1

FROM THE BOTTOM UP: VOLTAGES, CURRENTS, AND ELECTRICAL COMPONENTS

1.1 AN INTRODUCTION TO ELECTRIC CHARGES AND ATOMS

The ancient Greek philosophers knew that when amber was rubbed against wool, it would attract lightweight particles of other materials like small pieces of paper or lint. Also, little pieces of paper get attracted to a plastic comb when the weather is dry. These experiments reveal that electric charge exists. If we rub one end of a glass rod with silk, charges will move toward that end of the rod. Rubbing a second glass rod in the same fashion and placing it close to the rubbed end of the first glass rod will exhibit a repelling force between the rods. However, when a plastic rod is rubbed with fur and it is placed near the rubbed glass rod, the plastic and the glass rods will attract each other. These simple experiments prove the existence of two different types of charge. Benjamin Franklin* called one of them positive and the other one negative. Most charge in an everyday object appears to be nonexistent because there is an equal amount of positive and negative charge. The word *electron* is derived from the Greek word "elektron," which means amber. From the above experiments the following can be asserted:

> *Charges of the same sign repel each other, while charges of opposite signs attract each other.*

* Benjamin Franklin: American scientist, writer, and politician (1706–1790).

Electrical, Electronics, and Digital Hardware Essentials for Scientists and Engineers, First Edition. Ed Lipiansky.

All matter is made of the basic elements, those elements listed in the periodic table of chemical elements. As of 2006, there are 117 elements of which 94 are found naturally on the Earth. The remaining elements are synthesized in particle accelerators. Loosely speaking, all matter is made of some combination of atoms, where an atom is the basic unit of matter. An atom contains a nucleus surrounded by a cloud of electrons. The nucleus consists of positively charged protons and electrically neutral neutrons. Neutrons have no electrical charge, but their mass is about 1800 times the mass of electrons. The electronic cloud around the nucleus is negatively charged, and an atom with an equal number of protons and electrons is said to be neutral. Protons have a positive charge and a mass about 1800 times larger than the mass of electrons. Different element atoms are different from each other because of the different numbers and arrangements of the atom's basic particles: electrons, neutrons, and protons. Traditionally in elementary physics and chemistry, the atom was compared to our planetary system. The nucleus is in the center of the atom, like the sun is the center of our system. The electrons are like the planets, orbiting around the sun. Electrons occupy different layers or shells that are at different distances away from nucleus. The outermost shell is referred to as the valence shell. The valence shell electrons determine the electrical characteristics of an atom.

Table 1.1 presents the elementary charge, which has a positive sign for a proton and a negative sign for an electron. Values for the mass of the electron, proton, and neutron are also tabulated.

From an electrical point of view, there are four main types of materials: conductors, nonconductors or insulators, semiconductors, and superconductors. The fourth type of material, the superconductor, is beyond the scope of this book.

Conductors are materials through which charge can move quite freely, such as copper or gold. Insulators are materials through which charge cannot move freely such as plastic or rubber. Semiconductors are materials that have an intermediate behavior between that of conductors and insulators. More on semiconductors will be covered in Chapter 6.

Table 1.1 Some atomic constants

	Abbreviation	Mass Value	Units	Relative Mass to the Electron Mass (m_e)	Charge in C (coulombs)
Elementary charge	e				1.602×10^{-19}
Electron	m_e	9.109×10^{-31}	kg	1	-1.602×10^{-19}
Proton	m_p	1.673×10^{-27}	kg	1800 (approx.)	$+1.602 \times 10^{-19}$
Neutron	m_n	1.675×10^{-27}	kg	1800 (approx.)	0

1.2 ELECTRIC DC VOLTAGE AND CURRENT SOURCES

Two types of independent sources are available, voltage and current sources. A source is said to be independent when either its nominal voltage or current is constant and does not depend on any other voltage or current present in a circuit. In a later section, we will cover the concept of dependent sources. The ideal voltage source produces a constant voltage across its terminals, regardless of the current that is being drawn from it by a load. Conversely, an ideal current source produces a constant current to a load connected across its terminals regardless of the voltage that is developed across the load. Let us now address the concepts of electric current and voltage.

1.2.1 Electric Current and Voltage

A net flow of electric charges through a circuit establishes an electric current. Note that conductors in isolation, such as a piece of copper not connected to anything else, contain free electrons or conduction electrons that randomly move. Such electrons do not constitute an electric current since in any cross section of the copper wire, the net amount of charge moved through the wire is zero. The emphasis here is on the word "net"; the net flow of charge constitutes an electric current. Current is defined as

$$i(t) = \frac{dq}{dt}, \tag{1.1}$$

where $i(t)$ represents electric current as a function of time and dq/dt is the net variation of charge with respect to time. Traditionally, electric current was referred to as current intensity. In most places, the term "current" is used, which is a short form of current intensity. The letter i denotes current, while dq differential of charge over dt differential of time refers to the net passage of charge during a time interval through a cross section of the conductor. On the other hand, a voltage can be interpreted as the "pressure" that needs to be asserted in a circuit in order to cause electric current to flow.

Throughout the book, we will assume that a conductor or a wire is ideal and will have zero resistance to the flow of current, unless it is stated otherwise. The unit of resistance is the ohm (Ω). Electric components that have greater than 0 Ω resistance are called resistors. The current that flows through a resistor times the resistance value equals the voltage drop that is produced across such resistor. Conventional current in a resistor flows from higher voltages or potentials to lower voltages or potentials.

Figure 1.1 depicts a resistor, a current flowing through it, and the voltage with its polarities that is produced across the resistor. The current through the resistor times its resistance value equals the voltage obtained across the resistor terminals. Mathematically,

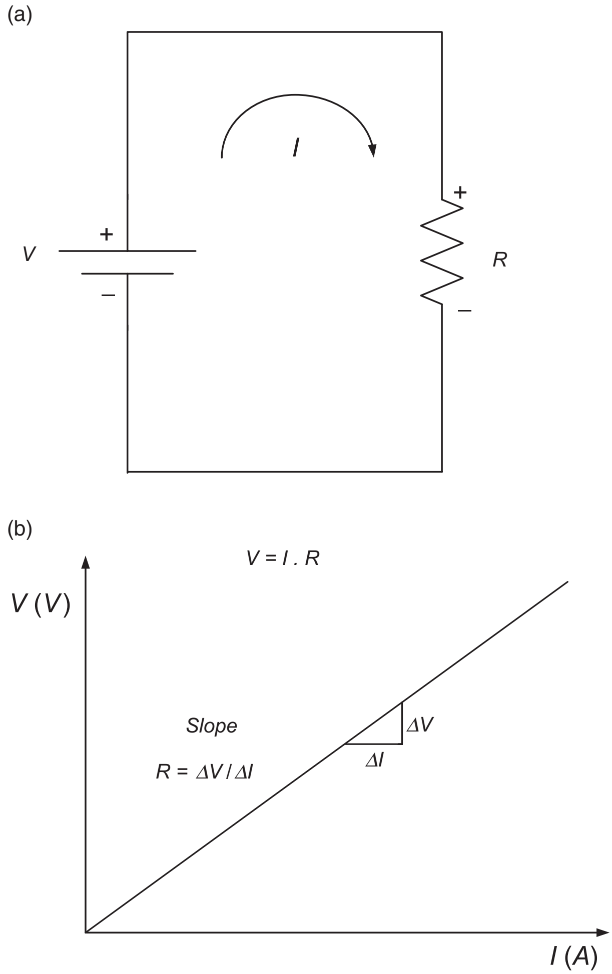


Figure 1.1 Ohm's Law: (a) DC voltage source powering a resistor; (b) linear variation of resistor voltage versus current.

$$V = IR. \tag{1.2}$$

Equation (1.2) states the voltage across a resistor is proportional to the current flowing through it. The constant of proportionality is the resistor value R. Equation (1.2) is *Ohm's Law*. In Figure 1.1a, a resistor powered by a DC source is shown; Figure 1.1b depicts the variation of resistor voltage versus current variation. The slope of the line $V = I\ R$ is the resistance value. Ohm's law in Equation (1.2) denotes a linear variation of the voltage across the resistor versus the current flowing through it.

Example 1.1 Given a 10-V DC voltage source that is connected across a 0.1 Ω resistor, calculate for the current that will flow through the resistor.

Solution to Example 1.1

From Equation (1.2),

$$V = IR.$$

And since

$$I = V/R,$$

$$I = 10\ \text{V}/0.1\ \Omega = 100\ \text{A}.$$

1.2.2 DC Voltage and Current Sources

We all have some familiarity with electricity and electronics. We have seen flashlights, batteries, battery chargers, lightbulbs, portable electronic devices, and electrical and electronic appliances such as toasters and microwave ovens.

Flashlight batteries, toy batteries, and automobile batteries are all examples of DC voltage sources. DC stands for direct current, and what this means is that the current polarity that the source supplies does not change; that is, the current always flows in the same direction through the load.

An idealization of the DC voltage source is that its DC voltage is always constant with respect to time and independent of the amount of current that it may supply. In practical devices such as batteries, that voltage is "somewhat" constant, and it varies based on factors such as temperature, environmental factors, mechanical vibration, age of the battery, and use of the battery. However, unless we state otherwise, the first-order approximation of a battery is that of a constant or DC voltage source.

Current sources are as also idealized like DC voltage sources. An everyday example of a current source is a battery charger. A battery charger provides a constant current to recharge a battery with rechargeable chemistry. Note that not all batteries are rechargeable. No attempt should be made to recharge batteries that are not of the rechargeable kind, since this causes a hazard to the user. Another example of a current source is that of a transistor hooked up to operate as a current source.

A DC voltage source may not always be a chemical battery. It may, for example, be built with electronic components that behave largely like a DC source. An example of this is a DC power supply (see Figure 1.2).

When a DC voltage source is not being used, it must be stored in an open-circuit condition (refer to Figure 1.3a). That means nothing is connected to the positive and the negative electrical terminals. Upon connecting an element

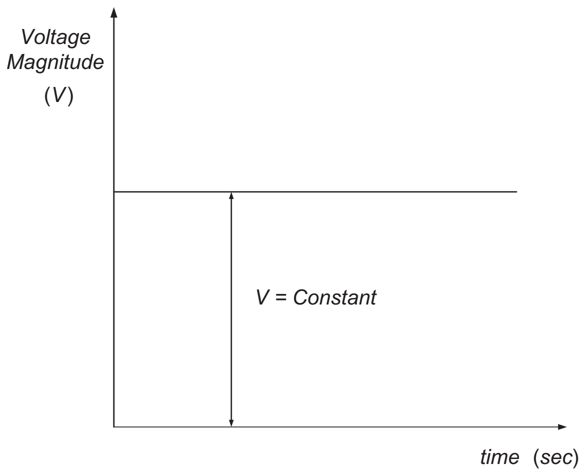


Figure 1.2 Mathematical representation of a DC voltage source as a function of time.

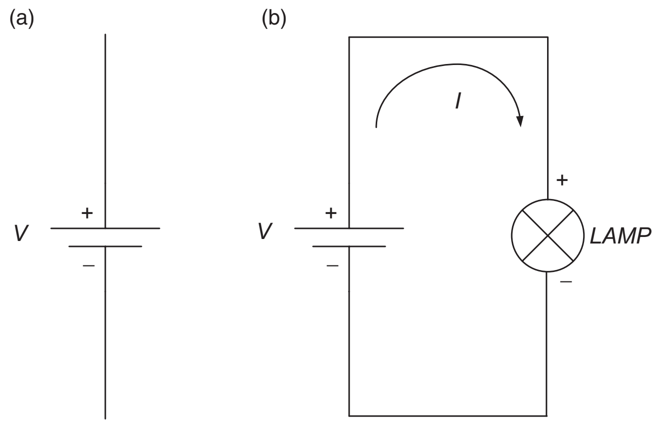


Figure 1.3 DC voltage source in (a) open-circuit condition and (b) loaded with a lightbulb.

such as a lightbulb across the voltage source terminals, a current flows through the circuit that was just established. Figure 1.3b shows a DC voltage source, which in this case is actually a battery connected with wires to a lightbulb.

The battery exerts "pressure" into the circuit by displacing charges. The net flow of charge with respect to time is called an electric current. Physically, an electric current consists of a net flow of electrons. That is, the electronic current leaves the negative terminal of the source, goes through the lightbulb, and returns back into the positive terminal of the source. However, the traditional interpretation is that current flows from the positive terminal of the source through the lightbulb and back into the negative terminal of the source. Throughout this book, the traditional or conventional current flow will be used. This is what most of the electrical engineering literature assumes.

The lightbulb depicted in Figure 1.3 is in effect a resistor. The voltage applied and the resistance of the lightbulb determines the current that will be present in the closed circuit. Resistance is the opposition that a resistor presents to the net flow of current. In other words, the DC voltage source voltage is basically constant, regardless of the amount of current that is being drawn form the source. Naturally, this is an idealization of what a DC voltage source is, or what we would like it to be. Real voltage sources do not behave that way; their output voltage is quite constant as long as the current flowing through the circuit is considerably less than what the total current pumping capability of the source is. More details on this topic will be provided when the internal resistance of a source is addressed, later in this chapter.

DC current sources, on the other hand, produce a constant current when a lightbulb or a resistive element establishes a closed loop circuit and the voltage across it will depend strictly on the resistive value placed across the current source and the current value. Just like with the DC voltage source, the DC current source is an idealization. Real current sources can provide a constant current as long as the voltage across the resistor does not produce an excessive voltage. Figure 1.4 depicts a constant DC current as a function of time.

Figure 1.5a depicts a DC current source in a standby condition, that is, with its terminals short-circuited to each other. A current source should not be left open-circuited because the voltage that gets developed across its terminals would grow without bound. A real or physical current source would self-destruct or become severely damaged if its terminals were left in an open-circuit condition. Figure 1.5b depicts a DC current source with a resistor connected across its terminals. In Figure 1.5a,b, both states of the current source are benign states or normal states. In both cases, the current supplied by the current source is identical.

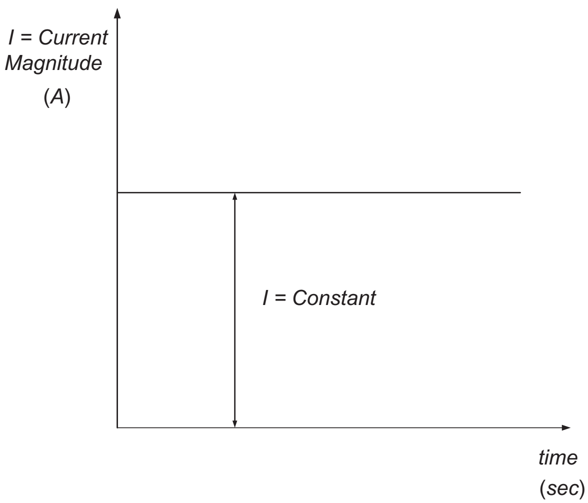


Figure 1.4 Mathematical representation of a DC current source as a function of time.

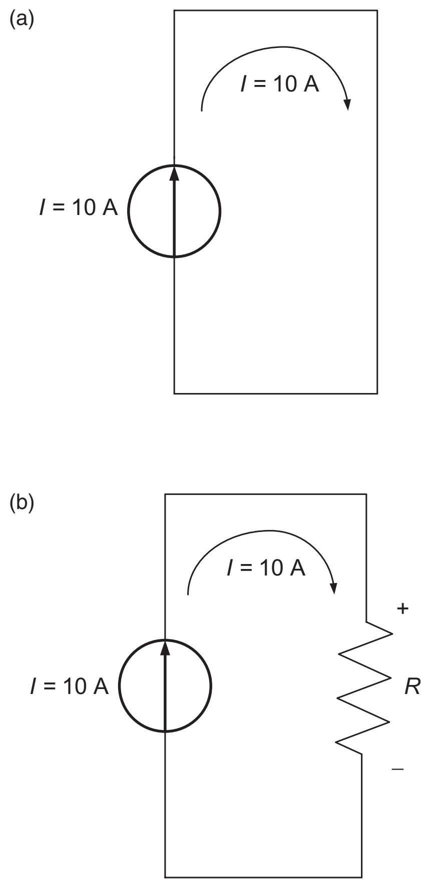


Figure 1.5 DC current source (a) in short-circuit condition and (b) loaded with a resistor.

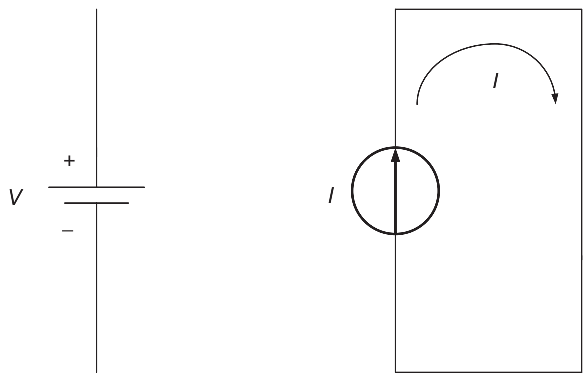
The ideal current source with a resistive element in a closed circuit (Fig. 1.5) provides a constant current, and the voltage across the terminals of the current source depends on the value of the resistor across the current source times the current supplied by the source. Changes of the resistor values across the current source will produce proportional changes of the voltage across the current source. Note that the resistor (or load) across a current source produces higher voltages as the load resistor increases in value, because the current remains constant. For the case that the load resistor is very large, the voltage across the current source will be very large. When current sources are in open-circuit condition, the voltage across its terminal grows without bound. Real current sources would self-destruct quickly under an open-circuit condition. The current source must always be short-circuited when not in use (refer to Figure 1.5a). The voltage across a shorted current source is zero because the wire across the current source has zero resistance. However, the current supplied by a shorted current source is its nominal value of current. For example, a 10 A current source must be shorted when not supplying any current to a load. This current has a value of 10 A. When the current source is loaded by a resistor (Fig. 1.5b), the 10 A times the value of the resistor determines the voltage across the resistor. Note that since the current supplied by a current source remains constant as the resistor becomes larger in value, the voltage becomes larger as well.

In the extreme case if the value of the resistor is an open circuit, or infinitely many ohms, the voltage developed across the ideal current source is infinite.

In mathematical terms, an open-circuit current source produces an indetermination. The voltage source on the other hand must never be short-circuited because if we did, the current that the voltage source would supply to the short circuit is infinitely large, that is, also an indetermination (unbounded current). A real or physical short-circuited voltage source would also self-destruct rather quickly. Figure 1.6 depicts an open-circuit voltage source and a short-circuited current source. These are the benign or idle states for the voltage and the current sources. Figure 1.7 depicts ill-defined or undesirable states for a voltage and current source, respectively. A voltage source cannot survive short-circuited conditions, like a current source cannot survive an open-circuited condition. If they did, their reliability would be severely affected after that. Finally, let us be 100% clear about it: Figure 1.7 depicts circuits that fall under the “do-not-do-this-at-home-or-at-work” category.

1.2.3 Sources Internal Resistance

No real voltage source has a capacity of generating an infinite current upon being short-circuited. Similarly, no real current source can produce an infinite voltage across its terminals if left open-circuited. Real sources have their



(a) *Open-Circuited Voltage Source* (b) *Short-Circuited Current Source*

Figure 1.6 Sources benign states: (a) for a voltage source; (b) for a current source.

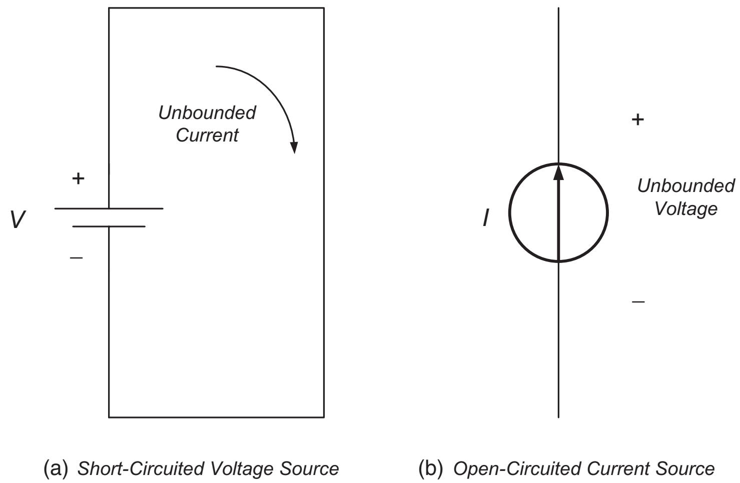


Figure 1.7 Ill-defined (hazardous) states: (a) for a voltage source; (b) for a current source.

physical limitations. To account for these limitations, a voltage source is modeled with a current-limiting resistor in series, and a current source is modeled with a voltage-limiting resistor in parallel with the current source. Figure 1.8a,b depicts models of physical voltage and current sources with their respective series and parallel resistors. The series resistance in the voltage source represents the current limitation characteristic that a voltage source has. Note that if the voltage source of Figure 1.8a is 10 V and has an internal resistance of 1 Ω, the total short-circuit current capability of this source is its open-circuit voltage V divided by its internal resistance $r_{internal}$:

$$I_{short\text{-}circuit} = V_{constant\_source\_open\text{-}circuit\_voltage} / r_{internal}. \tag{1.3}$$

In particular for the example stated above, this short-circuit current is 10 V/1 Ω = 1 A. The internal resistance distinguishes a real voltage source from an ideal voltage source, which is assumed to have an infinite capability of generating current. Its internal resistance limits the current that can be drawn from a real or physical voltage source. This limitation is stated by Equation (1.3). When referring to Figure 1.8a, it is important to say that the internal resistance is an integral part of the real DC source, and the real DC source is modeled by an ideal DC source in series with the source internal resistance in series.

Similarly for the DC current source (refer to Figure 1.8b), the paralleled resistor with the current source represents or models the finite voltage-generating capability that an open-circuited current source has. We can write the current voltage relationship for the real current source modeled in Figure 1.8b as follows:

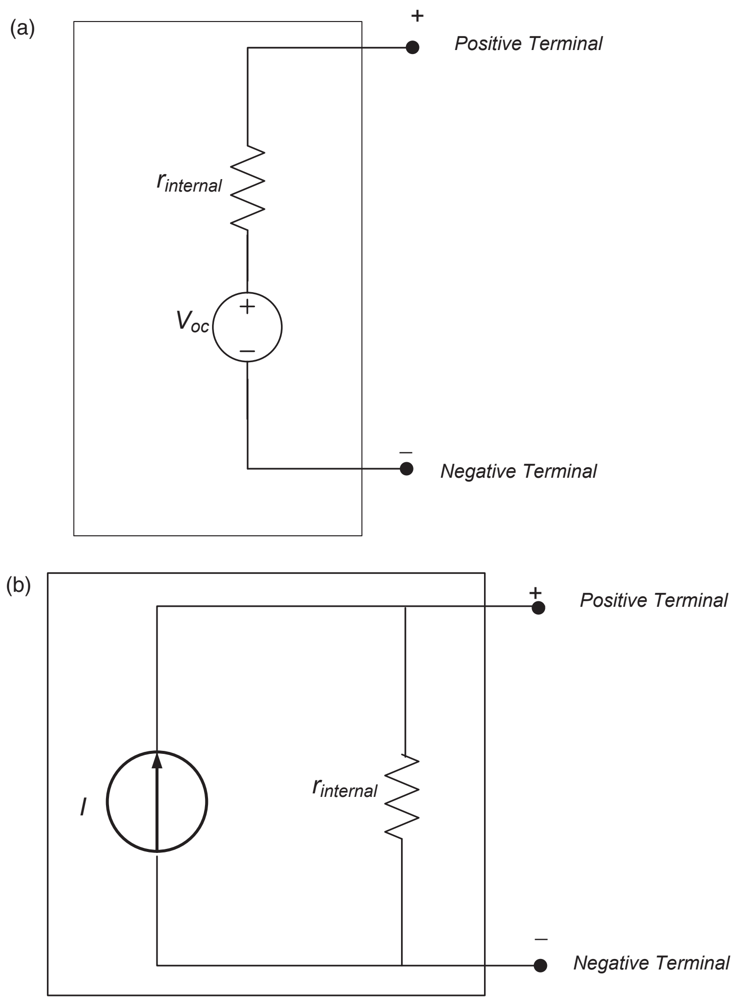


Figure 1.8 Modeling of real sources: (a) voltage source; (b) current source.

$$I_{constant\text{-}source\text{-}current} = V_{current\ source} / r_{internal} \tag{1.4}$$

Note that Equations (1.3) and (1.4) are governed by Ohm's law.

So, if we have a 10 A current source with its internal resistance of 10 Ω, using Equation (1.4), we determine that the maximum output voltage that this current source produces is 10 A·10 Ω = 100 V. Note that the open-circuit voltage of a real current source cannot exceed the limits imposed by Equation (1.4). The open-circuited ideal current source would produce a very large voltage across its terminals.

1.3 ELECTRIC COMPONENTS: RESISTORS, INDUCTORS, AND CAPACITORS

There are three fundamental circuit elements in electric circuits. These are resistors, inductors, and capacitors. From a circuit analysis point of view, we are interested in the voltage versus current as well as the current versus voltage relationships that exist for every one of these circuit elements. All three components, resistors, inductors, and capacitors, are said to be passive elements to differentiate them from active elements. Passive components do not have *gain*, while active components do. Active components will be covered in Chapters 5 and 6.

1.3.1 Resistors

The resistor is an electric component usually made with some of the resistive materials such as carbon, metal film, or paste. Other materials are used, but the ones mentioned are the most common. Resistor technologies vary, and the most common are metal thin film and thick film, metal strip, wire wound, foil, and composition.

A resistor opposes the flow of electric current. As the current flows through the resistor, a voltage is developed across such resistor. The voltage drop across the resistor follows Ohm's law, which states that the voltage across the resistor equals the current flowing through it, times the resistance value. Figure 1.9 shows the circuit symbol that represents a resistor. Figure 1.9 depicts a positive current and a positive voltage on the resistor (a), a negative current and a negative voltage (b), a positive current and a negative voltage (c), and a negative current and a positive voltage (d).

The current voltage and voltage current relationship on a resistor is given by Ohm's law, Equation (1.2), which we repeat for the reader's convenience:

$$V = IR = f(I),$$

where $f(I)$ is a linear function of I equal to $I\,R$.

And conversely,

$$I = V/R \tag{1.5}$$

$$I = g(V), \tag{1.6}$$

where $g(V)$ is a linear function of V and equals V/R. The term $1/R$, the inverse of the resistance, is called the conductance G. Using G in Equation (1.6), it becomes

$$I = GV. \tag{1.7}$$

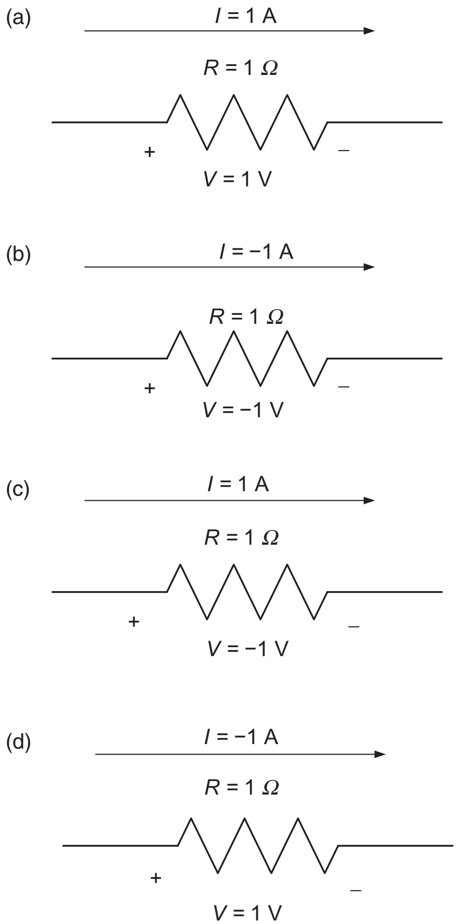


Figure 1.9 Circuit symbol of a resistor showing current and voltage polarities.

