Chapter 6

Electronic Instrumentation, Connecting Precautions and Signal Processing

The choice of electronic instruments is largely borrowed from a stock of commercially available apparatus used in dynamic tests of mechanical structures. However, experimenters, working in the domain of material characterization, will notice that these instruments often do not give them satisfactory answers. The main reason is that the same special measurements are not required compulsorily in the aforementioned domain. For example, the experimental problem of the measurement of wave dispersion at extremely high frequency is not necessarily something which concerns specialists in structural dynamics[1].

Each apparatus, each electrical or electronic component must be carefully selected so that its contribution to the whole dynamic behavior of the mechanical-electronic system constituted by the sample and the electronic equipments (Figure 6.1) does not surreptitiously introduce artifacts or errors to the measurement of the dynamic behavior of the sample. The second important reason for choosing a piece of equipment resides, to our mind (see [CHE 10]), in obtaining stationary vibrations.

The elastic (or viscoelastic) dynamic sample responses and the electric responses of each electronic apparatus must be taken into account.

Chapter written by Jean Tuong VINH

1 In this domain of structural dynamics, the frequency range does not exceed 10,000 Hertz.

Mechanical Characterization of Materials and Wave Dispersion: Instrumentation and Experiment Interpretation
Edited by Yvon Chevalier and Jean Tuong Vinh

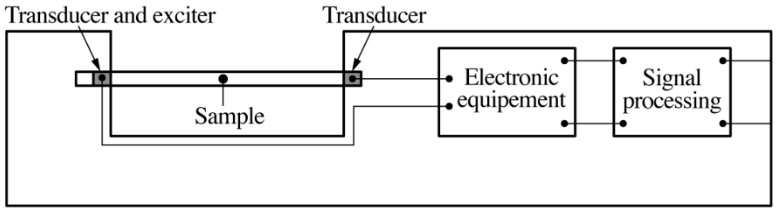


Figure 6.1. *The sample belongs to the complete electronic-mechanical set. The electronic equipment must not influence the interpretation of the sample's dynamic response*

6.1. Preamplifiers and signal conditioners following the transducers

Tables 6.1 and 6.2 present the four main classes of transducers with their principal electrical performances.

Electrical type	Preamplifier and conditioners	Mounting conditions
Piezoelectric (quartz, simple crystal or polarized ceramics) accelerometer and force transducers	Special charge preamplifier with high input impedance	Cable between transducer and preamplifier can produce pick up noise; cable length must be reduced to a minimum
	Line drive amplifier (inside the transducer case) Restriction on the dynamic range and frequency range	Connecting cable has no influence; Possible undetectable overload
Inductance (displacement transducers)	Self inductance Wheatstone bridge Carrier frequency of commercially available bridge: $f \cong 2{,}500$ Hz Working frequency: $f \cong 2{,}500$ Hz Low working frequency: $f_w = 250$ Hz Choose bridge with high carrier frequency: $f_c \cong 25{,}000$ Hz Vibrometer: $f_c \cong 100{,}000$ Hz	Envelope detector and amplifier; Low output impedance

Table 6.1. *Table presenting two electrical type transducers with special preamplifiers, with alternative intensity or voltage supply*

Electrical type	Preamplifier and conditioner	Mounting conditions
Eddy current (displacement transducers)	High frequency Wheatstone bridge, $f_c \gg 2$ MHz or High frequency Colpitts oscillator, $f_c \gg 2$ MHz High working frequency $f_w \gg 10^5$ Hz Necessary linearized circuit	Caution in cable length Caution in mounting transducer Shielding of moving target
Condenser (displacement transducers)	High carrier frequency Wheatstone bridge $f_c \gg 25$ to 100 KHz High working frequency f_w 2500 to 10,000 Hz Eventual linearized circuit (Check the voltage versus the variation law of condenser	Active condenser surface chosen according to the carrier frequency Pick up noise to be eliminated by transducer shielding Working at high frequency Target plate must be metallic

Table 6.2. *Two other electric-type transducers with special preamplifiers*

6.2. Cables and wiring considerations

In preceding section, we mentioned the influence of the cable length between the transducer and preamplifier. There are other electrical considerations concerning wiring which experimenters should be aware of.

6.2.1. *Spurious noise signals*

Many reasons can be given for taking extreme care about the choice of cables and their position in an experimental set-up. With the exception of the accelerometer-type transducer, which has a closed hermetic case which serves as a shield, many transducers are activated by open air moving targets; see Figures 6.1 and 6.2. As such, these targets play the role of antennas which capture parasitized signals from environment media, such as the numerous electrical sources around the test bench, the electric mains in the laboratory, acoustic emission of signals from electronic generators, etc. Shielding of all parts of the electrical equipment including the transducer is necessary to reduce the spurious noises.

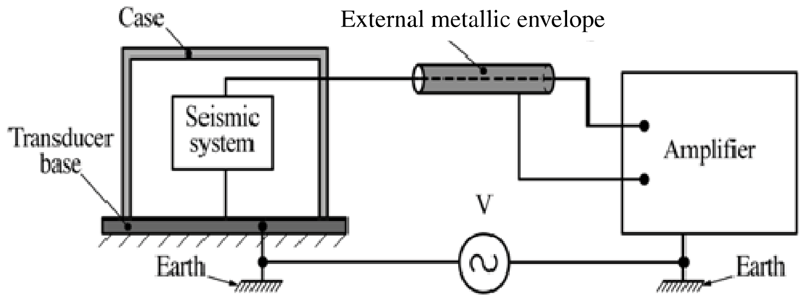


Figure 6.2. *Loop in grounding structures. A potential voltage difference V between them is a source of noise captured by electric equipment*

6.2.2. *Grounding*

In all laboratories, there is an electric cable called the earth. Often this earth can be found in any electric plug and the earth is used to ground all the electrical apparatus.

Unfortunately, the cable constitutes a main artery in which all possible electric noises are discharged. If the laboratory has a special earth independent of the classic earth for power cables, the spurious noises can be reduced. But this precaution is not sufficient. Figure 6.2 shows an experimental set-up with two separate grounded structures which gives rise to a noise voltage. It is necessary to set this voltage to zero by connecting the shield cable to the earth of the transducer and the earth of any electronic equipment. A coaxial conductor cable with a shield made of copper or aluminum braided strands is good. In some cases, for an electrostatic shield, these metallic strands are not sufficient; a second inner shield can be used to reduce the radiation of any electrostatic field into the central conductor. A magnetic shield is used to cut magnetic lines or flux from low reluctance paths. Magnetic interference is created by power lines, power transformers and its cancellation with respect to signal cables is difficult and sometimes impossible.[2]

The solutions for this problem are:

– keeping the signal cable as far as possible from the power sources;

– using a special magnetic shield with high permeability materials such as permalloy. This shield must be as thick as possible.

2 The problem of grounding loop constitutes a serious one when the experimenter has to envisage a campaign of testing in noisy ambiance. Some solutions are found for this problem by using special apparatus to increase the signal to noise ratio [BRU 98].

6.2.3. *Cable noise*

Cable noise can have a number of different origins:

– *triboelectricity.* Triboelectricity is electrostatic electricity created at the point of contact between the inner conductor and the insulated dielectric. When a cable is subjected to motion such as bending, torsion, squeeze or impact, friction takes place at the point of contact and an electric charge is produced across the capacitance constituted by the dielectric between the outer shield and the center conductor. This results in an electric noise voltage which varies with the application of the forces mentioned above;

– *variations of capacitance.* Mechanical motion creates variations of the capacitance constituted by the cable itself. In some special cables, leakage paths are foreseen along the cable, permitting the electric charge to flow into the shield before reaching the input of any preamplifiers;

– *cable fixing.* If possible, the cable must not be subjected to motion which is capable of creating noise. Collar attachments along the cable are useful. However, there is a specific problem in dynamic material tests. If seismic type transducers (accelerometers) are used on both ends of a sample, there are suspended cables and, if the non-attached parts of the cables are short, there is additional stiffness at both ends which can modify the sample motion. We have to deal with two incompatible and nearly contradictory conditions:

- preventing the motion of the cable by reducing the free part at the end of the cables; and

- leaving the cables to be as free as possible in the air so as to reduce the influence of the cables as supplementary boundary conditions or additional stiffness on the sample. The reason is that the cable is an integer with the vibrating sample, consequently the cable has an influence on the measurement of sample damping.

