Chapter 4

Exciters and Excitation Signals

There are many possibilities for creating force or pressure vibration in samples. The choice of an exciter depends on many factors which will be reviewed in this chapter. Exciters differ from each other by the means used to produce vibration, the frequency range, the available power, the peak force value, etc.

4.1. Frequency ranges

Frequency ranges are large, depending on the mechanical applications concerned. If the frequency intervals are brought together, they cover a very wide range and go from infrasound ($0.001 < f < 1$ Hz) up to ultrasound, and almost reach hyperfrequency in acoustics ($10^5 < f < 10^7$ Hz). This is the interval which is usually covered in mechanical applications. Hypersound, corresponding to frequencies beyond 10^8 Hz and used to study crystal lattice vibration, is not the frequency range used for dynamic mechanical analysis. No exciter covers the whole range from infrasound to ultrasound. Each exciter, in the majority of cases, hardly covers three frequency decades.

4.2. Power

Choosing an exciter according to its power can be important, particularly for an electromechanical shaker. The power range can go from a few watts to many

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kilowatts. Samples adopted for testing usually have a length not exceeding 0.5 m[1] and a maximum transverse dimension not exceeding 5 cm. The useful power range can be limited to some tenths of a watt as a lower limit. Commercially available electromagnetic transducer with no moving coils, acting as an exciter, generally deliver a limited power not exceeding a few watts.

Piezoelectric or ferromagnetic transducers have useful power depending on their contact surface. Taking into account the transverse dimensions of samples, the power of this class of emitter transducer is of the order of 10 to 100 watts.

The power, defined as the electric power of the exciter often concerns the use of a narrow frequency bandwidth. If white noise is used, particularly with a large bandwidth, the power absorbed is multiplied by a factor of ten or more, and the electronic power amplifier might be damaged or be under risk of being destroyed, unless there is a safety system which limits the excess power.

4.3. Nature and performance of various exciters

4.3.1. *Electrohydraulic exciters*

The main interest in this kind of exciter comes about because of its very low frequency range (0.01 Hz < f < 200 Hz). Its power, however, is not limited and can reach 10 kilowatts or more. An electrohydraulic exciter's disadvantage is the space required for its use, and thus, for material testing, this exciter is rarely adopted. Electrohydraulic shakers provide a large range of force depending on the type and size of shaker, with: 10 < Force < 1,000 Newtons.

4.3.2. *Electro-mechanical exciters*

Electro-mechanical exciters have a frequency range of 5 Hz < f < 5,000 Hz. For frequencies lower than 5 Hz, the delivered signal is non-linear and often strongly non-sinusoidal.

The available power varies from 1 watt to 1 kilowatt or more, depending on the type of exciter chosen. The weakness (the non-linearity of the signal) of this kind of exciter around the lower frequency limit is compensated by an additional system of spring and shaft linear bearings, mounted in parallel with the exciter's moving coil, which might improve the signal at the price of shifting the lower resonance frequency to a higher value, and reducing the stroke of the moving coil.

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1 Samples of a length exceeding 1 m are exceptionally adopted where very large frequency ranges up to 10 megaHz are required.

When using large bandwidth white noise, the effective power exceeds the normal power indicated by the manufacturer (in a prescribed limited frequency range and exciter working in sinusoidal monofrequency regime) by a factor of 5 or 10, and irreversible damage to the moving coil and to the electronic output power amplifier might occur. In usual applications for material testing, an electric shaker of the order of 10 to 200 watts is a reasonable choice.

The maximum effective force that can be used depends on the type and size of the shaker. When choosing an electro-mechanical shaker, the experimenter must avoid too low a value of force. In the working frequency bandwidth, a factor of 10 to 50 with respect to maximum force value is to be adopted.

4.3.3. *Electro-magnetic exciter*

This low-cost exciter has no moving coil. It is easy to build and is made up of a cylinder shaped ferromagnetic core around which is wound an electric coil, made by the winding of filament spirals whose cross-section area governs the admissible electric current.

4.3.3.1. *Working frequencies*

The mechanical force is created, via an air gap, in a thin, light, ferromagnetic blade whose alternative displacement is produced if an alternating electric signal is applied to the coil. The frequency bandwidth is usually $10 < f < 10{,}000$ Hz.

4.3.3.2. *Force*

This exciter is contactless and the force is not high, being just a few Newtons. It depends on the air gap between the exciter and the sample, and also on the admissible electric current furnished by an electronic amplifier with low output electric impedance.

The disadvantage of this exciter lies in the fact that the mechanical response is non-linear with respect to the air gap.

When using this kind of exciter including permanent magnet, the reader must be aware of the presence of a permanent electrodynamic force which influences damping measurements on samples, particularly for low damping material measurements.

4.3.3.3. *Power*

Usually, the electric power is reduced to several watts.

4.3.4. *Piezoelectric exciter*

The interest in this type of exciter lies not in its high force and high power[2], but rather in the possibility of building the emitter-transducer in a very large range of frequencies. Section 4.7 is devoted to this interesting problem.

4.3.5. *Capacitive exciter*

Capacitive exciters constitute a special class of contactless exciter which can be used in extreme conditions, for example in a special chamber when sample temperatures need to be high and exceed 100°C or more and, eventually, when an exciter must work in a vacuum to measure low and very low material damping.

This type of transducer can be used when tan $\delta \approx 10^{-3}$ to 10^{-2}.

This type of exciter creates low pressure via an air gap which serves as a medium between two condenser plates. Figure 4.1 is a schematic diagram of the electric circuits involved. Forced displacement is of the order of several micrometers (μm).

This exciter was used in the French apparatus, called Cabarat's instrument,[3] for measurement of Young's modulus and low damping coefficient at high temperature.

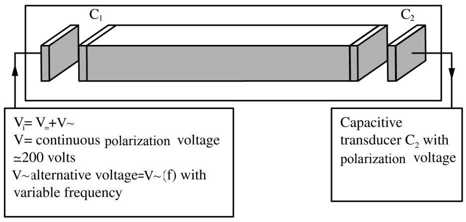

Figure 4.1. *Polarized capacitive exciter is used in extreme conditions in material testing: tests in vacuum atmosphere for very low damping measurement on materials (tan δ_E 10^{-3}) and the Young's modulus E, and tests at high temperatures exceeding 100 °C*

4.3.6. *Impact hammer*

This kind of exciter is popular for routine testing of mechanical structures. The hammer has a hard steel semi-spherical sphere, or conical head, which constitutes

2 However, industrial cleaning machines working with ultrasonic sound provide force and power which can be as high as hundreds of Newton and a number of kilowatts, respectively.

3 This instrument was used early in French laboratory on 1960 (École Centrale of Paris)

the important factor influencing the impact duration, and is chosen to be as short as possible (for example, a few milliseconds). The hammer momentum follows the approximate relationship:

$$F \text{ (force) } \times t \text{ (time duration) } = m \text{ (hammer weight) } \times V_s \text{ (sample velocity)}$$

V_s is the sample velocity at the beginning of the contact with the hammer. The shorter the contact duration between the hammer and the sample, the greater the force impulse.[4]

The impact duration depends on the geometry of the hammer head and also on the sample material itself.

Impact velocity is low, being about 50 meters per second. It is possible to obtain a higher value for the impact velocity by using a "slave hammer controlled by electronic circuit" [TAW 90].

4.4. Room required for exciter installation

The room dimensions required for an exciter are related to the power it delivers. An electrohydraulic exciter requires a large room and the sample must be adapted to the exciter's moving part(s).

Electromechanical shakers constitute the majority of exciters used in mechanical spectrometers devoted to material characterization. The power range is large, and so are the dimensions of the shaker.

A shaker of the order of 15 to 20 watts of power seems sufficient for dynamic tests on a sample of usual dimensions. However, the useful power applied to a sample is not the unique criterion for the choice of a shaker. The exciters whose power is indicated above are small in size and easy to connect to a sample. Experimenters should focus their attention on the moving coil with a cylinder core. Generally, small shakers are fragile to handle. They can be subjected to sudden transient motion of large displacement amplitude when sudden electric voltage is applied to them. This is the principal cause of deterioration of the moving coil. A shaker of 20 watts of power or more is to be preferred. The excess of power might prevent possible damage created by white noise excitation.

