Asd =
    44
%
%% Filter Response Plots
Hf_1 = figure('Units','inches','position',[1,1,6,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P7.17');
subplot(2,2,1); Hs_1= stem(n,hd,'filled'); set(Hs_1,'markersize',3);
title('Ideal Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h_d(n)')
subplot(2,2,2); Hs_1 = stem(n,w_han,'filled'); set(Hs_1,'markersize',3);
axis([-1,M,-0.1,1.1]); xlabel('n'); ylabel('w_{han}(n)'); title('Hann Window');
set(gca,'XTick',[0;M-1],'fontsize',8); set(gca,'YTick',[0;1],'fontsize',8)
subplot(2,2,3); Hs_1 = stem(n,h,'filled'); set(Hs_1,'markersize',3);
title('Actual Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h(n)')
subplot(2,2,4); plot(w/pi,db,'linewidth',1); title('Magnitude Response in dB');
axis([0,1,-As-30,5]); xlabel('\omega/\pi'); ylabel('Decibels')
```

```
set(gca,'XTick',[0;0.2;0.35;0.55;0.75;1])
set(gca,'XTickLabel',[' 0  ';'0.2 ';'0.35';'0.55';'0.75';' 1  '],'fontsize',8)
set(gca,'YTick',[-40;0]); set(gca,'YTickLabel',[' 40';' 0 ']);grid
```

The filter response plots are shown in Figure 7.10.

Figure 7.10: Filter design plots in Problem 7.17

P7.18 Repeat of the Problem P7.10 using the `fir1` function:

```
clc; close all;
%% Specifications:
wp1 = 0.3*pi;  % lower passband edge
ws1 = 0.4*pi;  % lower stopband edge
ws2 = 0.6*pi;  % upper stopband edge
wp2 = 0.7*pi;  % upper passband edge
 Rp = 0.2;     % passband ripple
 As = 50;      % stopband attenuation
%
% Select the min(delta1,delta2) since delta1=delta2 in windodow design
[delta1,delta2] = db2delta(Rp,As);
if (delta1 < delta2)
   delta2 = delta1; disp('Delta1 is smaller than delta2')
   [Rp,As] = delta2db(delta1,delta2)
end
%
tr_width = abs(min((wp1-ws1),(ws2-wp2)));
M = ceil(6.6*pi/tr_width); M = 2*floor(M/2)+1, % choose odd M
M =
    67
n = 0:M-1; w_ham = (hamming(M))';
wc1 = (ws1+wp1)/2; wc2 = (ws2+wp2)/2; hd = ideal_lp(pi,M)+ideal_lp(wc1,M)-ideal_lp(wc2,M);
h = fir1(M-1,[wc1,wc2]/pi,'stop',w_ham);
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
Asd = floor(-max(db(ceil(ws1/delta_w)+1:floor(ws2/delta_w)+1))), % Actual Attn
Asd =
    50
Rpd = -min(db(1:floor(wp1/delta_w)+1)),              % Actual passband ripple
Rpd =
    0.0435
%
%% Filter Response Plots
Hf_1 = figure('Units','inches','position',[1,1,6,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P7.18');
subplot(2,2,1); Hs_1= stem(n,hd,'filled'); set(Hs_1,'markersize',3);
title('Ideal Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h_d(n)')
subplot(2,2,2); Hs_1 = stem(n,w_ham,'filled'); set(Hs_1,'markersize',3);
axis([-1,M,-0.1,1.1]); xlabel('n'); ylabel('w_{ham}(n)'); title('Hamming Window');
set(gca,'XTick',[0;M-1],'fontsize',8); set(gca,'YTick',[0;1],'fontsize',8)
subplot(2,2,3); Hs_1 = stem(n,h,'filled'); set(Hs_1,'markersize',3);
title('Actual Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h(n)')
subplot(2,2,4); plot(w/pi,db,'linewidth',1); title('Magnitude Response in dB');
axis([0,1,-As-30,5]); xlabel('\omega/\pi'); ylabel('Decibels')
```

```
set(gca,'XTick',[0;0.3;0.4;0.6;0.7;1])
set(gca,'XTickLabel',[' 0 ';'0.3';'0.4';'0.6';'0.7';' 1 '],'fontsize',8)
set(gca,'YTick',[-50;0]); set(gca,'YTickLabel',[' 50';' 0 ']);grid
```

The filter response plots are shown in Figure 7.11.

Figure 7.11: Filter design plots in Problem 7.18

P7.19 Repeat of the Problem P7.11 using the `fir1` function:

```
clc; close all;
%% Specifications:
ws1 = 0.3*pi;  % lower stopband edge
wp1 = 0.4*pi;  % lower passband edge
wp2 = 0.5*pi;  % upper passband edge
ws2 = 0.6*pi;  % upper stopband edge
 Rp = 0.5;     % passband ripple
 As = 50;      % stopband attenuation
%
% Select the min(delta1,delta2) since delta1=delta2 in windodow design
[delta1,delta2] = db2delta(Rp,As);
if (delta1 < delta2)
   delta2 = delta1; disp('Delta1 is smaller than delta2')
   [Rp,As] = delta2db(delta1,delta2)
end
%
tr_width = abs(min((wp1-ws1),(ws2-wp2)));
M = ceil(6.6*pi/tr_width); M = 2*floor(M/2)+1, % choose odd M
M =
    67
n = 0:M-1; w_ham = (hamming(M))';
wc1 = (ws1+wp1)/2; wc2 = (ws2+wp2)/2; hd = ideal_lp(wc2,M)-ideal_lp(wc1,M);
h = fir1(M-1,[wc1,wc2]/pi,'bandpass',w_ham);
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
Rpd = -min(db(ceil(wp1/delta_w)+1:floor(wp2/delta_w)+1)), % Actual passband ripple
Rpd =
    0.0488
Asd = floor(-max(db(1:(ws1/delta_w)+1))),                 % Actual Attn
Asd =
    51
%
%% Filter Response Plots
Hf_1 = figure('Units','inches','position',[1,1,6,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P7.19');
subplot(2,2,1); Hs_1= stem(n,hd,'filled'); set(Hs_1,'markersize',3);
title('Ideal Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h_d(n)')
subplot(2,2,2); Hs_1 = stem(n,w_ham,'filled'); set(Hs_1,'markersize',3);
axis([-1,M,-0.1,1.1]); xlabel('n'); ylabel('w_{ham}(n)'); title('Hamming Window');
set(gca,'XTick',[0;M-1],'fontsize',8); set(gca,'YTick',[0;1],'fontsize',8)
subplot(2,2,3); Hs_1 = stem(n,h,'filled'); set(Hs_1,'markersize',3);
title('Actual Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h(n)')
subplot(2,2,4); plot(w/pi,db,'linewidth',1); title('Magnitude Response in dB');
axis([0,1,-As-30,5]); xlabel('\omega/\pi'); ylabel('Decibels')
```

```
set(gca,'XTick',[0;0.3;0.4;0.5;0.6;1])
set(gca,'XTickLabel',[' 0 ';'0.3';'0.4';'0.5';'0.6';' 1 '],'fontsize',8)
set(gca,'YTick',[-50;0]); set(gca,'YTickLabel',[' 50';' 0 ']);grid
```

The filter response plots are shown in Figure 7.12.

Figure 7.12: Filter design plots in Problem 7.19

P7.20 Repeat of the Problem P7.12 using the `fir1` function:

```
clc; close all;
%% Specifications:
ws = 0.4*pi;  % stopband edge
wp = 0.6*pi;  % passband edge
Rp = 0.004;   % passband ripple
As = 50;      % stopband attenuation
%
% Select the min(delta1,delta2) since delta1=delta2 in windodow design
[delta1,delta2] = db2delta(Rp,As);
if (delta1 < delta2)
    delta2 = delta1; disp('Delta1 is smaller than delta2')
Delta1 is smaller than delta2
    [Rp,As] = delta2db(delta1,delta2)
Rp =
    0.0040
As =
    72.7577
end
%
tr_width = abs(wp-ws);
M = ceil(11*pi/tr_width); M = 2*floor(M/2)+1, % choose odd M
M =
    55
n = 0:M-1; w_blk = (blackman(M))';
wc = (ws+wp)/2; hd = ideal_lp(pi,M)-ideal_lp(wc,M);
h = fir1(M-1,wc/pi,'high',w_blk);
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
Rpd = -min(db(ceil(wp/delta_w)+1:floor(pi/delta_w)+1)), % Actual passband ripple
Rpd =
    0.0039
Asd = floor(-max(db(1:(ws/delta_w)+1))),              % Actual Attn
Asd =
    71
%
%% Zoomed Filter Response Plot
Hf_1 = figure('Units','inches','position',[1,1,5,3],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,5,3]);
set(Hf_1,'NumberTitle','off','Name','P7.12');
plot(w(301:501)/pi,db(301:501),'linewidth',1);
title('Zoomed Magnitude Response in dB');
axis([0.6,1,-0.005,0.001]); xlabel('\omega/\pi'); ylabel('Decibels')
set(gca,'XTick',[0.6;1])
set(gca,'XTickLabel',['0.6';' 1 '],'fontsize',8)
set(gca,'YTick',[-0.004;0]); set(gca,'YTickLabel',['-0.004';'  0    ']);grid
```

The zoomed magnitude response plot is shown in Figure 7.13.

Zoomed Magnitude Response in dB

Figure 7.13: Filter design plots in Problem 7.20

P7.21 Repeat of the Problem P7.13 using the `fir1` function:

```
clc; close all;
%% Specifications:
wp1 = 0.25*pi;    % lower passband edge
ws1 = 0.35*pi;    % lower stopband edge
ws2 = 0.65*pi;    % upper stopband edge
wp2 = 0.75*pi;    % upper passband edge
delta1 = 0.025;   % passband ripple
delta2 = 0.005;   % stopband ripple
%
% Convert to decibels
[Rp,As] = delta2db(delta1,delta2)
Rp =
    0.4344
As =
   46.2351
%
tr_width = abs(min((wp1-ws1),(ws2-wp2))); M = ceil((As-7.95)/(2.285*tr_width)+1)+1;
M = 2*floor(M/2)+1, % choose odd M
M =
    57
n = [0:1:M-1]; beta = 0.1102*(As-8.7); w_kai = (kaiser(M,beta))'; % Kaiser Window
wc1 = (ws1+wp1)/2; wc2 = (ws2+wp2)/2;
hd = ideal_lp(pi,M)+ideal_lp(wc1,M)-ideal_lp(wc2,M);      % Ideal HP Filter
h = fir1(M-1,[wc1,wc2]/pi,'stop',w_kai);
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
Asd = floor(-max(db(ceil(ws1/delta_w)+1:floor(ws2/delta_w)+1))), % Actual Attn
Asd =
    49
Rpd = -min(db(1:floor(wp1/delta_w)+1)),              % Actual passband ripple
Rpd =
    0.0492
[Hr,w,P,L] = Ampl_res(h);
%
%% Filter Response Plots
Hf_1 = figure('Units','inches','position',[1,1,7,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,7,4]);
set(Hf_1,'NumberTitle','off','Name','P7.21');
subplot('position',[0.08,0.6,0.25,0.35]);
Hs_1= stem(n,hd,'filled'); set(Hs_1,'markersize',3);
title('Ideal Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h_d(n)');
subplot('position',[0.41,0.6,0.25,0.35]);
Hs_2 = stem(n,w_kai,'filled'); set(Hs_2,'markersize',3);
axis([-1,M,-0.1,1.1]); xlabel('n'); ylabel('w_{ham}(n)'); title('Kaiser Window');
set(gca,'XTick',[0;M-1],'fontsize',8); set(gca,'YTick',[0;1],'fontsize',8);
subplot('position',[0.74,0.6,0.25,0.35]);
```

```
Hs_3 = stem(n,h,'filled'); set(Hs_3,'markersize',3);
title('Actual Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8);
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h(n)');
subplot('position',[0.09,0.1,0.4,0.35]);
plot(w/pi,db,'linewidth',1); title('Magnitude Response in dB');
axis([0,1,-As-30,5]); xlabel('\omega/\pi'); ylabel('Decibels');
set(gca,'XTick',[0;0.25;0.35;0.65;0.75;1])
set(gca,'XTickLabel',[' 0  ';'0.25';'0.35';'0.65';'0.75';' 1  '],'fontsize',8)
set(gca,'YTick',[-Asd;0]); set(gca,'YTickLabel',[' 49';' 0 ']);grid;
subplot('position',[0.59,0.1,0.4,0.35]);
plot(w/pi,Hr,'linewidth',1); title('Apmlitude Response');
axis([0,1,-0.05,1.05]); xlabel('\omega/\pi'); ylabel('Amplitude');
set(gca,'XTick',[0;0.25;0.35;0.65;0.75;1]);
set(gca,'XTickLabel',[' 0  ';'0.25';'0.35';'0.65';'0.75';' 1  '],'fontsize',8)
set(gca,'YTick',[0;1]); grid;
```

The filter response plots are shown in Figure 7.14.

Figure 7.14: Filter design plots in Problem 7.21

P7.22 Repeat of the Problem P7.14 using the `fir1` function:

```
clc; close all;
%% Specifications:
ws1 = 0.25*pi;    % lower stopband edge
wp1 = 0.35*pi;    % lower passband edge
wp2 = 0.65*pi;    % upper passband edge
ws2 = 0.75*pi;    % upper stopband edge
delta1 = 0.05;    % passband ripple
delta2 = 0.01;    % stopband ripple
%
% Convert to decibels
[Rp,As] = delta2db(delta1,delta2)
Rp =
    0.8693
As =
   40.4238
%
tr_width = abs(min((wp1-ws1),(ws2-wp2)));
M = ceil((As-7.95)/(2.285*tr_width)+1)+1; M = 2*floor(M/2)+1, % choose odd M
M =
    49
n = [0:1:M-1]; beta = 0.1102*(As-8.7); w_kai = (kaiser(M,beta))'; % Kaiser Window
wc1 = (ws1+wp1)/2; wc2 = (ws2+wp2)/2;
hd = ideal_lp(wc2,M)-ideal_lp(wc1,M);
% Determine the Window Design Impulse Response and Frequency Response
h = fir1(M-1,[wc1,wc2]/pi,'bandpass',w_kai);
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
Rpd = -min(db(ceil(wp1/delta_w)+1:floor(wp2/delta_w))),   % Actual passband ripple
Rpd =
    0.1033
Asd = floor(-max(db(1:floor(ws1/delta_w)+1))),            % Actual Attn
Asd =
    42
[Hr,w,P,L] = Ampl_res(h);
%
%% Filter Response Plots
Hf_1 = figure('Units','inches','position',[1,1,7,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,7,4]);
set(Hf_1,'NumberTitle','off','Name','P7.22');
subplot('position',[0.08,0.6,0.25,0.35]);
Hs_1= stem(n,hd,'filled'); set(Hs_1,'markersize',3);
title('Ideal Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h_d(n)');
subplot('position',[0.41,0.6,0.25,0.35]);
Hs_2 = stem(n,w_kai,'filled'); set(Hs_2,'markersize',3);
axis([-1,M,-0.1,1.1]); xlabel('n'); ylabel('w_{ham}(n)'); title('Kaiser Window');
set(gca,'XTick',[0;M-1],'fontsize',8); set(gca,'YTick',[0;1],'fontsize',8);
```

```
subplot('position',[0.74,0.6,0.25,0.35]);
Hs_3 = stem(n,h,'filled'); set(Hs_3,'markersize',3);
title('Actual Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8);
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h(n)');
subplot('position',[0.09,0.1,0.4,0.35]);
plot(w/pi,db,'linewidth',1); title('Magnitude Response in dB');
axis([0,1,-As-30,5]); xlabel('\omega/\pi'); ylabel('Decibels');
set(gca,'XTick',[0;0.25;0.35;0.65;0.75;1])
set(gca,'XTickLabel',[' 0  ';'0.25';'0.35';'0.65';'0.75';' 1  '],'fontsize',8)
set(gca,'YTick',[-Asd;0]); set(gca,'YTickLabel',[' 42';' 0 ']);grid;
subplot('position',[0.59,0.1,0.4,0.35]);
plot(w/pi,Hr,'linewidth',1); title('Apmlitude Response');
axis([0,1,-0.05,1.05]); xlabel('\omega/\pi'); ylabel('Amplitude');
set(gca,'XTick',[0;0.25;0.35;0.65;0.75;1]);
set(gca,'XTickLabel',[' 0  ';'0.25';'0.35';'0.65';'0.75';' 1  '],'fontsize',8)
set(gca,'YTick',[0;1]); grid;
```

The filter response plots are shown in Figure 7.15.

Figure 7.15: Filter design plots in Problem 7.22

P7.23 Consider an ideal lowpass filter with the cutoff frequency $\omega_c = 0.3\pi$. We want to approximate this filter using a frequency sampling design in which we choose 40 samples.

(a) Choose the sample at ω_c equal to 0.5 and use the naive design method to compute $h(n)$. Determine the minimum stopband attenuation.

(b) Now vary the sample at ω_c and determine the optimum value to obtain the largest minimum stopband attenuation.

(c) Plot the magnitude responses in dB of the above two designs in one plot and comment on the results.

P7.24 Design of the bandstop filter of Problem P7.10 using the frequency sampling method with two optimum samples in the transition band and comparison of results with those obtained using the `fir2` function.

