Chapter 3

Mechanical Part of the Vibration Test Bench

The mechanical part of a material testing apparatus is the most important one and requires our attention. Sample holders play a major role and constitute boundary conditions of the sample. Consequently they can greatly influence the vibrations of the bounded medium.

For practical reasons, boundary conditions such as clamping, support and mass attachment are rarely concentrated in a line on a rod sample. Generally, each of them is distributed across the surface. These characteristics must be taken into account not only during dynamic tests but also at the beginning when the experimenter has to choose an appropriate test bench, or to design and build a bench themselves.

The three classic boundary conditions (clamping end, supported end, free end) are discussed below. In addition to these three, we suggest that a fourth, pseudo-clamping, merits the experimenter's attention.

3.1. Clamping end

Part of the sample is submitted to distributed pressure between gripping jaws. The corresponding sample surface must be firmly maintained. Its eventual sliding against the jaws modifies the sample length and consequently vibration characteristics of the rod.

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3.1.1. *Clamping length*

For practical purposes, the clamped sample surface must have a length at least two or three times the width of the sample if the cross-section is rectangular. The reason is to obtain a sample slenderness h/L (where h is width, and L length) of low value so that the influence of the sample length between the clamping jaws is weak compared to the whole length of the sample.

3.1.2. *Applied clamping forces*

Applied clamping forces are not mentioned in ideal boundary conditions. On practical ground, empirical formulae are referred to for corrected sample length which is different from effective measured length. The clamping force is taken as a parameter in the evaluation of sample elastic constants.

3.1.3. *Influence of external force applied to the sample on compressional stress*

Any additional compression stress, as a result of the influence of an external force applied, depends on the external force direction. Figure 3.1 shows a sample with three axes.

3.1.3.1. *Force applied at free end* $F = F_x$ *in direction x*

Here, we are dealing with bending force. The maximum stress at the clamping end is:

$$\sigma_{\text{max (bending)}} = \frac{6\,F\,L}{b\,h^2} \qquad [3.1]$$

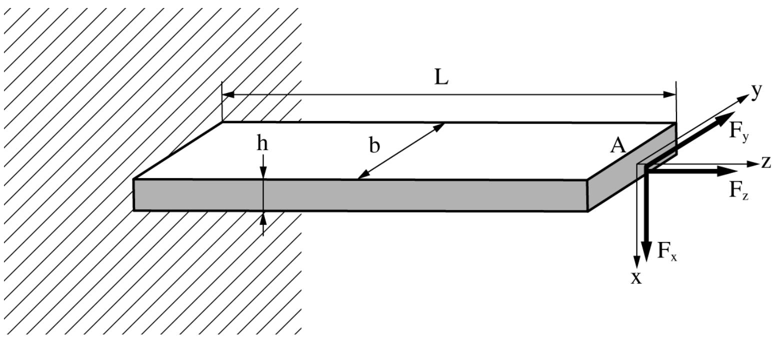

Figure 3.1. *Rod clamped at the left end and submitted to external force applied at the free end. The stress at the clamping end differs depending on the force direction*

And the displacement at the free end is:

$$x_{max} = \frac{4\,F\,L^3}{E\,b\,h^3} \qquad [3.2]$$

3.1.3.2. *Force applied at the free end* $F = F_x$ *and directed along the y axis*

Maximum stress:

$$\sigma_{max\,(bending)} = \frac{6\,F\,L}{h\,b^2} \qquad [3.3]$$

Displacement at the free end:

$$y_{max} = \frac{4\,F\,L^3}{E\,h\,b^3} \qquad [3.4]$$

3.1.3.3. *Force applied at the free end* $F = F_z$ *and directed along z axis*

Stress in rod is extensional:

$$\sigma_{max\,(extension)} = \frac{F}{b\,h} \qquad [3.5]$$

Displacement at the free end:

$$z = \frac{F\,L}{E\,b\,h} \qquad [3.6]$$

Comparing maximum stresses in the three cases [3.1], [3.3] and [3.5] we notice that the bending force F_x applied through the thickness (F_x) gives the highest maximum stress at the clamping end.

A practical conclusion concerning the simple calculations presented above shows that stress at the clamping end is the most important for bending force. Precautions must be taken to avoid the sample sliding, particularly when the sample length is great.