In Equation (1.5), V is the voltage across the resistor in volts, I is the current flowing through the resistor in amperes, and R is the resistor value in ohms. In Equation (1.7), I is in amperes, G in siemens (or mho), and V in volts.

The unit of resistance is the Ω, and R represents resistance. The inverse of the resistance ($1/R$) in DC circuit analysis is called conductance (G), and it is measured in siemens (S), where $1\ \mathrm{S} = \Omega^{-1}$, formerly also referred to as a mho (ohm spelled backward). Common values of resistors range from very small fractions of an ohm up to several megaohms ($1\ \mathrm{M}\Omega = 10^6\ \Omega$). This is a good point to introduce the most common prefixes used in the SI system of units.

Table 1.2 shows the internationally accepted power of 10 factors, their names, and their symbols. The most commonly used prefixes in electrical engineering and computer sciences range from 10^{24} (yotta) down to 10^{-24} (yocto). Within such range, some prefixes are more common than others.

Table 1.2 Prefixes used with the International System of Units (SI)

Factor	Name	Symbol	Factor	Name	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	fempto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deka	da	10^{-24}	yocto	y

When selecting a resistor from a data sheet, there are key parameters that cannot be ignored to make a good component selection. Such resistor parameters are

(a) Resistance value in Ω
(b) Resistor power rating in W
(c) Resistor tolerance in ±%
(d) Temperature coefficient of resistance in ±ppm/°C, which stands for parts per million per Celsius degree or 10^{-6}/°C.

Other defined parameters that characterize resistors may vary a little bit from manufacturer to manufacturer. For a more complete list of precision resistor parameters, access the first reference of the Further Reading section at the end of the chapter.

Resistance values range from a small fraction of an ohm all the way up to a few MΩ (1 M = 10^6 Ω). The electronic industry standardized the resistor values based on the E-series according to Table 1.3. For example, for 1% tolerance resistors, 96 values per decade are chosen in an equally spaced basis. The resistor values follow the following geometric progression:

$$N = 10^{n-1/k}. \tag{1.8}$$

In Equation (1.8), N is the nominal resistance value at position n, and k is 96 for the E96 series. For example, for series E96, using Equation (1.8) for n = 1, 2, 3 . . . , the progression of resistance values becomes

$$1.00, 1.02, 1.05\ldots \tag{1.9}$$

The resistor values of (1.9) have been rounded.

In Equation (1.8), k refers to a decade of resistors values such as 1 Ω, 10 Ω, 100 Ω, and so on, while “n” is the series number. For example, for the E96

Table 1.3 Common Values of Resistor Tolerances

Series	Tolerance	Comments
E3	50%	3 steps per decade. No longer used.
E6	20%	6 steps per decade. No longer used.
E12	10%	12 steps per decade. No longer used.
E24	5%	24 steps per decade. Not commonly used.
E48	2%	48 steps per decade. Commonly used.
E96	1%	96 steps per decade. Commonly used.
E192	0.5%, 0.25%, 0.1% or higher	192 steps per decade. Commonly used in high accuracy and precision designs.

series, there are 96 (i.e., the reason for Equation (1.8) is to obtain the same number of steps within each decade of resistor values). Ultimately, the goal is to limit the number of resistor values or the inventory that manufacturers and distributors would have to have to handle otherwise.

For example for the E48 series of standardized resistor values, using Equation (1.8), the first three values are 100, 105, and 110 Ω. E48 is the ±2% tolerance series. Note that each of the three values mentioned plus and minus their 2% tolerance are

$$98\,\Omega < 100\,\Omega < 102\,\Omega. \tag{1.10}$$

$$102.9\,\Omega < 105\,\Omega < 107.1\,\Omega. \tag{1.11}$$

$$107.8\,\Omega < 110\,\Omega < 112.2\,\Omega. \tag{1.12}$$

It is easy to see that none of the values including their tolerance overlap. The same applies to all values for all ohmic decades of all other series. The tolerance of a resistor is the deviation in percent that the actual value of a resistor can deviate from its nominal value at room temperature. Room temperature for some manufacturers is defined as 20°C, for others is 25°C. However, when the resistor is in use, the temperature surrounding the resistor will cause its nominal value to increase or decrease. Resistor manufacturers specify the maximum/minimum resistance variation of their components for a certain temperature span.

Example 1.2 Assume that a 1 MΩ resistor has a ±100 ppm/°C temperature coefficient within an operating temperature range of –25°C to +125°C. Further assume that the resistor is exactly 1 MΩ at 20°C (i.e., assume the resistor has zero tolerance ±0%):

(a) What will the resistor value range be for the above temperature range?

(b) What will the resistor value range be if in addition to the temperature range, a ±1% tolerance is assumed?

Solution to Example 1.2a

When the resistor operates at +125°C,

$$\begin{aligned}&1\,\text{M}\Omega\times[+100\text{ ppm/°C}]\times(125\text{°C}-20\text{°C})=\pm1\,\text{M}\Omega\times100\times10^{-6}\times105\\&\quad=\pm10{,}500\,\Omega=\pm1.05\%\text{ of the resistor nominal value }125\text{°C}.\end{aligned}\tag{1.13}$$

When the resistor operates at –25°C,

$$\begin{aligned}&1\,\text{M}\Omega\times[-100\text{ ppm/°C}]\times(-25\text{°C}-20\text{°C})=\pm1\,\text{M}\Omega\times100\times10^{-6}\times45\\&\quad=\pm4500\,\Omega=\pm0.45\%\text{ of the resistor nominal value}-25\text{°C}.\end{aligned}\tag{1.14}$$

From Equations (1.13) and (1.14), we conclude that for a –25°C to +125°C temperature range, the resistor varies from –0.45% up to 1.05% from its nominal value.

Solution to Example 1.2b

In Example 1.2a, the resistance variation was just due to the resistor temperature coefficient over the operating temperature range. If in addition the resistor nominal value will be its value ±tolerance (%). That means that if the resistor has a ±2% tolerance, its value can vary between –2% (or 980,000 Ω) to +2% (or 1,020,000 Ω); a total of 1.05% due to its temperature coefficient operating in the –25ºC to +125ºC temperature range. Thus, at the high end the resistor value can be +1% due to tolerance and +1.05% operating at +125ºC; that is a total of 2.05%. At the low end, the resistor value can be –1% due to tolerance and –0.45% operating at –25ºC, that is, a total of –1.45% from its nominal value.

Resistor Tolerances: The most commonly used resistor tolerances for electrical and electronic applications are ±1% and ±2%. Many years ago, ±5%, ±10%, and ±20% were commonly used. When dealing with high precision analog electronics, ±0.1% tolerance is available. When higher precision is required, metal foil resistors and ultrahigh precision metal film resistors are available. Metal foil resistor tolerances of up to ±0.005% tolerance are available, and special metal film resistors of up to ±0.01% are also available.

Example 1.3 Resistor Value, Tolerance, and Power Rating Selection

So let us assume that we need to select a resistor R to establish a constant current of 100 mA. The resistor will have 12 V DC ±120 mV applied across its terminals. Assume that the DC source and resistor R will always operate at 20°C and no temperature changes will occur. Determine (1) a reasonable resistor value, (2) its tolerance, and (3) its power rating that can keep the load current at 100 mA ±2% under all voltage variations and resistor variations due to its tolerance.

Solution to Example 1.3

If the voltage (V) were exactly 12 V without any variations and the resistor were exactly 120 Ω, the current would be, by virtue of Equation (1.2),

$$I = 12\ \text{V}/120\ \Omega = 100\ \text{mA}. \tag{1.15}$$

The statement that the resistor will always operate at 20°C is equivalent to saying that its temperature coefficient is zero or that there are no resistor variations due to temperature changes.

Let us start adding the real requirements to the problem. We are told that the voltage can vary ±120 mV or ±1% from its nominal value of V = 12 V. Thus,

$$11.88\ \text{V} < \text{V} < 12.12\ \text{V}. \tag{1.16}$$

If we assume that we have a perfect resistor of 120 Ω with a 0% tolerance, then

$$11.88\ \text{V}/120\ \Omega < 12\ \text{V}/120\ \Omega < 12.12\ \text{V}/120\ \Omega, \tag{1.17}$$

which leads to

$$99\ \text{mA}\Omega < \text{I} < 101\ \text{mA}. \tag{1.18}$$

From Equation (1.16) we can state that having a ±120 mV voltage variation and a perfect resistor of 120 Ω, the current I will be bounded between 99 mA and 101 mA, or 100 mA ±1%.

Now let us introduce the concept that the resistor R is not perfect, and let us assume that it has a tolerance of ±1%.

Then the resistor value will range from

$$118.80\ \Omega < R < 121.20\ \Omega. \tag{1.19}$$

Using the resistor R range obtained in Equation (1.19) and combining it with all possible variation of the voltage V, it yields

$$V_{\max}/R_{\max} \tag{1.20}$$

$$V_{\min}/R_{\min} \tag{1.21}$$

$$V_{\max}/R_{\min} \tag{1.22}$$

$$V_{\min}/R_{\max} \tag{1.23}$$

where

$$V_{\max} = V + (1\%\ \text{of}\ V) = 12\ \text{V} + 0.12\ \text{V} = 12.12\ \text{V}. \tag{1.24}$$

$$V_{\min} = V - (1\% \text{ of } V) = 12\ \text{V} - 0.12\ \text{V} = 11.88\ \text{V}. \tag{1.25}$$

$$R_{\max} = R + (1\% \text{ of } R) = (120\ \Omega + 1.2\ \Omega) = 121.2\ \Omega. \tag{1.26}$$

$$R_{\min} = R - (1\% \text{ of } R) = (120\ \Omega - 1.2\ \Omega) = 118.8\ \Omega. \tag{1.27}$$

Using Equations (1.24) through (1.27) in Equations (1.20) through (1.23), we obtain

$$V_{\max}/R_{\max} = 12.12\ \text{V}/121.2\ \Omega = 100.0\ \text{mA}. \tag{1.28}$$

$$V_{\min}/R_{\min} = 11.88\ \text{V}/118.8\ \Omega = 100.0\ \text{mA}. \tag{1.29}$$

$$V_{\max}/R_{\min} = 12.12\ \text{V}/118.8\ \Omega = 102.0\ \text{mA}. \tag{1.30}$$

$$V_{\min}/R_{\max} = 11.88\ \text{V}/121.2\ \Omega = 98.0\ \text{mA}. \tag{1.31}$$

Equations (1.28) through (1.31) provide all the possible extreme variations of current I. And from Equations (1.22) and (1.21), it can be seen that current I varies approximately ±2% from its nominal value of 100 mA.

Finally, the power dissipated by resistor R will be V^2/R. To account for voltage and resistor variations, we need to calculate

$$P_1 = V_{\max}^2/R_{\min} = 1.236\ \text{W (maximum)}. \tag{1.32}$$

$$P_2 = V_{\min}^2/R_{\max} = 1.164\ \text{W (minimum)}. \tag{1.33}$$

$$P_3 = V_{\max}^2/R_{\max} = 1.212\ \text{W}. \tag{1.34}$$

$$P_4 = V_{\min}^2/R_{\min} = 1.188\ \text{W}. \tag{1.35}$$

By inspection of Equations (1.32) through (1.35), Equation (1.32) shows the maximum dissipated power, and Equation (1.33) shows the smallest dissipated power.

Commonly available resistor power ratings are 0.063 W, 0.1 W, 0.125 W, 0.2 W, 0.25 W, 0.5 W, 1 W, 2 W, and 5 W for most electronic and some electrical applications. For special high-power electrical applications, the power ratings go well beyond 5 W, such as 10 W, 20 W, 50 W, 75 W, 100 W, 500 W, 1 kW, and above.

For our example, the most logical choice is to select a 2-W power-rated resistor.

Answers to Example 1.3

(a) $R = 120\ \Omega$, (b) ±1%, (c) 2 W.

R can be 120 Ω ±1%, 2 W. When the voltage varies from 12 V ±1%, current I remains within ±2% of its nominal value of 100 mA. Note that choosing a 1 W resistor is not an option since the minimum power dissipation exceeds 1 W; 2 W resistor is the next given resistor power rating that provides a headroom of 0.764 W or 76.4% more with respect to 1 W.

Figure 1.10 Resistors in series.

1.3.2 Resistors in Series and in Parallel

Two or more resistors are connected in series when the same current flows through all of them. Referring to Figure 1.10, resistors R_1, R_2, and R_3 are in series. Why? Because if a positive terminal of a voltage source is applied to the free end of one resistor in the series, and the negative terminal of the source is applied to the free terminal of the last resistor in the series, the current flowing through such circuit is identical for all resistors.

Generalizing the above concept, "n" resistors in series are equivalent to the sum of all n resistors. n is an integer and the total number of resistors in series.

$$R_1 + R_2 + R_3 + \ldots + R_n = \sum_i^n R_i. \tag{1.36}$$

Two resistors in series are equivalent to the sum of each of the resistors.

Example 1.4 Resistors in Series
Given a 1 kΩ, a 3 kΩ, and a 100 Ω resistor in series, find the series equivalent resistance.

Solution to Example 1.4

1000 Ω + 3000 Ω + 100 Ω = 4100 Ω = 4.1 kΩ.

Example 1.5 Resistors of Significantly Different Values
Given 1 MΩ and 1 kΩ resistors, find their series equivalent resistance.

Solution to Example 1.5

Not different from the previous example, the solution is 1,000,000 Ω + 1000 Ω = 1.001 MΩ.

Example 1.6 Let us assume that both resistors of the previous example have ±1% tolerance. What is the series equivalent resistance of both resistors with an error of approximately ±1%?

Solution to Example 1.6

With ±1% tolerance, it is easy to see that

$$\pm 1\% \text{ of } 1\,\mathrm{M\Omega} = \pm 10{,}000\,\Omega \tag{1.37}$$

and

$$\pm 1\% \text{ of } 1\,\mathrm{k\Omega} = \pm 10\,\Omega. \tag{1.38}$$

Since the value of the 1 kΩ resistor is much smaller than 1% of the value of the series equivalent resistor, found in the earlier part of this example to be 1.001 MΩ, the entire value of the small resistor can be neglected. The approximate answer is 1 MΩ ±10,000 kΩ, which is within a ±1% error. Note that a 1% error of 1 MΩ from Equation (1.37) is 10,000 Ω.

Resistors in parallel are those resistors that are connected such that the voltage across all of them is the same. Figure 1.11, depicts "n" resistors in

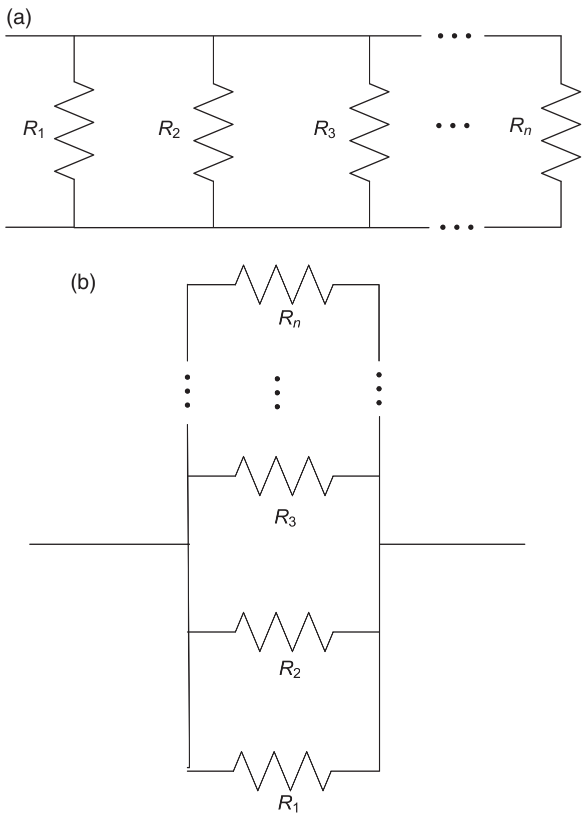


Figure 1.11 Resistors in parallel.

parallel. Note that part (a) and part (b) of the figure represent the exact same circuit.

Given two resistors R_1 and R_2 in parallel, the total parallel equivalent resistance ($R_{parallel\text{-}equiv}$) is

$$R_{parallel\text{-}equiv} = \frac{product-of-both-resistor-values}{sum-of-both-resistor-values}. \tag{1.39}$$

Example 1.7 Given two resistors in parallel, $R_1 = 3\ \Omega$ and $R_2 = 6\ \Omega$, find the total equivalent resistance.

Solution to Example 1.7

Applying Equation (1.39),

$$R_{parallel\text{-}equiv} = 3 \times 6/(3+6) = 2\ \Omega. \tag{1.40}$$

Equation (1.39) can be arithmetically expressed as follows:

$$\frac{1}{R_{parallel\text{-}equiv}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{1.41}$$

$$R_{parallel\text{-}equiv} = \frac{R_1 \cdot R_2}{R_1 + R_2}, \tag{1.42}$$

where $R_{parallel\text{-}equiv}$ refers to the parallel equivalent resistor of R_1 and R_2. Note that Equations (1.41) and (1.42) are equivalent.

Generalizing from Equation (1.41), the parallel equivalent resistance of n (where *n* is an integer) that represents the number of resistors equals

$$1/R_{parallel\text{-}equiv} = 1/R_1 + 1/R_2 + 1/R_3 + \ldots + 1/R_n. \tag{1.43}$$

Upon covering Kirchhoff's laws in the next section we will justify the computations to find series and parallel equivalent resistors.

Example 1.8 Given three resistors in parallel, where $R_1 = 3\ \Omega$, $R_2 = 6\ \Omega$, and $R_3 = 2\ \Omega$, calculate the parallel equivalent resistor.

Solution to Example 1.8

Using Equation (1.43), we obtain

$$\frac{1}{R_{parallel\text{-}equiv}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}, \tag{1.44}$$

and using the corresponding values for R_1, R_2, and R_3, we get that

$$1/R_{parallel\text{-}equiv} = 1/3 + 1/6 + 1/2, \tag{1.45}$$

from where

$$R_{equiv} = 1\ \Omega. \tag{1.46}$$

Example 1.9a Given 10 resistors in parallel of equal value, find the parallel equivalent resistor of the group of 10.

Solution to Example 1.9a

Using Equation (1.43) for "$n = 10$" resistors in parallel, we find that

$$1/R_{parallel\text{-}equiv} = 1/R + 1/R + 1/R + \ldots + 1/R, \tag{1.47}$$

where Equation (1.43) has 10 equal terms because all 10 resistors have the same value.

From Equation (1.43), we obtain

$$1/R_{parallel\text{-}equiv} = 10/R \tag{1.48}$$

or

$$R_{parallel\text{-}equiv} = R/10. \tag{1.49}$$

Example 1.9b Given two resistors in parallel where one is 1 kΩ and the other one is 1 Ω, find the total equivalent resistance.

Solution to Example 1.9b

Using Equation (1.43) one more time, we obtain

$$1/R_{parallel\text{-}equiv} = 1/1 + 1/1000, \tag{1.50}$$

from where we obtain that

$$R_{parallel\text{-}equiv} = 1000/1001 = 0.999001\ \Omega. \tag{1.51}$$

Corollary from Example 1.9

The parallel of one resistor with another one that is several orders of magnitude larger than the first one is approximately equal to the smaller resistor value.

Example 1.10 Given three resistors, where $R_1 = 1\ \Omega$, $R_2 = 27\ \Omega$, and $R_3 = 500\ \Omega$, calculate the parallel equivalent resistance of the three resistors.

Solution to Example 1.10

Using Equation (1.43) from above,

$$1/R_{parallel\text{-}equiv} = 1/R_1 + 1/R_2 + 1/R_3$$

$$1/R_{equiv} = 1/1 + 1/27 + 1/500 = 1/0.9624$$

$$R_{equiv} = 0.9624\ \Omega$$

Note that the parallel equivalent resistor of 0.9624 Ω is smaller than the smallest given resistor, which is 1 Ω.

Corollary from Example 1.10

The reader should be convinced that

> *Given* n *resistors,* $R_1, R_2, \ldots, R_n$ *where* $R_1 < R_2 < \ldots < R_n$, *the total parallel equivalent resistor is always smaller than* R_1. *In other words, a number of resistors in parallel has a parallel equivalent which is numerically smaller than the smallest resistor value.*

1.3.3 Resistivity: A Physical Interpretation

Figure 1.12 shows a conductor made of a conductive material. The most common conductive materials are metals; copper is the most abundant and the least expensive metal to mass-produce electrical wire. Although in many practical applications it is reasonable to assume that the total resistance of a conductor is close to 0 Ω, actually, it is not 0. So for some applications, the true resistance of a conductor needs to be taken into account.

The resistance of a conductor is given by

$$R = \rho L/A, \tag{1.52}$$

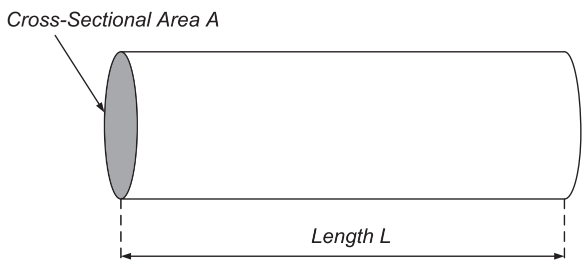


Figure 1.12 Conductor of length L and cross-section A.

Table 1.4 Resistivity of some materials at room temperature (20°C) [2]

Material	Resistivity ρ in (Ω·m) at 20°C	Conductor or Insulator?
Silver	1.62×10^{-8}	Conductor
Copper	1.69×10^{-8}	Conductor
Gold	2.35×10^{-8}	Conductor
Aluminum	2.75×10^{-8}	Conductor
Magnanin[a]	4.82×10^{-8}	Conductor
Tungsten	5.25×10^{-8}	Conductor
Iron	9.68×10^{-8}	Conductor
Platinum	10.6×10^{-8}	Conductor
Glass	10^{10} to 10^{14}	Insulator
Fused quartz	$\sim 10^{16}$	Insulator

[a] Magnanin is an alloy with a very small temperature coefficient of resistivity.

where ρ (lowercase Greek letter rho) is the material resistivity in ohm meter (Ωm), L is the length of the conductor in meters (m), and A is the cross-sectional area of the conductor in meters squared (m^2).

Figure 1.12 depicts a conductor of length L and cross-section A. Resistivity ρ is an electric characteristic of the material used, and it varies with temperature. Most commonly, resistivity is specified at room temperature of 20°C.

Table 1.4 lists some of the most common conductor and insulator materials and their resistivity.

Note that the range in resistivity between a conductor and an insulator such as glass minimally ranges from 10^{-8} to 10^{10}; this is 19 orders of magnitude!

Example 1.11 Assume that we have a DC voltage source that can produce 100 A of current at a constant 12 V. What voltage level will be present across the resistive load, without neglecting the voltage drop across the copper wires? Assume that you are using 10 mm^2 cross-section copper wires and that the one-way wire length (from source to load) is 1 m and the ambient temperature is 20°C. For illustration purposes of this problem, the reader is strongly referred to the circuit diagram of Figure 1.1 at the beginning of this chapter.

Solution to Example 1.11

If the resistance of the wires was 0 Ω, the load would see exactly 12 V at 100 A. We need to take into account the wire resistance, then:

From Equation (1.52), $R = \rho\, L/A$.

Since one-way length of the wire is 1 m, the round-trip length of the wire is 2 m. From Table 1.4, copper resistivity is 1.69×10^{-8} Ωm at a room temperature of 20°C. A cross section of 10 mm^2 equals 0.0001 m^2. Plugging all the values into Equation (1.52) yields

$$R_{wire} = (1.69 \times 10^{-8}\ \Omega\text{m}) \times 2\ \text{m}/10^{-4}\ \text{m}^2 = 338\ \mu\Omega. \tag{1.53}$$

Since the current flowing through the wires is 100 A, the voltage drop across the load wires is

$$V_{drop\text{-}across\text{-}wires} = IR_{\text{wire}} = 100 \times 338 \times 10^{-6}\ \text{A} \cdot \Omega = 33.8\ \text{mV}. \tag{1.54}$$

Answer to Example 1.11

The voltage that the load resistor will see is 12 V – 0.0338 V = 11.9662 V.

1.3.4 Resistance of Conductors

It is interesting to observe that for a given conductor material, for example, copper, the resistance of the conductor equals to its resistivity, which depends on the material, times the length of the conductor (L), and it is inversely proportional to the conductor cross-sectional area (A) according to Equation (1.52).

So if the length of a conductor is doubled, all other factors remaining equal, the resistance of such conductor doubles. If the thickness (i.e., cross section) of a conductor doubles, while all other factors remain equal, the resistance of the conductor becomes half of the original resistance.

Resistivity ρ, is a temperature-dependent parameter, and it is a characteristic of the material. A good empirical approximation of how resistivity varies with temperature is

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0), \tag{1.55}$$

where α is called the temperature coefficient of resistivity, ρ_0 is the resistivity at the reference temperature, usually 20°C (or 293K [kelvin degrees]), T_0 is the reference temperature (20°C in our case), and ρ and T are respectively the resistivity and the temperature of the conductor at the temperature of interest, or at the unknown temperature. Equation (1.55) is linear and remains linear for most engineering problems over a wide temperature range around 20°C. Table 1.5 lists the temperature coefficient of resistivity α for some metals.

1.4 OHM'S LAW, POWER DELIVERED AND POWER CONSUMED

A voltage source happens to behave very much like a constant pressure water pump. The voltage source pushes the current through the electric circuit very much like a water pump pushes a volume of water through the closed-loop hydraulic case as depicted in Figure 1.13a, which shows an electrical circuit with a DC voltage source, a conductor or wire and a resistor, and Figure 1.13b

Table 1.5 Temperature coefficients of resistivity for some metals [2]

Metal	(α) Temperature Coefficient of Resistivity [K^{-1}]
Silver	4.1×10^{-3}
Copper	4.3×10^{-3}
Gold	4.0×10^{-3}
Aluminum	4.4×10^{-3}
Manganin[a]	0.002×10^{-3}
Tungsten	4.5×10^{-3}
Iron	6.5×10^{-3}
Platinum	3.9×10^{-3}

[a] An alloy with an extremely low value of α.

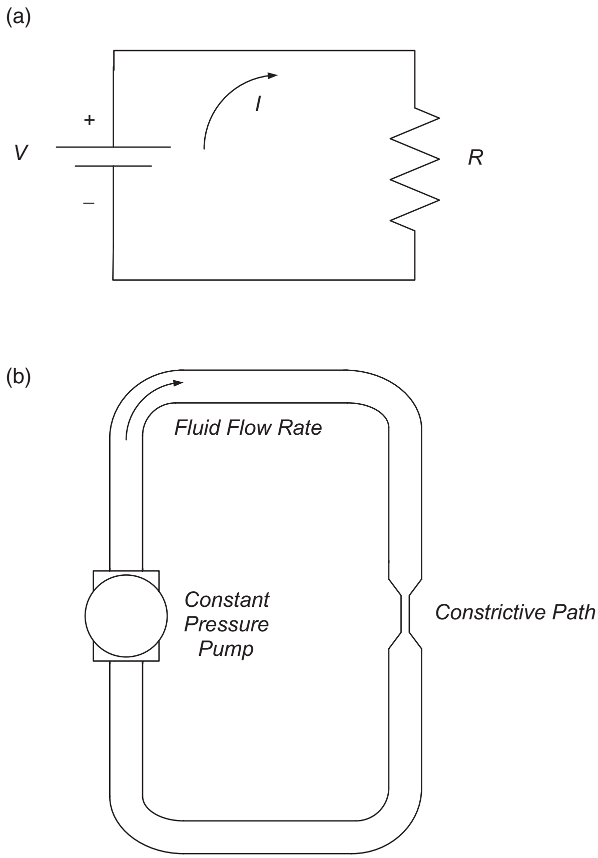


Figure 1.13 (a) An electric circuit and (b) its hydraulic analogy.

which depicts the hydraulic analogy of the electric circuit. The constant pressure pump is the analog to the electric voltage source. The pipe is analogous to the wiring, and the pipe with flow restriction is analogous to the resistor.