```matlab
clc; close all;
% Specifications:
wp1 = 0.3*pi;  % lower passband edge
ws1 = 0.4*pi;  % lower stopband edge
ws2 = 0.6*pi;  % upper stopband edge
wp2 = 0.7*pi;  % upper passband edge
 Rp = 0.2;       % passband ripple
 As = 50;        % stopband attenuation
Hf_1 = figure('Units','inches','position',[1,1,7,5],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,7,5]);
set(Hf_1,'NumberTitle','off','Name','P7.24');
%% Frequency Sampling Design
% Choose M = 65 to get two samples in the transition band
M = 65; alpha = (M-1)/2; k = 0:M-1; wk = (2*pi/M)*k;
% Samples of the Frequency Response
T1 = 0.58753; T2 = 0.10508;
Hrs = [ones(1,11),T1,T2,zeros(1,8),T2,T1,ones(1,20),...
    T1,T2,zeros(1,8),T2,T1,ones(1,10)];
% Ideal Amplitude Response for Plotting
fc1 = (wp1+ws1)/(2*pi); fc2 = (ws2+wp2)/(2*pi);
Hdr = [1,1,0,0,1,1]; fdr = [0,fc1,fc1,fc2,fc2,1];
% Compute the Impulse Response
k1 = 0:floor((M-1)/2); k2 = floor((M-1)/2)+1:M-1;
angH = [-alpha*(2*pi)/M*k1, alpha*(2*pi)/M*(M-k2)];
H = Hrs.*exp(j*angH); h = real(ifft(H,M));
% Compute Frequency Responses
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
% Actual Attn
Asd = floor(-max(db(ceil(ws1/delta_w)+1:floor(ws2/delta_w)+1)));
% Filter Response Plots
subplot('position',[0.09,0.6,0.4,0.35]);
Hp_1 = plot(wk(1:(M+1)/2)/pi,Hrs(1:(M+1)/2),'mo'); hold on;
set(Hp_1,'markersize',3);
plot(fdr,Hdr,'g','linewidth',2); axis([0,1,0,1.1]);
plot(w/pi,mag,'r','linewidth',1.5);
xlabel('\omega/\pi'); ylabel('Magnitude');
title('Frequency Sampling Design: Magnitude Plot');
set(gca,'xtick',[0,wp1,ws1,ws2,wp2,pi]/pi,'xgrid','on');
set(gca,'ytick',[0,1]); hold off;
subplot('position',[0.59,0.6,0.4,0.35]);
plot(w/pi,db,'g','linewidth',2); axis([0,1,-80,10]);
xlabel('\omega/\pi'); ylabel('Decibels');
title('Frequency Sampling Design: Log-Magnitude Plot');
set(gca,'xtick',[0,wp1,ws1,ws2,wp2,pi]/pi,'xgrid','on');
set(gca,'ytick',[-Asd,0],'ygrid','on');
```

```
%% Design Using the FIR2 Function
fc1 = (wp1+ws1)/(2*pi); fc2 = (ws2+wp2)/(2*pi);
Hdr = [1,1,0,0,1,1]; fdr = [0,fc1,fc1,fc2,fc2,1];
h = fir2(82,fdr,Hdr);
% Compute Frequency Responses
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
% Actual Attn
Asd = floor(-max(db(ceil(ws1/delta_w)+1:floor(ws2/delta_w)+1)));
% Filter Response Plots
subplot('position',[0.09,0.1,0.4,0.35]);
Hp_2 = plot(wk(1:(M+1)/2)/pi,Hrs(1:(M+1)/2),'mo'); hold on;set(Hp_2, 'markersize',3);
plot(fdr,Hdr,'g','linewidth',2); axis([0,1,0,1.1]);
plot(w/pi,mag,'r','linewidth',1.5);
xlabel('\omega/\pi'); ylabel('Magnitude');
title('FIR2 Function Design: Magnitude Plot');
set(gca,'xtick',[0,wp1,ws1,ws2,wp2,pi]/pi,'xgrid','on');
set(gca,'ytick',[0,1]); hold off;
subplot('position',[0.59,0.1,0.4,0.35]);
plot(w/pi,db,'g','linewidth',2); axis([0,1,-80,10]);
xlabel('\omega/\pi'); ylabel('Decibels');
title('FIR2 Function Design: Log-Magnitude Plot');
set(gca,'xtick',[0,wp1,ws1,ws2,wp2,pi]/pi,'xgrid','on');
set(gca,'ytick',[-Asd,0],'ygrid','on');
```

The filter response plots are shown in Figure 7.16.
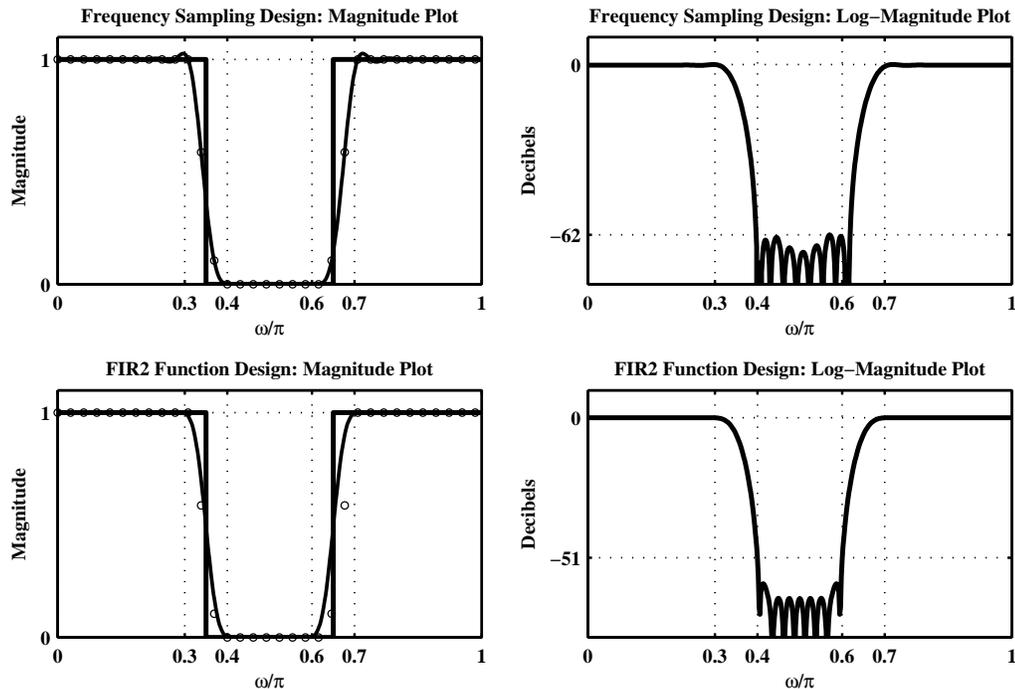
Figure 7.16: Filter design plots in Problem 7.24

P7.25 Design of the bandpass filter of Problem P7.10 using the frequency sampling method with two optimum samples in the transition band and comparison of results with those obtained using the `fir2` function.

```
clc; close all;
% Specifications:
ws1 = 0.3*pi;  % lower stopband edge
wp1 = 0.4*pi;  % lower passband edge
wp2 = 0.5*pi;  % upper passband edge
ws2 = 0.6*pi;  % upper stopband edge
 Rp = 0.5;       % passband ripple
 As = 50;        % stopband attenuation
Hf_1 = figure('Units','inches','position',[1,1,7,5],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,7,5]);
set(Hf_1,'NumberTitle','off','Name','P7.25');
%% Frequency Sampling Design
% Choose M = 60 to get two samples in the transition band
M = 60; alpha = (M-1)/2; k = 0:M-1; wk = (2*pi/M)*k;
% Samples of the Frequency Response
T0 = 1; T1 = 0.58753; T2 = 0.10508; T3 = 0;
TL = [T3,T2,T1,T0]; TR = fliplr(TL);
Hrs = [zeros(1,9),TL,ones(1,2),TR,zeros(1,23),...
    TL,ones(1,2),TR,zeros(1,8)];
% Ideal Amplitude Response for Plotting
fc1 = (wp1+ws1)/(2*pi); fc2 = (ws2+wp2)/(2*pi);
Hdr = [0,0,1,1,0,0]; fdr = [0,fc1,fc1,fc2,fc2,1];
% Compute the Impulse Response
k1 = 0:floor((M-1)/2); k2 = floor((M-1)/2)+1:M-1;
angH = [-alpha*(2*pi)/M*k1, alpha*(2*pi)/M*(M-k2)];
H = Hrs.*exp(j*angH); h = real(ifft(H,M));
% Compute Frequency Responses
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
% Actual Attn
Asd = floor(-max(db(1:floor(ws1/delta_w))));
% Filter Response Plots
subplot('position',[0.09,0.6,0.4,0.35]);
Hp_1 = plot(wk(1:(M+1)/2)/pi,Hrs(1:(M+1)/2),'mo'); set(Hp_1, 'markersize',3);
hold on; plot(fdr,Hdr,'g','linewidth',2); axis([0,1,0,1.1]);
plot(w/pi,mag,'r','linewidth',1.5);
xlabel('\omega/\pi'); ylabel('Magnitude');
title('Frequency Sampling Design: Magnitude Plot');
set(gca,'xtick',[0,ws1,wp1,wp2,ws2,pi]/pi,'xgrid','on','ygrid','on');
set(gca,'ytick',[0,1]); hold off;
subplot('position',[0.59,0.6,0.4,0.35]);
plot(w/pi,db,'g','linewidth',2); axis([0,1,-80,10]);
xlabel('\omega/\pi'); ylabel('Decibels');
title('Frequency Sampling Design: Log-Magnitude Plot');
set(gca,'xtick',[0,ws1,wp1,wp2,ws2,pi]/pi,'xgrid','on');
set(gca,'ytick',[-Asd,0],'ygrid','on');
```

```
%% Design Using the FIR2 Function
fc1 = (wp1+ws1)/(2*pi); fc2 = (ws2+wp2)/(2*pi);
Hdr = [0,0,1,1,0,0]; fdr = [0,fc1,fc1,fc2,fc2,1];
h = fir2(78,fdr,Hdr);
% Compute Frequency Responses
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
% Actual Attn
Asd = floor(-max(db(1:floor(ws1/delta_w))));
% Filter Response Plots
subplot('position',[0.09,0.1,0.4,0.35]);
Hp_2 = plot(wk(1:(M+1)/2)/pi,Hrs(1:(M+1)/2),'mo'); set(Hp_2,'markersize',3);
hold on; plot(fdr,Hdr,'g','linewidth',2); axis([0,1,0,1.1]);
plot(w/pi,mag,'r','linewidth',1.5);
xlabel('\omega/\pi'); ylabel('Magnitude');
title('FIR2 Function Design: Magnitude Plot');
set(gca,'xtick',[0,ws1,wp1,wp2,ws2,pi]/pi,'xgrid','on','ygrid','on');
set(gca,'ytick',[0,1]); hold off;
subplot('position',[0.59,0.1,0.4,0.35]);
plot(w/pi,db,'g','linewidth',2); axis([0,1,-80,10]);
xlabel('\omega/\pi'); ylabel('Decibels');
title('FIR2 Function Design: Log-Magnitude Plot');
set(gca,'xtick',[0,ws1,wp1,wp2,ws2,pi]/pi,'xgrid','on');
set(gca,'ytick',[-Asd,0],'ygrid','on');
```

The filter response plots are shown in Figure 7.17.

Figure 7.17: Filter design plots in Problem 7.25

P7.26 Design of the highpass filter of Problem P7.12 using the frequency sampling method with two optimum samples in the transition band and comparison of results with those obtained using the `fir2` function.