4 This is an approximation. The exact equation has, on one side, an integral of force with time as a variable. The second half is the product of the hammer mass with the difference between the velocities at the beginning and end of impact.

4.5. Details for electrodynamic shakers

As this is the most common type of exciter, we have gathered some technical information relating to it. The name derives from force generation by interaction between electric current in the moving coil and a strong magnetic field created by a permanent magnet, following the classical Laplace law of electrodynamics.

4.5.1. *Magnet circuits and the guiding system for the moving coil*

Figure 4.2 presents three possible magnet circuits. The magnetic force lines present closed loops with narrow gaps in which the moving coils are placed. Table and armature coil assemblies are supported by elastic means.

The alternating motion of the table is due to AC current in the moving coil. Figure 4.2(b) corresponds to the most used shaker. The moving coil is situated on top, while in Figure 4.2(b) the moving coil is at the bottom. Figure 4.2(c) presents systems of magnetic lines. This system improves the motion linearity.

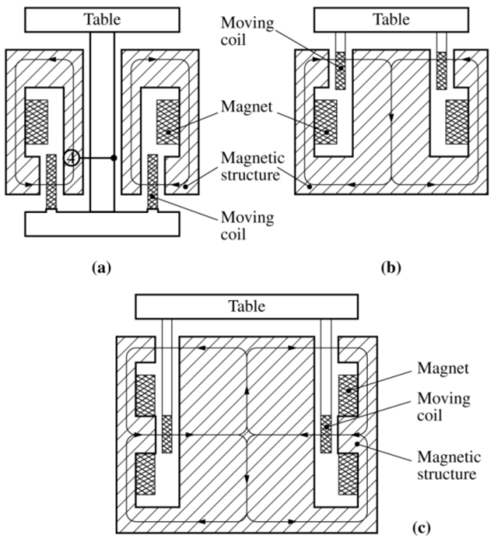

Figure 4.2. *Three main magnet circuits presented without elastic membrane springs*

These magnetic circuits represented by the loops and arrows are similar to a push–pull system in electronic circuits which use power transistors called differential amplifiers. A magnetic field in the range of 5 to 30 Gauss is normally used.

In Figures 4.2(a), 4.2(b) and 4.2(c), the table mass associated to the elastic membrane (considered as a spring) creates an elementary mechanical system and gives rise to a resonance at lower frequency.

The system, constituted by the sample and the holder, has its own mass which is to be added to the moving coil mass and consequently diminishes the lower resonance frequency.

Experimenters must be aware of the mechanical set-up when loading the table with excessive mass. Excessive mass might create a displacement of the moving coil to the bottom, beyond the allowed displacement limit, and thus damage the moving coil.

Figure 4.3 presents a section of a shaker. The guiding system is made up of a double rectilinear elastic leaf spring as shown in Figure 4.3(a). The leaf spring (a) is protected by a rubber membrane just above (b).

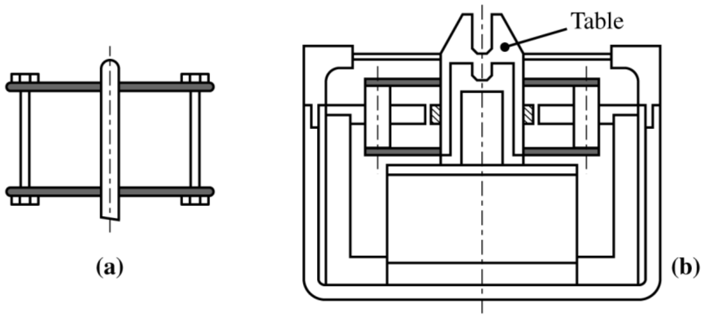

Figure 4.3. *Section view of an electrodynamic shaker: (a) leaf springs are used to guide the moving coil; (b) the shaker in detail*

4.5.2. *Magnetization of metallic sample*

The table itself is magnetized and any metallic samples or the transducer used for measurements fixed on the table might also be magnetized; a small magnetic field is required to degauss them. This degaussing effect may reduce the magnetic field to 2 to 5 Gauss. (Degaussing accessories are not shown in Figure 4.2(c)).

4.5.3. *Cooling system to reduce heat*

The presence of copper in the moving coil and ferromagnetic metal in the magnetic structure contribute to produce heat following Joule's effect and the eddy current effect. An air cooling circuit should be foreseen and, possibly, a cooling fan may eventually need to be connected.

4.5.4. *Stiffness of elastic suspension*

The stiffness (S_s) of the elastic suspension is of the order of 20 to 200 Newton/centimeters.

4.5.5. *Electrical performances*

Details of the characteristic electrical impedance are required in order to match the shaker impedance to that of the electrical output impedance of the power amplifier.

For a current shaker of 10 to 20 watts power, the moving coil impedance is:

$$1 < Z_c < 5 \text{ Ohm} \qquad [4.1]$$

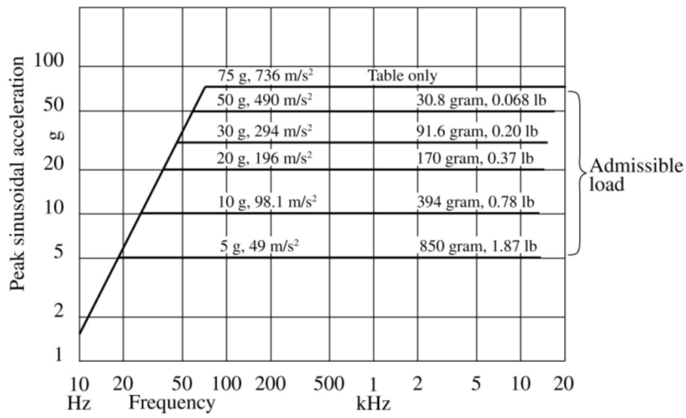

Figure 4.4. *Example of peak acceleration vs. frequency curve under sinusoidal regime with additional weight as a parameter (Brüel & Kjær documents) [BRU 60-10]*

This electrical impedance is specified at a given frequency (for example at 500 Hz). It is a complex impedance which varies with frequency.

The maximum output supply, P, depends naturally on the shaker size. A power range of 20 < P < 100 VA is a reasonable choice.

The amplitude of the frequency curves depend on the weight of the moving coil and also on the additional weight of the set (the sample and holder). Figure 4.4 presents an example of curve acceleration versus frequency with specified admissible weight, the input signal being a sinusoidal signal.

Figure 4.5 presents an example of a frequency response with a constant electric current as the excitation electrical signal, and with a constant electrical voltage respectively[5].

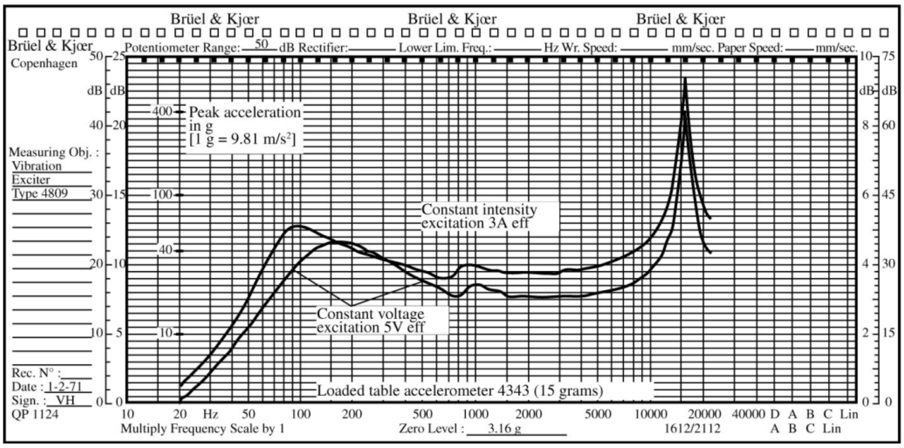

Figure 4.5. *Example of peak acceleration vs. frequency with constant electric intensity and constant electric voltage as parameters respectively. Brüel & Kjær documents [BRU 60-10]*

4.5.6. *Power amplifier for an electrodynamic shaker*

An electronic power amplifier is usually proposed by the company selling the shaker, at a prohibitive price. Another possible source is from audio high fidelity equipment suppliers. Audio power amplifiers of 100 to 200 watts with good performance of frequency response curves are available. The electrical performances mentioned above should be retained for the choice of power amplifier.