In Figure 1.13a, we can state that the current that flows through resistor R is proportional to the voltage applied across its terminals. Equation (1.2) is repeated here for the reader's convenience.

Ohm's Law

$$V = IR. \tag{1.56}$$

In Equation (1.56), V is the voltage across the resistor, I is the current flowing through resistor R, and R is the resistor value. Note from Figure 1.13a that V is the same as the voltage of the DC source applied.

The summary and a side-by-side comparison between the circuit elements and those of the hydraulic analogy in Figure 1.13 are presented in Table 1.6.

Rearranging terms of Equation (1.56), it yields

$$I = V/R \tag{1.57}$$

and

$$R = V/I. \tag{1.58}$$

SI units in Table 1.1 refer to the international system of units (in French *Le Système International d'Unités*) that was established worldwide by the General Conference of Weights and Measures in 1960. Note that Ohm's law can be expressed in any of the three forms given by Equations (1.56), (1.57), or (1.58), where in all three equations, I is the current flowing through resistor R, and V is the voltage across resistor R terminals.

Now back to Joule's law, a resistor will consume or dissipate in the form of heat an amount of power given in watts (W) in the SI system. This power

Table 1.6 Electrical and hydraulic analogies

Electric Circuit		Hydraulic Analogy	
Element	SI Units	Element	SI Units
DC voltage source	V (volts)	Constant pressure pump	Kg/m^2
Current	A (coulomb/s)	Volume flow rate	Liter/s
Resistor	Ω (ohms)	Flow constrictive pipe	There is no unit of resistance for a water flow constrictive pipe.

equals the product of the voltage across the resistor times the current flowing through it or

$$P = VI. \tag{1.59}$$

Plugging Ohm's law Equation 1.56 into power Equation 1.59, one can see that the power consumed by a resistor can also be expressed according to Equation (1.60):

$$P = I^2V. \tag{1.60}$$

Finally, using Ohm's law Equation (1.57) into power Equation (1.59) yields

$$P = V^2/R. \tag{1.61}$$

In terms of units, note that

$$[\text{watts}] = [\text{volts}] \cdot [\text{amperes}] = [\text{amperes}]^2 \cdot [\text{ohms}] = [\text{volts}]^2/[\text{ohms}], \tag{1.62}$$

and using the appropriate SI abbreviations for each unit in Equation (1.62) becomes

$$[\text{W}] = [\text{V}][\text{A}] = [\text{A}]^2[\Omega] = [\text{V}]^2/[\Omega]. \tag{1.63}$$

It is important to emphasize from Equations (1.60) and (1.61) that the power dissipated or consumed by a resistor increases with the square of the current flowing through it, and the power also increases with the square of the voltage applied directly across such resistor.

Example 1.12 Given the circuit of Figure 1.14 where the DC voltage source V = 10 V, the load resistor 0.1 Ω, find the current in the circuit.

Solution to Example 1.12

The current *I* in the circuit is calculated using Ohm's law from Equation (1.57), and it becomes

$$I = V/R = 10\ \text{V}/0.1\ \Omega = 100\ \text{A}. \tag{1.64}$$

Answer to Example 1.12

100 A

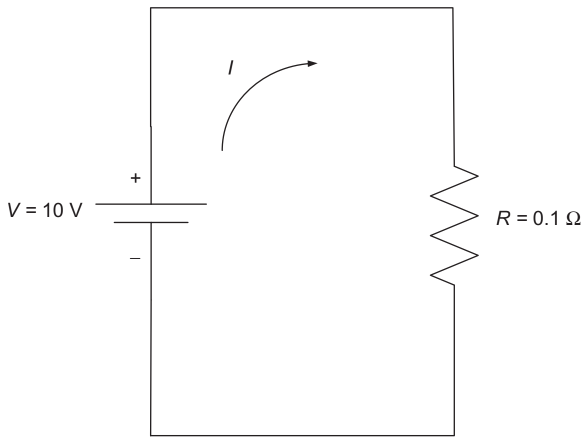


Figure 1.14 Circuit for Example 1.12 with a DC voltage source and a load resistor.

Example 1.13 Using the circuit of Figure 1.14, assume that the source is still 10 V, but the value of the resistor is now just 1 mΩ (or 0.001 Ω). Recalculate the current developed in the circuit.

Solution to Example 1.13

From Equation (2.3), the current is

$$I = V/R = 10\ \mathrm{V}/0.001\ \Omega = 10{,}000\ \mathrm{A}. \tag{1.65}$$

Answer to Example 1.13

10,000 A (ten thousand amperes!)

The purpose of Examples 1.12 and 1.13 is twofold. First, note that regardless of the value of the load resistance, which is 0.1 Ω in Example 1.12 and 0.001 Ω in Example 1.13, the voltage across the resistor is constant and equal to 10 V. Second, note that the ideal voltage source violates Ohm's law under the extreme case, that is, when the load resistor value is 0 Ω. The voltage source produces the same voltage at its output terminals regardless of the current being drawn from it. However, when the load is 0 Ω, meaning when the terminals of the voltage source are short-circuited, the current cannot be determined, because the resistance across its terminals is 0 Ω, and the voltage of the source is a finite number (10 V in our example); thus, 10 V divided by 0 is an undetermined quantity. In theory, the current approaches an infinitely large value. In actuality, the source will attempt to deliver a very large current, but it will only deliver a maximum number of amperes only for a short period of time. If this short-circuit condition is indefinitely applied, then the most likely outcome is at least one or more of the listed events: a damaged voltage source, burnt wires, smoke, even fire, and a serious hazard to people in the neighborhood. Do not try this at home or at any other place.

From Example 1.12 we find that the voltage source has to provide 100 A and maintain its 10 V across its terminals. From Example 1.13, the source has to provide 10,000 A and maintain its 10 V across its terminals. Clearly, the ideal model of a voltage source cannot hold up for extremely large currents because generating 10,000 A is an almost an unreal amount of current, too high for most standards.

The association of a resistor in series with the ideal voltage source adds a dose of reality to the modeling of a voltage source. This added resistor is referred to as the internal resistance of the voltage source.

1.4.1 Voltage Source Internal Resistance

No real battery or DC voltage source can generate an infinite amount of current for any length of time. Even if the current is not infinite, no battery can supply a fixed amount of current at a constant voltage indefinitely. These concepts seem pretty familiar because most of us probably had a flashlight or a car battery replaced, even if the battery is of the rechargeable type like a lead-acid car battery. Real batteries, unlike ideal DC voltage sources, have a finite lifetime. The ability of a fully charged and good battery to supply a given amount of energy, which means supplying a current at a voltage for a finite amount of time, depends on the battery construction, battery type, materials used, size, weight, discharge rate, temperature, resistance, time, and age of the battery. Clearly, real batteries neither have the capability of generating an infinite current nor that of generating a constant current indefinitely. The first-order approximation that we need to introduce into the ideal voltage source model is a non-zero Ohm internal resistance. In the real world, such resistance is a function of all of the factors mentioned such as temperature and discharge rate or usage of the battery. But for most practical purposes, it is reasonable enough to assume that the internal resistance of the battery is not zero, and under normal battery operating conditions, such internal resistance remains fairly constant. Figure 1.15 represents a circuit model of a real battery.

The real battery is depicted within the dotted box, and it is composed of two basic components, an ideal voltage source in series with a current limiting resistance or what we refer to as the battery internal resistance. Note that there is no access to the internal node, where the positive terminal of the battery connects to the internal resistance.

Note that the model in Figure 1.15 is a mathematical representation of the battery and is just a first-order approximation. The model represents the finite amount of current capability of the battery. Why is it only a first-order approximation? Because the model that includes the internal resistance does not account for the increase of its internal resistance as the battery looses

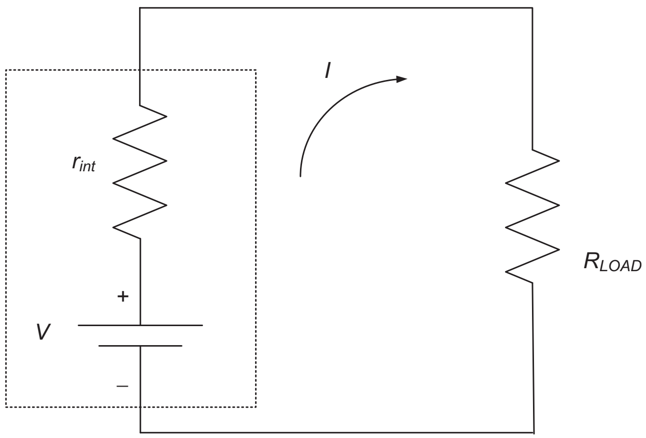


Figure 1.15 First-order approximation model of a real battery, showing its non-zero internal resistance.

current-generating capability due to usage, temperature, and any other factors over time.

As the battery ages or becomes discharged, its capacity of generating current diminishes and that can be modeled as the internal resistance increasing in value with respect to time. Note that as time goes by, the ampere-hour rating of the battery diminishes; the battery is being discharged. Figure 1.16 depicts an alkaline battery discharge characteristics. Ampere-hours versus power delivered by the battery are plotted for different battery output voltages.

Example 1.14 For example, let us assume we get a battery that is rated to provide 12 V in open-circuit mode, that is, no load. Let us further assume that we model such battery to have an internal resistance of 1 Ω. What does this mean?

Solution to Example 1.14

The internal resistance of the battery is in a way a figure of merit of the battery. It expresses what can conceivably be the absolute largest current that the battery can supply if its terminals are short-circuited. The battery short-circuit current is given by

$$\textit{Short-Circuit Current } I_{shc} = \textit{Open-Circuit Voltage } V_{oc} / \textit{Internal Resistance } r_{int}. \tag{1.66}$$

$$I_{shc} = V_{oc}/r_{int}. \tag{1.67}$$

$$I_{shc} = 12\text{ V}/1\,\Omega = 12\text{ A}. \tag{1.68}$$

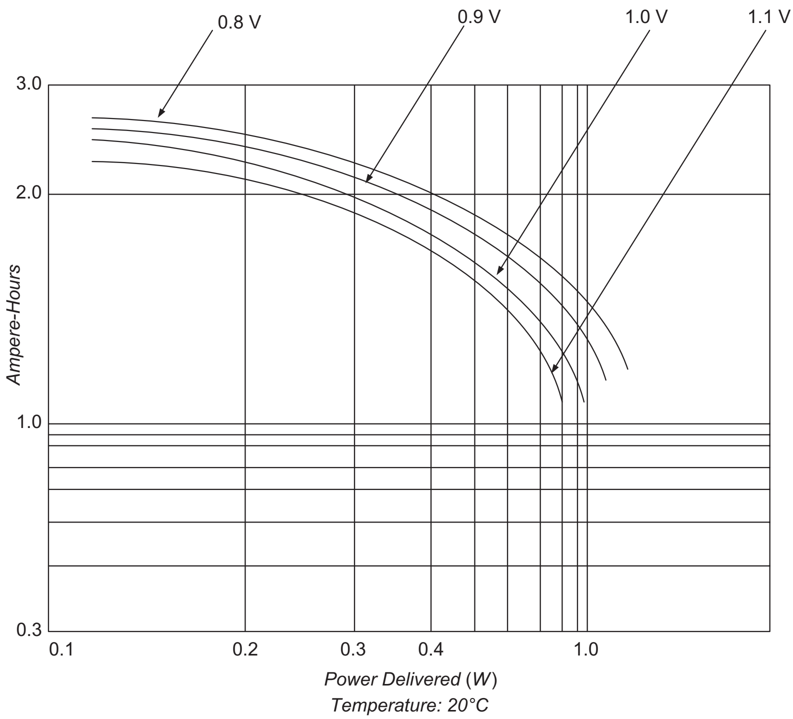


Figure 1.16 Alkaline battery discharge characteristic curves.

In practical terms, real batteries will not be able to supply their short-circuit current for very long (maybe for just a few seconds if that long). However, the internal resistance inclusion within the ideal voltage source model provides a touch of realism when modeling the battery. Note that modeling the battery with an ideal voltage source means that the short-circuit current that the source can supply is infinitely large. Inclusion of an internal resistance limits the current to a finite number. As the battery supplies power to a load, it becomes discharged; which is equivalent to the battery internal resistance to grow in ohmic value as time progresses.

Example 1.15 Assume that we have five different batteries all of which have a 10 V open-circuit voltage. However, each one of the batteries has a different internal resistance:

(a) 0.1 Ω, (b) 1 Ω, (c) 2 Ω, (d) 5 Ω, and (e) 10 Ω.
(b) What is the short-circuit current for each one of the batteries?

Solution to Example 1.15

Since the short-circuit current of a battery is its open-circuit voltage divided by its internal resistance from Equation (1.67), it yields that

(a) $I_{shc} = 10\ \text{V}/0.1\ \Omega = 100\ \text{A}$
(b) $I_{shc} = 10\ \text{V}/1\ \Omega = 10\ \text{A}$
(c) $I_{shc} = 10\ \text{V}/2\ \Omega = 5\ \text{A}$
(d) $I_{shc} = 10\ \text{V}/5\ \Omega = 2\ \text{A}$
(e) $I_{shc} = 10\ \text{V}/10\ \Omega = 1\ \text{A}$.

It is important to emphasize from Example 1.15 that the larger is the numerical value of the battery's internal resistance, the smaller is its short-circuit current. Shortly we will see that a battery with higher internal resistance also has less capacity of generating a voltage closer to its open-circuit voltage when a load is connected across the battery terminals. This example will be addressed again when we cover Kirchhoff's laws.

1.5 CAPACITORS

The most basic capacitor consists of two metallic or conducting plates in parallel, separated by a dielectric. A dielectric is an insulator, and it can be air, mica, polystyrene, transformer oil, glass, porcelain, or many others. Figure 1.17 depicts a parallel-plate capacitor with its two terminals. Each plate is connected to a capacitor terminal.

When we apply a DC voltage E across the capacitor plates, electrical charges (q) become accumulated on both plates of the capacitor. The positive side of the DC source will accumulate positive charges, and the negative side of the source will accumulate negative charges. Figure 1.18 depicts the parallel-plate capacitor, energized by a battery V. The voltage difference that exists across the capacitor is identical to the voltage V produced by the battery after all transients are over.

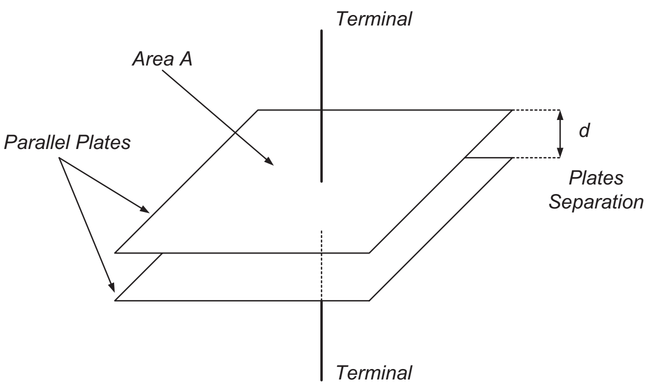


Figure 1.17 A parallel-plate capacitor: the area of the plates is *A*, and the separation between the plates is *d*; the dielectric used is air.

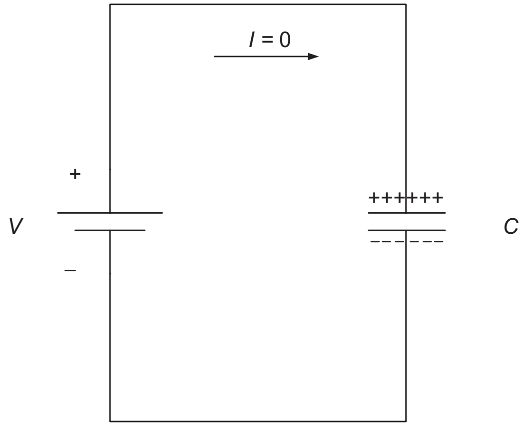


Figure 1.18 Capacitor with battery applied across its terminals.

It can be experimentally proven that the charge accumulated in the capacitor is proportional to the voltage applied across the capacitor. The constant of proportionality is referred to as C, the capacitance of the capacitor. The unit of capacitance is the farad (F), which practically speaking is a very large unit. More common values of everyday use capacitors are in μF, nF, or pF.

In mathematical terms,

$$q = CV, \tag{1.69}$$

where q is electric charge, C is capacitance, and V is the voltage impressed across the capacitor terminals. From units perspective from Equation (1.69), it can be seen that

$$[\text{coulombs}] = [\text{farads}][\text{volts}] \quad \text{or} \quad [\text{C}] = [\text{F}][\text{V}]. \tag{1.70}$$

The voltage V applied across the capacitor plates creates an electric field set up by the battery. Once the capacitor is charged, it takes some finite, but nonzero amount of time for the capacitor charging to occur; the plate connected to the negative of the battery accumulates negative charges or electrons, while the plate connected to the positive of the battery lacks negative charges. Should the wires and the battery be quickly removed, the charges on the plates of the capacitor will remain for as long as the capacitor is not discharged. Figure 1.19 shows two ways to discharge the capacitor after removing the battery across its terminals. Discharging the capacitor by short-circuiting its terminals may be a very hazardous operation, in particular when the capacitance value is large, and the voltage stored in the form of charge inside the capacitor is large. Discharging the capacitor, with an appropriately sized resistor, is safer. The discharge is more gradual and the resistor has to be sized to

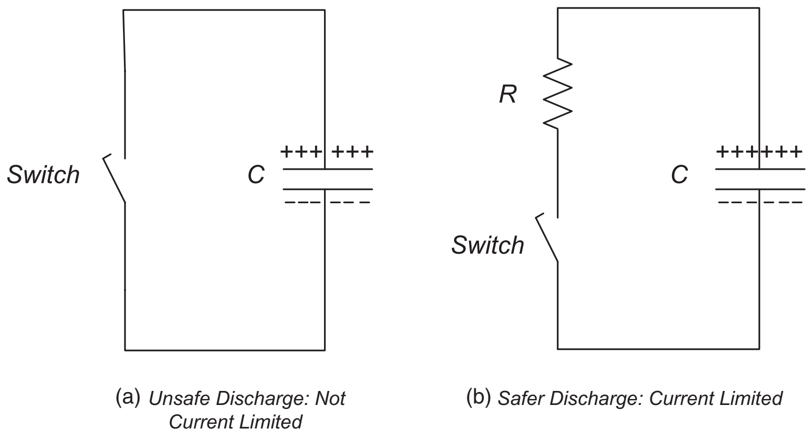


Figure 1.19 Discharging of a charged capacitor: (a) short-circuiting its terminals; (b) placing a resistor across its terminals.

handle the capacitor charge that will be dissipated by the resistor in the form of heat.

1.5.1 Physical Interpretation of a Parallel-Plate Capacitor Capacitance

It can be experimentally determined that the capacitance exhibited by the parallel-plate capacitor with air as dielectric is proportional to the dielectric constant of free space, to the area of the parallel plates, and inversely proportional to the separation d of the plates; see Equation (1.72). The dielectric constant of air is quite close to that of free space (1.0006 ε_o). And the dielectric constant of free space ε_o is a physical constant determined to be

$$\varepsilon_o = 8.85 \times 10^{-12} \text{ F/m} = 8.85 \text{ pF/m}. \tag{1.71}$$

The parallel-plate, air-dielectric, capacitor capacitance equals

$$C_{air} = \varepsilon_o \frac{A}{d}, \tag{1.72}$$

where C_{air} is capacitance in farads with air dielectric, A is the plate area in m^2, and d is plate's separation in meters. If instead of air or free space in between the capacitor plates we introduce a dielectric, the capacitance of the new capacitor with the dielectric different from air will be k times bigger than the capacitance of the original capacitor of capacitance given by Equation (1.74). What is the value of k? k is the relative dielectric constant of the used dielectric material.

For example, if the dielectric is FR4 material, frequently used to fabricate printed circuit boards, the relative dielectric constant is approximately

$$k = \varepsilon_{\mathrm{FR4}}/\varepsilon_{\mathrm{o}} = 4.5. \tag{1.73}$$

k is the relative dielectric constant of the material, FR4, in our example; $\varepsilon_{\mathrm{FR4}}$ is the dielectric absolute dielectric constant. k is always greater than one for dielectrics other than air and vacuum.

Combining Equation (1.72) with Equation (1.73), we obtain the capacitance of a parallel-plate capacitor with a dielectric in between its plates:

$$C = k\varepsilon_{\mathrm{o}}\frac{A}{d} = kC_{air}, \tag{1.74}$$

where k is the relative dielectric constant of the material used, the other parameters are the same ones described before. Looking at Equations (1.72) and (1.74), it is interesting to observe that with a dielectric other than air (or vacuum), a capacitor of the same structure (e.g., parallel-plate capacitor with area A and plate separation d) has a capacitance k times higher than the same capacitor with air as its dielectric.

Table 1.7 shows the dielectric constants for some materials relative to free space dielectric constant.

1.5.2 Capacitor Voltage Current Relationship

From Equation (1.69), charge in a capacitor is proportional to the voltage applied across its terminals.

Thus,

$$q = CV. \tag{1.75}$$

Table 1.7 Table of dielectric constants of some materials measured at room temperature (20°C) [2]

Material	Relative Dielectric Constant k
Vacuum (free space)	1
Air (at 1 atmosphere of pressure)	1.0006
Polystyrene	2.6
Paper	3.4
Porcelain	6.5
Silicon	12
Germanium	16
Standard FR4 Epoxy Glass[a]	4.5
Cyanate Ester[a]	3.8
Teflon[a]	2.2

[a] Dielectric material commonly used to fabricate printed circuit boards (PCBs).

Differentiating the above equation with respect to time yields

$$\frac{dq}{dt} = \frac{d}{dt}(CV), \tag{1.76}$$

and since the capacitance is a constant parameter, then

$$i(t) = \frac{dq}{dt} = C\frac{dV}{dt}, \tag{1.77}$$

where $i(t)$ is the electric current flowing through the capacitor, dq/dt is the definition of current, C is the capacitance value, and dV/dt is the variation of the voltage across the capacitor with respect to time.

Equation (1.77) is of utmost important equation and describes the voltage and current behavior on a capacitor. It is an experimentally determined expression and is valid for all current and voltage waveforms on a capacitor.

Example 1.16 Given the voltage across a capacitor is a sinusoidal function of time,

$$v(t) = V_{peak}\sin(2\pi ft - \varphi), \tag{1.78}$$

where:

- Frequency (f) is the inverse of the sinusoidal waveform period T.
- Phase angle (φ) is the angle with respect to the origin of the time axis that the waveform is shifted from. For a left-shifted sine waveform, the phase angle is positive; for a right-shifted waveform, the phase angle is negative.
- Amplitude (V_{peak}) is also called the *peak* value of the waveform.

Figure 1.20 depicts a sinusoidal voltage waveform and all of its parameters.

1.5.3 Capacitors in Series

Capacitors are said to be in series when they are in a circuit where the same current flows through all of them. Figure 1.21 depicts n capacitors in series, where n is an integer.

A DC voltage source is connected across the three capacitors in series. The positive terminal of the source is connected to the left-most terminal of the the left hand side capacitor; the negative terminal of the source is connected to the right-most terminal of right hand side capacitor.

Since the current is the same flowing through all three capacitors, they all have identical charge (q). The sum of all the voltage differences across each

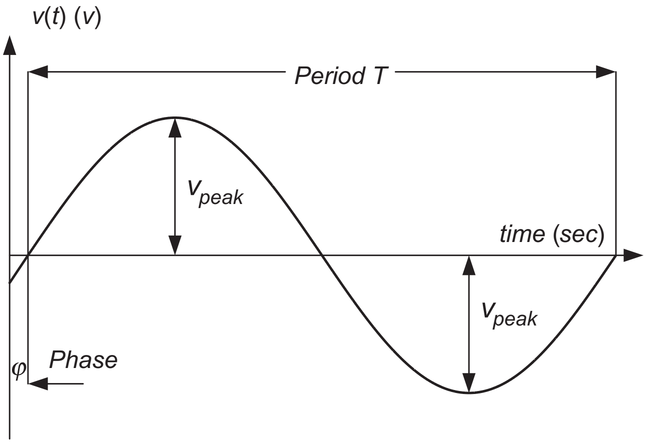


Figure 1.20 Sinusoidal voltage waveform as a function of time.

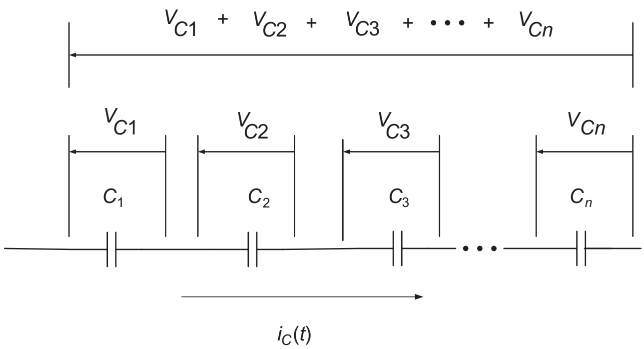


Figure 1.21 Capacitors in series.

capacitor equals to the DC source voltage whose positive terminal is placed across the first series capacitor free terminal and its negative terminal connected to the last capacitor free terminal:

$$V_{BATT} = \mathrm{V}_1 + \mathrm{V}_2 + \mathrm{V}_3 + \ldots + \mathrm{V}_\mathrm{n}. \tag{1.79}$$

We are now interested in finding the series equivalent value of all n capacitors connected in series. For each capacitor we have that

$$V_1 = q/C_1; V_2 = q/C_2; V_2 = q/C_3; \ldots; \quad ; V_n = q/C_n, \tag{1.80}$$

since all have the same amount of charge q. Plugging Equation (1.80) into Equation (1.79) we get

$$
\begin{aligned}
V_{BATT} &= V_1 + V_2 + V_3 \ldots + V_n \\
&= q/C_1 + q/C_2 + q/C_3 + \ldots + q/C_n \\
&= q(1/C_1 + 1/C_2 + 1/C_3 + \ldots + 1/C_n).
\end{aligned} \tag{1.81}
$$

From above we know that the series equivalent capacitance $C_{series\text{-}equiv}$ has the total voltage V_{BATT} across it and charge q within it. Thus, from Equation (1.81) we have

$$1/C_{series\text{-}equiv} = (1/C_1 + 1/C_2 + 1/C_3 + \ldots + 1/C_n) \tag{1.82}$$

or

$$\frac{1}{C_{series\text{-}equiv}} = \sum_{i=1}^{n} \frac{1}{C_i}. \tag{1.83}$$

Example 1.17 Capacitors in series: Given three capacitors in series, $C_1 = 2\ \mu\text{F}$, $C_2 = 3\ \mu\text{F}$, and $C_3 = 6\ \mu\text{F}$, find the series equivalent capacitance.

Solution to Example 1.17

Applying Equation (2.83) we obtain that

$$1/C_{series\text{-}equiv} = 1/2 + 1/3 + 1/6, \text{ which leads to } C_{series\text{-}equiv} = 1\ \mu\text{F}.$$

1.5.4 Capacitors in Parallel

Capacitors are said to be in parallel when they are in a circuit where the same voltage is applied to their terminals. Figure 1.22 depicts n capacitors in parallel.

A voltage source is connected across the n capacitors in parallel. All n capacitors have the exact same voltage applied across their terminals.

For each capacitor we have that

$$V = \mathrm{q}_1/C_1 = q_2/C_2 = q_3/C_3 = \ldots = q_n/C_n. \tag{1.84}$$

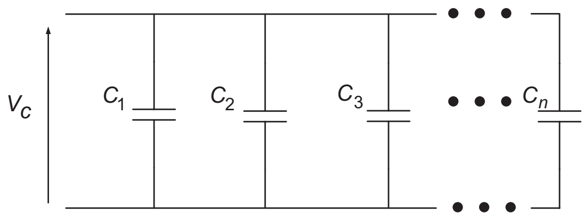


Figure 1.22 Capacitors in parallel.

