Chapter 6

# Observer Gain Determination Based on Parameter Sensitivity Analysis

## 6.1. Introduction

The major problem with the implementation of vector control in the induction motor involves controlling machine flux because of the problem in measuring this variable.

To avoid measuring the flux, it is possible to estimate it with the help of models using variables that are easier to acquire (stator current and voltage, mechanical speed). This large dependence on a model leads to major sensitivity problems and uncertainties for the control. These uncertainties are caused by the variations of stator and rotor resistances with temperature and skin effect and the variations of inductances with magnetic saturation.

Rotor resistance is the most difficult parameter to identify with precision, especially in the case of squirrel-cage machines, although it plays an important role in vector control. This parameter can vary by 100% with temperature.

Parameter uncertainties lead to errors of amplitude and flux orientation in the machine with the following consequences:

– the system can become unstable when the error of orientation becomes too large;

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Chapter written by Benoît ROBYNS.

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– an additional stator current is used to develop a given torque, increasing system losses.

In the first section, we suggest an overview of the principle of the two traditional rotor flux observers: a reduced order observer and a full order observer. The major problem linked to the development of a flux observer is the choice of observer gains that adjust the poles and thus the observer dynamics, and also greatly influences observer sensitivity to parametric uncertainties.

In the following sections, we detail a theoretical sensitivity study to determine observer gains constituting a good compromise between dynamics, sensitivity and simplicity. The theoretical study is applied to reduced and full order rotor flux observers. The theoretical sensitivity study method can however be applied systematically to other variants of rotor or stator flux estimators and observers.

## 6.2. Flux observers

### 6.2.1. *Rotor flux estimator*

To develop a flux observer, we must transpose the model of the induction machine in the form of state equations. The model of the induction machine is not intended for this transposition because the machine equations are non-linear. In fact, in these equations, there are products between currents and speed which are state variables. However, in most cases, we can accept that mechanical speed slowly varies in relation to electrical variables, that is, the larger electrical time constants are much smaller than the mechanical time constant. Speed is then considered as a parameter and not as a state variable, and the system becomes linear. From the induction machine model, it is possible to determine several state equation variants depending on what we choose as variables; currents, fluxes or a combination of both. Traditionally, we try to determine the rotor flux. We will choose as a state variable components $d$ and $q$ of this rotor flux. For the two remaining state variables, we will choose stator current components $d$ and $q$, because this current can easily be measured in practice. The model of the induction machine is then made up of equations [6.1a] and [6.1b] for the rotor, and [6.1c] and [6.1d] for the stator:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi_{\mathrm{r}d} = -\frac{R_{\mathrm{r}}}{L_{\mathrm{r}}}\Phi_{\mathrm{r}d} + \left(\omega_{\mathrm{p}} - P\Omega\right)\Phi_{\mathrm{r}q} + \frac{M_{\mathrm{sr}}R_{\mathrm{r}}}{L_{\mathrm{r}}}I_{\mathrm{s}d} \qquad [6.1a]$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi_{\mathrm{r}q} = -\left(\omega_{\mathrm{p}} - P\Omega\right)\Phi_{\mathrm{r}d} - \frac{R_{\mathrm{r}}}{L_{\mathrm{r}}}\Phi_{\mathrm{r}q} + \frac{M_{\mathrm{sr}}R_{\mathrm{r}}}{L_{\mathrm{r}}}I_{\mathrm{s}q} \qquad [6.1b]$$

$$\sigma L_\mathrm{s} \frac{\mathrm{d}}{\mathrm{d}t} I_{sd} = -R_\mathrm{s} I_{sd} + \omega_\mathrm{p} \sigma L_\mathrm{s} I_{sq} - \frac{M_\mathrm{sr}}{L_\mathrm{r}} \frac{d}{\mathrm{d}t} \Phi_{\mathrm{r}d} + \omega_\mathrm{p} \frac{M_\mathrm{sr}}{L_\mathrm{r}} \Phi_{\mathrm{r}q} + V_{sd} \quad [6.1c]$$

$$\sigma L_\mathrm{s} \frac{\mathrm{d}}{\mathrm{d}t} I_{sq} = -\omega_\mathrm{p} \sigma L_\mathrm{s} I_{sd} - R_\mathrm{s} I_{sq} - \omega_\mathrm{p} \frac{M_\mathrm{sr}}{L_\mathrm{r}} \Phi_{\mathrm{r}d} - \frac{M_\mathrm{sr}}{L_\mathrm{r}} \frac{\mathrm{d}}{\mathrm{d}t} \Phi_{\mathrm{r}q} + V_{sq} \quad [6.1d]$$

Angular frequency $\omega_\mathrm{p}$ takes a different value depending on the reference frame used:

– in a reference frame linked to stator $\omega_\mathrm{p} = 0$;

– in a reference frame linked to rotor $\omega_\mathrm{p} = P\Omega$;

– in a reference frame linked to rotating field $\omega_\mathrm{p} = \omega_\mathrm{s}$.

From equation [6.1], by eliminating the derivatives of the rotor flux components in [6.1c] and [6.1d] with the help of [6.1a] and [6.1b], we have in matrix form:

$$\begin{bmatrix} \dot{\Phi}_{\mathrm{r}d} \\ \dot{\Phi}_{\mathrm{r}q} \\ \dot{I}_{sd} \\ \dot{I}_{sq} \end{bmatrix} = \begin{bmatrix} -\dfrac{R_\mathrm{r}}{L_\mathrm{r}} & (\omega_\mathrm{p} - P\Omega) & \dfrac{M_\mathrm{sr} R_\mathrm{r}}{L_\mathrm{r}} & 0 \\ -(\omega_\mathrm{p} - P\Omega) & -\dfrac{R_\mathrm{r}}{L_\mathrm{r}} & 0 & \dfrac{M_\mathrm{sr} R_\mathrm{r}}{L_\mathrm{r}} \\ \dfrac{M_\mathrm{sr} R_\mathrm{r}}{\sigma L_\mathrm{s} L_\mathrm{r}^2} & \dfrac{M_\mathrm{sr} P\Omega}{\sigma L_\mathrm{s} L_\mathrm{r}} & -\dfrac{R_\mathrm{sr}}{\sigma L_\mathrm{s}} & \omega_\mathrm{p} \\ -\dfrac{M_\mathrm{sr} P\Omega}{\sigma L_\mathrm{s} L_\mathrm{r}} & \dfrac{M_\mathrm{sr} R_\mathrm{r}}{\sigma L_\mathrm{s} L_\mathrm{r}^2} & -\omega_\mathrm{p} & -\dfrac{R_\mathrm{sr}}{\sigma L_\mathrm{s}} \end{bmatrix} \begin{bmatrix} \Phi_{\mathrm{r}d} \\ \Phi_{\mathrm{r}q} \\ I_{sd} \\ I_{sq} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{1}{\sigma L_\mathrm{s}} & 0 \\ 0 & \dfrac{1}{\sigma L_\mathrm{s}} \end{bmatrix} \begin{bmatrix} V_{sd} \\ V_{sq} \end{bmatrix} \quad [6.2]$$

with:

$$R_\mathrm{sr} = R_\mathrm{s} + \frac{M_\mathrm{sr}^2}{L_\mathrm{r}^2} R_\mathrm{r}$$

Because of the fact that the equation system [6.2] is valid in any reference frame, it is not possible to differentiate the variable indices based on the referential. We have then chosen to use indices *d* and *q* in all cases.

From equation [6.2], we can develop rotor flux estimation as long as we assume that angular frequency $\Omega$ is a parameter. In practice, the mechanical speed varies; we must then refresh the value of $\Omega$ at each sampling period in the case of a digital implementation.

Even though the separation of electrical and mechanical modes is not possible, we still have to integrate the mechanical equation to system [6.2]. In this case, we must use deterministic non-linear observers or extended Kalman filters.

### 6.2.2. *Reduced order flux observer*

Insofar as we are attempting to estimate both rotor flux components, the simplest flux estimator that we can develop will be of the second order. From system [6.2], we deduct the following second order estimator:

$$\begin{bmatrix} \dot{\hat{\Phi}}_{rd} \\ \dot{\hat{\Phi}}_{rq} \end{bmatrix} = \begin{bmatrix} -\dfrac{R_r^*}{L_r^*} & (\omega_p - P\Omega) \\ -(\omega_p - P\Omega) & -\dfrac{R_r^*}{L_r^*} \end{bmatrix} \cdot \begin{bmatrix} \hat{\Phi}_{rd} \\ \hat{\Phi}_{rq} \end{bmatrix} + \begin{bmatrix} \dfrac{M_{sr}^* R_r^*}{L_r^*} & 0 \\ 0 & \dfrac{M_{sr}^* R_r^*}{L_r^*} \end{bmatrix} \cdot \begin{bmatrix} I_{sd} \\ I_{sq} \end{bmatrix} \quad [6.3]$$

or, in a more compact form:

$$\left[\dot{\hat{\Phi}}_r\right] = [A_r]\left[\hat{\Phi}_r\right] + [B_r][I_s] \quad [6.4]$$

By exposing the parameters, the star indicates that we are working with estimated parameters.

In order to obtain an observer from system [6.3], we must add a correction term to [6.3]

$$[K]\left([C]\left[\hat{\Phi}_r\right] - [Y]\right) \quad [6.5]$$

This term is deducted from stator equations that have not been used yet. We can deduce from these equations the following relations:

$$[C]\left[\hat{\Phi}_r\right] = \begin{bmatrix} \dfrac{M_{sr}^* R_r^*}{L_r^{*2}} & \dfrac{M_{sr}^* P\Omega}{L_r^*} \\ -\dfrac{M_{sr}^* P\Omega}{L_r^*} & \dfrac{M_{sr}^* R_r^*}{L_r^{*2}} \end{bmatrix} \begin{bmatrix} \hat{\Phi}_{rd} \\ \hat{\Phi}_{rq} \end{bmatrix} \quad [6.6]$$

$$\begin{aligned}[Y] &= -[D_1][I_s]+[D_2]\left[\dot{I}_s\right]-[V_s] \\ &= -\begin{bmatrix} -R_{sr}^* & \omega_p\sigma^* L_s^* \\ -\omega_p\sigma^* L_s^* & -R_{sr}^* \end{bmatrix}\begin{bmatrix} I_{sd} \\ I_{sq} \end{bmatrix}+\begin{bmatrix} \sigma^* L_s^* & 0 \\ 0 & \sigma^* L_s^* \end{bmatrix}\begin{bmatrix} \dot{I}_{sd} \\ \dot{I}_{sq} \end{bmatrix}-\begin{bmatrix} V_{sd} \\ V_{sq} \end{bmatrix}\end{aligned} \quad [6.7]$$

By associating [6.3], [6.6], and [6.7], we obtain the following observer:

$$\left[\dot{\hat{\Phi}}_r\right]=[A_r]\left[\hat{\Phi}_r\right]+[B_r][I_s]+[K]\left([C]\left[\hat{\Phi}_r\right]-\left(-[D_1][I_s]+[D_2]\left[\dot{I}_s\right]-[V_s]\right)\right) \quad [6.8]$$

with:

$$[K]=\begin{bmatrix} K_1 & -K_2 \\ K_2 & K_1 \end{bmatrix} \quad [6.9]$$

System [6.8] can be rewritten as follows:

$$\left[\dot{\hat{\Phi}}_r\right]=([A_r]+[K][C])\left[\hat{\Phi}_r\right]+([B_r]+[K][D_1])[I_s]-[K][D_2]\left[\dot{I}_s\right]+[K][V_s] \quad [6.10]$$

or, in a more compact form:

$$\left[\dot{\hat{\Phi}}_r\right]=[A_k]\left[\hat{\Phi}_r\right]+[B_k][I_s]-[K][D_2]\left[\dot{I}_s\right]+[K][V_s] \quad [6.11]$$

Matrices $[A_k]$ and $[B_k]$ have the following form:

$$[A_k]=\begin{bmatrix} A_{1k} & -A_{2k} \\ A_{2k} & A_{1k} \end{bmatrix},[B_k]=\begin{bmatrix} B_{1k} & -B_{2k} \\ B_{2k} & B_{1k} \end{bmatrix} \quad [6.12]$$

with:

$$A_{1k} = -\frac{R_r^*}{L_r^*}+K_1\frac{M_{sr}^* R_r^*}{L_r^{*2}}+K_2\frac{M_{sr}^* P\Omega}{L_r^*} \quad [6.13a]$$

$$A_{2k} = -\left(\omega_p - P\Omega\right)+K_1\frac{M_{sr}^* P\Omega}{L_r^*}+K_2\frac{M_{sr}^* R_r^*}{L_r^{*2}} \quad [6.13b]$$

$$B_{1k} = M_{\text{sr}}^* \frac{R_{\text{r}}^*}{L_{\text{r}}^*} - K_1 R_{\text{sr}}^* + K_2 \sigma^* L_{\text{s}}^* \omega_{\text{p}} \qquad [6.13c]$$

$$B_{2k} = -K_1 \sigma^* L_{\text{s}}^* \omega_{\text{p}} - K_2 R_{\text{sr}}^* \qquad [6.13d]$$

The estimation of the flux from observer [6.11] requires knowledge of the current derivative. It is not necessary, however, to derive the measured current to find out this value.