6.3. Transducer selection and mountings

Selection and mounting of transducers constitute two important problems to be correctly solved before any measurements can be carried out.

In the framework of dynamic characterization of materials, the problem, however, is more difficult compared to that concerning dynamic structural tests.

In this section, various choices of transducers, mountings and special precautions to avoid extraneous errors (introduced by environmental mediums surrounding the tested samples) will be examined.

6.3.1. *The choice of transducers*

We must distinguish the two main classes of tests:

(a) using two identical transducers to facilitate interpretation of experimental results;

(b) using transducers of different kinds to those commonly used in classical tests in structural dynamics.

Point (a) concerns complete experimental set-ups which are designed and realized by experimenters themselves. As presented in Chapter 3, our preference is for two identical transducers.

Point (b) concerns experimenters who utilize available industrial apparatus. Some of these apparatus are presented in Chapter 2. However, precautions have to be taken with transducer mountings as well as transducer calibrations.

6.3.1.1. *Main performances of the three classes of transducers*

Table 6.3 presents some characteristics of the three classes of transducers i.e. sensitivity and frequency advantages and disadvantages depending on the principles of measurement, with or without absolute reference for transducers.

	Acceleration	Velocity	Displacement
Sensitivity and frequency	High output signal at high frequency	Good sensitivity at medium frequency	High output signal at low and very low frequency
Weight	High frequency means light weight and low sensitivity	Velocity transducer mounted on sample might have non-negligible weight	High sensitivity at low and very low frequency
Advantages	Relative measurement does not necessitate absolute reference	Possible contactless measurement (proximity probes)	Possible contactless measurements (proximity probe); Absolute reference
Disadvantages	Influence of cable length on measurement (accelerometer mounted on sample)	Influence of permanent magnetic force on sample damping measurement	Calibration problem

Table 6.3. *Some indications on the performance of the three classes of transducers*

Special attention is focused on accelerometers which can have a high output signal level. At higher frequencies ($f > 10{,}000$ Hz), experimenters can adopt a low weight accelerometer and, additionally, weak sensitivity.

6.3.1.2. *Frequency considerations*

The frequency spectrum of a transducer, provided by the manufacturer, is an important factor in the choice of transducer. The flatness of the frequency spectrum is an important criterion.

6.3.1.3. *Resonance frequency*

This parameter, inherent to the transducer, should not be confused with the maximum working frequency of the sample. At the proximity of resonance frequency of a transducer, the amplitude and phase of an accelerometer-type transducer response vary a great deal and can give rise to large errors in measurements and interpretations.

6.3.1.4. *Carrier frequency*

The carrier frequency of an oscillator feeding a Wheatstone bridge depends on the frequency range in which this bridge works with the transducer. Often, in commercially available instruments, transducers belong to the apparatus itself. In practice, the bridge is followed by a low-pass filter[3] whose working frequency f_w is much lower than the frequency f_{osc} of the oscillator. In practice, the highest limit of working frequency f_w is:

$$f_w \leq \frac{1}{5} \text{ or } \frac{1}{10} f_{osc} \qquad [6.1]$$

Readers must refer to the whole response curve of the filter. The slope of the descending part of the filter versus frequency depends on the type of filter itself.

Measurements at frequencies corresponding to this slope give rise to large errors in phase and gain, which explain inequality [6.1].

6.3.2. *Accelerometer-type transducers and their mountings*

Mounting a transducer has its own importance and might influence the measurement result of the whole chain of electronic instruments.

3 For a low-pass filter, the so-called cut-off frequency is defined by an attenuation of 3 dB or 6 dB with respect to the level of signal (in principle "flat") at lower frequency, in the bandwidth.

As presented in Figure 6.1 concerning dynamic behavior of material samples, the transducer cross-section has the same size of that of the sample and indeed is often larger than it. The method of adopted mounting might affect the frequency range of the transducer by reducing the resonance frequency of the transducer itself. Table 6.3 presents factors to be taken into account. In general, in the four presented mounting methods, the nature of the mounting has its own influence on the frequency response of the transducer and also on the sensitivity of the transducer itself.

	Mounting performance	Remarks
Layer of wax, Self adhesive Tape or disc A	A viscoelastic layer can have a strong influence on the dynamic behavior of a sample: – additional damping in sample response – reduced frequency response of accelerometer – reduced transducer sensitivity – lack of stability – possible ungluing and detachment	Avoid this type of mounting
Stud mounting B	Used only for a large sized sample: – threaded stud and transducer constitute additional and partly distributed weight with respect to the sample – rigid mounting and no transducer detachment	To be adopted for short sample and/or small sample cross-section
Cement mounting C	Easy to operate: – cement layer must not be too thick (so as to reduce influence on sample response) – cement layer not too thin to permit good and uniform adhesion on the contact surface – can damage the base and the case of the transducer	To be adopted for short sample and/or small sample cross-section
Magnet (thick plate) D	For strong magnetic adhesion, a thick plate is used. – influence of magnet weight – influence on sample frequency response	To be used only for sample whose dimensions and weight are larger than those of the magnet

Table 6.4. *Comparative table for various transducer mountings*

Table 6.4 shows the influence of the mounting method on the frequency range of a transducer. Case (C) in Figure 6.4 presents excellent cemented mounting on condition that hard cement is used. There are two types of cements:

– cyanoacrylate is easy to use. Adhesion is rapidly obtained and does not require much time for mounting. However its hardness is not as high as that of other cements;

– epoxy resin requires mixing of two paste components and more time for resin hardening. Bonding with this resin is particularly interesting for surfaces which are not plane. Its disadvantage resides in the difficulty of debonding and it may damage the transducer.

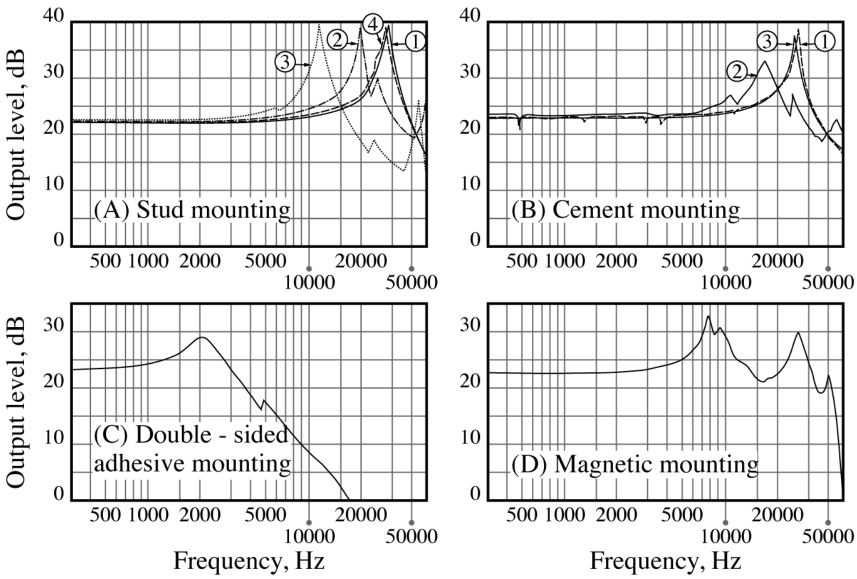


Figure 6.3. *Frequency response of accelerometer-type transducer mounted by various methods: (A) stud mounting: spanner tight (1), finger tight (2), with mica washer (for electrical insulation) (3), and a thinner mica washer (4); (B) cement mounted: cemented directly (1), cemented with soft adhesive (2), and with cementing stud (3); (C) double-sided adhesive mounting; (D) magnetic mounting*

6.3.3. *Fixed reference-type transducer*

This kind of transducer is interesting for dynamic tests on samples whose size is usually reduced. It does not require direct mounting at the sample ends and belongs to the class of contactless transducers (excepted velocity transducers with a permanent magnet which might modify the frequency response of a sample), particularly the measurement of low sample damping due to the presence of permanent attraction magnetic force.