```
clc; close all;
% Specifications:
ws = 0.3*pi;     % Stopband edge
wp = 0.4*pi;     % Passband edge
 Rp = 0.004;     % passband ripple
 As = 50;        % stopband attenuation
Hf_1 = figure('Units','inches','position',[1,1,7,5],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,7,5]);
set(Hf_1,'NumberTitle','off','Name','P7.26');

%% Frequency Sampling Design
% Choose M = 65 to get two samples in the transition band
M = 65; alpha = (M-1)/2; k = 0:M-1; wk = (2*pi/M)*k;
% Samples of the Frequency Response
T1 = 0.58753; T2 = 0.10508;
Hrs = [zeros(1,11),T2,T1,ones(1,40),T1,T2,zeros(1,10)];
% Ideal Amplitude Response for Plotting
fc = (wp+ws)/(2*pi); Hdr = [0,0,1,1]; fdr = [0,fc,fc,1];
% Compute the Impulse Response
k1 = 0:floor((M-1)/2); k2 = floor((M-1)/2)+1:M-1;
angH = [-alpha*(2*pi)/M*k1, alpha*(2*pi)/M*(M-k2)];
H = Hrs.*exp(j*angH); h = real(ifft(H,M));
% Compute Frequency Responses
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
% Actual Attn
Asd = floor(-max(db(1:floor(ws/delta_w))));
% Filter Response Plots
subplot('position',[0.09,0.6,0.4,0.35]);
Hp_1 = plot(wk(1:(M+1)/2)/pi,Hrs(1:(M+1)/2),'mo'); set(Hp_1,'markersize',3);
hold on; plot(fdr,Hdr,'g','linewidth',2); axis([0,1,0,1.1]);
plot(w/pi,mag,'r','linewidth',1.5);
xlabel('\omega/\pi'); ylabel('Magnitude');
title('Frequency Sampling Design: Magnitude Plot');
set(gca,'xtick',[0,ws,wp,pi]/pi,'xgrid','on','ygrid','on');
set(gca,'ytick',[0,1]); hold off;
subplot('position',[0.59,0.6,0.4,0.35]);
plot(w/pi,db,'g','linewidth',2); axis([0,1,-80,10]);
xlabel('\omega/\pi'); ylabel('Decibels');
title('Frequency Sampling Design: Log-Magnitude Plot');
set(gca,'xtick',[0,ws,wp,pi]/pi,'xgrid','on');
set(gca,'ytick',[-Asd,0],'ygrid','on');

%% Design Using the FIR2 Function
fc = (wp+ws)/(2*pi); Hdr = [0,0,1,1]; fdr = [0,fc,fc,1];
h = fir2(78,fdr,Hdr);
```

```
% Compute Frequency Responses
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
% Actual Attn
Asd = floor(-max(db(1:floor(ws/delta_w))));
% Filter Response Plots
subplot('position',[0.09,0.1,0.4,0.35]);
Hp_2 = plot(wk(1:(M+1)/2)/pi,Hrs(1:(M+1)/2),'mo'); set(Hp_2,'markersize',3);
hold on; plot(fdr,Hdr,'g','linewidth',2); axis([0,1,0,1.1]);
plot(w/pi,mag,'r','linewidth',1.5);
xlabel('\omega/\pi'); ylabel('Magnitude');
title('FIR2 Function Design: Magnitude Plot');
set(gca,'xtick',[0,ws,wp,pi]/pi,'xgrid','on','ygrid','on');
set(gca,'ytick',[0,1]); hold off;
subplot('position',[0.59,0.1,0.4,0.35]);
plot(w/pi,db,'g','linewidth',2); axis([0,1,-80,10]);
xlabel('\omega/\pi'); ylabel('Decibels');
title('FIR2 Function Design: Log-Magnitude Plot');
set(gca,'xtick',[0,ws,wp,pi]/pi,'xgrid','on');
set(gca,'ytick',[-Asd,0],'ygrid','on');
```

The filter response plots are shown in Figure 7.18.

Figure 7.18: Filter design plots in Problem 7.26

P7.27 We want to design a narrow bandpass filter to pass the center frequency at $\omega_0 = 0.5\pi$. The bandwidth should be no more than 0.1π.

(a) Use the frequency sampling technique and choose M so that there is one sample in the transition band. Use the optimum value for transition band samples and draw the frequency sampling structure.

(b) Use the Kaiser window technique so that the stopband attenuation is same as that of the above frequency sampling design. Determine the impulse response $h(n)$ and draw the linear-phase structure.

(c) Compare the above two filter designs in terms of their implementation and their filtering effectiveness.

P7.28 Frequency sampling design using the `fir2` function and a Hamming window.

```
clc; close all;
% Specifications:
fdr = [0,0.25,0.35,0.65,0.75,1]; Hdr = [0,1,2,2,1,0];
h = fir2(211,fdr,Hdr); [Hr,w,P,L] = ampl_res(h);
Hf_1 = figure('Units','inches','position',[1,1,5,3],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,5,3]);
set(Hf_1,'NumberTitle','off','Name','P7.28');
plot(w/pi,Hr,'r','linewidth',1.5); axis([0,1,-0.05,2.05]);
xlabel('\omega/\pi'); ylabel('Amplitude');
title('FIR2 Function Design');
set(gca,'xtick',fdr,'xgrid','on','ytick',[0,1,1.98,2.02],'ygrid','on');
```

The filter amplitude response plot is shown in Figure 7.19.
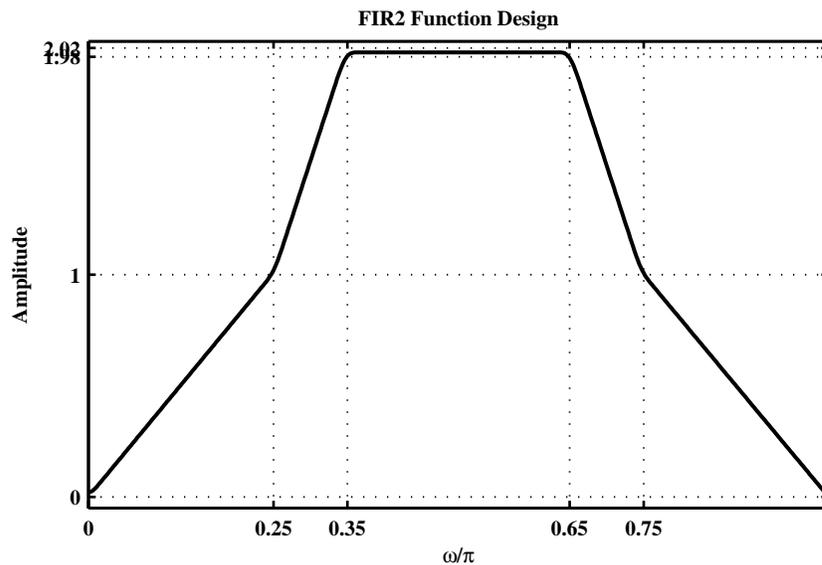
Figure 7.19: Filter design plots in Problem 7.28

P7.29 Design of a bandpass filter using the frequency sampling method with one optimum sample in the transition band and plot of the log-magnitude response in dB and `stem` plot of the impulse response.

```
clc; close all;
% Specifications:
ws1 = 0.3*pi;  % lower stopband edge
wp1 = 0.4*pi;  % lower passband edge
wp2 = 0.6*pi;  % upper passband edge
ws2 = 0.7*pi;  % upper stopband edge
 Rp = 0.5;      % passband ripple
 As = 40;       % stopband attenuation
Hf_1 = figure('Units','inches','position',[1,1,7,3],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,7,3]);
set(Hf_1,'NumberTitle','off','Name','P7.29');
%% Frequency Sampling Design
% Choose M = 40 to get one sample in the transition band
M = 40; alpha = (M-1)/2; k = 0:M-1; wk = (2*pi/M)*k;
% Samples of the Frequency Response
T1 = 0.405;
Hrs = [zeros(1,7),T1,ones(1,5),T1,zeros(1,13),T1,ones(1,5),T1,zeros(1,6)];
% Ideal Amplitude Response for Plotting
fc1 = (wp1+ws1)/(2*pi); fc2 = (ws2+wp2)/(2*pi);
Hdr = [0,0,1,1,0,0]; fdr = [0,fc1,fc1,fc2,fc2,1];
% Compute the Impulse Response
k1 = 0:floor((M-1)/2); k2 = floor((M-1)/2)+1:M-1;
angH = [-alpha*(2*pi)/M*k1, alpha*(2*pi)/M*(M-k2)];
H = Hrs.*exp(j*angH); h = real(ifft(H,M)); n = 0:M-1;
% Compute Frequency Responses
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
% Actual Attn
Asd = floor(-max(db(ceil(ws2/delta_w)+2:end)));
% Filter Response Plots
subplot('position',[0.09,0.1,0.4,0.8]);
Hs_1 = stem(n,h,'g','filled'); set(Hs_1,'markersize',3);
axis([-1,M,-0.2,0.25]); xlabel('n'); ylabel('Amplitude');
title('Impulse Response'); set(gca,'xtick',[0,M-1]);
subplot('position',[0.59,0.1,0.4,0.8]);
plot(w/pi,db,'g','linewidth',2); axis([0,1,-80,10]);
xlabel('\omega/\pi'); ylabel('Decibels');
title('Log-Magnitude Plot');
set(gca,'xtick',[0,ws1,wp1,wp2,ws2,pi]/pi,'xgrid','on');
set(gca,'ytick',[-Asd,0],'ygrid','on');
```

The filter response plots are shown in Figure 7.20.
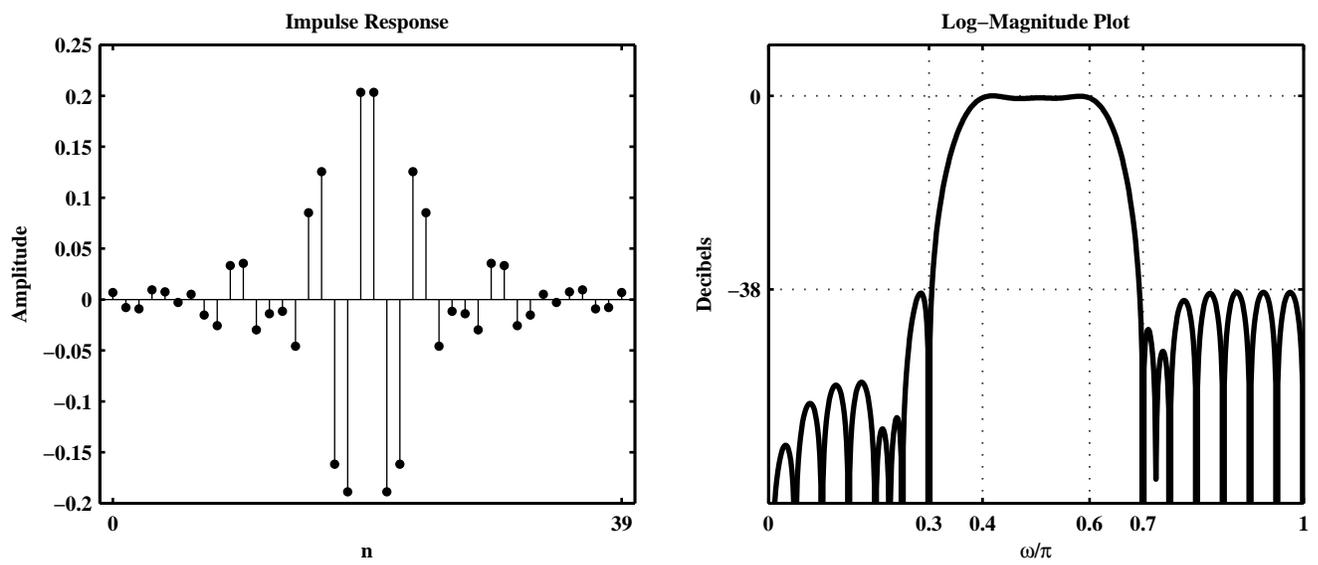
Figure 7.20: Filter design plots in Problem 7.29

P7.30 The frequency response of an ideal bandpass filter is given by

$$H_d(e^{j\omega}) = \begin{cases} 0, & 0 \leq |\omega| \leq \pi/3 \\ 1, & \pi/3 \leq |\omega| \leq 2\pi/3 \\ 0, & 2\pi/3 \leq |\omega| \leq \pi \end{cases}$$

(a) Design of a 25-tap filter based on the Parks-McClellan algorithm with stopband attenuation of 50 dB.

```
clc; close all;
%% Specifications
wc1 = pi/3;    % lower cutoff frequency
wc2 = 2*pi/3; % upper cuoff frequency
As = 50;       % stopband attenuation
M = 25;        % filter length
%
% (a) Design
tr_width = 2*pi*(As-13)/(14.36*(M-1)), % transition width in radians
ws1 = wc1-tr_width/2; wp1 = wc1+tr_width/2;
wp2 = wc2-tr_width/2; ws2 = wc2+tr_width/2;
f = [0,ws1/pi,wp1/pi,wp2/pi,ws2/pi,1];
m = [0,0,1,1,0,0];
n = 0:M-1;
h = remez(M-1,f,m);
[db,mag,pha,grd,w] = freqz_m(h,1);
delta_w = pi/500;
Asd = floor(-max(db([1:floor(ws1/delta_w)+1]))), % Actual Attn
Rpd = -min(db(ceil(wp1/delta_w)+1:floor(wp2/delta_w)+1)), % Actial ripple
% Filter Response Plots
Hf_1 = figure('Units','inches','position',[1,1,6,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P7.30a');
subplot(2,1,1); Hs_1 = stem(n,h,'g','filled'); set(Hs_1,'markersize',3);
title('Impulse Response: Bandpass'); axis([-1,M,min(h)-0.1,max(h)+0.1]);
xlabel('n','fontsize',10); ylabel('h(n)','fontsize',10);
set(gca,'XTick',[0;12;24]);
subplot(2,1,2); plot(w/pi,db,'g','linewidth',1.5);
title('Magnitude Response in dB','fontsize',10); axis([0,1,-80,5]);
xlabel('\omega/\pi','fontsize',10); ylabel('Decibels','fontsize',10);
set(gca,'XTick',f,'fontsize',10,'YTick',[-50;0]);
set(gca,'YTickLabel',['-50';' 0 ']);grid
```

The impulse and log-magnitude response plots are shown in Figure 7.21.

(b) Plot of the amplitude response of the filter.

```
Hf_2 = figure('Units','inches','position',[1,1,6,3],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,3]);
set(Hf_2,'NumberTitle','off','Name','P7.30b');

[Hr,w,a,L] = Hr_type1(h); plot(w/pi,Hr,'g','linewidth',1.5);
title('Amplitude Response','fontsize',10)
```
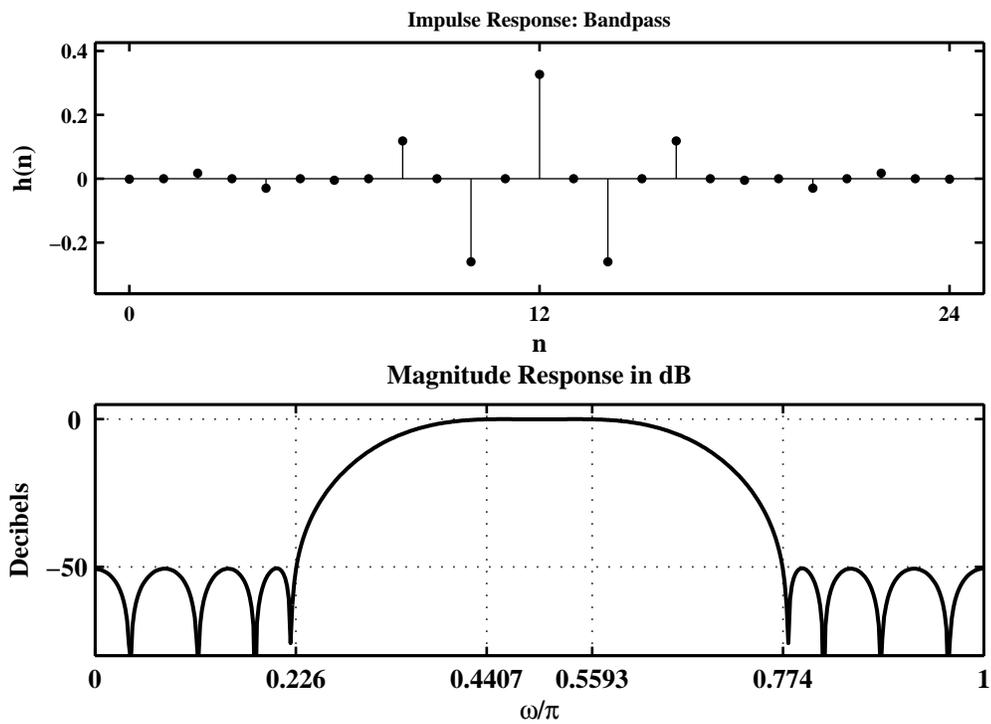
Figure 7.21: Filter impulse and log-magnitude response plots in Problem 7.30

```
xlabel('\omega/\pi','fontsize',10); ylabel('Amplitude','fontsize',10);
set(gca,'XTickMode','manual','XTick',f);axis([0,1,-0.1,1.1]);
```

The amplitude response plot is shown in Figure 7.22.
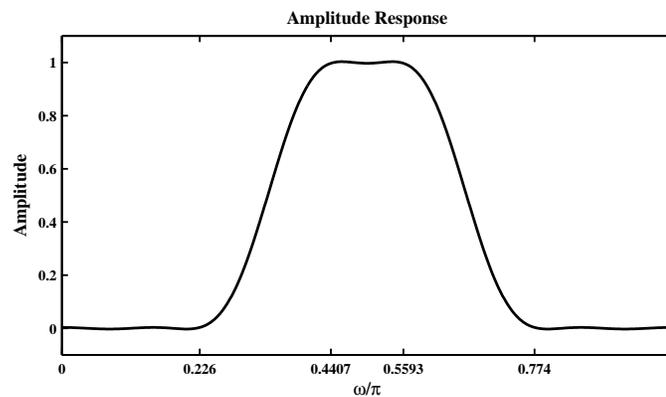
Figure 7.22: Filter amplitude response plot in Problem 7.30

P7.31 Design of the bandstop filter given in Problem P7.10.

(a) Design using the Parks-McClellan algorithm and a plots of the impulse response and the magnitude response in dB of the designed filter.