In fact, a simple variable change enables us to take into account the current derivative without directly calculating it [VER 88].

### 6.2.3. *Full order flux observer*

If we consider complete system [6.2] as an estimator, we estimate the components of the stator current in addition to the components of the rotor flux. By rewriting system [6.2] in a more compact form, we obtain:

$$\begin{bmatrix} \left[\dot{\hat{\Phi}}_{\text{r}}\right] \\ \left[\dot{\hat{I}}_{\text{s}}\right] \end{bmatrix} = \begin{bmatrix} [A_{c1}] & [A_{c2}] \\ [A_{c3}] & [A_{c4}] \end{bmatrix} \begin{bmatrix} \hat{\Phi}_{\text{r}} \\ \hat{I}_{\text{s}} \end{bmatrix} + \begin{bmatrix} [0] \\ [B_c] \end{bmatrix} [V_{\text{s}}] \qquad [6.14]$$

The full order observer is obtained by adding to [6.14] a correction term that is determined by the difference between the estimated current and measured current:

$$[K]\left([C]\left[\hat{X}\right] - [Y]\right) = \begin{bmatrix} [K_{12}] \\ [K_{34}] \end{bmatrix} \left(\left[\hat{I}_{\text{s}}\right] - [I_{\text{s}}]\right) \qquad [6.15]$$

with:

$$[K_{12}] = \begin{bmatrix} K_1 & -K_2 \\ K_2 & K_1 \end{bmatrix}, [K_{34}] = \begin{bmatrix} K_3 & -K_4 \\ K_4 & K_3 \end{bmatrix} \qquad [6.16]$$

By combining [6.14] and [6.15], we finally obtain:

$$\begin{bmatrix} \left[\dot{\hat{\Phi}}_{\text{r}}\right] \\ \left[\dot{\hat{I}}_{\text{s}}\right] \end{bmatrix} = \begin{bmatrix} [A_{c1}] & [A_{c2}] + [K_{12}] \\ [A_{c3}] & [A_{c4}] + [K_{34}] \end{bmatrix} \begin{bmatrix} \hat{\Phi}_{\text{r}} \\ \hat{I}_{\text{s}} \end{bmatrix} - \begin{bmatrix} [K_{12}] \\ [K_{34}] \end{bmatrix} [I_{\text{s}}] + \begin{bmatrix} 0 \\ B_c \end{bmatrix} [V_{\text{s}}] \qquad [6.17]$$

### 6.2.4. *Choice of observer gains*

Observer gains must be chosen in order to set the observer dynamics, and at a minimum, ensure its stability.

Gains are then functions of the polynomial poles characteristic of observer matrix [$A$].

Consider the reduced order observer [6.11]: the polynomial characteristic of matrix [$A_k$] is determined in the following way ([$I$] is the unit matrix):

$$P_c = det\left(s\left[I\right]-\left[A_k\right]\right) = s^2 - 2A_{1k}s + A_{1k}^2 + A_{2k}^2 \qquad [6.18]$$

The reduced order observer contains two complex combined poles in the form $(-a+jb)$ and $(-a-jb)$. The characteristic polynomial can be expressed according to these poles:

$$P_c = \left(s-\left(-a+jb\right)\right)\left(s-\left(-a-jb\right)\right) = s^2 + 2as + a^2 + b^2 \qquad [6.19]$$

By identifying the factors of both polynomials [6.18] and [6.19], we obtain:

$$-a = A_{1k} = -\frac{R_{\mathrm{r}}^*}{L_{\mathrm{r}}^*} + K_1 \frac{M_{\mathrm{sr}}^* R_{\mathrm{r}}^*}{L_{\mathrm{r}}^{*2}} + K_2 \frac{M_{\mathrm{sr}}^* P\Omega}{L_{\mathrm{r}}^*} \qquad [6.20a]$$

$$b = A_{2k} = -\left(\omega_{\mathrm{p}} - p\Omega\right) - K_1 \frac{M_{\mathrm{sr}}^* P\Omega}{L_{\mathrm{r}}^*} + K_2 \frac{M_{\mathrm{sr}}^* R_{\mathrm{r}}^*}{L_{\mathrm{r}}^{*2}} \qquad [6.20b]$$

From relations [6.20], we finally deduct the gain values according to the poles:

$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \frac{1}{\frac{M_{\mathrm{sr}}^*}{L_{\mathrm{r}}^*}\left(\left(\frac{R_{\mathrm{r}}^*}{L_{\mathrm{r}}^*}\right) + \left(P\Omega\right)^2\right)} \begin{bmatrix} \left(-a + \frac{R_{\mathrm{r}}^*}{L_{\mathrm{r}}^*}\right)\frac{R_{\mathrm{r}}^*}{L_{\mathrm{r}}^*} - \left(b + \left(\omega_{\mathrm{p}} - P\Omega\right)\right)P\Omega \\ P\Omega\left(-a + \frac{R_{\mathrm{r}}^*}{L_{\mathrm{r}}^*}\right) + \frac{R_{\mathrm{r}}^*}{L_{\mathrm{r}}^*}\left(b + \left(\omega_{\mathrm{p}} - P\Omega\right)\right) \end{bmatrix} \qquad [6.21]$$

In the case of the full order observer, the same methodology can be used in order to determine the relations between observer gains and poles. This observer, however, is characterized by four poles and four gains making these calculations fastidious. We use mathematical software to determine these poles.

Expression [6.21] shows that observer gains are functions of mechanical speed (and even the square of this speed). We must therefore recalculate these gains in real time at each sampling period. Since this operation is calculation time intensive, we often set the gains at constant values on certain ranges of speed. This methodology, generally empirical, is all the more complex as the gains greatly influence the sensitivity of the observer, and thus control, to parametric uncertainties. Experience shows that gains corresponding to good poles often lead to great sensitivity and vice versa. In the rest of the chapter, we will show that with the help of a theoretical study of parametric sensitivity, it is possible to easily determine gains constituting a good compromise between dynamics, sensitivity, and simplicity.

### 6.2.5. *Choice of the reference frame*

If the reference frame is linked to the stator, we must add $\omega_p = 0$ in equations [6.2]. This solution has the advantage that the only angular frequency involved in matrix [A] is angular frequency $\Omega$ linked to mechanical speed. Angular frequencies $\omega_s$ and $\omega_r$, which depend on flux orientation, are not involved. This solution has the disadvantage that flux, current, and voltage variations are sinusoidal angular frequency $\omega_s$ dimensions. Despite this drawback, this solution is often chosen in practice.

If the reference frame is linked to the rotor, we must add $\omega_p = P\Omega$ in equations [6.2]. Again, the only angular frequency involved in matrix [$A$] is $\Omega$. Flux, current and voltage variables are also sinusoidal angular frequency $\omega_r$ dimensions. We must note that in order to bring the components of stator current back in the reference frame linked to the rotor, a Park transformation is necessary, which is not the case when we work in the reference frame linked to the stator. This solution is not much used in practice.

Finally, if the reference frame is linked to the rotating field, we must add $\omega_p = \omega_s$ in equations [6.2]. The major advantage of this solution is that the different variables evolve like continuous variables, facilitating their real-time processing. On the other hand, angular frequencies $\omega_s$ and $\omega_r$ are involved in matrix [$A$]. These angular frequencies are linked to the flux orientation in the machine. This orientation is a tricky operation because it can be sensitive to parametric uncertainties. For this reason, this solution is not widely used in practice. In reference [ROB 00], we show that there is a way to obtain, in terms of sensitivity, similar properties for the observer developed in a reference frame linked to the rotating field, and for the observer developed in a reference frame linked to the stator.

In all cases, the amplitude of the estimated rotor flux is deducted from the following relation:

$$\left|\hat{\Phi}_{\mathrm{r}}\right| = \sqrt{\hat{\Phi}_{\mathrm{r}d}^{2} + \hat{\Phi}_{\mathrm{r}q}^{2}} \qquad [6.22]$$

On the other hand, the flux orientation depends on the reference frame in which the observer is developed [ROB 00].

## 6.3. Analysis method of the parametric sensitivity

### 6.3.1. *Flux amplitude and phase error estimation*

The flux observer depends on the different electrical parameters of the machine. The uncertainties concerning these parameters will therefore lead to errors in the estimation of the flux amplitude and phase. These errors can be determined in sinusoidal mode. To determine these errors, it is easier to manipulate the machine and observer equations in a complex form. Flux, current, and voltage vectors then take the following form:

$$\begin{aligned} \underline{\Phi}_{\mathrm{r}} &= \Phi_{\mathrm{r}d} + j\,\Phi_{\mathrm{r}q} \\ \underline{\hat{\Phi}}_{\mathrm{r}} &= \hat{\Phi}_{\mathrm{r}d} + j\,\hat{\Phi}_{\mathrm{r}q} \\ \underline{I}_{\mathrm{s}} &= I_{sd} + j\,I_{sq} \\ \underline{V}_{\mathrm{s}} &= V_{\mathrm{s}d} + j\,V_{\mathrm{s}q} \end{aligned} \qquad [6.23]$$

The sinusoidal mode can be treated, regardless of the reference frame in which the observer is developed, as long as the variables have a sinusoidal form when they are not continuous by replacing the derivatives with their complex form:

$$\dot{\underline{X}} = j\left(\omega_{\mathrm{s}} - \omega_{\mathrm{p}}\right)\underline{X} \qquad [6.24]$$

The equations of the flux observer can then take the following form:

$$\underline{A}_{\mathrm{o}}\underline{\Phi}_{\mathrm{r}} + \underline{B}_{\mathrm{o}}\underline{I}_{\mathrm{s}} + \underline{C}_{\mathrm{o}}\underline{V}_{\mathrm{s}} = 0 \qquad [6.25]$$

with:

$$\underline{A}_{\mathrm{o}} = A_{\mathrm{o}1} + j\,A_{\mathrm{o}2},\ \underline{B}_{\mathrm{o}} = B_{\mathrm{o}1} + j\,B_{\mathrm{o}2}$$

and:

$$\underline{C}_o = C_{o1} + j\, C_{o2}$$

Complex expressions $\underline{A}_o$, $\underline{B}_o$, and $\underline{C}_o$ depend on the type of observer considered. These expressions will be explained later in this chapter for reduced and full order observers.

