Appendix 5

Electric Dipole Formulas

A5.1. Complete formulas of the electric dipole

Let us consider a short wire of longitudinal dimension ΔL where a uniform and sine wave current I of ω angular frequency flows. An observer P is located at a distance r from the short wire in order to meet the $r >> \Delta L$ condition (Fresnel region).

With the assumptions previously established and taking into account the fact that the distance r is attached to a spherical coordinate system, whose origin coincides with the center of the wire, it is an *electric dipole*, which is represented with the notation conventions from Figure A5.1. Solving the wave equation in order to determine the radiated electromagnetic fields from this electric dipole, leads to an $\vec{E}$ electric field vector, including two projections according to the polar angle θ and the radial direction r respectively. A magnetic field vector $\vec{H}$ with symmetry of revolution including only one component is attached to the previous electric field vector. The magnetic field vector is directed according to the azimuth angle φ [DEM 05]:

$$\vec{E} = E_r\,\vec{u}_r + E_\theta \vec{u}_\theta \qquad \vec{H} = H_\phi \vec{u}_\phi \qquad \text{[A5.1]}$$

We find, in this relation, unit vectors which were omitted in Figure A5.1.

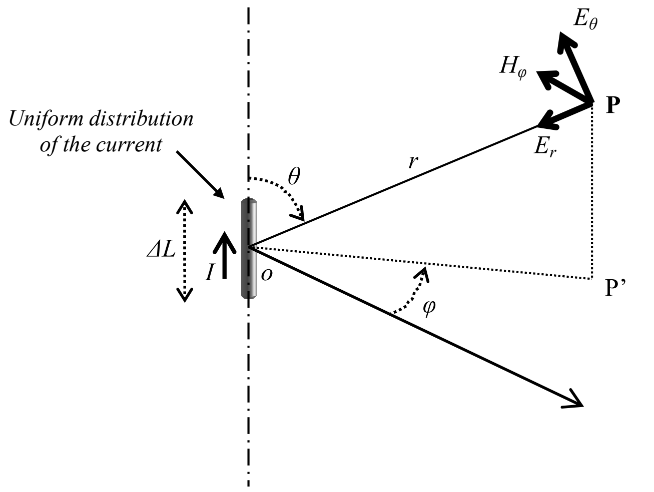

Figure A5.1. *The electric dipole in the spherical coordinate system*

Under the previous assumptions, we seek E_θ, E_r and H_φ in the analytical expressions below:

$$E_\theta = \frac{I\,\Delta L}{4\pi\, j\omega\varepsilon_0}\frac{\sin\theta}{r^3}\left[1 + jk\,r + (jk\,r)^2\right]e^{-jk\,r} \qquad \text{[A5.2]}$$

$$E_r = \frac{I\,\Delta L}{4\pi\, j\omega\varepsilon_0}\frac{2\cos\theta}{r^3}(1 + jk\,r)\,e^{-jk\,r} \qquad \text{[A5.3]}$$

$$H_\varphi = \frac{I\,\Delta L}{4\pi}\frac{\sin\theta}{r^2}(1 + jk\,r)\,e^{-jk\,r} \qquad \text{[A5.4]}$$

We find in these formulas the wave number in free space k, whose expressions linked to the ω angular frequency or to the λ wavelength will be recalled below.

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \qquad \text{[A5.5]}$$

In this formula c, the speed of light in vacuum, appears.

A5.2. Near-field formulas of the electric dipole

From the previous relationships, we take approximated formulas only valid for the distances r that are much lower than the λ wavelength. Under these conditions and taking into account equation [A5.5], the kr product being much lower than one, the use of the first term of the series expansion of equations [A5.2] to [A5.4] leads to the near-field formulae of E_θ, E_r and H_φ, i.e.:

$$kr << 1 \quad \rightarrow \quad E_\theta \cong \frac{p}{4\pi\,\varepsilon_0}\frac{\sin\theta}{r^3} \qquad \text{[A5.6]}$$

$$kr << 1 \quad \rightarrow \quad E_r \cong \frac{p}{4\pi\,\varepsilon_0}\frac{2\cos\theta}{r^3} \qquad \text{[A5.7]}$$

$$kr << 1 \quad \rightarrow \quad H_\varphi \cong j\omega\frac{p}{4\pi}\frac{\sin\theta}{r^2} \qquad \text{[A5.8]}$$

The p parameter is similar to the dipolar moment found in electrostatic theory:

$$p = \frac{I\,\Delta L}{j\omega} = q\,\Delta L \quad \text{with} \quad I = j\omega q \qquad \text{[A5.9]}$$

In equation [A5.9], q then represents the electric charges of a displacement current with an amplitude strictly similar to that of the I flowing current on the short wire in Figure A5.1.

A5.3. Far-field formulas of the electric dipole

The far-field of the electric dipole is produced at a distance r that is much longer than the wavelength. Under these conditions, the kr product takes a value that is much higher than one, which suggests use of the asymptomatic forms of equations [A5.2] to [A5.4]. In that case, we can show that the radial component of the electric field vanishes.

Consequently, only E_θ and H_φ remain:

$$kr >> 1 \quad \rightarrow \quad E_\theta \cong j\omega\mu_0\frac{I\,\Delta L}{4\pi}\frac{e^{-jkr}}{r} \qquad \text{[A5.10]}$$

$$kr >> 1 \quad \rightarrow \quad H_{\varphi} \cong \frac{E_{\theta}}{Z_w} \qquad [A5.11]$$

The remaining electric and magnetic field projections obey to the same law with the angular frequency ω and the distance r of the observer. The ratio of E_{θ} and H_{φ} corresponds to the impedance of the Z_w plane wave, which is recalled below:

$$Z_w = \sqrt{\frac{\mu_0}{\varepsilon_0}} \qquad [A5.12]$$

A5.4. Bibliography

[DEM 05] DEMOULIN B., Enseignement élémentaire sur la propagation des ondes, Volume II, Course Notes, ESEA bachelor, Lille 1 University, September 2005.