Chapter 11

Tuned Modified Transpose Jacobian Control of Robotic Systems

Control of robotic systems is a challenging task due to extreme nonlinearities and inherent coupling in system dynamics. Model-based (MB) control approaches can guarantee asymptotic convergence of the tracking error. However, implementing an MB algorithm, in addition to *a priori* knowledge of the system properties, requires computational power that may not be available. On the other hand, non-MB control approaches can provide simple alternatives for real-time implementations. The modified transpose Jacobian (MTJ) algorithm has been proposed based on an approximated feedback linearization approach, in which there is no need for *a priori* knowledge of the plant dynamics. This chapter presents a tuned MTJ (TMTJ) control algorithm that improves and facilitates the implementation of the MTJ algorithm by proper tuning of the switching gain matrix. This proposed control algorithm eliminates heuristic tuning for sensitivity thresholds of switching factor in the MTJ algorithm that will be useful for online control implementations. Obtained results show the merits of this new TMTJ controller as its performance is improved compared to that of the original MTJ algorithm even in the presence of significant disturbances and noises. Therefore, extremely low computational needs, without requiring *a priori* knowledge of the system dynamics, makes the proposed TMTJ

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algorithm a suitable candidate for position control of robotic systems in real-time practical implementations.

11.1. Introduction

Nonlinearity and complicated coupling in the system dynamics of mechanical manipulators have created challenging control problems. Hence, various algorithms has been proposed to tackle these problems, including adaptive control algorithms [SLO 87, TAI 00], time-delay control [YOU 87], motion-rate control [UME 89, KEL 05], artificial neural networks and fuzzy control [DOM 04, MBE 05, MEG 05, STE 04]. In practice, model-based (MB) algorithms can hardly yield a satisfactory solution due to their need to exact mathematical models of the system, which may be tremendously difficult to obtain in many real cases. On the other hand, among all these algorithms, those with fewer computational operations are more preferable for real-time implementations.

Transpose Jacobian (TJ) control is one of the simplest algorithms used to control motion of robotic manipulators, which has been arrived at intuitively [CRA 89]. In the case of using an approximate Jacobian, it has been shown that the damping matrix and the position gain matrix of this controller play an important role in system stability [MIY 88]. Apparently, the algorithm can be applied to redundant manipulators as shown by [ASA 93], and as discussed by [CHI 91] it does not fail when a singularity occurs. Hootsmans and Dubowsky [HOO 91] have developed an extended Jacobian transpose control algorithm to improve the performance of mobile manipulator systems. Subsequently, to fulfill simplicity requirements, Bevly *et al.* [BEV 00] have developed simplified Cartesian computed torque (SCCT) control algorithms for highly geared climbing robots.

The performance of TJ-based algorithms has been experimentally compared to those of different algorithms using unit quaternions on a direct-drive spherical wrist given by [GAR 02]. Papadopoulos and Moosavian [PAP 95] have compared the performance of this simple algorithm to those of various MB algorithms. Both experimental and simulation results show the merits of the TJ algorithm in controlling of highly nonlinear and complex systems with multiple degrees of freedom (DoFs), motivating further work on this algorithm. However, since the TJ is not dynamics-based, poor performance may result in fast trajectory tracking. Use of high gains can deteriorate performance seriously in the presence of feedback measurement noise. Another drawback is that there is no formal method of selecting its control gains, and a heuristic selection of gains makes it difficult to apply.

Hence, the modified transpose Jacobian (MTJ) has been proposed, which yields an improved performance over the standard algorithm, by employing stored data of the previous time step control command [MOO 97, MOO 07]. The MTJ algorithm is based on an approximation of feedback linearization methods, with no need for *a priori* knowledge of the plant dynamics terms. Its performance is comparable to that of MB algorithms, but requires reduced computational burden. Unlike the standard TJ, this algorithm works well in high-speed tracking tasks. In addition, controller gains can be selected in a systematic rather than a heuristic way, while higher gains and sensitivity to noise are avoided. The MTJ algorithm has been exploited for control of various systems; for instance, it has been recently implemented on a tractor-trailer wheeled mobile robot [KEY]. This is a modular robotic system that consists of a tractor module towing a passive trailer. Experimental results show the effectiveness of the proposed controller based on the MTJ algorithm. Nevertheless, the sensitivity thresholds of switching factor have to be assigned specifically for each robotic system before the tracking task. This can be a deficiency of the MTJ control law and needs to be resolved, which is the main focus of this study.

This chapter presents the tuned MTJ (TMTJ) control law that yields an improved applicability over the previously modified algorithm by tuning the switching gain matrix. Performance of the TMTJ algorithm will be compared to that of MTJ algorithm. Obtained results reveal the efficiency of the proposed algorithm even in the presence of influential disturbances, noises and substantial increase of trajectory frequency without requiring delicate selection of sensitivity thresholds.

11.2. TMTJ control law

11.2.1. *Feedback linearization approach*

Using the expressions for the kinetic and potential energy, and applying Lagrange's equations for a robotic system, the dynamics model can be obtained as follows:

$$H(q)\ddot{q} + C(q,\dot{q}) = Q \qquad [11.1]$$

where $q \in \Re^n$ is the vector of generalized coordinates, $H \in \Re^{n\times n}$ is the inertia matrix and $C \in \Re^n$ is the centripetal, coriolis and gravitational forces.