To activate this transducer, a light metallic blade (for example, a steel razor blade) is cemented to the end of the sample; see Figure 6.4. Symmetry considerations must be taken into account when mounting the metallic blade to avoid undesirable vibrations.

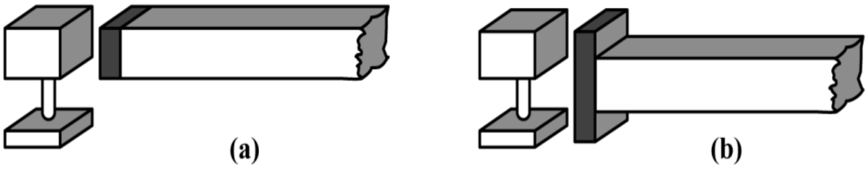


Figure 6.4. *Metallic blade glued on the sample tip to activate a contactless transducer. (a) Mounting a metallic blade at the tip of one end of the sample, with horizontal motion of the sample; (b) a large blade in front of a thick rod symmetrically fixed to the sample, with vertical motion of the sample tip*

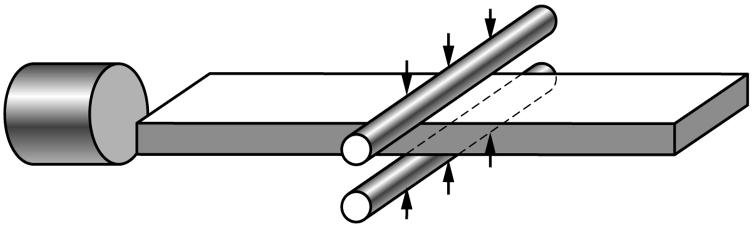


Figure 6.5. *Fixing a sample between two rigid cylinders which play the role of clamping in bending or torsion tests. Compressive clamping forces are applied on both cylinders*

Figure 6.5 shows a sample mounting. The sample has a static position required for the calibration of the transducer.

The interest of this mounting for a sample holder lies in the symmetry of the fixation at the middle of the sample, which does not require high compressive forces applied on the cylinders. Experience has shown that sample length correction is not necessary.

The plate mounted at the end of a sample must be electrically grounded by a light wire if the sample is non metallic. The wire weight must not influence the sample vibration.

6.4. Transducer calibration

Transducer calibration[4], with fixed or running reference, is an important problem which can be solved by two main groups of methods:

– separate calibration of each transducer;

– calibration of the complete measurement system.

6.4.1. *Separate calibration*

6.4.1.1. *Calibration of accelerometers by a special system*

In a case where an input signal and output signal are not of the same nature, the two corresponding transducers are of different types, for example a force transducer at the input and accelerometer at the output. A portable system is commercially available which has a reference accelerometer, signal generator and feedback control loop inside the case. A coil moving in the axis of a permanent magnet constitutes an electrodynamic exciter. Mechanical vibration is produced via a suspension spring. This furnishes an acceleration of 1 G. The accelerometer to be calibrated is mounted on the top of the calibrator, which is designed at one fixed frequency, as indicated by the manufacturer.

6.4.1.2. *Calibration by comparison method*

This method requires a standard reference which is mounted back to back with the transducer to be calibrated. Figure 6.6 shows a simple mechanical mounting which is submitted to different excitations at various frequencies.

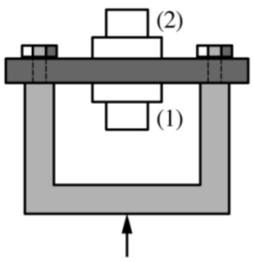


Figure 6.6. *Mechanical mounting for comparison calibration of a transducer, where (1) and (2) represent the reference transducer and test transducer*

4 See references in the bibliography of this chapter.

The sensitivity of the test transducer is:

$$S_t = \frac{e_t}{e_r} S_r \qquad [6.2]$$

where S_t and S_r designate sensitivities of the test transducer and the reference transducer, respectively, and e_t and e_r are the output voltages of the test transducer and the reference transducer. From the dynamic response of the reference transducer, the dynamic response of the test transducer is deduced on the condition that measurements are made far from the resonance frequencies of both transducers.

If an analyzer using a fast Fourier transform permits the transfer function of the calibration system to be obtained, relative phase information can be obtained (Figure 6.6) as well as its variation near to resonance frequency. Due to the special method of mounting used (mounting back to back), the relative phase in the usable frequency range is not zero but 180°. The transducers are mounted in opposite directions.

6.4.1.3. *Force transducer calibration*

Direct force calibration can be effected if a classical one degree of freedom equation of motion is adopted:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + k\,x = f(t) \qquad [6.3]$$

If the second and third terms in the first part of [6.3] are neglected:

$$m\frac{d^2x}{d\,t^2} = f\,(t) \qquad [6.4]$$

If a reference accelerometer is mounted back to back with the test force transducer, a double recording of acceleration and force f (t) permits the evaluation of the sensitivity of the force transducer; m is the weight of the whole mechanical system including the weights of the accelerometer and the force transducers. The calibrator weight must be taken into account.

Figure 6.7(a) shows a possible set-up to calibrate the force transducer. The pendulum is realized with eight strings (see Figure 6.7(b)), mounted in such a way that its oscillation vertical plane is imposed. A metallic spherical cap is mounted on the opposite side of the hammer. Impact is produced between the hammer cap and a thick rigid metallic plate fixed against a heavy concrete block. The hypothesis admitted in the equation can be verified by examination of the transient records of acceleration and force on an oscilloscope.

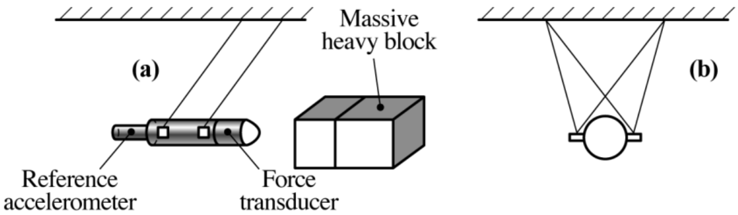


Figure 6.7. *(a) Calibration of force transducer by impact test; a heavy block whose mass is much larger than the pendulum system is necessary; (b) string suspension system prevents transversal pendulum oscillation and imposes a vertical oscillation plane to the pendulum*

6.4.2. *Calibration of the whole transducer set-up*

Two cases can be examined.

6.4.2.1. *Two identical accelerometers*

This constitutes the most favorable situation for calibration and effective measurement as well. Figure 6.8 presents an example of a set-up. A short metallic sample (a thick plate) joins the two transducers together for calibration; the transducers are placed on opposite sides of the plate[5], and the two accelerometer signals are in opposite phase. The disk is submitted to vibration either by inductance with a permanent magnet or piezoelectric exciter. A fast Fourier transform analyzer is used to obtain a transfer function. The gain (absolute value) is equal to 1 if the accelerometer sensitivities are identical. If not the ratio:

$$R = \frac{s_1}{s_2} \neq 1$$

is taken into account, where s_1 and s_2 are accelerometer sensitivities of the two accelerometers. The phase of the transfer function must be $\pm\,\pi$ over the whole frequency range.

If the resonance frequencies of the accelerometers are identical, utilization of a pair of transducers is possible beyond this frequency limit.

6.4.2.2. *Transducers of the same type and with fixed references*

This is the case of contactless transducers (capacitive type or eddy current type transducers). Mounting them as demonstrated in Figure 6.8(b) is a possible solution.

