Chapter 2

Modeling and Parameter Determination of the Saturated Synchronous Machine

The goal of this chapter is to introduce the rational treatment of saturation in synchronous machines. We will limit this study to the case of machines with sinusoidal coupling, the only case where the study of saturation has reached a certain level of maturity. We will not go into all the practical details of the tests involved; we instead refer the interested reader to standards such as [IEC 85] and [IEE 95]. This chapter is divided into three sections. In the first section, we describe the basic notions in a way that does not presume linearity. The second section is dedicated to conventional tests. The models used are mainly linear, but this part ends with the observation that the linear theory is insufficient and with the introduction of the Potier model for smooth pole machines. The third and last section of this chapter is dedicated to the treatment of non-linearities, as well as to the question of equation linearization.

2.1. Modeling of the synchronous machine: general theory

2.1.1. *Description of the machine studied and general modeling hypotheses*

The machine studied is a conventional machine. It contains two solid parts, stator and rotor, with the latter executing a rotation movement relative to the former.

Chapter written by Ernest MATAGNE and Emmanuel DE JAEGER.

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One of the parts that we can presume as being the stator without losing generality contains a three-phase winding, that is, a symmetrical system with three windings called “phases”. This system is called “armature winding”. The quantities relative to the three phases are identified by indices a, b, and c. We presume that the stator has smooth poles, that is, no effect from stator magnetic saliences occurs. This hypothesis can be formalized by saying that, in the absence of stator currents, the rotor's electric behavior does not depend on its position θ. It could, however, depend on the velocity $\mathrm{d}\theta/\mathrm{d}t$.

The, smooth or salient pole, rotor has only one winding called “excitation winding” or “field winding”. The quantities relative to this winding are indicated by index f (feld or field).

The mechanical connection between the rotor and the machine's exterior is characterized by angular position θ and applied torque C. Similarly, the electric connection between each winding and external electric circuit is characterized by voltage u and current i. For these quantities, we use reference directions such as product $C\,\mathrm{d}\theta/\mathrm{d}t$ and product $u\,i$ are input powers.

The connection between the machine and its environment is described by 10 variables, functions of time. The machine's behavior, from the outside, is described by the relations that exist between these 10 variables.

The movement follows a mechanical evolution equation, which is written with the reference directions indicated above:

$$\frac{\mathrm{d}\Im}{\mathrm{d}t} - C_{\mathrm{em}} + C_{\mathrm{p}} = C \qquad [2.1]$$

where the first term is the derivative of the kinetic moment, C_{em} being the electromagnetic torque and C_{p} the friction torque. The kinetic moment is in electrotechnic linked to the rotation speed by the constitutive relation:

$$\Im = J\frac{\mathrm{d}\theta}{\mathrm{d}t} \qquad [2.2a]$$

where J is mechanical inertia, so that the mechanical equation is often directly written in the form:

$$J\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} - C_{\mathrm{em}} + C_{\mathrm{p}} = C \qquad [2.2b]$$

To model the machine's electric behavior, we normally use "equivalent circuit" type models. Beside electrical ports, these models contain additional branches to account for the different physical phenomena.

The presence of grids or damping cages in the rotor or the possible damping effect of the steel in particular will be modeled by two equivalent circuits in short circuit. The quantities relative to these damping circuits are identified by the D and Q indices, resulting in three rotor circuits. We presume that the magnetic axis of winding D coincides with that of field winding f, and that winding Q is in magnetic quadrature with f and D (the notion of magnetic quadrature will be discussed in more detail later).

We again increase the number of branches of the model in order to simplify the equations of these branches. Whenever possible, the branches will be characterized by a relation that only involves quantities inherent in the branch involved, and connections between the different phenomena represented by branches are taken into consideration in the form of electric connections between the branches, which form a Kirchhoff network, that is, we consider that the network formed by the interconnection of the different branches, that belong to the machine studied or its environment, is done via the Kirchhoff laws (law of nodes between currents and mesh law between voltages).

The advantage with modeling with such electric circuits (also called Kirchhoff networks) mainly comes from a consequence of the Tellegen theorem, indicating that electric connections conserve power. This theorem can be written simply if we adopt the voltage and current reference direction as the "receiver" convention, that is, the product $u\ i$ represents an incoming electric power. By identifying the different branches of a circuit by index k, the desired property is written as:

$$\sum_{\mathrm{k}} u_{\mathrm{k}} i^{\mathrm{k}} = 0 \qquad [2.3]$$

By writing expression [2.3], we place the voltage index in lower position and the current index in upper position. A consequence of this choice is that, in the expressions with sums, we will systematically find in each term the summation index once in lower position and once in upper position. It is an interesting mnemonic method, but also with some physical significance.

2.1.2. *Fundamental circuit laws for the study of electrical machines*

Inductive circuits can be studied by using two internal variables, flux ψ and total electromotive force e, linked to voltage by the Faraday law:

$$u = \frac{\mathrm{d}\psi}{\mathrm{d}t} + e \qquad [2.4]$$

Physically, ψ is linked to magnetic induction field **B** whereas e is linked to electric field **E**.

Equations [2.1] and [2.4] are not enough to determine the machine's behavior. We must add relations, such as [2.2a], called constitutive relations. We will distinguish between electrical constitutive relations, connecting electromotive force e and current i, and magnetic constitutive relations, connecting flux ψ to current i. Constitutive relations take the different symmetries discussed in the previous section into consideration. These symmetries can then be formalized considering the form of these relations.

In order to simplify the study of elements, we distinguish:

1. purely resistive elements for which we have $\psi = 0$ and where e is a function of i only or $e = R\,i$ in the linear case;

2. and purely inductive elements, where e does not depend on i (it can be zero or depend on ψ, as we will see later).

Inductive elements are mainly characterized by a relation between fluxes and currents. A generally accepted hypothesis is supposing that this relation is single-valued, that is, the hysteresis and magnetic losses are either taken into consideration by resistive elements of the model or are ignored. We can then express the magnetic constitutive relation of a device with n circuit branches in the form of n functions with type:

$$\psi_{\mathrm{k}} = \psi_{\mathrm{k}}\,(i^1, i^2 \ldots\ldots i^{n-1}, i^n, \theta) \qquad [2.5]$$

In this case, we can define the incremental, mutual, and self-inductances by:

$$L_{\mathrm{jk}} = \frac{\partial \psi_{\mathrm{j}}}{\partial i^{\mathrm{k}}} \qquad [2.6]$$

In the case of non-linear devices, we can note that these inductances are, as fluxes, a function of the set of currents. In order for us to define the magnetic energy, and consequently use energy conservation, the following must occur:

$$L_{\mathrm{jk}} = L_{\mathrm{kj}} \qquad [2.7]$$

We always want to make sure that the choices made about the form of constitutive relations are rigorously compatible with the principle of energy conservation.

A consequence is that the expression of the electromagnetic torque C_{em} is tightly linked to that of constitutive relations.

Another consequence is that the models not respecting [2.7], even if they are popular, will not be outlined in this study.

Hypothesis [2.5] makes it possible to formalize the notion of magnetic quadrature. We say that a circuit is in magnetic quadrature relative to another if, all other currents being zero, the relation between the flux and current from the first circuit is an odd function, regardless of the current's value in the second. When the first current is zero, the corresponding flux must then be zero regardless of the current in the second circuit.

In the case of linear circuits, the quadrature condition is equivalent to the reduction of the mutual inductance to zero. However, in the case of non-linear circuits, it is no longer the case. This phenomenon, known as cross-saturation, is described in the website associated with [MAT 04]. We will study it in the third section of this chapter (section 2.3.2.2).

In the non-linear case, it is difficult to express the conditions on the fluxes [2.5] ensuring that condition [2.7] will be respected. A way to achieve it is to replace n expressions [2.5] by a given function called "coenergy":

$$w_{\mathrm{cm}} = w_{\mathrm{cm}}\left(i^1, i^2 \ldots i^{n-1}, i^{\mathrm{n}}, \theta\right) \qquad [2.8]$$

The expression of the different fluxes is then set by:

$$\psi_{\mathrm{k}} = \frac{\partial w_{\mathrm{cm}}}{\partial i^{\mathrm{k}}} \qquad [2.9]$$

and condition [2.7] is automatically satisfied. The use of coenergy also makes it possible to simplify the notion of quadrature: a circuit is in magnetic quadrature in relation to a second if the coenergy is an even function of the first current, regardless of the second.

2.1.3. *Equations of the machine in abc variables*

Circuits f, a, b, and c are made up of a winding of electric wire. The path of the current in these circuits is therefore set according to the matter (the electric wire) constituting them: they are said to be filiform circuits. We presume that it is the same for circuits D and Q, even though the hypothesis is less rigorous when these circuits relate to large-scale eddy currents. In the case of filiform circuits, total electromotive force e, introduced at formula [2.4], is often reduced to an ohmic term (in electrical engineering, we usually do not take into consideration the thermoelectric and electrochemical effects among others), assuming that this term is a linear function of currents:

$$e_{\mathrm{j}} = \sum_{\mathrm{k}} R_{\mathrm{jk}} i^{\mathrm{k}} \quad [2.10a]$$

The Faraday equation [2.4] is then often written directly combined with [2.10a]:

$$u_{\mathrm{j}} = \frac{\mathrm{d}\psi_{\mathrm{j}}}{\mathrm{d}t} + \sum_{\mathrm{k}} R_{\mathrm{jk}} i^{\mathrm{k}} \quad [2.10b]$$

where $\mathrm{j}, \mathrm{k} \in \{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$.

In the case of the circuits examined here, there is no mutual resistance because the currents use separate paths. Equation [2.10b] is then reduced to:

$$u_{\mathrm{k}} = \frac{\mathrm{d}\psi_{\mathrm{k}}}{\mathrm{d}t} + R_{\mathrm{kk}} i^{\mathrm{k}} \quad [2.11]$$

We suppose that the three stator phases form a three-phase system in which the machine is “well built” and “faultless”. This hypothesis can be formalized by a law of symmetry; there is an integer p, called “number of pole pairs”, such that a cyclic permutation of the three phases is equivalent to an angular shift of the rotor by an angle $2\pi/3p$.

A first consequence of this symmetry is that all three R_{aa}, R_{bb} and R_{cc} resistances are equal.

We now examine the constitutive relation linking fluxes ψ_k to currents. To simplify writings and graphical representations, we add the variable:

$$\theta_e = p\ \theta \qquad [2.12]$$

called "electric angle."

We can then formalize this hypothesis by noting the fact that permuting the phases amounts to a change of variable:

$$\psi_a = \psi_{c'} \text{ and } i^a = i^{c'} \qquad [2.13a]$$

$$\psi_b = \psi_{a'} \text{ and } i^b = i^{a'} \qquad [2.13b]$$

$$\psi_c = \psi_{b'} \text{ and } i^c = i^{b'} \qquad [2.13c]$$

$$\theta_e = \theta'_e + 2\pi/3 \qquad [2.13d]$$

where transformed variables were distinguished from original variables by an apostrophe assigned, due to a usual misuse of words, not to the variable but to its index.

We can admit that voltages are transformed like fluxes. Because of [2.4], electromotive forces of circuits a, b and c are also transformed like fluxes, whereas the variables relative to rotor circuits remain unchanged. The hypothesis made can be formalized by saying that, for any physically achievable evolution (i.e., one that verifies all equations of the model) of old variables, the same evolution applied to new variables defined by [2.13] is also a physically achievable evolution.

We can transform [2.13a], [2.13b] and [2.13c] in the following form:

$$i^{k'} = \sum_k \Pi(120^o)^{k'}{}_k\, i^k \text{ and } \psi_k = \sum_k \psi_{k'} \Pi(120^o)^{k'}{}_k \qquad [2.14]$$

where $k \in \{a, b, c\}$ and $k' \in \{a', b', c'\}$, and where factors $\Pi(120°)^{k'}{}_k$ are simply the components of the permutation matrix:

$$\mathbf{\Pi}(120^o) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad [2.15a]$$

By multiplying with itself, matrix [2.15a] generates two new matrices:

$$\mathbf{\Pi}(120^{o})\mathbf{\Pi}(120^{o}) = \mathbf{\Pi}(240^{o}) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \mathbf{\Pi}^{T}(120^{o}) \qquad [2.15b]$$

and:

$$\mathbf{\Pi}(120^{o})\mathbf{\Pi}(240^{o}) = \mathbf{\Pi}(360^{o}) = \mathbf{\Pi}(0^{o}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad [2.15c]$$

This last matrix is the identity matrix. [2.15c] also shows that matrices [2.15a] and [2.15b] are inverse of each other. The three matrices [2.15] form a group for multiplication. The same applies to corresponding transformations. The hypothesis above comes down to saying that this group is a symmetry group of the model.

Matrix $\mathbf{\Pi}(120°)$ can be diagonalized. From a method described in [LES 81], we obtain:

$$\mathbf{\Pi}(120^{o}) = \mathbf{F}\begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-j2\pi/3} & 0 \\ 0 & 0 & e^{j2\pi/3} \end{bmatrix}\mathbf{F}^{-1} \qquad [2.16]$$

where $\mathbf{F}$ is the normalized Fortescue matrix, that is, the Lyon matrix:

$$\mathbf{F} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \qquad [2.17]$$

and where a is the cube root of unit:

$$a = e^{j\frac{2\pi}{3}} \qquad [2.18]$$

The custom is to impose a stronger symmetry, known as “sinusoidal coupling hypothesis”. The corresponding symmetry group is made up of transformations that we will note by analogy with [2.14] and [2.13d]:

$$i^{k'} = \sum_{k} \Pi\left(\Delta\theta_e\right)^{k'}{}_{k}\, i^{k} \text{ and } \psi_k = \sum_{k} \psi_{k'} \Pi\left(\Delta\theta_e\right)^{k'}{}_{k} \qquad [2.19a]$$

$$\theta_e = \theta'_e + \Delta\theta_e \qquad [2.19b]$$

where matrices $\mathbf{\Pi}(\Delta\theta_e)$ form a group, parameterized by cyclic variable $\Delta\theta_e$, including group [2.15].

To find the expression of these matrices, we just need to replace equation [2.16] by:

$$\mathbf{\Pi}(\Delta\theta_e) = \mathbf{F}\begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-j\Delta\theta_e} & 0 \\ 0 & 0 & e^{j\Delta\theta_e} \end{bmatrix}\mathbf{F}^{-1} \qquad [2.20]$$

These matrices obviously form a group, because we immediately see that:

$$\mathbf{\Pi}(x)\,\mathbf{\Pi}(y) = \mathbf{\Pi}\,(x + y) \qquad [2.21]$$

By executing the member at the right of [2.20], we see that they are actually real matrices that can be written as:

$$\mathbf{\Pi}(\Delta\theta_e) = \begin{bmatrix} \frac{1}{3}+\frac{2}{3}\cos\left(\Delta\theta_e\right) & \frac{1}{3}+\frac{2}{3}\cos\left(\Delta\theta_e+\frac{2\pi}{3}\right) & \frac{1}{3}+\frac{2}{3}\cos\left(\Delta\theta_e-\frac{2\pi}{3}\right) \\ \frac{1}{3}+\frac{2}{3}\cos\left(\Delta\theta_e-\frac{2\pi}{3}\right) & \frac{1}{3}+\frac{2}{3}\cos\left(\Delta\theta_e\right) & \frac{1}{3}+\frac{2}{3}\cos\left(\Delta\theta_e+\frac{2\pi}{3}\right) \\ \frac{1}{3}+\frac{2}{3}\cos\left(\Delta\theta_e+\frac{2\pi}{3}\right) & \frac{1}{3}+\frac{2}{3}\cos\left(\Delta\theta_e-\frac{2\pi}{3}\right) & \frac{1}{3}+\frac{2}{3}\cos\left(\Delta\theta_e\right) \end{bmatrix} \qquad [2.22]$$

Among the interesting properties of these matrices, we can note the fact that their transpose is equal to their inverse, in other words, they are orthonormal:

$$\mathbf{\Pi}^T(x) = \mathbf{\Pi}^{-1}(x) = \mathbf{\Pi}\,(-x) \qquad [2.23]$$

In variables a, b, c, even with the simplifying [2.5], [2.6] and [2.7] hypothesis, finding the form that the constitutive relations must have for the imposed symmetries to the model to be satisfied remains generally difficult. Because of this, the problem is only handled in the linear case in the literature with the use of a very simplified local (field type) model (approximation said to be "of the first harmonic"). However, since the goal of this chapter is to study saturation in synchronous machines, we prefer to develop adequate mathematical tools before addressing the question of the form of constitutive equations.