The output velocities $\dot{\hat{q}}$ are obtained from the generalized velocities $\dot{q}$ using a Jacobian matrix, $J(q)$, as:

$$\dot{\hat{q}} = J(q)\dot{q} \tag{11.2}$$

Assuming that $J(q)$ is square and non-singular, equation [11.1] can be written in terms of the output variables as follows:

$$\hat{H}(q)\ddot{\hat{q}} + \hat{C}(q,\dot{q}) = \hat{Q} \tag{11.3a}$$

where

$$\hat{H} = J^{-T}HJ^{-1},\ \ \hat{C} = J^{-T}C - \hat{H}\dot{J}\dot{q},\ \ \hat{Q} = J^{-T}Q \tag{11.3b}$$

To control such a system, a model-based control law (MB algorithm), such as

$$Q = J^T(\hat{H}u + \hat{C}) \tag{11.4}$$

can be applied, where u is an auxiliary control signal. A usual assumption associated with this control law is that the system geometric and mass properties are known. This control law linearizes and decouples the system equations to a set of second-order differential equations:

$$\ddot{\hat{q}} = u \tag{11.5}$$

If u is computed such that

$$u = k_p e + k_d \dot{e} + \ddot{\hat{q}}_{des} \tag{11.6}$$

where k_p and k_d are positive definite gain matrices and e is the tracking error defined as:

$$e = \hat{q}_{des} - \hat{q} \tag{11.7}$$

then the control law given by equation [11.4] guarantees asymptotic convergence of the tracking error *e*. Implementing an MB algorithm, in addition to *a priori* knowledge of the system properties, requires computational power that may not be available.

11.2.2. *Transpose Jacobian algorithm*

The TJ control law is a computationally simple algorithm, which has been arrived at intuitively. The task error vector and its rate, both multiplied by relatively high gains and by the Jacobian transpose matrix, result in commands that push the end-effector (EE) in a direction that tends to reduce the tracking error. The TJ controller can be presented as follows [CRA 89]:

$$Q = J^T(q)\left\{k_p e + k_d \dot{e}\right\} \qquad [11.8]$$

However, since TJ control law is not dynamics-based, poor performance may occur in applications where high-speed tracking is required. Use of high gains makes this problem more serious, especially in the presence of noise.

11.2.3. *Modified transpose Jacobian algorithm*

To achieve both precision and simplicity, the TJ control law has been modified by inclusion of a term representing the system dynamics at the time steps when the system experiences low errors. As shown in Figure 11.1, the MTJ approximates a feedback linearization solution by using stored data of the control command in the previous time step.

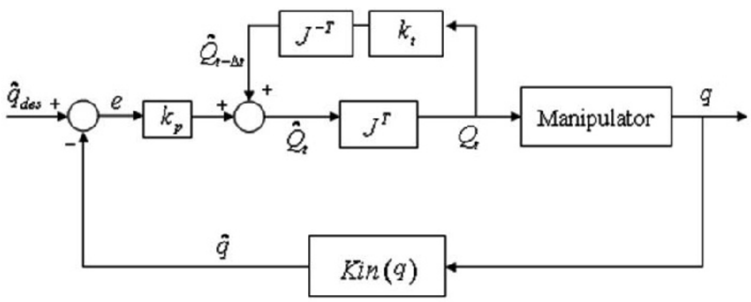

Figure 11.1. *The block diagram of the MTJ control law*

The MTJ control law is obtained as [MOO 07]:

$$Q = J^T(q)\left\{k_p e + k_d \dot{e} + h(t)\right\} \quad [11.9]$$

where $h(t)$ is the product of the control command in the previous time step and a switching gain matrix k_t that can be presented as follows:

$$h(t) = k_t \hat{Q}_{t-\Delta t} \quad [11.10]$$

$$k_{t\,ii} = \exp\left(-\left(\frac{|e_i|}{e_{\max_i}} + \frac{|\dot{e}_i|}{\dot{e}_{\max i}}\right)\right) \quad [11.11]$$

where $e_{\max}$ and $\dot{e}_{\max}$ represent sensitivity thresholds of the MTJ controller. Therefore, with proper selection of these sensitivity thresholds, so that the modifying term is properly activated (i.e. $k_t \approx 1.0$), and small time steps, the following error equation results [MOO 07]:

$$k_{d\,ii}\dot{e}_i + k_{p\,ii}e_i \cong 0 \quad [11.12]$$

which means the error dynamics will be governed by appropriate controller gains selection. It should be mentioned that the expression similar to equation [11.11] has been successfully employed to regulate sliding mode controllers in [MOO 04] for chattering elimination. Note that for better tracking, higher gains are required for the TJ algorithm and these lead to poor noise rejection characteristics. Also, high-frequency inputs can excite flexible system modes, and consequently decrease the accuracy, and the useful life of a system. Hence, it is confirmed that using high gains is not a viable option. On the other hand, the MTJ algorithm, by being an approximation of a feedback linearization algorithm, does not require high gains, or a high computational power, while its performance is comparable to that of the MB algorithms [MOO 07].