To determine the errors on the estimated flux, we must eliminate stator current and voltage in [6.25] by expressing them according to the flux, with the help of equation [6.1]. From equation [6.1] in sinusoidal form, we obtain the following expression in complex form:

– for stator current (deducted from [6.1a] and [6.1b]):

$$\underline{I}_s = \underline{G}\underline{\Phi}_r \tag{6.26}$$

with:

$$\underline{G} = G_1 + j\, G_2, \text{ where } G_1 = \frac{1}{M_{sr}}$$

and:

$$G_2 = \frac{L_r}{M_{sr} R_r}\omega_r$$

– for stator voltage (after elimination of the stator current in [6.1c] and [6.1d] with the help of [6.1a] and [6.1b]):

$$\underline{V}_s = \underline{H}\underline{\Phi}_r \tag{6.27}$$

with:

$$\underline{H} = H_1 + j\, H_2, \text{ where } H_1 = \frac{1}{M_{sr}}\left(R_s - \frac{\sigma L_s L_r}{R_r}\omega_s\omega_r\right)$$

and:

$$H_2 = \frac{1}{M_{sr}}\left(L_s\omega_s + \frac{R_s L_r}{R_r}\omega_r\right)$$

We must note that expressions $\underline{G}$ and $\underline{H}$ do not depend on angular frequency $\omega_p$ and thus on the reference frame in which we are working in sinusoidal mode.

By introducing [6.26] and [6.27] in [6.25], we obtain:

$$\underline{A}_o \underline{\hat{\Phi}}_r + \left(\underline{B}_o \underline{G} + \underline{C}_o \underline{H}\right)\underline{\Phi}_r = 0 \qquad [6.28]$$

From expression [6.28], we deduce a relation between the estimated and actual flux:

$$\underline{\hat{\Phi}}_r = \left(q_1 + j\, q_2\right)\underline{\Phi}_r \qquad [6.29]$$

with:

$$q_1 = \frac{-A_{o1}Z_1 - A_{o2}Z_2}{A_{o1}^2 + A_{o2}^2} \qquad [6.30a]$$

$$q_2 = \frac{A_{o2}Z_1 - A_{o1}Z_2}{A_{o1}^2 + A_{o2}^2} \qquad [6.30b]$$

where:

$$Z_1 = B_{o1}G_1 - B_{o2}G_2 + C_{o1}H_1 - C_{o2}H_2$$
$$Z_2 = B_{o2}G_1 + B_{o1}G_2 + C_{o2}H_1 + C_{o1}H_2$$

The expressions of factors $q_1$ and $q_2$ depend on the observer considered. These factors depend on real and estimated parameters, mechanical speed, and sliding angular frequency. Without error on parameters, $q_1 = 1$ and $q_2 = 0$.

From [6.29], we deduct the error in the estimation of the flux amplitude:

$$\left|\frac{\Phi_r}{\hat{\Phi}_r}\right| = \frac{1}{\sqrt{q_1^2 + q_2^2}} \qquad [6.31]$$

and the error in the estimation of the flux phase:

$$\rho_e = arctan\left(-\frac{q_2}{q_1}\right) \qquad [6.32]$$

When the estimated flux is controlled by a corrector containing an integral action, we have in sinusoidal mode $\left|\hat{\Phi}_{\mathrm{r}}\right| = \Phi_{\mathrm{ref}}$. We then directly obtain the error in the flux amplitude from [6.31]:

$$\left|\frac{\Phi_{\mathrm{r}}}{\Phi_{\mathrm{ref}}}\right| = \frac{1}{\sqrt{q_1^2 + q_2^2}} \qquad [6.33]$$

The orientation of flux $\rho_{\mathrm{o}}$, which is the gap between the real flux and the reference flux represented in Figure 6.1, depends on the technique of flux orientation used that is a function of the reference frame in which the observer is developed. In the presence of an integral action in the flux corrector, the electromagnetic torque developed by the machine equals:

$$C_{\mathrm{em}} = P\frac{\omega_{\mathrm{r}}}{R_{\mathrm{r}}}\left|\Phi_{\mathrm{r}}\right|^2 = P\frac{\omega_{\mathrm{r}}}{R_{\mathrm{r}}}\frac{1}{\left(q_1^2 + q_2^2\right)}\Phi_{\mathrm{ref}}^2 \qquad [6.34]$$

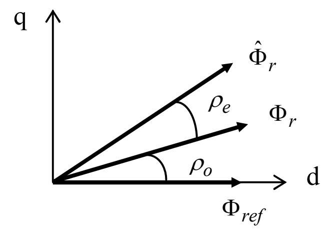



**Figure 6.1.** *Error in the flux phase $\rho_e$ and error in the flux orientation $\rho_o$ represented in a reference frame linked to the rotating field*

### 6.3.2. *Influence of the magnetic saturation*

Expressions [6.31] to [6.34] are calculated from a linear model that does not take into consideration magnetic saturation. Since errors in estimated values of stator and rotor resistance lead to errors in the value of the machine flux, and modify the value of the magnetization inductance, we introduce a simple model modeling the variations of parameter $M_{\mathrm{sr}}$ in the sensitivity study. This model involves two parameters, parameter $\beta$ to characterize the air-gap and parameter $s$ as exponent to characterize the saturation of the magnetic circuit [DEJ 80, KHA 87]:

$$I_{\mathrm{mn}} = \beta\Phi_{\mathrm{sn}} + \left(1-\beta\right)\Phi_{\mathrm{sn}}^{s} \qquad [6.35]$$

$$M_{\mathrm{n}} = \frac{\Phi_{\mathrm{sn}}}{I_{\mathrm{mn}}} \qquad [6.36]$$

where $I_{mn}$, $\Phi_{sn}$ and $M_n$ are standardized values of the magnetizing current $\left(\mathrm{I}_{\mathrm{m}} = \sqrt{\left(I_{sd} + I_{rd}\right)^2 + \left(I_{sq} + I_{rq}\right)^2}\right)$, of stator flux $\left(\Phi_s = \sqrt{\Phi_{sd}^2 + \Phi_{sq}^2}\right)$ and the value of static inductance $M_{sr}$ respectively.

Since the sensitivity study only considers sinusoidal situations, it is not necessary to consider the dynamic inductance [VAS 90].

Parameters $\beta$ and $s$ of expression [6.35] are determined with the help of the magnetic characteristic measured by a no load test [KHA 87, ROB 99].

### 6.3.3. *Calculation algorithm of errors in the estimated flux*

In order to obtain errors in the flux estimation, we must resolve, for each operation point determined by a value of speed $\Omega$ and electromagnetic torque $C_{em}$, a system of two equation [6.34] and [6.36] with two unknowns: slip angular frequency $\omega_r$ and inductance $M_{sr}$. Since this system is greatly non-linear, it must be resolved numerically. The algorithm is as follows:

1. an error is introduced in an estimated parameter;
2. a point of operation ($\Omega$, $C_{em}$) is set;
3. an initial value is set for $M_{sr}$;
4. equation [6.34] is resolved to find $\omega_r$;
5. with this value of $\omega_r$, [6.35] and [6.36] are calculated;
6. [6.36] results in a new value of $M_{sr}$: if this value is identical to the previous value, then [6.31] and [6.32] are calculated, otherwise, we start again at step 4 by considering a new value for $M_{sr}$ obtained by establishing an average between the last $M_{sr}$ value and the previous ones.

In the present study, it is assumed that the mechanical speed is correctly measured from a speed or position sensor. If the mechanical speed is estimated and not measured, it is possible to consider errors in speed estimation caused by parametric uncertainties in the sensitivity study, but we must then add an additional non-linear equation to the system to resolve.

### 6.3.4. *Variations of the stator current used*

Errors in the flux orientation and amplitude can hardly be measured in an experimental way. But these errors in flux lead to variations in the stator current used that can be measured. These variations enable the connection between the experimentation and theoretical predictions and thus validate the latter.

The variation of stator current is defined by:

$$\Delta I_{\mathrm{s}} = \frac{I_{\mathrm{s}} - I_{\mathrm{si}}}{I_{\mathrm{si}}} \tag{6.37}$$

where $I_{\mathrm{si}}$ is the ideal current used in the absence of error in the parameters, and $I_{\mathrm{s}}$ the current really used.

From equations [6.1a] and [6.1b] in sinusoidal mode, with $\Phi_{\mathrm{r}q} = 0$ and $\Phi_{\mathrm{r}d} = \Phi_{\mathrm{ref}}$, we obtain the following expression for current $I_{\mathrm{si}}$:

$$I_{\mathrm{si}} = \sqrt{I_{sdi}^2 + I_{sqi}^2} = \frac{\Phi_{\mathrm{ref}}}{\mathrm{M}_{\mathrm{sr}}^*} \sqrt{1 + \left( \omega_{\mathrm{ri}} \frac{L_{\mathrm{r}}^*}{R_{\mathrm{r}}^*} \right)^2} \tag{6.38}$$

$\omega_{\mathrm{ri}}$ is deducted from electromagnetic torque $C_{\mathrm{em}}$ [6.34] with $q_1 = 1$ and $q_2 = 0$:

$$\omega_{\mathrm{ri}} = \frac{C_{\mathrm{em}} R_{\mathrm{r}}^*}{P \Phi_{\mathrm{ref}}^2} \tag{6.39}$$

From equations [6.1a] and [6.1b], we deduce the expression of $I_{\mathrm{s}}$:

$$I_{\mathrm{s}} = \sqrt{I_{sd}^2 + I_{sq}^2} = \sqrt{1 + \left( \omega_{\mathrm{r}} \frac{L_{\mathrm{r}}}{R_{\mathrm{r}}} \right)^2} \frac{\sqrt{\Phi_{\mathrm{r}d}^2 + \Phi_{\mathrm{r}q}^2}}{M_{\mathrm{sr}}} \tag{6.40}$$

By considering expression [6.33], $I_{\mathrm{s}}$ becomes:

$$I_{\mathrm{s}} = \sqrt{1 + \left( \omega_{\mathrm{r}} \frac{L_{\mathrm{r}}}{R_{\mathrm{r}}} \right)^2} \frac{1}{\sqrt{q_1^2 + q_2^2}} \frac{\Phi_{\mathrm{ref}}}{M_{\mathrm{sr}}} \tag{6.41}$$

## 6.4. Choice of observer gains

### 6.4.1. *Pole placement and parametric sensitivity*

The sensitivity study presented in the previous section makes it possible to study the influence of gains and poles of the observer on its sensitivity to uncertainties with electrical parameters.

In section 6.2.4, we explained that gains could be chosen to set the observer poles. However, with this gain choice method, we often obtain a very sensitive observer with parametric uncertainties, or even unstable (even when poles have a really negative part), because of an excessive orientation error of the flux.

Figure 6.2 illustrates this problem by showing the evolution of errors in the flux phase and amplitude, estimated by the reduced order observer in the stator reference frame, according to the real part of poles in the presence of a 100% error in the estimation of rotor resistance $\left(R_r = 2R_r^*\right)$. Curves in Figure 6.2 were obtained by considering a zero mechanical speed and a torque close to the nominal torque (or 2.3 Nm for the test machine where the parameters are given in the section 6.8), as well as high dampening for the observer, or a ratio of the imaginary part on the real pole part (*b/a*) equaling 0.2. Straight line curves were obtained by not considering magnetic saturation, contrary to dotted line curves.