For now, we will settle for noting that when the coenergy function exists, the desired condition can be written as:

$$w_{\mathrm{cm}}\,(i^{\mathrm{a}},i^{\mathrm{b}},i^{\mathrm{c}},\,\theta) = w_{\mathrm{cm}}\,(i^{\mathrm{a}'},i^{\mathrm{b}'},i^{\mathrm{c}'},\,\theta') \qquad [2.24]$$

Even with coenergy, the notion of magnetic quadrature can only be formalized if the field system is smooth or is in particular locations relative to the phases of the stator. In practice, we choose, as origin $\theta = 0$, a position that will make the field system axis coincide with the magnetic axis of phase a, or, which amounts to the same thing, in magnetic quadrature with the circuit formed by connecting in series phases b and c.

In the case of a model constituted, as is the case in this section, of filiform circuits, the electromagnetic torque can also be expressed through the coenergy function.

We obtain expression:

$$C_{\mathrm{em}} = \frac{\partial\, w_{\mathrm{cm}}\left(i^{\mathrm{a}}, i^{\mathrm{b}}, i^{\mathrm{c}}, \theta\right)}{\partial \theta} \qquad [2.25]$$

The interested reader will find the demonstration of this well-known equation in the literature, for example, in [LOU 04], section 3.2.4.2.

2.1.4. *Concordia transformation: equations of the machine in $0\alpha\beta$ variables*

The three-phase symmetry [2.15] or [2.22] is not the only one found in machines. A simpler structure is with two-phase machines. These machines are identical to the ones described in section 2.1.1, except for the fact that they only have two stator windings. Designating these two phases by letters α and β, we define this symmetry by saying that a permutation of these phases, α and β, joined to the change of the reference direction of one of these phases, is equivalent to a rotation of 90°.

We can formalize this hypothesis by noting the fact that permuting the phases comes down to changing of variable:

$$\psi_\alpha = -\psi_{\beta'} \text{ and } i^\alpha = -i^{\beta'} \qquad [2.26b]$$

$$\psi_\beta = \psi_{\alpha'} \text{ and } i^\beta = i^{\alpha'} \qquad [2.26c]$$

$$\theta_e = \theta'_e + \pi/2 \qquad [2.26d]$$

In order to make it more general, we introduce a third stator circuit with index "o". The quantities of this circuit are not affected by transformation [2.26], or:

$$\psi_o = \psi_{o'} \text{ and } i^o = i^{o'} \qquad [2.26a]$$

Voltage and magnetomotive forces of circuits o, α, and β transform as flux, whereas the variables relative to rotor circuits remain unchanged. The hypothesis made can be formalized by saying that for any physically achievable evolution (i.e., that verifies all equations of the model) of old variables, the same evolution applied to new variables defined by [2.26] is also a physically achievable evolution.

We can put transformation [2.26a], [2.26b] and [2.26c] in the following form:

$$i^{k'} = \sum_k \Theta(90°)^{k'}{}_k \; i^k$$

and
$$\psi_k = \sum_k \psi_{k'} \; \Theta(90°)^{k'}{}_k \qquad [2.27]$$

where $k \in \{o, \alpha, \beta\}$ and $k' \in \{o', \alpha', \beta'\}$, and where factors $\Theta\,(90°)^{k'}{}_k$ are only the components of the matrix:

$$\mathbf{\Theta}(90°) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \qquad [2.28a]$$

By multiplying itself, matrix [2.28a] generates three new matrices:

$$\mathbf{\Theta}\,(90°)\,\mathbf{\Theta}\,(90°) = \mathbf{\Theta}\,(180°) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad [2.28b]$$

$$\boldsymbol{\Theta}\,(180°)\,\boldsymbol{\Theta}\,(90°) = \boldsymbol{\Theta}\,(270°) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \qquad [2.28c]$$

$$\boldsymbol{\Theta}\,(270°)\,\boldsymbol{\Theta}\,(90°) = \boldsymbol{\Theta}\,(360°) = \boldsymbol{\Theta}\,(0°) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad [2.28d]$$

This last matrix is the identity matrix. [2.28d] also shows that matrices [2.28a] and [2.28c] are inverse of each other. In addition, matrix [2.28b] is its own inverse. The four matrices [2.28] form a multiplying group. The same applies to corresponding transformations. The hypothesis above comes down to saying that this group is a symmetry group of the model. Matrix $\boldsymbol{\Theta}$ (90°) can be diagonalized. We get:

$$\boldsymbol{\Theta}(90°) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \mathrm{j}\frac{\sqrt{2}}{2} \\ 0 & -\mathrm{j}\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-\mathrm{j}\pi/2} & 0 \\ 0 & 0 & e^{\mathrm{j}\pi/2} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \mathrm{j}\frac{\sqrt{2}}{2} \\ 0 & -\mathrm{j}\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \qquad [2.29]$$

We can again impose a "sinusoidal coupling hypothesis". The corresponding group of symmetry is made up of transformations defined by:

$$i^{\mathrm{k'}} = \sum_{\mathrm{k}} \Theta\left(\Delta\theta_{\mathrm{e}}\right)^{\mathrm{k'}}{}_{\mathrm{k}}\, i^{\mathrm{k}} \quad \text{and} \quad \psi_{\mathrm{k}} = \sum_{\mathrm{k}} \psi_{\mathrm{k'}}\, \Theta\left(\Delta\theta_{\mathrm{e}}\right)^{\mathrm{k'}}{}_{\mathrm{k}} \qquad [2.30a]$$

$$\theta_{\mathrm{e}} = \theta'_{\mathrm{e}} + \Delta\theta_{\mathrm{e}} \qquad [2.30b]$$

where matrices $\Theta\,(\Delta\theta_e)$ form a group including group [2.28].

To find the expression of these matrices, we simply need to modify equation [2.29]. We get:

$$\boldsymbol{\Theta}(\Delta\theta_{\mathrm{e}}) = \begin{bmatrix} 1- & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \mathrm{j}\frac{\sqrt{2}}{2} \\ 0 & -\mathrm{j}\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 1- & 0 & 0 \\ 0 & \mathrm{e}^{-\mathrm{j}\Delta\theta_{\mathrm{e}}} & 0 \\ 0 & 0 & \mathrm{e}^{\mathrm{j}\Delta\theta_{\mathrm{e}}} \end{bmatrix} \begin{bmatrix} 1- & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \mathrm{j}\frac{\sqrt{2}}{2} \\ 0 & -\mathrm{j}\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \qquad [2.31]$$

The first and last matrices of the right member are identical, and this matrix is its own inverse. These matrices [2.31] obviously form a group, because we immediately see that:

$$\Theta(x)\,\Theta(y) = \Theta(x + y) \tag{2.32}$$

By computing the member at the right of [2.31], we see that they are actually real matrices that can be written as:

$$\mathbf{\Theta}\,(\Delta\theta_e) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Delta\theta_e & \sin\Delta\theta_e \\ 0 & -\sin\Delta\theta_e & \cos\Delta\theta_e \end{bmatrix} \tag{2.33}$$

Matrix [2.33] is simply a rotation matrix in the Euclidian plan (α, β).

We then expect that the symmetry based on [2.30] and [2.33] will be easier to use than the symmetry based on [2.19] and [2.22]. One way to simplify the equations of a three-phase machine will consist of replacing the three phases a, b, and c by an equivalent two-phase system o, α, and β in the model of this machine.

By designating **T** as the desired transformation matrix, we know that we must have:

$$\begin{bmatrix} i^a \\ i^b \\ i^c \end{bmatrix} = \mathbf{T} \begin{bmatrix} i^0 \\ i^\alpha \\ i^\beta \end{bmatrix} \tag{2.34a}$$

If we want to be able to interpret currents i^o, i^α, i^β as the currents circulating in circuits o, α, and β, implying that the expression of power in these circuits must keep the usual form $u\,i$, voltages, and thus fluxes, must transform according to law:

$$[\psi_0, \psi_\alpha, \psi_\beta] = [\psi_a, \psi_b, \psi_c]\,\mathbf{T} \tag{2.34b}$$

This transformation must make the three-phase transformation [2.19] and [2.22] and the two-phase transformation [2.30] and [2.33] correspond. Between transformed variables, we will then have the same connection [2.34a] than between original variables, or:

$$\begin{bmatrix} i^{a'} \\ i^{b'} \\ i^{c'} \end{bmatrix} = \mathbf{T} \begin{bmatrix} i^{0'} \\ i^{\alpha'} \\ i^{\beta'} \end{bmatrix} \tag{2.35}$$

By comparing [2.34a] and [2.35], and by considering transformation laws [2.19] and [2.30], the condition to satisfy matrix **T** is:

$$\mathbf{T} = \Pi(-\Delta\theta_e)\, \mathbf{T}\, \Pi\,(\Delta\theta_e) \qquad [2.36]$$

We arrive at the same conclusion on the basis of flux transformation [2.34b].

By using decompositions [2.20] and [2.31], this condition can be written as:

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{j\Delta\theta_e} & 0 \\ 0 & 0 & e^{-j\Delta\theta_e} \end{bmatrix} \mathbf{D} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-j\Delta\theta_e} & 0 \\ 0 & 0 & e^{j\Delta\theta_e} \end{bmatrix} \qquad [2.37]$$

where:

$$\mathbf{D} = \mathbf{F}^{-1}\, \mathbf{T} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & j\frac{\sqrt{2}}{2} \\ 0 & -j\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \qquad [2.38]$$

The condition for [2.37] to be verified regardless of the value of $\Delta\theta_e$ is that **D** be diagonal. In this case, [2.38] shows that we can write **T** in the form:

$$\mathbf{T} = \mathbf{F} \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & j\frac{\sqrt{2}}{2} \\ 0 & -j\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$$

$$= \frac{\sqrt{2}}{2\sqrt{3}} \begin{bmatrix} d_{11} & d_{22} - j\,d_{33} & j\,d_{22} - d_{33} \\ d_{11} & a^2\,d_{22} - j\,a\,d_{33} & j\,a^2\,d_{22} - a\,d_{33} \\ d_{11} & a\,d_{22} - j\,a^2\,d_{33} & j\,a\,d_{22} - a^2\,d_{33} \end{bmatrix} \qquad [2.39]$$

We can observe that the condition for $\mathbf{T}$ to be a real matrix is:

$$\mathrm{Im}(d_{11}) = 0 \qquad [2.40a]$$

$$d_{33} = j\, d_{22}^{*} \qquad [2.40b]$$

Change parameters by saying:

$$d_{11} = \sqrt{2}\, d_0 \qquad [2.41a]$$

$$d_{22} = d_\alpha\, e^{j\rho_e} \qquad [2.41c]$$

where parameters d_0, d_α and ρ_e are real.

We can then write $\mathbf{F}$ in the obviously real form:

$$\mathbf{T} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{\sqrt{2}}{2} & \cos \rho_e & -\sin \rho_e \\ \frac{\sqrt{2}}{2} & \cos\left(\rho_e - \frac{2\pi}{3}\right) & -\sin\left(\rho_e - \frac{2\pi}{3}\right) \\ \frac{\sqrt{2}}{2} & \cos\left(\rho_e + \frac{2\pi}{3}\right) & \sin\left(\rho_e + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} d_0 & 0 & 0 \\ 0 & d_\alpha & 0 \\ 0 & 0 & d_\alpha \end{bmatrix} \qquad [2.42]$$

In electrical engineering, we often mandate that variable transformation matrices be orthonormal. This condition actually makes it possible to change the current and voltage by using the same matrix formula. It comes down to impose in [2.42]:

$$d_0 = 1 \qquad [2.43a]$$

$$d_\alpha = 1 \qquad [2.43b]$$

Transformation $\mathbf{T}$ is then written:

$$\mathbf{T} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{\sqrt{2}}{2} & \cos \rho_e & -\sin \rho_e \\ \frac{\sqrt{2}}{2} & \cos\left(\rho_e - \frac{2\pi}{3}\right) & -\sin\left(\rho_e - \frac{2\pi}{3}\right) \\ \frac{\sqrt{2}}{2} & \cos\left(\rho_e + \frac{2\pi}{3}\right) & \sin\left(\rho_e + \frac{2\pi}{3}\right) \end{bmatrix} \qquad [2.44]$$

If in addition, we choose $\rho_e = 0$, the transformation matrix becomes:

$$\mathbf{T} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{\sqrt{2}}{2} & 1 & 0 \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \quad [2.45]$$

This matrix is well known in electrical engineering: it is the Concordia matrix (normalized Clarke).

This matrix is usually introduced as a way to simplify the expression of inductance matrices. The way in which we introduced this matrix is more general because it does not use the hypothesis of linearity.

The transformation of circuits abc in oαβ is often presented as a simple change in variables. However, the terminology used in practice talks of circuits o, α, and β, showing that the intuition of electrical engineers tends to be in changing circuits and not in changing variables. By adopting the point of view of changing circuits, we have the advantage of directly using the symmetries in the o, α, β group to write the form of the equations. We can then formalize the notion of changing circuits.

If we represent the original circuits by their conductor densities $\mathbf{N}_k$, the new circuits will be defined by densities:

$$\mathbf{N}_{k'} = \sum_k \mathrm{N}_k \mathrm{T}^k{}_{k'} \quad [2.46]$$

where factors $\mathrm{T}^k{}_{k'}$ are the components of the transformation matrix.

Circuit change must be accompanied by a change of current and flux variables such that local quantities **J** and **A**, that is, current density associated with a charge movement and vector potential, remain unchanged. By using the relations between local quantities and circuit type variables [MAT 06], that is:

$$\mathbf{J} = \sum_k \mathbf{N}_k \, i^k \quad [2.47a]$$

and:

$$\psi_k = \iiint \mathbf{A} \cdot \mathbf{N}_k \, \mathrm{d}V \quad [2.47b]$$

we then show that new circuit currents and fluxes are connected to the old ones through laws:

$$i^{k'} = \sum_k T^{k'}{}_k \, i^k \quad \text{and} \quad \psi_k = \sum_k \psi_{k'} \, T^{k'}{}_k \qquad [2.48]$$

Since transformation matrix **T** is constant, formula [2.4] shows that electromotive forces must transform (see [2.34b]) the same way as fluxes and voltage. The constitutive electric relation [2.10a] then becomes:

$$e_{j'} = \sum_k R_{j'k'} \, i^{k'} \qquad [2.49]$$

with:

$$R_{j',\,k'} = \sum_{j,\,k} R_{jk} \; T^{j}{}_{j'} \; T^{k}{}_{k'} \qquad [2.50a]$$

or, in matrix form:

$$\mathbf{R}_{o\alpha\beta} = \mathbf{T}^T \, \mathbf{R}_{abc} \, \mathbf{T} \qquad [2.50b]$$

The electric equations [2.49] then retain the same form [2.10a]: circuits o, α, β, as with original circuits, are filiform circuits. If **T** is orthonormal, we immediately obtain:

$$R_{oo} = R_{aa} \qquad [2.51a]$$

$$R_{\alpha\alpha} = R_{\beta\beta} = R_{aa} \qquad [2.51b]$$

mutual resistances being zero once again.

In terms of magnetic constitutive relations, modeling in the form of circuits o, α and β is still too complicated for us to be able to find the form of these relations by applying two-phase symmetry. We can note in particular that circuits α and β are not in magnetic quadrature (except if the rotor has smooth poles) for all values of angle θ.

In the linear case, on the other hand, this operation is possible. We can obtain the form of constitutive relations either directly, by applying two-phase symmetry to the general form written in variables o, α and β, or indirectly by applying the Concordia transformation to expressions already established in variables a, b, and c. The second method can be found in reference [SAR 04], section 2.3.