Since all have the same voltage across, a parallel equivalent capacitance $C_{parallel\text{-}equiv}$ such that its charge equals to the sum of the charges of each one of the capacitors in parallel, exists. Thus,

$$q_1 + q_2 + q_3 + \ldots + q_n = q_{parallel\text{-}equiv} = C_{parallell\text{-}equiv} V, \tag{1.85}$$

and since

$$q_1 = C_1 V;\, q_2 = C_2 V;\, q_3 = C_3 V; \ldots \text{and } q_n = C_n V. \tag{1.86}$$

Plugging Equation (1.86) into Equation (1.85),

$$C_1 V + C_2 V + C_3 V + \ldots + C_n V = C_{parallell\text{-}equiv} V, \tag{1.87}$$

which leads to

$$C_{parallell\text{-}equiv} = C_1 + C_2 + C_3 + \ldots + C_n. \tag{1.88}$$

Or in a more compact form,

$$C_{parallell\text{-}equiv} = \sum_{i=1}^{n} C_i. \tag{1.89}$$

Example 1.18 Given a 10 nF, 0.1 μF, and 140 pF capacitor, calculate the total parallel equivalent capacitance of all three capacitors in parallel.

Converting all capacitances to pF, we obtain
10 nF = 10,000 pF and 0.1 μF = 100,000 pF.
Applying Equation (1.89),

$$C_{parallell\_equiv} = 10{,}000 + 100{,}000 + 140 = 110{,}140 \text{ pF}.$$

1.5.5 Energy Stored in a Capacitor

The electrical energy is

$$\begin{aligned} Energy = {} & (capacitor\ average\ voltage) \times \\ & (average\ current\ through\ the\ capacitor) \times time. \end{aligned} \tag{1.90}$$

From Equation (1.75),

$$C = qV \tag{1.91}$$

and

$$q = It = I_{average}t, \tag{1.92}$$

where I is the average current through the capacitor, $I_{average}$.

From Equations (1.91) and (1.92) we obtain the average value of current through the capacitor:

$$I_{average} = CV/t. \tag{1.93}$$

When a capacitor is charged with a constant current, the voltage across it grows linearly with time from zero to V volts, so that the average voltage across the capacitor is

$$V_{average} = \frac{1}{2}V. \tag{1.94}$$

Plugging Equations (1.93) and (1.94) into Equation (1.90), the energy stored in the capacitor is

$$Energy_{capacitor} = \frac{1}{2}CV^2, \tag{1.95a}$$

where C is the capacitance value in farads, V is the voltage across the capacitor in volts, and the energy is in joules.

Example: Energy Stored by a Capacitor

A 100 μF capacitor has been connected to a 400-V DC source for a very long time. What amount of energy will the capacitor be holding upon removal of the 400-V source?

From Equation (1.95a),

$$Energy = \frac{1}{2}CV^2$$

Thus,

$$Energy = \frac{1}{2} \times 100 \times 10^{-6} \times 400^2 = 8\ \text{J}.$$

1.5.6 Real Capacitor Parameters and Capacitor Types

Real capacitors have the following parameters that characterize them:

C, their capacitance expressed in farads, this is the most important parameter of a capacitor; like the resistance is to a resistor.

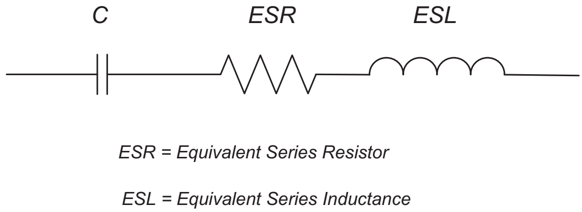


Figure 1.23 Simple model of a real capacitor.

The maximum constant voltage that can be applied to a capacitor dielectric is what capacitor manufacturers call the capacitor voltage rating. Depending on the type of capacitor dielectric the voltage can be from a few volts to hundreds of volts.

Some dielectric materials used are, for example, ceramics, mica, or polystyrene.

A real or physical capacitor does not behave entirely like a capacitor; it also has a small value of resistance in series referred to as the equivalent series resistance (ESR). This ESR is due to the fact that the dielectric is not a perfect insulator and inevitably has some resistance. Capacitors also have some unavoidable stray inductance because they have leads or terminals that have a small non-zero inductance. Figure 1.23 depicts a possible model used to describe the imperfections of a real capacitor.

The ESR accounts for the non-zero resistance of the dielectric material while the equivalent series inductance (ESL) mostly accounts for the inductive parasitic effects that the capacitor leads have. Of course other electrical capacitor models are possible. Capacitor manufacturers sometimes indicate the capacitor model they use for the data sheet specifications. Some capacitor manufacturers specify a *dissipation factor DF in %*. DF is defined as

$$DF = \tan\delta(\text{loss angle}) = \text{ESR}/X_C = 2\pi fC \times \text{ESR}. \tag{1.95b}$$

In Equation (1.95b), X_C is the absolute value of the capacitive reactance, which equals $1/2\pi fC$, where f is the frequency of the sinusoidal signal applied to the capacitor. Chapter 2 will cover capacitive reactance in more detail.

Table 1.8 lists some of the most popular types of capacitors, their dielectric, and some of their electrical characteristics.

Capacitor Component Selection To select capacitors, we need to start by finding out their key purpose in the application. Let us assume for simplicity that we look at only three basic kinds of applications: bulk decoupling, high frequency decoupling, and precise timing control. Bulk decoupling requires large capacitance values; generally, the exact value is not as important as the fact of obtaining large amounts of capacitance. Moreover, these capacitors need to operate with frequencies that are typically well under 100 kHz.

Table 1.8 Some capacitor types

Capacitor Type	Capacitance Range	Operating Voltage Range (V)	Temp. Range (°C)	Tolerance (%)	Typical Uses	Polarized (Yes/No)
Aluminum electrolytic	From low µF to ~68,000 µF	~16 to ~450	–40 to ~85	–10 to 150	Bulk decoupling	Yes
Tantalum electrolytic	From ~2.7 pF to ~1500 µF	~2 to ~125	–55 to ~105	–70 to ~20	Bulk decoupling	Yes
Poscap	4.7 µF to ~1500 µF	~2 to ~25	–55 to ~125	±20	Bulk decoupling, low ESR	Yes
Oscon	~10 µF to ~2700 µF	~2 to ~25	–55 to ~105	±20	Bulk decoupling, low ESR	Yes
Multilayer chip ceramic (MLCC) and ceramics	~1 pF to ~1 µF	10 to 700	–55 to 125, –55 to 150	±1, ±2, ±10, ±20	High frequency decoupling	No
Mylar	1 nF to 0.47 µF	50 to 630	–20 to 75	±2 to ±20	Timing	No
Polyester	1 nF to 10 µF				Timing	No
Polycarbonate	1 nF to 1 µF; 1 µF to 50 µF;	~30 to ~400	–55 to 125	±1, ±2, ±5, ±10, ±20	Timing	No
Polystyrene		various	up to 85	±1, ±2.5, ±5, ±10, ±20	Timing	No
Glass case, hermetically sealed with glass dielectric					Ultrareliable, ultrastable, and resistant to nuclear radiation. Typically used in space missions.	No
Oil impregnated paper	0.1 µF to 20 µF	100 to 10,000			Older technology for industrial controls	No

Practically speaking, one *farad* is an extremely large unit of capacitance. Large capacitance values are in the several thousands of microfarads (μF) and up.

The capacitors' ESR is usually not as important as it is to obtain a huge amount of capacitance for the bulk capacitance application. The operating voltage at which the capacitor will be subjected to in the application should, under no circumstances, be more than about 50% of the capacitor-rated voltage for good reliability. For example, if a number of capacitors will be connected in parallel across a 5-V source, the capacitors must be rated at a voltage of at least 10 V or more. The higher the voltage rating of the capacitor, the bigger the capacitor usually is, so if volume or real estate on the printed circuit board is a prime factor, a very large voltage-rated capacitor may not be suitable for such application. On the other hand, the closer the capacitor voltage rating is to the voltage that the capacitor will be withstanding (5 V in our example), the shorter will be the long-term reliability of the capacitor. As a good rule of thumb, people pick at least a rated voltage that is twice the maximum voltage at which the capacitor will be subjected to. Naturally, the higher is such voltage, the harder it is to obtain a more generous voltage margin. As another example, in electronic circuits, a very common voltage used is 3.3 V. Capacitance values of a few pF are easily obtained with voltage ratings of 50 V or higher. Microfarad-valued capacitors are easily obtained in 10 V, 16 V, and some higher voltage ratings. But when the capacitance values are many hundreds or even thousands of microfarads, the size of capacitor becomes very large with a high voltage rating such as 100 V and above.

High-frequency decoupling capacitors do not necessarily require as large a capacitance value, but they require good behavior of the capacitors at higher frequencies. High frequencies for decoupling capacitor mean frequencies of many thousands of kilohertz and above. The capacitors need to preferably be more stable in value and their ESR needs to be much smaller than those of capacitors used for bulk decoupling. Finally, in timing applications, capacitors are used as analog timing elements. It is required for the capacitance to be as accurate in value as possible; their capacitance value should also be stable under temperature variations, aging of the component, and changing operating conditions. And finally, it is also required that the ESR of the capacitor be small. This produces capacitor smaller losses and also plays a role in the capacitor accuracy characteristics.

1.6 INDUCTORS

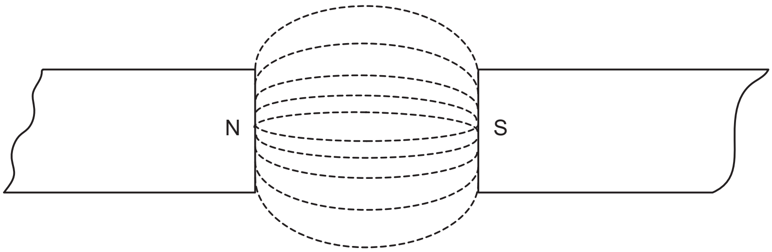
An inductor mainly consists of a copper wire wound around either a nonmagnetic core (air-core inductor) or a magnetic core. The core is the physical medium that holds the copper wire in place in relatively large inductors. Most importantly, the iron-core of an inductor allows one to obtain a much higher inductance per unit volume. For example, assume two inductors of the same dimensions, same wire gauge, same number of turns, and same volume. One has an air-core and the other one has an iron core. The iron-core inductor inductance value will be thousands of times the inductance of the same inductor built with an air-core. This directly depends on the relative permeability (μ_R) of the iron-core ferromagnetic material. We will cover more on magnetism and permeability of materials later in this chapter. Small air-core inductors generally do not need any media to hold the copper turns since they usually have a handful of turns. A thin layer of enamel electrically isolates the wound wire. The purpose of the insulating enamel is to keep multiple coil turns and multiple layers of turns from short-circuiting each other. Inductors used in power applications typically require higher inductance values than those inductors used in radio frequency (RF) applications. In RF, usually the inductor is used for the purpose of building a high-frequency tank circuit or tuned circuit and high frequencies (usually many megahertz) drive down the size of the inductor. On the other hand, on power applications, the high current requirements and the relatively low switching frequencies of the switching power supplies (in the order of hundreds of kilohertz) drive the size of the inductors up.

1.6.1 Magnetism

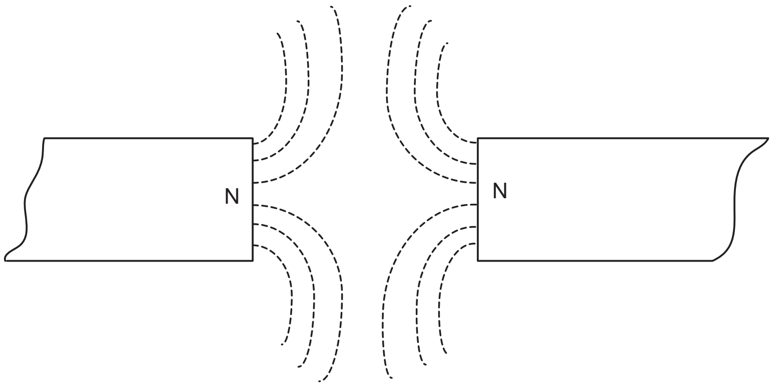
There are some minerals that have magnetic properties. Figure 1.24 shows lines of force created by the magnetic field produced by a naturally magnetized bar. The two ends of the magnetic bar are referred to as the north and the south poles. Note that opposite poles produce attracting lines of force (Fig. 1.24a), and like poles produce opposing lines of force (Fig. 1.24b).

When an electric current flows through a conductor, such current produces a magnetic field in the surroundings of the conductor. Figure 1.25 depicts a current-carrying conductor that penetrates a piece of cardboard in a perpendicular direction to the cardboard surface. Iron filings that are arbitrarily placed on the cardboard and in proximity to the current-carrying conductor will become rearranged in the shape of concentric rings around the conductor.

What is the direction of the concentric magnetic force field lines around the conductor? Let us refer one more time to Figure 1.25. When the current flows from top to bottom, the concentric magnetic field lines will be in a clockwise direction looking at the concentric rings from the top of the cardboard. This is also referred to as the right-hand rule. Referring now to Figure 1.26, a current is flowing from below the surface of this sheet of paper toward the reader. Concentric lines of magnetic force are formed in the surroundings of the current on all the planes, which are perpendicular to the current flow. Assume that you embrace with your right hand the current-carrying conductor with the right-hand side thumb pointing upward or toward the viewer. The right thumb represents the current direction, and the other four fingers embracing the current-carrying conductor represent the direction of the magnetic field.



(a) *Magnetic Lines of Force Produced by Unlike Poles*



(b) *Magnetic Lines of Force Produced by Like Poles*

Figure 1.24 (a) Magnetic bar with opposite polarity poles produces attracting lines of force; (b) magnetic bar with same polarity poles produces repelling lines of force.

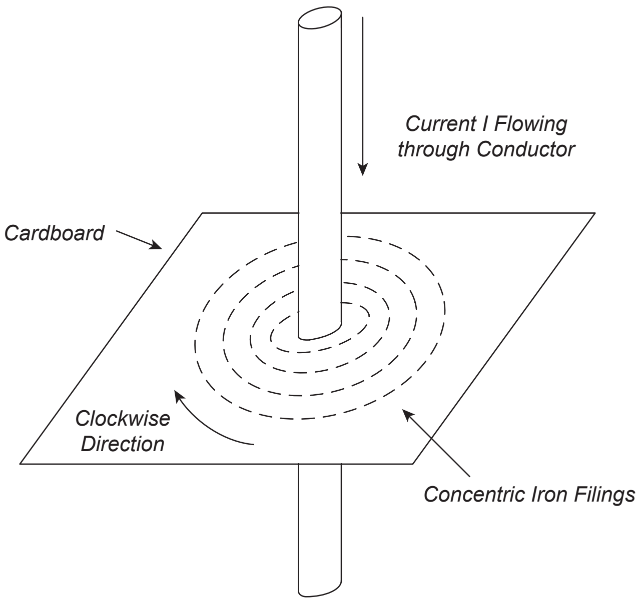


Figure 1.25 Magnetic field force produced around a current-carrying conductor.

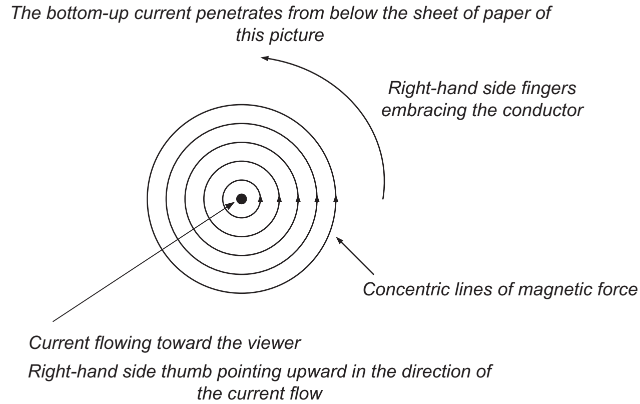


Figure 1.26 Right-hand rule.

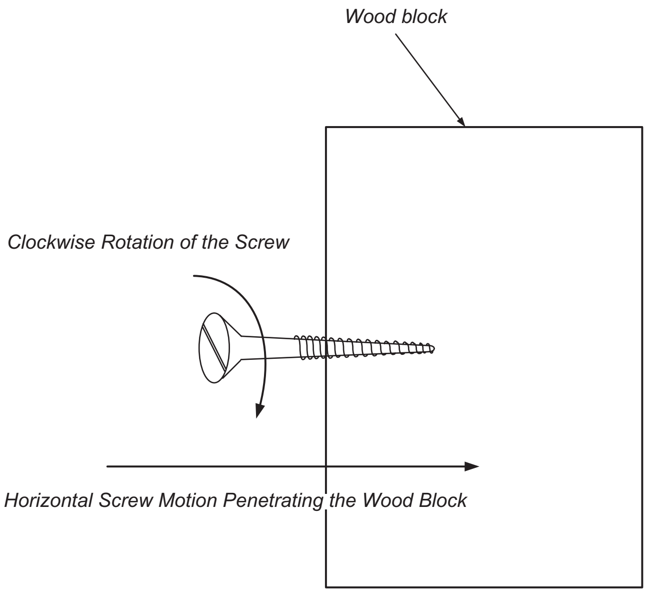


Figure 1.27 Right-hand screw rule.

The right-hand rule is analogous to the rotational and longitudinal motions of a screw into a piece of wood. Referring to Figure 1.27, when an observer looks at the head of the screw, and the screw is rotated in the clockwise direction, the screw length will penetrate the piece of wood. The analogy here is that the longitudinal displacement of the screw is analogous to the current flowing through a conductor, and the clockwise rotation of the screw is analogous to the direction of the magnetic field created around the current-carrying conductor.

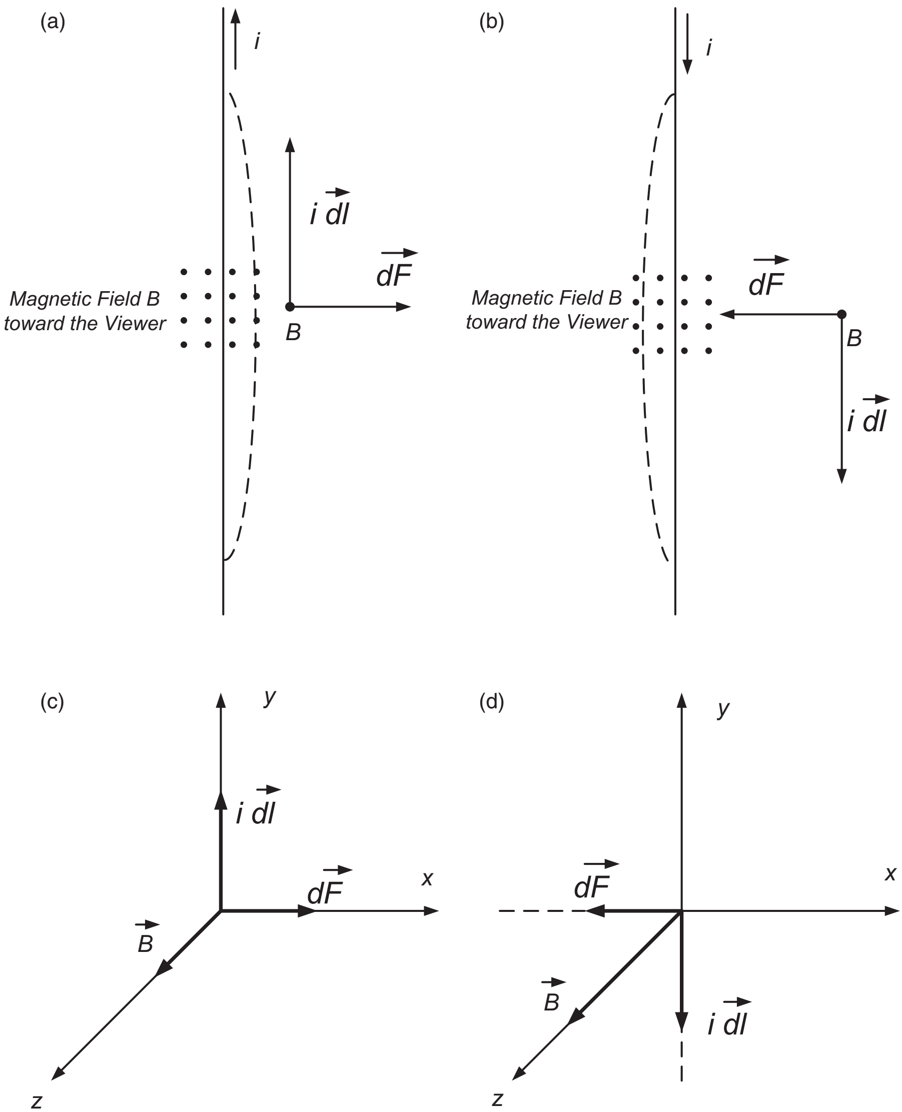


Figure 1.28 Magnetic forces produced by a current-carrying conductor: (a) current *i* flows from the bottom up; (b) current *i* flows from the top to the bottom; (c) isometric view of vectors *dF, idl, B* that corresponds to part (a); (d) isometric view of vectors *dF, idl, B* that corresponds to part (b).

Figure 1.28a shows a current "*i*" carrying conductor, the current flows from the bottom up, through a perpendicularly applied magnetic field ***B***. Magnetic field ***B*** is perpendicular to the surface of this sheet, coming toward the viewer. Small black circles represent the tips of the vector field arrows. A force on the conductor is exerted in the direction shown by ***dF***. Mathematically, the force equals the vector or cross product of ***idl*** with magnetic field ***B***. Figure 1.28a is associated with the Biot–Savart equation: $\boldsymbol{dF} = \boldsymbol{idl} \times \boldsymbol{B}$, and it is graphically represented in Figure 1.28c. Note that the three vectors, ***dF***, ***idl***, and ***B*** are shown in three perpendicular planes in three dimensions. For Figure 1.28a, its corresponding vectors are shown in Figure 1.28c: ***dF*** along the x-axis; ***idl*** along the y-axis, and ***B*** along the z-axis. The operator × stands for cross product or vectorial product, a mathematical operation among vectors.

Similarly, Figure 1.28b shows a conductor with the current flowing from top to bottom; the magnetic field ***B*** is perpendicular to the sheet of paper on this book. Figure 1.28d is associated with Figure 1.28b which graphically expresses the vector or cross product of ***idl*** and magnetic field ***B*** to generate force ***dF***.

Biot–Savart's law is fundamental in the understanding of electric motors and it is expressed by

Biot–Savart Law

$$\boldsymbol{dF} = \boldsymbol{idl} \times \boldsymbol{B}, \tag{1.96}$$

where ***dF*** is a differential of force, ***idl*** is current times differential of length, and ***B*** is the magnetic field. The × sign is the mathematical symbol for the cross product, also referred to as the vector product.

The units of force (***dF***) are newtons, units of ***idl*** in amperes-meters, and ***B*** in webers (Wb). Biot–Savart's Law can be expressed in its equivalent form, substituting ***idl*** with

q v, where q is the charge of the particle and v its velocity. This yields

Lorentz Law

$$\boldsymbol{dF} = \boldsymbol{qv} \times \boldsymbol{B}. \tag{1.97}$$

In Equation (1.97) ***dF***, ***v***, and ***B*** are vectors. Finally, it is important to emphasize that the vector or cross products graphically shown in Figure 1.28c,d follow the convention adopted by the right-hand rule, discussed earlier.

Looking one more time at Figure 1.28a,b, the dotted line drawn conductor is the deflection that the wire will experience due to the forces produced by the given current and magnetic field directions.

1.6.2 Magnetic Field around a Coil

An alternating current (AC) flowing through an inductor or coil (Fig. 1.29) produces a magnetic flux from the contribution of each one of its turns. An AC voltage or current is one whose polarity alternates between a positive and a negative direction at some constant frequency. Deeper coverage of AC circuits is addressed in Chapter 2.

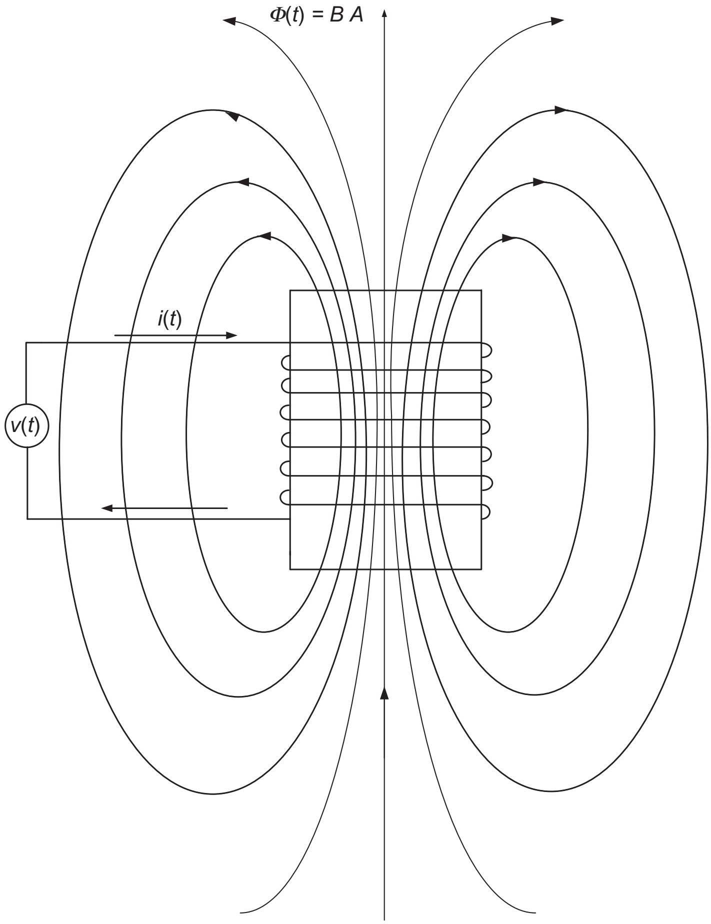


Figure 1.29 Iron-core inductor with AC excitation.

The larger the number of turns, the larger will be the magnetic flux. The magnetization effect from each one of the coil turns is additive, as if each turn was a small magnet. The contribution of all turns produces a magnetic field (B), also called the magnetic induction field B or flux density B. Flux density for a magnetic field perpendicular to a cross-sectional area is

$$B = \frac{\phi}{A}, \tag{1.98}$$

where B is the magnetic field or flux density in teslas (T) or weber/m^2 (Wb/m^2), ϕ is magnetic flux in webers, and A in m^2 is the cross-sectional area through which the lines of flux travel through.

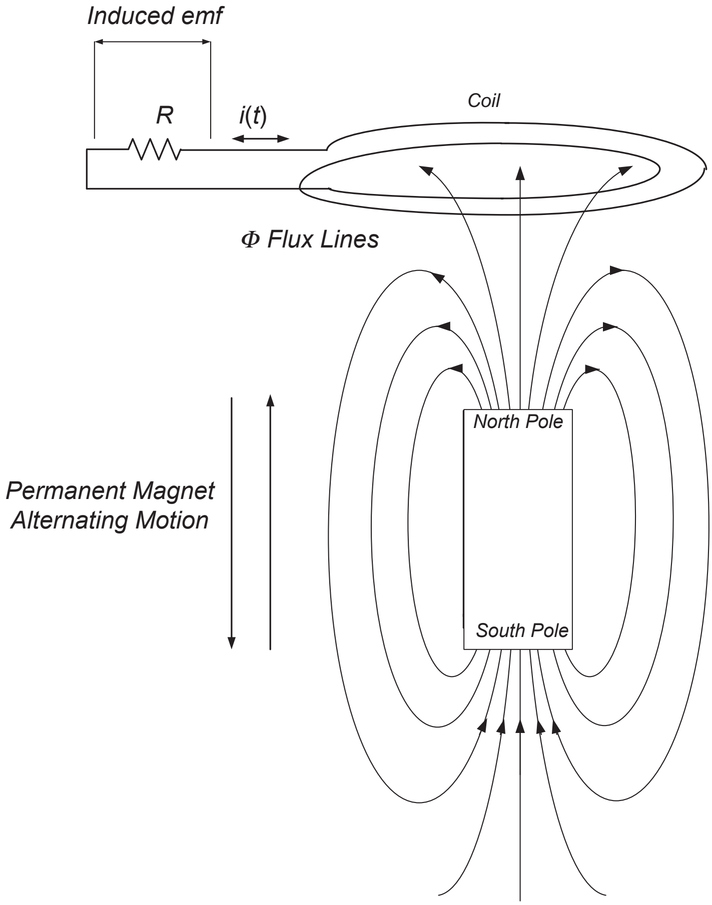


Figure 1.30 Magnetic induction produced by movement of permanent magnet bar.