The clamping submitted to extensional force is the least important and sample length does not intervene in the additional stress; see equation [3.5].

3.1.3.4. *Sliding influence on the sample length*

Particularly in the case of bending vibration, displacement at the free end is proportional to the cube of the rod length; see equations [3.2], [3.4].

This shows the necessity of creating a sufficient clamping force to avoid sample sliding, particularly at high temperature.

3.1.4. *Sample holder for clamping*

Figure 3.2 shows two examples of sample holders.

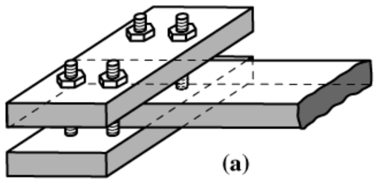

Figure 3.2. *Sample holders with gripping plates: (a) four nuts and bolts create gripping compressive force; (b) plates with two rectilinear grooves to maintain sample in position; nuts and bolts are used as in (a)*

Nuts and bolts constitute an elementary system of clamping. But this system's advantage resides in the possibility of exerting calibrated compression force on each nut and bolt couple. Use of a dynamometer key allows control of the torque applied to each nut and consequently the force applied to the sample.

Plates with rectilinear grooves are recommended and allow the sample to be maintained in the same position, preventing sample rotation.

Four couples of nuts and bolts are preferred to two, as the compression forces are better distributed on the sample.

In the following sections, we will show how to experimentally obtain correcting terms for the sample length.

Figure 3.3 shows clamping plates with springs or mobile jaws with clearance. These two types of mounting should be excluded from the design of a sample holder: springs do not maintain a sample in the same position during dynamic tests. The second disadvantage resides in a system constituted by mass (plates) and springs which create parasitic resonance at low frequency; a mobile jaw with clearance does not permit regular pressure to be distributed on a sample.

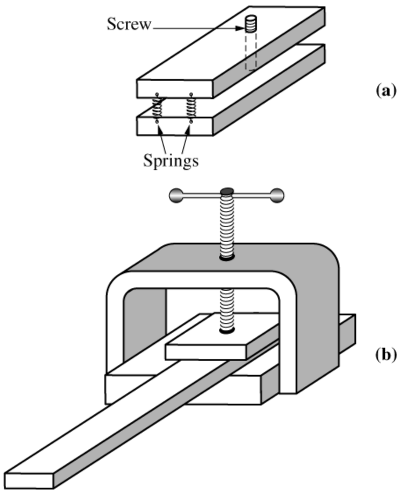

Figure 3.3. *(a) Jaws with springs; (b) mobile jaw with initial clearance*

3.2. Length correction

As mentioned above, real mechanical clamping does not produce ideal boundary conditions:

– displacement is reduced to zero;

– angular displacement is reduced to zero along a line;

– the clamping end necessitates distributed compressive stress in a finite surface. In the sample, near the clamping end, there is a three-dimensional stress state in the active part of the sample.

3.2.1. *Simple tests*

The effective sample length is not the measured length. To be convinced of this problem, the researcher may carry out a simple experiment, as follows:

– choose a sample whose material moduli are known (an iron sample, for example), with a Young's modulus which, in practice, does not vary with a frequency of $E \cong 210 \times 10^9$ Pascals;

– choose a length for a clamped-free sample;

– choose a vibration mode (extensional, torsional, bending);

– evaluate the Young's modulus versus the frequency;

– compare the evaluated value to the real value of the Young's modulus.

3.2.1.1. *Successive operations to evaluate length correction*

It is necessary to choose a clamping system which enables the clamping force to be evaluated (see section 3.1.1):

– use a dynamometric key and turn the bolt up to the predetermined value of the torque;

– for a given vibration mode (extensional, torsional, bending) use the appropriate equation for motion (see later chapters in Part I of this book). This equation can be a relationship between length and frequency, or length and beating period (Le Rolland-Sorin's double pendulum [CHE 10]);

– modify the sample L_i and conduct dynamic test to evaluate the eigenfrequency (or beating period);

– draw one of the following curves of the inverse of frequency versus. The sample length from which additional ΔL is obtained, see Figure 3.4

3.2.1.2. *Bending vibration*

The Bernoulli-Euler equation of motion is

$$E = \frac{\rho \, S \, \omega^2 \, L^4}{I \, \beta^4}$$

where ρ is density, S the cross-section area, ω angular frequency, I inertia moment, and β the solution of the eigen equation.

