Appendix A

Formulas from Vector Analysis

$$\mathbf{A}\cdot\mathbf{B}=\mathbf{B}\cdot\mathbf{A}$$

$$\mathbf{A}\times\mathbf{B}=-\mathbf{B}\times\mathbf{A}$$

$$(\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}=\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C})=(\mathbf{A}\cdot\mathbf{C})\,\mathbf{B}-(\mathbf{A}\cdot\mathbf{B})\,\mathbf{C}$$

$$\frac{d}{d\xi}(\mathbf{A}\cdot\mathbf{B})=\mathbf{A}\cdot\frac{d\mathbf{B}}{d\xi}+\frac{d\mathbf{A}}{d\xi}\cdot\mathbf{B}$$

$$\frac{d}{d\xi}(\mathbf{A}\times\mathbf{B})=\mathbf{A}\times\frac{d\mathbf{B}}{d\xi}+\frac{d\mathbf{A}}{d\xi}\times\mathbf{B}$$

Differential Operators

$$\text{curl grad } V=0$$

$$\text{div curl } \mathbf{A}=0$$

$$\text{lap } V=\text{div grad } V$$

$$\text{lap } \mathbf{A}=(\text{lap } A_x)\ \vec{e}_x+\left(\text{lap } A_y\right)\ \vec{e}_y+(\text{lap } A_z)\ \vec{e}_z$$

$$\text{curl}\,(\text{curl } \mathbf{A})=\text{grad div } \mathbf{A}-\text{lap } \mathbf{A}$$

$$\text{div}\,(V\mathbf{A})=V\ \text{div } \mathbf{A}+\text{grad } V\cdot\mathbf{A}$$

$$\text{curl}\,(V\mathbf{A})=V\ \text{rot } \mathbf{A}+\text{grad } V\times\mathbf{A}$$

$$\text{div}\,(\mathbf{A}\times\mathbf{B})=(\text{curl } \mathbf{A})\cdot\mathbf{B}-\mathbf{A}\cdot(\text{curl } \mathbf{B})$$

Electromagnetic Foundations of Electrical Engineering J. A. Brandão Faria
 ISBN: 978-0-470-72709-6

Rectangular coordinates (x, y, z):

$$\operatorname{grad} V = \frac{\partial V}{\partial x}\vec{e}_x + \frac{\partial V}{\partial y}\vec{e}_y + \frac{\partial V}{\partial z}\vec{e}_z$$

$$\operatorname{lap} V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\operatorname{div} \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\operatorname{curl} \mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{e}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{e}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{e}_z$$

Cylindrical coordinates (r, ϕ, z):

$$\operatorname{grad} V = \frac{\partial V}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial V}{\partial \phi}\vec{e}_\phi + \frac{\partial V}{\partial z}\vec{e}_z$$

$$\operatorname{lap} V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\operatorname{div} \mathbf{A} = \frac{1}{r}\left(\frac{\partial (rA_r)}{\partial r} + \frac{\partial A_\phi}{\partial \phi} + r\frac{\partial A_z}{\partial z}\right)$$

$$\operatorname{curl} \mathbf{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\vec{e}_r + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\vec{e}_\phi + \frac{1}{r}\left(\frac{\partial (rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi}\right)\vec{e}_z$$

Spherical coordinates (r, θ, ϕ):

$$\operatorname{grad} V = \frac{\partial V}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\vec{e}_\phi$$

$$\operatorname{lap} V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{(r\sin\theta)^2}\frac{\partial^2 V}{\partial \phi^2}$$

$$\operatorname{div} \mathbf{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\left(\frac{\partial (A_\theta \sin\theta)}{\partial \theta} + \frac{\partial A_\phi}{\partial \phi}\right)$$

$$\operatorname{curl} \mathbf{A} = \frac{1}{r\sin\theta}\left(\frac{\partial (A_\phi \sin\theta)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right)\vec{e}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial (rA_\phi)}{\partial r}\right)\vec{e}_\theta + \frac{1}{r}\left(\frac{\partial (rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\vec{e}_\phi$$

Curl Theorem (Stokes Theorem)

$$\int_{S_s} \operatorname{curl} \mathbf{A}\cdot\mathbf{n}_S \, dS = \oint_{\mathbf{s}} \mathbf{A}\cdot d\mathbf{s}$$

Divergence Theorem (Gauss Theorem)

$$\int_V \operatorname{div} \mathbf{A} \, dV = \int_{S_V} \mathbf{A}\cdot\mathbf{n}_o \, dS$$