Considering equations [11.9]–[11.11], it can be deduced that for an N DoF system, calculation of the MTJ law requires $3N^2 + N + 2$ multiplications and $3N^2 - N + 1$ additions. Compared to the results presented in Table 11.1, these are almost the same as those for the TJ law and still significantly less compared to those needed for implementing the MB laws. Note that it is assumed that the inverse of the Jacobian matrix and its time derivative, which are required for implementing MB algorithms, are available symbolically, and hence these computations are not counted in Table 11.1. However, this comparison in terms of required computational effort reveals the efficiency of the TJ and the MTJ algorithms.

Algorithm	Multiplication	Additions
TJ	$3N^2$	$3N^2 - 2N$
MTJ	$3N^2 + 2$	$3N^2 - N + 1$
MB	$2N^3 + 7N^2$	$2N + 5N^2 - 4N$

Table 11.1. *Comparison of the required computational operations*

The above analysis reveals the simplicity (concerning *a priori* the knowledge requirement of system dynamics) and efficiency (in terms of the required computational effort) of both the standard TJ and the new MTJ law compared to the MB algorithms. In addition, the MTJ yields approximately linearized error dynamics and therefore an improved performance over the standard TJ algorithm. Stability analysis of the developed MTJ algorithm, based on Lyapunov's theorems, shows that both the standard and the MTJ algorithms are globally asymptotically stable [MOO 97].

It should be noted that the MTJ law is a position control algorithm that yields an improved performance over the standard algorithm. However, to manipulate an object, application of force/impedance control laws will be required that are usually MB algorithms. For instance, the multiple impedance control (MIC) is an MB algorithm that requires knowledge of the system dynamics [MOO 05, MOO 10, RAS 10]. On the other hand, even if the system dynamics is perfectly known, its computation may require considerable process time at each step for implementing the control law. Based on the MTJ control approach proposed earlier, the MIC law has been recently modified to be implemented without using system dynamics as non-MB MIC (NMIC) [MOO 08]. Therefore, this NMIC law is a more realistic algorithm for online computations in cooperating robotic systems.

11.2.4. *Tuned modified transpose Jacobian algorithm*

The sensitivity thresholds of a switching gain matrix may have different values in each robotic system. Therefore, they must be determined by trial-and-error, prior to a tracking task being performed. This can be a deficiency of the MTJ control law that is improved here as a TMTJ algorithm. To overcome this weakness, the sensitivity thresholds of a switching gain matrix can be computed according to the proposition 11.1.

PROPOSITION 11.1.– *If* k_t *is computed as bellow, the feedback linearization can be smoothly approximated as:*

$$k_{t\,ii} = \exp\left(-\left|\frac{k_{p\ ii}e_i + k_{d\ ii}\dot{e}_i}{r\hat{Q}_{t-\Delta t\, i}}\right|^s\right) \qquad [11.13]$$

PROOF.– Considering the above switching gain matrix elements, $0 \le k_{t\ ii} \le 1$, where $k_{p\ ii}e_i + k_{d\ ii}\dot{e}_i$ can be determined from the MTJ law, equations [11.9] and [11.10], and substituting the result into [11.13], we obtain:

$$k_{t\,ii} = \exp\left(-\left|\frac{\hat{Q}_{t\,i} - k\hat{Q}_{t-\Delta t\, i}}{r\hat{Q}_{t-\Delta t\, i}}\right|^s\right) \qquad [11.14]$$

For simplicity, defining $\mu \ge 0$ as follows:

$$\mu = \left|\frac{\hat{Q}_{t\,i} - k_{t\,ii}\hat{Q}_{t-\Delta t\, i}}{\hat{Q}_{t-\Delta t\, i}}\right| \xrightarrow{(b)} k_{t\,ii} = \exp\left(-\frac{\mu^s}{r^s}\right) \qquad [11.15]$$

Then, two cases will exist as discussed below.

Case 1: $\dfrac{\hat{Q}_{t\,i} - k_{t\,ii}\hat{Q}_{t-\Delta t\, i}}{\hat{Q}_{t-\Delta t\, i}} \ge 0$

Relative difference of $\hat{Q}_{t\,i}$ is defined from the definition of μ, equation [11.15] and the aforementioned inequality.

$$E^{rel}_{\hat{Q}_{t\,i}} = \frac{\hat{Q}_{t\,i} - \hat{Q}_{t-\Delta t\, i}}{\hat{Q}_{t\,i}} = 1 - \frac{1}{\mu + k_{t\,ii}} \qquad [11.16]$$

Substituting equation [11.15] into [11.16], $E^{rel}_{\hat{Q}_{t\,i}}$ can be rewritten as:

$$E^{rel}_{\hat{Q}_{t\,i}} = 1 - \frac{1}{r(-\ln(k_{t\,ii}))^{1/s} + k_{t\,ii}} \qquad [11.17]$$