However, the experimenter's attention must be focused on an important factor: the electric shaker cannot be reduced to a simple audio loudspeaker. Power surges furnished to the shaker are frequent and precautions should be taken concerning the protection of the output circuit of the electronic power amplifier.

5 Electric current in the moving coil conditions the mechanical force after Laplace's law. However, the whole mechanical system must be taken into account. A control by direct measurement of the force at the excitation point is preferable.

When choosing such an amplifier, experimenters must ensure that the last stage of the power amplifier is electronically protected by an electronic breaker with a short time response of the order of one tenth of a second. A classical fuse cartridge has too long a time response and consequently is not appropriate to prevent damage to the power amplifier due to electric transients of high electric current amplitude.

When choosing an audio power amplifier to drive a shaker, an output electrical power of three or four times the maximum power delivered to the shaker is a useful precaution to ensure the longevity of the electronic equipment.

4.5.7. *Control system for an electrodynamic shaker*

The dynamic response of the sample often includes the response of the exciter. In Figure 4.5, above, two resonance frequencies are observed. When one rigidly connects samples to a shaker table, the sample and the external mass connected to the moving part of the shaker can modify the resonance frequency, which can be shifted to a lower frequency. The presence of this frequency must be taken into account in different test cases.

4.5.7.1. *Sinusoidal tests*

Figure 4.6 is a line diagram of a control system for sinusoidal tests. An input signal applied to the sample (displacement, acceleration, force) and obtained by an appropriate transducer is amplified and filtered by a tracking filter. Its average value, through a control signal selector and a computer program, pilots the power amplifier so as to make the input signal constant in the explored frequency range.

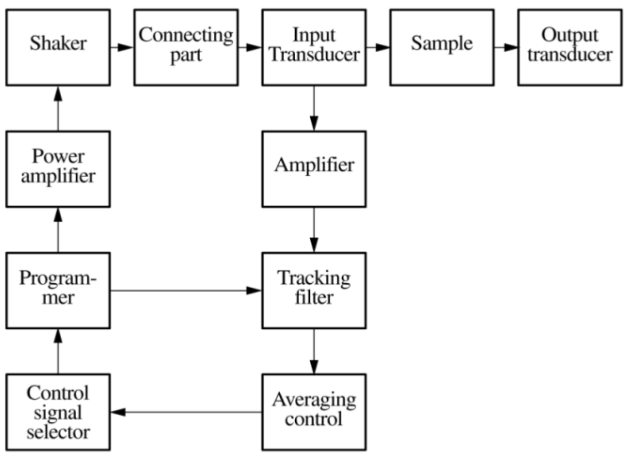

Figure 4.6. *Line diagram of an analog control system for sinusoidal tests*

The tracking filter is sensitive to the speed of frequency sweeping and particularly to the sharpness of the resonance peak of the sample, and also the resonance peak of the moving coil itself. When the sample material has a very low damping capacity (tan $\delta \approx 10^{-3}$) the sweeping speed must be very low so as to avoid a transient response from the tracking filter which can falsify the real frequency response of the sample. The amplitude of the electric signal sent to the shaker is adjusted so as to obtain the constant amplitude of the input signal applied to the sample.

4.5.7.2. *Random vibration tests*

Figure 4.7 shows a line diagram of a possible analog control system. The transducer input signal is introduced in an equalizer circuit with a number of fixed narrow band filters, each of which has an adjustable gain. The set of filters constitutes a contiguous narrow band filter. The output signal controls the amplitude of the input signal furnished to the power amplifier.

Experimenters are often tempted to use white noise with maximum bandwidth to save time. It is very important not be forget that this kind of input signal could be very harmful to power amplifiers and necessitates output power which is frequently beyond the maximum upper limit of power of an amplifier (often by a factor of ten), if the electronic breaker of the power amplifier is not efficient.

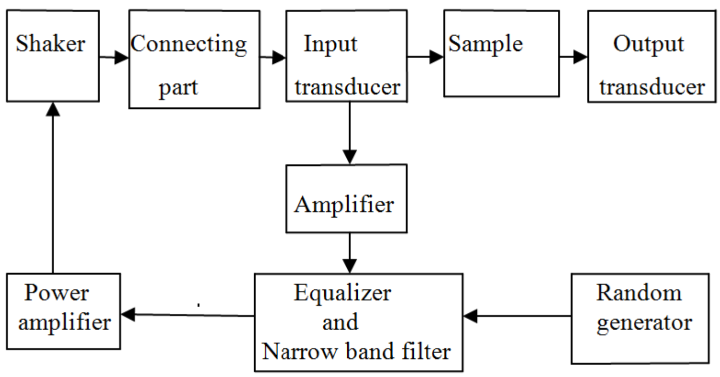

Figure 4.7. *Line diagram of an analog control system for random vibration test*

4.5.7.3. *Digital control system*

Similar to the aforementioned analog control system, a digital control system can be built. It is adopted in many commercially available instruments (Brüel and Kjær, Denmark, for example).

4.5.7.4. *Possible design of additional mechanical parts*

In dynamic tests devoted to material characterization, the amplitude of forced displacement or acceleration at the sample input (excitation) need not be large. One possibility is the use of an additional external stiffness mounted in parallel with the moving coil and table mass so as to reject the higher resonance frequency beyond the highest frequency range of interest. A dynamic test is necessary to ensure that the Fourier transform of the input signal is flat in the frequency range of interest.

4.5.7.5. *Low frequency tests*

Low frequency tests might present some difficulties, for example, if less than 10 Hz. Generally, in this frequency range, a signal produced is strongly non-linear. Signal distortion should be accounted for in the dynamics of a sample response.

4.5.7.6. *Interest of transfer function*

The transfer function (defined as the quotient of the output signal to the input signal) can be obtained by using two identical transducers at the input and the output of a sample. If the input and output signals are different in nature, separate calibrations are necessary for both transducers in the specified frequency range. The accuracy of the transfer function depends on both transducer calibrations. In Part II, two identical transducers are adopted. One of the main reasons for this is that the admissible frequency range might be larger than the specified frequency range of each transducer. There is a kind of compensation which enables measurements beyond the bandwidth of each transducer.

4.6. Low cost electromagnetic exciters with permanent magnet

Figure 4.8 shows a low-cost and contactless exciter which is composed of a static permanent magnet core around which is wound a certain number of electrical coils of wire. For non-magnetic samples, it is necessary to glue a ferromagnetic blade to one end of the sample. The presence of a permanent magnetic force constitutes the main disadvantage of this exciter for the measurement of very low damping of a sample material.

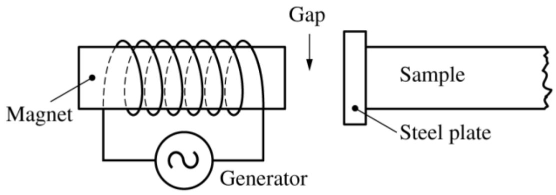

Figure 4.8. *Magnetic transducer transformed into mechanical exciter*

4.7. Piezoelectric and ferroelectric exciters

These exciters do not develop as much energy as the previously mentioned exciters[6]. However, they present real interest for dynamic tests, over a very wide frequency range, covering ultrasonic frequencies up to the realms of megaHz. In spite of the low magnitude of displacement, (of the order of a few micrometers) they deserve experimenters' attention, particularly at ultrasonic frequency. The experimenter can built a special exciter themselves, which is small in size and easy to realize. As these exciters are small and lightweight, they are well adapted for experimental study of the dispersion of waves which constitute the cornerstone of this book. Taking these advantages into account, the theories of piezoelectricity and piezoresistivity are presented in the following sections, together with practical considerations for making these transducers.