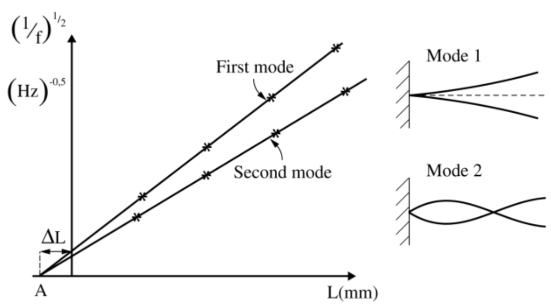

Figure 3.4. *Evaluation of length correction ΔL using bending vibration of rod*

For a narrow frequency interval, the following product is constant, ω circular frequency, f frequency:

$$L\omega^{0.5} = \text{constant} \quad \text{or} \quad Lf^{\,0.5} = \text{constant} \qquad [3.7]$$

The curve $\left(\frac{1}{f}\right)^{1/2}$ versus sample length L is drawn for different values of L.

Modes 1 and 2 (see Figure 3.4) are adopted. Two curves are obtained as straight lines which are concurrent at point A on the abscissa coordinate.

The additional length is Δ L and the effective length is:

$$L_{\text{ effective}} = L_{\text{ measured}} + \Delta L \qquad [3.8]$$

3.2.1.3. *Torsional vibration*

A rod clamped at one end with an inertia arm at the other end is subjected to a torsional vibration. The relationship between the eigenperiod T and sample length L is:

$$(T)^{1/2} \bullet L = \text{cst} \qquad [3.9]$$

Figure 3.5 gives the curve $T^{1/2} = T^{1/2}(L)$

The length correction is negative:

$$L_{\text{effective}} = L_{\text{measured}} - \Delta L \qquad [3.10]$$

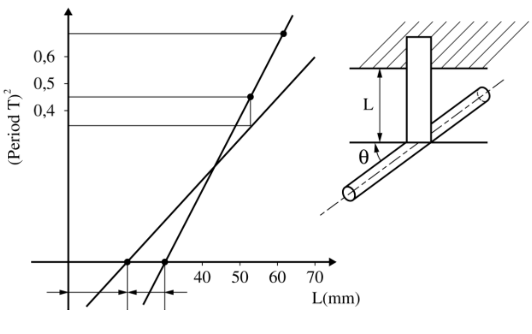

Figure 3.5. *Corrected length for torsional pendulum*

3.2.1.4. *Le Rolland-Sorin's double pendulum*

Chapter 8 gives details about equations of motion; see also [CHE 10]. The curve $\tau^{-1} = \tau^{-1}$ (L) is drawn for different values of length L; τ is the beating period.

Tables 3.1 and 3.2 concern torsional tests affected with aluminum and a fiberglass-epoxy composite, respectively.

τ (s)	12.45	12.09	10.80	9.78
$\frac{1}{\tau}$ (s^{-1})	0.080	0.0827	0.0925	0.1022
L (mm)	99	104.5	115.5	126.5

Table 3.1. *Inverse of beating period versus sample length – sample material: aluminum. The experimental table allows us to obtain the sample corrected length*

τ (s)	6.18	6.64	7.28	8.12
$\frac{1}{\tau}$ (s^{-1})	0.1618	0.1506	0.1373	0.1231
L (mm)	175	164	149	136

Table 3.2. *Inverse of beating period versus sample length. Sample material: fiberglass-epoxy composite*

Length corrections are 1 mm and 1.5 mm for these materials, respectively. Figure 3.6 shows that the length correction is negative:

$$L_{effective} = L_{measured} - \Delta L \tag{3.11}$$

The two examples presented above show that in some cases the length correction is not negligible, particularly when the sample length is short.