Claim I: If $s > 1$ and $r > \lambda_{(s)}s$, then:

$$\forall \ k_{tii} \in (0,1) \Rightarrow 0 < E^{rel}_{\hat{Q}_{ti}} (\text{as strictly decreasing}) < 1$$

$$\text{if} \quad k_{tii} \to 1^- \Rightarrow \frac{\partial k_{tii}}{\partial E^{rel}_{\hat{Q}_{ti}}} \to 0$$

PROOF.– Suppose $r,s > 0$, thus equation [11.17] yields:

$$\text{when } k_{tii} \to 0^+ \Rightarrow E^{rel}_{\hat{Q}_{ti}} \to 1^-$$

$$\text{when } k_{tii} \to 1^- \Rightarrow E^{rel}_{\hat{Q}_{ti}} \to 0^+$$

Therefore, it is sufficient to find a condition on which $\partial E^{rel}_{\hat{Q}_{ti}} \big/ \partial k_{tii} < 0$ is satisfied. Using derivation of $E^{rel}_{\hat{Q}_{ti}}$ in term of k_{tii}:

$$\frac{\partial E^{rel}_{\hat{Q}_{ti}}}{\partial k_{tii}} = \frac{-\frac{r}{s} \times \frac{1/k_{tii}}{\left(-\ln(k_{tii})\right)^{1-1/s}} + 1}{\left(r\left(-\ln(k_{tii})\right)^{1/s} + k_{tii}\right)^2} \qquad [11.18]$$

It is seen that the gradient of $E^{rel}_{\hat{Q}_{ti}}$ in term of k_{tii} is negative if: $\frac{r}{s} \times \frac{1/k_{tii}}{\left(-\ln(k_{tii})\right)^{1-1/s}} > 1$

The minimum value of an LHS of the above inequality is achieved when $k_{tii}(-\ln(k_{tii}))^{1-1/s}$ meets its maximum value as happened in the point $k_{tii} = \exp(1/s - 1)$. Substituting this point into the inequality results in $r > \exp(1/s - 1)(1 - 1/s)^{(1-1/s)} s$.

According to $r,s > 0$, when k_{tii} tends to 1^-, we can write:

$$\lim_{k_{tii} \to 1^-} \frac{\partial E^{rel}_{\hat{Q}_{ti}}}{\partial k_{tii}} = \frac{-\frac{r}{s} \times \frac{1/1}{\left(\ln(1^+)\right)^{1-1/s}} + 1}{\left(r\left(\ln(1^+)\right)^{1/s} + 1\right)^2} = -\frac{r}{s} \times \frac{1/1}{\left(0^+\right)^{1-1/s}} + 1 \qquad [11.19]$$

If $1-1/s>0$ or $s>1$, thus $\partial E^{rel}_{\hat{Q}_{ti}} \big/ \partial k_{tii} \to -\infty$ that leads to $\partial k_{tii} \big/ \partial E^{rel}_{\hat{Q}_{ti}} \to 0$, then claim I is proven.

So, using different r and s values to plot a $k_{tii}(E^{rel}_{\hat{Q}_{ti}})$ diagram, the suitable non-dimensional parameters can be determined. Some simulations reveal that choosing the conditions from a range approximately 0.5 for $E^{rel}_{\hat{Q}_{ti}}$ at $k_{tii}=0.5$ can be proper selections and their results are nearly the same.

Case 2: $\dfrac{\hat{Q}_{ti}-k_{tii}\hat{Q}_{t-\Delta t i}}{\hat{Q}_{t-\Delta t i}}<0$

Relative difference of $\hat{Q}_{t-\Delta t i}$ is defined from the definition of μ, equation [11.15] and the above-mentioned inequality.

$$E^{rel}_{\hat{Q}_{t-\Delta t i}}=\frac{\hat{Q}_{t-\Delta t i}-\hat{Q}_{ti}}{\hat{Q}_{t-\Delta t i}}=1+\mu-k_{tii} \qquad [11.20]$$

Substituting equation [11.15] into [11.20], $E^{rel}_{\hat{Q}_{t-\Delta t i}}$ can be rewritten as:

$$E^{rel}_{\hat{Q}_{t-\Delta t i}}=1+r\left(-\ln(k_{tii})\right)^{1/s}-k_{tii} \qquad [11.21]$$

Claim II: If $s,r>0$, then:

$$\forall \;\; k_{tii}\in(0,1) \Rightarrow E^{rel}_{\hat{Q}_{t-\Delta t i}}>0 \text{ is strictly decreasing.}$$

PROOF.– Suppose $r,s>0$, thus equation [11.20] yields:

when $k_{tii}\to 0^{+} \Rightarrow E^{rel}_{\hat{Q}_{t-\Delta t i}}\to +\infty$

when $k_{tii}\to 1^{-} \Rightarrow E^{rel}_{\hat{Q}_{t-\Delta t i}}\to 0^{+}$

Therefore, it is sufficient to find a condition on which $\partial E^{rel}_{\hat{Q}_{t-\Delta t i}} \big/ \partial k_{tii}<0$ is satisfied. Using derivation of $E^{rel}_{\hat{Q}_{t-\Delta t i}}$ in terms of k_{tii}:

$$\frac{\partial E^{rel}_{\hat{Q}_{t-\Delta t i}}}{\partial k_{tii}}=-\frac{r}{sk}\left(-\ln(k_{tii})\right)^{(1/s)-1}-1 \qquad [11.22]$$

It can be easily found that by the condition of *r,s* > 0, $\partial E^{rel}_{\hat{Q}_{t-\Delta t\, i}} / \partial k_{t\, ii}$ is negative and claim II is proven.

