Appendix D

Summary of the Formulae

This appendix gives an overview of the conservation, non-conservation and characteristic forms of the various conservation laws presented in this book.

Form	Equation	Equation number, section
Conservation form	$\frac{\partial}{\partial t}(AC) + \frac{\partial}{\partial x}(AuC) = 0$	[1.39], 1.3.1
Non-conservation form	$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = 0$	[1.48], 1.3.2
Characteristic form	$\frac{\mathrm{d}C}{\mathrm{d}t} = 0 \quad \text{for} \ \frac{\mathrm{d}x}{\mathrm{d}t} = u\,,\ C = \text{Const} \quad \text{for} \ \frac{\mathrm{d}x}{\mathrm{d}t} = u$	[1.49] and [1.50], 1.3.2

Table D.1. *The various forms of the linear advection equation*

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Form	Equation	Equation number, section
Conservation form	$\frac{\partial u}{\partial t}+\frac{\partial}{\partial x}\left(\frac{u^2}{2}\right)=0$	[1.69], 1.4.2
Non-conservation form	$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=0$	[1.66], 1.4.2
Characteristic form	$\frac{\mathrm{d}u}{\mathrm{d}t}=0 \quad \text{for} \frac{\mathrm{d}x}{\mathrm{d}t}=u\,,\ \ u=\text{Const} \quad \text{for} \frac{\mathrm{d}x}{\mathrm{d}t}=u$	[1.67] and [1.68], 1.4.2

Table D.2. *The various forms of the inviscid Burgers equation*

Form	Equation	Equation number, section
Conservation form	$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0\,,\ Q=K_{\text{Str}}\frac{A^{5/3}}{\chi^{2/3}}S_0^{1/2}$	[1.83-84], 1.5.1
Non-conservation form	$\frac{\partial A}{\partial t}+\lambda\frac{\partial A}{\partial x}=S'\,,\ \frac{\partial Q}{\partial t}+\lambda\frac{\partial Q}{\partial x}=\lambda S'\,,$ $\lambda=\frac{\partial Q}{\partial h}\left(\frac{\partial A}{\partial h}\right)^{-1}$	[1.87] and [1.91], 1.5.2 [1.93], 1.5.3
Characteristic form	$\frac{\mathrm{d}A}{\mathrm{d}t}=S' \ \text{for} \frac{\mathrm{d}x}{\mathrm{d}t}=\lambda\,,\ \frac{\mathrm{d}Q}{\mathrm{d}t}=\lambda S' \ \text{for} \frac{\mathrm{d}x}{\mathrm{d}t}=\lambda$	[1.88] and [1.92], 1.5.2

Table D.3. *The various forms of the kinematic wave equation*

Form	Equation	Equation number, section
Conservation form	$\frac{\partial s}{\partial t}+\frac{\partial F}{\partial x}=0\,,\ F=\frac{s^2}{s^2+(1-s)^2 b_{\mathrm{BL}}}V_d$	[1.111] and [1.115], 1.6.1
Non-conservation form	$\frac{\partial s}{\partial t}+\lambda\frac{\partial s}{\partial x}=0\,,$ $\lambda=2\frac{(1-s)s}{\left[s^2+(1-s)^2 b_{\mathrm{BL}}\right]^2}b_{\mathrm{BL}}V_d$	[1.116] and [1.117], 1.6.2
Characteristic form	$\frac{\mathrm{d}s}{\mathrm{d}t}=0\quad \text{for}\ \frac{\mathrm{d}x}{\mathrm{d}t}=\lambda\,,\ s=\mathrm{Const}\quad \text{for}\ \frac{\mathrm{d}x}{\mathrm{d}t}=\lambda$	[1.118] and [1.119], 1.6.2

Table D.4. *The various forms of the Buckley-Leverett equation*

Form	Equation	Equation number, section
Conservation form	$\frac{\partial M}{\partial t}+\frac{\partial}{\partial x}\left(\frac{V_d}{\theta R_F}M\right)=0\,,\ R_F=1+\frac{\rho_A C_A}{\theta C_T}$	[1.131] and [1.132], 1.7.1
Non-conservation form	$\frac{\partial M}{\partial t}+\lambda\frac{\partial M}{\partial x}=0\,,$ $\frac{\partial C_T}{\partial t}+\lambda\frac{\partial C_T}{\partial x}=0\ \lambda=\left(\frac{1}{R_F}-\frac{M}{R_F^2}\frac{\mathrm{d}R_F}{\mathrm{d}M}\right)\frac{V_d}{\theta}$	[1.136], [1.137] and [1.139], 1.7.2
Characteristic form	$\frac{\mathrm{d}M}{\mathrm{d}t}=\frac{\mathrm{d}C_T}{\mathrm{d}t}=0\quad \text{for}\ \frac{\mathrm{d}x}{\mathrm{d}t}=\lambda$	[1.140], 1.7.2

Table D.5. *The various forms of the advection equation with adsorption-desorption*