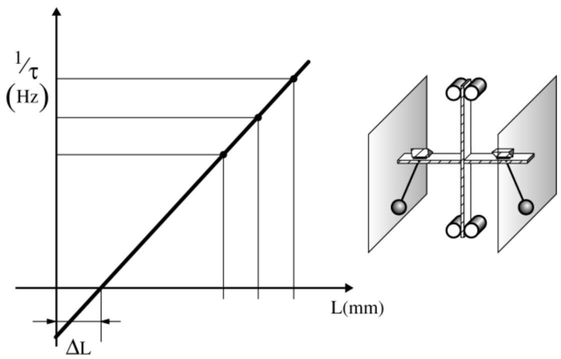

Figure 3.6. *Torsional Le Rolland-Sorin's double pendulum length correction curve gives a negative value of Δ L*

3.3. Supported end

The supported end is the most difficult boundary condition to realize in practice. Theoretically:

– displacement is set to zero; and

– angular displacement is free.

From a practical point of view, the supported end is supported by means of two opposite knife edges; Figure 3.7. The difficulty is that the supported end cannot be obtained at one end of the sample. A small part of the sample is necessary beyond the end; Figure 3.7.

The two knife edges must be maintained in position by two plates with rectangular grooves (see Figure 3.7) to ensure that the two edges are exactly positioned along two lines at the boundaries of a cross-section.

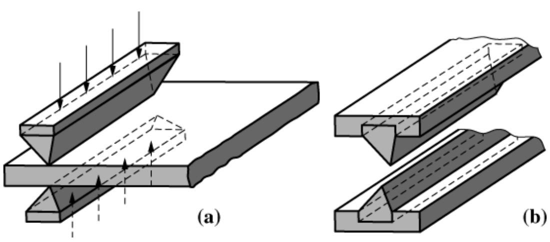

Figure 3.7. *(a) Rod with support at one end (two knife edges are used); (b) knife edges are adjusted by using rectilinear grooves so that the edges are at opposite sides of the cross-section*

3.4. Additional weight or additional torsion lever used as a boundary condition

An additional weight or torsion lever serves to produce lower resonance frequency of a system sample-weight where rod resonance amplitude is easy to measure.

For a sample maintained in a horizontal position, naturally any additional weight (or torsional lever) must not be too heavy to avoid sample bending.

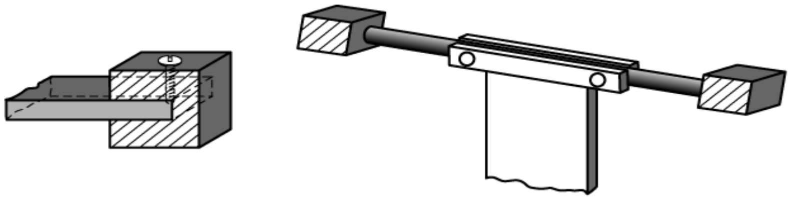

Figure 3.8. *(a) Attachment of additional mass in linear displacement on the left or (b) additional mass of torsion lever in angular displacement*

Except in the case where additional weight is glued to the end of the sample[1], a hole is pierced in the additional weight (or lever) to attach it to the sample by nuts and screws (Figure 3.8).

The additional weight is a distributed weight and the mass center must be determined so as to evaluate the effective sample length which is less than the measured length before weight attachment. The same remark can be applied to Figure 3.8(b), so as to determine the effective sample length.

3.5. Free end

Often the sample vibration amplitude is measured at the free end by magnetic transducer and a small thin steel blade which is glued to the free end.

The experimenter must take careful note of the weight of the blade and its possible influence on the eigenvalue of the sample.

1 Gluing is to be avoided when a sample is submitted for testing at high temperature ,although special glues for high temperature do exist.

3.6. Pseudo-clamping sample attachment

Figure 3.9 presents a pseudo-clamping sample holder. The sample is maintained between two cylinders submitted to a regularly distributed compressional force.

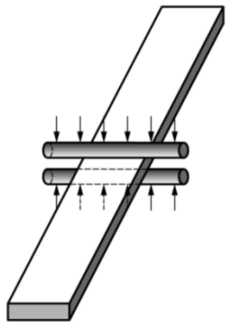

Figure 3.9. *Pseudo-clamping of a sample which is maintained between two metallic cylinders, where the excitation signal is applied in the middle of the sample*

Static compressional forces are imposed on the sample center line by two rigid iron cylinders with nuts and bolts (which are not represented in the Figure 3.9).