4.7.1. *Elements of piezoelectricity*

Some natural materials such as tourmaline have electrical properties which were known about in Asia a thousand years ago. In Ceylon (Sri Lanka) inhabitants remarked that tourmaline crystals attracted ash hot particles. In the 18th Century, these crystals were brought to Europe and, in 1824, Brewster first called this phenomenon pyroelectricity.

Years later, in 1880, the Curie brothers (Pierre and Jacques) studied piezoelectricity (piezo means "to press", in Greek). A variety of piezoelectric materials were examined (tourmaline, quartz, topaz, tartaric acid, seignette's salt). In 1881 Lippman (Nobel prize winner in 1908) predicted the existence of reciprocal piezoelectric effects, meaning that an applied electric voltage creates a mechanical pressure and a crystal deformation. This effect was confirmed by the Curie brothers (who were themselves to win a Nobel prize in 1903). From 1880 to 1914 piezoelectricity remained as an item of scientific curiosity, until its first application

6 This is related to the energy available, i.e. the surface of the ferroelectric plate.

on submarine detection was undertaken by Langevin in France during the First World War 1914-1918.

4.7.1.1. *Curie's symmetry principle*

Curie's symmetry principle states that a physical phenomenon has all the properties of the symmetry elements of the cause which produced it.

Dissymmetry of a phenomenon preexists in the causes. In real terms, this means that piezoelectricity only exists in a system which is a sub-group of the symmetry characterizing the phenomenon. Figure 4.9 shows a material's symmetry which gives rise to a ternary axis. Three cases are presented: a normal stress vector $\vec{\sigma}$ and an electrical polarization vector $\vec{D}$ are represented along the vertical axis, and in a plane perpendicular to the σ axis.

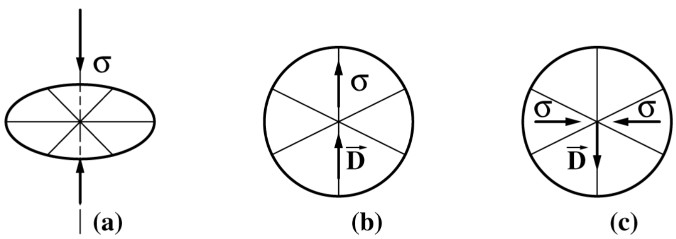

Figure 4.9. *Piezoelectric effect in a material with a ternary axis: (a) compression along ternary axis; (b) electrical polarization* ***D*** *collinear with normal stress* ***σ****;*
(c) electrical polarization ***D*** *perpendicular to the normal stress* ***σ***

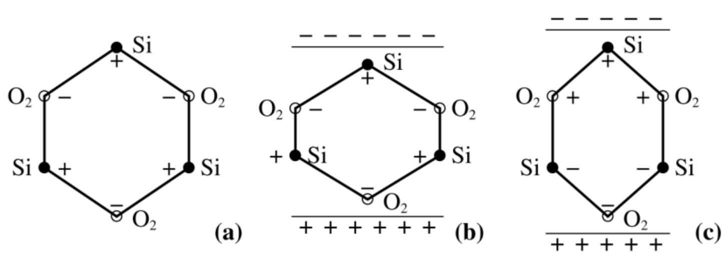

Figure 4.10. *Electrical charge production vs applied mechanical stress*

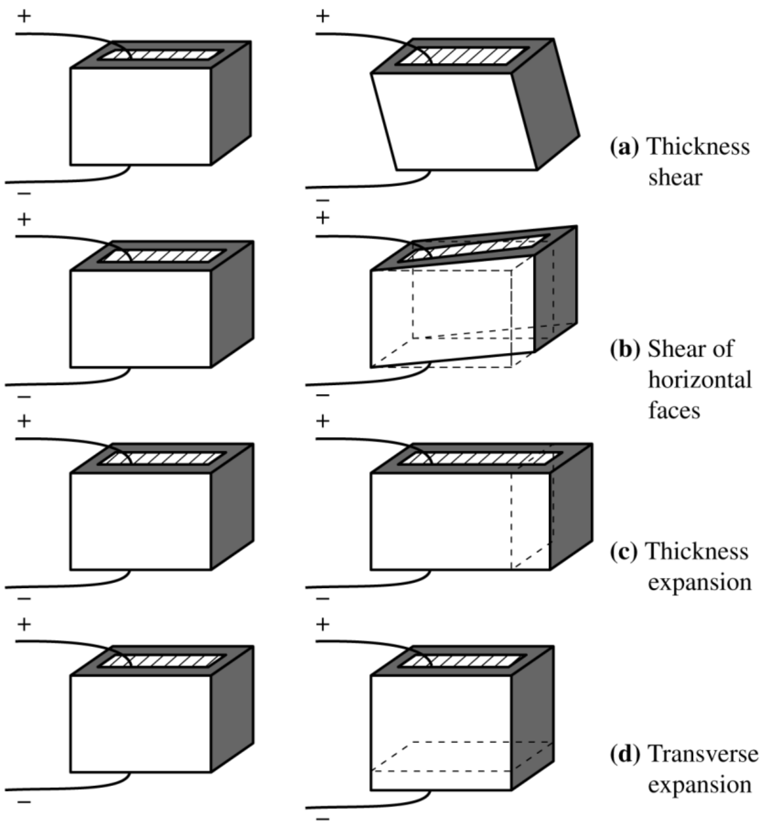

Figure 4.11. *Various deformation modes of piezoelectric parallelpiped: (a) thickness shear; (b) horizontal face shear; (c) thickness expansion; (d) transverse expansion*

An electrical charge is created by applying pressure to a piezoelectric disk. Quartz is represented as a hexagonal element with silicon (Si) and oxygen atoms (O_2) in Figure 4.10(a).

Figure 4.10(b) represents the application of mechanical pressure. A negative charge is produced at the upper surface and positive charge is produced at the lower surface.

A normal stress tension is applied in Figure 4.10(c). The electrical polarities of the silicon and oxygen atoms are reversed compared to the previous case.

4.7.1.2. *Various deformation modes*

Two deformation modes can be created. An expansion mode relates to volume variation and shear mode relates to equivolumic distortion, i.e. distortion without volume variation (see Figure 4.11). The four deformation modes as shown in figure 4.11 are exploited in transducers (both emitters and/or receivers) to create a shear and expansion modes corresponding to shear wave and longitudinal waves, respectively.

4.7.2. *A constitutive equation for piezoelectric materials*

Let σ_{ij}, ε_{ij} be stress and strain components, respectively. Constitutive equations must include piezoelectric effects which are portrayed by electric displacement (or polarization) vector $\vec{D}$ and an electric vector $\vec{E}_i$. These two vectors are introduced in constitutive equations with coupling coefficients. To avoid confusion between strain ε_{ij} and the dielectric tensor of the second order, we will adopt the notation e_{ij}^S with superscript "S" for the dielectric constant (instead of the "ε" usually adopted in electricity).

$$\left[e_{ij}^{s}\right]=\begin{pmatrix} e_{xx}^{s} & e_{xy}^{s} & e_{xz}^{s} \\ e_{yx}^{s} & e_{yy}^{s} & e_{yz}^{s} \\ e_{zx}^{s} & e_{zy}^{s} & e_{zz}^{s} \end{pmatrix} \qquad [4.2]$$

Constitutive equations are then:

$$\sigma_{jk} = C_{ijkl}^{E}\varepsilon_{lm} - e_{jkl}E_i \qquad [4.3]$$

$$D_{ij} = e_{ijk}\varepsilon_{jkl} + e_{ij}^{S} E_j \qquad [4.4]$$

In equation [4.3], a second tensor is related to a vector E (electric field), we have to use a third order tensor (e_{ijk}) which is defined as:

$$e_{ijk} = \left[-\partial\sigma_{jk}/\partial E_i\right]_S \qquad [4.5]$$

where S is constant strain.

In equation [4.4], polarization vector D_i is related to strain ε_{ij} by the same third order tensor as above in [4.5]; thus:

$$e_{ijk} = \left[-\partial D_i / \partial \varepsilon_{ij} \right]_E \qquad [4.6]$$

where E, electric field, is constant.

