A set is stronger than another set (and the other set is weaker) if (but not necessarily only if):

- it is a proper (similar-) superset of the other and none of its reasons weighs less than the similar reason in the other set (if there is such a similar reason); or
- all its elements are similar to elements of the other set and the other way round, none of its reasons weighs less than the similar reason in the other set, and at least one of its reasons weighs more than the similar reason in the other set.

A set is equal to another set if (but not necessarily only if):

- all its elements are similar to elements of the other set and the other way round; and
- all of its reasons have the same weight as the similar reasons in the other set.

2.4 Weak Transitivity

Often two sets of alternatives will not be comparable on logical grounds alone. Then additional decision making is necessary to establish which one is better. For instance, if a Volvo is a safer car then a Porsche, but a Porsche is better for one's social status, and these are the only relevant reasons, it is not possible to establish on logical grounds which brand is better. Suppose that a decision is made that a Volvo is better than a Porsche. Suppose, moreover, that a Mercedes is just as safe as a Volvo and is even better for one's social status, and there are no other relevant reasons, then it is possible to determine on logical grounds that a Mercedes is better than a Volvo. Since it has been established by decision making that a Volvo is preferable to a Porsche, it seems rational to assume that a Mercedes must, in the absence of special circumstances, be better than a Porsche too.

This can be generalized as follows: If alternative A is better than alternative B and if C is better than A, then, *by default*, C will be better than B too. Another way to say this is that the better than relation is *weakly transitive*. The weakness of the transitivity consists in the defeasibility of the application of transitivity. Analogously, the equal to-relation between alternatives is also weakly transitive.

Weak transitivity does not only hold for the better and worse than relation as applied to alternatives, but also for the stronger than, weaker than, and equal to relations as they hold between sets of reasons. For instance, if a Mercedes and a Volvo are both reliable and save cars, while the Volvo is safer, but the Mercedes is more reliable, the sets of pro-reasons for a

Mercedes and a Volvo cannot be compared purely on logical grounds. If it is decided that the pro-reasons for a Mercedes are stronger than those for a Volvo and if it is somehow (maybe on purely logical grounds) established that the pro-reasons for a Lexus are stronger than those for a Mercedes, it can by default be derived that the pro-reasons for a Lexus are stronger than those for a Volvo.

The theory of Qualitative Comparative Reasoning (QCR) formulated above is formalized and made more precise in the sections 8f. But first I will illustrate how QCR can be put to use in three important fields of legal reasoning, namely those of legal theory construction, case based reasoning, and of legal proof.

3. THEORY CONSTRUCTION

In a series of papers[8], culminating in his book *Law's Empire*, Dworkin has developed an intuitively attractive picture of legal theory construction. This picture recognizes three stages in constructing the law.[9] The first stage, the so-called pre-interpretative stage, consists of a preliminary identification of the rules, standards and (generalized) decisions that make up the law. In this connection one might think of an inventory of the rules and standards that can be found in statutes, cases and doctrinal literature. The second, interpretative, stage consists of an identification of the principles (including values and policies) that underlie (in the sense of explain), or are part of the legal phenomena identified in the first stage. The rules etc. identified in the first stage are to be seen as means to realize the principles identified in the second stage, but they are not necessarily the best way to realize them. The purpose of the third, reforming, stage is to formulate (relevant parts of) the set of rules, including (generalized) decisions of cases, that best realizes the principles identified in the second stage.

Abstracting a little from Dworkin's analysis, it is possible to distinguish within a theory of the law three subsets of elements. The first subset consists of the sources of the law, with a prominent place for legislation and for individual cases as decided by the judiciary. The second subset consists of the principles, policies, rights and values that underlie and form the inspiration for the law. And finally there is the law as a set of generic cases,

[8] The papers in question are in particular the paper 'Hard Cases', included in Dworkin 1978 and the papers in part two (Law as interpretation) of Dworkin 1985.

[9] Dworkin 1986, 65f.

with the legal consequences attached to them by the law.[10] Henceforth I will call the first subset the *legal sources*[11], the second subset the *legal goals*[12], and the third set the *normative system*.[13]

In the process of legal theory construction, the legal sources determine a rough first account of the normative system based upon them. This prima facie normative system forms, so to speak, the set of data that the theory must explain. As in empirical theories, it is possible that some of the data must be disregarded if they do not fit in the best theory that can be constructed from them. By means of inductive and abductive reasoning, a set of goals can be identified as underlying the prima facie version of the normative system. Given these goals it is possible to devise a normative system that realizes them best, and given such an ideal normative system, it is possible to devise an adapted set of sources (new legislation or decisions in upcoming cases) by means of which such an ideal normative system is realized.