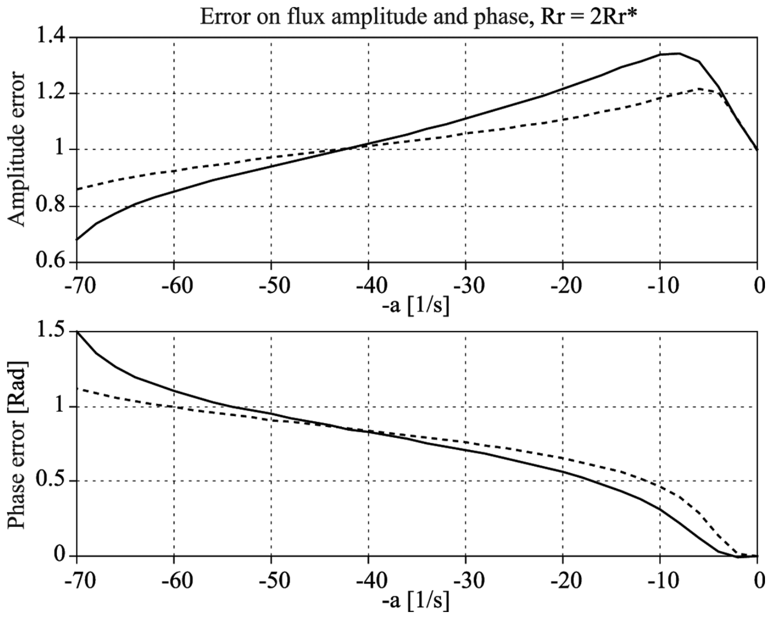



**Figure 6.2.** *Error in the estimation of flux phase and amplitude according to the real part of poles when $R_r = 2R_r^*$ (imaginary part over real part of poles b/a = 0.2; mechanical speed = 0 rpm and electromagnetic torque = 2.3 Nm). Straight line curves: without saturation. Dotted line curves: with saturation*

Figure 6.2 shows that the estimation error of the flux phase, and thus orientation error, increases when the real part of poles moves off from its origin, and leads toward π/2 destabilizing the observer. When observer gains are zero, the real part of poles equals $\left(-R_{\mathrm{r}}/L_{\mathrm{r}}\right)$ or –11.13 s$^{-1}$ for the test machine. In practice, we choose observer poles to increase its dynamics, which comes down to choosing poles with a real part that is lower than $\left(-R_{\mathrm{r}}/L_{\mathrm{r}}\right)$, but in this case, Figure 6.2 shows that we also increase observer sensitivity to uncertainties with rotor resistance.

Figure 6.2 also shows that magnetic saturation decreases error in the flux amplitude, but can increase error in the flux phase in certain cases.

### 6.4.2. *Optimal observer*

With the help of the sensitivity study, it is possible to choose gains greatly reducing the observer's parametric sensitivity, while obtaining satisfactory dynamics. It is also possible to simplify the evolution of gains based on the point of operation (torque, speed) of the machine, or even make them constant in the whole operation range, naturally simplifying their real-time calculation. The choice of gains can also be automated with the help of software in which we can enter different criteria on:

– the real part of poles ($-a$), in order to ensure the velocity and especially the stability of the observer;

– the ratio of the imaginary part over the real part ($b/a$) of poles, in order to set the minimal dampening desired for the flux estimation;

– error in flux amplitude;

– error in the flux orientation where the value will have to be limited in order to avoid instability risks caused by parametric uncertainties;

– variations of stator current used caused by parametric uncertainties.

With the method of choice of gains based on the study of sensitivity, we accept that observer poles vary according to the motor's point of operation in a delimited range of the pole plan set according to traditional stability range criteria (the poles then vary, e.g. in the ruled range in Figure 6.3).

We can consider that the method of gain determination proposed provides an optimal adjustment observer, in the sense that we optimize the observer according to certain criteria. These criteria can be of different types:

– *dynamics* criteria, constraints are imposed on real and imaginary pole parts;

– *sensitivity* criteria limiting acceptable errors in the flux orientation and amplitude;

– *energy* criteria on the variations of the stator current used, for example;

– *simplicity* criteria, the gains must remain constant or can vary in a linear or non-linear way according to the point of operation.

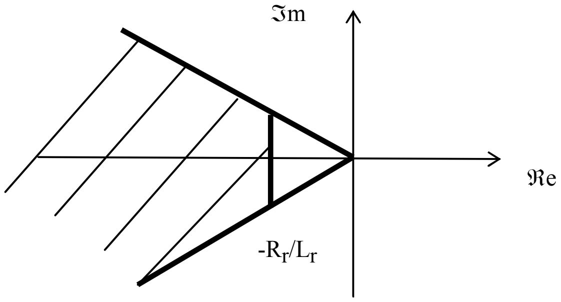



**Figure 6.3.** *Plan of poles*

This criteria list is not exhaustive. We can, for example, add criteria intended to minimize the noise affecting measured quantities, criteria that were taken into consideration in the Kalman filter; we would then obtain an optimal stochastic observer.

The traditional deterministic observer (Luenberger observer) only considers the dynamics criteria (pole placement).

The observers considered from now on are all deterministic. These will then be optimal deterministic observers. To simplify the text, we will simply speak of optimal observer.

## 6.5. Reduced order flux observer

### 6.5.1. *Control strategy*

Figure 6.4 represents the principle diagram of a direct vector control using the flux estimation obtained with the help of a reduced order observer developed in the stator reference frame.

When the flux is estimated from an observer developed in a reference frame linked to the stator, the cosine and sine of the $\theta_p$ angle involve the Park transformation can be directly deducted from the two components of the estimated flux:

$$\cos\theta_p = \frac{\hat{\Phi}_{r\alpha}}{\left|\hat{\Phi}_r\right|},\ \sin\theta_p = \frac{\hat{\Phi}_{r\beta}}{\left|\hat{\Phi}_r\right|} \qquad [6.42]$$

with:

$$\left|\hat{\Phi}_r\right| = \sqrt{\hat{\Phi}_{r\alpha}^2 + \hat{\Phi}_{r\beta}^2} \qquad [6.43]$$

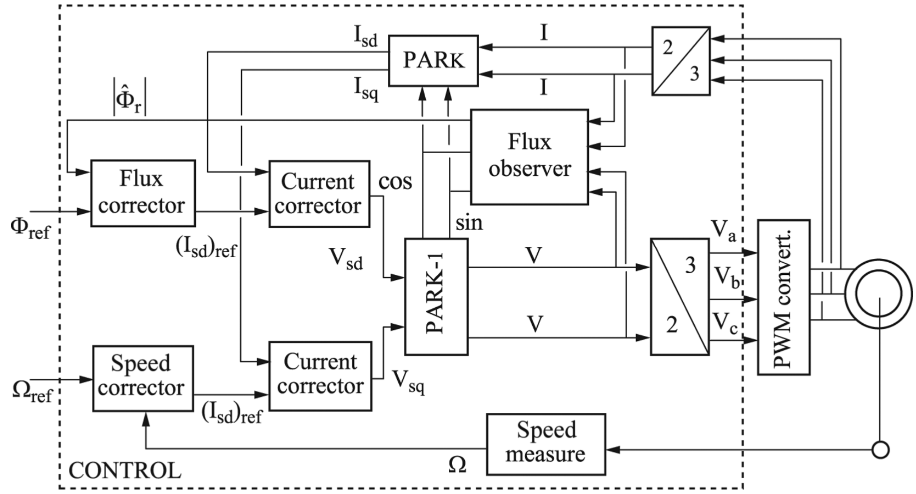



**Figure 6.4.** *Direct vector control with flux observer in the stator reference frame*

The different correctors generally include an integral action. However, insofar as the flux corrector contains an integral action, the nature of current correctors does not influence sensitivity to parametric uncertainties in sinusoidal mode. It is the same with the presence or absence of decoupling terms.

### 6.5.2. *Error in flux orientation and amplitude*

To determine the errors caused by the parametric uncertainties in flux amplitude when the flux is estimated by a reduced order observer, we must determine expressions [6.31] or [6.33]; the latter is only valid if the flux corrector contains an integral action. To determine these expressions, we must develop factors $q_1$ and $q_2$ in the case of the reduced order observer. These factors are determined by expressions [6.30] in which we must introduce complex factors $\underline{A}_o$, $\underline{B}_o$, and $\underline{C}_o$, characteristics of the observer considered.

In sinusoidal mode and in a complex form, system [6.11] takes the following form:

$$\begin{aligned} j\left(\omega-\omega_{\mathrm{p}}\right)\underline{\hat{\Phi}}_{\mathrm{r}} &= \left(A_{1k}+j\,A_{2k}\right)\underline{\hat{\Phi}}_{\mathrm{r}}+\left(B_{1k}+j\,B_{2k}\right)\underline{I}_{\mathrm{s}} \\ &\quad -\sigma^{*}L_{\mathrm{s}}^{*}\left(K_{1}+j\,K_{2}\right)j\left(\omega-\omega_{\mathrm{p}}\right)\underline{I}_{\mathrm{s}}+\left(K_{1}+j\,K_{2}\right)\underline{V}_{\mathrm{s}} \end{aligned} \qquad [6.44]$$

By identifying [6.44] with [6.25], and by considering expressions [6.13], we obtain:

$$A_{\mathrm{o}1}=A_{1k}=-\frac{R_{\mathrm{r}}^{*}}{L_{\mathrm{r}}^{*}}+K_{1}\frac{M_{\mathrm{sr}}^{*}R_{\mathrm{r}}^{*}}{\mathrm{L}_{\mathrm{r}}^{*2}}+K_{2}\frac{M_{\mathrm{sr}}^{*}P\Omega}{L_{\mathrm{r}}^{*}} \qquad [6.45a]$$

$$A_{\mathrm{o}2}=A_{2k}-\left(\omega_{\mathrm{p}}-P\Omega\right)=-\omega_{\mathrm{r}}-K_{1}\frac{M_{\mathrm{sr}}^{*}P\Omega}{L_{\mathrm{r}}^{*}}+K_{2}\frac{M_{\mathrm{sr}}^{*}R_{\mathrm{r}}^{*}}{\mathrm{L}_{\mathrm{r}}^{*2}} \qquad [6.45b]$$

$$B_{\mathrm{o}1}=B_{1k}+K_{2}\sigma^{*}L_{\mathrm{s}}^{*}(\omega_{\mathrm{s}}-\omega_{\mathrm{p}})=M_{\mathrm{sr}}^{*}\frac{R_{\mathrm{r}}^{*}}{L_{\mathrm{r}}^{*}}-K_{1}R_{\mathrm{sr}}^{*}+K_{2}\sigma^{*}L_{\mathrm{s}}^{*}\omega_{\mathrm{s}} \qquad [6.45c]$$

$$B_{\mathrm{o}2}=B_{2k}-K_{1}\sigma^{*}L_{\mathrm{s}}^{*}(\omega_{\mathrm{s}}-\omega_{\mathrm{p}})=-K_{1}\sigma^{*}L_{\mathrm{s}}^{*}\omega_{\mathrm{s}}-K_{2}R_{\mathrm{sr}}^{*} \qquad [6.45d]$$

$$C_{\mathrm{o}1}=K_{1} \qquad [6.45e]$$

$$C_{\mathrm{o}2}=K_{2} \qquad [6.45f]$$

We must note that expressions [6.45] only depend on angular frequency $\omega_{\mathrm{p}}$ (and thus the reference frame in which the observer is developed) by the intermediate of gains $K_1$ and $K_2$.