Figure 1.30 depicts a natural permanent magnet bar in proximity to an air-core coil. A back and forth motion of the permanent magnet causes the flux density to vary with respect to time. This time, varying flux induces an *electromotive force* (*emf*) on the upper coil of Figure 1.30. An important difference between an AC-induced magnetic field and the magnetic field of a permanent magnet is that the permanent magnet field is constant with respect to time when the bar is stationary, and in such case, we can call it a DC field. The AC-induced field B is alternating with respect to time (Fig. 1.29). The AC current produced by the induced *emf* (Fig. 1.30) is produced by a permanent magnet because its flux is varied with respect to time by moving the magnetic bar back and forth.

Referring again to the iron-core inductor of Figure 1.29, if we assume now that the iron-core now has an air-core of the same dimensions, there will also be a time-varying magnetic flux upon a current flowing through the coil, but the intensity or strength of such magnetic flux will be considerably smaller. How much smaller depends on the specifics of the iron material and the air-core. The reason is that magnetic flux or magnetic line forces travel much more easily through a ferromagnetic material than through air. The property or parameter that characterizes magnetic materials is called *permeability*. In practical terms, *relative permeability* is more frequently used, and relative permeability is defined as the magnetic material permeability divided by the permeability of vacuum or free space. Relative permeability is dimensionless. Permeability of air is close to that of vacuum.

$$\mu = \mu_o \cdot \mu_r. \tag{1.99}$$

The permeability of vacuum also referred to as the permeability of free space is a physical constant, and it was determined experimentally to be

Permeability of Free Space

$$\mu_o = 4\pi 10^{-7} \text{ Wb/A} \cdot \text{m}. \tag{1.100}$$

The relative permeability of free space is 1. The relative permeability of magnetic materials ranges from as little as 10 to as much as several hundreds of thousands. For example, the permeability of 3–6% FeSi ranges from 1000 to 10,000.

1.6.3 Magnetic Materials and Permeability

In relation to their capacity of allowing the passage of magnetic lines of force, materials can be classified into four categories:

- Nonmagnetic materials like vacuum, air, wood, paper, and plastic. These materials have no effect on the passage of magnetic lines of force.
- Diamagnetic materials which show a small opposition to magnetic lines of force. For most practical purposes are nonmagnetic materials. Examples are copper and silver. Their $\mu < \mu_o$ or their $\mu_r < 1$.
- Paramagnetic materials which somewhat assist the passage of magnetic lines of force. Examples are aluminum and platinum. Their permeability is $\mu > \mu_o$ or their relative permeability is $\mu_r > 1$.
- Ferromagnetic materials, sometimes simply called magnetic materials, which greatly assist the passage of magnetic lines of force through them. Some magnetic materials are iron, nickel, cobalt, steel (an iron alloy), and ferrites (ceramic composite materials). Ferromagnetic materials' permeability is usually much larger than that of free space μ_o. μ_r, the relative permeability of magnetic materials is usually several thousands

or hundreds of thousands larger than the relative permeability of free space. Note: By definition the relative permeability of free space or air is 1, because we are referring the relative permeability of free space to its permeability.

1.6.4 Electromagnetic Induction and Inductor Current–Voltage Relationship

We studied that an AC current induces a magnetic field. The converse is also true. An alternating magnetic field produces an AC current.

Michael Faraday* experimentally discovered electromagnetic induction. Given an electrical setup like the one shown in Figure 1.30, if we move a permanent magnet back and forth over the coil, an electromagnetic force (or *emf*) is generated across the coil terminals, and an electric current will be generated in the circuit with the coil and the resistor. It is very interesting to note that there is no physical or electrical contact between the magnetic bar and the coil. Alternatively, an *emf* can be generated by switching on and off the switch in the primary circuit as shown by Figure 1.31. The switching action will cause a time-varying magnetic flux produced by the iron-core inductor. This magnetic flux will be magnetically coupled onto the stationary secondary winding coil and an *emf* will be produced across the secondary inductor. Such *emf* will generate a current that will flow through resistor R.

Electromagnetic induction allows the generation of an electromagnetic force by the net movements of charges with respect to time.

In Figure 1.31, the opening and closure of the switch causes a time-varying current in the primary side of the circuit. This varying current produces a magnetic flux in the primary circuit loop. The varying magnetic flux due to the current in the primary coil induces an *emf* on the secondary coil. This *emf* produces a current in the secondary coil. The induced *emf* in the secondary coil is equal to the rate at which the flux changes with respect to time or in mathematical form,

$$emf = -d\Phi/dt. \tag{1.101}$$

Equation (1.101) is referred to as the Faraday–Lenz law, where Φ is the magnetic flux and t is the time variable. $d\Phi/dt$ is the derivative of Φ with respect to time.

The minus sign in front of $d\Phi/dt$ means that the induced *emf* opposes the change of flux. In other words, the induced current has a direction such that the magnetic field due to the current opposes the change in magnetic flux induced by the current in the primary coil.

* Michael Faraday was an English physicist and chemist (1791–1867). He is most known for having discovered electromagnetic induction and the laws of electrolysis.

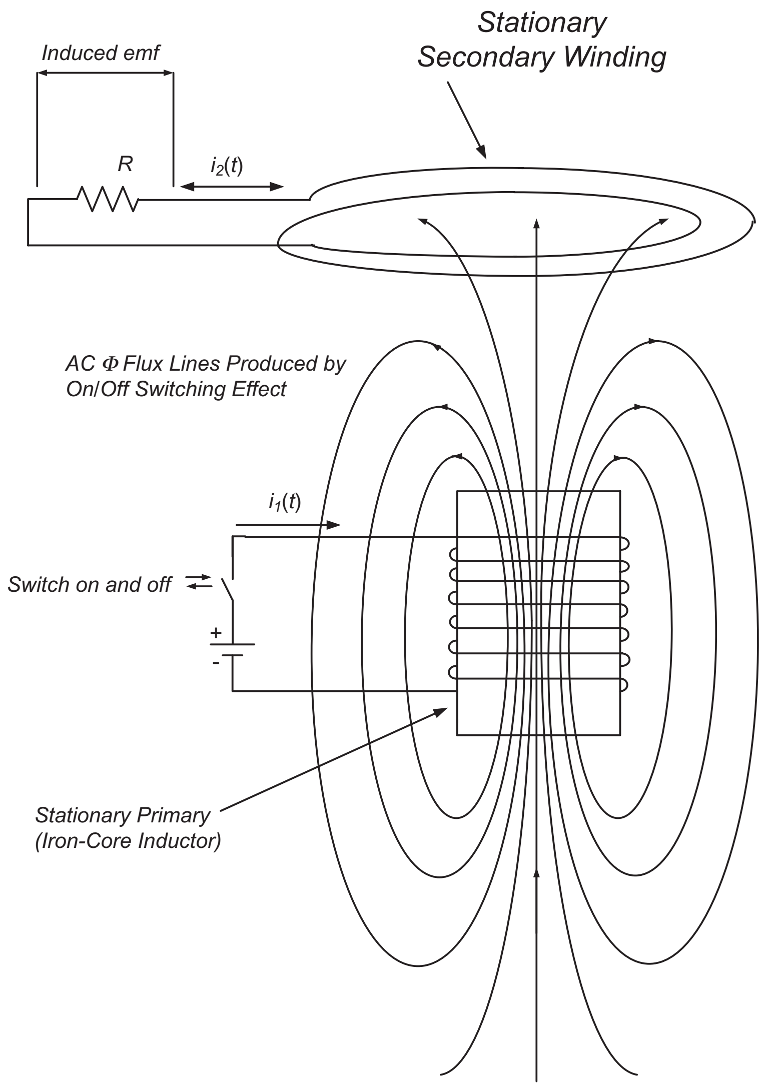


Figure 1.31 Magnetic induction generated electromotive force (*emf*) using an on/off switch.

The inductance (L) of a single turn coil is defined as

$$L = \Phi / i. \tag{1.102}$$

When the coil has N number of turns, the inductance L becomes

$$L = N\Phi / i. \tag{1.103}$$

In Equation (1.103), $N\,\Phi$ is defined as the magnetic flux linkage. The inductance of a coil is a measure of the flux linkage per unit of current. The unit of inductance is defined as a henry (H).

Rearranging terms in Equation (1.103), we obtain

$$N\Phi = Li. \tag{1.104}$$

Differentiating Equation (1.104) yields

$$d(N\Phi)/dt = Ldi/dt. \tag{1.105}$$

Since N, the number of inductor turns is constant, Equation (1.105) becomes

$$Nd\Phi/dt = Ldi/dt, \tag{1.106}$$

where $N\,d\Phi/dt$ is the voltage drop across the inductor L or $v_L(t)$. The magnitude of the voltage across an inductor depends on the rate of change of the current flowing through the inductor with respect to time.

Then, the fundamental voltage current relationship of an inductor is

$$v_L(t) = Ldi/dt. \tag{1.107}$$

The minus sign from Lenz law is dropped, since we are only interested in the magnitude of the voltage across the inductor. In circuit analysis, when a current enters the inductor, the voltage drop across the inductor has the polarity and direction shown based on the direction of the current as depicted in Figure 1.32.

Example 1.19 Determine the voltage waveform across an inductor knowing that its current waveform is 2 A DC and the inductance value is $L = 0.1$ mH.

Solution to Example 1.19

From Equation (1.107), the voltage–current relationship in an inductor is

$$v_L(t) = Ldi/dt.$$

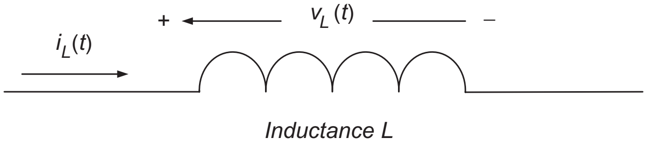


Figure 1.32 Inductor voltage drop polarity and current direction.

Since $i = 2$ A DC, the derivative of a constant is zero; thus, the voltage across the inductor is zero when a DC current flows through it. Idealized inductors have a zero-ohm winding resistance. Real inductors, however, have a winding resistance larger than zero ohm, whose value depends on the length of the wire used, its cross section, and the resistivity of the wire material used. (Refer again to Eq. 1.52 on resistivity.)

More generally, we can affirm that regardless of the inductance value, a DC current flowing through an inductor will develop 0 V of AC voltage across the inductor terminals.

Example 1.20 Determine the voltage developed across inductor terminals when a sinusoidal current that is a function of time (t) flows through it. The sinusoidal current is depicted in Figure 1.33, and three key parameters determine the sinusoidal current:

- Frequency (f): the inverse of the sinusoidal waveform period T.
- Phase angle (φ): the angle with respect to the origin of the time axis that the waveform is shifted.
- Amplitude: also called the peak value of the waveform.

Figure 1.33 depicts a sinusoidal current waveform that conforms to Equation (108):

$$i(t) = I_{peak} \sin(2\pi ft + \varphi). \tag{1.108}$$

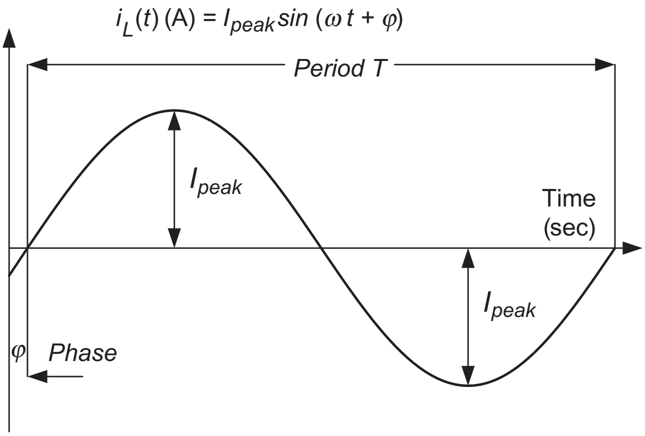


Figure 1.33 AC current waveform.

For this example we assume that

$$I_{peak} = 1\ \text{A},\ f = 1000\ \text{Hz, and } \varphi \text{ is zero degrees.} \tag{1.109}$$

Where the unit of peak current is the ampere, the unit of frequency is second^{-1} (s^{-1}), also called hertz and abbreviated Hz. The unit of phase angle is either degrees or radians. When the phase is expressed in degrees, the "°" has to be explicitly shown next to the number of degrees. If radians are used, then no units are indicated next to the phase angle in radians. The radian is considered to be dimensionless. From high school geometry, let us remember that 2π radians or simply 2π equals 360°. So, for example, a 45° angle equals to π/4 (i.e., π/4 radians).

Rewriting Equation (1.108) with the parameters given by Equation (1.109) leads to

$$i(t) = 1\sin(2000\pi t + 0°)\ \text{A}. \tag{1.110}$$

This can be simply stated as

$$i(t) = \sin(2000\pi t)\ \text{A}. \tag{1.111}$$

The argument of the sinusoidal waveform in Equation (1.111), $2\pi ft$ is also referred to as ωt; where ω (the Greek letter omega) is called the sinusoidal waveform angular frequency or pulsation. Its units are hertz or s^{-1}. In some electrical engineering literature, the units of ω are also referred to as radians per second (rad/s). Since the radian is a dimensionless unit, hertz and rad/s are basically the same thing.

Figure 1.33 depicts the waveform described by Equation (1.111) with a zero phase angle.

Solution to Example 1.20

$$v_L(t) = L\,di/dt. \tag{1.112}$$

The voltage across the inductor produced by the sinusoidal current given by Equation (1.112) is easily calculated plugging Equation (1.111) into Equation (1.112).

Thus,

$$\begin{aligned} v_L(t) &= L\frac{d}{dt}[\sin(2000\pi t)] \\ &= v_L(t) = 2000\pi L\cos(2000\pi t). \end{aligned} \tag{1.113}$$

Figure 1.34 shows both the sinusoidal current through the inductor and the sinusoidal voltage developed across the inductor terminals. It is important to note that the voltage waveform leads the current waveform by 90°. Note: Cosine and sine waveforms are both referred to as sinusoidal waveforms in a general sense.

1.6.5 Inductors in Series

When inductors are connected in series, and there is no mutual coupling among the magnetic fields produced by each inductor in the series, the current flowing through the series of inductor is the same. Upon the series of inductors being excited by a time-varying current, each inductor develops a voltage across its terminals given by Equation (1.107), repeated here for the reader's convenience:

$$v_L(t) = L di/dt.$$

Referring to Figure 1.35, the current in series with the inductors is the same, and each inductor develops a voltage across its terminals given by Equation (1.107). This voltage may be different for every inductor since individual inductances may be different.

Given L_1, L_2, L_3, . . . and L_n, current $i(t)$ develops the following voltages across each inductor:

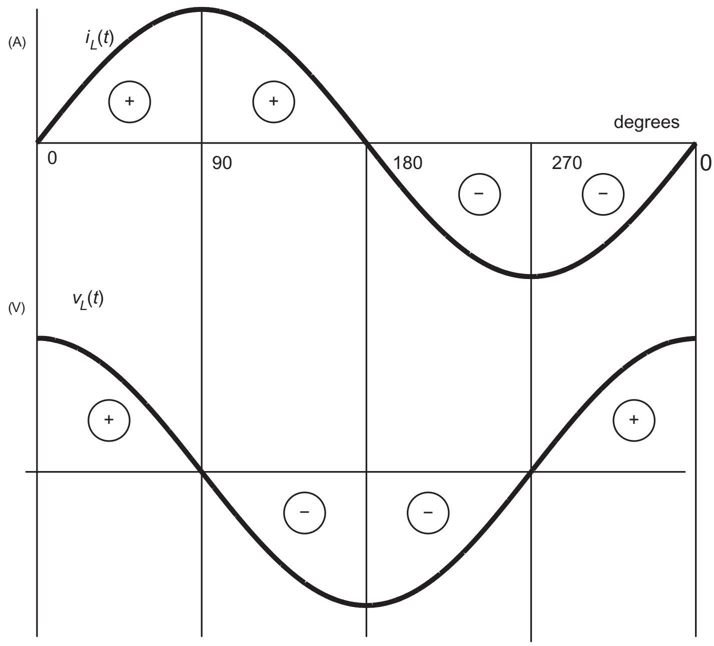


Figure 1.34 Sinusoidal current and voltage on an inductor.

Figure 1.35 Inductors in series.

$$v_{L1}(t) = L_1 di/dt \tag{1.114}$$

$$v_{L2}(t) = L_2 di/dt \tag{1.115}$$

$$v_{L3}(t) = L_3 di/dt \ldots \tag{1.116}$$

and

$$v_{Ln}(t) = L_n di/dt. \tag{1.117}$$

The sum of each inductor generated *emf* equals to the sum of the total *emf* applied across the complete series of inductors. Mathematically,

$$\begin{aligned} v_{total}(t) &= v_{L1}(t) + v_{L2}(t) + v_{L3}(t) + \ldots + v_{Ln}(t) \\ &= L_1 di/dt + L_2 di/dt + L_3 di/dt + \ldots + L_n di/dt, \end{aligned} \tag{1.118}$$

and since the current through the series of inductors is the same for all inductors, then

$$v_{total}(t) = (L_1 + L_2 + L_3 + \ldots + L_n) di/dt. \tag{1.119}$$

Thus, we can say that the equivalent series inductor equals the sum of each individual inductance in series:

$$L_{series\text{-}equivalent} = L_1 + L_2 + L_3 + \ldots + L_n. \tag{1.120}$$

Figure 1.35 depicts a series of inductors. Note that the current in the circuit is the same for every inductor.

Example 1.21 Inductors in series: Given three inductors $L_1 = 25$ nH, $L_2 = 75$ nH, and $L_3 = 50$ nH, determine the series equivalent inductor of L_1, L_2, and L_3. Assume that the magnetic field produced by each inductor does not couple with the magnetic field produced by any of the other inductors.

Solution to Example 1.21

From Equation (1.120) we can state that

$$L_{series\text{-}equivalent} = L_1 + L_2 + L_3. \tag{1.121}$$

Solving for the given values of inductance,

$$L_{series\text{-}equivalent} = 25\ \text{nH} + 75\ \text{nH} + 50\ \text{nH} = 150\ \text{nH}. \tag{1.122}$$

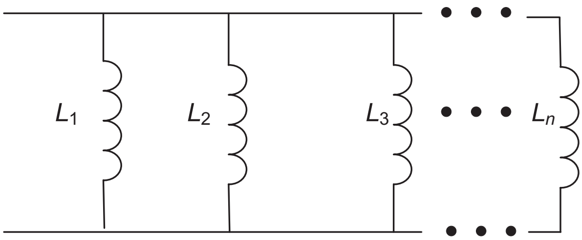


Figure 1.36 Inductors in parallel.

1.6.6 Inductors in Parallel

When inductors are connected in parallel, and there is no mutual coupling among the magnetic fields produced by each inductor in the parallel arrangement, the voltage across all the inductors in parallel is the same. Refer to Figure 1.36 for an arrangement of inductors in parallel. Upon the paralleled inductors being excited by a time-varying voltage, each inductor has across its terminals the same time-varying voltage. According to Equation (1.107), repeated here for convenience one more time,

$$v_L(t) = L di/dt.$$

Integrating the voltage across the inductor yields

$$i(t) = \frac{1}{L}\int v_L(t)dt. \tag{1.123}$$

Referring to Figure 1.36, the voltage is the same for all inductors, and since each inductance may be different (i.e., $L_1 \neq L_2 \neq \ldots \neq L_n$), the current through each inductor is

$$i_1(t) = \frac{1}{L_1}\int v_L(t)dt \tag{1.124}$$

$$i_2(t) = \frac{1}{L_2}\int v_L(t)dt \tag{1.125}$$

$$i_3(t) = \frac{1}{L_3}\int v_L(t)dt \ldots \tag{1.126}$$

$$i_N(t) = \frac{1}{L_N}\int v_L(t)dt. \tag{1.127}$$

From Equations (1.124) through (1.127), it can be seen that if we add the current of each of the inductors and name it $i_{total\text{-}parallel\text{-}equivalent\text{-}L}(t)$, there must exist a value of inductance that is equivalent to all of the inductances in parallel.

Thus, for

$$i_{total\text{-}parallel\text{-}equivalent\text{-}L}(t) = i_1(t) + i_2(t) + i_3(t) + \ldots + i_n(t). \tag{1.128}$$

There must exist a total parallel equivalent inductance such that

$$\begin{aligned} i_{total\text{-}parallel\text{-}equivalent\text{-}L}(t) &= \frac{1}{L_{parallel\text{-}equivalent}} \int v_L(t)dt \\ &= \frac{1}{L_1}\int v_L(t)dt + \frac{1}{L_2}\int v_L(t)dt \\ &\quad + \frac{1}{L_3}\int v_L(t)dt + \ldots + \frac{1}{L_n}\int v_L(t)dt. \end{aligned} \tag{1.129}$$

Now, since the voltage across each inductor is identical, we obtain from Equation (1.129) that

$$\frac{1}{L_{parallel\text{-}equivalent}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots + \frac{1}{L_n}. \tag{1.130}$$

Example 1.22 Given $L_1 = 5\ \mu H$ and $L_2 = 10\ \mu H$, determine the parallel equivalent inductor.

Solution to Example 1.22

Since

$$\begin{aligned} 1/L_{parallel\text{-}equivalent} &= 1/L_1 + 1/L_2, \\ L_{parallel\text{-}equivalent} &= (1/5\ \mu H + 1/10\ \mu H)^{-1} = 3.33\ \mu H. \end{aligned} \tag{1.131}$$

Example 1.23 Given five identical inductors whose value is 45 nH, determine the parallel equivalent inductor of all five 45 nH inductors in parallel. Assume that the magnetic field produced by each inductor does not couple with the magnetic field produced by any of the other inductors.

Solution to Example 1.23

From Equation (1.130) we have that

$$1/L_{parallel\text{-}equivalent} = 1/L_1 + 1/L_2 + 1/L_3 + 1/L_4 + 1/L_5. \tag{1.132}$$

But since

$$L_1 = L_2 = L_3 = L_4 = L_5 = 45\,\text{nH}, \tag{1.133}$$

$$1/L_{parallel\text{-}equivalent} = (1/45\,\text{nH} + 1/45\,\text{nH} + 1/45\,\text{nH} + 1/45\,\text{nH} + 1/45\,\text{nH}), \tag{1.134}$$

from where it is immediate to find that

$$L_{parallel\text{-}equivalent} = 45\,\text{nH}/5 = 9\,\text{nH}. \tag{1.135}$$

1.6.7 Mutual Inductance

Given an N-turn inductor and an AC voltage source excitation across its terminals, we know from Faraday's law of induction that

$$\textit{Flux Linkage}\ \lambda = N\Phi = Li(t) \tag{1.136}$$

and

$$v_L = Nd\Phi/dt = Ldi/dt, \tag{1.137}$$

where N is the number the inductor turns, Φ is the magnetic flux, and L is the self-inductance of the coil or simply the inductance. Refer to previously seen Figure 1.29 where the flux of the sole inductor is drawn. The term coil and inductor will be interchangeably used. However, an inductor is an ideal circuit element that only has inductance; a coil more commonly refers to a physical inductor, which is predominantly inductive, but it may also have some parasitic (usually undesirable) resistive and capacitive properties.

Applying the right-hand rule in Figure 1.37, we determine from the direction of the current flow the direction of the magnetic flux Φ. From Faraday's

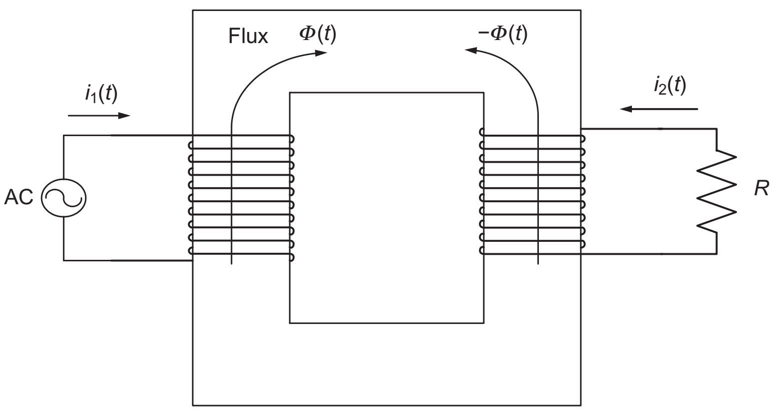
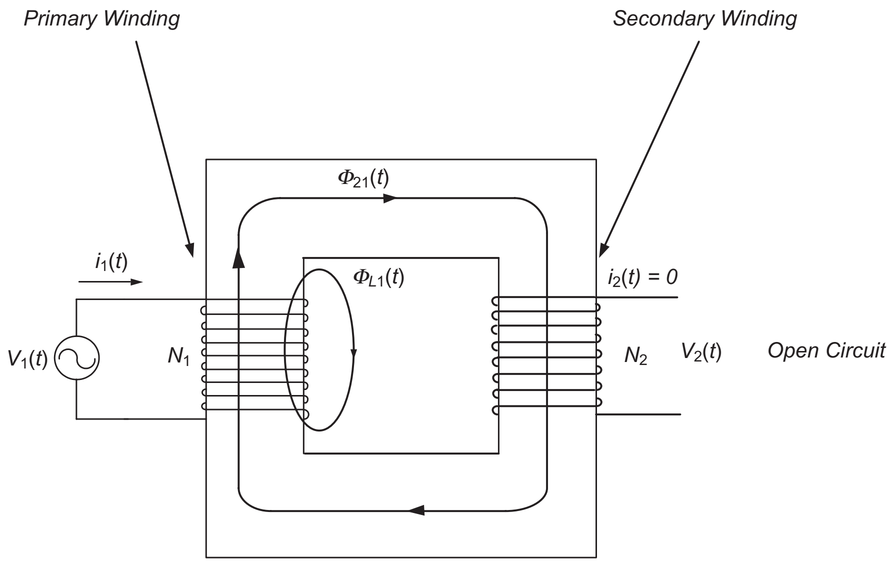


Figure 1.37 Inductors sharing same iron core.



$\Phi_{21}(t)$ *Magnetic flux coupled to the secondary generated by the primary current.*

$\Phi_{L1}(t)$ *Magnetic flux in the primary generated by the primary current. This flux does not couple to the secondary, that is why it referred to as primary leakage flux.*

Figure 1.38 Magnetically coupled primary and secondary coils: primary excitation, open-circuited secondary.

law, a voltage is induced on the inductor only if the flux Φ varies with respect to time. This flux variation need not be a sinusoidal time-varying flux; it just needs to be a time-varying flux. If the voltage exciting the inductor is DC or constant with respect to time, no voltage is induced across the inductor, and the inductor behaves as a virtual short circuit to the DC source. Why is it a virtual short? Real inductors or coils have a finite amount of DC resistance due to the resistance of its winding. However, such resistance is relatively small, typically some small fraction of an ohm, and is thus a virtual short circuit to DC.