Ideally, the new sources should deviate as little as possible from the actual sources that function as input to the theory[14], because a good theory of the law should be a theory of the *law as it actually is* and not merely a theory of ideal law. Yet, the goals underlying the law are also part of the law and the normative system should also reflect them. Obviously there is a certain tension here, and it depends on ones legal-philosophical disposition how the balance between the actual sources and the legal system that is ideal in the light of the principles underlying the sources is struck.[15]

Graphically the development of the relations between the three subsets of a legal theory can be depicted as follows:

[10] Cf. the discussion of CLCPs in chapter 1, section 3.2.

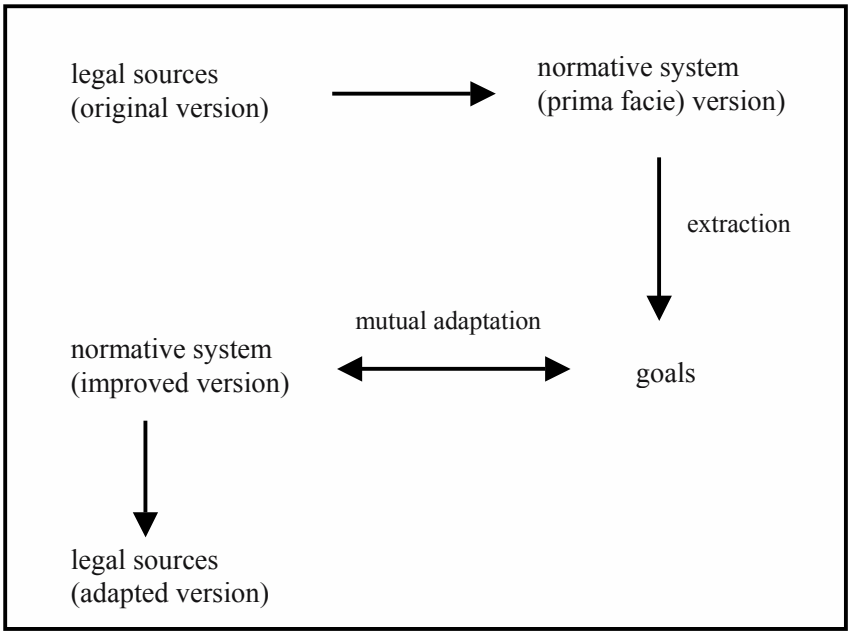
[11] Legal sources in the sense intended here are not statutes or case law in general, but individual regulations and individual cases.

[12] Notice that these goals include a broad spectrum of legally relevant entities, such as (human) rights, legal principles and policies. For the present purposes these different entities are all lumped together.

[13] The use of the expression 'normative system' for generic cases with their legal consequences was inspired by the use of this expression in Alchourrón and Bulygin 1971.

[14] Where case law is concerned it is not even possible to change the existing body of case law. Old case law can at most be discarded as outdated by new case law or legislation.

[15] This issue is discussed in more detail in chapter 2, section 8f.



4. COMPARING SOLUTIONS FOR A CASE TYPE

One aspect of the mutual adaptation of sources, principles and normative system is the determination of the ideal normative system, given a particular set of goals. Let us have a look at an extended example that illustrates the reasoning about whether a particular solution for a type of case should be part of the theory (should rationally be accepted) given the set of goals that are included in the theory in question. The example consists of variations on the so-called Lebach-case, which was made familiar by Alexy.[16]

The standard case runs as follows: A person, let us call him E, who was condemned for abduction and subsequent murder of his victim, is released from prison after ten years. A tabloid journal jumps on this news and uses the occasion to publish an article on the dangers of abduction in general. The article is illustrated with a photograph of E just after his release. E attempts to prevent circulation of the journal.

The judge who must decide this case should balance two goals. One goal is the freedom of the press; the other is that one should respect other persons' privacy. Let us assume that the judge decides that in cases like this, privacy protection outweighs freedom of the press.

[16] Alexy 1979 and 1996.

Now let us change the case a little bit by adding the fact that the news that E was to be released was given to the press on the condition that no photographs would be taken. This forms an additional reason against publication of the article, because the effects of the offence are even enlarged by publishing the photo that was illegally taken. As a consequence the decision that it is not allowed to publish photographs of recently released prisoners if the potential publisher undertook the obligation not to take photographs at all, has even more support and is in this sense better.