5 Accelerometers must be always glued or fixed at their socket and not on their case.

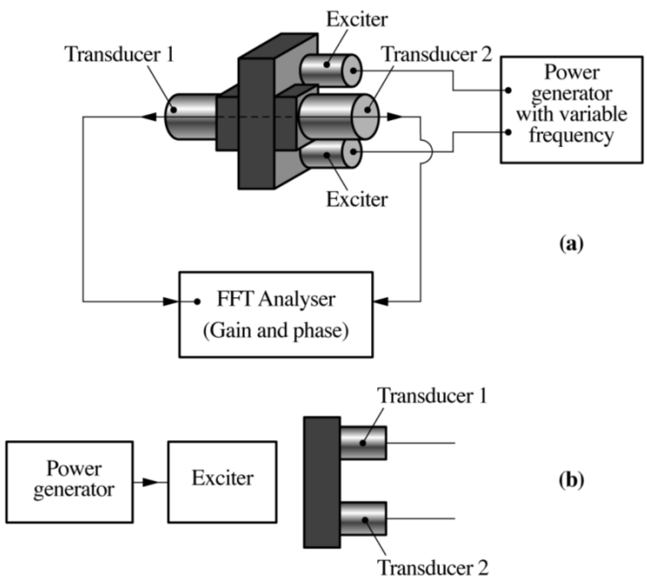


Figure 6.8. *(a) Calibration of both accelerometers in gain and phase using a pair of exciters. A fast Fourier transform analyzer is used to obtain calibration curves (gain and phase). With the accelerometer mounting presented, the phase difference between the two transducers must be ±π. (b) Calibration of a pair of transducers with fixed reference, and no phase difference between the two transducers*

If the transducers are of a displacement type (where the displacement concerns the gap x between the transducers and their targets), the variation law is of a hyperbolic type, with v being the transducer signal voltage:

$$v \approx \frac{1}{x}$$

Linearization is possible if displacement is small. At low frequency, displacement can be large and recourse to a special electronic linearizer is necessary.

If displacement is proportional to the surface, this is the case of transducer mounting for angular detection and working in rotation. It is of interest in torsion tests.

6.4.2.3. *Transducers of different types*

Transducers of different types are used, for example, in the case of inertia transfer functions (with force at the input and accelerometer at the output). Separate calibrations of the transducers are necessary (both for sensitivity and frequency response functions). Technical information supplied by manufacturers should be used. A transducer's technical characteristics can change after an intense period of utilization, and ageing can modify transducer performances, in either case. A complete recalibration is necessary.

6.5. Digital signal processing systems: an overview[6]

Experimenters, working in the domain of experimental electrodynamics or dynamics of structure, use all the resources of signal processing. In this section, the latest progress in electronic instrumentation is taken into account. Attention is particularly focused on the practical applications of signal processing. Indications concerning theoretical aspects are briefly presented.

6.5.1. *Presentation of a signal processing system giving transfer function*

Let us examine the system presented in Figure 6.9, which shows two signals: an input and an output. There are two chains of apparatus to process the input and output signals in parallel. Electrical signals are naturally obtained from transducers followed by signal conditioners.

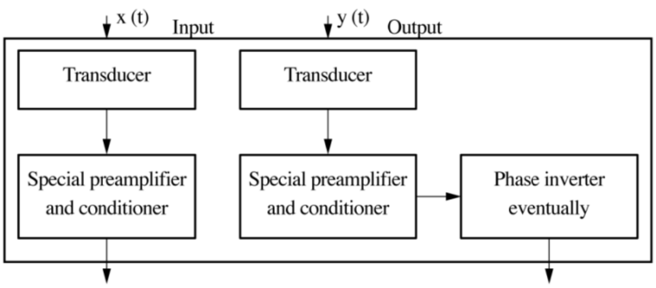


Figure 6.9. *Electronic part devoted to transducers and special preamplifiers and conditioners. A phase inverter is suggested for one signal in the case where one transducer is mounted in the opposite direction with respect to the other*

6.5.1.1. *First part of the analyzer working with analog signals*

It is important to note that there is always an electronic subsystem devoted to analog signal processing (Figure 6.10). A low-pass analog filter is necessary for both signals.

6 Readers unfamiliar with the theoretical and practical problems of signal processing can consult references in the bibliography at the end of this chapter.

There is, in digital signal processing, a troublesome phenomenon due to sampling. Appendix 6A provides a short explanation of this phenomenon, called the aliasing effect. The true frequency spectrum can be overlapped by an undesired spectrum. One remedy is to reduce the maximum frequency according to the Shannon theorem:

$$\text{For } f \geq f_c = \frac{1}{2T} \qquad [6.5]$$

T being the sampling period used to sample[7] the analog signal.

All the information in the frequency domain must disappear beyond the low-pass filter frequency f_c. This filter is efficient only for a continuous (analog) signal.

In practice, the maximum working frequency is taken as:

$$f_{max} = 0.7 \text{ or } 0.8\ f_{critical} \qquad [6.6]$$

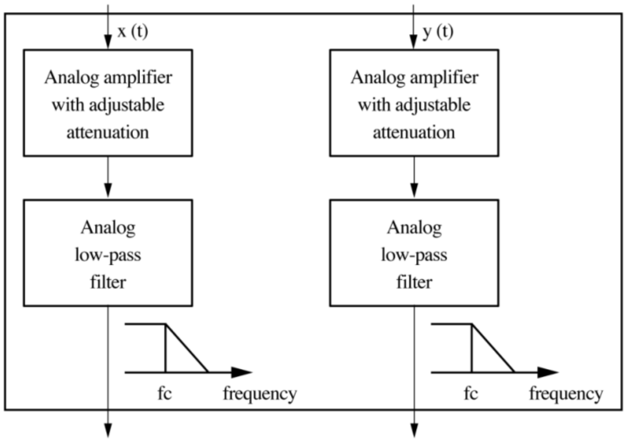


Figure 6.10. *At the entrance of a discrete Fourier transform (DFT) analyzer, there is an analog system made of amplifiers and lowpass filters for both signals*

6.5.1.2. *Digital signal processing part of the analyzer*

Figure 6.11 shows two chains of digital processing stages, including:

– a weighted function called a time window;

7 Sample in the framework of signal processing means cutting up the whole signal into a number of pieces that are named samples.

– a discrete Fourier transform (DFT);

– an averaging circuit;

– a special computer to evaluate various functions in time or frequency domain;

– computed functions, represented in different ways by graphic displays.

The following short comments concern various analyzer components.

6.5.1.3. *Role of weighted function (or window)*

The time window concerning signal processing is a mathematical Fourier transform of the continuous signal whose theoretical duration corresponds to the theoretical time interval $[-\infty, +\infty]$.

The time signal recording (which is finite) is subdivided into samples with finite length. Each sample is digitally processed, so that:

$x(t) \neq 0$ for $-nT < t < +nT$

$x(t) = 0$ outside the time interval $(-nT, nT)$

The discontinuities are then artificially created and they have an influence on the frequency spectrum. In Appendix 6B this problem is treated as a product of a continuous signal with a weighting function w(t), such as:

$$x(t).w(t) \qquad [6.7]$$

with $w(t) \neq 0$ for $-nT < t < +nT$, and $w(t) = 0$ outside the aforementioned time interval.