In this chapter, dedicated to non-linear models, we will develop other tools before looking for the form of constitutive relations. For now, we will settle for noting that, when the coenergy function exists, the desired condition can be written in a slightly simpler form than [2.24]:

$$w_{\mathrm{cm}}\left(i^{\alpha}, i^{\beta}, \theta\right) = w_{\mathrm{cm}}\left(i^{\alpha'}, i^{\beta'}, \theta'\right) \qquad [2.52]$$

Finally, since the circuits in this model are filiform, the electromagnetic torque can also be expressed with the help of the coenergy function. We have a formula similar to [2.25]:

$$C_{\mathrm{em}} = \frac{\partial\, w_{\mathrm{cm}}\left(i^{0}, i^{\alpha}, i^{\beta}, \theta\right)}{\partial \theta} \qquad [2.53]$$

2.1.5. *Park transformation: equations of the machine in 0dq variables*

In order to obtain simpler constitutive relations, we have to make a new circuit change. Circuits α and β are replaced by two circuits, d and q, defined from circuits α and β by the Park transformation:

$$\begin{bmatrix} i^{0} \\ i^{\alpha} \\ i^{\beta} \end{bmatrix} = \mathbf{P}(\rho_{\mathrm{e}}) \begin{bmatrix} i^{0} \\ i^{\mathrm{d}} \\ i^{\mathrm{q}} \end{bmatrix} \qquad [2.54a]$$

$$[\psi_0, \psi_{\mathrm{d}}, \psi_{\mathrm{q}}] = [\psi_0, \psi_{\alpha}, \psi_{\beta}]\ \mathbf{P}\left(\rho_{\mathrm{e}}\right) \qquad [2.54b]$$

where:

$$\mathbf{P}(\rho_{\mathrm{e}}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\rho_{\mathrm{e}} & -\sin\rho_{\mathrm{e}} \\ 0 & \sin\rho_{\mathrm{e}} & \cos\rho_{\mathrm{e}} \end{bmatrix} \qquad [2.55]$$

or, in terms of conductor density of the different circuits:

$$[\mathbf{N}_0, \mathbf{N}_{\mathrm{d}}, \mathbf{N}_{\mathrm{q}}] = [\mathbf{N}_0, \mathbf{N}_{\alpha}, \mathbf{N}_{\beta}]\ \mathbf{P}(\rho_{\mathrm{e}}) \qquad [2.56]$$

However, contrary to the transformations presented in the sections above, we no longer force variable ρ_e to be constant. We note:

$$\dot{\rho}_e = \frac{d\rho_e}{dt} \tag{2.57}$$

The result is that circuits d and q are not filiform circuits. Their voltage is therefore not given by law [2.11]. To obtain the expression of voltage, we derive [2.54b] in relation to time. We get:

$$\begin{aligned}\frac{d}{dt}[\Psi_0, \Psi_d, \Psi_q] &= \frac{d}{dt}\{[\Psi_0, \Psi_\alpha, \Psi_\beta]\,\mathbf{P}(\rho_e)\}\\ &= \left\{\frac{d}{dt}[\Psi_0, \Psi_\alpha, \Psi_\beta]\right\}\mathrm{P}(\rho_e) + \dot{\rho}_e[\Psi_0, \Psi_\alpha, \Psi_\beta]\begin{bmatrix}0 & 0 & 0\\ 0 & -\sin\rho_e & -\cos\rho_e\\ 0 & \cos\rho_e & -\sin\rho_e\end{bmatrix}\end{aligned} \tag{2.58}$$

By using [2.54b] once again, we have:

$$\begin{aligned}\frac{d}{dt}[\Psi_0, \Psi_d, \Psi_q] &= \left\{\frac{d}{dt}[\Psi_0, \Psi_\alpha, \Psi_\beta]\right\}\mathbf{P}(\rho_e)\\ &+ \dot{\rho}_e[\Psi_0, \Psi_d, \Psi_q]\,\mathbf{P}^{-1}(\rho_e)\begin{bmatrix}0 & 0 & 0\\ 0 & -\sin\rho_e & -\cos\rho_e\\ 0 & \cos\rho_e & -\sin\rho_e\end{bmatrix}\end{aligned} \tag{2.59}$$

or:

$$\begin{aligned}\frac{d}{dt}[\Psi_0, \Psi_d, \Psi_q] &= \left\{\frac{d}{dt}[\Psi_0, \Psi_\alpha, \Psi_\beta]\right\}\mathbf{P}(\rho_e)\\ &+ \dot{\rho}_e[\Psi_0, \Psi_d, \Psi_q]\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}\end{aligned} \tag{2.60}$$

By replacing temporal derivatives of fluxes by general law [2.4], and by accepting that voltages are transformed like fluxes, by [2.54b], we obtain:

$$[e_0, e_d, e_q] = [e_0, e_\alpha, e_\beta]\,\mathbf{P}(\rho_e) - \dot{\rho}_e[\Psi_0, \Psi_d, \Psi_q]\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix} \tag{2.61}$$

Finally, by using the constitutive relation [2.49] in variables α and β, we have:

$$[e_0, e_d, e_q] = [i^0, i^d, i^q]\begin{bmatrix} R_{00} & 0 & 0 \\ 0 & R_{dd} & 0 \\ 0 & 0 & R_{qq} \end{bmatrix} - \dot{\rho}_e[\psi_0, \psi_d, \psi_q]\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \quad [2.62a]$$

where:

$$\begin{bmatrix} R_{00} & 0 & 0 \\ 0 & R_{dd} & 0 \\ 0 & 0 & R_{qq} \end{bmatrix} = \mathbf{P}(-\rho_e)\begin{bmatrix} R_{00} & 0 & 0 \\ 0 & R_{\alpha\alpha} & 0 \\ 0 & 0 & R_{\beta\beta} \end{bmatrix}\mathbf{P}(\rho_e) \quad [2.63]$$

or, since $R_{\alpha\alpha} = R_{\beta\beta}$ and **P** is in the form [2.55]:

$$R_{dd} = R_{qq} = R_{\alpha\alpha} \quad [2.64]$$

Constitutive relation [2.62a] can be written in phasor form. In order to do this, we have to define phase electromotive force, current, and flux phasors by:

$$\overline{e} = e_d + j\, e_q, \quad \overline{i} = i^d + j\, i^q \quad \text{and} \quad \overline{\psi} = \psi_d + j\, \psi_q \quad [2.65]$$

We then have:

$$\overline{e} = R_{dd}\, \overline{i} + j\, \dot{\rho}_e\, \overline{\psi} \quad [2.62b]$$

The electric constitutive relation [2.62] has an additional term beside the ohmic term. We will show that this term can be interpreted as caused by circuits d and q sliding relative to the matter constituting them. To do this, we must introduce distinct variables ρ_{ed} and ρ_{eq} to describe the position of circuits d and q. The distinction is only formal because, at each moment, we have:

$$\rho_{ed} = \rho_{eq} = \rho_e \quad [2.66]$$

We can then define the coenergy function in terms of variables d and q as:

$$w_{cm}^* \left(i^d, i^q \ldots, \theta, \rho_{ed}, \rho_{eq}\right) = w_{cm} \left(i^\alpha, i^\beta, \theta\right) \tag{2.67}$$

with:

$$i^\alpha = \cos\rho_{ed}\, i^d - \sin\rho_{eq}\, i^q \tag{2.68a}$$

$$i^\beta = \sin\rho_{ed}\, i^d + \cos\rho_{eq}\, i^q \tag{2.68b}$$

We then have equality:

$$\begin{aligned}\frac{\partial\, w_{cm}^*}{\partial\, \rho_{ed}} &= \frac{\partial\, w_{cm}}{\partial\, i^\alpha}\left(-\sin \rho_{ed}\right) i^d + \frac{\partial\, w_{cm}}{\partial\, i^\beta} \cos \rho_{ed}\, i^d \\ &= \left(-\psi_\alpha \sin \rho_e + \psi_\beta \cos \rho_e\right) i^d = \psi_q\, i^d\end{aligned} \tag{2.69a}$$

and:

$$\begin{aligned}\frac{\partial\, w_{cm}^*}{\partial\, \rho_{eq}} &= \frac{\partial\, w_{cm}}{\partial\, i^\alpha}\left(-\cos \rho_{eq}\right) i^d + \frac{\partial\, w_{cm}}{\partial\, i^\beta} \sin \rho_{eq}\, i^d \\ &= \left(-\psi_\alpha \cos \rho_e - \psi_\beta \sin \rho_e\right) i^d = -\psi_d\, i^q\end{aligned} \tag{2.69b}$$

Putting aside the zero-sequence component, this makes it possible to write electromotive force [2.62a] in the form:

$$e_d = R_{dd}\, i^d - \dot{\rho}_{ed}\, \frac{1}{i^d} \frac{\partial\, w_{cm}^*}{\partial\, \rho_{ed}} \tag{2.70a}$$

$$e_q = R_{qq}\, i^q - \dot{\rho}_{eq}\, \frac{1}{i^d} \frac{\partial\, w_{cm}^*}{\partial\, \rho_{eq}} \tag{2.70b}$$

Because of the fact that the speed of matter constituting the stator is zero, [2.70] has the usual form of a sliding electromotive force, presented in [MAT 04], formula [1.21].

The development leading to [2.62] and [2.70] does not use two-phase symmetry. We could make the same development for any three-phase system. However, in order for the Park transformation to be interesting, the fact that the electric constitutive relation has become more complicated must be compensated by a simplification of the magnetic constitutive relation. This simplification is easily obtained in the case of a machine with sinusoidal coupling, in which case it is useless to transform from a three-phase system since this system is equivalent to a two-phase system.

We now consider the sinusoidal two-phase symmetry [2.30] and [2.33] and carry out a Park transformation such that:

$$\rho_e = \theta_e \qquad [2.71]$$

With this choice, we arrive at the conclusion that, in the case of a machine with sinusoidal coupling, constitutive relations expressed in terms of variables d and q in any position θ are identical to constitutive relations in variables α and β in position $\theta = 0$. Therefore, in variables d and q, constitutive relations do not depend on θ.

To obtain simpler constitutive relations, we must then choose the origin of θ as indicated at the end of section 2.1.3: circuits Q and q are in permanent magnetic quadrature with circuits f, D, and d.

We note that transformation **T P**, connecting circuits o, d, and q to original circuits a, b, and c, is simply transformation [2.44] linked to [2.71].

If constitutive relations were obtained in variables α and β, we can obtain constitutive relations in variables d and q by applying the Park transformation. This method is developed at reference [SAR 04], section 2.5.2, in the linear case.

However, since d and q are circuits, it is simpler, especially in the linear case, to express the constitutive relations directly in terms of circuits d and q. These relations will be expressed in the form of two equivalent circuits, one relating to quantities f, D, and d and the other to quantities Q and q.

If we want the elements of these equivalent circuits to be linked to distinct physical phenomena, we have to use circuits with several elements, such as those in Figure 2.1, only provided as a basis of reflection.

A third circuit, relating to zero-sequence quantities, should be added. Its structure is similar to the one in Figure 2.1b.

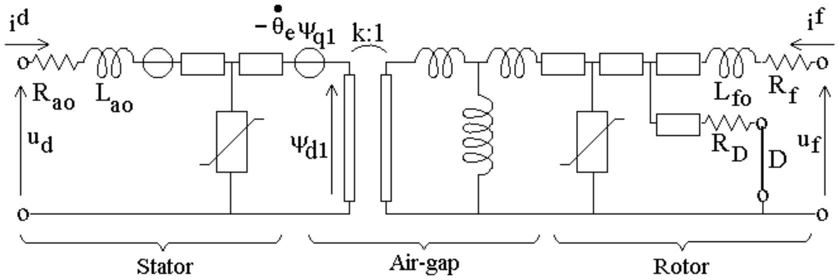

Figure 2.1a. *Example of direct axis equivalent circuit*

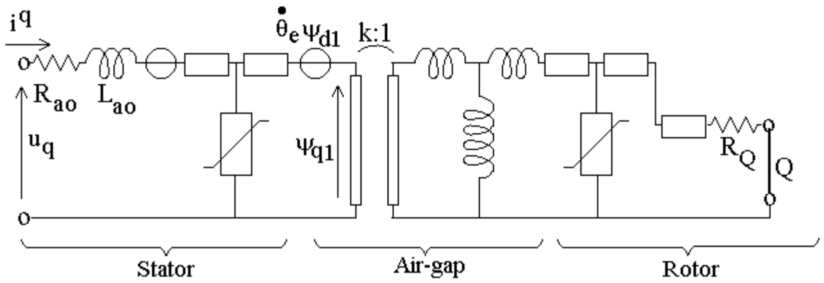

Figure 2.1b. *Example of quadrature axis equivalent circuit*

These circuits contain a k ratio ideal transformer. This ratio takes into account the number of turns of the armature and the field winding among others. We can note that it is not exactly equal to the turn ratio, because it also considers the different winding factors.

In accordance with [2.62], in order to obtain the correct voltage, we have placed a voltage source controlled by the flux of the equivalent element in series with each inductive element of the stator, including the primary of the ideal transformer.

The different elements of these schemas have a physical significance. The power absorbed by resistances corresponds to losses, and the power absorbed by inductances is stored as magnetic energy. In order for the energy balance to be correct, we must accept that the power converted into mechanical power is the one provided to voltage sources.

In these schemas, R_{ao} and R_f are ohmic resistances of the armature and field winding, whereas L_{ao} and L_{fo} are the parts of the series inductance caused by insulating material placed between the turns. These inductances, described by

[MAX 81], are small and often ignored compared to other series inductances. The elements represented by small rectangles are non-ideal inductances; they are accompanied by a resistance in order to account for magnetic losses. These rectangles will be modeled in the form of a inductance with a parallel or series resistance. In addition, in these rectangles related to the stator, the inductance is in series with a voltage source in accordance with [2.62].

Elements R_{ao}, R_f, L_{ao}, and L_{fo} slightly depend on frequency because of the skin effect. The frequency effect is more marked on non-ideal inductances, because of the proximity effect and magnetic losses linked to corresponding fluxes. In normal operation, the frequency effect is mostly definite in the stator because we do not have to consider the frequency of quantities d and q, but frequency of quantities a, b, and c.

We presume that only homologous elements are magnetically coupled. Since the circuits are in quadrature, the equivalent linear elements are not coupled. The only couplings relative to magnetic constitutive relations between direct axis and quadrature circuits are therefore only done via non-linear inductances. In Figure 2.1, we have only considered two inductances of this type, one in the stator and the other in the rotor.

2.1.6. *Connection between the machine and a three-phase link*

In practice, synchronous machines are connected by a three-phase three wire link.

With the hypothesis of the sinusoidal coupling, this results in the zero-sequence components being zero. Indeed, if the machine is star-connected, as the neutral is not linked, the zero-sequence current is zero. Similarly, if it is delta connected, the zero-sequence voltage is zero.

The zero-sequence circuit, however, with a structure that is similar to Figure 2.1b, can only be excited by its "terminals" because it is in quadrature with the two other circuits, and thus these circuits cannot induce non-zero quantities. The result is that all zero-sequence voltages and currents are zero. From then on, we will no longer consider the zero-sequence circuit. This simplification would not be achieved if we extended the study to the case of machines with non-sinusoidal coupling.

2.1.7. *Reduction of rotor circuits to the stator*

We should note that we can move inductances and resistances from one side to the other of the ideal transformer, as long as we multiply their value by k^2 when these elements go from side 2 to side 1, or if we divide by k^2 if they move from side 1 to side 2. It is therefore possible to group all the elements of the equivalent circuit on the stator side.

We obtain equivalent circuits such as those in Figure 2.2.

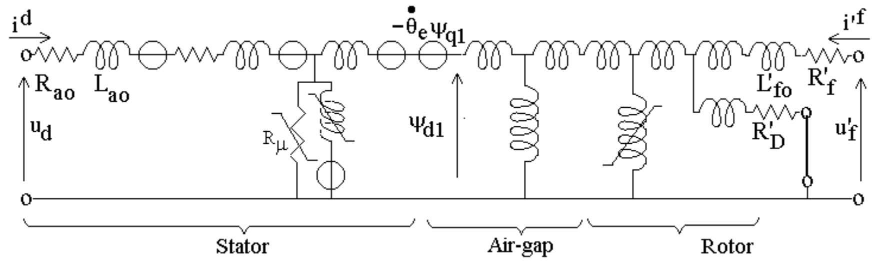

Figure 2.2a. *Axis d equivalent circuit reduced to the stator*

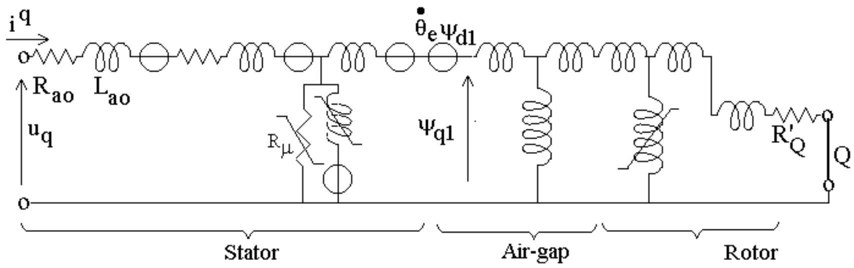

Figure 2.2b. *Axis q equivalent circuit reduced to the stator*

In Figure 2.2, we replace the series elements by pure inductances, except the one with stator port. This is an approximation, but it is satisfying insofar as, since losses are weak, we can represent stator losses *via* the three resistances retained in the stator, whereas we will represent all rotor losses by the resistance of circuits f, D, and Q.

By moving the ideal transformer to the right, we modify the value of rotor elements. They are noted differently in Figure 2.2 and, for example, we have:

$$i'^f = i^f/k \qquad [2.72a]$$

$$R'_f = k^2 R_f \qquad [2.72b]$$

$$L'_{f0} = k^2 L_{f0} \qquad [2.72c]$$

Because of these simplifications, inductances and voltage sources appear in series and can be combined to reduce the number of elements. We then obtain Figure 2.3.