Tensor e_{ijk} has 27 components which are reduced to 18 components by symmetry considerations on stress $\boldsymbol{\sigma}$ and strain ε. So we can write:

$$e_{ijk} = e_{ikj} \qquad [4.7a]$$

To simplify this, we can adopt the following notation:

jk	xx	yy	zz	yz = zy	xz = zx	xy = yz
J	1	2	3	4	5	6

[4.7b]

Thus, we can write $\left[e_{ijk} \right]$ in a simplified form as:

$$[e_{iJ}] = \begin{pmatrix} e_{x1} & e_{x2} & e_{x3} & e_{x4} & e_{x5} & e_{x6} \\ e_{y1} & e_{y2} & e_{y3} & e_{y4} & e_{y5} & e_{y6} \\ e_{z1} & e_{z2} & e_{z3} & e_{z4} & e_{z5} & e_{z6} \end{pmatrix} \qquad [4.8]$$

$[e_{iJ}]$ is also called the piezoelectric stress matrix. The polarization vector and electric field vectors have the following components:

$$[D_i] = \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix}, \quad [E_i] = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \qquad [4.9]$$

We can rewrite [4.3] and [4.4] in a condensed form, where superscript E designates a constant electric field:

$$\left. \begin{aligned} \sigma_J &= C_{Jk}{}^E \, \varepsilon_k - e_{Ji} \, E_i \\ D_i &= e_{iJ} \, \varepsilon_J + e_{iJ}{}^S \, E_j \end{aligned} \right\} \qquad [4.10]$$

Exponent S designates constant mechanical strain.

In [4.10] the Hooke tensor can be recognized as C_{JK}^E using abridged notation. [4.10] can be rewritten in matrix form; $[e]^T$ designates the transpose of matrix [e].

$$\left.\begin{aligned}\{\sigma\} &= \left[C^E\right]\{\varepsilon\} - [e]^T\{E\} \\ \{D\} &= [e]\ \{e\}\ +\ \left[e_d{}^S\right]\{E\}\end{aligned}\right\} \qquad [4.11]$$

Instead of writing stress versus strain and electric field, we can write $[d]^T$, being the transpose of matrix [d]:

$$\left.\begin{aligned}\{\varepsilon\} &= \left[S^E\right]\{\sigma\} + [d]^T\{E\} \\ \{D\} &= [d]\ \{\sigma\}\ +\ \left[\varepsilon_d{}^F\right]\{E\}\end{aligned}\right\} \qquad [4.12]$$

The exponent F in the matrix components in the second equation means constant force (stress).

As for the state vector in structural dynamics, we can use the state vector i.e. the mixed components vector which replaces [4.12]:

$$\begin{Bmatrix}\{\varepsilon_{ij}\} \\ \{D_i\}\end{Bmatrix} = \left(\begin{array}{c|c}[S]^E & [d]^T \\ \hline [d] & \left[e_d{}^F\right]\end{array}\right) g \begin{Bmatrix}\{\sigma\} \\ \{E\}\end{Bmatrix} \qquad [4.13]$$

$$[9x1] = \quad (9x9] \quad g\ (9x1)$$

For quartz, components S_{ij}^E are expressed in pm^2/N (where p = pico = 10^{-12}) (see Table 4.1). The quartz coupling coefficient d_{ij} components are expressed in pico Coulombs/Newton (pC/N) (see Table 4.2). Coefficient $d_{ijk} = d_{iJ}$ is the most and describes the coupling between stress components σ_{jk} and electric polarization D_i.

S_{11}^E	S_{33}^E	S_{12}^E	S_{13}^E	S_{44}^E	S_{14}^E	S_{66}^E
12.77	9.6	-1.79	-1.22	20.04	4.5	

Table 4.1. *Components of S_{ij}^E for quartz, expressed in pm^2/N (p = pico = 10^{-12})*

d_{x1}	d_{x4}	d_{x5}	d_{y2}	d_{z1}	d_{z3}
2.31	-0.723				

Table 4.2. *Components of the quartz coupling coefficient d_{ij} expressed in pico Coulombs/Newton (pC/N)*

4.7.3. *Quartz cutting*

Two cuttings are used to obtain piezoelectric disks functioning in expansion mode (X cutting) or in shear mode (Y cutting); see Figure 4.12.

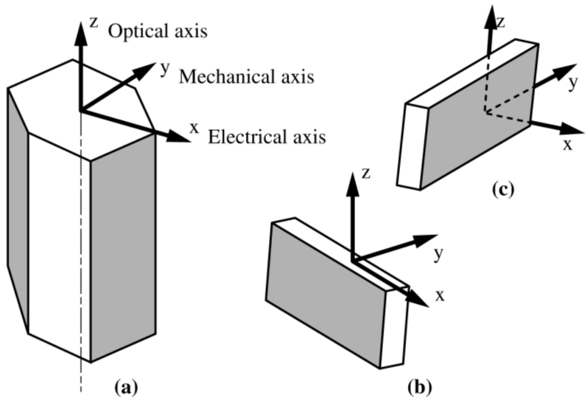

Figure 4.12. *Section of a quartz crystal showing: (a) reference axis-X axis joining a vertex of hexagonal cross-section. The Y axis is a mechanical axis joining the center of the hexagonal side; (b) cutting along the X direction for an extensional wave; (c) cutting along the Y direction for a shear wave*

The two faces of the disk are metallized and electric voltage is applied if the disk is used as an emitter. The same disk can function as a receiver: if it receives a mechanical signal, this is converted into an electric signal.

4.7.4. *Ferroelectricity elements*

Ferroelectricity concerns a class of artificial materials (ceramics) which are obtained by powder metallurgy. They are used in many industrial applications to replace quartz which cannot have any shape other than blades and disks.

Complex shapes can be obtained with ceramics, such as hollow cylinders used in hydrophones for submarine detection.

Strictly speaking, ferroelectric transducers should not appear under the piezoelectric material classification if the nature of the material is taken into consideration. However, constitutive equations with piezoelectric coupling terms are similar.

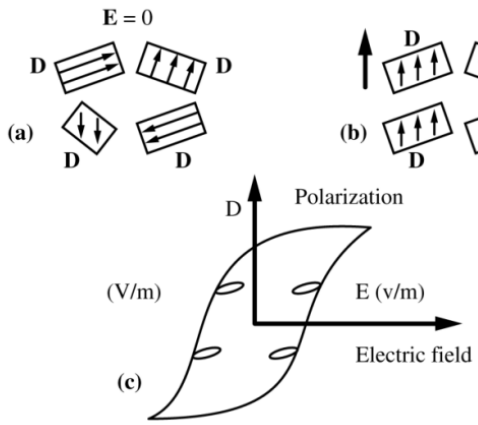

Figure 4.13. *Polarization $\vec{D}$ in an aggregate of ferroelectric ceramics: (a) absence of electric field $\vec{E}$; (b) presence of non-zero electric field $\vec{E}$; (c) ferroelectric hysteresis*

Figure 4.13 shows an aggregate representing a ferroelectric material. This element set has a spontaneous electric polarization $\vec{D}$ whose direction varies in a random manner from one grain to another, in the absence of an electric field $\vec{E}$ (Figure 4.13a). Application of an electric field $\vec{E}$ induces a modification of $\vec{D}$ in each element (Figure 4.13b).

There is consequently a mean orientation of $\vec{D}$ in the direction of $\vec{E}$ (Figure 4.13b), and the resultant of $\vec{D}$ particularly when the field $\vec{E}$ is strong. Figure 4.13c represents the curve $\vec{D} = \vec{D}(\vec{E})$ which should be compared to the magnetic hysteresis curve, $\mathcal{B}$ (induction) = $\mathcal{B}$ (H magnetic field).

Let us examine a barium titanate which is tetragonal in structure (see Figure 4.14).