The error of flux orientation (gap between the real flux and reference flux) is determined in the following manner:

$$\rho_{\mathrm{o}}=\arctan\left(\frac{\hat{\Phi}_{\mathrm{r}q}}{\Phi_{\mathrm{r}d}}\right) \qquad [6.46]$$

Expression [6.46] is determined from the components of the flux in a reference frame linked to the rotating field (ideally $\Phi_{\mathrm{r}q}=0$). The calculation of angle $\theta_{\mathrm{p}}$ involved in the Park transformation with the help of [6.42] imposes that the estimated value of the axis $q$ component of flux $(\hat{\Phi}_{\mathrm{r}q})$ in the reference frame linked to the rotating field be equal to zero. In this case, the orientation $\rho_o$ error is equal to the estimation error of flux $\rho_e$ phase:

$$\rho_{\mathrm{o}} = \arctan\left(\frac{\Phi_{\mathrm{r}q}}{\Phi_{\mathrm{r}d}}\right) = \arctan\left(\frac{-q_2}{q_1}\right) = \rho_{\mathrm{e}} \qquad [6.47]$$

### 6.5.3. *Theoretical results*

Figures 6.5 and 6.6 show flux orientation and amplitude errors, the variations of stator current, the evolution of inductance $M_{sr}$ with saturation, the real part of poles and the relation between the imaginary part and real part of these poles according to mechanical speed, and electromagnetic torque, in the presence of a 100% error in the rotor resistance $\left(R_{\mathrm{r}} = 2R_{\mathrm{r}}^{*}\right)$.

The results shown in Figure 6.5 correspond to zero gains. Errors in the flux, as well as the increase in stator current used, are then very important.

Figure 6.5 shows that relation I (pole)/R (pole) greatly increases when the speed increases, which makes the response of the observer oscillating.

The results shown in Figure 6.6 were obtained with gains $K_1$ and $K_2$ chosen with the help of the theoretical study of sensitivity by following the method presented in section 6.4.2, by imposing constraints in the flux orientation and amplitude errors, in the increase of stator current and in the real and imaginary parts of poles (*R (pole)* $\leq -R_r/L_r$ and *I (pole)/R (pole)* $\leq 2$).

These constraints can be completed with certain considerations deduced from experimental tests.

For example, it is advantageous that gains are small for reducing noise and that they evolve according to simple functions or are constants to simplify the physical implementation of the observer and reduce its calculation time.

From these considerations, we get the following gains:

$$\begin{aligned} K_1 &= -2.86\times10^{-3}\,\omega_{\mathrm{m}} + 0.3, \text{ when } \omega_{\mathrm{m}} \leq 1{,}000 \text{ rpm} \\ &= 0, \text{ when } \omega_{\mathrm{m}} > 1{,}000 \text{ rpm} \end{aligned} \qquad [6.48a]$$

$$\begin{aligned} K_2 &= -0.5, \text{ when } \omega_{\mathrm{m}} \geq 0 \\ &= 0.5, \text{ when } \omega_{\mathrm{m}} < 0 \end{aligned} \qquad [6.48b]$$

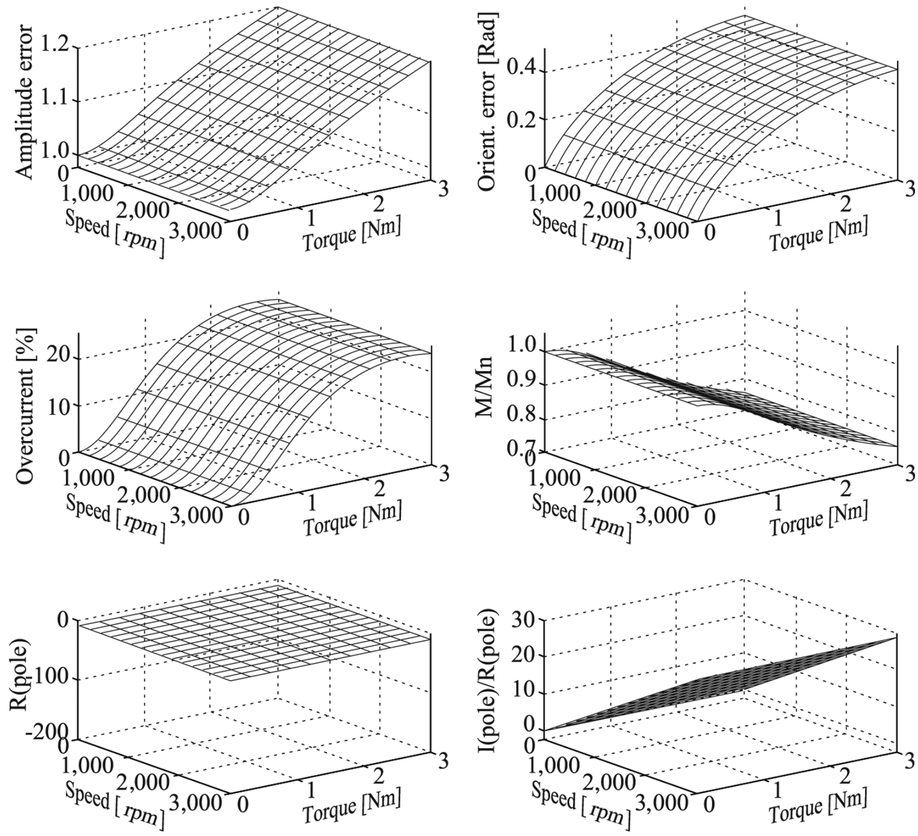



**Figure 6.5.** *Reduced order flux observer developed in a reference frame linked to the stator. Errors in the amplitude and orientation of the flux, used overcurrent, $M/M_n$, R (pole), and I(pole)/R(pole) when $K_1 = K_2 = 0$ and $\left(R_r = 2R_r^*\right)$*

The change of sign forced on $K_2$ according to speed ensures observer stability by imposing that the real part [6.20a] of poles always be negative.

The observer obtained with gains [6.48] will be called optimal observer from now on.

The comparison between Figures 6.5 and 6.6 shows that the gains chosen with the help of the sensitivity study greatly decrease errors in flux orientation and amplitude, as well as the increase of stator current used, while ensuring acceptable poles.

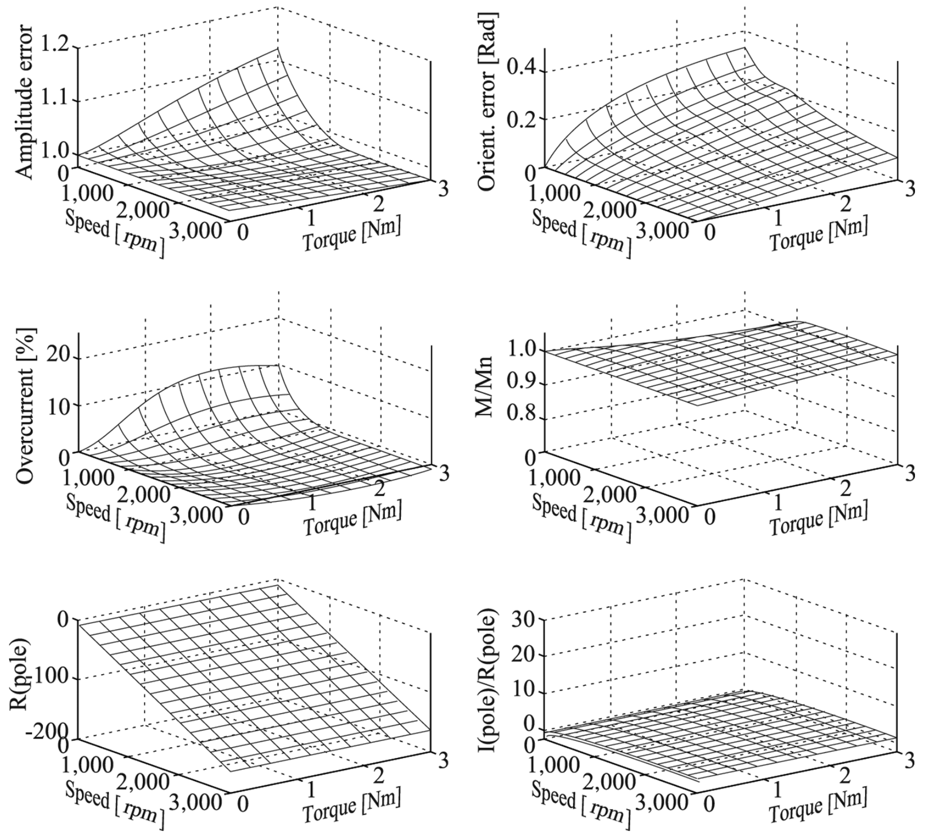



**Figure 6.6.** *Reduced order flux observer developed in a reference frame linked to the stator. Errors in the amplitude and orientation of the flux, used overcurrent, M/M*$_n$*, R (pole), and I(pole)/R(pole) obtained with the optimal observer when* $R_r = 2R_r^*$

When gains $K_1$ and $K_2$ are zero, the flux observer is not sensitive to errors in stator resistance, because $R_s$ is not involved in observer equations. Figure 6.7 shows the errors in the flux, the variations of stator current, the evolution of inductance $M_{sr}$ with the saturation and evolution of poles according to the mechanical speed and electromagnetic torque, in the presence of a 20% error in stator resistance $\left(R_s = 1,2R_s^*\right)$, when the gains are determined by [6.48]. Errors in fluxes are generally low and only become significant when the speed is slow.

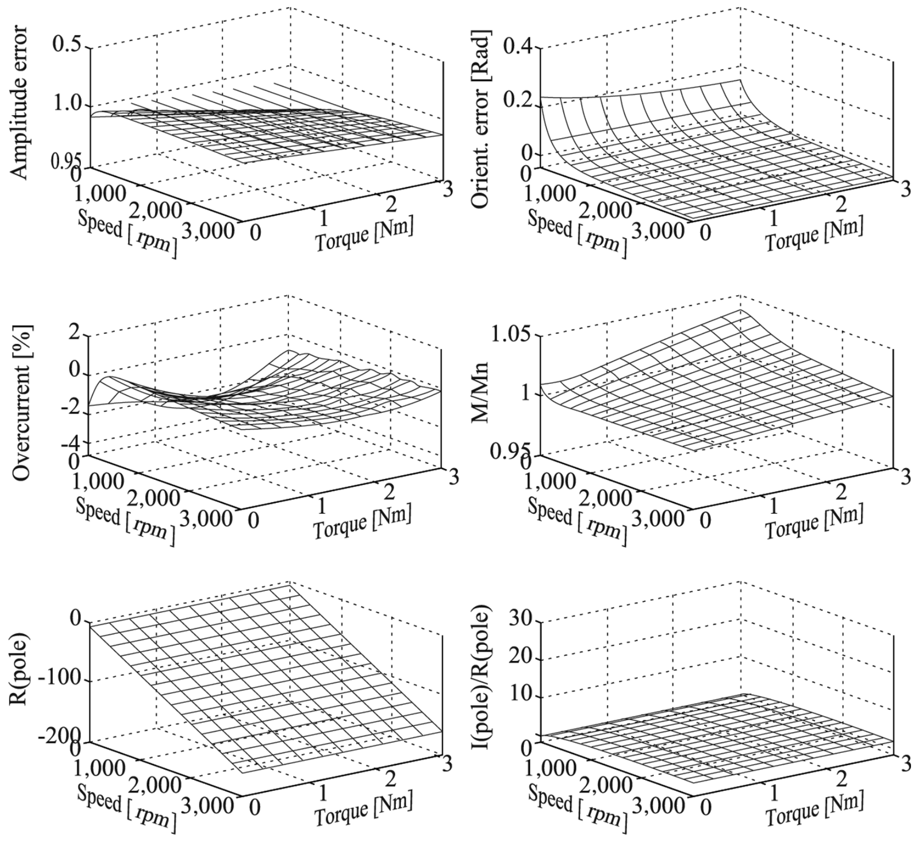



**Figure 6.7.** *Reduced order flux observer developed in a reference frame linked to the stator. Errors in the amplitude and orientation of the flux, used overcurrent, M/M*$_n$*, R (pole), and I(pole)/R(pole) obtained with the optimal observer when* $R_s = 1, 2R_s^*$

### 6.5.4. *Experimental results*

The experimental tests were done with the help of a benchmark with a signal processor TMS320C31. The inverter associated with the 750 W induction motor (parameters are provided in section 6.8) is equipped with MOSFET transistors with a commutation frequency of 30 kHz.