If the Fourier transform is applied to [6.7], the discontinuities create a *leakage effect* which is portrayed by a convolution, between the two Fourier transforms. For a discreticized[8] signal we write:

$$x(n).w(n) \rightarrow X(k) * W(k) \qquad [6.8]$$

where n and k are integers.[9]

8 It consists of transforming an analog continuous signal into a discontinuous signal.

9 Which represent discontinuous time n and discontinuous frequency k.

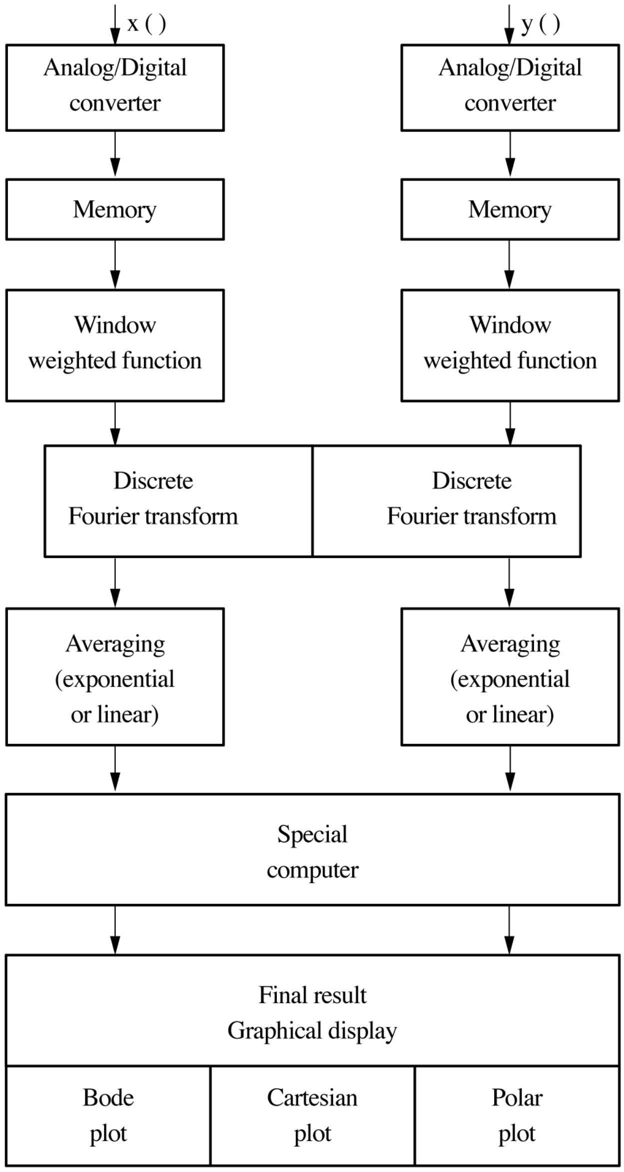


Figure 6.11. *Digital signal processing of a discrete Fourier transform analyzer*

Effecting discrete Fourier transform (DFT) we obtain, at the left of the arrow, the product of two time functions of discrete time variable n and, at the right of the arrow, the convolution of the two DFT of the two time functions, with the convolution operation represented by star symbol $*$. x(n) is the discretized time signal and w(n) the discretized time window. Capital letters at the right of the arrow X and W represent the DFT of the time functions indicated above

$$x(n).\ w(n) \rightarrow \sum_{-\infty}^{+\infty} X(i) * W(k-i) \qquad [6.9]$$

The profile of w(t) in the time domain looks like a bell-shaped curve. The two discontinuities presented in [6.7] are suppressed. The quality of the weighted function depends on its Fourier transform. It is difficult, in the frequency domain, to avoid the production of side lobes. The choice of the weighted function resorts to optimization. The main (central) lobe in the frequency domain must have the narrowest possible width, whereas the side lobes must have the lowest height (amplitudes). By using energy criteria, the two aforementioned conditions can be expressed as:

– secondary lobes have the lowest energy;

– the main lobe must have the maximum energy.

However, by adopting the narrowest frequency interval for the main lobe, the secondary lobes increase in height.

The two most familiar windows adopted are the Hamming and the Kaiser-Bessel windows presented in Appendix 6C.

6.5.1.4. *Discrete Fourier transform*

Various DFTs are commercially available and use different quick algorithms which reduce the number of arithmetic operations to a minimum. With today's generation of computers working at high speed, often at a frequency of several Gigahertz, the problem has become less acute than it was two decades ago, as the computation time has reduced at least by a factor of 100 with respect to the time required 20 years ago.

6.5.1.5. *Averaging*

Averaging is a necessary operation to obtain improved frequency responses. There are two types of averaging: exponential and linear ones. In dynamic material testing, samples are submitted to stationary waves and the sample behavior does not change with time, except in the case of temperature change. Linear averaging is preferred to exponential averaging.

Averaging can operate on a collection of frequency responses. With step sine sweeping or with white noise as generators for exciters, visually one can observe the improvement of frequency responses with time on an oscilloscope, i.e. the number of signal samples retained for averaging. Dynamic tests as well as the computing of frequency responses must be kept working until curve smoothing is obtained.

6.5.1.6. *Analyzer program*

Transfer functions with input and output signals of the same type are preferred, for the reasons presented in section 6.4.2. However, other types of transfer function can be used. In this section, the practical aspects of available programs are presented. Some experiments on material testing serve as application examples.

6.5.1.6.1. Transfer function

The main program is the calculation of a complex transfer function via the calculation of the autospectrum G_{xx} of the input signal and the cross spectrum G_{yx}, where x and y are the input and output signals, respectively:

$$\mathrm{TF(f)} = \frac{G_{yx}(f)}{G_{xx}(f)} = \frac{S_y(f)S_x^*(f)}{S_x(f)S_x^*(f)} \qquad [6.10]$$

where S_y and S_x designate the Fourier transform of the output signal and input signal, respectively. The star designates the conjugate quantity, and f the frequency. There are various representations of TF(f), as indicated in Figure 6.11. Three representations of results are used: Cartesian plotting is used for the computation of complex moduli; polar plotting is interesting for the computation of mathematical expression of complex modulus, and Bode plotting; see Figure 6.11

6.5.1.6.2. Autospectrum of input signal

In some circumstances, in the case of sinusoidal excitation, we may have to deal with the two following situations:

– the eventual non-linear response of a transducer;

– an exciter working at its resonance frequency.

A linear checking of transducer responses as well as the exciter is an important operation, particularly when the sample has low stiffness, such as is the case with rubber or soft viscoelastic materials. Using the autospectrum to examine the exciter and the two transducers allows any eventual non-linearities of this apparatus to be disclosed by examining the frequency spectrum and manually changing the exciter frequency, and observing the level of input signal.

In sinusoidal excitation, if harmonics are detected (outside of the excitation frequency) the excitation level must be reduced.

6.5.1.6.3. Coherence function

The coherence function is defined as:

$$\gamma_{yx}^{2}(f) = \frac{[G_{yx}(f)]^{2}}{G_{xx}(f)G_{yy}(f)} \quad [6.11]$$

where f is the frequency.

$G_{uv}(f)$ is defined as:

$$G_{yx}(f) = S_y . S_x^*(f)$$

$$G_{xx}(f) = S_x(f) . S_x^*(f)$$

$$G_{yy}(f) = S_y(f) . S_y^*(f) \quad [6.12]$$

By Schwartz inequality, it can be shown that:

$$[G_{yx}(f)]^2 \leq G_{xx}(f) . G_{yy}(f) \quad [6.13]$$

We then obtain:

$$0 \leq \gamma_{yx}^2 \leq 1 \quad [6.14]$$

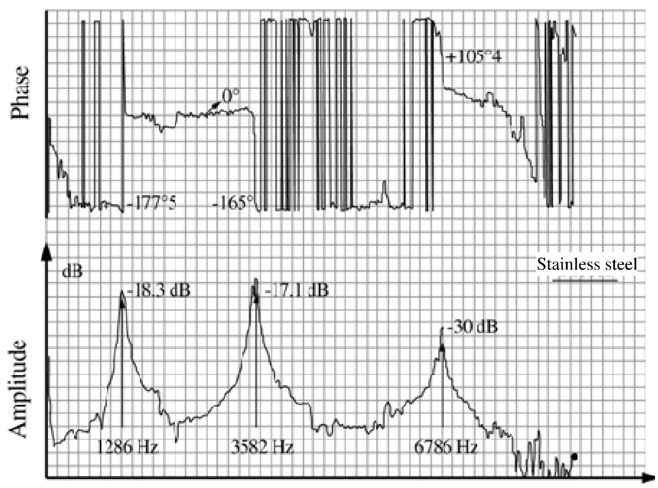


Figure 6.12. *Transfer function of a steel bar under bending impact created by hammer*

The first inequality $0 \leq \gamma_{yx}^2$ is obtained by observing that coherence is zero if there is no statistic parenthood between the input and the output signals.

The coherence function γ_{yx}^2 is considered to be a quality factor or a degree of confidence for the transfer function. If the coherence function is low and approaches zero, it means that the number n_d of signal samples used for averaging is not sufficient.