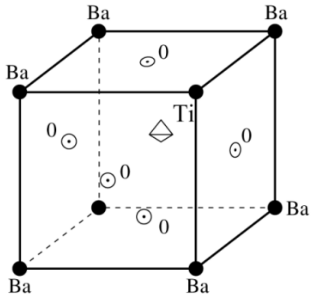

Figure 4.14. *Barium titanate structure $BaTiO_3$*

If this material is heated to 120°C, barium titanate, which is tetragonal, becomes cubic and loses its ferroelectric properties. To increase polarization stability, impurities are introduced (for example, lead titanate, calcium or yttrium titanate). Introduction of the impurities lowers the secondary transition point. The Curie point of these materials is between 320 and 350°C (remember that the Curie point is the temperature limit beyond which there is no ferroelectric effect).

There are a great variety of barium titanates. Fabrication of this material in disk, cylinder, ring or hollow cylinder form is effected by pressing, extrusion, and powder metallurgy, whilst the final mechanical operation is grinding. A polarized electric field of 50 KV/cm is applied in a number of minutes during hot pressing and temperature is progressively increased beyond the Curie temperature point. Then a freezing operation is effected.

Ferroelectric materials are crystals of the 6 m×m class and belong to the hexagonal system. The stiffness matrix has five independent coefficients. Equations [4.2] to [4.13] are valid for ferroelectric materials.

4.7.4.1. *Mechanical compliance matrix*

Equation [4.14] shows a mechanical compliance matrix in which non-zero coefficients are represented by little circles. A straight line connecting two terms equalizes these two coefficients. Dots correspond to zero coefficients.

$$[S] = \begin{pmatrix} 0 & 0 & 0 & \bullet & \bullet & \bullet \\ & 0 & 0 & \bullet & \bullet & \bullet \\ & & \bullet & \bullet & \bullet & \\ & \text{symmetric} & & 0 & \bullet & \bullet \\ & & & & 0 & \bullet \\ & & & & & 0 \end{pmatrix} \qquad [4.14]$$

4.7.4.2. *Dielectric matrix*

Polarization and electric field are related by the following matrix equations:

$$\{D\} = \left[e^{S}\right]\{E\} \qquad [4.15]$$

$$[S] = \begin{pmatrix} 0 & \bullet & \bullet \\ \bullet & 0 & \bullet \\ \bullet & \bullet & 0 \end{pmatrix} \qquad [4.16]$$

Equation 4.16 shows a dielectric matrix with three non-zero coefficients (represented by little circles) on the diagonal. Dots represent zero coefficients.

The dielectric matrix relates two vectors. Consequently it has (3×3) coefficients. For a hexagonal system, there are three non-zero coefficients, represented by circles as above.

4.7.4.3. *Comparative table for quartz and barium titanate*

Table 4.3 gives useful indications for these two materials.

	Quartz	Barium titanate
Advantages	High pressure measurements	High coupling coefficient d
	High frequency performances	High dielectric constant
	Weak temperature influence	High isolation performance Weak dielectric loss
Disadvantages	Weak dielectric constant	Temperature effect
	Influence of connecting cables	Weak conversion coefficient (electric energy/mechanical energy)

Table 4.3. *Comparison between quartz and barium titanate*

4.7.4.4. *Useful piezoelectric characteristics*

In Table 4.4, the coefficients of the four tensors (3×6) [d], [g], [e] and [h] are given, together with their units.

	Direct effect	Inverse effect
Charge Coefficient Unit	[Charge density on electrode normal to i axis] / [stress along j axis]* Coulomb. (Newton)$^{-1}$= CN^{-1} Constant E	[Strain along j axis]* [Electric field along i axis] Meter.(Volt)$^{-1}$ = $m.V^{-1}$ Constant stress
Coefficient g_{ij} Unit	[Electric field along axis i] / [Stress along j axis]* $V.m.N^{-1}$ Induction B constant	[Strain along j axis]* [charge density on electrode normal to i axis] m^2C^{-1} Constant stress
Coefficient q_{ij} Unit:	[Charge density on electrode normal to i axis] / [Strain along j axis]* Electric field constant $C.m^{-2}$	[Stress along j axis]* [Electric field along i axis] Constant strain $N.m^{-1}V^{-1}$
Coefficient h_{ij} h_{ij} Unit	[Electric field along i axis] / [Strain along j axis]* $V.m^{-1}$	[Stress along j axis]* Charge density on normal electrode to i axis] NC^{-1}

Table 4.4. *The four useful matrices [d] [g] [q] [h]used to characterize inverse or direct effect. (*)i,j axes in reality concern conventional double indexes, see the table presented as equation [4.7b] is contracted into one index to simplify the matrix.The quotient of two physical quantities are indicated by /*

Coefficients of these matrices are defined as follows[7]:

$$\left.\begin{aligned} d_{nj} &= e_{mn}{}^{T}\, g_{mj} = l_{ni} s_{ij}{}^{E} \\ q_{nj} &= e_{mn}{}^{S}\, h_{mj} = d_{ni} C_{ij}{}^{E} \\ g_{nj} &= b_{mn}{}^{T} d_{mj} = h_{ni} s_{ij}{}^{D} \\ h_{nj} &= b_{mn}{}^{S} k_{mj} = g_{ni} C_{ij}{}^{D} \end{aligned}\right\} \qquad [4.17]$$

7 For the definition of coefficients presented in [4.17], constitutive equations concerning coupling effects between piezoelectricity and mechanical effects is described by equations [4.2] to [4.12]. Coupling coefficients are presented in Tables 4.1 to 4.4

$(m, n) \in (1, 2, 3)$; [4.18]

$(i, j) \in (1, 2, 3)$ [4.19]

In the case where the ceramics are subjected to uniform stress on the surface, we have to deal with hydrostatic coefficients, which have the following value:

$g_h = \{g_{33} + 2g_{31}\}$ $\qquad d_h = d_{33} + 2d_{31}$ [4.20]

4.7.4.5. *Coupling between mechanical energy and electric energy – useful coefficients*

The coupling coefficient of a ceramic dynamically characterizes its ability to transform electric energy into mechanical energy, and vice versa. It is defined as:

$$k^2 = \frac{transformed\ energy}{furnished\ energy} \qquad [4.21]$$

Depending on the shape of the ceramics and the vibration mode, the following coefficients are possible k_{33}, k_{31}, k_p, k_t. The two first coefficients contain two indexes related, respectively, to the polarization D and the vibration motion.

k_p is the radial vibration of a thin disk. k_t is the vibration motion of this disk in the polarization direction.

Figure 4.15 gives the four indicated coupling coefficients.

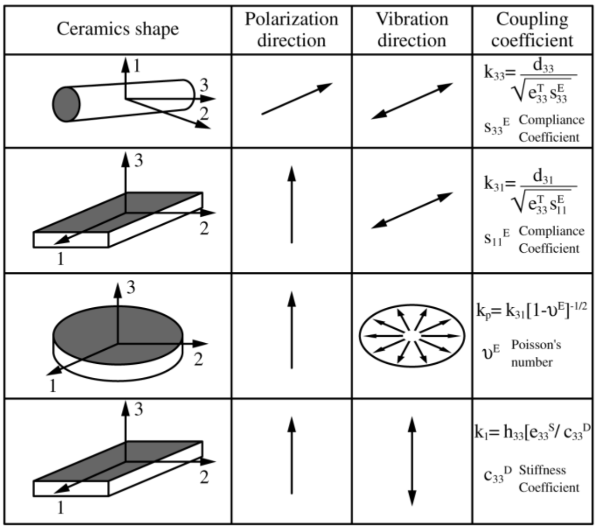

Figure 4.15. *Coupling coefficients versus polarization and vibration directions*

4.7.4.6. *Resonance frequency*

Figure 4.16 presents the most commonly used ceramic plates and disks with vibration modes (longitudinal, shear, transverse or radial) as well as useful coefficients and resonance frequencies. The frequencies are given with the constant N given by the constructor (unit: Hz.mm).