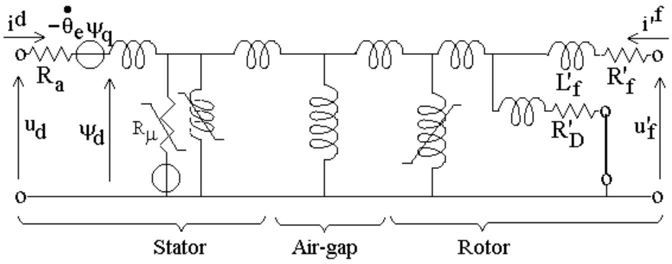

Figure 2.3a. *Axis d equivalent circuit after a first reduction of the number of elements*

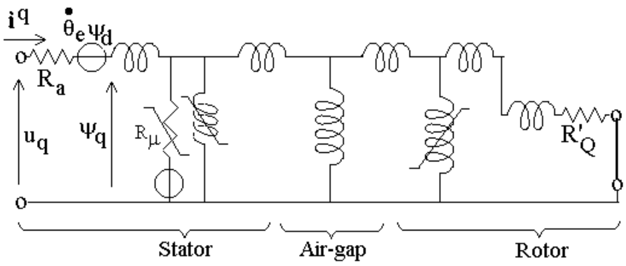

Figure 2.3b. *Axis q equivalent circuit after a first reduction of the number of elements*

It is possible through circuit manipulations to group the three parallel inductances into one, in the direct circuit (the inductance appearing in series with the current source can be eliminated) and in the quadrature circuit. In this way, the series inductances included between these parallel inductances are found in series with stator and rotor inductances to which we can combine them. We then obtain the circuits in Figure 2.4.

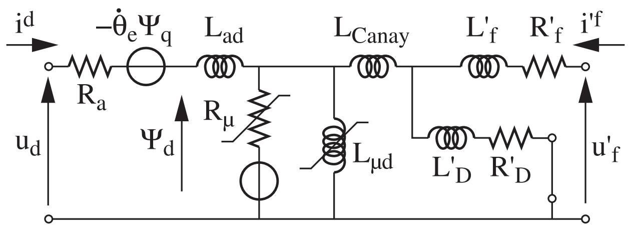

Figure 2.4a. *Axis d reference equivalent circuit*

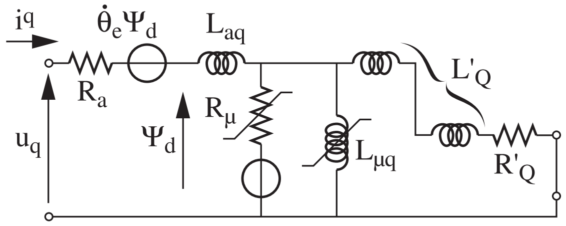

Figure 2.4b. *Axis q reference equivalent circuit*

We should note that, during the establishment of the circuits in Figure 2.4, we carry out transformations consisting of moving a series inductance upstream to downstream of a parallel inductance, and vice versa. During these transformations, both the value of the series inductance and the parallel inductance are modified. In addition, this transformation reveals new ideal transformers. These transformers can be eliminated as was done for the *k* ratio transformer, but the result is that the value of rotor elements is modified. In this way, if we designate the new transformation ratio by α (corresponding to the cascade connection of all ideal transformers introduced), instead of definitions [2.72], we will have definitions such as:

$$i'^{f} = i^{f}/\alpha \tag{2.73a}$$

$$R'_{f} = \alpha^2 R_f \tag{2.73b}$$

$$L'_{fo} = \alpha^2 L_{fo} \tag{2.73c}$$

Inductance L_{ad} is called Potier inductance, in view of the method usually used to determine it (see section 2.2.3.3). We must note that L_{ad} and L_{aq} are not stator leakage inductances, because they include a part of the leakage inductance of the air-gap, and even of the rotor. In fact, the distribution of series inductances between upstream and downstream of the equivalent circuit is not controlled by physical imperatives, but by the will to group all the non-linear phenomena in the only parallel element of the circuit.

Similarly, inductances $L_{\mu d}$ and $L_{\mu q}$, which we will call magnetization inductances, take into account the phenomena present in the air-gap, stator, and rotor. The flux of these inductances is called the main flux. Physically, we consider that the main flux goes through the air-gap and closes through the rotor on one side and the stator on the other side. On the one hand, $L_{\mu d}$ and $L_{\mu q}$ report the reluctances encountered by the main flux along the air-gap, stator, and rotor paths.

On the other hand, resistances R_a and R_μ only account for stator losses: the Joule effect in conductors and losses associated with the stator leakage fields on one side and magnetic losses associated with the main flux on the other side.

With these precisions, the equivalent circuits in Figure 2.4 correspond to the circuits presented as Figure 2.24 in [SAR 04], except the Canay inductance and resistance R_μ. This comes from the fact that reference [SAR 04] is limited to the linear case and ignores magnetic losses. In this case, the equivalent circuit in Figure 2.4 is redundant; we can obtain the same external behavior by deleting the Canay (implying an adaptation of the values of the other elements). In practice, magnetic losses are too low to justify the consideration of the Canay inductance. On the other hand, the non-linearities fully justify its introduction. We will explain this topic in more detail in the last part of this chapter, in section 2.3.2.1.

In sinusoidal mode, the frequency is zero so that the voltage on inductances cancels out.

The diagrams in Figure 2.5 are obtained by deleting the elements that have become useless.

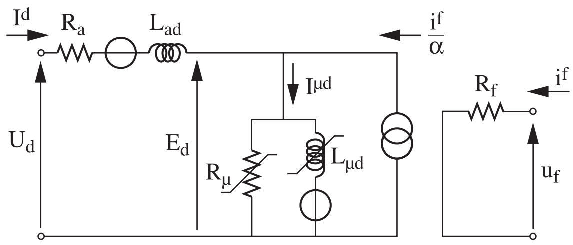

Figure 2.5a. *Sinusoidal mode direct axis reference circuit*

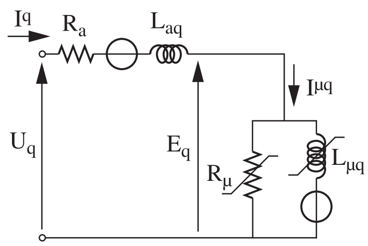

Figure 2.5b. *Sinusoidal mode quadrature axis reference circuit*

In Figure 2.5, we have redistributed the voltage source between the remaining series inductance and parallel inductance. In this model, the voltage appearing in inductive elements (made up of an inductance and a voltage source) only contains the voltage source, that is, the last term of [2.62], since in steady state the flux temporal derivatives are zero.

If the machine has smooth poles, we can combine both circuits into one owing to the use of phasors defined in [2.65].

This is obvious for linear elements because, since they have the same value in the direct and quadrature circuit, we can highlight this value.

For the non-linear element, we must introduce an additional hypothesis, that the "main flux" phasor and "magnetizing current" phasor have the same argument, and that there is a relation that involves no other quantity between their modules. In this case, $\overline{Z}_{\mu}$ is only a function of the modulus of $\overline{E}$ or, which amounts to the same, of the modulus of $\overline{I}^{\mu}$.

We then arrive at the equivalent circuit in Figure 2.6, the Potier equivalent circuit.

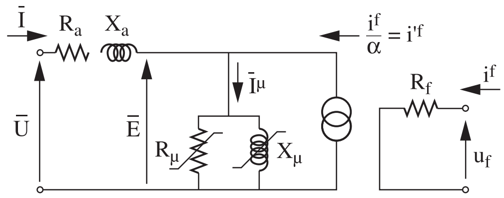

Figure 2.6. *Potier equivalent circuit, reference model for a smooth pole machine in balanced steady state*

Note that X_a and X_{μ} reactances do not replace the inductances of Figure 2.5, but the associated voltage sources. They are zero-energy, however, because the power absorbed by a couple of equivalent sources equals to:

$$\begin{aligned} -\dot{\theta}_e \psi_q i^d + \dot{\theta}_e \psi_d i^q &= \dot{\theta}_e \left(-L_q i^q i^d + L_d i^d i^q\right) \\ &= \dot{\theta}_e \left(L_d - L_q\right) i^d i^q = 0 \end{aligned} \qquad [2.74]$$

Please note that this conclusion is only valid for smooth pole machines!

In Figure 2.6, we presumed that the phase reference was taken in relation to the moment where $\theta = 0$, so that the current source has a zero argument. We can also take as phase reference the phase of the voltage, in which case the current source has a non-zero argument. We then write it as:

$$\overline{I}'_{\mathrm{f}} = \frac{i_{\mathrm{f}}}{\alpha}\,\mathrm{e}^{\mathrm{j}\left(\delta - \frac{\pi}{2}\right)} \qquad [2.75a]$$

where angular quantity δ is called internal angle.

We can also take as phase reference the phase of the current, in which case we will write the current source in the form:

$$\overline{I}'_{\mathrm{f}} = \frac{i_{\mathrm{f}}}{\alpha}\,e^{\mathrm{j}\left(\delta' - \frac{\pi}{2}\right)} \qquad [2.75b]$$

where angular δ' quantity is only:

$$\delta' = \delta + \varphi \qquad [2.76]$$

Because of transformations [2.45] and [2.55], in steady state, phasors $\overline{\mathrm{U}}$ and $\overline{\mathrm{I}}$ have the rms value of the phase variable multiplied by $\sqrt{3}$ as modulus. Especially, if we consider a star connection, the modulus of $\overline{\mathrm{U}}$ is simply the rms value of the line voltage. If we prefer to use the usual convention for the single-phase equivalent circuit, the impedances will remain the same, but ratio α will have to be divided by $\sqrt{3}$, and power corresponding to the left part of the equivalent circuit will have to be multiplied by 3 to provide the total three-phase power.

During tests, we usually measure line voltage and current circulating in a conductor. So we do not directly obtain the correct values of the two moduli in either of the two cases involved above. When required, we will designate these quantities as U_{L} and I_{L} (“L” for “line”).

The circuit in Figure 2.4 retains a physical sense: we can interpret the power dissipation in R_{a} as stator Joule losses, or:

$$p_{\mathrm{Js}} = 3\,R_{\mathrm{a}}\,I_{\mathrm{L}}^2 \qquad [2.77]$$

and the one dissipated in R_μ as stator magnetic losses, or:

$$p_\mu = 3\,E^2/R_\mu = E_L{}^2/R_\mu \qquad [2.78]$$

Since reactances X_a and X_μ are zero-energy, and the power dissipation in resistances corresponds to losses, the energy balance then shows that the power exchanged with the current source corresponds to the electromechanical power conversion.

This circuit can be simplified by replacing the current source and parallel impedance by a voltage source of value:

$$\overline{E}_0 = \overline{Z}_\mu\ \overline{I}'^{f} = E_0\, e^{\varphi_\mu + \delta - \frac{\pi}{2}} \qquad [2.79]$$

in series with an impedance of value $\overline{Z}_\mu$. It is a transformation similar to the one from Thevenin. We note that the argument of $\overline{E}_0$ is not exactly equal to δ, except if we ignore the effect of resistance R_μ, in which case $\varphi_\mu = \pi/2$.

We then obtain the equivalent circuit (see Figure 2.7).

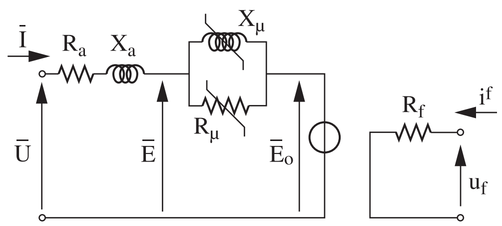

Figure 2.7. *Thevenin equivalent of the Potier model*

During the use of this circuit, we should be careful of the fact that elements R_μ, L_μ and $\overline{E}_0$ depend on the saturation level of the machine. This is not defined by the value of i^f, or by the value of E_0, or by the value of the voltage or current going through Z_μ in Figure 2.7. Voltage E once again determines the level of saturation.

In addition, even in the linear case, we should also be careful of the fact that magnetic losses are no longer represented by power dissipation in R_μ. They in fact retain the same expression as before, or as in [2.78]. A consequence of this fact is that the power exchanged with the voltage source does not correspond exactly to the mechanical power converted in electric power (except if magnetic losses are ignored).

We can carry on the simplification by combining all impedances into one, called synchronous impedance.

We then obtain the circuit in Figure 2.8, which is simply the equivalent Behn-Eschenburg circuit.

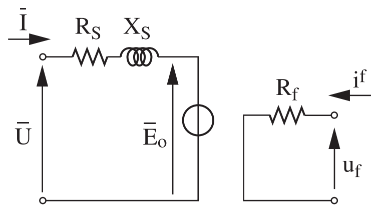

Figure 2.8. *Equivalent Behn-Eschenburg circuit*

In this equivalent circuit, R_s is usually very close to R_a and they are often confused. On the other hand, synchronous reactance X_s is usually much larger than X_a, since it takes into account magnetization reactance X_μ.

During the use of this circuit, we should be careful that elements $\overline{Z}_s$ and $\overline{E}_0$ depend on the level of machine saturation (in principle, the same applies to R_s but we can ignore it because R_s is close to R_a). This one is not defined by value i^f, or by E_0. Once again voltage E determines the level of saturation, even though it no longer explicitly appears in the equivalent circuit.

This problem disappears in the linear case.

There remains, however, the problem that resistance R_s does not account for all stator losses and that consequently, the power exchanged by the voltage source does not always correspond to the converted power (except if magnetic losses are ignored).

2.1.8. *Relative units (per-unit)*

Traditionally, in electrical engineering, we reason in terms of standardized variables (without a physical dimension) called "per-unit (p.u.) system". For this, the quantities are expressed in fraction of base quantities. Base quantities are chosen in such a way that reduced variables usually have a value close to 1.

The fundamental base quantities are power and voltage, from which current and impedance result.

For calculations relative to synchronous machines, two choices of base quantities are commonly practiced for stator quantities.

The first one is based on the choice of the *rated three-phase apparent power* as power base, combined with the *rated line voltage* as voltage base:

$$\begin{aligned} S_b &= S_N \\ U_b &= U_N \end{aligned} \qquad [2.80]$$

from which the current and impedance bases are deducted:

$$\begin{aligned} I_b &= \frac{S_b}{U_b} = \sqrt{3} I_N \\ Z_b &= \frac{U_b^2}{S_b} = \frac{U_b}{I_b} = \frac{U_N}{\sqrt{3} I_N} \end{aligned} \qquad [2.81]$$

The alternative is based on the choice of the *rated apparent power by phase* as power base, combined with the *rated line-to-neutral voltage* (phase-to-neutral voltage) as voltage base:

$$\begin{aligned} S_b &= \frac{S_N}{3} \\ U_b &= \frac{U_N}{\sqrt{3}} \end{aligned} \qquad [2.82]$$

from which the current and impedance bases also result:

$$\begin{aligned} I_b &= \frac{S_b}{U_b} = I_N \\ Z_b &= \frac{U_b^2}{S_b} = \frac{U_b}{I_b} = \frac{U_N}{\sqrt{3} I_N} \end{aligned} \qquad [2.83]$$

With the definition of an angular frequency p.u., ω_b, we can complete these set of base quantities and deduce, for example, inductance, flux, and torque p.u.:

$$
\begin{aligned}
L_b &= \frac{Z_b}{\omega_b} \\
\Psi_b &= L_b I_b = \frac{U_b}{\omega_b} \\
C_b &= \frac{S_b}{\omega_b}
\end{aligned}
\qquad [2.84]
$$

The choice of rotor base variables is trickier. A major constraint that is generally accepted in this problem is the choice of the *same angular frequency and power bases* for the rotor and stator. Since the stator power is generally much larger than that involved at the rotor level, the result will be that some rotor base variables will be numerically much larger and when this happens, certain rotor quantities, expressed in p.u., will be numerically very small.

A common methodology is based on the concept of the mutual coupled flux. The base rotor current is, for example, chosen in such a way that it produces the same air-gap flux as the base stator current in the direct axis. It is then defined by:

$$
i^f{}_b = \frac{L_{\mu d}}{L_{df0}} I_b \qquad [2.85]
$$

where L_{df0} is the stator-rotor mutual inductance (or the slope of the linear part of the characteristic described in Figure 2.9). Generally, we have:

$$
\begin{aligned}
i^f{}_b &= k_f \ I_b \\
u_{fb} &= \frac{1}{k_f} U_b
\end{aligned}
\qquad [2.86]
$$

If the stator leakage inductance is equal to the Potier inductance (see equivalent diagram in Figure 2.6), factor k_f is equal to $\frac{\alpha}{\sqrt{3}}$ or α depending on whether I_b is respectively equal to $\sqrt{3}\,I_N$ or I_N (i.e. following the choice of stator bases, discussed above).