Figure 6.12 shows the transfer function of a steel bar under impact bending excitation produced by a hammer. Figure 6.13 is the coherence function. Some remarks are needed. At resonance frequencies (300, 3,580 and 6,786) the coherence function has a low value (0.04, 0.247, 0.05) This is due to strong resonances with low damping capacity of the iron bar. These low coherence function values do not mean that the corresponding portion of rod response must be discarded. It can be explained by the fact that at and around resonance peaks, more the resonance is sharp less there are the number of points (signals being discretized) defining signals. Coherence function is consequently low, as shown in Figure 6.13.

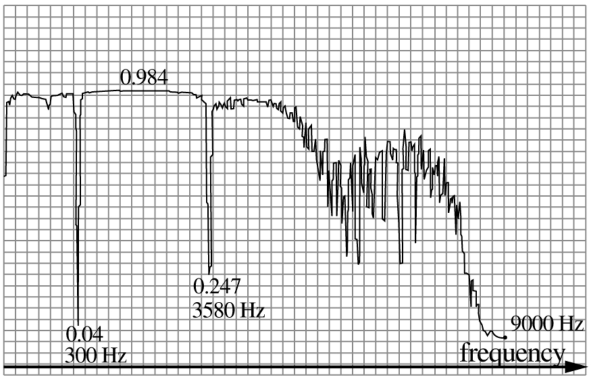


Figure 6.13. *Coherence function (f) related to transfer function in Figure 6.12*

At higher frequency (f > 5,000 Hz), the coherence function shows fast variations. This means that, at these frequencies, one other possibility is that the input signal has too low an amplitude, and the remedy is to improve the impulse excitation of the hammer or to adopt another kind of excitation, i.e. a piezoelectric exciter or sweep sine excitation.

Figure 6.14 shows the real and imaginary parts of the transfer function of a viscoelastic solid propellant bar. The coherence function is presented in Figure 6.15.

It has a good value ($\gamma_{yx}^2 \cong 1$) up to 10,000 Hz. Beyond this limit, the coherence function varies between 0.2 to 0.3. These values are low. A further improvement is envisaged by using sinusoidal excitation in this region.

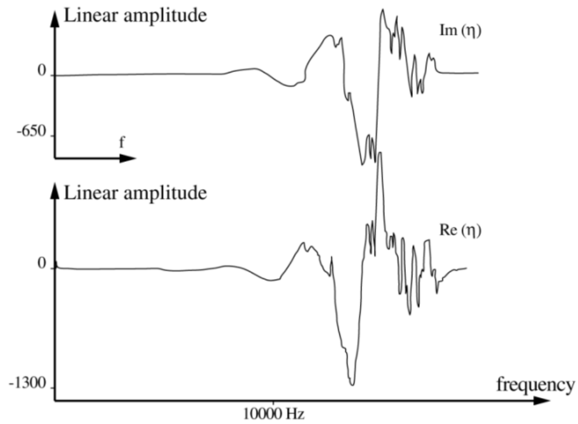


Figure 6.14. *Transfer function of a viscoelastic bar under random vibration produced by a piezoelectric exciter; material under test being a highly dampened solid propellant*

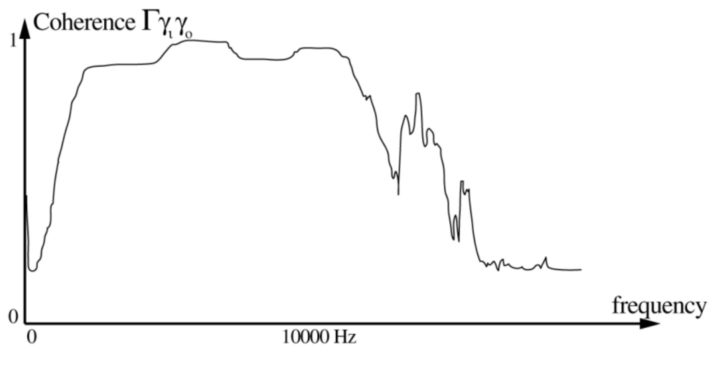


Figure 6.15. *Coherence function related to tests presented in Figure 6.14. Using random excitation, a filter must be used to concentrate excitation energy in the region f > 12,000 Hz*

6.6. Other signal processing programs

The coherence function gives us a method to statistically relate the output signal to the input signal. This constitutes a kind of quality factor which concerns a variety of symptoms (non-linearity of transducer responses, sharp resonance sample response, non-linearity of the amplifier response, etc.) which indicate that it is not possible to use it correctly. Another function exists which is closely related to the non-linear behavior of the output signal: the frequency Hilbert transform, which is presented in detail in Chapter 7, and applications of the transfer function.

6.7. Reasoned choice of excitation signals

This section is presented here, instead of in Chapter 4 (which is specifically devoted to exciters), because we cannot separate excitation signals from signal processing. Experimenters will find a profusion of commercially available generators and analyzers devoted essentially to audio acoustics.

The choice of an appropriate exciter signal for a specific application to dynamic material testing is not easy at first sight.

6.7.1. *Overview of excitation signals*

There are three types of signals: random signals, sinusoidal signals and impact signals.

6.7.1.1. *Random signals*

White noise is produced over a large frequency range. The upper limit of this signal is obtained by a low-pass filter. Often this limitation resides not in the generator but in the exciter itself. This kind of signal is popular in dynamic test on material sample. It permits a quick overview of frequency responses to be obtained.

However, one must be careful about the energy level of the power amplifier. If the input signal has a correct level for all frequencies, and taking into account the large desired frequency range, the power required might exceed the normal levels corresponding to an admissible monofrequency utilization. Consequently, energy limitation is necessary.

White noise over a narrow frequency bandwidth is a form of excitation used when one desires to explore a narrow frequency zone.

Pink noise (whose Fourier spectrum decreases with increasing frequency) is a popular signal in audio acoustics but is not used in material testing.

6.7.1.2. *Sinusoidal signals*

Various types of sinusoidal signals differ from each other by the kind of sweeping used or by the kind of frequency variation:

– *sinusoidal sweeping signals* take a long measurement time;

– *step sine signals* have a good resolution in frequency; however, the time devoted to measurement is long, the analysis frequency by frequency is very time consuming;

– quick results can be obtained from *high sweeping velocity signals* but unfortunately, damping measurement might be greatly affected.

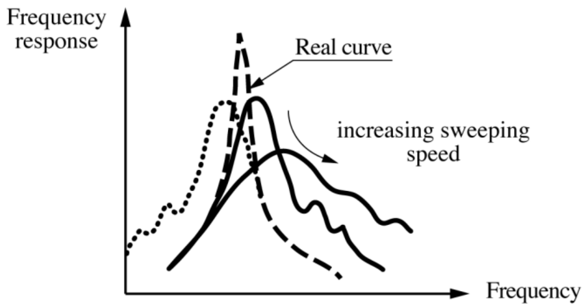


Figure 6.16. *A too fast sinusoidal sweeping gives rise to a false frequency response with resonance curve higher damping than the real curve*

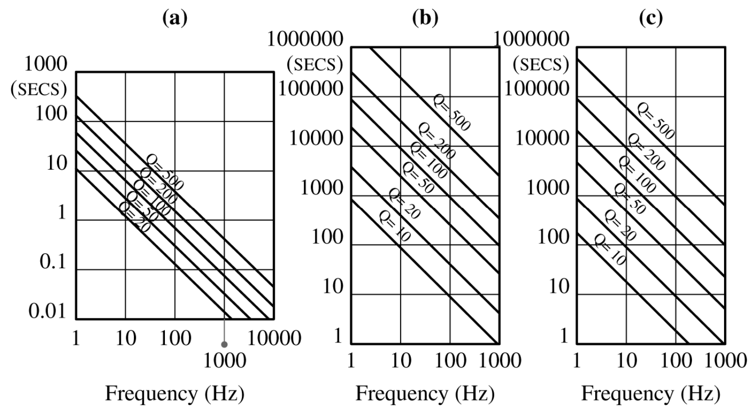


Figure 6.17. *(a) Time to permit output signal reaching 99% of its maximum amplitude, Q is the inverse of damping coefficient; (b) time for linear sweeping versus frequency and overvoltage factor* $Q = \frac{1}{tan\delta}$, *δ damping angle; (c) time for logarithmic sweeping versus frequency and factor Q*

Caution has to be taken in the choice of sweeping velocity. When dealing with low and very low damping of a material sample, it must be borne in mind that when the material damping is lower, then velocity must be much lower. Figure 6.16 presents sample responses versus frequency. If the velocity is great, the resonance part of the real curve is not obtained but rather a false one with much higher damping. In Europe and in the USA there are standards devoted to the choice of measurement time with linear or logarithmic sweeping time and a range of damping coefficients (see Figure 6.17).