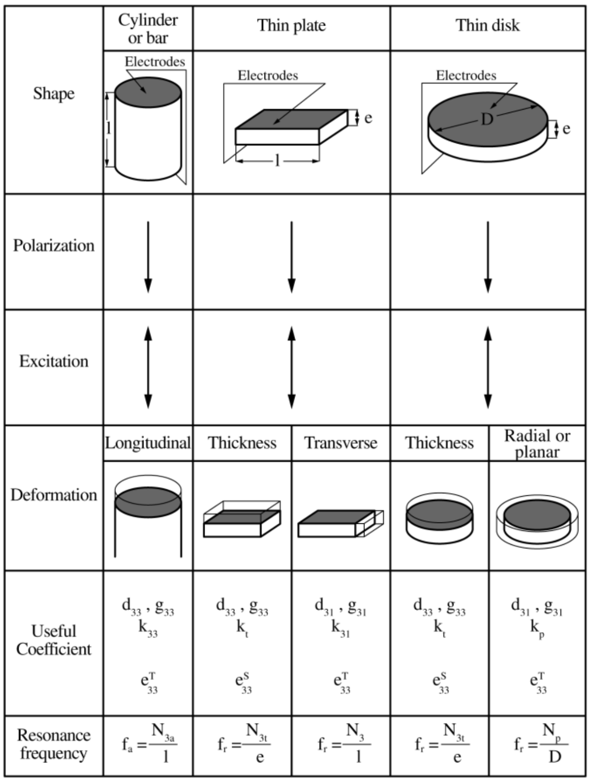

Figure 4.16. *Vibration modes of ferroelectric plates or disks*

4.8. Design of special ferroelectric transducers

Some characteristics of ferroelectric materials have been given above. The objective of the authors is to focus attention on the great interest of this material, which is not expensive and can easily be fabricated and adapted for dynamic material testing by experimenters themselves, particularly at high frequencies of 100,000 Hz. In Chapter 15, for example, which is devoted partly to experimental wave dispersion for higher elastodynamic modes, details of how to fabricate tiny ferroelectric emitters from large barium titanate plates are presented. Further indications for building these transducers, emitters and receivers are given below.

4.8.1. *Choice of ceramic plate*

For a rod with a rectangular cross-section, naturally ferroelectric plates with the same cross-section as the rod sample are more than adequate. Appendix 4A.1 gives an example of ferroelectric ceramics. Titanate or zirconate are to be preferred to barium-titanate. However, barium-titanate might be of interest in applications where excessive sensitivities of some technical parameters are not desired.

A wide frequency range should be chosen.

The following parameters should all be considered:

– *Thermal properties*

Curie point is important characteristic for tests at high temperature tests

– *Electric properties*

Dielectric constants e_{ij}^{T}, e_{ij}^{S}

Loss tangent: tan δ

– *Piezoelectric properties*

Charge coefficients d_{ij}

Tension stress coefficient g_{ij}

Coupling coefficient k_{ij}

Frequency constant N

– *Thermal dilatational coefficient*

α_1 (transverse), α_3 (longitudinal)

4.8.2. *Geometrical parameters*

The thickness of the ceramic plate plays an important role in determining the resonance frequency of the transducer. It is not the only parameter to take into account.

Often the shape of the cross-section does not correspond to the ceramic disk or plate. In such a case, acquire a large ceramic plate and cut a surface contour equal to the sample cross-section with a special knife (used to cut glass or ceramics).

Note. A thick transducer gives a higher resonance frequency than a thinner one. The transducer weight is, however, to be taken into account as additional weight, if it has to be glued to one end of the sample.

4.8.3. *Influence of connecting electric cable*

The transducers must have two metallized faces. The connecting cables can be glued to the two transducer faces. The glue must be electrically conducting. Gluing avoids soldering operations which might damage the transducer. Any temperature beyond the Curie point irreversibly damages the transducer. The weight of the electric cable might have an influence on the measurement of the material damping.

Figure 4.17 presents a schematic diagram of a ferroelectric exciter E_m (on the left). The transducer T_1 is identical to the emitter and furnishes the acoustical input signal. A ferroelectric transducer T_2 furnishes the output signal.

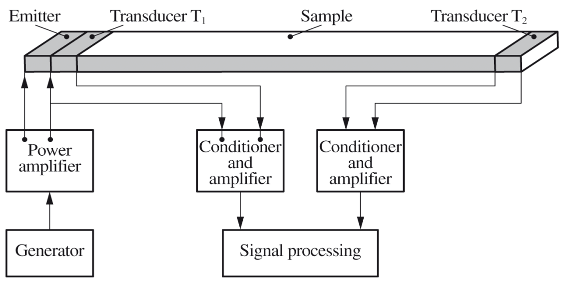

Figure 4.17. *Piezoelectric emitter–transducer specially tailored for dynamic tests on material samples. The exciter can work over an extremely large frequency range*

4.9. Power piezoelectric exciters

The piezo (or ferro) electric materials presented in the previous sections are used to make power exciters. In some special applications, a single plate or disk is often not sufficient to obtain enough power. To obtain large mechanical strain at the exciter input, it is necessary to increase the electrical charge by increasing the voltage gradient. For this purpose, multiple thin layers of piezoelectric materials and conducting materials, and alternating voltage polarities of the conducting separators, can be used; see Figure 4.18.

Figure 4.18a shows a stack of piezo (or ferro) electric disks with alternating electric voltage polarities. Alternating voltage polarity applied to the whole stack is converted into alternating mechanical compression and tension. Figure 4.18b shows the ferroelectric stack mounted in a case with a thick base which serves as a mechanical preload.

The only condition is that the stiffness of the mechanical preload system must be less than the stacked ferroelectric system. The inertia of the base mass gives rise to a force applied to the upper face of the case.

Displacement is not of such great amplitude as in an electrodynamic exciter (being only of the order of few hundredths of a millimeter). However, a ferroelectric exciter can cover a very large frequency range, i.e. from 0 to 60 kHz.

The exciter mass and the stack mass combined with the resulting stiffness give rise to high resonance frequency. That constitutes the upper limit of the frequency range.

For a piezoelectric exciter, the alternative voltage applied is of the order of 100 to 1,000 volts. The electric charge produced is of the order of 10 to 1,000 Nanofarads.

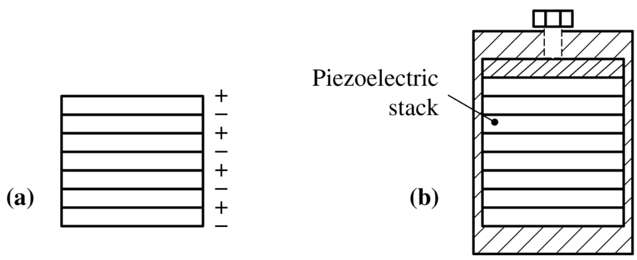

Figure 4.18. *(a) Stack of piezo (or ferro) electric disks (voltages are alternating); (b) mechanical mounting with preload*

4.10. Technical details concerning ultrasonic emitters for the measurement of material stiffness coefficients on ultrasonic test benches

Ultrasonic transducers are sensors which work in a direct sense (by transformation of an electric signal into an ultrasonic signal in a material) or in an inverse sense (by conversion of an acoustical signal into an electric signal).

Used in pairs (an emitter and a receiver) they are identical in ultrasonic benches whether used in direct contact or by immersion in water. The principal technical characteristics are:

– resonance frequency: the bandwidth is narrow so these transducers work practically in monofrequency;

– size of piezoelectric disk and diameter of the transducer: generally their cross-section area is much less than the surface of the material plate to avoid an edge effect in ultrasonic wave propagation;

– magnitude of electric pulses applied to the emitter: for piezoelectric crystals, this can be of the order of 10 to 100 Volts (peak value);

– output impedance of the electric generator: Z_g which generally serves as the power amplifier, is of the order of 100 Ohms (a special kind of power generator which delivers power to the order of 100 Watts is used). A value of Z_g of the order of 600 Ohms is usually adopted.