It is possible to think of another change in the case which leads to a different conclusion. As yet, the question whether the released prisoner objects to the publication has not been taken into consideration yet. It was tacitly assumed that he did object, but this needs not be the case, in particular not if he were to be compensated financially for the publication. A solution to the effect that publication is only allowed with explicit consent of the person concerned, would take a new goal into consideration, namely personal autonomy. This solution would have the pro- and the con-reasons of the first one, presumably with the same weights, but would have an additional pro-reason because it is supported by the goal of autonomy. As a consequence, the last solution is better than the first one.

A similar argument can be made for the case that a potential decision has similar reasons pleading for it as the old decision, but that the reasons pleading against it are a strict subset of the reasons pleading against the old decision. This would be illustrated by the case in which the tabloid journal has contracted with E that no publication of his photograph would be made. It is arguable that the freedom of the press is not infringed by a prohibition that was voluntarily undertaken by the journal. Since the freedom of the press was a reason against the prohibition, the balance of reasons is moved towards the prohibition if this con-reason is taken away. As a consequence the solution that publication is prohibited if the potential publisher has voluntarily undertaken the obligation not to publish, has even stronger support than the original prohibition.

It is possible that a set of reasons is strengthened by adding new reasons to it, but also by strengthening the reasons that occur in it. This is illustrated by the solution that not only forbids publication of the photograph, but also prescribes that the photograph is destroyed. This solution provides better protection of privacy and is therefore better than the simple prohibition.[17]

A similar argument can be made for the case that one or more of the reasons pleading against the new decision are weaker than the corresponding

[17] Arguably, this solution would infringe the property right of the journal, but for the sake of the example, this complication is ignored.

reasons pleading for the old decision. For instance, a solution that allows publication of photographs, as long as the persons on the photographs are not recognizable, makes a smaller infringement on the freedom of the press, while the protection of the privacy remains the same. Such a solution would therefore be better than a mere prohibition of publishing photographs.

The findings from the discussed examples can be summarized in the following global guidelines for the comparison of possible solutions for a type of case:

- A solution for a case that promotes a goal should pro tanto be adopted.
- A solution for a case that detracts from a goal should pro tanto be rejected.
- If a solution for a case promotes the more important goal and detracts from the less important goal, it should pro tanto be adopted.
- If a solution for a case detracts from the more important goal, and promotes the less important goal, it should pro tanto be rejected.
- If a solution for a case promotes a goal to a large extent, this is pro tanto a stronger reason to adopt this solution than if it only minimally promotes the goal.
- If a solution for a case detracts from a goal to a large extent, this is pro tanto a stronger reason to reject this solution than if it only minimally detracts from the goal.

These guidelines demonstrate how the comparison of solutions for case types, given a set of goals, can be constructed in the form of QCR.

5. COMPARING GOAL SETS

It is not only possible to compare competing solutions for a type of cases in the light of a given set of goals, it is also possible to compare competing sets of goals in the light of a given normative system, that is in the light of a set of actual case solutions. To show how this can be done, I must briefly return to the justification of case solutions on the basis of a set of goals.

Given a set of goals, the solution for a particular case will promote some (zero or more) of these goals, detract from some other goals, and will be neutral with regard to the rest. Every goal that the solution for this case promotes, provides a reason for (the rightness) of the solution for this case, while every goal from which the solution detracts, provides a reason against this solution. Whether the solution for a case is right all things considered depends on the balance of these reasons. If the reasons why the solution is right (the pro-reasons) outweigh the reasons why the solution is wrong (the con-reasons), the solution is *right*. If the balance of reasons goes the other

direction, the solution is *wrong*. If the reasons pro and con a solution are more or less in balance, the solution of the case is *indifferent*.

To make a decision about the rightness of the solution for a case, we must balance reasons and most often this will just be a matter of decision making. Such decisions about the relative weight of (sets of) reasons are expressed by what I will call *weighing knowledge.* Such weighing knowledge becomes also part of a theory of the law and I will include it in the goal part of the theory.

Improvements in the goal part of a theory can take three forms, then. One is by making modifications in the set of goals by adding new goals or removing old ones. The second is by making changes in the relative importance of the goals and the third consists of changes in the weighing knowledge. The issue to be dealt with is under what conditions one of these changes is an improvement in the goal part of the theory.