Forced vertical displacement vibrations imposed to the sample are obtained from an electromechanical exciter (not represented in the figure). The moving vertical cylinder of the exciter is integral with the horizontal lower cylinder represented in Figure 3.9.

Taking into account of the symmetry of the sample with respect to the vertical plane including the two arrays of arrows, two points of the sample on both sides of the sample vibrate with the same amplitude.

It is not necessary to apply high static compressional forces to the sample at the middle via the two horizontal cylinders to realize a pseudo-clamping which satisfies the two boundary conditions as a classical clamping. If the symmetry plane is maintained at the middle of the sample, the risk of sliding of the sample is negligible. This kind of pseudo-clamping is interesting for bending tests at high temperature. Details of the mountings are given in Part II of this book, in Chapters 9 and 10.

A second advantage of the pseudo-clamping presented in Figure 3.9, is that the length correction presented in section 3.1 is not necessary. These constitute the two main advantages of pseudo-clamping.

A counter example concerns the use of pseudo-clamping at the end of a rod. In this case, the symmetry of a sample with respect to pseudo-clamping is not satisfied and the clamping force must be increased to maintain the clamping holder in position. In this scenario, the pseudo-clamping system lies between a support system and a clamping system: the benefits of the pseudo-clamping system presented above are lost.

3.6.1. *Mechanical design of a pseudo-clamping system*

3.6.1.1. *For bending tests*

The axes of the two cylinders must be maintained parallel, and the two contact lines between cylinders and the sample must correspond to the two sides of the sample cross-section. If this is not the case for a bending test with a horizontal sample, the sample will not be horizontal and adjustment of the relative position of the two cylinders is then necessary.

Two parallel rectangular surfaces are obtained by milling at each end of each of the two cylinders (Figure 3.10). These surfaces serve as guides in two rectilinear grooves in a sample holder (Figure 3.11(a),

Figure 3.10. *Parallel rectangular surfaces at each end of each cylinder are obtained by milling*

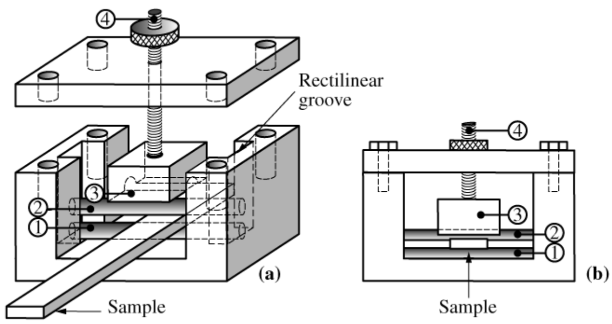

Figure 3.11. *(a) Bending test sample holder for pseudo-clamping by means of two cylinders* ❶ *and* ❷*. Compressional force is applied via a thick aluminum block* ❸ *with clearance serving as a jaw with a screw and knurled knob* ❹*. (b) Cross-section of the sample holder system*

The jaw is represented in the Figure 3.11(a) by part (3). To obtain good contact between the jaw (3) and the upper horizontal cylinder (2), a semi cylinder groove is realized on the lower face of the jaw by milling so as to obtain good contact between the jaw and the upper horizontal cylinder.

When using pseudo-clamping, the sample contact with the two horizontal cylinders (2) and (1) must be positioned exactly in the center of the sample. With respect to the two contact lines between cylinders (1) and (2), vibrations in the two parts of the sample are symmetrical. In cases where a lack of room does not permit a sample holding system (such as in Figure 3.11) to be adopted, the use of two pins and holes bored in the first and second cylinders constitute another possible solution.[2]

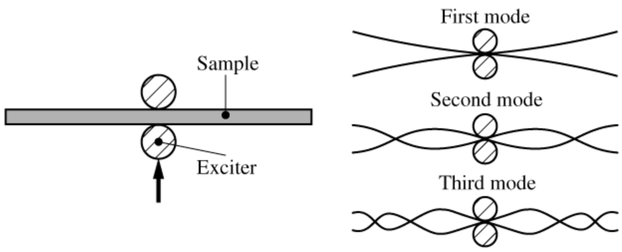

Figure 3.12. *Sample is held between two cylinders and symmetry line is the neutral line at the sample center; bending vibration modes are symmetrical with respect to the holder*

3.6.1.2. *Putting a sample into place*

A sample is placed on the first cylinder and its position checked (by horizontal equilibrium) and adjusted to ensure that the sample axis is perpendicular to the cylinder axes. The upper cylinder is then put into contact with the sample. A knob is turned to obtain compressive force via the jaw and the upper cylinder.