```
clc; close all;
%% Specifications
wp1 = 0.3*pi; % lower passband edge
ws1 = 0.4*pi; % lower stopband edge
ws2 = 0.6*pi; % upper stopband edge
wp2 = 0.7*pi; % upper passband edge
Rp = 0.25;      % passband ripple
As = 50;      % stopband attenuation
%
% 1. Design
delta1 = (10^(Rp/20)-1)/(10^(Rp/20)+1);
delta2 = (1+delta1)*(10^(-As/20));
weights = [delta2/delta1, 1, delta2/delta1];
delta_f =min((wp2-ws2)/(2*pi), (ws1-wp1)/(2*pi));
M = ceil((-20*log10(sqrt(delta1*delta2))-13)/(14.6*delta_f)+1);
M =
    43
M = 2*floor(M/2)+1
f = [0, wp1/pi, ws1/pi, ws2/pi, wp2/pi, 1];
m = [1 1 0 0 1 1];
h = remez(M-1,f,m,weights);
[db,mag,pha,grd,w] = freqz_m(h,[1]);
delta_w = pi/500;
Asd = floor(-max(db([floor(ws1/delta_w)+1:floor(ws2/delta_w)]))), % Actual Attn
Asd =
    46
M = M+2
M =
    45
h = remez(M-1,f,m,weights);
[db,mag,pha,grd,w] = freqz_m(h,[1]);
delta_w = pi/500;
Asd = floor(-max(db([floor(ws1/delta_w)+1:floor(ws2/delta_w)]))), % Actual Attn
Asd =
    52
n = 0:M-1;
% Filter Response Plots
Hf_1 = figure('Units','inches','position',[1,1,6,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P7.31a');
subplot(2,1,1); Hs_1 = stem(n,h,'g','filled'); set(Hs_1,'markersize',3);
title('Impulse Response: Bandpass','fontsize',10);
axis([-1,M,min(h)-0.1,max(h)+0.1]);
xlabel('n','fontsize',10); ylabel('h(n)','fontsize',10)
```

```
set(gca,'XTickMode','manual','XTick',[0;17;34])
subplot(2,1,2); plot(w/pi,db,'g','linewidth',1.5);
title('Magnitude Response in dB','fontsize',10); axis([0,1,-60,5]);
xlabel('\omega/\pi','fontsize',10); ylabel('Decibels','fontsize',10)
set(gca,'XTickMode','manual','XTick',f);
set(gca,'YTickMode','manual','YTick',[-50;0])
set(gca,'YTickLabel',[' 50';' 0ᵁ']);grid;
```

The filter response plots are shown in Figure 7.23.
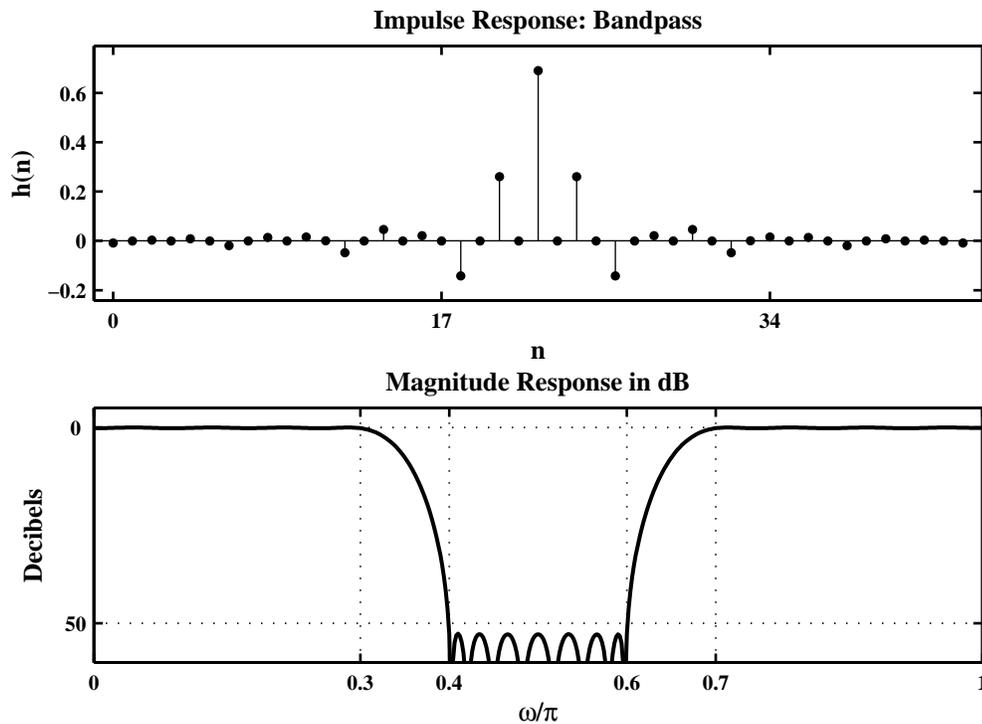
Figure 7.23: Filter impulse and log-magnitude response plots in Problem 7.31

(b) Plot of the amplitude response of the designed filter.

```
Hf_2 = figure('Units','inches','position',[1,1,6,3],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,3]);
set(Hf_2,'NumberTitle','off','Name','P7.31b');

[Hr,w,a,L] = Hr_type1(h);
plot(w/pi,Hr,'g','linewidth',1.5);
title('Amplitude Response','fontsize',10)
xlabel('\omega/\pi','fontsize',10); ylabel('Amplitude','fontsize',10)
axis([0,1,-0.1,1.1]); set(gca,'XTickMode','manual','XTick',f);
```

The amplitude response plot is shown in Figure 7.24.

From Figure 7.24 the total number of extrema in stopband and passbands are equal to 25. Since $M = 45$, $L = 22$. Then the extrema are $L + 2$ or $L + 3$. Hence this is a $L + 32 = 25$ equiripple design.
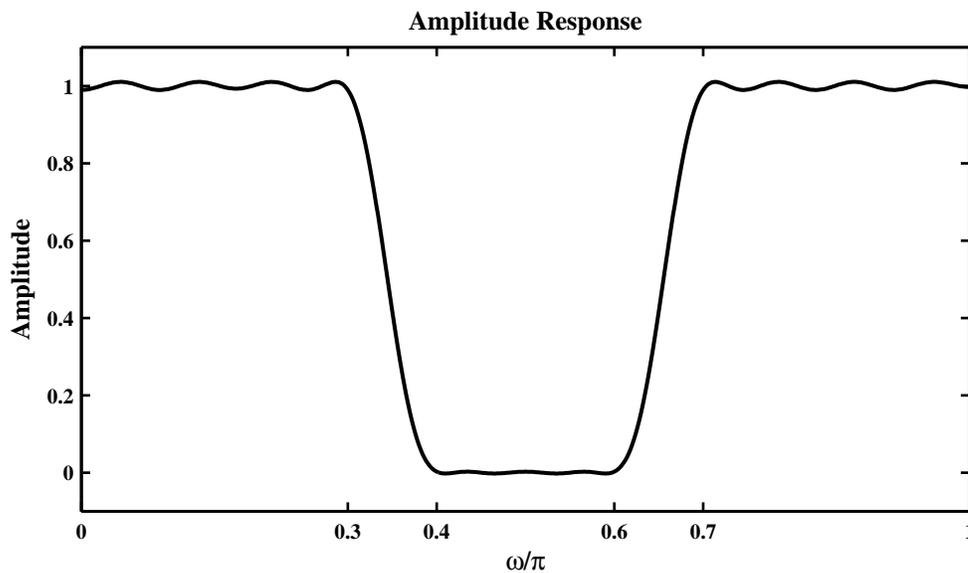
Figure 7.24: Filter amplitude response plot in Problem 7.31

(c) The filter order in Problem P7.10 is $67 - 1 = 66$ while that in P7.14 is $49 - 1 = 48$. The order using Parks-McClellan algorithm is 44.

(d) Operation of the designed filter on the following signal

$$x(n) = 5 - 5\cos\left(\frac{\pi n}{2}\right); \quad 0 \le n \le 300$$

```
n = 0:300; x = 5 - 5*cos(pi*n/2); y = filter(h,1,x);

Hf_3 = figure('Units','inches','position',[1,1,6,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_3,'NumberTitle','off','Name','P7.31c');
subplot(2,1,1); Hs_3 = stem(n(1:101),x(1:101),'g','filled');
axis([0,100,-1,11]); xlabel('n','fontsize',10); set(Hs_3,'markersize',3);
ylabel('x(n)','fontsize',10); title('Input Signal','fontsize',10);
subplot(2,1,2); Hs_4 = stem(n(1:101),y(1:101),'g','filled');
axis([0,100,-1,11]); xlabel('n','fontsize',10); set(Hs_4,'markersize',3);
ylabel('y(n)','fontsize',10); title('Output Signal','fontsize',10);
```

The input and output signal plots are shown in Figure 7.25.
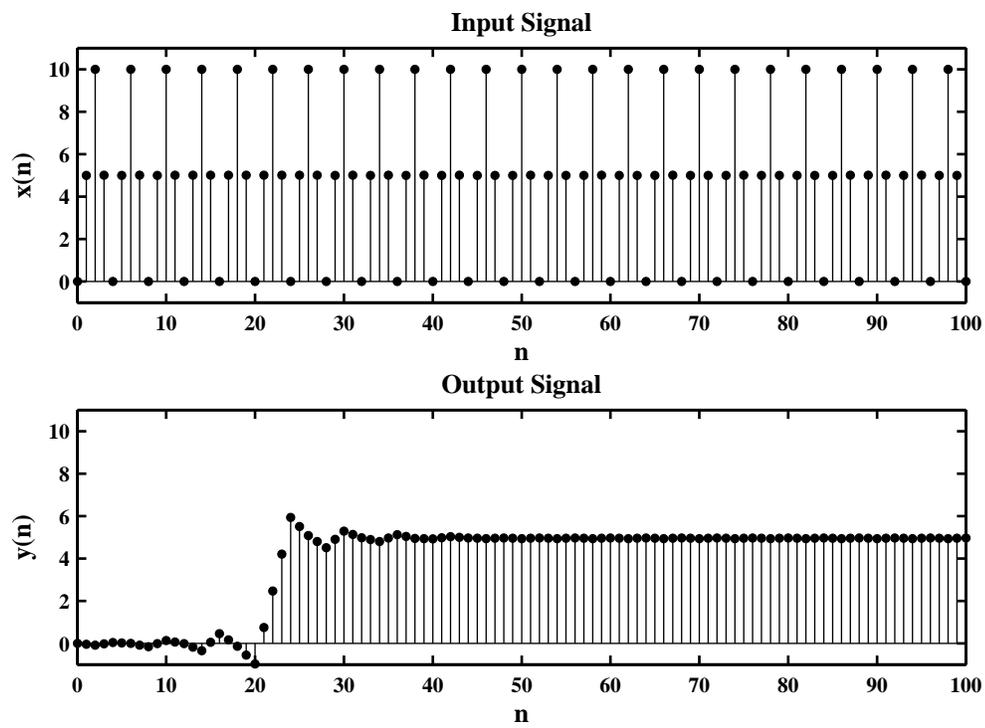
Figure 7.25: Input/output signal plots in Problem 7.31

P7.32 Design a 25-tap FIR differentiator using the Parks-McClellan algorithm, with slope equal to 1 sample/cycle.

(a) Frequency band of interest between 0.1π and 0.9π and plot of the impulse response and the amplitude response:

```
clc; close all;

%% Specifications
M = 25; w1 = 0.1*pi; w2 = 0.9*pi; % slope = 1 sam/cycle
%
% (a) Design
f = [w1/pi w2/pi]; m = [w1/(2*pi) w2/(2*pi)];
h = firpm(M-1,f,m,'differentiator'); [db,mag,pha,grd,w] = freqz_m(h,1);
Hf_1 = figure('Units','inches','position',[1,1,6,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P7.31a');
subplot(2,1,1); Hs_1 = stem([0:1:M-1],h,'g','filled');
set(Hs_1,'markersize',3); axis([-1,25,-0.2,.2]);
title('Impulse Response','fontsize',10);
xlabel('n','fontsize',10); ylabel('h(n)','fontsize',10);
set(gca,'XTick',[0;12;24],'fontsize',10);
subplot(2,1,2); [Hr,w,P,L] = ampl_res(h);
plot(w/(2*pi), Hr,'g','linewidth',1.5);
title('Amplitude Response','fontsize',10); grid; axis([0,0.5,0,0.5]);
set(gca,'XTick',[0;w1/(2*pi);w2/(2*pi);0.5]); set(gca,'YTick',[0;0.5]);
```

The filter impulse and amplitude response plots are shown in Figure 7.26.

(b) Verification of the filter performance using 100 samples of the sinusoid

$$x(n) = 3\sin(0.25\pi n), \quad n = 0, ..., 100$$