We will analyze the interaction of two magnetically coupled coils. One coil is referred to as the primary, and drawn on the left-hand side of the circuits; the right-hand side coil is referred to as the secondary in Figures 1.38 through 1.40. The primary is assumed to have N_1 turns and the secondary has N_2 turns. The coils are assumed to be physically located in close proximity and static with respect to each other. Three cases will be analyzed:

1. Primary is excited by a time-varying voltage source, secondary is open-circuited and there is no secondary current flow.
2. Primary is open-circuited, no primary current flows, and the secondary is excited by a time-varying voltage source.

3. Primary is excited by a time-varying voltage source, and current flows through the primary and secondary circuits.

Case 1 Let us analyze Case 1 by inspection of Figure 1.38.

The primary current $i_1(t)$ produced by voltage source $v_1(t)$ in Figure 1.38 generates a magnetic flux:

$$\Phi_{11} = \Phi_{Leakage1} + \Phi_{21}, \tag{1.138}$$

where $\Phi_{Leakage1}$ (Φ_{L1}) is the flux produced by primary current $i_1(t)$ that does not couple to the secondary inductor; it is also referred to as the leakage flux of the primary inductor. Φ_{21} is the flux produced by current $i_1(t)$ that couples between secondary inductor and the primary inductor. Φ_{21} is also called the mutual flux in the secondary due to the current in the primary. Since the secondary is an open circuit, there is no current in the secondary circuit; that is, $i_2(t) = 0$. Faraday's law gives the voltage induced in the secondary:

$$v_2(t) = d\lambda_2/dt = N_2 d\Phi_{21}/dt, \tag{1.139}$$

where λ_2 is the flux linkage on the secondary coil (refer to Figure 1.38).

If linearity* holds, then

$$N_2\Phi_{21} = M_{21}i_1(t), \tag{1.140}$$

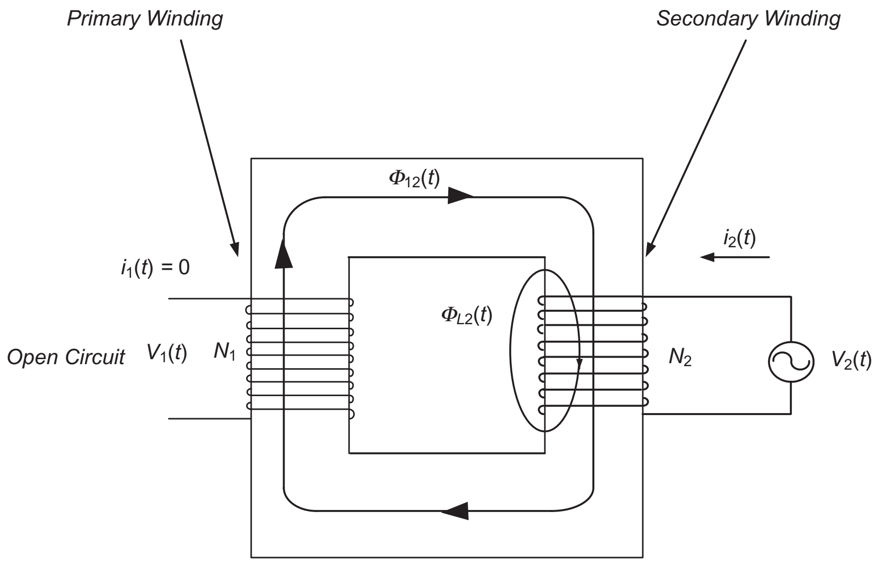
where the term M_{21} is referred to as the mutual inductance between inductor 2 and inductor 1. The unit of mutual inductance is the henry, the same unit used for inductance and self-inductance. Now we can rewrite Equation (1.139) in terms of mutual inductance Equation (1.140) as follows:

$$v_2(t) = N_2 d\Phi_{21}/dt = M_{21} di_1(t)/dt. \tag{1.141}$$

The mutual inductance term M_{21} is a constant because the relative position of the primary and secondary inductors is fixed. The inductance or self-inductance of the primary winding is

$$L_1 i_1(t) = N_1\Phi_{11}, \tag{1.142}$$

* Linearity in magnetically coupled inductors holds when the inductors have an air-core. When the inductors have ferromagnetic cores, linearity holds when the cores are operated with currents well below from their core saturation (nonlinear) regions.



$\Phi_{12}(t)$ Magnetic flux coupled to the primary generated by the secondary current.

$\Phi_{L2}(t)$ Magnetic flux in the secondary generated by the secondary current. This flux does not couple to the primary, that is why is referred to as secondary leakage flux.

Figure 1.39 Magnetically coupled primary and secondary coils: secondary excitation and open-circuited primary.

where Φ_{11} is given by Equation (1.138).
Using Equation (1.137), the voltage on the primary is then

$$v_1(t) = L_1 di_1(t)/dt = N_1 d\Phi_{11}/dt. \tag{1.143}$$

Case 2 Let us analyze Case 2 by inspection of Figure 1.39.

We see that the primary is open-circuited and no current flows through this circuit, and the secondary is excited by a time-varying voltage source.

The secondary current $i_2(t)$ produced by voltage source $v_2(t)$ in Figure 1.39 generates a magnetic flux:

$$\Phi_{22} = \Phi_{Leakage2} + \Phi_{12}, \tag{1.144}$$

where $\Phi_{Leakage2}$ is the flux produced by secondary current $i_2(t)$ that does not couple to the primary inductor; it is also referred to as the leakage flux of the secondary inductor. Φ_{12} is the flux produced by current $i_2(t)$ that couples between primary inductor and the secondary inductor. Φ_{12} is also called the mutual flux in the primary due to the current in the secondary. Since the primary is open circuit, there is no current in the primary circuit; that is, $i_1(t) = 0$. The flux linkage of the primary circuit is

$$\lambda_1 = N_1\Phi_{12}. \tag{1.145}$$

Using Equation (1.136) and Faraday's law lead to the voltage induced in the primary:

$$v_1(t) = N_1 d\Phi_{12}/dt. \tag{1.146}$$

If linearity holds then,

$$N_1\Phi_{12} = M_{12}i_2(t), \tag{1.147}$$

where the term M_{12} is referred to as the mutual inductance between inductor 1 and inductor 2, due to current $i_2(t)$. The unit of mutual inductance is the henry, the same unit used for inductance and self-inductance. Now we can rewrite Equation (1.146) in terms of mutual inductance Equation (1.147) as follows:

$$v_1(t) = N_1 d\Phi_{12}/dt = M_{12} di_2(t)/dt. \tag{1.148}$$

The mutual inductance term M_{12} is a constant because the relative position of the primary and secondary inductors is fixed.
The inductance or self-inductance of the primary winding is

$$L_2 i_1(t) = N_2\Phi_{22}, \tag{1.149}$$

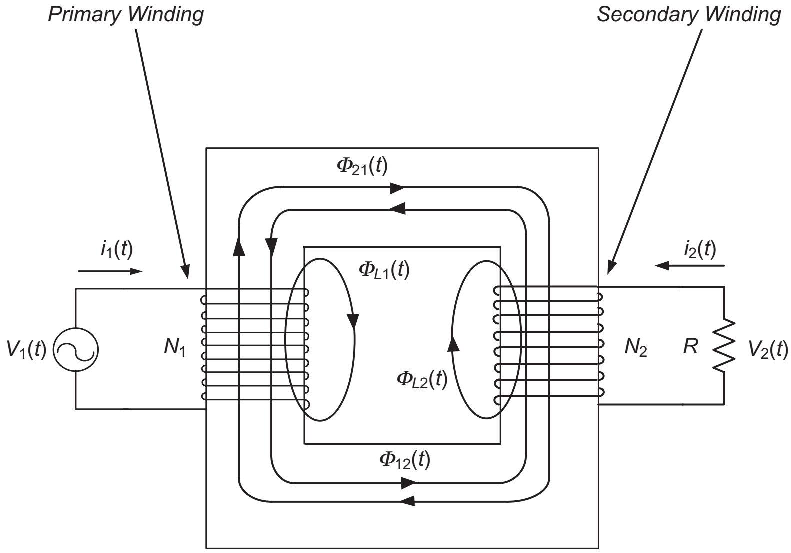
where Φ_{22} is given by Equation (1.144).

Case 3 This is the most general case, that is, when there is nonzero current on both the primary and secondary circuits. We assume for this case that the primary is excited by a time-varying source, and a resistor or load in the secondary allows the induced voltage on the secondary to produce a current that flows through R.

From Equations (1.138) and (1.44), repeated here for the reader's convenience,

$$\Phi_{11} = \Phi_{Leakage1} + \Phi_{21}$$
$$\Phi_{22} = \Phi_{Leakage2} + \Phi_{12}.$$

Referring now to Figure 1.40, we find that the fluxes in the primary and secondary are respectively,



$\Phi_{12}(t)$ *Magnetic flux coupled to the primary generated by the secondary current.*

$\Phi_{21}(t)$ *Magnetic flux coupled to the secondary generated by the primary current.*

$\Phi_{L1}(t)$ *Magnetic flux in the primary generated by the primary current. This flux does not couple to the secondary, that is why it is referred to as leakage flux.*

$\Phi_{L2}(t)$ *Magnetic flux in the secondary generated by the secondary current. This flux does not couple to the primary, that is why it is referred to as leakage flux.*

Figure 1.40 Magnetically coupled primary and secondary coils. Primary excitation and secondary with resistive load.

$$\boldsymbol{\Phi}_1 = \boldsymbol{\Phi}_{Leakage1} + \boldsymbol{\Phi}_{21} + \boldsymbol{\Phi}_{12} = \boldsymbol{\Phi}_{11} + \boldsymbol{\Phi}_{12}. \tag{1.150}$$

$$\boldsymbol{\Phi}_2 = \boldsymbol{\Phi}_{Leakage2} + \boldsymbol{\Phi}_{12} + \boldsymbol{\Phi}_{21} = \boldsymbol{\Phi}_{21} + \boldsymbol{\Phi}_{12}. \tag{1.151}$$

Then the flux linkages for the primary and secondary are

$$\lambda_1 = N_1\boldsymbol{\Phi}_{11} + N_1\boldsymbol{\Phi}_{12}. \tag{1.152}$$

$$\lambda_2 = N_2\boldsymbol{\Phi}_{21} + N_2\boldsymbol{\Phi}_{22}. \tag{1.153}$$

Finally, differentiating the flux linkages, the complete primary and secondary voltages for the basic transformer are obtained:

$$v_1 = L_1 di_1(t)/dt \pm M_{12} di_2(t)/dt \tag{1.154}$$

$$v_2 = M_{21} di_1(t)/dt \pm L_2 di_2(t)/dt, \tag{1.155}$$

where $M_{12} = M_{21} = M$.

Equations (1.154) and (1.155) have been derived assuming that the windings directions are not known. It is not always possible to know the windings directions of a pair of mutually coupled inductors; consequently, the dot convention is used.

The Dot Rule for Coupled Inductors

It is not always possible or practical to know the directions of the windings of a pair of mutually coupled inductors. Dots are assigned in the following manner: pick one inductor, say the primary, and place a dot where the current to be injected enters the winding. Determine the flux created by such current using the right hand rule. The flux generated in the secondary inductor from Lenz law has to oppose the direction of the primary flux. Remember that the secondary current also has to meet with the right-hand rule with respect to the secondary flux. Now assume you would load the secondary, place the dot on the secondary terminal where this natural current leaves the secondary winding.

Summary of the dot rules:

1. When both primary and secondary currents enter (or leave) the dotted inductor terminals, the sign on the M (mutual inductance) terms shall have positive sign so that the equations are

$$v_1 = L_1 di_1(t)/dt + M_{12} di_2(t)/dt. \tag{1.156}$$

$$v_2 = M_{21} di_1(t)/dt + L_2 di_2(t)/dt. \tag{1.157}$$

2. When one current enters the dotted terminal of one inductor and leaves the dotted terminal of the other inductor, the signs on the M terms shall have negative signs so that the equations are

$$v_1 = L_1 di_1(t)/dt - M_{12} di_2(t)/dt. \tag{1.158}$$

$$v_2 = -M_{21} di_1(t)/dt + L_2 di_2(t)/dt. \tag{1.159}$$

The dots on the end of each coil are markings, typically made by the transformer manufacturer to indicate the relative polarities of the windings mutual voltages. Given that we find a dot on the primary and a dot on the secondary, the dot on the primary inductor indicates that a current entering this side of the inductor produces an induced voltage with its positive sign on the dotted side of the secondary inductor.

Figure 1.41 shows a transformer with its respective windings dots. In Figure 1.41 $v_1(t)$ produces a current into the dot, and induced voltage $v_2(t)$ produces a current with the direction shown by $i_2(t)$.

Figure 1.42 shows the dotted primary and secondary inductors. As usual, the primary current enters the dotted primary winding. The dotted secondary

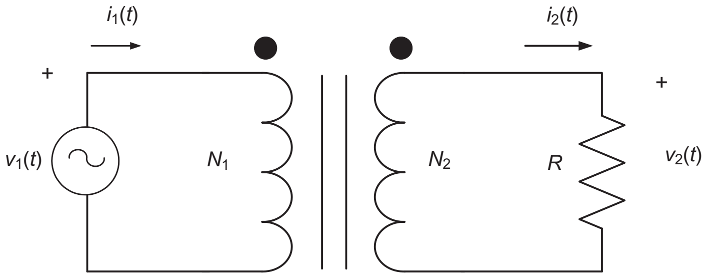


Figure 1.41 Dotted convention for two mutually coupled inductors (Case 1).

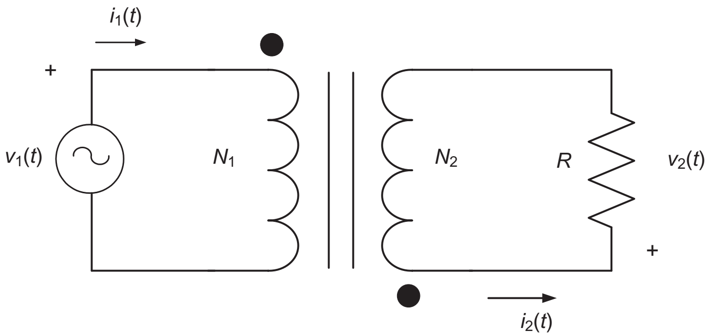


Figure 1.42 Dotted convention for two mutually coupled inductors (Case 2).

shows the positive polarity of the induced voltage and the direction of the secondary current [7].

1.6.8 Energy Stored by an Inductor

The energy held by an inductor between times t_0 and t_1 is given by

$$w_L(t) - w_L(0) = \int_{t0}^{t} v(t)i(t)dt, \tag{1.160}$$

where $v(t)$ is the voltage across the inductor, and $i(t)$ is the current through it.
Since

$$v(i) = Ldi(t)/dt, \tag{1.161}$$

plugging Equation (1.161) in Equation (1.160) leads to

$$w_L(t) - w_L(0) = \int_{t0}^{t} Li(t)di(t) \tag{1.162}$$

$$w_L(t) - w_L(0) = \frac{1}{2} L i^2(t) \mathrm{J}, \tag{1.163}$$

where $w_L(0)$ is the initial energy of the inductor, L is its inductance, and $i(t)$ the current flowing through it. In particular, for a pair of mutually coupled inductors (a transformer), the energy held by the transformer equals

$$w_L(t) - w_L(0) = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) \pm M i_1(t) i_2(t) \tag{1.164}$$

where in Equation (1.164), the plus sign applies if both currents enter or leave the dotted marked inductor terminals, and it is minus if one current enters a dotted terminal, while the other current leaves its dotted terminal [7].

1.6.9 Inductor Nonlinearity

The magnetic flux $\Phi(t)$ of an inductor of inductance L is proportional to the current flowing through such inductor, provided that the core used does not get saturated by the amount of current flowing through the inductor winding:

$$\Phi(t) = L i_L(t). \tag{1.165}$$

Equation (1.165) remains linear if an air-core is used. Similarly, Equation (1.165) remains linear when a ferromagnetic core is used, and the current does not exceed the linear limits of the magnetic flux to current relationship. This means, as long as the ferromagnetic core does not get saturated. L is the constant of proportionality in Equation (1.165) and in most practical cases is time independent and strongly depends on the geometry of the coil winding and the type of core used.

Figure 1.43 depicts the nonlinear flux–current relationship that exists on a ferromagnetic core. The magnetic flux Φ, also equal to the magnetic inductance field B per unit area, maintains proportionality to the voltage applied to the inductor terminals. The magnetic field intensity H is proportional to the current flowing through the inductor times the number of turns.

Alternatively, some manufacturers present the flux-current ($\Phi - i$) characteristic curve instead of the B-H characteristic curve.

Figure 1.43 graphically depicts the hysteresis phenomenon. Let us start at $i = 0$ and $\Phi = 0$, the origin of coordinates; as current i is increased, flux Φ follows curve A until it reaches point $i = i_1$ and $\Phi = \Phi_1$. Note that the magnetic flux saturates the core at this point. Core saturation means that as the current continues to increase, the flux will no longer increase in a significant fashion. As current i decreases, tracing over curve B, flux Φ becomes zero at $i = -i_2$. If we continue pushing current in the minus direction toward $-i_3$, the core will eventually saturate (flattened curve at $i = -i_3$ and $\Phi = -\Phi_3$). Upon increasing the current toward the positive direction, the flux increases, but it is still negative. When the current reaches a value of i_4, the flux becomes zero, and it is no

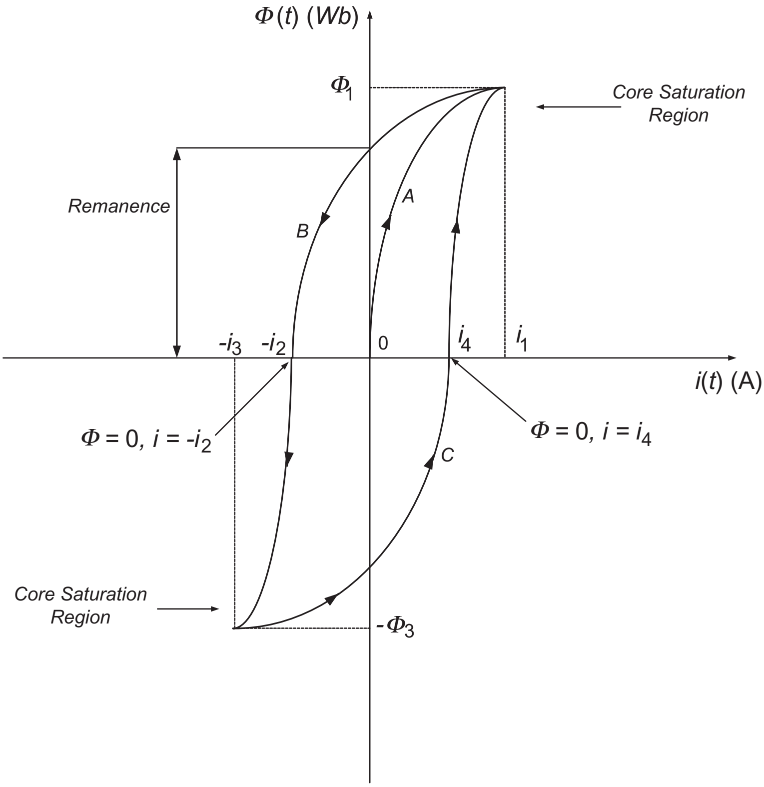


Figure 1.43 Magnetic characteristic (Φ-i) or B-H curve of an iron-core inductor.

longer negative (lower portion of curve C). If we continue to increase the current in the positive direction, curve C will finally meet point $i = i_1$ and $\Phi = \Phi_1$.

According to the best of our knowledge there are no close form equations describing the B-H $(\Phi - i)$ characteristics of ferromagnetic materials. Core manufacturers empirically obtain such magnetic curves.

The slope of the B-H curve at each point determines the permeability μ of the ferromagnetic material:

$$\mu = \mu_R \mu_o = B/H.$$

When the relative permeability of the ferromagnetic material is almost constant for all points throughout the B-H curve, the material behaves linearly.

1.6.10 Inductor Component Selection

Inductor selection requires not only a good understanding of the desired circuit performance but also the data sheet information available on the inductor

Table 1.9 Typical printed circuit board inductor data sheet excerpt

Part Number	L ± 20%[a] (μH)	DCR (max) (Ω)	SRF (MHz)	I_{sat}[b] (A)	I_{RMS}[c] (A)
Value[d]	100	0.020	10	10	7

[a] Inductance tested at 100 kHz, 0.1 V_{RMS}.
[b] Inductance drop = 10% typ. at I_{sat}.
[c] For 40°C temperature rise typ. at I_{RMS}.
[d] All parameters tested at 25°C.

provided by the manufacturer. The purpose of this section is to present some general guidelines of the key parameters and factors that need to be taken into consideration when an inductor is selected.

For example, Table 1.9 shows an excerpt from a typical inductor data sheet.

To use the data sheet parameters properly, one must have some understanding of what the parameters mean and how they were derived. Most manufacturers will not show all the performance parameters of their inductors under all possible and different sets of operational conditions. This would become practically prohibitive, since the testing and characterization cost of inductors would rise.

Following the entries of Table 1.9, after the part number entry, the inductance is provided. Inductance is the most basic parameter for selecting the inductor. This may be based on the energy storage capability required by the inductor or by the volt-second capacity, derived from equation $v_L = L\ di/dt$. Associated with the inductance value is the tolerance; it is common to see a wide spread tolerance, that is, ±20% is common.

The next parameter is the DC resistance (DCR) of the inductor wire. This resistance depends on the wire material, its gauge, and its length (or number of turns). DCR is almost always some fraction of an ohm for relatively small inductors used in the electronics industry. Industrial and heavy-duty inductors are not addressed by this example. The inductor's self-resonant frequency (SRF) is the frequency at which the inductor resonates naturally with its parasitic distributed capacitance. The resonating frequency is the frequency at which the inductive reactance equals the capacitive reactance. For all practical purposes, the inductor should be used at a frequency one order of magnitude lower than its self-resonating frequency. The topic inductive and capacitive reactances will be addressed in more depth in Chapter 2.

I_{sat} stands for the saturation current that the inductor can take just before its inductance goes down in value. Typically, manufacturers define I_{sat} as the current at which the inductance drops down by 10% of its nominal value at room temperature. Finally, the I_{RMS} current provides a measure of how much average current can continuously flow through the inductor while producing less than some specified temperature rise. This figure contains the ambient temperature at which it was measured and the temperature range at which the inductor will be used, taking into account self-heating effects of the inductor.

Shall any of the inductor manufacturers' parameters not be sufficiently explicit on the use of the inductor, it is the responsibility if the circuit designer to test the inductor and characterize its behavior for the desired operating conditions. As a conservative rule of thumb, it is wise to use an inductor up to some fraction, like 50% or 75% of the smaller of the two rated currents, that is, I_{sat} and I_{RMS}. The circuit designer should not be surprised that not all manufacturers specify both currents discussed.

1.7 KIRCHHOFF'S VOLTAGE LAW (KVL) AND KIRCHHOFF'S CURRENT LAW (KCL)

An electric circuit or network consists of a number of electric components, independent or dependent current, and voltage sources interconnected to each other. Figure 1.44a through f shows a variety of circuit topologies that are commonly seen in electrical and electronics engineering applications.

Note that the blank rectangles represent virtually any R, L, or C circuit series or parallel combination. However, in this section, and without loss of generality, we will apply Kirchhoff's laws when such rectangular shaped elements are resistors.

Note that when the Lattice Network of Figure 1.44f has an electrical element loading the two right-hand side terminals, the topologies of the Lattice and the Wheatstone bridge structures are identical.

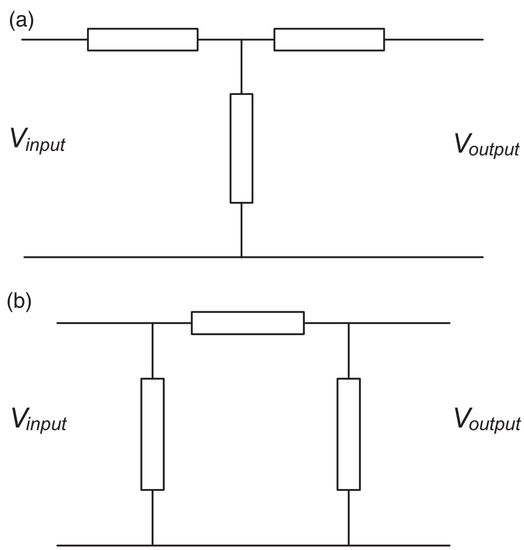


Figure 1.44 Various circuit topologies: (a) T-network or Y-network; (b) π-network or delta-network; (c) ladder network; (d) bridged-T-network; (e) bridge or Wheatstone bridge network; and (f) lattice network.

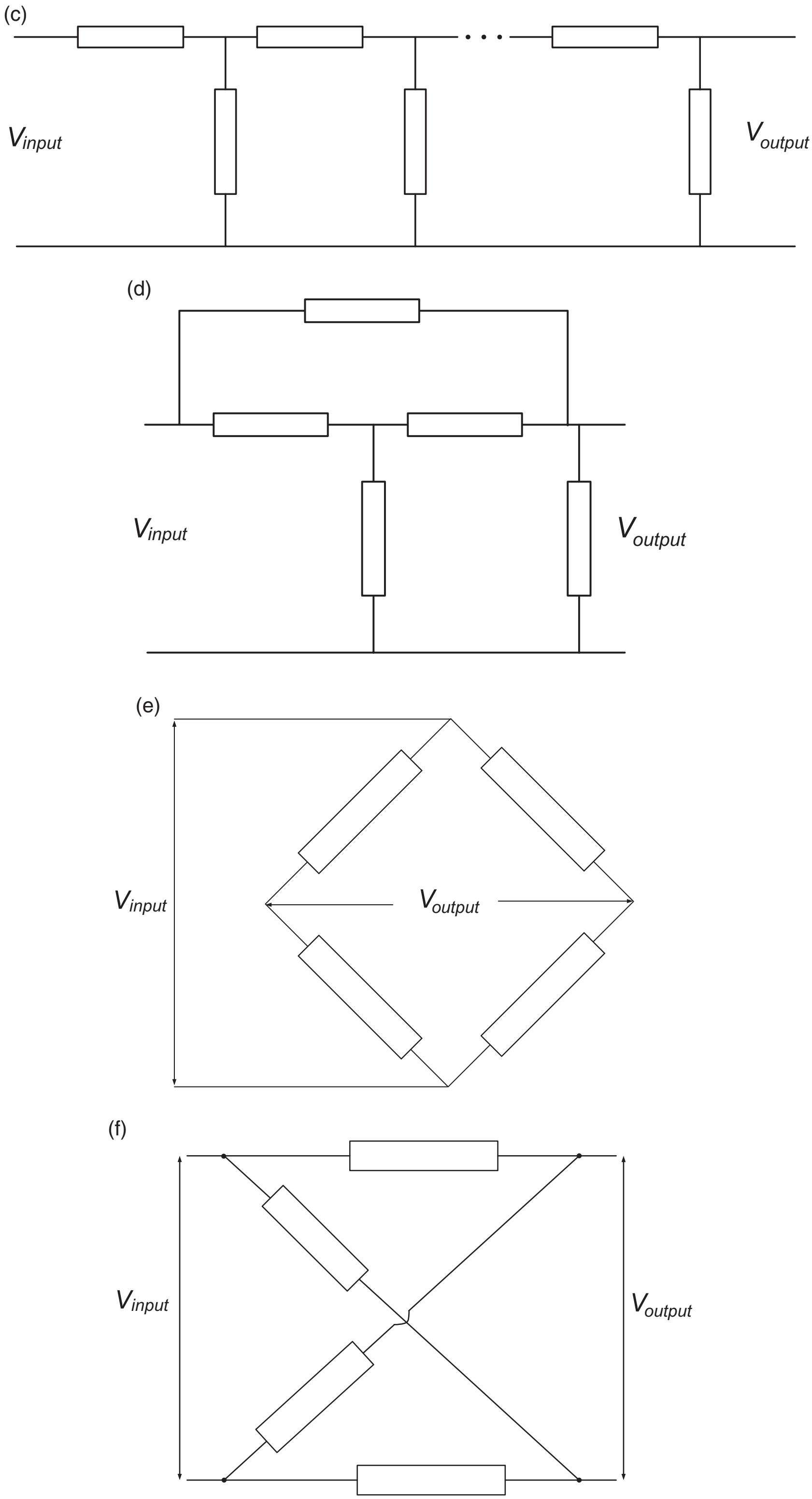


Figure 1.44 (*Continued*)

Student Exercise: Redraw a Lattice network with a resistor on its two right-hand side terminals and justify that it indeed matches the Wheatstone bridge topology.