Given a set of case solutions, it is possible to compare competing sets of goals (including relative importance and weighing knowledge) qualitatively. Every set of goals qualifies some of the actual case solutions as right, others as indifferent, and the rest as wrong. A set of goals A represents the actual case solutions better than another set B, if at least one of the actual case solutions is better in the light of A than it is in the light of B, while no actual solution is worse in the light of A than it is in the light of B. In other words, a change in the goal part of a theory is an improvement if at least one of the actual case solutions has changed from wrong into indifferent or right, or from indifferent into right, while no actual case decision has moved down one or more categories. If a solution that turns out to be wrong is seen as a reason against the goal set in the light of which this solution is wrong and if a right solution is seen as a reason for the goal set, this way of comparing goal sets qualitatively is an application of the general technique of QCR described above.

6. CASE-BASED REASONING AS A FORM OF COMPARATIVE REASONING

Cases can be used in legal reasoning in several ways. One way, prevalent in the civil law tradition, is to extract a kind of rule from a decided case, and use this rule like other rules stemming from other legal sources. Another way is to use the case as a point of reference for an argument by analogy, or an e contrario argument. By pointing out an analogy between the old case and a new case, it is possible to argue that the decision taken in the old case should also be taken in the new case. Or, by pointing out a crucial difference between the old case and a seemingly similar new case, it is possible to

distinguish the cases and to argue that there is no reason to copy the old decision in the new case, or even a reason to take a different decision.

The argument in which an analogy is drawn between two cases, in order to argue that the decision of the one case should be copied in the other case, can well be interpreted as a form of comparative reasoning. The way to do this is to compare cases with respect to their suitability for being decided in a particular way. If the old case was a suitable case for the decision that was actually taken in it, and the new case is just as suitable or even more suitable for such a decision, there is a reason to take this decision in the new case too. If the new case is less suitable than, or not well comparable to, the old case, this reason to decide the new case like the old one is lacking.

An example can illustrate this point. The following case was decided by the Dutch Supreme Court[18]:

Caustic soda case

Employees of a community centre placed a bag with household refuse along the street, in order to be taken away by the cleansing department. Unknown to the employees, the bag held a container with caustic soda. A cleaner put the bag into the dustcart and due to some malfunctioning of the cart's mechanism, part of the caustic soda was swept into his face, as a consequence of which he suffered serious damages to his eyes. The cleaner sued the operator of the community centre for the damages.

Even though the employees of the community centre were unaware of the presence of caustic soda in the bag, their behavior was held to be negligent. The court assumed a duty of care not to place a container with an unknown liquid in it, only protected by a cardboard box and a plastic bag, along the street to be taken away by the cleansing department, unless one has good reasons to assume that the liquid is not dangerous, or keeps the bag under control and warns those who want to handle the bag for its possibly dangerous contents.

Somewhat later the following case was brought before the Supreme Court[19]:

Yew case

Defendant's garden bordered on plaintiff's meadow, on which plaintiff held two horses. The meadow was fenced off by means of netting. Defendant had a heap of waste in his garden, near to plaintiff's meadow, on which he deposited a yew tree. Plaintiff's horses ate from the yew and died as a consequence. (Yew is poisonous for horses.) Plaintiff sues

[18] HR 8-1-1982, NJ 1982, 614.
[19] HR 22-4-1994, NJ 1994, 624.

defendant for the damages. Defendant argued that he neither knew nor should have known that the yew was poisonous for horses.

In both cases the defendant created a dangerous situation to which plaintiff fell victim. Moreover, in both cases the risk for defendant was quite high, while the costs defendant had to make to avoid the danger were low. And, finally, the cases have in common that defendant was not aware of the danger he created. Given these similarities, it is well arguable that the cases should have similar decisions and that therefore defendant in the second case should be held negligent as well.

This is not what actually happened, however. The Dutch Supreme Court held that in the yew-case, defendant was not expected to know that yew is poisonous for horses. Under these circumstances, defendant was not held negligent. Apparently there is a legally relevant difference between - on the one hand - card boxes with an unknown content and - on the other hand - yew. In connection with card boxes with an unknown content, one should assume that the content may be dangerous, unless there are positive indications to the contrary, while in connection with yew, one does not have to take possible risks into account. This difference may be summarized by saying that in the caustic soda case the creation of the danger was recognizable, while in the yew-case the creation of danger was not recognizable. By pointing out this difference, the cases can be distinguished, with the result that in the one case defendant was held negligent, and in the other case he was not held negligent.

Let us look at both lines of argument in terms of comparative reasoning. I will start from the assumption that both cases are similar. In the following table, the columns labeled with a plus-sign contain the reasons that plead for negligence, while the columns labeled with a minus-sign contain the reasons against negligence.