```
%% (b) Differentiator verification
Hf_1 = figure('Units','inches','position',[1,1,6,3],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,3]);
set(Hf_1,'NumberTitle','off','Name','P7.31b');
n=[0:1:100]; x = 3*sin(0.25*pi*n); y = conv(x,-h); m = [41:1:81];
plot(m,x(41:1:81),'g',m,y(41+12:1:81+12),'m',...
    'linewidth',1.5); grid; % add 12 sample delay to y
xlabel('n','fontsize',10); title('Input-Output Sequences','fontsize',10);
axis([40,82,-4,4]); set(gca,'XTick',[41;81]);
set(gca,'YTick',[-3;0;3]); ylabel('Amplitude','fontsize',10);
```

The filter input-output plots are shown in Figure 7.27. Since the slope is $\pi/2$ sam/rad, the gain at $\omega = 0.25\pi$ is equal to 0.125. Therefore, the output (when properly shifted) should be

$$y(n) = 3(0.125)\cos(0.25\pi n) = 0.375\cos(0.25\pi n)$$

From the Figure 7.27 we can verify that $y(n)$(the lower curve) is indeed a cosine waveform with amplitude ≈ 0.4.
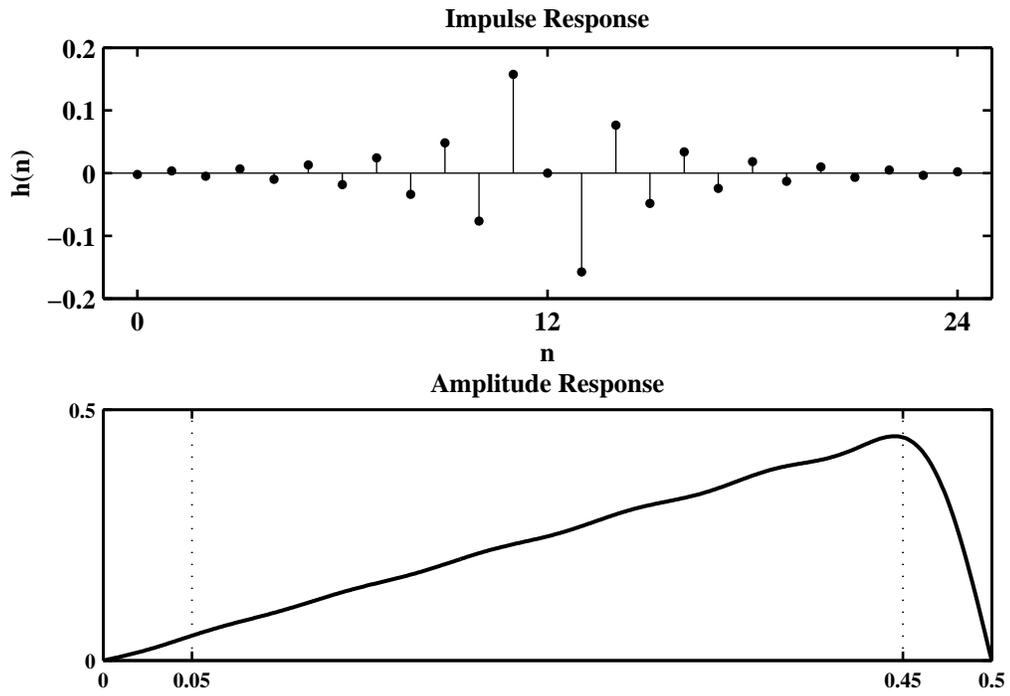
Impulse Response

Figure 7.26: Filter impulse and amplitude response plots in Problem 7.32

Input−Output Sequences
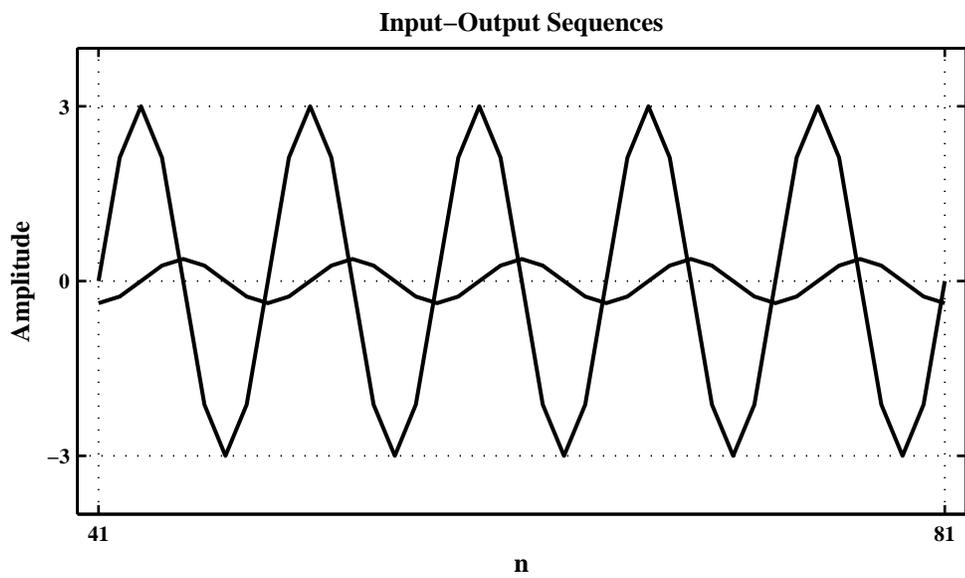
Figure 7.27: Filter input-output plots in Problem 7.32

P7.33 Design of a lowest order equiripple linear-phase FIR filter to satisfy the specifications given in Figure P7.33 and plots of its amplitude and impulse responses.

```
clc; close all;
%% Specifications
f = [0,0.4,0.5,0.7,0.8,1]; m = [0.4,0.4,0.95,0.95,0.025,0.025];
delta1 = 0.05; delta2 = 0.05; delta3 = 0.025;
weights = [delta3/delta2, delta3/delta2, delta3/delta3];
As = -20*log10(0.05)
As =
   26.0206
% Design
delta_f = 0.05; % Transition width in cycles per sample
M = ceil((-20*log10(sqrt(delta2*delta3))-13)/(14.6*delta_f)+1)
M =
   23
h = firpm(M-1,f,m,weights);
[db,mag,pha,grd,w] = freqz_m(h,1);
delta_w = pi/500;
Asd = floor(-max(db([(0.8*pi/delta_w)+1:501]))), % Actual Attn
Asd =
   24
M = M+2
M =
   25
h = firpm(M-1,f,m,weights);
[db,mag,pha,grd,w] = freqz_m(h,1);
Asd = floor(-max(db([(0.8*pi/delta_w)+1:501]))), % Actual Attn
Asd =
   25
M = M+2
M =
   27
h = firpm(M-1,f,m,weights);
[db,mag,pha,grd,w] = freqz_m(h,1);
Asd = floor(-max(db([(0.8*pi/delta_w)+1:501]))), % Actual Attn
Asd =
   26
n = 0:M-1;
% Filter Response Plots
Hf_1 = figure('Units','inches','position',[1,1,6,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P7.33');
subplot(2,1,1); Hs_1 = stem(n,h,'g','filled'); set(Hs_1,'markersize',3);
title('Impulse Response','fontsize',10);
axis([-1,M,min(h)-0.1,max(h)+0.1]); xlabel('n','fontsize',10);
ylabel('h(n)','fontsize',10); set(gca,'XTick',[0;13;26])
% Amplitude Response Plot
```

```
[Hr,w,a,L] = Hr_type1(h);
subplot(2,1,2); plot(w/pi,Hr,'g','linewidth',1.5);
title('Amplitude Response','fontsize',10); axis([0,1,0,1])
xlabel('\omega/\pi','fontsize',10); ylabel('Hr(w)','fontsize',10)
set(gca,'XTick',f,'YTick',[0;0.05;0.35;0.45;0.9;1]); grid
```

The filter amplitude and impulse response plots are shown in Figure 7.28.
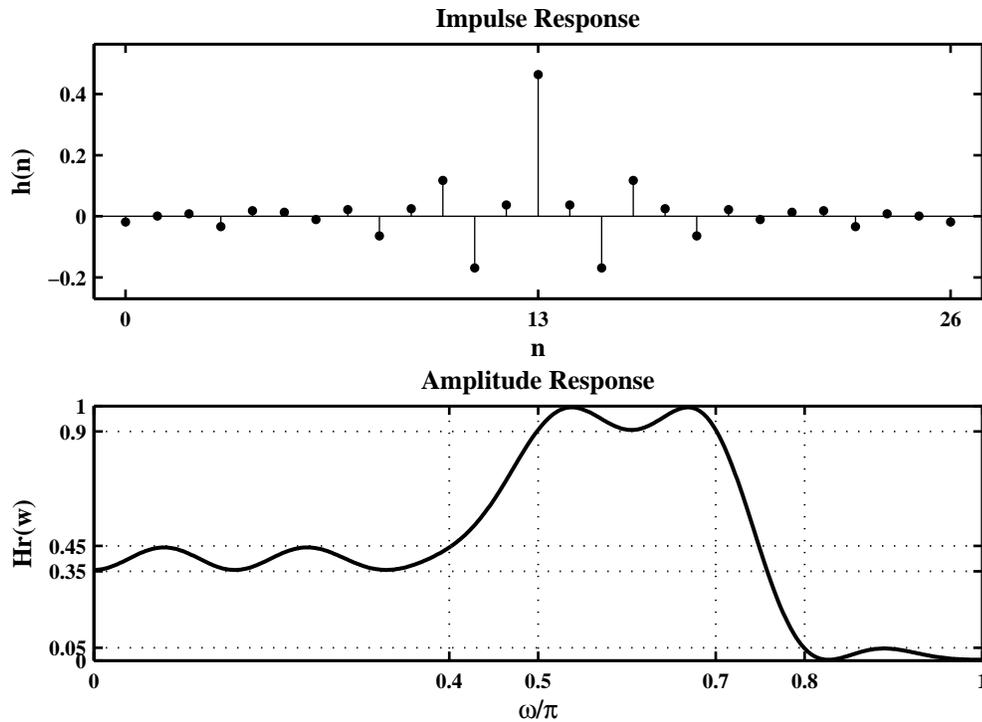
Figure 7.28: Filter amplitude and impulse response plots in Problem 7.33

P7.34 Narrow bandpass filter design using Parks-McClellan Algorithm: We want to design a 50th-order narrowband bandpass filter to filter out noise component with center frequency of $\omega_c = \pi/2$, bandwidth of 0.02π, and stopband attenuation of 30 dB.

(a) In this design we already know the order of the filter. The only parameters that we don't know are the stopband cutoff frequencies ω_{s_1} and ω_{s_2}. Let the transition bandwidth be $\Delta\omega$ and let the passband be symmetrical with respect to the center frequency ω_c. Then

$$\omega_{p_1} = \omega_c - 0.01\pi, \ \omega_{p_2} = \omega_c + 0.01\pi, \ \omega_{s_1} = \omega_{p_1} - \Delta\omega, \ \text{and} \ \omega_{s_2} = \omega_{p_2} + \Delta\omega$$

We will also assume that the tolerances in each band are equal. Now we will begin with initial value for $\Delta\omega = 0.2\pi$ and run the firpm algorithm to obtain the actual stopband attenuation. If it is smaller (larger) than the given 30 dB then we will increase (decrease) $\Delta\omega$ then iterate to obtain the desired solution. The desired solution was found for $\Delta\omega = 0.5\pi$. Matlab Script:

```
clear; close all;
%% Specifications
        N = 50;                              % Order of the filter
        w0 = 0.5*pi;                         % Center frequency
 Bandwidth = 0.02*pi;                        % Bandwidth
%
%   Deltaw = Transition bandwidth (iteration variable)
%
wp1 = w0-Bandwidth/2; wp2 = w0+Bandwidth/2;

% (a) Design
Deltaw = 0.02*pi;                            % Initial guess
ws1=wp1-Deltaw; ws2=wp2+Deltaw;
F=[0, ws1, wp1, wp2, ws2, pi]/pi;
m=[0,0,1,1,0,0];
h=remez(50,F,m);
[db,mag,pha,grd,w]=freqz_m(h,1);
delta_w = pi/500;
Asd = floor(-max(db([1:floor(ws1/delta_w)]))), % Actual Attn
Asd =
    13

% Next iteration
Deltaw = Deltaw+0.01*pi;
ws1=wp1-Deltaw; ws2=wp2+Deltaw;
F=[0, ws1, wp1, wp2, ws2, pi]/pi;
h=remez(50,F,m);
[db,mag,pha,grd,w]=freqz_m(h,1);
delta_w = pi/500;
Asd = floor(-max(db([1:floor(ws1/delta_w)]))), % Actual Attn
Asd =
    20

% Next iteration
```

```
Deltaw = Deltaw+0.01*pi;
ws1=wp1-Deltaw; ws2=wp2+Deltaw;
F=[0, ws1, wp1, wp2, ws2, pi]/pi;
h=remez(50,F,m);
[db,mag,pha,grd,w]=freqz_m(h,1);
delta_w = pi/500;
Asd = floor(-max(db([1:floor(ws1/delta_w)]))), % Actual Attn
Asd =
    26

% Next iteration
Deltaw = Deltaw+0.01*pi;
ws1=wp1-Deltaw; ws2=wp2+Deltaw;
F=[0, ws1, wp1, wp2, ws2, pi]/pi;
h=remez(50,F,m);
[db,mag,pha,grd,w]=freqz_m(h,1);
delta_w = pi/500;
Asd = floor(-max(db([1:floor(ws1/delta_w)]))), % Actual Attn
Asd =
    30

Hf_1 = figure('Units','inches','position',[1,1,6,3],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,3]);
set(Hf_1,'NumberTitle','off','Name','P7.34a');
plot(w/pi,db,'g','linewidth',1.5); axis([0,1,-50,0]);
title('Log-Magnitude Response','fontsize',10);
xlabel('\omega/\pi','fontsize',10); ylabel('DECIBELS','fontsize',10)
set(gca,'XTick',[0;ws1/pi;ws2/pi;1],'YTick',[-30;0])
set(gca,'YTickLabel',[' 30';' 0 ']);grid
```

The log-magnitude response is shown in Figure 7.29.

(b) The time-domain response of the filter. MATLAB script:

```
% (b) Time-domain Response
n = [0:1:200]; x = 2*cos(pi*n/2)+randn(1,201); y = filter(h,1,x);
Hf_1 = figure('Units','inches','position',[1,1,6,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P7.34b');
subplot(2,1,1); Hs_1 = stem(n(101:201),x(101:201),'g','filled');
title('Input sequence x(n)','fontsize',10); set(Hs_1,'markersize',3);
ylabel('Amplitude','fontsize',10);
subplot(2,1,2); Hs_2 = stem(n(101:201),y(101:201),'m','filled');
title('Output sequence y(n)','fontsize',10); set(Hs_2,'markersize',3);
xlabel('n','fontsize',10); ylabel('Amplitude','fontsize',10);
```

The time-domain response is shown in Figure 7.30.
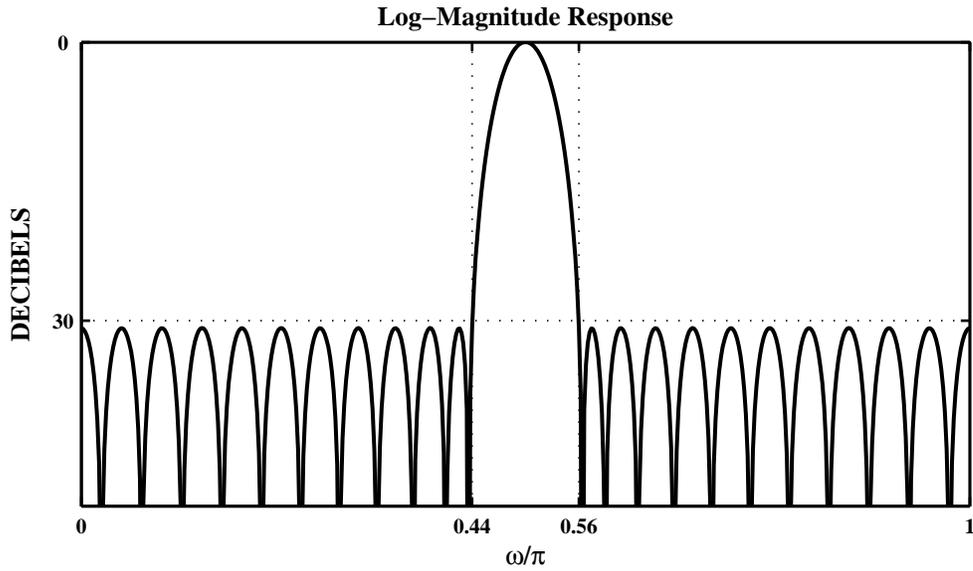
Figure 7.29: Log-Magnitude Response Plot in Problem P7.34a
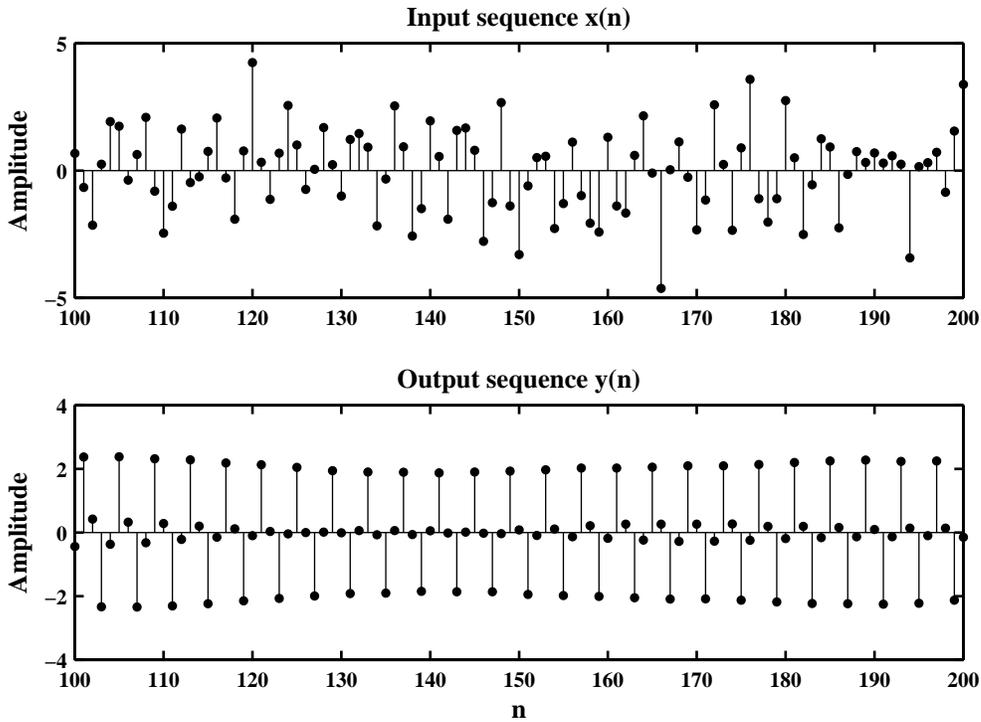
Figure 7.30: The Time-domain Response in Problem P7.34b

P7.35 Design an equiripple digital Hilbert transformer for the following specifications:

$$\text{passband} : 0.1\pi \leq |\omega| \leq 0.5\pi \quad \text{ripple } \delta_1 = 0.01$$
$$\text{stopband} : 0.5\pi \leq |\omega| \leq \pi \qquad \text{ripple } \delta_2 = 0.01$$

Plot the amplitude response over $-\pi \leq \omega \leq \pi$.

P7.36 Design of a minimum order linear-phase FIR filter, using the Parks-McClellan algorithm, to satisfy the requirements given in Figure P7.1.

(a) Design and plot of the amplitude response with grid-lines and axis labeling as shown in Figure P7.1:

```
clc; close all;
% Specifications
f = [0,0.25,0.35,0.65,0.75,1]; m = [0,1,2,2,1,0];
% Optimum Design
h = remez(46,f,m); [db,mag,pha,grd,w] = freqz_m(h,1);
Hf_1 = figure('Units','inches','position',[1,1,6,3],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,3]);
set(Hf_1,'NumberTitle','off','Name','P7.36a');
plot(w/pi,mag,'g','linewidth',1.5); axis([0,1,0,2.2]);
xlabel('\omega/\pi','fontsize',10); ylabel('Amplitude','fontsize',10);
title('Amplitude Response','fontsize',10);
set(gca,'xtick',f,'ytick',[1,2]); grid;
```

The amplitude response is shown in Figure 7.31.
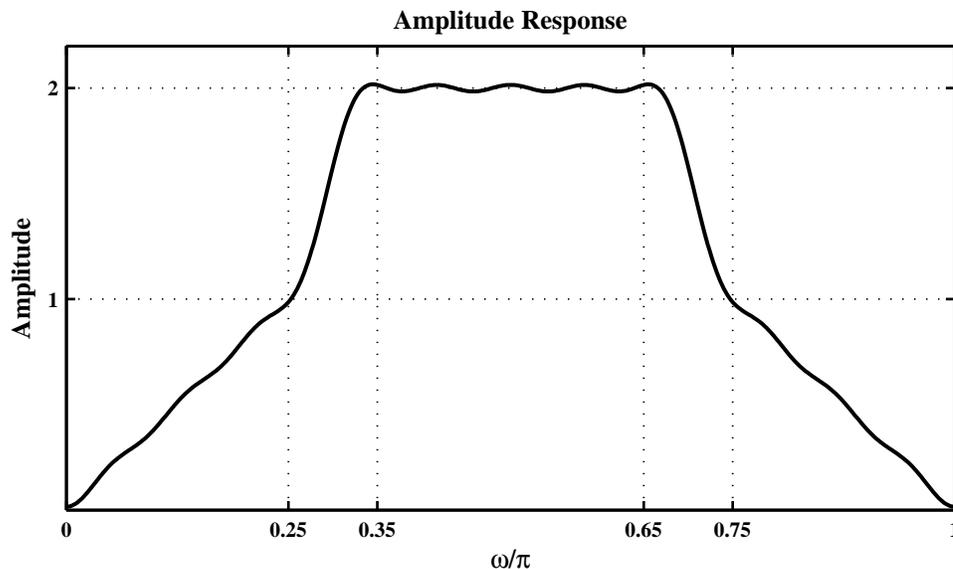
Figure 7.31: Amplitude Response Plot in Problem P7.36a

(b) Verification of the filter performance using the following signals

$$x_1(n) = \cos(0.25\pi n), \quad x_2(n) = \cos(0.5\pi n), \quad x_3(n) = \cos(0.75\pi n); \quad 0 \le n \le 100.$$

```
% Input/Output Responses
n = 0:100; x = 1*cos(0.25*pi*n)+0*cos(0.5*pi*n)+cos(pi*n);
y = filter(h,1,x);
Hf_1 = figure('Units','inches','position',[1,1,6,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,4]);
```

```
set(Hf_1,'NumberTitle','off','Name','P7.36a');
subplot(2,1,1); Hs_1 = stem(n,x,'g','filled'); set(Hs_1,'markersize',3);
title('Input Sequence','fontsize',10); ylabel('x(n)','fontsize',10);
subplot(2,1,2); Hs_2 = stem(n,y,'m','filled'); set(Hs_2,'markersize',3);
title('Output Sequence','fontsize',10); ylabel('y(n)','fontsize',10);
xlabel('n','fontsize',10);
```

The time-domain response is shown in Figure 7.32.
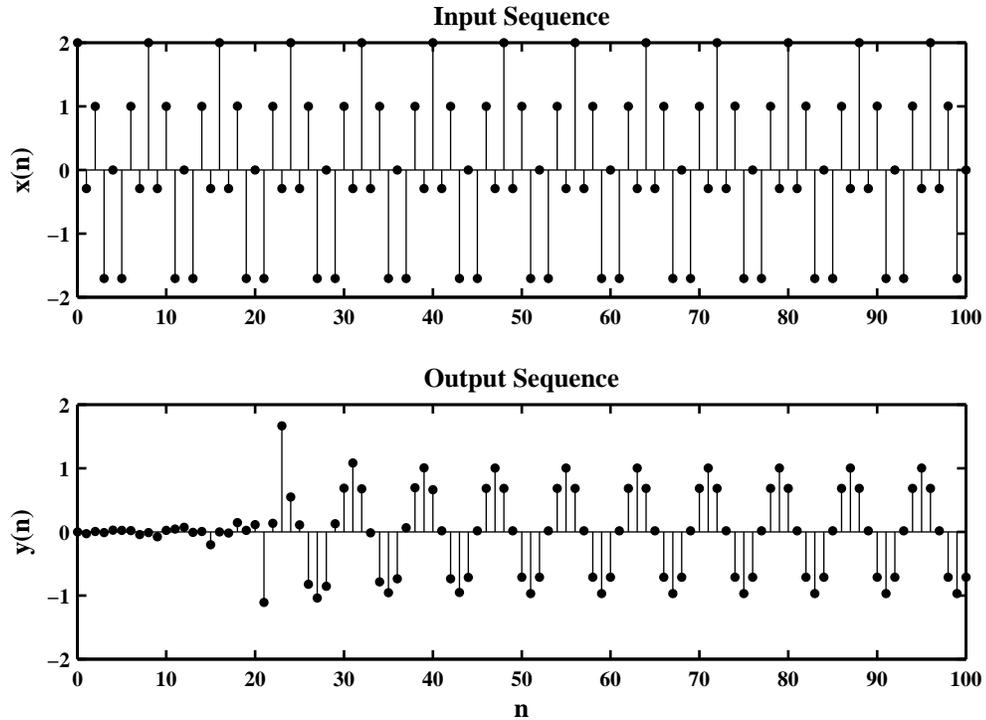
Figure 7.32: The Time-domain Response in Problem P7.36b

P7.37 Design of a minimum order linear-phase FIR filter, using the Parks-McClellan algorithm, to satisfy the require-
ments given in Figure P7.3 and a plot of the amplitude response with grid-lines and axis labeling as shown in
Figure P7.3.

```
clc; close all;
%Specifications
f = [0,0.2,0.25,0.45,0.55,0.7,0.75,1]; m = [2,2,0,0,3,3,1,1];
delta = [0.2,0.05,0.3,0.1]; YT = [1;-1]*delta; YT = (YT(:))'; YT = m+YT;
weight = delta(2)*ones(1,4)./delta;
% Optimum Design
M = 45;h = remez(M-1,f,m,weight); [Hr,w,a,L] = hr_type1(h); Hr_min = -min(Hr(126:226));
disp(sprintf('\n Achieved Tolerance in the stopband = %5.2f',Hr_min));
 Achieved Tolerance in the stopband =  0.04
Hf_1 = figure('Units','inches','position',[1,1,6,3],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,3]);
set(Hf_1,'NumberTitle','off','Name','P7.37');
plot(w/pi,Hr,'g','linewidth',1.5); axis([0,1,-0.1,3.5]); hold on; m = YT;
plot([f(1),f(2)],[m(1),m(1)],'y:',[f(1),f(2)],[m(2),m(2)],'y:');
plot([f(3),f(4)],[m(3),m(3)],'g:',[f(3),f(4)],[m(4),m(4)],'g:');
plot([f(5),f(6)],[m(5),m(5)],'m:',[f(5),f(6)],[m(6),m(6)],'m:');
plot([f(7),f(8)],[m(7),m(7)],'c:',[f(7),f(8)],[m(8),m(8)],'c:');
title('Amplitude Response','fontsize',10)
xlabel('\omega/\pi','fontsize',10);ylabel('H_{r}(\omega)','fontsize',10);
set(gca,'XTickMode','manual','XTick',f); set(gca,'YTickMode','manual','YTick',sort(YT));
hold off;%grid;
```

The amplitude response is shown in Figure 7.33.
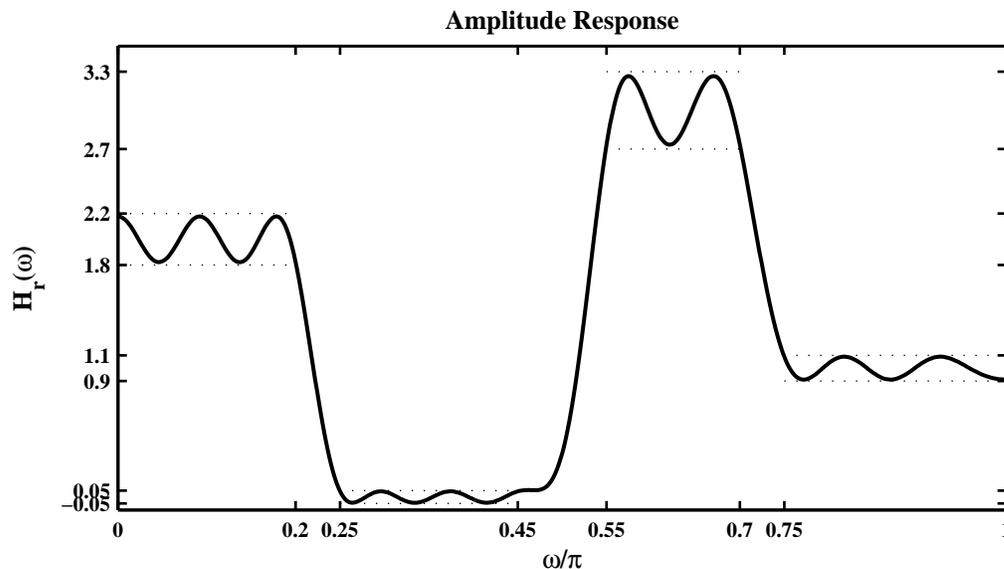


Figure 7.33: Amplitude Response Plot in Problem P7.37

**P7.38** The specifications on the amplitude response of an FIR filter are given in Figure P7.4.

```
clc; close all;
% Given specifications
f = [0,0.2,0.25,0.45,0.5,0.7,0.75,1];        % Bandedge Frequencies
m = [0,0,2,2,0,0,4.15,4.15];                 % Nominal (Ideal) band gains
d1 = 0.05; d2 = 0.1; d3 = 0.05; d4 = 0.15; % Band tolerances
% Stopband Attenuation
As = ceil(-20*log10(d1/(m(7)+d4)))
% Band-edge indices
I1 = 500*f(1)+1; % index into omega array for f(1) band-edge
I2 = 500*f(2)+1; % index into omega array for f(2) band-edge
I3 = 500*f(3)+1; % index into omega array for f(3) band-edge
I4 = 500*f(4)+1; % index into omega array for f(4) band-edge
I5 = 500*f(5)+1; % index into omega array for f(5) band-edge
I6 = 500*f(6)+1; % index into omega array for f(6) band-edge
I7 = 500*f(7)+1; % index into omega array for f(7) band-edge
I8 = 500*f(8)+1; % index into omega array for f(8) band-edge
```

(a) Minimum-length linear-phase FIR filter design using a *fixed* window function, to satisfy the given requirements and a plot of the amplitude response with grid-lines as shown in Figure P7.4:

```
% (a) Fixed window design: Stopband Attn <= 39db => Hanning Window
delta_f = f(3)-f(2); M = 6.2/delta_f; M = floor(M/2)*2+1;
% The above value of M is 125. The minimum M was found to be 119
M = 119; N = M-1;
w_han = hanning(M)';
h_ideal = m(7)*(ideal_lp(pi,M) - ideal_lp(0.725*pi,M))...
    + m(3)*(ideal_lp(0.475*pi,M) - ideal_lp(0.225*pi,M));
h = h_ideal.*w_han;
[Hr,w,a,L] = hr_type1(h);
% Computation of Exact band edges
w1 = 0;
w2 = w(max(find(Hr(I2:I3) <= 0.05)) + I2-1)/pi;
w3 = w(min(find(Hr(I2:I3) >= 1.90)) + I2-1)/pi;
w4 = w(max(find(Hr(I4:I5) >= 1.90)) + I4-1)/pi;
w5 = w(min(find(Hr(I4:I5) <= 0.05)) + I4-1)/pi;
w6 = w(max(find(Hr(I6:I7) <= 0.05)) + I6-1)/pi;
w7 = w(min(find(Hr(I6:I7) >= 4.00)) + I6-1)/pi;
w8 = 1;
% Computation of Exact tolerances
m1 = abs(min(Hr(I1:I2)));
m2 = max(Hr(I3:I4))-m(3);
m3 = abs(min(Hr(I5:I6)));
m4 = max(Hr(I7:I8))-m(7);
%
% Plot of Amplitude Response
Hf_1 = figure('paperunits','inches','paperposition',[0,0,6,3.5]);
set(Hf_1,'NumberTitle','off','Name','P7.38a');
```