Other circuit structures, which are common in engineering applications, are derivatives of the ladder networks. In general, they are referred to as window-pane topologies. Figure 1.45a through d shows a few examples of them.

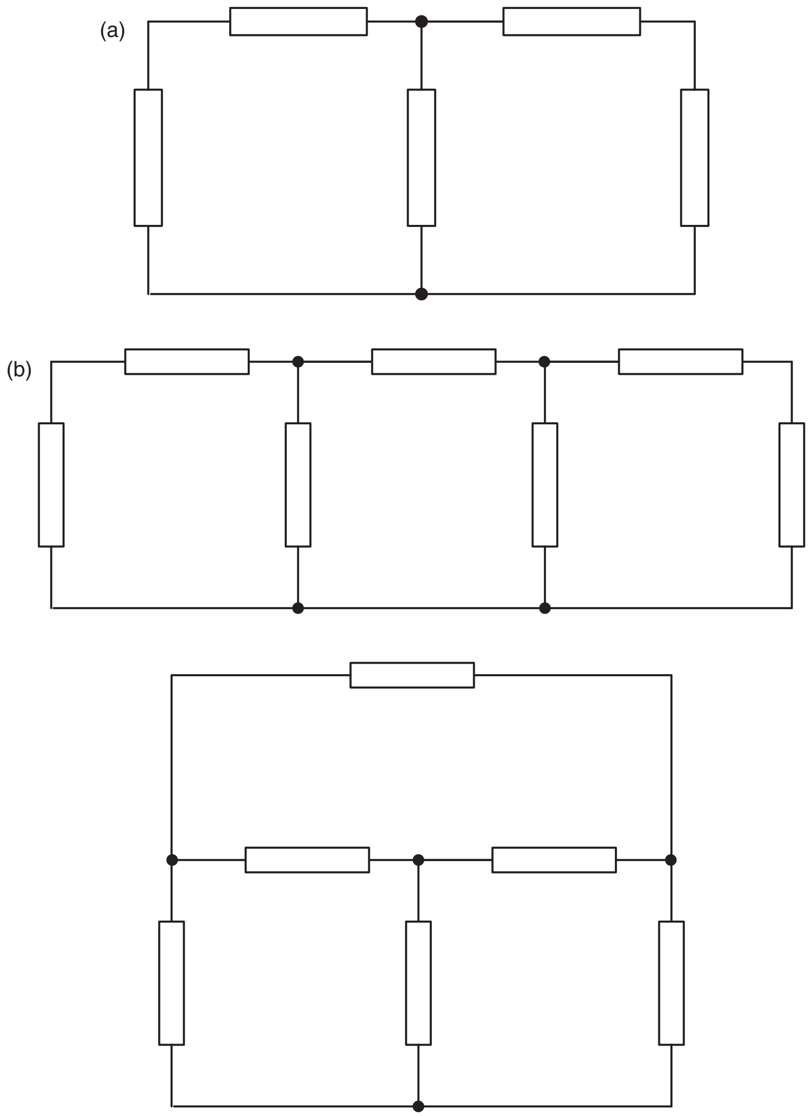


Figure 1.45 Window pane topologies networks: (a) 2-window; (b) 3-window; (c) 4-window; (d) 6-window.

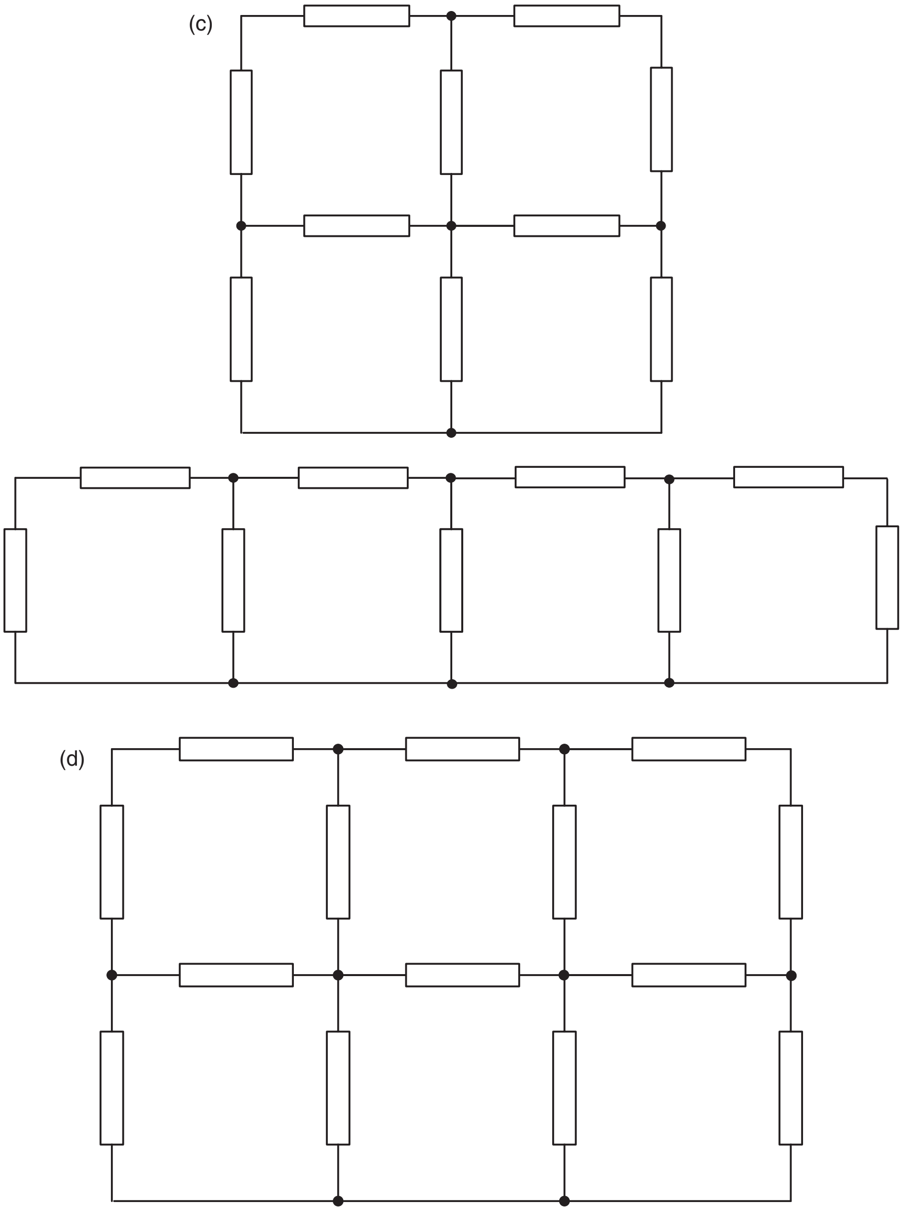


Figure 1.45 (*Continued*)

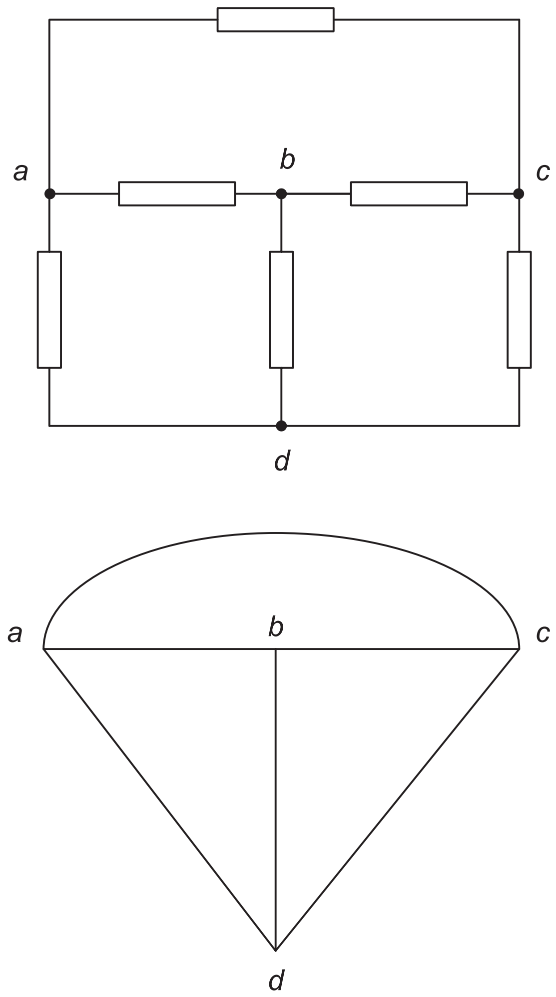


Figure 1.46 (a) 3-window network; (b) its topology.

The graph or the topology of a circuit or network consists in finding the *skeleton* of the circuit. Replacing every element in the circuit with a line does that. For example the graph or topology of one of the circuits of Figure 1.45b is shown in Figure 1.46.

Circuit Definitions: Branches, Nodes, and Loops

Referring to the circuit topology of Figure 1.47a, segments $\overline{ab}$, $\overline{bc}$, $\overline{cd}$, $\overline{da}$, $\overline{bd}$, and $\overline{ac}$ are the *branches* or links in the network. We will use the term *branch*. The junction of two or more branches is a *node*. A loop in a network is a closed path formed by a number of connected branches. For example, for the circuit

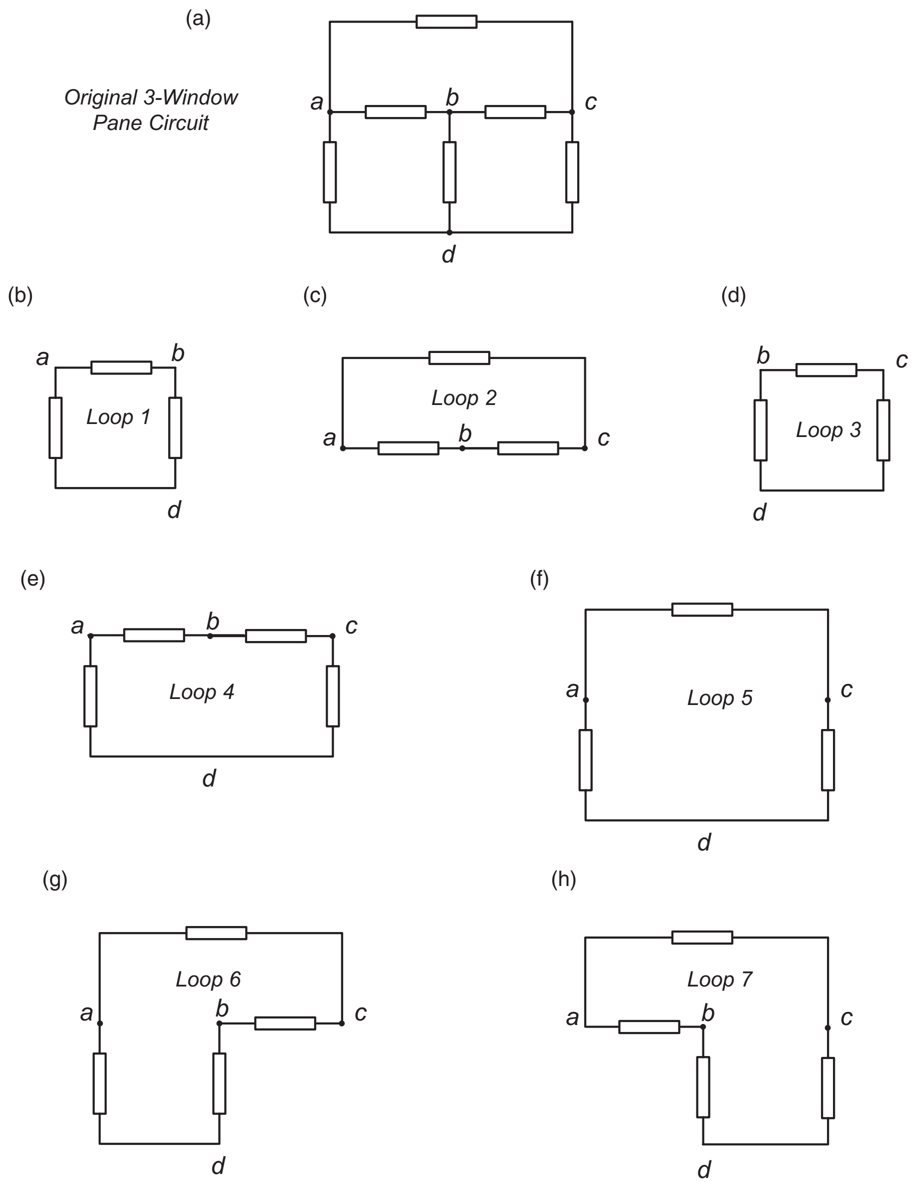


Figure 1.47 (a) Original 3-window pane circuit; (b) loop 1; (c) loop 2; (d) loop 3; (e) loop 4; (f) loop 5; (g) loop 6; (h) loop 7.

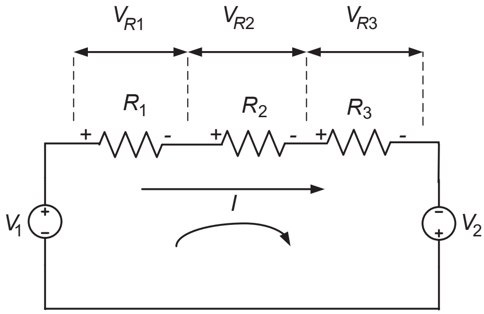


Figure 1.48 Single loop circuit, with two DC voltage sources.

whose topology is drawn in Figure 1.46, all the possible loops are shown in Figure 1.47.

Now let us assume a single loop circuit like the one depicted by Figure 1.48. V_1 and V_2 are DC voltage sources. R_1, R_2, and R_3 are all in series in the circuit shown.

KVL states that in any closed loop path in a network or circuit, the algebraic sum of all branch voltages equals to zero at all times.

An alternative way of stating KVL is

At any instant of time, in a closed loop path in a network, the sum of all the voltage rises must equal the sum of all the voltage drops.

V_1 and V_2 are both voltage rises, since they generate a rise in voltage. The current I developed in the series circuit produces voltage drops on each one of the resistors. $I\,R_1 = v_{R1}$ is the voltage drop across resistor R_1; $I\,R_2 = v_{R2}$ is the voltage drop across resistor R_2; and $I\,R_3 = v_{R3}$ is the voltage drop across resistor R_3.

Before using KVL, let us establish a direction we will be traveling around the loop. Arbitrarily, we will assume that we travel the loop in a clockwise direction. Voltages rises with their positive terminal before their negative terminal and in the direction of traveling the loop are negative. Whereas voltages rises with their negative terminal before their positive terminal and in the direction of traveling the loop are positive. Current I flows through the resistors from higher potentials to lower potentials. In this example, all three voltage drops have the same sign; however, in multiloop circuits, that may not be the case. Current I was arbitrarily assumed to flow from left to right as shown in the circuit of Figure 1.48. More examples will follow to explain this.

Applying KVL to the circuit of Figure 1.48, we obtain that

$$V_1 - V_2 - v_{R1} - v_{R2} - v_{R3} = 0, \tag{1.166}$$

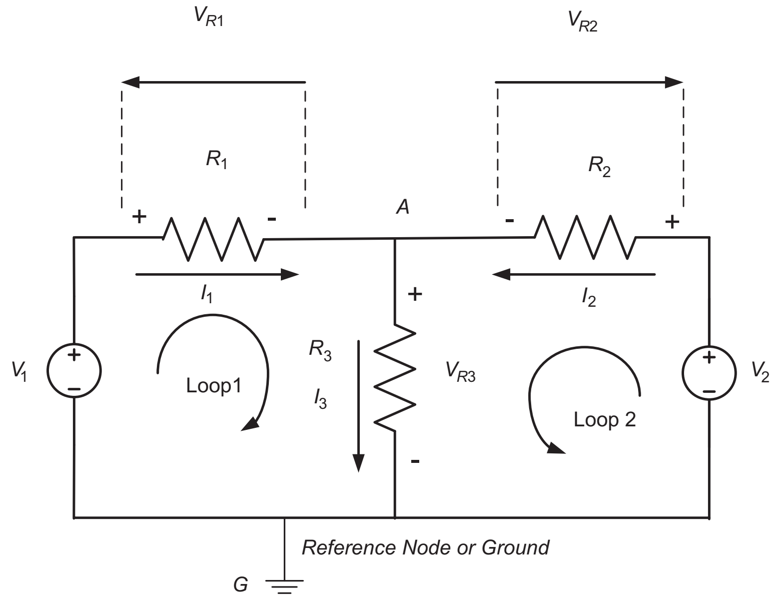


Figure 1.49 Circuit for KVL, Example 1.24.

Let us remember that an algebraic sum is a sum where every addend or sum member is taken into account with its respective sign. This is what Equation (1.166) shows.

Example 1.24 Given the circuit of Figure 1.49, determine the KVL equations for loops 1 and 2.

Solution to Example 1.24

Figure 1.49 indicates the two loops chosen. For the left-most loop, we have

$$V_1 = V_{R1} + V_{R3}. \tag{1.167}$$

And for the right-most loop, we have

$$V_2 = V_{R2} + V_{R3}. \tag{1.168}$$

Note that the loop directions (see arrows in Fig. 1.49) have been arbitrarily chosen.

Practice Problem Choosing different loop traveling directions, rewrite KVL for the circuit of Figure 1.49. Prove that your new equations are algebraically equivalent to Equations (1.167) and (1.168).

KCL states that the algebraic sum of all branch current at a node is zero at all instants of time.

Algebraic sum means to take into consideration the direction of flow of the current, that is, its sign. We can arbitrarily choose that any current entering a node is positive, and any current leaving a node is negative. The opposite can also be assumed. The important fact is to pick one current direction convention and keep such direction consistently throughout the solution of the complete circuit.

An alternative way of stating KCL is

The sum of currents entering a node must equal the sum of currents leaving such node at all instants of time.

Example 1.25 Apply KCL to the Two-Loop Circuit Given in Figure 1.49

Solution to Example 1.25

Let us apply KCL at node A of the circuit given in Figure 1.49. Node G (Ground) is assumed to be the reference node, which is the node with respect to which all other node voltages are referenced. Figure 1.49 shows branch current directions in the circuit, which can be arbitrarily assigned. Based on the current directions assumed for node A,

$$I_3 = I_1 + I_2. \tag{1.169}$$

Note that we may also write the KCL equations for node G and that leads to

$$I_1 + I_2 = I_3. \tag{1.170}$$

It should be clear that both Equations (1.169) and (1.170) are identical. Thus, just one of the equations is used.

Example 1.26 Using the circuit of Figure 1.49, assume that $R_1 = 6\ \Omega$, $R_2 = 3\ \Omega$, $R_3 = 10\ \Omega$, $V_1 = 5$ V, and $V_2 = 4$ V. Write the *KCL* equation for node A and *KVL* equations for loops 1 and 2 as shown on the figure.

For the reader's convenience Figure 1.49 is repeated here and referred to as Figure 1.50.

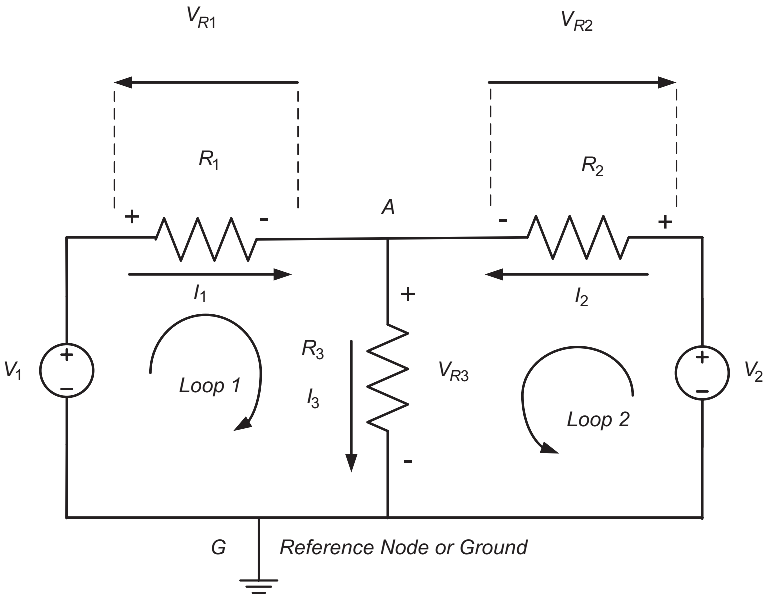


Figure 1.50 Circuit for KVL.

Solution to Example 1.26

By inspection of Figure 1.50 and applying KCL for node A,

$$I_1 + I_2 = I_3. \tag{1.171}$$

Applying KVL on loop 1,

$$V_1 = I_1R_1 + I_3R_3. \tag{1.172}$$

Applying KVL on loop 2,

$$V_2 = I_2R_2 + I_3R_3. \tag{1.173}$$

Equations (1.171) through (1.173) are three linearly independent equations, and since we have three unknowns which are $I_{1,}$ I_2, and I_3, we obtain unique solutions for each branch current.

Plugging the resistor and voltage sources values given into Equations (1.172) and (1.173) we obtain

$$5 = I_1 6 + I_3 10. \tag{1.174}$$

$$4 = I_2 3 + I_3 10. \tag{1.175}$$

$$I_3 = I_1 + I_2. \tag{1.176}$$

The solving of the three simultaneous Equations (1.174) through (1.176) is left as an exercise to the reader. As promised on the Preface, this book covers hardware essentials, and it is not intended to be a math book.

The solutions for the three currents are

$$I_1 = 0.2315\ \text{A}. \tag{1.177}$$

$$I_2 = 0.1296\ \text{A}. \tag{1.178}$$

$$I_3 = 0.3611\ \text{A}. \tag{1.179}$$

All three numerical results were rounded to the fourth decimal place. Let us plug results given by Equations (1.177) through (1.179) into Equations (1.174) through (1.176):

$$5 = 0.2315 \times 6 + 0.3611 \times 10. \tag{1.180}$$

$$4 = 0.1296 \times 3 + 0.3611 \times 10. \tag{1.181}$$

$$0.3611 = 0.1296 + 0.2315. \tag{1.182}$$

It is also easy to verify that the voltage at node A (Fig. 1.50) is

$$V_A = I_3 R_3. \tag{1.183}$$

$$V_A = 0.36111\ \text{A} \times 10\ \Omega = 3.6111\ \text{V}. \tag{1.184}$$

It is also instructive to realize that node voltage V_A also equals from Figure 1.50 using KVL:

$$V_A = V_1 - I_1 R_1. \tag{1.185}$$

Using the values for V_1, I_1, *and* R_1 in Equation (1.185) we find that

$$V_A = 5 - 0.2315 \times 6 = 3.6111\ \text{V}. \tag{1.186}$$

Applying KVL to loop 2 of the circuit of Figure 1.50,

$$V_A = V_2 - I_2 R_2. \tag{1.187}$$

And using the values known for V_2, I_2, and R_2 yields

$$V_A = 4 - 0.1296 \times 3 = 3.6112\ \text{V}. \tag{1.188}$$

In summary, the nodal voltage V_A was found using one set of KVL Equations (1.174) and (1.175) and found to be the same when using an alternate set of

KVL equations given by Equations (1.185) through (1.187). Keep in mind that due to the use of finite precision (4 decimal places in our example), the numbers may not be 100% exact. This is just due to numerical round-off errors and not to Kirchhoff's laws.

Practice Problem 1.27: Using the numerical Example 1.26, find solutions for the KVL and KCL equations expressed with rational numbers, instead of rounded or truncated decimal numbers to prove that KVL and KCL are exact.

Example 1.27 Given the three-mesh circuit of Figure 1.51, state KVL equations for meshes: ABG, BCG, and ABC, and KCL equations for node B.

Solution to Example 1.27

By inspection of the circuit of Figure 1.51 we write the following KVL equations:

Mesh ABG:

$$V_1 = I_1R_1 + I_4R_4 \tag{1.189}$$

Mesh BCG:

$$V_2 = I_2R_2 + I_4R_4 \tag{1.190}$$

Mesh ABC:

$$0 = I_1R_1 - I_2R_2 + I_3R_3 \tag{1.191}$$

Node B:

$$I_1 + I_2 = I_4 \tag{1.192}$$

Equations (1.189) through (1.192) are a set of linearly independent simultaneous equations and four unknowns. The unknowns are: I_1, I_2, I_3, and I_4.

Practice Problem 1.28: Using the circuit provided by Figure 1.51, assume the following resistor and voltage values:

$$R_1 = 1\ \Omega,\ R_2 = 2\ \Omega,\ R_3 = 3\ \Omega,\ R_4 = 4\ \Omega,\ V_1 = 5, \text{and } V_2 = 6\ \text{V}.$$

For the given numerical resistor and voltage values, find the numerical values of I_1 through I_4. Hint: Equations (1.189) through (1.191) constitute a set of linearly independent simultaneous equations. Solving, we obtain

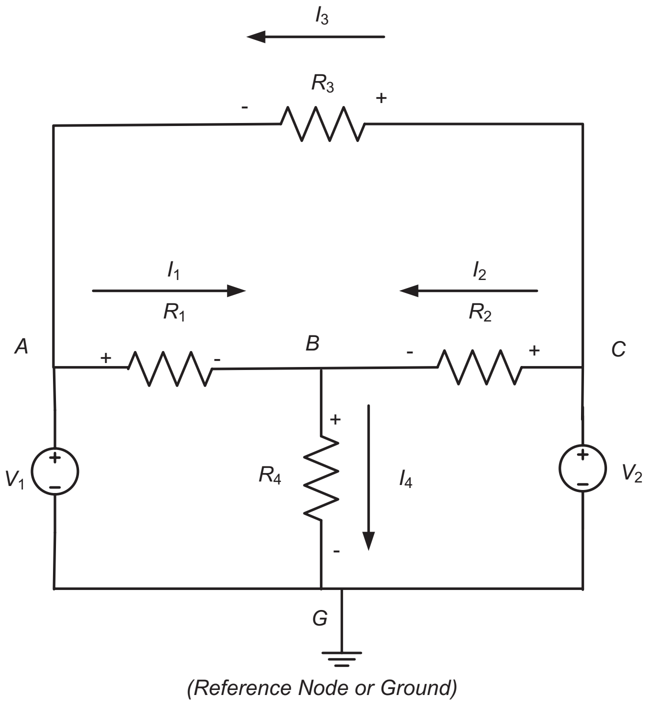


Figure 1.51 Three-mesh circuit with two voltage sources.

$$I_1 = 0.428571 \text{ A.} \tag{1.193}$$

$$I_2 = 0.714286 \text{ A.} \tag{1.194}$$

$$I_3 = 0.333333 \text{ A.} \tag{1.195}$$

$$I_4 = 1.14286 \text{ A.} \tag{1.196}$$

Using the circuit of Figure 1.51 is easy to verify that all currents comply with KCL.

State the remaining KCL equations not stated above (i.e., nodes A and C), and plug in the numerical values obtained in Equations (1.193) through (1.196) to validate KCL. Finally, using the found values of currents, validate KVL Equations (1.189) through (1.191).

Example 1.28 State the KVL and KCL of the loops and nodes found on the circuit of Figure 1.52. Assume that the value of each resistor (R_1 through R_4) is 1 Ω, V = 6 V, and I = 5 A.