Caustic Soda case			**Yew case**	
decision: defendant was negligent			**decision: ??**	
+	-		+	-
defendant created a dangerous situation to which plaintiff fell victim			defendant created a dangerous situation to which plaintiff fell victim	
it was easy and cheap to avoid the danger			it was easy and cheap to avoid the danger	
the potential damages were high			the potential damages were high	
	defendant was not aware that he created a danger			defendant was not aware that he created a danger

Since both cases have similar reasons pleading for and against the decision that defendant was negligent, they are prima facie equally suitable to support this decision. This would be different if the reasons in both cases had different weights. In the absence of evidence why this would be the case, one can work with the default assumption that similar reasons in different cases have equal weights. On this assumption, the cases are *equally suitable* to support the decision that defendant was negligent. In combination with the fact that in the caustic soda case defendant was actually held negligent, this is a reason why defendant should be held negligent in the yew case too.

Suppose, presumably counterfactually, that in the yew case the potential damages were even higher than in the caustic soda case. Then the reason based on the amount of damages in the yew case has a bigger weight than the similar reason in the caustic soda case. On the assumption that all other

similar reasons have the same weights in both cases, the Yew case is then even *more suitable* for assuming negligence than the caustic soda case. *A fortiori* it then holds that there is a reason to assume negligence in the yew case, given that there was such a reason in the caustic soda case.

Let us now have a look at the cases from the point of view of the Dutch Supreme Court, who found that in the caustic soda case, defendant should have taken the possible danger into account, while in the yew case this was not the case.

Caustic Soda case			**Yew case**	
decision: defendant was negligent			**decision: ??**	
+	-		+	-
defendant created a dangerous situation to which plaintiff fell victim			defendant created a dangerous situation to which plaintiff fell victim	
it was easy and cheap to avoid the danger			it was easy and cheap to avoid the danger	
the potential damages were high			the potential damages were high	
	defendant was not aware that he created a danger			defendant was not aware that he created a danger
defendant should have been aware that he created a danger				

On this reading of the cases, the caustic soda case has one reason to assume negligence that was lacking in the yew case. In all other respects the cases are similar. On this reading, the caustic soda case is more suitable for the assumption of negligence than the yew case. As a consequence the reason to decide the cases similarly that was present in the first reading of the cases, is lacking on this second reading.

Notice that this reason (based on similarity of the cases) would still be absent if the reasons pro negligence in the yew case would weigh more than the similar reasons in the caustic soda case, for instance because the potential damages were higher in the yew case. Then both cases would in one respect be more suitable for the assumption of negligence than the other case, and that makes comparison by means of qualitative reasoning impossible.[20]

As the above example illustrates, at least some form of case-based reasoning can be interpreted as a special case of QCR, namely as the comparison of cases with respect to their suitability for a particular solution. Obviously, theory construction is also relevant in connection with cases. When case-based reasoning is used as a technique, the solution is kept fixed, and cases are compared with regard to their suitability for *this solution*. When theory construction is used as a technique, a case is kept fixed as a point of reference, while solutions are compared with regard to their suitability for *this case*.

These two techniques can also be combined. Given a particular case, it is possible to compare possible solutions with regard to their suitability. When a particular solution has been adopted as, given the available information, the best one, it is possible to compare actual and hypothetical cases with regard to their suitability for this solution. In this way the best solution for one type of case can be transferred to other cases, thereby broadening the theory of the law that is under construction.

7. QUALITATIVE COMPARATIVE REASONING AND LEGAL PROOF

QCR can also play a role in connection with legal proof. If there are several competing accounts of the facts about a case, these accounts can be compared with regard to how well they fit the evidence. I cannot go deep into this issue here, but let me illustrate it by means of an example.

Suppose that Lord Hard was found in his room, murdered by means of a knife. The butler was seen by John, the Lord's son, when the butler allegedly entered Lord Hard's room at about the estimated time of the killing. Moreover, the butler had a motive to murder Lord Hard, because his Lordship had seduced the butler's daughter Harriet. However, there is also a witness, the gardener, who testifies that the butler was in the garden at the estimated time of the murder.

[20] At least, in the absence of additional relevant information.

There is also another suspect, the chamber maid Dorothea, who also had a motive to murder the Lord, because she had a relationship with the Lord before he fell in love with Harriet, and she suffered severely from jealousy. Dorothea was also seen by John when she allegedly entered Lord Hard's room at about the estimated time of the killing. The problem is, however, that Dorothea has an alibi too in the person of a visiting grocer who delivered some goods to Dorothea at the time in question.

