1

Basic Field Vectors

1.1 The Electric and Magnetic Field Vectors

A set of four vectors is needed to describe electromagnetic field phenomena. These are:

the electric field vector, **E** (units: V/m, volt per meter)
the magnetic induction vector, **B** (units: T, tesla)
the electric displacement vector, **D** (units: C/m^2, coulomb per square meter)
the magnetic field vector, **H** (units: A/m, ampere per meter)

Among these, the first two have special physical significance, since they can be determined experimentally and measured.

If you place a static charged particle $Q > 0$ in a region where **E** is to be determined, you will see that a force $\mathbf{F}_e$ is exerted on the charge (Figure 1.1(a)) from which **E** is obtained:

$$\mathbf{E} = \frac{1}{Q}\mathbf{F}_e \tag{1.1}$$

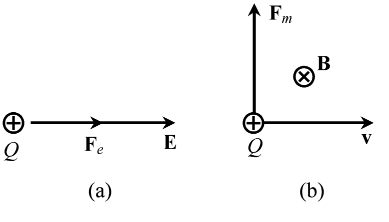

Figure 1.1 Actuating forces on a charged particle. (a) Electric force. (b) Magnetic force

Now consider that the same charged particle is moving with a prescribed velocity **v** in a region where **B** exists. You will notice that the trajectory of the moving particle may start changing due to the presence of a new force, $\mathbf{F}_m = Q(\mathbf{v} \times \mathbf{B})$. If you choose the velocity

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vector **v** perpendicular to **B** (Figure 1.1(b)) you will obtain the intensity of the magnetic induction vector

$$B = \frac{1}{vQ} F_m \tag{1.2}$$

If a moving charged particle is submitted to an electromagnetic field, the two forces will add and you get the so-called Laplace–Lorentz force

$$\mathbf{F} = Q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{1.3}$$

The fundamental vector pair (**E, B**) constitutes the electromagnetic field. Its intrinsic properties are defined by the following equations:

$$\text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1.4a}$$

$$\text{div } \mathbf{B} = 0 \tag{1.4b}$$

These equations have two very simple implications: the field lines of vector **B** are closed; and, moreover, if **B** happens to be a time-varying field then it will give rise to the presence of an electric field **E** (induction phenomena).

Electromagnetic fields cannot be created from a void. Their existence requires the presence of sources, namely charges, either at rest or moving in space. Charges at rest are usually characterized by their volume density (a scalar field); when they move, that is when currents are present, they are characterized by a current density (a vector field):

charge density, ρ (units: $\mathrm{C/m^3}$, coulomb per cubic meter)
current density vector, **J** (units: $\mathrm{A/m^2}$, ampere per square meter)

The connection between sources and fields is established using the two remaining Maxwell's equations:

$$\text{curl } \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{1.5a}$$

$$\text{div } \mathbf{D} = \rho \tag{1.5b}$$

Note that earlier theories of electromagnetism did not include the term $\partial \mathbf{D}/\partial t$ on the right-hand side of (1.5a); its inclusion due to Maxwell was revealed to be absolutely crucial. This is why the equations of electromagnetism are now termed Maxwell's equations.

1.2 Constitutive Relations

At this point you may feel that something is missing. On the one hand, we have a relationship between field sources (ρ, **J**) and the auxiliary fields (**D, H**) and, on the other hand, we have a relationship that dictates the properties of the electromagnetic field (**E, B**). The missing link is the one that relates the pair (**D, H**) to the pair (**E, B**) – see Figure 1.2.

The connections between (**D, H**) and (**E, B**) do not belong to the set of Maxwell's equations. They have to do with the interaction of the electromagnetic field with the material

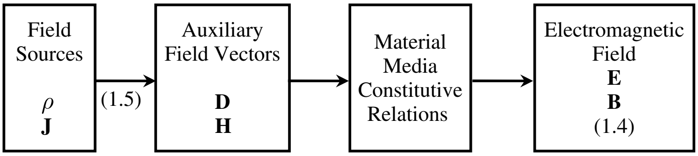

Figure 1.2 Connection among field sources, electromagnetic field vectors and material media properties

media where fields are impressed. Depending on the material medium, analysis of the field–matter interaction can be extremely complicated, requiring, for its understanding, contributions from atomic physics and quantum mechanics. Hopefully, you will learn about this subject in another discipline of your course.

Here, since we are going to deal with rather simple material media with linear isotropic characteristics, a pragmatic heuristic approach of the media macroscopic properties will be adopted. Except for a very few cases, these properties will be described by the following constitutive relations:

$$\mathbf{D} = \varepsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H} \tag{1.6}$$

where ε and μ denote, respectively, the permittivity and the permeability of the medium. Values for these parameters can be found in tabular form in many books devoted to the study of the electromagnetic properties of materials. In particular, for a vacuum, we have the following fundamental constants:

$$\varepsilon = \varepsilon_0 \approx \frac{1}{36\pi} \times 10^{-9} \text{ F/m (units: farad per meter)}$$
$$\mu = \mu_0 = 4\pi \times 10^{-7} \text{ H/m (units: henry per meter)}$$

from which you can see that

$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c = 3 \times 10^8 \text{ m/s}$$

where c denotes the speed of light in a vacuum.