6.7.1.3. *Transient excitation*

A hammer is the simplest instrument to produce impact on a sample. It should be tooled to produce the narrowest impulse force. Figure 6.18 represents a special hammer constituted of a spherical steel cap. A force transducer is screwed in place just behind the hammer head, between the cap and the center hammer cylinder. A counterweight is screwed on the opposite side.

6.7.1.3.1. Duration of contact

The following information might be useful for the design of the hammer. The hammer equation of motion[10] during impact is:

$$m\frac{d^2u}{dt^2} + Ku^{3/2} = 0 \qquad [6.15]$$

where K designates non-linear rigidity during the contact between hammer and sample, and m the mass of the hammer:

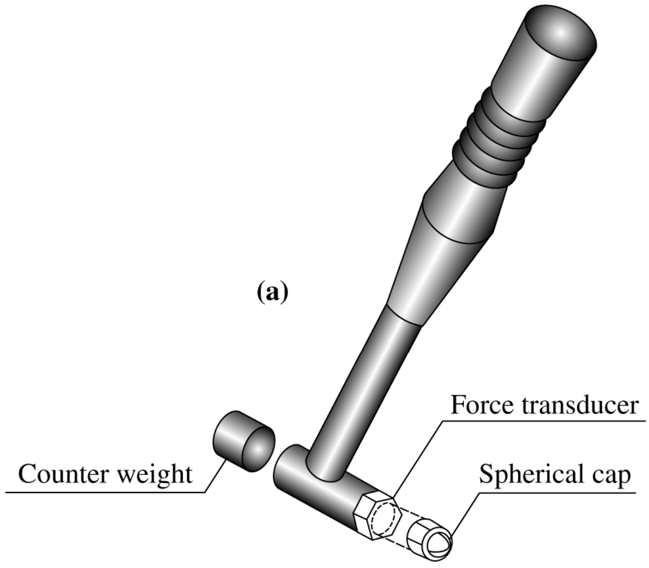
$$K = \frac{4R^{1/2}}{3\pi}\left[\frac{1-\gamma_s^2}{E_s} + \frac{1-\gamma_h^2}{E_h}\right] \qquad [6.16]$$

where (γ_s, E_s) and (γ_h, E_h) are, respectively, Poisson's number and the Young modulus of the sample (of index s) and the hammer (of index h), and *R* is the radius of the spherical cap.

Initial conditions are:

t = 0 du/dt = V_i (impact velocity); and displacement u = 0

10 Equation [6.15] concerns elastic impact. Often, a massive block undergoes elastoplastic deformation. If we are interested only in the duration of impact, the elastic impact might give an order of value; for more information see [VIN 69].



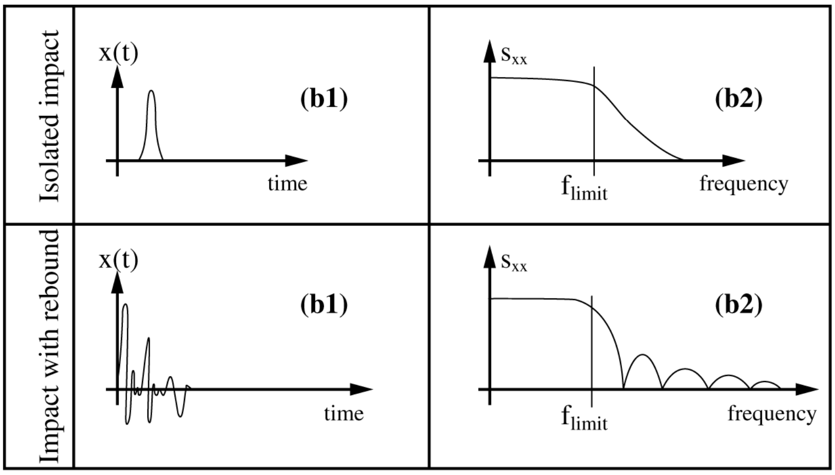


Figure 6.18. *(a) Hammer exciter with force transducer; (b) impacts created by hammer – time recording and corresponding autospectra: (b1) isolated impact with its autospectrum; (b2) impact with rebound and its force autospectrum*

Integration of [18.15] is possible, for an elastic contact, to obtain the duration contact time t, and where Γ is the Gamma function [VIN 69].

$$t = \frac{4}{5}\sqrt{\pi}\ \frac{u_{max}}{V_i}\ [\ \Gamma(2/5) / \Gamma(9/10)] \qquad [6.17]$$

Maximum displacement u_{max} is given by:

$$u_{\max} = [\frac{5\, V_i^2}{4K}\, m]^{\,2/5} \qquad [6.18]$$

From [6.17] and [6.18] the time duration is obtained. It is proportional to the non entire power of following parameters:

$$t_{contact} \cong V_i^{1/5} m^{2/5} E_h^{-2/5} R^{-1/5} \qquad [6.19]$$

Equation [6.19] shows that, to obtain the shortest possible contact duration, we must choose the hardest metal to obtain a high Young's modulus E_h, as well as the smallest radius R of a spherical cap, and a not too heavy hammer of mass m.

If one wants to obtain longer contact time, the cap should be made from a viscoelastic material. The upper limit for frequency is approximately:

$$f_{\lim} \cong \frac{1}{2t_{contact}} \qquad [6.20]$$

6.7.1.3.2. Advantages and disadvantages of the hammer

The hammer has many advantages, including its low cost, simplicity and robustness. However, experimenters must bear in mind its disadvantages:

– lack of accuracy in the direction of the applied impact force. From one impact to another one, it is difficult to maintain the same direction for the hammer;

– impact durations can change from one test to another. The frequency bandwidth of input signal consequently changes;

– the force level can be changed; the hand actuated hammer can be replaced by an electronic slave hammer which creates impact with the same intensity and in the same direction [TAW 05].

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6.9. Appendix 6A. The Shannon theorem and aliasing phenomenon

Let a time signal be x(t) which has a Fourier transform X(f). The time signal is discretized by an analog-digital converter:

$$x(t)\ xc(t) \qquad [6.A.1]$$

Figure 6A.1 shows a continuous time signal. Discretizing x(t) is equivalent to multiplying x(t) by a comb function δ_N (T):

n is an integer number, T the sampling time interval. The problem, by discrete Fourier transform, obtains:

$$\text{DFT}[x_c(nT)] = X_c\left(\frac{k}{T}\right) \qquad [6.A.2]$$

which is the nearest possible to the true Fourier transforms X(f).