Figures 6.8 and 6.9 show the system response at reference speed step of 0 to 1,500 rpm, followed by a load torque step of 0 to 2.3 Nm. For each test, Figures 6.8 and 6.9 show the reference values and mechanical speed measure, the rms value of stator current, and the estimated value of flux amplitude.

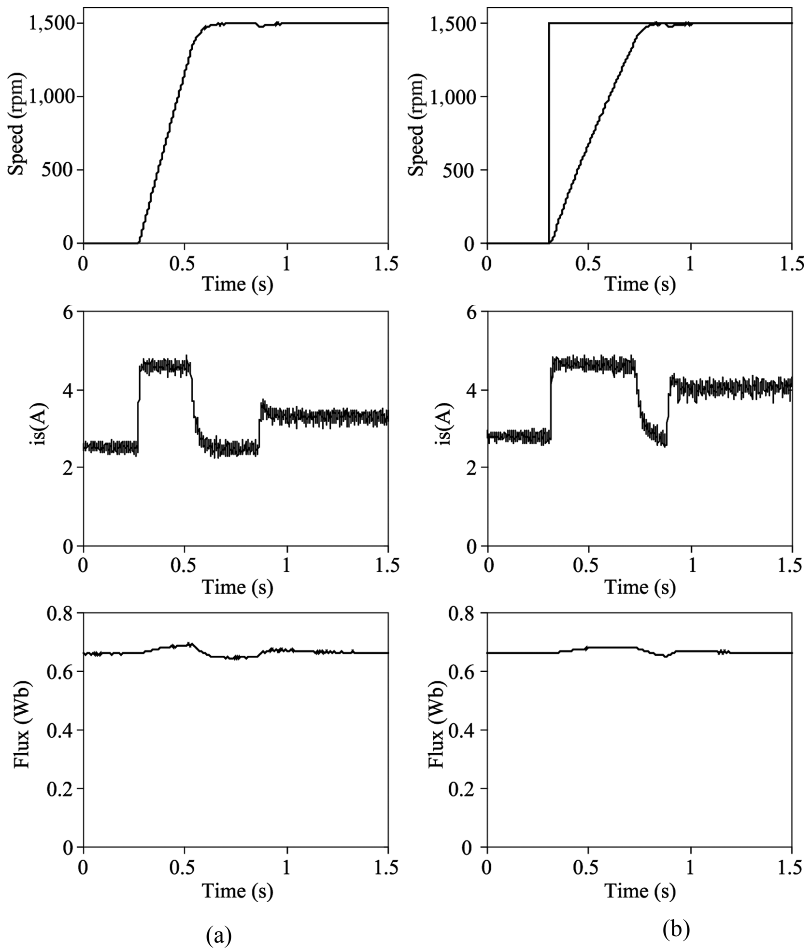



**Figure 6.8.** *System response at a reference speed step of 0 to 1,500 rpm followed by a load torque step. Flux observer developed in a reference frame linked to the stator with* $K_1 = K_2 = 0$. *Test (a) with optimized parameters; (b) with* $R_r = 2R_r^*$

In Figure 6.8 tests, $K_1 = K_2 = 0$. The test in Figure 6.8a was done with optimized parameters (it is obviously a hypothesis that is well verified in this case, e.g., by comparing the tests with simulations or by comparing the measured value of stator current used with the theoretically predicted value), whereas in the test in Figure 6.8b, a 100% error in rotor resistance was introduced in the control.

In Figure 6.9 tests, $K_1$ and $K_2$ are determined by [6.48]. The test in Figure 6.9a was done with optimized parameters, whereas in the test in Figure 6.9b a 100% error in rotor resistance was introduced in the control.

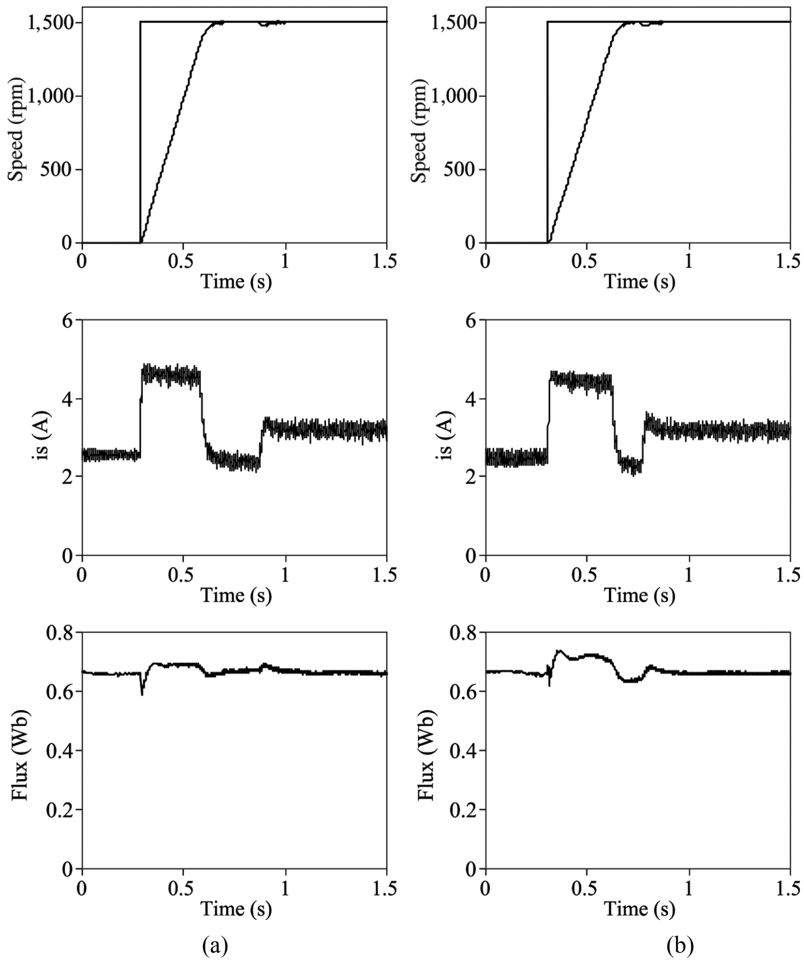



**Figure 6.9.** *System response at a reference speed step of 0 to 1,500 rpm followed by a load torque step. Optimal flux observer developed in a reference frame linked to the stator. Test (a) with optimized parameters; (b) with* $R_r = 2R_r^*$

The results of the two tests in Figures 6.8a and 6.9a are similar. On the other hand, the response of the system in Figure 6.9b is clearly superior to that of Figure 6.8b, because:

– the stator current used when the motor is loaded is clearly lower in the test in Figure 6.9b than in the test in Figure 6.8b, confirming that the flux is more correctly estimated with the optimal observer, and particularly that the flux orientation is correctly achieved;

– the response time with speed is lower in Figure 6.9b than in Figure 6.8b, because for the same used current during the acceleration phase, the developed torque is much closer to its maximum value in the command based on the optimal observer.

The variations of the estimated flux response represented in Figure 6.9b are slightly more significant than those represented in Figure 6.8b, because in the development of the optimal observer, a compromise between the dynamics and the sensitivity of the observer was desired. These variations remain acceptable in practice.

In [ROB 00], we show that we can obtain the same performance in terms of dynamics and sensitivity with an observer in the reference frame of the rotating field, through a judicious adaptation of the flux orientation method.

## 6.6. Full order flux observer

### 6.6.1. *Control strategy*

In this chapter, we will only discuss the full order observer developed in the stator reference frame. The control strategy is, in this case, identical to the one presented in section 6.5.1 and illustrated in Figure 6.4. The observer structure is the only one that changes, and input and output variables of the observer remain identical.

### 6.6.2. *Error in flux orientation and amplitude*

Error in the flux amplitude and orientation are determined by expressions [6.33] and [6.32] or [6.47]. To calculate these errors, we must develop the expressions of complex factors $\underline{A}_{\text{o}}$, $\underline{B}_{\text{o}}$, and $\underline{C}_{\text{o}}$, characteristic of the observer considered, in order to be able to determine factors $q_1$ and $q_2$ involved in [6.31], [6.33], and [6.47].

In sinusoidal mode, and in a complex form, system [6.17] takes the following form:

$$j\left(\omega_{\text{s}}-\omega_{\text{p}}\right)\underline{\hat{\Phi}}_{\text{r}}=\underline{A}_{c1}\underline{\hat{\Phi}}_{\text{r}}+\left(\underline{A}_{c2}+\underline{K}_{12}\right)\underline{\hat{I}}_{\text{s}}-\underline{K}_{12}\underline{I}_{\text{s}} \qquad [6.49a]$$

$$j\left(\omega_{\text{s}}-\omega_{\text{p}}\right)\underline{\hat{I}}_{\text{s}}=\underline{A}_{c3}\underline{\hat{\Phi}}_{\text{r}}+\left(\underline{A}_{c4}+\underline{K}_{34}\right)\underline{\hat{I}}_{\text{s}}-\underline{K}_{34}\underline{I}_{\text{s}}+\underline{B}_{c}\underline{V}_{\text{s}} \qquad [6.49b]$$

Estimated current $\underline{\hat{I}}_{\text{s}}$ can be deduced from [6.49a]:

$$\underline{\hat{I}}_{\text{s}}=\frac{j\left(\omega_{\text{s}}-\omega_{\text{p}}\right)-\underline{A}_{c1}}{\left(\underline{A}_{c2}+\underline{K}_{12}\right)}\underline{\hat{\Phi}}_{\text{r}}+\frac{\underline{K}_{12}}{\left(\underline{A}_{c2}+\underline{K}_{12}\right)}\underline{I}_{\text{s}} \qquad [6.50a]$$

or in a more compact form:

$$\underline{\hat{I}}_{\rm s} = \underline{E}\underline{\hat{\Phi}}_{\rm r} + \underline{F}\underline{I}_{\rm s} \quad [6.50b]$$

By introducing [6.50b] in [6.49b], we obtain:

$$\begin{aligned}&\left(\underline{A}_{c3} + \left(\left(\underline{A}_{c4} + \underline{K}_{34}\right) - j\left(\omega_{\rm s} - \omega_{\rm p}\right)\right)\underline{E}\right)\underline{\hat{\Phi}}_{\rm r}\\ &+\left(-K_{34} + \left(\left(\underline{A}_{c4} + \underline{K}_{34}\right) - j\left(\omega_{\rm s} - \omega_{\rm p}\right)\right)\underline{F}\right)\underline{I}_{\rm s} + \underline{B}_{\rm c}\underline{V}_{\rm s} = 0\end{aligned} \quad [6.51]$$

By identifying [6.51] and [6.25], we obtain factors $\underline{A}_{\rm o}$, $\underline{B}_{\rm o}$, and $\underline{C}_{\rm o}$.

### 6.6.3. *Theoretical results*

Figures 6.10 and 6.11 show errors of amplitude and orientation of the flux, stator current variations, and the evolution of inductance $M_{\rm sr}$ with the saturation according to the mechanical speed and electromagnetic torque, in the presence of a 100% error in rotor resistance.