Schematized, the two competing theories have the following reasons pleading for and against them:

The butler committed the murder			**The maid committed the murder**	
+	-		+	-
motive			motive	
witness that the butler had the opportunity			witness that the maid had the opportunity	
	alibi			alibi

At first sight the two theories are equally good. However, if the information is added that the gardener is the butler's brother, the value of the butler's alibi becomes considerably less. It may be assumed that if the gardener is the butler's brother, he may have lied about the presence of the butler in the garden at the estimated time of the murder. In terms of reasons, it may be said that the butler's alibi as a reason against the theory that he committed the murder, has less weight than the alibi of Dorothea, which was based on a more reliable witness. (I represented this in the above schema by giving the butler's alibi a smaller font.) Assuming that the motives of the butler and of Dorothea were equally strong (had the same weight) and that the testimony of John was equally reliable with regard to the butler and the maid, the theories that the butler killed the Lord and that the maid killed him are equally strong in pro-reasons. However, the theory that the butler committed the murder is weaker in con-reasons than its competitor and should therefore be preferred (in the absence of additional information).

Obviously, a similar result can be achieved by removing one of the reasons why the maid committed the murder. If John did not see the maid

enter Lord Hard's room, the theory that the maid killed Lord Hard is worse than the theory that the butler killed, even if we disregard the fact that the butler's alibi was provided by his brother.

In general, competing theories about what happened in a concrete case can be compared in terms of sets of reasons pleading for and against them. These reasons will (at least partly) be based on pieces of evidence that must be explained by the theory.

A piece of evidence that is explained provides a pro-reason for the theory. The better the explanation, the stronger the pro-reason. A piece of evidence is explained marginally if its existence is compatible with the theory. It is explained better if its existence is a plausible consequence of the theory, and it is explained still better if the truth of the theory necessitates its presence. For instance, the theory that the butler killed Lord Hard explains the testimony of John, because this theory explains why the butler entered the Lord's room, and thereby that John could see the butler entering the room, and thereby that John testified that he saw the Lord entering this room. The explanation would even be better if the theory entailed that John had to (instead of merely could) see the butler entering the room.

A pro-reason also becomes stronger if the evidence it explains is more reliable. If, for instance, John held a grudge against the butler, the pro-reason for the theory that the butler was the murderer would be weaker than it actually is.[21]

A piece of evidence that the theory fails to explain and that requires explanation[22] provides a con-reason for a theory. The more remarkable the lack of explanation, the stronger the con-reason is. What was written in connection with pro-reasons about the strength of the reasons holds *mutatis mutandis* also for con-reasons.

[21] This can also be accounted for by the observation that there is another explanation for John's testimony which makes the explanation by the theory that the butler killed the Lord less plausible.

[22] Not all facts of a case need to be explained by a theory about a case that is necessarily incomplete. However, some facts of a case are remarkable and seem to require a special explanation. It is these facts that lead to counterevidence for theories that do not explain them.

8. COMPARING SETS OF REASONS

Qualitative comparative reasoning as described above deals with the comparison of sets of reasons. It is therefore possible to use Reason-based Logic (RBL) the logic that was specially developed for reasoning with reasons to formalize the above account QCR. To this purpose, another extension of RBL is developed.[23]

SIMILAR STATES OF AFFAIRS AND REASONS

Sets of reasons that plead for the 'same' conclusion in different cases do not really contain the same reasons, but only reasons that are 'similar' to each other. The following definitions deal with similarity.

Similar states of affairs

Two concrete states of affairs are said to be similar, if and only if they are instantiations of the same abstract state of affairs:

```
∀*s1,*s2(Similar(*s1,*s2) ≡
∃*s3,i1,i2(s1 = instantiation(*s3, i1) &
   *s2 = instantiation(*s3, i2)))
```

Notice that identical states of affairs are also similar states of affairs.

Similar reasons

Analogous to the definition of similar states of affairs, two contributive reasons are similar, if and only if they are both instantiations of the same abstract state of affairs, and their conclusions are also similar states of affairs. Formally:

```
∀*a1,*a2(Similar_reasons(*a1, *a2) ≡
  ∃*c1,*c2(Cr(*a1,*c1) & Cr(*a2,*c2) &
    Similar(*a1,*a2) & Similar(*c1,*c2)))
```

[23] In chapter 3 the basic version of RBL was exposed and extended to deal with the logic of rules.