Interest in pseudo-clamping lies in the fact that, unlike in the case of classical clamping, a very high compressional force is not necessary.

Vibrations are symmetrical with respect to the lines of contact with the cylinders. No length correction is necessary.

3.6.1.3. *Torsion test*

For a torsion test, a similar sample holder as that used for a bending test can be adopted. However, because alternate angular motions are applied to the sample

2 Details of such a mounting system are given in Chapter 9, devoted to torsion tests.

holder, it is necessary that it must not be too heavy for an electromechanical exciter. Figure 3.13 shows a possible example of a torsion sample holder.

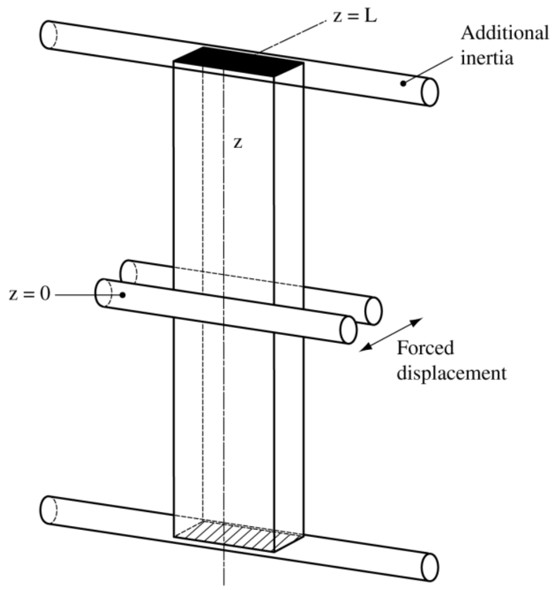

Figure 3.13. *Sample holder for a torsion test using a pseudo-clamping system. Pins and holes in the two cylinders allow the application of a symmetrical clamping force and also ensure that the pseudo-clamping system is at right-angles to the sample axis (the sample length is not to scale)*

3.7. Sample suspended by taut threads

Another method involves suspending a sample by taut threads. This simple system is interesting in the following situations:

– when measurement is required over a large frequency range ($10 < f < 10^5$ Hz) for which additional weight is avoided (because restricted by the frequency range);

– for possible viscoelastic measurements (but with precautions for the arrangement of the thread suspension);

– for special measurement of phase velocity for higher elastodynamic modes, which are equal to or greater than the second modes.

3.7.1. *Measurement over a large range of frequencies*

The range of frequencies depends naturally on the exciter itself; this problem will be examined in detail in the next chapters. Electrodynamic shakers have a limited frequency range which rarely exceeds 5,000 Hz. An electromagnetic exciter is preferred to other types of exciter.

A light steel blade (a shaving blade, for example) is glued to one end of the sample rod. The electromagnetic transducer is fixed in front of the steel blade with a sufficient gap to avoid contact between the blade and the natural magnetic coil of the exciter.

3.7.2. *Nature of the thread*

If possible, the thread selected for use must be as thin as possible but strong, such as a linen thread[3]. Thin nylon rope used for a fishing line can also be used.

For viscoelastic measurement of rod material, the absorption of energy by the thread should be taken into account. For this purpose, thin piano string is preferred.

3.7.3. *Suspension by two parallel threads*

Figure 3.14 shows a possible set-up for a system suspending a sample across two parallel threads. The distance, D, between the two threads has to be adjusted so as to ensure that it does not influence the resonance frequencies as well as the vibration amplitudes. Tension forces exerted on the strings can be adjusted by violin pegs.