According to claim II, it can be understood that the values employed in case 1 for *r* and *s* are appropriate for case 2 too.

It should be mentioned that it is possible that the difference between two consecutive switching factors can be naturally large due to the changing control law from TJ to MTJ algorithm and approaching $\hat{Q}_{t-\Delta t\, i}$ to zero. Hence, the following switching gain matrix elements are filtered by a Butterworth low-pass filter as:

$$k_{t\, ii}^{filtered} = \text{Butterworth (low-pass)} \times k_{t\, ii} \qquad [11.23]$$

where cutoff frequency is chosen relevant to the step size.

Note that factor k_t is initially taken equal to zero, resulting in a TJ control law at the first time step. Stability analysis of the MTJ algorithm, based on Lyapunov's theorems, shows that both the standard and the MTJ algorithms are globally asymptotically stable [MOO 97]. This analysis obviously justifies the same result for the proposed TMTJ algorithm.

11.3. Obtained results and discussions

In this section, the performance of the TMTJ control algorithm as given by equations [11.9], [11.10] and [11.13] is evaluated and compared to that of the MTJ algorithm, as given by equations [11.9], [11.10] and [11.11]. Two different systems as fixed and mobile base manipulators are considered for this comparison.

11.3.1. *Fixed base manipulator*

To focus on algorithmic aspects, a simple two-link planar manipulator as shown in Figure 11.2(a) is considered under various conditions. Performing high-speed tracking task, disturbance and noise rejection characteristics of the proposed TMTJ algorithm is investigated in these simulations and compared to that of alternative algorithm.

The task is tracking a trajectory, defined by:

$$x_{des} = \sqrt{l_1^2 + l_2^2}\, Cos(\omega t + \pi/4) + 0.1 \quad Sin(5\omega t)$$
$$y_{des} = \sqrt{l_1^2 + l_2^2} \quad Sin(\omega t + \pi/4) + 0.1 \quad Sin(5\omega t) \qquad [11.24]$$

This trajectory corresponds to a perturbed circular path (see Figure 11.2(b)). The motion speed along the path can be selected by setting the cyclical frequency ω.

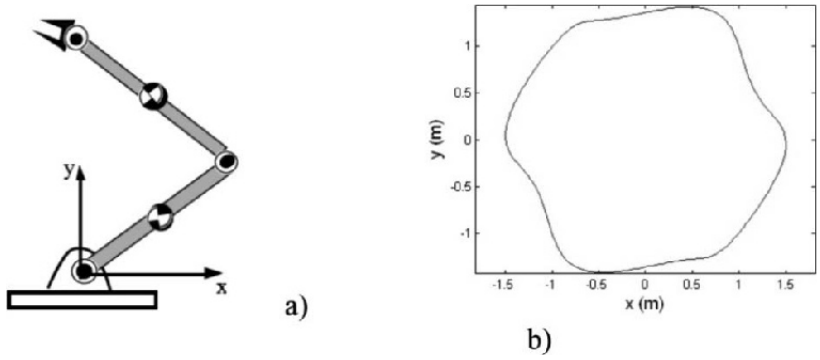

Figure 11.2. *a) The manipulator and b) the desired tracking path*

For the manipulator shown in Figure 11.2(a), the mass properties of the system are $m_1 = 4.0$ kg, $I_1 = 0.333$ kg/m^2, $m_2 = 3.0$ kg and $I_2 = 0.30$ kg/m^2, and the link lengths are $l_1 = l_2 = 1$ m. The initial conditions for joint angles and derivatives are:

$$(q_1(0), q_2(0), \dot{q}_1(0), \dot{q}_2(0)) = (0.03,\ \pi/2,\ 1.5, -1.0) \ \ (\text{rad}, \text{rad/s})$$

which correspond to some initial position and velocity errors. The sensitivity thresholds for the MTJ algorithm, $e_{\max}$ and $\dot{e}_{\max}$, in equation [11.11], are equal to 1 m and 10 m/s, respectively. Also, r and s parameters are set to 1.5 for both in TMTJ. The time step Δt_i is held constant and equal to 10.0 ms in this study. To establish a fair comparison, the gains for the algorithms under comparison should be selected such that the peaks of the required joint torques become approximately equal. Herein, choosing similar gains for both algorithms satisfies this condition. The fourth-order Runge–Kutta method for solving differential equations is used in all simulations.

The performance of the MTJ and TMTJ algorithms, in terms of the end-point error in relatively high-speed tracking task (ω = 1 rad/s), is compared in Figure 11.3. For both MTJ and TMTJ algorithms, k_p= *diag*(30,30) and k_d = *diag*(60,60) are properly selected based on error dynamics of equation [11.12], which are used for all simulations in this case.