Similar reasonsets

Sets that consist of pair wise similar reasons are called similar reason sets:

```
∀a,b(Similar_reasonsets(a, b) ≡
  ∀*s1((*s1 ∈ a) →
     ∃s2((*s2 ∈ b) & Similar_reasons(s1, s2))) &
  ∀*s2((*s2 ∈ b) →
     ∃s1((*s1 ∈ a) & Similar_reasons(s1, s2))))
```

Similar superset

Given the notion of similar reasons, it is also possible to define the notion of a 'similar superset'. A similar superset is like a proper superset, with the difference that all elements of the subset must have a *similar* reason in the superset. Formally:

```
∀a,b(Similar_superset(a, b) ≡
   ∀*s2((*s2 ∈ b) →
      ∃*s1((*s1 ∈ a) & Similar_reasons(*s1,*s2))) &
   ∃*s3 ((*s3 ∈ a) &
      ∀*s4((*s4 ∈ b) → ~Similar_reasons(*s4,*s3))))
```

Similar subset

A similar subset is like a proper subset, with the difference that all elements of the subset must have a *similar* reason in the superset. Formally:

```
∀a,b(Similar_subset(a, b) ≡
   ∀*s2((*s2 ∈ a) →
      ∃*s1((*s1 ∈ b) & Similar_reasons(*s1,*s2))) &
   ∃*s3 ((*s3 ∈ b) &
      ∀*s4((*s4 ∈ a) →  ~Similar_reasons(*s4,*s3))))
```

By means of the notions of a similar reason set, a similar superset and a similar subset it is possible to overcome the problems connected with the fact that the sets that must be compared do not contain identical, but merely 'similar' reasons.

Another issue that must be dealt with is that of comparing sets of reasons on their weights. The idea to be captured is that if a set contains a similar reason for every reason in the other set, while from each pair of similar reasons, the reason in the former set does not have a smaller weight than its counterpart in the latter set, while at least one reason has a bigger weight than its counterpart, the former set is *stronger in individual weight* than the latter.

Weight

The first step to take in this connection is to define a function that maps reasons on their weights. Let *r be a contributive reason for conclusion *c. Then weight(*r,*c) denotes the weight of *r as a contributive reason for *c.

Two similar reasons have in principle the same weight. This can be expressed as follows:

$$Ar(*cr(*r1,*c1) \ \&\ cr(*r2,*c2) \ \&\ similar(*r1,*r2) \ \&\ similar(*c1,*c2), *weight(*r1,*c1) = weight(*r2,*c2))$$

>/2 and </2

The second step is to assign a second meaning to the relations >/2 and </2.[24] These relations hold between the weights of two reasons if and only if the weight of the first reason is bigger, respectively smaller than the weight of the second reason. For instance:

$$weight(*r1,*c1) > weight(*r2, *c2).$$

Comparable reason sets

The third step is to define the relation *stronger in individual weight*, which can hold between sets of reasons. These sets must either both contain reasons that plead for a similar conclusion, or reasons that plead against a similar conclusion. I will call such sets *comparable reason sets*.

$$\forall s1,s2(Comparable\_reasonsets(s1,s2) \equiv \exists *c((s1 \subseteq r^{+}(*c) \ \&\ s2 \subseteq r^{+}(*c))) \vee (s1 \subseteq r^{-}(*c) \ \&\ s2 \subseteq r^{-}(*c)))$$

$>_w/2$

The relation *stronger in individual weight* ($>_w/2$)holds between two comparable reason sets, if and only if from the reasons which the two sets *have in common* at least one reason of the first set weighs more than the corresponding reason from the second set, while the opposite is not the case.

[24] The first meaning is that of the outweighs-relation that holds between sets of contributive reasons for and against the same conclusion. Cf. chapter 3, section 3.4.

Formally:

```
∀s1,s2((s1 >w s2) ≡
  Comparable_reasonsets(s1,s2) &
  ∃*r1,*r2,*c1,*c2(
     Cr(*r1, *c1) & (*r1 ∈ s1) &
     Cr(*r2, *c2) & (*r2 ∈ s2) &
     Similar_reasons(*r1, *r2) &
     (weight(*r1,*c1) > weight(*r2,*c2))) &
  ∀*r3,*r4,*c3,*c4(
     Cr(*r3, *c3) & (*r3 ∈ s1) &
     Cr(*r4, *c4) & (*r4 ∈ s2) &
     Similar_reasons(*r3, *r4) →
        ~(weight(*r4,*c4) > weight(*r3,*c3))))
```

$\leq_w/2$

The relation *weaker in individual weight* ($<_w/2$) holds between two comparable reasonsets, if and only if from the reasons which the two sets *have in common* at least one reason of the first set weighs less than the corresponding reason from the second set, while the opposite is not the case.