Other rotor p.u. choices are also often found in practice. They are motivated by the necessity of interfacing the equation of the field circuit with the equations of the excitation device and the voltage regulator among others. For example, the rotor current p.u. can be arbitrarily defined as the excitation current corresponding to the rated stator voltage in no-load or full-load condition.

The situation can be even more complicated because some authors, once they have introduced non-orthonormal transformations of variables, compensate for the resulting drawbacks with the choice of ad hoc base variables. Generally, we have to remain careful during the use of variables expressed in p.u.

2.2. Classical models and tests

2.2.1. *The synchronous non-saturated machine*

When the fluxes are low enough, we can consider that the machine is not saturated. We can then replace all inductances with linear inductances in the equivalent circuits introduced in section 2.1.7.

2.2.1.1. *Classical linear model (Park model)*

In the case of a non-saturated sinusoidal machine, the Park transformation separates the equations into two magnetically decoupled systems (direct and quadrature axis). In fact, since there is no cross-saturation in that case, there is no coupling between axis d and axis q circuit inductances either. The matrix techniques show that the Park transformation is the only one ensuring decoupling, except for scaling factors.

2.2.1.2. *Equivalent diagrams*

The most widely used equivalent diagram is shown in Figure 2.4, considering inductances as linear. When at standstill, both circuits are decoupled. Their order depends on the number of resistances they contain. According to the technology of the machine involved, we often have to introduce additional damping circuits in the equivalent circuit.

2.2.1.3. *Operational reactances in non-saturated mode*

During transients executed when the machine is rotating, stator resistances (R_a and R_μ) have different effects to rotor resistances (of the field winding and dampers). In order to separate the phenomena, we usually ignore the effect of resistance R_μ (an alternative would consist of placing it upstream of X_a). In this case, we can “isolate” R_a, R_μ, and the only remaining voltage source, and treat the rest of the circuit with techniques usual in the frequency domain by writing:

$$\psi_d = L_d(s)\, i^d + G(s)\, u_f \qquad [2.87]$$

$$\psi_q = L_q(s)\, i^q \qquad [2.88]$$

Factors $L_d(s)$ and $L_q(s)$ have the dimension of inductances. However, it is easy to give them the dimension of a reactance by multiplying them by angular frequency ω_e. We then define operational reactances as:

$$X_d\,(s) = \omega_e\, L_d(s) \qquad [2.89]$$

$$X_q(s) = \omega_e\, L_q\,(s) \qquad [2.90]$$

By isolating the stator resistance, the direct axis circuit has only two resistances. The response of this circuit, that is, $L_d(s)$ and $G(s)$, is then of the second order. The response of the quadrature circuit is of the first order, but we often introduce a second damper in this axis, making $L_q(s)$ of the second order also.

2.2.1.4. *Internal and external parameters in non-saturated mode*

By limiting to order two, the transfer functions of equations [2.87] and [2.88], we can write them in the form:

$$L_d\,(s) = L_d\,\frac{(1+T'_d\,s)(1+T''_d\,s)}{(1+T'_{d0}\,s)(1+T''_{d0}\,s)} \qquad [2.91]$$

$$G_d\,(s) = \frac{M}{R_f}\,\frac{\left(1+T_D s\right)}{\left(1+T'_{d0}\,s\right)\left(1+T''_{d0}\,s\right)} \qquad [2.92]$$

$$L_q\,(s) = L_q\,\frac{\left(1+T'_q\,s\right)\left(1+T''_q\,s\right)}{\left(1+T'_{q0}\,s\right)\left(1+T''_{q0}\,s\right)} \qquad [2.93]$$

where the parameters are called:

– L_d: d axis synchronous inductance;

– T'_d: direct-axis transient “short-circuit” time constant;

– T'_{d0}: direct-axis transient open-circuit time constant;

– T''_d: direct-axis subtransient “short-circuit” time constant;

– T''_{d0}: direct-axis subtransient open-circuit time constant;

We define similar parameters in axis q.

The names "short-circuit" and "open circuit" represent the situations in which the corresponding parameters play a dominant role. We should, however, not take these names literally.

Equations [2.91] and [2.93] can take the equivalent form:

$$Y_d(s) = \frac{1}{X_d(s)} = \frac{1}{X_d} + \left(\frac{1}{X'_d} - \frac{1}{X_d}\right)\frac{T'_d\,s}{1+T'_d\,s} + \left(\frac{1}{X''_d} - \frac{1}{X'_d}\right)\frac{T''_d\,s}{1+T''_d\,s} \qquad [2.94]$$

$$Y_q(s) = \frac{1}{X_q(s)} = \frac{1}{X_q} + \left(\frac{1}{X'_q} - \frac{1}{X_q}\right)\frac{T'_q\,s}{1+T'_q\,s} + \left(\frac{1}{X''_q} - \frac{1}{X'_q}\right)\frac{T''_q\,s}{1+T''_q\,s} \qquad [2.95]$$

where the new parameters are called:

– X_d: direct-axis synchronous reactance;

– X'_d: direct-axis transient reactance;

– X''_d: direct-axis subtransient reactance.

In the literature, expressions owing to go from T'_d and T''_d to X'_d and X''_d are often established with the approximation that:

$$T''_d \ll T'_d \qquad [2.96]$$

We must avoid considering these approximate formulas as definitions, because then it becomes impossible to return to the exact formulas.

The parameters introduced above are called external parameters. We can connect them to internal parameters (those of the equivalent circuit), but this link is simple only if we use approximation [2.96]. Qualitatively, we can then consider that each time constant is associated with one of the equivalent circuit's resistances, the others being presumed zero or infinite. In this way, the subtransient is associated with the damper's resistance, and the transient to the field winding or to the second damper planned in axis q. The reactance associated with each time constant is then that "seen" by the resistance involved, and is expressed as a series/parallel connection of inductances of the equivalent circuit.

When the order of the equivalent circuit is higher than two, other time constants appear (we sometimes speak of sub-subtransient). Expressions [2.91] to [2.95] are of the second order; therefore, they can only represent the behavior of the machine approximately. The external parameters discussed above make sense, however, and are still useful in characterizing the behavior of the machine, but their connection with internal parameters must be reviewed. The technique for obtaining them from the equivalent circuit is known as “special disturbances” in automatics. Its use in the context of synchronous machines is described in [GUE 95].

2.2.2. *General classical tests*

Classical tests are often done on standstill or in low-load steady state. This obviously presents the advantage that they can be done even if we do not have a powerful enough installation to treat the high power that some synchronous machines have. Another advantage, since exchanged power is often lost in the machine instead of being converted between electric and mechanical power, is that an energy balance makes it possible to obtain the value of losses with more precision.

We will not describe here the precautions required to carry out these tests with no danger for the machine. If the field winding contains much more turns than the stator, we will especially make sure that we make connections and carry out tests so as to never produce overvoltage on the field winding.

In order to carry out some tests, we must be able to precisely determine the rotor’s position. When in standstill, we can determine the reference position by feeding alternating current in the first phase of the rotor (or the circuit located between the first terminal and the two others interconnected) and by searching for a position such that the voltage induced in the field circuit is zero (we should be careful not to induce dangerous voltage in the field winding). The position found is at 90 electric degrees from the reference position. In the reference position, the induced voltage is in phase with the stator voltage, taking away the ambiguity.

An alternative that is not as strongly influenced by contact resistances consists of not connecting the first stator terminal and in feeding the stator between the two other terminals. The induced voltage is zero if the rotor is in the reference position (apart from 180°).

If the machine has a position sensor, the above tests make it possible to determine zero.

For the rotation tests, if we do not have a position sensor, we can use the stroboscopic method. The axis of the machine has a tag and its position is determined from the test above. We must be able to determine the position of lightning in relation to the zero crossing of the stator voltage, for example, by comparing this voltage to the output of a photoelectric cell.

Another useful determination consists of measuring the armature resistance at standstill, using a DC bridge, or by a voltamperometric method. Precautions must be taken to avoid contact and connection resistances. The measured value must be divided by two to obtain an estimation of the armature resistance, or R_{aDC}. This resistance is lower than R_a because it does not consider the additional resistance appearing in alternating current. We will use it only if a better estimation is not available. We must consider that resistances are sensitive to temperature variations, the fact that we will get back to in section 2.2.2.1.3.

2.2.2.1. *Test of the synchronous machine in low-load steady state*

The tests carried out in steady state only give access to a limited number of the model parameters, but this fact has the advantage of enabling a more precise determination of these parameters.

2.2.2.1.1. No-load test (taking down of the direct-axis magnetic characteristic)

Ideally, we will make a no-load test by driving the test machine by an auxiliary machine, at the speed of synchronism.

We then pick up the stator voltage (preferably the line voltage) according to the excitation current. In order to obtain results that can be reproduced despite the presence of hysteresis (which is not considered in the models examined here), we must follow a procedure consisting of starting with a high level of excitation current and pick up the characteristic with an excitation current decreasing in a monotonous way.

The tangent at the origin of this curve is called "air-gap characteristic". It is used in linear models instead of the real magnetic characteristic.

If it is not possible to carry out the no-load test at the synchronous speed, we can carry out the test at a different speed, because the induced voltage is practically proportional to the speed, so the correction is easy.

If we do not have a driving machine but we can still start the test machine as a motor, we can adjust the excitation current in order to make the reactive power zero. By repeating the test for different stator voltages, we can draw the magnetic characteristic. During such a test, the stator current is not quite zero since power supply must provide to the machine sufficient power to offset losses. An alternative consists of measuring the armature voltage only after disconnecting it, that is, during a speed decreasing test.

The magnetic characteristic is an important parameter in a synchronous machine model. In fact, during the no-load test, the armature voltage is simply q-axis electromotive force (voltage on magnetization inductance) whereas the excitation current corresponds to the magnetizing current, except for a factor α. We then define an efficient excitation current:

$$I^{fe} = \alpha\, I^{\mu} \qquad [2.97]$$

where I^{μ} is the norm of the magnetizing current; this allows for the reinterpretation of the diagram in Figure 2.9.

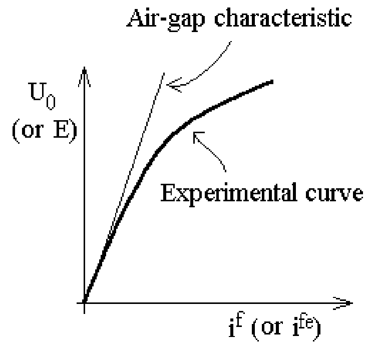

Figure 2.9. *Magnetic characteristic of direct axis $U_0 - i^f$ or $E - i^{fe}$*

2.2.2.1.2. Short-circuit test

Ideally, we will make a short-circuit test by driving the test machine with an auxiliary machine, at the synchronous speed. In order to not exceed the stator-rated current, we must use a reduced excitation current. If necessary, we will first demagnetize the field system by a current running through it that we will progressively decrease while carrying out inversions of direction (being careful to avoid overvoltage).

During the short-circuit test, we note the stator short-circuit current as a function of the excitation current. This characteristic is linear because the magnetic field is low in a short-circuit machine. In principle, one measure is sufficient, but a control measure makes it possible to eliminate the effect of a possible remanence.

If the test cannot be done exactly at the speed of synchronism, we can take advantage of the fact that, the current being mainly linked to the fluxes, the value of the current is independent from the speed as long as we can ignore the effect of armature resistance in comparison with reactances.

No-load and short-circuit test results enable us to determine direct axis synchronous reactance. In order to obtain this, we need to create the ratio between the slopes of the air-gap magnetic characteristic and the short-circuit characteristic. If the first one is drawn in line voltage, we should divide the quotient by $\sqrt{3}$ in order to respect the conventions relative to equivalent circuits.

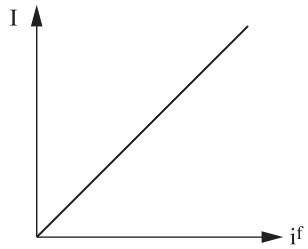

Figure 2.10. *Short-circuit line*

2.2.2.1.3. Loss determination (ohmic, magnetic, mechanical)

No-load and short-circuit tests also help us to determine losses in the test machine. In fact, during these tests, the power entering the machine corresponds to the losses. We must therefore determine the mechanical or electric power. The sum of these two powers (where one is usually zero) corresponds to the sum of mechanical and magnetic losses as well as Joule effect losses in the stator. The electric power consumed at the field winding is treated separately, because it is only made up of excitation losses.

As far as the determination of mechanical power is concerned, it comes down to the torque determination because the rotation speed is easy to determine. This is done ideally by a torque meter or a balance machine, but we can also use a calibrated auxiliary machine, or still force sensors or strain gauges placed at machine supports.

If the test is done using motor operation, the measure of the torque is replaced by a measure of the electric power. Finally, if the tests are speed decreasing, we must be able to measure its instant value and know the value of mechanical inertia because the reduction of kinetic energy offsets losses in this case. If we absolutely must, we can estimate the machine’s inertia by carrying out additional decreasing tests with adding known values to losses, for example by connecting known value resistances to the stator.

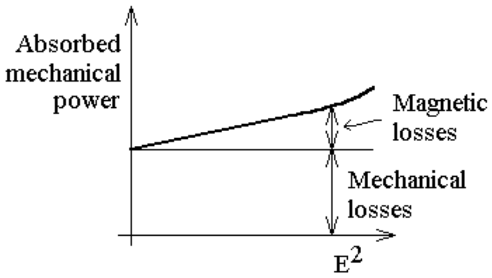

Figure 2.11. *Losses based on the square of the electromotive force*

Machine losses during no-load test are only mechanical and magnetic losses (if the test is done using motor operation, we will correct the value measured by deducting an estimation of ohmic losses, low in this case).

We can separate them because the mechanical losses do not depend on the level of the field present in the machine, whereas magnetic losses vary approximately as its square, thus as the square of the induced voltage. We can then draw on a diagram the value of mechanical and magnetic losses according to the square of the no-load voltage. The intersection of this curve with the axis of ordinates provides mechanical losses. Magnetic losses are obtained by subtracting them from the total.

Similarly, losses taken from the short-circuit test correspond to the sum of mechanical and Joule effect losses in the stator. In fact, during this test, the field is low as are magnetic losses, which are generally ignored. Losses can be separated from the fact that mechanical losses do not depend on the armature current, whereas Joule losses vary like the square of this current. On a diagram of losses based on the stator current, we can then easily separate both types of losses, as shown in Figure 2.12.

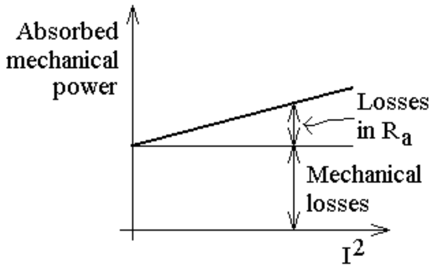

Figure 2.12. *Losses based on the square of the armature current*

Values of resistances (and thus of associated losses) are very sensitive to temperature variations. We should therefore bring their value back to a reference temperature. We will remember that the temperature factor characteristic of the material (copper, aluminum, iron, etc.) only applies to resistances directly corresponding to an ohmic effect. For example, the ohmic part of R_a behaves as expected (it increases with temperature), but the part of R_a corresponding to additional losses decreases with temperature because, when material resistivities increase, the eddy currents decrease along with the associated losses.

2.2.2.1.4. Low slip test (determination of longitudinal and transversal synchronous reactances)

The determination of the quadrature axis synchronous reactance is more difficult than the determination of X_d. In fact, it is not possible for this determination to excite the quadrature circuit by the field winding. We will then do a test during which the excitation current will be zero and the armature current reduced to its quadrature component. During this type of test, the torque is low and generally decreases the stability of the desired state.

One way to avoid this problem consists of measuring armature current and voltage while the unexcited machine rotates at a slightly different speed to the synchronous speed. We presume that the slip is low enough so that we can consider that we are in electric steady state at any time. Under these conditions, the extreme values of ratio $\dfrac{U_L}{\sqrt{3}\, I_L}$ provide X_d and X_q values. Such a test likely can only be done at reduced armature voltage so that the values obtained are non-saturated values. In fact, since X_d can be determined more precisely from no-load and steady state short-circuit tests, the low slip test is only used to determine the salience ratio X_d/X_q, used to determine X_q from the more precise X_d value.

If the sliding cannot be considered as minuscule, we should treat the results of this test by the methods discussed later involving transient tests.