Note that the actual value for the speed of light is $2.997\,925\ldots \times 10^8$ m/s, thus the correct value for ε_0 is $8.854188\ldots \times 10^{-12}$ F/m (but this is a seldom required fine detail).

1.3 Units and Notation

As you may have noticed in the preceding section, fields and a few other quantities have already been assigned units.

Throughout this book the rationalized International System of Units (SI) is adhered to. Table 1.1 contains a list of the main quantities that appear in this book, as well as their corresponding SI units.

As far as the computation of electromagnetic quantities is concerned, students are reminded that numerical results are incomplete until their units are made explicit.

Table 1.1 Quantities and units

Quantity	Quantity symbol	Unit designation	Unit symbol
Angular frequency	ω	radian per second	rad/s
Capacitance	C	farad	F
Charge density	ρ	coulomb per cubic meter	C/m^3
Conductance	G	siemens	S
Conductivity	σ	siemens per meter	S/m
Current density	$\mathbf{J}$	ampere per square meter	A/m^2
Current intensity	i, I	ampere	A
Electric charge	q, Q	coulomb	C
Electric displacement	$\mathbf{D}$	coulomb per square meter	C/m^2
Electric field	$\mathbf{E}$	volt per meter	V/m
Electric polarization	$\mathbf{P}$	coulomb per square meter	C/m^2
Electric potential	V	volt	V
Energy	W	joule	J
Force	$\mathbf{F}$	newton	N
Frequency	f	hertz	Hz
Inductance	L	henry	H
Length	l	meter	m
Magnetic induction	$\mathbf{B}$	tesla	T
Magnetic field	$\mathbf{H}$	ampere per meter	A/m
Magnetic flux	ϕ, Ψ	weber	Wb
Magnetic voltage	U_m	ampere	A
Magnetization	$\mathbf{M}$	ampere per meter	A/m
Permeability	μ	henry per meter	H/m
Permittivity	ε	farad per meter	F/m
Potential vector	$\mathbf{A}$	weber per meter	Wb/m
Power	p, P	watt	W
Poynting's vector	$\mathbf{S}$	watt per square meter	W/m^2
Reluctance	R_m	henry^{-1}	H^{-1}
Resistance	R	ohm	Ω
Time	t	second	s
Velocity	$\mathbf{v}$	meter per second	m/s
Voltage	u, U	volt	V

Regarding the writing of variables and quantities we use the following notation. Scalar quantities are in italics. Vectors are boldface. Time-invariant quantities are capitalized. Matrix quantities are identified with square brackets. Complex quantities are identified with overbars.

1.4 Fundamental Concepts of Voltage and Current Intensity

In this book the electromagnetic radiation aspects are only touched upon very superficially (Chapter 8); consequently, most of the phenomena we are going to analyze do not need, for their description, more then the key concepts of voltage and current intensity. These two quantities, with which you are certainly familiar, can be measured with the help of voltmeters and ammeters and can be visualized using oscilloscopes.

The concepts of voltage and current intensity, which are based on vector field integration, apply and are valid for any type of regime, either stationary or time varying.

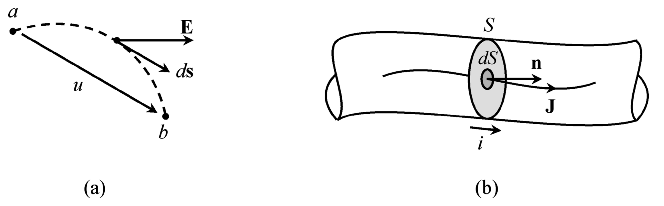

Figure 1.3 Basic definitions of electric voltage (a) and current intensity (b)

As shown in Figure 1.3(a), voltage u between two points a and b is a scalar quantity defined as the line integral of the electric field vector **E** between those points:

$$\text{Voltage}: u = \int_{\overrightarrow{ab}} \mathbf{E} \cdot d\mathbf{s} \tag{1.7}$$

where vector $d\mathbf{s}$ is an infinitesimal element of the path length between a and b.

The reference arrow shown in Figure 1.3(a), usually associated with the definition of u, does not mean that voltage is a vector quantity. The arrow is simply a reminder of the direction of the path length used in the line integral (1.7). Positive voltages signify that the field lines of **E** are predominantly oriented according to the $\overrightarrow{ab}$ path.

A key aspect to be kept in mind is that the evaluation of voltages may depend, or not, on the integration path specification.

As shown in Figure 1.3(b), the current intensity i flowing in a conductor is a scalar quantity defined as a surface integral corresponding to the flux of the current density vector **J** across a conductor section S:

$$\text{Current intensity}: i = \int_{S} \mathbf{J} \cdot \mathbf{n}\, dS \tag{1.8}$$

where dS is an infinitesimal element of area belonging to the section S, and **n** is a unit normal chosen arbitrarily.

The reference arrow shown in Figure 1.3(b), usually associated with the definition of i, does not mean that the current intensity is a vector quantity. The arrow is simply a reminder of the direction of the unit normal **n** used in the surface integral (1.8). Positive current intensities signify that field lines of **J** are predominantly oriented according to **n**.

Again, a key aspect to be kept in mind is that the evaluation of current intensities may depend, or not, on the specification of the conductor section S.