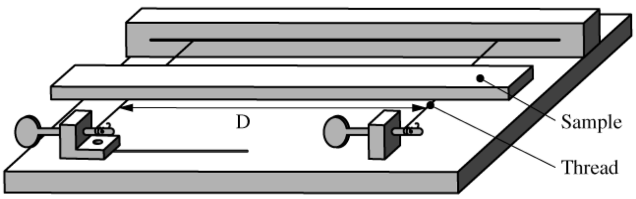

Figure 3.14. *A rod lying on two parallel taut threads. The rod has two free ends*

3 Used for sewing on buttons. In French this thread is called *fil chinois* (Chinese thread) or *lin au chinois*.

This mounting system is interesting when we want to explore a frequency domain which reaches ultrasonic frequencies say beyond 200,000 Hz.

3.7.3.1. *Errors in the evaluation of the Young's modulus using set-up as in Figure 3.14*

Using suspension by threads, errors are introduced in the evaluation of the absolute value of the Young's modulus as well as in the damping capacity. The following parameters should be taken into account.

3.7.3.1.1. Contact stiffness k_l

The contact stiffness depends on the nature of the threads. The weight of the sample contributes to the contact force between the thread and the sample.

3.7.3.1.2. Positioning of the threads

Positioning of the threads with respect to the sample has an importance particularly when measurement of damping capacity is envisaged. If the thread position corresponds to a vibration node, positioning errors are minimized. Consequently, if such a set up is adopted, the adjustment of the two threads (so that the contacts coincide with the two modes) is necessary.

3.7.3.1.3. Positioning of the threads with respect to the sample ratio of sample stiffness k_s on suspension stiffness k_l

Read & Dean [REA 78] suggested the following corrections for the apparent Young's E'_A modulus versus the real modulus E_r :

$$E'_A = E_r + \frac{96 k_l l^3}{b h^3 \beta_n^4} \bullet \frac{w_n^2(x)}{w_n^2(0)} \qquad [3.12]$$

where b is the sample width, h thickness, and L the sample length; β_n is the solution of the eigen equation in bending vibration (Bernouilli or Timoshenko's equation); w_n (x) is the bending displacement measured at the abscissa x; w_n (0) is the bending displacement at a node.

3.7.3.1.4. Damping due to suspensions d_s

Read & Dean proposed the following equation:

$$d_s \cong K\left[\rho^{3/2} b h f_{r\,n}^{1/2}\right]^{-1} \qquad [3.13]$$

where ρ is the sample density, $f_{r\,n}$ is the n^{th} eigenfrequency of the sample, b the width, and h the thickness of the sample. K is an empirical coefficient.

For a composite graphite-epoxy the damping coefficient due to suspension d_s is of the order of 3.4×10^{-4} and for an epoxy sample, $d_s \cong 1.9 \times 10^{-3}$.

3.8. Sample on foam rubber plate serving as a mattress

The set-up for this method is the simplest. It is currently used in the dynamics of structures to undertake modal analysis. It can be used for quick evaluation of the eigenfrequencies of a rod but can bring large errors to resonance amplitude measurements when calculating the damping capacity of the sample material.

3.9. Climatic chamber

Often, a sample must be tested over a large range of temperatures, which can be positive or negative. A climatic chamber is then necessary.

The mechanical set up must then be designed to withstand high and low temperatures. The problem of measurement at extreme temperatures by appropriate transducers will be treated in the next chapters.

It is necessary for the oven design to include a door with a glass aperture for eventual visual observation of the sample, during tests at high and low temperature.

When the climatic chamber runs at negative temperature, water present in the room can become frozen on the door glass, preventing visual observation; thus, the room must be equipped with a pump to drain off humid air before creating negative temperatures in the oven, to help prevent this problem.

3.10. Vacuum system

In some cases, when very low damping capacity of material is to be measured, the influence of air damping (which depends on the lateral surface of the sample) is predominant and is equal to or higher than the sample damping capacity.

To overcome this obstacle, it is necessary to introduce the sample in a vacuum chamber with an adjustable vacuum index. Without this precaution, measurements of the damping capacity are false in the following range:

$$\mathrm{tg}\,\delta \cong 10^{-4} \text{ to } 10^{-3} \qquad [3.14]$$

This damping interval is the result of experience. The scale of size [3.14] only shows that the air damping due to the resistance of air around the sample falls inside the interval and, consequently, correct measurements of sample damping requires, for those kinds of materials, a special vacuum chamber for the sample.

3.11. Bibliography

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