2.2.2.2. *Synchronous machine tests in transient state*

Tests carried out in transient state provide access to all the parameters of the model and allow us to determine many of them, but this fact has the disadvantage of making a precise determination of these parameters difficult. We will keep in mind that it is better not to question the value of parameters obtained by steady state tests, reducing the number of parameters to determine by transient tests. In the context of a purely linear study where R_μ is ignored, equivalent circuits such as those in

Figure 2.4 are redundant because leakage inductances can be moved downstream to upstream of the parallel branch with a redefinition of the α ratio. In this case, the Canay inductance must be deleted if we want to be able to determine L_{ad}. On the other hand, we will see later that if we consider magnetic saturation, it is possible to isolate the values of the leakage reactance of armature X_a by steady state tests, thus also the values of magnetization reactances $X_{\mu d}$ and $X_{\mu q}$, as well as ratio α. We will then prefer the methods retaining the value of these elements during the interpretation of transient tests.

In order to easily manipulate experimental results, it is desirable to acquire them in computer form.

The most general way to handle such results consists of considering a part of the variables recorded (e.g. voltage and position) such as data, identifying the value of unknown parameters by carrying out simulations on the basis of the chosen equivalent circuit, and by comparing the results of these simulations to experimental results.

This method directly provides the internal parameters of the machine. Unfortunately, it only converges if we are able to determine an initial value for the desired parameters that is close enough to the optimal value. The convergence is particularly difficult if we do not have a record of the position, so its evolution must be estimated by calculation.

We should, however, note that if we have a record of the angular position and stator variables of all phases, we can simplify the problem by comparing the calculated and experimental variables in system d q.

We often use a less-demanding approach in experimental data, presuming that quantities can be broken down into a frequency f_e pseudoperiodic component and an aperiodic component (there is also a frequency $2 f_e$ pseudoperiodic component but it is often ignored). We then search for the lower and upper shells of these variables. The half-difference between both shells provides the amplitude of the pseudo-periodic component, whereas the average provides the aperiodic component.

The curves obtained can then be broken down into a constant (steady state value) and a sum of exponential functions. This break down can be done graphically by using the fact that the diagram of a pure exponential function, in a logarithmic diagram, is a straight line, making it possible to use linear regression techniques. We can then determine a set of time constants and amplitudes.

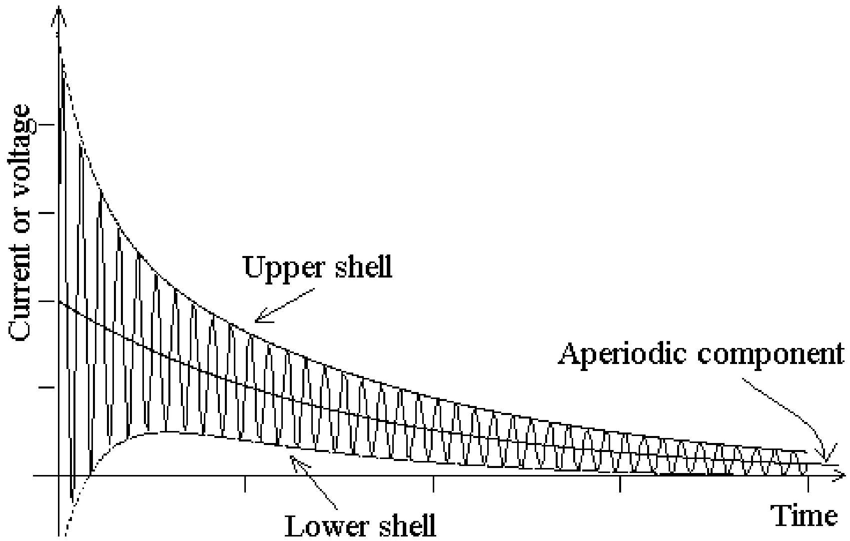

Figure 2.13. *Typical aspect of a transient carried out on a rotating machine*

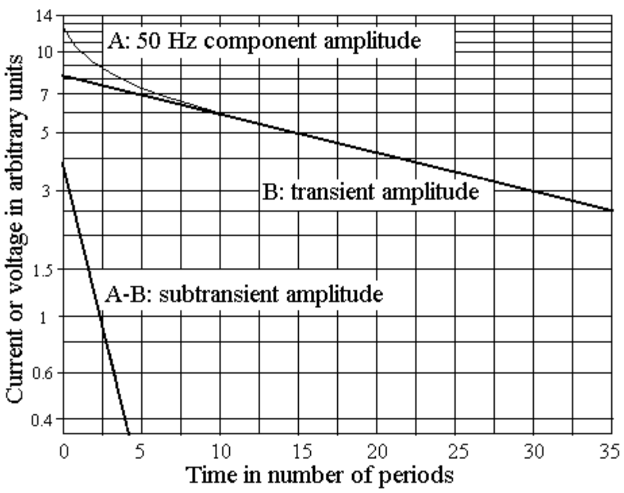

Figure 2.14. *Graphical identification of a sum of exponential functions*

In order for the transient and subtransient time constants to be equal to observed time constants, the electric circuit connected to the field winding must have zero impedance that we can approximately achieve with the use of batteries.

From these results, we can get the external parameters. Determining internal parameters from external parameters is, however, possible only if we have chosen a simple equivalent circuit. Even in this case, it will only be simple if we carry out simplifications resulting from the approximation [2.96].

2.2.2.2.1. Transient three-phase short-circuit test

The abrupt three-phase short-circuit test is interesting because it provides indications on the machine's behavior in a difficult situation. We do it at reduced excitation current so that the linearity hypothesis is verified. We can observe the evolution of armature and field winding currents.

The transient behavior of the machine during this test is influenced by resistance R_a. Contrary to what is sometimes asserted, this one is not insignificant compared to the other winding resistances. Thankfully, because of the rotation of the rotor relative to the stator, a sort of disconnection between the modes associated with rotor resistances and the mode associated with R_a occurs. This one is associated with a new time constant τ_a and results in a frequency f_e component in d q variables, mainly translated by an aperiodic component of the current in abc variables.

On the contrary, modes associated with rotor resistances are mainly aperiodic in d q variables, thus pseudoperiodic in abc variables. These two types of mode normally correspond to different frequency bands and they interact only slightly. We should note that the distinction made in this section is only approximate.

In this way, we have not considered the existence, even though it is real if the machine is not perfectly symmetrical, of a pseudoperiodic double frequency $2f_e$ component of the electric frequency in abc variables.

We can simplify the interpretation of results by using a source of voltage (battery) to power the field winding. In this case, the test makes it possible to immediately determine time constants T'_d, T''_d, T'_q, and T''_q.

2.2.2.2.2. Load shedding test

The load shedding test consists of abruptly opening the startor's circuit while it receives a non-zero current. In this test, R_a and voltage sources are not involved, making the interpretation of results easier. We notice that L_a is not involved either (except for a high-frequency transient during current breaking).

For a linear study, we must conduct this test with reduced excitation current, again using batteries to power the field winding, thus making the interpretation of results easier. In this case, the test makes it possible to immediately determine time constants T'_{d0}, T''_{d0}, T'_{q0}, and T''_{q0}.

2.2.2.3. *Transient standstill test*

In the standstill machine, voltage sources present in the equivalent circuit cancel out, making the processing of experimental data easier. In particular, there is no emergence of components at frequencies other than the one present in the excitation.

2.2.2.3.1. Direct current decay test

The decay test of a direct current in the armature is done by direct current powering of a circuit made up of two phases of the armature series connected, or the circuit obtained between one armature's terminal and the two other terminals together connected. We usually position the rotor in such a way that the resulting armature current has only the direct-axis component or only the quadrature-axis component. The test can also be done by the field winding.

We then short-circuit the power supply (most often presuming that we have connected a series additional resistance to limit its short-circuit current) and we can observe the evolution of the current in the short-circuited winding (or the voltage or current of the other winding). The observed quantity can be broken down into a sum of decreasing exponential functions. Unfortunately, since all components are aperiodic, the effect of resistance R_a do not separate during the transient three-phase short-circuit test as well. Time constants observed are therefore not identical to the ones we can identify by similar tests on the rotating machine!

2.2.2.3.2. Frequency response static tests

An interesting method consists of powering the standstill machine with a single-phase alternating voltage. By changing the frequency, we can directly get the frequency response of the machine. We can also obtain this frequency response by applying a signal that has numerous frequencies (white noise) to the machine and by analyzing the device response.

Knowing the frequency response, the circuit theory techniques help us to sequentially determine the equivalent circuit elements.

We shall avoid determining all the elements of the equivalent circuit with this method (which would, in fact, force the deletion of the Canay inductance since it makes no sense in a linear model). We would have to correct the frequency responses to eliminate the known parameters from the part that they characterize, which is not always easy.

Another possibility consists of identifying the parameters by non-linear regression by only looking at unknown parameters, which can be done by comparing the calculation results with the experience in the frequency domain (we will later see that the process can also be used in the temporal domain).

2.2.3. *Potier Method*

2.2.3.1. *Insufficiency of the linear theory*

Until now, we presumed that the machine was not saturated in our description of tests. This hypothesis is not realistic, as the magnetic characteristic (Figure 2.9) shows: the magnetic characteristic slightly deviates from the "air-gap characteristic" for voltage values neighboring normal use voltage. Because of this, when we calculate the excitation current and the internal angle necessary to achieve a given steady state with nominal voltage, the results obtained by using the linear model are significantly different from experimental values.

2.2.3.2. *Potier Model*

Saturation was first studied in the smooth pole machines. We know that in steady state and in the linear case we can use a single equivalent circuit, a combination of axis d and q circuits, in which the quantities are phasors. The Potier model (Figure 2.6) consists of presuming that this property remains valid if the magnetization impedance $\overline{Z}_{\mu}$ is non-linear, by accepting that the value of this impedance only depends on the modulus of its flux $\overline{\Psi}_{\mu}$ or current $\overline{i}^{\mu}$, and that these two last phasors have the same argument.

The practical advantage of the Potier model is that it only involves a function of a single variable to represent saturation, in other words:

$$\psi_{\mu} = \psi_{\mu}\,(i^{\mu}) \qquad [2.98]$$

with:

$$\psi_{\mu} = \sqrt{\psi_{\mu d}{}^{2} + \psi_{\mu q}{}^{2}} \qquad [2.99a]$$

and:

$$i^{\mu} = \sqrt{\left(i^{\mu d}\right)^{2} + \left(i^{\mu q}\right)^{2}} \qquad [2.99b]$$

Fluxes $\psi_{\mu d}$ and $\psi_{\mu q}$ depend on currents $i^{\mu d}$ and $i^{\mu q}$ by the relations:

$$\psi_{\mu d} = \frac{\psi_\mu}{i^\mu}\, i^{\mu d} \qquad [2.100]$$

$$\psi_{\mu q} = \frac{\psi_\mu}{i^\mu}\, i^{\mu q} \qquad [2.101]$$

By introducing saturation factor K_s ($0 \le K_s \le 1$) and non-saturated magnetization inductance $L_{\mu\,0}$, we have:

$$\frac{\psi_\mu}{i^\mu} = K_s\, L_{\mu 0} \qquad [2.102a]$$

We can also define the saturation factor in such a way that it will tend toward 0 when the magnetizing current tends toward the infinite. In order to do that, we must consider the value toward which the magnetization inductance tends for this limit, or $L_{\mu\,\infty}$, and treat it separately. We then write:

$$\frac{\psi_\mu}{i^\mu} = K_s \left(L_{\mu 0} - L_{\mu\infty} \right) + L_{\mu\infty} \qquad [2.102b]$$

We will see later that this second definition is more suited to the extension of the model to salient pole machines.

Finally, we can write the fluxes in the following form:

$$\psi_{\mu d} = K_s(i^\mu)\, L_{\mu 0}\, i^{\mu d} = L^*_{\mu s}\, i^{\mu d} \qquad [2.103a]$$

or:

$$\psi_{\mu d} = \left[K_s\left(i^\mu\right)\left(L_{\mu 0} - L_{\mu\infty}\right) + L_{\mu\infty} \right] i^{\mu d} = L^*_{\mu s}\, i^{\mu d} \qquad [2.103b]$$

$$\psi_{\mu q} = K_s\left(i^\mu\right) L_{\mu 0}\, i^{\mu q} = L^*_{\mu s}\, i^{\mu q} \qquad [2.104a]$$

or:

$$\psi_{\mu q} = \left[K_s\left(i^\mu\right)\left(L_{\mu 0} - L_{\mu\infty}\right) + L_{\mu\infty} \right] i^{\mu q} = L^*_{\mu s}\, i^{\mu q} \qquad [2.104b]$$

The saturated inductance factor $L^*_{\mu s}$ is called "static inductance", defined as the ratio between a saturated flux and a current. This static inductance is obtained by modifying its non-saturated $L_{\mu 0}$ value by a saturation K_s factor evaluated from the single non-linear characteristic of the machine. We must not conclude, however, that multiplying non-saturated inductances by the factor K_s is enough to obtain valid dynamic equations in saturated mode. In fact, incremental inductances $L_{\mu d}$ and $L_{\mu q}$ are not equal to $L^*_{\mu s}$, or even between themselves, and there is a mutual inductance between axes d and q in the Potier model that would not emerge this way. By using definition [2.6] of incremental inductances and after a few obvious developments, we obtain:

$$L_{\mu d} = \frac{\psi_\mu}{i^\mu} + \left(\frac{i^{\mu d}}{i^\mu}\right)^2 \left[\frac{\partial \psi_\mu}{\partial i^\mu} - \frac{\psi_\mu}{i^\mu}\right] \quad [2.105a]$$

$$L_{\mu q} = \frac{\psi_\mu}{i^\mu} + \left(\frac{i^{\mu q}}{i^\mu}\right)^2 \left[\frac{\partial \psi_\mu}{\partial i^\mu} - \frac{\psi_\mu}{i^\mu}\right] \quad [2.105b]$$

$$L_{\mu dq} = \frac{i^{\mu d}}{i^\mu}\,\frac{i^{\mu q}}{i^\mu} \left[\frac{\partial \psi_\mu}{\partial i^\mu} - \frac{\psi_\mu}{i^\mu}\right] \quad [2.105c]$$

We can immediately verify that the mutual inductance [2.105c] is cancelled in the linear case.

2.2.3.3. *Experimental determination of Potier model parameters*

The major part of the information necessary to determine the Potier model is already provided by the no-load test. In addition, values of R_a and R_μ (function of E) are also determined by classical tests. Only two parameters, X_a and α, remain to be determined.

We must note that this determination must involve additional tests when the machine is saturated. In fact, when the machine is not saturated, the number of parameters in the equivalent circuit of Potier is redundant because this circuit is equivalent to the Behn-Eschenburg circuit, which has less parameters. In order to determine X_a with precision, the additional tests must also involve a highly reactive current because the current's reactive component is the only one that leads to a significant difference between the value of voltage U and the value of electromotive force E because of the X_a effect. The steady-state short-circuit test can be counted among the useful tests as in this R_a is normally small in relation to X_a and the current is then highly reactive (in relation to $\overline{E}$).

For given X_a and α values, it is easy to calculate, from the values of the voltage and current (defined in norm and phase), the value of E and thus of the magnetizing current and finally the excitation current i^f necessary to achieve this mode. It is therefore possible, having recorded the value of the excitation current for a few modes satisfying the conditions described above, to find by non-linear regression the value of parameters X_a and α. The procedure followed can be graphically represented if the additional tests are done with a single armature current where the component of axis d is the only one that is different from 0 (by ignoring the effect of resistances R_a and R_μ, it is a purely reactive current). These tests are called "zero-power-factor" tests. In order to do this, we add to the same diagram as the magnetic characteristic the graph of relation $U - i^f$ corresponding to this armature current (a few points are sufficient). In this case, the difference between real and effective excitation currents is constant (let it be $\alpha\ I$). If the voltage drop $R_a\ I$ is insignificant compared to U, the difference between voltage and electromotive force (which then is equal to $X_a\ I$) is also insignificant. The second curve is therefore the translated curve of the magnetic characteristic. We can then graphically look for (e.g. by using a drawing) the value of this translation. We note that it is only defined uniquely if both graphs are curved, demanding that the zero-power-factor tests sufficiently cover the "saturation elbow". Once the translation for going from one diagram to the other is determined, X_a and α values are deducted immediately (Figure 2.13).

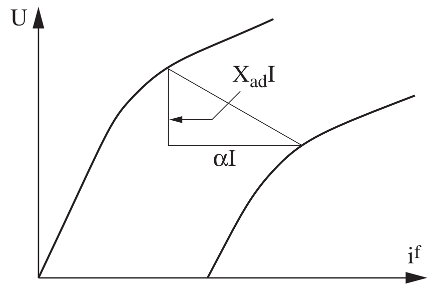

Figure 2.15. *Determination of X_a and α*

There is a conventional construction that only involves one zero-power-factor test, in addition to a point deducted from the short-circuit characteristic. We will not describe this procedure here because it is not as precise as the previous one as the hypothesis of a purely axis d current is less satisfied during the short-circuit test.