```
plot(w/pi,Hr,'g','linewidth',1.5);  set(gca,'fontsize',8);
axis([0,1,-0.05,4.3]);
title('Amplitude Response in Part 1','fontsize',10);
xlabel('\omega/\pi','fontsize',10);
ylabel('Hr(w)','fontsize',10);grid;
set(gca,'XTickMode','manual','XTick',f);
set(gca,'YTickMode','manual','YTick',[-0.05;0.05;1.9;2.1;4.0;4.3]);
% Printout of Order, Exact bandedges, and Exact tolerances
disp(sprintf('\n(a) Fixed Window Design:'));
disp(sprintf('           Order : %2i',N));
disp(sprintf(' Exact band-edges: %5.4f, %5.4f, %5.4f, %5.4f, %5.4f, %5.4f',...
    w2, w3, w4, w5, w6, w7));
disp(sprintf(' Exact tolerances: %5.4f, %5.4f, %5.4f, %5.4f,',...
    m1, m2, m3, m4));
```

A plot of the amplitude response with grid-lines and axis labeling is shown in Figure 7.34.
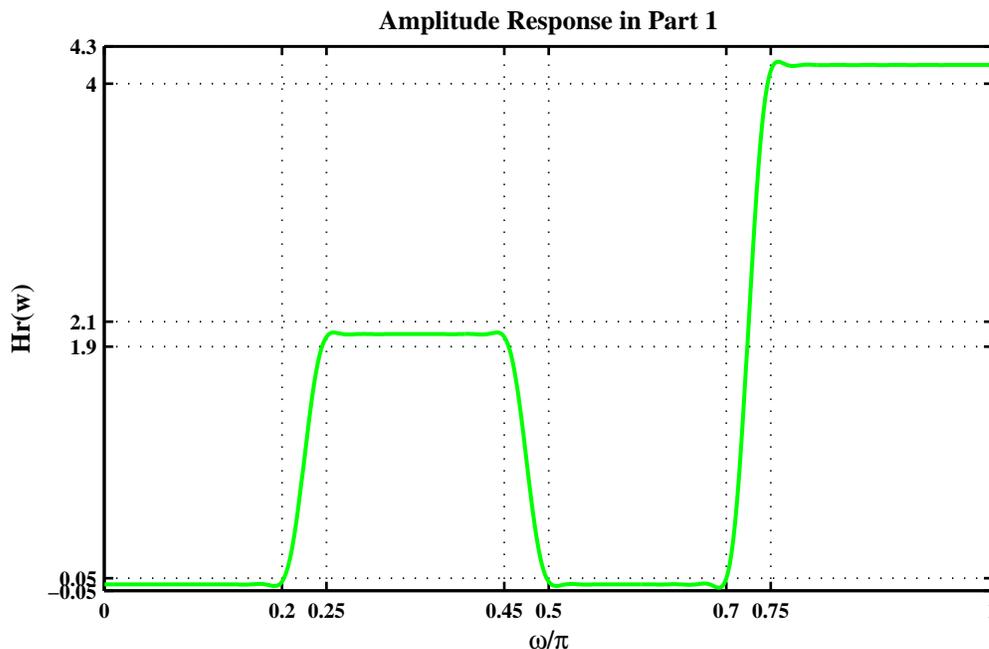
Figure 7.34: Amplitude Response Plot in Problem P7.38a

(b) Minimum-length linear-phase FIR filter design using the Kaiser window function to satisfy the given requirements and a plot of the amplitude response with grid-lines as shown in Figure P7.4:

```
% (b) Kaiser Window Design:
delta_f = (f(3)-f(2))/2;
M = ceil((As - 7.95)/(14.36*delta_f) + 1); M = floor(M/2)*2+1;
% The above value of M is 89. The minimum M was found to be 89
M = 89; N = M-1;
beta = 0.5842*(As-21)^0.4 + 0.07886*(As-21);
w_kai = kaiser(M,beta)';
```

```
h_ideal = m(7)*(ideal_lp(pi,M) - ideal_lp(0.725*pi,M))...
    + m(3)*(ideal_lp(0.475*pi,M) - ideal_lp(0.225*pi,M));
h = h_ideal.*w_kai;
[Hr,w,a,L] = hr_type1(h);
% Computation of Exact band edges
w1 = 0;
w2 = w(max(find(Hr(I2:I3) <= 0.05)) + I2-1)/pi;
w3 = w(min(find(Hr(I2:I3) >= 1.90)) + I2-1)/pi;
w4 = w(max(find(Hr(I4:I5) >= 1.90)) + I4-1)/pi;
w5 = w(min(find(Hr(I4:I5) <= 0.05)) + I4-1)/pi;
w6 = w(max(find(Hr(I6:I7) <= 0.05)) + I6-1)/pi;
w7 = w(min(find(Hr(I6:I7) >= 4.00)) + I6-1)/pi;
w8 = 1;
% Computation of Exact tolerances
m1 = abs(min(Hr(I1:I2)));
m2 = max(Hr(I3:I4))-m(3);
m3 = abs(min(Hr(I5:I6)));
m4 = max(Hr(I7:I8))-m(7);
%
% Plot of Amplitude Response
Hf_2 = figure('paperunits','inches','paperposition',[0,0,6,3.5]);
set(Hf_2,'NumberTitle','off','Name','P7.38b');
plot(w/pi,Hr,'g','linewidth',1.5);  set(gca,'fontsize',8);
axis([0,1,-0.05,4.3]);
title('Amplitude Response in Part 2','fontsize',10);
xlabel('\omega/\pi','fontsize',10);
ylabel('Hr(w)','fontsize',10);grid;
set(gca,'XTickMode','manual','XTick',f);
set(gca,'YTickMode','manual','YTick',[-0.05;0.05;1.9;2.1;4.0;4.3]);
%
% Printout of Order, Exact bandedges, and Exact tolerances
disp(sprintf('\n(b) Kaiser Window Design:'));
disp(sprintf('          Order : %2i',N));
disp(sprintf(' Exact band-edges: %5.4f, %5.4f, %5.4f, %5.4f, %5.4f, %5.4f',...
    w2, w3, w4, w5, w6, w7));
disp(sprintf(' Exact tolerances: %5.4f, %5.4f, %5.4f, %5.4f,',...
    m1, m2, m3, m4));
```

A plot of the amplitude response with grid-lines and axis labeling is shown in Figure 7.35.

(c) Minimum-length linear-phase FIR filter design using a frequency-sampling design approach with no more than two optimum samples in the transition bands to satisfy the given requirements and a plot of the amplitude response with grid-lines as shown in Figure P7.4:

```
% (c) Frequency Sampling Design: Choose M = 81
%
M = 81; N = M-1; alpha = (M-1)/2; l = 0:M-1; wl = (2*pi/M)*l;
% The following parameters are obtained by trial and error
T1 = 2*0.2; T2 = 2*0.9; T3 = 2*0.7; T4 = 2*0.1; T5 = 4.15*0.2; T6 = 4.15*0.85;
Hrs = [zeros(1,9),T1,T2, 2*ones(1,8),T3,T4,0.00,zeros(1,6),-0.02,T5,T6,4.15*ones(1,10)
```

Amplitude Response in Part 2
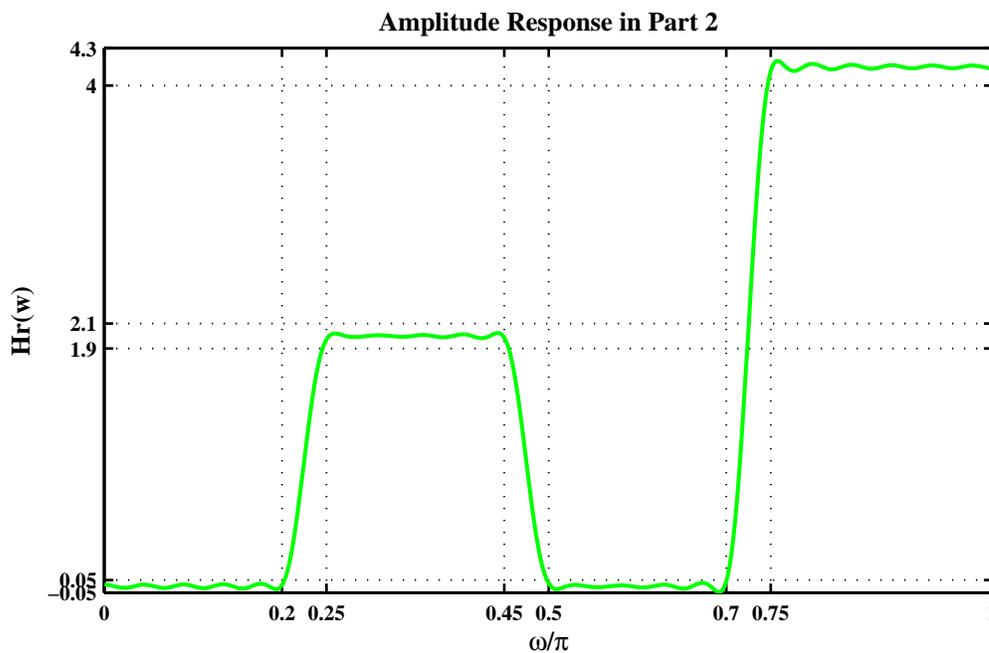
Figure 7.35: Amplitude Response Plot in Problem P7.38b

```
Hrs = [Hrs,fliplr(Hrs(2:end))];
k1 = 0:floor((M-1)/2); k2 = floor((M-1)/2)+1:M-1;
angH = [-alpha*(2*pi)/M*k1, alpha*(2*pi)/M*(M-k2)];
H = Hrs.*exp(j*angH);
h = real(ifft(H,M));
[Hr,w,a,L] = Hr_Type1(h);
%
% Computation of Exact band edges
w1 = 0;
w2 = w(max(find(Hr(I2:I3) <= 0.05)) + I2-1)/pi;
w3 = w(min(find(Hr(I2:I3) >= 1.90)) + I2-1)/pi;
w4 = w(max(find(Hr(I4:I5) >= 1.90)) + I4-1)/pi;
w5 = w(min(find(Hr(I4:I5+1) <= 0.05)) + I4-1)/pi;
w6 = w(max(find(Hr(I6:I7) <= 0.05)) + I6-1)/pi;
w7 = w(min(find(Hr(I6:I7) >= 4.00)) + I6-1)/pi;
w8 = 1;
%
% Computation of Exact tolerances
m1 = abs(min(Hr(I1:I2)));
m2 = max(Hr(I3:I4))-m(3);
m3 = abs(min(Hr(I5:I6)));
m4 = max(Hr(I7:I8))-m(7);
%
% Plot of Amplitude Response
Hf_3 = figure('paperunits','inches','paperposition',[0,0,6,3.5]);
```

```
set(Hf_3,'NumberTitle','off','Name','P7.38c');
plot(w/pi,Hr,'g',wl(1:41)/pi,Hrs(1:41),'ro','linewidth',1.5);
axis([0,1,-0.05,4.3]);
title('Amplitude Response in Part 3','fontsize',10);
xlabel('\omega/\pi','fontsize',10);
ylabel('Hr(w)','fontsize',10);grid;
set(gca,'XTickMode','manual','XTick',f);
set(gca,'YTickMode','manual','YTick',[-0.05;0.05;1.9;2.1;4.0;4.3]);
%
% Printout of Order, Exact bandedges, and Exact tolerances
disp(sprintf('\n(c) Frequency Sampling Design:'));
disp(sprintf('          Order : %2i',N));
disp(sprintf(' Exact band-edges: %5.4f, %5.4f, %5.4f, %5.4f, %5.4f, %5.4f',...
    w2, w3, w4, w5, w6, w7));
disp(sprintf(' Exact tolerances: %5.4f, %5.4f, %5.4f, %5.4f,',...
    m1, m2, m3, m4));
```

A plot of the amplitude response with grid-lines and axis labeling is shown in Figure 7.36.
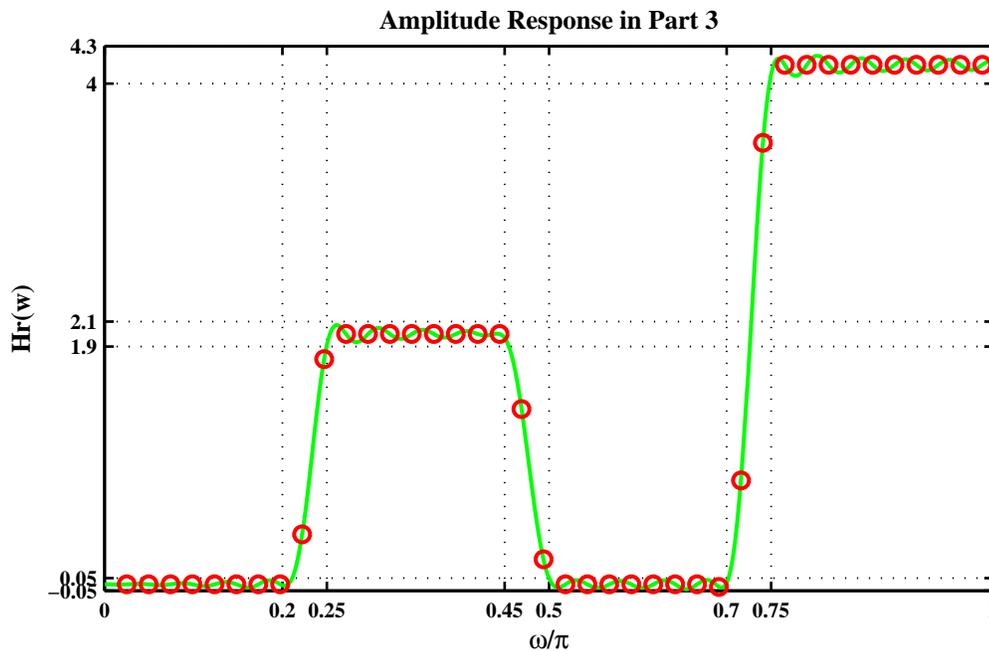
Figure 7.36: Amplitude Response Plot in Problem P7.38c

(d) Minimum-length linear-phase FIR filter design using the Parks-McClellan design approach to satisfy the given requirements and a plot of the amplitude response with grid-lines as shown in Figure P7.4:

```
% (d) Equiripple design: The minimum M was found to be 65
%
weights = [d1/d1,d1/d2,d1/d3,d1/d4];
M = 65; N = M-1; h = remez(M-1,f,m,weights);
[Hr,w,a,L] = hr_type1(h);
```

```
%
% Computation of Exact band edges
w1 = 0;
w2 = w(max(find(Hr(I2:I3) <= 0.05)) + I2-1)/pi;
w3 = w(min(find(Hr(I2:I3) >= 1.90)) + I2-1)/pi;
w4 = w(max(find(Hr(I4:I5) >= 1.90)) + I4-1)/pi;
w5 = w(min(find(Hr(I4:I5) <= 0.05)) + I4-1)/pi;
w6 = w(max(find(Hr(I6:I7) <= 0.05)) + I6-1)/pi;
w7 = w(min(find(Hr(I6:I7) >= 4.00)) + I6-1)/pi;
w8 = 1;
%
% Computation of Exact tolerances
m1 = abs(min(Hr(I1:I2)));
m2 = max(Hr(I3:I4))-m(3);
m3 = abs(min(Hr(I5:I6)));
m4 = max(Hr(I7:I8))-m(7);
%
% Plot of Amplitude Response
Hf_4 = figure('paperunits','inches','paperposition',[0,0,6,3.5]);
set(Hf_4,'NumberTitle','off','Name','P7.38d');
plot(w/pi,Hr,'g','linewidth',1.5); axis([0,1,-0.05,4.3]);
title('Amplitude Response in Part 4','fontsize',10);
xlabel('\omega/\pi','fontsize',10);
ylabel('Hr(w)','fontsize',10);grid;
set(gca,'XTickMode','manual','XTick',f);
set(gca,'YTickMode','manual','YTick',[-0.05;0.05;1.9;2.1;4.0;4.3]);
%
% Printout of Order, Exact bandedges, and Exact tolerances
disp(sprintf('\n(d) Parks-McClellan Design:'));
disp(sprintf('          Order : %2i',N));
disp(sprintf(' Exact band-edges: %5.4f, %5.4f, %5.4f, %5.4f, %5.4f, %5.4f',...
    w2, w3, w4, w5, w6, w7));
disp(sprintf(' Exact tolerances: %5.4f, %5.4f, %5.4f, %5.4f,',...
    m1, m2, m3, m4));
```

A plot of the amplitude response with grid-lines and axis labeling is shown in Figure 7.37.

(e) Comparison of the above four design methods in terms of

- the *order* of the filter
- the *exact* band-edge frequencies
- the *exact* tolerances in each band