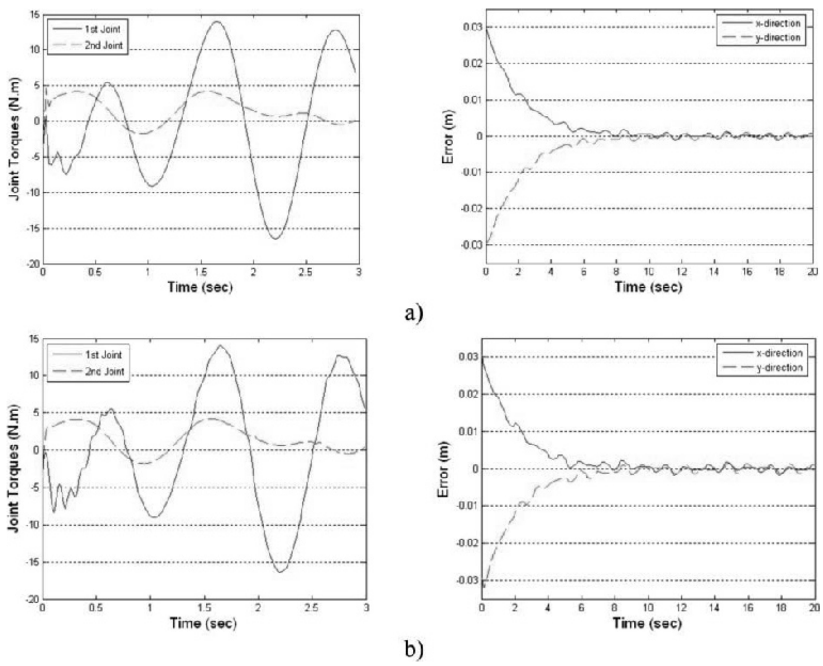

Figure 11.3. *(Left) Joint torques and (right) end-effector errors: a) MTJ and b) TMTJ*

It can be seen that the input torques in the TMTJ law have somewhat more fluctuation than that of MTJ control law. The end-point error is almost the same for both algorithms, while TMTJ does not need heuristic selection of the sensitivity thresholds, $e_{\max}$ and $\dot{e}_{\max}$ in equation [11.11]. Changing of *r* and *s* parameters in the range, pointed out in the proof of proposition 1, has made no considerable differences in the results. It should be mentioned that the total energy consumption of each algorithm for performing this task, given by the time integral of $\sum_{i=1}^{2}\left|Q_{a\,i}\dot{q}_i\right|$, is approximately the same.

11.3.1.1. *Disturbance rejection characteristics*

To show the capability of the proposed TMTJ control law for disturbance effect elimination, it is assumed that the input torques are perturbed by 20% with respect to the true values. Figure 11.4 shows that TMTJ and MTJ are similarly capable to preserve the trend of tracking errors elimination by more fluctuating in input torques in the presence of influential disturbances.

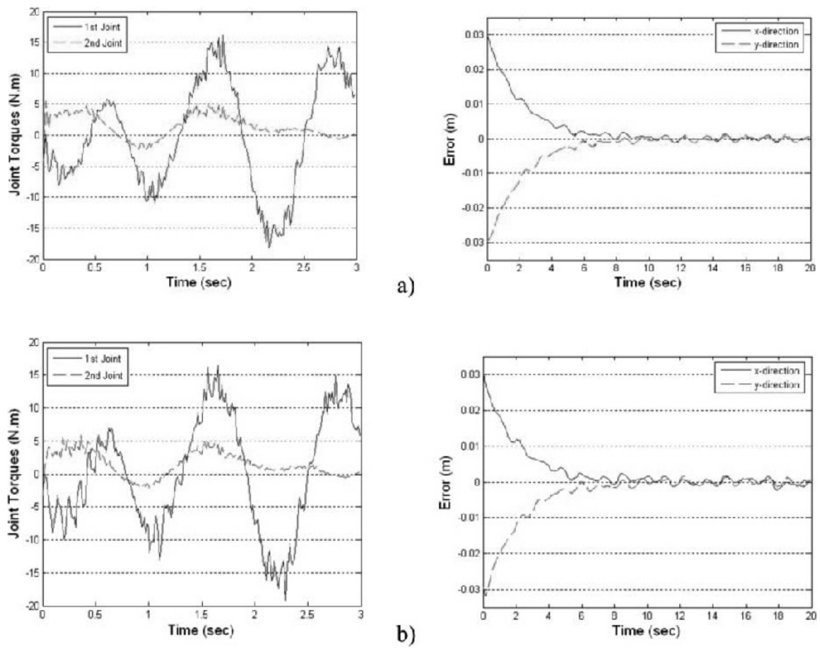

Figure 11.4. *(Left) Joint torques and (right) EE errors in the presence of disturbances: a) MTJ and b) TMTJ*

11.3.1.2. *Noise rejection characteristics*

In practice, noise effects deteriorate available feedback. Therefore, we should examine the noise rejection capabilities of would be implemented algorithms. Here, it is supposed that measurements of joint angles rates are corrupted by white noise whose amplitude is 1% of the signal magnitude. As shown in Figure 11.5, the tracking errors of TMTJ are comparable with those of an MTJ algorithm while the required torques for both algorithms are of reasonably equal magnitude.