2.2.3.4. *Necessity for a more general theory in the presence of saturation and magnetic saliences*

In the case of smooth pole machines, experience shows that the Potier model makes it possible to accurately determine the excitation current necessary to achieve a given operation mode and the corresponding internal angle. In the search for a theory to account for saturation in the presence of salient poles, it is normal to demand that this theory use the Potier theory as a limit in the case where the salience becomes insignificant. Some of the theories proposed, such as Blondel, do not satisfy this condition. Experience actually shows that these theories are not very accurate. We will attempt to rationally develop a satisfying theory in the next part of this chapter.

2.3. Advanced models: the synchronous machine in saturated mode

2.3.1. *Elements of the von der Embse theory of saturated electrical machines: inductive circuits in the presence of magnetic saturation*

In the general theory presented in section 2.1.2, the fluxes play a major role. However, it is often more useful to reason in terms of inductances. These were defined by [2.06]. Fluxes can be expressed in function of the inductances in the form [VON 68]:

$$\psi_j = \sum_k L_{jk}\, i^k + \psi_{j0} \qquad [2.106]$$

where ψ_{j0} verify the equation:

$$\frac{\partial \psi_{j0}}{\partial i^k} = -\sum_k \frac{\partial L_{jk}}{\partial i^k}\, i^k \qquad [2.107]$$

Magnetic coenergy can also be developed in function of the currents such as:

$$w_{cm} = \frac{1}{2}\sum_{jk} L_{jk} i^j i^k + \sum_j \psi_{j0}\, i^j + w_{00} \qquad [2.108]$$

where the last term verifies:

$$\frac{\partial w_{00}}{\partial i^m} = \frac{1}{2}\sum_{jk} \frac{\partial L_{jk}}{\partial i^m}\, i^j\, i^k \qquad [2.109]$$

Equations [2.106] to [2.109] are the generalization of classical expressions of coupled circuits considered as linear to saturated circuits.

By using definitions [2.6] of inductances, the induced voltage during a current variation is written as:

$$\frac{d\,\psi_j}{dt} = \sum_k L_{jk} \frac{d\,i^k}{dt} \qquad [2.110]$$

which is formally identical for saturated and non-saturated circuits.

2.3.2. *General study of magnetic coupling in the presence of saturation*

After a Park transformation, two types of coupling should be considered between the elements of the equivalent circuit of the synchronous machine. The first type involves the circuits aligned on the same axis, that is, circuits d, D, and f on one side, and q and Q on the other. The second type involves equivalent circuits located on different axes, that is, coupling between magnetization inductances of both axes.

The study of these two specific cases was detailed in [GAR 88]. We will also use elements from the website associated with [MAT 04] below.

2.3.2.1. *Transformer effect coupling*

We now consider a set of electric circuits highly coupled, such as the primary and the secondary of a transformer.

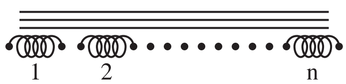

Figure 2.16. *N winding transformer*

Induced voltage is given by [2.110] where L_{jk} (i^1, i^2 . . . . i^n) must be obtained experimentally by incremental tests covering the whole range of the system operation.

This general approach requires a large number of tests and is not practical even for a small number of coupled circuits.

To handle such a problem, we try to describe parameters L_{jk} according to a single magnetic characteristic representing the non-linearities, along with a few constant parameters that can be determined with the help of a small number of tests.

This objective is satisfied if the system can be represented by an equivalent circuit containing a magnetization branch that considers saturation and n leakage inductances assumed linear, as illustrated in Figure 2.17.

We will now examine the conditions under which this possibility exists.

The model in Figure 2.17 presupposes the existence of a magnetizing current i^{μ}, which is a linear combination of circuit currents:

$$i^{\mu} = \sum_{k} \frac{i^{k}}{k_{k}} \tag{2.111}$$

such that the system's magnetic coenergy takes the form:

$$w_{cm} = f\left(i^{\mu}\right) + \sum_{jk} \ell_{jk}\, i^{j}\, i^{k} \tag{2.112}$$

where $\ell_{jk} = \ell_{jk}$ are constants.

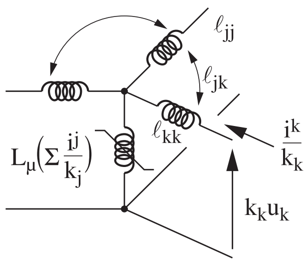

Figure 2.17. *Simplified model of an n winding transformer*

We are not losing generality by favoring one of the circuits and by presuming:

$$k_{1} = 1 \tag{2.113}$$

If we derive [2.112] along i^{j}, we obtain flux ψ_{j}:

$$\psi_{j} = \frac{\partial f}{\partial i^{\mu}} \frac{1}{k_{j}} + \sum_{k} \ell_{jk}\, i^{k} \tag{2.114}$$

If the equation relative to $k = 1$ is used to eliminate $\partial f / \partial i^{\mu}$, we get:

$$\psi_j - \frac{1}{k_j}\psi_1 = \sum_k \left(\ell_{jk} - \frac{1}{k_j}\ell_{1k} \right) i^k \qquad [2.115]$$

It is possible to write the second member in function of the inductance coefficients. In this way, deriving equation [2.115] along i^k, one gets:

$$\ell_{jk} - \frac{1}{k_j}\ell_{1k} = L_{jk} - \frac{1}{k_j}L_{1k} \qquad [2.116]$$

In this way, equation [2.115] becomes:

$$\psi_j - \frac{1}{k_j}\psi_1 = \sum_k \left(L_{jk} - \frac{1}{k_j}L_{1k} \right) \mathrm{i}^k \qquad [2.117]$$

It is a well-known fact that this equation is satisfied in linear circuits for any k_j value and that there is then no way to define uniquely the equivalent circuit in Figure 2.17.

However, for saturated circuits, according to [2.106], condition [2.117] is satisfied only if:

$$k_j = \frac{\psi_{10}}{\psi_{j0}} = \text{cste} \qquad [2.118]$$

which sets in a unique way the transformation ratios and leakage inductances of the model.

In practice, these conditions are well verified for highly coupled circuits as in a transformer. We will suppose that they are also well verified for circuits d and q of rotating machines taken separately.

The equivalent circuit in Figure 2.17 can easily be simplified in the case of a two windings transformer. In fact, it is possible to include mutual inductance $\ell_{12} = \ell_{21}$ in the magnetization branch and the equivalent circuit takes the form represented in Figure 2.18.

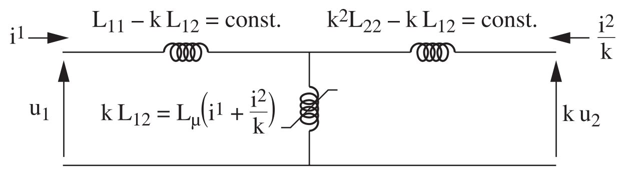

Figure 2.18. *Equivalent circuit of a saturated two windings transformer such that $k = \psi_{10}/\psi_{20}$ is a constant*

The fact that the saturation unequivocally determines the equivalent circuit of a two windings transformer clearly appears in the conventional Potier triangle of synchronous machines.

For a three windings transformer, it is generally not possible to include all the mutual inductance terms in the magnetization branch. However, if the mutual inductances between one of the circuits and the two others can be presumed identical, that is:

$$k_3\, L_{13} = k_2\, L_{12} \qquad [2.119]$$

some simplifications of the same type emerge and we obtain Canay's equivalent circuit of a three windings transformer.

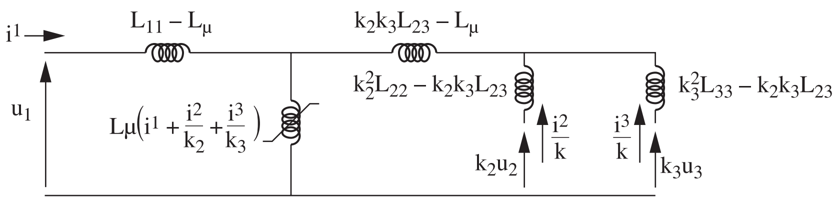

Figure 2.19. *Canay equivalent circuit*

The simplest model of saturated electrical machines is based on the presumption that condition [2.118] is satisfied.

That is not always the case and more sophisticated models, for example, saturable leak inductances, could be developed.

2.3.2.2. *Cross-saturation coupling*

2.3.2.2.1. General study

Saturation of electrical machines leads to a specific sort of magnetic coupling that disappears in linear operation: coupling between quadrature circuits where magnetic circuits occupy a common space.

Even though only the parallel branch is the basis of non-linearities in the synchronous machine models considered in this chapter, we will carry out a general study in this section. We will not use the "μ" index in the following study; it will have to be added to equations to customize them to the case of synchronous machine modeling. The situation involved in this section is the one represented in Figure 2.20.

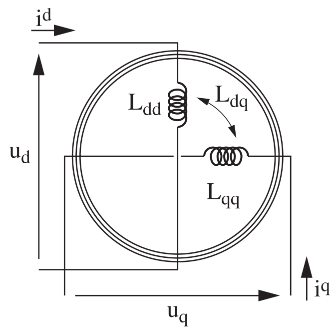

Figure 2.20. *Coupling between quadrature circuits by cross-saturation*

Condition [2.118] is not satisfied in this case and, in order to describe saturation according to a simple magnetic characteristic, very different techniques must be used.

In order to develop these techniques, we will first write the fluxes in the following form:

$$\psi_d\left(i^d, i^q\right) = \psi_d\left(i^d, 0\right) + \Delta\psi_d\left(i^d, i^q\right) \qquad [2.120a]$$

$$\psi_q\left(i^d, i^q\right) = \psi_q\left(0, i^q\right) + \Delta\psi_q\left(i^d, i^q\right) \qquad [2.120b]$$

where the last terms represent cross-saturation.

If the fluxes are presumed to be analytical functions, a serial development of the last terms is possible:

$$\Delta\psi_d\left(i^d, i^q\right) = \sum_m \frac{1}{m!}\left(\frac{\partial^m \psi_d}{(\partial i^q)^m}\right)_{i^q=0} (i^q)^m \qquad [2.121a]$$

$$\Delta\psi_q\left(i^d, i^q\right) = \sum_m \frac{1}{m!}\left(\frac{\partial^m \psi_q}{(\partial i^d)^m}\right)_{i^q=0} (i^d)^m \qquad [2.121b]$$

If d and q are the machine's axes of symmetry, the cross-saturation effect must satisfy certain symmetry conditions and certain limit conditions:

$$\begin{array}{ll} \Delta\psi_d\left(i^d, i^q\right) = \Delta\psi_d\left(i^d, -i^q\right) & \Delta\psi_q\left(i^d, i^q\right) = \Delta\psi_d\left(-i^d, i^q\right) \\ \Delta\psi_d\left(i^d, i^q\right) = -\Delta\psi_d\left(-i^d, i^q\right) & \Delta\psi_q\left(i^d, i^q\right) = -\Delta\psi_d\left(i^d, -i^q\right) \end{array} \qquad [2.122a]$$

$$\begin{array}{ll} \lim\limits_{i^d \to 0} \Delta\psi_d\left(i^d, i^q\right) = 0 & \lim\limits_{i^q \to 0} \Delta\psi_q\left(i^d, i^q\right) = 0 \\ \lim\limits_{i^d \to \infty} \Delta\psi_d\left(i^d, i^q\right) = 0 & \lim\limits_{i^q \to \infty} \Delta\psi_q\left(i^d, i^q\right) = 0 \end{array} \qquad [2.122b]$$

By using these conditions and by defining mutual inductances by [2.6], or:

$$L_{dq} = \frac{\partial \Delta\psi_d}{\partial\, i^q} = \frac{\partial \Delta\psi_q}{\partial\, i^d} = L_{qd} \qquad [2.123]$$

it is possible to conclude that:

$$L_{dq}\left(i^d, i^q\right) = -2A\, i^d\, i^q \left[1 - B_d\left(i^d\right)^2 - B_q\left(i^q\right)^2 + \ldots\right] \qquad [2.124]$$

where A, B_d, and B_q are constants and where the series present in [2.124] tends toward 0 faster than $1/i^d$ or $1/i^q$.

2.3.2.2.2. The UCL model

Some simplifications were used in the past to study cross-saturation.

The Blondel approximation ignores all the effects, including the first coupling term:

$$\mathrm{A} = 0 \qquad [2.125]$$

The armature reaction theory of conventional DC machines limits L_{dq} to the terms of the second order by saying:

$$\mathrm{B_d} = 0 \text{ and } \mathrm{B_q} = 0 \qquad [2.126]$$

We will expose a model that goes farther next, which we will call "model UCL" [DEJ 91, DEJ 92, GAR 88a, GAR 88b]. In order to continue the development of [2.124] beyond [2.126], we come up against the fact that polynomial developments like [2.124] are not well suited to represent magnetic saturation in a practical way. In fact, magnetic characteristics present a skewed asymptote, whereas polynomials behave very differently, except if they include an infinite number of terms with alternate signal factors. We will then abandon this parameterization, except for factor A, which has already been found interesting for characterizing cross-saturation. In order to characterize the mutual inductance [2.124], we will replace the polynomial between parentheses by a non-linear function. Since a two variable non-linear function is quite difficult to identify, we will hypothesize that we can represent this function with the help of a non-linear function of only one variable, and we will write:

$$L_{\mathrm{dq}}\left(i^{\mathrm{d}}, i^{\mathrm{q}}\right) = -2\,\mathrm{A}\, i^{\mathrm{d}}\, i^{\mathrm{q}}\, \mathrm{F''}\left[\mathrm{B_d}\left(i^{\mathrm{d}}\right)^2 + \mathrm{B_q}\left(i^{\mathrm{q}}\right)^2\right] \qquad [2.127]$$

where we impose the following conditions to function F"(x):

$$\mathrm{F''}(0) = 1 \qquad [2.128]$$

$$\left.\frac{\partial\, \mathrm{F''(x)}}{\partial\, \mathrm{x}}\right|_{x=0} = 1 \qquad [2.129]$$

in order to retain the definition of parameters A, $\mathrm{B_d}$, and $\mathrm{B_q}$.

Since L_{dq} can be obtained by deriving ψ_{d} along i^{q}, we can find the form of ψ_{d} by integrating [2.127] according to i^{q}. We get:

$$\psi_{\mathrm{d}}\left(i^{\mathrm{d}}, i^{\mathrm{q}}\right) = -\frac{\mathrm{A}}{\mathrm{B_q}}\, i^{\mathrm{d}} \left\{\mathrm{F'}\left[\mathrm{B_d}\left(i^{\mathrm{d}}\right)^2 + \mathrm{B_q}\left(i^{\mathrm{q}}\right)^2\right] - \mathrm{F'}\left[\mathrm{B_d}\left(i^{\mathrm{d}}\right)^2\right]\right\} + \mathrm{f}\left(i^{\mathrm{d}}\right) \qquad [2.130]$$

where F'(x) is the indefinite integral of F"(x) and f (i^d) a function of i^d alone to be determined. To completely define F'(x), we write:

$$\mathrm{F}'(\infty) = 0 \qquad [2.131]$$

To take it a step further, we will introduce a new physical hypothesis, which is that the magnetic circuits associated with currents i^d and i^q share the same domain in space. Mathematically, we can introduce this condition by saying that, when one of the currents tends toward the infinite, the system becomes linear.

When this hypothesis is made, function F"(x) does not settle for characterizing the mutual inductance; it almost completely defines the magnetic behavior of the device, as we will now show.