```
(a) Fixed Window Design:
          Order : 118
 Exact band-edges: 0.2020, 0.2460, 0.4540, 0.4980, 0.7000, 0.7480
 Exact tolerances: 0.0127, 0.0127, 0.0262, 0.0263,

(b) Kaiser Window Design:
          Order : 88
```
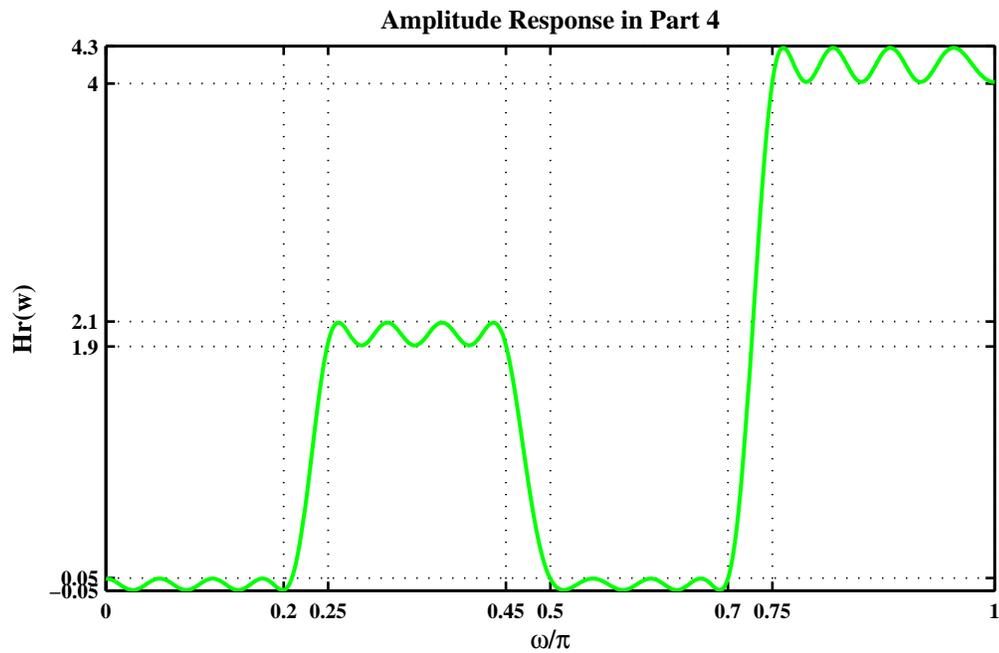
Figure 7.37: Amplitude Response Plot in Problem P7.38d

```
 Exact band-edges: 0.2020, 0.2460, 0.4540, 0.5000, 0.7000, 0.7480
 Exact tolerances: 0.0225, 0.0233, 0.0464, 0.0475,

(c) Frequency Sampling Design:
          Order : 80
 Exact band-edges: 0.2100, 0.2500, 0.4540, 0.5020, 0.7020, 0.7500
 Exact tolerances: 0.0262, 0.0706, 0.0387, 0.0731,

(d) Parks-McClellan Design:
          Order : 64
 Exact band-edges: 0.2080, 0.2500, 0.4500, 0.5000, 0.7000, 0.7500
 Exact tolerances: 0.0460, 0.0912, 0.0455, 0.1375,
```

P7.39 Design of a minimum-order linear-phase FIR filter, using the Parks-McClellan algorithm, to satisfy the requirements given in Figure P7.5 and a plot of the amplitude response with grid-lines as shown in Figure P7.5:

```
clc; close all;
% Specifications
f = [0,0.3,0.35,0.5,0.55,0.75,0.8,1];
m = [0.95,0.95,2,2,3.05,3.05,4.05,4.05];
d1 = 0.05; d2=0.1; d3=.05; d4=.05;
weights = [d4/d1,d4/d2,d4/d3,d4/d4];
% Optimum Design
h = remez(50,f,m,weights); [Hr,w,a,L] = hr_type1(h);
Hr_min = min(Hr(401:501));
disp(sprintf('\n Achieved Redsponse in the Band-4 = %5.2f',Hr_min));

 Achieved Redsponse in the Band-4 =  4.01
%
Hf_1 = figure('Units','inches','position',[1,1,6,3],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,3]);
set(Hf_1,'NumberTitle','off','Name','P7.39');
plot(w/pi,Hr,'g','linewidth',1.5); axis([0,1,0,5]);
title('Amplitude Response','fontsize',10);
xlabel('\omega/\pi','fontsize',10); ylabel('H_r(\omega)','fontsize',10)
set(gca,'XTickMode','manual','XTick',f); grid;
set(gca,'YTickMode','manual','YTick',[0.9,1.0,1.9,2.1,3.0,3.1,4.0,4.1]);
```

A plot of the amplitude response with grid-lines and axis labeling is shown in Figure 7.38.
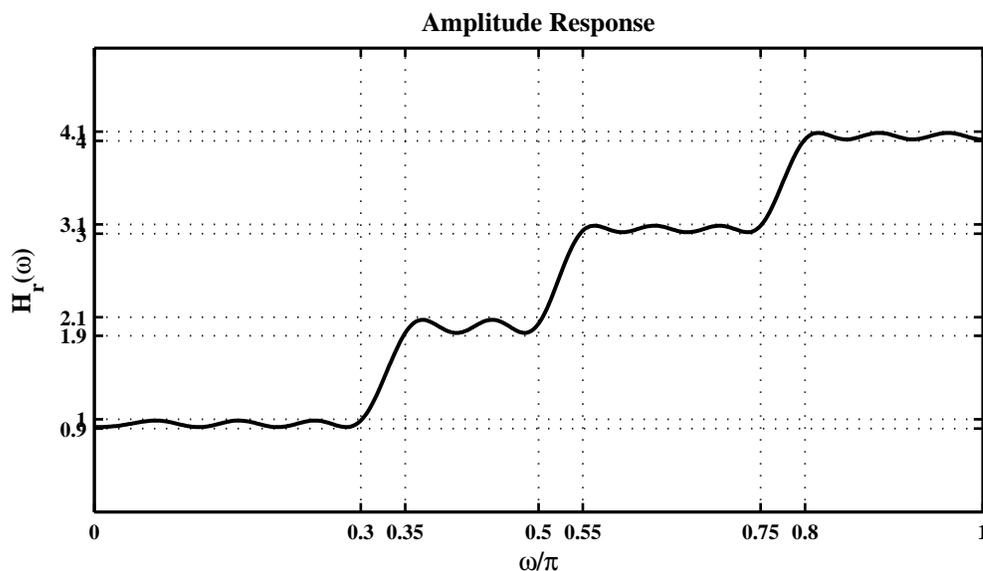
Figure 7.38: Amplitude Response Plot in Problem P7.39

P7.40 Design of a minimum-length linear-phase bandpass filter of Problem P7.9 using the Parks-McClellan algorithm:

```
clc; close all;
%% Specifications:
ws1 = 0.2*pi;  % lower stopband edge
wp1 = 0.35*pi; % lower passband edge
wp2 = 0.55*pi; % upper passband edge
ws2 = 0.75*pi; % upper stopband edge
 Rp = 0.25;    % passband ripple
 As = 40;      % stopband attenuation
%
% Compute Tolerances
[delta1,delta2] = db2delta(Rp,As);
% Assemble Design Parameters
f = [0,ws1,wp1,wp2,ws2,pi]/pi; m = [0,0,1,1,0,0];
weights = [1,delta2/delta1,1];
% Optimum Design
M = 26; h = firpm(M-1,f,m,weights); n= 0:M-1;
% Response Plots
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
Rpd = -min(db((wp1/delta_w)+1:(wp2/delta_w)+1)), % Actual passband ripple
Rpd =
    0.2170
Asd = floor(-max(db(1:(ws1/delta_w)+1))),        % Actual Attn
Asd =
    41
```

(a) Plot of the impulse response and the magnitude response in dB of the designed filter:

```
%% 1. Filter Impulse and Log-Magnitude Response Plots
Hf_1 = figure('Units','inches','position',[1,1,6,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P7.40a'); subplot(2,1,1);
Hs_1 = stem(n,h,'g','filled'); set(Hs_1,'markersize',3);
title('Impulse Response','fontsize',10);
set(gca,'XTick',[0;M-1]); axis([-1,M,min(h)-0.1,max(h)+0.1]);
xlabel('n','fontsize',10); ylabel('h(n)','fontsize',10)
subplot(2,1,2); plot(w/pi,db,'g','linewidth',1.5);
title('Magnitude Response in dB','fontsize',10); axis([0,1,-As-30,5]);
xlabel('\omega/\pi','fontsize',10); ylabel('Decibels','fontsize',10)
set(gca,'XTick',[0;0.2;0.35;0.55;0.75;1])
set(gca,'XTickLabel',[' 0  ';'0.2 ';'0.35';'0.55';'0.75';' 1  '],'fontsize',8)
set(gca,'YTick',[-40;0]); set(gca,'YTickLabel',[' 40';' 0 ']);grid
```

The plots are shown in Figure 7.39.

(b) Plot the amplitude response of the designed filter:

```
%% 2. Amplitude Response plot
[Hr,w,a,L] = hr_type2(h);
```

Impulse Response


Magnitude Response in dB
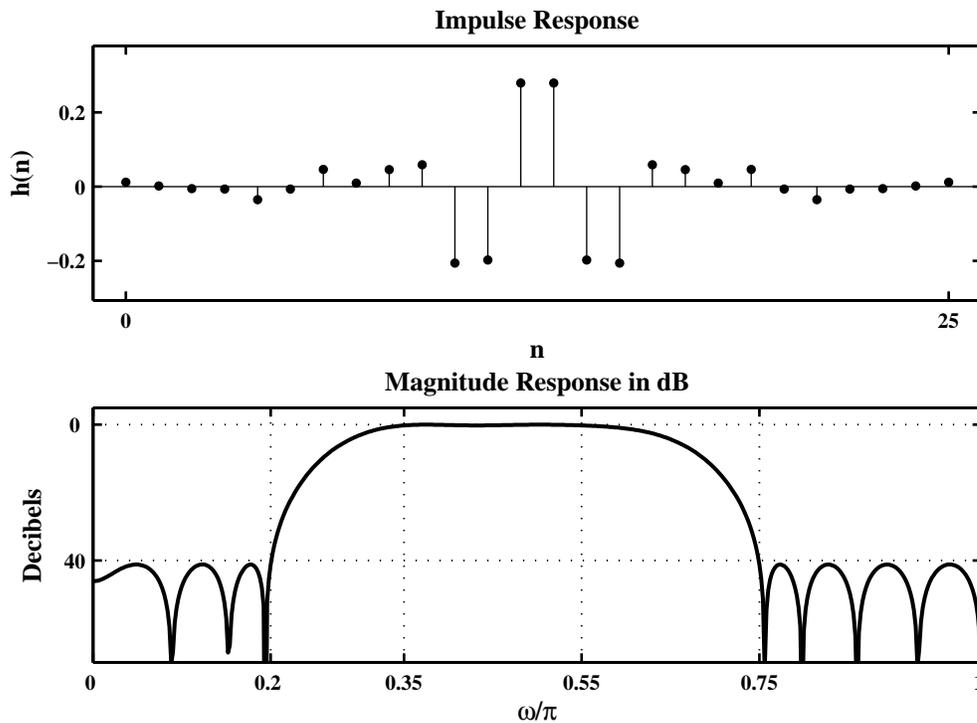
Figure 7.39: Amplitude Response Plot in Problem P7.40a

```
Hf_2 = figure('Units','inches','position',[1,1,6,3],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,3]);
set(Hf_2,'NumberTitle','off','Name','P7.40b');
plot(w/pi,Hr,'g','linewidth',1.5);
title('Amplitude Response','fontsize',10); axis([0,1,-0.2,1.2]);
xlabel('\omega/\pi','fontsize',10); ylabel('Amplitude','fontsize',10)
set(gca,'XTick',[0;0.2;0.35;0.55;0.75;1]);
set(gca,'XTickLabel',[' 0  ';'0.2 ';'0.35';'0.55';'0.75';' 1  '])
set(gca,'YTick',[-delta2;delta2;1-delta1;1+delta1]); grid;
```

The plot is shown in Figure 7.40. From Figure 7.40, the total number of extrema in the error function in the passband and stopbands are 14. Since $M = 26$ for this design, $L = M/2 - 1 = 12$. Then the extrema are $L + 2$ or $L + 3$. Hence this is a $L + 2 = 14$ equiripple design.

(c) The order of this filter is $M - 1 = 25$ while that of the filter designed in Problem P7.9 is $M - 1 = 43 - 1 = 42$. Thus this is an optimum design.

Amplitude Response
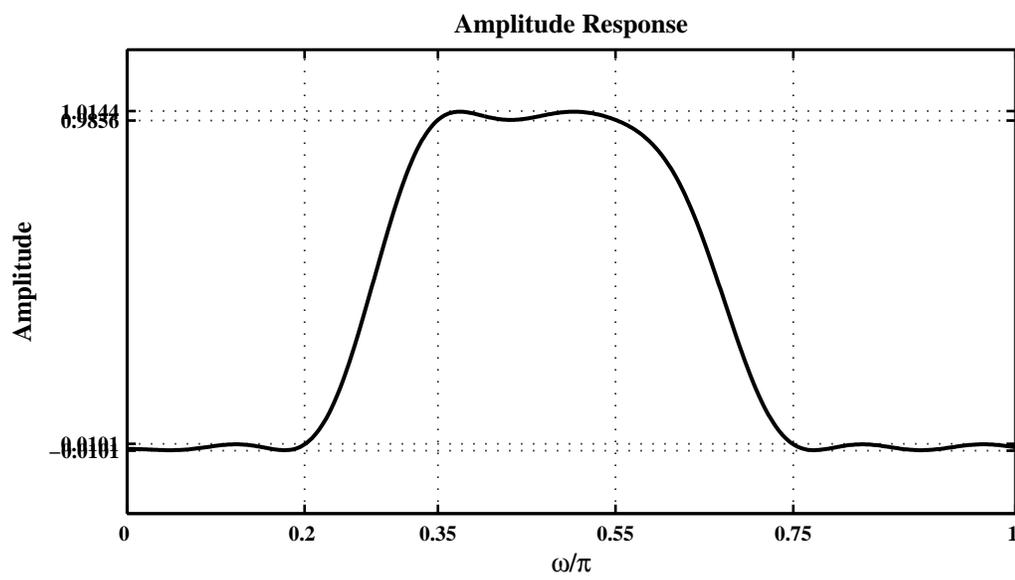
Figure 7.40: Amplitude Response Plot in Problem P7.40b