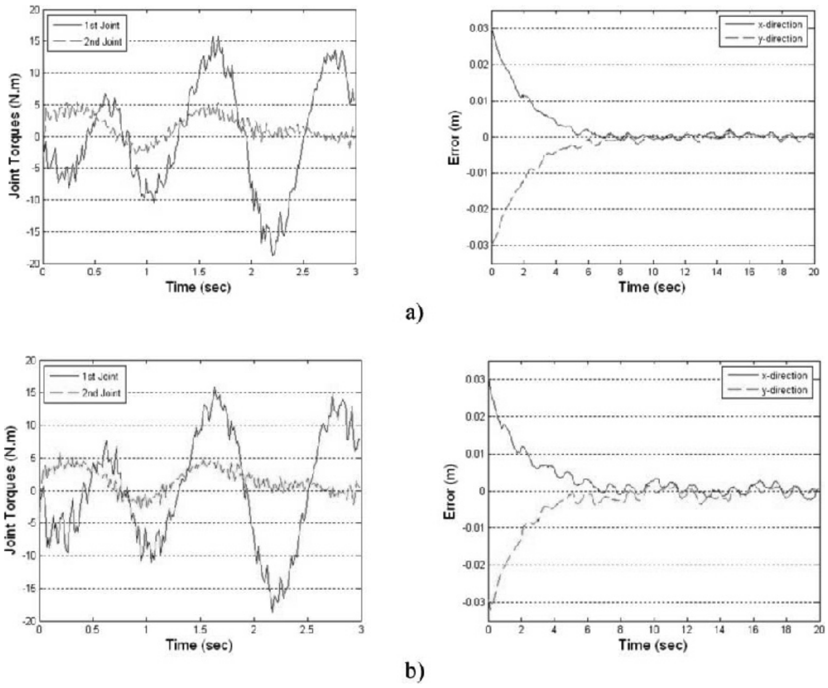

Figure 11.5. *(Left) Joint torques and (right) EE errors in the presence of noises: a) MTJ and b) TMTJ*

11.3.2. ***Mobile base manipulator***

To show the advantages of the proposed TMTJ compared to the MTJ algorithm, a mobile two-link planar manipulator is considered (Figure 11.6).

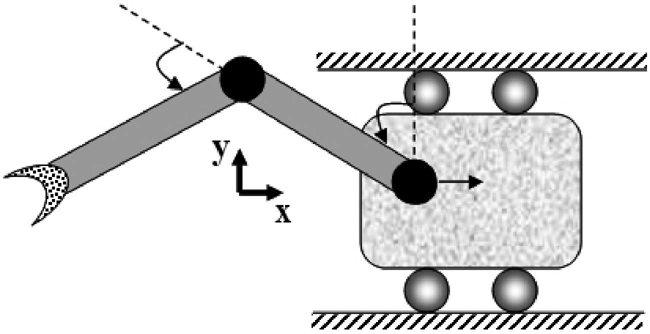

Figure 11.6. *The mobile base manipulator*

The task is defined for the end-effector as tracking a trajectory given by:

$$x_{des} = Cos(\omega t)$$
$$y_{des} = 0.1 \quad Sin(\omega t) \qquad [11.25]$$
$$\theta_{des} = 0.12 \quad Cos(\omega t) + \frac{5\pi}{6}$$

This trajectory corresponds to an elliptic path. For the manipulator shown in Figure 11.6, the mass of the base is $m_1 = 5.0$ kg and mechanical properties of the arms are the same as for the previous fixed base manipulator. The initial conditions for joint angles and derivatives are:

$$(q_1(0), q_2(0), q_3(0), \dot{q}_1(0), \dot{q}_2(0), \dot{q}_3(0)) = (0.03, \ \pi/2, \ 1.5, -1.0) \ \ (\text{rad}, \text{rad/s})$$

which correspond to some initial position and velocity errors.

The sensitivity thresholds for the MTJ algorithm, $e_{\max}$ and $\dot{e}_{\max}$ in equation [11.11], are equal to 1 m and 10 m/s for x and y directions, respectively, and 3 rad and 30 rad/s for θ. Also, r and s parameters are set equal to 1.5 as was priorly used for previous case. Choosing similar gains k_p = *diag*(30,30) and k_d = *diag*(60,60) for both algorithms satisfies a fair comparison condition, that is total energy consumption and the peaks of the required joint torques are approximately the same for both algorithms. First, the end-effector error is compared in Figure 11.7 for a tracking task with constant frequency of ω = 1 rad/s. It can be seen that tracking errors are similar for both algorithms, if not better for the TMTJ.