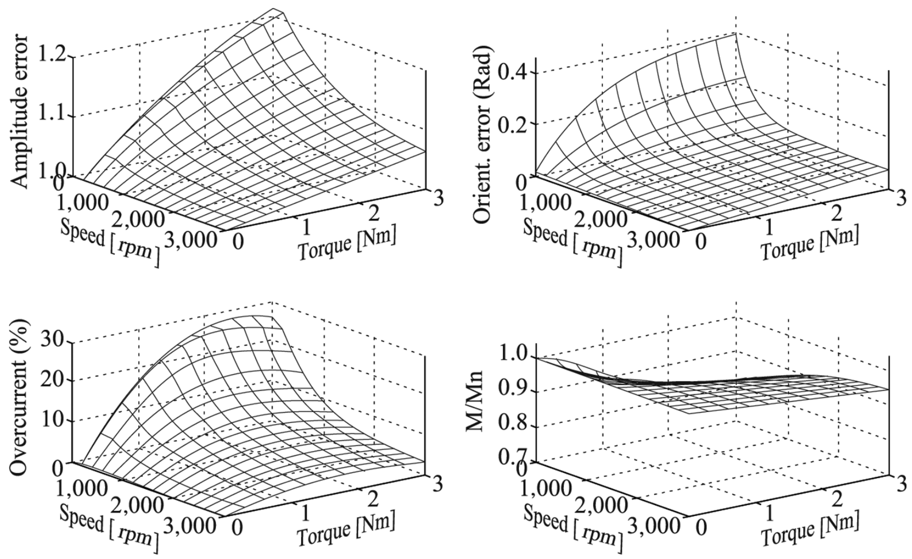



**Figure 6.10.** *Full order flux observer developed in a reference frame linked to the stator. Errors in flux orientation and amplitude, overcurrent used, M/M$_{\rm n}$ when $K_1 = K_2 = K_3 = K_4 = 0$, and $R_{\rm r} = 2R_{\rm r}^*$*

The results shown in Figure 6.10 correspond to zero gains. Errors in the flux, as well as the increase in stator current used, are then very important when the torque is high and the speed low. The comparison between Figures 6.10 and 6.5, however, show that sensitivity to uncertainties in $R_r$ of the full order observer with zero gains is generally clearly lower than the sensitivity of the reduced order observer with zero gains.

The results shown in Figure 6.11 were obtained with gains chosen with the help of the theoretical study of sensitivity by following the method presented in section 6.4.2, by imposing constraints in the flux orientation and amplitude errors, in the increase of stator current and in the real and imaginary parts of poles. These constraints are completed by certain considerations on the simplicity of observer implementation and its sensitivity to uncertainties on stator resistance. In fact, the full order observer is sensitive to uncertainties on $R_r$ and $R_s$ even when gains are zero.

Gains obtained from these constraints are as follows:

$$K_1 = 3 \quad [6.52a]$$

$$K_2 = 0 \quad [6.52b]$$

$$\mathrm{K}_3 = -70 \quad [6.52c]$$

$$K_4 = 0 \quad [6.52d]$$

The comparison between Figures 6.11 and 6.10 shows that the gains chosen with the help of the sensitivity study greatly decrease errors in flux orientation and amplitude, as well as an increase in stator current used. The comparison between Figures 6.11 and 6.6 shows that it is possible to obtain similar results, from the point of view of parametric sensitivity, between the two types of (reduced order and full order) observer.

Figures 6.12 and 6.13 represent the real parts of the poles and relations between real and imaginary parts of these poles according to the mechanical speed and electromagnetic torque, for zero gains and gains [6.52], respectively. These figures show that gains [6.52] determined by the sensitivity study provide acceptable poles.

Figures 6.14 and 6.15 show errors of amplitude and orientation of the flux, stator current variations and the evolution of inductance $M_{sr}$ with the saturation according to the mechanical speed and electromagnetic torque, in the presence of a 20% error in rotor resistance for zero gains and [6.52] gains, respectively.

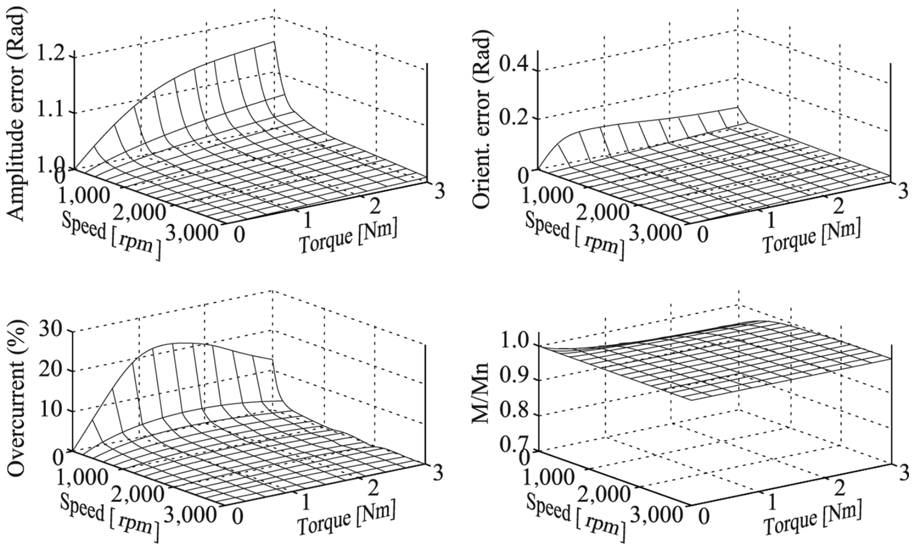



**Figure 6.11.** *Full order flux observer developed in a reference frame linked to the stator. Errors in the amplitude and orientation of the flux, used overcurrent, and $M/M_n$ obtained with the optimal observer when* $R_r = 2R_r^*$

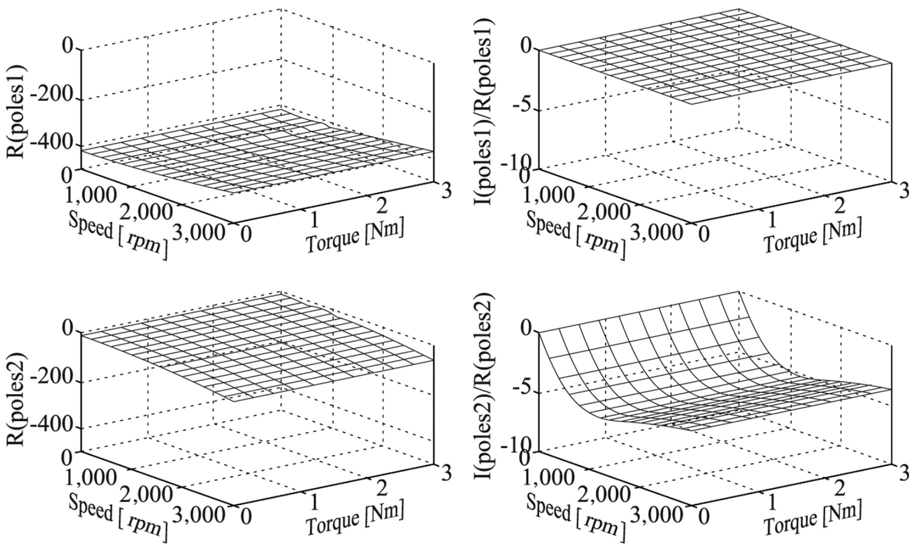



**Figure 6.12.** *Real part of poles and relations between real and imaginary parts of poles when $K_1 = K_2 = K_3 = K_4 = 0$*

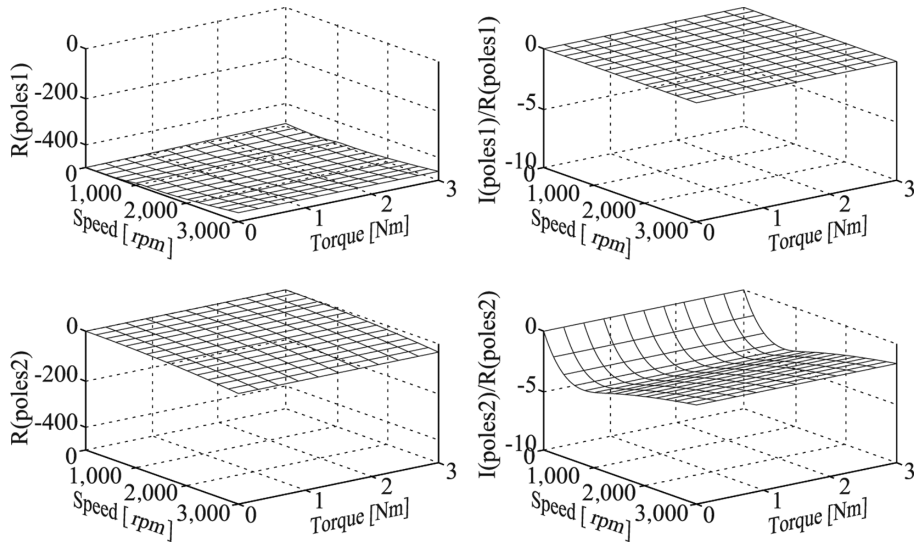



**Figure 6.13.** *Real parts of poles and relations between real and imaginary parts of poles of the optimal observer*

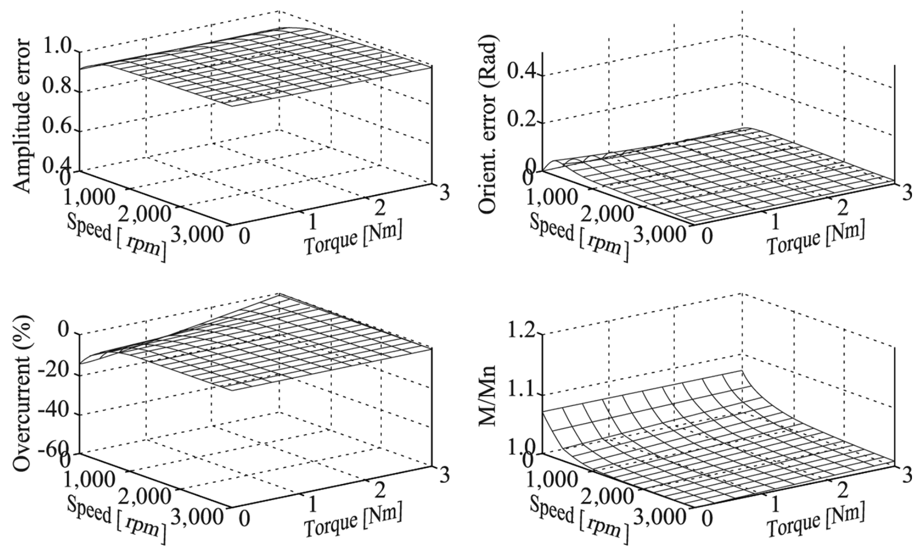



**Figure 6.14.** *Full order flux observer developed in a reference frame linked to the stator. Errors in flux orientation and amplitude, overcurrent used, $M/M_n$ when $K_1 = K_2 = K_3 = K_4 = 0$, and $R_s = 1.2R_s^*$*

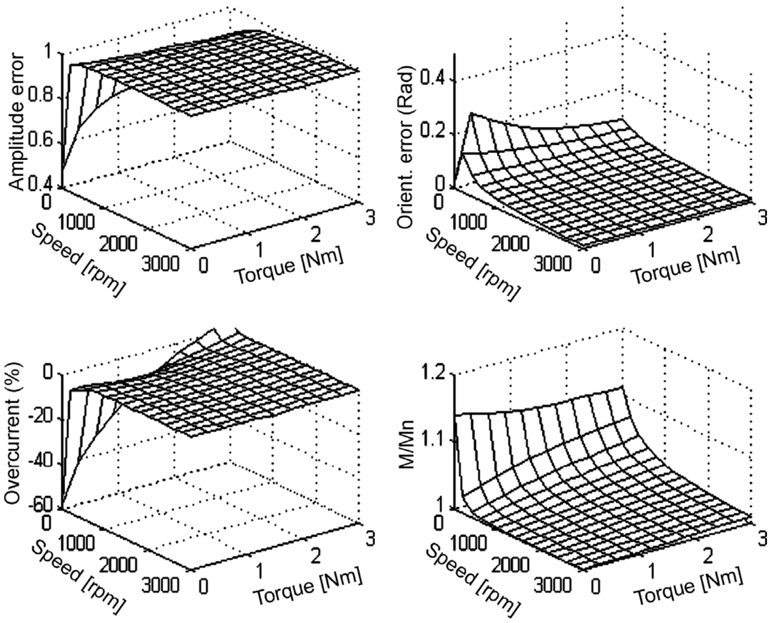



**Figure 6.15.** *Full order flux observer developed in a reference frame linked to the stator. Errors in the flux amplitude and orientation, overcurrent used, and $M/M_n$ obtained with the optimal observer when $R_s = 1.2R_s^*$*

These Figures show that gains chosen increase observer sensitivity to uncertainties with $R_S$, but this sensitivity remains low except at low speed and low torque. The choice of gains can be improved, however, by introducing a simple variation of these gains according to speed or torque, for example.