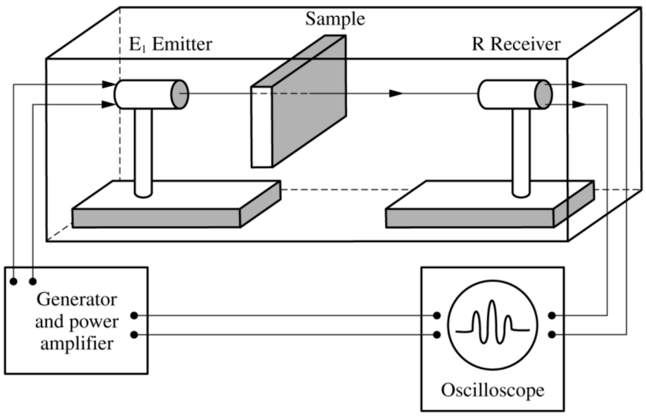

Figure 4.19. *Piezoelectric emitter E_1 sends a progressive wave under the form of a wave packet which propagates in a water immersion bench through the plate sample. The progressive wave is captured by piezoelectric receiver R identical to E_1. Oscillograph sweeping is triggered by a synchronous signal to control the emitter impulse*

4.10.1. *Nature of the electric signal applied to piezoelectric transducers*

An ultrasonic exciter works under an impulse regime. Figure 4.19 shows a schematic diagram of such an emitter.

4.10.1.1. *Pulse train*

In Figure 4.20, an isolated pulse of duration τ is repeated in time with a periodicity T. τ is adjusted so that the emitter transducer converts the electric pulse into a wave packet constituted by a reduced number of sinusoids whose envelope is Gaussian shaped. If only an isolated electric impulse is applied, an ultrasonic pulse train escapes as a transient signal and it is only captured by a camera with high sensitivity film.

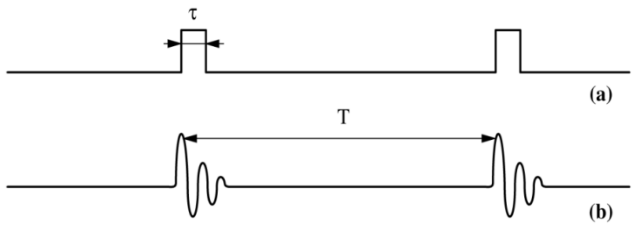

Figure 4.20. *(a)Electronic power pulse generator emits rectangular impulses of duration τ curve; (b) ultrasonic impulses are repeated with a periodicity of time T*

4.10.1.2. *Repeated pulse adjustment*

The repetition of this transient signal as periodic electric pulses permits an easy visual observation on an oscilloscope screen. The condition is that $\tau < T$ (τ being the impulse duration; T the period of repetition of the signal), so that there is no superposition of ultrasonic wave packets. An acoustic wave packet is easily observed on an oscilloscope screen. The position of the wave packet permits measurement of the time of flight of the ultrasonic signal after travelling through the sample plate. Details of measurements are presented in Chapter 14 (see also [CHE 10]).

4.10.1.3. *Utilization of a double pulse*

An impulse electric generator may have the ability to produce two impulses whose relative position in time can be adjusted. This ability is used to produce an ultrasonic wave packet, as in Figure 4.20b. The amplitudes and the shape of the sinusoids can be adjusted.

4.10.1.4. *Available electric power of the generator*

A power of the order of 100 V.A is sufficient to cover all ultrasonic test applications.

4.10.2. *Waterproof transducers*

Transducers used in a water immersion bench must be waterproof. The transducer piezoelectric crystals generally have a waterproof coating but this is not sufficient. Plugs and jacks for connecting cables must also be waterproof.

Under these conditions, the design of ultrasonic probes must include a waterproof tube and cylinder incorporating a transducer and connections protected by screws and tap washers (Figure 4.21).

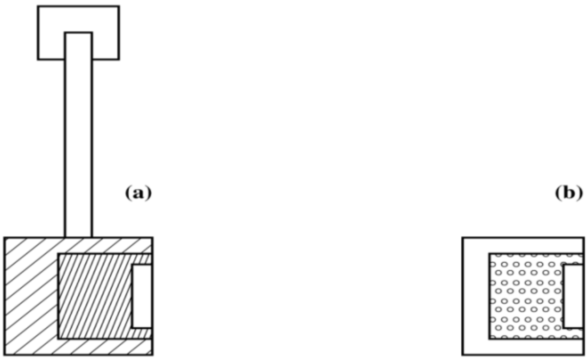

Figure 4.21. *Piezoelectric transducers used in ultrasonic immersion bench. A pair of transducers is used; one for emitter the other for receiver. (a) Waterproof transducer mounted in tube and in cylinder. (b) Detail of the transducer: piezoelectric disk is mounted with rear side filled with a mixture of lead grains and epoxy resin which annihilates emission of ultrasonic signal from the rear side of the transducer*

4.10.3. *Prevention of parasitic signals from the rear side of a transducer*

A piezoelectric crystal in a transducer can emit ultrasonic signals in both directions. Signals emitted from the rear are considered to be parasitic signals which scramble the true signal emitted or received on the surface in contact with water. To prevent the rear side of the transducer from emitting and receiving signals, a backing system should be used, made from a blend of epoxy resin and lead powder whose grain size is chosen to obtain maximum damping.

4.10.4. *Coupling liquid/special grease for direct transducer contact with a sample*

In a direct contact ultrasonic bench, a special coupling liquid between transducer and sample is used. For a longitudinal wave, liquid with low viscosity is sufficient. For a shear wave, a special coupling liquid or grease is used to permit an ultrasonic wave to be transmitted via the interface between sample and transducer. Details of these direct contact benches are presented in Chapter 2.

4.11. Bibliography

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4.12. Appendix 4A. Example of ferroelectric plates and disks

Various quartz and silica ferroelectric plates and disks taken from the catalog of a French company serve as illustrations of their possible applications in the building of transducers. Naturally readers can find other manufacturers in other countries.

Resonance frequencies related to a specified shape are given, together with a tolerance interval.

		P51	T57	P7-62	P1-60	P4-68	P1-63
Thin and Thick plates	Length and width (mm)	0.5 to 110		0.5 to 110			
	Thickness (mm) max-	max 60 min 0.2		max 20 min 0.2			
Mechanical properties	Units						
Density	10^3 kg /m^3	5.6	5.	7.4	7.3	7.1	7.4
Young's modulus	10^9 Pa (1)	140	100	109	89	87	104
	‘‘ (2)	79	88	56	43	51	67
	‘‘ (3)	100	100	75	60	55	92
Poisson's number		0.3	0.28	0.36	0.38	0.38	0.39
Curie's point	°C	125	120	340	340	165	400

Table 4.A.1. *Mechanical properties of some ferroelectric ceramics*

	Units	T51	T57	P7-62	P1-60	P4-68	P1-63
Piezoelectric properties							
Charge coefficient	10^{-12} m.V^{-1}						
	‘ ‘						
d_{33}	‘ ‘	190	130	280	400	480	120
d_{31}	‘ ‘	-75	-60	-100	-145	-200	-40
d_h (1)		40	10	80	110	80	40
Tension coefficient	10^{-3} V.m.N^{-1}						
g_{33}		18	21	25	29	20	27
g_{31}		-6	-9	-9	-10	-8	-9
g_{i1}		6	3	7	9	4	9

Table 4.A.2. *Piezoelectric of some ferroelectric ceramics-(1)* $d_h = d_{33}+2d_{31}$, $g_h = g_{33}+2g_{31}$

	Units	**T51**	**T57**	**P7-62**	**P1-60**	**P4-68**	**P1-63**
Frequency Constant	kHz•mm						
Longitudinal N_{3a}	“	2100	2140	1920	1740	1760	1870
Thickness N_{3t}	“	2500	2550	2050	1870	1780	2050
Transverse N_1	“	2160	2300	1550	1410	1400	1760
Radial or planar N_p	“						
Circumferential N_e		2800	2900	2250	2020	2030	2400
Mechanical quality factor Q_m				810	105	80	900

Table 4.A.3. *Mechanical properties of some ferroelectric ceramics*

The objective of this appendix is to present to readers how to design and build some special transducers which are useful for investigating the wave dispersion in samples of materials, over a large range of frequency if necessary. These transducers are not commercially available but their manufacture is not difficult.

Two barium-titanate ceramics and four titanate zirconate ceramics are shown above.

Each of the ceramics is furnished with a specified resonance frequency:

$\pm$ 10% of frequency f < 1 MHz

$\pm$ 15% of frequency 1 < f < 2 MHz

$\pm$ 20% of frequency 2 < f < 5 MHz