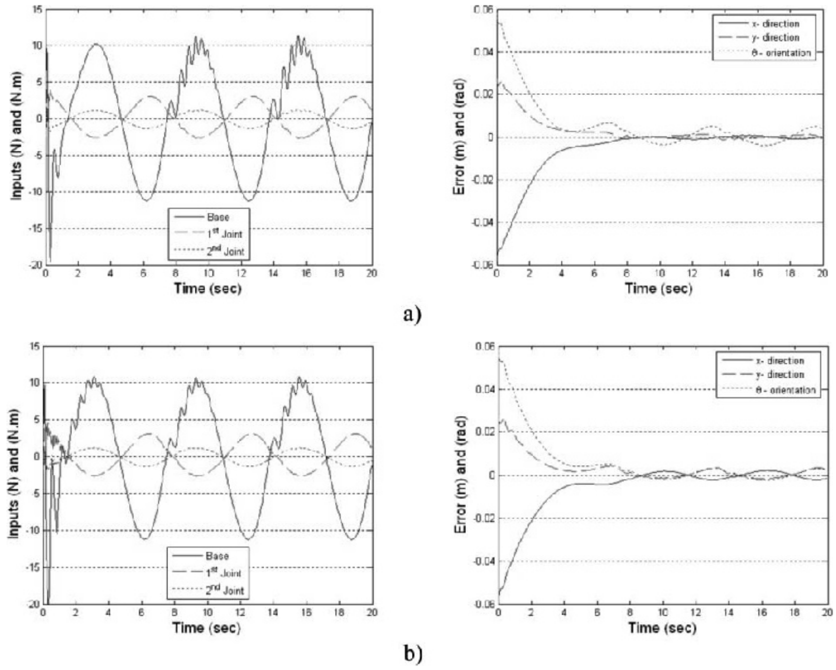

Figure 11.7. *(Left) Joint torques and (right) end-effector errors: a) MTJ and b) TMTJ*

11.3.2.1. *Increasing tracking frequency*

To show the merits of the proposed TMTJ algorithm, tracking frequency ω is increased from 1 to 3 rad/s, using a simple exponential function:

$$\omega = 3 - 2e^{-t} \qquad [11.26]$$

As it is shown in Figure 11.8, tracking errors for the MTJ are almost twice those of the TMTJ. The reason is that the sensitivity thresholds for the MTJ cannot be well tuned when the frequency ω is being developed. Also, it should be noted that r and s parameters in the TMTJ can be adapted with change in ω. As seen in the left side of Figure 11.8, the TMTJ algorithm demands slightly different input force and torques to preserve a better tracking performance.

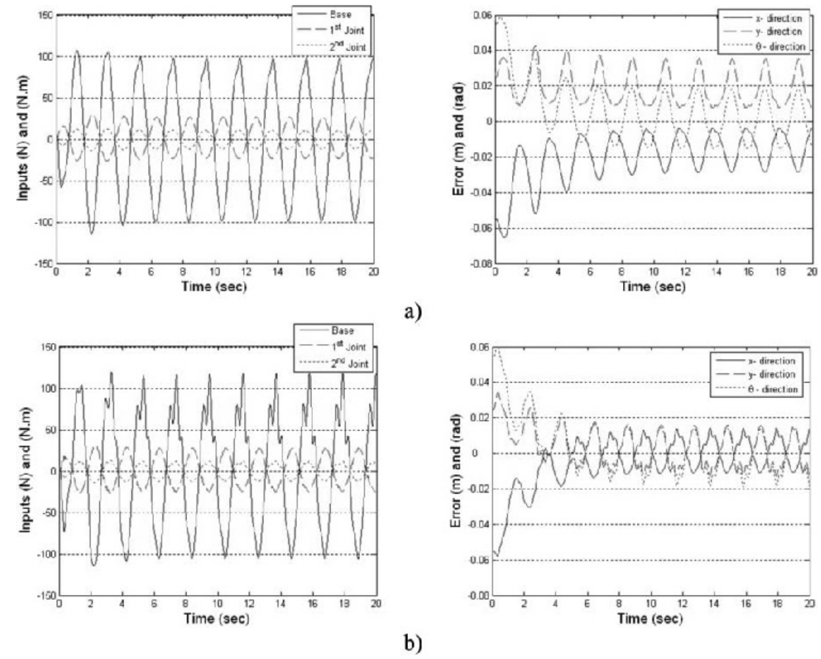

Figure 11.8. *(Left) Joint torques and (right) EE errors with the increased frequency: a) MTJ and b) TMTJ*

11.4. Conclusions

This chapter presented a new TMTJ control algorithm that improves the applicability of the MTJ algorithm, by eliminating the requirement of the delicate selection of the switching gain matrix for online control purposes. Obtained results reveal substantial merits of the proposed TMTJ controller as its performance is improved compared to the original MTJ algorithm even in the presence of significant disturbances and noises, and also in the increase of trajectory frequency. It was already shown that the performance of the MTJ controller is comparable to that of a perfect MB algorithm, with the advantage that less computational power is needed. Extremely low computational needs, without requiring *a priori* knowledge of the system dynamics, makes the proposed TMTJ algorithm a promising alternative for position control of robotic systems in practice. Therefore, the proposed TMTJ algorithm can be considered as a good candidate for the control of industrial robots, where simple efficient algorithms are vastly preferred to complicated theoretical algorithms that usually require huge computations.

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