### 6.6.4. *Experimental results*

Table 6.1 gives theoretical and experimental values of the increase in stator current caused by a 100% error in rotor resistance. Four different operation points are considered in this table corresponding to two mechanical speeds (750 and 1,500 rpm) and two electromagnetic torques (1.15 and 2.3 Nm). The experimental results confirm theoretical predictions. This table also confirms that the gains determined by the study of sensitivity decrease the sensitivity of the observer and thus of the control on uncertainties in $R_r$. Figures 6.16 and 6.17 show the system response at a reference speed step of 0 to 1,500 rpm, followed by a load torque step of 0 to 2.3 Nm. For each test, Figures 6.16 and 6.17 show the reference values and mechanical speed measure, the rms value of stator current, and the estimated value of flux amplitude.

| | | Variations of the stator current [%] | | | |
|---|---|---|---|---|---|
| | | Torque = 1.15 Nm | | Torque = 2.3 Nm | |
| Speed | Gains | Values | | Values | |
| rpm | | Experimental | Theoretical | Experimental | Theoretical |
| 750 | $K_1 = K_2 = 0$ $K_3 = K_4 = 0$ | 13.5 | 11.5 | 13.5 | 12.2 |
| | $K_1 = 3\ K_2 = 0$ $K_3 = -70$ $K_4 = 0$ | 2 | 1.2 | 1.5 | 0.9 |
| 1,500 | $K_1 = K_2 = 0$ $K_3 = K_4 = 0$ | 6 | 6.4 | 7 | 6.6 |
| | $K_1 = 3\ K_2 = 0$ $K_3 = -70$ $K_4 = 0$ | 0 | 0.5 | 0 | 0.4 |

**Table 6.1.** *Increase in the stator current used caused by an incorrect control of the flux caused by a 100% error in* $R_r$

In the tests in Figure 6.16, $K_1 = K_2 = K_3 = K_4 = 0$. The test in Figure 6.16a was done with optimized parameters, whereas in the test in Figure 6.16b, a 100% error in rotor resistance was introduced in the control. The comparison in Figure 6.16b with Figure 6.8b confirms that the full order observer is less sensitive to uncertainties in $R_r$ than the reduced order observer when the gains are zero, because the loaded current used is lower in Figure 6.16b than in Figure 6.8b.

In the tests in Figure 6.17, $K_1$, $K_2$, $K_3$, and $K_4$ are linear functions of the mechanical speed, evolved from 0 at zero speed to values [6.52] at 1,500 rpm. The test in Figure 6.17a was done with optimized parameters, whereas in the test in Figure 6.17b, a 100% error in rotor resistance was introduced in the control.

The comparison between the tests presented in Figures 6.16b and 6.17b shows that the consumption of stator current is slightly lower in the Figure 6.17b test, confirming theoretical predictions.

## 6.7. Conclusion

In this chapter, a method enabling the theoretical study of the parametric sensitivity of flux observers was developed. This method makes it possible to determine gains for these observers, constituting a good compromise between the dynamics, parametric sensitivity, and observer simplicity.

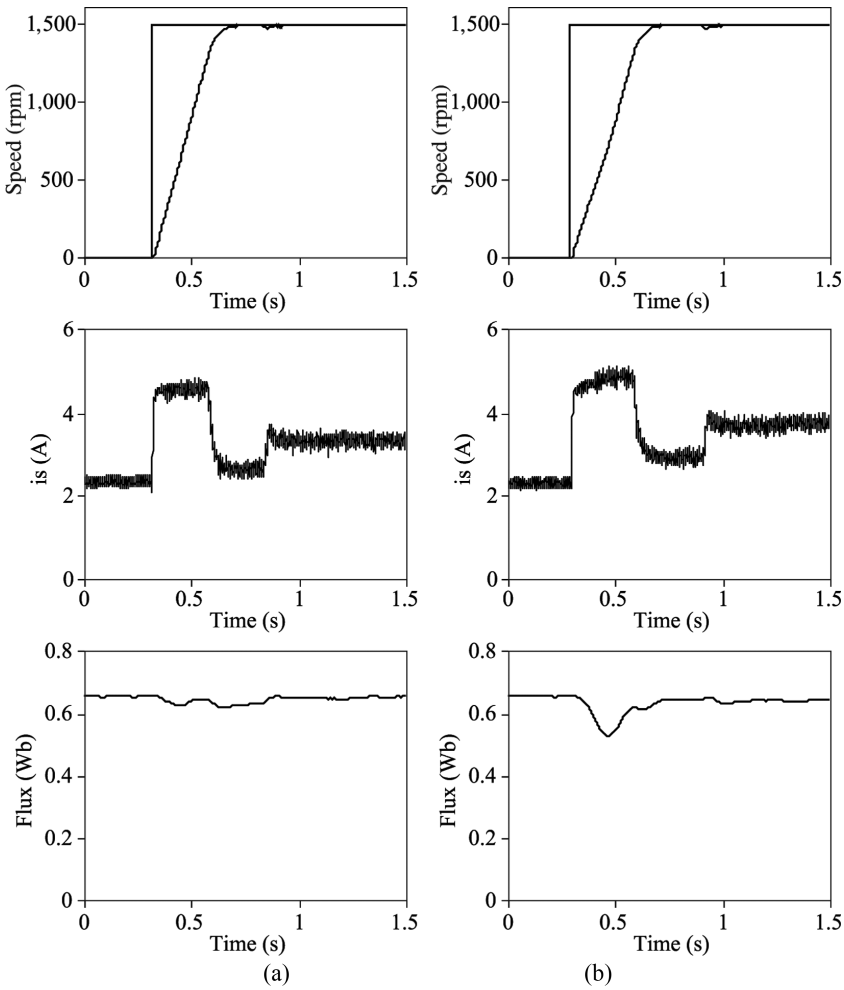
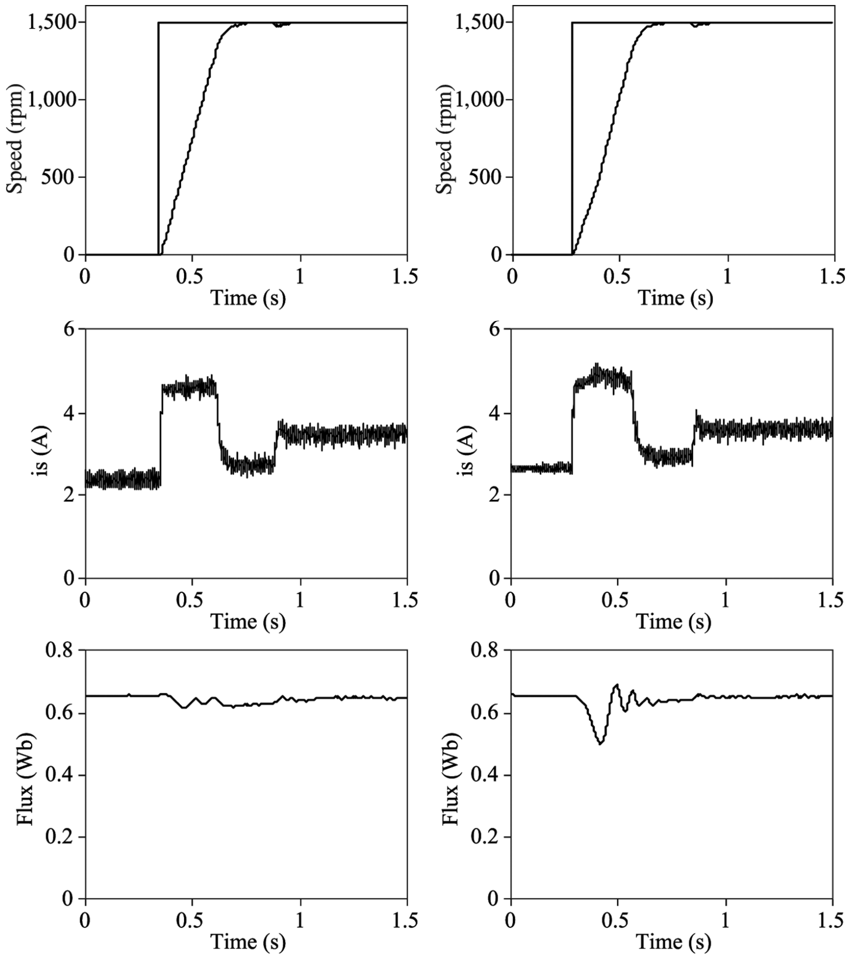



**Figure 6.16.** *System response at a reference speed step of 0 to 1,500 rpm followed by a load torque step. Full order flux observer with $K_1 = K_2 = K_3 = K_4 = 0$. Test (a) with optimized parameters; (b) with $R_r = 2R_r^*$*

The method was applied to a reduced order observer and a full order observer. The gains proposed for these observers improve their performances in terms of parametric sensitivity while ensuring an acceptable dynamics. These gains are characterized by their simplicity because they are constant or evolve in a linear way. Observer performance could still be improved if, in order to determine gains, we associate fuzzy logic to the study of sensitivity [ROB 02]. In fact, fuzzy logic will make it possible to obtain gains that can evolve following non-linear relations by compiling deduced information, not only theoretical study, as well as simulations and experimental tests.




(a) (b)

**Figure 6.17.** *System response at a reference speed step of 0 to 1,500 rpm followed by a load torque step. Optimal full order flux observer. Test (a): with optimized parameters, test (b): with* $R_r = 2R_r^*$

An extension of the theoretical sensitivity study on the consideration of the influence of the observer discretization in the estimation of the flux is presented in [DEL 01].

## 6.8. Appendix: parameters of the squirrel-cage induction machine

Rated output = 750 W
Rated speed = 2,900 rpm
Inertia = $7 \times 10^{-3}$ kg m$^2$
$R_s = 3\ \Omega$
$R_r = 1.78\ \Omega$
$M_{sr} = 0.1537$ H
$L_s = 0.16$ H
$L_r = 0.16$ H
$\beta = 0.78$
s = 8.8

## 6.9. Bibliography

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