Formally:

```
∀s1,s2((s1 <w s2) ≡
  Comparable_reasonsets(s1, s2) &
  ∃*r1,*r2,*c1,*c2(
     Cr(*r1, *c1) & (*r1 ∈ s1) &
     Cr(*r2, *c2) & (*r2 ∈ s2) &
     Similar_reasons(*r1, *r2) &
     (weight(*r1,*c1) < weight(*r2,*c2))) &
  ∀*r3,*r4,*c3,*c4(
     Cr(*r3, *c3) & (*r3 ∈ s1) &
     Cr(*r4, *c4) & (*r4 ∈ s2) &
     Similar_reasons(*r3, *r4) →
        ~(weight(*r4,*c4) < weight(*r3,*c3))))
```

$=_w/2$

The relation *equal in individual weight* ($=_w/2$) holds between two comparable reason sets, if and only if all the reasons which the two sets *have in common* pair wise have equal weights.

Formally:

$$\forall s1,s2((s1 =_w s2) \equiv$$
$$\text{Comparable\_reasonsets}(s1,s2) \;\&$$
$$\forall *r3,*r4,*c3,*c4($$
$$\text{Cr}(*r3, *c3) \;\&\; (*r3 \in s1) \;\&$$
$$\text{Cr}(*r4, *c4) \;\&(*r4 \in s2) \;\&$$
$$\text{Similar\_reasons}(*r3, *r4) \rightarrow$$
$$\text{weight}(*r4,*c4) = \text{weight}(*r3,*c3))))$$

Notice that between two comparable reason sets not necessarily one of the relations ‘stronger than’, ‘weaker than’, or ‘equal in individual weight’ holds.

Stronger

There may be several ways in which one set of reasons is overall stronger than another set. One way is that a set of reasons is stronger than another set *on logical grounds*. In this connection there are (at least) two possibilities:

1. the first set is a similar superset of the second, while the second is equal or weaker in individual weight;
2. the first set is stronger in individual weight than the second, while the second is either a similar reason set of a similar subset of the first.

Formally:

$$\forall s1,s2($$
$$(\text{similar\_superset}(s1, s2) \;\&$$
$$(s2 <_w s1) \vee (s2 =_w s1)) \rightarrow$$
$$\text{Stronger}(s1, s2))$$

$$\forall s1,s2($$
$$((s1 >_w s2) \;\&$$
$$\text{similar\_reasonset}(s2, s1) \vee$$
$$\text{similar\_subset}(s2, s1)) \rightarrow$$
$$\text{Stronger}(s1, s2))$$

Weaker

There are (at least) two logical grounds on which a set of reasons can overall be weaker than another set, namely:

1. the second set is a similar superset of the first, while the second is equal or stronger in individual weight;
2. the first set is weaker in individual weight than the second, while the first is not a similar superset of the second.

Formally:

```
∀s1,s2(
   (similar_superset(s2, s1) &
      (s2 >w s1) ∨ (s2 =w s1)) →
   Weaker(s1, s2))

∀s1,s2(
   ((s2 >w s1) &
      similar_reasonset(s1, s2) ∨
      similar_subset(s1, s2)) →
   Weaker(s1, s2))
```

Equal

A set of reasons is overall equal to another set on logical grounds if (but not necessarily only if):

1. they are similar sets, and
2. they are equal in individual weight.

Formally:

```
∀s1,s2(((s1 =w s2) & similar_reasonset(s1, s2)) →
   Equal(s1, s2))
```

9. COMPARATIVE REASONING ABOUT SETS OF CONTRIBUTIVE REASONS

The relations of strength between reason sets as defined in the previous section are quite tight, and will not hold very often. It may therefore seem that they are not very useful for practical reasoning purposes. However, it is thinkable that there are other logical grounds for the existence of one of the mentioned relations between reason sets than the ones discussed in section 8. Moreover, and more importantly, there may also be other than logical grounds on which a set is stronger than, weaker than, or equal to another set. In fact, the determination which of two sets is overall stronger or weaker than the other will most often be just a matter of decision making. But also then the following relations should hold:

```
∀s1,s2(Stronger(s1,s2) → ~Weaker(s1,s2))
∀s1,s2(Stronger(s1,s2) → ~Equal(s1,s2))
∀s1,s2(Weaker(s1,s2) → ~Stronger(s1,s2))
∀s1,s2(Weaker(s1,s2) → ~Equal(s1,s2))
∀s1,s2(Equal(s1,s2) → ~Stronger(s1,s2))
∀s1,s2(Equal(s1,s2) → ~Weaker(s1,s2))
```