Form	Equation	Equation number, section
Conservation form	$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial x}=S$, $U=\begin{bmatrix}\rho A\\ \rho Q\end{bmatrix}$, $F=\begin{bmatrix}\rho Q\\ Ap\end{bmatrix}$, $S=\begin{bmatrix}0\\ p\,\partial A/\partial x-\rho gA\sin\theta-k\lvert u\rvert u\end{bmatrix}$	[2.2] and [2.68], 2.4.2
Non-conservation form	$\frac{\partial U}{\partial t}+A\frac{\partial U}{\partial x}=S'$, $S'=S$ $A=\begin{bmatrix}0 & 1\\ c^2 & 0\end{bmatrix}$	[2.5] and [2.69], 2.4.3
Characteristic form	$\left.\begin{array}{ll}\frac{dp}{dt}-\frac{\rho c}{A}\frac{dQ}{dt}=\left(k\lvert u\rvert u+\rho gA\sin\theta\right)\frac{c}{A} & \text{for } \frac{dx}{dt}=-c\\ \frac{dp}{dt}+\frac{\rho c}{A}\frac{dQ}{dt}=\left(-k\lvert u\rvert u-\rho gA\sin\theta\right)\frac{c}{A} & \text{for } \frac{dx}{dt}=c\end{array}\right\}$	[2.79], 2.4.3

Table D.6. *The various forms of the water hammer equations*

Form	Equation	Equation number, section
Conservation form	$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial x}=S$, $U=\begin{bmatrix}A\\ Au\end{bmatrix}$, $F=\begin{bmatrix}Au\\ \frac{Q^2}{A}+\frac{P}{\rho}\end{bmatrix}$, $S=\begin{bmatrix}0\\ (S_0-S_f)gA+I_P\end{bmatrix}$	[2.2] and [2.118], 2.5.2.4
Non-conservation form	$\frac{\partial U}{\partial t}+A\frac{\partial U}{\partial x}=S'$, $S'=S$ $A=\begin{bmatrix}0 & 1\\ c^2-u^2 & 2u\end{bmatrix}$	[2.5] and [2.119], 2.5.3.1
Characteristic form	$\left.\begin{array}{ll}\frac{du}{dt}-\frac{c}{A}\frac{dA}{dt}=(S_0-S_f)g & \text{for } \frac{dx}{dt}=u-c\\ \frac{du}{dt}+\frac{c}{A}\frac{dA}{dt}=(S_0-S_f)g & \text{for } \frac{dx}{dt}=u+c\end{array}\right\}$	[2.140], 2.5.3.3

Table D.7. *The various forms of the Saint Venant equations*

Form	Equation	Equation number, section
Conservation form	$\dfrac{\partial \mathrm{U}}{\partial t}+\dfrac{\partial \mathrm{F}}{\partial x}=\mathrm{S}$, $\mathrm{U}=\begin{bmatrix}\rho\\ \rho u\\ E\end{bmatrix}$, $F=\begin{bmatrix}\rho u\\ \rho u^2+p\\ (E+p)u\end{bmatrix}$	[2.2] and [2.197], 2.6.2.5
Non-conservation form	$\dfrac{\partial \mathrm{U}}{\partial t}+\mathrm{A}\dfrac{\partial \mathrm{U}}{\partial x}=0$ $\mathrm{A}=\begin{bmatrix}0 & 1 & 0\\ c^2-u^2 & 2u & 0\\ uc^2-(E+p)u/\rho & (E+p)/\rho & u\end{bmatrix}$	[2.198] and [2.200], 2.6.3
Characteristic form	$\left.\begin{array}{ll}\dfrac{\mathrm{d}p}{\mathrm{d}t}-\rho c\dfrac{\mathrm{d}u}{\mathrm{d}t}=0 & \text{for } \dfrac{\mathrm{d}x}{\mathrm{d}t}=u-c\\ \dfrac{\mathrm{d}s}{\mathrm{d}t}=0 & \text{for } \dfrac{\mathrm{d}x}{\mathrm{d}t}=u\\ \dfrac{\mathrm{d}p}{\mathrm{d}t}+\rho c\dfrac{\mathrm{d}u}{\mathrm{d}t}=0 & \text{for } \dfrac{\mathrm{d}x}{\mathrm{d}t}=u+c\end{array}\right\}$ $\left.\begin{array}{ll}\dfrac{\mathrm{d}}{\mathrm{d}t}(u-\beta_1 p^{\beta_2})=0 & \text{for } \dfrac{\mathrm{d}x}{\mathrm{d}t}=u-c\\ \dfrac{\mathrm{d}s}{\mathrm{d}t}=0 & \text{for } \dfrac{\mathrm{d}x}{\mathrm{d}t}=u-c\\ \dfrac{\mathrm{d}}{\mathrm{d}t}(u+\beta_1 p^{\beta_2})=0 & \text{for } \dfrac{\mathrm{d}x}{\mathrm{d}t}=u+c\end{array}\right\}$ $\left.\begin{array}{l}\beta_1=\dfrac{2\gamma}{\gamma+1}\left(\dfrac{\gamma\rho_0}{p_0^{1/\gamma}}\right)^{1/2}\\ \beta_2=\dfrac{3\gamma+1}{2\gamma}\end{array}\right\}$	[2.209], [2.217], [2.215], 2.6.3

Table D.8. *The various forms of the Euler equations*