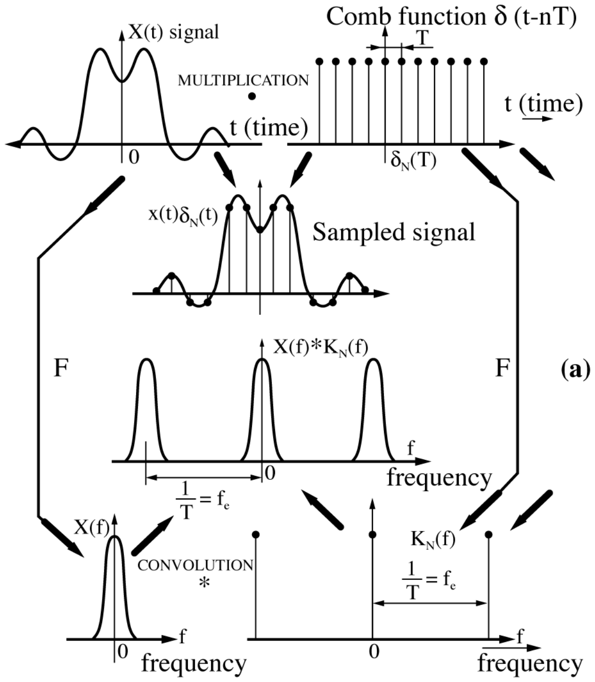

Figure 6A.1. *Influence of discretization on the Fourier transform of the time signal. Discretization with small T: no aliasing effect*

$$x_c\,(nT) = x(t).\ \delta_N\,(T) \qquad [6.A.3]$$

In the frequency domain, the product [6.A.3] becomes a convolution:

$$X(f)\ K_N\,(f) \qquad [6.A.4]$$

It can be shown that K_N (f), the DFT of δ_N (T), is also a comb frequency function with a sampling frequency interval:

$$f_c = \frac{1}{T} \qquad [6.A.5]$$

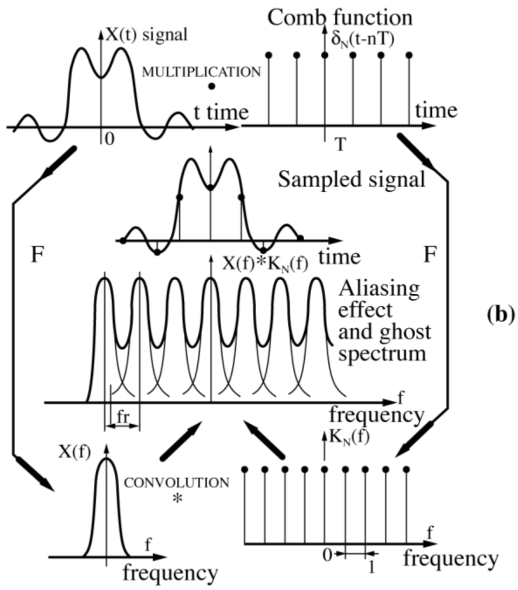

Figure 6A.2. *Influence of discretization on the Fourier transform of the signal. Discretization time T large, with an aliasing effect on the frequency spectrum*

The convolution [6.A.4] gives a periodic function in which the real spectrum is duplicated infinitely. Only the spectrum near the frequency origin is the true one and retained.

Figure 6A.2 shows the same time signal but, this time, the sampling period is larger than that adopted previously in Figure 6A.1. In the frequency domain, the convolution [6.A.4] is effected with a narrow sampling frequency in [6.A.5]. The aliasing phenomenon takes place by convolution and the true spectrum is mixed with "ghost" spectra on the right (for positive frequency) and on the left (for negative frequency as well). This spectrum folding is not acceptable.

The two principal remarks to be made are:

– there is an excess of information of the DFT from the discretized signal due to the use of comb function in discrete signal processing. This produces, by convolution, undesirable parts of the spectrum called "ghost spectra" which must be eliminated. There is an artificial periodicity introduced in the frequency domain;

– there is a possible distortion of the frequency spectrum which gives rise to the problem of spectrum folding with too large a sampling period, T.

The remedy is to arrange it so that no exceeding frequency information is available beyond:

$$f = f_c = \frac{1}{2T} \qquad [6.A.6]$$

where f_c is called the Nyquist frequency.

6.10. Appendix 6B. Time window (or weighting function)

In a Fourier analyzer, the time window plays an important role. The two discontinuities, at the beginning and at the end of a sample concerning the time signal x(t), must be replaced by an appropriate window.

The continuous (analog) time signal undergoes two transformations. The first is the discretization:

$$x(n) = \sum_{n=-\infty}^{+\infty} x(t)\delta(t - nT) \qquad [6.B.1]$$

where n is an integer; T is the time sample spacing. $\delta(nT)$ is the comb function.

The second discontinuity is due to truncating which corresponds to a rectangular window in [18.B.1]:

$$x_t(n) = x(n)\, w(n) \qquad [6.B.2]$$

with w(n) =1 $\quad 0 < n < N-1$

$$w(n) = 0 \text{ for } n \geq N \qquad [6.B.3]$$

A discrete Fourier transform is then applied to the discretized function:

$$x_c(n) = w(n) \sum_{-\infty}^{+\infty} x(t)\delta(t - nT) \qquad [6.B.4]$$

The second part of [6.B.4] is a simple product in the time domain. If W(k) is the direct Fourier transform of the series, then

$$X_t(k) = X(k) * W(k) \qquad [6.B.5]$$

The subscript t designates a truncated and discretized signal. The star designates a convolution.

Figure 6B.1 shows the convolution of X(k) with a Fourier Transform of the rectangular window, which is represented by a cardinal sine function. In the frequency domain this transform has a main lobe with a finite slope and secondary lobes of which the height is not negligible compared to that of the main lobe.

The remedy is to use, in lieu of a rectangular window, another window. The ideal window, after [6.B.5] is such that:

$$W(k) = \delta(k) \qquad [6.B.6]$$

This corresponds in the time domain to:

$$w(n) = 1 \qquad [6.B.7]$$

This signal must not be bounded; thus:

$$-\infty < n < +\infty$$

This means that the time record is indefinitely long, which is unfortunately not the case in practice.

There is consequently no ideal window. This problem resorts to an optimization one. Every window gives rise in the frequency domain (as for the rectangular window), to a main lobe and secondary lobes. The problem is to adopt the following criteria:

– keep the height of secondary lobes as low as possible;

– keep the amplitude of the main lobe as high as possible.

One of the difficulties in optimization is the *conflict between the two aforementioned criteria*. A compromise is then necessary.

Two popular windows are presented: the Kaiser-Bessel and the Hamming windows.

6B.1. *Kaiser-Bessel window*

The Bessel series $I_o(x)$ is troublesome to manipulate. Kaiser, however, proposed an approximate formula to use in a Fourier transform:

$$w(n) = \frac{\sin\left[\beta\sqrt{\left[\left(\frac{\omega}{\omega_\beta}\right)^2 - 1\right]}\right.}{\sqrt{\left[\left(\frac{\omega}{\omega_\beta}\right)^2 - 1\right]}} \qquad [6.B.8]$$

This expression is similar to the expression for a rectangular window except that two parameters are introduced:

– ω_β designates the width of the main lobe;

– β is a second parameter.

6B.2. *Hamming window*

There are many improved versions of this window. The Blackman version is presented here.

$$w(n) = \sum_{m=0}^{K}(-1)^{m} a_m \cos(2\pi\, mn / \mathbf{N}) \quad [6.B.9]$$

$$w(k) = \sum_{m=0}^{K}(-1)^{m} a_m [D(k+m)+D(k-m)] \quad [6.B.10]$$

with:

$$D(x) = e^{-j\pi x(1-\frac{1}{N})} \frac{\sin(\pi x)}{N\sin(\frac{\pi x}{N})} \quad [6.B.11]$$

With three terms (K=2) Blackman suggested in [6.B.9] the following values:

$$a_0 = 0.4, \quad a_1 = 0.5, \quad a_2 = 0.08 \quad [6.B.12]$$

Window	Height (first secondary lobe) h_r (dB)	Width (main lobe) l_p (Bin)*	Slope (main lobe) dB/octave
Rectangular	-13	0.89	-6
Hamming	-43	1.3	-12
Kaiser-Bessel	-68	1.69	-6

**width defined with a 3 dB attenuation*

Table 6B.1. *Characteristics of three time windows*

Table 6B.1 gives the characteristics of this window. The three retained parameters are the relative height of the first secondary lobe (with respect to the main lobe height), the relative width of the main lobe, and the slope of the main lobe.

Let us recall that the cardinal sine (which is the Fourier transform of the rectangular window) is:

$$\text{Card. sine} = \frac{\sin(\omega T_p)}{(\omega T_p\ \ )}$$

where T_p is the period of the sine function.