Part III

Slow Time-Varying Fields

Electromagnetic Foundations of Electrical Engineering J. A. Brandão Faria
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Introduction

For time-varying electromagnetic phenomena the time variable t is ubiquitous, hence you need to consider the original Maxwell's equations exactly as they are:

$$\begin{cases} \text{curl } \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \text{div } \mathbf{B} = 0 \\ \text{curl } \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t} \\ \text{div } \mathbf{D} = \rho \end{cases} \tag{PIII.1}$$

Does this mean that the subjects treated in Part II have the recycle bin as their destiny? Well, it depends on how fast or slow the fields vary with time.

It makes sense that for slow time-varying fields – the so-called quasi-stationary regime – most of the results obtained in Part II should hold true and provide very good approximations. The strategy ordinarily used to deal with quasi-stationary regimes consists of treating the capacitive effects associated with $\partial\mathbf{D}/\partial t$ (electric induction phenomena) separately from the inductive effects associated with $\partial\mathbf{B}/\partial t$ (magnetic induction phenomena).

Before we can continue, a crucial question needs to be clarified. What is a slow time-varying regime? How do you draw the line between slow and fast fields?

Although we are going to prove this in Part IV, you probably have already heard that electromagnetic waves propagate in free space with a velocity $c = 3 \times 10^8\,\text{m/s}$. In the particular case of space–time sinusoidal waves, the time periodicity and the space periodicity are characterized, respectively, by the period T and the wavelength λ, these two parameters being correlated through $\lambda = cT$.

Consider a sinusoidal voltage generator connected to a load through a pair of perfectly conducting wires of length l. Let u_G and i_G be the generator voltage and current; likewise let u_L and i_L be the load voltage and current. Voltages u_G and u_L share the same shape and the same period T, but, in general, they do not coincide with each other, $u_G(t) \neq u_L(t)$, because, in fact, the electromagnetic wave originating at the generator site takes some time to reach the distant load; the corresponding time delay (or propagation time) is given by $\tau = l/c$.

The same argument applies equally to the generator and load currents, that is $i_G(t) \neq i_L(t)$.

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You can say that the regime is a quasi-stationary regime whenever the delay time is negligibly small compared to T, in which case you can employ the very good approximations $u_G = u_L$ and $i_G = i_L$. Note, in addition, that the inequality $\tau \ll T$ can be rewritten as

$$\underbrace{(l/c)}_{\tau} \ll \underbrace{(\lambda/c)}_{T} \rightarrow l \ll \lambda \qquad \text{(PIII.2)}$$

In short, quasi-stationary regimes are those for which the length of the structure under analysis is much shorter than the lowest wavelength characterizing the field dynamics. If the opposite happens then you will be dealing with a rapid time-varying phenomenon. This is how you draw a line between the problems we are going to analyze in Part III and Part IV.

Part III is subdivided into three chapters. Chapter 5 is concerned with magnetic induction phenomena where electric fields $\mathbf{E}$ are produced by $\mathbf{B}$ fields originated by time-varying conduction currents $\mathbf{J}(t)$. In Chapter 5 we assume that the magnitude of $\partial \mathbf{D}/\partial t$ is small compared to the magnitude of $\mathbf{J}$ (neglecting of capacitive effects). Hence, the key equations for Chapter 5 are

$$\begin{cases} \text{curl } \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \text{div } \mathbf{B} = 0 \\ \text{curl } \mathbf{H} \approx \mathbf{J} \end{cases} \qquad \text{(PIII.3)}$$

Chapter 6 is concerned with electric induction phenomena, where magnetic fields $\mathbf{H}$ are produced by $\mathbf{E}$ fields originated by time-varying charge distributions $\rho(t)$. Magnetic induction phenomena from Chapter 5 are assumed to be negligibly small. Key equations for Chapter 6 are

$$\begin{cases} \text{curl } \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t} \\ \text{div } \mathbf{D} = \rho \\ \text{curl } \mathbf{E} \approx 0 \end{cases} \qquad \text{(PIII.4)}$$

Chapter 7 deals, to a great extent, with electrical engineering applications of the theoretical results presented in Chapters 5 and 6. There we will address the steady-state harmonic regime and the transient regime for circuit analysis using the standard lumped parameters approach, which applies to slow time-varying phenomena.

A final word is in order concerning the notation employed in the analysis of time-varying phenomena: While in Part II upper-case italic symbols have been used to denote stationary quantities, from now on we will utilize lower-case italic symbols for time-varying quantities.