Provide numerical answers for currents I_1 through I_4, node voltages V_B and V_C. Finally, determine the current that the voltage source V supplies, that is, I_{V1}

Writing KVL and KCL for above circuit,

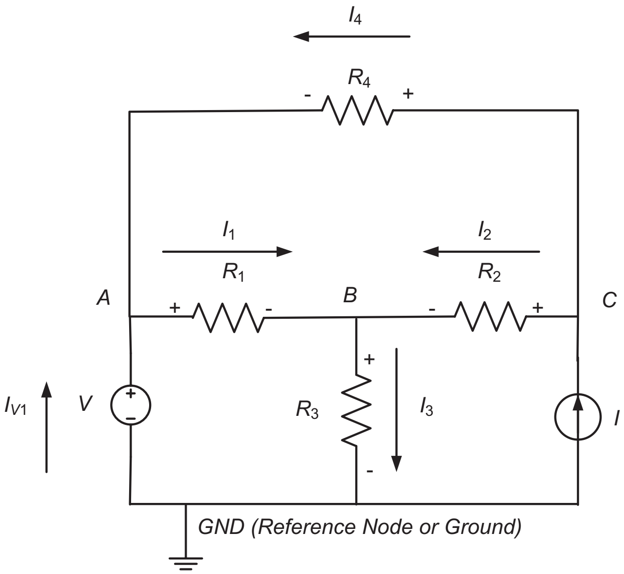


Figure 1.52 Circuit for Example 1.29: apply KVL and KCL.

Mesh ABG:

$$V = I_1R_1 + I_3R_3 \tag{1.197}$$

Mesh BCG:

$$V_{CG} = I_2R_2 + I_3R_3 \tag{1.198}$$

Mesh ABC:

$$0 = I_1R_1 - I_2R_2 + I_4R_4 \tag{1.199}$$

And for the nodes:
Node A:

$$I_{V1} + I_4 = I_1 \tag{1.200}$$

Node B:

$$I_1 + I_2 = I_3 \tag{1.201}$$

Node C:

$$I = I_2 + I_4 \tag{1.202}$$

Node G:

$$I_{V1} + I = I_3 \tag{1.203}$$

Solving a set of linearly independent equations we obtain

$$V_B = 4.6\ \text{V}.$$

$$V_C = 7.8\ \text{V}.$$

$$I_1 = 1.4\ \text{A}.$$

$$I_2 = 3.2\ \text{A}.$$

$$I_3 = 4.6\ \text{A}.$$

$$I_4 = 1.8\ \text{A}.$$

$$I_{V1} = -0.4\ \text{A}.$$

Practice Problem 1.30: Verify that all answers given above meet Kirchhoff's Equations (1.197) through (1.203).

1.8 SUMMARY

This chapter covers the essentials of DC circuits. It starts with resistors, capacitors, and inductors, all three passive circuit elements, and voltage and current sources. The emphasis on this chapter is on DC or direct current circuits. The fundamental laws of circuit analysis are presented: Ohm's law and Kirchhoff's voltage and current laws are the pillars to solve simple as well as complicated circuits. Magnetics basics were also presented.

It is important that the reader works out as many problems as possible reading and reading the chapter as often as it is necessary.

FURTHER READING

1. Vishay Intertechnology, Inc., *Tech Note TN0015: Design Guide for Use of Precision Resistors*, 2005.
2. David Halliday, Robert Resnick, and Jearl Walker, *Fundamentals of Physics*, 7th ed., Wiley, Hoboken, NJ, 2004.
3. Paul Horowitz and Windfield Hill, *The Art of Electronics*, 2nd ed., Cambridge University Press, New York, 1989.
4. Sanjaya Maniktala, *Switching Power Supply Design & Optimization*, McGraw-Hill, New York, 2004.
5. Arthur F. Kip, *Fundamentals of Electricity and Magnetism*, McGraw-Hill Inc., New York, 1962.
6. David A. Bell, *Fundamentals of Electric Circuits*, 4th ed., Prentice Hall, Upper Saddle River, NJ, 1988.
7. Mahmood Navhi and Joseph Edminister, *Electric Circuits*, 4th ed., McGraw-Hill, New York, 2003.

PROBLEMS

1.1 What is electrical conductivity of a material?

1.2 Which atomic particles have zero charge?

1.3 Which atomic particles have a mass about 1800 times larger than the mass of an electron?

1.4 Name the four types of materials based on their electrical characteristics.

1.5 How is an electric current defined?

1.6 Assume you have an automobile battery that has a 12-V nominal output voltage and a nominal internal resistance of 20 mΩ. We safely apply a short circuit across the battery terminals. Assume that the internal resistance of the battery does not change in a significant manner during the first few seconds after applying the short circuit. What is the current that the battery will deliver within the first couple of seconds?

1.7 We are given two DC voltage sources; one of them has a 12-V open-circuit voltage and an internal resistance of 1 Ω, the second source also has a 12-V open-circuit voltage and an internal resistance of 0.5 Ω. Which of the two sources is capable of delivering more current? Calculate the short circuit current of each source.

1.8 A resistor has 150 V applied across it and the current through it is 10 A. Find the value of resistance.

1.9 Given the circuit of Figure 1.53, calculate the current through the resistor for Figure 1.53a–c. What conclusion can you make as the resistor across the DC voltage source increases? What is the voltage across the resistor for all three cases?

1.10 Given the circuit of Figure 1.54, calculate the voltage across the resistor for Figure 1.54a–c. What conclusion can you make as the resistor across the DC current source increases? What is the current through the resistor for all three cases?

1.11 Given 1 MΩ nominal valued resistor, of a ±5% accuracy rating and a temperature coefficient of ±300 ppm/°C, calculate the resistance range that the resistor will span for a temperature range of –50°C to +150°C. Assume that the resistor nominal value provided is such, at 25°C.

1.12 Repeat Problem 1.11 for 1 MΩ resistor of ±5% accuracy rating, with a ±50 ppm/°C temperature coefficient.

1.13 Referring to the circuits of Figure 1.55a through l, which circuits are meaningless from a circuit analysis perspective? For each case, explicitly state the reason why each circuit is meaningless or not. In all cases, make an attempt to calculate the voltage and/or current in all the circuit

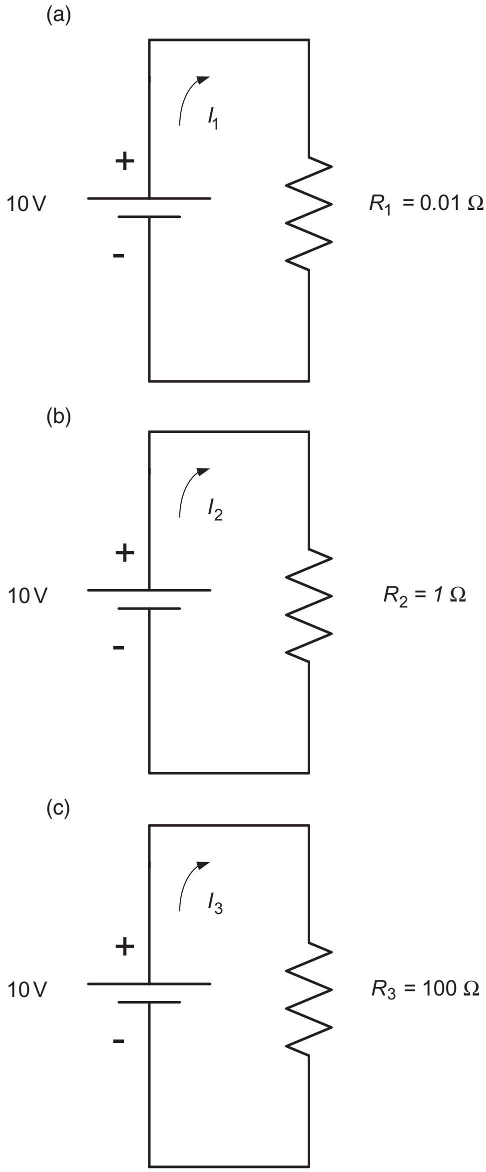


Figure 1.53 Circuit for Problem 1.9.

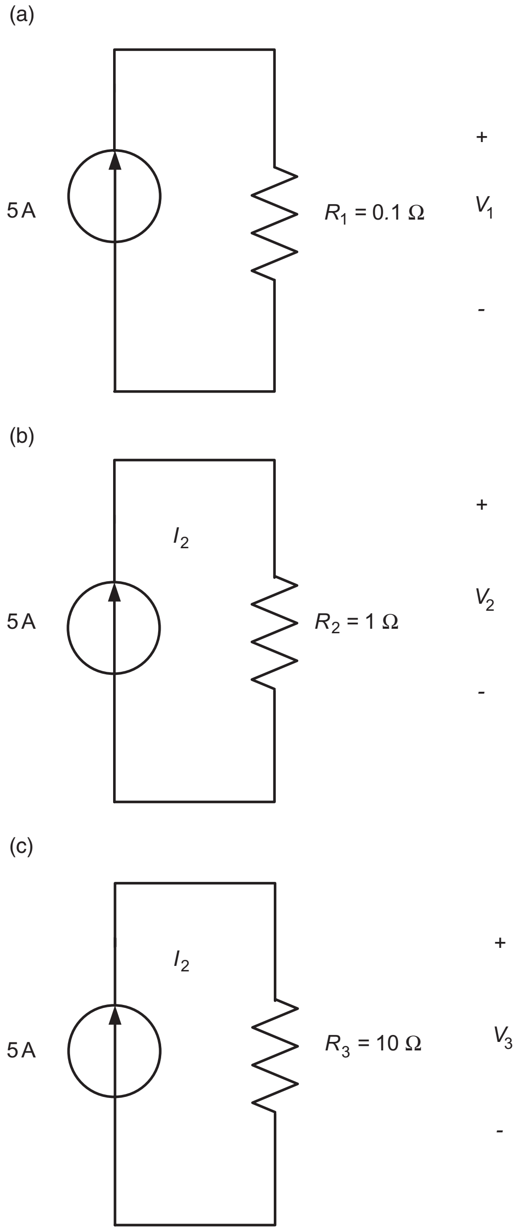


Figure 1.54 Circuit for Problem 1.10.

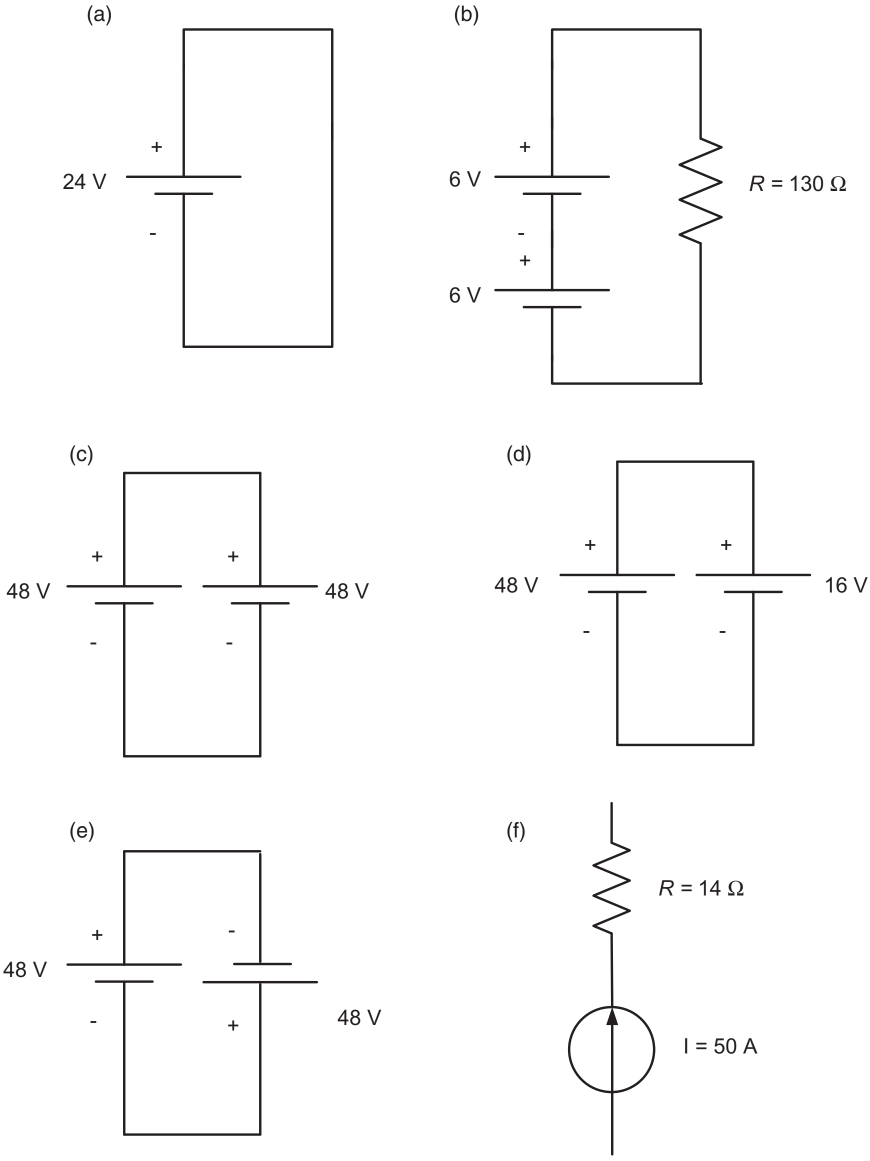


Figure 1.55 Circuit for Problem 1.13.

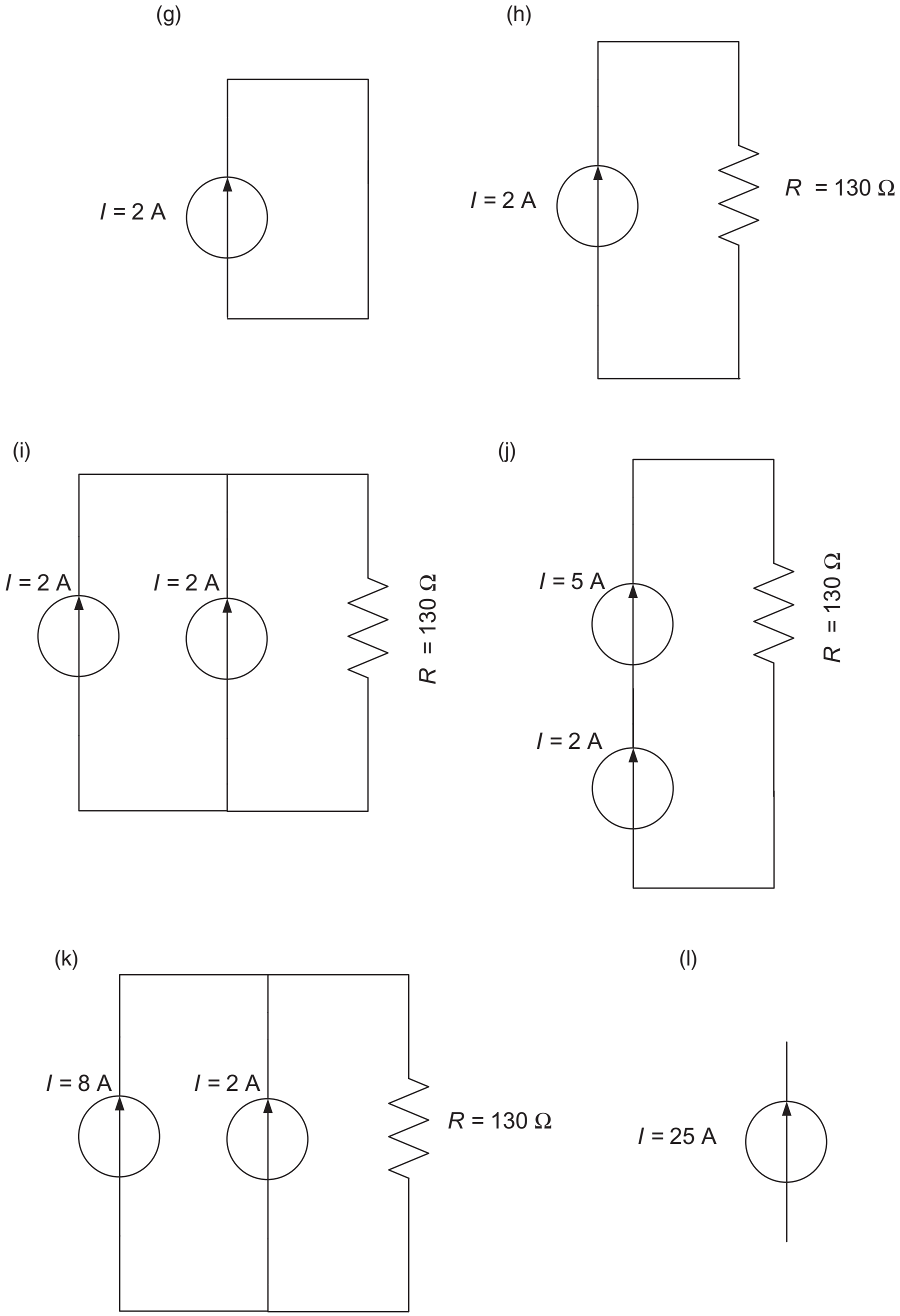


Figure 1.55 (*Continued*)

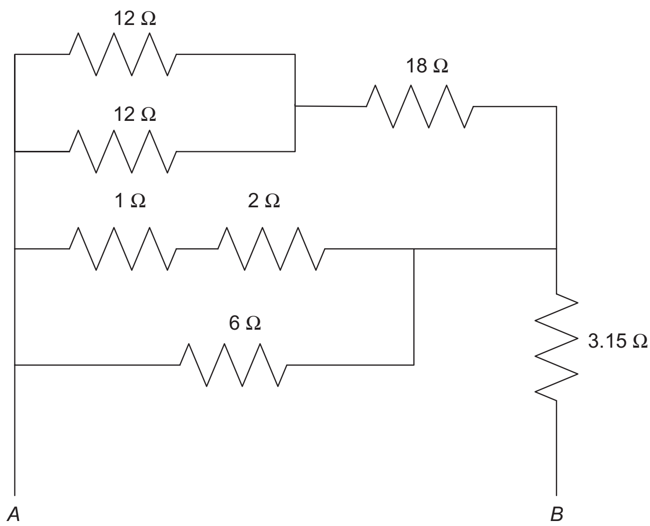


Figure 1.56 Circuit for Problems 1.14 through 1.18.

elements and sources whenever possible. If it is not possible, justify why it is not.

1.14 Given the circuit of Figure 1.56, calculate the equivalent resistance between points A and B.

1.15 Given the same circuit of Figure 1.56, assume that the series of the 1 Ω and 2 Ω resistors is replaced with a 0 Ω resistance. Calculate the equivalent resistance between points A and B.

1.16 Given the same circuit of Figure 1.56, assume that the 18 Ω resistor is removed from the circuit and replaced with a short circuit. Calculate the equivalent resistance between points A and B.

1.17 Given the same circuit of Figure 1.56, assume that only one of the 12 Ω resistors is replaced with a short circuit. Calculate the equivalent resistance between points A and B.

1.18 Given the same circuit of Figure 1.56, assume that the 3.15 Ω resistor is removed from the circuit and replaced with a short circuit. Calculate the equivalent resistance between points A and B.

1.19 Calculate the resistance of a 35-mm^2 cross-section copper wire that is 1 km long. Assume the room temperature is 20°C.

1.20 Repeat Problem 1.19 using silver for the wire material.

1.21 Repeat Problem 1.19 using gold for the wire material.

1.22 This question is not addressed by the material covered in this chapter. (Just for fun.) If the resistance of a gold wire is higher than that of a copper or a silver wire of the same cross section, length, and temperature, why do you believe that in some applications, for example, integrated circuit manufacturing, gold is used over copper and silver?

1.23 Which is the resistance value of the copper wire of Problem 1.19 if the ambient temperature of the wire is 100°C?

1.24 What is the resistance value of a 110 V, 100 W-rated lightbulb? At what temperature and current value do you believe that you have information to provide the answer? Justify your answers.

1.25 An ideal voltage source has an infinite capacity of generating current. Explain what is the point in paralleling two ideal voltage sources of the same voltage?

1.26 How should a real current source be connected so as not to damage the real device or create huge voltages?

1.27 Given two parallel-plate capacitors of the same plate area, and the same plate separation, assume that one has air as dielectric while the other one has Teflon. Which capacitor has a higher capacitance value?

1.28 If we apply a sinusoidal current source to a capacitor, draw the given current waveform and the voltage waveform developed across the capacitor.

1.29 A capacitor of value C is completely discharged. If a constant DC current source is indefinitely applied across the capacitor terminals, what does the voltage waveform as a function of time look like? Why?

1.30 Calculate the total equivalent capacitance between points A and B of the circuit of Figure 1.57.

1.31 Calculate the energy stored in a 100 μF capacitor with 1 kV applied across its terminals after a long period of time.

1.32 How much charge does the capacitor in Problem 1.31 have when it is fully charged?

1.33 Assume that we want to build a gigantic 1-F capacitor. We want to implement it with two square parallel metallic plates separated by a 10 mm air dielectric. Knowing that $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, determine the area of the plates needed. Translate your area result into acres. Ignore practical difficulties building the capacitor.

1.34 What can you conclude about the size of the capacitor from Problem 1.33?

1.35 Research problem: Using the World Wide Web, find the following features:

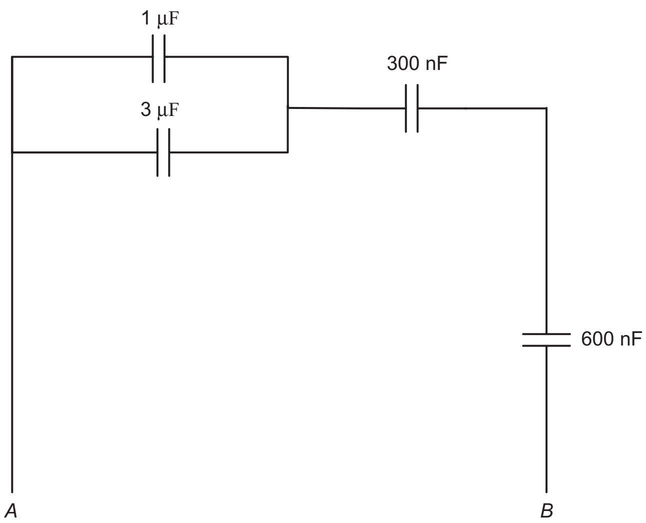


Figure 1.57 Circuit for Problem 1.30.

(1) Operational temperature range and (2) standard capacitance tolerances of the following capacitor dielectrics:

NPO/COG
X5R
X7R
Y5U
Z5U

1.36 Find at least five ferromagnetic materials, which can be either natural elements or man-made. Annotate the approximate relative permeability of each one. Hint: Access the World Wide Web.

1.37 Review the inductors section of this chapter; derive an equation for straight coil inductance which is a function of the core length, the loop effective area, the number of wire turns, and the core permeability.

Refer to Figure 1.58 for a graphical representation of the inductor.

1.38 Calculate the total equivalent inductance of the circuit of Figure 1.59 between points A and B.

1.39 Figure 1.41d depicts a transformer. Two coupled inductors constitute a transformer. The input inductor on the left is the primary; the output inductor on the right is the secondary. An ideal transformer receives AC current and voltage at a power level and produces at its output the same power level, but voltage and current are transformed by the following idealized equations:

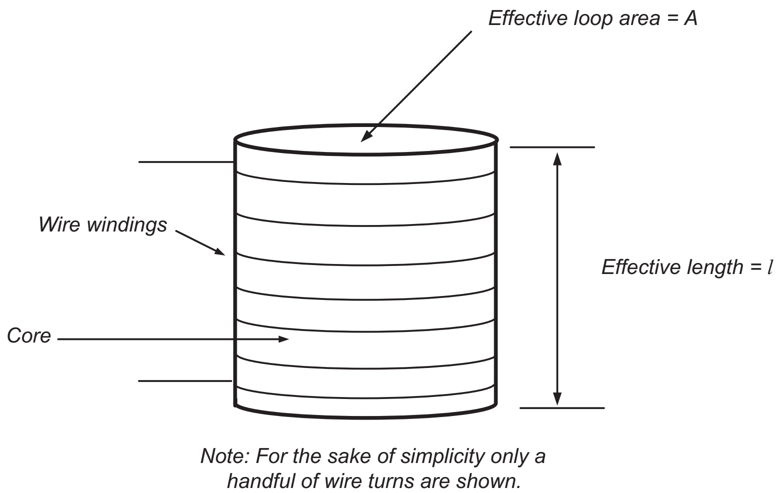


Figure 1.58 Inductor graphical representation.

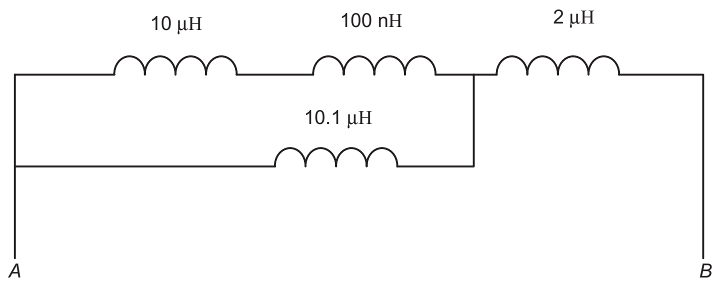


Figure 1.59 Inductor combination.

$$v_1 i_1 = v_2 i_2$$

$$v_1 N_2 = v_2 N_1$$

$$i_1 N_1 = i_2 N_2$$

In the given equations, v_1 is the primary voltage or excitation, i_1 is the primary current, v_2 is the output or secondary voltage, i_2 is the secondary current, N_1 is the primary number of wire turns, and N_2 is the secondary number of wire turns.

An ideal transformer differs from a real one in that the ideal transformer coupling between primary and secondary is assumed to be 100%; there is no leakage flux. High power, 50/60 Hz transformers work very close to such model. Radio-frequency (RF) transformers do not follow that closely the ideal transformer model.

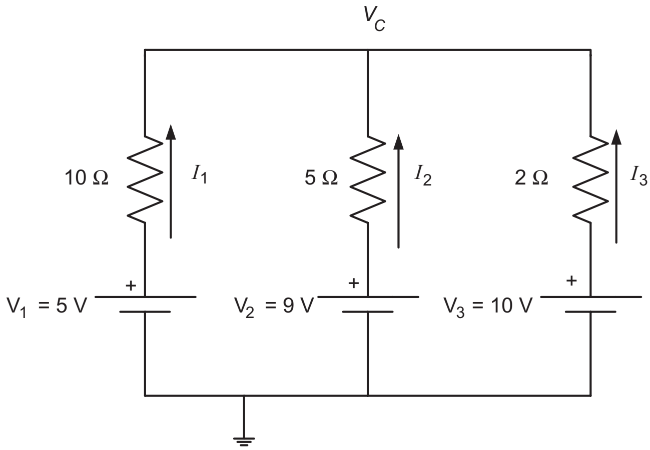


Figure 1.60 KVL/KCL circuit for Problem 1.42.

If the primary voltage is 240 V and the primary current is 10 A, determine the secondary voltage and secondary current. Assume that the transformer is ideal and its N_1/N_2 turns ratio is 4/1.

1.40 Based on your knowledge of an air-core inductor and an iron-core inductor, which one will store a larger amount of magnetic energy if the same current flows through both of them, and why?

1.41 An iron-core inductor becomes saturated because a current higher than its maximum saturation current is applied to it. (a) Which is the value of inductance L after the inductor is fully saturated? (b) Is the saturated inductor behaving in linear mode? (c) Which will be the current value through the inductor after it becomes fully saturated? (d) Explain one way by which the core saturation can be eliminated.

1.42 Using KVL and KCL equations for the circuit depicted in Figure 1.60, (a) Find the voltage and current on each resistor. (b) Find the current that each voltage source provides. (c) Verify that the sum of the powers produced by all sources equals to the power consumed by all resistors.