In fact, according to this hypothesis, when i^q tends toward the infinite and because of [2.131], [2.130] becomes:

$$L_{d\infty}\, i^d = \frac{\mathrm{A}}{\mathrm{B_q}}\, i^d\, \mathrm{F}'\left[\mathrm{B_d}\left(i^d\right)^2\right] + \mathrm{f}\left(i^d\right) \qquad [2.132]$$

By extracting from [2.132] the expression of f(i^d) and by substituting it in [2.130], we get an expression of ψ_d that only has one more parameter than [2.126] which is:

$$\psi_d\left(i^d, i^q\right) = \left\{L_{d\infty} - \frac{\mathrm{A}}{\mathrm{B_q}}\,\mathrm{F}'\left[\mathrm{B_d}\left(i^d\right)^2 + \mathrm{B_q}\left(i^q\right)^2\right]\right\} i^d \qquad [2.133]$$

We can of course create the same development for circuit "q", leading to:

$$\psi_q\left(i^d, i^q\right) = \left\{L_{q\infty} - \frac{\mathrm{A}}{\mathrm{B_d}}\,\mathrm{F}'\left[\mathrm{B_d}(i^d)^2 + \mathrm{B_q}(i^q)^2\right]\right\} i^q \qquad [2.134]$$

The magnetic characteristics inherent to each circuit are therefore:

$$\psi_d\left(i^d, 0\right) = \left\{L_{d\infty} - \frac{\mathrm{A}}{\mathrm{B_q}}\,\mathrm{F}'\left[\mathrm{B_d}(i^d)^2\right]\right\} i^d \qquad [2.135]$$

$$\psi_q\left(0, i^q\right) = \left\{L_{q\infty} - \frac{\mathrm{A}}{\mathrm{B_d}}\,\mathrm{F}'\left[\mathrm{B_q}(i^q)^2\right]\right\} i^q \qquad [2.136]$$

The differential inductances in terms of new parameters are easily obtained by deriving [2.133] and [2.134] in relation to i^d and i^q, respectively, or:

$$\begin{aligned} \mathrm{L_d}\left(\mathrm{i^d},\mathrm{i^q}\right) = \mathrm{L_{d\infty}} - \frac{\mathrm{A}}{\mathrm{B_q}}\,\mathrm{F'}\left[\mathrm{B_d}\left(\mathrm{i^d}\right)^2 + \mathrm{B_q}\left(\mathrm{i^q}\right)^2\right] \\ -\frac{2\,\mathrm{B_d A}}{\mathrm{B_q}}\left(\mathrm{i^d}\right)^2 \mathrm{F''}\left[\mathrm{B_d}\left(\mathrm{i^d}\right)^2 + \mathrm{B_q}\left(\mathrm{i^q}\right)^2\right] \end{aligned} \qquad [2.137]$$

$$\begin{aligned} L_{\mathrm{q}}\left(i^{\mathrm{d}},i^{\mathrm{q}}\right) = L_{\mathrm{q}\infty} - \frac{\mathrm{A}}{\mathrm{B_d}}\,\mathrm{F'}\left[\mathrm{B_d}\left(i^{\mathrm{d}}\right)^2 + \mathrm{B_q}\left(i^{\mathrm{q}}\right)^2\right] \\ -\frac{2\,\mathrm{B_q A}}{\mathrm{B_d}}\left(i^{\mathrm{q}}\right)^2 \mathrm{F''}\left[\mathrm{B_d}\left(i^{\mathrm{d}}\right)^2 + \mathrm{B_q}\left(i^{\mathrm{q}}\right)^2\right] \end{aligned} \qquad [2.138]$$

to which we add [2.127]:

$$L_{\mathrm{dq}}\left(i^{\mathrm{d}},i^{\mathrm{q}}\right) = -\,2\,\mathrm{A}\,i^{\mathrm{d}}\,i^{\mathrm{q}}\,\mathrm{F''}\left[\mathrm{B_d}\left(i^{\mathrm{d}}\right)^2 + \mathrm{B_q}\left(i^{\mathrm{q}}\right)^2\right] \qquad [2.127]$$

If we have experimental values of quantities [2.133] and [2.134] or [2.135] and [2.136] or even [2.137], [2.138], and [2.127], and a non-linear regression program, we can identify parameters $L_{\mathrm{d}\infty}$, $L_{\mathrm{q}\infty}$, A, $\mathrm{B_d}$, and $\mathrm{B_q}$, as well as function F'(x) and its derivative F"(x). In order for the asymptotic behavior of fluxes [2.135] and [2.136] to be correct, the last term of these expressions must tend toward a constant when the current in the circuit involved tends toward the infinite. This condition forces an additional condition on function F'(x). By combining this condition and [2.128] and [2.129] previously encountered, we have:

$$\mathrm{F''}(0) = 1 \qquad [2.128]$$

$$\left.\frac{\partial\,\mathrm{F''(x)}}{\partial\,x}\right|_{x=0} = 1 \qquad [2.129]$$

$$\lim_{x\to\infty} \sqrt{x}\;\mathrm{F'(x)} = C \qquad [2.139]$$

where C is a parameter inherent to function F'(x).

Condition [2.139] leads to the validity of [2.131], which we do not have to consider anymore. It also results in function F"(x) tending toward 0 in the infinite as $\mathrm{x}^{-3/2}$.

If we have reasonable knowledge of both characteristics [2.135] and [2.136], we can determine three of the parameters, $L_{d\infty}$, $L_{q\infty}$, and:

$$K = \sqrt{\frac{B_d}{B_q}} \qquad [2.140]$$

without having to determine the others. In fact, from [2.135] and [2.136] we can deduce:

$$\psi_d\left(i^d, 0\right) - L_{d\infty}\, i^d = -\frac{A}{B_q} F'\left[B_d\left(i^d\right)^2\right] i^d \qquad [2.141]$$

$$\psi_q\left(0, i^q\right) - L_{q\infty}\, i^q = -\frac{A}{B_d} F'\left[B_q\left(i^q\right)^2\right] i^q \qquad [2.142]$$

The comparison of right members of [2.141] and [2.142] shows that, with definition [2.140], we have:

$$K[\psi_q(0, K\, i) - L_{q\infty}\, i^q] = \psi_d(i^d, 0) - L_{d\infty}\, i^d \qquad [2.143]$$

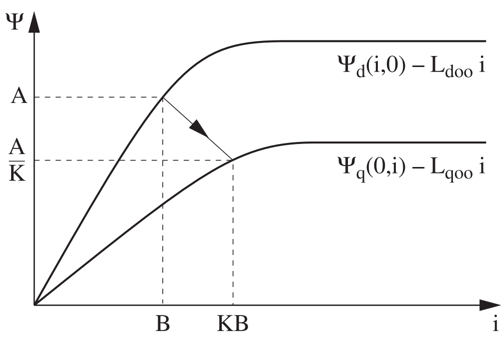

Figure 2.21. *Relation between the magnetic characteristics of axis d and q*

This relation means that the two magnetic characteristics, after subtraction from a linear term, can be led in coincidence by a multiplication of the abscissas of factor K and a division of ordinates by the same factor K, as shown in Figure 2.21.

Reciprocally, if we know one of the magnetic characteristics and the three parameters $L_{d\infty}$, $L_{q\infty}$ and K, it is possible to determine the other magnetic characteristic. We should also note the interesting relation that we can deduce from [2.137], [2.138], and [2.127]:

$$\frac{L_{\mathrm{d}}(0,0)-L_{\mathrm{d}\infty}}{L_{\mathrm{q}}(0,0)-L_{\mathrm{q}\infty}}=K^2 \qquad [2.144]$$

that can be used to determine K once $L_{\mathrm{d}\infty}$ and $L_{\mathrm{q}\infty}$ are known.

2.3.2.2.3. Isotropic inductance

We have seen in section 2.2.3.2 that the Potier theory helped us in defining a single magnetic characteristic function of a single current i^{μ} (or i since we do not use indices μ in this section).

Similarly, in the UCL model, the non-linear function used is a function of current $[B_{\mathrm{d}}\,(i^{\mathrm{d}})^2 + B_{\mathrm{q}}\,(i^{\mathrm{q}})^2]^{1/2}$. We can make this expression more symmetrical and get closer to the Potier model by replacing circuit "q" by circuit "q'" defined by the variable change:

$$i^{\mathrm{q}'}=\frac{i^{\mathrm{q}}}{K} \qquad [2.145\mathrm{a}]$$

that will be accompanied by a redefinition of the flux so that we can speak of a circuit change as defined in section 2.1:

$$\psi_{\mathrm{q}'}=K\,\psi_{\mathrm{q}} \qquad [2.145\mathrm{b}]$$

In formula [2.145], K is the salience factor defined by [2.140].

The current that controls magnetic saturation can then be replaced by:

$$i'=\left[\left(i^{\mathrm{d}}\right)^2+\frac{1}{K^2}\left(i^{\mathrm{q}}\right)^2\right]^{1/2}=\left[\left(i^{\mathrm{d}}\right)^2+\left(i^{\mathrm{q}'}\right)^2\right]^{1/2} \qquad [2.146\mathrm{a}]$$

and the flux associated with this current is:

$$\psi'=\left[\ \psi_{\mathrm{d}}{}^2+K^2\,\psi_{\mathrm{q}}{}^2\right]^{1/2}=\left[\psi_{\mathrm{d}}{}^2+\psi_{\mathrm{q}'}{}^2\right]^{1/2} \qquad [2.146\mathrm{b}]$$

Another way to obtain a symmetrical expression, dual from the previous one, consists of keeping the axis q circuit unchanged but replacing circuit "d" by circuit "d"" defined by:

$$i^{d''} = K\, i^{d} \tag{2.147a}$$

which will be accompanied by a redefinition of the flux:

$$\psi_{d''} = \frac{\psi_d}{K} \tag{2.147b}$$

where K is once more the salience factor defined by [2.140].

The current that controls magnetic saturation can then be replaced by:

$$i'' = \left[\, K^2 \left(i^{d}\right)^2 + \left(i^{q}\right)^2 \right]^{1/2} = \left[\left(i^{d''}\right)^2 + \left(i^{q}\right)^2\right]^{1/2} \tag{2.148a}$$

and the flux associated with this current is:

$$\psi'' = \left[\frac{1}{K^2}\,\psi_d{}^2 + \psi_q{}^2\right]^{1/2} = \left[\psi_{d''}{}^2 + \psi_q{}^2\right]^{1/2} \tag{2.148b}$$

We can then get an isotropic inductance in two ways by considering the non-linear relation between ψ' and i', or between ψ'', and i''.

As in the Potier theory, the behavior of this inductance is completely defined by the data of its non-saturated values, of its completely saturated values, and of a single saturation factor that we can considered as a function of one of the variables ψ', i', ψ'', or i'', the choice being one of convenience.

In the first case, we arrive at:

$$\psi_d = [K_s(i')\,(L_{d0} - L_{d\infty}) + L_{d\infty}]\, i^{d} \tag{2.149}$$

$$\psi_{q'} = [K_s(i')\,(L_{d0} - L_{d\infty}) + L_{d\infty}]\, i^{q'} \tag{2.150}$$

Or, by going back to original quantities:

$$\psi_{\mathrm{d}} = [K_{\mathrm{s}}(i')\,(L_{\mathrm{d0}} - L_{\mathrm{d}\infty}) + L_{\mathrm{d}\infty}]\, i^{\mathrm{d}} \quad [2.151]$$

$$\psi_{\mathrm{q}} = [K_{\mathrm{s}}(i')\,(L_{\mathrm{q0}} - L_{\mathrm{d}\infty}) + L_{\mathrm{q}\infty}]\, i^{\mathrm{q}} \quad [2.152]$$

We get the same result by the dual transformation, except that K_{s} then appears as a function of i''.

In this way, the UCL model makes it possible, as in the Potier model, to express the saturated inductances according to non-saturated inductances by using a common saturation factor.

2.3.3. *Implementation of the model*

In order for the models above to be useful in practice, we must show that we can determine their parameters by experimental tests, and that they can be used to predict the behavior of the machine in common operation modes.

2.3.3.1. *Leakage flux*

We have seen in section 2.2.2.1 that considering non-linearities makes it possible to separate leakage inductances from magnetization inductances. We can apply the same principle to direct and quadrature circuits. There is still a degree of arbitrary that comes from the existence of the mutual coupling between leakage inductances (e.g. for circuit d, between the leakage inductances of stator, of damper, and of field winding). This degree of freedom can be used to cancel one of the mutual inductances, for example, the mutual inductance between the leakage inductance of the stator and of the field winding. In this case, there will still be two non-zero mutual inductances. We usually presume that cancelling the mutual inductance between stator and field winding leakages simultaneously cancels the mutual inductance between the stator and damper leakages. The only remaining mutual inductance is the mutual inductance between damper and field winding leakages. This mutual inductance can be considered by introducing the Canay inductance in the circuit (see Figure 2.4).

The Canay inductance thus has a well-defined meaning in the case of a non-linear model. On the other hand, we have already mentioned, in the linear case, that the Canay inductance cannot be defined by experimental tests only because there is a circuit equivalence allowing grouping this inductance and the stator leakage inductance into a single inductance that we can put either upstream or downstream from the magnetization inductance.

The determination of the stator leakage inductance L_{ad} can be done easily by using "zero-power-factor" tests and the method seen in section 2.1.4.3. We simultaneously determine relation α.

The stator quadrature leakage inductance L_{aq} raises more problems. There is actually no physical reason to suppose that it is equal to L_{ad} because leakage inductances L_{ad} and L_{aq} both contain a contribution from the air-gap's leaks, and these leaks are not identical according to both axes because the air-gap width is not the same in a salient pole machine.

In order to determine L_{aq}, we must use tests with a significant quadrature component for the current because this component can produce a significant voltage drop in this inductance. In fact, these are tests to get the quadrature magnetic characteristic. From these tests, we can determine the non-linear part of this characteristic, but in steady state, inductance L_{aq} is combined with another linear inductance or in other words $L_{\mu d\infty}$. To separate L_{aq}, we must use either tests where the current is not aligned with the axes or transient tests.

This program was never completely executed. However, if we simplify the problem by saying $L_{aq} = L_{ad}$ and by ignoring inductances $L_{\mu d\infty}$ and $L_{\mu q\infty}$ (since this last hypothesis only has consequences for the study of modes where the magnetizing current uses abnormally high values), the determination of the model is no longer problematic. Figures 2.22 and 2.23, obtained by carrying out several tests on an 11.4 MVA salient pole alternator, [DEJ 92] show that the resulting model, despite simplifications, is very superior to usual models.

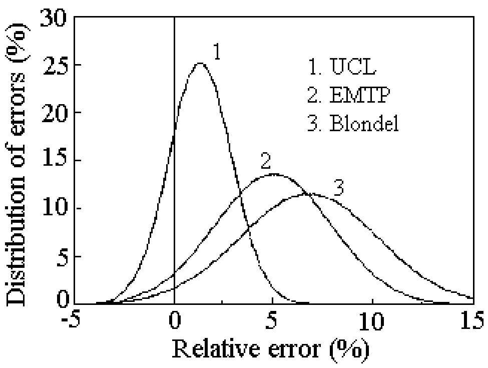

Figure 2.22. *Distribution of errors in the excitation current*

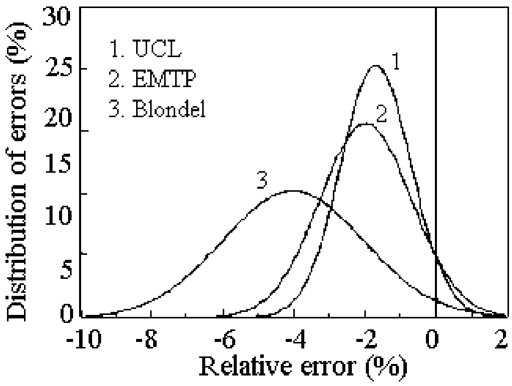

Figure 2.23. *Distribution of errors on the internal angle*

2.3.3.2. *Operational reactances in saturated mode*

Even when we consider non-linearities, it can be useful to obtain a linear model by the application of "small disturbance" techniques. Such a model is actually important for the study of the operational stability or the creation of an efficient control algorithm.

If the non-linear model of parallel elements and the values of the other elements (leakage inductances, resistances, α ratio) are known, we can easily deduct a small signal model of these elements. The elements that are not involved in steady-state operation are determined as was done in the first part of this chapter. We should note, however, that the behavior of the saturated machine presents an intrinsic difference to its behavior in non-saturated mode. In fact, cross-saturation reveals magnetic coupling between axes d and q, so that there is only one system, but in a double order.

Reciprocally, the determination of small signal parameters from tests where a disturbance is superimposed to a saturated steady state also comes up against this problem since the machine's external parameters are influenced by this phenomenon. But we sometimes want to determine the elements necessary to modeling transients with this type of test, because they can be done without interrupting the normal machine operation, or because we want to obtain the value of these elements by considering the influence of saturation on their value (phenomenon ignored in the previous study). In this case, it is better to make those tests from modes where only one component of steady-state magnetizing current is non-zero, at the risk of complicating the interpretation of results.

2.3.3.3. *Large-amplitude transients*

Another situation where the methods used in the case of a non-saturated machine are not generalized is large-amplitude transients. The use of these transients is tempting because it will help to determine all the elements in principle, including the non-linear elements, from a limited number of tests. In addition, we can reach levels of saturation in transient mode that would not be acceptable in steady state. Unfortunately in this case, it is no longer possible to identify parameters linked to their respective modes because the superimposition principle, and thus the decomposition in eigenmodes as well, are no longer valid. We cannot use an intermediate step of establishing a frequency response because this response no longer makes sense in the non-linear case. There is still the more general method, which is the identification of parameters by comparing simulation results to experimental results in the temporal field.

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